Computer Algebra
Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.1-Hyperbolic-sine/163-6.1.5-Hyperbolic-sine-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [369]. This is test number [163].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (369)	0.00 (0)
Mathematica	100.00 (369)	0.00 (0)
Fricas	95.66 (353)	4.34 (16)
Maple	89.43 (330)	10.57 (39)
Giac	75.61 (279)	24.39 (90)
Maxima	72.09 (266)	27.91 (103)
Mupad	59.89 (221)	40.11 (148)
Sympy	33.06 (122)	66.94 (247)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

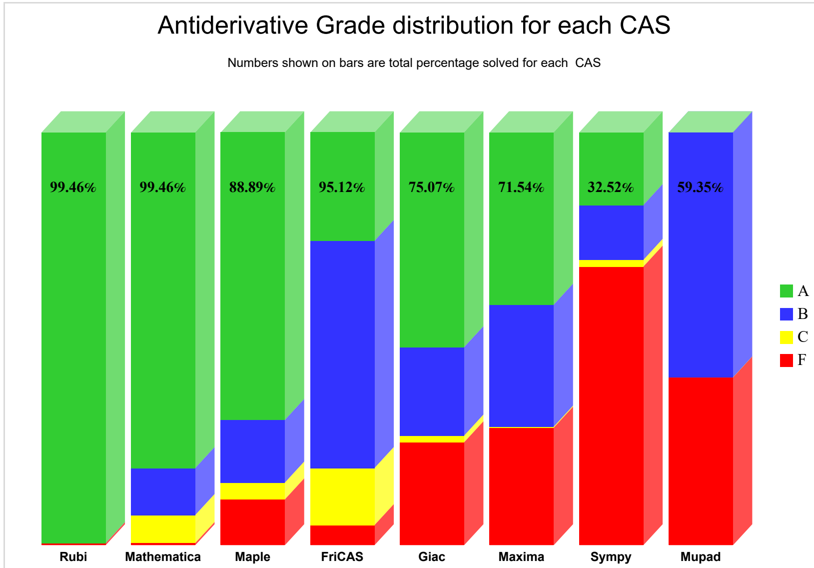
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

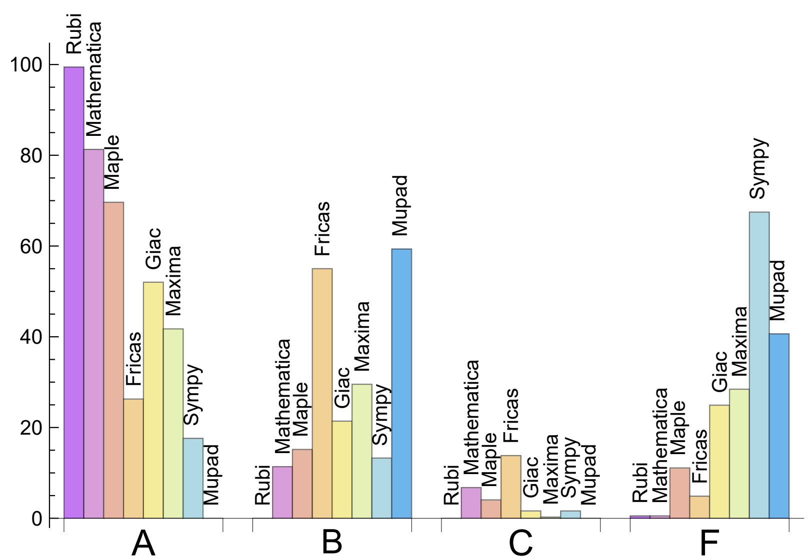
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.458	0.000	0.000	0.542
Mathematica	81.301	11.382	6.775	0.542
Maple	69.648	15.176	4.065	11.111
Giac	52.033	21.409	1.626	24.932
Maxima	41.734	29.539	0.271	28.455
Fricas	26.287	55.014	13.821	4.878
Sympy	17.615	13.279	1.626	67.480
Mupad	0.000	59.350	0.000	40.650

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	16	100.00	0.00	0.00
Maple	39	100.00	0.00	0.00
Giac	90	97.78	2.22	0.00
Maxima	103	98.06	0.00	1.94
Mupad	148	0.00	100.00	0.00
Sympy	247	80.57	19.43	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.08
Maxima	0.24
Fricas	0.27
Giac	0.55
Mathematica	0.63
Mupad	1.56
Sympy	3.52
Maple	8.78

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	86.31	1.00	66.00	1.00
Mathematica	114.50	1.35	72.00	1.00
Maple	133.82	1.43	79.50	1.16
Sympy	134.72	2.38	59.50	1.61
Maxima	137.13	1.96	94.00	1.50
Mupad	173.40	2.48	74.00	1.58
Giac	185.76	1.79	81.00	1.37
Fricas	422.76	3.96	156.00	2.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

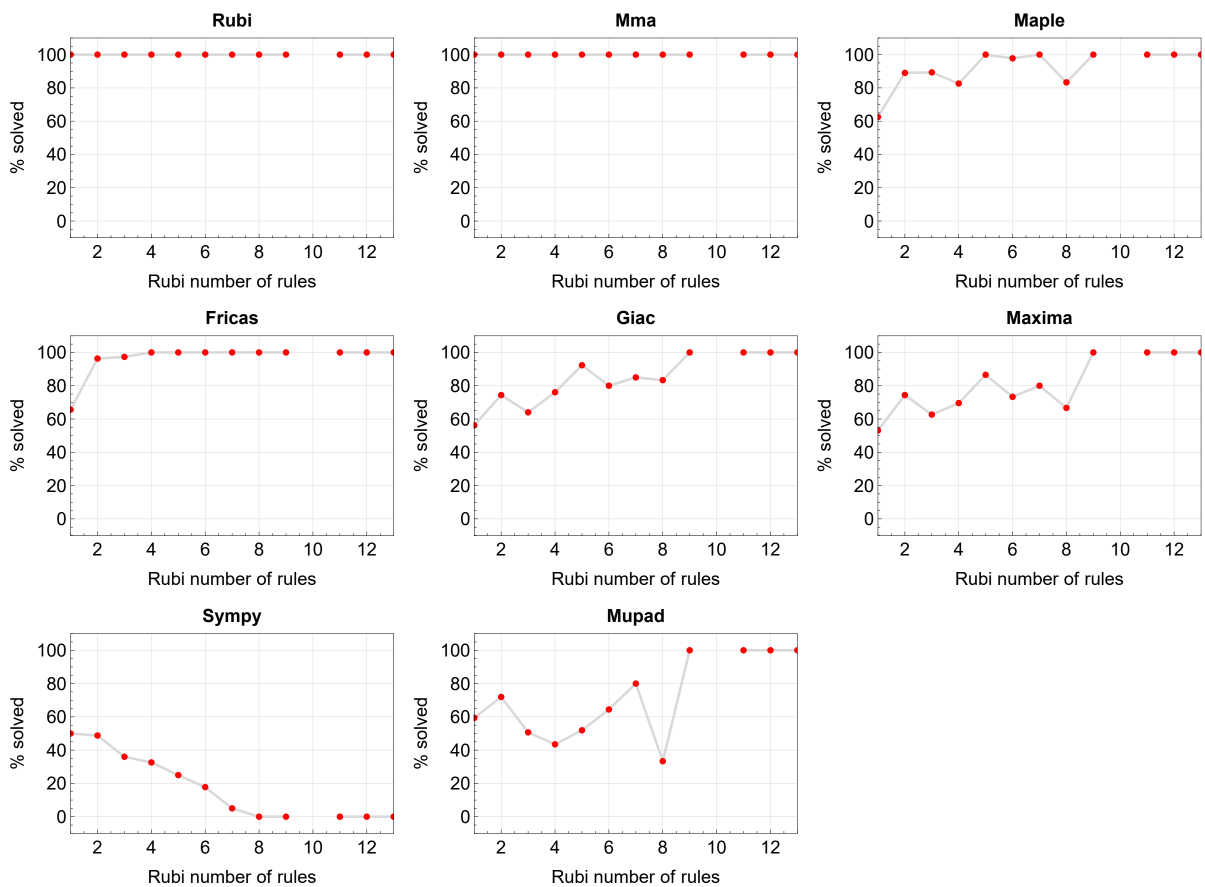


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

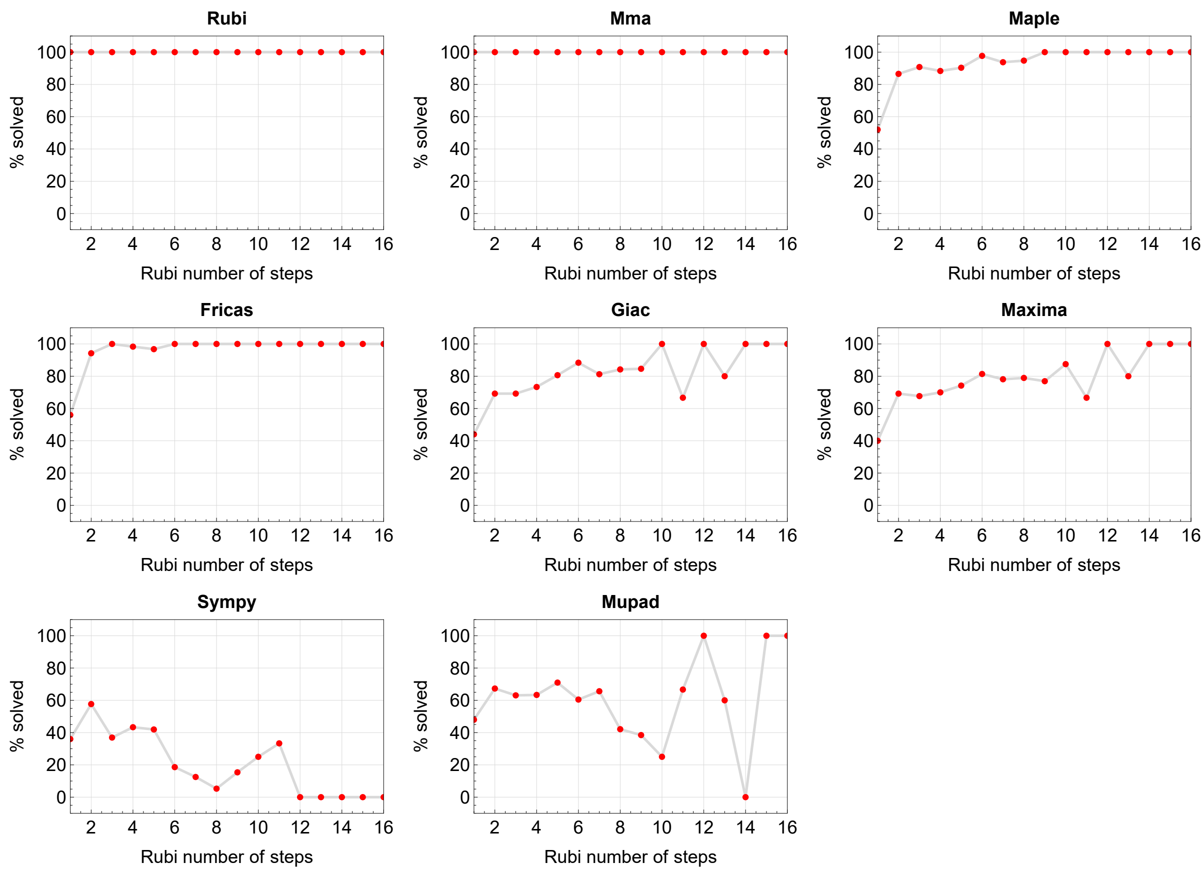


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

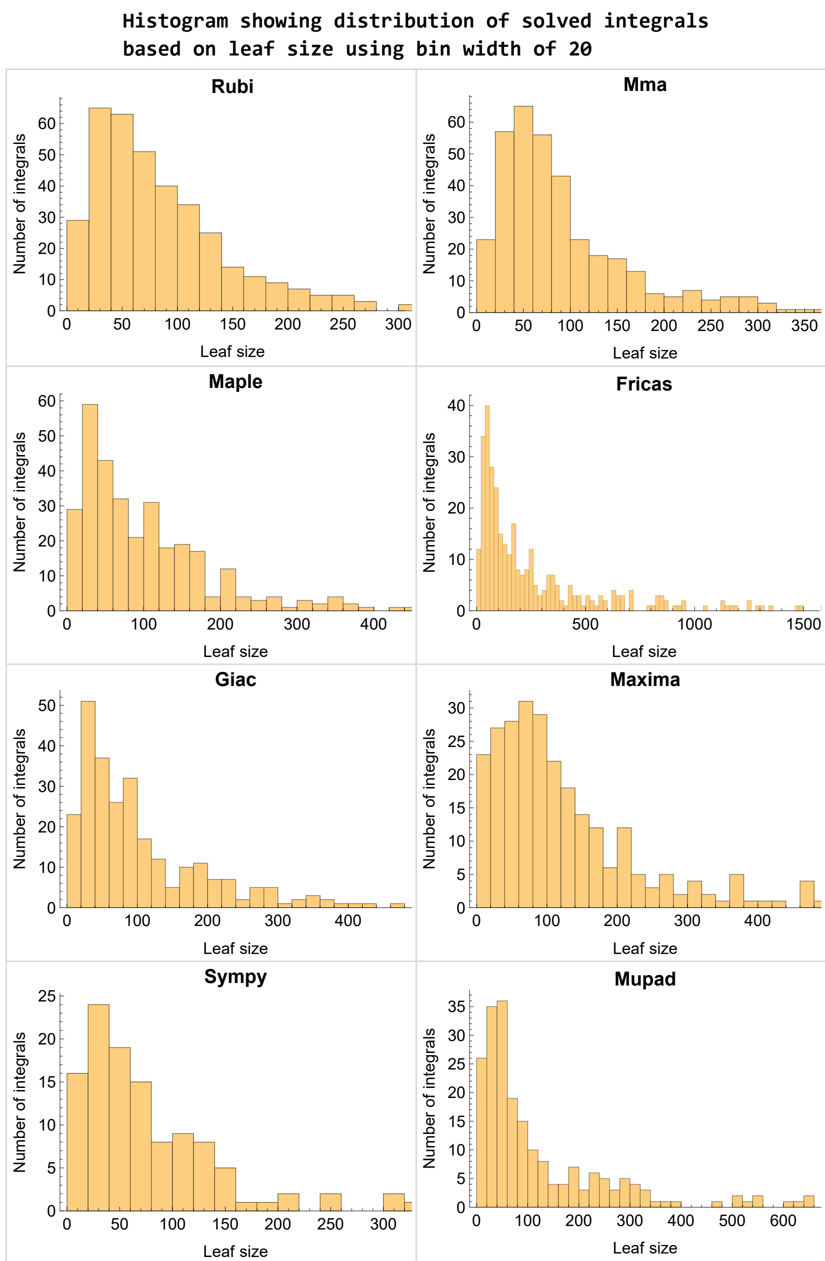


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

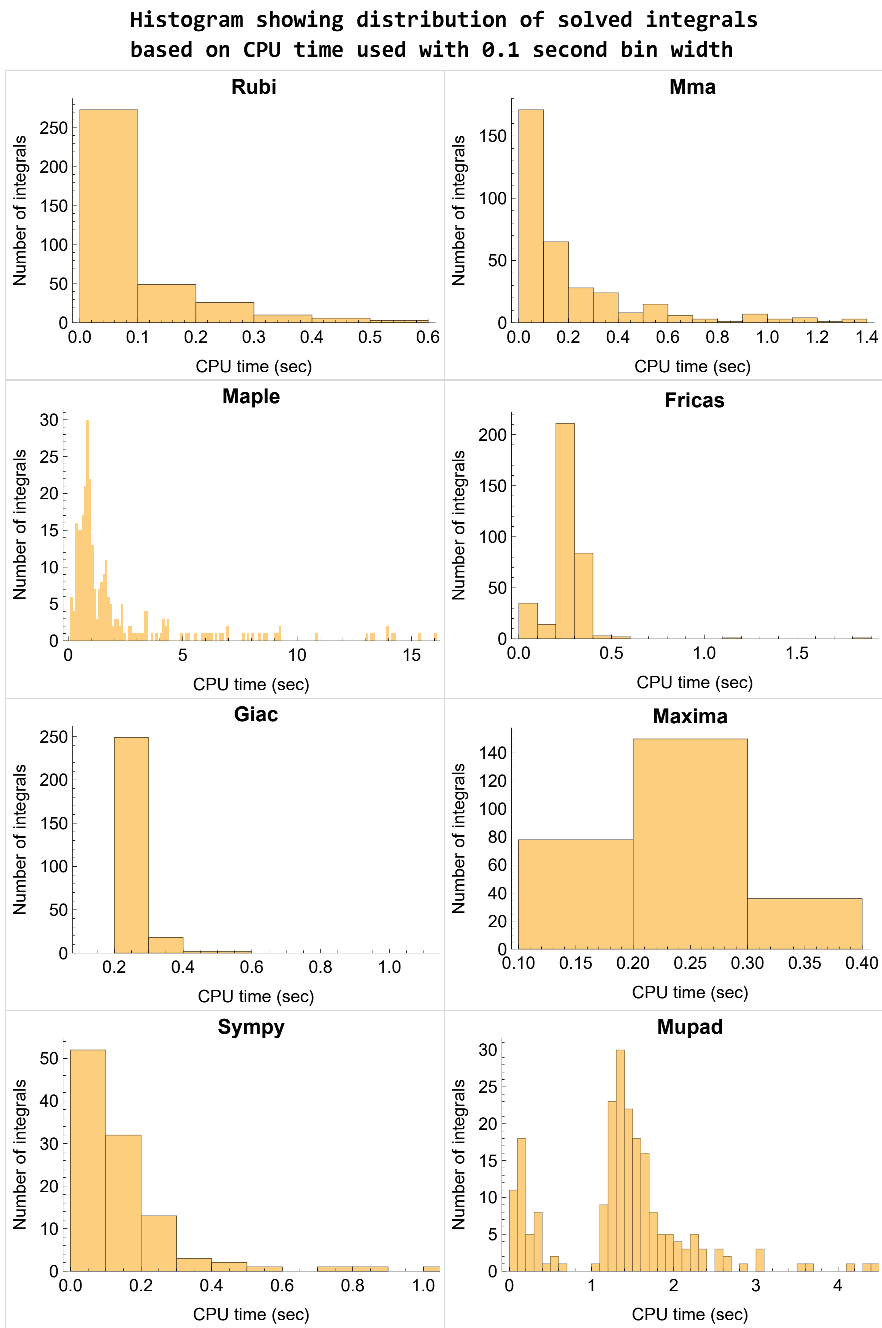


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

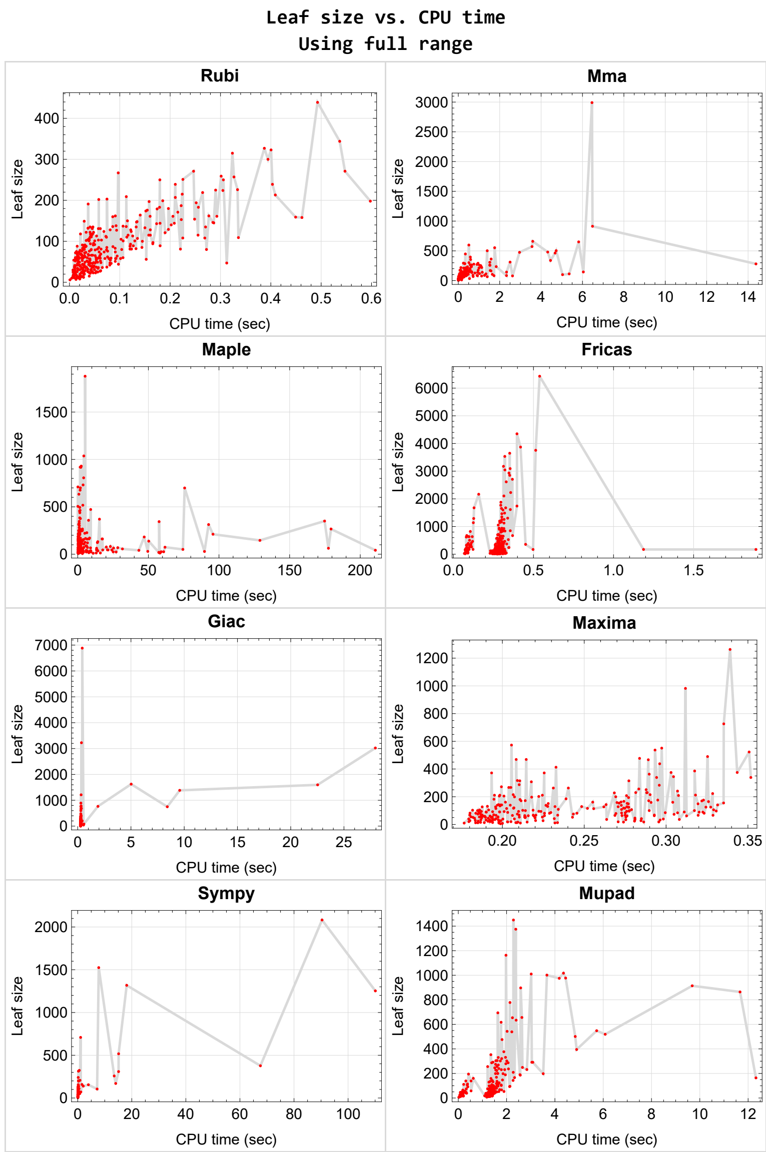


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{264, 265\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{363, 364\}$

Maple $\{329, 330, 331, 332, 333, 334\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

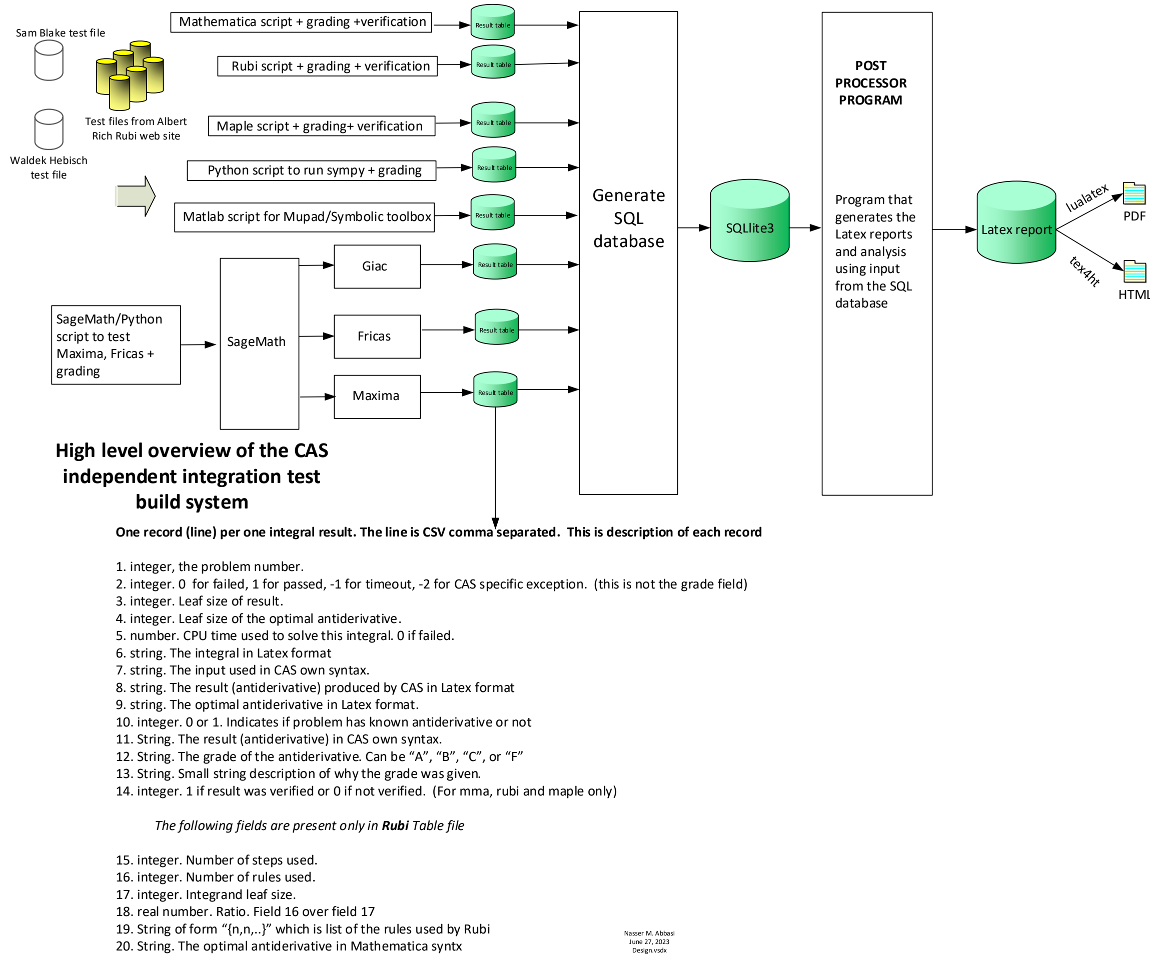
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 44, 45, 46, 47, 49, 50, 51, 52, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 189, 191, 193, 195, 196, 197, 199, 201, 203, 204, 205, 206, 207, 209, 211, 212, 214, 216, 219, 221, 222, 224, 226, 228, 230, 232, 233, 234, 235, 236, 238, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 279, 281, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 294, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 318, 319, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 366, 367, 369 }

B grade { 1, 40, 42, 43, 48, 53, 54, 55, 68, 92, 93, 94, 95, 115, 158, 160, 162, 164, 171, 175, 194, 198, 208, 210, 213, 215, 217, 218, 220, 223, 225, 227, 248, 274, 295, 296, 297, 298, 299, 300, 327, 365 }

C grade { 9, 13, 17, 21, 25, 29, 148, 188, 190, 192, 200, 202, 229, 231, 237, 239, 249, 280, 284, 285, 316, 317, 320, 321, 368 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 108, 110, 111, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 136, 137, 140, 141, 142, 145, 152, 153, 155, 156, 157, 159, 161, 163, 165, 166, 167, 168, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 196, 200, 201, 202, 203, 204, 205, 206, 207, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 223, 224, 226, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 260, 266, 267, 268, 269, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 289, 290, 291, 292, 293, 294, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 318, 319, 322, 323, 324, 335, 336, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369 }

B grade { 24, 64, 65, 68, 69, 72, 73, 103, 104, 105, 106, 107, 109, 123, 126, 127, 128, 131, 132, 135, 138, 139, 143, 144, 154, 158, 160, 162, 164, 169, 170, 171, 179, 180, 188, 197, 198, 199, 208, 209, 215, 216, 217, 225, 227, 255, 256, 257, 258, 259, 295, 296, 297, 298, 299, 300 }

C grade { 244, 245, 312, 313, 316, 317, 320, 321, 329, 330, 331, 332, 333, 334, 339 }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 66, 67, 70, 71, 112, 113, 114, 124, 125, 146, 147, 148, 149, 150, 151, 261, 262, 263, 270, 271, 272, 273, 285, 286, 287, 288, 325, 326, 327, 328 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 6, 40, 41, 43, 51, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 88, 89, 90, 92, 93, 96, 97, 98, 99, 100, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 163, 167, 171, 173, 175, 176, 178, 182, 185, 194, 212, 231, 232, 247, 248, 260, 266, 267, 268, 269, 270, 271, 274, 275, 276, 277, 285, 286, 287, 288, 289, 290, 295, 298, 301, 303, 305, 311, 316, 329, 330, 331, 332, 336, 337, 338, 339, 340, 341, 349, 366, 367 }

B grade { 5, 42, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 94, 95, 101, 102, 103, 104, 123, 124, 125, 129, 130, 131, 132, 133, 135, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 180, 181, 183, 184, 186, 187, 188, 189, 190, 191, 192, 193, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 272, 273, 278, 291, 292, 293, 294, 296, 297, 299, 300, 302, 304, 306, 307, 308, 309, 310, 312, 313, 314, 315, 317, 318, 319, 322, 323, 324, 333, 334, 335, 342, 343, 344, 345, 346, 347, 348, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

C grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 105, 106, 107, 108, 109, 110, 111, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 279, 280, 281, 282, 283, 284, 320, 321 }

F normal fail { 31, 32, 33, 34, 35, 36, 37, 38, 39, 261, 262, 263, 325, 326, 327, 328 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 4, 6, 40, 41, 43, 44, 48, 49, 50, 56, 57, 60, 61, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 84, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 119, 136, 140, 141, 142, 143, 144, 145, 152, 153, 154, 155, 165, 171, 173, 175, 176, 181, 183, 184, 186, 190, 192, 193, 194, 195, 196, 202, 203, 204, 229, 230, 231, 233, 238, 239, 246, 247, 248, 249, 250, 251, 260, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 301, 302, 303, 304, 305, 306, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 329, 330, 331, 332, 333, 334, 337, 338, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade { 3, 5, 42, 45, 46, 47, 51, 52, 53, 54, 55, 58, 59, 62, 63, 78, 83, 85, 86, 87, 103, 104, 116, 117, 118, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 156, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 172, 174, 177, 178, 179, 180, 182, 185, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 234, 235, 236, 237, 240, 241, 242, 243, 252, 253, 254, 255, 256, 276, 278, 307, 309, 335 }

C grade { 339 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 368 }

F(-1) timedout fail { }

F(-2) exception fail { 336, 369 }

Giac

A grade { 2, 4, 6, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 136, 143, 144, 145, 152, 153, 154, 155, 156, 157, 167, 171, 173, 175, 176, 178, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 195, 197, 202, 203, 205, 210, 212, 220, 222, 226, 228, 230, 231, 232, 233, 236, 238, 241, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 260, 266, 267, 276, 277, 278, 285, 287, 288, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 340, 341, 342, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367 }

B grade { 1, 3, 5, 45, 100, 104, 131, 132, 135, 140, 141, 142, 158, 159, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 174, 177, 179, 188, 196, 198, 199, 200, 201, 204, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 227, 229, 234, 235, 237, 239, 240, 242, 255, 256, 268, 269, 270, 271, 272, 273, 274, 275, 289, 290, 291, 295, 296, 297, 298, 299, 300, 312 }

C grade { 322, 323, 324, 339, 343, 346 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 68, 69, 70, 71, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 137, 138, 139, 146, 147, 148, 149, 150, 151, 244, 245, 257, 258, 259, 261, 262, 263, 279, 280, 281, 282, 283, 284, 292, 293, 294, 325, 326, 327, 328, 368, 369 }

F(-1) timedout fail { 24, 286 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 68, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 133, 134, 135, 136, 142, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 253, 254, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 331, 333, 334, 335, 336, 366, 367 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 64, 65, 66, 67, 69, 70, 71, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 123, 124, 125, 126, 127, 128, 131, 132, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 244, 245, 255, 256, 257, 258, 259, 260, 261, 262, 263, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 325, 326, 327, 328, 329, 330, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 368, 369 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 5, 40, 41, 42, 43, 48, 49, 50, 51, 56, 57, 58, 59, 60, 61, 62, 63, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 115, 116, 117, 118, 119, 120, 121, 122, 134, 136, 142, 165, 171, 173, 175, 181, 184, 185, 193, 203, 211, 212, 220, 221, 224, 247, 248, 278, 311, 315, 319, 366, 367 }

B grade { 2, 4, 6, 74, 102, 158, 159, 160, 161, 162, 163, 164, 172, 174, 176, 182, 183, 186, 192, 201, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 222, 223, 225, 226, 227, 274, 276, 301, 302, 303, 304, 310, 314, 318, 322, 323, 324, 331, 336 }

C grade { 75, 101, 129, 133, 246, 253 }

F normal fail { 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 52, 53, 54, 64, 65, 67, 68, 69, 70, 76, 77, 78, 79, 84, 85, 86, 87, 105, 106, 107, 108, 109, 110, 111, 113, 114, 123, 124, 127, 128, 137, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 166, 167, 168, 169, 170, 177, 178, 179, 180, 194, 195, 196, 197, 198, 199, 204, 205, 206, 207, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 249, 250, 251, 252, 257, 258, 259, 260, 261, 262, 263, 266, 267, 268, 269, 270, 271, 272, 273, 275, 277, 280, 281, 282, 283, 286, 287, 289, 290, 292, 293, 295, 296, 298, 305, 306, 307, 308, 309, 312, 313, 316, 317, 320, 321, 325, 326, 327, 328, 332, 333, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 368, 369 }

F(-1) timedout fail { 7, 15, 23, 47, 55, 66, 71, 72, 73, 80, 81, 82, 83, 103, 104, 112, 125, 126, 130, 131, 132, 135, 138, 139, 187, 188, 189, 190, 191, 200, 202, 254, 255, 256, 279, 284, 285, 288, 291, 294, 297, 299, 300, 329, 330, 334, 335, 365 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00
time (sec)	N/A	0.004	0.004	0.329	0.177	0.271	0.049	0.272	0.057

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	32	23	46	32	18
N.S.	1	1.00	0.92	0.80	1.28	0.92	1.84	1.28	0.72
time (sec)	N/A	0.007	0.014	0.477	0.190	0.248	0.077	0.278	1.091

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	23	54	38	36	54	24
N.S.	1	1.00	1.07	0.85	2.00	1.41	1.33	2.00	0.89
time (sec)	N/A	0.010	0.004	1.493	0.186	0.244	0.100	0.273	0.061

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	33	31	60	49	95	60	32
N.S.	1	1.00	0.72	0.67	1.30	1.07	2.07	1.30	0.70
time (sec)	N/A	0.016	0.041	1.428	0.188	0.244	0.145	0.286	0.089

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	33	82	79	58	82	31
N.S.	1	1.00	1.07	0.80	2.00	1.93	1.41	2.00	0.76
time (sec)	N/A	0.012	0.030	1.520	0.180	0.244	0.200	0.275	1.144

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	43	42	86	90	139	88	42
N.S.	1	1.00	0.64	0.63	1.28	1.34	2.07	1.31	0.63
time (sec)	N/A	0.024	0.057	1.643	0.186	0.240	0.311	0.270	0.148

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	75	116	0	326	0	0	0
N.S.	1	1.00	0.73	1.13	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.039	0.116	0.853	0.000	0.082	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	68	164	0	202	0	0	0
N.S.	1	1.00	0.85	2.05	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.027	0.064	0.749	0.000	0.077	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	83	100	0	103	0	0	0
N.S.	1	1.00	1.04	1.25	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.026	0.071	0.777	0.000	0.080	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	108	0	37	0	0	0
N.S.	1	1.00	0.93	2.00	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.015	0.083	0.891	0.000	0.071	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	48	87	0	24	0	0	0
N.S.	1	1.00	0.89	1.61	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.015	0.097	0.594	0.000	0.073	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	57	154	0	152	0	0	0
N.S.	1	1.00	0.75	2.03	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.049	0.875	0.000	0.073	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	101	0	314	0	0	0
N.S.	1	1.00	1.08	1.26	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	0.030	0.067	0.780	0.000	0.083	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	73	192	0	621	0	0	0
N.S.	1	1.00	0.71	1.86	0.00	6.03	0.00	0.00	0.00
time (sec)	N/A	0.037	0.126	0.802	0.000	0.080	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	76	122	0	394	0	0	0
N.S.	1	1.00	0.66	1.05	0.00	3.40	0.00	0.00	0.00
time (sec)	N/A	0.045	0.217	1.058	0.000	0.093	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	68	170	0	249	0	0	0
N.S.	1	1.00	0.77	1.93	0.00	2.83	0.00	0.00	0.00
time (sec)	N/A	0.033	0.102	0.783	0.000	0.084	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	106	0	115	0	0	0
N.S.	1	1.00	1.00	1.20	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.032	0.103	0.891	0.000	0.076	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	111	0	42	0	0	0
N.S.	1	1.00	0.93	1.98	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.017	0.052	1.246	0.000	0.072	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	89	0	27	0	0	0
N.S.	1	1.00	0.96	1.59	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.018	0.038	0.731	0.000	0.068	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	62	159	0	169	0	0	0
N.S.	1	1.00	0.72	1.85	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.030	0.059	0.746	0.000	0.074	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	84	114	0	347	0	0	0
N.S.	1	1.00	0.93	1.27	0.00	3.86	0.00	0.00	0.00
time (sec)	N/A	0.028	0.081	0.875	0.000	0.086	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	79	205	0	675	0	0	0
N.S.	1	1.00	0.67	1.74	0.00	5.72	0.00	0.00	0.00
time (sec)	N/A	0.046	0.134	0.934	0.000	0.084	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	65	122	0	104	0	0	0
N.S.	1	1.00	0.71	1.34	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.031	0.134	0.849	0.000	0.085	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	55	169	0	94	0	0	0
N.S.	1	1.00	0.89	2.73	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.016	0.053	0.868	0.000	0.079	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	94	104	0	79	0	0	0
N.S.	1	1.00	1.52	1.68	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.017	0.107	1.084	0.000	0.077	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	91	0	53	0	0	0
N.S.	1	1.00	0.93	3.03	0.00	1.77	0.00	0.00	0.00
time (sec)	N/A	0.007	0.025	1.485	0.000	0.076	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	68	0	20	0	0	0
N.S.	1	1.00	0.93	2.27	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.008	0.031	0.641	0.000	0.074	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	50	159	0	87	0	0	0
N.S.	1	1.00	0.86	2.74	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.015	0.081	0.973	0.000	0.082	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	83	112	0	132	0	0	0
N.S.	1	1.00	1.34	1.81	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.018	0.074	0.934	0.000	0.086	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	204	0	178	0	0	0
N.S.	1	1.00	0.88	2.24	0.00	1.96	0.00	0.00	0.00
time (sec)	N/A	0.026	0.100	0.977	0.000	0.076	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	57	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	65	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	67	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	0.033	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	134	51	59	67	58	50	50
N.S.	1	1.00	2.91	1.11	1.28	1.46	1.26	1.09	1.09
time (sec)	N/A	0.045	0.148	3.487	0.189	0.246	0.092	0.279	1.262

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	39	45	55	41	38	38
N.S.	1	1.00	1.14	1.08	1.25	1.53	1.14	1.06	1.06
time (sec)	N/A	0.032	0.100	2.369	0.191	0.239	0.078	0.259	1.165

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	79	25	33	37	20	26	24
N.S.	1	1.00	3.59	1.14	1.50	1.68	0.91	1.18	1.09
time (sec)	N/A	0.041	0.086	1.780	0.187	0.254	0.061	0.261	1.144

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	43	13	12	16	8	10	12
N.S.	1	1.00	3.07	0.93	0.86	1.14	0.57	0.71	0.86
time (sec)	N/A	0.020	0.052	1.029	0.184	0.244	0.040	0.260	1.177

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	30	21	29	33	0	24	35
N.S.	1	1.00	1.58	1.11	1.53	1.74	0.00	1.26	1.84
time (sec)	N/A	0.028	0.017	1.635	0.183	0.241	0.000	0.274	0.193

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	36	35	51	77	0	44	51
N.S.	1	1.00	1.57	1.52	2.22	3.35	0.00	1.91	2.22
time (sec)	N/A	0.042	0.034	2.717	0.182	0.250	0.000	0.269	1.405

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	49	53	75	126	0	51	70
N.S.	1	1.00	1.32	1.43	2.03	3.41	0.00	1.38	1.89
time (sec)	N/A	0.053	0.152	3.497	0.185	0.245	0.000	0.260	1.446

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	53	71	103	174	0	58	85
N.S.	1	1.00	1.13	1.51	2.19	3.70	0.00	1.23	1.81
time (sec)	N/A	0.051	0.201	4.335	0.182	0.257	0.000	0.276	1.486

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	147	52	71	89	70	50	97
N.S.	1	1.00	2.53	0.90	1.22	1.53	1.21	0.86	1.67
time (sec)	N/A	0.070	0.146	3.043	0.187	0.246	0.102	0.259	1.296

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	45	38	59	74	54	38	79
N.S.	1	1.00	1.02	0.86	1.34	1.68	1.23	0.86	1.80
time (sec)	N/A	0.097	0.104	2.362	0.187	0.250	0.088	0.263	1.339

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	55	26	40	50	41	22	71
N.S.	1	1.00	1.72	0.81	1.25	1.56	1.28	0.69	2.22
time (sec)	N/A	0.044	0.112	1.661	0.179	0.242	0.069	0.262	1.301

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	23	81	32	37	20	25
N.S.	1	1.00	0.71	0.74	2.61	1.03	1.19	0.65	0.81
time (sec)	N/A	0.021	0.009	1.400	0.184	0.229	0.062	0.260	1.296

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	68	36	55	78	0	34	41
N.S.	1	1.00	2.00	1.06	1.62	2.29	0.00	1.00	1.21
time (sec)	N/A	0.057	0.717	2.332	0.181	0.251	0.000	0.267	0.311

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	88	58	79	130	0	46	85
N.S.	1	1.00	2.10	1.38	1.88	3.10	0.00	1.10	2.02
time (sec)	N/A	0.080	1.267	3.171	0.186	0.255	0.000	0.295	1.483

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	140	72	105	174	0	59	79
N.S.	1	1.00	2.41	1.24	1.81	3.00	0.00	1.02	1.36
time (sec)	N/A	0.092	1.158	4.161	0.192	0.266	0.000	0.258	1.582

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	143	88	127	226	0	84	189
N.S.	1	1.00	2.23	1.38	1.98	3.53	0.00	1.31	2.95
time (sec)	N/A	0.095	2.331	5.817	0.188	0.276	0.000	0.278	1.950

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	42	18	20	16	15	15	17
N.S.	1	1.00	1.56	0.67	0.74	0.59	0.56	0.56	0.63
time (sec)	N/A	0.009	0.188	0.862	0.180	0.254	0.059	0.266	0.210

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	61	28	94	50	61	25	29
N.S.	1	1.00	1.03	0.47	1.59	0.85	1.03	0.42	0.49
time (sec)	N/A	0.020	0.278	0.945	0.187	0.246	0.099	0.255	1.318

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	81	40	211	85	109	36	40
N.S.	1	1.00	0.92	0.45	2.40	0.97	1.24	0.41	0.45
time (sec)	N/A	0.032	0.295	0.984	0.195	0.240	0.152	0.265	1.508

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	87	51	372	120	155	47	53
N.S.	1	1.00	0.74	0.44	3.18	1.03	1.32	0.40	0.45
time (sec)	N/A	0.042	0.184	1.061	0.193	0.238	0.214	0.271	1.891

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	42	18	20	16	17	15	17
N.S.	1	1.00	1.56	0.67	0.74	0.59	0.63	0.56	0.63
time (sec)	N/A	0.009	0.190	0.968	0.192	0.262	0.059	0.261	0.162

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	28	94	50	61	25	29
N.S.	1	1.00	1.00	0.47	1.59	0.85	1.03	0.42	0.49
time (sec)	N/A	0.020	0.184	0.963	0.210	0.237	0.101	0.276	1.362

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	81	40	211	85	109	36	40
N.S.	1	1.00	0.92	0.45	2.40	0.97	1.24	0.41	0.45
time (sec)	N/A	0.031	0.338	0.956	0.224	0.237	0.156	0.260	1.421

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	87	51	372	120	155	47	52
N.S.	1	1.00	0.74	0.44	3.18	1.03	1.32	0.40	0.44
time (sec)	N/A	0.043	0.189	1.137	0.226	0.231	0.216	0.275	1.678

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	75	108	0	76	0	0	0
N.S.	1	1.00	1.32	1.89	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.041	0.252	4.206	0.000	0.276	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	76	108	0	76	0	0	0
N.S.	1	1.00	1.33	1.89	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.040	0.278	4.369	0.000	0.275	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	145	0	0	101	0	0	0
N.S.	1	1.00	1.39	0.00	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.037	6.041	0.000	0.000	0.261	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	113	0	0	63	0	0	0
N.S.	1	1.00	1.64	0.00	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.023	5.357	0.000	0.000	0.251	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	74	89	0	27	0	0	53
N.S.	1	1.00	2.39	2.87	0.00	0.87	0.00	0.00	1.71
time (sec)	N/A	0.011	0.033	2.191	0.000	0.243	0.000	0.000	1.528

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	84	160	0	93	0	0	0
N.S.	1	1.00	1.62	3.08	0.00	1.79	0.00	0.00	0.00
time (sec)	N/A	0.017	0.169	6.702	0.000	0.243	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	156	0	0	235	0	0	0
N.S.	1	1.00	1.79	0.00	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	0.032	0.283	0.000	0.000	0.276	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	210	0	0	348	0	0	0
N.S.	1	1.00	1.72	0.00	0.00	2.85	0.00	0.00	0.00
time (sec)	N/A	0.048	0.259	0.000	0.000	0.296	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	105	201	158	799	0	156	199
N.S.	1	1.00	0.97	1.86	1.46	7.40	0.00	1.44	1.84
time (sec)	N/A	0.225	1.643	0.670	0.270	0.295	0.000	0.280	1.539

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	152	118	459	0	117	159
N.S.	1	1.00	1.00	1.85	1.44	5.60	0.00	1.43	1.94
time (sec)	N/A	0.132	0.926	0.579	0.274	0.304	0.000	0.271	1.353

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	92	84	238	1253	86	129
N.S.	1	1.00	1.07	1.61	1.47	4.18	21.98	1.51	2.26
time (sec)	N/A	0.080	0.253	0.534	0.275	0.346	110.085	0.279	1.268

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	63	65	134	170	67	99
N.S.	1	1.00	1.11	1.34	1.38	2.85	3.62	1.43	2.11
time (sec)	N/A	0.042	0.029	0.405	0.269	0.258	14.053	0.276	1.276

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	67	49	83	156	0	82	287
N.S.	1	1.00	1.34	0.98	1.66	3.12	0.00	1.64	5.74
time (sec)	N/A	0.053	0.238	0.536	0.273	0.292	0.000	0.269	1.382

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	90	73	100	345	0	98	292
N.S.	1	1.00	1.53	1.24	1.69	5.85	0.00	1.66	4.95
time (sec)	N/A	0.082	0.709	0.679	0.277	0.293	0.000	0.273	1.435

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	136	108	154	929	0	137	617
N.S.	1	1.00	1.68	1.33	1.90	11.47	0.00	1.69	7.62
time (sec)	N/A	0.220	0.655	0.820	0.271	0.315	0.000	0.272	1.773

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	211	151	194	1676	0	171	694
N.S.	1	1.00	1.94	1.39	1.78	15.38	0.00	1.57	6.37
time (sec)	N/A	0.336	0.999	0.879	0.277	0.326	0.000	0.286	1.638

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	118	218	256	1769	0	235	305
N.S.	1	1.00	0.73	1.35	1.58	10.92	0.00	1.45	1.88
time (sec)	N/A	0.277	1.024	0.803	0.283	0.296	0.000	0.277	1.609

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	95	161	208	1053	0	184	274
N.S.	1	1.00	0.83	1.40	1.81	9.16	0.00	1.60	2.38
time (sec)	N/A	0.157	0.429	0.684	0.276	0.280	0.000	0.282	1.543

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	86	127	149	521	0	131	228
N.S.	1	1.00	1.04	1.53	1.80	6.28	0.00	1.58	2.75
time (sec)	N/A	0.091	0.202	0.530	0.277	0.286	0.000	0.280	1.551

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	97	117	341	0	99	142
N.S.	1	1.00	1.13	1.62	1.95	5.68	0.00	1.65	2.37
time (sec)	N/A	0.053	0.144	0.454	0.275	0.261	0.000	0.281	1.408

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	100	115	162	672	0	142	1001
N.S.	1	1.00	1.18	1.35	1.91	7.91	0.00	1.67	11.78
time (sec)	N/A	0.144	0.400	0.727	0.272	0.369	0.000	0.285	3.669

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	128	141	251	1740	0	205	1017
N.S.	1	1.00	1.11	1.23	2.18	15.13	0.00	1.78	8.84
time (sec)	N/A	0.255	0.803	0.803	0.288	0.397	0.000	0.285	4.355

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	174	175	363	3754	0	203	977
N.S.	1	1.00	1.10	1.11	2.30	23.76	0.00	1.28	6.18
time (sec)	N/A	0.462	0.959	0.969	0.290	0.514	0.000	0.270	4.444

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	232	219	477	6430	0	236	975
N.S.	1	1.00	1.17	1.11	2.41	32.47	0.00	1.19	4.92
time (sec)	N/A	0.598	1.111	1.023	0.284	0.538	0.000	0.300	4.181

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	81	36	19	28	31	32	39
N.S.	1	1.00	1.11	0.49	0.26	0.38	0.42	0.44	0.53
time (sec)	N/A	0.019	0.128	2.055	0.271	0.279	0.171	0.270	0.345

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	142	74	79	103	75	67	106
N.S.	1	1.00	1.39	0.73	0.77	1.01	0.74	0.66	1.04
time (sec)	N/A	0.033	0.385	1.168	0.269	0.292	0.211	0.271	1.776

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	204	100	124	193	138	89	147
N.S.	1	1.00	1.56	0.76	0.95	1.47	1.05	0.68	1.12
time (sec)	N/A	0.057	0.566	1.778	0.277	0.316	0.263	0.275	1.830

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	265	123	167	283	197	111	237
N.S.	1	1.00	1.66	0.77	1.04	1.77	1.23	0.69	1.48
time (sec)	N/A	0.086	0.574	2.047	0.274	0.291	0.326	0.276	2.076

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	171	32	36	26	31	28	32
N.S.	1	1.00	4.62	0.86	0.97	0.70	0.84	0.76	0.86
time (sec)	N/A	0.010	0.127	1.754	0.264	0.295	0.118	0.268	1.342

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	183	73	64	103	82	65	102
N.S.	1	1.00	2.77	1.11	0.97	1.56	1.24	0.98	1.55
time (sec)	N/A	0.025	0.374	1.071	0.273	0.296	0.155	0.267	1.780

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	277	96	108	193	141	87	143
N.S.	1	1.00	2.92	1.01	1.14	2.03	1.48	0.92	1.51
time (sec)	N/A	0.048	0.750	1.342	0.273	0.291	0.205	0.272	2.269

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	308	119	152	283	202	109	232
N.S.	1	1.00	2.48	0.96	1.23	2.28	1.63	0.88	1.87
time (sec)	N/A	0.068	1.522	1.323	0.276	0.311	0.258	0.274	2.839

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	138	155	272	223	314	269	160
N.S.	1	1.00	0.75	0.85	1.49	1.22	1.72	1.47	0.87
time (sec)	N/A	0.179	0.938	4.184	0.199	0.273	0.272	0.283	0.621

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	108	119	182	146	240	200	114
N.S.	1	1.00	0.79	0.87	1.33	1.07	1.75	1.46	0.83
time (sec)	N/A	0.106	0.480	2.837	0.195	0.291	0.188	0.274	0.381

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	71	77	115	91	128	135	75
N.S.	1	1.00	0.77	0.84	1.25	0.99	1.39	1.47	0.82
time (sec)	N/A	0.050	0.252	2.133	0.203	0.279	0.135	0.293	1.212

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	48	48	55	46	78	76	41
N.S.	1	1.00	0.92	0.92	1.06	0.88	1.50	1.46	0.79
time (sec)	N/A	0.012	0.143	0.787	0.197	0.283	0.099	0.270	1.234

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	31	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	2.07	1.00
time (sec)	N/A	0.007	0.037	0.589	0.187	0.276	0.066	0.270	1.150

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	52	43	67	162	155	67	55
N.S.	1	1.00	1.18	0.98	1.52	3.68	3.52	1.52	1.25
time (sec)	N/A	0.026	0.063	0.780	0.275	0.281	3.950	0.276	1.508

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	85	118	138	423	2082	119	200
N.S.	1	1.00	1.08	1.49	1.75	5.35	26.35	1.51	2.53
time (sec)	N/A	0.043	0.238	0.862	0.273	0.282	90.367	0.289	1.632

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	117	280	315	1347	0	231	0
N.S.	1	1.00	0.92	2.20	2.48	10.61	0.00	1.82	0.00
time (sec)	N/A	0.094	0.245	1.037	0.277	0.302	0.000	0.273	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	159	494	551	2934	0	357	0
N.S.	1	1.00	0.91	2.84	3.17	16.86	0.00	2.05	0.00
time (sec)	N/A	0.152	0.542	1.439	0.297	0.347	0.000	0.296	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	178	917	0	464	0	0	0
N.S.	1	1.00	0.99	5.12	0.00	2.59	0.00	0.00	0.00
time (sec)	N/A	0.174	0.329	2.230	0.000	0.120	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	139	676	0	263	0	0	0
N.S.	1	1.00	0.93	4.51	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.115	0.271	1.682	0.000	0.113	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	65	262	0	173	0	0	0
N.S.	1	1.00	1.08	4.37	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.026	0.137	1.923	0.000	0.098	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	125	0	61	0	0	0
N.S.	1	1.00	1.00	2.08	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.028	0.150	0.857	0.000	0.080	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	81	456	0	407	0	0	0
N.S.	1	1.00	0.86	4.85	0.00	4.33	0.00	0.00	0.00
time (sec)	N/A	0.042	0.126	1.302	0.000	0.112	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	166	438	0	1291	0	0	0
N.S.	1	1.00	0.84	2.22	0.00	6.55	0.00	0.00	0.00
time (sec)	N/A	0.158	0.477	1.504	0.000	0.127	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	101	218	0	174	0	0	0
N.S.	1	1.00	0.79	1.70	0.00	1.36	0.00	0.00	0.00
time (sec)	N/A	0.081	0.344	1.536	0.000	0.123	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	112	112	100	0	0	126	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	1.12	0.00	0.00	0.00
time (sec)	N/A	0.072	5.037	0.000	0.000	0.315	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	81	83	0	0	82	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	1.01	0.00	0.00	0.00
time (sec)	N/A	0.056	1.725	0.000	0.000	0.308	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	66	0	0	49	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.038	0.154	0.000	0.000	0.313	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	53	26	26	23	15	17	21
N.S.	1	1.00	2.30	1.13	1.13	1.00	0.65	0.74	0.91
time (sec)	N/A	0.026	0.189	0.703	0.193	0.293	0.065	0.281	0.123

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	32	36	141	43	53	32	39
N.S.	1	1.00	0.74	0.84	3.28	1.00	1.23	0.74	0.91
time (sec)	N/A	0.029	0.025	0.857	0.200	0.308	0.130	0.268	1.315

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	50	51	267	70	82	46	52
N.S.	1	1.00	0.74	0.75	3.93	1.03	1.21	0.68	0.76
time (sec)	N/A	0.037	0.034	1.048	0.204	0.310	0.242	0.261	1.567

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	67	66	469	95	110	60	66
N.S.	1	1.00	0.74	0.73	5.15	1.04	1.21	0.66	0.73
time (sec)	N/A	0.047	0.040	1.217	0.208	0.317	0.453	0.267	1.791

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	27	27	24	15	18	21
N.S.	1	1.00	1.26	1.00	1.00	0.89	0.56	0.67	0.78
time (sec)	N/A	0.030	0.541	0.754	0.199	0.322	0.068	0.257	0.118

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	32	36	141	43	51	32	37
N.S.	1	1.00	0.65	0.73	2.88	0.88	1.04	0.65	0.76
time (sec)	N/A	0.031	0.023	0.922	0.199	0.330	0.130	0.272	1.311

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	49	51	267	70	82	46	52
N.S.	1	1.00	0.64	0.67	3.51	0.92	1.08	0.61	0.68
time (sec)	N/A	0.040	0.085	1.023	0.205	0.303	0.240	0.265	1.492

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	63	66	469	95	109	60	68
N.S.	1	1.00	0.62	0.65	4.64	0.94	1.08	0.59	0.67
time (sec)	N/A	0.050	0.041	1.280	0.215	0.280	0.452	0.276	1.712

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	85	126	0	188	0	0	0
N.S.	1	1.00	1.29	1.91	0.00	2.85	0.00	0.00	0.00
time (sec)	N/A	0.046	0.219	5.189	0.000	0.319	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	105	0	0	264	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	3.34	0.00	0.00	0.00
time (sec)	N/A	0.051	0.335	0.000	0.000	0.306	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	110	184	0	0	347	0	0	0
N.S.	1	1.00	1.67	0.00	0.00	3.15	0.00	0.00	0.00
time (sec)	N/A	0.073	0.316	0.000	0.000	0.301	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	241	1878	0	1139	0	0	0
N.S.	1	1.00	0.93	7.25	0.00	4.40	0.00	0.00	0.00
time (sec)	N/A	0.301	0.623	5.224	0.000	0.125	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	196	1037	0	635	0	0	0
N.S.	1	1.00	0.95	5.01	0.00	3.07	0.00	0.00	0.00
time (sec)	N/A	0.210	0.436	4.315	0.000	0.118	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	151	731	0	324	0	0	0
N.S.	1	1.00	0.92	4.46	0.00	1.98	0.00	0.00	0.00
time (sec)	N/A	0.141	0.350	3.805	0.000	0.110	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	61	72	124	147	309	75	269
N.S.	1	1.00	1.11	1.31	2.25	2.67	5.62	1.36	4.89
time (sec)	N/A	0.053	0.148	0.528	0.277	0.313	15.112	0.288	1.657

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	82	113	229	444	0	119	223
N.S.	1	1.00	1.11	1.53	3.09	6.00	0.00	1.61	3.01
time (sec)	N/A	0.059	0.145	0.559	0.288	0.308	0.000	0.282	1.593

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	131	314	537	1614	0	279	0
N.S.	1	1.00	1.02	2.45	4.20	12.61	0.00	2.18	0.00
time (sec)	N/A	0.122	0.220	0.656	0.293	0.355	0.000	0.279	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	189	633	982	3870	0	477	0
N.S.	1	1.00	1.01	3.39	5.25	20.70	0.00	2.55	0.00
time (sec)	N/A	0.222	0.326	0.832	0.312	0.419	0.000	0.287	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	66	80	128	154	258	82	331
N.S.	1	1.00	1.10	1.33	2.13	2.57	4.30	1.37	5.52
time (sec)	N/A	0.058	0.110	0.533	0.276	0.289	13.506	0.282	1.955

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	128	6	3	6	6
N.S.	1	1.00	1.00	1.17	21.33	1.00	0.50	1.00	1.00
time (sec)	N/A	0.001	0.000	0.393	0.271	0.275	0.074	0.276	0.023

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	26	230	58	0	30	49
N.S.	1	1.00	1.00	2.17	19.17	4.83	0.00	2.50	4.08
time (sec)	N/A	0.022	0.162	0.495	0.281	0.306	0.000	0.283	1.262

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	33	34	42	51	33	48
N.S.	1	1.00	0.82	0.97	1.00	1.24	1.50	0.97	1.41
time (sec)	N/A	0.027	0.067	0.340	0.275	0.279	0.767	0.273	1.270

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	109	266	0	183	0	0	0
N.S.	1	1.00	0.80	1.96	0.00	1.35	0.00	0.00	0.00
time (sec)	N/A	0.086	0.394	3.352	0.000	0.088	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	159	517	0	633	0	0	0
N.S.	1	1.00	0.90	2.94	0.00	3.60	0.00	0.00	0.00
time (sec)	N/A	0.155	0.507	3.628	0.000	0.098	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	236	806	0	2167	0	0	0
N.S.	1	1.00	0.94	3.21	0.00	8.63	0.00	0.00	0.00
time (sec)	N/A	0.225	0.623	4.202	0.000	0.158	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	53	511	0	120	0
N.S.	1	1.00	0.68	0.60	1.00	9.64	0.00	2.26	0.00
time (sec)	N/A	0.025	0.061	1.195	0.275	0.281	0.000	0.281	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	35	226	0	70	0
N.S.	1	1.00	0.76	0.71	1.03	6.65	0.00	2.06	0.00
time (sec)	N/A	0.016	0.052	0.714	0.281	0.287	0.000	0.253	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	71	15	34	21
N.S.	1	1.00	1.00	1.15	1.31	5.46	1.15	2.62	1.62
time (sec)	N/A	0.008	0.022	0.875	0.274	0.284	0.113	0.267	1.170

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	30	49	24	110	0	1	0
N.S.	1	1.00	1.76	2.88	1.41	6.47	0.00	0.06	0.00
time (sec)	N/A	0.009	0.034	0.906	0.281	0.323	0.000	0.275	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	71	62	327	0	37	0
N.S.	1	1.00	1.26	1.69	1.48	7.79	0.00	0.88	0.00
time (sec)	N/A	0.016	0.062	0.742	0.276	0.302	0.000	0.278	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	77	89	96	875	0	52	0
N.S.	1	1.00	1.26	1.46	1.57	14.34	0.00	0.85	0.00
time (sec)	N/A	0.025	0.160	0.920	0.277	0.311	0.000	0.285	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	67	0	0	823	0	0	0
N.S.	1	1.00	0.50	0.00	0.00	6.10	0.00	0.00	0.00
time (sec)	N/A	0.041	0.158	0.000	0.000	0.106	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	83	57	0	0	317	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	3.82	0.00	0.00	0.00
time (sec)	N/A	0.028	0.065	0.000	0.000	0.102	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	60	0	0	60	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	0.031	0.082	0.000	0.000	0.084	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	42	0	0	97	0	0	0
N.S.	1	1.00	0.70	0.00	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.027	0.041	0.000	0.000	0.090	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	53	0	0	639	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	7.34	0.00	0.00	0.00
time (sec)	N/A	0.029	0.077	0.000	0.000	0.094	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	69	0	0	1676	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	12.41	0.00	0.00	0.00
time (sec)	N/A	0.047	0.165	0.000	0.000	0.129	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	53	171	100	1597	0	114	0
N.S.	1	1.00	0.40	1.30	0.76	12.10	0.00	0.86	0.00
time (sec)	N/A	0.037	0.149	8.387	0.290	0.310	0.000	0.267	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	38	125	63	659	0	50	0
N.S.	1	1.00	0.49	1.60	0.81	8.45	0.00	0.64	0.00
time (sec)	N/A	0.021	0.089	1.635	0.293	0.299	0.000	0.260	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	84	27	180	0	26	0
N.S.	1	1.00	0.67	2.33	0.75	5.00	0.00	0.72	0.00
time (sec)	N/A	0.011	0.050	1.826	0.285	0.285	0.000	0.275	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	18	122	0	13	38
N.S.	1	1.00	1.00	1.81	1.12	7.62	0.00	0.81	2.38
time (sec)	N/A	0.010	0.030	1.552	0.286	0.262	0.000	0.276	1.141

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	34	48	171	1163	0	27	48
N.S.	1	1.00	0.50	0.71	2.51	17.10	0.00	0.40	0.71
time (sec)	N/A	0.016	0.056	1.527	0.289	0.300	0.000	0.289	1.246

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	47	60	467	3093	0	39	256
N.S.	1	1.00	0.40	0.51	3.96	26.21	0.00	0.33	2.17
time (sec)	N/A	0.022	0.078	1.671	0.289	0.354	0.000	0.274	1.217

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	219	186	90	91	124	86	93
N.S.	1	1.00	4.38	3.72	1.80	1.82	2.48	1.72	1.86
time (sec)	N/A	0.037	0.126	0.480	0.198	0.288	0.143	0.266	1.510

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	42	39	75	72	100	71	77
N.S.	1	1.00	0.98	0.91	1.74	1.67	2.33	1.65	1.79
time (sec)	N/A	0.029	0.024	0.421	0.209	0.292	0.133	0.277	1.391

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	131	138	66	67	82	62	67
N.S.	1	1.00	3.45	3.63	1.74	1.76	2.16	1.63	1.76
time (sec)	N/A	0.032	0.185	0.354	0.196	0.272	0.110	0.269	1.304

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	28	26	51	48	63	47	51
N.S.	1	1.00	0.85	0.79	1.55	1.45	1.91	1.42	1.55
time (sec)	N/A	0.026	0.018	0.360	0.193	0.307	0.093	0.263	1.284

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	93	42	42	41	48	38	41
N.S.	1	1.00	3.58	1.62	1.62	1.58	1.85	1.46	1.58
time (sec)	N/A	0.027	0.133	210.698	0.196	0.294	0.079	0.266	0.128

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	12	13	27	22	27	23	31
N.S.	1	1.00	0.80	0.87	1.80	1.47	1.80	1.53	2.07
time (sec)	N/A	0.022	0.010	16.026	0.198	0.292	0.068	0.267	1.168

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	34	16	14	17	14	14	7
N.S.	1	1.00	4.25	2.00	1.75	2.12	1.75	1.75	0.88
time (sec)	N/A	0.020	0.036	7.892	0.194	0.274	0.054	0.281	1.177

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	5	11	8	11	10
N.S.	1	1.00	1.00	1.00	0.71	1.57	1.14	1.57	1.43
time (sec)	N/A	0.013	0.008	3.332	0.199	0.284	0.040	0.269	1.216

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	18	30	41	52	0	51	46
N.S.	1	1.00	0.75	1.25	1.71	2.17	0.00	2.12	1.92
time (sec)	N/A	0.024	0.021	8.522	0.194	0.321	0.000	0.273	0.212

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	24	53	26	0	29	63
N.S.	1	1.00	0.88	0.96	2.12	1.04	0.00	1.16	2.52
time (sec)	N/A	0.027	0.028	27.193	0.197	0.307	0.000	0.266	1.283

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	61	63	92	143	0	92	115
N.S.	1	1.00	1.17	1.21	1.77	2.75	0.00	1.77	2.21
time (sec)	N/A	0.035	0.035	177.479	0.194	0.306	0.000	0.270	1.660

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	93	205	62	0	53	231
N.S.	1	1.00	0.95	2.51	5.54	1.68	0.00	1.43	6.24
time (sec)	N/A	0.029	0.051	0.434	0.198	0.266	0.000	0.266	1.784

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	94	137	140	245	0	118	249
N.S.	1	1.00	1.18	1.71	1.75	3.06	0.00	1.48	3.11
time (sec)	N/A	0.043	0.057	2.349	0.197	0.305	0.000	0.270	2.656

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	121	112	54	55	65	50	54
N.S.	1	1.00	3.02	2.80	1.35	1.38	1.62	1.25	1.35
time (sec)	N/A	0.049	0.144	0.379	0.194	0.317	0.103	0.279	0.170

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	12	39	34	44	35	37
N.S.	1	1.00	1.29	0.86	2.79	2.43	3.14	2.50	2.64
time (sec)	N/A	0.024	0.016	0.393	0.200	0.297	0.085	0.263	0.103

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	29	30	31	29	26	28
N.S.	1	1.00	1.53	0.97	1.00	1.03	0.97	0.87	0.93
time (sec)	N/A	0.042	0.060	89.743	0.194	0.279	0.073	0.261	1.203

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	25	23	26	26	21	24
N.S.	1	1.00	1.00	1.79	1.64	1.86	1.86	1.50	1.71
time (sec)	N/A	0.025	0.011	25.100	0.191	0.296	0.090	0.277	1.262

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	69	13	12	16	8	10	12
N.S.	1	1.00	4.93	0.93	0.86	1.14	0.57	0.71	0.86
time (sec)	N/A	0.021	0.045	13.097	0.185	0.309	0.051	0.262	1.316

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	8	16	19	10	12
N.S.	1	1.00	1.00	1.00	0.80	1.60	1.90	1.00	1.20
time (sec)	N/A	0.013	0.023	10.888	0.200	0.290	0.054	0.261	1.247

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	26	45	70	103	0	66	86
N.S.	1	1.00	0.76	1.32	2.06	3.03	0.00	1.94	2.53
time (sec)	N/A	0.026	0.033	24.110	0.195	0.313	0.000	0.265	1.457

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	30	117	44	0	41	109
N.S.	1	1.00	0.84	0.81	3.16	1.19	0.00	1.11	2.95
time (sec)	N/A	0.048	0.018	49.599	0.207	0.282	0.000	0.279	1.386

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	116	120	201	0	105	198
N.S.	1	1.00	1.13	1.93	2.00	3.35	0.00	1.75	3.30
time (sec)	N/A	0.035	0.037	0.625	0.207	0.304	0.000	0.274	1.852

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	47	116	317	80	0	65	139
N.S.	1	1.00	0.96	2.37	6.47	1.63	0.00	1.33	2.84
time (sec)	N/A	0.056	0.032	0.408	0.209	0.284	0.000	0.272	0.528

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	47	26	33	50	31	21	41
N.S.	1	1.00	1.68	0.93	1.18	1.79	1.11	0.75	1.46
time (sec)	N/A	0.028	0.076	60.849	0.192	0.299	0.090	0.277	0.189

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	16	19	53	28	34	16	19
N.S.	1	1.00	0.80	0.95	2.65	1.40	1.70	0.80	0.95
time (sec)	N/A	0.021	0.046	57.380	0.210	0.293	0.072	0.275	1.407

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	37	12	16
N.S.	1	1.00	0.88	0.81	0.62	1.88	2.31	0.75	1.00
time (sec)	N/A	0.014	0.039	58.272	0.198	0.282	0.072	0.261	1.406

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	43	26	33	50	31	21	39
N.S.	1	1.00	1.65	1.00	1.27	1.92	1.19	0.81	1.50
time (sec)	N/A	0.026	0.073	59.068	0.198	0.292	0.089	0.258	0.173

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	16	19	53	28	34	16	19
N.S.	1	1.00	0.80	0.95	2.65	1.40	1.70	0.80	0.95
time (sec)	N/A	0.021	0.051	57.560	0.214	0.286	0.073	0.269	1.269

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	13	10	30	36	12	16
N.S.	1	1.00	0.88	0.81	0.62	1.88	2.25	0.75	1.00
time (sec)	N/A	0.014	0.038	58.086	0.211	0.279	0.073	0.263	0.214

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	121	146	308	2105	0	254	287
N.S.	1	1.00	0.88	1.06	2.23	15.25	0.00	1.84	2.08
time (sec)	N/A	0.093	0.124	128.980	0.218	0.315	0.000	0.274	2.063

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	463	344	283	1486	0	288	302
N.S.	1	1.00	3.19	2.37	1.95	10.25	0.00	1.99	2.08
time (sec)	N/A	0.287	4.677	57.610	0.296	0.320	0.000	0.282	2.007

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	82	180	865	0	139	169
N.S.	1	1.00	0.94	1.01	2.22	10.68	0.00	1.72	2.09
time (sec)	N/A	0.063	0.067	23.125	0.211	0.298	0.000	0.273	1.580

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	553	162	170	569	0	168	200
N.S.	1	1.00	5.70	1.67	1.75	5.87	0.00	1.73	2.06
time (sec)	N/A	0.166	1.757	8.007	0.322	0.307	0.000	0.282	1.633

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	81	221	0	61	77
N.S.	1	1.00	1.00	0.97	2.13	5.82	0.00	1.61	2.03
time (sec)	N/A	0.045	0.030	2.992	0.246	0.295	0.000	0.278	1.388

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	396	93	81	171	377	83	87
N.S.	1	1.00	7.33	1.72	1.50	3.17	6.98	1.54	1.61
time (sec)	N/A	0.081	0.581	1.122	0.323	0.319	67.566	0.274	1.316

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	22	11
N.S.	1	1.00	1.00	1.09	1.00	2.45	1.27	2.00	1.00
time (sec)	N/A	0.017	0.006	0.365	0.232	0.301	0.073	0.279	0.070

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	99	64	66	57	0	89	93
N.S.	1	1.00	2.06	1.33	1.38	1.19	0.00	1.85	1.94
time (sec)	N/A	0.040	0.075	2.237	0.320	0.295	0.000	0.264	2.124

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	67	71	89	259	0	87	321
N.S.	1	1.00	1.14	1.20	1.51	4.39	0.00	1.47	5.44
time (sec)	N/A	0.059	0.153	6.923	0.320	0.297	0.000	0.275	1.797

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	161	159	652	0	214	291
N.S.	1	1.00	0.89	1.85	1.83	7.49	0.00	2.46	3.34
time (sec)	N/A	0.086	0.140	17.266	0.291	0.311	0.000	0.281	3.045

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	102	182	230	1142	0	180	634
N.S.	1	1.00	1.02	1.82	2.30	11.42	0.00	1.80	6.34
time (sec)	N/A	0.145	0.293	47.077	0.288	0.290	0.000	0.280	2.395

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	284	313	345	2707	0	369	548
N.S.	1	1.00	2.10	2.32	2.56	20.05	0.00	2.73	4.06
time (sec)	N/A	0.133	0.316	92.654	0.305	0.368	0.000	0.278	5.729

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	146	350	438	3175	0	323	1010
N.S.	1	1.00	1.00	2.40	3.00	21.75	0.00	2.21	6.92
time (sec)	N/A	0.285	0.384	174.745	0.296	0.313	0.000	0.281	3.019

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	660	161	176	833	0	178	256
N.S.	1	1.00	7.02	1.71	1.87	8.86	0.00	1.89	2.72
time (sec)	N/A	0.166	3.597	17.491	0.295	0.308	0.000	0.290	1.602

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	41	102	370	133	82	60
N.S.	1	1.00	0.92	1.02	2.55	9.25	3.32	2.05	1.50
time (sec)	N/A	0.041	0.049	6.092	0.212	0.288	0.273	0.269	1.449

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	502	101	100	362	0	97	132
N.S.	1	1.00	8.10	1.63	1.61	5.84	0.00	1.56	2.13
time (sec)	N/A	0.076	1.398	2.178	0.317	0.308	0.000	0.290	1.437

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	51	19	22	14
N.S.	1	1.00	1.00	1.08	1.00	3.92	1.46	1.69	1.08
time (sec)	N/A	0.018	0.020	1.356	0.234	0.293	0.189	0.271	0.106

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	121	123	149	383	0	186	186
N.S.	1	1.00	1.53	1.56	1.89	4.85	0.00	2.35	2.35
time (sec)	N/A	0.074	0.326	13.925	0.320	0.314	0.000	0.267	2.567

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	138	215	802	0	167	302
N.S.	1	1.00	1.01	1.48	2.31	8.62	0.00	1.80	3.25
time (sec)	N/A	0.121	0.197	50.269	0.290	0.299	0.000	0.301	1.741

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	260	211	375	2615	0	295	519
N.S.	1	1.00	1.91	1.55	2.76	19.23	0.00	2.17	3.82
time (sec)	N/A	0.113	1.519	95.750	0.303	0.330	0.000	0.276	6.083

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	137	266	490	3044	0	287	476
N.S.	1	1.00	0.95	1.85	3.40	21.14	0.00	1.99	3.31
time (sec)	N/A	0.208	0.354	179.319	0.325	0.322	0.000	0.283	1.813

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	96	63	413	86	107	53	231
N.S.	1	1.00	3.10	2.03	13.32	2.77	3.45	1.71	7.45
time (sec)	N/A	0.050	0.133	13.366	0.233	0.290	0.100	0.263	1.922

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	42	67	95	151	99	92	113
N.S.	1	1.00	1.17	1.86	2.64	4.19	2.75	2.56	3.14
time (sec)	N/A	0.057	0.066	9.066	0.222	0.287	0.112	0.263	1.621

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	67	37	109	38	46	29	80
N.S.	1	1.00	2.91	1.61	4.74	1.65	2.00	1.26	3.48
time (sec)	N/A	0.047	0.090	5.965	0.234	0.327	0.072	0.275	1.403

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	20	31	42	55	32	53	29
N.S.	1	1.00	0.77	1.19	1.62	2.12	1.23	2.04	1.12
time (sec)	N/A	0.036	0.017	4.032	0.232	0.289	0.066	0.271	0.177

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	21	28	17	19	23	24
N.S.	1	1.00	0.89	1.11	1.47	0.89	1.00	1.21	1.26
time (sec)	N/A	0.021	0.010	4.119	0.220	0.275	0.090	0.278	1.313

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	41	23	27	37	22	24	28
N.S.	1	1.00	3.42	1.92	2.25	3.08	1.83	2.00	2.33
time (sec)	N/A	0.028	0.076	6.122	0.225	0.276	0.071	0.264	0.187

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	24	67	31	32	23	25
N.S.	1	1.00	1.00	1.60	4.47	2.07	2.13	1.53	1.67
time (sec)	N/A	0.039	0.010	9.270	0.219	0.297	0.065	0.277	1.300

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	111	46	61	90	61	44	74
N.S.	1	1.00	4.27	1.77	2.35	3.46	2.35	1.69	2.85
time (sec)	N/A	0.053	0.053	13.286	0.219	0.293	0.100	0.263	0.330

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	33	45	205	63	70	51	44
N.S.	1	1.00	1.43	1.96	8.91	2.74	3.04	2.22	1.91
time (sec)	N/A	0.052	0.010	17.705	0.201	0.263	0.091	0.279	1.344

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	175	65	91	144	100	62	124
N.S.	1	1.00	4.86	1.81	2.53	4.00	2.78	1.72	3.44
time (sec)	N/A	0.065	0.080	25.379	0.198	0.269	0.127	0.282	1.692

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	112	69	573	104	128	65	395
N.S.	1	1.00	2.38	1.47	12.19	2.21	2.72	1.38	8.40
time (sec)	N/A	0.080	0.165	28.339	0.206	0.271	0.134	0.270	4.900

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	52	76	115	197	129	102	209
N.S.	1	1.00	0.79	1.15	1.74	2.98	1.95	1.55	3.17
time (sec)	N/A	0.044	0.105	20.248	0.196	0.277	0.131	0.263	2.275

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	84	43	197	56	66	41	139
N.S.	1	1.00	2.27	1.16	5.32	1.51	1.78	1.11	3.76
time (sec)	N/A	0.074	0.144	13.904	0.218	0.262	0.095	0.276	0.313

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	29	36	61	94	58	66	99
N.S.	1	1.00	0.81	1.00	1.69	2.61	1.61	1.83	2.75
time (sec)	N/A	0.024	0.028	9.282	0.192	0.267	0.099	0.261	1.608

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	31	48	54	36	33	49
N.S.	1	1.00	1.00	1.24	1.92	2.16	1.44	1.32	1.96
time (sec)	N/A	0.025	0.022	8.642	0.220	0.271	0.088	0.266	0.312

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	66	35	53	78	49	48	60
N.S.	1	1.00	2.54	1.35	2.04	3.00	1.88	1.85	2.31
time (sec)	N/A	0.045	0.173	14.276	0.224	0.310	0.102	0.273	1.483

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	45	63	70	53	54	56
N.S.	1	1.00	1.00	1.55	2.17	2.41	1.83	1.86	1.93
time (sec)	N/A	0.032	0.020	21.470	0.216	0.286	0.109	0.271	0.271

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	107	56	67	100	66	50	111
N.S.	1	1.00	3.82	2.00	2.39	3.57	2.36	1.79	3.96
time (sec)	N/A	0.062	0.092	31.665	0.220	0.280	0.122	0.266	0.223

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	41	171	59	65	38	40
N.S.	1	1.00	1.00	1.52	6.33	2.19	2.41	1.41	1.48
time (sec)	N/A	0.027	0.011	43.296	0.211	0.277	0.102	0.277	1.344

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	175	74	103	160	114	74	246
N.S.	1	1.00	3.65	1.54	2.15	3.33	2.38	1.54	5.12
time (sec)	N/A	0.062	0.107	61.777	0.219	0.280	0.156	0.274	1.509

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	108	171	241	1199	0	197	654
N.S.	1	1.00	0.87	1.38	1.94	9.67	0.00	1.59	5.27
time (sec)	N/A	0.123	0.315	1.907	0.307	0.289	0.000	0.293	2.242

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	153	166	160	655	0	211	291
N.S.	1	1.00	1.74	1.89	1.82	7.44	0.00	2.40	3.31
time (sec)	N/A	0.120	0.144	0.911	0.304	0.302	0.000	0.292	3.086

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	84	89	257	0	87	330
N.S.	1	1.00	1.00	1.22	1.29	3.72	0.00	1.26	4.78
time (sec)	N/A	0.062	0.187	0.753	0.322	0.283	0.000	0.283	1.655

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	56	73	66	57	0	89	95
N.S.	1	1.00	1.17	1.52	1.38	1.19	0.00	1.85	1.98
time (sec)	N/A	0.048	0.046	0.507	0.329	0.298	0.000	0.273	2.140

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	33	46	40	0	39	195
N.S.	1	1.00	1.00	1.65	2.30	2.00	0.00	1.95	9.75
time (sec)	N/A	0.030	0.009	0.484	0.281	0.278	0.000	0.280	0.422

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	89	81	97	228	0	95	304
N.S.	1	1.00	1.59	1.45	1.73	4.07	0.00	1.70	5.43
time (sec)	N/A	0.152	0.238	0.684	0.331	0.296	0.000	0.273	1.555

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	45	98	116	427	0	125	1163
N.S.	1	1.00	0.87	1.88	2.23	8.21	0.00	2.40	22.37
time (sec)	N/A	0.061	0.046	0.885	0.252	0.285	0.000	0.274	1.975

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	197	169	212	1303	0	194	778
N.S.	1	1.00	1.82	1.56	1.96	12.06	0.00	1.80	7.20
time (sec)	N/A	0.269	0.369	1.472	0.318	0.339	0.000	0.274	2.140

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	144	245	523	3534	0	292	543
N.S.	1	1.00	0.64	1.09	2.33	15.78	0.00	1.30	2.42
time (sec)	N/A	0.305	0.306	2.697	0.351	0.321	0.000	0.306	2.022

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	150	237	375	2850	0	307	501
N.S.	1	1.00	1.11	1.76	2.78	21.11	0.00	2.27	3.71
time (sec)	N/A	0.270	0.519	1.413	0.343	0.352	0.000	0.296	4.842

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	100	142	223	900	0	181	377
N.S.	1	1.00	0.69	0.99	1.55	6.25	0.00	1.26	2.62
time (sec)	N/A	0.180	0.237	0.862	0.328	0.287	0.000	0.318	1.885

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	146	136	155	423	0	199	190
N.S.	1	1.00	1.72	1.60	1.82	4.98	0.00	2.34	2.24
time (sec)	N/A	0.074	0.179	0.679	0.335	0.288	0.000	0.286	2.554

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	27	57	75	158	0	75	240
N.S.	1	1.00	0.84	1.78	2.34	4.94	0.00	2.34	7.50
time (sec)	N/A	0.036	0.039	1.114	0.243	0.290	0.000	0.274	1.679

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	112	118	165	1257	0	148	897
N.S.	1	1.00	1.40	1.48	2.06	15.71	0.00	1.85	11.21
time (sec)	N/A	0.272	0.447	2.055	0.326	0.316	0.000	0.283	2.583

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	73	142	202	1463	0	190	1375
N.S.	1	1.00	0.96	1.87	2.66	19.25	0.00	2.50	18.09
time (sec)	N/A	0.076	0.151	3.468	0.222	0.303	0.000	0.288	2.381

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	235	221	339	3648	0	242	1450
N.S.	1	1.00	1.48	1.39	2.13	22.94	0.00	1.52	9.12
time (sec)	N/A	0.449	0.637	4.995	0.294	0.352	0.000	0.292	2.284

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	21	0	356	0	0	0
N.S.	1	1.00	1.00	0.57	0.00	9.62	0.00	0.00	0.00
time (sec)	N/A	0.045	0.017	1.857	0.000	0.451	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	17	0	370	0	0	0
N.S.	1	1.00	1.00	0.71	0.00	15.42	0.00	0.00	0.00
time (sec)	N/A	0.039	0.011	0.914	0.000	0.333	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	59	49	68	170	517	87	198
N.S.	1	1.00	1.16	0.96	1.33	3.33	10.14	1.71	3.88
time (sec)	N/A	0.091	0.078	1.651	0.293	0.300	15.127	0.274	3.516

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	48	23	19	36	20	22	24
N.S.	1	1.00	1.92	0.92	0.76	1.44	0.80	0.88	0.96
time (sec)	N/A	0.056	0.131	1.126	0.203	0.270	0.082	0.269	0.139

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	81	24	20	35	20	21	23
N.S.	1	1.00	3.00	0.89	0.74	1.30	0.74	0.78	0.85
time (sec)	N/A	0.061	0.383	0.809	0.205	0.263	0.081	0.259	0.120

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	149	117	125	172	0	123	914
N.S.	1	1.00	1.67	1.31	1.40	1.93	0.00	1.38	10.27
time (sec)	N/A	0.127	0.649	0.581	0.327	1.187	0.000	0.287	9.686

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	65	73	106	183	0	102	164
N.S.	1	1.00	1.08	1.22	1.77	3.05	0.00	1.70	2.73
time (sec)	N/A	0.106	1.399	0.885	0.329	0.293	0.000	0.304	12.324

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	93	102	125	172	0	123	864
N.S.	1	1.00	1.04	1.15	1.40	1.93	0.00	1.38	9.71
time (sec)	N/A	0.180	0.271	1.880	0.324	1.889	0.000	0.282	11.666

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	77	58	141	172	0	90	539
N.S.	1	1.00	1.33	1.00	2.43	2.97	0.00	1.55	9.29
time (sec)	N/A	0.101	1.127	0.934	0.331	0.497	0.000	0.296	2.216

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	85	111	176	249	1318	127	656
N.S.	1	1.00	1.05	1.37	2.17	3.07	16.27	1.57	8.10
time (sec)	N/A	0.120	2.328	2.624	0.323	0.285	18.115	0.291	2.634

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	113	151	339	570	0	170	279
N.S.	1	1.00	1.00	1.34	3.00	5.04	0.00	1.50	2.47
time (sec)	N/A	0.126	1.338	3.279	0.352	0.269	0.000	0.309	1.969

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	170	370	726	1880	0	405	0
N.S.	1	1.00	0.94	2.06	4.03	10.44	0.00	2.25	0.00
time (sec)	N/A	0.197	1.542	15.393	0.335	0.297	0.000	0.327	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	235	699	1263	4350	0	685	0
N.S.	1	1.00	0.94	2.80	5.05	17.40	0.00	2.74	0.00
time (sec)	N/A	0.306	1.832	75.691	0.339	0.397	0.000	0.347	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	598	919	0	1655	0	0	0
N.S.	1	1.00	1.36	2.09	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.493	0.510	1.735	0.000	0.300	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	448	710	0	1247	0	0	0
N.S.	1	1.00	1.37	2.17	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	0.387	0.343	0.133	0.000	0.302	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	294	505	0	837	0	0	0
N.S.	1	1.00	1.37	2.35	0.00	3.89	0.00	0.00	0.00
time (sec)	N/A	0.224	0.247	0.112	0.000	0.322	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	42	0
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.89	0.00
time (sec)	N/A	0.312	0.097	3.306	0.285	0.254	0.000	0.279	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	57	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	0.031	0.000	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.023	0.000	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	35	34	48	34	35
N.S.	1	1.00	1.06	0.94	1.03	1.00	1.41	1.00	1.03
time (sec)	N/A	0.028	12.930	0.880	0.329	0.272	10.068	0.431	1.398

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	32	187	34	51	34	35
N.S.	1	1.00	1.06	0.89	5.19	0.94	1.42	0.94	0.97
time (sec)	N/A	0.055	44.088	0.520	0.346	0.258	34.708	0.600	1.524

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	41	42	52	44	0	47	43
N.S.	1	1.00	0.76	0.78	0.96	0.81	0.00	0.87	0.80
time (sec)	N/A	0.009	0.045	0.920	0.243	0.261	0.000	0.269	1.376

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	55	58	67	91	0	169	53
N.S.	1	1.00	0.62	0.66	0.76	1.03	0.00	1.92	0.60
time (sec)	N/A	0.015	0.067	2.711	0.234	0.278	0.000	0.283	1.371

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	166	114	200	0	665	93
N.S.	1	1.00	0.81	1.11	0.77	1.34	0.00	4.46	0.62
time (sec)	N/A	0.029	0.337	5.521	0.256	0.271	0.000	0.302	1.408

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	167	132	129	294	0	777	102
N.S.	1	1.00	0.87	0.69	0.68	1.54	0.00	4.07	0.53
time (sec)	N/A	0.037	0.307	14.180	0.232	0.263	0.000	0.322	1.449

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	74	54	0	64	98	0	235	56
N.S.	1	1.01	0.74	0.00	0.88	1.34	0.00	3.22	0.77
time (sec)	N/A	0.020	0.093	0.000	0.214	0.265	0.000	0.294	1.448

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	122	89	0	87	248	0	758	74
N.S.	1	1.02	0.74	0.00	0.72	2.07	0.00	6.32	0.62
time (sec)	N/A	0.033	0.198	0.000	0.203	0.264	0.000	0.293	1.502

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	203	203	292	0	138	585	0	3225	118
N.S.	1	1.00	1.44	0.00	0.68	2.88	0.00	15.89	0.58
time (sec)	N/A	0.074	0.952	0.000	0.227	0.273	0.000	0.351	1.576

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	266	267	311	0	161	1125	0	6884	136
N.S.	1	1.00	1.17	0.00	0.61	4.23	0.00	25.88	0.51
time (sec)	N/A	0.097	2.502	0.000	0.255	0.316	0.000	0.424	1.602

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	34	40	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	1.89	2.22	1.00
time (sec)	N/A	0.012	0.008	1.714	0.203	0.255	0.211	0.263	1.415

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	30	49	40	0	81	32
N.S.	1	1.00	0.92	0.77	1.26	1.03	0.00	2.08	0.82
time (sec)	N/A	0.021	0.017	1.704	0.214	0.263	0.000	0.269	1.447

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	45	36	86	65	71	81	37
N.S.	1	1.00	1.05	0.84	2.00	1.51	1.65	1.88	0.86
time (sec)	N/A	0.026	0.008	6.646	0.230	0.271	1.163	0.280	1.472

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	46	93	84	0	114	51
N.S.	1	1.00	0.70	0.63	1.27	1.15	0.00	1.56	0.70
time (sec)	N/A	0.033	0.030	21.055	0.201	0.275	0.000	0.277	1.553

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	51	130	130	105	115	49
N.S.	1	1.00	1.05	0.78	2.00	2.00	1.62	1.77	0.75
time (sec)	N/A	0.030	0.011	74.458	0.228	0.271	7.172	0.291	1.589

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	96	227	0	331	0	0	0
N.S.	1	1.00	0.86	2.05	0.00	2.98	0.00	0.00	0.00
time (sec)	N/A	0.043	0.056	6.447	0.000	0.104	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	114	143	0	171	0	0	0
N.S.	1	1.00	1.03	1.29	0.00	1.54	0.00	0.00	0.00
time (sec)	N/A	0.043	0.094	0.998	0.000	0.095	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	68	146	0	58	0	0	0
N.S.	1	1.00	0.94	2.03	0.00	0.81	0.00	0.00	0.00
time (sec)	N/A	0.030	0.021	0.805	0.000	0.091	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	66	120	0	39	0	0	0
N.S.	1	1.00	0.92	1.67	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.030	0.026	0.709	0.000	0.090	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	80	212	0	246	0	0	0
N.S.	1	1.00	0.75	1.98	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.040	0.036	0.823	0.000	0.099	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	122	144	0	504	0	0	0
N.S.	1	1.00	1.10	1.30	0.00	4.54	0.00	0.00	0.00
time (sec)	N/A	0.041	0.079	0.856	0.000	0.101	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	209	209	86	0	0	162	0	203	0
N.S.	1	1.00	0.41	0.00	0.00	0.78	0.00	0.97	0.00
time (sec)	N/A	0.113	0.227	0.000	0.000	0.288	0.000	0.423	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	74	0	0	117	0	0	0
N.S.	1	1.00	0.72	0.00	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.064	0.189	0.000	0.000	0.268	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	61	0	0	68	0	41	0
N.S.	1	1.00	1.42	0.00	0.00	1.58	0.00	0.95	0.00
time (sec)	N/A	0.038	0.097	0.000	0.000	0.263	0.000	0.543	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	121	0	0	128	0	81	0
N.S.	1	1.00	1.17	0.00	0.00	1.24	0.00	0.79	0.00
time (sec)	N/A	0.058	0.160	0.000	0.000	0.260	0.000	0.579	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	48	0	102	0
N.S.	1	1.00	1.00	1.06	0.00	1.33	0.00	2.83	0.00
time (sec)	N/A	0.034	0.016	1.643	0.000	0.275	0.000	0.284	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	37	50	0	73	0	97	0
N.S.	1	1.00	0.95	1.28	0.00	1.87	0.00	2.49	0.00
time (sec)	N/A	0.045	0.033	0.461	0.000	0.260	0.000	0.328	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	74	0	118	0	167	0
N.S.	1	1.00	0.92	1.25	0.00	2.00	0.00	2.83	0.00
time (sec)	N/A	0.061	0.046	0.531	0.000	0.277	0.000	0.365	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	93	113	0	253	0	0	0
N.S.	1	1.00	1.26	1.53	0.00	3.42	0.00	0.00	0.00
time (sec)	N/A	0.091	0.177	1.087	0.000	0.274	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	115	120	0	277	0	0	0
N.S.	1	1.00	1.44	1.50	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.104	0.233	6.919	0.000	0.272	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	232	250	0	701	0	0	0
N.S.	1	1.00	1.62	1.75	0.00	4.90	0.00	0.00	0.00
time (sec)	N/A	0.173	0.440	3.347	0.000	0.278	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	302	347	0	171	0	764	0
N.S.	1	1.00	2.99	3.44	0.00	1.69	0.00	7.56	0.00
time (sec)	N/A	0.130	0.512	1.076	0.000	0.283	0.000	1.899	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	475	358	0	370	0	749	0
N.S.	1	1.00	4.44	3.35	0.00	3.46	0.00	7.00	0.00
time (sec)	N/A	0.135	2.958	7.666	0.000	0.264	0.000	8.411	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	651	700	0	717	0	1383	0
N.S.	1	1.00	3.36	3.61	0.00	3.70	0.00	7.13	0.00
time (sec)	N/A	0.252	5.816	1.886	0.000	0.282	0.000	9.575	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	363	463	0	202	0	1624	0
N.S.	1	1.00	3.00	3.83	0.00	1.67	0.00	13.42	0.00
time (sec)	N/A	0.190	1.587	1.664	0.000	0.286	0.000	5.020	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	572	472	0	477	0	1596	0
N.S.	1	1.00	4.43	3.66	0.00	3.70	0.00	12.37	0.00
time (sec)	N/A	0.197	3.562	9.178	0.000	0.277	0.000	22.527	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	913	930	0	942	0	3021	0
N.S.	1	1.00	4.04	4.12	0.00	4.17	0.00	13.37	0.00
time (sec)	N/A	0.334	6.484	2.496	0.000	0.285	0.000	27.948	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	62	45	68	113	139	60	58
N.S.	1	1.00	0.75	0.54	0.82	1.36	1.67	0.72	0.70
time (sec)	N/A	0.026	0.037	1.637	0.180	0.276	2.058	0.263	0.528

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	45	47	53	95	207	57	42
N.S.	1	1.00	0.79	0.82	0.93	1.67	3.63	1.00	0.74
time (sec)	N/A	0.025	0.030	0.707	0.214	0.278	0.835	0.272	1.513

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	39	35	40	54	78	34	36
N.S.	1	1.00	0.80	0.71	0.82	1.10	1.59	0.69	0.73
time (sec)	N/A	0.020	0.019	0.326	0.207	0.311	0.393	0.259	1.339

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	50	63	24	18
N.S.	1	1.00	1.00	0.83	1.04	2.17	2.74	1.04	0.78
time (sec)	N/A	0.011	0.012	0.173	0.185	0.275	0.184	0.305	0.072

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	27	30	0	24	16
N.S.	1	1.00	1.00	0.89	1.42	1.58	0.00	1.26	0.84
time (sec)	N/A	0.013	0.011	0.124	0.180	0.263	0.000	0.275	0.073

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	25	52	157	0	48	52
N.S.	1	1.00	0.88	0.60	1.24	3.74	0.00	1.14	1.24
time (sec)	N/A	0.020	0.042	0.369	0.190	0.275	0.000	0.271	1.347

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	24	68	88	0	31	31
N.S.	1	1.00	0.94	0.77	2.19	2.84	0.00	1.00	1.00
time (sec)	N/A	0.019	0.014	1.562	0.210	0.269	0.000	0.264	1.307

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	75	37	100	705	0	75	135
N.S.	1	1.00	0.74	0.37	0.99	6.98	0.00	0.74	1.34
time (sec)	N/A	0.039	0.048	2.311	0.214	0.276	0.000	0.272	1.320

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	44	35	172	233	0	42	42
N.S.	1	1.00	0.67	0.53	2.61	3.53	0.00	0.64	0.64
time (sec)	N/A	0.038	0.038	3.417	0.214	0.262	0.000	0.270	1.297

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	17	47	42	17	17
N.S.	1	1.00	1.00	0.65	0.65	1.81	1.62	0.65	0.65
time (sec)	N/A	0.013	0.013	1.589	0.209	0.279	0.156	0.264	0.101

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	26	20	13	12
N.S.	1	1.00	0.84	0.74	0.68	1.37	1.05	0.68	0.63
time (sec)	N/A	0.008	0.008	0.346	0.209	0.279	0.096	0.257	1.280

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	34	18	25	0	19	26
N.S.	1	1.00	1.00	3.09	1.64	2.27	0.00	1.73	2.36
time (sec)	N/A	0.009	0.011	0.874	0.296	0.283	0.000	0.259	0.187

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	46	32	182	0	33	36
N.S.	1	1.00	1.00	1.44	1.00	5.69	0.00	1.03	1.12
time (sec)	N/A	0.016	0.056	0.647	0.296	0.281	0.000	0.259	1.434

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	17	67	42	17	17
N.S.	1	1.00	1.00	0.65	0.65	2.58	1.62	0.65	0.65
time (sec)	N/A	0.015	0.013	0.556	0.215	0.272	0.152	0.262	1.318

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	14	13	40	20	13	12
N.S.	1	1.00	0.84	0.74	0.68	2.11	1.05	0.68	0.63
time (sec)	N/A	0.009	0.012	0.635	0.208	0.268	0.095	0.277	0.057

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	22	79	73	83	0	43	65
N.S.	1	1.00	0.41	1.46	1.35	1.54	0.00	0.80	1.20
time (sec)	N/A	0.041	0.011	0.630	0.305	0.287	0.000	0.269	0.189

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	34	148	85	560	0	86	91
N.S.	1	1.00	0.32	1.41	0.81	5.33	0.00	0.82	0.87
time (sec)	N/A	0.102	0.020	0.740	0.300	0.283	0.000	0.270	0.394

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	17	87	42	17	17
N.S.	1	1.00	1.00	0.65	0.65	3.35	1.62	0.65	0.65
time (sec)	N/A	0.014	0.014	0.638	0.198	0.281	0.152	0.266	0.081

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	46	20	13	13
N.S.	1	1.00	1.00	0.74	0.68	2.42	1.05	0.68	0.68
time (sec)	N/A	0.008	0.011	0.229	0.189	0.273	0.097	0.258	1.289

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	22	56	95	107	0	96	106
N.S.	1	1.00	0.19	0.50	0.84	0.95	0.00	0.85	0.94
time (sec)	N/A	0.054	0.010	0.934	0.276	0.281	0.000	0.262	0.370

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	34	68	107	855	0	108	120
N.S.	1	1.00	0.26	0.52	0.82	6.53	0.00	0.82	0.92
time (sec)	N/A	0.061	0.025	0.745	0.279	0.298	0.000	0.273	1.646

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	157	147	134	2228	1525	1211	166
N.S.	1	1.00	0.78	0.73	0.66	11.03	7.55	6.00	0.82
time (sec)	N/A	0.058	0.508	1.609	0.199	0.358	7.771	0.318	2.330

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	86	90	94	703	709	890	97
N.S.	1	1.00	0.65	0.68	0.71	5.33	5.37	6.74	0.73
time (sec)	N/A	0.037	0.128	0.850	0.224	0.284	1.070	0.297	1.647

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	50	51	63	244	323	598	73
N.S.	1	1.00	0.67	0.68	0.84	3.25	4.31	7.97	0.97
time (sec)	N/A	0.014	0.075	0.161	0.226	0.287	0.580	0.293	1.383

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	79	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	2.619	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	87	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.908	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	281	0	0	0	0	0	0
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	14.380	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	163	0	0	0	0	0	0
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.045	1.579	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	250	109	88	90	218	0	269	0
N.S.	1	1.00	0.44	0.35	0.36	0.87	0.00	1.08	0.00
time (sec)	N/A	0.180	0.078	6.257	0.311	0.293	0.000	0.286	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	162	76	75	62	126	0	195	0
N.S.	1	1.00	0.47	0.46	0.38	0.78	0.00	1.20	0.00
time (sec)	N/A	0.091	0.044	0.571	0.313	0.294	0.000	0.303	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	74	48	60	36	66	158	71	77
N.S.	1	1.00	0.65	0.81	0.49	0.89	2.14	0.96	1.04
time (sec)	N/A	0.069	0.033	0.588	0.308	0.273	1.568	0.285	1.369

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	46	46	44	29	39	42	0	85	0
N.S.	1	1.00	0.96	0.63	0.85	0.91	0.00	1.85	0.00
time (sec)	N/A	0.083	0.042	0.639	0.298	0.263	0.000	0.285	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	58	58	46	42	84	121	0	87	76
N.S.	1	1.00	0.79	0.72	1.45	2.09	0.00	1.50	1.31
time (sec)	N/A	0.092	0.056	1.311	0.307	0.269	0.000	0.289	1.289

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	147	147	72	66	209	315	0	122	89
N.S.	1	1.00	0.49	0.45	1.42	2.14	0.00	0.83	0.61
time (sec)	N/A	0.133	0.050	0.681	0.308	0.283	0.000	0.302	1.353

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	84	91	386	592	0	161	353
N.S.	1	1.00	0.42	0.46	1.94	2.97	0.00	0.81	1.77
time (sec)	N/A	0.185	0.059	0.871	0.317	0.291	0.000	0.308	1.349

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	28	0	42	99	32	45
N.S.	1	1.00	0.68	0.68	0.00	1.02	2.41	0.78	1.10
time (sec)	N/A	0.010	0.038	0.674	0.000	0.267	0.230	0.274	0.099

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	80	72	65	103	0	73	0
N.S.	1	1.00	0.94	0.85	0.76	1.21	0.00	0.86	0.00
time (sec)	N/A	0.058	0.067	0.847	0.206	0.271	0.000	0.266	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	92	97	81	129	0	91	0
N.S.	1	1.00	0.91	0.96	0.80	1.28	0.00	0.90	0.00
time (sec)	N/A	0.107	0.123	0.943	0.226	0.279	0.000	0.259	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	C	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	72	0	45	0
N.S.	1	1.00	0.78	0.80	0.69	1.11	0.00	0.69	0.00
time (sec)	N/A	0.051	0.050	1.089	0.183	0.281	0.000	0.277	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	72	48	47	75	0	49	0
N.S.	1	1.00	1.11	0.74	0.72	1.15	0.00	0.75	0.00
time (sec)	N/A	0.060	0.080	1.554	0.187	0.281	0.000	0.259	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	123	105	89	164	0	101	0
N.S.	1	1.00	1.07	0.91	0.77	1.43	0.00	0.88	0.00
time (sec)	N/A	0.116	0.285	1.300	0.190	0.280	0.000	0.272	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	103	100	90	213	0	106	0
N.S.	1	1.00	0.94	0.91	0.82	1.94	0.00	0.96	0.00
time (sec)	N/A	0.114	0.112	0.677	0.190	0.294	0.000	0.271	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	356	0
N.S.	1	1.00	1.01	0.85	0.86	1.88	0.00	2.41	0.00
time (sec)	N/A	0.134	0.559	0.770	0.262	0.309	0.000	0.291	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	287	207	200	445	0	223	0
N.S.	1	1.00	1.20	0.87	0.84	1.86	0.00	0.93	0.00
time (sec)	N/A	0.210	0.299	1.541	0.269	0.316	0.000	0.292	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	124	126	102	253	0	132	0
N.S.	1	1.00	1.08	1.10	0.89	2.20	0.00	1.15	0.00
time (sec)	N/A	0.161	0.194	0.492	0.186	0.295	0.000	0.269	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	143	334	0	388	0
N.S.	1	1.00	1.37	0.98	0.89	2.07	0.00	2.41	0.00
time (sec)	N/A	0.206	0.448	0.892	0.263	0.300	0.000	0.285	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	354	265	228	541	0	281	0
N.S.	1	1.00	1.38	1.03	0.89	2.11	0.00	1.09	0.00
time (sec)	N/A	0.327	0.594	1.379	0.268	0.296	0.000	0.299	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	217	0	132	0
N.S.	1	1.00	0.78	0.88	0.79	1.63	0.00	0.99	0.00
time (sec)	N/A	0.150	0.127	0.427	0.186	0.298	0.000	0.266	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	244	0	150	0
N.S.	1	1.00	0.81	0.86	0.81	1.52	0.00	0.93	0.00
time (sec)	N/A	0.160	0.196	0.353	0.191	0.347	0.000	0.279	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	427	0	264	0
N.S.	1	1.00	0.79	0.86	0.78	1.58	0.00	0.97	0.00
time (sec)	N/A	0.247	0.346	0.978	0.198	0.306	0.000	0.274	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	76	70	69	146	0	75	0
N.S.	1	1.00	0.94	0.86	0.85	1.80	0.00	0.93	0.00
time (sec)	N/A	0.123	0.254	0.192	0.188	0.305	0.000	0.270	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	0
N.S.	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	0.00
time (sec)	N/A	0.161	0.383	0.309	0.185	0.291	0.000	0.277	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	272	144	143	492	0	155	0
N.S.	1	1.00	1.59	0.84	0.84	2.88	0.00	0.91	0.00
time (sec)	N/A	0.215	0.905	0.799	0.198	0.286	0.000	0.287	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	166	147	127	322	0	172	0
N.S.	1	1.00	1.19	1.05	0.91	2.30	0.00	1.23	0.00
time (sec)	N/A	0.202	0.520	0.271	0.196	0.310	0.000	0.264	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	422	0	198	0
N.S.	1	1.00	1.41	0.97	0.88	2.31	0.00	1.08	0.00
time (sec)	N/A	0.256	1.026	0.398	0.202	0.277	0.000	0.279	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	480	302	263	848	0	352	0
N.S.	1	1.00	1.60	1.01	0.88	2.83	0.00	1.17	0.00
time (sec)	N/A	0.394	4.321	1.171	0.231	0.355	0.000	0.296	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	135	162	129	263	0	167	0
N.S.	1	1.00	0.88	1.06	0.84	1.72	0.00	1.09	0.00
time (sec)	N/A	0.222	0.231	0.339	0.248	0.308	0.000	0.282	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	183	210	185	343	0	223	0
N.S.	1	1.00	0.84	0.96	0.84	1.57	0.00	1.02	0.00
time (sec)	N/A	0.265	0.412	0.420	0.239	0.280	0.000	0.292	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	263	326	263	527	0	339	0
N.S.	1	1.00	0.83	1.03	0.83	1.67	0.00	1.08	0.00
time (sec)	N/A	0.324	0.641	0.997	0.240	0.343	0.000	0.284	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	179	160	139	325	0	181	0
N.S.	1	1.00	1.16	1.04	0.90	2.11	0.00	1.18	0.00
time (sec)	N/A	0.249	0.574	0.230	0.205	0.303	0.000	0.272	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	0
N.S.	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	0.00
time (sec)	N/A	0.290	1.530	0.405	0.207	0.321	0.000	0.295	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	503	326	287	852	0	369	0
N.S.	1	1.00	1.56	1.01	0.89	2.64	0.00	1.14	0.00
time (sec)	N/A	0.400	4.729	1.095	0.210	0.302	0.000	0.291	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	252	186	151	363	0	207	0
N.S.	1	1.00	1.57	1.16	0.94	2.25	0.00	1.29	0.00
time (sec)	N/A	0.293	1.084	0.236	0.228	0.295	0.000	0.282	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	215	516	0	271	0
N.S.	1	1.00	1.42	1.04	0.90	2.16	0.00	1.13	0.00
time (sec)	N/A	0.403	4.455	0.602	0.207	0.293	0.000	0.296	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	315	940	0	427	0
N.S.	1	1.00	8.69	1.12	0.92	2.73	0.00	1.24	0.00
time (sec)	N/A	0.537	6.459	1.403	0.211	0.303	0.000	0.297	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	22	41	35	24
N.S.	1	1.00	1.00	0.83	1.17	0.73	1.37	1.17	0.80
time (sec)	N/A	0.027	0.044	0.344	0.195	0.277	0.060	0.250	1.287

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	48	81	58	85	75	46
N.S.	1	1.00	0.86	0.86	1.45	1.04	1.52	1.34	0.82
time (sec)	N/A	0.054	0.063	0.416	0.229	0.277	0.089	0.252	0.085

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	167	212	0	316	0	0	0
N.S.	1	1.00	0.78	1.00	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.409	0.547	0.796	0.000	0.303	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	220	376	0	671	0	0	0
N.S.	1	1.00	0.81	1.39	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	0.547	1.131	1.880	0.000	0.289	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [320] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	4	3	1.00	10	0.300
8	A	3	3	1.00	10	0.300
9	A	3	3	1.00	10	0.300
10	A	2	2	1.00	10	0.200
11	A	2	2	1.00	10	0.200
12	A	3	3	1.00	10	0.300
13	A	3	3	1.00	10	0.300
14	A	4	3	1.00	10	0.300
15	A	4	3	1.00	12	0.250
16	A	3	3	1.00	12	0.250
17	A	3	3	1.00	12	0.250
18	A	2	2	1.00	12	0.167
19	A	2	2	1.00	12	0.167
20	A	3	3	1.00	12	0.250
21	A	3	3	1.00	12	0.250
22	A	4	3	1.00	12	0.250
23	A	3	2	1.00	14	0.143
24	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	2	2	1.00	14	0.143
26	A	1	1	1.00	14	0.071
27	A	1	1	1.00	14	0.071
28	A	2	2	1.00	14	0.143
29	A	2	2	1.00	14	0.143
30	A	3	2	1.00	14	0.143
31	A	1	1	1.00	12	0.083
32	A	1	1	1.00	12	0.083
33	A	1	1	1.00	12	0.083
34	A	1	1	1.00	12	0.083
35	A	1	1	1.00	12	0.083
36	A	1	1	1.00	12	0.083
37	A	1	1	1.00	10	0.100
38	A	1	1	1.00	12	0.083
39	A	1	1	1.00	12	0.083
40	A	6	5	1.00	13	0.385
41	A	2	2	1.00	13	0.154
42	A	3	3	1.00	13	0.231
43	A	2	2	1.00	11	0.182
44	A	3	3	1.00	11	0.273
45	A	5	5	1.00	13	0.385
46	A	6	6	1.00	13	0.462
47	A	6	5	1.00	13	0.385
48	A	3	3	1.00	13	0.231
49	A	6	6	1.00	13	0.462
50	A	3	3	1.00	13	0.231
51	A	2	2	1.00	11	0.182
52	A	4	4	1.00	11	0.364
53	A	6	6	1.00	13	0.462
54	A	7	7	1.00	13	0.538
55	A	7	6	1.00	13	0.462
56	A	1	1	1.00	14	0.071
57	A	2	2	1.00	14	0.143
58	A	3	2	1.00	14	0.143
59	A	4	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	1	1	1.00	14	0.071
61	A	2	2	1.00	14	0.143
62	A	3	2	1.00	14	0.143
63	A	4	2	1.00	14	0.143
64	A	3	3	1.00	16	0.188
65	A	3	3	1.00	16	0.188
66	A	3	2	1.00	17	0.118
67	A	2	2	1.00	17	0.118
68	A	1	1	1.00	17	0.059
69	A	2	2	1.00	17	0.118
70	A	3	3	1.00	17	0.176
71	A	4	3	1.00	17	0.176
72	A	7	7	1.00	13	0.538
73	A	6	6	1.00	13	0.462
74	A	6	6	1.00	13	0.462
75	A	4	4	1.00	11	0.364
76	A	5	5	1.00	11	0.454
77	A	7	7	1.00	13	0.538
78	A	7	7	1.00	13	0.538
79	A	8	7	1.00	13	0.538
80	A	7	7	1.00	13	0.538
81	A	6	6	1.00	13	0.462
82	A	5	5	1.00	13	0.385
83	A	5	5	1.00	11	0.454
84	A	6	6	1.00	11	0.546
85	A	7	7	1.00	13	0.538
86	A	8	7	1.00	13	0.538
87	A	9	7	1.00	13	0.538
88	A	4	3	1.00	14	0.214
89	A	6	5	1.00	14	0.357
90	A	7	6	1.00	14	0.429
91	A	8	6	1.00	14	0.429
92	A	1	1	1.00	14	0.071
93	A	3	3	1.00	14	0.214
94	A	4	4	1.00	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	5	4	1.00	14	0.286
96	A	4	3	1.00	12	0.250
97	A	3	3	1.00	12	0.250
98	A	2	2	1.00	12	0.167
99	A	1	1	1.00	12	0.083
100	A	2	1	1.00	10	0.100
101	A	3	3	1.00	12	0.250
102	A	5	5	1.00	12	0.417
103	A	6	6	1.00	12	0.500
104	A	7	6	1.00	12	0.500
105	A	7	7	1.00	10	0.700
106	A	6	6	1.00	10	0.600
107	A	2	2	1.00	10	0.200
108	A	2	2	1.00	10	0.200
109	A	4	4	1.00	10	0.400
110	A	7	7	1.00	10	0.700
111	A	5	5	1.00	13	0.385
112	A	4	3	1.00	20	0.150
113	A	3	3	1.00	20	0.150
114	A	2	2	1.00	20	0.100
115	A	2	2	1.00	15	0.133
116	A	2	2	1.00	15	0.133
117	A	3	3	1.00	15	0.200
118	A	4	3	1.00	15	0.200
119	A	2	2	1.00	17	0.118
120	A	2	2	1.00	17	0.118
121	A	3	3	1.00	17	0.176
122	A	4	3	1.00	17	0.176
123	A	3	3	1.00	20	0.150
124	A	3	3	1.00	20	0.150
125	A	4	4	1.00	20	0.200
126	A	8	6	1.00	17	0.353
127	A	7	6	1.00	17	0.353
128	A	6	6	1.00	17	0.353
129	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
130	A	5	5	1.00	15	0.333
131	A	6	5	1.00	15	0.333
132	A	7	5	1.00	15	0.333
133	A	4	4	1.00	20	0.200
134	A	2	2	1.00	20	0.100
135	A	2	2	1.00	16	0.125
136	A	2	2	1.00	13	0.154
137	A	5	5	1.00	17	0.294
138	A	6	6	1.00	17	0.353
139	A	7	6	1.00	17	0.353
140	A	4	3	1.00	10	0.300
141	A	3	3	1.00	10	0.300
142	A	2	2	1.00	10	0.200
143	A	2	2	1.00	10	0.200
144	A	3	3	1.00	10	0.300
145	A	4	3	1.00	10	0.300
146	A	7	4	1.00	10	0.400
147	A	5	4	1.00	10	0.400
148	A	4	4	1.00	10	0.400
149	A	4	4	1.00	10	0.400
150	A	5	4	1.00	10	0.400
151	A	7	4	1.00	10	0.400
152	A	7	3	1.00	10	0.300
153	A	5	3	1.00	10	0.300
154	A	3	3	1.00	10	0.300
155	A	3	3	1.00	10	0.300
156	A	3	2	1.00	10	0.200
157	A	3	2	1.00	10	0.200
158	A	5	3	1.00	13	0.231
159	A	3	2	1.00	13	0.154
160	A	4	3	1.00	13	0.231
161	A	3	2	1.00	13	0.154
162	A	3	3	1.00	13	0.231
163	A	2	1	1.00	13	0.077
164	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	2	2	1.00	11	0.182
166	A	4	3	1.00	11	0.273
167	A	3	3	1.00	13	0.231
168	A	4	3	1.00	13	0.231
169	A	3	2	1.00	13	0.154
170	A	4	3	1.00	13	0.231
171	A	4	4	1.00	13	0.308
172	A	2	2	1.00	13	0.154
173	A	3	3	1.00	13	0.231
174	A	3	2	1.00	13	0.154
175	A	2	2	1.00	13	0.154
176	A	2	2	1.00	11	0.182
177	A	4	3	1.00	11	0.273
178	A	4	3	1.00	13	0.231
179	A	4	3	1.00	13	0.231
180	A	4	2	1.00	13	0.154
181	A	3	2	1.00	15	0.133
182	A	1	1	1.00	15	0.067
183	A	2	2	1.00	13	0.154
184	A	3	2	1.00	15	0.133
185	A	1	1	1.00	15	0.067
186	A	2	2	1.00	13	0.154
187	A	3	2	1.00	13	0.154
188	A	7	6	1.00	13	0.462
189	A	3	2	1.00	13	0.154
190	A	6	6	1.00	13	0.462
191	A	3	2	1.00	13	0.154
192	A	5	5	1.00	13	0.385
193	A	2	2	1.00	11	0.182
194	A	6	6	1.00	11	0.546
195	A	5	5	1.00	13	0.385
196	A	7	6	1.00	13	0.462
197	A	6	6	1.00	13	0.462
198	A	8	7	1.00	13	0.538
199	A	7	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	6	6	1.00	13	0.462
201	A	3	2	1.00	13	0.154
202	A	5	5	1.00	13	0.385
203	A	2	2	1.00	11	0.182
204	A	7	6	1.00	11	0.546
205	A	6	6	1.00	13	0.462
206	A	7	6	1.00	13	0.462
207	A	7	6	1.00	13	0.462
208	A	6	5	1.00	13	0.385
209	A	6	5	1.00	13	0.385
210	A	5	4	1.00	13	0.308
211	A	5	5	1.00	11	0.454
212	A	4	4	1.00	11	0.364
213	A	4	4	1.00	13	0.308
214	A	5	4	1.00	13	0.308
215	A	5	5	1.00	13	0.385
216	A	5	4	1.00	13	0.308
217	A	6	5	1.00	13	0.385
218	A	10	6	1.00	13	0.462
219	A	4	3	1.00	13	0.231
220	A	10	5	1.00	13	0.385
221	A	4	3	1.00	11	0.273
222	A	3	2	1.00	11	0.182
223	A	7	5	1.00	13	0.385
224	A	3	2	1.00	13	0.154
225	A	9	6	1.00	13	0.462
226	A	3	2	1.00	13	0.154
227	A	11	5	1.00	13	0.385
228	A	13	9	1.00	13	0.692
229	A	7	6	1.00	13	0.462
230	A	8	7	1.00	13	0.538
231	A	6	5	1.00	11	0.454
232	A	4	4	1.00	11	0.364
233	A	7	7	1.00	13	0.538
234	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	7	7	1.00	13	0.538
236	A	16	9	1.00	13	0.692
237	A	7	6	1.00	13	0.462
238	A	13	9	1.00	13	0.692
239	A	6	5	1.00	11	0.454
240	A	3	2	1.00	11	0.182
241	A	8	7	1.00	13	0.538
242	A	3	2	1.00	13	0.154
243	A	8	7	1.00	13	0.538
244	A	4	4	1.00	13	0.308
245	A	3	3	1.00	13	0.231
246	A	7	6	1.00	15	0.400
247	A	5	4	1.00	15	0.267
248	A	5	4	1.00	17	0.235
249	A	11	9	1.00	15	0.600
250	A	9	8	1.00	15	0.533
251	A	12	11	1.00	15	0.733
252	A	6	6	1.00	15	0.400
253	A	7	7	1.00	31	0.226
254	A	8	8	1.00	31	0.258
255	A	9	8	1.00	31	0.258
256	A	10	8	1.00	31	0.258
257	A	13	8	1.00	14	0.571
258	A	11	7	1.00	14	0.500
259	A	9	6	1.00	12	0.500
260	A	13	5	1.00	20	0.250
261	A	5	3	1.00	36	0.083
262	A	4	3	1.00	36	0.083
263	A	2	2	1.00	34	0.059
264	N/A	0	0	1.00	34	0.000
265	N/A	0	0	1.00	36	0.000
266	A	1	1	1.00	11	0.091
267	A	2	2	1.00	13	0.154
268	A	2	2	1.00	13	0.154
269	A	3	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
270	A	1	1	1.01	15	0.067
271	A	2	2	1.02	17	0.118
272	A	2	2	1.00	17	0.118
273	A	3	2	1.00	17	0.118
274	A	2	1	1.00	15	0.067
275	A	3	2	1.00	17	0.118
276	A	3	1	1.00	17	0.059
277	A	4	2	1.00	17	0.118
278	A	3	1	1.00	17	0.059
279	A	4	3	1.00	19	0.158
280	A	4	3	1.00	19	0.158
281	A	3	2	1.00	19	0.105
282	A	3	2	1.00	19	0.105
283	A	4	3	1.00	19	0.158
284	A	4	3	1.00	19	0.158
285	A	8	8	1.00	18	0.444
286	A	6	6	1.00	18	0.333
287	A	3	3	1.00	18	0.167
288	A	4	4	1.00	18	0.222
289	A	4	4	1.00	10	0.400
290	A	5	5	1.00	12	0.417
291	A	6	4	1.00	12	0.333
292	A	5	5	1.00	11	0.454
293	A	6	6	1.00	13	0.462
294	A	9	5	1.00	13	0.385
295	A	5	5	1.00	14	0.357
296	A	6	6	1.00	16	0.375
297	A	9	5	1.00	16	0.312
298	A	6	6	1.00	17	0.353
299	A	7	7	1.00	19	0.368
300	A	10	6	1.00	19	0.316
301	A	4	3	1.00	16	0.188
302	A	5	4	1.00	16	0.250
303	A	4	3	1.00	16	0.188
304	A	4	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	3	3	1.00	14	0.214
306	A	4	4	1.00	16	0.250
307	A	3	3	1.00	16	0.188
308	A	6	5	1.00	16	0.312
309	A	5	4	1.00	16	0.250
310	A	4	3	1.00	10	0.300
311	A	4	3	1.00	8	0.375
312	A	5	5	1.00	8	0.625
313	A	6	6	1.00	10	0.600
314	A	4	3	1.00	10	0.300
315	A	4	3	1.00	8	0.375
316	A	9	9	1.00	8	1.125
317	A	13	9	1.00	10	0.900
318	A	4	3	1.00	10	0.300
319	A	4	3	1.00	8	0.375
320	A	15	12	1.00	8	1.500
321	A	16	13	1.00	10	1.300
322	A	2	2	1.00	18	0.111
323	A	2	2	1.00	18	0.111
324	A	1	1	1.00	16	0.062
325	A	1	1	1.00	16	0.062
326	A	1	1	1.00	18	0.056
327	A	2	2	1.00	18	0.111
328	A	2	2	1.00	18	0.111
329	A	6	5	1.00	25	0.200
330	A	6	5	1.00	25	0.200
331	A	5	4	1.00	25	0.160
332	A	4	4	1.00	25	0.160
333	A	4	4	1.00	25	0.160
334	A	6	5	1.00	25	0.200
335	A	6	5	1.00	25	0.200
336	A	1	1	1.00	10	0.100
337	A	6	4	1.00	12	0.333
338	A	6	4	1.00	15	0.267
339	A	6	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	4	2	1.00	14	0.143
341	A	6	3	1.00	17	0.176
342	A	8	5	1.00	16	0.312
343	A	9	6	1.00	18	0.333
344	A	14	5	1.00	18	0.278
345	A	8	5	1.00	19	0.263
346	A	9	6	1.00	21	0.286
347	A	14	5	1.00	21	0.238
348	A	8	4	1.00	16	0.250
349	A	9	4	1.00	18	0.222
350	A	14	4	1.00	18	0.222
351	A	6	4	1.00	18	0.222
352	A	7	4	1.00	20	0.200
353	A	10	4	1.00	20	0.200
354	A	8	5	1.00	21	0.238
355	A	9	5	1.00	23	0.217
356	A	14	5	1.00	23	0.217
357	A	8	4	1.00	19	0.210
358	A	10	4	1.00	21	0.190
359	A	14	4	1.00	21	0.190
360	A	8	5	1.00	21	0.238
361	A	10	5	1.00	23	0.217
362	A	14	5	1.00	23	0.217
363	A	8	5	1.00	24	0.208
364	A	10	5	1.00	26	0.192
365	A	14	5	1.00	26	0.192
366	A	6	5	1.00	6	0.833
367	A	9	6	1.00	6	1.000
368	A	8	4	1.00	16	0.250
369	A	8	4	1.00	19	0.210

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sinh(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \sinh(a+bx)\,dx = \frac{\cosh(a+bx)}{b}$$

```
[Out] cosh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2718}

$$\int \sinh(a+bx)\,dx = \frac{\cosh(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x],x]
```

```
[Out] Cosh[a + b*x]/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\cosh(a+bx)}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \sinh(a+bx)\,dx = \frac{\cosh(a)\cosh(bx)}{b} + \frac{\sinh(a)\sinh(bx)}{b}$$

```
[In] Integrate[Sinh[a + b*x],x]
[Out] (Cosh[a]*Cosh[b*x])/b + (Sinh[a]*Sinh[b*x])/b
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\cosh(bx+a)}{b}$	11
default	$\frac{\cosh(bx+a)}{b}$	11
parallelrisch	$\frac{1+\cosh(bx+a)}{b}$	13
risch	$\frac{e^{bx+a}}{2b}+\frac{e^{-bx-a}}{2b}$	27
meijerg	$\frac{\sinh(a)\sinh(bx)}{b}-\frac{\cosh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(bx)}{\sqrt{\pi}}\right)}{b}$	35

```
[In] int(sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] cosh(b*x+a)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\,dx = \frac{\cosh\left(bx+a\right)}{b}$$

```
[In] integrate(sinh(b*x+a),x, algorithm="fricas")
[Out] cosh(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \sinh(a+bx)\,dx = \begin{cases} \frac{\cosh{(a+bx)}}{b} & \text{for } b \neq 0 \\ x\sinh{(a)} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(b*x+a),x)
[Out] Piecewise((cosh(a + b*x)/b, Ne(b, 0)), (x*sinh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\,dx = \frac{\cosh{(bx+a)}}{b}$$

```
[In] integrate(sinh(b*x+a),x, algorithm="maxima")
[Out] cosh(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \sinh(a+bx)\,dx = \frac{e^{(bx+a)}}{2\,b} + \frac{e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(sinh(b*x+a),x, algorithm="giac")
[Out] 1/2*e^(b*x + a)/b + 1/2*e^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sinh(a+bx)\,dx = \frac{\cosh(a+b\,x)}{b}$$

```
[In] int(sinh(a + b*x),x)
[Out] cosh(a + b*x)/b
```

3.2 $\int \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \sinh^2(a+bx)\,dx = -\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}$$

```
[Out] -1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\int \sinh^2(a+bx)\,dx = \frac{\sinh(a+bx)\cosh(a+bx)}{2b}-\frac{x}{2}$$

```
[In] Int[Sinh[a + b*x]^2,x]
```

```
[Out] -1/2*x + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 2715</u>

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\cosh(a+bx)\sinh(a+bx)}{2b} - \frac{\int 1\,dx}{2}\\ &= -\frac{x}{2} + \frac{\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sinh^2(a+bx)\,dx = \frac{-2(a+bx)+\sinh(2(a+bx))}{4b}$$

```
[In] Integrate[Sinh[a + b*x]^2,x]
[Out] (-2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{-2bx+\sinh(2bx+2a)}{4b}$	20
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}-\frac{bx}{2}-\frac{a}{2}}{b}$	27
risch	$-\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}$	33

```
[In] int(sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(-2*b*x+sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \sinh^2(a+bx)\,dx = -\frac{bx-\cosh(bx+a)\sinh(bx+a)}{2\,b}$$

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/2*(b*x - cosh(b*x + a)*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \sinh^2(a+bx)\,dx = \begin{cases} \frac{x\sinh^2(a+bx)}{2} - \frac{x\cosh^2(a+bx)}{2} + \frac{\sinh(a+bx)\cosh(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sinh^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(b*x+a)**2,x)
[Out] Piecewise((x*sinh(a + b*x)**2/2 - x*cosh(a + b*x)**2/2 + sinh(a + b*x)*cosh
(a + b*x)/(2*b), Ne(b, 0)), (x*sinh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \sinh^2(a+bx)\,dx = -\frac{1}{2}\,x + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \sinh^2(a+bx)\,dx = -\frac{1}{2}\,x + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b}$$

```
[In] integrate(sinh(b*x+a)^2,x, algorithm="giac")
[Out] -1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \sinh^2(a+bx)\,dx = \frac{\sinh(2\,a+2\,b\,x)}{4\,b} - \frac{x}{2}$$

```
[In] int(sinh(a + b*x)^2,x)
[Out] sinh(2*a + 2*b*x)/(4*b) - x/2
```

3.3 $\int \sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sinh^3(a+bx)\,dx=-\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}$$

```
[Out] -cosh(b*x+a)/b+1/3*cosh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = $\{2713\}$

$$\int \sinh^3(a+bx)\,dx=\frac{\cosh^3(a+bx)}{3b}-\frac{\cosh(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]^3,x]
```

```
[Out] -(Cosh[a + b*x]/b) + Cosh[a + b*x]^3/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=-\frac{\text{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cosh(a+bx)\right)}{b}\\&=-\frac{\cosh(a+bx)}{b}+\frac{\cosh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sinh^3(a+bx)\,dx = -\frac{3\cosh(a+bx)}{4b} + \frac{\cosh(3(a+bx))}{12b}$$

```
[In] Integrate[Sinh[a + b*x]^3,x]
[Out] (-3*Cosh[a + b*x])/(4*b) + Cosh[3*(a + b*x)]/(12*b)
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}$	23
default	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)}{b}$	23
parallelrisch	$\frac{\cosh(3bx+3a)-9\cosh(bx+a)-8}{12b}$	25
risch	$\frac{e^{3bx+3a}}{24b}-\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}+\frac{e^{-3bx-3a}}{24b}$	55

```
[In] int(sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \sinh^3(a+bx)\,dx = \frac{\cosh(bx+a)^3 + 3\,\cosh(bx+a)\sinh(bx+a)^2 - 9\,\cosh(bx+a)}{12\,b}$$

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/12*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 9*cosh(b*x + a))/
b
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sinh^3(a+bx)\,dx = \begin{cases} \frac{\sinh^2(a+bx)\cosh(a+bx)}{b} - \frac{2\cosh^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sinh^3(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((sinh(a + b*x)**2*cosh(a + b*x)/b - 2*cosh(a + b*x)**3/(3*b), Ne(
b, 0)), (x*sinh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} + \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(-
3*b*x - 3*a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \sinh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} - \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} + \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

```
[In] integrate(sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b - 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b + 1/24*e^(-
3*b*x - 3*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \sinh^3(a+bx)\,dx = -\frac{3\cosh(a+b\,x)-\cosh(a+b\,x)^3}{3\,b}$$

```
[In] int(sinh(a + b*x)^3,x)
[Out] -(3*cosh(a + b*x) - cosh(a + b*x)^3)/(3*b)
```

3.4 $\int \sinh^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \sinh^4(a+bx)\,dx = \frac{3x}{8}-\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}$$

```
[Out] 3/8*x-3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)*sinh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\int \sinh^4(a+bx)\,dx = \frac{\sinh^3(a+bx)\cosh(a+bx)}{4b}-\frac{3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x}{8}$$

```
[In] Int[Sinh[a + b*x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]*Sinh[a + b
*x]^3)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

*n]

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\cosh(a+bx)\sinh^3(a+bx)}{4b} - \frac{3}{4}\int \sinh^2(a+bx)\,dx \\
&= -\frac{3\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh(a+bx)\sinh^3(a+bx)}{4b} + \frac{3\int 1\,dx}{8} \\
&= \frac{3x}{8} - \frac{3\cosh(a+bx)\sinh(a+bx)}{8b} + \frac{\cosh(a+bx)\sinh^3(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \sinh^4(a+bx)\,dx = \frac{12(a+bx) - 8\sinh(2(a+bx)) + \sinh(4(a+bx))}{32b}$$

```
[In] Integrate[Sinh[a + b*x]^4,x]
[Out] (12*(a + b*x) - 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{12bx+\sinh(4bx+4a)-8\sinh(2bx+2a)}{32b}$	31
derivativedivides	$\frac{\left(\frac{\sinh(bx+a)^3}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$	39
default	$\frac{\left(\frac{\sinh(bx+a)^3}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$	39
risch	$\frac{3x}{8}+\frac{e^{4bx+4a}}{64b}-\frac{e^{2bx+2a}}{8b}+\frac{e^{-2bx-2a}}{8b}-\frac{e^{-4bx-4a}}{64b}$	61

```
[In] int(sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/32*(12*b*x+sinh(4*b*x+4*a)-8*sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \sinh^4(a+bx)\,dx = \frac{\cosh(bx+a)\sinh(bx+a)^3 + 3\,bx + \left(\cosh(bx+a)^3 - 4\,\cosh(bx+a)\right)\sinh(bx+a)}{8\,b}$$

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 - 4*cosh(b*x
+ a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \sinh^4(a+bx)\,dx = \begin{cases} \frac{3x\sinh^4(a+bx)}{8} - \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4} + \frac{3x\cosh^4(a+bx)}{8} + \frac{5\sinh^3(a+bx)\cosh(a+bx)}{8b} - \frac{3\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{for} \\ x\sinh^4(a) & \text{oth} \end{cases}$$

```
[In] integrate(sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/4
 + 3*x*cosh(a + b*x)**4/8 + 5*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) - 3*sinh
(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*sinh(a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \sinh^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \sinh^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} - \frac{e^{(2\,bx+2\,a)}}{8\,b} + \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

```
[In] integrate(sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/64*e^(4*b*x + 4*a)/b - 1/8*e^(2*b*x + 2*a)/b + 1/8*e^(-2*b*x - 2*
a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \sinh^4(a+bx)\,dx = \frac{3\,x}{8} - \frac{\frac{\sinh(2\,a+2\,b\,x)}{4} - \frac{\sinh(4\,a+4\,b\,x)}{32}}{b}$$

```
[In] int(sinh(a + b*x)^4,x)
```

```
[Out] (3*x)/8 - (sinh(2*a + 2*b*x)/4 - sinh(4*a + 4*b*x)/32)/b
```

3.5 $\int \sinh^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \sinh^5(a+bx)\,dx=\frac{\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}$$

```
[Out] cosh(b*x+a)/b-2/3*cosh(b*x+a)^3/b+1/5*cosh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$\int \sinh^5(a+bx)\,dx=\frac{\cosh^5(a+bx)}{5b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh(a+bx)}{b}$$

```
[In] Int[Sinh[a + b*x]^5,x]
```

```
[Out] Cosh[a + b*x]/b - (2*Cosh[a + b*x]^3)/(3*b) + Cosh[a + b*x]^5/(5*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cosh(a+bx)\right)}{b}\\&=\frac{\cosh(a+bx)}{b}-\frac{2\cosh^3(a+bx)}{3b}+\frac{\cosh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \sinh^5(a+bx)\,dx=\frac{5\cosh(a+bx)}{8b}-\frac{5\cosh(3(a+bx))}{48b}+\frac{\cosh(5(a+bx))}{80b}$$

```
[In] Integrate[Sinh[a + b*x]^5,x]
[Out] (5*Cosh[a + b*x])/(8*b) - (5*Cosh[3*(a + b*x)])/(48*b) + Cosh[5*(a + b*x)]/
(80*b)
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)}{b}$	33
default	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)}{b}$	33
parallelrisch	$\frac{128-25\cosh(3bx+3a)+150\cosh(bx+a)+3\cosh(5bx+5a)}{240b}$	38
risch	$\frac{e^{5bx+5a}}{160b}-\frac{5\,e^{3bx+3a}}{96b}+\frac{5\,e^{bx+a}}{16b}+\frac{5\,e^{-bx-a}}{16b}-\frac{5\,e^{-3bx-3a}}{96b}+\frac{e^{-5bx-5a}}{160b}$	83

```
[In] int(sinh(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(37) = 74.

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.93

$$\int \sinh^5(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^5+15\cosh(bx+a)\sinh(bx+a)^4-25\cosh(bx+a)^3+15\left(2\cosh(bx+a)^3-5\cosh(bx\right.}{240\,b}$$

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="fricas")
[Out] 1/240*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 - 25*cosh(b*x +
 a)^3 + 15*(2*cosh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a)^2 + 150*cosh
(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \sinh^5(a+bx)\,dx = \begin{cases} \frac{\sinh^4(a+bx)\cosh(a+bx)}{b} - \frac{4\sinh^2(a+bx)\cosh^3(a+bx)}{3b} + \frac{8\cosh^5(a+bx)}{15b} & \text{for } b \neq 0 \\ x\sinh^5(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(b*x+a)**5,x)
```

```
[Out] Piecewise((sinh(a + b*x)**4*cosh(a + b*x)/b - 4*sinh(a + b*x)**2*cosh(a + b
*x)**3/(3*b) + 8*cosh(a + b*x)**5/(15*b), Ne(b, 0)), (x*sinh(a)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(37) = 74.

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \sinh^5(a+bx)\,dx = \frac{e^{(5bx+5a)}}{160b} - \frac{5e^{(3bx+3a)}}{96b} + \frac{5e^{(bx+a)}}{16b} + \frac{5e^{(-bx-a)}}{16b} - \frac{5e^{(-3bx-3a)}}{96b} + \frac{e^{(-5bx-5a)}}{160b}$$

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(37) = 74.

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \sinh^5(a+bx)\,dx = \frac{e^{(5bx+5a)}}{160b} - \frac{5e^{(3bx+3a)}}{96b} + \frac{5e^{(bx+a)}}{16b} + \frac{5e^{(-bx-a)}}{16b} - \frac{5e^{(-3bx-3a)}}{96b} + \frac{e^{(-5bx-5a)}}{160b}$$

```
[In] integrate(sinh(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b - 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b + 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b + 1/160*e^(-5*b*x - 5*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \sinh^5(a+bx)\,dx=\frac{\frac{\cosh(a+b\,x)^5}{5}-\frac{2\cosh(a+b\,x)^3}{3}+\cosh(a+b\,x)}{b}$$

```
[In] int(sinh(a + b*x)^5,x)
[Out] (cosh(a + b*x) - (2*cosh(a + b*x)^3)/3 + cosh(a + b*x)^5/5)/b
```

3.6 $\int \sinh^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \sinh^6(a+bx)\,dx = -\frac{5x}{16}+\frac{5\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b}+\frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}$$

```
[Out] -5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b-5/24*cosh(b*x+a)*sinh(b*x+a)^3/b+1/6
*cosh(b*x+a)*sinh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\int \sinh^6(a+bx)\,dx = \frac{\sinh^5(a+bx)\cosh(a+bx)}{6b}-\frac{5\sinh^3(a+bx)\cosh(a+bx)}{24b} + \frac{5\sinh(a+bx)\cosh(a+bx)}{16b}-\frac{5x}{16}$$

```
[In] Int[Sinh[a + b*x]^6,x]
```

```
[Out] (-5*x)/16 + (5*Cosh[a + b*x]*Sinh[a + b*x])/(16*b) - (5*Cosh[a + b*x]*Sinh[
a + b*x]^3)/(24*b) + (Cosh[a + b*x]*Sinh[a + b*x]^5)/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b} - \frac{5}{6}\int \sinh^4(a+bx)\,dx \\
&= -\frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b} + \frac{5}{8}\int \sinh^2(a+bx)\,dx \\
&= \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} - \frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b} \\
&\quad + \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b} - \frac{5\int 1\,dx}{16} \\
&= -\frac{5x}{16} + \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} \\
&\quad - \frac{5\cosh(a+bx)\sinh^3(a+bx)}{24b} + \frac{\cosh(a+bx)\sinh^5(a+bx)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \sinh^6(a+bx)\,dx = \frac{-60a-60bx+45\sinh(2(a+bx))-9\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

```
[In] Integrate[Sinh[a + b*x]^6,x]
```

```
[Out] (-60*a - 60*b*x + 45*Sinh[2*(a + b*x)] - 9*Sinh[4*(a + b*x)] + Sinh[6*(a +
b*x)])/(192*b)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{-60bx+\sinh(6bx+6a)-9\sinh(4bx+4a)+45\sinh(2bx+2a)}{192b}$	42
derivativedivides	$\frac{\left(\frac{\sinh(bx+a)^5}{6}-\frac{5\sinh(bx+a)^3}{24}+\frac{5\sinh(bx+a)}{16}\right)\cosh(bx+a)-\frac{5bx}{16}-\frac{5a}{16}}{b}$	49
default	$\frac{\left(\frac{\sinh(bx+a)^5}{6}-\frac{5\sinh(bx+a)^3}{24}+\frac{5\sinh(bx+a)}{16}\right)\cosh(bx+a)-\frac{5bx}{16}-\frac{5a}{16}}{b}$	49
risch	$-\frac{5x}{16}+\frac{e^{6bx+6a}}{384b}-\frac{3e^{4bx+4a}}{128b}+\frac{15e^{2bx+2a}}{128b}-\frac{15e^{-2bx-2a}}{128b}+\frac{3e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$	89

```
[In] int(sinh(b*x+a)^6,x,method=_RETURNVERBOSE)
[Out] 1/192*(-60*b*x+sinh(6*b*x+6*a)-9*sinh(4*b*x+4*a)+45*sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \sinh^6(a+bx)\,dx = \frac{3\cosh(bx+a)\sinh(bx+a)^5+2\left(5\cosh(bx+a)^3-9\cosh(bx+a)\right)\sinh(bx+a)^3-30\,bx+3\left(\cosh(b\right.}{96\,b}$$

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="fricas")
[Out] 1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 - 9*cosh(b*x +
 a))*sinh(b*x + a)^3 - 30*b*x + 3*(cosh(b*x + a)^5 - 6*cosh(b*x + a)^3 + 15
*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(61) = 122.

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \sinh^6(a+bx)\,dx = \begin{cases} \frac{5x\sinh^6(a+bx)}{16}-\frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16}+\frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16}-\frac{5x\cosh^6(a+bx)}{16}+\frac{11\sinh^5(a+bx)\cosh(a+bx)}{16b} \\ x\sinh^6(a) \end{cases}$$

```
[In] integrate(sinh(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sinh(a + b*x)**6/16 - 15*x*sinh(a + b*x)**4*cosh(a + b*x)**2
/16 + 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 - 5*x*cosh(a + b*x)**6/16 +
 11*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a + b*x
)**3/(6*b) + 5*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*sinh(a)
**6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \sinh^6(a+bx)\,dx = -\frac{\left(9\,e^{(-2\,bx-2\,a)} - 45\,e^{(-4\,bx-4\,a)} - 1\right)e^{(6\,bx+6\,a)}}{384\,b} - \frac{5\,(bx+a)}{16\,b} - \frac{45\,e^{(-2\,bx-2\,a)} - 9\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/384*(9*e^(-2*b*x - 2*a) - 45*e^(-4*b*x - 4*a) - 1)*e^(6*b*x + 6*a)/b - 5
/16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) - 9*e^(-4*b*x - 4*a) + e^(-6*b
*x - 6*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \sinh^6(a+bx)\,dx = -\frac{5}{16}\,x + \frac{e^{(6\,bx+6\,a)}}{384\,b} - \frac{3\,e^{(4\,bx+4\,a)}}{128\,b} + \frac{15\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{15\,e^{(-2\,bx-2\,a)}}{128\,b} + \frac{3\,e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

```
[In] integrate(sinh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -5/16*x + 1/384*e^(6*b*x + 6*a)/b - 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*b
*x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b + 3/128*e^(-4*b*x - 4*a)/b - 1/384*
e^(-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \sinh^6(a+bx)\,dx = \frac{\frac{15\sinh(2\,a+2\,b\,x)}{64} - \frac{3\sinh(4\,a+4\,b\,x)}{64} + \frac{\sinh(6\,a+6\,b\,x)}{192}}{b} - \frac{5\,x}{16}$$

```
[In] int(sinh(a + b*x)^6,x)
```

```
[Out] ((15*sinh(2*a + 2*b*x))/64 - (3*sinh(4*a + 4*b*x))/64 + sinh(6*a + 6*b*x)/1
92)/b - (5*x)/16
```

3.7 $\int \sinh^{\frac{7}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 103

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx=-\frac{10i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{21b\sqrt{\sinh(a+bx)}}-\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b}+\frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b}$$

```
[Out] 2/7*cosh(b*x+a)*sinh(b*x+a)^(5/2)/b+10/21*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^
2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b
*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sinh(b*x+a)^(1/2)-10/21*cosh(b*x+a)*si
nh(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2715, 2721, 2720}

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx=\frac{2\sinh^{\frac{5}{2}}(a+bx)\cosh(a+bx)}{7b}-\frac{10\sqrt{\sinh(a+bx)}\cosh(a+bx)}{21b}-\frac{10i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{21b\sqrt{\sinh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(
b*Sqrt[Sinh[a + b*x]]) - (10*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(21*b) + (2
*Cosh[a + b*x]*Sinh[a + b*x]^(5/2))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} - \frac{5}{7}\int \sinh^{\frac{3}{2}}(a+bx)\,dx \\
&= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} + \frac{5}{21}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx \\
&= -\frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b} \\
&\quad + \frac{\left(5\sqrt{i\sinh(a+bx)}\right)\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{21\sqrt{\sinh(a+bx)}} \\
&= -\frac{10i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{21b\sqrt{\sinh(a+bx)}} \\
&\quad - \frac{10\cosh(a+bx)\sqrt{\sinh(a+bx)}}{21b} + \frac{2\cosh(a+bx)\sinh^{\frac{5}{2}}(a+bx)}{7b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.73

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \frac{40i\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{i\sinh(a+bx)}-26\sinh(2(a+bx))+3\sinh(4(a+bx))}{84b\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[Sinh[a + b*x]^(7/2),x]
```

```
[Out] ((40*I)*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] -
 26*Sinh[2*(a + b*x)] + 3*Sinh[4*(a + b*x)])/(84*b*Sqrt[Sinh[a + b*x]])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

method	result
default	$\frac{\frac{5i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{21}+\frac{2\cosh(bx+a)^{4}\sinh(bx+a)}{7}-\frac{16\cosh(bx+a)^{2}\sinh(bx+a)}{21}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

```
[In] int(sinh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] (5/21*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/7*cosh(b*x+a)^4
*sinh(b*x+a)-16/21*cosh(b*x+a)^2*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)
/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.17

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \frac{40\left(\sqrt{2}\cosh(bx+a)^{3}+3\sqrt{2}\cosh(bx+a)^{2}\sinh(bx+a)+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^{2}+\sqrt{2}\sinh(bx\right.}{}$$

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(40*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a)
 + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3)*weier
strassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a)) + (3*cosh(b*x + a)^6 +
```

```
18*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2
- 23)*sinh(b*x + a)^4 - 23*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 - 23*cos
h(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 - 138*cosh(b*x + a)^2 - 2
3)*sinh(b*x + a)^2 - 23*cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 - 46*cosh(b*
x + a)^3 - 23*cosh(b*x + a))*sinh(b*x + a) + 3)*sqrt(sinh(b*x + a)))/(b*cos
h(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b
*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^{\frac{7}{2}}(a + bx)\, dx = \text{Timed out}$$

```
[In] integrate(sinh(b*x+a)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sinh^{\frac{7}{2}}(a + bx)\, dx = \int \sinh\left(bx + a\right)^{\frac{7}{2}}\, dx$$

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \sinh^{\frac{7}{2}}(a + bx)\, dx = \int \sinh\left(bx + a\right)^{\frac{7}{2}}\, dx$$

```
[In] integrate(sinh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{7}{2}}(a+bx)\,dx = \int \sinh(a+b\,x)^{7/2}\,dx$$

```
[In] int(sinh(a + b*x)^(7/2),x)
[Out] int(sinh(a + b*x)^(7/2), x)
```

3.8 $\int \sinh^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}$$

```
[Out] 2/5*cosh(b*x+a)*sinh(b*x+a)^(3/2)/b-6/5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)
^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x
),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2715, 2721, 2719}

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{2\sinh^{\frac{3}{2}}(a+bx)\cosh(a+bx)}{5b} + \frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(5/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sinh[a + b*x]^(3/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b} - \frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx \\ &= \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b} - \frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{i\sinh(a+bx)}} \\ &= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sinh^{\frac{3}{2}}(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{-6E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|2\right)\sqrt{i\sinh(a+bx)}+\sinh(a+bx)\sinh(2(a+bx))}{5b\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[Sinh[a + b*x]^(5/2),x]
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sinh[a + b*x]] + Sin
h[a + b*x]*Sinh[2*(a + b*x)])/(5*b*Sqrt[Sinh[a + b*x]])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.05

method	result
default	$\frac{-\frac{6\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a}}{5}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

```
[In] int(sinh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-6/5*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a
))^(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+3/5*(1-I*sinh(b*x+a
))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1
-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/5*cosh(b*x+a)^4-2/5*cosh(b*x+a)^2)/cos
h(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.52

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \frac{12\left(\sqrt{2}\cosh(bx+a)^2+2\sqrt{2}\cosh(bx+a)\sinh(bx+a)+\sqrt{2}\sinh(bx+a)^2\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weiers}}{}$$

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(12*(sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) +
 sqrt(2)*sinh(b*x + a)^2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, c
osh(b*x + a) + sinh(b*x + a))) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*
x + a)^3 + sinh(b*x + a)^4 + 6*(cosh(b*x + a)^2 + 2)*sinh(b*x + a)^2 + 12*c
osh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + 6*cosh(b*x + a))*sinh(b*x + a) - 1)*s
qrt(sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) +
b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh^{\frac{5}{2}}(a+bx)\,dx$$

```
[In] integrate(sinh(b*x+a)**(5/2),x)
[Out] Integral(sinh(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{5}{2}}(a+bx)\,dx = \int \sinh(a+b\,x)^{5/2}\,dx$$

```
[In] int(sinh(a + b*x)^(5/2),x)
[Out] int(sinh(a + b*x)^(5/2), x)
```

3.9 $\int \sinh^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}$$

```
[Out] -2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x
)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/
sinh(b*x+a)^(1/2)+2/3*cosh(b*x+a)*sinh(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2715, 2721, 2720}

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{2\sqrt{\sinh(a+bx)}\cosh(a+bx)}{3b} + \frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(3/2),x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*S
qrt[Sinh[a + b*x]]) + (2*Cosh[a + b*x]*Sqrt[Sinh[a + b*x]])/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b} - \frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx \\
&= \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b} - \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}} \\
&= \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}} + \frac{2\cosh(a+bx)\sqrt{\sinh(a+bx)}}{3b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \frac{\sinh(2(a+bx)) - 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+bx))+\sinh(2(a+bx))\right)\sqrt{1-\cosh(2a+2bx)}}{3b\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[Sinh[a + b*x]^(3/2),x]
[Out] (Sinh[2*(a + b*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] +
 Sinh[2*(a + b*x)]]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b*x]])/(3*b*S
qrt[Sinh[a + b*x]])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

method	result	size
default	$\frac{-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\cosh(bx+a)^{2}\sinh(bx+a)}{3}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	100

```
[In] int(sinh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/3*I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x
+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))+2/3*cosh(b*x+a)^2
*sinh(b*x+a))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\left(\sqrt{2}\cosh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)\right)\mathrm{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))-(\cos}{3\left(b\cosh\left(bx+a\right)+b\sinh\left(bx\right.\right.}$$

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))*weierstrassPInverse
(4, 0, cosh(b*x + a) + sinh(b*x + a)) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 + 1)*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)
+ b*sinh(b*x + a))
```

Sympy [F]

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh^{\frac{3}{2}}\left(a+bx\right)\,dx$$

```
[In] integrate(sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sinh(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{3}{2}}(a+bx)\,dx = \int \sinh(a+b\,x)^{3/2}\,dx$$

```
[In] int(sinh(a + b*x)^(3/2),x)
[Out] int(sinh(a + b*x)^(3/2), x)
```

3.10 $\int \sqrt{\sinh(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \sqrt{\sinh(a+bx)}\,dx=-\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2721, 2719}

$$\int \sqrt{\sinh(a+bx)}\,dx=-\frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

```
[In] Int[Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I
*Sinh[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{\sinh(a+bx)}\int\sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int\sqrt{\sinh(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-i(a+bx)\right)\middle|\,2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[Sqrt[Sinh[a + b*x]],x]
[Out] (2*EllipticE[(Pi/2 - I*(a + b*x))/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh
[a + b*x]])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.00

method	result
default	$\frac{\sqrt{-i(\sinh(bx+a)+i)}\sqrt{2}\sqrt{-i(-\sinh(bx+a)+i)}\sqrt{i\sinh(bx+a)}\left(2\,\text{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\text{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\right)}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$
risch	$\frac{\sqrt{2}\sqrt{\left(e^{2bx+2a}-1\right)e^{-bx-a}}}{b}-\frac{\left(\frac{2\,e^{2bx+2a}-2}{\sqrt{\left(e^{2bx+2a}-1\right)e^{bx+a}}}-\frac{\sqrt{e^{bx+a}+1}\sqrt{-2\,e^{bx+a}+2}\sqrt{-e^{bx+a}}\left(-2\,\text{EllipticE}\left(\sqrt{e^{bx+a}+1},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{e^{b \ldots}}\right.\right.}{\sqrt{e^{3bx+3a}-e^{bx+a}}}\right.}{b\left(e^{2bx+2a}-1\right)}$

```
[In] int(sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*x+
a))^(1/2)*(2*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-EllipticF((1-I*
sinh(b*x+a))^(1/2),1/2*2^(1/2)))/cosh(b*x+a)/sinh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \sqrt{\sinh(a+bx)}\,dx = -\frac{2\left(\sqrt{2}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))+\sqrt{\sinh\left(bx+a\right)}\right)}{b}$$

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*x + a) +
 sinh(b*x + a))) + sqrt(sinh(b*x + a)))/b
```

Sympy [F]

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh\left(a+bx\right)}\,dx$$

```
[In] integrate(sinh(b*x+a)**(1/2),x)
[Out] Integral(sqrt(sinh(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh\left(bx+a\right)}\,dx$$

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh\left(bx+a\right)}\,dx$$

```
[In] integrate(sinh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sinh(a+bx)}\,dx = \int \sqrt{\sinh\left(a+b\,x\right)}\,dx$$

```
[In] int(sinh(a + b*x)^(1/2),x)
[Out] int(sinh(a + b*x)^(1/2), x)
```

3.11 $\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sin
h(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2721, 2720}

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{b\sqrt{\sinh(a+bx)}}$$

```
[In] Int[1/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt
[Sinh[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{\sqrt{\sinh(a+bx)}}$$
$$= -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{b\sqrt{\sinh(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ibx),2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

```
[In] Integrate[1/Sqrt[Sinh[a + b*x]],x]
```

```
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqr
t[I*Sinh[a + b*x]])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{i\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\text{EllipticF}\left(\sqrt{-i(\sinh(bx+a)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$	87

```
[In] int(1/sinh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x+a)+I))^(1/2)*(I*sinh(b*
x+a))^(1/2)*EllipticF((-I*(sinh(b*x+a)+I))^(1/2),1/2*2^(1/2))/cosh(b*x+a)/s
inh(b*x+a)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)**(1/2),x)
[Out] Integral(1/sqrt(sinh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh(bx+a)}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sinh(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh(bx+a)}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sinh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\sinh\left(a+b\,x\right)}}\,dx$$

```
[In] int(1/sinh(a + b*x)^(1/2),x)
[Out] int(1/sinh(a + b*x)^(1/2), x)
```

3.12 $\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 76

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)} dx = -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)+2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1
/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2
^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2716, 2721, 2719}

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)} dx = -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} - \frac{2i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{b\sqrt{i\sinh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(-3/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(b*Sqrt[Sinh[a + b*x]]) - ((2*I)*EllipticE[(I*a - Pi/2 +
 I*b*x)/2, 2]*Sqrt[Sinh[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} + \int \sqrt{\sinh(a+bx)}\,dx \\ &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} + \frac{\sqrt{\sinh(a+bx)}\int \sqrt{i\sinh(a+bx)}\,dx}{\sqrt{i\sinh(a+bx)}} \\ &= -\frac{2\cosh(a+bx)}{b\sqrt{\sinh(a+bx)}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{\sinh(a+bx)}}{b\sqrt{i\sinh(a+bx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\left(\cosh(a+bx) - E\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|\,2\right)\sqrt{i\sinh(a+bx)}\right)}{b\sqrt{\sinh(a+bx)}}$$

```
[In] Integrate[Sinh[a + b*x]^(-3/2),x]
```

```
[Out] (-2*(Cosh[a + b*x] - EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*Sqrt[I*Sin
h[a + b*x]]))/(b*Sqrt[Sinh[a + b*x]])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.03

method	result
default	$\frac{2\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}}{\cosh(bx+a)\sqrt{\sinh(bx+a)}\,b}$

```
[In] int(1/sinh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^
(1/2)*EllipticE((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))-(1-I*sinh(b*x+a))^(1/2
)*2^(1/2)*(1+I*sinh(b*x+a))^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh
(b*x+a))^(1/2),1/2*2^(1/2))-2*cosh(b*x+a)^2)/cosh(b*x+a)/sinh(b*x+a)^(1/2)/
b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^{2}+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^{2}-\sqrt{2}\right)\text{weierstrassZeta}(4,\right.}{b\cosh\left(bx+a\right.}$$

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt
(2)*sinh(b*x + a)^2 - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2)*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^2 +
2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

```
[In] integrate(1/sinh(b*x+a)**(3/2),x)
```

```
[Out] Integral(sinh(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh{(bx+a)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{{\mathrm{sinh}\left(a+b\,x\right)}^{3/2}}\,dx$$

```
[In] int(1/sinh(a + b*x)^(3/2),x)
[Out] int(1/sinh(a + b*x)^(3/2), x)
```

3.13 $\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}$$

```
[Out] -2/3*cosh(b*x+a)/b/sinh(b*x+a)^(3/2)-2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2
)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*
x),2^(1/2))*(I*sinh(b*x+a))^(1/2)/b/sinh(b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2716, 2721, 2720}

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2i\sqrt{i\sinh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right),2\right)}{3b\sqrt{\sinh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(3*b*Sinh[a + b*x]^(3/2)) + (((2*I)/3)*EllipticF[(I*a -
Pi/2 + I*b*x)/2, 2]*Sqrt[I*Sinh[a + b*x]])/(b*Sqrt[Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{1}{3}\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx\\ &= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{i\sinh(a+bx)}\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx}{3\sqrt{\sinh(a+bx)}}\\ &= -\frac{2\cosh(a+bx)}{3b\sinh^{\frac{3}{2}}(a+bx)} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right),2\right)\sqrt{i\sinh(a+bx)}}{3b\sqrt{\sinh(a+bx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(\cosh(a+bx)+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+bx))+\sinh(2(a+bx))\right)\sinh(a+bx)\sqrt{1-\text{co}}\right.}{3b\sinh^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[Sinh[a + b*x]^(-5/2),x]
```

```
[Out] (-2*(Cosh[a + b*x] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*x)] + S
inh[2*(a + b*x)]]*Sinh[a + b*x]*Sqrt[1 - Cosh[2*a + 2*b*x] - Sinh[2*a + 2*b
*x]]))/(3*b*Sinh[a + b*x]^(3/2))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{i\sqrt{1-i\sinh(bx+a)}\,\sqrt{2}\,\sqrt{1+i\sinh(bx+a)}\,\sqrt{i\sinh(bx+a)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(bx+a)},\frac{\sqrt{2}}{2}\right)\sinh(bx+a)+2\cosh(bx+a)^{2}}{3\sinh(bx+a)^{\frac{3}{2}}\cosh(bx+a)b}$	101

```
[In] int(1/sinh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3/sinh(b*x+a)^(3/2)*(I*(1-I*sinh(b*x+a))^(1/2)*2^(1/2)*(1+I*sinh(b*x+a))
^(1/2)*(I*sinh(b*x+a))^(1/2)*EllipticF((1-I*sinh(b*x+a))^(1/2),1/2*2^(1/2))
*sinh(b*x+a)+2*cosh(b*x+a)^2)/cosh(b*x+a)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.92

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^{4}+4\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}+\sqrt{2}\sinh\left(bx+a\right)^{4}+2\left(3\sqrt{2}\cosh\left(bx+a\right)^{2}\right.\right.\right.}{}$$

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="fricas")
[Out] -2/3*((sqrt(2)*cosh(b*x + a)^4 + 4*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^3 +
sqrt(2)*sinh(b*x + a)^4 + 2*(3*sqrt(2)*cosh(b*x + a)^2 - sqrt(2))*sinh(b*x
+ a)^2 - 2*sqrt(2)*cosh(b*x + a)^2 + 4*(sqrt(2)*cosh(b*x + a)^3 - sqrt(2)*c
osh(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassPInverse(4, 0, cosh(b*x +
 a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2
 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 + 1)*sinh(b*x + a) + cosh(b*x + a))
*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^
3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*s
inh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/sinh(b*x+a)**(5/2),x)
[Out] Integral(sinh(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh(a+b\,x)^{5/2}}\,dx$$

```
[In] int(1/sinh(a + b*x)^(5/2),x)
[Out] int(1/sinh(a + b*x)^(5/2), x)
```

3.14 $\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 103

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}$$

```
[Out] -2/5*cosh(b*x+a)/b/sinh(b*x+a)^(5/2)+6/5*cosh(b*x+a)/b/sinh(b*x+a)^(1/2)-6/
5*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*x)^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*x)*E
llipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*x),2^(1/2))*sinh(b*x+a)^(1/2)/b/(I*sinh
(b*x+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2716, 2721, 2719}

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)}+\frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} + \frac{6i\sqrt{\sinh(a+bx)}E\left(\frac{1}{2}\left(ia+ibx-\frac{\pi}{2}\right)\middle|2\right)}{5b\sqrt{i\sinh(a+bx)}}$$

```
[In] Int[Sinh[a + b*x]^(-7/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(5*b*Sinh[a + b*x]^(5/2)) + (6*Cosh[a + b*x])/(5*b*Sqrt[
Sinh[a + b*x]]) + (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*x)/2, 2]*Sqrt[Sinh
[a + b*x]])/(b*Sqrt[I*Sinh[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} - \frac{3}{5}\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx \\
&= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} - \frac{3}{5}\int \sqrt{\sinh(a+bx)}\,dx \\
&= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} - \frac{\left(3\sqrt{\sinh(a+bx)}\right)\int \sqrt{i\sinh(a+bx)}\,dx}{5\sqrt{i\sinh(a+bx)}} \\
&= -\frac{2\cosh(a+bx)}{5b\sinh^{\frac{5}{2}}(a+bx)} + \frac{6\cosh(a+bx)}{5b\sqrt{\sinh(a+bx)}} + \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ibx\right)\middle|\,2\right)\sqrt{\sinh(a+bx)}}{5b\sqrt{i\sinh(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \frac{-2\coth(a+bx) + 6iE\left(\frac{1}{4}(-2ia+\pi-2ibx)\middle|\,2\right)(i\sinh(a+bx))^{3/2} + 3\sinh(2(a+bx))}{5b\sinh^{\frac{3}{2}}(a+bx)}$$

```
[In] Integrate[Sinh[a + b*x]^(-7/2),x]
```

```
[Out] (-2*Coth[a + b*x] + (6*I)*EllipticE[((-2*I)*a + Pi - (2*I)*b*x)/4, 2]*(I*Si
nh[a + b*x])^(3/2) + 3*Sinh[2*(a + b*x)])/(5*b*Sinh[a + b*x]^(3/2))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.86

method	result
default	$-\frac{6\sqrt{-i(\sinh(bx+a)+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(bx+a)+i)}\,\sqrt{i\sinh(bx+a)}\,\sinh(bx+a)^2\,\mathrm{EllipticE}\left(\sqrt{-i(\sinh(bx+a)+i)},\frac{\sqrt{2}}{2}\right)-3\sqrt{-i(\sinh(}}{5\sinh(bx-}$

```
[In] int(1/sinh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/sinh(b*x+a)^(5/2)*(6*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*x
+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticE((-I*(sinh(b*x+a
)+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(b*x+a)+I))^(1/2)*2^(1/2)*(-I*(-sinh(b*
x+a)+I))^(1/2)*(I*sinh(b*x+a))^(1/2)*sinh(b*x+a)^2*EllipticF((-I*(sinh(b*x+
a)+I))^(1/2),1/2*2^(1/2))-6*sinh(b*x+a)^4-4*sinh(b*x+a)^2+2)/cosh(b*x+a)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 621, normalized size of antiderivative = 6.03

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \frac{2\left(3\left(\sqrt{2}\cosh\left(bx+a\right)^6+6\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5+\sqrt{2}\sinh\left(bx+a\right)^6+3\left(5\sqrt{2}\cosh\left(bx+a\right)^2\right.\right.\right.}{}$$

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*(sqrt(2)*cosh(b*x + a)^6 + 6*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^5 +
 sqrt(2)*sinh(b*x + a)^6 + 3*(5*sqrt(2)*cosh(b*x + a)^2 - sqrt(2))*sinh(b*x
 + a)^4 - 3*sqrt(2)*cosh(b*x + a)^4 + 4*(5*sqrt(2)*cosh(b*x + a)^3 - 3*sqrt
(2)*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*sqrt(2)*cosh(b*x + a)^4 - 6*sqrt(
2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x + a)^2 + 3*sqrt(2)*cosh(b*x + a)^2 +
 6*(sqrt(2)*cosh(b*x + a)^5 - 2*sqrt(2)*cosh(b*x + a)^3 + sqrt(2)*cosh(b*x
+ a))*sinh(b*x + a) - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(4,
 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(3*cosh(b*x + a)^6 + 18*cosh(b*x +
a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2 - 8)*sinh(b*x
+ a)^4 - 8*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 - 8*cosh(b*x + a))*sinh(
b*x + a)^3 + (45*cosh(b*x + a)^4 - 48*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2
+ cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 - 16*cosh(b*x + a)^3 + cosh(b*x +
a))*sinh(b*x + a))*sqrt(sinh(b*x + a)))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x +
 a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh
(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x +
a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*co
```

```
sh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x +
a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/sinh(b*x+a)**(7/2),x)
[Out] Integral(sinh(a + b*x)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh(bx+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/sinh(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\sinh(a+b\,x)^{7/2}}\,dx$$

```
[In] int(1/sinh(a + b*x)^(7/2),x)
[Out] int(1/sinh(a + b*x)^(7/2), x)
```

3.15 $\int (b\sinh(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 116

$$\int (b\sinh(c+dx))^{7/2}\,dx = -\frac{10ib^4 \operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{21d\sqrt{b\sinh(c+dx)}} - \frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}$$

```
[Out] 2/7*b*cosh(d*x+c)*(b*sinh(d*x+c))^(5/2)/d+10/21*I*b^4*(sin(1/2*I*c+1/4*Pi+1
/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*
Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/d/(b*sinh(d*x+c))^(1/2)-10/21*
b^3*cosh(d*x+c)*(b*sinh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2720}

$$\int (b\sinh(c+dx))^{7/2}\,dx = -\frac{10ib^4\sqrt{i\sinh(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{21d\sqrt{b\sinh(c+dx)}} - \frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}$$

```
[In] Int[(b*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*b^4*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]
])/(d*Sqrt[b*Sinh[c + d*x]]) - (10*b^3*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])
/(21*d) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(5/2))/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} - \frac{1}{7}\left(5b^2\right)\int (b\sinh(c+dx))^{3/2}\,dx \\
&= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} \\
&\quad + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} + \frac{1}{21}\left(5b^4\right)\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx \\
&= -\frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d} \\
&\quad + \frac{\left(5b^4\sqrt{i\sinh(c+dx)}\right)\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{21\sqrt{b\sinh(c+dx)}} \\
&= -\frac{10ib^4\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{21d\sqrt{b\sinh(c+dx)}} \\
&\quad - \frac{10b^3\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{21d} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{5/2}}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int (b\sinh(c+dx))^{7/2}\,dx = \frac{b^3\left(-23\cosh(c+dx)+3\cosh(3(c+dx))-\frac{20\,\text{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)}{\sqrt{i\sinh(c+dx)}}\right)\sqrt{b\sinh(c+dx)}}{42d}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (b^3*(-23*Cosh[c + d*x] + 3*Cosh[3*(c + d*x)] - (20*EllipticF[((-2*I)*c + P
i - (2*I)*d*x)/4, 2])/Sqrt[I*Sinh[c + d*x]])*Sqrt[b*Sinh[c + d*x]])/(42*d)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^4\left(5i\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\text{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)+6\cosh(dx+c)^4\sinh(dx+c)-16\cosh\right.}{21\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

```
[In] int((b*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*b^4*(5*I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*si
nh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+6*cosh(d*x+
c)^4*sinh(d*x+c)-16*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.40

$$\int (b\sinh(c+dx))^{7/2}\,dx = \frac{40\left(\sqrt{2}b^3\cosh(dx+c)^3+3\sqrt{2}b^3\cosh(dx+c)^2\sinh(dx+c)+3\sqrt{2}b^3\cosh(dx+c)\sinh(dx\right.}{}$$

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(40*(sqrt(2)*b^3*cosh(d*x + c)^3 + 3*sqrt(2)*b^3*cosh(d*x + c)^2*sinh(
d*x + c) + 3*sqrt(2)*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + sqrt(2)*b^3*sinh(d
*x + c)^3)*sqrt(b)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))
```

```
 + (3*b^3*cosh(d*x + c)^6 + 18*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 3*b^3*si
nh(d*x + c)^6 - 23*b^3*cosh(d*x + c)^4 - 23*b^3*cosh(d*x + c)^2 + (45*b^3*c
osh(d*x + c)^2 - 23*b^3)*sinh(d*x + c)^4 + 4*(15*b^3*cosh(d*x + c)^3 - 23*b
^3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b^3 + (45*b^3*cosh(d*x + c)^4 - 138*b
^3*cosh(d*x + c)^2 - 23*b^3)*sinh(d*x + c)^2 + 2*(9*b^3*cosh(d*x + c)^5 - 4
6*b^3*cosh(d*x + c)^3 - 23*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*
x + c)))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(
d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*sinh(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\sinh(c+dx))^{7/2}\,dx = \int (b\sinh(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{7/2}\,dx = \int (b\sinh(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{7/2}\,dx = \int (b\sinh(c+d\,x))^{7/2}\,dx$$

```
[In] int((b*sinh(c + d*x))^(7/2),x)
[Out] int((b*sinh(c + d*x))^(7/2), x)
```

3.16 $\int (b \sinh(c+dx))^{5/2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int (b \sinh(c+dx))^{5/2} \, dx = \frac{6ib^2 E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b \sinh(c+dx)}}{5d\sqrt{i \sinh(c+dx)}} + \frac{2b \cosh(c+dx)(b \sinh(c+dx))^{3/2}}{5d}$$

```
[Out] 2/5*b*cosh(d*x+c)*(b*sinh(d*x+c))^(3/2)/d-6/5*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2
*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi
+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2719}

$$\int (b \sinh(c+dx))^{5/2} \, dx = \frac{2b \cosh(c+dx)(b \sinh(c+dx))^{3/2}}{5d} + \frac{6ib^2 E\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b \sinh(c+dx)}}{5d\sqrt{i \sinh(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((6*I)/5)*b^2*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/
(d*Sqrt[I*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(3/2))/(5*
d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d} - \frac{1}{5}\left(3b^2\right)\int\sqrt{b\sinh(c+dx)}\,dx \\
&= \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d} - \frac{\left(3b^2\sqrt{b\sinh(c+dx)}\right)\int\sqrt{i\sinh(c+dx)}\,dx}{5\sqrt{i\sinh(c+dx)}} \\
&= \frac{6ib^2E\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5d\sqrt{i\sinh(c+dx)}} + \frac{2b\cosh(c+dx)(b\sinh(c+dx))^{3/2}}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int(b\sinh(c+dx))^{5/2}\,dx = \frac{b^2\sqrt{b\sinh(c+dx)}\left(-\frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)}{\sqrt{i\sinh(c+dx)}}+\sinh(2(c+dx))\right)}{5d}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(5/2),x]
[Out] (b^2*Sqrt[b*Sinh[c + d*x]]*(((-6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4
, 2])/Sqrt[I*Sinh[c + d*x]] + Sinh[2*(c + d*x)]))/(5*d)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.93

method	result
default	$-\frac{b^{3}\left(6\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\ \mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-3\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(}\right.}{5\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

```
[In] int((b*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*b^3*(6*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-3*(1-I*sinh(d
*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*Elliptic
F((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^4+2*cosh(d*x+c)^2)/cos
h(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.83

$$\int (b\sinh(c+dx))^{5/2}\,dx = \frac{12\left(\sqrt{2}b^{2}\cosh\left(dx+c\right)^{2}+2\sqrt{2}b^{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sqrt{2}b^{2}\sinh\left(dx+c\right)^{2}\right)\sqrt{b}\mathrm{weierstr}}{}$$

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/10*(12*(sqrt(2)*b^2*cosh(d*x + c)^2 + 2*sqrt(2)*b^2*cosh(d*x + c)*sinh(d*
x + c) + sqrt(2)*b^2*sinh(d*x + c)^2)*sqrt(b)*weierstrassZeta(4, 0, weierst
rassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + (b^2*cosh(d*x + c)^4 +
 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 12*b^2*cosh(d*
x + c)^2 + 6*(b^2*cosh(d*x + c)^2 + 2*b^2)*sinh(d*x + c)^2 - b^2 + 4*(b^2*c
osh(d*x + c)^3 + 6*b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)))
/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh{(c+dx)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))**(5/2),x)
[Out] Integral((b*sinh(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{5/2}\,dx = \int (b\sinh(c+d\,x))^{5/2}\,dx$$

```
[In] int((b*sinh(c + d*x))^(5/2),x)
[Out] int((b*sinh(c + d*x))^(5/2), x)
```

3.17 $\int (b\sinh(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int (b\sinh(c+dx))^{3/2}\,dx = \frac{2ib^2 \operatorname{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3d\sqrt{b\sinh(c+dx)}} + \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}$$

```
[Out] -2/3*I*b^2*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I
*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2
)/d/(b*sinh(d*x+c))^(1/2)+2/3*b*cosh(d*x+c)*(b*sinh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2720}

$$\int (b\sinh(c+dx))^{3/2}\,dx = \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d} + \frac{2ib^2\sqrt{i\sinh(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{b\sinh(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((2*I)/3)*b^2*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/
(d*Sqrt[b*Sinh[c + d*x]]) + (2*b*Cosh[c + d*x]*Sqrt[b*Sinh[c + d*x]])/(3*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d} - \frac{1}{3}b^2\int\frac{1}{\sqrt{b\sinh(c+dx)}}\,dx$$

$$= \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d} - \frac{\left(b^2\sqrt{i\sinh(c+dx)}\right)\int\frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3\sqrt{b\sinh(c+dx)}}$$

$$= \frac{2ib^2\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3d\sqrt{b\sinh(c+dx)}} + \frac{2b\cosh(c+dx)\sqrt{b\sinh(c+dx)}}{3d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int(b\sinh(c+dx))^{3/2}\,dx = \frac{b^2\left(\sinh(2(c+dx)) - 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{1-\right.}{3d\sqrt{b\sinh(c+dx)}}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (b^2*(Sinh[2*(c + d*x)] - 2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cosh[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/
(3*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20

method	result	siz
default	$-\frac{b^{2}\left(i\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\ \mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-2\cosh(dx+c)^{2}\sinh(dx+c)\right)}{3\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$	10

```
[In] int((b*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*b^2*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh
(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)
^2*sinh(d*x+c))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\int (b\sinh(c+dx))^{3/2}\,dx =$$
$$-\frac{2\left(\sqrt{2}b\cosh\left(dx+c\right)+\sqrt{2}b\sinh\left(dx+c\right)\right)\sqrt{b}\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))-(b\,c}{3\left(d\cosh\left(dx+c\right)+d\sinh\left(d\right.\right.}$$

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(sqrt(2)*b*cosh(d*x + c) + sqrt(2)*b*sinh(d*x + c))*sqrt(b)*weierst
rassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c)) - (b*cosh(d*x + c)^2 + 2*
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(b*sinh(d*x + c)
))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int \left(b\sinh\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int (b\sinh{(dx+c)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int (b\sinh{(dx+c)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{3/2}\,dx = \int (b\sinh(c+d\,x))^{3/2}\,dx$$

```
[In] int((b*sinh(c + d*x))^(3/2),x)
[Out] int((b*sinh(c + d*x))^(3/2), x)
```

3.18 $\int \sqrt{b \sinh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \sqrt{b \sinh(c+dx)}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b \sinh(c+dx)}}{d\sqrt{i \sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/d/(I*
sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2721, 2719}

$$\int \sqrt{b \sinh(c+dx)}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b \sinh(c+dx)}}{d\sqrt{i \sinh(c+dx)}}$$

```
[In] Int[Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(d*Sqrt
[I*Sinh[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{b\sinh(c+dx)}\int\sqrt{i\sinh(c+dx)}\,dx}{\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int\sqrt{b\sinh(c+dx)}\,dx = \frac{2iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{b\sinh(c+dx)}}{d\sqrt{i\sinh(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Sinh[c + d*x]],x]
[Out] ((2*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[b*Sinh[c + d*x]])/(
d*Sqrt[I*Sinh[c + d*x]])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.98

method	result
default	$\frac{b\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\cdots\right)\right)}{\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$
risch	$\frac{\sqrt{2}\,\sqrt{b\left(\mathrm{e}^{2dx+2c}-1\right)\mathrm{e}^{-dx-c}}}{d}-\frac{\left(\frac{2b\,\mathrm{e}^{2dx+2c}-2b}{b\sqrt{\mathrm{e}^{dx+c}\left(b\,\mathrm{e}^{2dx+2c}-b\right)}}-\frac{\sqrt{\mathrm{e}^{dx+c}+1}\,\sqrt{-2\,\mathrm{e}^{dx+c}+2}\,\sqrt{-\mathrm{e}^{dx+c}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{dx+c}+1},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticF}\cdots\right)}{\sqrt{b\,\mathrm{e}^{3dx+3c}-b\,\mathrm{e}^{dx+c}}}\cdots\right)\cdots}{d\left(\mathrm{e}^{2dx+2c}-1\right)}$

```
[In] int((b*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] b*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*x
+c))^(1/2)*(2*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-I
*sinh(d*x+c))^(1/2),1/2*2^(1/2)))/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \sqrt{b\sinh(c+dx)}\,dx = -\frac{2\left(\sqrt{2}\sqrt{b}\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right)))+\sqrt{b\sinh\left(dx+c\right)}\right)}{d}$$

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(2)*sqrt(b)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(d*
x + c) + sinh(d*x + c))) + sqrt(b*sinh(d*x + c)))/d
```

Sympy [F]

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh\left(c+dx\right)}\,dx$$

```
[In] integrate((b*sinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\sinh(c+dx)}\,dx = \int \sqrt{b\sinh\left(c+d\,x\right)}\,dx$$

```
[In] int((b*sinh(c + d*x))^(1/2),x)
[Out] int((b*sinh(c + d*x))^(1/2), x)
```

3.19 $\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx=-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/d/(b*
sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2721, 2720}

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx=-\frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{d\sqrt{b\sinh(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt
[b*Sinh[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{i\sinh(c+dx)}\int\frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{\sqrt{b\sinh(c+dx)}}$$
$$= -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int\frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)\sqrt{i\sinh(c+dx)}}{d\sqrt{b\sinh(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Sinh[c + d*x]],x]
```

```
[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2]*Sqrt[I*Sinh[c + d*x]])/(d*Sqrt[
b*Sinh[c + d*x]])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

method	result	size
default	$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\,\text{EllipticF}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$	89

```
[In] int(1/(b*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(I*sinh(d*x
+c))^(1/2)*EllipticF((-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/(b
*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{\sqrt{b}d}$$

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(d*x + c) + sinh(d*x + c))/(sqrt(b)
*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(b*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(dx+c\right)}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(dx+c\right)}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(c+d\,x\right)}}\,dx$$

```
[In] int(1/(b*sinh(c + d*x))^(1/2),x)
[Out] int(1/(b*sinh(c + d*x))^(1/2), x)
```

3.20 $\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} - \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(1/2)+2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*
d*x),2^(1/2))*(b*sinh(d*x+c))^(1/2)/b^2/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2716, 2721, 2719}

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} - \frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(b*d*Sqrt[b*Sinh[c + d*x]]) - ((2*I)*EllipticE[(I*c - Pi
/2 + I*d*x)/2, 2]*Sqrt[b*Sinh[c + d*x]])/(b^2*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} + \frac{\int \sqrt{b\sinh(c+dx)}\,dx}{b^2}\\ &= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} + \frac{\sqrt{b\sinh(c+dx)}\int \sqrt{i\sinh(c+dx)}\,dx}{b^2\sqrt{i\sinh(c+dx)}}\\ &= -\frac{2\cosh(c+dx)}{bd\sqrt{b\sinh(c+dx)}} - \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)\sqrt{b\sinh(c+dx)}}{b^2d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = -\frac{2\left(\cosh(c+dx) - E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|\,2\right)\sqrt{i\sinh(c+dx)}\right)}{bd\sqrt{b\sinh(c+dx)}}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(-3/2),x]
[Out] (-2*(Cosh[c + d*x] - EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sin
h[c + d*x]]))/(b*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.85

method	result
default	$\frac{2\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c}}{b\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$

```
[In] int(1/(b*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^
(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(1/2
)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh
(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/b/cosh(d*x+c)/(b*sinh(d*x+c))^
(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.97

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx =$$
$$-\frac{2\left(\left(\sqrt{2}\cosh\left(dx+c\right)^{2}+2\sqrt{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sqrt{2}\sinh\left(dx+c\right)^{2}-\sqrt{2}\right)\sqrt{b}\mathrm{weierstrassZeta}(4,0\ldots}{b^{2}d\cosh\left(dx+c\right)^{2}+\ldots}$$

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(d*x + c)^2 + 2*sqrt(2)*cosh(d*x + c)*sinh(d*x + c) + sqrt
(2)*sinh(d*x + c)^2 - sqrt(2))*sqrt(b)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + 2*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(b*sinh(d*x + c)))/(b^2*d*cosh
(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 -
 b^2*d)
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\sinh\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sinh(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

```
[In] int(1/(b*sinh(c + d*x))^(3/2),x)
[Out] int(1/(b*sinh(c + d*x))^(3/2), x)
```

3.21 $\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3b^2d\sqrt{b\sinh(c+dx)}}$$

```
[Out] -2/3*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(3/2)-2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*
d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/
2*I*d*x),2^(1/2))*(I*sinh(d*x+c))^(1/2)/b^2/d/(b*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2720}

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} + \frac{2i\sqrt{i\sinh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3b^2d\sqrt{b\sinh(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(3*b*d*(b*Sinh[c + d*x])^(3/2)) + (((2*I)/3)*EllipticF[(
I*c - Pi/2 + I*d*x)/2, 2]*Sqrt[I*Sinh[c + d*x]])/(b^2*d*Sqrt[b*Sinh[c + d*x
]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\int \frac{1}{\sqrt{b\sinh(c+dx)}}\,dx}{3b^2}\\ &= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} - \frac{\sqrt{i\sinh(c+dx)}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx}{3b^2\sqrt{b\sinh(c+dx)}}\\ &= -\frac{2\cosh(c+dx)}{3bd(b\sinh(c+dx))^{3/2}} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)\sqrt{i\sinh(c+dx)}}{3b^2d\sqrt{b\sinh(c+dx)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(\coth(c+dx)+\sqrt{2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2(c+dx))\right)\sqrt{-\left((1+\coth(c+\right.}\right.}{3b^2d\sqrt{b\sinh(c+dx)}}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (-2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d
*x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3
*b^2*d*Sqrt[b*Sinh[c + d*x]])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{i\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)\sinh(dx+c)+2\cosh(dx+c)^{2}}{3b^{2}\sinh(dx+c)\cosh(dx+c)\sqrt{b\sinh(dx+c)}\,d}$	114

```
[In] int(1/(b*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/b^2/sinh(d*x+c)*(I*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(
1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*s
inh(d*x+c)+2*cosh(d*x+c)^2)/cosh(d*x+c)/(b*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.86

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx =$$

$$-\frac{2\left(\left(\sqrt{2}\cosh\left(dx+c\right)^{4}+4\sqrt{2}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+\sqrt{2}\sinh\left(dx+c\right)^{4}+2\left(3\sqrt{2}\cosh\left(dx+c\right)^{2}-\right.\right.\right.}{3\left(b^{3}\right.}$$

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*((sqrt(2)*cosh(d*x + c)^4 + 4*sqrt(2)*cosh(d*x + c)*sinh(d*x + c)^3 +
sqrt(2)*sinh(d*x + c)^4 + 2*(3*sqrt(2)*cosh(d*x + c)^2 - sqrt(2))*sinh(d*x
+ c)^2 - 2*sqrt(2)*cosh(d*x + c)^2 + 4*(sqrt(2)*cosh(d*x + c)^3 - sqrt(2)*c
osh(d*x + c))*sinh(d*x + c) + sqrt(2))*sqrt(b)*weierstrassPInverse(4, 0, co
sh(d*x + c) + sinh(d*x + c)) + 2*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d
*x + c))*sqrt(b*sinh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 - 2*b^3*d*cosh(d*x + c)^2 + b^3
*d + 2*(3*b^3*d*cosh(d*x + c)^2 - b^3*d)*sinh(d*x + c)^2 + 4*(b^3*d*cosh(d*
x + c)^3 - b^3*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(c+dx)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))**(5/2),x)
[Out] Integral((b*sinh(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(c+d\,x)})^{5/2}}\,dx$$

```
[In] int(1/(b*sinh(c + d*x))^(5/2),x)
[Out] int(1/(b*sinh(c + d*x))^(5/2), x)
```

3.22 $\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 118

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2/5*cosh(d*x+c)/b/d/(b*sinh(d*x+c))^(5/2)+6/5*cosh(d*x+c)/b^3/d/(b*sinh(d*
x+c))^(1/2)-6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*P
i+1/2*I*d*x)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))*(b*sinh(d*x+c
))^(1/2)/b^4/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2719}

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}}$$

```
[In] Int[(b*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Cosh[c + d*x])/(5*b*d*(b*Sinh[c + d*x])^(5/2)) + (6*Cosh[c + d*x])/(5*b
^3*d*Sqrt[b*Sinh[c + d*x]]) + (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2,
2]*Sqrt[b*Sinh[c + d*x]])/(b^4*d*Sqrt[I*Sinh[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} - \frac{3\int \frac{1}{(b\sinh(c+dx))^{3/2}}\,dx}{5b^2}\\
&= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{3\int \sqrt{b\sinh(c+dx)}\,dx}{5b^4}\\
&= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} - \frac{\left(3\sqrt{b\sinh(c+dx)}\right)\int \sqrt{i\sinh(c+dx)}\,dx}{5b^4\sqrt{i\sinh(c+dx)}}\\
&= -\frac{2\cosh(c+dx)}{5bd(b\sinh(c+dx))^{5/2}} + \frac{6\cosh(c+dx)}{5b^3d\sqrt{b\sinh(c+dx)}} + \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)\sqrt{b\sinh(c+dx)}}{5b^4d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.67

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = -\frac{2\left(-3\cosh(c+dx)+\coth(c+dx)\text{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{5b^3d\sqrt{b\sinh(c+dx)}}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(-7/2),x]
[Out] (-2*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[((-2*I)*c
 + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(5*b^3*d*Sqrt[b*Sinh[c + d
*x]])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.74

method	result
default	$-\frac{6\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\,\sinh(dx+c)^2\,\text{EllipticE}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)-3\sqrt{-i(\sinh(dx+c}}{5b^3\sinh(dx+c)^2\cosh(}$

```
[In] int(1/(b*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/b^3/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*
x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)
+I))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*
x+c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+c)
+I))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(b*s
inh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 675, normalized size of antiderivative = 5.72

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \frac{2\left(3\left(\sqrt{2}\cosh(dx+c)^6+6\sqrt{2}\cosh(dx+c)\sinh(dx+c)^5+\sqrt{2}\sinh(dx+c)^6+\right.\right.}{}$$

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*(sqrt(2)*cosh(d*x + c)^6 + 6*sqrt(2)*cosh(d*x + c)*sinh(d*x + c)^5 +
 sqrt(2)*sinh(d*x + c)^6 + 3*(5*sqrt(2)*cosh(d*x + c)^2 - sqrt(2))*sinh(d*x
 + c)^4 - 3*sqrt(2)*cosh(d*x + c)^4 + 4*(5*sqrt(2)*cosh(d*x + c)^3 - 3*sqrt
(2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*sqrt(2)*cosh(d*x + c)^4 - 6*sqrt(
2)*cosh(d*x + c)^2 + sqrt(2))*sinh(d*x + c)^2 + 3*sqrt(2)*cosh(d*x + c)^2 +
 6*(sqrt(2)*cosh(d*x + c)^5 - 2*sqrt(2)*cosh(d*x + c)^3 + sqrt(2)*cosh(d*x
+ c))*sinh(d*x + c) - sqrt(2))*sqrt(b)*weierstrassZeta(4, 0, weierstrassPIn
verse(4, 0, cosh(d*x + c) + sinh(d*x + c))) + 2*(3*cosh(d*x + c)^6 + 18*cos
h(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x + c)^6 + (45*cosh(d*x + c)^2 - 8)*s
inh(d*x + c)^4 - 8*cosh(d*x + c)^4 + 4*(15*cosh(d*x + c)^3 - 8*cosh(d*x + c
))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 - 48*cosh(d*x + c)^2 + 1)*sinh(d*x
 + c)^2 + cosh(d*x + c)^2 + 2*(9*cosh(d*x + c)^5 - 16*cosh(d*x + c)^3 + cos
h(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 +
6*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 - 3*b^4*d*cos
h(d*x + c)^4 + 3*b^4*d*cosh(d*x + c)^2 - b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2
 - b^4*d)*sinh(d*x + c)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 - 3*b^4*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 3*(5*b^4*d*cosh(d*x + c)^4 - 6*b^4*d*cosh(d*x + c)^2
 + b^4*d)*sinh(d*x + c)^2 + 6*(b^4*d*cosh(d*x + c)^5 - 2*b^4*d*cosh(d*x + c
)^3 + b^4*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sinh(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))**(7/2),x)
[Out] Integral((b*sinh(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\sinh(c+d\,x))^{7/2}}\,dx$$

```
[In] int(1/(b*sinh(c + d*x))^(7/2),x)
[Out] int(1/(b*sinh(c + d*x))^(7/2), x)
```

3.23 $\int (i\sinh(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int (i\sinh(c+dx))^{7/2}\,dx = -\frac{10i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{21d} + \frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d} + \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d}$$

```
[Out] 10/21*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*
x)*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/7*I*cosh(d*x+c)*(I*
sinh(d*x+c))^(5/2)/d+10/21*I*cosh(d*x+c)*(I*sinh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2715, 2720}

$$\int (i\sinh(c+dx))^{7/2}\,dx = -\frac{10i\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{21d} + \frac{2i(i\sinh(c+dx))^{5/2}\cosh(c+dx)}{7d} + \frac{10i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{21d}$$

```
[In] Int[(I*Sinh[c + d*x])^(7/2),x]
```

```
[Out] (((-10*I)/21)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((10*I)/21)*Cosh[c
 + d*x]*Sqrt[I*Sinh[c + d*x]])/d + (((2*I)/7)*Cosh[c + d*x]*(I*Sinh[c + d*x
])^(5/2))/d
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d} + \frac{5}{7}\int (i\sinh(c+dx))^{3/2}\,dx \\
&= \frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d} \\
&\quad + \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d} + \frac{5}{21}\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx \\
&= -\frac{10i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{21d} + \frac{10i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{21d} \\
&\quad + \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{5/2}}{7d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int (i\sinh(c+dx))^{7/2}\,dx = \frac{i\left(20\,\text{EllipticF}\left(\frac{1}{4}(-2ic+\pi-2idx),2\right)+(23\cosh(c+dx)-3\cosh(3(c+dx)))\sqrt{i\sinh(c+d$$

$$42d$$

```
[In] Integrate[(I*Sinh[c + d*x])^(7/2),x]
[Out] ((I/42)*(20*EllipticF[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + (23*Cosh[c + d*x]
 - 3*Cosh[3*(c + d*x)])*Sqrt[I*Sinh[c + d*x]]))/d
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.34

method	result
default	$\frac{i\left(-6i\cosh(dx+c)^4\sinh(dx+c)+5\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)+16i\cosh\right.}{21\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

```
[In] int((I*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*I*(-6*I*cosh(d*x+c)^4*sinh(d*x+c)+5*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1
+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/
2),1/2*2^(1/2))+16*I*cosh(d*x+c)^2*sinh(d*x+c))/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int (i\sinh(c+dx))^{7/2}\,dx = \frac{\left(\sqrt{\frac{1}{2}}\left(-3i\,e^{(6\,dx+6\,c)}+23i\,e^{(4\,dx+4\,c)}+23i\,e^{(2\,dx+2\,c)}-3i\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-40i\,\sqrt{2}\sqrt{i}\right.}{84\,d}$$

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/84*(sqrt(1/2)*(-3*I*e^(6*d*x + 6*c) + 23*I*e^(4*d*x + 4*c) + 23*I*e^(2*d*
x + 2*c) - 3*I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) - 40*I*sqr
t(2)*sqrt(I)*e^(3*d*x + 3*c)*weierstrassPInverse(4, 0, e^(d*x + c)))*e^(-3*
d*x - 3*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (i \sinh(c+dx))^{7/2} \, dx = \int (i \sinh(dx+c))^{\frac{7}{2}} \, dx$$

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (i \sinh(c+dx))^{7/2} \, dx = \int (i \sinh(dx+c))^{\frac{7}{2}} \, dx$$

```
[In] integrate((I*sinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (i \sinh(c+dx))^{7/2} \, dx = \int (\sinh(c+d\,x)\;1\mathrm{i})^{7/2} \, dx$$

```
[In] int((sinh(c + d*x)*1i)^(7/2),x)
[Out] int((sinh(c + d*x)*1i)^(7/2), x)
```

3.24 $\int (i \sinh(c+dx))^{5/2}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int (i \sinh(c+dx))^{5/2}\, dx = -\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{5d}+\frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}$$

```
[Out] 6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c)*(I*si
nh(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2715, 2719}

$$\int (i \sinh(c+dx))^{5/2}\, dx = \frac{2i(i\sinh(c+dx))^{3/2}\cosh(c+dx)}{5d}-\frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|\,2\right)}{5d}$$

```
[In] Int[(I*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d
*x]*(I*Sinh[c + d*x])^(3/2))/d
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d} + \frac{3}{5}\int\sqrt{i\sinh(c+dx)}\,dx\\ &= -\frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{5d} + \frac{2i\cosh(c+dx)(i\sinh(c+dx))^{3/2}}{5d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int(i\sinh(c+dx))^{5/2}\,dx = \frac{6iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|\,2\right)-\sqrt{i\sinh(c+dx)}\sinh(2(c+dx))}{5d}$$

```
[In] Integrate[(I*Sinh[c + d*x])^(5/2),x]
[Out] ((6*I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] - Sqrt[I*Sinh[c + d*x]]*
Sinh[2*(c + d*x)])/(5*d)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(82) = 164.

Time = 0.87 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.73

method	result
default	$-\frac{i\left(3\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\ \text{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-6\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(}\right.}{5\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

```
[In] int((I*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/5*I*(3*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d
*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-6*(1-I*sinh(d*x
+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticE(
(1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+2*cosh(d*x+c)^4-2*cosh(d*x+c)^2)/cosh(
d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int (i\sinh(c+dx))^{5/2}\,dx =$$

$$-\frac{\left(\sqrt{\frac{1}{2}}\left(e^{(4\,dx+4\,c)}+12\,e^{(2\,dx+2\,c)}-1\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+12\,\sqrt{2}\sqrt{i}e^{(2\,dx+2\,c)}\mathrm{weierstrassZeta}\left(4,0,\mathrm{weie}\right.\right.}{10\,d}$$

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -1/10*(sqrt(1/2)*(e^(4*d*x + 4*c) + 12*e^(2*d*x + 2*c) - 1)*sqrt(I*e^(2*d*x
 + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 12*sqrt(2)*sqrt(I)*e^(2*d*x + 2*c)*weie
rstrassZeta(4, 0, weierstrassPInverse(4, 0, e^(d*x + c))))*e^(-2*d*x - 2*c)
/d
```

Sympy [F]

$$\int (i\sinh(c+dx))^{5/2}\,dx = \int (i\sinh{(c+dx)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((I*sinh(d*x+c))**(5/2),x)
[Out] Integral((I*sinh(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (i\sinh(c+dx))^{5/2}\,dx = \int (i\,\sinh{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate((I*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{5/2}\,dx = \int (\sinh(c+d\,x)\ 1\mathrm{i})^{5/2}\,dx$$

```
[In] int((sinh(c + d*x)*1i)^(5/2),x)
[Out] int((sinh(c + d*x)*1i)^(5/2), x)
```

3.25 $\int (i \sinh(c+dx))^{3/2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int (i\sinh(c+dx))^{3/2}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{3d}+\frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}$$

```
[Out] 2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/3*I*cosh(d*x+c)*(I*si
nh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2715, 2720}

$$\int (i\sinh(c+dx))^{3/2}\,dx = \frac{2i\sqrt{i\sinh(c+dx)}\cosh(c+dx)}{3d}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d}$$

```
[In] Int[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x]*Sqrt[I*Sinh[c + d*x]])/d
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}+\frac{1}{3}\int\frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{3d}+\frac{2i\cosh(c+dx)\sqrt{i\sinh(c+dx)}}{3d}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int(i\sinh(c+dx))^{3/2}\,dx = -\frac{2i\sqrt{i\sinh(c+dx)}\Big(-\cosh(c+dx)+\text{csch}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2(}{3d}$$

```
[In] Integrate[(I*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[I*Sinh[c + d*x]]*(-Cosh[c + d*x] + Csch[c + d*x]*Hypergeom
etric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]]*Sqrt[1 - Cos
h[2*c + 2*d*x] - Sinh[2*c + 2*d*x]]))/d
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

method	result	size
default	$\frac{i\left(\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\text{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)+2i\cosh(dx+c)^2\sinh(dx+c)\right)}{3\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$	104

```
[In] int((I*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I*((1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+
c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))+2*I*cosh(d*x+c)^2*
sinh(d*x+c))/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int (i\sinh(c+dx))^{3/2}\,dx = \frac{\left(\sqrt{\frac{1}{2}}\left(i\,e^{(2\,dx+2\,c)}+i\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-2i\,\sqrt{2}\sqrt{i}e^{(dx+c)}\mathrm{weierstrassPInverse}\left(4,0,e^{(dx}\right.\right.}{3\,d}$$

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(1/2)*(I*e^(2*d*x + 2*c) + I)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*
d*x - 1/2*c) - 2*I*sqrt(2)*sqrt(I)*e^(d*x + c)*weierstrassPInverse(4, 0, e^
(d*x + c)))*e^(-d*x - c)/d
```

Sympy [F]

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (i\sinh(c+dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((I*sinh(d*x+c))**(3/2),x)
[Out] Integral((I*sinh(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (i\,\sinh(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (i\,\sinh\left(dx+c\right))^{\frac{3}{2}}\,dx$$

```
[In] integrate((I*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^{3/2}\,dx = \int (\mathrm{sinh}(c+d\,x)\ 1\mathrm{i})^{3/2}\,dx$$

```
[In] int((sinh(c + d*x)*1i)^(3/2),x)
[Out] int((sinh(c + d*x)*1i)^(3/2), x)
```

3.26 $\int \sqrt{i \sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \sqrt{i \sinh(c+dx)}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2719}

$$\int \sqrt{i \sinh(c+dx)}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}$$

```
[In] Int[Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \sqrt{i \sinh(c+dx)}\, dx = \frac{2iE\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right)\middle|\, 2\right)}{d}$$

```
[In] Integrate[Sqrt[I*Sinh[c + d*x]],x]
[Out] ((2*I)*EllipticE[(Pi/2 - I*(c + d*x))/2, 2])/d
```

Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.03

method	result
default	$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)\right)}{\cosh(dx+c)d}$
risch	$\frac{\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{2dx+2c}-1\right)\mathrm{e}^{-dx-c}}}{d}-\frac{\left(-\frac{2i\left(-i+i\mathrm{e}^{2dx+2c}\right)}{\sqrt{\mathrm{e}^{dx+c}\left(-i+i\mathrm{e}^{2dx+2c}\right)}}-\frac{\sqrt{\mathrm{e}^{dx+c}+1}\,\sqrt{-2\,\mathrm{e}^{dx+c}+2}\,\sqrt{-\mathrm{e}^{dx+c}}\,\left(-2\,\mathrm{EllipticE}\left(\sqrt{\mathrm{e}^{dx+c}+1},\frac{\sqrt{2}}{2}\right)+\mathrm{Elliptic}\right.}{\sqrt{i\mathrm{e}^{3dx+3c}-i\mathrm{e}^{dx+c}}}}{d\left(\mathrm{e}^{2dx+2c}-1\right)}$

```
[In] int((I*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*(2*Elliptic
E((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(d*x+c))^(1/2),1/
2*2^(1/2)))/cosh(d*x+c)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \sqrt{i \sinh(c+dx)}\, dx = -\frac{2\left(\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+\sqrt{2}\sqrt{i}\mathrm{weierstrassZeta}\left(4,0,\mathrm{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)\right)\right)}{d}$$

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*(sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + sqrt(2)*sq
rt(I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, e^(d*x + c))))/d
```

Sympy [F]

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{i\sinh\left(c+dx\right)}\,dx$$

```
[In] integrate((I*sinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{i\,\sinh\left(dx+c\right)}\,dx$$

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*sinh(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{i\,\sinh\left(dx+c\right)}\,dx$$

```
[In] integrate((I*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{i\sinh(c+dx)}\,dx = \int \sqrt{\sinh\left(c+d\,x\right)\,1\mathrm{i}}\,dx$$

```
[In] int((sinh(c + d*x)*1i)^(1/2),x)
[Out] int((sinh(c + d*x)*1i)^(1/2), x)
```

3.27 $\int \frac{1}{\sqrt{i \sinh(c+dx)}} dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{1}{\sqrt{i \sinh(c+dx)}} dx = -\frac{2i \operatorname{EllipticF}\left(\frac{1}{2}\left(ic - \frac{\pi}{2} + idx\right), 2\right)}{d}$$

```
[Out] 2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*E
llipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2720}

$$\int \frac{1}{\sqrt{i \sinh(c+dx)}} dx = -\frac{2i \operatorname{EllipticF}\left(\frac{1}{2}\left(ic + idx - \frac{\pi}{2}\right), 2\right)}{d}$$

```
[In] Int[1/Sqrt[I*Sinh[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{2i \operatorname{EllipticF}\left(\frac{1}{2}\left(ic - \frac{\pi}{2} + idx\right), 2\right)}{d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(\frac{\pi}{2}-i(c+dx)\right),2\right)}{d}$$

```
[In] Integrate[1/Sqrt[I*Sinh[c + d*x]],x]
[Out] ((2*I)*EllipticF[(Pi/2 - I*(c + d*x))/2, 2])/d
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

method	result	size
default	$\frac{i\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\text{EllipticF}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)}{\cosh(dx+c)d}$	68

```
[In] int(1/(I*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+c)))^(1/2)*EllipticF((
-I*(sinh(d*x+c)+I))^(1/2),1/2*2^(1/2))/cosh(d*x+c)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = -\frac{2i\,\sqrt{2}\sqrt{i}\text{weierstrassPInverse}\left(4,0,e^{(dx+c)}\right)}{d}$$

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] -2*I*sqrt(2)*sqrt(I)*weierstrassPInverse(4, 0, e^(d*x + c))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\sinh{(c+dx)}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(I*sinh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\,\sinh{(dx+c)}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\,\sinh{(dx+c)}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(I*sinh(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{i\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{\sinh{(c+d\,x)}\ 1\mathrm{i}}}\,dx$$

```
[In] int(1/(sinh(c + d*x)*1i)^(1/2),x)
[Out] int(1/(sinh(c + d*x)*1i)^(1/2), x)
```

3.28 $\int \frac{1}{(i \sinh(c+dx))^{3/2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 58

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -2*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)*
EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2*I*cosh(d*x+c)/d/(I*sin
h(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2716, 2719}

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \frac{2iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{d}+\frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}$$

```
[In] Int[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((2*I)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + ((2*I)*Cosh[c + d*x])/(d*S
qrt[I*Sinh[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}} - \int \sqrt{i\sinh(c+dx)}\,dx \\
&= \frac{2iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|\,2\right)}{d} + \frac{2i\cosh(c+dx)}{d\sqrt{i\sinh(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \frac{2\left(-iE\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|\,2\right)+\coth(c+dx)\sqrt{i\sinh(c+dx)}\right)}{d}$$

```
[In] Integrate[(I*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] (2*((-I)*EllipticE[((-2*I)*c + Pi - (2*I)*d*x)/4, 2] + Coth[c + d*x]*Sqrt[I
*Sinh[c + d*x]]))/d
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.74

method	result
default	$-\frac{i\left(2\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\,\operatorname{EllipticE}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)-\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(\ldots}{\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$

```
[In] int(1/(I*sinh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*(2*(1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c
))^(1/2)*EllipticE((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))-(1-I*sinh(d*x+c))^(
1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*s
inh(d*x+c))^(1/2),1/2*2^(1/2))-2*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))
^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \frac{2\left(2\sqrt{\frac{1}{2}}\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}+\left(\sqrt{2}\sqrt{i}e^{(2\,dx+2\,c)}-\sqrt{2}\sqrt{i}\right)\text{weierstrassZeta}\left(4,\right.}{de^{(2\,dx+2\,c)}-d}$$

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 2*(2*sqrt(1/2)*sqrt(I*e^(2*d*x + 2*c) - I)*e^(3/2*d*x + 3/2*c) + (sqrt(2)*s
qrt(I)*e^(2*d*x + 2*c) - sqrt(2)*sqrt(I))*weierstrassZeta(4, 0, weierstrass
PInverse(4, 0, e^(d*x + c))))/(d*e^(2*d*x + 2*c) - d)
```

Sympy [F]

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\sinh{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))**(3/2),x)
[Out] Integral((I*sinh(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\sinh{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(i\sinh{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(i\sinh(c+dx))^{3/2}}\,dx=\int \frac{1}{{\left(\mathrm{sinh}\left(c+d\,x\right)\,1{}\mathrm{i}\right)}^{3/2}} \,d x$$

```
[In] int(1/(sinh(c + d*x)*1i)^(3/2),x)
[Out] int(1/(sinh(c + d*x)*1i)^(3/2), x)
```

3.29 $\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 62

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{3d}+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}$$

```
[Out] 2/3*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x)
*EllipticF(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/3*I*cosh(d*x+c)/d/(I*
sinh(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2716, 2720}

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right),2\right)}{3d}$$

```
[In] Int[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/3)*Cosh[c + d
*x])/(d*(I*Sinh[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}+\frac{1}{3}\int\frac{1}{\sqrt{i\sinh(c+dx)}}\,dx\\ &= -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right),2\right)}{3d}+\frac{2i\cosh(c+dx)}{3d(i\sinh(c+dx))^{3/2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.34

$$\int\frac{1}{(i\sinh(c+dx))^{5/2}}\,dx=\frac{2\Big(\coth(c+dx)+\sqrt{2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(c+dx))+\sinh(2(c+\right.}{3d\sqrt{i\sinh(c+dx)}}$$

```
[In] Integrate[(I*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (2*(Coth[c + d*x] + Sqrt[2]*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]]*Sqrt[-((1 + Coth[c + d*x])*Sinh[c + d*x]^2)]))/(3*
d*Sqrt[I*Sinh[c + d*x]])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

method	result	size
default	$\frac{i\left(\sqrt{1-i\sinh(dx+c)}\,\sqrt{2}\,\sqrt{1+i\sinh(dx+c)}\,\sqrt{i\sinh(dx+c)}\ \text{EllipticF}\left(\sqrt{1-i\sinh(dx+c)},\frac{\sqrt{2}}{2}\right)\sinh(dx+c)-2i\cosh(dx+c)^2\right)}{3\sinh(dx+c)\cosh(dx+c)\sqrt{i\sinh(dx+c)}\,d}$	112

```
[In] int(1/(I*sinh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*I/sinh(d*x+c)*((1-I*sinh(d*x+c))^(1/2)*2^(1/2)*(1+I*sinh(d*x+c))^(1/2)*
(I*sinh(d*x+c))^(1/2)*EllipticF((1-I*sinh(d*x+c))^(1/2),1/2*2^(1/2))*sinh(d
*x+c)-2*I*cosh(d*x+c)^2)/cosh(d*x+c)/(I*sinh(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx =$$
$$-\frac{2\left(2\sqrt{\frac{1}{2}}\left(i\,e^{(3\,dx+3\,c)}+i\,e^{(dx+c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+\left(i\,\sqrt{2}\sqrt{i}e^{(4\,dx+4\,c)}-2i\,\sqrt{2}\sqrt{i}e^{(2\,dx+2\,c)}+i\,\sqrt{2}\sqrt{}\right.\right.}{3\left(de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d\right)}$$

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(1/2)*(I*e^(3*d*x + 3*c) + I*e^(d*x + c))*sqrt(I*e^(2*d*x + 2*c
) - I)*e^(-1/2*d*x - 1/2*c) + (I*sqrt(2)*sqrt(I)*e^(4*d*x + 4*c) - 2*I*sqrt
(2)*sqrt(I)*e^(2*d*x + 2*c) + I*sqrt(2)*sqrt(I))*weierstrassPInverse(4, 0,
e^(d*x + c)))/(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(i\sinh\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))**(5/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(i\,\sinh\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*sinh(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,\sinh\,(dx+c))^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(I*sinh(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((I*sinh(d*x + c))^(-5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(i\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(\sinh\,(c+d\,x)\ 1\mathrm{i})^{5/2}}\,dx$$

[In] int(1/(sinh(c + d*x)*1i)^(5/2),x)

[Out] int(1/(sinh(c + d*x)*1i)^(5/2), x)

3.30 $\int \frac{1}{(i \sinh(c+dx))^{7/2}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d} + \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}} + \frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}$$

```
[Out] -6/5*I*(sin(1/2*I*c+1/4*Pi+1/2*I*d*x)^2)^(1/2)/sin(1/2*I*c+1/4*Pi+1/2*I*d*x
)*EllipticE(cos(1/2*I*c+1/4*Pi+1/2*I*d*x),2^(1/2))/d+2/5*I*cosh(d*x+c)/d/(I
*sinh(d*x+c))^(5/2)+6/5*I*cosh(d*x+c)/d/(I*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2716, 2719}

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \frac{6iE\left(\frac{1}{2}\left(ic+idx-\frac{\pi}{2}\right)\middle|2\right)}{5d} + \frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}} + \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}$$

```
[In] Int[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*c - Pi/2 + I*d*x)/2, 2])/d + (((2*I)/5)*Cosh[c + d*
x])/(d*(I*Sinh[c + d*x])^(5/2)) + (((6*I)/5)*Cosh[c + d*x])/(d*Sqrt[I*Sinh[
c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{3}{5}\int\frac{1}{(i\sinh(c+dx))^{3/2}}\,dx\\ &= \frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}-\frac{3}{5}\int\sqrt{i\sinh(c+dx)}\,dx\\ &= \frac{6iE\left(\frac{1}{2}\left(ic-\frac{\pi}{2}+idx\right)\middle|2\right)}{5d}+\frac{2i\cosh(c+dx)}{5d(i\sinh(c+dx))^{5/2}}+\frac{6i\cosh(c+dx)}{5d\sqrt{i\sinh(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int\frac{1}{(i\sinh(c+dx))^{7/2}}\,dx=$$

$$-\frac{2i\left(-3\cosh(c+dx)+\coth(c+dx)\operatorname{csch}(c+dx)+3E\left(\frac{1}{4}(-2ic+\pi-2idx)\middle|2\right)\sqrt{i\sinh(c+dx)}\right)}{5d\sqrt{i\sinh(c+dx)}}$$

```
[In] Integrate[(I*Sinh[c + d*x])^(-7/2),x]
```

```
[Out] (((-2*I)/5)*(-3*Cosh[c + d*x] + Coth[c + d*x]*Csch[c + d*x] + 3*EllipticE[(
(-2*I)*c + Pi - (2*I)*d*x)/4, 2]*Sqrt[I*Sinh[c + d*x]]))/(d*Sqrt[I*Sinh[c +
 d*x]])
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.24

method	result
default	$-\frac{i\left(6\sqrt{-i(\sinh(dx+c)+i)}\,\sqrt{2}\,\sqrt{-i(i-\sinh(dx+c))}\,\sqrt{i\sinh(dx+c)}\,\sinh(dx+c)^2\,\mathrm{EllipticE}\left(\sqrt{-i(\sinh(dx+c)+i)},\frac{\sqrt{2}}{2}\right)-3\sqrt{-i(\sinh(dx}{5\sinh(dx+c)^2\cosh(}$

```
[In] int(1/(I*sinh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*I/sinh(d*x+c)^2*(6*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+
c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticE((-I*(sinh(d*x+c)+I
))^(1/2),1/2*2^(1/2))-3*(-I*(sinh(d*x+c)+I))^(1/2)*2^(1/2)*(-I*(I-sinh(d*x+
c)))^(1/2)*(I*sinh(d*x+c))^(1/2)*sinh(d*x+c)^2*EllipticF((-I*(sinh(d*x+c)+I
))^(1/2),1/2*2^(1/2))-6*sinh(d*x+c)^4-4*sinh(d*x+c)^2+2)/cosh(d*x+c)/(I*sin
h(d*x+c))^(1/2)/d
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.96

$$\int\frac{1}{(i\sinh(c+dx))^{7/2}}\,dx=\frac{2\left(2\sqrt{\frac{1}{2}}\left(3\,e^{(6\,dx+6\,c)}-8\,e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}\right)\sqrt{i\,e^{(2\,dx+2\,c)}-i}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+3\left(\sqrt{2}\right.\right.}{5\,(de^{(}}$$

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(2*sqrt(1/2)*(3*e^(6*d*x + 6*c) - 8*e^(4*d*x + 4*c) + e^(2*d*x + 2*c))*
sqrt(I*e^(2*d*x + 2*c) - I)*e^(-1/2*d*x - 1/2*c) + 3*(sqrt(2)*sqrt(I)*e^(6*
d*x + 6*c) - 3*sqrt(2)*sqrt(I)*e^(4*d*x + 4*c) + 3*sqrt(2)*sqrt(I)*e^(2*d*x
 + 2*c) - sqrt(2)*sqrt(I))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
e^(d*x + c))))/(d*e^(6*d*x + 6*c) - 3*d*e^(4*d*x + 4*c) + 3*d*e^(2*d*x + 2*
c) - d)
```

Sympy [F]

$$\int\frac{1}{(i\sinh(c+dx))^{7/2}}\,dx=\int\frac{1}{\left(i\sinh\left(c+dx\right)\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))**(7/2),x)
```

```
[Out] Integral((I*sinh(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(i\,\sinh\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(i\,\sinh\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(I*sinh(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(i\sinh(c+dx))^{7/2}}\,dx = \int \frac{1}{(\sinh\,(c+d\,x)\;1\mathrm{i})^{7/2}}\,dx$$

```
[In] int(1/(sinh(c + d*x)*1i)^(7/2),x)
[Out] int(1/(sinh(c + d*x)*1i)^(7/2), x)
```

3.31 $\int (b\sinh(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int (b\sinh(c+dx))^{4/3}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{7/3}}{7bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/7*cosh(d*x+c)*hypergeom([1/2, 7/6],[13/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(7/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int (b\sinh(c+dx))^{4/3}\,dx = \frac{3\cosh(c+dx)(b\sinh(c+dx))^{7/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},-\sinh^2(c+dx)\right)}{7bd\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(4/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(7/3))/(7*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{7/3}}{7bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int(b\sinh(c+dx))^{4/3}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}\tanh(c+dx)}{7d}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(4/3),x]
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 7/6, 13/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(4/3)*Tanh[c + d*x])/(7*d)
```

Maple [F]

$$\int (b\sinh(dx+c))^{\frac{4}{3}}\,dx$$

```
[In] int((b*sinh(d*x+c))^(4/3),x)
[Out] int((b*sinh(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int(b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh(dx+c))^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c))^(1/3)*b*sinh(d*x + c), x)
```

Sympy [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh{(c+dx)})^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))**(4/3),x)
[Out] Integral((b*sinh(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh{(dx+c)})^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh{(dx+c)})^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{4/3}\,dx = \int (b\sinh(c+d\,x))^{4/3}\,dx$$

```
[In] int((b*sinh(c + d*x))^(4/3),x)
[Out] int((b*sinh(c + d*x))^(4/3), x)
```

3.32 $\int (b\sinh(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int (b\sinh(c+dx))^{2/3}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{5/3}}{5bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/5*cosh(d*x+c)*hypergeom([1/2, 5/6],[11/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))
^(5/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int (b\sinh(c+dx))^{2/3}\,dx = \frac{3\cosh(c+dx)(b\sinh(c+dx))^{5/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\sinh^2(c+dx)\right)}{5bd\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2]*(b*Sin
h[c + d*x])^(5/3))/(5*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{5/3}}{5bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int (b\sinh(c+dx))^{2/3}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}\tanh(c+d}{5d}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(2/3),x]
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 5/6, 11/6, -Sinh[c + d*x]^2
]*(b*Sinh[c + d*x])^(2/3)*Tanh[c + d*x])/(5*d)
```

Maple [F]

$$\int (b\sinh(dx+c))^{\frac{2}{3}}\,dx$$

```
[In] int((b*sinh(d*x+c))^(2/3),x)
[Out] int((b*sinh(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh(dx+c))^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c))^(2/3), x)
```

Sympy [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh(c+dx))^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))**(2/3),x)
[Out] Integral((b*sinh(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh(dx+c))^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

Giac [F]

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh(dx+c))^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^{2/3}\,dx = \int (b\sinh(c+d\,x))^{2/3}\,dx$$

```
[In] int((b*sinh(c + d*x))^(2/3),x)
[Out] int((b*sinh(c + d*x))^(2/3), x)
```

3.33 $\int \sqrt[3]{b \sinh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \sqrt[3]{b \sinh(c+dx)}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}}{4bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/4*cosh(d*x+c)*hypergeom([1/2, 2/3],[5/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^
(4/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \sqrt[3]{b \sinh(c+dx)}\,dx = \frac{3\cosh(c+dx)(b\sinh(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\sinh^2(c+dx)\right)}{4bd\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(4/3))/(4*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{4/3}}{4bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}\tanh(c+dx)}{4d}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(1/3),x]
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -Sinh[c + d*x]^2]
*(b*Sinh[c + d*x])^(1/3)*Tanh[c + d*x])/(4*d)
```

Maple [F]

$$\int (b\sinh(dx+c))^{\frac{1}{3}}\,dx$$

```
[In] int((b*sinh(d*x+c))^(1/3),x)
[Out] int((b*sinh(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int (b\sinh(dx+c))^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c))^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int \sqrt[3]{b\sinh{(c+dx)}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))**(1/3),x)
[Out] Integral((b*sinh(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int (b\sinh{(dx+c)})^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int (b\sinh{(dx+c)})^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*sinh(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\sinh(c+dx)}\,dx = \int (b\sinh(c+d\,x))^{1/3}\,dx$$

```
[In] int((b*sinh(c + d*x))^(1/3),x)
[Out] int((b*sinh(c + d*x))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}}{2bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3/2*cosh(d*x+c)*hypergeom([1/3, 1/2],[4/3],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^
(2/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{3\cosh(c+dx)(b\sinh(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\sinh^2(c+dx)\right)}{2bd\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(-1/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(2/3))/(2*b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{2/3}}{2bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\sinh^2(c+dx)\right)\tanh(c+dx)}{2d\sqrt[3]{b\sinh(c+dx)}}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(-1/3),x]
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/3, 1/2, 4/3, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(2*d*(b*Sinh[c + d*x])^(1/3))
```

Maple [F]

$$\int \frac{1}{(b\sinh(dx+c))^{\frac{1}{3}}}dx$$

```
[In] int(1/(b*sinh(d*x+c))^(1/3),x)
[Out] int(1/(b*sinh(d*x+c))^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c))^(2/3)/(b*sinh(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\sinh\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))**(1/3),x)
[Out] Integral((b*sinh(c + d*x))**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\sinh(c+dx)}}\,dx = \int \frac{1}{{\left(b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

```
[In] int(1/(b*sinh(c + d*x))^(1/3),x)
[Out] int(1/(b*sinh(c + d*x))^(1/3), x)
```

3.35 $\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}}{bd\sqrt{\cosh^2(c+dx)}}$$

```
[Out] 3*cosh(d*x+c)*hypergeom([1/6, 1/2],[7/6],-sinh(d*x+c)^2)*(b*sinh(d*x+c))^(1
/3)/b/d/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \frac{3\cosh(c+dx)\sqrt[3]{b\sinh(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(-2/3),x]
```

```
[Out] (3*Cosh[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]*(b*Sinh
[c + d*x])^(1/3))/(b*d*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\sinh^2(c+dx)\right)\sqrt[3]{b\sinh(c+dx)}}{bd\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},-\sinh^2(c+dx)\right)\tanh(c+dx)}{d(b\sinh(c+dx))^{2/3}}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(-2/3),x]
[Out] (3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/6, 1/2, 7/6, -Sinh[c + d*x]^2]
*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{2}{3}}}dx$$

```
[In] int(1/(b*sinh(d*x+c))^(2/3),x)
[Out] int(1/(b*sinh(d*x+c))^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c))^(1/3)/(b*sinh(d*x + c)), x)
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh(c+dx))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))**(2/3),x)
[Out] Integral((b*sinh(c + d*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh(dx+c))^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\sinh(c+d\,x))^{2/3}}\,dx$$

```
[In] int(1/(b*sinh(c + d*x))^(2/3),x)
[Out] int(1/(b*sinh(c + d*x))^(2/3), x)
```

3.36 $\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

```
[Out] -3*cosh(d*x+c)*hypergeom([-1/6, 1/2],[5/6],-sinh(d*x+c)^2)/b/d/(b*sinh(d*x+
c))^(1/3)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^(-4/3),x]
```

```
[Out] (-3*Cosh[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^2])/(b*d
*Sqrt[Cosh[c + d*x]^2]*(b*Sinh[c + d*x])^(1/3))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{3\cosh(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\sinh^2(c+dx)\right)}{bd\sqrt{\cosh^2(c+dx)}\sqrt[3]{b\sinh(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = -\frac{3\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},-\sinh^2(c+dx)\right)\tanh(c+dx)}{d(b\sinh(c+dx))^{4/3}}$$

```
[In] Integrate[(b*Sinh[c + d*x])^(-4/3),x]
[Out] (-3*Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[-1/6, 1/2, 5/6, -Sinh[c + d*x]^
2]*Tanh[c + d*x])/(d*(b*Sinh[c + d*x])^(4/3))
```

Maple [F]

$$\int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{4}{3}}}dx$$

```
[In] int(1/(b*sinh(d*x+c))^(4/3),x)
[Out] int(1/(b*sinh(d*x+c))^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{\left(b\sinh\left(dx+c\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c))^(2/3)/(b^2*sinh(d*x + c)^2), x)
```

Sympy [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh{(c+dx)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))**(4/3),x)
[Out] Integral((b*sinh(c + d*x))**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh{(dx+c)})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*sinh(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\sinh(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\sinh{(c+d\,x)})^{4/3}}\,dx$$

```
[In] int(1/(b*sinh(c + d*x))^(4/3),x)
[Out] int(1/(b*sinh(c + d*x))^(4/3), x)
```

3.37 $\int (b\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int (b\sinh(c+dx))^n\,dx = \frac{\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{1+n}}{bd(1+n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(b*sinh(
d*x+c))^(1+n)/b/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2722}

$$\int (b\sinh(c+dx))^n\,dx = \frac{\cosh(c+dx)(b\sinh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\sinh^2(c+dx)\right)}{bd(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(b*Sinh[c + d*x])^n,x]
```

```
[Out] (Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x]^
2]*(b*Sinh[c + d*x])^(1 + n))/(b*d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^{1+n}}{bd(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (b\sinh(c+dx))^n\,dx = \frac{\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(b\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

```
[In] Integrate[(b*Sinh[c + d*x])^n,x]
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(b*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F]

$$\int (b\sinh(dx+c))^n\,dx$$

```
[In] int((b*sinh(d*x+c))^n,x)
[Out] int((b*sinh(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\sinh(c+dx))^n\,dx = \int (b\sinh(dx+c))^n\,dx$$

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sinh(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\sinh(c+dx))^n\,dx = \int (b\sinh{(c+dx)})^n\,dx$$

```
[In] integrate((b*sinh(d*x+c))**n,x)
[Out] Integral((b*sinh(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\sinh(c+dx))^n\,dx = \int (b\sinh{(dx+c)})^n\,dx$$

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sinh(d*x + c))^n, x)
```

Giac [F]

$$\int (b\sinh(c+dx))^n\,dx = \int (b\sinh{(dx+c)})^n\,dx$$

```
[In] integrate((b*sinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sinh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\sinh(c+dx))^n\,dx = \int (b\sinh(c+d\,x))^n\,dx$$

```
[In] int((b*sinh(c + d*x))^n,x)
[Out] int((b*sinh(c + d*x))^n, x)
```

3.38 $\int (i\sinh(c+dx))^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int (i\sinh(c+dx))^n\,dx = -\frac{i\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] -I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(I*si
nh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\int (i\sinh(c+dx))^n\,dx = -\frac{i\cosh(c+dx)(i\sinh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[(I*Sinh[c + d*x])^n,x]
```

```
[Out] ((-I)*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c +
d*x]^2]*(I*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{i\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int (i\sinh(c+dx))^n\,dx = \frac{\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(i\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

```
[In] Integrate[(I*Sinh[c + d*x])^n,x]
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*(I*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F]

$$\int (i\sinh(dx+c))^n\,dx$$

```
[In] int((I*sinh(d*x+c))^n,x)
[Out] int((I*sinh(d*x+c))^n,x)
```

Fricas [F]

$$\int (i\sinh(c+dx))^n\,dx = \int (i\ \sinh(dx+c))^n\ dx$$

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((1/2*(I*e^(2*d*x + 2*c) - I)*e^(-d*x - c))^n, x)
```

Sympy [F]

$$\int (i\sinh(c+dx))^n\,dx = \int (i\sinh(c+dx))^n\,dx$$

```
[In] integrate((I*sinh(d*x+c))**n,x)
[Out] Integral((I*sinh(c + d*x))**n, x)
```

Maxima [F]

$$\int (i\sinh(c+dx))^n\,dx = \int (i\sinh(dx+c))^n\,dx$$

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((I*sinh(d*x + c))^n, x)
```

Giac [F]

$$\int (i\sinh(c+dx))^n\,dx = \int (i\sinh(dx+c))^n\,dx$$

```
[In] integrate((I*sinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((I*sinh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (i\sinh(c+dx))^n\,dx = \int (\sinh(c+d\,x)\ 1\mathrm{i})^n\,dx$$

```
[In] int((sinh(c + d*x)*1i)^n,x)
[Out] int((sinh(c + d*x)*1i)^n, x)
```

3.39 $\int (-i\sinh(c+dx))^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int (-i\sinh(c+dx))^n \, dx = \frac{i\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

```
[Out] I*cosh(d*x+c)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],-sinh(d*x+c)^2)*(-I*si
nh(d*x+c))^(1+n)/d/(1+n)/(cosh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2722}

$$\int (-i\sinh(c+dx))^n \, dx = \frac{i\cosh(c+dx)(-i\sinh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},-\sinh^2(c+dx)\right)}{d(n+1)\sqrt{\cosh^2(c+dx)}}$$

```
[In] Int[((-I)*Sinh[c + d*x])^n,x]
```

```
[Out] (I*Cosh[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c + d*x
]^2]*((-I)*Sinh[c + d*x])^(1 + n))/(d*(1 + n)*Sqrt[Cosh[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{i\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^{1+n}}{d(1+n)\sqrt{\cosh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int(-i\sinh(c+dx))^n\,dx$$
$$= \frac{\sqrt{\cosh^2(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},-\sinh^2(c+dx)\right)(-i\sinh(c+dx))^n\tanh(c+dx)}{d(1+n)}$$

```
[In] Integrate[((-I)*Sinh[c + d*x])^n,x]
[Out] (Sqrt[Cosh[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, -Sinh[c
 + d*x]^2]*((-I)*Sinh[c + d*x])^n*Tanh[c + d*x])/(d*(1 + n))
```

Maple [F]

$$\int(-i\sinh(dx+c))^n\,dx$$

```
[In] int((-I*sinh(d*x+c))^n,x)
[Out] int((-I*sinh(d*x+c))^n,x)
```

Fricas [F]

$$\int(-i\sinh(c+dx))^n\,dx = \int(-i\,\sinh(dx+c))^n\,dx$$

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((1/2*(-I*e^(2*d*x + 2*c) + I)*e^(-d*x - c))^n, x)
```

Sympy [F]

$$\int (-i\sinh(c+dx))^n\,dx = \int \left(-i\sinh\left(c+dx\right)\right)^n\,dx$$

```
[In] integrate((-I*sinh(d*x+c))**n,x)
[Out] Integral((-I*sinh(c + d*x))**n, x)
```

Maxima [F]

$$\int (-i\sinh(c+dx))^n\,dx = \int \left(-i\,\sinh\left(dx+c\right)\right)^n\,dx$$

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((-I*sinh(d*x + c))^n, x)
```

Giac [F]

$$\int (-i\sinh(c+dx))^n\,dx = \int \left(-i\,\sinh\left(dx+c\right)\right)^n\,dx$$

```
[In] integrate((-I*sinh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((-I*sinh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (-i\sinh(c+dx))^n\,dx = \int (-\sinh(c+d\,x)\ 1\mathrm{i})^n\,dx$$

```
[In] int((-sinh(c + d*x)*1i)^n,x)
[Out] int((-sinh(c + d*x)*1i)^n, x)
```

3.40 $\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3ix}{2} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}$$

```
[Out] 3/2*I*x-4*cosh(x)+4/3*cosh(x)^3-3/2*I*cosh(x)*sinh(x)-cosh(x)*sinh(x)^3/(I+
sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2846, 2827, 2715, 8, 2713}

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3ix}{2} + \frac{4\cosh^3(x)}{3} - 4\cosh(x) - \frac{\sinh^3(x)\cosh(x)}{\sinh(x)+i} - \frac{3}{2}i\sinh(x)\cosh(x)$$

```
[In] Int[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((3*I)/2)*x - 4*Cosh[x] + (4*Cosh[x]^3)/3 - ((3*I)/2)*Cosh[x]*Sinh[x] - (Co
sh[x]*Sinh[x]^3)/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} + \int \sinh^2(x)(-3i+4\sinh(x))\,dx \\
&= -\frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} - 3i\int \sinh^2(x)\,dx + 4\int \sinh^3(x)\,dx \\
&= -\frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)} + \frac{3}{2}i\int 1\,dx - 4\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh(x)\right) \\
&= \frac{3ix}{2} - 4\cosh(x) + \frac{4\cosh^3(x)}{3} - \frac{3}{2}i\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{i+\sinh(x)}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 134 vs. 2(46) = 92.

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.91

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx$$
$$= \frac{\cosh(x)\left(-16i\left(\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+\sqrt{\cosh^2(x)}\right)-\left(16\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)+7\sqrt{\cosh^2(x)}\right)\sinh(x)}{6\sqrt{\cosh^2(x)}(i+\sinh(x)}$$

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]*((-16*I)*(ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + Sqrt[Cosh[x]^2]) -
 (16*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]] + 7*Sqrt[Cosh[x]^2])*Sinh[x] - I*S
qrt[Cosh[x]^2]*Sinh[x]^2 + 2*Sqrt[Cosh[x]^2]*Sinh[x]^3 + I*ArcSinh[Sinh[x]]
*(I + Sinh[x])))/(6*Sqrt[Cosh[x]^2]*(I + Sinh[x]))
```

Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

method	result
risch	$\frac{3ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}-\frac{7\,e^{x}}{8}-\frac{7\,e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}-\frac{2}{e^{x}+i}$
default	$-\frac{2i}{\tanh(\frac{x}{2})+i}-\frac{3i\ln(\tanh(\frac{x}{2})-1)}{2}+\frac{\frac{3}{2}-\frac{i}{2}}{\tanh(\frac{x}{2})-1}+\frac{-\frac{1}{2}-\frac{i}{2}}{(\tanh(\frac{x}{2})-1)^2}-\frac{1}{3(\tanh(\frac{x}{2})-1)^3}+\frac{3i\ln(\tanh(\frac{x}{2})+1)}{2}+\frac{}{(\tan}$
parallelrisch	$\frac{(-36i\cosh(x)+36\sinh(x)+36i)\ln(1-\coth(x)+\operatorname{csch}(x))+(36i\cosh(x)-36i-36\sinh(x))\ln(\coth(x)-\operatorname{csch}(x)+1)-3i\sinh(3x)+}{24i\sinh(x)+24\cosh(x)-24}$

```
[In] int(sinh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I*x+1/24*exp(x)^3-1/8*I*exp(x)^2-7/8*exp(x)-7/8/exp(x)+1/8*I/exp(x)^2+1
/24/exp(x)^3-2/(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = -\frac{3\,(-12i\,x+7i)e^{(4\,x)}+3\,(12\,x+23)e^{(3\,x)}-e^{(7\,x)}+2i\,e^{(6\,x)}+18\,e^{(5\,x)}+18i\,e^{(2\,x)}+2\,e^{x}-i}{24\,\left(e^{(4\,x)}+i\,e^{(3\,x)}\right)}$$

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(-12*I*x + 7*I)*e^(4*x) + 3*(12*x + 23)*e^(3*x) - e^(7*x) + 2*I*e^
(6*x) + 18*e^(5*x) + 18*I*e^(2*x) + 2*e^x - I)/(e^(4*x) + I*e^(3*x))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}-\frac{7e^{x}}{8}-\frac{7e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}-\frac{2}{e^{x}+i}$$

```
[In] integrate(sinh(x)**4/(I+sinh(x)),x)
```

```
[Out] 3*I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 - 7*exp(x)/8 - 7*exp(-x)/8 + I*exp(-2*
x)/8 + exp(-3*x)/24 - 2/(exp(x) + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3}{2}i\,x-\frac{2\,e^{(-x)}-18i\,e^{(-2\,x)}+69\,e^{(-3\,x)}+i}{8\,\left(-3i\,e^{(-3\,x)}+3\,e^{(-4\,x)}\right)}-\frac{7}{8}\,e^{(-x)}+\frac{1}{8}i\,e^{(-2\,x)}+\frac{1}{24}\,e^{(-3\,x)}$$

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 3/2*I*x - 1/8*(2*e^(-x) - 18*I*e^(-2*x) + 69*e^(-3*x) + I)/(-3*I*e^(-3*x) +
 3*e^(-4*x)) - 7/8*e^(-x) + 1/8*I*e^(-2*x) + 1/24*e^(-3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{3}{2}i\,x - \frac{\left(69\,e^{(3\,x)} + 18i\,e^{(2\,x)} + 2\,e^{x} - i\right)e^{(-3\,x)}}{24\,(e^{x}+i)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} - \frac{7}{8}\,e^{x}$$

```
[In] integrate(sinh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 3/2*I*x - 1/24*(69*e^(3*x) + 18*I*e^(2*x) + 2*e^x - I)*e^(-3*x)/(e^x + I) +
 1/24*e^(3*x) - 1/8*I*e^(2*x) - 7/8*e^x
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^4(x)}{i+\sinh(x)}\,dx = \frac{x\,3\mathrm{i}}{2} - \frac{7\,\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8} + \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} - \frac{7\,\mathrm{e}^{x}}{8} - \frac{2}{\mathrm{e}^{x}+1\mathrm{i}}$$

```
[In] int(sinh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] (x*3i)/2 - (7*exp(-x))/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/2
4 + exp(3*x)/24 - (7*exp(x))/8 - 2/(exp(x) + 1i)
```

3.41 $\int \frac{\sinh^3(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx=-\frac{3x}{2}-2i\cosh(x)+\frac{3}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}$$

```
[Out] -3/2*x-2*I*cosh(x)+3/2*cosh(x)*sinh(x)-cosh(x)*sinh(x)^2/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2846, 2813}

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx=-\frac{3x}{2}-2i\cosh(x)-\frac{\sinh^2(x)\cosh(x)}{\sinh(x)+i}+\frac{3}{2}\sinh(x)\cosh(x)$$

```
[In] Int[Sinh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (-3*x)/2 - (2*I)*Cosh[x] + (3*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^2)/(I +
 Sinh[x])
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2846

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Dist[d/(a*b), Int[(c + d*S
in[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[e
 + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] || E
qQ[c, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)} + \int \sinh(x)(-2i+3\sinh(x))\,dx \\ &= -\frac{3x}{2} - 2i\cosh(x) + \frac{3}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^2(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\cosh(x)\left(-\frac{3\operatorname{arcsinh}(\sinh(x))}{\sqrt{\cosh^2(x)}} + \frac{4-i\sinh(x)+\sinh^2(x)}{i+\sinh(x)}\right)$$

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x]),x]
[Out] (Cosh[x]*((-3*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (4 - I*Sinh[x] + Sinh[x]^
2)/(I + Sinh[x])))/2
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{ie^{x}}{2}-\frac{ie^{-x}}{2}-\frac{e^{-2x}}{8}-\frac{2i}{e^{x}+i}$
default	$\frac{2}{\tanh(\frac{x}{2})+i}+\frac{\frac{1}{2}-i}{\tanh(\frac{x}{2})+1}-\frac{1}{2(\tanh(\frac{x}{2})+1)^{2}}-\frac{3\ln(\tanh(\frac{x}{2})+1)}{2}+\frac{\frac{1}{2}+i}{\tanh(\frac{x}{2})-1}+\frac{1}{2(\tanh(\frac{x}{2})-1)^{2}}+\frac{3\ln(\tanh(\frac{x}{2}}{2}$
parallelrisch	$\frac{(12i\sinh(x)+12\cosh(x)-12)\ln(1-\coth(x)+\operatorname{csch}(x))+(-12i\sinh(x)-12\cosh(x)+12)\ln(\coth(x)-\operatorname{csch}(x)+1)+19i\cosh(x)-4}{8i\sinh(x)+8\cosh(x)-8}$

```
[In] int(sinh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -3/2*x+1/8*exp(x)^2-1/2*I*exp(x)-1/2*I/exp(x)-1/8/exp(x)^2-2*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{4\,(3\,x-1)e^{(3\,x)}+4\,(3i\,x+5i)e^{(2\,x)}-e^{(5\,x)}+3i\,e^{(4\,x)}-3\,e^{x}+i}{8\,(e^{(3\,x)}+i\,e^{(2\,x)})}$$

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(3*x - 1)*e^(3*x) + 4*(3*I*x + 5*I)*e^(2*x) - e^(5*x) + 3*I*e^(4*x)
 - 3*e^x + I)/(e^(3*x) + I*e^(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{3x}{2}+\frac{e^{2x}}{8}-\frac{ie^{x}}{2}-\frac{ie^{-x}}{2}-\frac{e^{-2x}}{8}-\frac{2i}{e^{x}+i}$$

```
[In] integrate(sinh(x)**3/(I+sinh(x)),x)
```

```
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x)/2 - I*exp(-x)/2 - exp(-2*x)/8 - 2*I/(exp(x)
+ I)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{3}{2}\,x-\frac{3\,e^{(-x)}+20i\,e^{(-2\,x)}+i}{8\,(-i\,e^{(-2\,x)}+e^{(-3\,x)})}-\frac{1}{2}i\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x - 1/8*(3*e^(-x) + 20*I*e^(-2*x) + I)/(-I*e^(-2*x) + e^(-3*x)) - 1/2*
I*e^(-x) - 1/8*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = -\frac{3}{2}\,x - \frac{\left(20i\,e^{(2\,x)} - 3\,e^{x} + i\right)e^{(-2\,x)}}{8\,(e^{x}+i)} + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{2}i\,e^{x}$$

```
[In] integrate(sinh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -3/2*x - 1/8*(20*I*e^(2*x) - 3*e^x + I)*e^(-2*x)/(e^x + I) + 1/8*e^(2*x) -
1/2*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sinh^3(x)}{i+\sinh(x)}\,dx = \frac{e^{2\,x}}{8} - \frac{e^{-x}\,1i}{2} - \frac{e^{-2\,x}}{8} - \frac{3\,x}{2} - \frac{e^{x}\,1i}{2} - \frac{2i}{e^{x}+1i}$$

```
[In] int(sinh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] exp(2*x)/8 - (exp(-x)*1i)/2 - exp(-2*x)/8 - (3*x)/2 - (exp(x)*1i)/2 - 2i/(e
xp(x) + 1i)
```

3.42 $\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -ix+\cosh(x)+\frac{i\cosh(x)}{i+\sinh(x)}$$

```
[Out] -I*x+cosh(x)+I*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2825, 2814, 2727}

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -ix+\cosh(x)+\frac{i\cosh(x)}{\sinh(x)+i}$$

```
[In] Int[Sinh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I)*x + Cosh[x] + (I*Cosh[x])/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \cosh(x) - i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx \\ &= -ix + \cosh(x) - \int \frac{1}{i+\sinh(x)}\,dx \\ &= -ix + \cosh(x) + \frac{i\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 79 vs. 2(22) = 44.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.59

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = \frac{\cosh(x)\left(2i + \frac{2i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}} + \sinh(x) + \frac{2\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sinh(x)}{\sqrt{\cosh^2(x)}}\right)}{i+\sinh(x)}$$

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x]),x]
[Out] (Cosh[x]*(2*I + ((2*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2]
 + Sinh[x] + (2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sinh[x])/Sqrt[Cosh[x]^2
]))/(I + Sinh[x])
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result	size
risch	$-ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{2}{e^{x}+i}$	25
parallelrisch	$\frac{\cosh(2x)+i\sinh(2x)+(6i+2x)\sinh(x)-2i\cosh(x)x+2ix-1}{2i\sinh(x)+2\cosh(x)-2}$	47
default	$\frac{2i}{\tanh(\frac{x}{2})+i}+i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh(\frac{x}{2})-1}-i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{\tanh(\frac{x}{2})+1}$	52

```
[In] int(sinh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -I*x+1/2*exp(x)+1/2*exp(-x)+2/(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(16) = 32.

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = \frac{(-2i\,x+i)e^{(2\,x)}+(2\,x+5)e^{x}+e^{(3\,x)}+i}{2\left(e^{(2\,x)}+i\,e^{x}\right)}$$

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/2*((-2*I*x + I)*e^(2*x) + (2*x + 5)*e^x + e^(3*x) + I)/(e^(2*x) + I*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -ix+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+\frac{2}{e^{x}+i}$$

```
[In] integrate(sinh(x)**2/(I+sinh(x)),x)
[Out] -I*x + exp(x)/2 + exp(-x)/2 + 2/(exp(x) + I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -i\,x+\frac{5\,e^{(-x)}-i}{2\left(-i\,e^{(-x)}+e^{(-2\,x)}\right)}+\frac{1}{2}\,e^{(-x)}$$

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="maxima")
[Out] -I*x + 1/2*(5*e^(-x) - I)/(-I*e^(-x) + e^(-2*x)) + 1/2*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = -i\,x + \frac{(5\,e^{x}+i)e^{(-x)}}{2\,(e^{x}+i)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] -I*x + 1/2*(5*e^x + I)*e^(-x)/(e^x + I) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh^2(x)}{i+\sinh(x)}\,dx = \frac{\mathrm{e}^{-x}}{2} - x\,1\mathrm{i} + \frac{\mathrm{e}^{x}}{2} + \frac{2}{\mathrm{e}^{x}+1\mathrm{i}}$$

```
[In] int(sinh(x)^2/(sinh(x) + 1i),x)
[Out] exp(-x)/2 - x*1i + exp(x)/2 + 2/(exp(x) + 1i)
```

3.43 $\int \frac{\sinh(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sinh(x)}{i+\sinh(x)} \, dx = x - \frac{\cosh(x)}{i+\sinh(x)}$$

```
[Out] x-cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2814, 2727}

$$\int \frac{\sinh(x)}{i+\sinh(x)} \, dx = x - \frac{\cosh(x)}{\sinh(x)+i}$$

```
[In] Int[Sinh[x]/(I + Sinh[x]),x]
```

```
[Out] x - Cosh[x]/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = x - i\int \frac{1}{i+\sinh(x)}\,dx$$
$$= x - \frac{\cosh(x)}{i+\sinh(x)}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.07

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = i\text{sech}(x)\left(1+2\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{\cosh^2(x)}+i\sinh(x)\right)$$

```
[In] Integrate[Sinh[x]/(I + Sinh[x]),x]
```

```
[Out] I*Sech[x]*(1 + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2] + I*Si
nh[x])
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
risch	$x+\frac{2i}{{\rm e}^{x}+i}$	13
parallelrisch	$\frac{-2+ix+x\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)+i}$	23
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	29

```
[In] int(sinh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x+2*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = \frac{xe^{x}+i\,x+2i}{e^{x}+i}$$

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] (x*e^x + I*x + 2*I)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x+\frac{2i}{e^{x}+i}$$

```
[In] integrate(sinh(x)/(I+sinh(x)),x)
[Out] x + 2*I/(exp(x) + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x+\frac{2i}{e^{(-x)}-i}$$

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] x + 2*I/(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x+\frac{2i}{e^{x}+i}$$

```
[In] integrate(sinh(x)/(I+sinh(x)),x, algorithm="giac")
[Out] x + 2*I/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh(x)}{i+\sinh(x)}\,dx = x + \frac{2\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

```
[In] int(sinh(x)/(sinh(x) + 1i),x)
[Out] x + 2i/(exp(x) + 1i)
```

3.44 $\int \frac{\text{csch}(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\text{csch}(x)}{i+\sinh(x)}\,dx = i\text{arctanh}(\cosh(x)) + \frac{\cosh(x)}{i+\sinh(x)}$$

```
[Out] I*arctanh(cosh(x))+cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2826, 2727, 3855}

$$\int \frac{\text{csch}(x)}{i+\sinh(x)}\,dx = i\text{arctanh}(\cosh(x)) + \frac{\cosh(x)}{\sinh(x)+i}$$

```
[In] Int[Csch[x]/(I + Sinh[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] + Cosh[x]/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
```

```
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -(i\int \operatorname{csch}(x)\,dx) + i\int \frac{1}{i+\sinh(x)}\,dx\\ &= i\text{arctanh}(\cosh(x)) + \frac{\cosh(x)}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = \operatorname{sech}(x)\left(-i + i\text{arctanh}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)} + \sinh(x)\right)$$

```
[In] Integrate[Csch[x]/(I + Sinh[x]),x]
[Out] Sech[x]*(-I + I*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + Sinh[x])
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2}{\tanh\left(\frac{x}{2}\right)+i}$	21
risch	$-\frac{2i}{e^{x}+i}+i\ln\left(e^{x}+1\right)-i\ln\left(e^{x}-1\right)$	28
parallelrisch	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)\tanh\left(\frac{x}{2}\right)+2}{\tanh\left(\frac{x}{2}\right)+i}$	30

```
[In] int(csch(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -I*ln(tanh(1/2*x))+2/(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(15) = 30$.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = \frac{(i\,e^{x}-1)\log\left(e^{x}+1\right)+(-i\,e^{x}+1)\log\left(e^{x}-1\right)-2i}{e^{x}+i}$$

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] ((I*e^x - 1)*log(e^x + 1) + (-I*e^x + 1)*log(e^x - 1) - 2*I)/(e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}\left(x\right)}{\sinh\left(x\right)+i}\,dx$$

```
[In] integrate(csch(x)/(I+sinh(x)),x)
[Out] Integral(csch(x)/(sinh(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = -\frac{2i}{e^{(-x)}-i}+i\log\left(e^{(-x)}+1\right)-i\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] -2*I/(e^(-x) - I) + I*log(e^(-x) + 1) - I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = -\frac{2i}{e^{x}+i}+i\log\left(e^{x}+1\right)-i\log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x)/(I+sinh(x)),x, algorithm="giac")
[Out] -2*I/(e^x + I) + I*log(e^x + 1) - I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\operatorname{csch}(x)}{i+\sinh(x)}\,dx = -\ln\left(\mathrm{e}^x\,2\mathrm{i}-2\mathrm{i}\right)\,1\mathrm{i}+\ln\left(\mathrm{e}^x\,2\mathrm{i}+2\mathrm{i}\right)\,1\mathrm{i}-\frac{2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

```
[In] int(1/(sinh(x)*(sinh(x) + 1i)),x)
[Out] log(exp(x)*2i + 2i)*1i - log(exp(x)*2i - 2i)*1i - 2i/(exp(x) + 1i)
```

3.45 $\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = -\operatorname{arctanh}(\cosh(x)) + 2i\coth(x) + \frac{\coth(x)}{i+\sinh(x)}$$

```
[Out] -arctanh(cosh(x))+2*I*coth(x)+coth(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2847, 2827, 3852, 8, 3855}

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = -\operatorname{arctanh}(\cosh(x)) + 2i\coth(x) + \frac{\coth(x)}{\sinh(x)+i}$$

```
[In] Int[Csch[x]^2/(I + Sinh[x]),x]
[Out] -ArcTanh[Cosh[x]] + (2*I)*Coth[x] + Coth[x]/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth(x)}{i+\sinh(x)} + \int \operatorname{csch}^2(x)(-2i+\sinh(x))\,dx \\
&= \frac{\coth(x)}{i+\sinh(x)} - 2i\int \operatorname{csch}^2(x)\,dx + \int \operatorname{csch}(x)\,dx \\
&= -\operatorname{arctanh}(\cosh(x)) + \frac{\coth(x)}{i+\sinh(x)} - 2\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right) \\
&= -\operatorname{arctanh}(\cosh(x)) + 2i\coth(x) + \frac{\coth(x)}{i+\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \operatorname{sech}(x)\left(1-\operatorname{arctanh}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)} + i\operatorname{csch}(x) + 2i\sinh(x)\right)$$

```
[In] Integrate[Csch[x]^2/(I + Sinh[x]),x]
[Out] Sech[x]*(1 - ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] + I*Csch[x] + (2*I)*S
inh[x])
```

Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}$	35
risch	$\frac{-4+2ie^{x}+2\,e^{2x}}{\left(e^{2x}-1\right)\left(e^{x}+i\right)}+\ln\left(e^{x}-1\right)-\ln\left(e^{x}+1\right)$	42
parallelrisch	$\frac{\left(2\tanh\left(\frac{x}{2}\right)+2i\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+i\tanh\left(\frac{x}{2}\right)^{2}+6i-\coth\left(\frac{x}{2}\right)}{2\tanh\left(\frac{x}{2}\right)+2i}$	46

```
[In] int(csch(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*I*tanh(1/2*x)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))+2*I/(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(19) = 38.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.35

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = -\frac{\left(e^{(3x)}+i\,e^{(2x)}-e^{x}-i\right)\log\left(e^{x}+1\right)-\left(e^{(3x)}+i\,e^{(2x)}-e^{x}-i\right)\log\left(e^{x}-1\right)-2\,e^{(2x)}-2i\,e^{x}+4}{e^{(3x)}+i\,e^{(2x)}-e^{x}-i}$$

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] -((e^(3*x) + I*e^(2*x) - e^x - I)*log(e^x + 1) - (e^(3*x) + I*e^(2*x) - e^x
 - I)*log(e^x - 1) - 2*e^(2*x) - 2*I*e^x + 4)/(e^(3*x) + I*e^(2*x) - e^x -
I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{\sinh(x)+i}\,dx$$

```
[In] integrate(csch(x)**2/(I+sinh(x)),x)
[Out] Integral(csch(x)**2/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(19) = 38.

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = -\frac{2\left(-i\,e^{(-x)}+e^{(-2\,x)}-2\right)}{e^{(-x)}+i\,e^{(-2\,x)}-e^{(-3\,x)}-i} - \log\left(e^{(-x)}+1\right) + \log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*(-I*e^(-x) + e^(-2*x) - 2)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) - log(e^
(-x) + 1) + log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \frac{2\left(e^{(2\,x)}+i\,e^{x}-2\right)}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i} - \log\left(e^{x}+1\right) + \log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2*(e^(2*x) + I*e^x - 2)/(e^(3*x) + I*e^(2*x) - e^x - I) - log(e^x + 1) + lo
g(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{csch}^2(x)}{i+\sinh(x)}\,dx = \ln\left(2-2\,\mathrm{e}^{x}\right) - \ln\left(-2\,\mathrm{e}^{x}-2\right) + \frac{2\,\mathrm{e}^{2\,x}-4+\mathrm{e}^{x}\,2\mathrm{i}}{\mathrm{e}^{2\,x}\,1\mathrm{i}+\mathrm{e}^{3\,x}-\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(1/(sinh(x)^2*(sinh(x) + 1i)),x)
```

```
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + (2*exp(2*x) + exp(x)*2i - 4)/(exp
(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

3.46 $\int \frac{\text{csch}^3(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\text{csch}^3(x)}{i+\sinh(x)}\,dx=-\frac{3}{2}i\text{arctanh}(\cosh(x))-2\coth(x)+\frac{3}{2}i\coth(x)\text{csch}(x)+\frac{\coth(x)\text{csch}(x)}{i+\sinh(x)}$$

```
[Out] -3/2*I*arctanh(cosh(x))-2*coth(x)+3/2*I*coth(x)*csch(x)+coth(x)*csch(x)/(I+
sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2847, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\text{csch}^3(x)}{i+\sinh(x)}\,dx=-\frac{3}{2}i\text{arctanh}(\cosh(x))-2\coth(x)+\frac{3}{2}i\coth(x)\text{csch}(x)+\frac{\coth(x)\text{csch}(x)}{\sinh(x)+i}$$

```
[In] Int[Csch[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/2)*ArcTanh[Cosh[x]] - 2*Coth[x] + ((3*I)/2)*Coth[x]*Csch[x] + (Coth
[x]*Csch[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth(x)\text{csch}(x)}{i+\sinh(x)} + \int \text{csch}^3(x)(-3i+2\sinh(x))\,dx \\
&= \frac{\coth(x)\text{csch}(x)}{i+\sinh(x)} - 3i\int \text{csch}^3(x)\,dx + 2\int \text{csch}^2(x)\,dx \\
&= \frac{3}{2}i\coth(x)\text{csch}(x) + \frac{\coth(x)\text{csch}(x)}{i+\sinh(x)} + \frac{3}{2}i\int \text{csch}(x)\,dx - 2i\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right) \\
&= -\frac{3}{2}i\text{arctanh}(\cosh(x)) - 2\coth(x) + \frac{3}{2}i\coth(x)\text{csch}(x) + \frac{\coth(x)\text{csch}(x)}{i+\sinh(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = \frac{1}{2}i\left(4i+3\operatorname{csch}(x)-3\operatorname{arctanh}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)}\operatorname{csch}(x)+2i\operatorname{csch}^2(x)+\operatorname{csch}^3(x)\right)\tanh(x)$$

```
[In] Integrate[Csch[x]^3/(I + Sinh[x]),x]
[Out] (I/2)*(4*I + 3*Csch[x] - 3*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2]*Csch[x]
 + (2*I)*Csch[x]^2 + Csch[x]^3)*Tanh[x]
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{i\tanh\left(\frac{x}{2}\right)^2}{8}-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{8\tanh\left(\frac{x}{2}\right)^2}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}$	53
risch	$\frac{i\left(3\,\mathrm{e}^{4x}-5\,\mathrm{e}^{2x}+3i\mathrm{e}^{3x}+4-i\mathrm{e}^{x}\right)}{\left(\mathrm{e}^{2x}-1\right)^2\left(\mathrm{e}^{x}+i\right)}+\frac{3i\ln\left(\mathrm{e}^{x}-1\right)}{2}-\frac{3i\ln\left(\mathrm{e}^{x}+1\right)}{2}$	62
parallelrisch	$\frac{\left(12i\tanh\left(\frac{x}{2}\right)-12\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)-i\tanh\left(\frac{x}{2}\right)^3-3i\coth\left(\frac{x}{2}\right)-\coth\left(\frac{x}{2}\right)^2-3\tanh\left(\frac{x}{2}\right)^2-24}{8\tanh\left(\frac{x}{2}\right)+8i}$	62

```
[In] int(csch(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*tanh(1/2*x)-1/8*I*tanh(1/2*x)^2-2/(tanh(1/2*x)+I)+1/8*I/tanh(1/2*x)^2+
3/2*I*ln(tanh(1/2*x))-1/2/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(27) = 54.

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.41

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = -\frac{3\left(i\,e^{(5\,x)}-e^{(4\,x)}-2i\,e^{(3\,x)}+2\,e^{(2\,x)}+i\,e^{x}-1\right)\log\left(e^{x}+1\right)+3\left(-i\,e^{(5\,x)}+e^{(4\,x)}+2i\,e^{(3\,x)}-2\,e^{(2\,x)}-i\,e^{x}\cdots\right.}{2\left(e^{(5\,x)}+i\,e^{(4\,x)}-2\,e^{(3\,x)}-2i\,e^{(2\,x)}+e^{x}+i\right)}$$

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(3*(I*e^(5*x) - e^(4*x) - 2*I*e^(3*x) + 2*e^(2*x) + I*e^x - 1)*log(e^x
 + 1) + 3*(-I*e^(5*x) + e^(4*x) + 2*I*e^(3*x) - 2*e^(2*x) - I*e^x + 1)*log(
e^x - 1) - 6*I*e^(4*x) + 6*e^(3*x) + 10*I*e^(2*x) - 2*e^x - 8*I)/(e^(5*x) +
 I*e^(4*x) - 2*e^(3*x) - 2*I*e^(2*x) + e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{\sinh(x)+i}\,dx$$

```
[In] integrate(csch(x)**3/(I+sinh(x)),x)
```

```
[Out] Integral(csch(x)**3/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = -\frac{e^{(-x)} + 5i\,e^{(-2x)} - 3\,e^{(-3x)} - 3i\,e^{(-4x)} - 4i}{e^{(-x)} + 2i\,e^{(-2x)} - 2\,e^{(-3x)} - i\,e^{(-4x)} + e^{(-5x)} - i} - \frac{3}{2}i\log\left(e^{(-x)}+1\right) + \frac{3}{2}i\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) - 4*I)/(e^(-x) + 2*I*e^
(-2*x) - 2*e^(-3*x) - I*e^(-4*x) + e^(-5*x) - I) - 3/2*I*log(e^(-x) + 1) +
3/2*I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = \frac{i\,e^{(3x)} - 2\,e^{(2x)} + i\,e^{x} + 2}{\left(e^{(2x)}-1\right)^2} + \frac{2i}{e^x+i} - \frac{3}{2}i\log\left(e^x+1\right) + \frac{3}{2}i\log\left(\left|e^x-1\right|\right)$$

```
[In] integrate(csch(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] (I*e^(3*x) - 2*e^(2*x) + I*e^x + 2)/(e^(2*x) - 1)^2 + 2*I/(e^x + I) - 3/2*I
*log(e^x + 1) + 3/2*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{csch}^3(x)}{i+\sinh(x)}\,dx = -\frac{\ln\left(-\mathrm{e}^x\,3\mathrm{i}-3\mathrm{i}\right)\,3\mathrm{i}}{2}+\frac{\ln\left(-\mathrm{e}^x\,3\mathrm{i}+3\mathrm{i}\right)\,3\mathrm{i}}{2}+\frac{2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}+\frac{\mathrm{e}^x\,2\mathrm{i}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1}+\frac{-2+\mathrm{e}^x\,1\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

```
[In] int(1/(sinh(x)^3*(sinh(x) + 1i)),x)
```

```
[Out] (log(3i - exp(x)*3i)*3i)/2 - (log(- exp(x)*3i - 3i)*3i)/2 + 2i/(exp(x) + 1i
) + (exp(x)*2i)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(x)*1i - 2)/(exp(2*x) - 1
)
```

3.47 $\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{3}{2}\operatorname{arctanh}(\cosh(x)) - 4i\coth(x) + \frac{4}{3}i\coth^3(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}$$

```
[Out] 3/2*arctanh(cosh(x))-4*I*coth(x)+4/3*I*coth(x)^3-3/2*coth(x)*csch(x)+coth(x
)*csch(x)^2/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2847, 2827, 3852, 3853, 3855}

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{3}{2}\operatorname{arctanh}(\cosh(x)) + \frac{4}{3}i\coth^3(x) - 4i\coth(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{\sinh(x)+i}$$

```
[In] Int[Csch[x]^4/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTanh[Cosh[x]])/2 - (4*I)*Coth[x] + ((4*I)/3)*Coth[x]^3 - (3*Coth[x]*C
sch[x])/2 + (Coth[x]*Csch[x]^2)/(I + Sinh[x])
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2847

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Dist[d/(a*(b*c - a*d))
, Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c
^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)} + \int \operatorname{csch}^4(x)(-4i+3\sinh(x))\,dx \\
&= \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)} - 4i\int \operatorname{csch}^4(x)\,dx + 3\int \operatorname{csch}^3(x)\,dx \\
&= -\frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)} - \frac{3}{2}\int \operatorname{csch}(x)\,dx \\
&\quad + 4\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\coth(x)\right) \\
&= \frac{3}{2}\operatorname{arctanh}(\cosh(x)) - 4i\coth(x) + \frac{4}{3}i\coth^3(x) - \frac{3}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{i+\sinh(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{1}{6}\operatorname{sech}(x)\left(-9+9\operatorname{arctanh}\left(\sqrt{\cosh^2(x)}\right)\sqrt{\cosh^2(x)}-8i\operatorname{csch}(x) - 3\operatorname{csch}^2(x)+2i\operatorname{csch}^3(x)-16i\sinh(x)\right)$$

```
[In] Integrate[Csch[x]^4/(I + Sinh[x]),x]
```

```
[Out] (Sech[x]*(-9 + 9*ArcTanh[Sqrt[Cosh[x]^2]]*Sqrt[Cosh[x]^2] - (8*I)*Csch[x] -
 3*Csch[x]^2 + (2*I)*Csch[x]^3 - (16*I)*Sinh[x]))/6
```

Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

method	result
default	$-\frac{7i\tanh\left(\frac{x}{2}\right)}{8}+\frac{i\tanh\left(\frac{x}{2}\right)^3}{24}+\frac{\tanh\left(\frac{x}{2}\right)^2}{8}-\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{24\tanh\left(\frac{x}{2}\right)^3}-\frac{7i}{8\tanh\left(\frac{x}{2}\right)}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^2}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2}$
risch	$-\frac{9i e^{5x}-24\,e^{4x}+9\,e^{6x}-24i e^{3x}+39\,e^{2x}+7i e^{x}-16}{3(e^{2x}-1)^3(e^{x}+i)}-\frac{3\ln(e^{x}-1)}{2}+\frac{3\ln(e^{x}+1)}{2}$
parallelrisch	$\frac{\left(-36i-36\tanh\left(\frac{x}{2}\right)\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+i\tanh\left(\frac{x}{2}\right)^4-2i\coth\left(\frac{x}{2}\right)^2-18i\tanh\left(\frac{x}{2}\right)^2-\coth\left(\frac{x}{2}\right)^3+2\tanh\left(\frac{x}{2}\right)^3-90i+18\coth\left(\frac{x}{2}\right)}{24\tanh\left(\frac{x}{2}\right)+24i}$

```
[In] int(csch(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -7/8*I*tanh(1/2*x)+1/24*I*tanh(1/2*x)^3+1/8*tanh(1/2*x)^2-2*I/(tanh(1/2*x)+
I)+1/24*I/tanh(1/2*x)^3-7/8*I/tanh(1/2*x)-1/8/tanh(1/2*x)^2-3/2*ln(tanh(1/2
*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(35) = 70.

Time = 0.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.70

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{9\left(e^{(7x)}+i\,e^{(6x)}-3\,e^{(5x)}-3i\,e^{(4x)}+3\,e^{(3x)}+3i\,e^{(2x)}-e^{x}-i\right)\log\left(e^{x}+1\right)-9\left(e^{(7x)}+i\,e^{(6x)}-3\,e^{(5x)}\right.}{6\left(e^{(7x)}+i\,e^{(6x)}-3\,e^{(5x)}\right.}$$

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(9*(e^(7*x) + I*e^(6*x) - 3*e^(5*x) - 3*I*e^(4*x) + 3*e^(3*x) + 3*I*e^(
2*x) - e^x - I)*log(e^x + 1) - 9*(e^(7*x) + I*e^(6*x) - 3*e^(5*x) - 3*I*e^(
4*x) + 3*e^(3*x) + 3*I*e^(2*x) - e^x - I)*log(e^x - 1) - 18*e^(6*x) - 18*I*
e^(5*x) + 48*e^(4*x) + 48*I*e^(3*x) - 78*e^(2*x) - 14*I*e^x + 32)/(e^(7*x)
+ I*e^(6*x) - 3*e^(5*x) - 3*I*e^(4*x) + 3*e^(3*x) + 3*I*e^(2*x) - e^x - I)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(x)}{i + \sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(csch(x)**4/(I+sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(35) = 70.

Time = 0.18 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.19

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{i + \sinh(x)}\,dx &= \frac{-7i\,e^{(-x)} + 39\,e^{(-2\,x)} + 24i\,e^{(-3\,x)} - 24\,e^{(-4\,x)} - 9i\,e^{(-5\,x)} + 9\,e^{(-6\,x)} - 16}{3\left(e^{(-x)} + 3i\,e^{(-2\,x)} - 3\,e^{(-3\,x)} - 3i\,e^{(-4\,x)} + 3\,e^{(-5\,x)} + i\,e^{(-6\,x)} - e^{(-7\,x)} - i\right)} \\ &\quad + \frac{3}{2}\,\log\left(e^{(-x)} + 1\right) - \frac{3}{2}\,\log\left(e^{(-x)} - 1\right)\end{aligned}$$

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(-7*I*e^(-x) + 39*e^(-2*x) + 24*I*e^(-3*x) - 24*e^(-4*x) - 9*I*e^(-5*x)
 + 9*e^(-6*x) - 16)/(e^(-x) + 3*I*e^(-2*x) - 3*e^(-3*x) - 3*I*e^(-4*x) + 3*
e^(-5*x) + I*e^(-6*x) - e^(-7*x) - I) + 3/2*log(e^(-x) + 1) - 3/2*log(e^(-x
) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = -\frac{2}{e^x+i} - \frac{3\,e^{(5\,x)}+6i\,e^{(4\,x)}-24i\,e^{(2\,x)}-3\,e^x+10i}{3\,\left(e^{(2\,x)}-1\right)^3} + \frac{3}{2}\,\log\left(e^x+1\right) - \frac{3}{2}\,\log\left(\left|e^x-1\right|\right)$$

```
[In] integrate(csch(x)^4/(I+sinh(x)),x, algorithm="giac")
[Out] -2/(e^x + I) - 1/3*(3*e^(5*x) + 6*I*e^(4*x) - 24*I*e^(2*x) - 3*e^x + 10*I)/
(e^(2*x) - 1)^3 + 3/2*log(e^x + 1) - 3/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^4(x)}{i+\sinh(x)}\,dx = \frac{3\,\ln\left(3\,\mathrm{e}^x+3\right)}{2} - \frac{3\,\ln\left(3\,\mathrm{e}^x-3\right)}{2} - \frac{\mathrm{e}^x}{\mathrm{e}^{2\,x}-1} - \frac{2\,\mathrm{e}^x}{\left(\mathrm{e}^{2\,x}-1\right)^2} - \frac{2}{\mathrm{e}^x+1\mathrm{i}} - \frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1} + \frac{4\mathrm{i}}{\left(\mathrm{e}^{2\,x}-1\right)^2} + \frac{8\mathrm{i}}{3\,\left(\mathrm{e}^{2\,x}-1\right)^3}$$

```
[In] int(1/(sinh(x)^4*(sinh(x) + 1i)),x)
[Out] (3*log(3*exp(x) + 3))/2 - (3*log(3*exp(x) - 3))/2 - exp(x)/(exp(2*x) - 1) -
 (2*exp(x))/(exp(2*x) - 1)^2 - 2/(exp(x) + 1i) - 2i/(exp(2*x) - 1) + 4i/(ex
p(2*x) - 1)^2 + 8i/(3*(exp(2*x) - 1)^3)
```

3.48 $\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} dx = -\frac{7x}{2} - \frac{16}{3}i\cosh(x) + \frac{7}{2}\cosh(x)\sinh(x) - \frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2} - \frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}$$

```
[Out] -7/2*x-16/3*I*cosh(x)+7/2*cosh(x)*sinh(x)-1/3*cosh(x)*sinh(x)^3/(I+sinh(x))
^2-8/3*cosh(x)*sinh(x)^2/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2844, 3056, 2813}

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2} dx = -\frac{7x}{2} - \frac{16}{3}i\cosh(x) - \frac{\sinh^3(x)\cosh(x)}{3(\sinh(x)+i)^2} - \frac{8\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)} + \frac{7}{2}\sinh(x)\cosh(x)$$

```
[In] Int[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-7*x)/2 - ((16*I)/3)*Cosh[x] + (7*Cosh[x]*Sinh[x])/2 - (Cosh[x]*Sinh[x]^3)
/(3*(I + Sinh[x])^2) - (8*Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x]))
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
```

```
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int\frac{\sinh^2(x)(-3i+5\sinh(x))}{i+\sinh(x)}\,dx\\
&= -\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}-\frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}-\frac{1}{3}i\int(16+21i\sinh(x))\sinh(x)\,dx\\
&= -\frac{7x}{2}-\frac{16}{3}i\cosh(x)+\frac{7}{2}\cosh(x)\sinh(x)-\frac{\cosh(x)\sinh^3(x)}{3(i+\sinh(x))^2}-\frac{8\cosh(x)\sinh^2(x)}{3(i+\sinh(x))}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 147 vs. 2(58) = 116.

Time = 0.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.53

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{5i\cosh(x)}{6(1-i\sinh(x))^2} - \frac{31i\cosh(x)}{6(1-i\sinh(x))} - \frac{i\sqrt{2}\cosh(x)\sqrt{1+\frac{1}{2}(-1+i\sinh(x))}}{\sqrt{1+i\sinh(x)}} - \frac{7i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\cosh(x)}{\sqrt{1-i\sinh(x)}\sqrt{1+i\sinh(x)}} - \frac{\cosh(x)\sinh^3(x)}{2(1-i\sinh(x))^2}$$

```
[In] Integrate[Sinh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (((5*I)/6)*Cosh[x])/(1 - I*Sinh[x])^2 - (((31*I)/6)*Cosh[x])/(1 - I*Sinh[x]
) - (I*Sqrt[2]*Cosh[x]*Sqrt[1 + (-1 + I*Sinh[x])/2])/Sqrt[1 + I*Sinh[x]] -
((7*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Cosh[x])/(Sqrt[1 - I*Sinh[x]]*Sq
rt[1 + I*Sinh[x]]) - (Cosh[x]*Sinh[x]^3)/(2*(1 - I*Sinh[x])^2)
```

Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{7x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}-\frac{2i(21ie^{x}+12e^{2x}-11)}{3(e^{x}+i)^{3}}$
default	$\frac{\frac{1}{2}+2i}{\tanh(\frac{x}{2})-1}+\frac{1}{2(\tanh(\frac{x}{2})-1)^{2}}+\frac{7\ln(\tanh(\frac{x}{2})-1)}{2}+\frac{2i}{(\tanh(\frac{x}{2})+i)^{2}}+\frac{4}{3(\tanh(\frac{x}{2})+i)^{3}}+\frac{6}{\tanh(\frac{x}{2})+i}+\frac{\frac{1}{2}-2i}{\tanh(\frac{x}{2})+1}$
parallelrisch	$\frac{-993i+420(\cosh(2x)+4i\sinh(x)-3-i\sinh(2x)+2\cosh(x))\ln(1-\coth(x)+\operatorname{csch}(x))+420(-\cosh(2x)-4i\sinh(x)+3+i\sinh(2x)-}{120\cosh(2x)+480i s}$

```
[In] int(sinh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -7/2*x+1/8*exp(x)^2-I*exp(x)-I/exp(x)-1/8/exp(x)^2-2/3*I*(21*I*exp(x)+12*ex
p(x)^2-11)/(exp(x)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(42) = 84$.

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{21\,(4\,x-3)e^{(5\,x)}+21\,(12i\,x+7i)e^{(4\,x)}-3\,(84\,x+127)e^{(3\,x)}-(84i\,x+239i)e^{(2\,x)}-3\,e^{(7\,x)}+15i\,e^{(6\,x)}+15\,e^{x}-3i}{24\,(e^{(5\,x)}+3i\,e^{(4\,x)}-3\,e^{(3\,x)}-i\,e^{(2\,x)})}$$

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/24*(21*(4*x - 3)*e^(5*x) + 21*(12*I*x + 7*I)*e^(4*x) - 3*(84*x + 127)*e^
(3*x) - (84*I*x + 239*I)*e^(2*x) - 3*e^(7*x) + 15*I*e^(6*x) + 15*e^x - 3*I)
/(e^(5*x) + 3*I*e^(4*x) - 3*e^(3*x) - I*e^(2*x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{7x}{2}+\frac{-24ie^{2x}+42e^{x}+22i}{3e^{3x}+9ie^{2x}-9e^{x}-3i}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$$

```
[In] integrate(sinh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] -7*x/2 + (-24*I*exp(2*x) + 42*exp(x) + 22*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9
*exp(x) - 3*I) + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{7}{2}\,x+\frac{15\,e^{(-x)}+239i\,e^{(-2\,x)}-405\,e^{(-3\,x)}-216i\,e^{(-4\,x)}+3i}{8\,(3i\,e^{(-2\,x)}-9\,e^{(-3\,x)}-9i\,e^{(-4\,x)}+3\,e^{(-5\,x)})}-i\,e^{(-x)}-\frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -7/2*x + 1/8*(15*e^(-x) + 239*I*e^(-2*x) - 405*e^(-3*x) - 216*I*e^(-4*x) +
3*I)/(3*I*e^(-2*x) - 9*e^(-3*x) - 9*I*e^(-4*x) + 3*e^(-5*x)) - I*e^(-x) - 1
/8*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{7}{2}\,x - \frac{\left(216i\,e^{(4\,x)} - 405\,e^{(3\,x)} - 239i\,e^{(2\,x)} + 15\,e^{x} - 3i\right)e^{(-2\,x)}}{24\,(e^{x}+i)^{3}} + \frac{1}{8}\,e^{(2\,x)} - i\,e^{x}$$

```
[In] integrate(sinh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -7/2*x - 1/24*(216*I*e^(4*x) - 405*e^(3*x) - 239*I*e^(2*x) + 15*e^x - 3*I)*
e^(-2*x)/(e^x + I)^3 + 1/8*e^(2*x) - I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67

$$\int \frac{\sinh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{2\,x}}{8} - e^{-x}\,1i - \frac{e^{-2\,x}}{8} - \frac{7\,x}{2} - e^{x}\,1i - \frac{-2+\frac{e^{x}\,8i}{3}}{e^{2\,x}-1+e^{x}\,2i} + \frac{4\,e^{x} - \frac{e^{2\,x}\,8i}{3} + \frac{8}{3}i}{e^{2\,x}\,3i + e^{3\,x} - 3\,e^{x} - i} - \frac{8i}{3\,\left(e^{x}+1i\right)}$$

```
[In] int(sinh(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (7*x)/2 - exp(x)*1i - ((exp(x)*8i)/
3 - 2)/(exp(2*x) + exp(x)*2i - 1) + (4*exp(x) - (exp(2*x)*8i)/3 + 8i/3)/(ex
p(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 8i/(3*(exp(x) + 1i))
```

3.49 $\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} dx = -2ix + \frac{4\cosh(x)}{3} - \frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2} + \frac{2i\cosh(x)}{i+\sinh(x)}$$

```
[Out] -2*I*x+4/3*cosh(x)-1/3*cosh(x)*sinh(x)^2/(I+sinh(x))^2+2*I*cosh(x)/(I+sinh(
x))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2844, 3047, 3102, 12, 2814, 2727}

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2} dx = -2ix + \frac{4\cosh(x)}{3} - \frac{\sinh^2(x)\cosh(x)}{3(\sinh(x)+i)^2} + \frac{2i\cosh(x)}{\sinh(x)+i}$$

```
[In] Int[Sinh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*x + (4*Cosh[x])/3 - (Cosh[x]*Sinh[x]^2)/(3*(I + Sinh[x])^2) + ((2*I)
*Cosh[x])/(I + Sinh[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2844

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[1/(a*b*
(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*S
imp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
+ b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &
& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int\frac{\sinh(x)(-2i+4\sinh(x))}{i+\sinh(x)}\,dx\\
&= -\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}-\frac{1}{3}i\int\frac{2\sinh(x)+4i\sinh^2(x)}{i+\sinh(x)}\,dx\\
&= \frac{4\cosh(x)}{3}-\frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int-\frac{6i\sinh(x)}{i+\sinh(x)}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{4\cosh(x)}{3} - \frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2} - 2i\int \frac{\sinh(x)}{i+\sinh(x)}\,dx \\
&= -2ix + \frac{4\cosh(x)}{3} - \frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2} - 2\int \frac{1}{i+\sinh(x)}\,dx \\
&= -2ix + \frac{4\cosh(x)}{3} - \frac{\cosh(x)\sinh^2(x)}{3(i+\sinh(x))^2} + \frac{2i\cosh(x)}{i+\sinh(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$
\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{3}\cosh(x)\left(-\frac{6i\text{arcsinh}(\sinh(x))}{\sqrt{\cosh^2(x)}} + \frac{-10+14i\sinh(x)+3\sinh^2(x)}{(i+\sinh(x))^2}\right)
$$

```
[In] Integrate[Sinh[x]^3/(I + Sinh[x])^2,x]
[Out] (Cosh[x]*(((-6*I)*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2] + (-10 + (14*I)*Sinh[x]
 + 3*Sinh[x]^2)/(I + Sinh[x])^2))/3
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
risch	$-2ix + \frac{e^{x}}{2} + \frac{e^{-x}}{2} + \frac{10ie^{x}+6\,e^{2x}-\frac{16}{3}}{\left(e^{x}+i\right)^{3}}$
default	$\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}} + \frac{4i}{\tanh\left(\frac{x}{2}\right)+i} - \frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}} + 2i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{1}{\tanh\left(\frac{x}{2}\right)-1} - 2i\ln\left(\tanh\left(\frac{x}{2}\right)\right.$
parallelrisch	$\frac{24+12(-3i+i\cosh(2x)-4\sinh(x)+2i\cosh(x)+\sinh(2x))\ln(1-\coth(x)+\text{csch}(x))+12(3i-2i\cosh(x)-i\cosh(2x)+4\sinh(x)-\text{si}}{6\cosh(2x)+24i\sinh(x)-18-6i\sin}$

```
[In] int(sinh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*I*x+1/2*exp(x)+1/2*exp(-x)+2/3*(15*I*exp(x)+9*exp(2*x)-8)/(exp(x)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(32) = 64.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.68

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{3\,(4i\,x-3i)e^{(4\,x)}-6\,(6\,x+5)e^{(3\,x)}+6\,(-6i\,x-11i)e^{(2\,x)}+(12\,x+41)e^{x}-3\,e^{(5\,x)}+3i}{6\,(e^{(4\,x)}+3i\,e^{(3\,x)}-3\,e^{(2\,x)}-i\,e^{x})}$$

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*(4*I*x - 3*I)*e^(4*x) - 6*(6*x + 5)*e^(3*x) + 6*(-6*I*x - 11*I)*e^(
2*x) + (12*x + 41)*e^x - 3*e^(5*x) + 3*I)/(e^(4*x) + 3*I*e^(3*x) - 3*e^(2*x
) - I*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = -2ix + \frac{18e^{2x}+30ie^{x}-16}{3e^{3x}+9ie^{2x}-9e^{x}-3i} + \frac{e^{x}}{2} + \frac{e^{-x}}{2}$$

```
[In] integrate(sinh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] -2*I*x + (18*exp(2*x) + 30*I*exp(x) - 16)/(3*exp(3*x) + 9*I*exp(2*x) - 9*ex
p(x) - 3*I) + exp(x)/2 + exp(-x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\,x - \frac{41\,e^{(-x)}+69i\,e^{(-2\,x)}-39\,e^{(-3\,x)}-3i}{2\,(3i\,e^{(-x)}-9\,e^{(-2\,x)}-9i\,e^{(-3\,x)}+3\,e^{(-4\,x)})} + \frac{1}{2}\,e^{(-x)}$$

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*I*x - 1/2*(41*e^(-x) + 69*I*e^(-2*x) - 39*e^(-3*x) - 3*I)/(3*I*e^(-x) -
9*e^(-2*x) - 9*I*e^(-3*x) + 3*e^(-4*x)) + 1/2*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\,x + \frac{\left(39\,e^{(3\,x)} + 69i\,e^{(2\,x)} - 41\,e^{x} - 3i\right)e^{(-x)}}{6\,(e^{x}+i)^3} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(sinh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*I*x + 1/6*(39*e^(3*x) + 69*I*e^(2*x) - 41*e^x - 3*I)*e^(-x)/(e^x + I)^3
+ 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{\sinh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{\mathrm{e}^{-x}}{2} - x\,2\mathrm{i} + \frac{\mathrm{e}^{x}}{2} + \frac{2\,\mathrm{e}^{x} + \frac{4}{3}\mathrm{i}}{\mathrm{e}^{2\,x} - 1 + \mathrm{e}^{x}\,2\mathrm{i}} + \frac{2\,\mathrm{e}^{2\,x} - 2 + \frac{\mathrm{e}^{x}\,8\mathrm{i}}{3}}{\mathrm{e}^{2\,x}\,3\mathrm{i} + \mathrm{e}^{3\,x} - 3\,\mathrm{e}^{x} - \mathrm{i}} + \frac{2}{\mathrm{e}^{x} + 1\mathrm{i}}$$

```
[In] int(sinh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(-x)/2 - x*2i + exp(x)/2 + (2*exp(x) + 4i/3)/(exp(2*x) + exp(x)*2i - 1)
+ (2*exp(2*x) + (exp(x)*8i)/3 - 2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)
 + 2/(exp(x) + 1i)
```

3.50 $\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x+\frac{i\cosh(x)}{3(i+\sinh(x))^2}-\frac{5\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] x+1/3*I*cosh(x)/(I+sinh(x))^2-5/3*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2837, 2814, 2727}

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x-\frac{5\cosh(x)}{3(\sinh(x)+i)}+\frac{i\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[In] Int[Sinh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] x + ((I/3)*Cosh[x])/(I + Sinh[x])^2 - (5*Cosh[x])/(3*(I + Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2837

```
Int[sin[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_),
x_Symbol] :> Simp[b*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(a*m - b*(2*m
 + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
&& LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\cosh(x)}{3(i+\sinh(x))^2}+\frac{1}{3}\int\frac{-2i+3\sinh(x)}{i+\sinh(x)}\,dx\\ &= x+\frac{i\cosh(x)}{3(i+\sinh(x))^2}-\frac{5}{3}i\int\frac{1}{i+\sinh(x)}\,dx\\ &= x+\frac{i\cosh(x)}{3(i+\sinh(x))^2}-\frac{5\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int\frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx=-\frac{1}{3}i\cosh(x)\left(-\frac{6\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}}+\frac{4-5i\sinh(x)}{(i+\sinh(x))^2}\right)$$

```
[In] Integrate[Sinh[x]^2/(I + Sinh[x])^2,x]
[Out] (-1/3*I)*Cosh[x]*((-6*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2]
+ (4 - (5*I)*Sinh[x])/(I + Sinh[x])^2)
```

Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result
risch	$x+\frac{2i\left(9i\,\mathrm{e}^{x}+6\,\mathrm{e}^{2x}-5\right)}{3\left(\mathrm{e}^{x}+i\right)^{3}}$
default	$-\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{2}{\tanh\left(\frac{x}{2}\right)+i}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$
parallelrisch	$\frac{\left(-3\tanh\left(\frac{x}{2}\right)^{3}-9i\tanh\left(\frac{x}{2}\right)^{2}+9\tanh\left(\frac{x}{2}\right)+3i\right)\ln\left(1-\tanh\left(\frac{x}{2}\right)\right)+\left(3\tanh\left(\frac{x}{2}\right)^{3}+9i\tanh\left(\frac{x}{2}\right)^{2}-9\tanh\left(\frac{x}{2}\right)-3i\right)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-}{3\tanh\left(\frac{x}{2}\right)^{3}+9i\tanh\left(\frac{x}{2}\right)^{2}-9\tanh\left(\frac{x}{2}\right)-3i}$

```
[In] int(sinh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] x+2/3*I*(9*I*exp(x)+6*exp(2*x)-5)/(exp(x)+I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(22) = 44.

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{3\,xe^{(3\,x)} - 3\,(-3i\,x - 4i)e^{(2\,x)} - 9\,(x+2)e^{x} - 3i\,x - 10i}{3\,(e^{(3\,x)} + 3i\,e^{(2\,x)} - 3\,e^{x} - i)}$$

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
[Out] 1/3*(3*x*e^(3*x) - 3*(-3*I*x - 4*I)*e^(2*x) - 9*(x + 2)*e^x - 3*I*x - 10*I)
/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{12ie^{2x} - 18e^{x} - 10i}{3e^{3x} + 9ie^{2x} - 9e^{x} - 3i}$$

```
[In] integrate(sinh(x)**2/(I+sinh(x))**2,x)
[Out] x + (12*I*exp(2*x) - 18*exp(x) - 10*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x
) - 3*I)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x - \frac{2\left(9\,e^{(-x)} + 6i\,e^{(-2\,x)} - 5i\right)}{3\,(3\,e^{(-x)} + 3i\,e^{(-2\,x)} - e^{(-3\,x)} - i)}$$

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
[Out] x - 2/3*(9*e^(-x) + 6*I*e^(-2*x) - 5*I)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x)
 - I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x - \frac{2\left(-6i\,e^{(2\,x)} + 9\,e^x + 5i\right)}{3\,(e^x+i)^3}$$

```
[In] integrate(sinh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
[Out] x - 2/3*(-6*I*e^(2*x) + 9*e^x + 5*I)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \frac{\sinh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{-\frac{2}{3} + \frac{e^x\,4i}{3}}{e^{2\,x} - 1 + e^x\,2i} - \frac{\frac{4\,e^x}{3} - \frac{e^{2\,x}\,4i}{3} + \frac{4}{3}i}{e^{2\,x}\,3i + e^{3\,x} - 3\,e^x - i} + \frac{4i}{3\,(e^x+1i)}$$

```
[In] int(sinh(x)^2/(sinh(x) + 1i)^2,x)
[Out] x + ((exp(x)*4i)/3 - 2/3)/(exp(2*x) + exp(x)*2i - 1) - ((4*exp(x))/3 - (exp
(2*x)*4i)/3 + 4i/3)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 4i/(3*(exp(x
) + 1i))
```

3.51 $\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{2i\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] -1/3*cosh(x)/(I+sinh(x))^2-2/3*I*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2829, 2727}

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2i\cosh(x)}{3(\sinh(x)+i)} - \frac{\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[In] Int[Sinh[x]/(I + Sinh[x])^2,x]
```

```
[Out] -1/3*Cosh[x]/(I + Sinh[x])^2 - (((2*I)/3)*Cosh[x])/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
```

```
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cosh(x)}{3(i+\sinh(x))^2}+\frac{2}{3}\int\frac{1}{i+\sinh(x)}\,dx\\ &= -\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{2i\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int\frac{\sinh(x)}{(i+\sinh(x))^2}\,dx=\frac{\cosh(x)(1-2i\sinh(x))}{3(i+\sinh(x))^2}$$

```
[In] Integrate[Sinh[x]/(I + Sinh[x])^2,x]
[Out] (Cosh[x]*(1 - (2*I)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{2\left(3i\mathrm{e}^{x}+3\,\mathrm{e}^{2x}-2\right)}{3\left(\mathrm{e}^{x}+i\right)^{3}}$	23
default	$\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$	25
parallelrisch	$\frac{2i+6\tanh\left(\frac{x}{2}\right)}{3\tanh\left(\frac{x}{2}\right)^{3}+9i\tanh\left(\frac{x}{2}\right)^{2}-9\tanh\left(\frac{x}{2}\right)-3i}$	39

```
[In] int(sinh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2/3*(3*I*exp(x)+3*exp(2*x)-2)/(exp(x)+I)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,e^{(2\,x)}+3i\,e^{x}-2\right)}{3\,\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="fricas")
[Out] -2/3*(3*e^(2*x) + 3*I*e^x - 2)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = \frac{-6e^{2x}-6ie^{x}+4}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

```
[In] integrate(sinh(x)/(I+sinh(x))**2,x)
[Out] (-6*exp(2*x) - 6*I*exp(x) + 4)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x) - 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(21) = 42.

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.61

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2i\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i} - \frac{4}{3\,\left(3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i\right)}$$

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -2*I*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) + 2*e^(-2*x)/(3*e^(-x)
 + 3*I*e^(-2*x) - e^(-3*x) - I) - 4/3/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) -
 I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,e^{(2\,x)}+3i\,e^{x}-2\right)}{3\left(e^{x}+i\right)^{3}}$$

```
[In] integrate(sinh(x)/(I+sinh(x))^2,x, algorithm="giac")
[Out] -2/3*(3*e^(2*x) + 3*I*e^x - 2)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,\mathrm{e}^{x}-\mathrm{e}^{2\,x}\,3\mathrm{i}+2\mathrm{i}\right)}{3\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)^{3}}$$

```
[In] int(sinh(x)/(sinh(x) + 1i)^2,x)
[Out] -(2*(3*exp(x) - exp(2*x)*3i + 2i))/(3*(exp(x)*1i - 1)^3)
```

3.52 $\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx=\operatorname{arctanh}(\cosh(x))+\frac{\cosh(x)}{3(i+\sinh(x))^2}-\frac{4i\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] arctanh(cosh(x))+1/3*cosh(x)/(I+sinh(x))^2-4/3*I*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2845, 3057, 12, 3855}

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx=\operatorname{arctanh}(\cosh(x))-\frac{4i\cosh(x)}{3(\sinh(x)+i)}+\frac{\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[In] Int[Csch[x]/(I + Sinh[x])^2,x]
```

```
[Out] ArcTanh[Cosh[x]] + Cosh[x]/(3*(I + Sinh[x])^2) - (((4*I)/3)*Cosh[x])/(I + S
inh[x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
```

```
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\operatorname{csch}(x)(3i-\sinh(x))}{i+\sinh(x)}\,dx \\
&= \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{4i\cosh(x)}{3(i+\sinh(x))} + \frac{1}{3}i\int 3i\operatorname{csch}(x)\,dx \\
&= \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{4i\cosh(x)}{3(i+\sinh(x))} - \int \operatorname{csch}(x)\,dx \\
&= \operatorname{arctanh}(\cosh(x)) + \frac{\cosh(x)}{3(i+\sinh(x))^2} - \frac{4i\cosh(x)}{3(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \log\left(\cosh\left(\frac{x}{2}\right)\right) - \log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{i}{3i+3\sinh(x)} - \frac{2\sinh\left(\frac{x}{2}\right)(5i+4\sinh(x))}{3\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[Csch[x]/(I + Sinh[x])^2,x]
```

```
[Out] Log[Cosh[x/2]] - Log[Sinh[x/2]] - I/(3*I + 3*Sinh[x]) - (2*Sinh[x/2]*(5*I +
 4*Sinh[x]))/(3*(Cosh[x/2] - I*Sinh[x/2])^3)
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{2(9i\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}-4)}{3(\mathrm{e}^{x}+i)^{3}}+\ln\left(\mathrm{e}^{x}+1\right)-\ln\left(\mathrm{e}^{x}-1\right)$	36
default	$-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{4i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{4i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}$	44
parallelrisch	$\frac{\left(-3\tanh\left(\frac{x}{2}\right)^{3}-9i\tanh\left(\frac{x}{2}\right)^{2}+9\tanh\left(\frac{x}{2}\right)+3i\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+4\tanh\left(\frac{x}{2}\right)^{3}+6i+6\tanh\left(\frac{x}{2}\right)}{3\tanh\left(\frac{x}{2}\right)^{3}+9i\tanh\left(\frac{x}{2}\right)^{2}-9\tanh\left(\frac{x}{2}\right)-3i}$	79

```
[In] int(csch(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(9*I*exp(x)+3*exp(2*x)-4)/(exp(x)+I)^3+ln(exp(x)+1)-ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \frac{3\left(e^{(3x)}+3i\,e^{(2x)}-3\,e^{x}-i\right)\log\left(e^{x}+1\right)-3\left(e^{(3x)}+3i\,e^{(2x)}-3\,e^{x}-i\right)\log\left(e^{x}-1\right)-6\,e^{(2x)}-18i\,e^{x}+8}{3\left(e^{(3x)}+3i\,e^{(2x)}-3\,e^{x}-i\right)}$$

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)*log(e^x + 1) - 3*(e^(3*x) + 3*I*
e^(2*x) - 3*e^x - I)*log(e^x - 1) - 6*e^(2*x) - 18*I*e^x + 8)/(e^(3*x) + 3*
I*e^(2*x) - 3*e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}(x)}{(\sinh(x)+i)^2}\,dx$$

```
[In] integrate(csch(x)/(I+sinh(x))**2,x)
```

```
[Out] Integral(csch(x)/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(24) = 48$.

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \frac{2\left(-9i\,e^{(-x)}+3\,e^{(-2\,x)}-4\right)}{3\left(3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i\right)} + \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(-9*I*e^(-x) + 3*e^(-2*x) - 4)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I)
 + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,e^{(2\,x)}+9i\,e^{x}-4\right)}{3\left(e^{x}+i\right)^3} + \log\left(e^{x}+1\right) - \log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^(2*x) + 9*I*e^x - 4)/(e^x + I)^3 + log(e^x + 1) - log(abs(e^x - 1
))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{csch}(x)}{(i+\sinh(x))^2}\,dx = \ln\left(\mathrm{e}^x+1\right)-\ln\left(\mathrm{e}^x-1\right)-\frac{2}{\mathrm{e}^x+1\mathrm{i}}-\frac{2\mathrm{i}}{\left(\mathrm{e}^x+1\mathrm{i}\right)^2}-\frac{4}{3\left(\mathrm{e}^x+1\mathrm{i}\right)^3}$$

```
[In] int(1/(sinh(x)*(sinh(x) + 1i)^2),x)
```

```
[Out] log(exp(x) + 1) - log(exp(x) - 1) - 2/(exp(x) + 1i) - 2i/(exp(x) + 1i)^2 -
4/(3*(exp(x) + 1i)^3)
```

3.53 $\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2} dx = 2i\operatorname{arctanh}(\cosh(x)) + \frac{10\coth(x)}{3} + \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)}$$

```
[Out] 2*I*arctanh(cosh(x))+10/3*coth(x)+1/3*coth(x)/(I+sinh(x))^2-2*I*coth(x)/(I+
sinh(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2845, 3057, 2827, 3852, 8, 3855}

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2} dx = 2i\operatorname{arctanh}(\cosh(x)) + \frac{10\coth(x)}{3} - \frac{2i\coth(x)}{\sinh(x)+i} + \frac{\coth(x)}{3(\sinh(x)+i)^2}$$

```
[In] Int[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + (10*Coth[x])/3 + Coth[x]/(3*(I + Sinh[x])^2) - ((2
*I)*Coth[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

```
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\operatorname{csch}^2(x)(4i-2\sinh(x))}{i+\sinh(x)}\,dx \\
&= \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)} + \frac{1}{3}\int \operatorname{csch}^2(x)(-10-6i\sinh(x))\,dx \\
&= \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)} - 2i\int \operatorname{csch}(x)\,dx - \frac{10}{3}\int \operatorname{csch}^2(x)\,dx
\end{aligned}$$

$$= 2i\text{arctanh}(\cosh(x)) + \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)} + \frac{10}{3} i\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)$$

$$= 2i\text{arctanh}(\cosh(x)) + \frac{10\coth(x)}{3} + \frac{\coth(x)}{3(i+\sinh(x))^2} - \frac{2i\coth(x)}{i+\sinh(x)}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.

Time = 1.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.10

$$\int \frac{\text{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{6}\left(3\coth\left(\frac{x}{2}\right) + 12i\log\left(\cosh\left(\frac{x}{2}\right)\right) - 12i\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{2}{i+\sinh(x)} - \frac{4\sinh\left(\frac{x}{2}\right)(8i+7\sinh(x))}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3} + 3\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (3*Coth[x/2] + (12*I)*Log[Cosh[x/2]] - (12*I)*Log[Sinh[x/2]] + 2/(I + Sinh[
x]) - (4*Sinh[x/2]*(8*I + 7*Sinh[x]))/(I*Cosh[x/2] + Sinh[x/2])^3 + 3*Tanh[
x/2])/6
```

Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^2} - \frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^3} + \frac{6}{\tanh\left(\frac{x}{2}\right)+i} - 2i\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{2\tanh\left(\frac{x}{2}\right)}$
risch	$-\frac{4i\left(9i\,e^{3x}+3\,e^{4x}-12i\,e^{x}-11\,e^{2x}+5\right)}{3\left(e^{2x}-1\right)\left(e^{x}+i\right)^3} + 2i\ln\left(e^x+1\right) - 2i\ln\left(e^x-1\right)$
parallelrisch	$\frac{\left(-12i\tanh\left(\frac{x}{2}\right)^3+36\tanh\left(\frac{x}{2}\right)^2+36i\tanh\left(\frac{x}{2}\right)-12\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)+19i\tanh\left(\frac{x}{2}\right)^3+3\tanh\left(\frac{x}{2}\right)^4-3i\coth\left(\frac{x}{2}\right)+36i\tanh\left(\frac{x}{2}\right)-31}{6\tanh\left(\frac{x}{2}\right)^3+18i\tanh\left(\frac{x}{2}\right)^2-18\tanh\left(\frac{x}{2}\right)-6i}$

```
[In] int(csch(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tanh(1/2*x)-2*I/(tanh(1/2*x)+I)^2-4/3/(tanh(1/2*x)+I)^3+6/(tanh(1/2*x)+
I)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(30) = 60.

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.10

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\left(-i\,e^{(5\,x)}+3\,e^{(4\,x)}+4i\,e^{(3\,x)}-4\,e^{(2\,x)}-3i\,e^{x}+1\right)\log\left(e^{x}+1\right)+3\left(i\,e^{(5\,x)}-3\,e^{(4\,x)}-4i\,e^{(3\,x)}+4\,e^{(2}\right.}{3\left(e^{(5\,x)}+3i\,e^{(4\,x)}-4\,e^{(3\,x)}-4i\,e^{(2\,x)}+\right.}$$

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*(-I*e^(5*x) + 3*e^(4*x) + 4*I*e^(3*x) - 4*e^(2*x) - 3*I*e^x + 1)*lo
g(e^x + 1) + 3*(I*e^(5*x) - 3*e^(4*x) - 4*I*e^(3*x) + 4*e^(2*x) + 3*I*e^x -
 1)*log(e^x - 1) + 6*I*e^(4*x) - 18*e^(3*x) - 22*I*e^(2*x) + 24*e^x + 10*I)
/(e^(5*x) + 3*I*e^(4*x) - 4*e^(3*x) - 4*I*e^(2*x) + 3*e^x + I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^2(x)}{(\sinh(x)+i)^2}\,dx$$

```
[In] integrate(csch(x)**2/(I+sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**2/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(30) = 60.

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{4\left(12\,e^{(-x)}+11i\,e^{(-2\,x)}-9\,e^{(-3\,x)}-3i\,e^{(-4\,x)}-5i\right)}{3\left(3\,e^{(-x)}+4i\,e^{(-2\,x)}-4\,e^{(-3\,x)}-3i\,e^{(-4\,x)}+e^{(-5\,x)}-i\right)} + 2i\,\log\left(e^{(-x)}+1\right)-2i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 4/3*(12*e^(-x) + 11*I*e^(-2*x) - 9*e^(-3*x) - 3*I*e^(-4*x) - 5*I)/(3*e^(-x)
 + 4*I*e^(-2*x) - 4*e^(-3*x) - 3*I*e^(-4*x) + e^(-5*x) - I) + 2*I*log(e^(-x
) + 1) - 2*I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{e^{(2x)}-1} - \frac{2\left(6i\,e^{(2x)}-15\,e^{x}-7i\right)}{3\left(e^{x}+i\right)^3} + 2i\,\log\left(e^{x}+1\right) - 2i\,\log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/(e^(2*x) - 1) - 2/3*(6*I*e^(2*x) - 15*e^x - 7*I)/(e^x + I)^3 + 2*I*log(e^
x + 1) - 2*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{csch}^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{e^{2x}-1+e^{x}\,2i} + \frac{2}{e^{2x}-1} - \ln\left(e^{x}\,4i-4i\right)\,2i + \ln\left(e^{x}\,4i+4i\right)\,2i - \frac{4i}{e^{x}+1i} - \frac{4i}{3\,\left(e^{2x}\,3i+e^{3x}-3\,e^{x}-i\right)}$$

```
[In] int(1/(sinh(x)^2*(sinh(x) + 1i)^2),x)
```

```
[Out] log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + 2/(exp(2*x) + exp(x)*2i -
 1) - 4i/(exp(x) + 1i) - 4i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)) +
2/(exp(2*x) - 1)
```

3.54 $\int \frac{\text{csch}^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{\text{csch}^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{7}{2}\text{arctanh}(\cosh(x)) + \frac{16}{3} i \coth(x) + \frac{7}{2} \coth(x)\text{csch}(x) + \frac{\coth(x)\text{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i \coth(x)\text{csch}(x)}{3(i+\sinh(x))}$$

```
[Out] -7/2*arctanh(cosh(x))+16/3*I*coth(x)+7/2*coth(x)*csch(x)+1/3*coth(x)*csch(x
)/(I+sinh(x))^2-8/3*I*coth(x)*csch(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2845, 3057, 2827, 3853, 3855, 3852, 8}

$$\int \frac{\text{csch}^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{7}{2}\text{arctanh}(\cosh(x)) + \frac{16}{3} i \coth(x) + \frac{7}{2} \coth(x)\text{csch}(x) - \frac{8i \coth(x)\text{csch}(x)}{3(\sinh(x)+i)} + \frac{\coth(x)\text{csch}(x)}{3(\sinh(x)+i)^2}$$

```
[In] Int[Csch[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-7*ArcTanh[Cosh[x]])/2 + ((16*I)/3)*Coth[x] + (7*Coth[x]*Csch[x])/2 + (Cot
h[x]*Csch[x])/(3*(I + Sinh[x])^2) - (((8*I)/3)*Coth[x]*Csch[x])/(I + Sinh[x
])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\operatorname{csch}^3(x)(5i-3\sinh(x))}{i+\sinh(x)}\,dx \\
&= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} + \frac{1}{3}\int \operatorname{csch}^3(x)(-21-16i\sinh(x))\,dx \\
&= \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} - \frac{16}{3}i\int \operatorname{csch}^2(x)\,dx - 7\int \operatorname{csch}^3(x)\,dx \\
&= \frac{7}{2}\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))} \\
&\quad + \frac{7}{2}\int \operatorname{csch}(x)\,dx - \frac{16}{3}\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right) \\
&= -\frac{7}{2}\operatorname{arctanh}(\cosh(x)) + \frac{16}{3}i\coth(x) + \frac{7}{2}\coth(x)\operatorname{csch}(x) \\
&\quad + \frac{\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))^2} - \frac{8i\coth(x)\operatorname{csch}(x)}{3(i+\sinh(x))}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.

Time = 1.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.41

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\left(24i\coth\left(\frac{x}{2}\right) + 3\operatorname{csch}^2\left(\frac{x}{2}\right) - 84\log\left(\cosh\left(\frac{x}{2}\right)\right) + 84\log\left(\sinh\left(\frac{x}{2}\right)\right) + 3\operatorname{sech}^2\left(\frac{x}{2}\right) + \frac{8}{\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^2} + \frac{160i\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)} + \frac{16\sinh\left(\frac{x}{2}\right)}{\left(i\cosh\left(\frac{x}{2}\right) + \sinh\left(\frac{x}{2}\right)\right)^3} + 24i\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] ((24*I)*Coth[x/2] + 3*Csch[x/2]^2 - 84*Log[Cosh[x/2]] + 84*Log[Sinh[x/2]] +
 3*Sech[x/2]^2 + 8/(Cosh[x/2] - I*Sinh[x/2])^2 + ((160*I)*Sinh[x/2])/(Cosh[
x/2] - I*Sinh[x/2]) + (16*Sinh[x/2])/(I*Cosh[x/2] + Sinh[x/2])^3 + (24*I)*T
anh[x/2])/24
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

method	result
risch	$\frac{-98\,\mathrm{e}^{4x}+63i\mathrm{e}^{5x}+97\,\mathrm{e}^{2x}-126i\mathrm{e}^{3x}+21\,\mathrm{e}^{6x}-32+75i\mathrm{e}^{x}}{3(\mathrm{e}^{2x}-1)^{2}(\mathrm{e}^{x}+i)^{3}}-\frac{7\ln(\mathrm{e}^{x}+1)}{2}+\frac{7\ln(\mathrm{e}^{x}-1)}{2}$
default	$i\tanh\left(\frac{x}{2}\right)-\frac{\tanh(\frac{x}{2})^{2}}{8}+\frac{i}{\tanh(\frac{x}{2})}+\frac{1}{8\tanh(\frac{x}{2})^{2}}+\frac{7\ln(\tanh(\frac{x}{2}))}{2}+\frac{8i}{\tanh(\frac{x}{2})+i}-\frac{4i}{3(\tanh(\frac{x}{2})+i)^{3}}+\frac{2}{(\tanh(\frac{x}{2}$
parallelrisch	$\frac{\left(84\tanh(\frac{x}{2})^{3}+252i\tanh(\frac{x}{2})^{2}-252\tanh(\frac{x}{2})-84i\right)\ln(\tanh(\frac{x}{2}))+15i\tanh(\frac{x}{2})^{4}-3\tanh(\frac{x}{2})^{5}-3i\coth(\frac{x}{2})^{2}-112\tanh(\frac{x}{2})^{3}-1}{24\tanh(\frac{x}{2})^{3}+72i\tanh(\frac{x}{2})^{2}-72\tanh(\frac{x}{2})-24i}$

```
[In] int(csch(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-98*exp(x)^4+63*I*exp(x)^5+97*exp(x)^2-126*I*exp(x)^3+21*exp(x)^6-32+7
5*I*exp(x))/(exp(x)^2-1)^2/(exp(x)+I)^3-7/2*ln(exp(x)+1)+7/2*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(40) = 80.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{21\left(e^{(7x)}+3i\,e^{(6x)}-5\,e^{(5x)}-7i\,e^{(4x)}+7\,e^{(3x)}+5i\,e^{(2x)}-3\,e^{x}-i\right)\log\left(e^{x}+1\right)-21\left(e^{(7x)}+3i\,e^{(6x)}\right.}{6\left(e^{(7x)}+3i\,e^{(6x)}\right.}$$

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(21*(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x) - 7*I*e^(4*x) + 7*e^(3*x) + 5*I
*e^(2*x) - 3*e^x - I)*log(e^x + 1) - 21*(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x)
- 7*I*e^(4*x) + 7*e^(3*x) + 5*I*e^(2*x) - 3*e^x - I)*log(e^x - 1) - 42*e^(6
*x) - 126*I*e^(5*x) + 196*e^(4*x) + 252*I*e^(3*x) - 194*e^(2*x) - 150*I*e^x
 + 64)/(e^(7*x) + 3*I*e^(6*x) - 5*e^(5*x) - 7*I*e^(4*x) + 7*e^(3*x) + 5*I*e
^(2*x) - 3*e^x - I)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^3(x)}{(\sinh(x)+i)^2}\,dx$$

```
[In] integrate(csch(x)**3/(I+sinh(x))**2,x)
[Out] Integral(csch(x)**3/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(40) = 80.

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{-75i\,e^{(-x)} + 97\,e^{(-2\,x)} + 126i\,e^{(-3\,x)} - 98\,e^{(-4\,x)} - 63i\,e^{(-5\,x)} + 21\,e^{(-6\,x)} - 32}{3\,(3\,e^{(-x)} + 5i\,e^{(-2\,x)} - 7\,e^{(-3\,x)} - 7i\,e^{(-4\,x)} + 5\,e^{(-5\,x)} + 3i\,e^{(-6\,x)} - e^{(-7\,x)} - i)} - \frac{7}{2}\log\left(e^{(-x)}+1\right) + \frac{7}{2}\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -1/3*(-75*I*e^(-x) + 97*e^(-2*x) + 126*I*e^(-3*x) - 98*e^(-4*x) - 63*I*e^(-
5*x) + 21*e^(-6*x) - 32)/(3*e^(-x) + 5*I*e^(-2*x) - 7*e^(-3*x) - 7*I*e^(-4*
x) + 5*e^(-5*x) + 3*I*e^(-6*x) - e^(-7*x) - I) - 7/2*log(e^(-x) + 1) + 7/2*
log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{(3\,x)} + 4i\,e^{(2\,x)} + e^x - 4i}{\left(e^{(2\,x)}-1\right)^2} + \frac{2\left(9\,e^{(2\,x)} + 21i\,e^x - 10\right)}{3\,(e^x+i)^3} - \frac{7}{2}\log\left(e^x+1\right) + \frac{7}{2}\log\left(|e^x-1|\right)$$

```
[In] integrate(csch(x)^3/(I+sinh(x))^2,x, algorithm="giac")
[Out] (e^(3*x) + 4*I*e^(2*x) + e^x - 4*I)/(e^(2*x) - 1)^2 + 2/3*(9*e^(2*x) + 21*I
*e^x - 10)/(e^x + I)^3 - 7/2*log(e^x + 1) + 7/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{\operatorname{csch}^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^x}{e^{2x}-1} - \frac{7\ln\left(e^x+1\right)}{2} - \frac{7\ln\left(\frac{1}{e^x-1}\right)}{2} + \frac{2\,e^x}{\left(e^{2x}-1\right)^2} + \frac{6}{e^x+1i} + \frac{2i}{\left(e^x+1i\right)^2} + \frac{4}{3\left(e^x+1i\right)^3} + \frac{4i}{e^{2x}-1}$$

```
[In] int(1/(sinh(x)^3*(sinh(x) + 1i)^2),x)
```

```
[Out] exp(x)/(exp(2*x) - 1) - (7*log(exp(x) + 1))/2 - (7*log(1/(exp(x) - 1)))/2 +
 (2*exp(x))/(exp(2*x) - 1)^2 + 6/(exp(x) + 1i) + 2i/(exp(x) + 1i)^2 + 4/(3*
(exp(x) + 1i)^3) + 4i/(exp(2*x) - 1)
```

3.55 $\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -5i\operatorname{arctanh}(\cosh(x)) - 12\coth(x) + 4\coth^3(x) + 5i\coth(x)\operatorname{csch}(x) + \frac{\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(i+\sinh(x))}$$

```
[Out] -5*I*arctanh(cosh(x))-12*coth(x)+4*coth(x)^3+5*I*coth(x)*csch(x)+1/3*coth(x
)*csch(x)^2/(I+sinh(x))^2-10/3*I*coth(x)*csch(x)^2/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2845, 3057, 2827, 3852, 3853, 3855}

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -5i\operatorname{arctanh}(\cosh(x)) + 4\coth^3(x) - 12\coth(x) + 5i\coth(x)\operatorname{csch}(x) - \frac{10i\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)} + \frac{\coth(x)\operatorname{csch}^2(x)}{3(\sinh(x)+i)^2}$$

```
[In] Int[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*I)*ArcTanh[Cosh[x]] - 12*Coth[x] + 4*Coth[x]^3 + (5*I)*Coth[x]*Csch[x]
+ (Coth[x]*Csch[x]^2)/(3*(I + Sinh[x])^2) - (((10*I)/3)*Coth[x]*Csch[x]^2)/
(I + Sinh[x])
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2845

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^
m*((c + d*Sin[e + f*x])^(n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{1}{3}\int \frac{\text{csch}^4(x)(6i-4\sinh(x))}{i+\sinh(x)}\,dx \\
&= \frac{\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))} + \frac{1}{3}\int \text{csch}^4(x)(-36-30i\sinh(x))\,dx \\
&= \frac{\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))} - 10i\int \text{csch}^3(x)\,dx - 12\int \text{csch}^4(x)\,dx \\
&= 5i\coth(x)\text{csch}(x) + \frac{\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))} \\
&\quad + 5i\int \text{csch}(x)\,dx - 12i\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\coth(x)\right) \\
&= -5i\text{arctanh}(\cosh(x)) - 12\coth(x) + 4\coth^3(x) \\
&\quad + 5i\coth(x)\text{csch}(x) + \frac{\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))^2} - \frac{10i\coth(x)\text{csch}^2(x)}{3(i+\sinh(x))}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 143 vs. $2(64) = 128$.

Time = 2.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.23

$$
\begin{aligned}
\int \frac{\text{csch}^4(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\Bigg(&-44\coth\left(\frac{x}{2}\right) + 6i\text{csch}^2\left(\frac{x}{2}\right) + \frac{1}{2}\text{csch}^4\left(\frac{x}{2}\right)\sinh(x) \\
&+ 2\Bigg(-60i\log\left(\cosh\left(\frac{x}{2}\right)\right) + 60i\log\left(\sinh\left(\frac{x}{2}\right)\right) + 3i\text{sech}^2\left(\frac{x}{2}\right) \\
&- 4\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right) - \frac{4}{i+\sinh(x)} + \frac{8\sinh\left(\frac{x}{2}\right)(14i+13\sinh(x))}{\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)^3} \\
&- 22\tanh\left(\frac{x}{2}\right)\Bigg)\Bigg)
\end{aligned}
$$

```
[In] Integrate[Csch[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-44*Coth[x/2] + (6*I)*Csch[x/2]^2 + (Csch[x/2]^4*Sinh[x])/2 + 2*((-60*I)*L
og[Cosh[x/2]] + (60*I)*Log[Sinh[x/2]] + (3*I)*Sech[x/2]^2 - 4*Csch[x]^3*Sin
h[x/2]^4 - 4/(I + Sinh[x]) + (8*Sinh[x/2]*(14*I + 13*Sinh[x]))/(I*Cosh[x/2]
 + Sinh[x/2])^3 - 22*Tanh[x/2]))/24
```

Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38

method	result
risch	$\frac{2i\left(45i\,\mathrm{e}^{7x}+15\,\mathrm{e}^{8x}-135i\,\mathrm{e}^{5x}-85\,\mathrm{e}^{6x}+155i\,\mathrm{e}^{3x}+153\,\mathrm{e}^{4x}-57i\,\mathrm{e}^{x}-99\,\mathrm{e}^{2x}+24\right)}{3\left(\mathrm{e}^{2x}-1\right)^{3}\left(\mathrm{e}^{x}+i\right)^{3}}+5i\ln\left(\mathrm{e}^{x}-1\right)-5i\ln\left(\mathrm{e}^{x}+1\right)$
default	$-\frac{15\tanh\left(\frac{x}{2}\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)^{3}}{24}-\frac{i\tanh\left(\frac{x}{2}\right)^{2}}{4}+\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{10}{\tanh\left(\frac{x}{2}\right)+i}+\frac{i}{4\tanh\left(\frac{x}{2}\right)^{2}}+5i\,\mathrm{l}$
parallelrisch	$\frac{290+120\left(i\tanh\left(\frac{x}{2}\right)^{3}-3i\tanh\left(\frac{x}{2}\right)-3\tanh\left(\frac{x}{2}\right)^{2}+1\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)-3i\tanh\left(\frac{x}{2}\right)^{5}+\tanh\left(\frac{x}{2}\right)^{6}-i\coth\left(\frac{x}{2}\right)^{3}-170i\tanh\left(\frac{x}{2}\right)^{3}-30\,\mathrm{t}}{24\tanh\left(\frac{x}{2}\right)^{3}+72i\tanh\left(\frac{x}{2}\right)^{2}-72\tanh\left(\frac{x}{2}\right)-24i}$

```
[In] int(csch(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(45*I*exp(x)^7+15*exp(x)^8-135*I*exp(x)^5-85*exp(x)^6+155*I*exp(x)^3+
153*exp(x)^4-57*I*exp(x)-99*exp(x)^2+24)/(exp(x)^2-1)^3/(exp(x)+I)^3+5*I*ln
(exp(x)-1)-5*I*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(50) = 100.

Time = 0.28 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.53

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{15\left(i\,e^{(9x)}-3\,e^{(8x)}-6i\,e^{(7x)}+10\,e^{(6x)}+12i\,e^{(5x)}-12\,e^{(4x)}-10i\,e^{(3x)}+6\,e^{(2x)}+3i\,e^{x}-1\right)\log\left(e^{x}+\right.}{}$$

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(15*(I*e^(9*x) - 3*e^(8*x) - 6*I*e^(7*x) + 10*e^(6*x) + 12*I*e^(5*x) -
 12*e^(4*x) - 10*I*e^(3*x) + 6*e^(2*x) + 3*I*e^x - 1)*log(e^x + 1) + 15*(-I
*e^(9*x) + 3*e^(8*x) + 6*I*e^(7*x) - 10*e^(6*x) - 12*I*e^(5*x) + 12*e^(4*x)
 + 10*I*e^(3*x) - 6*e^(2*x) - 3*I*e^x + 1)*log(e^x - 1) - 30*I*e^(8*x) + 90
*e^(7*x) + 170*I*e^(6*x) - 270*e^(5*x) - 306*I*e^(4*x) + 310*e^(3*x) + 198*
I*e^(2*x) - 114*e^x - 48*I)/(e^(9*x) + 3*I*e^(8*x) - 6*e^(7*x) - 10*I*e^(6*
x) + 12*e^(5*x) + 12*I*e^(4*x) - 10*e^(3*x) - 6*I*e^(2*x) + 3*e^x + I)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(csch(x)**4/(I+sinh(x))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(50) = 100.

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.98

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(57\,e^{(-x)}+99i\,e^{(-2\,x)}-155\,e^{(-3\,x)}-153i\,e^{(-4\,x)}+135\,e^{(-5\,x)}+85i\,e^{(-6\,x)}-45\,e^{(-7\,x)}-15i\,e^{(-8\,x)}-24\right.}{3\left(3\,e^{(-x)}+6i\,e^{(-2\,x)}-10\,e^{(-3\,x)}-12i\,e^{(-4\,x)}+12\,e^{(-5\,x)}+10i\,e^{(-6\,x)}-6\,e^{(-7\,x)}-3i\,e^{(-8\,x)}+e^{(-9\,x)}-i\right.} - 5i\,\log\left(e^{(-x)}+1\right)+5i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -2/3*(57*e^(-x) + 99*I*e^(-2*x) - 155*e^(-3*x) - 153*I*e^(-4*x) + 135*e^(-5
*x) + 85*I*e^(-6*x) - 45*e^(-7*x) - 15*I*e^(-8*x) - 24*I)/(3*e^(-x) + 6*I*e
^(-2*x) - 10*e^(-3*x) - 12*I*e^(-4*x) + 12*e^(-5*x) + 10*I*e^(-6*x) - 6*e^(
-7*x) - 3*I*e^(-8*x) + e^(-9*x) - I) - 5*I*log(e^(-x) + 1) + 5*I*log(e^(-x)
 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(-15i\,e^{(8\,x)}+45\,e^{(7\,x)}+85i\,e^{(6\,x)}-135\,e^{(5\,x)}-153i\,e^{(4\,x)}+155\,e^{(3\,x)}+99i\,e^{(2\,x)}-57\,e^{x}-24i\right)}{3\left(e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i\right)^3} - 5i\,\log\left(e^{x}+1\right)+5i\,\log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x)^4/(I+sinh(x))^2,x, algorithm="giac")
[Out] -2/3*(-15*I*e^(8*x) + 45*e^(7*x) + 85*I*e^(6*x) - 135*e^(5*x) - 153*I*e^(4*
x) + 155*e^(3*x) + 99*I*e^(2*x) - 57*e^x - 24*I)/(e^(3*x) + I*e^(2*x) - e^x
 - I)^3 - 5*I*log(e^x + 1) + 5*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{csch}^4(x)}{(i+\sinh(x))^2}\,dx = -\ln\left(-e^x\,10i-10i\right)\,5i+\ln\left(-e^x\,10i+10i\right)\,5i - \frac{\frac{16\,e^x}{3}-\frac{e^{2\,x}\,32i}{3}+\frac{16}{3}i}{12\,e^{5\,x}-10\,e^{3\,x}+e^{4\,x}\,12i-e^{2\,x}\,6i-e^{6\,x}\,10i-6\,e^{7\,x}+e^{8\,x}\,3i+e^{9\,x}+3\,e^x+1i} + \frac{\frac{20\,e^{2\,x}}{3}-\frac{44}{3}+\frac{e^x\,16i}{3}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1-e^{3\,x}\,4i+e^{5\,x}\,2i+e^x\,2i}-\frac{10\,e^x-e^{2\,x}\,10i+\frac{20}{3}i}{e^{2\,x}\,1i+e^{3\,x}-e^x-i}$$

```
[In] int(1/(sinh(x)^4*(sinh(x) + 1i)^2),x)
```

```
[Out] log(10i - exp(x)*10i)*5i - log(- exp(x)*10i - 10i)*5i - ((16*exp(x))/3 - (e
xp(2*x)*32i)/3 + 16i/3)/(exp(4*x)*12i - 10*exp(3*x) - exp(2*x)*6i + 12*exp(
5*x) - exp(6*x)*10i - 6*exp(7*x) + exp(8*x)*3i + exp(9*x) + 3*exp(x) + 1i)
+ ((20*exp(2*x))/3 + (exp(x)*16i)/3 - 44/3)/(3*exp(2*x) - exp(3*x)*4i - 3*e
xp(4*x) + exp(5*x)*2i + exp(6*x) + exp(x)*2i - 1) - (10*exp(x) - exp(2*x)*1
0i + 20i/3)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

3.56 $\int \frac{1}{1+i\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

```
[Out] I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{2727\}$

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

```
[In] Int[(1 + I*Sinh[c + d*x])^(-1),x]
```

```
[Out] (I*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral} = \frac{i\cosh(c+dx)}{d(1+i\sinh(c+dx))}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{2\sinh\left(\frac{1}{2}(c+dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-1),x]
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{2i}{d(\mathrm{e}^{dx+c}-i)}$	18
derivativedivides	$\frac{2}{d\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
default	$\frac{2}{d\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
parallelrisch	$-\frac{2}{d\left(i-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	22

```
[In] int(1/(1+I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 2*I/d/(exp(d*x+c)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx = \frac{2i}{de^{(dx+c)}-i\,d}$$

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="fricas")
[Out] 2*I/(d*e^(d*x + c) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx=\frac{2i}{de^{c}e^{dx}-id}$$

```
[In] integrate(1/(1+I*sinh(d*x+c)),x)
[Out] 2*I/(d*exp(c)*exp(d*x) - I*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx=-\frac{2}{d(i\,e^{(-dx-c)}-1)}$$

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="maxima")
[Out] -2/(d*(I*e^(-d*x - c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx=\frac{2i}{d(e^{(dx+c)}-i)}$$

```
[In] integrate(1/(1+I*sinh(d*x+c)),x, algorithm="giac")
[Out] 2*I/(d*(e^(d*x + c) - I))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{1+i\sinh(c+dx)}\,dx=\frac{2\mathrm{i}}{d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

```
[In] int(1/(sinh(c + d*x)*1i + 1),x)
[Out] 2i/(d*(exp(c + d*x) - 1i))
```

3.57 $\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2} + \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))}$$

```
[Out] 1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+1/3*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c
))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2729, 2727}

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))} + \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2}$$

```
[In] Int[(1 + I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((I/3)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^2) + ((I/3)*Cosh[c + d*x])/(
d*(1 + I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2} + \frac{1}{3}\int \frac{1}{1+i\sinh(c+dx)}\,dx \\ &= \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))^2} + \frac{i\cosh(c+dx)}{3d(1+i\sinh(c+dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{3i-4i\cosh(c+dx)-i\cosh(2(c+dx))-4\sinh(c+dx)+\sinh(2(c+dx))}{6d(-i+\sinh(c+dx))^2}$$

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-2),x]
[Out] (3*I - (4*I)*Cosh[c + d*x] - I*Cosh[2*(c + d*x)] - 4*Sinh[c + d*x] + Sinh[2
*(c + d*x)])/(6*d*(-I + Sinh[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{-\frac{2i}{3}+2\,\mathrm{e}^{dx+c}}{\left(\mathrm{e}^{dx+c}-i\right)^{3} d}$	28
derivativedivides	$\frac{\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55
default	$\frac{\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}-\frac{4}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}}{d}$	55
parallelrisch	$\frac{6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4}{3d\left(3i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-i+3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	76

```
[In] int(1/(1+I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-I+3*exp(d*x+c))/(exp(d*x+c)-I)^3/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{2\left(3\,e^{(dx+c)}-i\right)}{3\left(de^{(3\,dx+3\,c)}-3i\,de^{(2\,dx+2\,c)}-3\,de^{(dx+c)}+i\,d\right)}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(3*e^(d*x + c) - I)/(d*e^(3*d*x + 3*c) - 3*I*d*e^(2*d*x + 2*c) - 3*d*e^
(d*x + c) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{6e^ce^{dx}-2i}{3de^{3c}e^{3dx}-9ide^{2c}e^{2dx}-9de^ce^{dx}+3id}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))**2,x)
```

```
[Out] (6*exp(c)*exp(d*x) - 2*I)/(3*d*exp(3*c)*exp(3*d*x) - 9*I*d*exp(2*c)*exp(2*d
*x) - 9*d*exp(c)*exp(d*x) + 3*I*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}-3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}+i)} + \frac{2i}{3\,d(3\,e^{(-dx-c)}-3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}+i)}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 + I)) + 2/3*I/(d*(3*e^(-d*x - c) - 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 + I))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = \frac{2\left(3\,e^{(dx+c)}-i\right)}{3\,d\left(e^{(dx+c)}-i\right)^3}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^2,x, algorithm="giac")
[Out] 2/3*(3*e^(d*x + c) - I)/(d*(e^(d*x + c) - I)^3)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1+i\sinh(c+dx))^2}\,dx = -\frac{\frac{2}{3}+\mathrm{e}^{c+d\,x}\,2\mathrm{i}}{d\,\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^3}$$

```
[In] int(1/(sinh(c + d*x)*1i + 1)^2,x)
[Out] -(exp(c + d*x)*2i + 2/3)/(d*(exp(c + d*x)*1i + 1)^3)
```

3.58 $\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2} + \frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}$$

```
[Out] 1/5*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^3+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x+
c))^2+2/15*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2729, 2727}

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = \frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))} + \frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2} + \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}$$

```
[In] Int[(1 + I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((I/5)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/15)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^2) + (((2*I)/15)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2}{5}\int\frac{1}{(1+i\sinh(c+dx))^2}\,dx\\
&= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2}{15}\int\frac{1}{1+i\sinh(c+dx)}\,dx\\
&= \frac{i\cosh(c+dx)}{5d(1+i\sinh(c+dx))^3}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{15d(1+i\sinh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int\frac{1}{(1+i\sinh(c+dx))^3}\,dx = \frac{10-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))+15i\sinh(c+dx)-6i\sinh(2(c+dx))-i\sin}{30d(-i+\sinh(c+dx))^3}$$

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-3),x]
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] + (15*I)*S
inh[c + d*x] - (6*I)*Sinh[2*(c + d*x)] - I*Sinh[3*(c + d*x)])/(30*d*(-I + S
inh[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result	size
risch	$-\frac{4i(-5i\mathrm{e}^{dx+c}+10\,\mathrm{e}^{2dx+2c}-1)}{15d(\mathrm{e}^{dx+c}-i)^5}$	40
derivativedivides	$\frac{\frac{8}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	88
default	$\frac{\frac{8}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{3\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	88
parallelrisch	$\frac{\frac{14}{15}-\frac{16\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3}+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\frac{8i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{3}}{d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5-5i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-10\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+10i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$	128

```
[In] int(1/(1+I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] -4/15*I*(-5*I*exp(d*x+c)+10*exp(2*d*x+2*c)-1)/d/(exp(d*x+c)-I)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = -\frac{4\left(10i\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}-i\right)}{15\left(de^{(5\,dx+5\,c)}-5i\,de^{(4\,dx+4\,c)}-10\,de^{(3\,dx+3\,c)}+10i\,de^{(2\,dx+2\,c)}+5\,de^{(dx+c)}-i\,d\right)}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="fricas")
[Out] -4/15*(10*I*e^(2*d*x + 2*c) + 5*e^(d*x + c) - I)/(d*e^(5*d*x + 5*c) - 5*I*d
*e^(4*d*x + 4*c) - 10*d*e^(3*d*x + 3*c) + 10*I*d*e^(2*d*x + 2*c) + 5*d*e^(d
*x + c) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = \frac{-40ie^{2c}e^{2dx}-20e^{c}e^{dx}+4i}{15de^{5c}e^{5dx}-75ide^{4c}e^{4dx}-150de^{3c}e^{3dx}+150ide^{2c}e^{2dx}+75de^{c}e^{dx}-15id}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))**3,x)
[Out] (-40*I*exp(2*c)*exp(2*d*x) - 20*exp(c)*exp(d*x) + 4*I)/(15*d*exp(5*c)*exp(5
*d*x) - 75*I*d*exp(4*c)*exp(4*d*x) - 150*d*exp(3*c)*exp(3*d*x) + 150*I*d*ex
p(2*c)*exp(2*d*x) + 75*d*exp(c)*exp(d*x) - 15*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(70) = 140.

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx$$
$$= \frac{20i\,e^{(-dx-c)}}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}$$
$$+\frac{40\,e^{(-2\,dx-2\,c)}}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}$$
$$-\frac{4}{-15\,d(-5i\,e^{(-dx-c)}-10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}+1)}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) + 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) - 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) - 15)) - 4/(d*(75*I*e
^(-d*x - c) + 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) - 75*e^(-4*d*x
- 4*c) + 15*I*e^(-5*d*x - 5*c) - 15))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = -\frac{4i\left(10\,e^{(2\,dx+2\,c)}-5i\,e^{(dx+c)}-1\right)}{15\,d\left(e^{(dx+c)}-i\right)^5}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -4/15*I*(10*e^(2*d*x + 2*c) - 5*I*e^(d*x + c) - 1)/(d*(e^(d*x + c) - I)^5)
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{1}{(1+i\sinh(c+dx))^3}\,dx = -\frac{\frac{4}{15}-\frac{8\,\mathrm{e}^{2\,c+2\,d\,x}}{3}+\frac{\mathrm{e}^{c+d\,x}\,4\mathrm{i}}{3}}{d\,{\left(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)}^5}$$

```
[In] int(1/(sinh(c + d*x)*1i + 1)^3,x)
```

```
[Out] -((exp(c + d*x)*4i)/3 - (8*exp(2*c + 2*d*x))/3 + 4/15)/(d*(exp(c + d*x)*1i
+ 1)^5)
```

3.59 $\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2} + \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}$$

```
[Out] 1/7*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^4+3/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x+
c))^3+2/35*I*cosh(d*x+c)/d/(1+I*sinh(d*x+c))^2+2/35*I*cosh(d*x+c)/d/(1+I*si
nh(d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2729, 2727}

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))} + \frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2} + \frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3} + \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}$$

```
[In] Int[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((I/7)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x])^4) + (((3*I)/35)*Cosh[c + d*
x])/(d*(1 + I*Sinh[c + d*x])^3) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh
[c + d*x])^2) + (((2*I)/35)*Cosh[c + d*x])/(d*(1 + I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3}{7}\int\frac{1}{(1+i\sinh(c+dx))^3}\,dx\\
&= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}+\frac{6}{35}\int\frac{1}{(1+i\sinh(c+dx))^2}\,dx\\
&= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}\\
&\quad+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{2}{35}\int\frac{1}{1+i\sinh(c+dx)}\,dx\\
&= \frac{i\cosh(c+dx)}{7d(1+i\sinh(c+dx))^4}+\frac{3i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^3}\\
&\quad+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))^2}+\frac{2i\cosh(c+dx)}{35d(1+i\sinh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int\frac{1}{(1+i\sinh(c+dx))^4}\,dx = \frac{21i\cosh\left(\frac{3}{2}(c+dx)\right)-i\cosh\left(\frac{7}{2}(c+dx)\right)+35\sinh\left(\frac{1}{2}(c+dx)\right)-7\sinh\left(\frac{5}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

```
[In] Integrate[(1 + I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((21*I)*Cosh[(3*(c + d*x))/2] - I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x)
/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/
2])^7)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{4\left(-7\,\mathrm{e}^{dx+c}-21i\mathrm{e}^{2dx+2c}+35\,\mathrm{e}^{3dx+3c}+i\right)}{35\left(\mathrm{e}^{dx+c}-i\right)^{7}d}$
derivativedivides	$\frac{\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{72}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{16}{7\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{8i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{12}{\left(-i+\tanh\left(\right.\right.}}{d}$
default	$\frac{\frac{2}{-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{16i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{72}{5\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{16}{7\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}+\frac{8i}{\left(-i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{12}{\left(-i+\tanh\left(\right.\right.}}{d}$
parallelrisch	$-\frac{2\left(6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+7\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-21i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-21\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+7i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{35d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}-7i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-21\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+35i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-21i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$

```
[In] int(1/(1+I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -4/35*(-7*exp(d*x+c)-21*I*exp(2*d*x+2*c)+35*exp(3*d*x+3*c)+I)/(exp(d*x+c)-I
)^7/d
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}-21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}+i\right)}{35\left(de^{(7\,dx+7\,c)}-7i\,de^{(6\,dx+6\,c)}-21\,de^{(5\,dx+5\,c)}+35i\,de^{(4\,dx+4\,c)}+35\,de^{(3\,dx+3\,c)}-21i\,de^{(2\,dx+2\,c)}-7\,de^{(dx+c}\right.}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) - 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) + I)/(d*e^
(7*d*x + 7*c) - 7*I*d*e^(6*d*x + 6*c) - 21*d*e^(5*d*x + 5*c) + 35*I*d*e^(4*
d*x + 4*c) + 35*d*e^(3*d*x + 3*c) - 21*I*d*e^(2*d*x + 2*c) - 7*d*e^(d*x + c
) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx$$
$$= \frac{-140e^{3c}e^{3dx} + 84ie^{2c}e^{2dx} + 28e^{c}e^{dx} - 4i}{35de^{7c}e^{7dx} - 245ide^{6c}e^{6dx} - 735de^{5c}e^{5dx} + 1225ide^{4c}e^{4dx} + 1225de^{3c}e^{3dx} - 735ide^{2c}e^{2dx} - 245de^{c}e^{dx} + 35}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))**4,x)
```

```
[Out] (-140*exp(3*c)*exp(3*d*x) + 84*I*exp(2*c)*exp(2*d*x) + 28*exp(c)*exp(d*x) -
 4*I)/(35*d*exp(7*c)*exp(7*d*x) - 245*I*d*exp(6*c)*exp(6*d*x) - 735*d*exp(5
*c)*exp(5*d*x) + 1225*I*d*exp(4*c)*exp(4*d*x) + 1225*d*exp(3*c)*exp(3*d*x)
- 735*I*d*exp(2*c)*exp(2*d*x) - 245*d*exp(c)*exp(d*x) + 35*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(93) = 186.

Time = 0.19 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.18

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx$$
$$= \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)} - 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} + 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-7\,c}}$$
$$- \frac{12i\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)} - 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} + 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-}}$$
$$- \frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)} - 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} + 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-7}}$$
$$+ \frac{4i}{35\,d(7\,e^{(-dx-c)} - 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} + 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx}}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x
- 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c)
 - e^(-7*d*x - 7*c) + I)) - 12/5*I*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) - 21
*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-
5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) + I)) - 4*e^(-3*d*x
- 3*c)/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) + 3
5*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*x - 6*c) - e^(-7*d
*x - 7*c) + I)) + 4/35*I/(d*(7*e^(-d*x - c) - 21*I*e^(-2*d*x - 2*c) - 35*e^
(-3*d*x - 3*c) + 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*I*e^(-6*d*
x - 6*c) - e^(-7*d*x - 7*c) + I))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)} - 21i\,e^{(2\,dx+2\,c)} - 7\,e^{(dx+c)} + i\right)}{35\,d(e^{(dx+c)} - i)^7}$$

```
[In] integrate(1/(1+I*sinh(d*x+c))^4,x, algorithm="giac")
[Out] -4/35*(35*e^(3*d*x + 3*c) - 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) + I)/(d*(e
^(d*x + c) - I)^7)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int \frac{1}{(1+i\sinh(c+dx))^4}\,dx = -\frac{\left(7\,\mathrm{e}^{c+d\,x} + \mathrm{e}^{2\,c+2\,d\,x}\,21\mathrm{i} - 35\,\mathrm{e}^{3\,c+3\,d\,x} - \mathrm{i}\right)\,4\mathrm{i}}{35\,d\,(1+\mathrm{e}^{c+d\,x}\,1\mathrm{i})^7}$$

```
[In] int(1/(sinh(c + d*x)*1i + 1)^4,x)
[Out] -((7*exp(c + d*x) + exp(2*c + 2*d*x)*21i - 35*exp(3*c + 3*d*x) - 1i)*4i)/(3
5*d*(exp(c + d*x)*1i + 1)^7)
```

3.60 $\int \frac{1}{1-i\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx=-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

```
[Out] -I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2727}

$$\int \frac{1}{1-i\sinh(c+dx)}\,dx=-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

```
[In] Int[(1 - I*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((-I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\text{integral}=-\frac{i\cosh(c+dx)}{d(1-i\sinh(c+dx))}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{1}{1 - i\sinh(c + dx)}\,dx = \frac{2\sinh\left(\frac{1}{2}(c + dx)\right)}{d\left(\cosh\left(\frac{1}{2}(c + dx)\right) - i\sinh\left(\frac{1}{2}(c + dx)\right)\right)}$$

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-1),x]
[Out] (2*Sinh[(c + d*x)/2])/(d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)/2]))
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{2i}{d(e^{dx+c}+i)}$	18
derivativedivides	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
default	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20
parallelrisch	$\frac{2}{d\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	20

```
[In] int(1/(1-I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -2*I/d/(exp(d*x+c)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{1 - i\sinh(c + dx)}\,dx = -\frac{2i}{de^{(dx+c)} + i\,d}$$

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="fricas")
[Out] -2*I/(d*e^(d*x + c) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{1 - i \sinh(c + dx)}\, dx = -\frac{2i}{de^{c}e^{dx} + id}$$

```
[In] integrate(1/(1-I*sinh(d*x+c)),x)
[Out] -2*I/(d*exp(c)*exp(d*x) + I*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{1}{1 - i \sinh(c + dx)}\, dx = \frac{2}{d(i\, e^{(-dx-c)} + 1)}$$

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="maxima")
[Out] 2/(d*(I*e^(-d*x - c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{1}{1 - i \sinh(c + dx)}\, dx = -\frac{2i}{d(e^{(dx+c)} + i)}$$

```
[In] integrate(1/(1-I*sinh(d*x+c)),x, algorithm="giac")
[Out] -2*I/(d*(e^(d*x + c) + I))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{1 - i \sinh(c + dx)}\, dx = -\frac{2\mathrm{i}}{d\, \left(\mathrm{e}^{c+d\,x} + 1\mathrm{i}\right)}$$

```
[In] int(-1/(sinh(c + d*x)*1i - 1),x)
[Out] -2i/(d*(exp(c + d*x) + 1i))
```

3.61 $\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2} - \frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}$$

```
[Out] -1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-1/3*I*cosh(d*x+c)/d/(1-I*sinh(d*x+
c))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2729, 2727}

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))} - \frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}$$

```
[In] Int[(1 - I*Sinh[c + d*x])^(-2),x]
```

```
[Out] ((-1/3*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^2) - ((I/3)*Cosh[c + d*x]
)/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}+\frac{1}{3}\int\frac{1}{1-i\sinh(c+dx)}\,dx\\ &= -\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))^2}-\frac{i\cosh(c+dx)}{3d(1-i\sinh(c+dx))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int\frac{1}{(1-i\sinh(c+dx))^2}\,dx=-\frac{\cosh\left(\frac{3}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)}{3d\left(i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-2),x]
[Out] -1/3*(Cosh[(3*(c + d*x))/2] + (3*I)*Sinh[(c + d*x)/2])/(d*(I*Cosh[(c + d*x)
/2] + Sinh[(c + d*x)/2])^3)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{\frac{2i}{3}+2\,e^{dx+c}}{d\left(e^{dx+c}+i\right)^3}$	28
derivativedivides	$\frac{-\frac{4}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{d}$	55
default	$\frac{-\frac{4}{3\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}}{d}$	55
parallelrisch	$\frac{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4}{3d\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+3i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$	74

```
[In] int(1/(1-I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 2/3*(I+3*exp(d*x+c))/d/(exp(d*x+c)+I)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = \frac{2\left(3\,e^{(dx+c)}+i\right)}{3\left(de^{(3\,dx+3\,c)}+3i\,de^{(2\,dx+2\,c)}-3\,de^{(dx+c)}-i\,d\right)}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(3*e^(d*x + c) + I)/(d*e^(3*d*x + 3*c) + 3*I*d*e^(2*d*x + 2*c) - 3*d*e^
(d*x + c) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = \frac{6e^{c}e^{dx}+2i}{3de^{3c}e^{3dx}+9ide^{2c}e^{2dx}-9de^{c}e^{dx}-3id}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))**2,x)
```

```
[Out] (6*exp(c)*exp(d*x) + 2*I)/(3*d*exp(3*c)*exp(3*d*x) + 9*I*d*exp(2*c)*exp(2*d
*x) - 9*d*exp(c)*exp(d*x) - 3*I*d)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.59

$$\int \frac{1}{(1-i\sinh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}+3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}-i)} - \frac{2i}{3\,d(3\,e^{(-dx-c)}+3i\,e^{(-2\,dx-2\,c)}-e^{(-3\,dx-3\,c)}-i)}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 - I)) - 2/3*I/(d*(3*e^(-d*x - c) + 3*I*e^(-2*d*x - 2*c) - e^(-3*d*x - 3*c)
 - I))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.42

$$\int \frac{1}{(1 - i\sinh(c + dx))^2}\, dx = \frac{2\left(3\, e^{(dx+c)} + i\right)}{3\, d(e^{(dx+c)} + i)^3}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^2,x, algorithm="giac")
[Out] 2/3*(3*e^(d*x + c) + I)/(d*(e^(d*x + c) + I)^3)
```

Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1 - i\sinh(c + dx))^2}\, dx = -\frac{2\left(-1 + \mathrm{e}^{c+d\,x}\, 3\mathrm{i}\right)}{3\, d\left(-1 + \mathrm{e}^{c+d\,x}\, 1\mathrm{i}\right)^3}$$

```
[In] int(1/(sinh(c + d*x)*1i - 1)^2,x)
[Out] -(2*(exp(c + d*x)*3i - 1))/(3*d*(exp(c + d*x)*1i - 1)^3)
```

3.62 $\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}$$

```
[Out] -1/5*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^3-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*x
+c))^2-2/15*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2729, 2727}

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))} - \frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2} - \frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}$$

```
[In] Int[(1 - I*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-1/5*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/15)*Cosh[c +
 d*x])/(d*(1 - I*Sinh[c + d*x])^2) - (((2*I)/15)*Cosh[c + d*x])/(d*(1 - I*S
inh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}+\frac{2}{5}\int\frac{1}{(1-i\sinh(c+dx))^2}\,dx\\
&= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}+\frac{2}{15}\int\frac{1}{1-i\sinh(c+dx)}\,dx\\
&= -\frac{i\cosh(c+dx)}{5d(1-i\sinh(c+dx))^3}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))^2}-\frac{2i\cosh(c+dx)}{15d(1-i\sinh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int\frac{1}{(1-i\sinh(c+dx))^3}\,dx = \frac{10-15\cosh(c+dx)-6\cosh(2(c+dx))+\cosh(3(c+dx))-15i\sinh(c+dx)+6i\sinh(2(c+dx))+i\,\mathrm{s}}{30d(i+\sinh(c+dx))^3}$$

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-3),x]
[Out] (10 - 15*Cosh[c + d*x] - 6*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)] - (15*I)*S
inh[c + d*x] + (6*I)*Sinh[2*(c + d*x)] + I*Sinh[3*(c + d*x)])/(30*d*(I + Si
nh[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

method	result	size
risch	$\frac{4i(5i\,e^{dx+c}+10\,e^{2dx+2c}-1)}{15d(e^{dx+c}+i)^5}$	40
derivativedivides	$\frac{\frac{2}{i+\tanh(\frac{dx}{2}+\frac{c}{2})}+\frac{8}{5(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^5}-\frac{16}{3(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^3}+\frac{4i}{(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^4}-\frac{4i}{(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^2}}{d}$	88
default	$\frac{\frac{2}{i+\tanh(\frac{dx}{2}+\frac{c}{2})}+\frac{8}{5(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^5}-\frac{16}{3(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^3}+\frac{4i}{(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^4}-\frac{4i}{(i+\tanh(\frac{dx}{2}+\frac{c}{2}))^2}}{d}$	88
parallelrisch	$\frac{\frac{14}{15}+2\tanh(\frac{dx}{2}+\frac{c}{2})^4+4i\tanh(\frac{dx}{2}+\frac{c}{2})^3-\frac{16\tanh(\frac{dx}{2}+\frac{c}{2})^2}{3}-\frac{8i\tanh(\frac{dx}{2}+\frac{c}{2})}{3}}{d\left(\tanh(\frac{dx}{2}+\frac{c}{2})^5+5i\tanh(\frac{dx}{2}+\frac{c}{2})^4-10\tanh(\frac{dx}{2}+\frac{c}{2})^3-10i\tanh(\frac{dx}{2}+\frac{c}{2})^2+5\tanh(\frac{dx}{2}+\frac{c}{2})+i\right)}$	128

```
[In] int(1/(1-I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 4/15*I*(5*I*exp(d*x+c)+10*exp(2*d*x+2*c)-1)/d/(exp(d*x+c)+I)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{4\left(-10i\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}+i\right)}{15\left(de^{(5\,dx+5\,c)}+5i\,de^{(4\,dx+4\,c)}-10\,de^{(3\,dx+3\,c)}-10i\,de^{(2\,dx+2\,c)}+5\,de^{(dx+c)}+i\,d\right)}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="fricas")
[Out] -4/15*(-10*I*e^(2*d*x + 2*c) + 5*e^(d*x + c) + I)/(d*e^(5*d*x + 5*c) + 5*I*
d*e^(4*d*x + 4*c) - 10*d*e^(3*d*x + 3*c) - 10*I*d*e^(2*d*x + 2*c) + 5*d*e^(
d*x + c) + I*d)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = \frac{40ie^{2c}e^{2dx}-20e^{c}e^{dx}-4i}{15de^{5c}e^{5dx}+75ide^{4c}e^{4dx}-150de^{3c}e^{3dx}-150ide^{2c}e^{2dx}+75de^{c}e^{dx}+15id}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))**3,x)
[Out] (40*I*exp(2*c)*exp(2*d*x) - 20*exp(c)*exp(d*x) - 4*I)/(15*d*exp(5*c)*exp(5*
d*x) + 75*I*d*exp(4*c)*exp(4*d*x) - 150*d*exp(3*c)*exp(3*d*x) - 150*I*d*exp
(2*c)*exp(2*d*x) + 75*d*exp(c)*exp(d*x) + 15*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(70) = 140.

Time = 0.22 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx$$
$$= \frac{20i\,e^{(-dx-c)}}{-15\,d(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1)}$$
$$-\frac{40\,e^{(-2\,dx-2\,c)}}{-15\,d(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1)}$$
$$+\frac{4}{-15\,d(-5i\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10i\,e^{(-3\,dx-3\,c)}-5\,e^{(-4\,dx-4\,c)}-i\,e^{(-5\,dx-5\,c)}-1)}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 20*I*e^(-d*x - c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-
3*d*x - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) - 40*e^(-
2*d*x - 2*c)/(d*(75*I*e^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x
 - 3*c) + 75*e^(-4*d*x - 4*c) + 15*I*e^(-5*d*x - 5*c) + 15)) + 4/(d*(75*I*e
^(-d*x - c) - 150*e^(-2*d*x - 2*c) - 150*I*e^(-3*d*x - 3*c) + 75*e^(-4*d*x
- 4*c) + 15*I*e^(-5*d*x - 5*c) + 15))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.41

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = \frac{4i\left(10\,e^{(2\,dx+2\,c)}+5i\,e^{(dx+c)}-1\right)}{15\,d(e^{(dx+c)}+i)^5}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/15*I*(10*e^(2*d*x + 2*c) + 5*I*e^(d*x + c) - 1)/(d*(e^(d*x + c) + I)^5)
```

Mupad [B] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{1}{(1-i\sinh(c+dx))^3}\,dx = -\frac{4\left(10\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,5\mathrm{i}\right)}{15\,d\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^5}$$

```
[In] int(-1/(sinh(c + d*x)*1i - 1)^3,x)
[Out] -(4*(exp(c + d*x)*5i + 10*exp(2*c + 2*d*x) - 1))/(15*d*(exp(c + d*x)*1i - 1
)^5)
```

3.63 $\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}$$

```
[Out] -1/7*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^4-3/35*I*cosh(d*x+c)/d/(1-I*sinh(d*x
+c))^3-2/35*I*cosh(d*x+c)/d/(1-I*sinh(d*x+c))^2-2/35*I*cosh(d*x+c)/d/(1-I*s
inh(d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2729, 2727}

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))} - \frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2} - \frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3} - \frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}$$

```
[In] Int[(1 - I*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-1/7*I)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x])^4) - (((3*I)/35)*Cosh[c +
 d*x])/(d*(1 - I*Sinh[c + d*x])^3) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*S
inh[c + d*x])^2) - (((2*I)/35)*Cosh[c + d*x])/(d*(1 - I*Sinh[c + d*x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}+\frac{3}{7}\int\frac{1}{(1-i\sinh(c+dx))^3}\,dx\\
&= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}+\frac{6}{35}\int\frac{1}{(1-i\sinh(c+dx))^2}\,dx\\
&= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}\\
&\quad-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}+\frac{2}{35}\int\frac{1}{1-i\sinh(c+dx)}\,dx\\
&= -\frac{i\cosh(c+dx)}{7d(1-i\sinh(c+dx))^4}-\frac{3i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^3}\\
&\quad-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))^2}-\frac{2i\cosh(c+dx)}{35d(1-i\sinh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

$$\int\frac{1}{(1-i\sinh(c+dx))^4}\,dx = \frac{-21i\cosh\left(\frac{3}{2}(c+dx)\right)+i\cosh\left(\frac{7}{2}(c+dx)\right)+35\sinh\left(\frac{1}{2}(c+dx)\right)-7\sinh\left(\frac{5}{2}(c+dx)\right)}{70d\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^7}$$

```
[In] Integrate[(1 - I*Sinh[c + d*x])^(-4),x]
[Out] ((-21*I)*Cosh[(3*(c + d*x))/2] + I*Cosh[(7*(c + d*x))/2] + 35*Sinh[(c + d*x
)/2] - 7*Sinh[(5*(c + d*x))/2])/(70*d*(Cosh[(c + d*x)/2] - I*Sinh[(c + d*x)
/2])^7)
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{4\left(-7\,\mathrm{e}^{dx+c}+21i\,\mathrm{e}^{2dx+2c}+35\,\mathrm{e}^{3dx+3c}-i\right)}{35d\left(\mathrm{e}^{dx+c}+i\right)^{7}}$
derivativedivides	$\frac{\frac{16i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{72}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{12}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{16}{7\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}-\frac{8i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
default	$\frac{\frac{16i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{2}{i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{72}{5\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}}-\frac{12}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}}-\frac{16}{7\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}}-\frac{8i}{\left(i+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{d}$
parallelrisch	$\frac{-\frac{12}{35}+\frac{12i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{35}-\frac{6i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5}-\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{5}+\frac{2i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}+\frac{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{5}}{d\left(-21\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}+7i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}+35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-35i\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-7\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right.}$

```
[In] int(1/(1-I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -4/35*(-7*exp(d*x+c)+21*I*exp(2*d*x+2*c)+35*exp(3*d*x+3*c)-I)/d/(exp(d*x+c)
+I)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}+21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}-i\right)}{35\left(de^{(7\,dx+7\,c)}+7i\,de^{(6\,dx+6\,c)}-21\,de^{(5\,dx+5\,c)}-35i\,de^{(4\,dx+4\,c)}+35\,de^{(3\,dx+3\,c)}+21i\,de^{(2\,dx+2\,c)}-7\,de^{(dx\right.}}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) + 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) - I)/(d*e^
(7*d*x + 7*c) + 7*I*d*e^(6*d*x + 6*c) - 21*d*e^(5*d*x + 5*c) - 35*I*d*e^(4*
d*x + 4*c) + 35*d*e^(3*d*x + 3*c) + 21*I*d*e^(2*d*x + 2*c) - 7*d*e^(d*x + c
) - I*d)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = \frac{-140e^{3c}e^{3dx} - 84ie^{2c}e^{2dx} + 28e^{c}e^{dx} + 4i}{35de^{7c}e^{7dx} + 245ide^{6c}e^{6dx} - 735de^{5c}e^{5dx} - 1225ide^{4c}e^{4dx} + 1225de^{3c}e^{3dx} + 735ide^{2c}e^{2dx} - 245de^{c}e^{dx} - 35ic}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))**4,x)
```

```
[Out] (-140*exp(3*c)*exp(3*d*x) - 84*I*exp(2*c)*exp(2*d*x) + 28*exp(c)*exp(d*x) +
 4*I)/(35*d*exp(7*c)*exp(7*d*x) + 245*I*d*exp(6*c)*exp(6*d*x) - 735*d*exp(5
*c)*exp(5*d*x) - 1225*I*d*exp(4*c)*exp(4*d*x) + 1225*d*exp(3*c)*exp(3*d*x)
+ 735*I*d*exp(2*c)*exp(2*d*x) - 245*d*exp(c)*exp(d*x) - 35*I*d)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(93) = 186.

Time = 0.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.18

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-7\,c)}}$$
$$+ \frac{12i\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-7}}$$
$$- \frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-7\,c)}}$$
$$- \frac{4i}{35\,d(7\,e^{(-dx-c)} + 21i\,e^{(-2\,dx-2\,c)} - 35\,e^{(-3\,dx-3\,c)} - 35i\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} + 7i\,e^{(-6\,dx-6\,c)} - e^{(-7\,dx-}}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x
- 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c)
 - e^(-7*d*x - 7*c) - I)) + 12/5*I*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) + 21
*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-
5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c) - e^(-7*d*x - 7*c) - I)) - 4*e^(-3*d*x
- 3*c)/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^(-3*d*x - 3*c) - 3
5*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*x - 6*c) - e^(-7*d
*x - 7*c) - I)) - 4/35*I/(d*(7*e^(-d*x - c) + 21*I*e^(-2*d*x - 2*c) - 35*e^
(-3*d*x - 3*c) - 35*I*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*I*e^(-6*d*
x - 6*c) - e^(-7*d*x - 7*c) - I))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}+21i\,e^{(2\,dx+2\,c)}-7\,e^{(dx+c)}-i\right)}{35\,d\left(e^{(dx+c)}+i\right)^7}$$

```
[In] integrate(1/(1-I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -4/35*(35*e^(3*d*x + 3*c) + 21*I*e^(2*d*x + 2*c) - 7*e^(d*x + c) - I)/(d*(e
^(d*x + c) + I)^7)
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \frac{1}{(1-i\sinh(c+dx))^4}\,dx = -\frac{4\left(21\,\mathrm{e}^{2\,c+2\,d\,x}-1+\mathrm{e}^{c+d\,x}\,7\mathrm{i}-\mathrm{e}^{3\,c+3\,d\,x}\,35\mathrm{i}\right)}{35\,d\,\left(-1+\mathrm{e}^{c+d\,x}\,1\mathrm{i}\right)^7}$$

```
[In] int(1/(sinh(c + d*x)*1i - 1)^4,x)
```

```
[Out] -(4*(exp(c + d*x)*7i + 21*exp(2*c + 2*d*x) - exp(3*c + 3*d*x)*35i - 1))/(35
*d*(exp(c + d*x)*1i - 1)^7)
```

3.64 $\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = -\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

```
[Out] -arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))*2^(1/2)/a^(1/2)
+2*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2830, 2728, 212}

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}$$

```
[In] Int[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}} + i\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx \\
&= \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}} - 2\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right) \\
&= -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}} + \frac{2\cosh(x)}{\sqrt{a+ia\sinh(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{2\left((1+i)\sqrt[4]{-1}\arctan\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

```
[In] Integrate[Sinh[x]/Sqrt[a + I*a*Sinh[x]],x]
[Out] (2*((1 + I)*(-1)^(1/4)*ArcTan[(I + Tanh[x/4])/Sqrt[2]] + Cosh[x/2] - I*Sinh
[x/2])*(Cosh[x/2] + I*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(44) = 88.

Time = 4.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{(e^x-i)^2\sqrt{2}\,e^{-x}}{\sqrt{a(ie^{2x}+2\,e^x-i)\,e^{-x}}}-\frac{2i(-e^x+i)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{ia\,e^x}}{\sqrt{a}}\right)a\sqrt{ia\,e^x}\right)\sqrt{2}\,e^{-x}}{a^{\frac{3}{2}}\sqrt{a(ie^{2x}+2\,e^x-i)\,e^{-x}}}$	108

```
[In] int(sinh(x)/(a+I*a*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (exp(x)-I)^2*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/exp(x))^(1/2)/exp(x)-2*I*(-
exp(x)+I)*(a^(3/2)+arctan((I*a*exp(x))^(1/2)/a^(1/2))*a*(I*a*exp(x))^(1/2))
/a^(3/2)*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/exp(x))^(1/2)/exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = -\frac{\sqrt{2}\sqrt{a}\log\left(\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\sqrt{a}\log\left(-\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}\right)+2\sqrt{\frac{1}{2}i\,ae^{(-x)}}(i\,e^x-1)}{a}$$

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) - sqrt(2)
*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(1/2*I*a*e^(-x))) + 2*sqrt(1/2*I*a*
e^(-x))*(I*e^x - 1))/a
```

Sympy [F]

$$\int \frac{\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{ia(\sinh(x)-i)}}\,dx$$

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(I*a*(sinh(x) - I)), x)
```

Maxima [F]

$$\int \frac{\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{i\,a\sinh(x) + a}}\,dx$$

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{i\,a\sinh(x) + a}}\,dx$$

```
[In] integrate(sinh(x)/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sinh(x)/sqrt(I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\sqrt{a + ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{a + a\sinh(x)\ 1\mathrm{i}}}\,dx$$

```
[In] int(sinh(x)/(a + a*sinh(x)*1i)^(1/2),x)
[Out] int(sinh(x)/(a + a*sinh(x)*1i)^(1/2), x)
```

3.65 $\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx=-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}$$

```
[Out] -arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a-I*a*sinh(x))^(1/2))*2^(1/2)/a^(1/2)
+2*cosh(x)/(a-I*a*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2830, 2728, 212}

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx=\frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}}$$

```
[In] Int[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a - I*a*Sinh[x]])])/Sqrt
[a]) + (2*Cosh[x])/Sqrt[a - I*a*Sinh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}} - i\int\frac{1}{\sqrt{a-ia\sinh(x)}}\,dx\\ &= \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}} + 2\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx, x, -\frac{a\cosh(x)}{\sqrt{a-ia\sinh(x)}}\right)\\ &= -\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a-ia\sinh(x)}}\right)}{\sqrt{a}} + \frac{2\cosh(x)}{\sqrt{a-ia\sinh(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int\frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx = \frac{2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\left(\cosh\left(\frac{x}{2}\right)+i\left((1+i)(-1)^{3/4}\arctan\left(\frac{-i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+\sinh\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a-ia\sinh(x)}}$$

```
[In] Integrate[Sinh[x]/Sqrt[a - I*a*Sinh[x]],x]
```

```
[Out] (2*(Cosh[x/2] - I*Sinh[x/2])*(Cosh[x/2] + I*((1 + I)*(-1)^(3/4)*ArcTan[(-I
+ Tanh[x/4])/Sqrt[2]] + Sinh[x/2])))/Sqrt[a - I*a*Sinh[x]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(44) = 88.

Time = 4.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.89

method	result	size
risch	$\frac{(\mathrm{e}^{x}+i)^{2}\sqrt{2}\,\mathrm{e}^{-x}}{\sqrt{-a(i\mathrm{e}^{2x}-2\,\mathrm{e}^{x}-i)\mathrm{e}^{-x}}}-\frac{2i(\mathrm{e}^{x}+i)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{-ia\,\mathrm{e}^{x}}}{\sqrt{a}}\right)a\sqrt{-ia\,\mathrm{e}^{x}}\right)\sqrt{2}\,\mathrm{e}^{-x}}{a^{\frac{3}{2}}\sqrt{-a(i\mathrm{e}^{2x}-2\,\mathrm{e}^{x}-i)\mathrm{e}^{-x}}}$	108

```
[In] int(sinh(x)/(a-I*a*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (exp(x)+I)^2*2^(1/2)/(-a*(I*exp(x)^2-2*exp(x)-I)/exp(x))^(1/2)/exp(x)-2*I*(
exp(x)+I)*(a^(3/2)+arctan((-I*a*exp(x))^(1/2)/a^(1/2))*a*(-I*a*exp(x))^(1/2
))/a^(3/2)*2^(1/2)/(-a*(I*exp(x)^2-2*exp(x)-I)/exp(x))^(1/2)/exp(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx = -\frac{\sqrt{2}\sqrt{a}\log\left(\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)-\sqrt{2}\sqrt{a}\log\left(-\frac{1}{2}\sqrt{2}\sqrt{a}+\sqrt{-\frac{1}{2}i\,ae^{(-x)}}\right)+2\sqrt{-\frac{1}{2}i\,ae^{(-x)}}(-i\,e^{x}-\cdots}{a}$$

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(a)*log(1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) - sqrt(2
)*sqrt(a)*log(-1/2*sqrt(2)*sqrt(a) + sqrt(-1/2*I*a*e^(-x))) + 2*sqrt(-1/2*I
*a*e^(-x))*(-I*e^x - 1))/a
```

Sympy [F]

$$\int \frac{\sinh(x)}{\sqrt{a-ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{-ia\left(\sinh(x)+i\right)}}\,dx$$

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(-I*a*(sinh(x) + I)), x)
```

Maxima [F]

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x) + a}}\,dx$$

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{-i\,a\sinh(x) + a}}\,dx$$

```
[In] integrate(sinh(x)/(a-I*a*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sinh(x)/sqrt(-I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\sqrt{a - ia\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{a - a\sinh(x)\ 1\mathrm{i}}}\,dx$$

```
[In] int(sinh(x)/(a - a*sinh(x)*1i)^(1/2),x)
[Out] int(sinh(x)/(a - a*sinh(x)*1i)^(1/2), x)
```

3.66 $\int (a+ia\sinh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 104

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx = \frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}} + \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d} + \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

```
[Out] 2/5*I*a*cosh(d*x+c)*(a+I*a*sinh(d*x+c))^(3/2)/d+64/15*I*a^3*cosh(d*x+c)/d/(
a+I*a*sinh(d*x+c))^(1/2)+16/15*I*a^2*cosh(d*x+c)*(a+I*a*sinh(d*x+c))^(1/2)/
d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2726, 2725}

$$\int (a+ia\sinh(c+dx))^{5/2}\,dx = \frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}} + \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d} + \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}$$

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (((64*I)/15)*a^3*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((16*I)/
15)*a^2*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d + (((2*I)/5)*a*Cosh[c
+ d*x]*(a + I*a*Sinh[c + d*x])^(3/2))/d
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}+\frac{1}{5}(8a)\int(a+ia\sinh(c+dx))^{3/2}\,dx\\
&= \frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}\\
&\quad+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}+\frac{1}{15}\left(32a^2\right)\int\sqrt{a+ia\sinh(c+dx)}\,dx\\
&= \frac{64ia^3\cosh(c+dx)}{15d\sqrt{a+ia\sinh(c+dx)}}+\frac{16ia^2\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{15d}\\
&\quad+\frac{2ia\cosh(c+dx)(a+ia\sinh(c+dx))^{3/2}}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.04 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.39

$$\int(a+ia\sinh(c+dx))^{5/2}\,dx=\frac{a^2(-i+\sinh(c+dx))^2\sqrt{a+ia\sinh(c+dx)}\left(-150i\cosh\left(\frac{1}{2}(c+dx)\right)-25i\cosh\left(\frac{3}{2}(c+dx)\right)\right.}{30d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+\right.}$$

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (a^2*(-I + Sinh[c + d*x])^2*Sqrt[a + I*a*Sinh[c + d*x]]*((-150*I)*Cosh[(c +
 d*x)/2] - (25*I)*Cosh[(3*(c + d*x))/2] + (3*I)*Cosh[(5*(c + d*x))/2] - 150
*Sinh[(c + d*x)/2] + 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(
30*d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^5)
```

Maple [F]

$$\int (a + ia \sinh (dx + c))^{\frac{5}{2}} \, dx$$

```
[In] int((a+I*a*sinh(d*x+c))^(5/2),x)
[Out] int((a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int (a + ia \sinh(c + dx))^{5/2} \, dx =$$

$$-\frac{\left(3\,a^2 e^{(5\,dx+5\,c)} - 25i\,a^2 e^{(4\,dx+4\,c)} - 150\,a^2 e^{(3\,dx+3\,c)} - 150i\,a^2 e^{(2\,dx+2\,c)} - 25\,a^2 e^{(dx+c)} + 3i\,a^2\right)\sqrt{\frac{1}{2}i\,a e^{(-dx-c)}}e^{(-}}{30\,d}$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -1/30*(3*a^2*e^(5*d*x + 5*c) - 25*I*a^2*e^(4*d*x + 4*c) - 150*a^2*e^(3*d*x
+ 3*c) - 150*I*a^2*e^(2*d*x + 2*c) - 25*a^2*e^(d*x + c) + 3*I*a^2)*sqrt(1/2
*I*a*e^(-d*x - c))*e^(-2*d*x - 2*c)/d
```

Sympy [F(-1)]

Timed out.

$$\int (a + ia \sinh(c + dx))^{5/2} \, dx = \text{Timed out}$$

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (a + ia \sinh(c + dx))^{5/2} \, dx = \int (i\,a \sinh (dx + c) + a)^{\frac{5}{2}} \, dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

Giac [F]

$$\int (a + ia\sinh(c + dx))^{5/2} \, dx = \int (i\,a\sinh\left(dx + c\right) + a)^{\frac{5}{2}} \, dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a + ia\sinh(c + dx))^{5/2} \, dx = \int \left(a + a\sinh(c + d\,x)\ 1\mathrm{i}\right)^{5/2} \, dx$$

```
[In] int((a + a*sinh(c + d*x)*1i)^(5/2),x)
[Out] int((a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.67 $\int (a + ia\sinh(c + dx))^{3/2}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (a + ia\sinh(c + dx))^{3/2}\, dx = \frac{8ia^2\cosh(c + dx)}{3d\sqrt{a + ia\sinh(c + dx)}} + \frac{2ia\cosh(c + dx)\sqrt{a + ia\sinh(c + dx)}}{3d}$$

```
[Out] 8/3*I*a^2*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)+2/3*I*a*cosh(d*x+c)*(a+I*
a*sinh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2726, 2725}

$$\int (a + ia\sinh(c + dx))^{3/2}\, dx = \frac{8ia^2\cosh(c + dx)}{3d\sqrt{a + ia\sinh(c + dx)}} + \frac{2ia\cosh(c + dx)\sqrt{a + ia\sinh(c + dx)}}{3d}$$

```
[In] Int[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] (((8*I)/3)*a^2*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((2*I)/3)*
a*Cosh[c + d*x]*Sqrt[a + I*a*Sinh[c + d*x]])/d
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d} + \frac{1}{3}(4a)\int\sqrt{a+ia\sinh(c+dx)}\,dx\\ &= \frac{8ia^2\cosh(c+dx)}{3d\sqrt{a+ia\sinh(c+dx)}} + \frac{2ia\cosh(c+dx)\sqrt{a+ia\sinh(c+dx)}}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 5.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.64

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx =$$

$$-\frac{a(-i+\sinh(c+dx))\sqrt{a+ia\sinh(c+dx)}\left(9\cosh\left(\frac{1}{2}(c+dx)\right)+\cosh\left(\frac{3}{2}(c+dx)\right)-9i\sinh\left(\frac{1}{2}(c+dx)\right)+}{3d\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^3}$$

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(3/2),x]
```

```
[Out] -1/3*(a*(-I + Sinh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]]*(9*Cosh[(c + d*x)/
2] + Cosh[(3*(c + d*x))/2] - (9*I)*Sinh[(c + d*x)/2] + I*Sinh[(3*(c + d*x))
/2]))/(d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^3)
```

Maple [F]

$$\int (a+ia\sinh(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx = \frac{\left(i\,ae^{(3\,dx+3\,c)}+9\,ae^{(2\,dx+2\,c)}+9i\,ae^{(dx+c)}+a\right)\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}e^{(-dx-c)}}{3\,d}$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/3*(I*a*e^(3*d*x + 3*c) + 9*a*e^(2*d*x + 2*c) + 9*I*a*e^(d*x + c) + a)*sqr
t(1/2*I*a*e^(-d*x - c))*e^(-d*x - c)/d
```

Sympy [F]

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx = \int \left(ia\sinh\left(c+dx\right)+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))**(3/2),x)
[Out] Integral((I*a*sinh(c + d*x) + a)**(3/2), x)
```

Maxima [F]

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx = \int \left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx = \int \left(i\,a\sinh\left(dx+c\right)+a\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\sinh(c+dx))^{3/2}\,dx=\int (a+a\sinh(c+d\,x)\ 1\mathrm{i})^{3/2}\,dx$$

```
[In] int((a + a*sinh(c + d*x)*1i)^(3/2),x)
[Out] int((a + a*sinh(c + d*x)*1i)^(3/2), x)
```

3.68 $\int \sqrt{a+ia\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

```
[Out] 2*I*a*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2725}

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

```
[In] Int[Sqrt[a + I*a*Sinh[c + d*x]],x]
[Out] ((2*I)*a*Cosh[c + d*x])/(d*Sqrt[a + I*a*Sinh[c + d*x]])
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2ia\cosh(c+dx)}{d\sqrt{a+ia\sinh(c+dx)}}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.39

$$\int \sqrt{a + ia\sinh(c + dx)}\,dx = \frac{2\left(i\cosh\left(\frac{1}{2}(c + dx)\right) + \sinh\left(\frac{1}{2}(c + dx)\right)\right)\sqrt{a + ia\sinh(c + dx)}}{d\left(\cosh\left(\frac{1}{2}(c + dx)\right) + i\sinh\left(\frac{1}{2}(c + dx)\right)\right)}$$

```
[In] Integrate[Sqrt[a + I*a*Sinh[c + d*x]],x]
[Out] (2*(I*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])*Sqrt[a + I*a*Sinh[c + d*x]])/(
d*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(27) = 54$.

Time = 2.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.87

method	result	size
risch	$\frac{i\sqrt{2}\,\sqrt{a\left(i\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{dx+c}-i\right)\mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}+i\right)\left(\mathrm{e}^{dx+c}-i\right)}{\left(i\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{dx+c}-i\right)d}$	89

```
[In] int((a+I*a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] I*2^(1/2)*(a*(I*exp(2*d*x+2*c)+2*exp(d*x+c)-I)*exp(-d*x-c))^(1/2)/(I*exp(2*
d*x+2*c)+2*exp(d*x+c)-I)*(exp(d*x+c)+I)*(exp(d*x+c)-I)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \sqrt{a + ia\sinh(c + dx)}\,dx = \frac{2\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\left(e^{(dx+c)} + i\right)}{d}$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(1/2*I*a*e^(-d*x - c))*(e^(d*x + c) + I)/d
```

Sympy [F]

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \int \sqrt{ia\sinh{(c+dx)}+a}\,dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(I*a*sinh(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \int \sqrt{i\,a\sinh{(dx+c)}+a}\,dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \int \sqrt{i\,a\sinh{(dx+c)}+a}\,dx$$

```
[In] integrate((a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(I*a*sinh(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \sqrt{a+ia\sinh(c+dx)}\,dx = \frac{\sqrt{2}\,\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,\sqrt{a\,\mathrm{e}^{-c-d\,x}\,{\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}^2\,1\mathrm{i}}}{d\,\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)}$$

```
[In] int((a + a*sinh(c + d*x)*1i)^(1/2),x)
[Out] (2^(1/2)*(exp(c + d*x) + 1i)*(a*exp(- c - d*x)*(exp(c + d*x) - 1i)^2*1i)^(1
/2))/(d*(exp(c + d*x) - 1i))
```

3.69 $\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx=\frac{i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] I*arctanh(1/2*cosh(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2728, 212}

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx=\frac{i\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
```

```
[Out] (I*Sqrt[2]*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d
*x]])])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(2i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{d}$$
$$= \frac{i\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx$$
$$= \frac{(2+2i)\sqrt[4]{-1}\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{-1}\left(1-i\tanh\left(\frac{1}{4}(c+dx)\right)\right)\right)\left(-i\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a+ia\sinh(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a + I*a*Sinh[c + d*x]],x]
[Out] ((2 + 2*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4
])]*((-I)*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]))/(d*Sqrt[a + I*a*Sinh[c +
d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(41) = 82.

Time = 6.70 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.08

method	result	size
risch	$-\frac{2\left(\mathrm{e}^{dx+c}-i\right)\sqrt{2}\,\mathrm{e}^{-dx-c}}{d\sqrt{a\left(i\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{dx+c}-i\right)\mathrm{e}^{-dx-c}}}-\frac{2\left(-\mathrm{e}^{dx+c}+i\right)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{i\mathrm{e}^{dx+c}a}}{\sqrt{a}}\right)a\sqrt{i\mathrm{e}^{dx+c}a}\right)\sqrt{2}\,\mathrm{e}^{-dx-c}}{d\,a^{\frac{3}{2}}\sqrt{a\left(i\mathrm{e}^{2dx+2c}+2\,\mathrm{e}^{dx+c}-i\right)\mathrm{e}^{-dx-c}}}$	160

```
[In] int(1/(a+I*a*sinh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d*(exp(d*x+c)-I)*2^(1/2)/(a*(I*exp(d*x+c)^2+2*exp(d*x+c)-I)/exp(d*x+c))^
(1/2)/exp(d*x+c)-2/d*(-exp(d*x+c)+I)*(a^(3/2)+arctan((I*exp(d*x+c)*a)^(1/2)
/a^(1/2))*a*(I*exp(d*x+c)*a)^(1/2))/a^(3/2)*2^(1/2)/(a*(I*exp(d*x+c)^2+2*ex
p(d*x+c)-I)/exp(d*x+c))^(1/2)/exp(d*x+c)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(39) = 78$.

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(\frac{1}{2}\sqrt{2}ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right) - i\sqrt{2}\sqrt{\frac{1}{ad^2}}\log\left(-\frac{1}{2}\sqrt{2}ad\sqrt{\frac{1}{ad^2}}+\sqrt{\frac{1}{2}i\,ae^{(-dx-c)}}\right)$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] I*sqrt(2)*sqrt(1/(a*d^2))*log(1/2*sqrt(2)*a*d*sqrt(1/(a*d^2)) + sqrt(1/2*I*
a*e^(-d*x - c))) - I*sqrt(2)*sqrt(1/(a*d^2))*log(-1/2*sqrt(2)*a*d*sqrt(1/(a
*d^2)) + sqrt(1/2*I*a*e^(-d*x - c)))
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{ia\sinh(c+dx)+a}}\,dx$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(I*a*sinh(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx = \int \frac{1}{\sqrt{i\,a\sinh(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx=\int \frac{1}{\sqrt{i\,a\sinh\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(I*a*sinh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+a\sinh\left(c+d\,x\right)\,1\mathrm{i}}}\,dx$$

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(1/2),x)
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(1/2), x)
```

3.70 $\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

```
[Out] 1/2*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(3/2)+1/4*I*arctanh(1/2*cosh(d*x+c)
*a^(1/2)*2^(1/2)/(a+I*a*sinh(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2729, 2728, 212}

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \frac{i\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}$$

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((I/2)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c + d*x]]
)])/(Sqrt[2]*a^(3/2)*d) + ((I/2)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d*x])^
(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{4a}\\
&= \frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}+\frac{i\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,x,\frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{2ad}\\
&= \frac{i\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{i\cosh(c+dx)}{2d(a+ia\sinh(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx=\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\cosh\left(\frac{1}{2}(c+dx)\right)-i\left((1-i)\sqrt[4]{-1}\,a\right.\right.}{2ad(-}$$

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-3/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*(Cosh[(c + d*x)/2] - I*((1 - I)*
(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + Sinh[(c + d*x)/2])))/(2*a*d*(-I + Si
nh[c + d*x])*Sqrt[a + I*a*Sinh[c + d*x]])
```

Maple [F]

$$\int \frac{1}{(a+ia\sinh{(dx+c)})^{\frac{3}{2}}}dx$$

[In] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)

[Out] int(1/(a+I*a*sinh(d*x+c))^(3/2),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.70

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(i\,a^2de^{(2\,dx+2\,c)}+2\,a^2de^{(dx+c)}-i\,a^2d\right)\sqrt{\frac{1}{a^3d^2}}\log\left(\sqrt{\frac{1}{2}}a^2d\sqrt{\frac{1}{a^3d^2}}+\sqrt{\frac{1}{2}i\,c\cdots}\right.}{\cdots}$$

[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/2*(sqrt(1/2)*(I*a^2*d*e^(2*d*x + 2*c) + 2*a^2*d*e^(d*x + c) - I*a^2*d)*sqrt(1/(a^3*d^2))*log(sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) + sqrt(1/2)*(-I*a^2*d*e^(2*d*x + 2*c) - 2*a^2*d*e^(d*x + c) + I*a^2*d)*sqrt(1/(a^3*d^2))*log(-sqrt(1/2)*a^2*d*sqrt(1/(a^3*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) - 2*sqrt(1/2*I*a*e^(-d*x - c))*(I*e^(2*d*x + 2*c) - e^(d*x + c)))/(a^2*d*e^(2*d*x + 2*c) - 2*I*a^2*d*e^(d*x + c) - a^2*d)

Sympy [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx = \int \frac{1}{(ia\sinh{(c+dx)}+a)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(a+I*a*sinh(d*x+c))**(3/2),x)

[Out] Integral((I*a*sinh(c + d*x) + a)**(-3/2), x)

Maxima [F]

$$\int \frac{1}{(a + ia\sinh(c + dx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(dx + c\right) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + ia\sinh(c + dx))^{3/2}}\,dx = \int \frac{1}{(i\,a\sinh\left(dx + c\right) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + ia\sinh(c + dx))^{3/2}}\,dx = \int \frac{1}{\left(a + a\sinh\left(c + d\,x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(3/2),x)
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(3/2), x)
```

3.71 $\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 122

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}$$

```
[Out] 1/4*I*cosh(d*x+c)/d/(a+I*a*sinh(d*x+c))^(5/2)+3/16*I*cosh(d*x+c)/a/d/(a+I*a
*sinh(d*x+c))^(3/2)+3/32*I*arctanh(1/2*cosh(d*x+c)*a^(1/2)*2^(1/2)/(a+I*a*s
inh(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2729, 2728, 212}

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{3i\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}}$$

```
[In] Int[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] (((3*I)/16)*ArcTanh[(Sqrt[a]*Cosh[c + d*x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[c +
d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((I/4)*Cosh[c + d*x])/(d*(a + I*a*Sinh[c + d
*x])^(5/2)) + (((3*I)/16)*Cosh[c + d*x])/(a*d*(a + I*a*Sinh[c + d*x])^(3/2)
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3\int \frac{1}{(a+ia\sinh(c+dx))^{3/2}}\,dx}{8a} \\
&= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{a+ia\sinh(c+dx)}}\,dx}{32a^2} \\
&= \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}} \\
&\quad + \frac{(3i)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(c+dx)}{\sqrt{a+ia\sinh(c+dx)}}\right)}{16a^2d} \\
&= \frac{3i\text{arctanh}\left(\frac{\sqrt{a}\cosh(c+dx)}{\sqrt{2}\sqrt{a+ia\sinh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{i\cosh(c+dx)}{4d(a+ia\sinh(c+dx))^{5/2}} + \frac{3i\cosh(c+dx)}{16ad(a+ia\sinh(c+dx))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.72

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(4i\cosh\left(\frac{1}{2}(c+dx)\right)+(3-3i)\sqrt[4]{-1}\right.}{}$$

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(-5/2),x]
```

```
[Out] ((Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])*((4*I)*Cosh[(c + d*x)/2] + (3 -
3*I)*(-1)^(1/4)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 - I*Tanh[(c + d*x)/4])]*(C
osh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^4 + 4*Sinh[(c + d*x)/2] + 6*(Cosh[(
c + d*x)/2] + I*Sinh[(c + d*x)/2])^2*Sinh[(c + d*x)/2] + 3*((-I)*Cosh[(c +
d*x)/2] + Sinh[(c + d*x)/2])^3))/(16*d*(a + I*a*Sinh[c + d*x])^(5/2))
```

Maple [F]

$$\int \frac{1}{(a+ia\sinh(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(1/(a+I*a*sinh(d*x+c))^(5/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(91) = 182.

Time = 0.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.85

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = -\frac{3\sqrt{\frac{1}{2}}\left(-i\,a^3de^{(4\,dx+4\,c)}-4\,a^3de^{(3\,dx+3\,c)}+6i\,a^3de^{(2\,dx+2\,c)}+4\,a^3de^{(dx+c)}-i\,a^3d\right)\sqrt{\frac{1}{a^5d^2}}\log\left(\sqrt{\frac{1}{2}}a^3d\sqrt{\frac{1}{a^5d^2}}+\cdots\right.}{\cdots}$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*sqrt(1/2)*(-I*a^3*d*e^(4*d*x + 4*c) - 4*a^3*d*e^(3*d*x + 3*c) + 6*
I*a^3*d*e^(2*d*x + 2*c) + 4*a^3*d*e^(d*x + c) - I*a^3*d)*sqrt(1/(a^5*d^2))*
log(sqrt(1/2)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) + 3*sqr
t(1/2)*(I*a^3*d*e^(4*d*x + 4*c) + 4*a^3*d*e^(3*d*x + 3*c) - 6*I*a^3*d*e^(2*
d*x + 2*c) - 4*a^3*d*e^(d*x + c) + I*a^3*d)*sqrt(1/(a^5*d^2))*log(-sqrt(1/2
)*a^3*d*sqrt(1/(a^5*d^2)) + sqrt(1/2*I*a*e^(-d*x - c))) - 2*sqrt(1/2*I*a*e^
(-d*x - c))*(-3*I*e^(4*d*x + 4*c) - 11*e^(3*d*x + 3*c) - 11*I*e^(2*d*x + 2*
c) - 3*e^(d*x + c)))/(a^3*d*e^(4*d*x + 4*c) - 4*I*a^3*d*e^(3*d*x + 3*c) - 6
*a^3*d*e^(2*d*x + 2*c) + 4*I*a^3*d*e^(d*x + c) + a^3*d)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(i\,a\sinh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((I*a*sinh(d*x + c) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+ia\sinh(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+a\sinh(c+d\,x)\,1\mathrm{i})^{5/2}}\,dx$$

```
[In] int(1/(a + a*sinh(c + d*x)*1i)^(5/2),x)
[Out] int(1/(a + a*sinh(c + d*x)*1i)^(5/2), x)
```

3.72 $\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a(2a^2-b^2)\,x}{2b^4} - \frac{2a^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}} - \frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b}$$

```
[Out] -1/2*a*(2*a^2-b^2)*x/b^4-1/3*(2-3*a^2/b^2)*cosh(x)/b-1/2*a*cosh(x)*sinh(x)/
b^2+1/3*cosh(x)*sinh(x)^2/b-2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^4/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2872, 3128, 3102, 2814, 2739, 632, 212}

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = -\frac{\left(2-\frac{3a^2}{b^2}\right)\cosh(x)}{3b} - \frac{ax(2a^2-b^2)}{2b^4} - \frac{2a^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}} - \frac{a\sinh(x)\cosh(x)}{2b^2} + \frac{\sinh^2(x)\cosh(x)}{3b}$$

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 - b^2)*x)/b^4 - (2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^4*Sqrt[a^2 + b^2]) - ((2 - (3*a^2)/b^2)*Cosh[x])/(3*b) - (a*Cosh[
x]*Sinh[x])/(2*b^2) + (Cosh[x]*Sinh[x]^2)/(3*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{\sinh(x)\left(2a+2b\sinh(x)+3a\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{3b} \\
&= -\frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{\int \frac{-3a^2+ab\sinh(x)-2\left(3a^2-2b^2\right)\sinh^2(x)}{a+b\sinh(x)}\,dx}{6b^2} \\
&= \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{i\int \frac{3ia^2b-3ia\left(2a^2-b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{6b^3} \\
&= -\frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} \\
&\quad + \frac{\cosh(x)\sinh^2(x)}{3b} + \frac{a^4\int \frac{1}{a+b\sinh(x)}\,dx}{b^4} \\
&= -\frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} \\
&\quad + \frac{\cosh(x)\sinh^2(x)}{3b} + \frac{(2a^4)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^4} \\
&= -\frac{a(2a^2-b^2)\,x}{2b^4} + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} \\
&\quad + \frac{\cosh(x)\sinh^2(x)}{3b} - \frac{(4a^4)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^4} \\
&= -\frac{a(2a^2-b^2)\,x}{2b^4} - \frac{2a^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}} \\
&\quad + \frac{(3a^2-2b^2)\cosh(x)}{3b^3} - \frac{a\cosh(x)\sinh(x)}{2b^2} + \frac{\cosh(x)\sinh^2(x)}{3b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.97

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{3b(4a^2-3b^2)\cosh(x)+b^3\cosh(3x)+3a\left(-4a^2x+2b^2x+\frac{8a^3\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-b^2\sinh(2x)\right)}{12b^4}$$

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (3*b*(4*a^2 - 3*b^2)*Cosh[x] + b^3*Cosh[3*x] + 3*a*(-4*a^2*x + 2*b^2*x + (8
*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - b^2*Sin
h[2*x]))/(12*b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(94) = 188.

Time = 0.67 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{a^3x}{b^4}+\frac{ax}{2b^2}+\frac{e^{3x}}{24b}-\frac{a\,e^{2x}}{8b^2}+\frac{e^{x}a^2}{2b^3}-\frac{3\,e^{x}}{8b}+\frac{e^{-x}a^2}{2b^3}-\frac{3\,e^{-x}}{8b}+\frac{a\,e^{-2x}}{8b^2}+\frac{e^{-3x}}{24b}+\frac{a^4\ln\left(e^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^4}-\ a^4$
default	$-\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{a+b}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{2a^2+ab-b^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\left(2a^2-b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^4}+\frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\ b-\dots$

```
[In] int(sinh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*x/b^4+1/2*a*x/b^2+1/24/b*exp(x)^3-1/8*a/b^2*exp(x)^2+1/2/b^3*exp(x)*a^
2-3/8/b*exp(x)+1/2/b^3/exp(x)*a^2-3/8/b/exp(x)+1/8*a/b^2/exp(x)^2+1/24/b/ex
p(x)^3+1/(a^2+b^2)^(1/2)*a^4/b^4*ln(exp(x)+(a*(a^2+b^2)^(1/2)-a^2-b^2)/(a^2
+b^2)^(1/2)/b)-1/(a^2+b^2)^(1/2)*a^4/b^4*ln(exp(x)+(a*(a^2+b^2)^(1/2)+a^2+b
^2)/(a^2+b^2)^(1/2)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 799 vs. $2(96) = 192$.

Time = 0.30 (sec) , antiderivative size = 799, normalized size of antiderivative = 7.40

$$\int \frac{\sinh^4(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*((a^2*b^3 + b^5)*cosh(x)^6 + (a^2*b^3 + b^5)*sinh(x)^6 - 3*(a^3*b^2 +
a*b^4)*cosh(x)^5 - 3*(a^3*b^2 + a*b^4 - 2*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^
5 + a^2*b^3 + b^5 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4*b +
 a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b
^5)*cosh(x)^2 - 5*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b^3 + b
^5)*cosh(x)^3 - 15*(a^3*b^2 + a*b^4)*cosh(x)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4
)*x + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 + 3*(4*a^4*b + a^2*b
^3 - 3*b^5)*cosh(x)^2 + 3*(4*a^4*b + a^2*b^3 - 3*b^5 + 5*(a^2*b^3 + b^5)*co
sh(x)^4 - 10*(a^3*b^2 + a*b^4)*cosh(x)^3 - 12*(2*a^5 + a^3*b^2 - a*b^4)*x*c
osh(x) + 6*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^4*cosh(
x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*sinh(x)^3)*s
qrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 +
 b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sin
h(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sin
h(x) - b)) + 3*(a^3*b^2 + a*b^4)*cosh(x) + 3*(2*(a^2*b^3 + b^5)*cosh(x)^5 +
 a^3*b^2 + a*b^4 - 5*(a^3*b^2 + a*b^4)*cosh(x)^4 - 12*(2*a^5 + a^3*b^2 - a*
b^4)*x*cosh(x)^2 + 4*(4*a^4*b + a^2*b^3 - 3*b^5)*cosh(x)^3 + 2*(4*a^4*b + a
^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^2*b^4 +
 b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 + b^6)*cosh(x)*sinh(x)^2 + (a^2*b^4 +
b^6)*sinh(x)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(x)}{a + b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.46

$$\int \frac{\sinh^4(x)}{a + b\sinh(x)}\,dx = \frac{a^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^4} - \frac{\left(3\,abe^{(-x)} - b^2 - 3\,(4\,a^2 - 3\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,b^3} + \frac{3\,abe^{(-2\,x)} + b^2e^{(-3\,x)} + 3\,(4\,a^2 - 3\,b^2)e^{(-x)}}{24\,b^3} - \frac{(2\,a^3 - ab^2)x}{2\,b^4}$$

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^4) - 1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 - 3*b^2)*e^(-2*
x))*e^(3*x)/b^3 + 1/24*(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 - 3*b^2)*e
^(-x))/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^4(x)}{a + b\sinh(x)}\,dx = \frac{a^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4} + \frac{b^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^2e^x - 9\,b^2e^x}{24\,b^3} - \frac{(2\,a^3 - ab^2)x}{2\,b^4} + \frac{\left(3\,ab^2e^x + b^3 + 3\,(4\,a^2b - 3\,b^3)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4}$$

```
[In] integrate(sinh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*
a^2*e^x - 9*b^2*e^x)/b^3 - 1/2*(2*a^3 - a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x +
b^3 + 3*(4*a^2*b - 3*b^3)*e^(2*x))*e^(-3*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.84

$$\int \frac{\sinh^4(x)}{a+b\sinh(x)}\,dx = \frac{\mathrm{e}^{-3x}}{24b}+\frac{\mathrm{e}^{3x}}{24b}+\frac{x\,(a\,b^2-2\,a^3)}{2\,b^4}+\frac{\mathrm{e}^{x}\,(4\,a^2-3\,b^2)}{8\,b^3} + \frac{a\,\mathrm{e}^{-2x}}{8\,b^2}-\frac{a\,\mathrm{e}^{2x}}{8\,b^2}+\frac{\mathrm{e}^{-x}\,(4\,a^2-3\,b^2)}{8\,b^3} - \frac{a^4\,\ln\left(-\frac{2\,a^4\,\mathrm{e}^{x}}{b^5}-\frac{2\,a^4\,(b-a\,\mathrm{e}^{x})}{b^5\,\sqrt{a^2+b^2}}\right)}{b^4\,\sqrt{a^2+b^2}}+\frac{a^4\,\ln\left(\frac{2\,a^4\,(b-a\,\mathrm{e}^{x})}{b^5\,\sqrt{a^2+b^2}}-\frac{2\,a^4\,\mathrm{e}^{x}}{b^5}\right)}{b^4\,\sqrt{a^2+b^2}}$$

```
[In] int(sinh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*b) + exp(3*x)/(24*b) + (x*(a*b^2 - 2*a^3))/(2*b^4) + (exp(x)*
(4*a^2 - 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) + (
exp(-x)*(4*a^2 - 3*b^2))/(8*b^3) - (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^4*(
b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(b^4*(a^2 + b^2)^(1/2)) + (a^4*log
((2*a^4*(b - a*exp(x)))/(b^5*(a^2 + b^2)^(1/2)) - (2*a^4*exp(x))/b^5))/(b^4
*(a^2 + b^2)^(1/2))
```

3.73 $\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx=\frac{(2a^2-b^2)\,x}{2b^3}+\frac{2a^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\cosh(x)\sinh(x)}{2b}$$

```
[Out] 1/2*(2*a^2-b^2)*x/b^3-a*cosh(x)/b^2+1/2*cosh(x)*sinh(x)/b+2*a^3*arctanh((b-
a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^3/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2872, 3102, 2814, 2739, 632, 212}

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx=\frac{x(2a^2-b^2)}{2b^3}+\frac{2a^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}}-\frac{a\cosh(x)}{b^2}+\frac{\sinh(x)\cosh(x)}{2b}$$

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2
]])/(b^3*Sqrt[a^2 + b^2]) - (a*Cosh[x])/b^2 + (Cosh[x]*Sinh[x])/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2872

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Dist[1/(d*(m
 + n)), Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d*
(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m]
 || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0] &
& NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\cosh(x)\sinh(x)}{2b} - \frac{\int \frac{a+b\sinh(x)+2a\sinh^2(x)}{a+b\sinh(x)}\,dx}{2b}$$

$$= -\frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{i\int \frac{-iab+i(2a^2-b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^2}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{a^3\int\frac{1}{a+b\sinh(x)}\,dx}{b^3}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} - \frac{(2a^3)\,\mathrm{Subst}\left(\int\frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b} + \frac{(4a^3)\,\mathrm{Subst}\left(\int\frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^3}$$

$$= \frac{(2a^2-b^2)\,x}{2b^3} + \frac{2a^3\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}} - \frac{a\cosh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int\frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \frac{4a^2x-2b^2x-\frac{8a^3\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-4ab\cosh(x)+b^2\sinh(2x)}{4b^3}$$

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x]),x]
[Out] (4*a^2*x - 2*b^2*x - (8*a^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqr
t[-a^2 - b^2] - 4*a*b*Cosh[x] + b^2*Sinh[2*x])/(4*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(72) = 144.

Time = 0.58 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.85

method	result
default	$-\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-b+2a}{2b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(2a^2-b^2)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b^3}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-b-2a}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-2a^2+b^2)\,\ln}{2}$
risch	$\frac{x\,a^2}{b^3}-\frac{x}{2b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}-\frac{a\,e^{-x}}{2b^2}-\frac{e^{-2x}}{8b}+\frac{a^3\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^3}-\frac{a^3\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^3}$

```
[In] int(sinh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2/b/(tanh(1/2*x)+1)^2-1/2*(-b+2*a)/b^2/(tanh(1/2*x)+1)+1/2*(2*a^2-b^2)/b
^3*ln(tanh(1/2*x)+1)+1/2/b/(tanh(1/2*x)-1)^2-1/2*(-b-2*a)/b^2/(tanh(1/2*x)-
```

```
1)+1/2/b^3*(-2*a^2+b^2)*ln(tanh(1/2*x)-1)-2*a^3/b^3/(a^2+b^2)^(1/2)*arctanh
(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(74) = 148$.

Time = 0.30 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.60

$$\int \frac{\sinh^3(x)}{a + b\sinh(x)}\,dx = \frac{(a^2b^2 + b^4)\cosh(x)^4 + (a^2b^2 + b^4)\sinh(x)^4 - a^2b^2 - b^4 + 4(2a^4 + a^2b^2 - b^4)x\cosh(x)^2 - 4(a^3b + ab^3)\cosh(x)^3 \cdots}{\cdots}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^2 + b^4)*sinh(x)^4 - a^2*b^2 - b^4
+ 4*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x)^3 - 4*(
a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 + b^4)*c
osh(x)^2 + 2*(2*a^4 + a^2*b^2 - b^4)*x - 6*(a^3*b + a*b^3)*cosh(x))*sinh(x)
^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a^2 + b
^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b
^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/
(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b))
- 4*(a^3*b + a*b^3)*cosh(x) - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(x)^3
- 2*(2*a^4 + a^2*b^2 - b^4)*x*cosh(x) + 3*(a^3*b + a*b^3)*cosh(x)^2)*sinh(x
))/((a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^2*b^3 + b^5)*cosh(x)*sinh(x) + (a^2*b^
3 + b^5)*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a + b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = -\frac{a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^3} - \frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2} - \frac{4\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^2} + \frac{(2\,a^2-b^2)x}{2\,b^3}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^3) - 1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x)
 + b*e^(-2*x))/b^2 + 1/2*(2*a^2 - b^2)*x/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.43

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = -\frac{a^3\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^3} + \frac{be^{(2\,x)}-4\,ae^x}{8\,b^2} + \frac{(2\,a^2-b^2)x}{2\,b^3} - \frac{(4\,abe^x+b^2)e^{(-2\,x)}}{8\,b^3}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*b^3) + 1/8*(b*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2
*a^2 - b^2)*x/b^3 - 1/8*(4*a*b*e^x + b^2)*e^(-2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.94

$$\int \frac{\sinh^3(x)}{a+b\sinh(x)}\,dx = \frac{e^{2\,x}}{8\,b} - \frac{e^{-2\,x}}{8\,b} + \frac{x\,(2\,a^2-b^2)}{2\,b^3} - \frac{a\,e^x}{2\,b^2} - \frac{a\,e^{-x}}{2\,b^2} - \frac{a^3\,\ln\left(\frac{2\,a^3\,e^x}{b^4} - \frac{2\,a^3\,(b-a\,e^x)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,\sqrt{a^2+b^2}} + \frac{a^3\,\ln\left(\frac{2\,a^3\,e^x}{b^4} + \frac{2\,a^3\,(b-a\,e^x)}{b^4\,\sqrt{a^2+b^2}}\right)}{b^3\,\sqrt{a^2+b^2}}$$

```
[In] int(sinh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] exp(2*x)/(8*b) - exp(-2*x)/(8*b) + (x*(2*a^2 - b^2))/(2*b^3) - (a*exp(x))/(
2*b^2) - (a*exp(-x))/(2*b^2) - (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b - a*
exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(a^2 + b^2)^(1/2)) + (a^3*log((2*a^
3*exp(x))/b^4 + (2*a^3*(b - a*exp(x)))/(b^4*(a^2 + b^2)^(1/2))))/(b^3*(a^2
+ b^2)^(1/2))
```

3.74 $\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx=-\frac{ax}{b^2}-\frac{2a^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{\cosh(x)}{b}$$

```
[Out] -a*x/b^2+cosh(x)/b-2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^
2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2825, 12, 2814, 2739, 632, 212}

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx=-\frac{2a^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt
[a^2 + b^2]) + Cosh[x]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2825

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Dist[1/d, I
nt[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\cosh(x)}{b} - \frac{\int \frac{a\sinh(x)}{a+b\sinh(x)}\,dx}{b} \\
&= \frac{\cosh(x)}{b} - \frac{a\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} + \frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} - \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}
\end{aligned}
$$

$$= -\frac{ax}{b^2} - \frac{2a^2 \operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{\cosh(x)}{b}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{a\left(-x+\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)+b\cosh(x)}{b^2}$$

```
[In] Integrate[Sinh[x]^2/(a + b*Sinh[x]),x]
[Out] (a*(-x + (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])
 + b*Cosh[x])/b^2
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result	size
default	$-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}+\frac{2a^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}$	92
risch	$-\frac{ax}{b^2}+\frac{e^x}{2b}+\frac{e^{-x}}{2b}+\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}$	132

```
[In] int(sinh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] -1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)+1/b/(tanh(1/2*x)+1)-a/b^2*ln(t
anh(1/2*x)+1)+2*a^2/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(53) = 106$.

Time = 0.35 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.18

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{a^2b+b^3-2\left(a^3+ab^2\right)x\cosh\left(x\right)+\left(a^2b+b^3\right)\cosh\left(x\right)^2+\left(a^2b+b^3\right)\sinh\left(x\right)^2+2\left(a^2\cosh\left(x\right)+a^2\sinh\left(x\right.\right.}{2}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b + b^3 - 2*(a^3 + a*b^2)*x*cosh(x) + (a^2*b + b^3)*cosh(x)^2 + (a
^2*b + b^3)*sinh(x)^2 + 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 + b^2)*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x
) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x
)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*((a^3
 + a*b^2)*x - (a^2*b + b^3)*cosh(x))*sinh(x))/((a^2*b^2 + b^4)*cosh(x) + (a
^2*b^2 + b^4)*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(49) = 98$.

Time = 110.09 (sec) , antiderivative size = 1253, normalized size of antiderivative = 21.98

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*cosh(x), Eq(a, 0) & Eq(b, 0)), (cosh(x)/b, Eq(a, 0)), (b*x*t
anh(x/2)**2/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2)*tanh(x/2)**
2 + b*sqrt(-b**2)) - b*x/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2
)*tanh(x/2)**2 + b*sqrt(-b**2)) - 2*b*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*t
anh(x/2) - b*sqrt(-b**2)*tanh(x/2)**2 + b*sqrt(-b**2)) + x*sqrt(-b**2)*tanh
(x/2)**3/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2)*tanh(x/2)**2 +
 b*sqrt(-b**2)) - x*sqrt(-b**2)*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/
2) - b*sqrt(-b**2)*tanh(x/2)**2 + b*sqrt(-b**2)) - 2*sqrt(-b**2)*tanh(x/2)*
*2/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**2)*tanh(x/2)**2 + b*sqr
t(-b**2)) + 4*sqrt(-b**2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) - b*sqrt(-b**
2)*tanh(x/2)**2 + b*sqrt(-b**2)), Eq(a, -sqrt(-b**2))), (b*x*tanh(x/2)**2/(
b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b
**2)) - b*x/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**
```

```
2 - b*sqrt(-b**2)) - 2*b*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*
sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b**2)) - x*sqrt(-b**2)*tanh(x/2)**3/(b**
2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b**2
)) + x*sqrt(-b**2)*tanh(x/2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-
b**2)*tanh(x/2)**2 - b*sqrt(-b**2)) + 2*sqrt(-b**2)*tanh(x/2)**2/(b**2*tanh
(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)**2 - b*sqrt(-b**2)) - 4
*sqrt(-b**2)/(b**2*tanh(x/2)**3 - b**2*tanh(x/2) + b*sqrt(-b**2)*tanh(x/2)*
*2 - b*sqrt(-b**2)), Eq(a, sqrt(-b**2))), ((x*sinh(x)**2/2 - x*cosh(x)**2/2
 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), (-a**2*log(tanh(x/2) - b/a - sqrt(a**2
+ b**2)/a)*tanh(x/2)**2/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2 - b**2*sqrt(a*
*2 + b**2)) + a**2*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b**2*sqrt(a*
*2 + b**2)*tanh(x/2)**2 - b**2*sqrt(a**2 + b**2)) + a**2*log(tanh(x/2) - b/
a + sqrt(a**2 + b**2)/a)*tanh(x/2)**2/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2
- b**2*sqrt(a**2 + b**2)) - a**2*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)
/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2 - b**2*sqrt(a**2 + b**2)) - a*x*sqrt(
a**2 + b**2)*tanh(x/2)**2/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2 - b**2*sqrt(
a**2 + b**2)) + a*x*sqrt(a**2 + b**2)/(b**2*sqrt(a**2 + b**2)*tanh(x/2)**2
- b**2*sqrt(a**2 + b**2)) - 2*b*sqrt(a**2 + b**2)/(b**2*sqrt(a**2 + b**2)*t
anh(x/2)**2 - b**2*sqrt(a**2 + b**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2} - \frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{a^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} - \frac{ax}{b^2} + \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.26

$$\int \frac{\sinh^2(x)}{a+b\sinh(x)}\,dx = \frac{e^{-x}}{2\,b}+\frac{e^{x}}{2\,b}-\frac{a\,x}{b^2}-\frac{a^2\,\ln\left(-\frac{2\,a^2\,e^{x}}{b^3}-\frac{2\,a^2\,(b-a\,e^{x})}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}+\frac{a^2\,\ln\left(\frac{2\,a^2\,(b-a\,e^{x})}{b^3\,\sqrt{a^2+b^2}}-\frac{2\,a^2\,e^{x}}{b^3}\right)}{b^2\,\sqrt{a^2+b^2}}$$

```
[In] int(sinh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] exp(-x)/(2*b) + exp(x)/(2*b) - (a*x)/b^2 - (a^2*log(- (2*a^2*exp(x))/b^3 -
(2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2)) +
(a^2*log((2*a^2*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2)) - (2*a^2*exp(x))/b^
3))/(b^2*(a^2 + b^2)^(1/2))
```

3.75 $\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx=\frac{x}{b}+\frac{2a\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] x/b+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2814, 2739, 632, 212}

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx=\frac{2a\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{x}{b}$$

```
[In] Int[Sinh[x]/(a + b*Sinh[x]),x]
```

```
[Out] x/b + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x}{b} - \frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{x}{b} - \frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{b} + \frac{(4a)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\
&= \frac{x}{b} + \frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \frac{x - \frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}$$

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x]),x]
[Out] (x - (2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])/b
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{2a\ \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$	63
risch	$\frac{x}{b}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b}$	110

```
[In] int(sinh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b*ln(tanh(1/2*x)-1)+1/b*ln(tanh(1/2*x)+1)-2*a/b/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.85

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)+(a^2+}{a^2b+b^3}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) + (a^2 + b^2)*x)/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.62

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{\cosh(x)}{a} & \text{for } b=0 \\ \frac{x\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{ix}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{2}{b\tanh\left(\frac{x}{2}\right)-ib} & \text{for } a=-ib \\ \frac{x\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib} + \frac{ix}{b\tanh\left(\frac{x}{2}\right)+ib} - \frac{2}{b\tanh\left(\frac{x}{2}\right)+ib} & \text{for } a=ib \\ \frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{a\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (cosh(x)/a, Eq(b,
0)), (x*tanh(x/2)/(b*tanh(x/2) - I*b) - I*x/(b*tanh(x/2) - I*b) - 2/(b*tanh
(x/2) - I*b), Eq(a, -I*b)), (x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*x/(b*tanh(
x/2) + I*b) - 2/(b*tanh(x/2) + I*b), Eq(a, I*b)), (a*log(tanh(x/2) - b/a -
sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) - a*log(tanh(x/2) - b/a + sqrt(a
**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) + x/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} + \frac{x}{b}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*b) + x/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{a\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b}+\frac{x}{b}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*b) + x/b
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx = \frac{x}{b}-\frac{a\,\ln\left(\frac{2\,a\,e^{x}}{b^2}-\frac{2\,a\,(b-a\,e^{x})}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,\sqrt{a^2+b^2}}+\frac{a\,\ln\left(\frac{2\,a\,e^{x}}{b^2}+\frac{2\,a\,(b-a\,e^{x})}{b^2\,\sqrt{a^2+b^2}}\right)}{b\,\sqrt{a^2+b^2}}$$

```
[In] int(sinh(x)/(a + b*sinh(x)),x)
```

```
[Out] x/b - (a*log((2*a*exp(x))/b^2 - (2*a*(b - a*exp(x)))/(b^2*(a^2 + b^2)^(1/2)
)))/(b*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(x))/b^2 + (2*a*(b - a*exp(x)))/
(b^2*(a^2 + b^2)^(1/2))))/(b*(a^2 + b^2)^(1/2))
```

3.76 $\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{a}+\frac{2b\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] -arctanh(cosh(x))/a+2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/(a^2+b
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2826, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx=\frac{2b\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{\operatorname{arctanh}(\cosh(x))}{a}$$

```
[In] Int[Csch[x]/(a + b*Sinh[x]),x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a) + (2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a
*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \operatorname{csch}(x)\,dx}{a} - \frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{a} - \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{a} + \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{a} + \frac{2b\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)}{a}$$

```
[In] Integrate[Csch[x]/(a + b*Sinh[x]),x]
[Out] ((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - Log[C
osh[x/2]] + Log[Sinh[x/2]])/a
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2b\ \mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$	49
risch	$-\frac{\ln(\mathrm{e}^{x}+1)}{a}+\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\ln(\mathrm{e}^{x}-1)}{a}$	124

```
[In] int(csch(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*ln(tanh(1/2*x))-2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(46) = 92$.

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.12

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(a^2}{a^3+ab^2}$$

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="fricas")
[Out] (sqrt(a^2 + b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
```

```
*sinh(x) - b)) - (a^2 + b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 + b^2)*log(c
osh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(csch(x)/(a+b*sinh(x)),x)
[Out] Integral(csch(x)/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = -\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a} - \frac{\log\left(e^{(-x)}+1\right)}{a} + \frac{\log\left(e^{(-x)}-1\right)}{a}$$

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(
sqrt(a^2 + b^2)*a) - log(e^(-x) + 1)/a + log(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = -\frac{b\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a} - \frac{\log\left(e^{x}+1\right)}{a} + \frac{\log\left(|e^{x}-1|\right)}{a}$$

```
[In] integrate(csch(x)/(a+b*sinh(x)),x, algorithm="giac")
[Out] -b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^
2 + b^2)))/(sqrt(a^2 + b^2)*a) - log(e^x + 1)/a + log(abs(e^x - 1))/a
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.74

$$\int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{\ln\left(32\,a-32\,a\,e^x\right)}{a}-\frac{\ln\left(32\,a+32\,a\,e^x\right)}{a} + \frac{b\,\ln\left(128\,a^5\,e^x-64\,a^2\,b^3-64\,a^4\,b-128\,a^4\,e^x\,\sqrt{a^2+b^2}+32\,a\,b^4\,e^x+160\,a^3\,b^2\,e^x+32\,a\,b^3\,\sqrt{a^2+b^2}+6\right.}{a^3+a\,b^2} - \frac{b\,\ln\left(64\,a^4\,b+64\,a^2\,b^3-128\,a^5\,e^x-128\,a^4\,e^x\,\sqrt{a^2+b^2}-32\,a\,b^4\,e^x-160\,a^3\,b^2\,e^x+32\,a\,b^3\,\sqrt{a^2+b^2}+6\right.}{a^3+a\,b^2}$$

```
[In] int(1/(sinh(x)*(a + b*sinh(x))),x)
```

```
[Out] log(32*a - 32*a*exp(x))/a - log(32*a + 32*a*exp(x))/a + (b*log(128*a^5*exp(
x) - 64*a^2*b^3 - 64*a^4*b - 128*a^4*exp(x)*(a^2 + b^2)^(1/2) + 32*a*b^4*ex
p(x) + 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2 + b^2)^(1/2) + 64*a^3*b*(a^2 + b^
2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a*b^2 +
 a^3) - (b*log(64*a^4*b + 64*a^2*b^3 - 128*a^5*exp(x) - 128*a^4*exp(x)*(a^2
 + b^2)^(1/2) - 32*a*b^4*exp(x) - 160*a^3*b^2*exp(x) + 32*a*b^3*(a^2 + b^2)
^(1/2) + 64*a^3*b*(a^2 + b^2)^(1/2) - 96*a^2*b^2*exp(x)*(a^2 + b^2)^(1/2))*
(a^2 + b^2)^(1/2))/(a*b^2 + a^3)
```

3.77 $\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx=\frac{b\operatorname{arctanh}(\cosh(x))}{a^2}-\frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\coth(x)}{a}$$

```
[Out] b*arctanh(cosh(x))/a^2-coth(x)/a-2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^
(1/2))/a^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 12, 2826, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx=-\frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{b\operatorname{arctanh}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[In] Int[Csch[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/(a^2*Sqrt[a^2 + b^2]) - Coth[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2826

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\coth(x)}{a} - \frac{\int \frac{b \operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}$$

$$= -\frac{\coth(x)}{a} - \frac{b \int \frac{\operatorname{csch}(x)}{a+b\sinh(x)}\,dx}{a}$$

$$= -\frac{\coth(x)}{a} - \frac{b \int \operatorname{csch}(x)\,dx}{a^2} + \frac{b^2 \int \frac{1}{a+b\sinh(x)}\,dx}{a^2}$$

$$= \frac{b\operatorname{arctanh}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} + \frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

$$= \frac{b\operatorname{arctanh}(\cosh(x))}{a^2} - \frac{\coth(x)}{a} - \frac{(4b^2)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2}$$

$$= \frac{b\operatorname{arctanh}(\cosh(x))}{a^2} - \frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} - \frac{\coth(x)}{a}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = -\frac{a\coth\left(\frac{x}{2}\right) + 2b\left(-\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)\right) + a\tanh\left(\frac{x}{2}\right)}{2a^2}$$

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x]),x]
[Out] -1/2*(a*Coth[x/2] + 2*b*((-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] - Log[Cosh[x/2]] + Log[Sinh[x/2]]) + a*Tanh[x/2])/a^2
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a} - \frac{1}{2a\tanh\left(\frac{x}{2}\right)} - \frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2} + \frac{2b^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}$	73
risch	$-\frac{2}{a(e^{2x}-1)} - \frac{b\ln(e^x-1)}{a^2} + \frac{b\ln(e^x+1)}{a^2} + \frac{b^2\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^2} - \frac{b^2\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^2}$	143

```
[In] int(csch(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*tanh(1/2*x)-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))+2*b^2/a^2/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(55) = 110.

Time = 0.29 (sec) , antiderivative size = 345, normalized size of antiderivative = 5.85

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{2a^3+2ab^2-\left(b^2\cosh(x)^2+2b^2\cosh(x)\sinh(x)+b^2\sinh(x)^2-b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\cdots}{b\cosh\cdots}\right.}{}$$

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 -
 b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3
)*cosh(x)*sinh(x) - (a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) + 1) - (
a^2*b + b^3 - (a^2*b + b^3)*cosh(x)^2 - 2*(a^2*b + b^3)*cosh(x)*sinh(x) - (
a^2*b + b^3)*sinh(x)^2)*log(cosh(x) + sinh(x) - 1))/(a^4 + a^2*b^2 - (a^4 +
 a^2*b^2)*cosh(x)^2 - 2*(a^4 + a^2*b^2)*cosh(x)*sinh(x) - (a^4 + a^2*b^2)*s
inh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^2} + \frac{b\log\left(e^{(-x)}+1\right)}{a^2} - \frac{b\log\left(e^{(-x)}-1\right)}{a^2} + \frac{2}{ae^{(-2x)}-a}$$

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^2) + b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + 2/(
a*e^(-2*x) - a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.66

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2} + \frac{b\log\left(e^x+1\right)}{a^2} - \frac{b\log\left(\left|e^x-1\right|\right)}{a^2} - \frac{2}{a(e^{(2x)}-1)}$$

```
[In] integrate(csch(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1)
)/a^2 - 2/(a*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.95

$$\int \frac{\operatorname{csch}^2(x)}{a+b\sinh(x)}\,dx = \frac{2}{a-a\,\mathrm{e}^{2\,x}} - \frac{b\,\ln\left(32\,\mathrm{e}^x-32\right)}{a^2} + \frac{b\,\ln\left(32\,\mathrm{e}^x+32\right)}{a^2} + \frac{b^2\,\ln\left(128\,a^4\,\mathrm{e}^x-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^x+128\,a^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x-64\,a^2\right.}{a^4+a^2\,b^2} - \frac{b^2\,\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x+64\,a^2\right.}{a^4+a^2\,b^2}$$

```
[In] int(1/(sinh(x)^2*(a + b*sinh(x))),x)

[Out] 2/(a - a*exp(2*x)) - (b*log(32*exp(x) - 32))/a^2 + (b*log(32*exp(x) + 32))/
a^2 + (b^2*log(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1
/2) + 32*b^4*exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x)
 - 64*a^2*b*(a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b
^2)^(1/2))/(a^4 + a^2*b^2) - (b^2*log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3 -
 64*a^3*b + 128*a^4*exp(x) + 32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^2)^(1/
2) + 160*a^2*b^2*exp(x) + 64*a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(x)*(a^2
 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)
```

3.78 $\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = \frac{(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{2a^3} + \frac{2b^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} + \frac{b\coth(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a}$$

```
[Out] 1/2*(a^2-2*b^2)*arctanh(cosh(x))/a^3+b*coth(x)/a^2-1/2*coth(x)*csch(x)/a+2*
b^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = \frac{b\coth(x)}{a^2} + \frac{(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{2a^3} + \frac{2b^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{\coth(x)\operatorname{csch}(x)}{2a}$$

```
[In] Int[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 - 2*b^2)*ArcTanh[Cosh[x]])/(2*a^3) + (2*b^3*ArcTanh[(b - a*Tanh[x/2])
/Sqrt[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) + (b*Coth[x])/a^2 - (Coth[x]*Csch[
x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
```

```
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\coth(x)\text{csch}(x)}{2a}+\frac{i\int\frac{\text{csch}^2(x)\left(2ib+ia\sinh(x)+ib\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{2a}\\
&=\frac{b\coth(x)}{a^2}-\frac{\coth(x)\text{csch}(x)}{2a}-\frac{\int\frac{\text{csch}(x)\left(a^2-2b^2+ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{2a^2}\\
&=\frac{b\coth(x)}{a^2}-\frac{\coth(x)\text{csch}(x)}{2a}-\frac{b^3\int\frac{1}{a+b\sinh(x)}\,dx}{a^3}-\frac{\left(a^2-2b^2\right)\int\text{csch}(x)\,dx}{2a^3}\\
&=\frac{\left(a^2-2b^2\right)\text{arctanh}(\cosh(x))}{2a^3}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\text{csch}(x)}{2a}\\
&\quad-\frac{\left(2b^3\right)\text{Subst}\left(\int\frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&=\frac{\left(a^2-2b^2\right)\text{arctanh}(\cosh(x))}{2a^3}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\text{csch}(x)}{2a}\\
&\quad+\frac{\left(4b^3\right)\text{Subst}\left(\int\frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^3}\\
&=\frac{\left(a^2-2b^2\right)\text{arctanh}(\cosh(x))}{2a^3}+\frac{2b^3\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}+\frac{b\coth(x)}{a^2}-\frac{\coth(x)\text{csch}(x)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = -\frac{\frac{16b^3\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab\coth\left(\frac{x}{2}\right) + a^2\operatorname{csch}^2\left(\frac{x}{2}\right) - 4(a^2-2b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4(a^2-2b^2)\log\left(\sinh\right.}{8a^3}$$

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -1/8*((16*b^3*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2]
- 4*a*b*Coth[x/2] + a^2*Csch[x/2]^2 - 4*(a^2 - 2*b^2)*Log[Cosh[x/2]] + 4*(a
^2 - 2*b^2)*Log[Sinh[x/2]] + a^2*Sech[x/2]^2 - 4*a*b*Tanh[x/2])/a^3
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^2 a}{2}+2b\tanh\left(\frac{x}{2}\right)}{4a^2} - \frac{1}{8a\tanh\left(\frac{x}{2}\right)^2} + \frac{(-2a^2+4b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^3} + \frac{b}{2a^2\tanh\left(\frac{x}{2}\right)} - \frac{2b^3\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}}$
risch	$-\frac{a\,e^{3x}-2b\,e^{2x}+e^x a+2b}{(e^{2x}-1)^2 a^2} + \frac{b^3\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{b^3\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^3} - \frac{\ln(e^x-1)}{2a} + \frac{\ln(e^x-1)b^2}{a^3} + \frac{\ln(e}{2}$

```
[In] int(csch(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/2*tanh(1/2*x)^2*a+2*b*tanh(1/2*x))-1/8/a/tanh(1/2*x)^2+1/4/a^3*(
-2*a^2+4*b^2)*ln(tanh(1/2*x))+1/2/a^2*b/tanh(1/2*x)-2/a^3*b^3/(a^2+b^2)^(1/
2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(73) = 146$.

Time = 0.32 (sec) , antiderivative size = 929, normalized size of antiderivative = 11.47

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*s
inh(x)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^
```

```
2*b^2)*cosh(x))*sinh(x)^2 - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^
3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 +
 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^
2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh
(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(a^4 + a^2*b^2)*c
osh(x) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cos
h(x)*sinh(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4
- 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 -
 a^2*b^2 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)
^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) +
 ((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(x)*sin
h(x)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(x)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4
 - a^2*b^2 - 2*b^4)*cosh(x)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2
 - 2*b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(x)^3 - (a^
4 - a^2*b^2 - 2*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(a^4
+ a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(x)^2 - 4*(a^3*b + a*b^3)*cosh(x))*sinh(x
))/(a^5 + a^3*b^2 + (a^5 + a^3*b^2)*cosh(x)^4 + 4*(a^5 + a^3*b^2)*cosh(x)*s
inh(x)^3 + (a^5 + a^3*b^2)*sinh(x)^4 - 2*(a^5 + a^3*b^2)*cosh(x)^2 - 2*(a^5
 + a^3*b^2 - 3*(a^5 + a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + a^3*b^2)*co
sh(x)^3 - (a^5 + a^3*b^2)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(csch(x)**3/(a+b*sinh(x)),x)
[Out] Integral(csch(x)**3/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(73) = 146$.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx=-\frac{b^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3}+\frac{ae^{(-x)}+2\,be^{(-2\,x)}+ae^{(-3\,x)}-2\,b}{2\,a^2e^{(-2\,x)}-a^2e^{(-4\,x)}-a^2}+\frac{(a^2-2\,b^2)\log\left(e^{(-x)}+1\right)}{2\,a^3}-\frac{(a^2-2\,b^2)\log\left(e^{(-x)}-1\right)}{2\,a^3}$$

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -b^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*a^3) + (a*e^(-x) + 2*b*e^(-2*x) + a*e^(-3*x) - 2*b)/(2*a^
2*e^(-2*x) - a^2*e^(-4*x) - a^2) + 1/2*(a^2 - 2*b^2)*log(e^(-x) + 1)/a^3 -
1/2*(a^2 - 2*b^2)*log(e^(-x) - 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.69

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = -\frac{b^3\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3} + \frac{(a^2-2\,b^2)\log\left(e^x+1\right)}{2\,a^3} - \frac{(a^2-2\,b^2)\log\left(\left|e^x-1\right|\right)}{2\,a^3} - \frac{ae^{(3\,x)}-2\,be^{(2\,x)}+ae^x+2\,b}{a^2\left(e^{(2\,x)}-1\right)^2}$$

```
[In] integrate(csch(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(
a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3) + 1/2*(a^2 - 2*b^2)*log(e^x + 1)/a^3 - 1
/2*(a^2 - 2*b^2)*log(abs(e^x - 1))/a^3 - (a*e^(3*x) - 2*b*e^(2*x) + a*e^x +
 2*b)/(a^2*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 617, normalized size of antiderivative = 7.62

$$\int \frac{\operatorname{csch}^3(x)}{a+b\sinh(x)}\,dx = \frac{\mathrm{e}^x}{a-a\,\mathrm{e}^{2\,x}} - \frac{2\,\mathrm{e}^x}{a-2\,a\,\mathrm{e}^{2\,x}+a\,\mathrm{e}^{4\,x}} - \frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,\mathrm{e}^x-24\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a} + \frac{\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,\mathrm{e}^x+24\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a} + \frac{2\,b}{a^2\,\mathrm{e}^{2\,x}-a^2} + \frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,\mathrm{e}^x-24\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{a^3} - \frac{b^2\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,\mathrm{e}^x+24\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{a^3} - \frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\,\sqrt{a^2+b^2}-32\,a^3\,b^3-32\,a^6\,\mathrm{e}^x+24\,b^6\,\mathrm{e}^x-40\,a^2\,b^3\,\sqrt{a^2+b^2}-32\,a^5\,\mathrm{e}^x\,\sqrt{\cdots}\right.}{a^5\cdots} + \frac{b^3\,\ln\left(24\,b^5\,\sqrt{a^2+b^2}-48\,a\,b^5+16\,a^5\,b-32\,a^3\,b^3-32\,a^6\,\mathrm{e}^x+24\,b^6\,\mathrm{e}^x+40\,a^2\,b^3\,\sqrt{a^2+b^2}+32\,a^5\,\mathrm{e}^x\,\sqrt{\cdots}\right.}{a^5\cdots}$$

```
[In] int(1/(sinh(x)^3*(a + b*sinh(x))),x)
```

```
[Out] exp(x)/(a - a*exp(2*x)) - (2*exp(x))/(a - 2*a*exp(2*x) + a*exp(4*x)) - log(
4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp
(x))/(2*a) + log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x)
 - 20*a^2*b^2*exp(x))/(2*a) + (2*b)/(a^2*exp(2*x) - a^2) + (b^2*log(4*a^4 +
 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/a
^3 - (b^2*log(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) -
20*a^2*b^2*exp(x)))/a^3 - (b^3*log(16*a^5*b - 48*a*b^5 - 24*b^5*(a^2 + b^2)
^(1/2) - 32*a^3*b^3 - 32*a^6*exp(x) + 24*b^6*exp(x) - 40*a^2*b^3*(a^2 + b^2
)^(1/2) - 32*a^5*exp(x)*(a^2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 56*a^4*b^2
*exp(x) + 16*a^4*b*(a^2 + b^2)^(1/2) + 72*a*b^4*exp(x)*(a^2 + b^2)^(1/2) +
72*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) + (
b^3*log(24*b^5*(a^2 + b^2)^(1/2) - 48*a*b^5 + 16*a^5*b - 32*a^3*b^3 - 32*a^
6*exp(x) + 24*b^6*exp(x) + 40*a^2*b^3*(a^2 + b^2)^(1/2) + 32*a^5*exp(x)*(a^
2 + b^2)^(1/2) + 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 16*a^4*b*(a^2 + b
^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^2*exp(x)*(a^2 + b^
2)^(1/2))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2)
```

3.79 $\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx=-\frac{b(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{2a^4}-\frac{2b^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{(2a^2-3b^2)\coth(x)}{3a^3}+\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

```
[Out] -1/2*b*(a^2-2*b^2)*arctanh(cosh(x))/a^4+1/3*(2*a^2-3*b^2)*coth(x)/a^3+1/2*b
*coth(x)*csch(x)/a^2-1/3*coth(x)*csch(x)^2/a-2*b^4*arctanh((b-a*tanh(1/2*x)
)/(a^2+b^2)^(1/2))/a^4/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx=\frac{b\coth(x)\operatorname{csch}(x)}{2a^2}-\frac{b(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{2a^4}-\frac{2b^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}+\frac{(2a^2-3b^2)\coth(x)}{3a^3}-\frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

```
[In] Int[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*(a^2 - 2*b^2)*ArcTanh[Cosh[x]])/a^4 - (2*b^4*ArcTanh[(b - a*Tanh[x/
2])/Sqrt[a^2 + b^2]])/(a^4*Sqrt[a^2 + b^2]) + ((2*a^2 - 3*b^2)*Coth[x])/(3*
a^3) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
```

```
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth(x)\text{csch}^2(x)}{3a} + \frac{i\int \frac{\text{csch}^3(x)(3ib+2ia\sinh(x)+2ib\sinh^2(x))}{a+b\sinh(x)}\,dx}{3a} \\
&= \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} - \frac{\int \frac{\text{csch}^2(x)(2(2a^2-3b^2)+ab\sinh(x)-3b^2\sinh^2(x))}{a+b\sinh(x)}\,dx}{6a^2} \\
&= \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} \\
&\quad - \frac{\coth(x)\text{csch}^2(x)}{3a} - \frac{i\int \frac{\text{csch}(x)(3ib(a^2-2b^2)+3iab^2\sinh(x))}{a+b\sinh(x)}\,dx}{6a^3} \\
&= \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} \\
&\quad + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{a^4} + \frac{(b(a^2-2b^2))\int \text{csch}(x)\,dx}{2a^4} \\
&= -\frac{b(a^2-2b^2)\,\text{arctanh}(\cosh(x))}{2a^4} + \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} \\
&\quad - \frac{\coth(x)\text{csch}^2(x)}{3a} + \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^4} \\
&= -\frac{b(a^2-2b^2)\,\text{arctanh}(\cosh(x))}{2a^4} + \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} \\
&\quad - \frac{\coth(x)\text{csch}^2(x)}{3a} - \frac{(4b^4)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^4}
\end{aligned}
$$

$$= -\frac{b(a^2-2b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} - \frac{2b^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} + \frac{(2a^2-3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\frac{48b^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 4a(2a^2-3b^2)\coth\left(\frac{x}{2}\right) + 3a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) - 12a^2b\log\left(\cosh\left(\frac{x}{2}\right)\right) + 24b^3\log\left(\cosh\left(\frac{x}{2}\right)\right)}{}$$

```
[In] Integrate[Csch[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((48*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 4*a
*(2*a^2 - 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 - 12*a^2*b*Log[Cosh[x/2]]
+ 24*b^3*Log[Cosh[x/2]] + 12*a^2*b*Log[Sinh[x/2]] - 24*b^3*Log[Sinh[x/2]] +
 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[
x])/2 + 8*a^3*Tanh[x/2] - 12*a*b^2*Tanh[x/2])/(24*a^4)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^3a^2}{3}+ab\tanh\left(\frac{x}{2}\right)^2-3a^2\tanh\left(\frac{x}{2}\right)+4b^2\tanh\left(\frac{x}{2}\right)}{8a^3}+\frac{2b^4\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{1}{24a\tanh\left(\frac{x}{2}\right)^3}-\frac{-3a^2+4b^2}{8a^3\tanh\left(\frac{x}{2}\right)}+$
risch	$-\frac{-3ab\,e^{5x}+6b^2e^{4x}+12a^2e^{2x}-12b^2e^{2x}+3b\,e^{x}a-4a^2+6b^2}{3a^3(e^{2x}-1)^3}+\frac{b^4\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^4}-\frac{b^4\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a^4}+$

```
[In] int(csch(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/a^3*(1/3*tanh(1/2*x)^3*a^2+a*b*tanh(1/2*x)^2-3*a^2*tanh(1/2*x)+4*b^2*t
anh(1/2*x))+2/a^4*b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^
2+b^2)^(1/2))-1/24/a/tanh(1/2*x)^3-1/8/a^3*(-3*a^2+4*b^2)/tanh(1/2*x)+1/8/a
^2*b/tanh(1/2*x)^2+1/2/a^4*b*(a^2-2*b^2)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1676 vs. $2(97) = 194$.

Time = 0.33 (sec) , antiderivative size = 1676, normalized size of antiderivative = 15.38

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*(a^4*b + a^2*b^3)*cosh(x)^5 + 6*(a^4*b + a^2*b^3)*sinh(x)^5 + 8*a^5
- 4*a^3*b^2 - 12*a*b^4 - 12*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(2*a^3*b^2 + 2*
a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x))*sinh(x)^4 + 12*(5*(a^4*b + a^2*b^3)*co
sh(x)^2 - 4*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 24*(a^5 - a*b^4)*cosh(x)
^2 - 12*(2*a^5 - 2*a*b^4 - 5*(a^4*b + a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 + a*b
^4)*cosh(x)^2)*sinh(x)^2 + 6*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4
*sinh(x)^6 - 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*
sinh(x)^4 - b^4 + 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*
cosh(x)^4 - 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cos
h(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(a^4*b + a^2*b^3)*cosh(x) -
3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2*b^5
 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 - 5*(
a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 - 2*b
^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b -
 a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a^2*b
^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 +
6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x
)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1
) + 3*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^6 + 6*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)*sinh(x)^5 + (a^4*b - a^2*b^3 - 2*b^5)*sinh(x)^6 - a^4*b + a^2*b^3 + 2
*b^5 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^4 - 3*(a^4*b - a^2*b^3 - 2*b^5 -
 5*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - a^2*b^3 -
 2*b^5)*cosh(x)^3 - 3*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4
*b - a^2*b^3 - 2*b^5)*cosh(x)^2 + 3*(a^4*b - a^2*b^3 - 2*b^5 + 5*(a^4*b - a
^2*b^3 - 2*b^5)*cosh(x)^4 - 6*(a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^
2 + 6*((a^4*b - a^2*b^3 - 2*b^5)*cosh(x)^5 - 2*(a^4*b - a^2*b^3 - 2*b^5)*co
sh(x)^3 + (a^4*b - a^2*b^3 - 2*b^5)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 - 1) - 6*(a^4*b + a^2*b^3 - 5*(a^4*b + a^2*b^3)*cosh(x)^4 + 8*(a^3*b^2 + a
*b^4)*cosh(x)^3 + 8*(a^5 - a*b^4)*cosh(x))*sinh(x))/((a^6 + a^4*b^2)*cosh(x
)^6 + 6*(a^6 + a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 + a^4*b^2)*sinh(x)^6 - a^6
 - a^4*b^2 - 3*(a^6 + a^4*b^2)*cosh(x)^4 - 3*(a^6 + a^4*b^2 - 5*(a^6 + a^4*
```

```
b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + a^4*b^2)*cosh(x)^3 - 3*(a^6 + a^4*b
^2)*cosh(x))*sinh(x)^3 + 3*(a^6 + a^4*b^2)*cosh(x)^2 + 3*(a^6 + a^4*b^2 + 5
*(a^6 + a^4*b^2)*cosh(x)^4 - 6*(a^6 + a^4*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a
^6 + a^4*b^2)*cosh(x)^5 - 2*(a^6 + a^4*b^2)*cosh(x)^3 + (a^6 + a^4*b^2)*cos
h(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(csch(x)**4/(a+b*sinh(x)),x)
[Out] Integral(csch(x)**4/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.78

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^4} - \frac{3\,abe^{(-x)} - 6\,b^2e^{(-4\,x)} - 3\,abe^{(-5\,x)} + 4\,a^2 - 6\,b^2 - 12\,(a^2-b^2)e^{(-2\,x)}}{3\,(3\,a^3e^{(-2\,x)} - 3\,a^3e^{(-4\,x)} + a^3e^{(-6\,x)} - a^3)} - \frac{(a^2b-2\,b^3)\log\left(e^{(-x)}+1\right)}{2\,a^4} + \frac{(a^2b-2\,b^3)\log\left(e^{(-x)}-1\right)}{2\,a^4}$$

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
[Out] b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(sqrt(a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(-x) - 6*b^2*e^(-4*x) - 3*a*b*e^(-5*x)
 + 4*a^2 - 6*b^2 - 12*(a^2 - b^2)*e^(-2*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x
) + a^3*e^(-6*x) - a^3) - 1/2*(a^2*b - 2*b^3)*log(e^(-x) + 1)/a^4 + 1/2*(a^
2*b - 2*b^3)*log(e^(-x) - 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4} - \frac{(a^2b-2\,b^3)\log\left(e^x+1\right)}{2\,a^4} + \frac{(a^2b-2\,b^3)\log\left(|e^x-1|\right)}{2\,a^4} + \frac{3\,abe^{(5\,x)}-6\,b^2e^{(4\,x)}-12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2-6\,b^2}{3\,a^3\left(e^{(2\,x)}-1\right)^3}$$

```
[In] integrate(csch(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) - 1/2*(a^2*b - 2*b^3)*log(e^x + 1)/a^4 +
1/2*(a^2*b - 2*b^3)*log(abs(e^x - 1))/a^4 + 1/3*(3*a*b*e^(5*x) - 6*b^2*e^(4
*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 - 6*b^2)/(a^3*(e^
(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 694, normalized size of antiderivative = 6.37

$$\int \frac{\operatorname{csch}^4(x)}{a+b\sinh(x)}\,dx = \frac{8}{3\,\left(a-3\,a\,\mathrm{e}^{2\,x}+3\,a\,\mathrm{e}^{4\,x}-a\,\mathrm{e}^{6\,x}\right)} - \frac{4}{a-2\,a\,\mathrm{e}^{2\,x}+a\,\mathrm{e}^{4\,x}} - \frac{2\,b^2}{a^3\,\mathrm{e}^{2\,x}-a^3} + \frac{b\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,\mathrm{e}^x-24\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a^2} - \frac{b\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,\mathrm{e}^x+24\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a^2} - \frac{b^3\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2-4\,a^4\,\mathrm{e}^x-24\,b^4\,\mathrm{e}^x+20\,a^2\,b^2\,\mathrm{e}^x\right)}{a^4} + \frac{b^3\,\ln\left(4\,a^4+24\,b^4-20\,a^2\,b^2+4\,a^4\,\mathrm{e}^x+24\,b^4\,\mathrm{e}^x-20\,a^2\,b^2\,\mathrm{e}^x\right)}{a^4} + \frac{2\,b\,\mathrm{e}^x}{a^2\,\mathrm{e}^{4\,x}-2\,a^2\,\mathrm{e}^{2\,x}+a^2} + \frac{b\,\mathrm{e}^x}{a^2\,\mathrm{e}^{2\,x}-a^2} + \frac{b^4\,\ln\left(16\,a^5\,b^2-48\,a\,b^6-32\,a^3\,b^4-24\,b^6\,\sqrt{a^2+b^2}+24\,b^7\,\mathrm{e}^x-40\,a^2\,b^4\,\sqrt{a^2+b^2}+16\,a^4\,b^2\,\sqrt{a^2+b^2}-\right.}{a} - \frac{b^4\,\ln\left(24\,b^6\,\sqrt{a^2+b^2}-48\,a\,b^6-32\,a^3\,b^4+16\,a^5\,b^2+24\,b^7\,\mathrm{e}^x+40\,a^2\,b^4\,\sqrt{a^2+b^2}-16\,a^4\,b^2\,\sqrt{a^2+b^2}-\right.}{a}$$

```
[In] int(1/(sinh(x)^4*(a + b*sinh(x))),x)
```

```
[Out] 8/(3*(a - 3*a*exp(2*x) + 3*a*exp(4*x) - a*exp(6*x))) - 4/(a - 2*a*exp(2*x)
+ a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) - a^3) + (b*log(4*a^4 + 24*b^4 - 20*a
^2*b^2 - 4*a^4*exp(x) - 24*b^4*exp(x) + 20*a^2*b^2*exp(x)))/(2*a^2) - (b*lo
g(4*a^4 + 24*b^4 - 20*a^2*b^2 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b^2*e
xp(x)))/(2*a^2) - (b^3*log(4*a^4 + 24*b^4 - 20*a^2*b^2 - 4*a^4*exp(x) - 24*
b^4*exp(x) + 20*a^2*b^2*exp(x)))/a^4 + (b^3*log(4*a^4 + 24*b^4 - 20*a^2*b^2
 + 4*a^4*exp(x) + 24*b^4*exp(x) - 20*a^2*b^2*exp(x)))/a^4 + (2*b*exp(x))/(a
^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) + (b*exp(x))/(a^2*exp(2*x) - a^2) + (b^
4*log(16*a^5*b^2 - 48*a*b^6 - 32*a^3*b^4 - 24*b^6*(a^2 + b^2)^(1/2) + 24*b^
7*exp(x) - 40*a^2*b^4*(a^2 + b^2)^(1/2) + 16*a^4*b^2*(a^2 + b^2)^(1/2) - 32
*a^6*b*exp(x) + 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) + 72*a*b^5*exp(x)*(a
^2 + b^2)^(1/2) - 32*a^5*b*exp(x)*(a^2 + b^2)^(1/2) + 72*a^3*b^3*exp(x)*(a^
2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/(a^6 + a^4*b^2) - (b^4*log(24*b^6*(a^2 +
 b^2)^(1/2) - 48*a*b^6 - 32*a^3*b^4 + 16*a^5*b^2 + 24*b^7*exp(x) + 40*a^2*b
^4*(a^2 + b^2)^(1/2) - 16*a^4*b^2*(a^2 + b^2)^(1/2) - 32*a^6*b*exp(x) + 112
*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a*b^5*exp(x)*(a^2 + b^2)^(1/2) + 3
2*a^5*b*exp(x)*(a^2 + b^2)^(1/2) - 72*a^3*b^3*exp(x)*(a^2 + b^2)^(1/2))*(a^
2 + b^2)^(1/2))/(a^6 + a^4*b^2)
```

3.80 $\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 162

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(6a^2-b^2)\,x}{2b^4} + \frac{2a^3(3a^2+4b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}} - \frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}$$

```
[Out] 1/2*(6*a^2-b^2)*x/b^4+2*a^3*(3*a^2+4*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^
2)^(1/2))/b^4/(a^2+b^2)^(3/2)-a*(3*a^2+2*b^2)*cosh(x)/b^3/(a^2+b^2)+1/2*(3*
a^2+b^2)*cosh(x)*sinh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)^2/b/(a^2+b^2)/(a
+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2871, 3128, 3102, 2814, 2739, 632, 212}

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a^2\sinh^2(x)\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))} + \frac{(3a^2+b^2)\sinh(x)\cosh(x)}{2b^2\,(a^2+b^2)} + \frac{x(6a^2-b^2)}{2b^4} - \frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)} + \frac{2a^3(3a^2+4b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2+b^2)^{3/2}}$$

```
[In] Int[Sinh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^2 - b^2)*x)/(2*b^4) + (2*a^3*(3*a^2 + 4*b^2)*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/(b^4*(a^2 + b^2)^(3/2)) - (a*(3*a^2 + 2*b^2)*Cosh[x])/(
b^3*(a^2 + b^2)) + ((3*a^2 + b^2)*Cosh[x]*Sinh[x])/(2*b^2*(a^2 + b^2)) - (a
^2*Cosh[x]*Sinh[x]^2)/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}+\frac{\int \frac{\sinh(x)\left(2a^2-ab\sinh(x)+(3a^2+b^2)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{b\,(a^2+b^2)}\\
&= \frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)}-\frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}\\
&\quad+\frac{\int \frac{-a(3a^2+b^2)+b(a^2-b^2)\sinh(x)-2a(3a^2+2b^2)\sinh^2(x)}{a+b\sinh(x)}\,dx}{2b^2\,(a^2+b^2)}\\
&= -\frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)}+\frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)}\\
&\quad-\frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}+\frac{i\int \frac{iab(3a^2+b^2)-i(6a^4+5a^2b^2-b^4)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3\,(a^2+b^2)}\\
&= \frac{(6a^2-b^2)\,x}{2b^4}-\frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)}+\frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)}\\
&\quad-\frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}-\frac{(a^3(3a^2+4b^2))\int \frac{1}{a+b\sinh(x)}\,dx}{b^4\,(a^2+b^2)}\\
&= \frac{(6a^2-b^2)\,x}{2b^4}-\frac{a(3a^2+2b^2)\cosh(x)}{b^3\,(a^2+b^2)}+\frac{(3a^2+b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2+b^2)}\\
&\quad-\frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}-\frac{(2a^3(3a^2+4b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^4\,(a^2+b^2)}
\end{aligned}$$

$$= \frac{(6a^2 - b^2)\,x}{2b^4} - \frac{a(3a^2 + 2b^2)\cosh(x)}{b^3\,(a^2 + b^2)} + \frac{(3a^2 + b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2 + b^2)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2 + b^2)\,(a + b\sinh(x))} + \frac{(4a^3(3a^2 + 4b^2))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b - 2a\tanh\left(\frac{x}{2}\right)\right)}{b^4\,(a^2 + b^2)}$$

$$= \frac{(6a^2 - b^2)\,x}{2b^4} + \frac{2a^3(3a^2 + 4b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4\,(a^2 + b^2)^{3/2}} - \frac{a(3a^2 + 2b^2)\cosh(x)}{b^3\,(a^2 + b^2)} + \frac{(3a^2 + b^2)\cosh(x)\sinh(x)}{2b^2\,(a^2 + b^2)} - \frac{a^2\cosh(x)\sinh^2(x)}{b\,(a^2 + b^2)\,(a + b\sinh(x))}$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.73

$$\int \frac{\sinh^4(x)}{(a + b\sinh(x))^2}\,dx = \frac{-2(-6a^2 + b^2)\,x + \frac{8a^3(3a^2+4b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} - 8ab\cosh(x) - \frac{4a^4b\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} + b^2\sinh(2x)}{4b^4}$$

```
[In] Integrate[Sinh[x]^4/(a + b*Sinh[x])^2,x]
[Out] (-2*(-6*a^2 + b^2)*x + (8*a^3*(3*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) - 8*a*b*Cosh[x] - (4*a^4*b*Cosh[x])/((a^2
 + b^2)*(a + b*Sinh[x])) + b^2*Sinh[2*x])/(4*b^4)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

method	result
default	$\frac{2a^3\left(\frac{\frac{b^2\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{ab}{a^2+b^2}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(3a^2+4b^2\right)\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{b^4} - \frac{1}{2b^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{-b+4a}{2b^3\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{(6a^2-b^2)\ln\ldots}{2\ldots}$
risch	$\frac{3x\,a^2}{b^4} - \frac{x}{2b^2} + \frac{e^{2x}}{8b^2} - \frac{a\,e^{x}}{b^3} - \frac{a\,e^{-x}}{b^3} - \frac{e^{-2x}}{8b^2} + \frac{2a^4(e^{x}a-b)}{b^4(a^2+b^2)(b\,e^{2x}+2\,e^{x}a-b)} + \frac{3a^5\ln\left(e^{x}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}\,b^4} + \frac{4a\ldots}{\ldots}$

```
[In] int(sinh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^3/b^4*((b^2/(a^2+b^2)*tanh(1/2*x)+a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*t
anh(1/2*x)-a)-(3*a^2+4*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*
b)/(a^2+b^2)^(1/2)))-1/2/b^2/(tanh(1/2*x)+1)^2-1/2*(-b+4*a)/b^3/(tanh(1/2*x
)+1)+1/2*(6*a^2-b^2)/b^4*ln(tanh(1/2*x)+1)+1/2/b^2/(tanh(1/2*x)-1)^2-1/2*(-
b-4*a)/b^3/(tanh(1/2*x)-1)+1/2/b^4*(-6*a^2+b^2)*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1769 vs. $2(154) = 308$.

Time = 0.30 (sec) , antiderivative size = 1769, normalized size of antiderivative = 10.92

$$\int \frac{\sinh^4(x)}{(a + b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(a^4*b^3 + 2*a^2*b^5 + b^7 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^6 + (a
^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^6 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(
x)^5 - 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6 - (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)
)*sinh(x)^5 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a
^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^4 - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^
5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^2 - 4*(6*a^6*b + 11*a^4*b^
3 + 4*a^2*b^5 - b^7)*x + 30*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^
4 + 8*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh
(x)^3 + 4*(4*a^7 + 4*a^5*b^2 + 5*(a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^3 - 15
*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^2 + 2*(6*a^7 + 11*a^5*b^2 + 4*a^3*b^
4 - a*b^6)*x - (16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*
a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x)^3 - (32*a^6*b + 49*a^4*b^3 +
 18*a^2*b^5 + b^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x)^2
 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b^7 - 15*(a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x)^4 + 60*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 6*(16*a^6*b + 3
3*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x
)*cosh(x)^2 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x - 24*(2*a^7 + 2*
a^5*b^2 + (6*a^7 + 11*a^5*b^2 + 4*a^3*b^4 - a*b^6)*x)*cosh(x))*sinh(x)^2 +
8*((3*a^5*b + 4*a^3*b^3)*cosh(x)^4 + (3*a^5*b + 4*a^3*b^3)*sinh(x)^4 + 2*(3
*a^6 + 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 + 4*a^4*b^2 + 2*(3*a^5*b + 4*a^3*b^3
)*cosh(x))*sinh(x)^3 - (3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - (3*a^5*b + 4*a^3*b
^3 - 6*(3*a^5*b + 4*a^3*b^3)*cosh(x)^2 - 6*(3*a^6 + 4*a^4*b^2)*cosh(x))*sin
h(x)^2 + 2*(2*(3*a^5*b + 4*a^3*b^3)*cosh(x)^3 + 3*(3*a^6 + 4*a^4*b^2)*cosh(
x)^2 - (3*a^5*b + 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cos
h(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b
)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 6*(a^5*b^2 + 2
```

```
*a^3*b^4 + a*b^6)*cosh(x) + 2*(3*a^5*b^2 + 6*a^3*b^4 + 3*a*b^6 + 3*(a^4*b^3
 + 2*a^2*b^5 + b^7)*cosh(x)^5 - 15*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^4
- 2*(16*a^6*b + 33*a^4*b^3 + 18*a^2*b^5 + b^7 - 4*(6*a^6*b + 11*a^4*b^3 + 4
*a^2*b^5 - b^7)*x)*cosh(x)^3 + 12*(2*a^7 + 2*a^5*b^2 + (6*a^7 + 11*a^5*b^2
+ 4*a^3*b^4 - a*b^6)*x)*cosh(x)^2 - (32*a^6*b + 49*a^4*b^3 + 18*a^2*b^5 + b
^7 + 4*(6*a^6*b + 11*a^4*b^3 + 4*a^2*b^5 - b^7)*x)*cosh(x))*sinh(x))/((a^4*
b^5 + 2*a^2*b^7 + b^9)*cosh(x)^4 + (a^4*b^5 + 2*a^2*b^7 + b^9)*sinh(x)^4 +
2*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x)^3 + 2*(a^5*b^4 + 2*a^3*b^6 + a*b^8
+ 2*(a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 - (a^4*b^5 + 2*a^2*b^7 +
 b^9)*cosh(x)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9 - 6*(a^4*b^5 + 2*a^2*b^7 + b^9
)*cosh(x)^2 - 6*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 + 2*(2*(a^
4*b^5 + 2*a^2*b^7 + b^9)*cosh(x)^3 + 3*(a^5*b^4 + 2*a^3*b^6 + a*b^8)*cosh(x
)^2 - (a^4*b^5 + 2*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(3\,a^2+4\,b^2)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{a^2b^3+b^5-6\,(a^3b^2+ab^4)e^{(-x)}-(32\,a^4b+17\,a^2b^3+b^5)e^{(-2\,x)}-8\,(2\,a^5-a^3b^2-ab^4)e^{(-3\,x)}}{8\,((a^2b^5+b^7)e^{(-2\,x)}+2\,(a^3b^4+ab^6)e^{(-3\,x)}-(a^2b^5+b^7)e^{(-4\,x)})} - \frac{8\,ae^{(-x)}+be^{(-2\,x)}}{8\,b^3} + \frac{(6\,a^2-b^2)x}{2\,b^4}$$

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(3*a^2 + 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/8*(a^2*b^3 + b^5 - 6
*(a^3*b^2 + a*b^4)*e^(-x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(-2*x) - 8*(2*a
^5 - a^3*b^2 - a*b^4)*e^(-3*x))/((a^2*b^5 + b^7)*e^(-2*x) + 2*(a^3*b^4 + a*
b^6)*e^(-3*x) - (a^2*b^5 + b^7)*e^(-4*x)) - 1/8*(8*a*e^(-x) + b*e^(-2*x))/b
^3 + 1/2*(6*a^2 - b^2)*x/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.45

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= -\frac{(3\,a^5+4\,a^3b^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{(6\,a^2-b^2)x}{2\,b^4} + \frac{b^2e^{(2\,x)}-8\,abe^x}{8\,b^4}$$
$$+ \frac{(a^2b^3+b^5+8\,(2\,a^5-a^3b^2-ab^4)e^{(3\,x)}-(32\,a^4b+17\,a^2b^3+b^5)e^{(2\,x)}+6\,(a^3b^2+ab^4)e^x)e^{(-2\,x)}}{8\,(a^2+b^2)(be^{(2\,x)}+2\,ae^x-b)b^4}$$

```
[In] integrate(sinh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(3*a^5 + 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/2*(6*a^2
 - b^2)*x/b^4 + 1/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(a^2*b^3 + b^5 + 8*
(2*a^5 - a^3*b^2 - a*b^4)*e^(3*x) - (32*a^4*b + 17*a^2*b^3 + b^5)*e^(2*x) +
 6*(a^3*b^2 + a*b^4)*e^x)*e^(-2*x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b
^4)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.88

$$\int \frac{\sinh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\mathrm{e}^{2\,x}}{8\,b^2} - \frac{\mathrm{e}^{-2\,x}}{8\,b^2} - \frac{\frac{2\,a^4}{b^2\,(a^2\,b+b^3)} - \frac{2\,a^5\,\mathrm{e}^x}{b^3\,(a^2\,b+b^3)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}} + \frac{x\,(6\,a^2-b^2)}{2\,b^4} - \frac{a\,\mathrm{e}^x}{b^3} - \frac{a\,\mathrm{e}^{-x}}{b^3}$$
$$- \frac{a^3\,\ln\left(\frac{2\,\mathrm{e}^x\,(3\,a^5+4\,a^3\,b^2)}{a^2\,b^5+b^7} - \frac{2\,a^3\,(b-a\,\mathrm{e}^x)\,(3\,a^2+4\,b^2)}{b^5\,(a^2+b^2)^{3/2}}\right)\,(3\,a^2+4\,b^2)}{b^4\,(a^2+b^2)^{3/2}}$$
$$+ \frac{a^3\,\ln\left(\frac{2\,\mathrm{e}^x\,(3\,a^5+4\,a^3\,b^2)}{a^2\,b^5+b^7} + \frac{2\,a^3\,(b-a\,\mathrm{e}^x)\,(3\,a^2+4\,b^2)}{b^5\,(a^2+b^2)^{3/2}}\right)\,(3\,a^2+4\,b^2)}{b^4\,(a^2+b^2)^{3/2}}$$

```
[In] int(sinh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*a^4)/(b^2*(a^2*b + b^3)) - (2*a^
5*exp(x))/(b^3*(a^2*b + b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2 -
b^2))/(2*b^4) - (a*exp(x))/b^3 - (a*exp(-x))/b^3 - (a^3*log((2*exp(x)*(3*a^
5 + 4*a^3*b^2))/(b^7 + a^2*b^5) - (2*a^3*(b - a*exp(x))*(3*a^2 + 4*b^2))/(b
^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b^2)^(3/2)) + (a^3*log(
(2*exp(x)*(3*a^5 + 4*a^3*b^2))/(b^7 + a^2*b^5) + (2*a^3*(b - a*exp(x))*(3*a
^2 + 4*b^2))/(b^5*(a^2 + b^2)^(3/2)))*(3*a^2 + 4*b^2))/(b^4*(a^2 + b^2)^(3/
2))
```

3.81 $\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2ax}{b^3} - \frac{2a^2(2a^2+3b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\left(a^2+b^2\right)^{3/2}} + \frac{(2a^2+b^2)\cosh(x)}{b^2\left(a^2+b^2\right)} - \frac{a^2\cosh(x)\sinh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}$$

```
[Out] -2*a*x/b^3-2*a^2*(2*a^2+3*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b
^3/(a^2+b^2)^(3/2)+(2*a^2+b^2)*cosh(x)/b^2/(a^2+b^2)-a^2*cosh(x)*sinh(x)/b/
(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2871, 3102, 2814, 2739, 632, 212}

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a^2(2a^2+3b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3\left(a^2+b^2\right)^{3/2}} + \frac{(2a^2+b^2)\cosh(x)}{b^2\left(a^2+b^2\right)} - \frac{a^2\sinh(x)\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{2ax}{b^3}$$

```
[In] Int[Sinh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*x)/b^3 - (2*a^2*(2*a^2 + 3*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 +
b^2]])/(b^3*(a^2 + b^2)^(3/2)) + ((2*a^2 + b^2)*Cosh[x])/(b^2*(a^2 + b^2))
- (a^2*Cosh[x]*Sinh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2871

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*
(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[
e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^(n + 1)*Simp[b*(m - 2)*(b*c - a*d)^2
 + a*d*(n + 1)*(c*(a^2 + b^2) - 2*a*b*d) + (b*(n + 1)*(a*b*c^2 + c*d*(a^2 +
 b^2) - 3*a*b*d^2) - a*(n + 2)*(b*c - a*d)^2)*Sin[e + f*x] + b*(b^2*(c^2 -
d^2) - m*(b*c - a*d)^2 + d*n*(2*a*b*c - d*(a^2 + b^2)))*Sin[e + f*x]^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && LtQ[n, -1] && (IntegerQ[m] || I
ntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
```

```
&&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))} + \frac{\int \frac{a^2-ab\sinh(x)+(2a^2+b^2)\sinh^2(x)}{a+b\sinh(x)}\,dx}{b(a^2+b^2)}$$

$$= \frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)} - \frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))} + \frac{i\int \frac{-ia^2b+2ia(a^2+b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{b^2(a^2+b^2)}$$

$$= -\frac{2ax}{b^3} + \frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)} - \frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))} + \frac{(a^2(2a^2+3b^2))\int \frac{1}{a+b\sinh(x)}\,dx}{b^3(a^2+b^2)}$$

$$= -\frac{2ax}{b^3} + \frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)} - \frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))} + \frac{(2a^2(2a^2+3b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3(a^2+b^2)}$$

$$= -\frac{2ax}{b^3} + \frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)} - \frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))} - \frac{(4a^2(2a^2+3b^2))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^3(a^2+b^2)}$$

$$= -\frac{2ax}{b^3} - \frac{2a^2(2a^2+3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^3(a^2+b^2)^{3/2}} + \frac{(2a^2+b^2)\cosh(x)}{b^2(a^2+b^2)} - \frac{a^2\cosh(x)\sinh(x)}{b(a^2+b^2)(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{-2ax - \frac{2a^2(2a^2+3b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + \cosh(x)\left(b + \frac{a^3b}{(a^2+b^2)(a+b\sinh(x))}\right)}{b^3}$$

```
[In] Integrate[Sinh[x]^3/(a + b*Sinh[x])^2,x]
[Out] (-2*a*x - (2*a^2*(2*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]]
)/(-a^2 - b^2)^(3/2) + Cosh[x]*(b + (a^3*b)/((a^2 + b^2)*(a + b*Sinh[x]))))
/b^3
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.40

method	result
default	$-\frac{1}{b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{2a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{3}}+\frac{1}{b^{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^{3}}-\frac{4a^{2}\left(\frac{\frac{b^{2}\tanh\left(\frac{x}{2}\right)}{2a^{2}+2b^{2}}+\frac{ab}{2a^{2}+2b^{2}}}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(2a^{2}+3b\right.}{}\right.}{b^{3}}$
risch	$-\frac{2ax}{b^{3}}+\frac{e^{x}}{2b^{2}}+\frac{e^{-x}}{2b^{2}}-\frac{2a^{3}\left(e^{x}a-b\right)}{b^{3}\left(a^{2}+b^{2}\right)\left(b\,e^{2x}+2\,e^{x}a-b\right)}+\frac{2a^{4}\ln\left(e^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}-2a^{2}b^{2}-b^{4}}{b\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}b^{3}}+\frac{3a^{2}\ln\left(e^{x}+\frac{a\left(a^{2}+b^{2}\right)^{\frac{3}{2}}-a^{4}}{b\left(a^{2}+b^{2}\right.}\right.}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}b}$

```
[In] int(sinh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/(tanh(1/2*x)-1)+2/b^3*a*ln(tanh(1/2*x)-1)+1/b^2/(tanh(1/2*x)+1)-2/b^
3*a*ln(tanh(1/2*x)+1)-4/b^3*a^2*((1/2*b^2/(a^2+b^2)*tanh(1/2*x)+1/2*a*b/(a^
2+b^2))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-1/2*(2*a^2+3*b^2)/(a^2+b^2)^(3/
2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. 2(111) = 222.

Time = 0.28 (sec) , antiderivative size = 1053, normalized size of antiderivative = 9.16

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 - (
a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^
5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^3 - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*x + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x)^2 + 2*(2*
a^6 + 2*a^4*b^2 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 + 4*(a^6 + 2*a^4*
b^2 + a^2*b^4)*x - 3*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^5*b + 2*a^3*b^3 + a*
b^5)*x)*cosh(x))*sinh(x)^2 - 2*((2*a^4*b + 3*a^2*b^3)*cosh(x)^3 + (2*a^4*b
+ 3*a^2*b^3)*sinh(x)^3 + 2*(2*a^5 + 3*a^3*b^2)*cosh(x)^2 + (4*a^5 + 6*a^3*b
^2 + 3*(2*a^4*b + 3*a^2*b^3)*cosh(x))*sinh(x)^2 - (2*a^4*b + 3*a^2*b^3)*cos
h(x) - (2*a^4*b + 3*a^2*b^3 - 3*(2*a^4*b + 3*a^2*b^3)*cosh(x)^2 - 4*(2*a^5
+ 3*a^3*b^2)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^5
```

```
 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x) - 2*(3*a^5*b + 4*a^3*b^3 + a*b^
5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^5*b + 2*a^3*b^3 + a*b^5
- 2*(a^5*b + 2*a^3*b^3 + a*b^5)*x)*cosh(x)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5
)*x - 4*(a^6 + a^4*b^2 + 2*(a^6 + 2*a^4*b^2 + a^2*b^4)*x)*cosh(x))*sinh(x))
/((a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x)^3 + (a^4*b^4 + 2*a^2*b^6 + b^8)*sinh(
x)^3 + 2*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x)^2 + (2*a^5*b^3 + 4*a^3*b^5 +
 2*a*b^7 + 3*(a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x))*sinh(x)^2 - (a^4*b^4 + 2*
a^2*b^6 + b^8)*cosh(x) - (a^4*b^4 + 2*a^2*b^6 + b^8 - 3*(a^4*b^4 + 2*a^2*b^
6 + b^8)*cosh(x)^2 - 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.81

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(2\,a^2+3\,b^2)a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^3+b^5)\sqrt{a^2+b^2}} + \frac{a^2b^2+b^4+2\,(3\,a^3b+ab^3)e^{(-x)}+(4\,a^4-a^2b^2-b^4)e^{(-2\,x)}}{2\,((a^2b^4+b^6)e^{(-x)}+2\,(a^3b^3+ab^5)e^{(-2\,x)}-(a^2b^4+b^6)e^{(-3\,x)})} - \frac{2\,ax}{b^3} + \frac{e^{(-x)}}{2\,b^2}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2 + 3*b^2)*a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) + 1/2*(a^2*b^2 + b^4 + 2*
(3*a^3*b + a*b^3)*e^(-x) + (4*a^4 - a^2*b^2 - b^4)*e^(-2*x))/((a^2*b^4 + b^
6)*e^(-x) + 2*(a^3*b^3 + a*b^5)*e^(-2*x) - (a^2*b^4 + b^6)*e^(-3*x)) - 2*a*
x/b^3 + 1/2*e^(-x)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.60

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(2\,a^4+3\,a^2b^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b^3+b^5)\sqrt{a^2+b^2}} - \frac{2\,ax}{b^3} + \frac{e^x}{2\,b^2} - \frac{(a^2b^2+b^4+(4\,a^4-a^2b^2-b^4)e^{(2\,x)}-2\,(3\,a^3b+ab^3)e^x)e^{(-x)}}{2\,(a^2+b^2)(be^{(2\,x)}+2\,ae^x-b)b^3}$$

```
[In] integrate(sinh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^4 + 3*a^2*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^2*b^3 + b^5)*sqrt(a^2 + b^2)) - 2*a*x/b^3 +
 1/2*e^x/b^2 - 1/2*(a^2*b^2 + b^4 + (4*a^4 - a^2*b^2 - b^4)*e^(2*x) - 2*(3*
a^3*b + a*b^3)*e^x)*e^(-x)/((a^2 + b^2)*(b*e^(2*x) + 2*a*e^x - b)*b^3)
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.38

$$\int \frac{\sinh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{e^{-x}}{2\,b^2} + \frac{\frac{2\,a^3}{b\,(a^2\,b+b^3)} - \frac{2\,a^4\,e^x}{b^2\,(a^2\,b+b^3)}}{2\,a\,e^x-b+b\,e^{2\,x}} + \frac{e^x}{2\,b^2} - \frac{2\,a\,x}{b^3} - \frac{a^2\,\ln\left(-\frac{2\,e^x\,(2\,a^4+3\,a^2\,b^2)}{a^2\,b^4+b^6} - \frac{2\,a^2\,(b-a\,e^x)\,(2\,a^2+3\,b^2)}{b^4\,(a^2+b^2)^{3/2}}\right)\,(2\,a^2+3\,b^2)}{b^3\,(a^2+b^2)^{3/2}} + \frac{a^2\,\ln\left(\frac{2\,a^2\,(b-a\,e^x)\,(2\,a^2+3\,b^2)}{b^4\,(a^2+b^2)^{3/2}} - \frac{2\,e^x\,(2\,a^4+3\,a^2\,b^2)}{a^2\,b^4+b^6}\right)\,(2\,a^2+3\,b^2)}{b^3\,(a^2+b^2)^{3/2}}$$

```
[In] int(sinh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] exp(-x)/(2*b^2) + ((2*a^3)/(b*(a^2*b + b^3)) - (2*a^4*exp(x))/(b^2*(a^2*b +
 b^3)))/(2*a*exp(x) - b + b*exp(2*x)) + exp(x)/(2*b^2) - (2*a*x)/b^3 - (a^2
*log(- (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4) - (2*a^2*(b - a*exp(x
))*(2*a^2 + 3*b^2))/(b^4*(a^2 + b^2)^(3/2)))*(2*a^2 + 3*b^2))/(b^3*(a^2 + b
^2)^(3/2)) + (a^2*log((2*a^2*(b - a*exp(x))*(2*a^2 + 3*b^2))/(b^4*(a^2 + b^
2)^(3/2)) - (2*exp(x)*(2*a^4 + 3*a^2*b^2))/(b^6 + a^2*b^4))*(2*a^2 + 3*b^2)
)/(b^3*(a^2 + b^2)^(3/2))
```

3.82 $\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{x}{b^2}+\frac{2a(a^2+2b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\,(a^2+b^2)^{3/2}}-\frac{a^2\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}$$

```
[Out] x/b^2+2*a*(a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b^2/(a^2+b
^2)^(3/2)-a^2*cosh(x)/b/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2869, 2814, 2739, 632, 212}

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2a(a^2+2b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\,(a^2+b^2)^{3/2}}-\frac{a^2\cosh(x)}{b\,(a^2+b^2)\,(a+b\sinh(x))}+\frac{x}{b^2}$$

```
[In] Int[Sinh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2
*(a^2 + b^2)^(3/2)) - (a^2*Cosh[x])/(b*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2869

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*Cos[e
+ f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[
1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m + 1)*
(2*b*c*d - a*(c^2 + d^2)) + (a^2*d^2 - 2*a*b*c*d*(m + 2) + b^2*(d^2*(m + 1)
 + c^2*(m + 2)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{i\int\frac{-iab+i\left(a^2+b^2\right)\sinh(x)}{a+b\sinh(x)}\,dx}{b\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(a(a^2+2b^2)\right)\int\frac{1}{a+b\sinh(x)}\,dx}{b^2\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(2a(a^2+2b^2)\right)\text{Subst}\left(\int\frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\left(4a(a^2+2b^2)\right)\text{Subst}\left(\int\frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2\left(a^2+b^2\right)}\\
&= \frac{x}{b^2}+\frac{2a(a^2+2b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{a^2\cosh(x)}{b\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{x+\frac{2a(a^2+2b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}-\frac{a^2b\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}}{b^2}$$

[In] Integrate[Sinh[x]^2/(a + b*Sinh[x])^2,x]

[Out] (x + (2*a*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) - (a^2*b*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])))/b^2

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

method	result
default	$\frac{2a\left(\frac{\frac{b^2\tanh\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{ab}{a^2+b^2}}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}$
risch	$\frac{x}{b^2}+\frac{2a^2(\mathrm{e}^x a-b)}{b^2(a^2+b^2)(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^x a-b)}+\frac{a^3\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}b^2}+\frac{2a\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{a^3\ln\left(\mathrm{e}\right.}{}$

[In] int(sinh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] 2*a/b^2*((b^2/(a^2+b^2)*tanh(1/2*x)+a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*tan h(1/2*x)-a)-(a^2+2*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/b^2*ln(tanh(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 521, normalized size of antiderivative = 6.28

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{2a^4b+2a^2b^3-(a^4b+2a^2b^3+b^5)x\cosh(x)^2-(a^4b+2a^2b^3+b^5)x\sinh(x)^2+(a^3b+2ab^3-(a^3b+2ab\ldots}{}$$

[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")

```
[Out] (2*a^4*b + 2*a^2*b^3 - (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x)^2 - (a^4*b + 2*a
^2*b^3 + b^5)*x*sinh(x)^2 + (a^3*b + 2*a*b^3 - (a^3*b + 2*a*b^3)*cosh(x)^2
- (a^3*b + 2*a*b^3)*sinh(x)^2 - 2*(a^4 + 2*a^2*b^2)*cosh(x) - 2*(a^4 + 2*a^
2*b^2 + (a^3*b + 2*a*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x
)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*s
inh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*si
nh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (a^4*b + 2*a^2*b^
3 + b^5)*x - 2*(a^5 + a^3*b^2 + (a^5 + 2*a^3*b^2 + a*b^4)*x)*cosh(x) - 2*(a
^5 + a^3*b^2 + (a^4*b + 2*a^2*b^3 + b^5)*x*cosh(x) + (a^5 + 2*a^3*b^2 + a*b
^4)*x)*sinh(x))/(a^4*b^3 + 2*a^2*b^5 + b^7 - (a^4*b^3 + 2*a^2*b^5 + b^7)*co
sh(x)^2 - (a^4*b^3 + 2*a^2*b^5 + b^7)*sinh(x)^2 - 2*(a^5*b^2 + 2*a^3*b^4 +
a*b^6)*cosh(x) - 2*(a^5*b^2 + 2*a^3*b^4 + a*b^6 + (a^4*b^3 + 2*a^2*b^5 + b^
7)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.80

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^2+2\,b^2)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^3e^{(-x)}+a^2b\right)}{a^2b^3+b^5+2\,(a^3b^2+ab^4)e^{(-x)}-(a^2b^3+b^5)e^{(-2\,x)}} + \frac{x}{b^2}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(a^2 + 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(
a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^3*e^(-x) + a^2*b)/(a^
2*b^3 + b^5 + 2*(a^3*b^2 + a*b^4)*e^(-x) - (a^2*b^3 + b^5)*e^(-2*x)) + x/b^
2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^3+2\,ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\,(a^3e^x-a^2b)}{(a^2b^2+b^4)(be^{(2\,x)}+2\,ae^x-b)} + \frac{x}{b^2}$$

```
[In] integrate(sinh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3 + 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*e^x - a
^2*b)/((a^2*b^2 + b^4)*(b*e^(2*x) + 2*a*e^x - b)) + x/b^2
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.75

$$\int \frac{\sinh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{x}{b^2} - \frac{\frac{2\,a^2}{a^2\,b+b^3}-\frac{2\,a^3\,\mathrm{e}^x}{b\,(a^2\,b+b^3)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}} - \frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^3+2\,a\,b^2)}{b^3\,(a^2+b^2)}-\frac{2\,a\,(a^2+2\,b^2)\,(b-a\,\mathrm{e}^x)}{b^3\,{(a^2+b^2)}^{3/2}}\right)\,(a^2+2\,b^2)}{b^2\,{(a^2+b^2)}^{3/2}} + \frac{a\,\ln\left(\frac{2\,\mathrm{e}^x\,(a^3+2\,a\,b^2)}{b^3\,(a^2+b^2)}+\frac{2\,a\,(a^2+2\,b^2)\,(b-a\,\mathrm{e}^x)}{b^3\,{(a^2+b^2)}^{3/2}}\right)\,(a^2+2\,b^2)}{b^2\,{(a^2+b^2)}^{3/2}}$$

```
[In] int(sinh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] x/b^2 - ((2*a^2)/(a^2*b + b^3) - (2*a^3*exp(x))/(b*(a^2*b + b^3)))/(2*a*exp
(x) - b + b*exp(2*x)) - (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^2 + b^2))
 - (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a^2 + 2*b^2
))/(b^2*(a^2 + b^2)^(3/2)) + (a*log((2*exp(x)*(2*a*b^2 + a^3))/(b^3*(a^2 +
b^2)) + (2*a*(a^2 + 2*b^2)*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(3/2)))*(a^2 +
2*b^2))/(b^2*(a^2 + b^2)^(3/2))
```

3.83 $\int \frac{\sinh(x)}{(a+b\sinh(x))^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2} dx = -\frac{2b\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} + \frac{a\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}$$

```
[Out] -2*b*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+a*cosh(x)/(
a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2833, 12, 2739, 632, 212}

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2} dx = \frac{a\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} - \frac{2b\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

```
[In] Int[Sinh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*b*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (a*Co
sh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{\int \frac{b}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{b\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}+\frac{(2b)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}-\frac{(4b)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2b\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2b\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + \frac{a\cosh(x)}{(a^2+b^2)\,(a+b\sinh(x))}$$

```
[In] Integrate[Sinh[x]/(a + b*Sinh[x])^2,x]
[Out] (-2*b*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (a*C
osh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result	size
default	$\frac{8b\tanh\left(\frac{x}{2}\right)+8a}{(-4a^2-4b^2)\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}-\frac{8b\ \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(-4a^2-4b^2)\sqrt{a^2+b^2}}$	97
risch	$-\frac{2a(e^x a-b)}{b(a^2+b^2)(b\,e^{2x}+2\,e^x a-b)}+\frac{b\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{b\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	155

```
[In] int(sinh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] 4*(2*b*tanh(1/2*x)+2*a)/(-4*a^2-4*b^2)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-
8*b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(56) = 112.

Time = 0.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.68

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx =$$

$$-\frac{2\,a^3b+2\,ab^3+\left(b^3\cosh\left(x\right)^2+b^3\sinh\left(x\right)^2+2\,ab^2\cosh\left(x\right)-b^3+2\left(b^3\cosh\left(x\right)+ab^2\right)\sinh\left(x\right)\right)\sqrt{a^2+b}}{a^4b^2+2\,a^2b^4+b^6-\left(a^4b^2+2\,a^2b^4+b^6\right)\cosh\left(x\right)^2-\left(a^4b^2+2\,a^2b^4+b^6\right.}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b + 2*a*b^3 + (b^3*cosh(x)^2 + b^3*sinh(x)^2 + 2*a*b^2*cosh(x) - b^
3 + 2*(b^3*cosh(x) + a*b^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b
^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x)
- 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4 + a^2*b^2)*cosh(x
) - 2*(a^4 + a^2*b^2)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2 + 2*a^
2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2 + 2*a
^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.95

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(a^2e^{(-x)}+ab\right)}{a^2b^2+b^4+2\,(a^3b+ab^3)e^{(-x)}-(a^2b^2+b^4)e^{(-2\,x)}}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(a
^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)/(a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*e
^(-x) - (a^2*b^2 + b^4)*e^(-2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{b\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\,(a^2e^x-ab)}{(a^2b+b^3)(be^{(2\,x)}+2\,ae^x-b)}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^2*e^x - a*b)/((a^2*b + b^3)*(b*e^(2*x) +
 2*a*e^x - b))
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.37

$$\int \frac{\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2\,a\,b}{a^2\,b+b^3}-\frac{2\,a^2\,e^x}{a^2\,b+b^3}}{2\,a\,e^x-b+b\,e^{2\,x}} - \frac{b\,\ln\left(-\frac{2\,e^x}{a^2+b^2}-\frac{2\,(b-a\,e^x)}{(a^2+b^2)^{3/2}}\right)}{(a^2+b^2)^{3/2}} + \frac{b\,\ln\left(\frac{2\,(b-a\,e^x)}{(a^2+b^2)^{3/2}}-\frac{2\,e^x}{a^2+b^2}\right)}{(a^2+b^2)^{3/2}}$$

```
[In] int(sinh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*a*b)/(a^2*b + b^3) - (2*a^2*exp(x))/(a^2*b + b^3))/(2*a*exp(x) - b + b*
exp(2*x)) - (b*log(- (2*exp(x))/(a^2 + b^2) - (2*(b - a*exp(x)))/(a^2 + b^2
)^(3/2)))/(a^2 + b^2)^(3/2) + (b*log((2*(b - a*exp(x)))/(a^2 + b^2)^(3/2) -
 (2*exp(x))/(a^2 + b^2)))/(a^2 + b^2)^(3/2)
```

3.84 $\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))}{a^2}+\frac{2b(2a^2+b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

```
[Out] -arctanh(cosh(x))/a^2+2*b*(2*a^2+b^2)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(
1/2))/a^2/(a^2+b^2)^(3/2)+b^2*cosh(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {2881, 3080, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b(2a^2+b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}}-\frac{\operatorname{arctanh}(\cosh(x))}{a^2}+\frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

```
[In] Int[Csch[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -(ArcTanh[Cosh[x]]/a^2) + (2*b*(2*a^2 + b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2*(a^2 + b^2)^(3/2)) + (b^2*Cosh[x])/(a*(a^2 + b^2)*(a + b
*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)} + \frac{\int \frac{\operatorname{csch}(x)\left(a^2+b^2-ab\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)} + \frac{\int \operatorname{csch}(x)\,dx}{a^2} - \frac{\left(b\left(2a^2+b^2\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{\operatorname{arctanh}(\cosh(x))}{a^2} + \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)} - \frac{\left(2b\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{\operatorname{arctanh}(\cosh(x))}{a^2} + \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)} + \frac{\left(4b\left(2a^2+b^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{\operatorname{arctanh}(\cosh(x))}{a^2} + \frac{2b\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{3/2}} + \frac{b^2\cosh(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{csch}(x)}{\left(a+b\sinh(x)\right)^2}\,dx = \frac{\frac{2b\left(2a^2+b^2\right)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\left(-a^2-b^2\right)^{3/2}} - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{ab^2\cosh(x)}{\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}}{a^2}$$

```
[In] Integrate[Csch[x]/(a + b*Sinh[x])^2,x]
[Out] ((2*b*(2*a^2 + b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2
)^(3/2) - Log[Cosh[x/2]] + Log[Sinh[x/2]] + (a*b^2*Cosh[x])/((a^2 + b^2)*(a
 + b*Sinh[x])))/a^2
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

method	result
default	$\frac{4b\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{2\left(a^2+b^2\right)}-\frac{ab}{2\left(a^2+b^2\right)}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$
risch	$-\frac{2b(\mathrm{e}^x a-b)}{a(a^2+b^2)(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^x a-b)}-\frac{\ln(\mathrm{e}^x+1)}{a^2}+\frac{2b\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{b^3\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a^2}$ -

```
[In] int(csch(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^2*b*((-1/2*b^2/(a^2+b^2)*tanh(1/2*x)-1/2*a*b/(a^2+b^2))/(tanh(1/2*x)^2*
a-2*b*tanh(1/2*x)-a)-1/2*(2*a^2+b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(
1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/a^2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(81) = 162$.

Time = 0.37 (sec) , antiderivative size = 672, normalized size of antiderivative = 7.91

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx =$$
$$-\frac{2\,a^3b^2+2\,ab^4-\left(2\,a^2b^2+b^4-(2\,a^2b^2+b^4)\cosh\left(x\right)^2-(2\,a^2b^2+b^4)\sinh\left(x\right)^2-2\,(2\,a^3b+ab^3)\cosh\left(x\right)\right.}{}$$

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^3*b^2 + 2*a*b^4 - (2*a^2*b^2 + b^4 - (2*a^2*b^2 + b^4)*cosh(x)^2 - (2
*a^2*b^2 + b^4)*sinh(x)^2 - 2*(2*a^3*b + a*b^3)*cosh(x) - 2*(2*a^3*b + a*b^
3 + (2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
+ b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x
)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(a^4*b + a^2*b^3)*c
osh(x) + (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (
a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) -
2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))*lo
g(cosh(x) + sinh(x) + 1) - (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 +
b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 +
 a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*co
sh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(a^4*b + a^2*b^3)*sinh(x))/(
```

```
a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x)^2 - (a^
6*b + 2*a^4*b^3 + a^2*b^5)*sinh(x)^2 - 2*(a^7 + 2*a^5*b^2 + a^3*b^4)*cosh(x
) - 2*(a^7 + 2*a^5*b^2 + a^3*b^4 + (a^6*b + 2*a^4*b^3 + a^2*b^5)*cosh(x))*s
inh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(csch(x)/(a+b*sinh(x))**2,x)
[Out] Integral(csch(x)/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx=-\frac{(2\,a^2b+b^3)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}}+\frac{2\left(abe^{(-x)}+b^2\right)}{a^3b+ab^3+2\,(a^4+a^2b^2)e^{(-x)}-(a^3b+ab^3)e^{(-2\,x)}}-\frac{\log\left(e^{(-x)}+1\right)}{a^2}+\frac{\log\left(e^{(-x)}-1\right)}{a^2}$$

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] -(2*a^2*b + b^3)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(
a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) + 2*(a*b*e^(-x) + b^2)/(a^3*
b + a*b^3 + 2*(a^4 + a^2*b^2)*e^(-x) - (a^3*b + a*b^3)*e^(-2*x)) - log(e^(-
x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx=-\frac{(2\,a^2b+b^3)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+a^2b^2)\sqrt{a^2+b^2}}-\frac{2\,(abe^x-b^2)}{(a^3+ab^2)(be^{(2\,x)}+2\,ae^x-b)}-\frac{\log\left(e^x+1\right)}{a^2}+\frac{\log\left(|e^x-1|\right)}{a^2}$$

```
[In] integrate(csch(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*b + b^3)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2
*a + 2*sqrt(a^2 + b^2)))/((a^4 + a^2*b^2)*sqrt(a^2 + b^2)) - 2*(a*b*e^x - b
^2)/((a^3 + a*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - log(e^x + 1)/a^2 + log(abs(
e^x - 1))/a^2
```

Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 1001, normalized size of antiderivative = 11.78

$$\int \frac{\operatorname{csch}(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2\,b^5}{a\,(a^2\,b^3+b^5)} - \frac{2\,b^4\,e^x}{a^2\,b^3+b^5}}{2\,a\,e^x - b + b\,e^{2\,x}} + \frac{\ln\left(e^x-1\right)}{a^2} - \frac{\ln\left(e^x+1\right)}{a^2}$$
$$- \frac{b\,\ln\left(\frac{32\,\left(-8\,e^x\,a^5+4\,a^4\,b-10\,e^x\,a^3\,b^2+6\,a^2\,b^3-3\,e^x\,a\,b^4+2\,b^5\right)}{a\,\left(a^4\,b^3+a^2\,b^5\right)\,\left(a^2\,b+b^3\right)} + \frac{b\left(\frac{32\,\left(-4\,e^x\,a^7+2\,a^6\,b-11\,e^x\,a^5\,b^2+9\,a^4\,b^3-10\,e^x\,a^3\,b^4+8\,a^2\,b^5-3\,e^x\,a\,b^6+2 \cdots\right.}{a\,b^5\,\left(a^5+2\,a^3\,b^2+a\,b^4\right)} \cdots\right.}{\cdots}\right.}{\cdots}$$
$$+ \frac{b\,\ln\left(\frac{32\,\left(-8\,e^x\,a^5+4\,a^4\,b-10\,e^x\,a^3\,b^2+6\,a^2\,b^3-3\,e^x\,a\,b^4+2\,b^5\right)}{a\,\left(a^4\,b^3+a^2\,b^5\right)\,\left(a^2\,b+b^3\right)} - \frac{b\left(\frac{32\,\left(-4\,e^x\,a^7+2\,a^6\,b-11\,e^x\,a^5\,b^2+9\,a^4\,b^3-10\,e^x\,a^3\,b^4+8\,a^2\,b^5-3\,e^x\,a\,b^6+2 \cdots\right.}{a\,b^5\,\left(a^5+2\,a^3\,b^2+a\,b^4\right)} \cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] int(1/(sinh(x)*(a + b*sinh(x))^2),x)
```

```
[Out] ((2*b^5)/(a*(b^5 + a^2*b^3)) - (2*b^4*exp(x))/(b^5 + a^2*b^3))/(2*a*exp(x)
- b + b*exp(2*x)) + log(exp(x) - 1)/a^2 - log(exp(x) + 1)/a^2 - (b*log((32*
(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*exp(x) - 10*a^3*b^2*e
xp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) + (b*((32*(2*a^6*b + 2*b^7 +
8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x) - 10*a^3*b^4*exp(x) -
 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2)) - (b*(2*a^2 + b^2)*(
(a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^4*exp(x) - 4*a^2*b^2*exp
(x)))/(b*(b^5 + a^2*b^3)) + (32*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)*(3*a^4*
b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^4*(a^8 + a^2*b^6 + 3*a
^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a^2 + b^
2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a^2 +
```

```
 b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2) + (b*l
og((32*(4*a^4*b + 2*b^5 + 6*a^2*b^3 - 8*a^5*exp(x) - 3*a*b^4*exp(x) - 10*a^
3*b^2*exp(x)))/(a*(a^2*b^5 + a^4*b^3)*(a^2*b + b^3)) - (b*((32*(2*a^6*b + 2
*b^7 + 8*a^2*b^5 + 9*a^4*b^3 - 4*a^7*exp(x) - 3*a*b^6*exp(x) - 10*a^3*b^4*e
xp(x) - 11*a^5*b^2*exp(x)))/(a*b^5*(a*b^4 + a^5 + 2*a^3*b^2)) + (b*(2*a^2 +
 b^2)*((a^2 + b^2)^3)^(1/2)*((32*(2*a*b^3 + 4*a^3*b - 7*a^4*exp(x) - 4*a^2*
b^2*exp(x)))/(b*(b^5 + a^2*b^3)) - (32*(2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2)*
(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b^4*(a^8 + a^2*b^
6 + 3*a^4*b^4 + 3*a^6*b^2))))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(2*a
^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2))*(
2*a^2 + b^2)*((a^2 + b^2)^3)^(1/2))/(a^8 + a^2*b^6 + 3*a^4*b^4 + 3*a^6*b^2)
```

3.85 $\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b\operatorname{arctanh}(\cosh(x))}{a^3} - \frac{2b^2(3a^2+2b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}} - \frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} + \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

```
[Out] 2*b*arctanh(cosh(x))/a^3-2*b^2*(3*a^2+2*b^2)*arctanh((b-a*tanh(1/2*x))/(a^2
+b^2)^(1/2))/a^3/(a^2+b^2)^(3/2)-(a^2+2*b^2)*coth(x)/a^2/(a^2+b^2)+b^2*coth
(x)/a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b\operatorname{arctanh}(\cosh(x))}{a^3} - \frac{(a^2+2b^2)\coth(x)}{a^2(a^2+b^2)} + \frac{b^2\coth(x)}{a(a^2+b^2)(a+b\sinh(x))} - \frac{2b^2(3a^2+2b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}}$$

```
[In] Int[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*b^2*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2
])/Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)) - ((a^2 + 2*b^2)*Coth[x])/(a^2
*(a^2 + b^2)) + (b^2*Coth[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
```

```
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int\frac{\operatorname{csch}^2(x)\left(a^2+2b^2-ab\sinh(x)+b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}\\
&= -\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{i\int\frac{\operatorname{csch}(x)\left(2ib\left(a^2+b^2\right)-iab^2\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}\\
&= -\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\\
&\quad -\frac{(2b)\int\operatorname{csch}(x)\,dx}{a^3}+\frac{\left(b^2\left(3a^2+2b^2\right)\right)\int\frac{1}{a+b\sinh(x)}\,dx}{a^3\left(a^2+b^2\right)}\\
&= \frac{2b\operatorname{arctanh}(\cosh(x))}{a^3}-\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\\
&\quad +\frac{\left(2b^2\left(3a^2+2b^2\right)\right)\operatorname{Subst}\left(\int\frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^3\left(a^2+b^2\right)}\\
&= \frac{2b\operatorname{arctanh}(\cosh(x))}{a^3}-\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\\
&\quad -\frac{\left(4b^2\left(3a^2+2b^2\right)\right)\operatorname{Subst}\left(\int\frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^3\left(a^2+b^2\right)}\\
&= \frac{2b\operatorname{arctanh}(\cosh(x))}{a^3}-\frac{2b^2\left(3a^2+2b^2\right)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\left(a^2+b^2\right)^{3/2}}\\
&\quad -\frac{\left(a^2+2b^2\right)\coth(x)}{a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\frac{4b^2(3a^2+2b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + a\coth\left(\frac{x}{2}\right) - 4b\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4b\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{2ab^3\cosh(x)}{(a^2+b^2)(a+b\sinh(x))} + a\tanh\left(\frac{x}{2}\right)}{2a^3}$$

```
[In] Integrate[Csch[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*((4*b^2*(3*a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-
a^2 - b^2)^(3/2) + a*Coth[x/2] - 4*b*Log[Cosh[x/2]] + 4*b*Log[Sinh[x/2]] +
(2*a*b^3*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])) + a*Tanh[x/2])/a^3
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.23

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a^2} - \frac{1}{2a^2\tanh\left(\frac{x}{2}\right)} - \frac{2b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3} - \frac{2b^2\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{ab}{a^2+b^2}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a} - \frac{(3a^2+2b^2)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}\right)}{a^3}$
risch	$-\frac{2(-a\,b^2 e^{3x}+a^2 b\,e^{2x}+2b^3 e^{2x}+2a^3 e^x+3a\,b^2 e^x-a^2 b-2b^3)}{a^2(a^2+b^2)(b\,e^{2x}+2\,e^x a-b)(e^{2x}-1)} + \frac{2b\ln(e^x+1)}{a^3} - \frac{2b\ln(e^x-1)}{a^3} + \frac{3b^2\ln\left(e^x+\frac{a(a^2+b^2)^{\frac{3}{2}}-a^4-2a^2b^2-b}{b(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}a}$

```
[In] int(csch(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*tanh(1/2*x)-1/2/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))-2/a^3*b^2*
((-b^2/(a^2+b^2)*tanh(1/2*x)-a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x
)-a)-(3*a^2+2*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b
^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1740 vs. $2(111) = 222$.

Time = 0.40 (sec) , antiderivative size = 1740, normalized size of antiderivative = 15.13

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^5*b + 6*a^3*b^3 + 4*a*b^5 + 2*(a^4*b^2 + a^2*b^4)*cosh(x)^3 + 2*(a^4*b
^2 + a^2*b^4)*sinh(x)^3 - 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 - 2*(a^
5*b + 3*a^3*b^3 + 2*a*b^5 - 3*(a^4*b^2 + a^2*b^4)*cosh(x))*sinh(x)^2 + (3*a
^2*b^3 + 2*b^5 + (3*a^2*b^3 + 2*b^5)*cosh(x)^4 + (3*a^2*b^3 + 2*b^5)*sinh(x
)^4 + 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 2*(3*a^3*b^2 + 2*a*b^4 + 2*(3*a^2
*b^3 + 2*b^5)*cosh(x))*sinh(x)^3 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 2*(3*a
^2*b^3 + 2*b^5 - 3*(3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3*(3*a^3*b^2 + 2*a*b^4)*
cosh(x))*sinh(x)^2 - 2*(3*a^3*b^2 + 2*a*b^4)*cosh(x) - 2*(3*a^3*b^2 + 2*a*b
^4 - 2*(3*a^2*b^3 + 2*b^5)*cosh(x)^3 - 3*(3*a^3*b^2 + 2*a*b^4)*cosh(x)^2 +
2*(3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
+ b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x
)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(2*a^6 + 5*a^4*b^2
+ 3*a^2*b^4)*cosh(x) + 2*(a^4*b^2 + 2*a^2*b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4
+ b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2
*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 -
2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*
(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b
^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*
cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^
4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*(a^4*b^2 + 2*a^2*
b^4 + b^6 + (a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^4 + (a^4*b^2 + 2*a^2*b^4 +
b^6)*sinh(x)^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x)^3 + 2*(a^5*b + 2*a^3
*b^3 + a*b^5 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 - 2*(a^4*b^
2 + 2*a^2*b^4 + b^6)*cosh(x)^2 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6 - 3*(a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x))*sinh(
x)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5
 - 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x)^3 - 3*(a^5*b + 2*a^3*b^3 + a*b^5)*
cosh(x)^2 + 2*(a^4*b^2 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))*log(cosh(x) + s
inh(x) - 1) - 2*(2*a^6 + 5*a^4*b^2 + 3*a^2*b^4 - 3*(a^4*b^2 + a^2*b^4)*cosh
(x)^2 + 2*(a^5*b + 3*a^3*b^3 + 2*a*b^5)*cosh(x))*sinh(x))/(a^7*b + 2*a^5*b^
3 + a^3*b^5 + (a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^4 + (a^7*b + 2*a^5*b^3
+ a^3*b^5)*sinh(x)^4 + 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x)^3 + 2*(a^8 + 2
*a^6*b^2 + a^4*b^4 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x)^3 - 2
```

```
*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5 -
 3*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^2 - 3*(a^8 + 2*a^6*b^2 + a^4*b^4)*
cosh(x))*sinh(x)^2 - 2*(a^8 + 2*a^6*b^2 + a^4*b^4)*cosh(x) - 2*(a^8 + 2*a^6
*b^2 + a^4*b^4 - 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x)^3 - 3*(a^8 + 2*a^6
*b^2 + a^4*b^4)*cosh(x)^2 + 2*(a^7*b + 2*a^5*b^3 + a^3*b^5)*cosh(x))*sinh(x
))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\operatorname{csch}^2{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(111) = 222$.

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.18

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx=\frac{(3\,a^2b^2+2\,b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}}+\frac{2\left(ab^2e^{(-3\,x)}-a^2b-2\,b^3-(2\,a^3+3\,ab^2)e^{(-x)}+(a^2b+2\,b^3)e^{(-2\,x)}\right)}{a^4b+a^2b^3+2\,(a^5+a^3b^2)e^{(-x)}-2\,(a^4b+a^2b^3)e^{(-2\,x)}-2\,(a^5+a^3b^2)e^{(-3\,x)}+(a^4b+a^2b^3)e^{(-4\,x)}}+\frac{2\,b\log\left(e^{(-x)}+1\right)}{a^3}-\frac{2\,b\log\left(e^{(-x)}-1\right)}{a^3}$$

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (3*a^2*b^2 + 2*b^4)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^(-3*x) - a^2
*b - 2*b^3 - (2*a^3 + 3*a*b^2)*e^(-x) + (a^2*b + 2*b^3)*e^(-2*x))/(a^4*b +
a^2*b^3 + 2*(a^5 + a^3*b^2)*e^(-x) - 2*(a^4*b + a^2*b^3)*e^(-2*x) - 2*(a^5
+ a^3*b^2)*e^(-3*x) + (a^4*b + a^2*b^3)*e^(-4*x)) + 2*b*log(e^(-x) + 1)/a^3
 - 2*b*log(e^(-x) - 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.78

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{(3\,a^2b^2+2\,b^4)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^5+a^3b^2)\sqrt{a^2+b^2}} + \frac{2\left(ab^2e^{(3\,x)}-a^2be^{(2\,x)}-2\,b^3e^{(2\,x)}-2\,a^3e^x-3\,ab^2e^x+a^2b+2\,b^3\right)}{(a^4+a^2b^2)(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^x+b)} + \frac{2\,b\log\left(e^x+1\right)}{a^3} - \frac{2\,b\log\left(|e^x-1|\right)}{a^3}$$

```
[In] integrate(csch(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^2*b^2 + 2*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^5 + a^3*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^2*e^
(3*x) - a^2*b*e^(2*x) - 2*b^3*e^(2*x) - 2*a^3*e^x - 3*a*b^2*e^x + a^2*b + 2
*b^3)/((a^4 + a^2*b^2)*(b*e^(4*x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b
)) + 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3
```

Mupad [B] (verification not implemented)

Time = 4.36 (sec) , antiderivative size = 1017, normalized size of antiderivative = 8.84

$$\int \frac{\operatorname{csch}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2\,(25\,a^8\,b^6+90\,a^6\,b^8+96\,a^4\,b^{10}+32\,a^2\,b^{12})}{a^4\,b^2\,(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9)} - \frac{2\,e^x\,(50\,a^9\,b^6+155\,a^7\,b^8+152\,a^5\,b^{10}+48\,a^3\,b^{12})}{a^4\,b^3\,(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9)} - \frac{2\,e^{2\,x}\,(25\,a^8\,b^6+90\,a^6\,b^8+96\,a^4\,b^{10}+32\,a^2\,b^{12})}{a^4\,b^2\,(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9)}}{b-2\,a\,e^x+2\,a\,e^{3\,x}-2\,b\,e^{2\,x}+b\,e^{4\,x}} - \frac{2\,b\,\ln\left(e^x-1\right)}{a^3} + \frac{2\,b\,\ln\left(e^x+1\right)}{a^3} + \frac{b^2\,\ln\left(-\frac{64\,(3\,a^2+2\,b^2)\,(-8\,e^x\,a^3+4\,a^2\,b-7\,e^x\,a\,b^2+4\,b^3)}{a^6\,b\,(a^2+b^2)^2} - \frac{32\,(3\,a^2+2\,b^2)\left(8\,a^9\,b-8\,b^7\,\sqrt{(a^2+b^2)^3}+3\,a^3\,b^7+13\,a^5\,b^5+18\,a^7\,b^3-16\,a^{10}\,e\right.}{}\right.}{} - \frac{b^2\,\ln\left(\frac{32\,(3\,a^2+2\,b^2)\left(8\,a^9\,b+8\,b^7\,\sqrt{(a^2+b^2)^3}+3\,a^3\,b^7+13\,a^5\,b^5+18\,a^7\,b^3-16\,a^{10}\,e^x+24\,a^2\,b^5\,\sqrt{(a^2+b^2)^3}+18\,a^4\,b^3\,\sqrt{(a^2+b^2)^3}-9\,a^4\,b^6\,e\right.}{a^6\,b\,\sqrt{(a^2+b^2)^3}\,(a^2+b^2)^4}\right.}{}$$

```
[In] int(1/(sinh(x)^2*(a + b*sinh(x))^2),x)
```

```
[Out] ((2*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^8 + 25*a^8*b^6))/(a^4*b^2*(16*b^9
 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (2*exp(x)*(48*a^3*b^12 + 152*a^
```

```
5*b^10 + 155*a^7*b^8 + 50*a^9*b^6))/(a^4*b^3*(16*b^9 + 56*a^2*b^7 + 65*a^4*
b^5 + 25*a^6*b^3)) - (2*exp(2*x)*(32*a^2*b^12 + 96*a^4*b^10 + 90*a^6*b^8 +
25*a^8*b^6))/(a^4*b^2*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) + (2
*exp(3*x)*(16*a^3*b^12 + 40*a^5*b^10 + 25*a^7*b^8))/(a^4*b^3*(16*b^9 + 56*a
^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) + 2*a*exp(3*x) - 2*b*ex
p(2*x) + b*exp(4*x)) - (2*b*log(exp(x) - 1))/a^3 + (2*b*log(exp(x) + 1))/a^
3 + (b^2*log(- (64*(3*a^2 + 2*b^2)*(4*a^2*b + 4*b^3 - 8*a^3*exp(x) - 7*a*b^
2*exp(x)))/(a^6*b*(a^2 + b^2)^2) - (32*(3*a^2 + 2*b^2)*(8*a^9*b - 8*b^7*((a
^2 + b^2)^3)^(1/2) + 3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^10*exp(x) -
 24*a^2*b^5*((a^2 + b^2)^3)^(1/2) - 18*a^4*b^3*((a^2 + b^2)^3)^(1/2) - 9*a^
4*b^6*exp(x) - 33*a^6*b^4*exp(x) - 40*a^8*b^2*exp(x) + 41*a^3*b^4*exp(x)*((
a^2 + b^2)^3)^(1/2) + 30*a^5*b^2*exp(x)*((a^2 + b^2)^3)^(1/2) + 14*a*b^6*ex
p(x)*((a^2 + b^2)^3)^(1/2)))/(a^6*b*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4))*(
(a^2 + b^2)^3)^(1/2)*(3*a^2 + 2*b^2))/(a^9 + a^3*b^6 + 3*a^5*b^4 + 3*a^7*b^
2) - (b^2*log((32*(3*a^2 + 2*b^2)*(8*a^9*b + 8*b^7*((a^2 + b^2)^3)^(1/2) +
3*a^3*b^7 + 13*a^5*b^5 + 18*a^7*b^3 - 16*a^10*exp(x) + 24*a^2*b^5*((a^2 + b
^2)^3)^(1/2) + 18*a^4*b^3*((a^2 + b^2)^3)^(1/2) - 9*a^4*b^6*exp(x) - 33*a^6
*b^4*exp(x) - 40*a^8*b^2*exp(x) - 41*a^3*b^4*exp(x)*((a^2 + b^2)^3)^(1/2) -
 30*a^5*b^2*exp(x)*((a^2 + b^2)^3)^(1/2) - 14*a*b^6*exp(x)*((a^2 + b^2)^3)^
(1/2)))/(a^6*b*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4) - (64*(3*a^2 + 2*b^2)*(
4*a^2*b + 4*b^3 - 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^6*b*(a^2 + b^2)^2))*((
a^2 + b^2)^3)^(1/2)*(3*a^2 + 2*b^2))/(a^9 + a^3*b^6 + 3*a^5*b^4 + 3*a^7*b^2
)
```

3.86 $\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 158

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-6b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} + \frac{2b^3(4a^2+3b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4(a^2+b^2)^{3/2}} + \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

```
[Out] 1/2*(a^2-6*b^2)*arctanh(cosh(x))/a^4+2*b^3*(4*a^2+3*b^2)*arctanh((b-a*tanh(
1/2*x))/(a^2+b^2)^(1/2))/a^4/(a^2+b^2)^(3/2)+b*(2*a^2+3*b^2)*coth(x)/a^3/(a
^2+b^2)-1/2*(a^2+3*b^2)*coth(x)*csch(x)/a^2/(a^2+b^2)+b^2*coth(x)*csch(x)/a
/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{(a^2-6b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} + \frac{2b^3(4a^2+3b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4(a^2+b^2)^{3/2}} + \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)}$$

```
[In] Int[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - 6*b^2)*ArcTanh[Cosh[x]])/(2*a^4) + (2*b^3*(4*a^2 + 3*b^2)*ArcTanh[(
b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^4*(a^2 + b^2)^(3/2)) + (b*(2*a^2 + 3*
b^2)*Coth[x])/(a^3*(a^2 + b^2)) - ((a^2 + 3*b^2)*Coth[x]*Csch[x])/(2*a^2*(a
^2 + b^2)) + (b^2*Coth[x]*Csch[x])/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}+\frac{\int\frac{\operatorname{csch}^3(x)\left(a^2+3b^2-ab\sinh(x)+2b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}\\
&= -\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\\
&\quad+\frac{i\int\frac{\operatorname{csch}^2(x)\left(2ib\left(2a^2+3b^2\right)+ia\left(a^2-b^2\right)\sinh(x)+ib\left(a^2+3b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{2a^2\left(a^2+b^2\right)}\\
&= \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)}-\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)}\\
&\quad+\frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\int\frac{\operatorname{csch}(x)\left(a^4-5a^2b^2-6b^4+ab\left(a^2+3b^2\right)\sinh(x)\right)}{a+b\sinh(x)}\,dx}{2a^3\left(a^2+b^2\right)}\\
&= \frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)}-\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}\\
&\quad-\frac{\left(a^2-6b^2\right)\int\operatorname{csch}(x)\,dx}{2a^4}-\frac{\left(b^3\left(4a^2+3b^2\right)\right)\int\frac{1}{a+b\sinh(x)}\,dx}{a^4\left(a^2+b^2\right)}\\
&= \frac{\left(a^2-6b^2\right)\operatorname{arctanh}(\cosh(x))}{2a^4}+\frac{b\left(2a^2+3b^2\right)\coth(x)}{a^3\left(a^2+b^2\right)}-\frac{\left(a^2+3b^2\right)\coth(x)\operatorname{csch}(x)}{2a^2\left(a^2+b^2\right)}\\
&\quad+\frac{b^2\coth(x)\operatorname{csch}(x)}{a\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{\left(2b^3\left(4a^2+3b^2\right)\right)\operatorname{Subst}\left(\int\frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^4\left(a^2+b^2\right)}
\end{aligned}$$

$$= \frac{(a^2-6b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} + \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{(4b^3(4a^2+3b^2))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^4(a^2+b^2)}$$

$$= \frac{(a^2-6b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} + \frac{2b^3(4a^2+3b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4(a^2+b^2)^{3/2}} + \frac{b(2a^2+3b^2)\coth(x)}{a^3(a^2+b^2)} - \frac{(a^2+3b^2)\coth(x)\operatorname{csch}(x)}{2a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{\frac{16b^3(4a^2+3b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}} + 8ab\coth\left(\frac{x}{2}\right) - a^2\operatorname{csch}^2\left(\frac{x}{2}\right) + 4(a^2-6b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right) - 4(a^2-6b^2)\log\left(s\right.}{8a^4}$$

```
[In] Integrate[Csch[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((16*b^3*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2
- b^2)^(3/2) + 8*a*b*Coth[x/2] - a^2*Csch[x/2]^2 + 4*(a^2 - 6*b^2)*Log[Cosh
[x/2]] - 4*(a^2 - 6*b^2)*Log[Sinh[x/2]] - a^2*Sech[x/2]^2 + (8*a*b^4*Cosh[x
])/((a^2 + b^2)*(a + b*Sinh[x])) + 8*a*b*Tanh[x/2])/(8*a^4)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^2 a}{2}+4b\tanh\left(\frac{x}{2}\right)}{4a^3} + \frac{4b^3\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{2(a^2+b^2)}-\frac{ab}{2(a^2+b^2)}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a} - \frac{(4a^2+3b^2)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{\frac{3}{2}}}\right)}{a^4} - \frac{1}{8a^2\tanh\left(\frac{x}{2}\right)^2} + \frac{(-2a^2+12\ldots}{}$
risch	$-\frac{a^3b\,e^{5x}+3a\,b^3e^{5x}+2a^4e^{4x}-2a^2b^2e^{4x}-6b^4e^{4x}-8a^3b\,e^{3x}-12a\,b^3e^{3x}+2a^4e^{2x}+10a^2b^2e^{2x}+12b^4e^{2x}+7a^3b\,e^{x}+9b^3e^{x}a-4a^2b^2-6b^4}{a^3(e^{2x}-1)^2(a^2+b^2)(b\,e^{2x}+2\,e^{x}a-b)} + \ldots$

```
[In] int(csch(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^3*(1/2*tanh(1/2*x)^2*a+4*b*tanh(1/2*x))+4/a^4*b^3*((-1/2*b^2/(a^2+b^2
)*tanh(1/2*x)-1/2*a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-1/2*(4
*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/
2)))-1/8/a^2/tanh(1/2*x)^2+1/4/a^4*(-2*a^2+12*b^2)*ln(tanh(1/2*x))+1/a^3*b/
tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3754 vs. $2(150) = 300$.

Time = 0.51 (sec) , antiderivative size = 3754, normalized size of antiderivative = 23.76

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a^5*b^2 + 20*a^3*b^4 + 12*a*b^6 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)
*cosh(x)^5 - 2*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*sinh(x)^5 - 4*(a^7 - 4*a^3*b
^4 - 3*a*b^6)*cosh(x)^4 - 2*(2*a^7 - 8*a^3*b^4 - 6*a*b^6 + 5*(a^6*b + 4*a^4
*b^3 + 3*a^2*b^5)*cosh(x))*sinh(x)^4 + 8*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*
cosh(x)^3 + 4*(4*a^6*b + 10*a^4*b^3 + 6*a^2*b^5 - 5*(a^6*b + 4*a^4*b^3 + 3*
a^2*b^5)*cosh(x)^2 - 4*(a^7 - 4*a^3*b^4 - 3*a*b^6)*cosh(x))*sinh(x)^3 - 4*(
a^7 + 6*a^5*b^2 + 11*a^3*b^4 + 6*a*b^6)*cosh(x)^2 - 4*(a^7 + 6*a^5*b^2 + 11
*a^3*b^4 + 6*a*b^6 + 5*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*cosh(x)^3 + 6*(a^7 -
 4*a^3*b^4 - 3*a*b^6)*cosh(x)^2 - 6*(2*a^6*b + 5*a^4*b^3 + 3*a^2*b^5)*cosh(
x))*sinh(x)^2 + 2*((4*a^2*b^4 + 3*b^6)*cosh(x)^6 + (4*a^2*b^4 + 3*b^6)*sinh
(x)^6 - 4*a^2*b^4 - 3*b^6 + 2*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 2*(4*a^3*b^
3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^5 - 3*(4*a^2*b^4 + 3*b
^6)*cosh(x)^4 - (12*a^2*b^4 + 9*b^6 - 15*(4*a^2*b^4 + 3*b^6)*cosh(x)^2 - 10
*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^4 - 4*(4*a^3*b^3 + 3*a*b^5)*cosh(x)
^3 - 4*(4*a^3*b^3 + 3*a*b^5 - 5*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 5*(4*a^3*b^
3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x)^3 + 3*(4*a^
2*b^4 + 3*b^6)*cosh(x)^2 + (12*a^2*b^4 + 9*b^6 + 15*(4*a^2*b^4 + 3*b^6)*cos
h(x)^4 + 20*(4*a^3*b^3 + 3*a*b^5)*cosh(x)^3 - 18*(4*a^2*b^4 + 3*b^6)*cosh(x
)^2 - 12*(4*a^3*b^3 + 3*a*b^5)*cosh(x))*sinh(x)^2 + 2*(4*a^3*b^3 + 3*a*b^5)
*cosh(x) + 2*(4*a^3*b^3 + 3*a*b^5 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x)^5 + 5*(4*
a^3*b^3 + 3*a*b^5)*cosh(x)^4 - 6*(4*a^2*b^4 + 3*b^6)*cosh(x)^3 - 6*(4*a^3*b
^3 + 3*a*b^5)*cosh(x)^2 + 3*(4*a^2*b^4 + 3*b^6)*cosh(x))*sinh(x))*sqrt(a^2
+ b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2
*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a
))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b
)) - 2*(7*a^6*b + 16*a^4*b^3 + 9*a^2*b^5)*cosh(x) - (a^6*b - 4*a^4*b^3 - 11
*a^2*b^5 - 6*b^7 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^
```

6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x)^5 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3
*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4
*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b
^5 - 18*b^7 - 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a
^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*
b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6
*a*b^6 - 5*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*
a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b
^5 - 6*b^7)*cosh(x))*sinh(x)^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)
*cosh(x)^2 - (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^
4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^3 - 18*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2
- 12*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 -
4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^
4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7
- 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*
a^2*b^5 - 6*b^7)*cosh(x)^3 - 6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cos
h(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(c
osh(x) + sinh(x) + 1) + (a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7 - (a^6*b -
4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^6 - (a^6*b - 4*a^4*b^3 - 11*a^2*b^5
- 6*b^7)*sinh(x)^6 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^5
- 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2
*b^5 - 6*b^7)*cosh(x))*sinh(x)^5 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^
7)*cosh(x)^4 + (3*a^6*b - 12*a^4*b^3 - 33*a^2*b^5 - 18*b^7 - 15*(a^6*b - 4*
a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 10*(a^7 - 4*a^5*b^2 - 11*a^3*b^4
- 6*a*b^6)*cosh(x))*sinh(x)^4 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*
cosh(x)^3 + 4*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 - 5*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 - 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b
^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x)
^3 - 3*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - (3*a^6*b - 12*a
^4*b^3 - 33*a^2*b^5 - 18*b^7 + 15*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*
cosh(x)^4 + 20*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^3 - 18*(a^6
*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^2 - 12*(a^7 - 4*a^5*b^2 - 11*a
^3*b^4 - 6*a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*
a*b^6)*cosh(x) - 2*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6 + 3*(a^6*b - 4*a
^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^5 + 5*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 -
6*a*b^6)*cosh(x)^4 - 6*(a^6*b - 4*a^4*b^3 - 11*a^2*b^5 - 6*b^7)*cosh(x)^3 -
6*(a^7 - 4*a^5*b^2 - 11*a^3*b^4 - 6*a*b^6)*cosh(x)^2 + 3*(a^6*b - 4*a^4*b^
3 - 11*a^2*b^5 - 6*b^7)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(7
*a^6*b + 16*a^4*b^3 + 9*a^2*b^5 + 5*(a^6*b + 4*a^4*b^3 + 3*a^2*b^5)*cosh(x)
^4 + 8*(a^7 - 4*a^3*b^4 - 3*a*b^6)*cosh(x)^3 - 12*(2*a^6*b + 5*a^4*b^3 + 3*
a^2*b^5)*cosh(x)^2 + 4*(a^7 + 6*a^5*b^2 + 11*a^3*b^4 + 6*a*b^6)*cosh(x))*si
nh(x))/(a^8*b + 2*a^6*b^3 + a^4*b^5 - (a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)
^6 - (a^8*b + 2*a^6*b^3 + a^4*b^5)*sinh(x)^6 - 2*(a^9 + 2*a^7*b^2 + a^5*b^4

```
)*cosh(x)^5 - 2*(a^9 + 2*a^7*b^2 + a^5*b^4 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5
)*cosh(x))*sinh(x)^5 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^4 + (3*a^8*b
 + 6*a^6*b^3 + 3*a^4*b^5 - 15*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^2 - 10*
(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x))*sinh(x)^4 + 4*(a^9 + 2*a^7*b^2 + a^5*b
^4)*cosh(x)^3 + 4*(a^9 + 2*a^7*b^2 + a^5*b^4 - 5*(a^8*b + 2*a^6*b^3 + a^4*b
^5)*cosh(x)^3 - 5*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x)^2 + 3*(a^8*b + 2*a^6*
b^3 + a^4*b^5)*cosh(x))*sinh(x)^3 - 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)
^2 - (3*a^8*b + 6*a^6*b^3 + 3*a^4*b^5 + 15*(a^8*b + 2*a^6*b^3 + a^4*b^5)*co
sh(x)^4 + 20*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x)^3 - 18*(a^8*b + 2*a^6*b^3
+ a^4*b^5)*cosh(x)^2 - 12*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x))*sinh(x)^2 -
2*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(x) - 2*(a^9 + 2*a^7*b^2 + a^5*b^4 + 3*(a
^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^5 + 5*(a^9 + 2*a^7*b^2 + a^5*b^4)*cosh(
x)^4 - 6*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x)^3 - 6*(a^9 + 2*a^7*b^2 + a^5
*b^4)*cosh(x)^2 + 3*(a^8*b + 2*a^6*b^3 + a^4*b^5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(csch(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(csch(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(150) = 300$.

Time = 0.29 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.30

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(4\,a^2b^3+3\,b^5)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+a^4b^2)\sqrt{a^2+b^2}} + \frac{4\,a^2b^2+6\,b^4+(7\,a^3b+9\,ab^3)e^{(-x)}-2\,(a^4+5\,a^2b^2+6\,b^4)e^{(-2\,x)}-4\,(2\,a^3b+3\,ab^3)e^{(-3\,x)}-2\,(a^4-a^2b}{a^5b+a^3b^3+2\,(a^6+a^4b^2)e^{(-x)}-3\,(a^5b+a^3b^3)e^{(-2\,x)}-4\,(a^6+a^4b^2)e^{(-3\,x)}+3\,(a^5b+a^3b^3)e^{(-4\,x)}+2\,(} + \frac{(a^2-6\,b^2)\log\left(e^{(-x)}+1\right)}{2\,a^4} - \frac{(a^2-6\,b^2)\log\left(e^{(-x)}-1\right)}{2\,a^4}$$

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -(4*a^2*b^3 + 3*b^5)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) + (4*a^2*b^2 + 6*b^4 + (
7*a^3*b + 9*a*b^3)*e^(-x) - 2*(a^4 + 5*a^2*b^2 + 6*b^4)*e^(-2*x) - 4*(2*a^3
*b + 3*a*b^3)*e^(-3*x) - 2*(a^4 - a^2*b^2 - 3*b^4)*e^(-4*x) + (a^3*b + 3*a*
```

```
b^3)*e^(-5*x))/(a^5*b + a^3*b^3 + 2*(a^6 + a^4*b^2)*e^(-x) - 3*(a^5*b + a^3
*b^3)*e^(-2*x) - 4*(a^6 + a^4*b^2)*e^(-3*x) + 3*(a^5*b + a^3*b^3)*e^(-4*x)
+ 2*(a^6 + a^4*b^2)*e^(-5*x) - (a^5*b + a^3*b^3)*e^(-6*x)) + 1/2*(a^2 - 6*b
^2)*log(e^(-x) + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(e^(-x) - 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(4a^2b^3+3b^5)\log\left(\frac{|2be^x+2a-2\sqrt{a^2+b^2}|}{|2be^x+2a+2\sqrt{a^2+b^2}|}\right)}{(a^6+a^4b^2)\sqrt{a^2+b^2}} - \frac{2(ab^3e^x-b^4)}{(a^5+a^3b^2)(be^{(2x)}+2ae^x-b)} + \frac{(a^2-6b^2)\log(e^x+1)}{2a^4} - \frac{(a^2-6b^2)\log(|e^x-1|)}{2a^4} - \frac{ae^{(3x)}-4be^{(2x)}+ae^x+4b}{a^3(e^{(2x)}-1)^2}$$

```
[In] integrate(csch(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -(4*a^2*b^3 + 3*b^5)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
 + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + a^4*b^2)*sqrt(a^2 + b^2)) - 2*(a*b^3*e
^x - b^4)/((a^5 + a^3*b^2)*(b*e^(2*x) + 2*a*e^x - b)) + 1/2*(a^2 - 6*b^2)*l
og(e^x + 1)/a^4 - 1/2*(a^2 - 6*b^2)*log(abs(e^x - 1))/a^4 - (a*e^(3*x) - 4*
b*e^(2*x) + a*e^x + 4*b)/(a^3*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 977, normalized size of antiderivative = 6.18

$$\int \frac{\operatorname{csch}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{4b}{a^3}-\frac{e^x}{a^2}}{e^{2x}-1} + \frac{\frac{2b^7}{a^3(a^2b^3+b^5)}-\frac{2b^6e^x}{a^2(a^2b^3+b^5)}}{2ae^x-b+be^{2x}} - \frac{\ln(e^x-1)\,(a^2-6b^2)}{2a^4} + \frac{\ln(e^x+1)\,(a^2-6b^2)}{2a^4} - \frac{2e^x}{a^2\,(e^{4x}-2e^{2x}+1)} + \frac{b^3\ln\left(\frac{8(4a^2+3b^2)\left(20a^9b^5-72b^{11}\sqrt{(a^2+b^2)^3}-9a^3b^{11}-30a^5b^9-18a^7b^7-2a^{13}b+15a^{11}b^3+4a^{14}e^x-192a^2b^9\sqrt{(a^2+b^2)^3}-128a^4b^7\right.}{}\right.}{} - \frac{b^3\ln\left(\frac{8(4a^2+3b^2)\left(2a^{13}b-72b^{11}\sqrt{(a^2+b^2)^3}+9a^3b^{11}+30a^5b^9+18a^7b^7-20a^9b^5-15a^{11}b^3-4a^{14}e^x-192a^2b^9\sqrt{(a^2+b^2)^3}-128a^4b^7\right.}{}\right.}{}$$

```
[In] int(1/(sinh(x)^3*(a + b*sinh(x))^2),x)
```

```
[Out] ((4*b)/a^3 - exp(x)/a^2)/(exp(2*x) - 1) + ((2*b^7)/(a^3*(b^5 + a^2*b^3)) -
(2*b^6*exp(x))/(a^2*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2*x)) - (log(
exp(x) - 1)*(a^2 - 6*b^2))/(2*a^4) + (log(exp(x) + 1)*(a^2 - 6*b^2))/(2*a^4
) - (2*exp(x))/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) + (b^3*log((8*(4*a^2 + 3*b
^2)*(20*a^9*b^5 - 72*b^11*((a^2 + b^2)^3)^(1/2) - 9*a^3*b^11 - 30*a^5*b^9 -
 18*a^7*b^7 - 2*a^13*b + 15*a^11*b^3 + 4*a^14*exp(x) - 192*a^2*b^9*((a^2 +
b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^2)^3)^(1/2) + 27*a^4*b^10*exp(x) + 72
*a^6*b^8*exp(x) + 30*a^8*b^6*exp(x) - 48*a^10*b^4*exp(x) - 29*a^12*b^2*exp(
x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 206*a^5*b^6*exp(x)*((a^2 +
b^2)^3)^(1/2) + 8*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) + 118*a*b^10*exp(x)*((
a^2 + b^2)^3)^(1/2)))/(a^9*b^2*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4) - (8*(1
8*b^4 - 4*a^4 + 21*a^2*b^2)*(2*a^4*b - 12*b^5 - 10*a^2*b^3 - 4*a^5*exp(x) +
 21*a*b^4*exp(x) + 19*a^3*b^2*exp(x)))/(a^9*b^2*(a^2 + b^2)^2))*((a^2 + b^2
)^3)^(1/2)*(4*a^2 + 3*b^2))/(a^10 + a^4*b^6 + 3*a^6*b^4 + 3*a^8*b^2) - (b^3
*log((8*(4*a^2 + 3*b^2)*(2*a^13*b - 72*b^11*((a^2 + b^2)^3)^(1/2) + 9*a^3*b
^11 + 30*a^5*b^9 + 18*a^7*b^7 - 20*a^9*b^5 - 15*a^11*b^3 - 4*a^14*exp(x) -
192*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 128*a^4*b^7*((a^2 + b^2)^3)^(1/2) - 27*
a^4*b^10*exp(x) - 72*a^6*b^8*exp(x) - 30*a^8*b^6*exp(x) + 48*a^10*b^4*exp(x
) + 29*a^12*b^2*exp(x) + 312*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 206*a^5
*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 8*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) +
118*a*b^10*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^9*b^2*((a^2 + b^2)^3)^(1/2)*(a
^2 + b^2)^4) - (8*(18*b^4 - 4*a^4 + 21*a^2*b^2)*(2*a^4*b - 12*b^5 - 10*a^2*
b^3 - 4*a^5*exp(x) + 21*a*b^4*exp(x) + 19*a^3*b^2*exp(x)))/(a^9*b^2*(a^2 +
b^2)^2))*((a^2 + b^2)^3)^(1/2)*(4*a^2 + 3*b^2))/(a^10 + a^4*b^6 + 3*a^6*b^4
 + 3*a^8*b^2)
```

3.87 $\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 198

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = &-\frac{b(a^2-4b^2)\operatorname{arctanh}(\cosh(x))}{a^5} \\ &-\frac{2b^4(5a^2+4b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5\left(a^2+b^2\right)^{3/2}} \\ &+\frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4\left(a^2+b^2\right)}+\frac{b(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3\left(a^2+b^2\right)} \\ &-\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a^2\left(a^2+b^2\right)}+\frac{b^2\coth(x)\operatorname{csch}^2(x)}{a\left(a^2+b^2\right)\left(a+b\sinh(x)\right)}\end{aligned}$$

```
[Out] -b*(a^2-4*b^2)*arctanh(cosh(x))/a^5-2*b^4*(5*a^2+4*b^2)*arctanh((b-a*tanh(1
/2*x))/(a^2+b^2)^(1/2))/a^5/(a^2+b^2)^(3/2)+1/3*(2*a^4-7*a^2*b^2-12*b^4)*co
th(x)/a^4/(a^2+b^2)+b*(a^2+2*b^2)*coth(x)*csch(x)/a^3/(a^2+b^2)-1/3*(a^2+4*
b^2)*coth(x)*csch(x)^2/a^2/(a^2+b^2)+b^2*coth(x)*csch(x)^2/a/(a^2+b^2)/(a+b
*sinh(x))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used

= {2881, 3134, 3080, 3855, 2739, 632, 212}

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^2+4b^2)\coth(x)\operatorname{csch}^2(x)}{3a^2(a^2+b^2)} + \frac{b^2\coth(x)\operatorname{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))} - \frac{b(a^2-4b^2)\operatorname{arctanh}(\cosh(x))}{a^5} - \frac{2b^4(5a^2+4b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5(a^2+b^2)^{3/2}} + \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3(a^2+b^2)}$$

```
[In] Int[Csch[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] -((b*(a^2 - 4*b^2)*ArcTanh[Cosh[x]])/a^5) - (2*b^4*(5*a^2 + 4*b^2)*ArcTanh[
(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^5*(a^2 + b^2)^(3/2)) + ((2*a^4 - 7*a
^2*b^2 - 12*b^4)*Coth[x])/(3*a^4*(a^2 + b^2)) + (b*(a^2 + 2*b^2)*Coth[x]*Cs
ch[x])/(a^3*(a^2 + b^2)) - ((a^2 + 4*b^2)*Coth[x]*Csch[x]^2)/(3*a^2*(a^2 +
b^2)) + (b^2*Coth[x]*Csch[x]^2)/(a*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2881

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
+ 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Sin
```

```
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[2
*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*n
] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^2\coth(x)\text{csch}^2(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))}+\frac{\int\frac{\text{csch}^4(x)\left(a^2+4b^2-ab\sinh(x)+3b^2\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\,(a^2+b^2)}\\
&= -\frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2\,(a^2+b^2)}+\frac{b^2\coth(x)\text{csch}^2(x)}{a\,(a^2+b^2)\,(a+b\sinh(x))}\\
&\quad+\frac{i\int\frac{\text{csch}^3(x)\left(6ib(a^2+2b^2)+ia(2a^2-b^2)\sinh(x)+2ib(a^2+4b^2)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{3a^2\,(a^2+b^2)}
\end{aligned}
$$

$$= \frac{b(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2(a^2+b^2)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))} - \frac{\int \frac{\text{csch}^2(x)(2(2a^4-7a^2b^2-12b^4)-2ab(a^2-2b^2)\sinh(x)-6b^2(a^2+2b^2)\sinh^2(x))}{a+b\sinh(x)}\,dx}{6a^3(a^2+b^2)}$$

$$= \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2(a^2+b^2)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))} - \frac{i\int \frac{\text{csch}(x)(6ib(a^4-3a^2b^2-4b^4)+6iab^2(a^2+2b^2)\sinh(x))}{a+b\sinh(x)}\,dx}{6a^4(a^2+b^2)}$$

$$= \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2(a^2+b^2)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{(b(a^2-4b^2))\int \text{csch}(x)\,dx}{a^5} + \frac{(b^4(5a^2+4b^2))\int \frac{1}{a+b\sinh(x)}\,dx}{a^5(a^2+b^2)}$$

$$= -\frac{b(a^2-4b^2)\text{arctanh}(\cosh(x))}{a^5} + \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2(a^2+b^2)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))} + \frac{(2b^4(5a^2+4b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^5(a^2+b^2)}$$

$$= -\frac{b(a^2-4b^2)\text{arctanh}(\cosh(x))}{a^5} + \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2(a^2+b^2)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))} - \frac{(4b^4(5a^2+4b^2))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^5(a^2+b^2)}$$

$$= -\frac{b(a^2-4b^2)\text{arctanh}(\cosh(x))}{a^5} - \frac{2b^4(5a^2+4b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5(a^2+b^2)^{3/2}} + \frac{(2a^4-7a^2b^2-12b^4)\coth(x)}{3a^4(a^2+b^2)} + \frac{b(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3(a^2+b^2)} - \frac{(a^2+4b^2)\coth(x)\text{csch}^2(x)}{3a^2(a^2+b^2)} + \frac{b^2\coth(x)\text{csch}^2(x)}{a(a^2+b^2)(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{-\frac{48b^4(5a^2+4b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+4a(2a^2-9b^2)\coth\left(\frac{x}{2}\right)+6a^2b\operatorname{csch}^2\left(\frac{x}{2}\right)-24b(a^2-4b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)+2}{}$$

[In] Integrate[Csch[x]^4/(a + b*Sinh[x])^2,x]

[Out] ((-48*b^4*(5*a^2 + 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + 4*a*(2*a^2 - 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 - 24*b*(a^2 - 4*b^2)*Log[Cosh[x/2]] + 24*b*(a^2 - 4*b^2)*Log[Sinh[x/2]] + 6*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - (24*a*b^5*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x])) + 4*a*(2*a^2 - 9*b^2)*Tanh[x/2])/(24*a^5)

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^3 a^2}{3}+2ab\tanh\left(\frac{x}{2}\right)^2-3a^2\tanh\left(\frac{x}{2}\right)+12b^2\tanh\left(\frac{x}{2}\right)}{8a^4}-\frac{2b^4\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{ab}{a^2+b^2}}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(5a^2+4b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right.}{a^5}$
risch	$-\frac{2\left(-3a^3b^2e^{7x}-6ab^4e^{7x}-6a^4be^{6x}+3a^2b^3e^{6x}+12b^5e^{6x}+21a^3b^2e^{5x}+30ab^4e^{5x}+6a^4be^{4x}-21a^2b^3e^{4x}-36b^5e^{4x}+12a^5e^{3x}-21a^3b^2e^{3x}\right.}{3a^4\left(e^{2x}-1\right)^3\left(a^2+b^2\right)\left(b e^{2x}+2e^x a-b\right)}$

[In] int(csch(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] -1/8/a^4*(1/3*tanh(1/2*x)^3*a^2+2*a*b*tanh(1/2*x)^2-3*a^2*tanh(1/2*x)+12*b^2*tanh(1/2*x))-2/a^5*b^4*((-b^2/(a^2+b^2)*tanh(1/2*x)-a*b/(a^2+b^2))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-(5*a^2+4*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/24/a^2/tanh(1/2*x)^3-1/8/a^4*(-3*a^2+12*b^2)/tanh(1/2*x)+1/4/a^3*b/tanh(1/2*x)^2+1/a^5*b*(a^2-4*b^2)*ln(tanh(1/2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6430 vs. $2(190) = 380$.

Time = 0.54 (sec) , antiderivative size = 6430, normalized size of antiderivative = 32.47

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{csch}^4{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

```
[In] integrate(csch(x)**4/(a+b*sinh(x))**2,x)
[Out] Integral(csch(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(190) = 380$.

Time = 0.28 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.41

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(5a^2b^4+4b^6)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^7+a^5b^2)\sqrt{a^2+b^2}} + \frac{2\left(2a^4b-7a^2b^3-12b^5+(4a^5-11a^3b^2-18ab^4)e^{(-x)}-(2a^4b-25a^2b^3-36b^5)e^{(-2x)}-3(4a^5-7\right.}{3\left(a^6b+a^4b^3+2(a^7+a^5b^2)e^{(-x)}-4(a^6b+a^4b^3)e^{(-2x)}-6(a^7+a^5b^2)e^{(-3x)}\right.} - \frac{(a^2b-4b^3)\log\left(e^{(-x)}+1\right)}{a^5} + \frac{(a^2b-4b^3)\log\left(e^{(-x)}-1\right)}{a^5}$$

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] (5*a^2*b^4 + 4*b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sq
rt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2/3*(2*a^4*b - 7*a^2*b^
3 - 12*b^5 + (4*a^5 - 11*a^3*b^2 - 18*a*b^4)*e^(-x) - (2*a^4*b - 25*a^2*b^3
 - 36*b^5)*e^(-2*x) - 3*(4*a^5 - 7*a^3*b^2 - 14*a*b^4)*e^(-3*x) + 3*(2*a^4*
b - 7*a^2*b^3 - 12*b^5)*e^(-4*x) - 3*(7*a^3*b^2 + 10*a*b^4)*e^(-5*x) - 3*(2
*a^4*b - a^2*b^3 - 4*b^5)*e^(-6*x) + 3*(a^3*b^2 + 2*a*b^4)*e^(-7*x))/(a^6*b
 + a^4*b^3 + 2*(a^7 + a^5*b^2)*e^(-x) - 4*(a^6*b + a^4*b^3)*e^(-2*x) - 6*(a
^7 + a^5*b^2)*e^(-3*x) + 6*(a^6*b + a^4*b^3)*e^(-4*x) + 6*(a^7 + a^5*b^2)*e
^(-5*x) - 4*(a^6*b + a^4*b^3)*e^(-6*x) - 2*(a^7 + a^5*b^2)*e^(-7*x) + (a^6*
b + a^4*b^3)*e^(-8*x)) - (a^2*b - 4*b^3)*log(e^(-x) + 1)/a^5 + (a^2*b - 4*b
^3)*log(e^(-x) - 1)/a^5
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(5\,a^2b^4+4\,b^6)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^7+a^5b^2)\sqrt{a^2+b^2}} + \frac{2\,(ab^4e^x-b^5)}{(a^6+a^4b^2)(be^{(2\,x)}+2\,ae^x-b)} - \frac{(a^2b-4\,b^3)\log\left(e^x+1\right)}{a^5} + \frac{(a^2b-4\,b^3)\log\left(|e^x-1|\right)}{a^5} + \frac{2\left(3\,abe^{(5\,x)}-9\,b^2e^{(4\,x)}-6\,a^2e^{(2\,x)}+18\,b^2e^{(2\,x)}-3\,abe^x+2\,a^2-9\,b^2\right)}{3\,a^4\left(e^{(2\,x)}-1\right)^3}$$

```
[In] integrate(csch(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (5*a^2*b^4 + 4*b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/((a^7 + a^5*b^2)*sqrt(a^2 + b^2)) + 2*(a*b^4*e^
x - b^5)/((a^6 + a^4*b^2)*(b*e^(2*x) + 2*a*e^x - b)) - (a^2*b - 4*b^3)*log(
e^x + 1)/a^5 + (a^2*b - 4*b^3)*log(abs(e^x - 1))/a^5 + 2/3*(3*a*b*e^(5*x) -
 9*b^2*e^(4*x) - 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b*e^x + 2*a^2 - 9*b^2
)/(a^4*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 4.18 (sec) , antiderivative size = 975, normalized size of antiderivative = 4.92

$$\int \frac{\operatorname{csch}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)\,\left(a^2\,b-4\,b^3\right)}{a^5} - \frac{8}{3\,a^2\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)} - \frac{\frac{4}{a^2}-\frac{4\,b\,\mathrm{e}^x}{a^3}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1} - \frac{\frac{6\,b^2}{a^4}-\frac{2\,b\,\mathrm{e}^x}{a^3}}{\mathrm{e}^{2\,x}-1} - \frac{\frac{2\,b^8}{a^4\,\left(a^2\,b^3+b^5\right)}-\frac{2\,b^7\,\mathrm{e}^x}{a^3\,\left(a^2\,b^3+b^5\right)}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}} - \frac{\ln\left(\mathrm{e}^x+1\right)\,\left(a^2\,b-4\,b^3\right)}{a^5} - \frac{b^4\,\ln\left(\frac{32\,b\,\left(-5\,a^4+16\,a^2\,b^2+16\,b^4\right)\,\left(-4\,\mathrm{e}^x\,a^5+2\,a^4\,b+11\,\mathrm{e}^x\,a^3\,b^2-6\,a^2\,b^3+14\,\mathrm{e}^x\,a\,b^4-8\,b^5\right)}{a^{12}\,\left(a^2+b^2\right)^2} - \frac{32\,b\,\left(5\,a^2+4\,b^2\right)\,\left(5\,a^5\,b^9-32\,b^{11}\,\sqrt{\left(a^2+b^2\right)^3}\cdots\right.}{\cdots}\right.}{\cdots} + \frac{b^4\,\ln\left(\frac{32\,b\,\left(-5\,a^4+16\,a^2\,b^2+16\,b^4\right)\,\left(-4\,\mathrm{e}^x\,a^5+2\,a^4\,b+11\,\mathrm{e}^x\,a^3\,b^2-6\,a^2\,b^3+14\,\mathrm{e}^x\,a\,b^4-8\,b^5\right)}{a^{12}\,\left(a^2+b^2\right)^2} - \frac{32\,b\,\left(5\,a^2+4\,b^2\right)\,\left(2\,a^{13}\,b-32\,b^{11}\,\sqrt{\left(a^2+b^2\right)^3}\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] int(1/(sinh(x)^4*(a + b*sinh(x))^2),x)
```

```
[Out] (log(exp(x) - 1)*(a^2*b - 4*b^3))/a^5 - 8/(3*a^2*(3*exp(2*x) - 3*exp(4*x) +
 exp(6*x) - 1)) - (4/a^2 - (4*b*exp(x))/a^3)/(exp(4*x) - 2*exp(2*x) + 1) -
((6*b^2)/a^4 - (2*b*exp(x))/a^3)/(exp(2*x) - 1) - ((2*b^8)/(a^4*(b^5 + a^2*
b^3)) - (2*b^7*exp(x))/(a^3*(b^5 + a^2*b^3)))/(2*a*exp(x) - b + b*exp(2*x))
 - (log(exp(x) + 1)*(a^2*b - 4*b^3))/a^5 - (b^4*log((32*b*(16*b^4 - 5*a^4 +
 16*a^2*b^2)*(2*a^4*b - 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*exp(x)
+ 11*a^3*b^2*exp(x)))/(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(5*a^5*b
^9 - 32*b^11*((a^2 + b^2)^3)^(1/2) - 2*a^13*b + 20*a^7*b^7 + 24*a^9*b^5 + 7
*a^11*b^3 + 4*a^14*exp(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*a^4*b^7*(
(a^2 + b^2)^3)^(1/2) - 15*a^6*b^8*exp(x) - 50*a^8*b^6*exp(x) - 52*a^10*b^4*
exp(x) - 13*a^12*b^2*exp(x) + 127*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 79
*a^5*b^6*exp(x)*((a^2 + b^2)^3)^(1/2) + 5*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2
) + 51*a*b^10*exp(x)*((a^2 + b^2)^3)^(1/2)))/(a^12*((a^2 + b^2)^3)^(1/2)*(a
^2 + b^2)^4))*((a^2 + b^2)^3)^(1/2)*(5*a^2 + 4*b^2))/(a^11 + a^5*b^6 + 3*a^
7*b^4 + 3*a^9*b^2) + (b^4*log((32*b*(16*b^4 - 5*a^4 + 16*a^2*b^2)*(2*a^4*b
- 8*b^5 - 6*a^2*b^3 - 4*a^5*exp(x) + 14*a*b^4*exp(x) + 11*a^3*b^2*exp(x)))/
(a^12*(a^2 + b^2)^2) - (32*b*(5*a^2 + 4*b^2)*(2*a^13*b - 32*b^11*((a^2 + b^
2)^3)^(1/2) - 5*a^5*b^9 - 20*a^7*b^7 - 24*a^9*b^5 - 7*a^11*b^3 - 4*a^14*exp
(x) - 80*a^2*b^9*((a^2 + b^2)^3)^(1/2) - 50*a^4*b^7*((a^2 + b^2)^3)^(1/2) +
 15*a^6*b^8*exp(x) + 50*a^8*b^6*exp(x) + 52*a^10*b^4*exp(x) + 13*a^12*b^2*e
xp(x) + 127*a^3*b^8*exp(x)*((a^2 + b^2)^3)^(1/2) + 79*a^5*b^6*exp(x)*((a^2
+ b^2)^3)^(1/2) + 5*a*b^4*exp(x)*((a^2 + b^2)^3)^(3/2) + 51*a*b^10*exp(x)*(
(a^2 + b^2)^3)^(1/2)))/(a^12*((a^2 + b^2)^3)^(1/2)*(a^2 + b^2)^4))*((a^2 +
b^2)^3)^(1/2)*(5*a^2 + 4*b^2))/(a^11 + a^5*b^6 + 3*a^7*b^4 + 3*a^9*b^2)
```

3.88 $\int \frac{1}{3+5i\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[Out] 1/4*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d-1/4*I*ln(cosh(1/2*d
*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2739, 630, 31}

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((I/4)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - ((I/4)*Log[Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2i)\text{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\
&= -\frac{(3i)\text{Subst}\left(\int \frac{1}{1+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{4d} + \frac{(3i)\text{Subst}\left(\int \frac{1}{9+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{4d}\\
&= \frac{i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d} - \frac{i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.11

$$\begin{aligned}
\int \frac{1}{3+5i\sinh(c+dx)}\,dx &= \frac{\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)}{4d} + \frac{\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}\\
&\quad - \frac{i\log(4-5\cosh(c+dx))}{8d} + \frac{i\log(4+5\cosh(c+dx))}{8d}
\end{aligned}$$

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ArcTan[3*Coth[(c + d*x)/2]]/(4*d) + ArcTan[3*Tanh[(c + d*x)/2]]/(4*d) - ((I
/8)*Log[4 - 5*Cosh[c + d*x]])/d + ((I/8)*Log[4 + 5*Cosh[c + d*x]])/d
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

method	result	size
risch	$-\frac{i \ln\left(e^{d x+c}-\frac{4}{5}-\frac{3 i}{5}\right)}{4 d}+\frac{i \ln\left(e^{d x+c}+\frac{4}{5}-\frac{3 i}{5}\right)}{4 d}$	36
derivativedivides	$\frac{-\frac{i \ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{4}+\frac{i \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}{4}}{d}$	40
default	$\frac{-\frac{i \ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{4}+\frac{i \ln\left(\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}{4}}{d}$	40
parallelrisch	$-\frac{i\left(-\ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-9 i\right)+\ln\left(3 \tanh\left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)\right)}{4 d}$	40

```
[In] int(1/(3+5*I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/4*I/d*ln(exp(d*x+c)-4/5-3/5*I)+1/4*I/d*ln(exp(d*x+c)+4/5-3/5*I)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.38

$$\int \frac{1}{3+5 i \sinh(c+dx)}\,dx=\frac{i \log\left(e^{(dx+c)}-\frac{3}{5} i+\frac{4}{5}\right)-i \log\left(e^{(dx+c)}-\frac{3}{5} i-\frac{4}{5}\right)}{4\,d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(I*log(e^(d*x + c) - 3/5*I + 4/5) - I*log(e^(d*x + c) - 3/5*I - 4/5))/d
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

$$\int \frac{1}{3+5 i \sinh(c+dx)}\,dx=\frac{\operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(\frac{(-16ii-3i)e^{-c}}{5}+e^{dx}\right)\right)\right)}{d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x)
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log((-16*_i*I - 3*I)*exp(-c)/5 + exp(d*
x))))/d
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.26

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = \frac{\arctan\left(\frac{5}{4}i\,e^{(-dx-c)}-\frac{3}{4}\right)}{2\,d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="maxima")
[Out] 1/2*arctan(5/4*I*e^(-d*x - c) - 3/4)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = -\frac{-i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right)+i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{4\,d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c)),x, algorithm="giac")
[Out] -1/4*(-I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) + I*log(-(2*I - 1)*e^(d*x + c)
 + I - 2))/d
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{1}{3+5i\sinh(c+dx)}\,dx = -\frac{\ln\left(-\frac{5}{2}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2-\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}+\frac{\ln\left(\frac{5}{2}+\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(2+\frac{3}{2}\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,d}$$

```
[In] int(1/(sinh(c + d*x)*5i + 3),x)
[Out] (log(exp(d*x)*exp(c)*(2 + 3i/2) + 5/2)*1i)/(4*d) - (log(exp(d*x)*exp(c)*(2
- 3i/2) - 5/2)*1i)/(4*d)
```

3.89 $\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}$$

```
[Out] -3/64*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d+3/64*I*ln(cosh(1/
2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/16*I*cosh(d*x+c)/d/(3+5*I*sinh(d*
x+c))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2743, 12, 2739, 630, 31}

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} - \frac{3i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{3i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}$$

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (((-3*I)/64)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((3*I)/64
)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/16)*Cosh[c
+ d*x])/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{1}{16}\int -\frac{3}{3+5i\sinh(c+dx)}\,dx \\
&= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} - \frac{3}{16}\int \frac{1}{3+5i\sinh(c+dx)}\,dx \\
&= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{(3i)\text{Subst}\left(\int \frac{1}{3+10x+3x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{8d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))} + \frac{(9i)\text{Subst}\left(\int \frac{1}{1+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{64d} \\
&\quad - \frac{(9i)\text{Subst}\left(\int \frac{1}{9+3x}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{64d} \\
&= -\frac{3i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} \\
&\quad + \frac{3i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} + \frac{5i\cosh(c+dx)}{16d(3+5i\sinh(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.39

$$
\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{-9\left(2\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)+2\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)-i\log(4-5\cosh(c+dx))+i\log(4+5\right.}{384d}
$$

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-9*(2*ArcTan[3*Coth[(c + d*x)/2]] + 2*ArcTan[3*Tanh[(c + d*x)/2]] - I*Log[
4 - 5*Cosh[c + d*x]] + I*Log[4 + 5*Cosh[c + d*x]]) + 40*((3*Cosh[(c + d*x)/
2] + I*Sinh[(c + d*x)/2])^(-1) + 3/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x
)/2]))*Sinh[(c + d*x)/2])/(384*d)
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{64}+\frac{5}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}+\frac{3i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{64}+\frac{5}{48\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}}{d}$	74
default	$\frac{-\frac{3i\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}{64}+\frac{5}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-3i\right)}+\frac{3i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{64}+\frac{5}{48\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}}{d}$	74
risch	$\frac{i\left(3\,\mathrm{e}^{dx+c}-5i\right)}{8d\left(5\,\mathrm{e}^{2dx+2c}-5-6i\mathrm{e}^{dx+c}\right)}+\frac{3i\ln\left(\mathrm{e}^{dx+c}-\frac{4}{5}-\frac{3i}{5}\right)}{64d}-\frac{3i\ln\left(\mathrm{e}^{dx+c}+\frac{4}{5}-\frac{3i}{5}\right)}{64d}$	77
parallelrisch	$\frac{(-9i+15\sinh(dx+c))\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-9i\right)+(9i-15\sinh(dx+c))\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+20i\cosh(dx+c)}{320id\sinh(dx+c)+192d}$	82

```
[In] int(1/(3+5*I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-3/64*I*ln(tanh(1/2*d*x+1/2*c)-3*I)+5/16/(tanh(1/2*d*x+1/2*c)-3*I)+3/6
4*I*ln(3*tanh(1/2*d*x+1/2*c)-I)+5/48/(3*tanh(1/2*d*x+1/2*c)-I))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.01

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\frac{3\left(5i\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}-5i\right)\log\left(e^{(dx+c)}-\frac{3}{5}i+\frac{4}{5}\right)+3\left(-5i\,e^{(2\,dx+2\,c)}-6\,e^{(dx+c)}+5i\right)\log\left(e^{(dx+c)}-\right.}{64\left(5\,de^{(2\,dx+2\,c)}-6i\,de^{(dx+c)}-5\,d\right)}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/64*(3*(5*I*e^(2*d*x + 2*c) + 6*e^(d*x + c) - 5*I)*log(e^(d*x + c) - 3/5*
I + 4/5) + 3*(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I)*log(e^(d*x + c) -
 3/5*I - 4/5) - 24*I*e^(d*x + c) - 40)/(5*d*e^(2*d*x + 2*c) - 6*I*d*e^(d*x
+ c) - 5*d)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{3ie^{c}e^{dx}+5}{40de^{2c}e^{2dx}-48ide^{c}e^{dx}-40d} + \frac{\operatorname{RootSum}\left(4096z^2+9,\left(i\mapsto i\log\left(\frac{(256ii-9i)e^{-c}}{15}+e^{dx}\right)\right)\right)}{d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**2,x)
```

```
[Out] (3*I*exp(c)*exp(d*x) + 5)/(40*d*exp(2*c)*exp(2*d*x) - 48*I*d*exp(c)*exp(d*x
) - 40*d) + RootSum(4096*_z**2 + 9, Lambda(_i, _i*log((256*_i*I - 9*I)*exp(
-c)/15 + exp(d*x))))/d
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = \frac{3i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{64\,d} + \frac{3i\,e^{(-dx-c)}-5}{-8\,d(-6i\,e^{(-dx-c)}-5\,e^{(-2\,dx-2\,c)}+5)}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 3/64*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d + (3*I*
e^(-d*x - c) - 5)/(d*(48*I*e^(-d*x - c) + 40*e^(-2*d*x - 2*c) - 40))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\frac{\frac{8\left(-3i\,e^{(dx+c)}-5\right)}{5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5} + 3i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right) - 3i\,\log\left(-(2i-1)\,e^{(dx+c)}+i-2\right)}{64\,d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*(-3*I*e^(d*x + c) - 5)/(5*e^(2*d*x + 2*c) - 6*I*e^(d*x + c) - 5) +
 3*I*log(-(I - 2)*e^(d*x + c) - 2*I + 1) - 3*I*log(-(2*I - 1)*e^(d*x + c) +
 I - 2))/d
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3+5i\sinh(c+dx))^2}\,dx = -\frac{5}{8\,\left(5\,d-5\,d\,e^{2\,c+2\,d\,x}+d\,e^{c+d\,x}\,6\mathrm{i}\right)} - \frac{\ln\left(-\frac{15}{4}+e^{d\,x}\,e^{c}\,\left(-3-\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d} + \frac{\ln\left(\frac{15}{4}+e^{d\,x}\,e^{c}\,\left(-3+\frac{9}{4}\mathrm{i}\right)\right)\,3\mathrm{i}}{64\,d} - \frac{e^{c+d\,x}\,3\mathrm{i}}{8\,\left(5\,d-5\,d\,e^{2\,c+2\,d\,x}+d\,e^{c+d\,x}\,6\mathrm{i}\right)}$$

```
[In] int(1/(sinh(c + d*x)*5i + 3)^2,x)
```

```
[Out] (log(15/4 - exp(d*x)*exp(c)*(3 - 9i/4))*3i)/(64*d) - (log(- exp(d*x)*exp(c)
*(3 + 9i/4) - 15/4)*3i)/(64*d) - 5/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(2*
c + 2*d*x))) - (exp(c + d*x)*3i)/(8*(5*d + d*exp(c + d*x)*6i - 5*d*exp(2*c
+ 2*d*x)))
```

3.90 $\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 131

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = \frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}$$

```
[Out] 43/2048*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d-43/2048*I*ln(co
sh(1/2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/32*I*cosh(d*x+c)/d/(3+5*I*si
nh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2743, 2833, 12, 2739, 630, 31}

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = -\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))} + \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2} + \frac{43i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{43i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}$$

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (((43*I)/2048)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d - (((43*I)
/2048)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/32)*Co
sh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) - (((45*I)/512)*Cosh[c + d*x])
/(d*(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}+\frac{1}{32}\int\frac{-6+5i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,dx\\
&=\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{1}{512}\int\frac{43}{3+5i\sinh(c+dx)}\,dx\\
&=\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}+\frac{43}{512}\int\frac{1}{3+5i\sinh(c+dx)}\,dx\\
&=\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}\\
&\quad-\frac{(43i)\text{Subst}\left(\int\frac{1}{3+10x+3x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{256d}\\
&=\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}\\
&\quad-\frac{(129i)\text{Subst}\left(\int\frac{1}{1+3x}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{2048d}\\
&\quad+\frac{(129i)\text{Subst}\left(\int\frac{1}{9+3x}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{2048d}\\
&=\frac{43i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}-\frac{43i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d}\\
&\quad+\frac{5i\cosh(c+dx)}{32d(3+5i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.56

$$\int\frac{1}{(3+5i\sinh(c+dx))^3}\,dx$$
$$=\frac{86\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right)+86\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right)-43i\log(4-5\cosh(c+dx))+43i\log(4}{}$$

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (86*ArcTan[3*Coth[(c + d*x)/2]] + 86*ArcTan[3*Tanh[(c + d*x)/2]] - (43*I)*L
og[4 - 5*Cosh[c + d*x]] + (43*I)*Log[4 + 5*Cosh[c + d*x]] - (80*I)/(3*Cosh[
(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 + (80*I)/(Cosh[(c + d*x)/2] + (3*I)*S
inh[(c + d*x)/2])^2 + (-120/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) - 3
60/(Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(4096*
d)
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{i\left(-387 i \mathrm{e}^{2 d x+2 c}+215 \mathrm{e}^{3 d x+3 c}+225 i-325 \mathrm{e}^{d x+c}\right)}{256 d\left(5 \mathrm{e}^{2 d x+2 c}-5-6 i \mathrm{e}^{d x+c}\right)^{2}}-\frac{43 i \ln \left(\mathrm{e}^{d x+c}-\frac{4}{5}-\frac{3 i}{5}\right)}{2048 d}+\frac{43 i \ln \left(\mathrm{e}^{d x+c}+\frac{4}{5}-\frac{3 i}{5}\right)}{2048 d}$
derivativedivides	$\frac{\frac{25 i}{128\left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)^{2}}+\frac{43 i \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}{2048}+\frac{15}{512\left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}-\frac{43 i \ln \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{2048}-\frac{25 i}{1152\left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-\right.}}{d}$
default	$\frac{\frac{25 i}{128\left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)^{2}}+\frac{43 i \ln \left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}{2048}+\frac{15}{512\left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-3 i\right)}-\frac{43 i \ln \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}{2048}-\frac{25 i}{1152\left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-\right.}}{d}$
parallelrisch	$38700 i \ln \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-9 i\right)-38700 i \ln \left(3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)+(9675 i \cosh (2 d x+2 c)+22059 i+23220 \sinh (d x+c)) \ln \left(\tan\right.$

```
[In] int(1/(3+5*I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/256*I*(-387*I*exp(2*d*x+2*c)+215*exp(3*d*x+3*c)+225*I-325*exp(d*x+c))/d/
(5*exp(2*d*x+2*c)-5-6*I*exp(d*x+c))^2-43/2048*I/d*ln(exp(d*x+c)-4/5-3/5*I)+
43/2048*I/d*ln(exp(d*x+c)+4/5-3/5*I)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.47

$$\int \frac{1}{(3+5 i \sinh (c+d x))^{3}} d x = -\frac{43\left(-25 i e^{(4 d x+4 c)}-60 e^{(3 d x+3 c)}+86 i e^{(2 d x+2 c)}+60 e^{(d x+c)}-25 i\right) \log \left(e^{(d x+c)}-\frac{3}{5} i+\frac{4}{5}\right)+43\left(25 i e^{(4 d x\right.}}{2048\left(25 d e^{(4 d x+4 c)}-6\right.}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2048*(43*(-25*I*e^(4*d*x + 4*c) - 60*e^(3*d*x + 3*c) + 86*I*e^(2*d*x + 2
*c) + 60*e^(d*x + c) - 25*I)*log(e^(d*x + c) - 3/5*I + 4/5) + 43*(25*I*e^(4
*d*x + 4*c) + 60*e^(3*d*x + 3*c) - 86*I*e^(2*d*x + 2*c) - 60*e^(d*x + c) +
25*I)*log(e^(d*x + c) - 3/5*I - 4/5) + 1720*I*e^(3*d*x + 3*c) + 3096*e^(2*d
*x + 2*c) - 2600*I*e^(d*x + c) - 1800)/(25*d*e^(4*d*x + 4*c) - 60*I*d*e^(3*
d*x + 3*c) - 86*d*e^(2*d*x + 2*c) + 60*I*d*e^(d*x + c) + 25*d)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = \frac{-215ie^{3c}e^{3dx}-387e^{2c}e^{2dx}+325ie^{c}e^{dx}+225}{6400de^{4c}e^{4dx}-15360ide^{3c}e^{3dx}-22016de^{2c}e^{2dx}+15360ide^{c}e^{dx}+6400d} + \frac{\operatorname{RootSum}\left(4194304z^2+1849,\left(i\mapsto i\log\left(\frac{(-8192ii-129i)e^{-c}}{215}+e^{dx}\right)\right)\right)}{d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**3,x)
```

```
[Out] (-215*I*exp(3*c)*exp(3*d*x) - 387*exp(2*c)*exp(2*d*x) + 325*I*exp(c)*exp(d*
x) + 225)/(6400*d*exp(4*c)*exp(4*d*x) - 15360*I*d*exp(3*c)*exp(3*d*x) - 220
16*d*exp(2*c)*exp(2*d*x) + 15360*I*d*exp(c)*exp(d*x) + 6400*d) + RootSum(41
94304*_z**2 + 1849, Lambda(_i, _i*log((-8192*_i*I - 129*I)*exp(-c)/215 + ex
p(d*x))))/d
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = -\frac{43i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{2048\,d} - \frac{-325i\,e^{(-dx-c)}-387\,e^{(-2\,dx-2\,c)}+215i\,e^{(-3\,dx-3\,c)}+225}{-256\,d(60i\,e^{(-dx-c)}+86\,e^{(-2\,dx-2\,c)}-60i\,e^{(-3\,dx-3\,c)}-25\,e^{(-4\,dx-4\,c)}-25)}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -43/2048*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d - (
-325*I*e^(-d*x - c) - 387*e^(-2*d*x - 2*c) + 215*I*e^(-3*d*x - 3*c) + 225)/
(d*(-15360*I*e^(-d*x - c) - 22016*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x - 3*
c) + 6400*e^(-4*d*x - 4*c) + 6400))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = -\frac{\frac{8\left(-215i\,e^{(3\,dx+3\,c)}-387\,e^{(2\,dx+2\,c)}+325i\,e^{(dx+c)}+225\right)}{\left(-5i\,e^{(2\,dx+2\,c)}-6\,e^{(dx+c)}+5i\right)^2} - 43i\,\log\left(-(i-2)\,e^{(dx+c)}-2i+1\right) + 43i\,\log\left(-(2i-1)\,e^{(dx}\right.}{2048\,d}$$

[In] integrate(1/(3+5*I*sinh(d*x+c))^3,x, algorithm="giac")

[Out] -1/2048*(8*(-215*I*e^(3*d*x + 3*c) - 387*e^(2*d*x + 2*c) + 325*I*e^(d*x + c
) + 225)/(-5*I*e^(2*d*x + 2*c) - 6*e^(d*x + c) + 5*I)^2 - 43*I*log(-(I - 2)
*e^(d*x + c) - 2*I + 1) + 43*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

$$\int \frac{1}{(3+5i\sinh(c+dx))^3}\,dx = \frac{\frac{129}{6400\,d}+\frac{\mathrm{e}^{c+d\,x}\,43\mathrm{i}}{1280\,d}}{1-\mathrm{e}^{2\,c+2\,d\,x}+\frac{\mathrm{e}^{c+d\,x}\,6\mathrm{i}}{5}} - \frac{\ln\left(-\frac{215}{4}+\mathrm{e}^{c+d\,x}\left(43-\frac{129}{4}\mathrm{i}\right)\right)\,43\mathrm{i}}{2048\,d} + \frac{\ln\left(\frac{215}{4}+\mathrm{e}^{c+d\,x}\left(43+\frac{129}{4}\mathrm{i}\right)\right)\,43\mathrm{i}}{2048\,d} - \frac{-\frac{3}{200\,d}+\frac{\mathrm{e}^{c+d\,x}\,7\mathrm{i}}{1000\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-\frac{86\,\mathrm{e}^{2\,c+2\,d\,x}}{25}+1+\frac{\mathrm{e}^{c+d\,x}\,12\mathrm{i}}{5}-\frac{\mathrm{e}^{3\,c+3\,d\,x}\,12\mathrm{i}}{5}}$$

[In] int(1/(sinh(c + d*x)*5i + 3)^3,x)

[Out] ((exp(c + d*x)*43i)/(1280*d) + 129/(6400*d))/((exp(c + d*x)*6i)/5 - exp(2*c
 + 2*d*x) + 1) - (log(exp(c + d*x)*(43 - 129i/4) - 215/4)*43i)/(2048*d) + (
log(exp(c + d*x)*(43 + 129i/4) + 215/4)*43i)/(2048*d) - ((exp(c + d*x)*7i)/
(1000*d) - 3/(200*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/25 - (e
xp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)

3.91 $\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 160

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = -\frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}$$

```
[Out] -279/32768*I*ln(3*cosh(1/2*d*x+1/2*c)+I*sinh(1/2*d*x+1/2*c))/d+279/32768*I*
ln(cosh(1/2*d*x+1/2*c)+3*I*sinh(1/2*d*x+1/2*c))/d+5/48*I*cosh(d*x+c)/d/(3+5
*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))^2+995/24576*I*
cosh(d*x+c)/d/(3+5*I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used

= {2743, 2833, 12, 2739, 630, 31}

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} + \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{279i\log\left(3\cosh\left(\frac{1}{2}(c+dx)\right)+i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279i\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)+3i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d}$$

```
[In] Int[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (((-279*I)/32768)*Log[3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]])/d + (((27
9*I)/32768)*Log[Cosh[(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2]])/d + (((5*I)/4
8)*Cosh[c + d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^3) - (((25*I)/512)*Cosh[c +
d*x])/(d*(3 + (5*I)*Sinh[c + d*x])^2) + (((995*I)/24576)*Cosh[c + d*x])/(d*
(3 + (5*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
```

```
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}+\frac{1}{48}\int\frac{-9+10i\sinh(c+dx)}{(3+5i\sinh(c+dx))^3}\,dx\\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}+\frac{\int\frac{154-75i\sinh(c+dx)}{(3+5i\sinh(c+dx))^2}\,dx}{1536}\\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}\\
&\quad+\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}+\frac{\int-\frac{837}{3+5i\sinh(c+dx)}\,dx}{24576}\\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}\\
&\quad+\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}-\frac{279\int\frac{1}{3+5i\sinh(c+dx)}\,dx}{8192}\\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}\\
&\quad+\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}+\frac{(279i)\text{Subst}\left(\int\frac{1}{3+10x+3x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{4096d}\\
&= \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2}\\
&\quad+\frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}+\frac{(837i)\text{Subst}\left(\int\frac{1}{1+3x}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{32768d}\\
&\quad-\frac{(837i)\text{Subst}\left(\int\frac{1}{9+3x}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{32768d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{279i\log\left(3+i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} + \frac{279i\log\left(1+3i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32768d} \\
&\quad + \frac{5i\cosh(c+dx)}{48d(3+5i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(3+5i\sinh(c+dx))^2} \\
&\quad + \frac{995i\cosh(c+dx)}{24576d(3+5i\sinh(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.66

$$
\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx
$$

$$
= \frac{-5022\arctan\left(3\coth\left(\frac{1}{2}(c+dx)\right)\right) - 5022\arctan\left(3\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 2511i\log(4-5\cosh(c+dx)) - 2}{}
$$

```
[In] Integrate[(3 + (5*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (-5022*ArcTan[3*Coth[(c + d*x)/2]] - 5022*ArcTan[3*Tanh[(c + d*x)/2]] + (25
11*I)*Log[4 - 5*Cosh[c + d*x]] - (2511*I)*Log[4 + 5*Cosh[c + d*x]] + (4640*
I)/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])^2 - (1440*I)/(Cosh[(c + d*x)
/2] + (3*I)*Sinh[(c + d*x)/2])^2 + 40*(80/(3*Cosh[(c + d*x)/2] + I*Sinh[(c
+ d*x)/2])^3 + 199/(3*Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]) + 240/(Cosh[
(c + d*x)/2] + (3*I)*Sinh[(c + d*x)/2])^3 + 597/(Cosh[(c + d*x)/2] + (3*I)*
Sinh[(c + d*x)/2]))*Sinh[(c + d*x)/2])/(589824*d)
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

method	result
risch	$\frac{i\left(-62775i\,e^{4dx+4c}+20925\,e^{5dx+5c}+119310i\,e^{2dx+2c}-111042\,e^{3dx+3c}-24875i+68625\,e^{dx+c}\right)}{12288d\left(5\,e^{2dx+2c}-5-6i\,e^{dx+c}\right)^{3}} + \frac{279i\ln\left(e^{dx+c}-\frac{4}{5}-\frac{3i}{5}\right)}{32768d}$
derivativedivides	$\frac{\frac{275i}{27648\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{279i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{32768}-\frac{125}{20736\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}+\frac{3505}{221184\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{279i\ln\left(\tanh\left(\right.\right.}{327}}{d}$
default	$\frac{\frac{275i}{27648\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{279i\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}{32768}-\frac{125}{20736\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}+\frac{3505}{221184\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}-\frac{279i\ln\left(\tanh\left(\right.\right.}{327}}{d}$
parallelrisch	$\frac{\left(10169550i\cosh(2dx+2c)-12610242i+20678085\sinh(dx+c)-2824875\sinh(3dx+3c)\right)\ln\left(3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-9i\right)+(-10169}{}$

```
[In] int(1/(3+5*I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12288*I*(-62775*I*exp(4*d*x+4*c)+20925*exp(5*d*x+5*c)+119310*I*exp(2*d*x+
2*c)-111042*exp(3*d*x+3*c)-24875*I+68625*exp(d*x+c))/d/(5*exp(2*d*x+2*c)-5-
6*I*exp(d*x+c))^3+279/32768*I/d*ln(exp(d*x+c)-4/5-3/5*I)-279/32768*I/d*ln(e
xp(d*x+c)+4/5-3/5*I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(126) = 252.

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.77

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = -\frac{837\left(125i\,e^{(6\,dx+6\,c)}+450\,e^{(5\,dx+5\,c)}-915i\,e^{(4\,dx+4\,c)}-1116\,e^{(3\,dx+3\,c)}+915i\,e^{(2\,dx+2\,c)}+450\,e^{(dx+c)}-125\right.}{}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/98304*(837*(125*I*e^(6*d*x + 6*c) + 450*e^(5*d*x + 5*c) - 915*I*e^(4*d*x
 + 4*c) - 1116*e^(3*d*x + 3*c) + 915*I*e^(2*d*x + 2*c) + 450*e^(d*x + c) -
125*I)*log(e^(d*x + c) - 3/5*I + 4/5) + 837*(-125*I*e^(6*d*x + 6*c) - 450*e
^(5*d*x + 5*c) + 915*I*e^(4*d*x + 4*c) + 1116*e^(3*d*x + 3*c) - 915*I*e^(2*
d*x + 2*c) - 450*e^(d*x + c) + 125*I)*log(e^(d*x + c) - 3/5*I - 4/5) - 1674
00*I*e^(5*d*x + 5*c) - 502200*e^(4*d*x + 4*c) + 888336*I*e^(3*d*x + 3*c) +
954480*e^(2*d*x + 2*c) - 549000*I*e^(d*x + c) - 199000)/(125*d*e^(6*d*x + 6
*c) - 450*I*d*e^(5*d*x + 5*c) - 915*d*e^(4*d*x + 4*c) + 1116*I*d*e^(3*d*x +
 3*c) + 915*d*e^(2*d*x + 2*c) - 450*I*d*e^(d*x + c) - 125*d)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.23

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{20925ie^{5c}e^{5dx}+62775e^{4c}e^{4dx}-111042ie^{3c}e^{3dx}-119310e^{2c}e^{2dx}+68625ie^{c}e^{dx}+248}{1536000de^{6c}e^{6dx}-5529600ide^{5c}e^{5dx}-11243520de^{4c}e^{4dx}+13713408ide^{3c}e^{3dx}+11243520de^{2c}e^{2dx}-5529} + \frac{\operatorname{RootSum}\left(1073741824z^2+77841,\left(i\mapsto i\log\left(\frac{(131072ii-837i)e^{-c}}{1395}+e^{dx}\right)\right)\right)}{d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))**4,x)
```

```
[Out] (20925*I*exp(5*c)*exp(5*d*x) + 62775*exp(4*c)*exp(4*d*x) - 111042*I*exp(3*c
)*exp(3*d*x) - 119310*exp(2*c)*exp(2*d*x) + 68625*I*exp(c)*exp(d*x) + 24875
)/(1536000*d*exp(6*c)*exp(6*d*x) - 5529600*I*d*exp(5*c)*exp(5*d*x) - 112435
```

```
20*d*exp(4*c)*exp(4*d*x) + 13713408*I*d*exp(3*c)*exp(3*d*x) + 11243520*d*ex
p(2*c)*exp(2*d*x) - 5529600*I*d*exp(c)*exp(d*x) - 1536000*d) + RootSum(1073
741824*_z**2 + 77841, Lambda(_i, _i*log((131072*_i*I - 837*I)*exp(-c)/1395
+ exp(d*x))))/d
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{279i\,\log\left(\frac{5\,e^{(-dx-c)}+3i-4}{5\,e^{(-dx-c)}+3i+4}\right)}{32768\,d} + \frac{68625i\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}-111042i\,e^{(-3\,dx-3\,c)}-62775\,e^{(-4\,dx-4\,c)}+20925i\,e^{(-5\,dx-5\,c)}}{-12288\,d(-450i\,e^{(-dx-c)}-915\,e^{(-2\,dx-2\,c)}+1116i\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}-450i\,e^{(-5\,dx-5\,c)}-125\,e^{(}}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 279/32768*I*log((5*e^(-d*x - c) + 3*I - 4)/(5*e^(-d*x - c) + 3*I + 4))/d +
(68625*I*e^(-d*x - c) + 119310*e^(-2*d*x - 2*c) - 111042*I*e^(-3*d*x - 3*c)
 - 62775*e^(-4*d*x - 4*c) + 20925*I*e^(-5*d*x - 5*c) - 24875)/(d*(5529600*I
*e^(-d*x - c) + 11243520*e^(-2*d*x - 2*c) - 13713408*I*e^(-3*d*x - 3*c) - 1
1243520*e^(-4*d*x - 4*c) + 5529600*I*e^(-5*d*x - 5*c) + 1536000*e^(-6*d*x -
 6*c) - 1536000))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.69

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx = \frac{\frac{8\,(20925i\,e^{(5\,dx+5\,c)}+62775\,e^{(4\,dx+4\,c)}-111042i\,e^{(3\,dx+3\,c)}-119310\,e^{(2\,dx+2\,c)}+68625i\,e^{(dx+c)}+24875)}{\left(5\,e^{(2\,dx+2\,c)}-6i\,e^{(dx+c)}-5\right)^3} - 837i\,\log\left(-(i-2)\,e^{(dx+c)}-\right.}{98304\,d}$$

```
[In] integrate(1/(3+5*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/98304*(8*(20925*I*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) - 111042*I*e^(3
*d*x + 3*c) - 119310*e^(2*d*x + 2*c) + 68625*I*e^(d*x + c) + 24875)/(5*e^(2
*d*x + 2*c) - 6*I*e^(d*x + c) - 5)^3 - 837*I*log(-(I - 2)*e^(d*x + c) - 2*I
 + 1) + 837*I*log(-(2*I - 1)*e^(d*x + c) + I - 2))/d
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.48

$$\int \frac{1}{(3+5i\sinh(c+dx))^4}\,dx$$
$$= -\frac{\frac{837}{102400\,d}+\frac{e^{c+d\,x}\,279\mathrm{i}}{20480\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,6\mathrm{i}}{5}} + \frac{\frac{7}{3750\,d}+\frac{e^{c+d\,x}\,39\mathrm{i}}{6250\,d}}{\frac{183\,e^{4\,c+4\,d\,x}}{25}-\frac{183\,e^{2\,c+2\,d\,x}}{25}-e^{6\,c+6\,d\,x}+1+\frac{e^{c+d\,x}\,18\mathrm{i}}{5}-\frac{e^{3\,c+3\,d\,x}\,1116\mathrm{i}}{125}+\frac{e^{5\,c+5\,d\,x}\,18\mathrm{i}}{5}} - \frac{\ln\left(-\frac{1395}{4}+e^{c+d\,x}\left(-279-\frac{837}{4}\mathrm{i}\right)\right)\,279\mathrm{i}}{32768\,d} + \frac{\ln\left(\frac{1395}{4}+e^{c+d\,x}\left(-279+\frac{837}{4}\mathrm{i}\right)\right)\,279\mathrm{i}}{32768\,d} - \frac{\frac{791}{80000\,d}+\frac{e^{c+d\,x}\,93\mathrm{i}}{16000\,d}}{e^{4\,c+4\,d\,x}-\frac{86\,e^{2\,c+2\,d\,x}}{25}+1+\frac{e^{c+d\,x}\,12\mathrm{i}}{5}-\frac{e^{3\,c+3\,d\,x}\,12\mathrm{i}}{5}}$$

```
[In] int(1/(sinh(c + d*x)*5i + 3)^4,x)
```

```
[Out] ((exp(c + d*x)*39i)/(6250*d) + 7/(3750*d))/((exp(c + d*x)*18i)/5 - (183*exp
(2*c + 2*d*x))/25 - (exp(3*c + 3*d*x)*1116i)/125 + (183*exp(4*c + 4*d*x))/2
5 + (exp(5*c + 5*d*x)*18i)/5 - exp(6*c + 6*d*x) + 1) - ((exp(c + d*x)*279i)
/(20480*d) + 837/(102400*d))/((exp(c + d*x)*6i)/5 - exp(2*c + 2*d*x) + 1) -
 (log(- exp(c + d*x)*(279 + 837i/4) - 1395/4)*279i)/(32768*d) + (log(1395/4
 - exp(c + d*x)*(279 - 837i/4))*279i)/(32768*d) - ((exp(c + d*x)*93i)/(1600
0*d) + 791/(80000*d))/((exp(c + d*x)*12i)/5 - (86*exp(2*c + 2*d*x))/25 - (e
xp(3*c + 3*d*x)*12i)/5 + exp(4*c + 4*d*x) + 1)
```

3.92 $\int \frac{1}{5+3i\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{x}{4} - \frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

```
[Out] 1/4*x-1/2*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2736}

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{x}{4} - \frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] x/4 - ((I/2)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\text{integral} = \frac{x}{4} - \frac{i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{2d}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 171 vs. 2(37) = 74.

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.62

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = -\frac{i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{4d} + \frac{i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)}{4d} - \frac{\log(5\cosh(c+dx)-4\sinh(c+dx))}{8d} + \frac{\log(5\cosh(c+dx)+4\sinh(c+dx))}{8d}$$

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-1),x]
```

```
[Out] ((-1/4*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/
2] - 2*Sinh[(c + d*x)/2])])/d + ((I/4)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])])/d - Log[5*Cosh[c +
 d*x] - 4*Sinh[c + d*x]]/(8*d) + Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]]/(8*
d)
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{\ln\left(-\frac{i}{3}+e^{dx+c}\right)}{4d}-\frac{\ln\left(e^{dx+c}-3i\right)}{4d}$	32
parallelrisch	$\frac{-\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)+\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{4d}$	41
derivativedivides	$\frac{-\frac{\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{4}+\frac{\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{4}}{d}$	42
default	$\frac{-\frac{\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{4}+\frac{\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{4}}{d}$	42

```
[In] int(1/(5+3*I*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d*ln(-1/3*I+exp(d*x+c))-1/4/d*ln(exp(d*x+c)-3*I)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{\log\left(e^{(dx+c)} - \frac{1}{3}i\right) - \log\left(e^{(dx+c)} - 3i\right)}{4\,d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="fricas")
[Out] 1/4*(log(e^(d*x + c) - 1/3*I) - log(e^(d*x + c) - 3*I))/d
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{-\frac{\log\left(e^{dx}-3ie^{-c}\right)}{4} + \frac{\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{4}}{d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x)
[Out] (-log(exp(d*x) - 3*I*exp(-c))/4 + log(exp(d*x) - I*exp(-c)/3)/4)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{\log\left(-\frac{6\left(-i\,e^{(-dx-c)}+3\right)}{6i\,e^{(-dx-c)}-2}\right)}{4\,d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="maxima")
[Out] 1/4*log(-6*(-I*e^(-d*x - c) + 3)/(6*I*e^(-d*x - c) - 2))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = \frac{\log\left(3\,e^{(dx+c)}-i\right)-\log\left(e^{(dx+c)}-3i\right)}{4\,d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(3*e^(d*x + c) - I) - log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{5+3i\sinh(c+dx)}\,dx = -\frac{\ln\left(-\frac{e^{d\,x}\,e^{c}}{2}+\frac{3}{2}\mathrm{i}\right)-\ln\left(\frac{9\,e^{d\,x}\,e^{c}}{2}-\frac{3}{2}\mathrm{i}\right)}{4\,d}$$

```
[In] int(1/(sinh(c + d*x)*3i + 5),x)
```

```
[Out] -(log(3i/2 - (exp(d*x)*exp(c))/2) - log((9*exp(d*x)*exp(c))/2 - 3i/2))/(4*d
)
```

3.93 $\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{5x}{64} - \frac{5i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d} - \frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}$$

```
[Out] 5/64*x-5/32*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/16*I*cosh(d*x+c)/d/
(5+3*I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2743, 12, 2736}

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\frac{5i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d} - \frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))} + \frac{5x}{64}$$

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (5*x)/64 - (((5*I)/32)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d - (((
3*I)/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}-\frac{1}{16}\int -\frac{5}{5+3i\sinh(c+dx)}\,dx\\ &= -\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}+\frac{5}{16}\int \frac{1}{5+3i\sinh(c+dx)}\,dx\\ &= \frac{5x}{64}-\frac{5i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{32d}-\frac{3i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 183 vs. $2(66) = 132$.

Time = 0.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{24i-50i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)+50i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)-25\log(5\cosh(c}{640d}$$

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-2),x]
```

```
[Out] (24*I - (50*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c +
d*x)/2] - 2*Sinh[(c + d*x)/2])] + (50*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh
[(c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 25*Log[5*Cosh[c
 + d*x] - 4*Sinh[c + d*x]] + 25*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] - (1
20*Cosh[c + d*x])/(-5*I + 3*Sinh[c + d*x]))/(640*d)
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{i\left(5\,\mathrm{e}^{dx+c}-3i\right)}{8d\left(3\,\mathrm{e}^{2dx+2c}-3-10i\mathrm{e}^{dx+c}\right)}-\frac{5\ln\left(\mathrm{e}^{dx+c}-3i\right)}{64d}+\frac{5\ln\left(-\frac{i}{3}+\mathrm{e}^{dx+c}\right)}{64d}$
derivativedivides	$\frac{\frac{-\frac{9}{80}-\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{64}+\frac{-\frac{9}{80}+\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i}-\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{64}}{d}$
default	$\frac{\frac{-\frac{9}{80}-\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{64}+\frac{-\frac{9}{80}+\frac{3i}{20}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i}-\frac{5\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)}{64}}{d}$
parallelrisch	$\frac{125\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)-125\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)-60i+75i\sinh(dx+c)\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)-75i\sin}{960id\sinh(dx+c)+1600d}$

```
[In] int(1/(5+3*I*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -1/8*I*(5*exp(d*x+c)-3*I)/d/(3*exp(2*d*x+2*c)-3-10*I*exp(d*x+c))-5/64/d*ln(
exp(d*x+c)-3*I)+5/64/d*ln(-1/3*I+exp(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.56

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{5\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-5\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)\log\left(e^{(dx+c)}-3i\right)-40i}{64\left(3\,de^{(2\,dx+2\,c)}-10i\,de^{(dx+c)}-3\,d\right)}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="fricas")
[Out] 1/64*(5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 1/3*I)
 - 5*(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3)*log(e^(d*x + c) - 3*I) - 40
*I*e^(d*x + c) - 24)/(3*d*e^(2*d*x + 2*c) - 10*I*d*e^(d*x + c) - 3*d)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{-5ie^{c}e^{dx}-3}{24de^{2c}e^{2dx}-80ide^{c}e^{dx}-24d}+\frac{-\frac{5\log\left(e^{dx}-3ie^{-c}\right)}{64}+\frac{5\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{64}}{d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**2,x)
```

```
[Out] (-5*I*exp(c)*exp(d*x) - 3)/(24*d*exp(2*c)*exp(2*d*x) - 80*I*d*exp(c)*exp(d*
x) - 24*d) + (-5*log(exp(d*x) - 3*I*exp(-c))/64 + 5*log(exp(d*x) - I*exp(-c
)/3)/64)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\frac{5i\,\arctan\left(\frac{3}{4}\,e^{(-dx-c)}+\frac{5}{4}i\right)}{32\,d} - \frac{5i\,e^{(-dx-c)}-3}{-8\,d(-10i\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+3)}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -5/32*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (5*I*e^(-d*x - c) - 3)/(d*(80*
I*e^(-d*x - c) + 24*e^(-2*d*x - 2*c) - 24))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = -\frac{\frac{8\,(5i\,e^{(dx+c)}+3)}{3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3} - 5\,\log\left(3\,e^{(dx+c)}-i\right) + 5\,\log\left(e^{(dx+c)}-3i\right)}{64\,d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/64*(8*(5*I*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) - 10*I*e^(d*x + c) - 3) -
 5*log(3*e^(d*x + c) - I) + 5*log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int \frac{1}{(5+3i\sinh(c+dx))^2}\,dx = \frac{3}{8\,\left(3\,d-3\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}+d\,{\mathrm{e}}^{c+d\,x}\,10\mathrm{i}\right)}-\frac{5\,\ln\left(-\frac{5\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}}{4}+\frac{15}{4}\mathrm{i}\right)}{64\,d}+\frac{5\,\ln\left(\frac{45\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^{c}}{4}-\frac{15}{4}\mathrm{i}\right)}{64\,d}+\frac{{\mathrm{e}}^{c+d\,x}\,5\mathrm{i}}{8\,\left(3\,d-3\,d\,{\mathrm{e}}^{2\,c+2\,d\,x}+d\,{\mathrm{e}}^{c+d\,x}\,10\mathrm{i}\right)}$$

```
[In] int(1/(sinh(c + d*x)*3i + 5)^2,x)

[Out] 3/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*x))) - (5*log(15i/4 - (5
*exp(d*x)*exp(c))/4))/(64*d) + (5*log((45*exp(d*x)*exp(c))/4 - 15i/4))/(64*
d) + (exp(c + d*x)*5i)/(8*(3*d + d*exp(c + d*x)*10i - 3*d*exp(2*c + 2*d*x))
)
```

3.94 $\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 95

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{59x}{2048} - \frac{59i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d} - \frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}$$

```
[Out] 59/2048*x-59/1024*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-3/32*I*cosh(d*x
+c)/d/(5+3*I*sinh(d*x+c))^2-45/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2743, 2833, 12, 2736}

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = -\frac{59i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d} - \frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))} - \frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2} + \frac{59x}{2048}$$

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] (59*x)/2048 - (((59*I)/1024)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])])/d
 - (((3*I)/32)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((45*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{1}{32}\int\frac{-10+3i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}\,dx\\
&= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}+\frac{1}{512}\int\frac{59}{5+3i\sinh(c+dx)}\,dx\\
&= -\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}+\frac{59}{512}\int\frac{1}{5+3i\sinh(c+dx)}\,dx\\
&= \frac{59x}{2048}-\frac{59i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{1024d}-\frac{3i\cosh(c+dx)}{32d(5+3i\sinh(c+dx))^2}-\frac{45i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 277 vs. 2(95) = 190.

Time = 0.75 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.92

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{-118i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)+118i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)-59\log(5\cosh(c+$$

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-3),x]
```

```
[Out] ((-118*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)/
2] - 2*Sinh[(c + d*x)/2])] + (118*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(c
+ d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 59*Log[5*Cosh[c + d
*x] - 4*Sinh[c + d*x]] + 59*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] + 48/((1
 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2 + 48/((2 + I)*Cosh
[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^2 - (144*Sinh[(c + d*x)/2]*((-
3*I)*Cosh[(c + d*x)/2] + 5*Sinh[(c + d*x)/2]))/(-5*I + 3*Sinh[c + d*x]))/(4
096*d)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{3i\left(-295i\,e^{2dx+2c}+59\,e^{3dx+3c}+45i-241\,e^{dx+c}\right)}{256d\left(3\,e^{2dx+2c}-3-10i\,e^{dx+c}\right)^2}+\frac{59\ln\left(-\frac{i}{3}+e^{dx+c}\right)}{2048d}-\frac{59\ln\left(e^{dx+c}-3i\right)}{2048d}$
derivativedivides	$\frac{\frac{\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)^2}+\frac{-\frac{963}{12800}-\frac{123i}{1600}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{59\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{2048}+\frac{-\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^2}+\frac{-\frac{963}{12800}+\frac{123i}{1600}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4}}{d}$
default	$\frac{\frac{\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)^2}+\frac{-\frac{963}{12800}-\frac{123i}{1600}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i}+\frac{59\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4-3i\right)}{2048}+\frac{-\frac{63}{3200}-\frac{27i}{400}}{\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)^2}+\frac{-\frac{963}{12800}+\frac{123i}{1600}}{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4}}{d}$
parallelrisch	$\frac{(-88500i\sinh(dx+c)+13275\cosh(2dx+2c)-87025)\ln\left(5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-4-3i\right)+(88500i\sinh(dx+c)-13275\cosh(2dx+}{51200d(59-9\cos}$

```
[In] int(1/(5+3*I*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/256*I*(-295*I*exp(2*d*x+2*c)+59*exp(3*d*x+3*c)+45*I-241*exp(d*x+c))/d/(3
*exp(2*d*x+2*c)-3-10*I*exp(d*x+c))^2+59/2048/d*ln(-1/3*I+exp(d*x+c))-59/204
8/d*ln(exp(d*x+c)-3*I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(75) = 150.

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.03

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{59\left(9\,e^{(4\,dx+4\,c)}-60i\,e^{(3\,dx+3\,c)}-118\,e^{(2\,dx+2\,c)}+60i\,e^{(dx+c)}+9\right)\log\left(e^{(dx+c)}-\frac{1}{3}i\right)-59\left(9\,e^{(4\,dx+4\,c)}-60i\,e^{(3}\right.}{2048\left(9\,de^{(4\,dx+4\,c)}-60i\,de^{(3\,dx+}\right.}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2048*(59*(9*e^(4*d*x + 4*c) - 60*I*e^(3*d*x + 3*c) - 118*e^(2*d*x + 2*c)
+ 60*I*e^(d*x + c) + 9)*log(e^(d*x + c) - 1/3*I) - 59*(9*e^(4*d*x + 4*c) -
60*I*e^(3*d*x + 3*c) - 118*e^(2*d*x + 2*c) + 60*I*e^(d*x + c) + 9)*log(e^(d
*x + c) - 3*I) - 1416*I*e^(3*d*x + 3*c) - 7080*e^(2*d*x + 2*c) + 5784*I*e^(
d*x + c) + 1080)/(9*d*e^(4*d*x + 4*c) - 60*I*d*e^(3*d*x + 3*c) - 118*d*e^(2
*d*x + 2*c) + 60*I*d*e^(d*x + c) + 9*d)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.48

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{-177ie^{3c}e^{3dx}-885e^{2c}e^{2dx}+723ie^{c}e^{dx}+135}{2304de^{4c}e^{4dx}-15360ide^{3c}e^{3dx}-30208de^{2c}e^{2dx}+15360ide^{c}e^{dx}+2304d} + \frac{-\frac{59\log\left(e^{dx}-3ie^{-c}\right)}{2048}+\frac{59\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{2048}}{d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**3,x)
```

```
[Out] (-177*I*exp(3*c)*exp(3*d*x) - 885*exp(2*c)*exp(2*d*x) + 723*I*exp(c)*exp(d*
x) + 135)/(2304*d*exp(4*c)*exp(4*d*x) - 15360*I*d*exp(3*c)*exp(3*d*x) - 302
08*d*exp(2*c)*exp(2*d*x) + 15360*I*d*exp(c)*exp(d*x) + 2304*d) + (-59*log(e
xp(d*x) - 3*I*exp(-c))/2048 + 59*log(exp(d*x) - I*exp(-c)/3)/2048)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.14

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx \\ = -\frac{59i\arctan\left(\frac{3}{4}e^{(-dx-c)}+\frac{5}{4}i\right)}{1024\,d} \\ + \frac{3\left(241i\,e^{(-dx-c)}+295\,e^{(-2\,dx-2\,c)}-59i\,e^{(-3\,dx-3\,c)}-45\right)}{-256\,d(60i\,e^{(-dx-c)}+118\,e^{(-2\,dx-2\,c)}-60i\,e^{(-3\,dx-3\,c)}-9\,e^{(-4\,dx-4\,c)}-9)}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -59/1024*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d + 3*(241*I*e^(-d*x - c) + 295
*e^(-2*d*x - 2*c) - 59*I*e^(-3*d*x - 3*c) - 45)/(d*(-15360*I*e^(-d*x - c) -
 30208*e^(-2*d*x - 2*c) + 15360*I*e^(-3*d*x - 3*c) + 2304*e^(-4*d*x - 4*c)
+ 2304))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx \\ = -\frac{\frac{24\left(-59i\,e^{(3\,dx+3\,c)}-295\,e^{(2\,dx+2\,c)}+241i\,e^{(dx+c)}+45\right)}{\left(-3i\,e^{(2\,dx+2\,c)}-10\,e^{(dx+c)}+3i\right)^2}-59\log\left(3\,e^{(dx+c)}-i\right)+59\log\left(e^{(dx+c)}-3i\right)}{2048\,d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2048*(24*(-59*I*e^(3*d*x + 3*c) - 295*e^(2*d*x + 2*c) + 241*I*e^(d*x + c
) + 45)/(-3*I*e^(2*d*x + 2*c) - 10*e^(d*x + c) + 3*I)^2 - 59*log(3*e^(d*x +
 c) - I) + 59*log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.51

$$\int \frac{1}{(5+3i\sinh(c+dx))^3}\,dx = \frac{\frac{295}{2304\,d}+\frac{e^{c+d\,x}\,59\mathrm{i}}{768\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,10\mathrm{i}}{3}} - \frac{59\,\ln\left(-\frac{59\,e^{c+d\,x}}{4}+\frac{177}{4}\mathrm{i}\right)}{2048\,d} + \frac{59\,\ln\left(\frac{531\,e^{c+d\,x}}{4}-\frac{177}{4}\mathrm{i}\right)}{2048\,d} - \frac{\frac{5}{72\,d}+\frac{e^{c+d\,x}\,41\mathrm{i}}{216\,d}}{e^{4\,c+4\,d\,x}-\frac{118\,e^{2\,c+2\,d\,x}}{9}+1+\frac{e^{c+d\,x}\,20\mathrm{i}}{3}-\frac{e^{3\,c+3\,d\,x}\,20\mathrm{i}}{3}}$$

```
[In] int(1/(sinh(c + d*x)*3i + 5)^3,x)
```

```
[Out] ((exp(c + d*x)*59i)/(768*d) + 295/(2304*d))/((exp(c + d*x)*10i)/3 - exp(2*c
 + 2*d*x) + 1) - (59*log(177i/4 - (59*exp(c + d*x))/4))/(2048*d) + (59*log(
(531*exp(c + d*x))/4 - 177i/4))/(2048*d) - ((exp(c + d*x)*41i)/(216*d) + 5/
(72*d))/((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3*d*x
)*20i)/3 + exp(4*c + 4*d*x) + 1)
```

3.95 $\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 124

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d} - \frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}$$

```
[Out] 385/32768*x-385/16384*I*arctan(cosh(d*x+c)/(3+I*sinh(d*x+c)))/d-1/16*I*cosh
(d*x+c)/d/(5+3*I*sinh(d*x+c))^3-25/512*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))^
2-311/8192*I*cosh(d*x+c)/d/(5+3*I*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2743, 2833, 12, 2736}

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = -\frac{385i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d} - \frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} + \frac{385x}{32768}$$

```
[In] Int[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] (385*x)/32768 - (((385*I)/16384)*ArcTan[Cosh[c + d*x]/(3 + I*Sinh[c + d*x])
])/d - ((I/16)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^3) - (((25*I)/51
2)*Cosh[c + d*x])/(d*(5 + (3*I)*Sinh[c + d*x])^2) - (((311*I)/8192)*Cosh[c
+ d*x])/(d*(5 + (3*I)*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{1}{48}\int\frac{-15+6i\sinh(c+dx)}{(5+3i\sinh(c+dx))^3}\,dx\\
&= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}+\frac{\int\frac{186-75i\sinh(c+dx)}{(5+3i\sinh(c+dx))^2}\,dx}{1536}\\
&= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3}-\frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2}\\
&\quad-\frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}-\frac{\int-\frac{1155}{5+3i\sinh(c+dx)}\,dx}{24576}
\end{aligned}$$

$$= -\frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))} + \frac{385\int\frac{1}{5+3i\sinh(c+dx)}\,dx}{8192}$$

$$= \frac{385x}{32768} - \frac{385i\arctan\left(\frac{\cosh(c+dx)}{3+i\sinh(c+dx)}\right)}{16384d} - \frac{i\cosh(c+dx)}{16d(5+3i\sinh(c+dx))^3} - \frac{25i\cosh(c+dx)}{512d(5+3i\sinh(c+dx))^2} - \frac{311i\cosh(c+dx)}{8192d(5+3i\sinh(c+dx))}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 308 vs. 2(124) = 248.

Time = 1.52 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.48

$$\int\frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{-3850i\arctan\left(\frac{2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)}{\cosh\left(\frac{1}{2}(c+dx)\right)-2\sinh\left(\frac{1}{2}(c+dx)\right)}\right)+3850i\arctan\left(\frac{\cosh\left(\frac{1}{2}(c+dx)\right)+2\sinh\left(\frac{1}{2}(c+dx)\right)}{2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)}\right)-1925\log(5\cosh}{}$$

```
[In] Integrate[(5 + (3*I)*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((-3850*I)*ArcTan[(2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2])/(Cosh[(c + d*x)
/2] - 2*Sinh[(c + d*x)/2])] + (3850*I)*ArcTan[(Cosh[(c + d*x)/2] + 2*Sinh[(
c + d*x)/2])/(2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] - 1925*Log[5*Cosh[c
 + d*x] - 4*Sinh[c + d*x]] + 1925*Log[5*Cosh[c + d*x] + 4*Sinh[c + d*x]] +
(2656 - 192*I)/((1 + 2*I)*Cosh[(c + d*x)/2] - (2 + I)*Sinh[(c + d*x)/2])^2
+ (2656 + 192*I)/((2 + I)*Cosh[(c + d*x)/2] + (1 + 2*I)*Sinh[(c + d*x)/2])^
2 + (2*(-235150 + 166615*Cosh[c + d*x] + 82530*Cosh[2*(c + d*x)] - 13995*Co
sh[3*(c + d*x)] - (298563*I)*Sinh[c + d*x] + (89364*I)*Sinh[2*(c + d*x)] +
(8397*I)*Sinh[3*(c + d*x)]))/(-5*I + 3*Sinh[c + d*x])^3)/(327680*d)
```

Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{i\left(-86625 i \mathrm{e}^{4 d x+4 c}+10395 \,\mathrm{e}^{5 d x+5 c}+218466 i \mathrm{e}^{2 d x+2 c}-239470 \,\mathrm{e}^{3 d x+3 c}-8397 i+73575 \,\mathrm{e}^{d x+c}\right)}{12288 d \left(3 \,\mathrm{e}^{2 d x+2 c}-3-10 i \mathrm{e}^{d x+c}\right)^{3}}+\frac{385 \ln \left(-\frac{i}{3}+\mathrm{e}^{d x+c}\right)}{32768 d}-$
derivativedivides	$\frac{\frac{\frac{1053}{32000}-\frac{99 i}{8000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{3}}+\frac{\frac{783}{128000}-\frac{3753 i}{64000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{2}}+\frac{-\frac{39933}{1024000}-\frac{8361 i}{256000}}{5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i}+\frac{385 \ln \left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{32768}+\frac{\frac{1053}{32000}+\frac{99 i}{8000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4\right.}}{d}$
default	$\frac{\frac{\frac{1053}{32000}-\frac{99 i}{8000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{3}}+\frac{\frac{783}{128000}-\frac{3753 i}{64000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)^{2}}+\frac{-\frac{39933}{1024000}-\frac{8361 i}{256000}}{5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i}+\frac{385 \ln \left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)+4-3 i\right)}{32768}+\frac{\frac{1053}{32000}+\frac{99 i}{8000}}{\left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4\right.}}{d}$
parallelrisch	$\frac{\left(-47210625 i \sinh \left(d x+c\right)+1299375 i \sinh \left(3 d x+3 c\right)+12993750 \cosh \left(2 d x+2 c\right)-37056250\right) \ln \left(5 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)-4-3 i\right)+\left(472\right.}{}$

```
[In] int(1/(5+3*I*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12288*I*(-86625*I*exp(4*d*x+4*c)+10395*exp(5*d*x+5*c)+218466*I*exp(2*d*x
+2*c)-239470*exp(3*d*x+3*c)-8397*I+73575*exp(d*x+c))/d/(3*exp(2*d*x+2*c)-3-
10*I*exp(d*x+c))^3+385/32768/d*ln(-1/3*I+exp(d*x+c))-385/32768/d*ln(exp(d*x
+c)-3*I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(98) = 196$.

Time = 0.31 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.28

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{1155\left(27\,e^{(6\,dx+6\,c)}-270i\,e^{(5\,dx+5\,c)}-981\,e^{(4\,dx+4\,c)}+1540i\,e^{(3\,dx+3\,c)}+981\,e^{(2\,dx+2\,c)}-270i\,e^{(dx+c)}-27\right)\log}{}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/98304*(1155*(27*e^(6*d*x + 6*c) - 270*I*e^(5*d*x + 5*c) - 981*e^(4*d*x +
4*c) + 1540*I*e^(3*d*x + 3*c) + 981*e^(2*d*x + 2*c) - 270*I*e^(d*x + c) - 2
7)*log(e^(d*x + c) - 1/3*I) - 1155*(27*e^(6*d*x + 6*c) - 270*I*e^(5*d*x + 5
*c) - 981*e^(4*d*x + 4*c) + 1540*I*e^(3*d*x + 3*c) + 981*e^(2*d*x + 2*c) -
270*I*e^(d*x + c) - 27)*log(e^(d*x + c) - 3*I) - 83160*I*e^(5*d*x + 5*c) -
693000*e^(4*d*x + 4*c) + 1915760*I*e^(3*d*x + 3*c) + 1747728*e^(2*d*x + 2*c
) - 588600*I*e^(d*x + c) - 67176)/(27*d*e^(6*d*x + 6*c) - 270*I*d*e^(5*d*x
+ 5*c) - 981*d*e^(4*d*x + 4*c) + 1540*I*d*e^(3*d*x + 3*c) + 981*d*e^(2*d*x
+ 2*c) - 270*I*d*e^(d*x + c) - 27*d)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.63

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = \frac{-10395ie^{5c}e^{5dx} - 86625e^{4c}e^{4dx} + 239470ie^{3c}e^{3dx} + 218466e^{2c}e^{2dx} - 73575ie^{c}e^{dx} - 839}{331776de^{6c}e^{6dx} - 3317760ide^{5c}e^{5dx} - 12054528de^{4c}e^{4dx} + 18923520ide^{3c}e^{3dx} + 12054528de^{2c}e^{2dx} - 33177} + \frac{-\frac{385\log\left(e^{dx}-3ie^{-c}\right)}{32768} + \frac{385\log\left(e^{dx}-\frac{ie^{-c}}{3}\right)}{32768}}{d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))**4,x)
```

```
[Out] (-10395*I*exp(5*c)*exp(5*d*x) - 86625*exp(4*c)*exp(4*d*x) + 239470*I*exp(3*
c)*exp(3*d*x) + 218466*exp(2*c)*exp(2*d*x) - 73575*I*exp(c)*exp(d*x) - 8397
)/(331776*d*exp(6*c)*exp(6*d*x) - 3317760*I*d*exp(5*c)*exp(5*d*x) - 1205452
8*d*exp(4*c)*exp(4*d*x) + 18923520*I*d*exp(3*c)*exp(3*d*x) + 12054528*d*exp
(2*c)*exp(2*d*x) - 3317760*I*d*exp(c)*exp(d*x) - 331776*d) + (-385*log(exp(
d*x) - 3*I*exp(-c))/32768 + 385*log(exp(d*x) - I*exp(-c)/3)/32768)/d
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = -\frac{385i\,\arctan\left(\frac{3}{4}\,e^{(-dx-c)} + \frac{5}{4}i\right)}{16384\,d} - \frac{73575i\,e^{(-dx-c)} + 218466\,e^{(-2\,dx-2\,c)} - 239470i\,e^{(-3\,dx-3\,c)} - 86625\,e^{(-4\,dx-4\,c)} + 10395i\,e^{(-5\,dx-5\,c)}}{-12288\,d(-270i\,e^{(-dx-c)} - 981\,e^{(-2\,dx-2\,c)} + 1540i\,e^{(-3\,dx-3\,c)} + 981\,e^{(-4\,dx-4\,c)} - 270i\,e^{(-5\,dx-5\,c)} - 27\,e^{(}}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -385/16384*I*arctan(3/4*e^(-d*x - c) + 5/4*I)/d - (73575*I*e^(-d*x - c) + 2
18466*e^(-2*d*x - 2*c) - 239470*I*e^(-3*d*x - 3*c) - 86625*e^(-4*d*x - 4*c)
 + 10395*I*e^(-5*d*x - 5*c) - 8397)/(d*(3317760*I*e^(-d*x - c) + 12054528*e
^(-2*d*x - 2*c) - 18923520*I*e^(-3*d*x - 3*c) - 12054528*e^(-4*d*x - 4*c) +
 3317760*I*e^(-5*d*x - 5*c) + 331776*e^(-6*d*x - 6*c) - 331776))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx = -\frac{\frac{8\left(10395i\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}-239470i\,e^{(3\,dx+3\,c)}-218466\,e^{(2\,dx+2\,c)}+73575i\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}-10i\,e^{(dx+c)}-3\right)^3}-1155\log\left(3\,e^{(dx+c)}-i\right)+ \cdots}{98304\,d}$$

```
[In] integrate(1/(5+3*I*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/98304*(8*(10395*I*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) - 239470*I*e^(
3*d*x + 3*c) - 218466*e^(2*d*x + 2*c) + 73575*I*e^(d*x + c) + 8397)/(3*e^(2
*d*x + 2*c) - 10*I*e^(d*x + c) - 3)^3 - 1155*log(3*e^(d*x + c) - I) + 1155*
log(e^(d*x + c) - 3*I))/d
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \frac{1}{(5+3i\sinh(c+dx))^4}\,dx &= \frac{\frac{1925}{36864\,d}+\frac{e^{c+d\,x}\,385i}{12288\,d}}{1-e^{2\,c+2\,d\,x}+\frac{e^{c+d\,x}\,10i}{3}} \\ &+ \frac{\frac{41}{486\,d}+\frac{e^{c+d\,x}\,365i}{1458\,d}}{\frac{109\,e^{4\,c+4\,d\,x}}{3}-\frac{109\,e^{2\,c+2\,d\,x}}{3}-e^{6\,c+6\,d\,x}+1+e^{c+d\,x}\,10i-\frac{e^{3\,c+3\,d\,x}\,1540i}{27}+e^{5\,c+5\,d\,x}\,10i} \\ &- \frac{385\,\ln\left(-\frac{385\,e^{c+d\,x}}{4}+\frac{1155}{4}i\right)}{32768\,d}+\frac{385\,\ln\left(\frac{3465\,e^{c+d\,x}}{4}-\frac{1155}{4}i\right)}{32768\,d} \\ &- \frac{\frac{3461}{31104\,d}+\frac{e^{c+d\,x}\,385i}{10368\,d}}{e^{4\,c+4\,d\,x}-\frac{118\,e^{2\,c+2\,d\,x}}{9}+1+\frac{e^{c+d\,x}\,20i}{3}-\frac{e^{3\,c+3\,d\,x}\,20i}{3}}\end{aligned}$$

```
[In] int(1/(sinh(c + d*x)*3i + 5)^4,x)
```

```
[Out] ((exp(c + d*x)*385i)/(12288*d) + 1925/(36864*d))/((exp(c + d*x)*10i)/3 - ex
p(2*c + 2*d*x) + 1) + ((exp(c + d*x)*365i)/(1458*d) + 41/(486*d))/(exp(c +
d*x)*10i - (109*exp(2*c + 2*d*x))/3 - (exp(3*c + 3*d*x)*1540i)/27 + (109*ex
p(4*c + 4*d*x))/3 + exp(5*c + 5*d*x)*10i - exp(6*c + 6*d*x) + 1) - (385*log
(1155i/4 - (385*exp(c + d*x))/4))/(32768*d) + (385*log((3465*exp(c + d*x))/
4 - 1155i/4))/(32768*d) - ((exp(c + d*x)*385i)/(10368*d) + 3461/(31104*d))/
((exp(c + d*x)*20i)/3 - (118*exp(2*c + 2*d*x))/9 - (exp(3*c + 3*d*x)*20i)/3
 + exp(4*c + 4*d*x) + 1)
```

3.96 $\int (a+b\sinh(c+dx))^5\,dx$

Optimal result

Integrand size = 12, antiderivative size = 183

$$\int (a+b\sinh(c+dx))^5\,dx = \frac{1}{8}a\left(8a^4-40a^2b^2+15b^4\right)x + \frac{b(107a^4-192a^2b^2+16b^4)\cosh(c+dx)}{30d} + \frac{7ab^2(22a^2-23b^2)\cosh(c+dx)\sinh(c+dx)}{120d} + \frac{b(47a^2-16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} + \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}$$

```
[Out] 1/8*a*(8*a^4-40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4-192*a^2*b^2+16*b^4)*cosh(
d*x+c)/d+7/120*a*b^2*(22*a^2-23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47*a
^2-16*b^2)*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d+9/20*a*b*cosh(d*x+c)*(a+b*sinh
(d*x+c))^3/d+1/5*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{2735, 2832, 2813\}$

$$\int (a+b\sinh(c+dx))^5\,dx = \frac{b(47a^2-16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} + \frac{7ab^2(22a^2-23b^2)\sinh(c+dx)\cosh(c+dx)}{120d} + \frac{b(107a^4-192a^2b^2+16b^4)\cosh(c+dx)}{30d} + \frac{1}{8}ax\left(8a^4-40a^2b^2+15b^4\right) + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} + \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d}$$

```
[In] Int[(a + b*Sinh[c + d*x])^5,x]
```

```
[Out] (a*(8*a^4 - 40*a^2*b^2 + 15*b^4)*x)/8 + (b*(107*a^4 - 192*a^2*b^2 + 16*b^4)
*Cosh[c + d*x])/(30*d) + (7*a*b^2*(22*a^2 - 23*b^2)*Cosh[c + d*x]*Sinh[c +
d*x])/(120*d) + (b*(47*a^2 - 16*b^2)*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)
/(60*d) + (9*a*b*Cosh[c + d*x]*(a + b*Sinh[c + d*x])^3)/(20*d) + (b*Cosh[c
+ d*x]*(a + b*Sinh[c + d*x])^4)/(5*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} \\
&\quad + \frac{1}{5}\int (a+b\sinh(c+dx))^3 \left(5a^2-4b^2+9ab\sinh(c+dx)\right)\,dx \\
&= \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} \\
&\quad + \frac{1}{20}\int (a+b\sinh(c+dx))^2 \left(a\left(20a^2-43b^2\right)+b\left(47a^2-16b^2\right)\sinh(c+dx)\right)\,dx \\
&= \frac{b(47a^2-16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} \\
&\quad + \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d} \\
&\quad + \frac{1}{60}\int (a+b\sinh(c+dx)) \left(60a^4-223a^2b^2+32b^4 \right.\\
&\qquad \left. +7ab\left(22a^2-23b^2\right)\sinh(c+dx)\right)\,dx \\
&= \frac{1}{8}a\left(8a^4-40a^2b^2+15b^4\right)x + \frac{b(107a^4-192a^2b^2+16b^4)\cosh(c+dx)}{30d} \\
&\quad + \frac{7ab^2(22a^2-23b^2)\cosh(c+dx)\sinh(c+dx)}{120d} \\
&\quad + \frac{b(47a^2-16b^2)\cosh(c+dx)(a+b\sinh(c+dx))^2}{60d} \\
&\quad + \frac{9ab\cosh(c+dx)(a+b\sinh(c+dx))^3}{20d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^4}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.75

$$\int (a+b\sinh(c+dx))^5\,dx$$
$$= \frac{300b(8a^4-12a^2b^2+b^4)\cosh(c+dx)+50(8a^2b^3-b^5)\cosh(3(c+dx))+6b^5\cosh(5(c+dx))+15a(4(8a}{480d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^5,x]
```

```
[Out] (300*b*(8*a^4 - 12*a^2*b^2 + b^4)*Cosh[c + d*x] + 50*(8*a^2*b^3 - b^5)*Cosh
[3*(c + d*x)] + 6*b^5*Cosh[5*(c + d*x)] + 15*a*(4*(8*a^4 - 40*a^2*b^2 + 15*
b^4)*(c + d*x) + 40*(2*a^2*b^2 - b^4)*Sinh[2*(c + d*x)] + 5*b^4*Sinh[4*(c +
 d*x)]))/(480*d)
```

Maple [A] (verified)

Time = 4.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{b^5\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+5a\,b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2b^3\left(\right.}{d}$
default	$\frac{b^5\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)+5a\,b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2b^3\left(\right.}{d}$
parallelrisch	$\frac{(400a^2b^3-50b^5)\cosh(3dx+3c)+(1200a^3b^2-600a\,b^4)\sinh(2dx+2c)+6b^5\cosh(5dx+5c)+75a\,b^4\sinh(4dx+4c)+(2400a^4b}{480d}$
parts	$a^5x+\frac{b^5\left(\frac{8}{15}+\frac{\sinh(dx+c)^4}{5}-\frac{4\sinh(dx+c)^2}{15}\right)\cosh(dx+c)}{d}+\frac{5a^4b\cosh(dx+c)}{d}+\frac{10a^3b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\right.}{d}$
risch	$a^5x-5a^3b^2x+\frac{15a\,b^4x}{8}+\frac{b^5e^{5dx+5c}}{160d}+\frac{5a\,b^4e^{4dx+4c}}{64d}+\frac{5b^3e^{3dx+3c}a^2}{12d}-\frac{5b^5e^{3dx+3c}}{96d}+\frac{5a^3b^2e^{2dx+2c}}{4d}-$

```
[In] int((a+b*sinh(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^5*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+5*a*b^4*((
1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+10*a^2*b^3*(-
2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+10*a^3*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-
1/2*d*x-1/2*c)+5*a^4*b*cosh(d*x+c)+a^5*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.22

$$\int (a+b\sinh(c+dx))^5\,dx$$
$$=\frac{3\,b^5\cosh{(dx+c)}^5+15\,b^5\cosh{(dx+c)}\sinh{(dx+c)}^4+150\,ab^4\cosh{(dx+c)}\sinh{(dx+c)}^3+25\,(8\,a^2b^3-}{}$$

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^5*cosh(d*x + c)^5 + 15*b^5*cosh(d*x + c)*sinh(d*x + c)^4 + 150*a
*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 25*(8*a^2*b^3 - b^5)*cosh(d*x + c)^3 +
 30*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*d*x + 15*(2*b^5*cosh(d*x + c)^3 + 5*(8*
a^2*b^3 - b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + 150*(8*a^4*b - 12*a^2*b^3 +
 b^5)*cosh(d*x + c) + 150*(a*b^4*cosh(d*x + c)^3 + 4*(2*a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int (a + b\sinh(c + dx))^5\,dx$$
$$= \begin{cases} a^5x + \frac{5a^4b\cosh(c+dx)}{d} + 5a^3b^2x\sinh^2(c + dx) - 5a^3b^2x\cosh^2(c + dx) + \frac{5a^3b^2\sinh(c+dx)\cosh(c+dx)}{d} + \frac{10a^2b^3\sin}{} \\ x(a + b\sinh(c))^5 \end{cases}$$

```
[In] integrate((a+b*sinh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*cosh(c + d*x)/d + 5*a**3*b**2*x*sinh(c + d*x)*
*2 - 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 10*a**2*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 20*a**2*b**3*cosh(c
+ d*x)**3/(3*d) + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 + 25*a*b**4*sinh(c
 + d*x)**3*cosh(c + d*x)/(8*d) - 15*a*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(
8*d) + b**5*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b**5*sinh(c + d*x)**2*cosh
(c + d*x)**3/(3*d) + 8*b**5*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*s
inh(c))**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.49

$$\int (a + b\sinh(c + dx))^5\,dx$$
$$= \frac{5}{64}ab^4\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{5}{4}a^3b^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + a^5x + \frac{1}{480}b^5\left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d}\right) + \frac{5}{12}a^2b^3\left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right) + \frac{5a^4b\cosh(dx+c)}{d}$$

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 5/4*a^3*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2
*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c
)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^
(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 5*a^4*b*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.47

$$\int (a+b\sinh(c+dx))^5\,dx = \frac{b^5e^{(5\,dx+5\,c)}}{160\,d} + \frac{5\,ab^4e^{(4\,dx+4\,c)}}{64\,d} - \frac{5\,ab^4e^{(-4\,dx-4\,c)}}{64\,d} + \frac{b^5e^{(-5\,dx-5\,c)}}{160\,d} + \frac{1}{8}\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)x + \frac{5\left(8\,a^2b^3-b^5\right)e^{(3\,dx+3\,c)}}{96\,d} + \frac{5\left(2\,a^3b^2-ab^4\right)e^{(2\,dx+2\,c)}}{8\,d} + \frac{5\left(8\,a^4b-12\,a^2b^3+b^5\right)e^{(dx+c)}}{16\,d} + \frac{5\left(8\,a^4b-12\,a^2b^3+b^5\right)e^{(-dx-c)}}{16\,d} - \frac{5\left(2\,a^3b^2-ab^4\right)e^{(-2\,dx-2\,c)}}{8\,d} + \frac{5\left(8\,a^2b^3-b^5\right)e^{(-3\,dx-3\,c)}}{96\,d}$$

```
[In] integrate((a+b*sinh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^(
-4*d*x - 4*c)/d + 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 - 40*a^3*b^2 +
15*a*b^4)*x + 5/96*(8*a^2*b^3 - b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2 - a
*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(d*x + c)/d +
 5/16*(8*a^4*b - 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 - a*b^4)
*e^(-2*d*x - 2*c)/d + 5/96*(8*a^2*b^3 - b^5)*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int (a+b\sinh(c+dx))^5\,dx = \frac{75\,b^5\cosh(c+d\,x) - \frac{25\,b^5\cosh(3\,c+3\,d\,x)}{2} + \frac{3\,b^5\cosh(5\,c+5\,d\,x)}{2} - 900\,a^2\,b^3\cosh(c+d\,x) - 150\,a\,b^4\sinh(2\,c+2\,d\,x)}{}$$

```
[In] int((a + b*sinh(c + d*x))^5,x)
```

```
[Out] (75*b^5*cosh(c + d*x) - (25*b^5*cosh(3*c + 3*d*x))/2 + (3*b^5*cosh(5*c + 5*
d*x))/2 - 900*a^2*b^3*cosh(c + d*x) - 150*a*b^4*sinh(2*c + 2*d*x) + (75*a*b
^4*sinh(4*c + 4*d*x))/4 + 100*a^2*b^3*cosh(3*c + 3*d*x) + 300*a^3*b^2*sinh(
2*c + 2*d*x) + 600*a^4*b*cosh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x - 600*
a^3*b^2*d*x)/(120*d)
```

3.97 $\int (a+b\sinh(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{1}{8}\left(8a^4-24a^2b^2+3b^4\right)x+\frac{ab(19a^2-16b^2)\cosh(c+dx)}{6d} + \frac{b^2(26a^2-9b^2)\cosh(c+dx)\sinh(c+dx)}{24d} + \frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}$$

```
[Out] 1/8*(8*a^4-24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2-16*b^2)*cosh(d*x+c)/d+1/24*b
^2*(26*a^2-9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*cosh(d*x+c)*(a+b*sinh(
d*x+c))^2/d+1/4*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2735, 2832, 2813}

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{ab(19a^2-16b^2)\cosh(c+dx)}{6d} + \frac{b^2(26a^2-9b^2)\sinh(c+dx)\cosh(c+dx)}{24d} + \frac{1}{8}x\left(8a^4-24a^2b^2+3b^4\right)+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d} + \frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d}$$

```
[In] Int[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] ((8*a^4 - 24*a^2*b^2 + 3*b^4)*x)/8 + (a*b*(19*a^2 - 16*b^2)*Cosh[c + d*x])/
(6*d) + (b^2*(26*a^2 - 9*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(24*d) + (7*a*b*
Cosh[c + d*x]*(a + b*Sinh[c + d*x])^2)/(12*d) + (b*Cosh[c + d*x]*(a + b*Sin
h[c + d*x])^3)/(4*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d} \\
&\quad + \frac{1}{4}\int (a+b\sinh(c+dx))^2\left(4a^2-3b^2+7ab\sinh(c+dx)\right)\,dx \\
&= \frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d} \\
&\quad + \frac{1}{12}\int (a+b\sinh(c+dx))\left(a\left(12a^2-23b^2\right)+b\left(26a^2-9b^2\right)\sinh(c+dx)\right)\,dx \\
&= \frac{1}{8}\left(8a^4-24a^2b^2+3b^4\right)x + \frac{ab(19a^2-16b^2)\cosh(c+dx)}{6d} \\
&\quad + \frac{b^2(26a^2-9b^2)\cosh(c+dx)\sinh(c+dx)}{24d} \\
&\quad + \frac{7ab\cosh(c+dx)(a+b\sinh(c+dx))^2}{12d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^3}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int (a+b\sinh(c+dx))^4\,dx$$
$$=\frac{96ab(4a^2-3b^2)\cosh(c+dx)+32ab^3\cosh(3(c+dx))+3(4(8a^4-24a^2b^2+3b^4)(c+dx)+8(6a^2b^2-b^4)}{96d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^4,x]
```

```
[Out] (96*a*b*(4*a^2 - 3*b^2)*Cosh[c + d*x] + 32*a*b^3*Cosh[3*(c + d*x)] + 3*(4*(
8*a^4 - 24*a^2*b^2 + 3*b^4)*(c + d*x) + 8*(6*a^2*b^2 - b^4)*Sinh[2*(c + d*x
)] + b^4*Sinh[4*(c + d*x)]))/(96*d)
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4b^3a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+6a^2b^2\left(\frac{\cosh(dx+c)\sinh}{2}\right.}{d}$
default	$\frac{b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4b^3a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+6a^2b^2\left(\frac{\cosh(dx+c)\sinh}{2}\right.}{d}$
parts	$a^4x+\frac{b^4\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{4a^3b\cosh(dx+c)}{d}+\frac{6a^2b^2\left(\frac{\cosh(dx+c)\sinh(dx+c}{2}\right.}{d}$
parallelrisch	$\frac{96a^4dx-288a^2b^2dx+36b^4dx+32b^3a\cosh(3dx+3c)+384a^3b\cosh(dx+c)-288b^3a\cosh(dx+c)+3b^4\sinh(4dx+4c)+144a^2}{96d}$
risch	$a^4x-3a^2b^2x+\frac{3x\,b^4}{8}+\frac{b^4e^{4dx+4c}}{64d}+\frac{b^3a\,e^{3dx+3c}}{6d}+\frac{3b^2e^{2dx+2c}a^2}{4d}-\frac{b^4e^{2dx+2c}}{8d}+\frac{2a^3b\,e^{dx+c}}{d}-\frac{3a\,b^3e^{d}}{2d}$

```
[In] int((a+b*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+4*
b^3*a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+6*a^2*b^2*(1/2*cosh(d*x+c)*sinh(
d*x+c)-1/2*d*x-1/2*c)+4*a^3*b*cosh(d*x+c)+a^4*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int (a + b\sinh(c + dx))^4 \, dx$$
$$= \frac{3\, b^4 \cosh\left(dx + c\right) \sinh\left(dx + c\right)^3 + 8\, ab^3 \cosh\left(dx + c\right)^3 + 24\, ab^3 \cosh\left(dx + c\right) \sinh\left(dx + c\right)^2 + 3\left(8\, a^4 - 24\, a\right.}{}$$

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^3 + 24*a*
b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*d*x + 24
*(4*a^3*b - 3*a*b^3)*cosh(d*x + c) + 3*(b^4*cosh(d*x + c)^3 + 4*(6*a^2*b^2
- b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.75

$$\int (a + b\sinh(c + dx))^4 \, dx$$
$$= \begin{cases} a^4x + \frac{4a^3b\cosh\left(c+dx\right)}{d} + 3a^2b^2x\sinh^2\left(c + dx\right) - 3a^2b^2x\cosh^2\left(c + dx\right) + \frac{3a^2b^2\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{d} + \frac{4ab^3\sinh^2\left(\right.}{} \\ x(a + b\sinh\left(c\right))^4 \end{cases}$$

```
[In] integrate((a+b*sinh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*cosh(c + d*x)/d + 3*a**2*b**2*x*sinh(c + d*x)*
*2 - 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c + d*
x)/d + 4*a*b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 8*a*b**3*cosh(c + d*x)**
3/(3*d) + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 + 5*b**4*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) - 3*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a
+ b*sinh(c))**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{1}{64}b^4\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right) - \frac{3}{4}a^2b^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^4x + \frac{1}{6}ab^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{4\,a^3b\cosh\left(dx+c\right)}{d}$$

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/64*b^4*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 3/4*a^2*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e
^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 4*a^3*b*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.46

$$\int (a+b\sinh(c+dx))^4\,dx = \frac{b^4e^{(4\,dx+4\,c)}}{64\,d}+\frac{ab^3e^{(3\,dx+3\,c)}}{6\,d}+\frac{ab^3e^{(-3\,dx-3\,c)}}{6\,d} - \frac{b^4e^{(-4\,dx-4\,c)}}{64\,d}+\frac{1}{8}\left(8\,a^4-24\,a^2b^2+3\,b^4\right)x + \frac{(6\,a^2b^2-b^4)e^{(2\,dx+2\,c)}}{8\,d}+\frac{(4\,a^3b-3\,ab^3)e^{(dx+c)}}{2\,d} + \frac{(4\,a^3b-3\,ab^3)e^{(-dx-c)}}{2\,d}-\frac{(6\,a^2b^2-b^4)e^{(-2\,dx-2\,c)}}{8\,d}$$

```
[In] integrate((a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d + 1/6*a*b^3*e^(-3*
d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 - 24*a^2*b^2 + 3*b^
4)*x + 1/8*(6*a^2*b^2 - b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b - 3*a*b^3)*e^
(d*x + c)/d + 1/2*(4*a^3*b - 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 - b^4
)*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int (a + b\sinh(c + dx))^4 \, dx$$
$$= \frac{\frac{3\,b^4\,\sinh(4\,c+4\,d\,x)}{4} - 6\,b^4\,\sinh(2\,c+2\,d\,x) + 8\,a\,b^3\,\cosh(3\,c+3\,d\,x) + 36\,a^2\,b^2\,\sinh(2\,c+2\,d\,x) - 72\,a\,b^3\,\cosh(c + \dots}{24\,d}$$

```
[In] int((a + b*sinh(c + d*x))^4,x)
```

```
[Out] ((3*b^4*sinh(4*c + 4*d*x))/4 - 6*b^4*sinh(2*c + 2*d*x) + 8*a*b^3*cosh(3*c +
 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) - 72*a*b^3*cosh(c + d*x) + 96*a^3*b*
cosh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x - 72*a^2*b^2*d*x)/(24*d)
```

3.98 $\int (a+b\sinh(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 92

$$\int (a+b\sinh(c+dx))^3\,dx=\frac{1}{2}a\left(2a^2-3b^2\right)x+\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

```
[Out] 1/2*a*(2*a^2-3*b^2)*x+2/3*b*(4*a^2-b^2)*cosh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c)
*sinh(d*x+c)/d+1/3*b*cosh(d*x+c)*(a+b*sinh(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2735, 2813}

$$\int (a+b\sinh(c+dx))^3\,dx=\frac{2b(4a^2-b^2)\cosh(c+dx)}{3d}+\frac{1}{2}ax\left(2a^2-3b^2\right)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{6d}+\frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}$$

```
[In] Int[(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 - 3*b^2)*x)/2 + (2*b*(4*a^2 - b^2)*Cosh[c + d*x])/(3*d) + (5*a*b^
2*Cosh[c + d*x]*Sinh[c + d*x])/(6*d) + (b*Cosh[c + d*x]*(a + b*Sinh[c + d*x
])^2)/(3*d)
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2813

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d} \\ &\quad + \frac{1}{3}\int(a+b\sinh(c+dx))\left(3a^2-2b^2+5ab\sinh(c+dx)\right)\,dx \\ &= \frac{1}{2}a\left(2a^2-3b^2\right)x + \frac{2b(4a^2-b^2)\cosh(c+dx)}{3d} \\ &\quad + \frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d} + \frac{b\cosh(c+dx)(a+b\sinh(c+dx))^2}{3d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int(a+b\sinh(c+dx))^3\,dx = \frac{6a(2a^2-3b^2)(c+dx)-9b(-4a^2+b^2)\cosh(c+dx)+b^3\cosh(3(c+dx))+9ab^2\sinh(2(c+dx))}{12d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^3,x]
[Out] (6*a*(2*a^2 - 3*b^2)*(c + d*x) - 9*b*(-4*a^2 + b^2)*Cosh[c + d*x] + b^3*Cos
h[3*(c + d*x)] + 9*a*b^2*Sinh[2*(c + d*x)])/(12*d)
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{b^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\cosh(dx+c)+a^3(dx+c)}{d}$
default	$\frac{b^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\cosh(dx+c)+a^3(dx+c)}{d}$
parts	$a^3x+\frac{b^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)}{d}+\frac{3a\,b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{3a^2b\cosh(dx+c)}{d}$
parallelrisch	$\frac{b^3\cosh(3dx+3c)+9a\,b^2\sinh(2dx+2c)+(36a^2b-9b^3)\cosh(dx+c)+12a^3dx-18a\,b^2dx+36a^2b-8b^3}{12d}$
risch	$a^3x-\frac{3a\,b^2x}{2}+\frac{b^3\mathrm{e}^{3dx+3c}}{24d}+\frac{3a\,b^2\mathrm{e}^{2dx+2c}}{8d}+\frac{3b\,\mathrm{e}^{dx+c}a^2}{2d}-\frac{3b^3\mathrm{e}^{dx+c}}{8d}+\frac{3b\,\mathrm{e}^{-dx-c}a^2}{2d}-\frac{3b^3\mathrm{e}^{-dx-c}}{8d}-\frac{3a\,b^2}{}$

```
[In] int((a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh
(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*cosh(d*x+c)+a^3*(d*x+c))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.99

$$\int (a+b\sinh(c+dx))^3\,dx$$
$$=\frac{b^3\cosh\left(dx+c\right)^3+3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+18\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+6\left(2\,a^3-3\,ab^2\right)c}{12\,d}$$

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b^3*cosh(d*x + c)^3 + 3*b^3*cosh(d*x + c)*sinh(d*x + c)^2 + 18*a*b^2*
cosh(d*x + c)*sinh(d*x + c) + 6*(2*a^3 - 3*a*b^2)*d*x + 9*(4*a^2*b - b^3)*c
osh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.39

$$\int (a+b\sinh(c+dx))^3\,dx$$
$$= \begin{cases} a^3x + \frac{3a^2b\cosh{(c+dx)}}{d} + \frac{3ab^2x\sinh^2{(c+dx)}}{2} - \frac{3ab^2x\cosh^2{(c+dx)}}{2} + \frac{3ab^2\sinh{(c+dx)}\cosh{(c+dx)}}{2d} + \frac{b^3\sinh^2{(c+dx)}\cosh{(c+dx)}}{d} - \\ x(a+b\sinh{(c)})^3 \end{cases}$$

```
[In] integrate((a+b*sinh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*cosh(c + d*x)/d + 3*a*b**2*x*sinh(c + d*x)**2/
2 - 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/(2
*d) + b**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b**3*cosh(c + d*x)**3/(3*d)
, Ne(d, 0)), (x*(a + b*sinh(c))**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int (a+b\sinh(c+dx))^3\,dx = -\frac{3}{8}\,ab^2\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + a^3x + \frac{1}{24}\,b^3\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{3\,a^2b\cosh{(dx+c)}}{d}$$

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^
3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c
)/d) + 3*a^2*b*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.47

$$\int (a+b\sinh(c+dx))^3\,dx = \frac{b^3e^{(3\,dx+3\,c)}}{24\,d} + \frac{3\,ab^2e^{(2\,dx+2\,c)}}{8\,d} - \frac{3\,ab^2e^{(-2\,dx-2\,c)}}{8\,d} + \frac{b^3e^{(-3\,dx-3\,c)}}{24\,d} + \frac{1}{2}\left(2\,a^3 - 3\,ab^2\right)x + \frac{3\,(4\,a^2b - b^3)e^{(dx+c)}}{8\,d} + \frac{3\,(4\,a^2b - b^3)e^{(-dx-c)}}{8\,d}$$

```
[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2*
d*x - 2*c)/d + 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 - 3*a*b^2)*x + 3/8*
(4*a^2*b - b^3)*e^(d*x + c)/d + 3/8*(4*a^2*b - b^3)*e^(-d*x - c)/d
```

Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int (a + b\sinh(c + dx))^3\, dx$$
$$= \frac{6\,d\,x\,a^3 + 18\,a^2\,b\cosh(c + d\,x) + 9\sinh(c + d\,x)\ a\,b^2\cosh(c + d\,x) - 9\,d\,x\,a\,b^2 + 2\,b^3\cosh(c + d\,x)^3 - 6\,b^3}{6\,d}$$

```
[In] int((a + b*sinh(c + d*x))^3,x)
```

```
[Out] (2*b^3*cosh(c + d*x)^3 - 6*b^3*cosh(c + d*x) + 18*a^2*b*cosh(c + d*x) + 6*a
^3*d*x + 9*a*b^2*cosh(c + d*x)*sinh(c + d*x) - 9*a*b^2*d*x)/(6*d)
```

3.99 $\int (a+b\sinh (c+dx))^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 52

$$\int (a+b\sinh (c+dx))^2 \, dx=\frac{1}{2}\left(2a^2-b^2\right) x+\frac{2ab\cosh (c+dx)}{d}+\frac{b^2\cosh (c+dx)\sinh (c+dx)}{2d}$$

```
[Out] 1/2*(2*a^2-b^2)*x+2*a*b*cosh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2723}

$$\int (a+b\sinh (c+dx))^2 \, dx=\frac{1}{2}x\left(2a^2-b^2\right)+\frac{2ab\cosh (c+dx)}{d}+\frac{b^2\sinh (c+dx)\cosh (c+dx)}{2d}$$

```
[In] Int[(a + b*Sinh[c + d*x])^2,x]
```

```
[Out] ((2*a^2 - b^2)*x)/2 + (2*a*b*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 2723

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\text{integral}=\frac{1}{2}\left(2a^2-b^2\right) x+\frac{2ab\cosh (c+dx)}{d}+\frac{b^2\cosh (c+dx)\sinh (c+dx)}{2d}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int (a+b\sinh(c+dx))^2\,dx=\frac{2(2a^2-b^2)\,(c+dx)+8ab\cosh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^2,x]
[Out] (2*(2*a^2 - b^2)*(c + d*x) + 8*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(
4*d)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{4a^2dx-2b^2dx+8ab\cosh(dx+c)+b^2\sinh(2dx+2c)+8ab}{4d}$	48
parts	$a^2x+\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)}{d}+\frac{2ab\cosh(dx+c)}{d}$	49
derivativedivides	$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\cosh(dx+c)+a^2(dx+c)}{d}$	51
default	$\frac{b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\cosh(dx+c)+a^2(dx+c)}{d}$	51
risch	$a^2x-\frac{b^2x}{2}+\frac{b^2e^{2dx+2c}}{8d}+\frac{ab\,e^{dx+c}}{d}+\frac{ab\,e^{-dx-c}}{d}-\frac{b^2e^{-2dx-2c}}{8d}$	74

```
[In] int((a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*(4*a^2*d*x-2*b^2*d*x+8*a*b*cosh(d*x+c)+b^2*sinh(2*d*x+2*c)+8*a*b)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int (a+b\sinh(c+dx))^2\,dx=\frac{b^2\cosh(dx+c)\sinh(dx+c)+(2\,a^2-b^2)dx+4\,ab\cosh(dx+c)}{2\,d}$$

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*(b^2*cosh(d*x + c)*sinh(d*x + c) + (2*a^2 - b^2)*d*x + 4*a*b*cosh(d*x +
 c))/d
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int (a+b\sinh(c+dx))^2\,dx = \begin{cases} a^2x + \frac{2ab\cosh{(c+dx)}}{d} + \frac{b^2x\sinh^2{(c+dx)}}{2} - \frac{b^2x\cosh^2{(c+dx)}}{2} + \frac{b^2\sinh{(c+dx)}\cosh{(c+dx)}}{2d} & \text{for } d\neq 0 \\ x(a+b\sinh{(c)})^2 & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*cosh(c + d*x)/d + b**2*x*sinh(c + d*x)**2/2 - b**
2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)),
 (x*(a + b*sinh(c))**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int (a+b\sinh(c+dx))^2\,dx = -\frac{1}{8}\,b^2\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + a^2x + \frac{2\,ab\cosh{(dx+c)}}{d}$$

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*cos
h(d*x + c)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int (a+b\sinh(c+dx))^2\,dx = \frac{1}{2}\left(2\,a^2 - b^2\right)x + \frac{b^2e^{(2\,dx+2\,c)}}{8\,d} + \frac{abe^{(dx+c)}}{d} + \frac{abe^{(-dx-c)}}{d} - \frac{b^2e^{(-2\,dx-2\,c)}}{8\,d}$$

```
[In] integrate((a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2 - b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d + a*b*e
^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int (a+b\sinh(c+dx))^2\,dx=a^2\,x-\frac{b^2\,x}{2}+\frac{\frac{\sinh(2\,c+2\,d\,x)\,b^2}{4}+2\,a\cosh(c+d\,x)\,b}{d}$$

```
[In] int((a + b*sinh(c + d*x))^2,x)
[Out] a^2*x - (b^2*x)/2 + ((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*cosh(c + d*x))/d
```

3.100 $\int (a+b\sinh(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a+b\sinh(c+dx))\,dx=ax+\frac{b\cosh(c+dx)}{d}$$

```
[Out] a*x+b*cosh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2718}

$$\int (a+b\sinh(c+dx))\,dx=ax+\frac{b\cosh(c+dx)}{d}$$

```
[In] Int[a + b*Sinh[c + d*x],x]
```

```
[Out] a*x + (b*Cosh[c + d*x])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=ax+b\int \sinh(c+dx)\,dx\\&=ax+\frac{b\cosh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a + b\sinh(c + dx))\,dx = ax + \frac{b\cosh(c)\cosh(dx)}{d} + \frac{b\sinh(c)\sinh(dx)}{d}$$

```
[In] Integrate[a + b*Sinh[c + d*x],x]
[Out] a*x + (b*Cosh[c]*Cosh[d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax + \frac{b\cosh(dx+c)}{d}$	16
parts	$ax + \frac{b\cosh(dx+c)}{d}$	16
parallelrisch	$\frac{b(1+\cosh(dx+c))}{d} + ax$	18
derivativedivides	$\frac{(dx+c)a+b\cosh(dx+c)}{d}$	21
risch	$ax + \frac{b\,e^{dx+c}}{2d} + \frac{b\,e^{-dx-c}}{2d}$	32

```
[In] int(a+b*sinh(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+b*cosh(d*x+c)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + b\sinh(c + dx))\,dx = \frac{adx + b\cosh(dx + c)}{d}$$

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="fricas")
[Out] (a*d*x + b*cosh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a + b\sinh(c + dx))\,dx = ax + b\left(\begin{cases} \frac{\cosh{(c+dx)}}{d} & \text{for } d \neq 0 \\ x\sinh{(c)} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*sinh(d*x+c),x)
[Out] a*x + b*Piecewise((cosh(c + d*x)/d, Ne(d, 0)), (x*sinh(c), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b\sinh(c + dx))\,dx = ax + \frac{b\cosh{(dx + c)}}{d}$$

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="maxima")
[Out] a*x + b*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int (a + b\sinh(c + dx))\,dx = ax + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right)$$

```
[In] integrate(a+b*sinh(d*x+c),x, algorithm="giac")
[Out] a*x + 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d)
```

Mupad [B] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a + b\sinh(c + dx))\,dx = a\,x + \frac{b\cosh(c + d\,x)}{d}$$

```
[In] int(a + b*sinh(c + d*x),x)
[Out] a*x + (b*cosh(c + d*x))/d
```

3.101 $\int \frac{1}{a+b\sinh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 44

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

```
[Out] -2*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2739, 632, 210}

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[In] Int[(a + b*Sinh[c + d*x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(2i)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\ &= \frac{(4i)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{d}\\ &= -\frac{2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{2\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-1),x]
[Out] (2*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,d}$	111

```
[In] int(1/(a+b*sinh(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.68

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx$$
$$= \frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2+b^2+2\,(b^2\cosh(dx+c)+ab)\sinh(dx+c)-2\,\sqrt{a^2+b^2}(b\cosh(dx+c)+b\sinh(dx+c)+}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)-b}\right.}{\sqrt{a^2+b^2}d}$$

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^
2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*
cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b))/(sqrt(a
^2 + b^2)*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.52

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\sinh(c)} & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd} & \text{for } a=0 \\ \frac{x}{a+b\sinh(c)} & \text{for } d=0 \\ \frac{2i}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-ibd} & \text{for } a=-ib \\ -\frac{2i}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+ibd} & \text{for } a=ib \\ -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}}+\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*sinh(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x/sinh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tanh(c/2 +
d*x/2))/(b*d), Eq(a, 0)), (x/(a + b*sinh(c)), Eq(d, 0)), (2*I/(b*d*tanh(c/2
 + d*x/2) - I*b*d), Eq(a, -I*b)), (-2*I/(b*d*tanh(c/2 + d*x/2) + I*b*d), Eq
(a, I*b)), (-log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(d*sqrt(a**
2 + b**2)) + log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(d*sqrt(a**
2 + b**2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2 +
 b^2)))/(sqrt(a^2 + b^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

```
[In] integrate(1/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{1}{a+b\sinh(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{\sqrt{-a^2\,d^2-b^2\,d^2}}\right)}{\sqrt{-a^2\,d^2-b^2\,d^2}}$$

```
[In] int(1/(a + b*sinh(c + d*x)),x)
```

```
[Out] (2*atan((a*d + b*d*exp(d*x)*exp(c))/(- a^2*d^2 - b^2*d^2)^(1/2)))/(- a^2*d^
2 - b^2*d^2)^(1/2)
```

3.102 $\int \frac{1}{(a+b\sinh(c+dx))^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{(a+b\sinh(c+dx))^2} dx = -\frac{2a\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{b\cosh(c+dx)}{(a^2+b^2)d(a+b\sinh(c+dx))}$$

```
[Out] -2*a*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-b
*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {2743, 12, 2739, 632, 210}

$$\int \frac{1}{(a+b\sinh(c+dx))^2} dx = -\frac{2a\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}} - \frac{b\cosh(c+dx)}{d(a^2+b^2)(a+b\sinh(c+dx))}$$

```
[In] Int[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] (-2*a*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2
)*d) - (b*Cosh[c + d*x])/((a^2 + b^2)*d*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}+\frac{\int \frac{a}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}+\frac{a\int \frac{1}{a+b\sinh(c+dx)}\,dx}{a^2+b^2}\\
&= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}-\frac{(2ia)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)\,d}\\
&= -\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}\\
&\quad+\frac{(4ia)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx,x,-2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)\,d}\\
&= -\frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b\cosh(c+dx)}{(a^2+b^2)\,d(a+b\sinh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = -\frac{\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{(-a^2-b^2)^{3/2}}+\frac{b\cosh(c+dx)}{(a^2+b^2)(a+b\sinh(c+dx))}}{d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-2),x]
```

```
[Out] -(((2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3
/2) + (b*Cosh[c + d*x])/((a^2 + b^2)*(a + b*Sinh[c + d*x])))/d)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\ \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(a^2+b^2\right)}-\frac{b}{a^2+b^2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\ \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{2a\,e^{dx+c}-2b}{d(a^2+b^2)\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)}+\frac{a\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{a\ln\left(e^{dx+c}+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right.}{(a^2+b^2)^{\frac{3}{2}}d}$

```
[In] int(1/(a+b*sinh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-b^2/a/(a^2+b^2)*tanh(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tanh(1/2*d*x+1/
2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*ta
nh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(76) = 152.

Time = 0.28 (sec) , antiderivative size = 423, normalized size of antiderivative = 5.35

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = -\frac{2\,a^2b+2\,b^3-\left(ab\cosh\left(dx+c\right)^2+ab\sinh\left(dx+c\right)^2+2\,a^2\cosh\left(dx+c\right)-ab+2\left(ab\cosh\left(dx+c\right)+a\right.\right.}{\left(a^4b+2\,a^2b^3+b^5\right)d\cosh\left(dx+c\right)^2+\left(a^4b+2\,a^2b^3+b^5\right)d\sin}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b + 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cosh
(d*x + c) - a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 + b^2
)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(
b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(
a^3 + a*b^2)*cosh(d*x + c) - 2*(a^3 + a*b^2)*sinh(d*x + c))/((a^4*b + 2*a^2
*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2
 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) - (a^4*b + 2*a^2*b^3 + b^5)*
d + 2*((a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4
)*d)*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2082 vs. 2(68) = 136.

Time = 90.37 (sec) , antiderivative size = 2082, normalized size of antiderivative = 26.35

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-tanh(c/2 +
 d*x/2)/(2*d) - 1/(2*d*tanh(c/2 + d*x/2)))/b**2, Eq(a, 0)), (-6*b*tanh(c/2
+ d*x/2)**2/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*tanh(c/2 + d*x/2) - 9
*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 + 3*b**2*d*sqrt(-b**2)) + 4*b/(3*b
**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*tanh(c/2 + d*x/2) - 9*b**2*d*sqrt(-b*
*2)*tanh(c/2 + d*x/2)**2 + 3*b**2*d*sqrt(-b**2)) + 6*sqrt(-b**2)*tanh(c/2 +
 d*x/2)/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*tanh(c/2 + d*x/2) - 9*b**
2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 + 3*b**2*d*sqrt(-b**2)), Eq(a, -sqrt(-
b**2))), (-6*b*tanh(c/2 + d*x/2)**2/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3
*d*tanh(c/2 + d*x/2) + 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 - 3*b**2*d
```

```
*sqrt(-b**2)) + 4*b/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*tanh(c/2 + d*
x/2) + 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 - 3*b**2*d*sqrt(-b**2)) -
6*sqrt(-b**2)*tanh(c/2 + d*x/2)/(3*b**3*d*tanh(c/2 + d*x/2)**3 - 9*b**3*d*t
anh(c/2 + d*x/2) + 9*b**2*d*sqrt(-b**2)*tanh(c/2 + d*x/2)**2 - 3*b**2*d*sqr
t(-b**2)), Eq(a, sqrt(-b**2))), (x/(a + b*sinh(c))**2, Eq(d, 0)), (-a**3*lo
g(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(a**4
*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a**2 + b**2) - 2*a*
*3*b*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2) + a**2*b**2*d*sqrt(a**2 + b**2)*
tanh(c/2 + d*x/2)**2 - a**2*b**2*d*sqrt(a**2 + b**2) - 2*a*b**3*d*sqrt(a**2
 + b**2)*tanh(c/2 + d*x/2)) + a**3*log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2
+ b**2)/a)/(a**4*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a**
2 + b**2) - 2*a**3*b*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2) + a**2*b**2*d*sq
rt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**2*b**2*d*sqrt(a**2 + b**2) - 2*a*
b**3*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)) + a**3*log(tanh(c/2 + d*x/2) -
b/a + sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)**2/(a**4*d*sqrt(a**2 + b**2)*t
anh(c/2 + d*x/2)**2 - a**4*d*sqrt(a**2 + b**2) - 2*a**3*b*d*sqrt(a**2 + b**
2)*tanh(c/2 + d*x/2) + a**2*b**2*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 -
 a**2*b**2*d*sqrt(a**2 + b**2) - 2*a*b**3*d*sqrt(a**2 + b**2)*tanh(c/2 + d*
x/2)) - a**3*log(tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**4*d*sqr
t(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a**2 + b**2) - 2*a**3*b*d
*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2) + a**2*b**2*d*sqrt(a**2 + b**2)*tanh(c
/2 + d*x/2)**2 - a**2*b**2*d*sqrt(a**2 + b**2) - 2*a*b**3*d*sqrt(a**2 + b**
2)*tanh(c/2 + d*x/2)) + 2*a**2*b*log(tanh(c/2 + d*x/2) - b/a - sqrt(a**2 +
b**2)/a)*tanh(c/2 + d*x/2)/(a**4*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 -
 a**4*d*sqrt(a**2 + b**2) - 2*a**3*b*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)
+ a**2*b**2*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**2*b**2*d*sqrt(a**
2 + b**2) - 2*a*b**3*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)) - 2*a**2*b*log(
tanh(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)*tanh(c/2 + d*x/2)/(a**4*d*sq
rt(a**2 + b**2)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a**2 + b**2) - 2*a**3*b*
d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2) + a**2*b**2*d*sqrt(a**2 + b**2)*tanh(
c/2 + d*x/2)**2 - a**2*b**2*d*sqrt(a**2 + b**2) - 2*a*b**3*d*sqrt(a**2 + b*
*2)*tanh(c/2 + d*x/2)) + 2*a*b*sqrt(a**2 + b**2)/(a**4*d*sqrt(a**2 + b**2)*
tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a**2 + b**2) - 2*a**3*b*d*sqrt(a**2 + b*
*2)*tanh(c/2 + d*x/2) + a**2*b**2*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)**2
- a**2*b**2*d*sqrt(a**2 + b**2) - 2*a*b**3*d*sqrt(a**2 + b**2)*tanh(c/2 + d
*x/2)) + 2*b**2*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)/(a**4*d*sqrt(a**2 + b**
2)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a**2 + b**2) - 2*a**3*b*d*sqrt(a**2 +
 b**2)*tanh(c/2 + d*x/2) + a**2*b**2*d*sqrt(a**2 + b**2)*tanh(c/2 + d*x/2)*
*2 - a**2*b**2*d*sqrt(a**2 + b**2) - 2*a*b**3*d*sqrt(a**2 + b**2)*tanh(c/2
+ d*x/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.75

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \frac{a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}d} - \frac{2\left(ae^{(-dx-c)}+b\right)}{(a^2b+b^3+2\,(a^3+ab^2)e^{(-dx-c)}-(a^2b+b^3)e^{(-2\,dx-2\,c)})d}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c) - a + sqrt(a^2
 + b^2)))/((a^2 + b^2)^(3/2)*d) - 2*(a*e^(-d*x - c) + b)/((a^2*b + b^3 + 2*
(a^3 + a*b^2)*e^(-d*x - c) - (a^2*b + b^3)*e^(-2*d*x - 2*c))*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \frac{\frac{a\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\,(ae^{(dx+c)}-b)}{(a^2+b^2)(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b)}}{d}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +
 2*a + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*e^(d*x + c) - b)/((a^2
+ b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b)))/d
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+b\sinh(c+dx))^2}\,dx = \frac{a\,\ln\left(\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b\,{\left(a^2+b^2\right)}^{3/2}}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,\left(a^2+b^2\right)}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{a\,\ln\left(-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,\left(a^2+b^2\right)}-\frac{2\,a\,\left(b-a\,\mathrm{e}^{c+d\,x}\right)}{b\,{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}} - \frac{\frac{2\,b^2}{d\,\left(a^2\,b+b^3\right)}-\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(a^2\,b+b^3\right)}}{2\,a\,\mathrm{e}^{c+d\,x}-b+b\,\mathrm{e}^{2\,c+2\,d\,x}}$$

```
[In] int(1/(a + b*sinh(c + d*x))^2,x)
```

```
[Out] (a*log((2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2)) - (2*a*exp(c + d*x)
)/(b*(a^2 + b^2))))/(d*(a^2 + b^2)^(3/2)) - (a*log(- (2*a*exp(c + d*x))/(b*
(a^2 + b^2)) - (2*a*(b - a*exp(c + d*x)))/(b*(a^2 + b^2)^(3/2))))/(d*(a^2 +
 b^2)^(3/2)) - ((2*b^2)/(d*(a^2*b + b^3)) - (2*a*b*exp(c + d*x))/(d*(a^2*b
+ b^3)))/(2*a*exp(c + d*x) - b + b*exp(2*c + 2*d*x))
```

3.103 $\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(2a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} - \frac{b\cosh(c+dx)}{2(a^2+b^2)\,d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2(a^2+b^2)^2\,d(a+b\sinh(c+dx))}$$

```
[Out] -(2*a^2-b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(
5/2)/d-1/2*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^2-3/2*a*b*cosh(d*x+c
)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2743, 2833, 12, 2739, 632, 210}

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = -\frac{(2a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{5/2}} - \frac{3ab\cosh(c+dx)}{2d(a^2+b^2)^2(a+b\sinh(c+dx))} - \frac{b\cosh(c+dx)}{2d(a^2+b^2)(a+b\sinh(c+dx))^2}$$

```
[In] Int[(a + b*Sinh[c + d*x])^(-3),x]
```

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2
+ b^2)^(5/2)*d)) - (b*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x])
^2) - (3*a*b*Cosh[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{b\cosh(c+dx)}{2(a^2+b^2)\,d(a+b\sinh(c+dx))^2} - \frac{\int \frac{-2a+b\sinh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx}{2(a^2+b^2)}$$

$$= -\frac{b\cosh(c+dx)}{2(a^2+b^2)\,d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2(a^2+b^2)^2\,d(a+b\sinh(c+dx))} + \frac{\int \frac{2a^2-b^2}{a+b\sinh(c+dx)}\,dx}{2(a^2+b^2)^2}$$

$$= -\frac{b\cosh(c+dx)}{2(a^2+b^2)\,d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2(a^2+b^2)^2\,d(a+b\sinh(c+dx))} + \frac{(2a^2-b^2)\int \frac{1}{a+b\sinh(c+dx)}\,dx}{2(a^2+b^2)^2}$$

$$= -\frac{b\cosh(c+dx)}{2(a^2+b^2)\,d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2(a^2+b^2)^2\,d(a+b\sinh(c+dx))} - \frac{(i(2a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)^2\,d}$$

$$= -\frac{b\cosh(c+dx)}{2(a^2+b^2)\,d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2(a^2+b^2)^2\,d(a+b\sinh(c+dx))} + \frac{(2i(2a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)^2\,d}$$

$$= -\frac{(2a^2-b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} - \frac{b\cosh(c+dx)}{2(a^2+b^2)\,d(a+b\sinh(c+dx))^2} - \frac{3ab\cosh(c+dx)}{2(a^2+b^2)^2\,d(a+b\sinh(c+dx))}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \frac{\frac{2(2a^2-b^2)\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \frac{b\cosh(c+dx)(4a^2+b^2+3ab\sinh(c+dx))}{(a+b\sinh(c+dx))^2}}{2(a^2+b^2)^2\,d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-3),x]
[Out] ((2*(2*a^2 - b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[
-a^2 - b^2] - (b*Cosh[c + d*x]*(4*a^2 + b^2 + 3*a*b*Sinh[c + d*x]))/(a + b*
Sinh[c + d*x])^2)/(2*(a^2 + b^2)^2*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(118) = 236$.

Time = 1.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.20

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+}{d}$
risch	$\frac{2a^2b\,e^{3dx+3c}-b^3e^{3dx+3c}+6\,e^{2dx+2c}a^3-3\,e^{2dx+2c}a\,b^2-10\,e^{dx+c}a^2b-e^{dx+c}b^3+3a\,b^2}{d(a^2+b^2)^2\left(b\,e^{2dx+2c}+2a\,e^{dx+c}-b\right)^2}+\frac{\ln\left(e^{dx+c}+\frac{(a^2+b^2)^{\frac{5}{2}}a-a^6-3a^4}{b(a^2+b^2)}\right.}{(a^2+b^2)^{\frac{5}{2}}d}$

```
[In] int(1/(a+b*sinh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*d*x+1/2*c)^3
-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tanh(1/2*d*x+1/2*c)^
2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tanh(1/2*d*x+1/2*c)+1/2*b*(4
*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/
2*c)-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. $2(120) = 240$.

Time = 0.30 (sec) , antiderivative size = 1347, normalized size of antiderivative = 10.61

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*b^2 + 6*a*b^4 + 2*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^3 + 2*
(2*a^4*b + a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 + a^3*b^2 - a*b^4)*cos
h(d*x + c)^2 + 6*(2*a^5 + a^3*b^2 - a*b^4 + (2*a^4*b + a^2*b^3 - b^5)*cosh(
d*x + c))*sinh(d*x + c)^2 - ((2*a^2*b^2 - b^4)*cosh(d*x + c)^4 + (2*a^2*b^2
 - b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 - b^4 + 4*(2*a^3*b - a*b^3)*cosh(d*x +
c)^3 + 4*(2*a^3*b - a*b^3 + (2*a^2*b^2 - b^4)*cosh(d*x + c))*sinh(d*x + c)^
3 + 2*(4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 - 4*a^2*b^2 + b^
4 + 3*(2*a^2*b^2 - b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b - a*b^3)*cosh(d*x + c)
```

```
)*sinh(d*x + c)^2 - 4*(2*a^3*b - a*b^3)*cosh(d*x + c) - 4*(2*a^3*b - a*b^3
- (2*a^2*b^2 - b^4)*cosh(d*x + c)^3 - 3*(2*a^3*b - a*b^3)*cosh(d*x + c)^2 -
 (4*a^4 - 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*lo
g((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2
+ b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*co
sh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(10*a
^4*b + 11*a^2*b^3 + b^5)*cosh(d*x + c) - 2*(10*a^4*b + 11*a^2*b^3 + b^5 - 3
*(2*a^4*b + a^2*b^3 - b^5)*cosh(d*x + c)^2 - 6*(2*a^5 + a^3*b^2 - a*b^4)*co
sh(d*x + c))*sinh(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^4 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*sinh(d*x + c)^4 + 4
*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^3 + 2*(2*a^8 + 5*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c)^2 + 4*((a^6*b^2 + 3*a^4
*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c) + (a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
a*b^7)*d)*sinh(d*x + c)^3 - 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos
h(d*x + c) + 2*(3*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh(d*x + c)^2
 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c) + (2*a^8 + 5*a
^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d)*sinh(d*x + c)^2 + (a^6*b^2 + 3*a^4*b
^4 + 3*a^2*b^6 + b^8)*d + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d*cosh
(d*x + c)^3 + 3*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cosh(d*x + c)^2 +
 (2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d*cosh(d*x + c) - (a^7*b +
 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(120) = 240$.

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.48

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \frac{(2a^2-b^2)\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{2(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}d} - \frac{3ab^2+(10a^2b+b^3)e^{(-dx-c)}+3(2a^3-ab^2)e^{(-2dx-2c)}-(2a^2b-}{(a^4b^2+2a^2b^4+b^6+4(a^5b+2a^3b^3+ab^5)e^{(-dx-c)}+2(2a^6+3a^4b^2-b^6)e^{(-2dx-2c)}-4(a^5b+2a^3b^3}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2 - b^2)*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x - c
) - a + sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*d) - (3*
a*b^2 + (10*a^2*b + b^3)*e^(-d*x - c) + 3*(2*a^3 - a*b^2)*e^(-2*d*x - 2*c)
- (2*a^2*b - b^3)*e^(-3*d*x - 3*c))/((a^4*b^2 + 2*a^2*b^4 + b^6 + 4*(a^5*b
+ 2*a^3*b^3 + a*b^5)*e^(-d*x - c) + 2*(2*a^6 + 3*a^4*b^2 - b^6)*e^(-2*d*x -
 2*c) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*d*x - 3*c) + (a^4*b^2 + 2*a^2*b
^4 + b^6)*e^(-4*d*x - 4*c))*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \frac{\frac{(2\,a^2-b^2)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\,(2\,a^2be^{(3\,dx+3\,c)}-b^3e^{(3\,dx+3\,c)}+6\,a^3e^{(2\,dx+2\,c)}-3\,ab^2e^{(2\,dx+2\,c)}-10\,a^2be^{(dx+c)}-b^3e^{(dx+c)}+3\,ab}{(a^4+2\,a^2b^2+b^4)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}-b\right)^2}}{2\,d}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 - b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2
*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^
2 + b^2)) + 2*(2*a^2*b*e^(3*d*x + 3*c) - b^3*e^(3*d*x + 3*c) + 6*a^3*e^(2*d
*x + 2*c) - 3*a*b^2*e^(2*d*x + 2*c) - 10*a^2*b*e^(d*x + c) - b^3*e^(d*x + c
) + 3*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c)
- b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(c+dx))^3}\,dx = \int \frac{1}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^3} \,d x$$

```
[In] int(1/(a + b*sinh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x))^3, x)
```

3.104 $\int \frac{1}{(a+b\sinh(c+dx))^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 174

$$\int \frac{1}{(a+b\sinh(c+dx))^4} dx = -\frac{a(2a^2-3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}\,d} - \frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2} - \frac{b(11a^2-4b^2)\cosh(c+dx)}{6(a^2+b^2)^3\,d(a+b\sinh(c+dx))}$$

```
[Out] -a*(2*a^2-3*b^2)*arctanh((b-a*tanh(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(7/2)/d-1/3*b*cosh(d*x+c)/(a^2+b^2)/d/(a+b*sinh(d*x+c))^3-5/6*a*b*cosh(d
*x+c)/(a^2+b^2)^2/d/(a+b*sinh(d*x+c))^2-1/6*b*(11*a^2-4*b^2)*cosh(d*x+c)/(a
^2+b^2)^3/d/(a+b*sinh(d*x+c))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{2743, 2833, 12, 2739, 632, 210\}$

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = -\frac{a(2a^2-3b^2)\,\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{7/2}} - \frac{b(11a^2-4b^2)\cosh(c+dx)}{6d\,(a^2+b^2)^3\,(a+b\sinh(c+dx))} - \frac{5ab\cosh(c+dx)}{6d\,(a^2+b^2)^2\,(a+b\sinh(c+dx))^2} - \frac{b\cosh(c+dx)}{3d\,(a^2+b^2)\,(a+b\sinh(c+dx))^3}$$

```
[In] Int[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((
a^2 + b^2)^(7/2)*d)) - (b*Cosh[c + d*x])/(3*(a^2 + b^2)*d*(a + b*Sinh[c + d
*x])^3) - (5*a*b*Cosh[c + d*x])/(6*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x])^2)
 - (b*(11*a^2 - 4*b^2)*Cosh[c + d*x])/(6*(a^2 + b^2)^3*d*(a + b*Sinh[c + d*
x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3}-\frac{\int\frac{-3a+2b\sinh(c+dx)}{(a+b\sinh(c+dx))^3}\,dx}{3(a^2+b^2)}\\
&= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3}\\
&\quad -\frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2}+\frac{\int\frac{2(3a^2-2b^2)-5ab\sinh(c+dx)}{(a+b\sinh(c+dx))^2}\,dx}{6(a^2+b^2)^2}\\
&= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3}-\frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2}\\
&\quad -\frac{b(11a^2-4b^2)\cosh(c+dx)}{6(a^2+b^2)^3\,d(a+b\sinh(c+dx))}-\frac{\int-\frac{3a(2a^2-3b^2)}{a+b\sinh(c+dx)}\,dx}{6(a^2+b^2)^3}\\
&= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3}-\frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2}\\
&\quad -\frac{b(11a^2-4b^2)\cosh(c+dx)}{6(a^2+b^2)^3\,d(a+b\sinh(c+dx))}+\frac{(a(2a^2-3b^2))\int\frac{1}{a+b\sinh(c+dx)}\,dx}{2(a^2+b^2)^3}\\
&= -\frac{b\cosh(c+dx)}{3(a^2+b^2)\,d(a+b\sinh(c+dx))^3}-\frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2\,d(a+b\sinh(c+dx))^2}\\
&\quad -\frac{b(11a^2-4b^2)\cosh(c+dx)}{6(a^2+b^2)^3\,d(a+b\sinh(c+dx))}\\
&\quad -\frac{(ia(2a^2-3b^2))\,\text{Subst}\left(\int\frac{1}{a-2ibx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)^3\,d}
\end{aligned}
$$

$$= -\frac{b\cosh(c+dx)}{3(a^2+b^2)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2 d(a+b\sinh(c+dx))^2} - \frac{b(11a^2-4b^2)\cosh(c+dx)}{6(a^2+b^2)^3 d(a+b\sinh(c+dx))} + \frac{(2ia(2a^2-3b^2))\,\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2}\,dx, x, -2ib+2a\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)^3 d}$$

$$= -\frac{a(2a^2-3b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2} d} - \frac{b\cosh(c+dx)}{3(a^2+b^2)d(a+b\sinh(c+dx))^3} - \frac{5ab\cosh(c+dx)}{6(a^2+b^2)^2 d(a+b\sinh(c+dx))^2} - \frac{b(11a^2-4b^2)\cosh(c+dx)}{6(a^2+b^2)^3 d(a+b\sinh(c+dx))}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \frac{\frac{6a(2a^2-3b^2)\arctan\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{b\cosh(c+dx)\left(-18a^4-5a^2b^2-2b^4+3ab(-9a^2+b^2)\sinh(c+dx)+(-11a^2b^2+4b^4)\sinh^2(c+dx)\right)}{(a+b\sinh(c+dx))^3}}{6(a^2+b^2)^3 d}$$

```
[In] Integrate[(a + b*Sinh[c + d*x])^(-4),x]
```

```
[Out] ((6*a*(2*a^2 - 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/S
qrt[-a^2 - b^2] + (b*Cosh[c + d*x]*(-18*a^4 - 5*a^2*b^2 - 2*b^4 + 3*a*b*(-9
*a^2 + b^2)*Sinh[c + d*x] + (-11*a^2*b^2 + 4*b^4)*Sinh[c + d*x]^2))/(a + b*
Sinh[c + d*x])^3)/(6*(a^2 + b^2)^3*d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(163) = 326.

Time = 1.44 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.84

method	result
derivativedivides	$-\frac{2\left(-\frac{b^2\left(9a^4+6a^2b^2+2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{b\left(6a^6-27a^4b^2-12a^2b^4-4b^6\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b^2\left(54a^6-21a^4b^2-4a^2b^4-4b^6\right)\tanh(}{3a^3\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\,\mathrm{t}\right.}$
default	$-\frac{2\left(-\frac{b^2\left(9a^4+6a^2b^2+2b^4\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{b\left(6a^6-27a^4b^2-12a^2b^4-4b^6\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b^2\left(54a^6-21a^4b^2-4a^2b^4-4b^6\right)\tanh(}{3a^3\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\,\mathrm{t}\right.}$
risch	$\frac{6a^3b^2\mathrm{e}^{5dx+5c}-9a\,b^4\mathrm{e}^{5dx+5c}+30a^4b\,\mathrm{e}^{4dx+4c}-45a^2b^3\mathrm{e}^{4dx+4c}+44a^5\mathrm{e}^{3dx+3c}-82a^3b^2\mathrm{e}^{3dx+3c}+24a\,b^4\mathrm{e}^{3dx+3c}-102a^4b\,\mathrm{e}^{2}}{3d(a^2+b^2)^3\left(b\,\mathrm{e}^{2dx+2c}+2a\,\mathrm{e}^{dx+c}-b\right)^3}$

```
[In] int(1/(a+b*sinh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
anh(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*a
^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2-
4*a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*d*x+1/2*c)^3+1/a^2*
b*(6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*d*x+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b
^6)*tanh(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2
*b^4+b^6))/(tanh(1/2*d*x+1/2*c)^2*a-2*b*tanh(1/2*d*x+1/2*c)-a)^3+a*(2*a^2-3
*b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2934 vs. $2(165) = 330$.

Time = 0.35 (sec) , antiderivative size = 2934, normalized size of antiderivative = 16.86

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*c
osh(d*x + c)^5 - 6*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*sinh(d*x + c)^5 - 30*(2*
a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^4 - 30*(2*a^6*b - a^4*b^3 - 3*a^
2*b^5 + (2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*
(22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^3 - 4*(22*a^7 -
 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6 + 15*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)*co
sh(d*x + c)^2 + 30*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 12*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c)^2 + 1
```

```
2*(17*a^6*b + 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7 - 5*(2*a^5*b^2 - a^3*b^4 - 3*a
*b^6)*cosh(d*x + c)^3 - 15*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^2
- (22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c))*sinh(d*x + c
)^2 + 3*((2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^6 + (2*a^3*b^3 - 3*a*b^5)*sinh
(d*x + c)^6 - 2*a^3*b^3 + 3*a*b^5 + 6*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)
^5 + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c))*sinh(d
*x + c)^5 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^4 + 3*(8*a^5*b
 - 14*a^3*b^3 + 3*a*b^5 + 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^2 + 10*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^6 - 12*a^4*b^2
+ 9*a^2*b^4)*cosh(d*x + c)^3 + 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4 + 5*(2*a^3
*b^3 - 3*a*b^5)*cosh(d*x + c)^3 + 15*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^
2 + 3*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 - 3*(8*a^5*b - 14*a^3*b^3 +
 3*a*b^5 - 5*(2*a^3*b^3 - 3*a*b^5)*cosh(d*x + c)^4 - 20*(2*a^4*b^2 - 3*a^2*
b^4)*cosh(d*x + c)^3 - 6*(8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^2 -
 4*(4*a^6 - 12*a^4*b^2 + 9*a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a
^4*b^2 - 3*a^2*b^4)*cosh(d*x + c) + 6*(2*a^4*b^2 - 3*a^2*b^4 + (2*a^3*b^3 -
 3*a*b^5)*cosh(d*x + c)^5 + 5*(2*a^4*b^2 - 3*a^2*b^4)*cosh(d*x + c)^4 + 2*(
8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c)^3 + 2*(4*a^6 - 12*a^4*b^2 + 9
*a^2*b^4)*cosh(d*x + c)^2 - (8*a^5*b - 14*a^3*b^3 + 3*a*b^5)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c
)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(
d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c)
+ a)*sinh(d*x + c) - b)) - 30*(4*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(d*x + c)
 - 6*(20*a^5*b^2 + 15*a^3*b^4 - 5*a*b^6 + 5*(2*a^5*b^2 - a^3*b^4 - 3*a*b^6)
*cosh(d*x + c)^4 + 20*(2*a^6*b - a^4*b^3 - 3*a^2*b^5)*cosh(d*x + c)^3 + 2*(
22*a^7 - 19*a^5*b^2 - 29*a^3*b^4 + 12*a*b^6)*cosh(d*x + c)^2 - 4*(17*a^6*b
+ 11*a^4*b^3 - 4*a^2*b^5 + 2*b^7)*cosh(d*x + c))*sinh(d*x + c))/((a^8*b^3 +
 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^6 + (a^8*b^3 + 4
*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*sinh(d*x + c)^6 + 6*(a^9*b^2 + 4
*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c)^5 + 3*(4*a^10*b
+ 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x + c)^4 + 6*((a^8*
b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c) + (a^9*b^2
+ 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d)*sinh(d*x + c)^5 + 4*(2*a^1
1 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d*cosh(d*x + c)^3 + 3*(
5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*x + c)^2 +
10*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh(d*x + c) +
 (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d)*sinh(d*x + c)^
4 - 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*cosh(d*x +
 c)^2 + 4*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d*cosh(d*
x + c)^3 + 15*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cosh
(d*x + c)^2 + 3*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b^11)*d*
cosh(d*x + c) + (2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 - 3*a*b^10)*d
)*sinh(d*x + c)^3 + 6*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10
```

```
)*d*cosh(d*x + c) + 3*(5*(a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^1
1)*d*cosh(d*x + c)^4 + 20*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*
b^10)*d*cosh(d*x + c)^3 + 6*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^
7 - b^11)*d*cosh(d*x + c)^2 + 4*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b
^8 - 3*a*b^10)*d*cosh(d*x + c) - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a
^4*b^7 - b^11)*d)*sinh(d*x + c)^2 - (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^
2*b^9 + b^11)*d + 6*((a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d
*cosh(d*x + c)^5 + 5*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)
*d*cosh(d*x + c)^4 + 2*(4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*b^7 - b
^11)*d*cosh(d*x + c)^3 + 2*(2*a^11 + 5*a^9*b^2 - 10*a^5*b^6 - 10*a^3*b^8 -
3*a*b^10)*d*cosh(d*x + c)^2 - (4*a^10*b + 15*a^8*b^3 + 20*a^6*b^5 + 10*a^4*
b^7 - b^11)*d*cosh(d*x + c) + (a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8
+ a*b^10)*d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \text{Timed out}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. 2(165) = 330.

Time = 0.30 (sec) , antiderivative size = 551, normalized size of antiderivative = 3.17

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \frac{(2\,a^2-3\,b^2)a\log\left(\frac{be^{(-dx-c)}-a-\sqrt{a^2+b^2}}{be^{(-dx-c)}-a+\sqrt{a^2+b^2}}\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}d}$$

$$-\frac{11\,a^2b^3-4\,b^5+15\,(4\,a^3b^2-ab^4)e^{(\cdots}}{3\,(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9+6\,(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8)e^{(-dx-c)}+3\,(4\,a^8b+11\,a^6b^3+9\,a^4b^5+\cdots}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2 - 3*b^2)*a*log((b*e^(-d*x - c) - a - sqrt(a^2 + b^2))/(b*e^(-d*x
 - c) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)*d) - 1/3*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-d*x - c)
+ 6*(17*a^4*b - 6*a^2*b^3 + 2*b^5)*e^(-2*d*x - 2*c) + 2*(22*a^5 - 41*a^3*b^
2 + 12*a*b^4)*e^(-3*d*x - 3*c) - 15*(2*a^4*b - 3*a^2*b^3)*e^(-4*d*x - 4*c)
+ 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*d*x - 5*c))/((a^6*b^3 + 3*a^4*b^5 + 3*a^2*b
^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-d*x - c) + 3*(4*
a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-2*d*x - 2*c) + 4*(2*a^9
```

```
 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*d*x - 3*c) - 3*(4*a^8
*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*d*x - 4*c) + 6*(a^7*b^2
+ 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*d*x - 5*c) - (a^6*b^3 + 3*a^4*b^5 +
3*a^2*b^7 + b^9)*e^(-6*d*x - 6*c))*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(165) = 330$.

Time = 0.30 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.05

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \frac{\frac{3\,(2\,a^3-3\,ab^2)\log\left(\frac{\left|2\,be^{(dx+c)}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{(dx+c)}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\,(6\,a^3b^2e^{(5\,dx+5\,c)}-9\,ab^4e^{(5\,dx+5\,c)}+30\,a^4be^{(4\,dx+4\,c)}-45\,a^2b^3e^{(4\,dx+4\,c)}+44\,a^5e^{(3\,dx+3\,c)}-8}{(a^6-}}{6\,d}$$

```
[In] integrate(1/(a+b*sinh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(6*a^3*b^2*e^(5*d*x + 5*c) - 9*a*b^4*e^(5*d
*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) - 45*a^2*b^3*e^(4*d*x + 4*c) + 44*a^5*
e^(3*d*x + 3*c) - 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a*b^4*e^(3*d*x + 3*c) - 1
02*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2*c) - 12*b^5*e^(2*d*x + 2
*c) + 60*a^3*b^2*e^(d*x + c) - 15*a*b^4*e^(d*x + c) - 11*a^2*b^3 + 4*b^5)/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) -
b)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(c+dx))^4}\,dx = \int \frac{1}{{\left(a+b\,\mathrm{sinh}\left(c+d\,x\right)\right)}^4} \,d x$$

```
[In] int(1/(a + b*sinh(c + d*x))^4,x)
```

```
[Out] int(1/(a + b*sinh(c + d*x))^4, x)
```

3.105 $\int (a+b\sinh(x))^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 179

$$\int (a+b\sinh(x))^{5/2}\,dx = \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i(23a^2-9b^2)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{16ia(a^2+b^2)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{15\sqrt{a+b\sinh(x)}}$$

```
[Out] 2/5*b*cosh(x)*(a+b*sinh(x))^(3/2)+16/15*a*b*cosh(x)*(a+b*sinh(x))^(1/2)+2/1
5*I*(23*a^2-9*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ellipt
icE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/((a+
b*sinh(x))/(a-I*b))^(1/2)-16/15*I*a*(a^2+b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)
/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2
))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used

= {2735, 2832, 2831, 2742, 2740, 2734, 2732}

$$\int (a+b\sinh(x))^{5/2}\,dx = -\frac{16ia(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{15\sqrt{a+b\sinh(x)}} + \frac{2i(23a^2-9b^2)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}$$

```
[In] Int[(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (16*a*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*b*Cosh[x]*(a + b*Sinh[x])^(3/2
))/5 + (((2*I)/15)*(23*a^2 - 9*b^2)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a +
b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (((16*I)/15)*a*(
a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{5}\int\sqrt{a+b\sinh(x)}\left(\frac{1}{2}\left(5a^2-3b^2\right)+4ab\sinh(x)\right)\,dx\\
&= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}\\
&\quad+\frac{4}{15}\int\frac{\frac{1}{4}a\left(15a^2-17b^2\right)+\frac{1}{4}b\left(23a^2-9b^2\right)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx\\
&= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}\\
&\quad+\frac{1}{15}\left(23a^2-9b^2\right)\int\sqrt{a+b\sinh(x)}\,dx-\frac{1}{15}\left(8a\left(a^2+b^2\right)\right)\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx\\
&= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2}\\
&\quad+\frac{\left(\left(23a^2-9b^2\right)\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\
&\quad-\frac{\left(8a\left(a^2+b^2\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{15\sqrt{a+b\sinh(x)}}
\end{aligned}$$

$$= \frac{16}{15}ab\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}b\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i(23a^2-9b^2)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{15\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{16ia(a^2+b^2)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{15\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99

$$\int (a+b\sinh(x))^{5/2}\,dx = \frac{2(23ia^3+23a^2b-9iab^2-9b^3)\,E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}} - 16ia(a^2+b^2)\,\text{Elliptic}}{15\sqrt{a+}}$$

```
[In] Integrate[(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (2*((23*I)*a^3 + 23*a^2*b - (9*I)*a*b^2 - 9*b^3)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (16*I)*a*(a^2 + b
^2)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/
(a - I*b)] + b*Cosh[x]*(22*a^2 - 3*b^2 + 3*b^2*Cosh[2*x] + 28*a*b*Sinh[x]))
/(15*Sqrt[a + b*Sinh[x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 916 vs. 2(203) = 406.

Time = 2.23 (sec) , antiderivative size = 917, normalized size of antiderivative = 5.12

method	result
default	$\frac{\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^3b}{15}+\frac{16i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\text{E}}{15}}{}$

```
[In] int((a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a
)/(I*b+a))^(1/2))*a^3*b+8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-
a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^3+15*(-(a+b*sinh(x))/(I*b-a))^(1/2)
```

```
*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a
+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4+6*(-(a+b*sinh(x))/
(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*
EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2-
9*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))
*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*b^4-23*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/
2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(
-(I*b-a)/(I*b+a))^(1/2))*a^4-14*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))
*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(
I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2+9*(-(a+b*sinh(x))/(I*b-a))^
(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE
((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^4+3*b^4*sinh(x)
^4+14*a*b^3*sinh(x)^3+11*a^2*b^2*sinh(x)^2+3*b^4*sinh(x)^2+14*a*b^3*sinh(x)
+11*a^2*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.59

$$\int (a+b\sinh(x))^{5/2}\,dx =$$

$$-\frac{4\left(\sqrt{2}(a^3+33\,ab^2)\cosh{(x)}^2+2\,\sqrt{2}(a^3+33\,ab^2)\cosh{(x)}\sinh{(x)}+\sqrt{2}(a^3+33\,ab^2)\sinh{(x)}^2\right)\sqrt{b}\text{weierstr}}{}$$

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/90*(4*(sqrt(2)*(a^3 + 33*a*b^2)*cosh(x)^2 + 2*sqrt(2)*(a^3 + 33*a*b^2)*c
osh(x)*sinh(x) + sqrt(2)*(a^3 + 33*a*b^2)*sinh(x)^2)*sqrt(b)*weierstrassPIn
verse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x
) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(23*a^2*b - 9*b^3)*cosh(x)^2 + 2*sq
rt(2)*(23*a^2*b - 9*b^3)*cosh(x)*sinh(x) + sqrt(2)*(23*a^2*b - 9*b^3)*sinh(
x)^2)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b
^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^
2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*b^3*cosh(x)^4 + 3*
b^3*sinh(x)^4 + 22*a*b^2*cosh(x)^3 + 22*a*b^2*cosh(x) + 2*(6*b^3*cosh(x) +
11*a*b^2)*sinh(x)^3 - 3*b^3 - 4*(23*a^2*b - 9*b^3)*cosh(x)^2 + 2*(9*b^3*cos
h(x)^2 + 33*a*b^2*cosh(x) - 46*a^2*b + 18*b^3)*sinh(x)^2 + 2*(6*b^3*cosh(x)
^3 + 33*a*b^2*cosh(x)^2 + 11*a*b^2 - 4*(23*a^2*b - 9*b^3)*cosh(x))*sinh(x))
*sqrt(b*sinh(x) + a))/(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)
```

Sympy [F]

$$\int (a+b\sinh(x))^{5/2}\,dx = \int (a+b\sinh{(x)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))**(5/2),x)
[Out] Integral((a + b*sinh(x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\sinh(x))^{5/2}\,dx = \int (b\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\sinh(x))^{5/2}\,dx = \int (b\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((b*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{5/2}\,dx = \int (a+b\sinh(x))^{5/2}\,dx$$

```
[In] int((a + b*sinh(x))^(5/2),x)
[Out] int((a + b*sinh(x))^(5/2), x)
```

3.106 $\int (a+b\sinh(x))^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 150

$$\int (a+b\sinh(x))^{3/2}\,dx = \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)} + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(a^2+b^2)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3\sqrt{a+b\sinh(x)}}$$

```
[Out] 2/3*b*cosh(x)*(a+b*sinh(x))^(1/2)+8/3*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin
(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(
a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(a^2+b^2)*(sin(1/4*P
i+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/
2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2735, 2831, 2742, 2740, 2734, 2732}

$$\int (a+b\sinh(x))^{3/2}\,dx = -\frac{2i(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{3\sqrt{a+b\sinh(x)}} + \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)} + \frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[In] Int[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x
, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (
((2*I)/3)*(a^2 + b^2)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
b*Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2735

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[1/n, Int[(a + b*S
in[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x],
 x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{3}\int\frac{\frac{1}{2}(3a^2-b^2)+2ab\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx\\
&= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{1}{3}(4a)\int\sqrt{a+b\sinh(x)}\,dx+\frac{1}{3}(-a^2-b^2)\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx\\
&= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{\left(4a\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\
&\quad+\frac{\left((-a^2-b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{3\sqrt{a+b\sinh(x)}}\\
&= \frac{2}{3}b\cosh(x)\sqrt{a+b\sinh(x)}+\frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\
&\quad-\frac{2i(a^2+b^2)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3\sqrt{a+b\sinh(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$
\int(a+b\sinh(x))^{3/2}\,dx=\frac{2b\cosh(x)(a+b\sinh(x))+8a(ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}-2i(a^2+b^2)\,\text{El}}{3\sqrt{a+b\sinh(x)}}
$$

```
[In] Integrate[(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*Cosh[x]*(a + b*Sinh[x]) + 8*a*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((
-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*Ell
ipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*
b)])/(3*Sqrt[a + b*Sinh[x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(178) = 356$.

Time = 1.68 (sec) , antiderivative size = 676, normalized size of antiderivative = 4.51

method	result
default	$\dfrac{\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2b}{3}+\frac{2i\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{Elli}}{3}}{}$

```
[In] int((a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sin
h(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(
I*b+a))^(1/2))*a^2*b+I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a
))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(
1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^3+3*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sin
h(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(
x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3+3*(-(a+b*sinh(x))/(I*b-a))
^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Elliptic
F((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^2-4*(-(a+b*s
inh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a)
)^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*
a^3-4*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I
*b+a))^(1/2))*a*b^2+b^3*sinh(x)^3+a*b^2*sinh(x)^2+b^3*sinh(x)+a*b^2)/b/cosh
(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.75

$$\int (a+b\sinh(x))^{3/2}\,dx = \frac{2\left(\sqrt{2}(a^2-3\,b^2)\cosh(x)+\sqrt{2}(a^2-3\,b^2)\sinh(x)\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8}{}\right.}{}$$

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*(sqrt(2)*(a^2 - 3*b^2)*cosh(x) + sqrt(2)*(a^2 - 3*b^2)*sinh(x))*sqrt
(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^
3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 24*(sqrt(2)*a*b*cosh(x) + sqr
```

```
t(2)*a*b*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8
*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*
a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*(b^2*cosh
(x)^2 + b^2*sinh(x)^2 - 8*a*b*cosh(x) + b^2 + 2*(b^2*cosh(x) - 4*a*b)*sinh(
x))*sqrt(b*sinh(x) + a))/(b*cosh(x) + b*sinh(x))
```

Sympy [F]

$$\int (a+b\sinh(x))^{3/2}\,dx=\int (a+b\sinh{(x)})^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))**(3/2),x)
[Out] Integral((a + b*sinh(x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\sinh(x))^{3/2}\,dx=\int (b\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\sinh(x))^{3/2}\,dx=\int (b\sinh{(x)}+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((b*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{3/2}\,dx=\int (a+b\sinh(x))^{3/2}\,dx$$

```
[In] int((a + b*sinh(x))^(3/2),x)
[Out] int((a + b*sinh(x))^(3/2), x)
```

3.107 $\int \sqrt{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \sqrt{a+b\sinh(x)}\,dx=\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/((a+b*sinh(x))/(a-I
*b))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2734, 2732}

$$\int \sqrt{a+b\sinh(x)}\,dx=\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[In] Int[Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/Sqrt
[(a + b*Sinh[x])/(a - I*b)]
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+b\sinh(x)}\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

$$= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int\sqrt{a+b\sinh(x)}\,dx = \frac{2(ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

```
[In] Integrate[Sqrt[a + b*Sinh[x]],x]
[Out] (2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(78) = 156.

Time = 1.92 (sec) , antiderivative size = 262, normalized size of antiderivative = 4.37

method	result
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\left(i\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-}{ib+}}\right.\right.}{b\cosh(x)\sqrt{a+b\sinh(x)}}$
risch	$\sqrt{2}\,\sqrt{(b\,e^{2x}+2\,e^{x}a-b)\,e^{-x}}+\frac{\left(\frac{4a\left(a+\sqrt{a^2+b^2}\right)\sqrt{\frac{\left(e^x+\frac{a+\sqrt{a^2+b^2}}{b}\right)b}{a+\sqrt{a^2+b^2}}}\sqrt{\frac{e^x-\frac{-a+\sqrt{a^2+b^2}}{b}}{-\frac{a+\sqrt{a^2+b^2}}{b}-\frac{-a+\sqrt{a^2+b^2}}{b}}}\sqrt{-\frac{e^x b}{a+\sqrt{a^2+b^2}}}\ \mathrm{EllipticF}}{b\sqrt{e^{3x}b+2\,e^{2x}a-e^x b}}\right.}{}$

```
[In] int((a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)/b*(I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-
(I*b-a)/(I*b+a))^(1/2))*b-I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b
-a)/(I*b+a))^(1/2))*b+EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I
*b+a))^(1/2))*a-EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))
^(1/2))*a)/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.88

$$\int \sqrt{a+b\sinh(x)}\,dx$$
$$=\frac{2\left(\sqrt{2}a\sqrt{b}\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)-3\,\sqrt{2}b^{\frac{3}{2}}\,\mathrm{weierstrassZeta}\left(\right.\right.}{}$$

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(
8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 3*sqrt(2)*
b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^
3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3
, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*sqrt(b*sinh(x) + a)*b)/b
```

Sympy [F]

$$\int \sqrt{a + b\sinh(x)}\, dx = \int \sqrt{a + b\sinh{(x)}}\, dx$$

```
[In] integrate((a+b*sinh(x))**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a + b\sinh(x)}\, dx = \int \sqrt{b\sinh{(x)} + a}\, dx$$

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\sinh(x)}\, dx = \int \sqrt{b\sinh{(x)} + a}\, dx$$

```
[In] integrate((a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b\sinh(x)}\, dx = \int \sqrt{a + b\sinh{(x)}}\, dx$$

```
[In] int((a + b*sinh(x))^(1/2),x)
[Out] int((a + b*sinh(x))^(1/2), x)
```

3.108 $\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a+b*sinh
(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2742, 2740}

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{\sqrt{a+b\sinh(x)}}$$

```
[In] Int[1/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a -
 I*b)])/Sqrt[a + b*Sinh[x]]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{\sqrt{a+b\sinh(x)}}$$

$$= \frac{2i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{\sqrt{a+b\sinh(x)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/Sqrt[a + b*Sinh[x]]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.08

method	result	size
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)}{b\cosh(x)\sqrt{a+b\sinh(x)}}$	125

```
[In] int(1/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \frac{2\,\sqrt{2}\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},\,-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\,\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)}{\sqrt{b}}$$

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b
^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)/sqrt(b)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\sinh{(x)}}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\sinh{(x)}+a}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{b\sinh\left(x\right)+a}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

```
[In] int(1/(a + b*sinh(x))^(1/2),x)
[Out] int(1/(a + b*sinh(x))^(1/2), x)
```

3.109 $\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 94

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx=-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[Out] -2*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2
)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/
2))*(a+b*sinh(x))^(1/2)/(a^2+b^2)/((a+b*sinh(x))/(a-I*b))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2743, 21, 2734, 2732}

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx=-\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}}+\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[In] Int[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*EllipticE[Pi/4 -
(I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)*Sqrt[(a + b*Sin
h[x])/(a - I*b)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} - \frac{2\int \frac{-\frac{a}{2}-\frac{1}{2}b\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2} \\
&= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{\int \sqrt{a+b\sinh(x)}\,dx}{a^2+b^2} \\
&= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} \\
&= -\frac{2b\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \frac{-2b\cosh(x)+2(ia+b)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

```
[In] Integrate[(a + b*Sinh[x])^(-3/2),x]
```

```
[Out] (-2*b*Cosh[x] + 2*(I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b
)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(110) = 220.

Time = 1.30 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.85

method	result
default	$\frac{2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2+2\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}}{}$

```
[In] int(1/(a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x)
)*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a^2+(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)
*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*b^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I
*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a
))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-s
inh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sin
h(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^2-b^2*sinh(x)^2-b^2)/(a^2+
b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.33

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2\left(\left(\sqrt{2}ab\cosh\left(x\right)^{2}+\sqrt{2}ab\sinh\left(x\right)^{2}+2\sqrt{2}a^{2}\cosh\left(x\right)-\sqrt{2}ab+2\left(\sqrt{2}ab\cosh\left(x\right)+\sqrt{2}a^{2}\right)\sinh\left(x\right)\right)\sqrt{b}\mathrm{w}\right.}{}$$

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
[Out] -2/3*((sqrt(2)*a*b*cosh(x)^2 + sqrt(2)*a*b*sinh(x)^2 + 2*sqrt(2)*a^2*cosh(x
) - sqrt(2)*a*b + 2*(sqrt(2)*a*b*cosh(x) + sqrt(2)*a^2)*sinh(x))*sqrt(b)*we
ierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3
*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 3*(sqrt(2)*b^2*cosh(x)^2 + sqrt(2)*
b^2*sinh(x)^2 + 2*sqrt(2)*a*b*cosh(x) - sqrt(2)*b^2 + 2*(sqrt(2)*b^2*cosh(x
) + sqrt(2)*a*b)*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2,
-8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -
8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 6*(
b^2*cosh(x)^2 + b^2*sinh(x)^2 + a*b*cosh(x) + (2*b^2*cosh(x) + a*b)*sinh(x)
)*sqrt(b*sinh(x) + a))/(a^2*b^2 + b^4 - (a^2*b^2 + b^4)*cosh(x)^2 - (a^2*b^
2 + b^4)*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 + (a^2*b^
2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{\left(a+b\sinh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))**(3/2),x)
[Out] Integral((a + b*sinh(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{\left(b\sinh\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((b*sinh(x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\sinh(x))^{3/2}}\,dx$$

```
[In] int(1/(a + b*sinh(x))^(3/2),x)
[Out] int(1/(a + b*sinh(x))^(3/2), x)
```

3.110 $\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 197

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

```
[Out] -2/3*b*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-8/3*a*b*cosh(x)/(a^2+b^2)^2/(a
+b*sinh(x))^(1/2)+8/3*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)
*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/
2)/(a^2+b^2)^2/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(sin(1/4*Pi+1/2*I*x)^2)^
(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))
^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/(a^2+b^2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {2743, 2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{8ab\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} - \frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{2i\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{3(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{8ia\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[In] Int[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (-2*b*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (8*a*b*Cosh[x])/(3*(
a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((8*I)/3)*a*EllipticE[Pi/4 - (I/2)*x,
(2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/((a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/3)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a
+ b*Sinh[x])/(a - I*b)])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{2\int \frac{-\frac{3a}{2}+\frac{1}{2}b\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3(a^2+b^2)} \\
&= -\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{4\int \frac{\frac{1}{4}(3a^2-b^2)+ab\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{3(a^2+b^2)^2} \\
&= -\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} \\
&\quad + \frac{(4a)\int \sqrt{a+b\sinh(x)}\,dx}{3(a^2+b^2)^2} - \frac{\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{3(a^2+b^2)} \\
&= -\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} \\
&\quad + \frac{\left(4a\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{3(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{\sqrt{\frac{a+b\sinh(x)}{a-ib}}\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{3(a^2+b^2)\sqrt{a+b\sinh(x)}} \\
&= -\frac{2b\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{8ab\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} \\
&\quad + \frac{8iaE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3(a^2+b^2)\sqrt{a+b\sinh(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \frac{\frac{8iaE\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)(a+b\sinh(x))^2}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - 2i(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)(a+b\sin}{3\left(a^2+b^2\right)^2(a+b\sinh(x))^{3/}}$$

```
[In] Integrate[(a + b*Sinh[x])^(-5/2),x]
```

```
[Out] (((8*I)*a*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])
^2)/Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(a^2 + b^2)*EllipticF[(Pi - (2*
I)*x)/4, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*
b)] - 2*b*Cosh[x]*(5*a^2 + b^2 + 4*a*b*Sinh[x]))/(3*(a^2 + b^2)^2*(a + b*Si
nh[x])^(3/2))
```

Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{\cosh(x)^2(a+b\sinh(x))}\left(-\frac{2\sqrt{\cosh(x)^2(a+b\sinh(x))}}{3b\left(a^2+b^2\right)\left(\sinh(x)+\frac{a}{b}\right)^2}-\frac{8b\cosh(x)^2 a}{3\left(a^2+b^2\right)^2\sqrt{\cosh(x)^2(a+b\sinh(x))}}+\frac{2\left(3a^2-b^2\right)\left(\frac{a}{b}-i\right)\sqrt{\frac{-a-b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))}{ib+a}}}{\left(3a^4+6a^2b^2+3b^4\right.}}{}$

```
[In] int(1/(a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*(-2/3/b/(a^2+b^2)*(cosh(x)^2*(a+b*sinh(x)))
^(1/2)/(sinh(x)+a/b)^2-8/3*b*cosh(x)^2/(a^2+b^2)^2*a/(cosh(x)^2*(a+b*sinh(x
)))^(1/2)+2*(3*a^2-b^2)/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-a-b*sinh(x))/(I*
b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(co
sh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I
*b)/(I*b+a))^(1/2))+8/3*a*b/(a^2+b^2)^2*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1
/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*
(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(1/2),((a
-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(
I*b+a))^(1/2))))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1291, normalized size of antiderivative = 6.55

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((sqrt(2)*(a^2*b^2 - 3*b^4)*cosh(x)^4 + sqrt(2)*(a^2*b^2 - 3*b^4)*sinh(
x)^4 + 4*sqrt(2)*(a^3*b - 3*a*b^3)*cosh(x)^3 + 4*(sqrt(2)*(a^2*b^2 - 3*b^4)
*cosh(x) + sqrt(2)*(a^3*b - 3*a*b^3))*sinh(x)^3 + 2*sqrt(2)*(2*a^4 - 7*a^2*
b^2 + 3*b^4)*cosh(x)^2 + 2*(3*sqrt(2)*(a^2*b^2 - 3*b^4)*cosh(x)^2 + 6*sqrt(
2)*(a^3*b - 3*a*b^3)*cosh(x) + sqrt(2)*(2*a^4 - 7*a^2*b^2 + 3*b^4))*sinh(x)
^2 - 4*sqrt(2)*(a^3*b - 3*a*b^3)*cosh(x) + 4*(sqrt(2)*(a^2*b^2 - 3*b^4)*cos
h(x)^3 + 3*sqrt(2)*(a^3*b - 3*a*b^3)*cosh(x)^2 + sqrt(2)*(2*a^4 - 7*a^2*b^2
 + 3*b^4)*cosh(x) - sqrt(2)*(a^3*b - 3*a*b^3))*sinh(x) + sqrt(2)*(a^2*b^2 -
 3*b^4))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3
+ 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 12*(sqrt(2)*a*b^
3*cosh(x)^4 + sqrt(2)*a*b^3*sinh(x)^4 + 4*sqrt(2)*a^2*b^2*cosh(x)^3 - 4*sqr
t(2)*a^2*b^2*cosh(x) + sqrt(2)*a*b^3 + 4*(sqrt(2)*a*b^3*cosh(x) + sqrt(2)*a
^2*b^2)*sinh(x)^3 + 2*sqrt(2)*(2*a^3*b - a*b^3)*cosh(x)^2 + 2*(3*sqrt(2)*a*
b^3*cosh(x)^2 + 6*sqrt(2)*a^2*b^2*cosh(x) + sqrt(2)*(2*a^3*b - a*b^3))*sinh
(x)^2 + 4*(sqrt(2)*a*b^3*cosh(x)^3 + 3*sqrt(2)*a^2*b^2*cosh(x)^2 - sqrt(2)*
a^2*b^2 + sqrt(2)*(2*a^3*b - a*b^3)*cosh(x))*sinh(x))*sqrt(b)*weierstrassZe
ta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInvers
e(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) +
3*b*sinh(x) + 2*a)/b)) - 6*(4*a*b^3*cosh(x)^4 + 4*a*b^3*sinh(x)^4 + (13*a^2
*b^2 + b^4)*cosh(x)^3 + (16*a*b^3*cosh(x) + 13*a^2*b^2 + b^4)*sinh(x)^3 + 4
*(2*a^3*b - a*b^3)*cosh(x)^2 + (24*a*b^3*cosh(x)^2 + 8*a^3*b - 4*a*b^3 + 3*
(13*a^2*b^2 + b^4)*cosh(x))*sinh(x)^2 - (3*a^2*b^2 - b^4)*cosh(x) + (16*a*b
^3*cosh(x)^3 - 3*a^2*b^2 + b^4 + 3*(13*a^2*b^2 + b^4)*cosh(x)^2 + 8*(2*a^3*
b - a*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(a^4*b^3 + 2*a^2*b^5 + b^
7 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*sin
h(x)^4 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2 + 2*a^3*b^4
 + a*b^6 + (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b + 3*
a^4*b^3 - b^7)*cosh(x)^2 + 2*(2*a^6*b + 3*a^4*b^3 - b^7 + 3*(a^4*b^3 + 2*a^
2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^2
 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cosh(x) - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6
 - (a^4*b^3 + 2*a^2*b^5 + b^7)*cosh(x)^3 - 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*
cosh(x)^2 - (2*a^6*b + 3*a^4*b^3 - b^7)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\sinh{(x)})^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))**(5/2),x)
[Out] Integral((a + b*sinh(x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(x)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(b\sinh{(x)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((b*sinh(x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\sinh{(x)})^{5/2}}\,dx$$

```
[In] int(1/(a + b*sinh(x))^(5/2),x)
[Out] int(1/(a + b*sinh(x))^(5/2), x)
```

3.111 $\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 128

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2ia\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a
-I*b))^(1/2)-2*I*a*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*Ellipt
icF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^
(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\frac{2i\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2ia\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

```
[In] Int[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*S
qrt[(a + b*Sinh[x])/(a - I*b)]) - ((2*I)*a*EllipticF[Pi/4 - (I/2)*x, (2*b)/
(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \sqrt{a+b\sinh(x)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b} \\
&= \frac{\sqrt{a+b\sinh(x)}\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{\left(a\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{b\sqrt{a+b\sinh(x)}}
\end{aligned}
$$

$$= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2ia\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$
$$= \frac{2\left((ia+b)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)-ia\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[In] Integrate[Sinh[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (2*((I*a + b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - I*a*Ellip
ticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[x])/(a - I*b
)])/(b*Sqrt[a + b*Sinh[x]])
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.70

method	result
default	$\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\left(i\,\text{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\text{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-}{ib+}}\right.\right.}{b^2\cosh(x)\sqrt{a+b\sinh(x)}}$
risch	$\frac{\left(b\,e^{2x}+2\,e^{x}a-b\right)\sqrt{2}\,e^{-x}}{b\sqrt{\left(b\,e^{2x}+2\,e^{x}a-b\right)e^{-x}}}+\frac{\left(-\frac{4\left(b\,e^{2x}+2\,e^{x}a-b\right)}{b\sqrt{\left(b\,e^{2x}+2\,e^{x}a-b\right)e^{x}}}+\frac{4\left(a+\sqrt{a^2+b^2}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)b}{a+\sqrt{a^2+b^2}}}\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^2+b^2}}{b}}{-\frac{a+\sqrt{a^2+b^2}}{b}-\frac{-a+\sqrt{a^2+b^2}}{b}}}\sqrt{-\frac{e^{x}b}{a+\sqrt{a^2}}}}{}\right.}{}$

```
[In] int(sinh(x)/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+
sinh(x))*b/(I*b-a))^(1/2)*(I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*
b-a)/(I*b+a))^(1/2))*b-I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)
```

```
/(I*b+a))^(1/2))*b+EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.36

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\left(2\sqrt{2}a\sqrt{b}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+3\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZe}\right.}{}$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/2
7*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqrt(
2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)
/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*sqrt(b*sinh(x) + a)*b)/b
^2
```

Sympy [F]

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(sinh(x)/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{b\sinh(x)+a}}\,dx$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sinh\left(x\right)}{\sqrt{b\sinh\left(x\right)+a}}\,dx$$

```
[In] integrate(sinh(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sinh(x)/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\sinh(x)}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

```
[In] int(sinh(x)/(a + b*sinh(x))^(1/2),x)
[Out] int(sinh(x)/(a + b*sinh(x))^(1/2), x)
```

3.112 $\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 112

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\frac{64a^3(7iA+5B)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

```
[Out] 2/35*a*(7*I*A+5*B)*cosh(x)*(a+I*a*sinh(x))^(3/2)+2/7*B*cosh(x)*(a+I*a*sinh(
x))^(5/2)+64/105*a^3*(7*I*A+5*B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+16/105*a^2*(
7*I*A+5*B)*cosh(x)*(a+I*a*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2830, 2726, 2725}

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\frac{64a^3(5B+7iA)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(5B+7iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(5B+7iA)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}$$

```
[In] Int[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (64*a^3*((7*I)*A + 5*B)*Cosh[x])/(105*Sqrt[a + I*a*Sinh[x]]) + (16*a^2*((7*
I)*A + 5*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/105 + (2*a*((7*I)*A + 5*B)*Cosh[
x]*(a + I*a*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(5/2))/7
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}+\frac{1}{7}(7A-5iB)\int(a+ia\sinh(x))^{5/2}\,dx\\
&= \frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2}\\
&\quad+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}+\frac{1}{35}(8a(7A-5iB))\int(a+ia\sinh(x))^{3/2}\,dx\\
&= \frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2}\\
&\quad+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}+\frac{1}{105}\left(32a^2(7A-5iB)\right)\int\sqrt{a+ia\sinh(x)}\,dx\\
&= \frac{64a^3(7iA+5B)\cosh(x)}{105\sqrt{a+ia\sinh(x)}}+\frac{16}{105}a^2(7iA+5B)\cosh(x)\sqrt{a+ia\sinh(x)}\\
&\quad+\frac{2}{35}a(7iA+5B)\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+ia\sinh(x))^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx = \frac{a^2\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}(1246iA+1040B+(-42iA-120B)\cosh(2x)+}{210\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

```
[In] Integrate[(a + I*a*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
[Out] (a^2*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((1246*I)*A + 1040*B +
 ((-42*I)*A - 120*B)*Cosh[2*x] + (-392*A + (505*I)*B)*Sinh[x] - (15*I)*B*Si
nh[3*x]))/(210*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [F]

$$\int (a+ia\sinh(x))^{\frac{5}{2}}(A+B\sinh(x))\,dx$$

```
[In] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
[Out] int((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx =$$

$$-\frac{1}{420}\left(15\,Ba^2e^{(7\,x)}+21\,(2\,A-5i\,B)a^2e^{(6\,x)}+35\,(-10i\,A-11\,B)a^2e^{(5\,x)}-525\,(4\,A-3i\,B)a^2e^{(4\,x)}+525\,(-\right.$$

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
[Out] -1/420*(15*B*a^2*e^(7*x) + 21*(2*A - 5*I*B)*a^2*e^(6*x) + 35*(-10*I*A - 11*
B)*a^2*e^(5*x) - 525*(4*A - 3*I*B)*a^2*e^(4*x) + 525*(-4*I*A - 3*B)*a^2*e^(
3*x) - 35*(10*A - 11*I*B)*a^2*e^(2*x) + 21*(2*I*A + 5*B)*a^2*e^x - 15*I*B*a
^2)*sqrt(1/2*I*a*e^(-x))*e^(-3*x)
```

Sympy [F(-1)]

Timed out.

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\text{Timed out}$$

```
[In] integrate((a+I*a*sinh(x))**(5/2)*(A+B*sinh(x)),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)(i\,a\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)(i\,a\sinh{(x)}+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+I*a*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\ \left(a+a\sinh(x)\ 1\mathrm{i}\right)^{5/2}\,dx$$

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2),x)
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(5/2), x)
```

3.113 $\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\frac{8a^2(5iA+3B)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

```
[Out] 2/5*B*cosh(x)*(a+I*a*sinh(x))^(3/2)+8/15*a^2*(5*I*A+3*B)*cosh(x)/(a+I*a*sin
h(x))^(1/2)+2/15*a*(5*I*A+3*B)*cosh(x)*(a+I*a*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2830, 2726, 2725}

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\frac{8a^2(3B+5iA)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(3B+5iA)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}$$

```
[In] Int[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (8*a^2*((5*I)*A + 3*B)*Cosh[x])/(15*Sqrt[a + I*a*Sinh[x]]) + (2*a*((5*I)*A
+ 3*B)*Cosh[x]*Sqrt[a + I*a*Sinh[x]])/15 + (2*B*Cosh[x]*(a + I*a*Sinh[x])^(
3/2))/5
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2, 0]
```

Rule 2726

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[a*((2*n - 1)/n),
Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a
^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}+\frac{1}{5}(5A-3iB)\int(a+ia\sinh(x))^{3/2}\,dx\\ &= \frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}\\ &\quad+\frac{1}{15}(4a(5A-3iB))\int\sqrt{a+ia\sinh(x)}\,dx\\ &= \frac{8a^2(5iA+3B)\cosh(x)}{15\sqrt{a+ia\sinh(x)}}+\frac{2}{15}a(5iA+3B)\cosh(x)\sqrt{a+ia\sinh(x)}\\ &\quad+\frac{2}{5}B\cosh(x)(a+ia\sinh(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int(a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = -\frac{a\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}(-50iA-39B+3B\cosh(2x)+2(5A-9iB)\sinh(x))}{15\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

```
[In] Integrate[(a + I*a*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] -1/15*(a*(Cosh[x/2] - I*Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*((-50*I)*A - 39*B
+ 3*B*Cosh[2*x] + 2*(5*A - (9*I)*B)*Sinh[x]))/(Cosh[x/2] + I*Sinh[x/2])
```

Maple [F]

$$\int (a+ia\sinh(x))^{\frac{3}{2}}(A+B\sinh(x))\,dx$$

[In] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)

[Out] int((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x)

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\frac{1}{30}\left(3iBae^{(5x)}-5(-2iA-3B)ae^{(4x)}+30(3A-2iB)ae^{(3x)}-30(-3iA-2B)ae^{(2x)}+\right.$$

[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")

[Out] 1/30*(3*I*B*a*e^(5*x) - 5*(-2*I*A - 3*B)*a*e^(4*x) + 30*(3*A - 2*I*B)*a*e^(3*x) - 30*(-3*I*A - 2*B)*a*e^(2*x) + 5*(2*A - 3*I*B)*a*e^x - 3*B*a)*sqrt(1/2*I*a*e^(-x))*e^(-2*x)

Sympy [F]

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\int (ia(\sinh(x)-i))^{\frac{3}{2}}(A+B\sinh(x))\,dx$$

[In] integrate((a+I*a*sinh(x))**(3/2)*(A+B*sinh(x)),x)

[Out] Integral((I*a*(sinh(x) - I))**(3/2)*(A + B*sinh(x)), x)

Maxima [F]

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx=\int (B\sinh(x)+A)(ia\sinh(x)+a)^{\frac{3}{2}}\,dx$$

[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")

[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)

Giac [F]

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (B\sinh(x)+A)(i\,a\sinh(x)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+I*a*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*(I*a*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+ia\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (A+B\sinh(x))\ (a+a\sinh(x)\ 1\mathrm{i})^{3/2}\,dx$$

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2),x)
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(3/2), x)
```

3.114 $\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 20, antiderivative size = 48

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx=\frac{2a(3iA+B)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

```
[Out] 2/3*a*(3*I*A+B)*cosh(x)/(a+I*a*sinh(x))^(1/2)+2/3*B*cosh(x)*(a+I*a*sinh(x))
^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2830, 2725}

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx=\frac{2a(B+3iA)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}$$

```
[In] Int[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*a*((3*I)*A + B)*Cosh[x])/(3*Sqrt[a + I*a*Sinh[x]]) + (2*B*Cosh[x]*Sqrt[a
 + I*a*Sinh[x]])/3
```

Rule 2725

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)}+\frac{1}{3}(3A-iB)\int\sqrt{a+ia\sinh(x)}\,dx \\ &= \frac{2a(3iA+B)\cosh(x)}{3\sqrt{a+ia\sinh(x)}}+\frac{2}{3}B\cosh(x)\sqrt{a+ia\sinh(x)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int\sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx = \frac{2\left(i\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\sqrt{a+ia\sinh(x)}(3A-2iB+B\sinh(x))}{3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

```
[In] Integrate[Sqrt[a + I*a*Sinh[x]]*(A + B*Sinh[x]),x]
[Out] (2*(I*Cosh[x/2] + Sinh[x/2])*Sqrt[a + I*a*Sinh[x]]*(3*A - (2*I)*B + B*Sinh[
x]))/(3*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [F]

$$\int\sqrt{a+ia\sinh\left(x\right)}\left(A+B\sinh\left(x\right)\right)dx$$

```
[In] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
[Out] int((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx$$
$$=\frac{1}{3}\left(Be^{(3\,x)}+3\,(2\,A-i\,B)e^{(2\,x)}-3\,(-2i\,A-B)e^{x}-i\,B\right)\sqrt{\frac{1}{2}i\,ae^{(-x)}}e^{(-x)}$$

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
[Out] 1/3*(B*e^(3*x) + 3*(2*A - I*B)*e^(2*x) - 3*(-2*I*A - B)*e^x - I*B)*sqrt(1/2
*I*a*e^(-x))*e^(-x)
```

Sympy [F]

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx=\int \sqrt{ia\left(\sinh\left(x\right)-i\right)}\left(A+B\sinh\left(x\right)\right)\,dx$$

```
[In] integrate((a+I*a*sinh(x))**(1/2)*(A+B*sinh(x)),x)
[Out] Integral(sqrt(I*a*(sinh(x) - I))*(A + B*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx=\int \left(B\sinh\left(x\right)+A\right)\sqrt{i\,a\sinh\left(x\right)+a}\,dx$$

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx=\int \left(B\sinh\left(x\right)+A\right)\sqrt{i\,a\sinh\left(x\right)+a}\,dx$$

```
[In] integrate((a+I*a*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*sqrt(I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+ia\sinh(x)}(A+B\sinh(x))\,dx = \int (A+B\sinh(x))\ \sqrt{a+a\sinh(x)\ 1\mathrm{i}}\,dx$$

```
[In] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2),x)
[Out] int((A + B*sinh(x))*(a + a*sinh(x)*1i)^(1/2), x)
```

3.115 $\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = Bx - \frac{(iA+B)\cosh(x)}{i+\sinh(x)}$$

```
[Out] B*x-(I*A+B)*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2814, 2727}

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = Bx - \frac{(B+iA)\cosh(x)}{\sinh(x)+i}$$

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x]),x]
```

```
[Out] B*x - ((I*A + B)*Cosh[x])/(I + Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = Bx - (-A + iB)\int \frac{1}{i + \sinh(x)}\,dx$$
$$= Bx - \frac{(iA + B)\cosh(x)}{i + \sinh(x)}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\,dx = \cosh(x)\left(\frac{2iB\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)}{\sqrt{\cosh^2(x)}} - \frac{iA + B}{i + \sinh(x)}\right)$$

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x]),x]
[Out] Cosh[x]*(((2*I)*B*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]])/Sqrt[Cosh[x]^2] - (I
*A + B)/(I + Sinh[x]))
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result	size
risch	$Bx - \frac{2A}{e^{x}+i} + \frac{2iB}{e^{x}+i}$	26
parallelrisch	$\frac{iBx+x\tanh\left(\frac{x}{2}\right)B-2iA-2B}{\tanh\left(\frac{x}{2}\right)+i}$	31
default	$B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) - \frac{2i(-iB+A)}{\tanh\left(\frac{x}{2}\right)+i} - B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	39

```
[In] int((A+B*sinh(x))/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] B*x-2/(exp(x)+I)*A+2*I/(exp(x)+I)*B
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\,dx = \frac{Bxe^x + i\,Bx - 2\,A + 2i\,B}{e^x + i}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="fricas")
[Out] (B*x*e^x + I*B*x - 2*A + 2*I*B)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\,dx = Bx + \frac{-2A + 2iB}{e^x + i}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x)
[Out] B*x + (-2*A + 2*I*B)/(exp(x) + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\,dx = B\left(x + \frac{2i}{e^{(-x)} - i}\right) - \frac{2\,A}{e^{(-x)} - i}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="maxima")
[Out] B*(x + 2*I/(e^(-x) - I)) - 2*A/(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{A + B\sinh(x)}{i + \sinh(x)}\,dx = Bx - \frac{2\,(A - i\,B)}{e^x + i}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x)),x, algorithm="giac")
[Out] B*x - 2*(A - I*B)/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{A+B\sinh(x)}{i+\sinh(x)}\,dx = B\,x-\frac{2\,A-B\,2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i),x)
[Out] B*x - (2*A - B*2i)/(exp(x) + 1i)
```

3.116 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx=-\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}-\frac{(A+2iB)\cosh(x)}{3(i+\sinh(x))}$$

```
[Out] -1/3*(I*A+B)*cosh(x)/(I+sinh(x))^2-1/3*(A+2*I*B)*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2829, 2727}

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx=-\frac{(A+2iB)\cosh(x)}{3(\sinh(x)+i)}-\frac{(B+iA)\cosh(x)}{3(\sinh(x)+i)^2}$$

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
```

```
[Out] -1/3*((I*A + B)*Cosh[x])/(I + Sinh[x])^2 - ((A + (2*I)*B)*Cosh[x])/(3*(I +
Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
```

```
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}+\frac{1}{3}(-iA+2B)\int\frac{1}{i+\sinh(x)}\,dx\\&= -\frac{(iA+B)\cosh(x)}{3(i+\sinh(x))^2}-\frac{(A+2iB)\cosh(x)}{3(i+\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int\frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx=\frac{\cosh(x)(-2iA+B-(A+2iB)\sinh(x))}{3(i+\sinh(x))^2}$$

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^2,x]
[Out] (Cosh[x]*((-2*I)*A + B - (A + (2*I)*B)*Sinh[x]))/(3*(I + Sinh[x])^2)
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{2\left(3A\,\mathrm{e}^{x}+3iB\,\mathrm{e}^{x}+3B\,\mathrm{e}^{2x}+iA-2B\right)}{3\left(\mathrm{e}^{x}+i\right)^{3}}$	36
default	$-\frac{-2iA-2B}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2A}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2(2iB-2A)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}$	52
parallelrisch	$\frac{(3iA-3B)\cosh(2x)+(-iB+A)\sinh(2x)+(-2iB-10A)\sinh(x)-3iA+3B}{-3i\sinh(2x)+12i\sinh(x)+6\cosh(x)+3\cosh(2x)-9}$	71

```
[In] int((A+B*sinh(x))/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2/3*(3*A*exp(x)+3*I*B*exp(x)+3*B*exp(x)^2+I*A-2*B)/(exp(x)+I)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,(A+i\,B)e^{x}+i\,A-2\,B\right)}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*B*e^(2*x) + 3*(A + I*B)*e^x + I*A - 2*B)/(e^(3*x) + 3*I*e^(2*x) - 3
*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = \frac{-2iA-6Be^{2x}+4B+(-6A-6iB)\,e^{x}}{3e^{3x}+9ie^{2x}-9e^{x}-3i}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**2,x)
```

```
[Out] (-2*I*A - 6*B*exp(2*x) + 4*B + (-6*A - 6*I*B)*exp(x))/(3*exp(3*x) + 9*I*exp
(2*x) - 9*exp(x) - 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.28

$$\begin{aligned}&\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx\\&= -\frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}-\frac{i}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}\right)\\&\quad-\frac{2}{3}\,B\left(\frac{3i\,e^{(-x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+\frac{2}{3\,e^{(-x)}+3i\,e^{(-2\,x)}-e^{(-3\,x)}-i}\right)\end{aligned}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*A*(3*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) - I/(3*e^(-x) + 3
*I*e^(-2*x) - e^(-3*x) - I)) - 2/3*B*(3*I*e^(-x)/(3*e^(-x) + 3*I*e^(-2*x) -
 e^(-3*x) - I) - 3*e^(-2*x)/(3*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I) + 2/(3
*e^(-x) + 3*I*e^(-2*x) - e^(-3*x) - I))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}+3i\,Be^{x}+i\,A-2\,B\right)}{3\left(e^{x}+i\right)^{3}}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^2,x, algorithm="giac")
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x + 3*I*B*e^x + I*A - 2*B)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^2}\,dx = -\frac{\frac{2\,A}{3}+\frac{B\,4\mathrm{i}}{3}-\mathrm{e}^{x}\left(-2\,B+A\,2\mathrm{i}\right)-B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)^{3}}$$

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^2,x)
[Out] -((2*A)/3 + (B*4i)/3 - exp(x)*(A*2i - 2*B) - B*exp(2*x)*2i)/(exp(x)*1i - 1)
^3
```

3.117 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3} - \frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2} + \frac{(2iA-3B)\cosh(x)}{15(i+\sinh(x))}$$

```
[Out] -1/5*(I*A+B)*cosh(x)/(I+sinh(x))^3-1/15*(2*A+3*I*B)*cosh(x)/(I+sinh(x))^2+1
/15*(2*I*A-3*B)*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2829, 2729, 2727}

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \frac{(-3B+2iA)\cosh(x)}{15(\sinh(x)+i)} - \frac{(2A+3iB)\cosh(x)}{15(\sinh(x)+i)^2} - \frac{(B+iA)\cosh(x)}{5(\sinh(x)+i)^3}$$

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
```

```
[Out] -1/5*((I*A + B)*Cosh[x])/(I + Sinh[x])^3 - ((2*A + (3*I)*B)*Cosh[x])/(15*(I
 + Sinh[x])^2) + (((2*I)*A - 3*B)*Cosh[x])/(15*(I + Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3} + \frac{1}{5}(-2iA+3B)\int \frac{1}{(i+\sinh(x))^2}\,dx \\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3} - \frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2} + \frac{1}{15}(-2A-3iB)\int \frac{1}{i+\sinh(x)}\,dx \\
&= -\frac{(iA+B)\cosh(x)}{5(i+\sinh(x))^3} - \frac{(2A+3iB)\cosh(x)}{15(i+\sinh(x))^2} + \frac{(2iA-3B)\cosh(x)}{15(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \frac{\cosh(x)\left(-7iA+3B-3(2A+3iB)\sinh(x)+(2iA-3B)\sinh^2(x)\right)}{15(i+\sinh(x))^3}$$

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^3,x]
[Out] (Cosh[x]*((-7*I)*A + 3*B - 3*(2*A + (3*I)*B)*Sinh[x] + ((2*I)*A - 3*B)*Sinh
[x]^2))/(15*(I + Sinh[x])^3)
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{2\left(15B\,e^{3x}-15B\,e^{x}+15iB\,e^{2x}-2A+10iA\,e^{x}-3iB+20A\,e^{2x}\right)}{15\left(e^{x}+i\right)^{5}}$
default	$-\frac{2iB-4A}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2(8iA+6B)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{-8iB+8A}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{2(-4iA-4B)}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}+\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}$
parallelrisch	$\frac{\operatorname{sech}(x)(-10iA\sinh(x)+2iA\sinh(3x)-28iA\sinh(2x)+15iB\cosh(x)-12iB\cosh(2x)-3iB\cosh(3x)+35A\cosh(x)-8A\cosh(2x)-}{120i\sinh(x)+30\cosh(2x)-90}$

```
[In] int((A+B*sinh(x))/(I+sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(15*B*exp(x)^3-15*B*exp(x)+15*I*B*exp(x)^2-2*A+10*I*A*exp(x)-3*I*B+20
*A*exp(x)^2)/(exp(x)+I)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = -\frac{2\left(15\,Be^{(3\,x)}+5\,(4\,A+3i\,B)e^{(2\,x)}+5\,(2i\,A-3\,B)e^{x}-2\,A-3i\,B\right)}{15\left(e^{(5\,x)}+5i\,e^{(4\,x)}-10\,e^{(3\,x)}-10i\,e^{(2\,x)}+5\,e^{x}+i\right)}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -2/15*(15*B*e^(3*x) + 5*(4*A + 3*I*B)*e^(2*x) + 5*(2*I*A - 3*B)*e^x - 2*A -
 3*I*B)/(e^(5*x) + 5*I*e^(4*x) - 10*e^(3*x) - 10*I*e^(2*x) + 5*e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \frac{4A-30Be^{3x}+6iB+(-40A-30iB)\,e^{2x}+(-20iA+30B)\,e^{x}}{15e^{5x}+75ie^{4x}-150e^{3x}-150ie^{2x}+75e^{x}+15i}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))**3,x)
```

```
[Out] (4*A - 30*B*exp(3*x) + 6*I*B + (-40*A - 30*I*B)*exp(2*x) + (-20*I*A + 30*B)
*exp(x))/(15*exp(5*x) + 75*I*exp(4*x) - 150*exp(3*x) - 150*I*exp(2*x) + 75*
exp(x) + 15*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(50) = 100.

Time = 0.20 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.93

$$\int \frac{A + B\sinh(x)}{(i + \sinh(x))^3}\,dx =$$
$$-\frac{2}{5}\,B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)} + 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} - 5i\,e^{(-4\,x)} + e^{(-5\,x)} - i} + \frac{5i\,e^{(-2\,x)}}{5\,e^{(-x)} + 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} - 5i\,e^{(-}}\right.$$
$$-\frac{4}{15}\,A\left(-\frac{5i\,e^{(-x)}}{5\,e^{(-x)} + 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} - 5i\,e^{(-4\,x)} + e^{(-5\,x)} - i} + \frac{10\,e^{(-2\,x)}}{5\,e^{(-x)} + 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} - 5i\,e}\right.$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -2/5*B*(5*e^(-x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e
^(-5*x) - I) + 5*I*e^(-2*x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e
^(-4*x) + e^(-5*x) - I) - 5*e^(-3*x)/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x
) - 5*I*e^(-4*x) + e^(-5*x) - I) - I/(5*e^(-x) + 10*I*e^(-2*x) - 10*e^(-3*x
) - 5*I*e^(-4*x) + e^(-5*x) - I)) - 4/15*A*(-5*I*e^(-x)/(5*e^(-x) + 10*I*e^
(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I) + 10*e^(-2*x)/(5*e^(-x)
 + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I) - 1/(5*e^(-x)
 + 10*I*e^(-2*x) - 10*e^(-3*x) - 5*I*e^(-4*x) + e^(-5*x) - I))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{A + B\sinh(x)}{(i + \sinh(x))^3}\,dx = -\frac{2\,\left(15\,Be^{(3\,x)} + 20\,Ae^{(2\,x)} + 15i\,Be^{(2\,x)} + 10i\,Ae^{x} - 15\,Be^{x} - 2\,A - 3i\,B\right)}{15\,\left(e^{x} + i\right)^{5}}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^3,x, algorithm="giac")
```

```
[Out] -2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*I*B*e^(2*x) + 10*I*A*e^x - 15*B*e^x
 - 2*A - 3*I*B)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^3}\,dx = \frac{\frac{A\,4\mathrm{i}}{15}-\frac{2\,B}{5}-\frac{A\,\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}+\mathrm{e}^{x}\,\left(\frac{4\,A}{3}+B\,2\mathrm{i}\right)+2\,B\,\mathrm{e}^{2\,x}-B\,\mathrm{e}^{3\,x}\,2\mathrm{i}}{\left(-1+\mathrm{e}^{x}\,1\mathrm{i}\right)^5}$$

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^3,x)
```

```
[Out] ((A*4i)/15 - (2*B)/5 - (A*exp(2*x)*8i)/3 + exp(x)*((4*A)/3 + B*2i) + 2*B*ex
p(2*x) - B*exp(3*x)*2i)/(exp(x)*1i - 1)^5
```

3.118 $\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4} - \frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3} + \frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2} + \frac{2(3A+4iB)\cosh(x)}{105(i+\sinh(x))}$$

```
[Out] -1/7*(I*A+B)*cosh(x)/(I+sinh(x))^4-1/35*(3*A+4*I*B)*cosh(x)/(I+sinh(x))^3+2
/105*(3*I*A-4*B)*cosh(x)/(I+sinh(x))^2+2/105*(3*A+4*I*B)*cosh(x)/(I+sinh(x)
)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2829, 2729, 2727}

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = \frac{2(3A+4iB)\cosh(x)}{105(\sinh(x)+i)} + \frac{2(-4B+3iA)\cosh(x)}{105(\sinh(x)+i)^2} - \frac{(3A+4iB)\cosh(x)}{35(\sinh(x)+i)^3} - \frac{(B+iA)\cosh(x)}{7(\sinh(x)+i)^4}$$

```
[In] Int[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
```

```
[Out] -1/7*((I*A + B)*Cosh[x])/(I + Sinh[x])^4 - ((3*A + (4*I)*B)*Cosh[x])/(35*(I
 + Sinh[x])^3) + (2*((3*I)*A - 4*B)*Cosh[x])/(105*(I + Sinh[x])^2) + (2*(3*
A + (4*I)*B)*Cosh[x])/(105*(I + Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}+\frac{1}{7}(-3iA+4B)\int\frac{1}{(i+\sinh(x))^3}\,dx\\
&= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}-\frac{1}{35}(2(3A+4iB))\int\frac{1}{(i+\sinh(x))^2}\,dx\\
&= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}\\
&\quad+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{1}{105}(2(3iA-4B))\int\frac{1}{i+\sinh(x)}\,dx\\
&= -\frac{(iA+B)\cosh(x)}{7(i+\sinh(x))^4}-\frac{(3A+4iB)\cosh(x)}{35(i+\sinh(x))^3}+\frac{2(3iA-4B)\cosh(x)}{105(i+\sinh(x))^2}+\frac{2(3A+4iB)\cosh(x)}{105(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int\frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = \frac{\cosh(x)\left(-36iA+13B-13(3A+4iB)\sinh(x)+8i(3A+4iB)\sinh^2(x)+(6A+8iB)\sinh^3(x)\right)}{105(i+\sinh(x))^4}$$

```
[In] Integrate[(A + B*Sinh[x])/(I + Sinh[x])^4,x]
[Out] (Cosh[x]*((-36*I)*A + 13*B - 13*(3*A + (4*I)*B)*Sinh[x] + (8*I)*(3*A + (4*I
)*B)*Sinh[x]^2 + (6*A + (8*I)*B)*Sinh[x]^3))/(105*(I + Sinh[x])^4)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{4\left(4B-28iB\,\mathrm{e}^{x}+63iA\,\mathrm{e}^{2x}+70iB\,\mathrm{e}^{3x}-3iA+105A\,\mathrm{e}^{3x}+70B\,\mathrm{e}^{4x}-84B\,\mathrm{e}^{2x}-21A\,\mathrm{e}^{x}\right)}{105\left(\mathrm{e}^{x}+i\right)^{7}}$
default	$-\frac{2(-10iB+18A)}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{6iA+2B}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{-32iA-24B}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{24iA+24B}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}+\frac{2A}{\tanh\left(\frac{x}{2}\right)+i}-\frac{2(32iB-36A)}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2(-\cdots}{7(\tan\cdots}$
parallelrisch	$\frac{(1092iA-476B)\cosh(2x)+(-168iA+14B)\cosh(3x)+(-42iA+21B)\cosh(4x)+(-42iB+336A)\sinh(2x)+(152iB+324A)\sinh\cdots}{840i\sinh(3x)-5880i\sinh(x)+1470i\sinh(2x)-105i\sinh(4x)-1470\cosh(x)+6\cdots}$

[In] int((A+B*sinh(x))/(I+sinh(x))^4,x,method=_RETURNVERBOSE)

[Out] -4/105*(4*B-28*I*B*exp(x)+63*I*A*exp(x)^2+70*I*B*exp(x)^3-3*I*A+105*A*exp(x
)^3+70*B*exp(x)^4-84*B*exp(x)^2-21*A*exp(x))/(exp(x)+I)^7

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)}+35\,(3\,A+2i\,B)e^{(3\,x)}+21\,(3i\,A-4\,B)e^{(2\,x)}-7\,(3\,A+4i\,B)e^{x}-3i\,A+4\,B\right)}{105\left(e^{(7\,x)}+7i\,e^{(6\,x)}-21\,e^{(5\,x)}-35i\,e^{(4\,x)}+35\,e^{(3\,x)}+21i\,e^{(2\,x)}-7\,e^{x}-i\right)}$$

[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="fricas")

[Out] -4/105*(70*B*e^(4*x) + 35*(3*A + 2*I*B)*e^(3*x) + 21*(3*I*A - 4*B)*e^(2*x)
- 7*(3*A + 4*I*B)*e^x - 3*I*A + 4*B)/(e^(7*x) + 7*I*e^(6*x) - 21*e^(5*x) -
35*I*e^(4*x) + 35*e^(3*x) + 21*I*e^(2*x) - 7*e^x - I)

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = \frac{12iA-280Be^{4x}-16B+(-420A-280iB)\,e^{3x}+(84A+112iB)\,e^{x}+(-252iA+336B)\,e^{2x}}{105e^{7x}+735ie^{6x}-2205e^{5x}-3675ie^{4x}+3675e^{3x}+2205ie^{2x}-735e^{x}-105i}$$

[In] integrate((A+B*sinh(x))/(I+sinh(x))**4,x)

[Out] (12*I*A - 280*B*exp(4*x) - 16*B + (-420*A - 280*I*B)*exp(3*x) + (84*A + 112
*I*B)*exp(x) + (-252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) + 735*I*exp(6*x)
- 2205*exp(5*x) - 3675*I*exp(4*x) + 3675*exp(3*x) + 2205*I*exp(2*x) - 735*e
xp(x) - 105*I)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(67) = 134.

Time = 0.21 (sec) , antiderivative size = 469, normalized size of antiderivative = 5.15

$$\int \frac{A + B\sinh(x)}{(i + \sinh(x))^4}\,dx$$
$$= \frac{4}{35}A\left(\frac{7\,e^{(-x)}}{7\,e^{(-x)} + 21i\,e^{(-2\,x)} - 35\,e^{(-3\,x)} - 35i\,e^{(-4\,x)} + 21\,e^{(-5\,x)} + 7i\,e^{(-6\,x)} - e^{(-7\,x)} - i} + \frac{}{7\,e^{(-x)} + 21i\,e^{(-2x}}\right.$$
$$- \frac{8}{105}B\left(-\frac{14i\,e^{(-x)}}{7\,e^{(-x)} + 21i\,e^{(-2\,x)} - 35\,e^{(-3\,x)} - 35i\,e^{(-4\,x)} + 21\,e^{(-5\,x)} + 7i\,e^{(-6\,x)} - e^{(-7\,x)} - i} + \frac{}{7\,e^{(-x)} + 21i}\right.$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 4/35*A*(7*e^(-x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) +
21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) + 21*I*e^(-2*x)/(7*e^(-x) + 21*I
*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-
7*x) - I) - 35*e^(-3*x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-
4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) - I/(7*e^(-x) + 21*I*e^(-
2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x)
- I)) - 8/105*B*(-14*I*e^(-x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*
I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) + 42*e^(-2*x)/(7*e^
(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-
6*x) - e^(-7*x) - I) + 35*I*e^(-3*x)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x
) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I) - 35*e^(-4*x
)/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 35*I*e^(-4*x) + 21*e^(-5*x) + 7
*I*e^(-6*x) - e^(-7*x) - I) - 2/(7*e^(-x) + 21*I*e^(-2*x) - 35*e^(-3*x) - 3
5*I*e^(-4*x) + 21*e^(-5*x) + 7*I*e^(-6*x) - e^(-7*x) - I))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{A + B\sinh(x)}{(i + \sinh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)} + 105\,Ae^{(3\,x)} + 70i\,Be^{(3\,x)} + 63i\,Ae^{(2\,x)} - 84\,Be^{(2\,x)} - 21\,Ae^{x} - 28i\,Be^{x} - 3i\,A + 4\,B\right)}{105\,(e^{x} + i)^{7}}$$

```
[In] integrate((A+B*sinh(x))/(I+sinh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*I*B*e^(3*x) + 63*I*A*e^(2*x) - 84
*B*e^(2*x) - 21*A*e^x - 28*I*B*e^x - 3*I*A + 4*B)/(e^x + I)^7
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int \frac{A+B\sinh(x)}{(i+\sinh(x))^4}\,dx = -\frac{\frac{16\,B}{105}+4\,A\,\mathrm{e}^{3\,x}-\mathrm{e}^{x}\left(\frac{4\,A}{5}+\frac{B\,16\mathrm{i}}{15}\right)-\frac{16\,B\,\mathrm{e}^{2\,x}}{5}+\frac{8\,B\,\mathrm{e}^{4\,x}}{3}-\frac{A\,4\mathrm{i}}{35}+\frac{A\,\mathrm{e}^{2\,x}\,12\mathrm{i}}{5}+\frac{B\,\mathrm{e}^{3\,x}\,8\mathrm{i}}{3}}{\left(\mathrm{e}^{x}+1\mathrm{i}\right)^{7}}$$

```
[In] int((A + B*sinh(x))/(sinh(x) + 1i)^4,x)
```

```
[Out] -((16*B)/105 - (A*4i)/35 + (A*exp(2*x)*12i)/5 + 4*A*exp(3*x) - exp(x)*((4*A
)/5 + (B*16i)/15) - (16*B*exp(2*x))/5 + (B*exp(3*x)*8i)/3 + (8*B*exp(4*x))/
3)/(exp(x) + 1i)^7
```

3.119 $\int \frac{A+B\sinh(x)}{i-\sinh(x)} dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)} dx = -Bx + \frac{(iA-B)\cosh(x)}{i-\sinh(x)}$$

```
[Out] -B*x+(I*A-B)*cosh(x)/(I-sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2814, 2727}

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)} dx = -Bx + \frac{(-B+iA)\cosh(x)}{-\sinh(x)+i}$$

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x]),x]
```

```
[Out] -(B*x) + ((I*A - B)*Cosh[x])/(I - Sinh[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -Bx + (A+iB)\int \frac{1}{i-\sinh(x)}\,dx \\ &= -Bx + \frac{(iA-B)\cosh(x)}{i-\sinh(x)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx = \cosh(x)\left(-\frac{B\text{arcsinh}(\sinh(x))}{\sqrt{\cosh^2(x)}} + \frac{-iA+B}{-i+\sinh(x)}\right)$$

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x]),x]
[Out] Cosh[x]*(-((B*ArcSinh[Sinh[x]])/Sqrt[Cosh[x]^2]) + ((-I)*A + B)/(-I + Sinh[
x]))
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
risch	$-Bx+\frac{2A}{e^{x}-i}+\frac{2iB}{e^{x}-i}$	27
parallelrisch	$\frac{iBx-x\tanh\left(\frac{x}{2}\right)B-2iA+2B}{-i+\tanh\left(\frac{x}{2}\right)}$	32
default	$-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2i(iB+A)}{-i+\tanh\left(\frac{x}{2}\right)}+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	39

```
[In] int((A+B*sinh(x))/(I-sinh(x)),x,method=_RETURNVERBOSE)
[Out] -B*x+2/(exp(x)-I)*A+2*I/(exp(x)-I)*B
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{A + B\sinh(x)}{i - \sinh(x)}\,dx = -\frac{Bxe^{x} - i\,Bx - 2\,A - 2i\,B}{e^{x} - i}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="fricas")
[Out] -(B*x*e^x - I*B*x - 2*A - 2*I*B)/(e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

$$\int \frac{A + B\sinh(x)}{i - \sinh(x)}\,dx = -Bx + \frac{2A + 2iB}{e^{x} - i}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x)
[Out] -B*x + (2*A + 2*I*B)/(exp(x) - I)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{A + B\sinh(x)}{i - \sinh(x)}\,dx = -B\left(x - \frac{2i}{e^{(-x)} + i}\right) + \frac{2\,A}{e^{(-x)} + i}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="maxima")
[Out] -B*(x - 2*I/(e^(-x) + I)) + 2*A/(e^(-x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{A + B\sinh(x)}{i - \sinh(x)}\,dx = -Bx + \frac{2\,(A + i\,B)}{e^{x} - i}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x)),x, algorithm="giac")
[Out] -B*x + 2*(A + I*B)/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{A+B\sinh(x)}{i-\sinh(x)}\,dx=-B\,x+\frac{2\,A+B\,2\mathrm{i}}{\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(-(A + B*sinh(x))/(sinh(x) - 1i),x)
[Out] (2*A + B*2i)/(exp(x) - 1i) - B*x
```

3.120 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 49

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = \frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2} + \frac{(A-2iB)\cosh(x)}{3(i-\sinh(x))}$$

```
[Out] 1/3*(I*A-B)*cosh(x)/(I-sinh(x))^2+1/3*(A-2*I*B)*cosh(x)/(I-sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2829, 2727}

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = \frac{(A-2iB)\cosh(x)}{3(-\sinh(x)+i)} + \frac{(-B+iA)\cosh(x)}{3(-\sinh(x)+i)^2}$$

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(3*(I - Sinh[x])^2) + ((A - (2*I)*B)*Cosh[x])/(3*(I - S
inh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
```

```
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{1}{3}(-iA-2B)\int\frac{1}{i-\sinh(x)}\,dx\\ &= \frac{(iA-B)\cosh(x)}{3(i-\sinh(x))^2}+\frac{(A-2iB)\cosh(x)}{3(i-\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int\frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx=\frac{\cosh(x)(2iA+B-(A-2iB)\sinh(x))}{3(-i+\sinh(x))^2}$$

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^2,x]
[Out] (Cosh[x]*((2*I)*A + B - (A - (2*I)*B)*Sinh[x]))/(3*(-I + Sinh[x])^2)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
risch	$-\frac{2\left(3A\,{\rm e}^{x}-3iB\,{\rm e}^{x}+3B\,{\rm e}^{2x}-iA-2B\right)}{3\left({\rm e}^{x}-i\right)^{3}}$	36
default	$-\frac{2iA-2B}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{2A}{-i+\tanh\left(\frac{x}{2}\right)}-\frac{2(-2iB-2A)}{3\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{3}}$	52
parallelrisch	$\frac{(3iA+3B)\cosh(2x)+(-iB-A)\sinh(2x)+(-2iB+10A)\sinh(x)-3iA-3B}{12i\sinh(x)-3i\sinh(2x)-6\cosh(x)-3\cosh(2x)+9}$	73

```
[In] int((A+B*sinh(x))/(I-sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2/3*(3*A*exp(x)-3*I*B*exp(x)+3*B*exp(x)^2-I*A-2*B)/(exp(x)-I)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,(A-i\,B)e^x-i\,A-2\,B\right)}{3\left(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^x+i\right)}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="fricas")
[Out] -2/3*(3*B*e^(2*x) + 3*(A - I*B)*e^x - I*A - 2*B)/(e^(3*x) - 3*I*e^(2*x) - 3
*e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = \frac{2iA-6Be^{2x}+4B+(-6A+6iB)\,e^x}{3e^{3x}-9ie^{2x}-9e^x+3i}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**2,x)
[Out] (2*I*A - 6*B*exp(2*x) + 4*B + (-6*A + 6*I*B)*exp(x))/(3*exp(3*x) - 9*I*exp(
2*x) - 9*exp(x) + 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(31) = 62.

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.88

$$\begin{aligned}&\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx\\&= -\frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}+\frac{i}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}\right)\\&\quad-\frac{2}{3}\,B\left(-\frac{3i\,e^{(-x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+i}+\frac{2}{3\,e^{(-x)}-3i\,e^{(-2\,x)}-e^{(-3\,x)}+\cdots}\right.\end{aligned}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="maxima")
[Out] -2/3*A*(3*e^(-x)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I) + I/(3*e^(-x) - 3
*I*e^(-2*x) - e^(-3*x) + I)) - 2/3*B*(-3*I*e^(-x)/(3*e^(-x) - 3*I*e^(-2*x)
- e^(-3*x) + I) - 3*e^(-2*x)/(3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I) + 2/(
3*e^(-x) - 3*I*e^(-2*x) - e^(-3*x) + I))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}-3i\,Be^{x}-i\,A-2\,B\right)}{3\left(e^{x}-i\right)^{3}}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^2,x, algorithm="giac")
[Out] -2/3*(3*B*e^(2*x) + 3*A*e^x - 3*I*B*e^x - I*A - 2*B)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^2}\,dx = \frac{\frac{2\,A}{3}-\frac{B\,4\mathrm{i}}{3}+\mathrm{e}^{x}\,(2\,B+A\,2\mathrm{i})+B\,\mathrm{e}^{2\,x}\,2\mathrm{i}}{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)^{3}}$$

```
[In] int((A + B*sinh(x))/(sinh(x) - 1i)^2,x)
[Out] ((2*A)/3 - (B*4i)/3 + exp(x)*(A*2i + 2*B) + B*exp(2*x)*2i)/(exp(x)*1i + 1)^
3
```

3.121 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 76

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3} + \frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2} - \frac{(2iA+3B)\cosh(x)}{15(i-\sinh(x))}$$

```
[Out] 1/5*(I*A-B)*cosh(x)/(I-sinh(x))^3+1/15*(2*A-3*I*B)*cosh(x)/(I-sinh(x))^2-1/
15*(2*I*A+3*B)*cosh(x)/(I-sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2829, 2729, 2727}

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = -\frac{(3B+2iA)\cosh(x)}{15(-\sinh(x)+i)} + \frac{(2A-3iB)\cosh(x)}{15(-\sinh(x)+i)^2} + \frac{(-B+iA)\cosh(x)}{5(-\sinh(x)+i)^3}$$

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(5*(I - Sinh[x])^3) + ((2*A - (3*I)*B)*Cosh[x])/(15*(I
- Sinh[x])^2) - (((2*I)*A + 3*B)*Cosh[x])/(15*(I - Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{1}{5}(-2iA-3B)\int\frac{1}{(i-\sinh(x))^2}\,dx\\
&= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}+\frac{1}{15}(-2A+3iB)\int\frac{1}{i-\sinh(x)}\,dx\\
&= \frac{(iA-B)\cosh(x)}{5(i-\sinh(x))^3}+\frac{(2A-3iB)\cosh(x)}{15(i-\sinh(x))^2}-\frac{(2iA+3B)\cosh(x)}{15(i-\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int\frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx=\frac{\cosh(x)\left(-7iA-3B+(6A-9iB)\sinh(x)+(2iA+3B)\sinh^2(x)\right)}{15(-i+\sinh(x))^3}$$

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^3,x]
[Out] (Cosh[x]*((-7*I)*A - 3*B + (6*A - (9*I)*B)*Sinh[x] + ((2*I)*A + 3*B)*Sinh[x
]^2))/(15*(-I + Sinh[x])^3)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{2B\,e^{3x}-2B\,e^{x}-2iB\,e^{2x}-\frac{4A}{15}-\frac{4iA\,e^{x}}{3}+\frac{2iB}{5}+\frac{8A\,e^{2x}}{3}}{(e^{x}-i)^{5}}$	51
default	$\frac{2iA}{-i+\tanh(\frac{x}{2})}-\frac{2iB+4A}{(-i+\tanh(\frac{x}{2}))^{2}}-\frac{-8iB-8A}{2(-i+\tanh(\frac{x}{2}))^{4}}-\frac{2(-4iA+4B)}{5(-i+\tanh(\frac{x}{2}))^{5}}-\frac{2(8iA-6B)}{3(-i+\tanh(\frac{x}{2}))^{3}}$	91
parallelrisch	$\frac{(3iB-6A)\tanh(\frac{x}{2})^{5}+15B\tanh(\frac{x}{2})^{4}+20i\tanh(\frac{x}{2})^{2}A+(-15iB+10A)\tanh(\frac{x}{2})-8iA-3B}{150\tanh(\frac{x}{2})^{3}+75i\tanh(\frac{x}{2})^{4}-15\tanh(\frac{x}{2})^{5}-75\tanh(\frac{x}{2})-150i\tanh(\frac{x}{2})^{2}+15i}$	102

```
[In] int((A+B*sinh(x))/(I-sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(15*B*exp(x)^3-15*B*exp(x)-15*I*B*exp(x)^2-2*A-10*I*A*exp(x)+3*I*B+20*
A*exp(x)^2)/(exp(x)-I)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{2\left(15\,Be^{(3\,x)}+5\,(4\,A-3i\,B)e^{(2\,x)}-5\,(2i\,A+3\,B)e^{x}-2\,A+3i\,B\right)}{15\,(e^{(5\,x)}-5i\,e^{(4\,x)}-10\,e^{(3\,x)}+10i\,e^{(2\,x)}+5\,e^{x}-i)}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="fricas")
```

```
[Out] 2/15*(15*B*e^(3*x) + 5*(4*A - 3*I*B)*e^(2*x) - 5*(2*I*A + 3*B)*e^x - 2*A +
3*I*B)/(e^(5*x) - 5*I*e^(4*x) - 10*e^(3*x) + 10*I*e^(2*x) + 5*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx = \frac{-4A+30Be^{3x}+6iB+(40A-30iB)\,e^{2x}+(-20iA-30B)\,e^{x}}{15e^{5x}-75ie^{4x}-150e^{3x}+150ie^{2x}+75e^{x}-15i}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**3,x)
```

```
[Out] (-4*A + 30*B*exp(3*x) + 6*I*B + (40*A - 30*I*B)*exp(2*x) + (-20*I*A - 30*B)
*exp(x))/(15*exp(5*x) - 75*I*exp(4*x) - 150*exp(3*x) + 150*I*exp(2*x) + 75*
exp(x) - 15*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(50) = 100$.

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.51

$$\int \frac{A + B\sinh(x)}{(i - \sinh(x))^3}\,dx$$
$$= \frac{2}{5}B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)} - 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} + 5i\,e^{(-4\,x)} + e^{(-5\,x)} + i} - \frac{5i\,e^{(-2\,x)}}{5\,e^{(-x)} - 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} + 5i\,e^{(-4\,x)}}\right.$$
$$+ \frac{4}{15}A\left(\frac{5i\,e^{(-x)}}{5\,e^{(-x)} - 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} + 5i\,e^{(-4\,x)} + e^{(-5\,x)} + i} + \frac{10\,e^{(-2\,x)}}{5\,e^{(-x)} - 10i\,e^{(-2\,x)} - 10\,e^{(-3\,x)} + 5i\,e^{(-}}\right.$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 2/5*B*(5*e^(-x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^
(-5*x) + I) - 5*I*e^(-2*x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^
(-4*x) + e^(-5*x) + I) - 5*e^(-3*x)/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x)
 + 5*I*e^(-4*x) + e^(-5*x) + I) + I/(5*e^(-x) - 10*I*e^(-2*x) - 10*e^(-3*x)
 + 5*I*e^(-4*x) + e^(-5*x) + I)) + 4/15*A*(5*I*e^(-x)/(5*e^(-x) - 10*I*e^(-
2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I) + 10*e^(-2*x)/(5*e^(-x) -
 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I) - 1/(5*e^(-x) -
 10*I*e^(-2*x) - 10*e^(-3*x) + 5*I*e^(-4*x) + e^(-5*x) + I))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \frac{A + B\sinh(x)}{(i - \sinh(x))^3}\,dx = \frac{2\left(15\,Be^{(3\,x)} + 20\,Ae^{(2\,x)} - 15i\,Be^{(2\,x)} - 10i\,Ae^{x} - 15\,Be^{x} - 2\,A + 3i\,B\right)}{15\,(e^{x} - i)^{5}}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*I*B*e^(2*x) - 10*I*A*e^x - 15*B*e^x
- 2*A + 3*I*B)/(e^x - I)^5
```

Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^3}\,dx=\frac{2\,B\,\mathrm{e}^{2\,x}-\frac{2\,B}{5}+\frac{A\,\mathrm{e}^{2\,x}\,8\mathrm{i}}{3}+\mathrm{e}^{x}\,\left(\frac{4\,A}{3}-B\,2\mathrm{i}\right)-\frac{A\,4\mathrm{i}}{15}+B\,\mathrm{e}^{3\,x}\,2\mathrm{i}}{{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)}^5}$$

```
[In] int(-(A + B*sinh(x))/(sinh(x) - 1i)^3,x)
```

```
[Out] ((A*exp(2*x)*8i)/3 - (2*B)/5 - (A*4i)/15 + exp(x)*((4*A)/3 - B*2i) + 2*B*ex
p(2*x) + B*exp(3*x)*2i)/(exp(x)*1i + 1)^5
```

3.122 $\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 101

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3} - \frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}-\frac{2(3A-4iB)\cosh(x)}{105(i-\sinh(x))}$$

```
[Out] 1/7*(I*A-B)*cosh(x)/(I-sinh(x))^4+1/35*(3*A-4*I*B)*cosh(x)/(I-sinh(x))^3-2/
105*(3*I*A+4*B)*cosh(x)/(I-sinh(x))^2-2/105*(3*A-4*I*B)*cosh(x)/(I-sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2829, 2729, 2727}

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = -\frac{2(3A-4iB)\cosh(x)}{105(-\sinh(x)+i)}-\frac{2(4B+3iA)\cosh(x)}{105(-\sinh(x)+i)^2} +\frac{(3A-4iB)\cosh(x)}{35(-\sinh(x)+i)^3}+\frac{(-B+iA)\cosh(x)}{7(-\sinh(x)+i)^4}$$

```
[In] Int[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
```

```
[Out] ((I*A - B)*Cosh[x])/(7*(I - Sinh[x])^4) + ((3*A - (4*I)*B)*Cosh[x])/(35*(I
- Sinh[x])^3) - (2*((3*I)*A + 4*B)*Cosh[x])/(105*(I - Sinh[x])^2) - (2*(3*A
 - (4*I)*B)*Cosh[x])/(105*(I - Sinh[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{1}{7}(-3iA-4B)\int\frac{1}{(i-\sinh(x))^3}\,dx\\
&= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{1}{35}(2(3A-4iB))\int\frac{1}{(i-\sinh(x))^2}\,dx\\
&= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}\\
&\quad -\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}+\frac{1}{105}(2(3iA+4B))\int\frac{1}{i-\sinh(x)}\,dx\\
&= \frac{(iA-B)\cosh(x)}{7(i-\sinh(x))^4}+\frac{(3A-4iB)\cosh(x)}{35(i-\sinh(x))^3}-\frac{2(3iA+4B)\cosh(x)}{105(i-\sinh(x))^2}-\frac{2(3A-4iB)\cosh(x)}{105(i-\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int\frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{\cosh(x)\left(36iA+13B+(-39A+52iB)\sinh(x)+(-24iA-32B)\sinh^2(x)+(6A-8iB)\sinh^3(x)\right)}{105(-i+\sinh(x))^4}$$

```
[In] Integrate[(A + B*Sinh[x])/(I - Sinh[x])^4,x]
[Out] (Cosh[x]*((36*I)*A + 13*B + (-39*A + (52*I)*B)*Sinh[x] + ((-24*I)*A - 32*B)
*Sinh[x]^2 + (6*A - (8*I)*B)*Sinh[x]^3))/(105*(-I + Sinh[x])^4)
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{4\left(4B-70iB\,\mathrm{e}^{3x}-63iA\,\mathrm{e}^{2x}+28iB\,\mathrm{e}^{x}+3iA-84B\,\mathrm{e}^{2x}+105A\,\mathrm{e}^{3x}+70B\,\mathrm{e}^{4x}-21A\,\mathrm{e}^{x}\right)}{105\left(\mathrm{e}^{x}-i\right)^{7}}$
default	$-\frac{2(8iB+8A)}{7\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{7}}-\frac{32iA-24B}{2\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{4}}-\frac{2(-32iB-36A)}{5\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{5}}-\frac{-24iA+24B}{3\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{6}}-\frac{-6iA+2B}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{2A}{-i+\tanh\left(\frac{x}{2}\right.}$
parallelrisch	$\frac{(1092iA+476B)\cosh(2x)+(-168iA-14B)\cosh(3x)+(-42iA-21B)\cosh(4x)+(-42iB-336A)\sinh(2x)+(152iB-324A)\sinh}{-5880i\sinh(x)+1470i\sinh(2x)+840i\sinh(3x)-105i\sinh(4x)+2940\cosh(2x)-}$

```
[In] int((A+B*sinh(x))/(I-sinh(x))^4,x,method=_RETURNVERBOSE)
[Out] -4/105*(4*B-70*I*B*exp(x)^3-63*I*A*exp(x)^2+28*I*B*exp(x)+3*I*A-84*B*exp(x)
^2+105*A*exp(x)^3+70*B*exp(x)^4-21*A*exp(x))/(exp(x)-I)^7
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)}+35\,(3\,A-2i\,B)e^{(3\,x)}+21\,(-3i\,A-4\,B)e^{(2\,x)}-7\,(3\,A-4i\,B)e^{x}+3i\,A+4\,B\right)}{105\left(e^{(7\,x)}-7i\,e^{(6\,x)}-21\,e^{(5\,x)}+35i\,e^{(4\,x)}+35\,e^{(3\,x)}-21i\,e^{(2\,x)}-7\,e^{x}+i\right)}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="fricas")
[Out] -4/105*(70*B*e^(4*x) + 35*(3*A - 2*I*B)*e^(3*x) + 21*(-3*I*A - 4*B)*e^(2*x)
 - 7*(3*A - 4*I*B)*e^x + 3*I*A + 4*B)/(e^(7*x) - 7*I*e^(6*x) - 21*e^(5*x) +
 35*I*e^(4*x) + 35*e^(3*x) - 21*I*e^(2*x) - 7*e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{-12iA-280Be^{4x}-16B+(-420A+280iB)\,e^{3x}+(84A-112iB)\,e^{x}+(252iA+336B)\,e^{2x}}{105e^{7x}-735ie^{6x}-2205e^{5x}+3675ie^{4x}+3675e^{3x}-2205ie^{2x}-735e^{x}+105i}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))**4,x)
[Out] (-12*I*A - 280*B*exp(4*x) - 16*B + (-420*A + 280*I*B)*exp(3*x) + (84*A - 11
2*I*B)*exp(x) + (252*I*A + 336*B)*exp(2*x))/(105*exp(7*x) - 735*I*exp(6*x)
- 2205*exp(5*x) + 3675*I*exp(4*x) + 3675*exp(3*x) - 2205*I*exp(2*x) - 735*e
xp(x) + 105*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(67) = 134.

Time = 0.21 (sec) , antiderivative size = 469, normalized size of antiderivative = 4.64

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx$$
$$= \frac{4}{35}A\left(\frac{7\,e^{(-x)}}{7\,e^{(-x)}-21i\,e^{(-2\,x)}-35\,e^{(-3\,x)}+35i\,e^{(-4\,x)}+21\,e^{(-5\,x)}-7i\,e^{(-6\,x)}-e^{(-7\,x)}+i}-\frac{}{7\,e^{(-x)}-21i\,e^{(-2x}}\right.$$
$$-\frac{8}{105}B\left(\frac{14i\,e^{(-x)}}{7\,e^{(-x)}-21i\,e^{(-2\,x)}-35\,e^{(-3\,x)}+35i\,e^{(-4\,x)}+21\,e^{(-5\,x)}-7i\,e^{(-6\,x)}-e^{(-7\,x)}+i}+\frac{}{7\,e^{(-x)}-21i\,e}\right.$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 4/35*A*(7*e^(-x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) +
21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) - 21*I*e^(-2*x)/(7*e^(-x) - 21*I
*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-
7*x) + I) - 35*e^(-3*x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-
4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) + I/(7*e^(-x) - 21*I*e^(-
2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x)
+ I)) - 8/105*B*(14*I*e^(-x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I
*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) + 42*e^(-2*x)/(7*e^(
-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6
*x) - e^(-7*x) + I) - 35*I*e^(-3*x)/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x)
 + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I) - 35*e^(-4*x)
/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35*I*e^(-4*x) + 21*e^(-5*x) - 7*
I*e^(-6*x) - e^(-7*x) + I) - 2/(7*e^(-x) - 21*I*e^(-2*x) - 35*e^(-3*x) + 35
*I*e^(-4*x) + 21*e^(-5*x) - 7*I*e^(-6*x) - e^(-7*x) + I))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.59

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx =$$
$$-\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}-70i\,Be^{(3\,x)}-63i\,Ae^{(2\,x)}-84\,Be^{(2\,x)}-21\,Ae^{x}+28i\,Be^{x}+3i\,A+4\,B\right)}{105\,(e^{x}-i)^{7}}$$

```
[In] integrate((A+B*sinh(x))/(I-sinh(x))^4,x, algorithm="giac")
```

```
[Out] -4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*I*B*e^(3*x) - 63*I*A*e^(2*x) - 84
*B*e^(2*x) - 21*A*e^x + 28*I*B*e^x + 3*I*A + 4*B)/(e^x - I)^7
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{A+B\sinh(x)}{(i-\sinh(x))^4}\,dx = \frac{\frac{12\,A\,\mathrm{e}^{2\,x}}{5}+\frac{B\,16\mathrm{i}}{105}-\frac{4\,A}{35}+A\,\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\left(\frac{16\,B}{15}+\frac{A\,4\mathrm{i}}{5}\right)-\frac{B\,\mathrm{e}^{2\,x}\,16\mathrm{i}}{5}+\frac{8\,B\,\mathrm{e}^{3\,x}}{3}+\frac{B\,\mathrm{e}^{4\,x}\,8\mathrm{i}}{3}}{\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)^7}$$

```
[In] int((A + B*sinh(x))/(sinh(x) - 1i)^4,x)
```

```
[Out] ((B*16i)/105 - (4*A)/35 + (12*A*exp(2*x))/5 + A*exp(3*x)*4i - exp(x)*((A*4i
)/5 + (16*B)/15) - (B*exp(2*x)*16i)/5 + (8*B*exp(3*x))/3 + (B*exp(4*x)*8i)/
3)/(exp(x)*1i + 1)^7
```

3.123 $\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=\frac{\sqrt{2}(iA-B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

```
[Out] (I*A-B)*arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))*2^(1/2)/
a^(1/2)+2*B*cosh(x)/(a+I*a*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2830, 2728, 212}

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=\frac{\sqrt{2}(-B+iA)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}$$

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (Sqrt[2]*(I*A - B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]]
)])/Sqrt[a] + (2*B*Cosh[x])/Sqrt[a + I*a*Sinh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2830

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e
+ f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}+(A+iB)\int\frac{1}{\sqrt{a+ia\sinh(x)}}\,dx\\
&= \frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}+(2(iA-B))\text{Subst}\left(\int\frac{1}{2a-x^2}\,dx,x,\frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)\\
&= \frac{\sqrt{2}(iA-B)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{\sqrt{a}}+\frac{2B\cosh(x)}{\sqrt{a+ia\sinh(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

$$\int\frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{2\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left((1+i)\sqrt[4]{-1}(-iA+B)\arctan\left(\frac{i+\tanh\left(\frac{x}{4}\right)}{\sqrt{2}}\right)+B\cosh\left(\frac{x}{2}\right)-iB\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a+ia\sinh(x)}}$$

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + I*a*Sinh[x]],x]
```

```
[Out] (2*(Cosh[x/2] + I*Sinh[x/2])*((1 + I)*(-1)^(1/4)*((-I)*A + B)*ArcTan[(I + T
anh[x/4])/Sqrt[2]] + B*Cosh[x/2] - I*B*Sinh[x/2]))/Sqrt[a + I*a*Sinh[x]]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(52) = 104.

Time = 5.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.91

method	result	size
risch	$\frac{(-2A-iB+B\,e^{x})(e^{x}-i)\sqrt{2}\,e^{-x}}{\sqrt{a(ie^{2x}+2\,e^{x}-i)e^{-x}}}+\frac{i(2iA-2B)(-e^{x}+i)\left(a^{\frac{3}{2}}+\arctan\left(\frac{\sqrt{ia\,e^{x}}}{\sqrt{a}}\right)a\sqrt{ia\,e^{x}}\right)\sqrt{2}\,e^{-x}}{a^{\frac{3}{2}}\sqrt{a(ie^{2x}+2\,e^{x}-i)e^{-x}}}$	126

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2*A-I*B+B*exp(x))*(exp(x)-I)*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/exp(x))^(
1/2)/exp(x)+I*(2*I*A-2*B)*(-exp(x)+I)*(a^(3/2)+arctan((I*a*exp(x))^(1/2)/a^
(1/2))*a*(I*a*exp(x))^(1/2))/a^(3/2)*2^(1/2)/(a*(I*exp(x)^2+2*exp(x)-I)/exp
(x))^(1/2)/exp(x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(49) = 98.

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.85

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx = \frac{\sqrt{2}a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}\log\left(-\frac{2\left(\sqrt{2}a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}+2\sqrt{\frac{1}{2}i\,ae^{(-x)}}(i\,A-B)\right)}{-4i\,A+4\,B}\right)-\sqrt{2}a\sqrt{-\frac{A^2+2i\,AB-B^2}{a}}\log\left(\frac{2\left(\sqrt{2}a\sqrt{-\right.}{}\right.}{}\right.}{a}$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log(-2*(sqrt(2)*a*sqrt(-(A^2 + 2*
I*A*B - B^2)/a) + 2*sqrt(1/2*I*a*e^(-x))*(I*A - B))/(-4*I*A + 4*B)) - sqrt(
2)*a*sqrt(-(A^2 + 2*I*A*B - B^2)/a)*log(2*(sqrt(2)*a*sqrt(-(A^2 + 2*I*A*B -
 B^2)/a) - 2*sqrt(1/2*I*a*e^(-x))*(I*A - B))/(-4*I*A + 4*B)) - 2*sqrt(1/2*I
*a*e^(-x))*(I*B*e^x - B))/a
```

Sympy [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=\int \frac{A+B\sinh{(x)}}{\sqrt{ia\left(\sinh{(x)}-i\right)}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(1/2),x)
[Out] Integral((A + B*sinh(x))/sqrt(I*a*(sinh(x) - I)), x)
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=\int \frac{B\sinh{(x)}+A}{\sqrt{i\,a\sinh{(x)}+a}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=\int \frac{B\sinh{(x)}+A}{\sqrt{i\,a\sinh{(x)}+a}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/sqrt(I*a*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{\sqrt{a+ia\sinh(x)}}\,dx=\int \frac{A+B\sinh(x)}{\sqrt{a+a\sinh{(x)}\,1\mathrm{i}}}\,dx$$

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(1/2), x)
```

3.124 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx=\frac{(iA+3B)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

```
[Out] 1/2*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(3/2)+1/4*(I*A+3*B)*arctanh(1/2*cosh(x)
*a^(1/2)*2^(1/2)/(a+I*a*sinh(x))^(1/2))/a^(3/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2829, 2728, 212}

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx=\frac{(3B+iA)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(-B+iA)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}$$

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

```
[Out] ((I*A + 3*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])])/(2
*Sqrt[2]*a^(3/2)) + ((I*A - B)*Cosh[x])/(2*(a + I*a*Sinh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(A-3iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx}{4a} \\
&= \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}} + \frac{(iA+3B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)}{2a} \\
&= \frac{(iA+3B)\operatorname{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{2\sqrt{2}a^{3/2}} + \frac{(iA-B)\cosh(x)}{2(a+ia\sinh(x))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(i(A+iB)\cosh\left(\frac{x}{2}\right)+(A+iB)\sinh\left(\frac{x}{2}\right)+(1+i)\sqrt[4]{-1}\right.}{2(a+ia\sinh(x))^{3/2}}$$

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(3/2),x]
```

```
[Out] ((Cosh[x/2] + I*Sinh[x/2])*(I*(A + I*B)*Cosh[x/2] + (A + I*B)*Sinh[x/2] + (
1 + I)*(-1)^(1/4)*(A - (3*I)*B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(-I + Sinh[
x])))/(2*(a + I*a*Sinh[x])^(3/2))
```

Maple [F]

$$\int \frac{A+B\sinh (x)}{(a+ia\sinh (x))^{\frac{3}{2}}}dx$$

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

```
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(54) = 108$.

Time = 0.31 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.34

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a^2e^{(2\,x)}-2i\,a^2e^x-a^2\right)\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}}\log\left(\frac{\sqrt{\frac{1}{2}}a^2\sqrt{-\frac{A^2-6i\,AB-9\,B^2}{a^3}}+\sqrt{\frac{1}{2}i\,ae^{(-x)}}(i}{i\,A+3\,B}\right.}{}$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*e^x - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B
^2)/a^3)*log((sqrt(1/2)*a^2*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3) + sqrt(1/2*I
*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)) - sqrt(1/2)*(a^2*e^(2*x) - 2*I*a^2*e^x
 - a^2)*sqrt(-(A^2 - 6*I*A*B - 9*B^2)/a^3)*log(-(sqrt(1/2)*a^2*sqrt(-(A^2 -
 6*I*A*B - 9*B^2)/a^3) - sqrt(1/2*I*a*e^(-x))*(I*A + 3*B))/(I*A + 3*B)) - 2
*((I*A - B)*e^(2*x) - (A + I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/(a^2*e^(2*x) - 2
*I*a^2*e^x - a^2)
```

Sympy [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{A+B\sinh (x)}{(ia(\sinh (x)-i))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(3/2),x)
```

```
[Out] Integral((A + B*sinh(x))/(I*a*(sinh(x) - I))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{B\sinh\left(x\right)+A}{\left(i\,a\sinh\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{B\sinh\left(x\right)+A}{\left(i\,a\sinh\left(x\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{3/2}}\,dx = \int \frac{A+B\sinh(x)}{\left(a+a\sinh\left(x\right)\,1\mathrm{i}\right)^{3/2}}\,dx$$

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(3/2), x)
```

3.125 $\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 110

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \frac{(3iA+5B)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}$$

```
[Out] 1/4*(I*A-B)*cosh(x)/(a+I*a*sinh(x))^(5/2)+1/16*(3*I*A+5*B)*cosh(x)/a/(a+I*a
*sinh(x))^(3/2)+1/32*(3*I*A+5*B)*arctanh(1/2*cosh(x)*a^(1/2)*2^(1/2)/(a+I*a
*sinh(x))^(1/2))/a^(5/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2829, 2729, 2728, 212}

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \frac{(5B+3iA)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(5B+3iA)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(-B+iA)\cosh(x)}{4(a+ia\sinh(x))^{5/2}}$$

```
[In] Int[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

```
[Out] (((3*I)*A + 5*B)*ArcTanh[(Sqrt[a]*Cosh[x])/(Sqrt[2]*Sqrt[a + I*a*Sinh[x]])]
)/(16*Sqrt[2]*a^(5/2)) + ((I*A - B)*Cosh[x])/(4*(a + I*a*Sinh[x])^(5/2)) +
(((3*I)*A + 5*B)*Cosh[x])/(16*a*(a + I*a*Sinh[x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2728

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2729

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2829

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3A-5iB)\int \frac{1}{(a+ia\sinh(x))^{3/2}}\,dx}{8a} \\
&= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3A-5iB)\int \frac{1}{\sqrt{a+ia\sinh(x)}}\,dx}{32a^2} \\
&= \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}} + \frac{(3iA+5B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, \frac{a\cosh(x)}{\sqrt{a+ia\sinh(x)}}\right)}{16a^2} \\
&= \frac{(3iA+5B)\text{arctanh}\left(\frac{\sqrt{a}\cosh(x)}{\sqrt{2}\sqrt{a+ia\sinh(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(iA-B)\cosh(x)}{4(a+ia\sinh(x))^{5/2}} + \frac{(3iA+5B)\cosh(x)}{16a(a+ia\sinh(x))^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.67

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(4i(A+iB)\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)+(3iA+5B)\left(\cosh\right.\right.}{}$$

```
[In] Integrate[(A + B*Sinh[x])/(a + I*a*Sinh[x])^(5/2),x]
```

```
[Out] ((Cosh[x/2] + I*Sinh[x/2])*((4*I)*(A + I*B)*(Cosh[x/2] + I*Sinh[x/2]) + ((3
*I)*A + 5*B)*(Cosh[x/2] + I*Sinh[x/2])^3 + (1 - I)*(-1)^(1/4)*(3*A - (5*I)*
B)*ArcTan[(I + Tanh[x/4])/Sqrt[2]]*(Cosh[x/2] + I*Sinh[x/2])^4 + 8*(A + I*B
)*Sinh[x/2] + 2*((3*I)*A + 5*B)*Sinh[x/2]*(-I + Sinh[x])))/(16*(a + I*a*Sin
h[x])^(5/2))
```

Maple [F]

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{\frac{5}{2}}}dx$$

```
[In] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

```
[Out] int((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(77) = 154.

Time = 0.30 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.15

$$\int \frac{A+B\sinh(x)}{(a+ia\sinh(x))^{5/2}}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a^3e^{(4x)}-4i\,a^3e^{(3x)}-6\,a^3e^{(2x)}+4i\,a^3e^{x}+a^3\right)\sqrt{-\frac{9A^2-30i\,AB-25B^2}{a^5}}\log\left(\frac{\sqrt{\frac{1}{2}}a^3\cdots}{}\right.}{}$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(1/2)*(a^3*e^(4*x) - 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a^3*e^
x + a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log((sqrt(1/2)*a^3*sqrt(-(9
*A^2 - 30*I*A*B - 25*B^2)/a^5) + sqrt(1/2*I*a*e^(-x))*(3*I*A + 5*B))/(3*I*A
 + 5*B)) - sqrt(1/2)*(a^3*e^(4*x) - 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a
^3*e^x + a^3)*sqrt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5)*log(-(sqrt(1/2)*a^3*sq
rt(-(9*A^2 - 30*I*A*B - 25*B^2)/a^5) - sqrt(1/2*I*a*e^(-x))*(3*I*A + 5*B))/
(3*I*A + 5*B)) + 2*((-3*I*A - 5*B)*e^(4*x) - (11*A + 3*I*B)*e^(3*x) + (-11*
I*A + 3*B)*e^(2*x) - (3*A - 5*I*B)*e^x)*sqrt(1/2*I*a*e^(-x)))/(a^3*e^(4*x)
- 4*I*a^3*e^(3*x) - 6*a^3*e^(2*x) + 4*I*a^3*e^x + a^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{5/2}}\,dx = \int \frac{B\sinh(x) + A}{(i\,a\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{5/2}}\,dx = \int \frac{B\sinh(x) + A}{(i\,a\sinh(x) + a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+I*a*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/(I*a*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + ia\sinh(x))^{5/2}}\,dx = \int \frac{A + B\sinh(x)}{(a + a\sinh(x)\ 1\mathrm{i})^{5/2}}\,dx$$

```
[In] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2),x)
[Out] int((A + B*sinh(x))/(a + a*sinh(x)*1i)^(5/2), x)
```

3.126 $\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 259

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}+\frac{2i(161a^2Ab-63Ab^3+15a^3B-145ab^2B)\,E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{105b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}-\frac{2i(a^2+b^2)\left(56aAb+15a^2B-25b^2B\right)\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{105b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2/35*(7*A*b+5*B*a)*cosh(x)*(a+b*sinh(x))^(3/2)+2/7*B*cosh(x)*(a+b*sinh(x))^
(5/2)+2/105*(56*A*a*b+15*B*a^2-25*B*b^2)*cosh(x)*(a+b*sinh(x))^(1/2)+2/105*
I*(161*A*a^2*b-63*A*b^3+15*B*a^3-145*B*a*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)
/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2
))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/105*I*(a^2+b^2)*(5
6*A*a*b+15*B*a^2-25*B*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x
)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(
a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\begin{aligned}\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx &= \frac{2}{105}\cosh(x)\left(15a^2B\right.\\&\left.+56aAb-25b^2B\right)\sqrt{a+b\sinh(x)}\\&-\frac{2i(a^2+b^2)\left(15a^2B+56aAb-25b^2B\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{105b\sqrt{a+b\sinh(x)}}\\&+\frac{2i(15a^3B+161a^2Ab-145ab^2B-63Ab^3)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{105b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\&+\frac{2}{35}\cosh(x)(5aB+7Ab)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2}\end{aligned}$$

```
[In] Int[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*(56*a*A*b + 15*a^2*B - 25*b^2*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/105 + (2*(
7*A*b + 5*a*B)*Cosh[x]*(a + b*Sinh[x])^(3/2))/35 + (2*B*Cosh[x]*(a + b*Sinh
[x])^(5/2))/7 + (((2*I)/105)*(161*a^2*A*b - 63*A*b^3 + 15*a^3*B - 145*a*b^2
*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt
[(a + b*Sinh[x])/(a - I*b)]) - (((2*I)/105)*(a^2 + b^2)*(56*a*A*b + 15*a^2*
B - 25*b^2*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x
])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} \\
&\quad+\frac{2}{7}\int(a+b\sinh(x))^{3/2}\left(\frac{1}{2}(7aA-5bB)+\frac{1}{2}(7Ab+5aB)\sinh(x)\right)\,dx \\
&= \frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} \\
&\quad+\frac{4}{35}\int\sqrt{a+b\sinh(x)}\left(\frac{1}{4}\left(35a^2A-21Ab^2-40abB\right)+\frac{1}{4}\left(56aAb+15a^2B-25b^2B\right)\sinh(x)\right)\,dx \\
&= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} \\
&\quad+\frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} \\
&\quad+\frac{8}{105}\int\frac{\frac{1}{8}\left(105a^3A-119aAb^2-135a^2bB+25b^3B\right)+\frac{1}{8}\left(161a^2Ab-63Ab^3+15a^3B-145ab^2B\right)\mathrm{s}}{\sqrt{a+b\sinh(x)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} \\
&\quad+\frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} \\
&\quad-\frac{\left(\left(a^2+b^2\right)\left(56aAb+15a^2B-25b^2B\right)\right)\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{105b} \\
&\quad+\frac{\left(161a^2Ab-63Ab^3+15a^3B-145ab^2B\right)\int\sqrt{a+b\sinh(x)}\,dx}{105b}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} \\
&\quad+\frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} \\
&\quad+\frac{\left(\left(161a^2Ab-63Ab^3+15a^3B-145ab^2B\right)\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{105b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} \\
&\quad-\frac{\left(\left(a^2+b^2\right)\left(56aAb+15a^2B-25b^2B\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{105b\sqrt{a+b\sinh(x)}}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2}{105}\left(56aAb+15a^2B-25b^2B\right)\cosh(x)\sqrt{a+b\sinh(x)} \\
&\quad+\frac{2}{35}(7Ab+5aB)\cosh(x)(a+b\sinh(x))^{3/2}+\frac{2}{7}B\cosh(x)(a+b\sinh(x))^{5/2} \\
&\quad+\frac{2i\left(161a^2Ab-63Ab^3+15a^3B-145ab^2B\right)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{105b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} \\
&\quad-\frac{2i\left(a^2+b^2\right)\left(56aAb+15a^2B-25b^2B\right)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{105b\sqrt{a+b\sinh(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.93

$$
\int(a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\frac{2i\left(b\left(105a^3A-119aAb^2-135a^2bB+25b^3B\right)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+\left(161a^2Ab-63Ab^3+15a^3B-145ab^2B\right)\left((a-ib)E\right.\right.}{b}
$$

```
[In] Integrate[(a + b*Sinh[x])^(5/2)*(A + B*Sinh[x]),x]
```

```
[Out] (((2*I)*(b*(105*a^3*A - 119*a*A*b^2 - 135*a^2*b*B + 25*b^3*B)*EllipticF[(Pi
 - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + (161*a^2*A*b - 63*A*b^3 + 15*a^3*B -
 145*a*b^2*B)*((a - I*b)*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]
```

```
- a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x]
)/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x])*(154*a*A*b + 90*a^2*B - 65*b^2*B
+ 15*b^2*B*Cosh[2*x] + 6*b*(7*A*b + 15*a*B)*Sinh[x]))/(105*Sqrt[a + b*Sinh[
x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1877 vs. 2(279) = 558.

Time = 5.22 (sec) , antiderivative size = 1878, normalized size of antiderivative = 7.25

method	result	size
parts	Expression too large to display	1878
default	Expression too large to display	1893

```
[In] int((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*A*(8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((
I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b
-a)/(I*b+a))^(1/2))*a^3*b+8*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b
/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*
b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^3+15*(-(a+b*sinh(x))/(I*b-a))^(1/
2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-
(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4+6*(-(a+b*sinh(x)
)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2
)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^
2-9*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x
))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b
+a))^(1/2))*b^4-23*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(
1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2)
,(-(I*b-a)/(I*b+a))^(1/2))*a^4-14*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x
))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))
/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b^2+9*(-(a+b*sinh(x))/(I*b-a)
)^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Ellipti
cE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b^4+3*b^4*sinh(
x)^4+14*a*b^3*sinh(x)^3+11*a^2*b^2*sinh(x)^2+3*b^4*sinh(x)^2+14*a*b^3*sinh(
x)+11*a^2*b^2)/b/cosh(x)/(a+b*sinh(x))^(1/2)+2/21*B*(3*I*(-(a+b*sinh(x))/(I
*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*El
lipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^4*b-2*I*
(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b
/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))
^(1/2))*a^2*b^3-5*I*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^
(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2
),(-(I*b-a)/(I*b+a))^(1/2))*b^5+3*b^5*sinh(x)^5-24*(-(a+b*sinh(x))/(I*b-a))
```

```
^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*Elliptic
F((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3*b^2-24*(-(a+
b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b
-a))^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2
))*a*b^4-3*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-
a)/(I*b+a))^(1/2))*a^5+26*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*
b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a)
)^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^3*b^2+29*(-(a+b*sinh(x))/(I*b-a))^(1/2)
*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a
+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a*b^4+12*a*b^4*sinh(x)
^4+18*a^2*b^3*sinh(x)^3-2*b^5*sinh(x)^3+9*a^3*b^2*sinh(x)^2+7*a*b^4*sinh(x)
^2+18*a^2*b^3*sinh(x)-5*b^5*sinh(x)+9*a^3*b^2-5*a*b^4)/b^2/cosh(x)/(a+b*sin
h(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 1139, normalized size of antiderivative = 4.40

$$\int (a + b\sinh(x))^{5/2}(A + B\sinh(x))\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/1260*(8*(sqrt(2)*(30*B*a^4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*A*a*b^3 - 7
5*B*b^4)*cosh(x)^3 + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*
A*a*b^3 - 75*B*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(30*B*a^4 + 7*A*a^3*b + 1
15*B*a^2*b^2 + 231*A*a*b^3 - 75*B*b^4)*cosh(x)*sinh(x)^2 + sqrt(2)*(30*B*a^
4 + 7*A*a^3*b + 115*B*a^2*b^2 + 231*A*a*b^3 - 75*B*b^4)*sinh(x)^3)*sqrt(b)*
weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1
/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 24*(sqrt(2)*(15*B*a^3*b + 161*A*a
^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)^3 + 3*sqrt(2)*(15*B*a^3*b + 161*A*
a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)^2*sinh(x) + 3*sqrt(2)*(15*B*a^3*b
 + 161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)*sinh(x)^2 + sqrt(2)*(15*
B*a^3*b + 161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*sinh(x)^3)*sqrt(b)*weiers
trassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrass
PInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cos
h(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(15*B*b^4*cosh(x)^6 + 15*B*b^4*sinh(x)^6
+ 6*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^5 + 6*(15*B*b^4*cosh(x) + 15*B*a*b^3 + 7
*A*b^4)*sinh(x)^5 + 15*B*b^4 + (180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*co
sh(x)^4 + (225*B*b^4*cosh(x)^2 + 180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4 +
30*(15*B*a*b^3 + 7*A*b^4)*cosh(x))*sinh(x)^4 - 8*(15*B*a^3*b + 161*A*a^2*b^
2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x)^3 + 4*(75*B*b^4*cosh(x)^3 - 30*B*a^3*b
- 322*A*a^2*b^2 + 290*B*a*b^3 + 126*A*b^4 + 15*(15*B*a*b^3 + 7*A*b^4)*cosh(
```

```
x)^2 + (180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x))*sinh(x)^3 + (180*
B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x)^2 + (225*B*b^4*cosh(x)^4 + 180
*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4 + 60*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^3
+ 6*(180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x)^2 - 24*(15*B*a^3*b +
161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(x))*sinh(x)^2 - 6*(15*B*a*b^3
+ 7*A*b^4)*cosh(x) + 2*(45*B*b^4*cosh(x)^5 - 45*B*a*b^3 - 21*A*b^4 + 15*(15
*B*a*b^3 + 7*A*b^4)*cosh(x)^4 + 2*(180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)
*cosh(x)^3 - 12*(15*B*a^3*b + 161*A*a^2*b^2 - 145*B*a*b^3 - 63*A*b^4)*cosh(
x)^2 + (180*B*a^2*b^2 + 308*A*a*b^3 - 115*B*b^4)*cosh(x))*sinh(x))*sqrt(b*s
inh(x) + a))/(b^2*cosh(x)^3 + 3*b^2*cosh(x)^2*sinh(x) + 3*b^2*cosh(x)*sinh(
x)^2 + b^2*sinh(x)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\text{Timed out}$$

```
[In] integrate((a+b*sinh(x))**(5/2)*(A+B*sinh(x)),x)
[Out] Timed out
```

Maxima [F]

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (B\sinh(x)+A)(b\sinh(x)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (B\sinh(x)+A)(b\sinh(x)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(5/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{5/2}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\ (a+b\sinh(x))^{5/2}\,dx$$

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(5/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(5/2), x)
```

3.127 $\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 207

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i(20aAb+3a^2B-9b^2B)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(a^2+b^2)\,(5Ab+3aB)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{15b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2/5*B*cosh(x)*(a+b*sinh(x))^(3/2)+2/15*(5*A*b+3*B*a)*cosh(x)*(a+b*sinh(x))^
(1/2)+2/15*I*(20*A*a*b+3*B*a^2-9*B*b^2)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1
/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+
b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/15*I*(a^2+b^2)*(5*A*b+3*
B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi
+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*s
inh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2832, 2831, 2742, 2740, 2734, 2732}

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = -\frac{2i(a^2+b^2)\,(3aB+5Ab)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{15b\sqrt{a+b\sinh(x)}} + \frac{2i(3a^2B+20aAb-9b^2B)\,\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2}{15}\cosh(x)(3aB+5Ab)\sqrt{a+b\sinh(x)}+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2}$$

```
[In] Int[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*(5*A*b + 3*a*B)*Cosh[x]*Sqrt[a + b*Sinh[x]])/15 + (2*B*Cosh[x]*(a + b*Si
nh[x])^(3/2))/5 + (((2*I)/15)*(20*a*A*b + 3*a^2*B - 9*b^2*B)*EllipticE[Pi/4
 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])/(
a - I*b)]) - (((2*I)/15)*(a^2 + b^2)*(5*A*b + 3*a*B)*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} \\
&\quad+\frac{2}{5}\int\sqrt{a+b\sinh(x)}\left(\frac{1}{2}(5aA-3bB)+\frac{1}{2}(5Ab+3aB)\sinh(x)\right)\,dx \\
&= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} \\
&\quad+\frac{4}{15}\int\frac{\frac{1}{4}(15a^2A-5Ab^2-12abB)+\frac{1}{4}(20aAb+3a^2B-9b^2B)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx \\
&= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} \\
&\quad-\frac{\left((a^2+b^2)\,(5Ab+3aB)\right)\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{15b} \\
&\quad+\frac{(20aAb+3a^2B-9b^2B)\int\sqrt{a+b\sinh(x)}\,dx}{15b}
\end{aligned}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{\left((20aAb+3a^2B-9b^2B)\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{\left((a^2+b^2)(5Ab+3aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{15b\sqrt{a+b\sinh(x)}}$$

$$= \frac{2}{15}(5Ab+3aB)\cosh(x)\sqrt{a+b\sinh(x)}+\frac{2}{5}B\cosh(x)(a+b\sinh(x))^{3/2} + \frac{2i(20aAb+3a^2B-9b^2B)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{15b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(a^2+b^2)(5Ab+3aB)\,\mathrm{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{15b\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.95

$$\int(a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \frac{2\left(\frac{i\left(b(15a^2A-5Ab^2-12abB)\,\mathrm{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+(20aAb+3a^2B-9b^2B)\left((a-ib)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)-a\,\mathrm{Elli}\ldots\right.\right.}{b}\right.}{15\sqrt{a+b\ldots}}$$

```
[In] Integrate[(a + b*Sinh[x])^(3/2)*(A + B*Sinh[x]),x]
```

```
[Out] (2*((I*(b*(15*a^2*A - 5*A*b^2 - 12*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*
I)*b)/(a - I*b)] + (20*a*A*b + 3*a^2*B - 9*b^2*B)*((a - I*b)*EllipticE[(Pi
- (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)
*b)/(a - I*b)]))*Sqrt[(a + b*Sinh[x])/(a - I*b)])/b + Cosh[x]*(a + b*Sinh[x
])*(5*A*b + 6*a*B + 3*b*B*Sinh[x])))/(15*Sqrt[a + b*Sinh[x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1036 vs. 2(231) = 462.

Time = 4.32 (sec) , antiderivative size = 1037, normalized size of antiderivative = 5.01

method	result	size
default	Expression too large to display	1037
parts	Expression too large to display	1489

```
[In] int((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*(2*a^2*A*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(
1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2
*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*
b+a))^(1/2))+B*b^2*(2/5/b*sinh(x)*(cosh(x)^2*(a+b*sinh(x)))^(1/2)-8/15*a/b^
2*(cosh(x)^2*(a+b*sinh(x)))^(1/2)-4/15*a/b*(a/b-I)*((-a-b*sinh(x))/(I*b-a))
^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)
^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(
I*b+a))^(1/2))+2*(-3/5+8/15*a^2/b^2)*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)
*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+
b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*
b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b
+a))^(1/2))))+(A*b^2+2*B*a*b)*(2/3/b*(cosh(x)^2*(a+b*sinh(x)))^(1/2)-2/3*(a
/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh
(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh
(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))-4/3*a/b*(a/b-I)*((-a-b*sinh(x)
)/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2
)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-a-b*sinh(x))/(I*b-
a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/
2),((a-I*b)/(I*b+a))^(1/2))))+2*(2*A*a*b+B*a^2)*(a/b-I)*((-a-b*sinh(x))/(I*
b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(co
sh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(
1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((
a-I*b)/(I*b+a))^(1/2))))/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 635, normalized size of antiderivative = 3.07

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx =$$
$$-\frac{4\left(\sqrt{2}(6\,Ba^3-5\,Aa^2b+18\,Bab^2+15\,Ab^3)\cosh(x)^2+2\,\sqrt{2}(6\,Ba^3-5\,Aa^2b+18\,Bab^2+15\,Ab^3)\cosh(x)\right.}{}$$

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/90*(4*(sqrt(2)*(6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*cosh(x)^2 +
 2*sqrt(2)*(6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*cosh(x)*sinh(x) +
sqrt(2)*(6*B*a^3 - 5*A*a^2*b + 18*B*a*b^2 + 15*A*b^3)*sinh(x)^2)*sqrt(b)*we
ierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3
*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 12*(sqrt(2)*(3*B*a^2*b + 20*A*a*b^2
 - 9*B*b^3)*cosh(x)^2 + 2*sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x
)*sinh(x) + sqrt(2)*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*sinh(x)^2)*sqrt(b)*w
eierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weiers
trassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*
b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*B*b^3*cosh(x)^4 + 3*B*b^3*sinh(x)
^4 - 3*B*b^3 + 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^3 + 2*(6*B*b^3*cosh(x) + 6*B
*a*b^2 + 5*A*b^3)*sinh(x)^3 - 4*(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x)^
2 + 2*(9*B*b^3*cosh(x)^2 - 6*B*a^2*b - 40*A*a*b^2 + 18*B*b^3 + 3*(6*B*a*b^2
 + 5*A*b^3)*cosh(x))*sinh(x)^2 + 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x) + 2*(6*B*b
^3*cosh(x)^3 + 6*B*a*b^2 + 5*A*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^2 - 4*
(3*B*a^2*b + 20*A*a*b^2 - 9*B*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(
b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2)
```

Sympy [F]

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (A+B\sinh(x))\,(a+b\sinh(x))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))**(3/2)*(A+B*sinh(x)),x)
```

```
[Out] Integral((A + B*sinh(x))*(a + b*sinh(x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (B\sinh(x)+A)(b\sinh(x)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (B\sinh(x)+A)(b\sinh(x)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*sinh(x))^(3/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*(b*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\sinh(x))^{3/2}(A+B\sinh(x))\,dx = \int (A+B\sinh(x))\,(a+b\sinh(x))^{3/2}\,dx$$

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(3/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(3/2), x)
```

3.128 $\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 164

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2i(3Ab+aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(a^2+b^2)\,B\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2/3*B*cosh(x)*(a+b*sinh(x))^(1/2)+2/3*I*(3*A*b+B*a)*(sin(1/4*Pi+1/2*I*x)^2)
^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b)
)^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/(a-I*b))^(1/2)-2/3*I*(a^2+b^2
)*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+
1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2)/b/(a+b*si
nh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used

$= \{2832, 2831, 2742, 2740, 2734, 2732\}$

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = -\frac{2iB(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{3b\sqrt{a+b\sinh(x)}} + \frac{2i(aB+3Ab)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}|\frac{2b}{ia+b}\right)}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)}$$

```
[In] Int[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*B*Cosh[x]*Sqrt[a + b*Sinh[x]])/3 + (((2*I)/3)*(3*A*b + a*B)*EllipticE[Pi
/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*Sqrt[(a + b*Sinh[x])
/(a - I*b)]) - (((2*I)/3)*(a^2 + b^2)*B*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*
a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
```

```
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2832

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d
*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
0] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2}{3}\int \frac{\frac{1}{2}(3aA-bB)+\frac{1}{2}(3Ab+aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} - \frac{((a^2+b^2)B)\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{3b} + \frac{(3Ab+aB)\int \sqrt{a+b\sinh(x)}\,dx}{3b}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{\left((3Ab+aB)\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{\left((a^2+b^2)B\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{3b\sqrt{a+b\sinh(x)}}$$

$$= \frac{2}{3}B\cosh(x)\sqrt{a+b\sinh(x)} + \frac{2i(3Ab+aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(a^2+b^2)B\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3b\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx$$
$$= \frac{2bB\cosh(x)(a+b\sinh(x))+2(ia+b)(3Ab+aB)E\left(\frac{1}{4}(\pi-2ix)|-\frac{2ib}{a-ib}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}-2i(a^2+b^2)\,B\,\mathrm{Ellipt}}{3b\sqrt{a+b\sinh(x)}}$$

```
[In] Integrate[Sqrt[a + b*Sinh[x]]*(A + B*Sinh[x]),x]
```

```
[Out] (2*b*B*Cosh[x]*(a + b*Sinh[x]) + 2*(I*a + b)*(3*A*b + a*B)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)] - (2*I)*(
a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[(a + b*
Sinh[x])/(a - I*b)])/(3*b*Sqrt[a + b*Sinh[x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. 2(192) = 384.

Time = 3.80 (sec) , antiderivative size = 731, normalized size of antiderivative = 4.46

method	result
parts	$-\frac{2A(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\left(i\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-i\,\mathrm{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{i}{i}}\right.\right.}{b\cosh(x)\sqrt{a+b\sinh(x)}}$
default	$\frac{\frac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2b}{3}+\frac{2iB\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}}{3}}{}$

```
[In] int((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*A*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*(
(I+sinh(x))*b/(I*b-a))^(1/2)/b*(I*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),
(-(I*b-a)/(I*b+a))^(1/2))*b-I*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I
*b-a)/(I*b+a))^(1/2))*b+EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/
(I*b+a))^(1/2))*a-EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a
))^(1/2))*a)/cosh(x)/(a+b*sinh(x))^(1/2)+2/3*B*(I*(-(a+b*sinh(x))/(I*b-a))^
(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticF
((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*a^2*b+I*(-(a+b*si
nh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))
^(1/2)*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+a))^(1/2))*b
^3-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x)
)*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I*b+
```

```
a))^(1/2))*a^3-(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)
*((I+sinh(x))*b/(I*b-a))^(1/2)*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(
I*b-a)/(I*b+a))^(1/2))*a*b^2+b^3*sinh(x)^3+a*b^2*sinh(x)^2+b^3*sinh(x)+a*b^
2)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.98

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = -\frac{2\left(\sqrt{2}(2\,Ba^2-3\,Aab+3\,Bb^2)\cosh\left(x\right)+\sqrt{2}(2\,Ba^2-3\,Aab+3\,Bb^2)\sinh\left(x\right)\right)\sqrt{b}\text{weierstrassPInverse}\Big(}{}$$

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(sqrt(2)*(2*B*a^2 - 3*A*a*b + 3*B*b^2)*cosh(x) + sqrt(2)*(2*B*a^2 -
 3*A*a*b + 3*B*b^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2
)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b
) + 6*(sqrt(2)*(B*a*b + 3*A*b^2)*cosh(x) + sqrt(2)*(B*a*b + 3*A*b^2)*sinh(x
))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)
/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/
b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(B*b^2*cosh(x)^2 + B*b^2
*sinh(x)^2 + B*b^2 - 2*(B*a*b + 3*A*b^2)*cosh(x) + 2*(B*b^2*cosh(x) - B*a*b
 - 3*A*b^2)*sinh(x))*sqrt(b*sinh(x) + a))/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [F]

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx = \int \left(A+B\sinh\left(x\right)\right)\sqrt{a+b\sinh\left(x\right)}\,dx$$

```
[In] integrate((a+b*sinh(x))**(1/2)*(A+B*sinh(x)),x)
```

```
[Out] Integral((A + B*sinh(x))*sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)\sqrt{b\sinh{(x)}+a}\,dx$$

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx=\int (B\sinh{(x)}+A)\sqrt{b\sinh{(x)}+a}\,dx$$

```
[In] integrate((a+b*sinh(x))^(1/2)*(A+B*sinh(x)),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)*sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\sinh(x)}(A+B\sinh(x))\,dx=\int (A+B\sinh(x))\ \sqrt{a+b\sinh{(x)}}\,dx$$

```
[In] int((A + B*sinh(x))*(a + b*sinh(x))^(1/2),x)
[Out] int((A + B*sinh(x))*(a + b*sinh(x))^(1/2), x)
```

3.129 $\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}-\frac{2(Ab-aB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] B*x/b-2*(A*b-B*a)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/b/(a^2+b^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2814, 2739, 632, 212}

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}-\frac{2(Ab-aB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b - (2*(A*b - a*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b*Sqr
t[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{Bx}{b} - \frac{(i(iAb - iaB)) \int \frac{1}{a+b\sinh(x)}\,dx}{b} \\
&= \frac{Bx}{b} - \frac{(2i(iAb - iaB))\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b} \\
&= \frac{Bx}{b} + \frac{(4i(iAb - iaB))\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b - 2a\tanh\left(\frac{x}{2}\right)\right)}{b} \\
&= \frac{Bx}{b} - \frac{2(Ab - aB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11

$$
\int \frac{A + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{Bx + \frac{2(Ab-aB)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{b}
$$

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x]),x]
[Out] (B*x + (2*(A*b - a*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2])/b
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

method	result
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{2(-Ab+aB)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$
risch	$\frac{Bx}{b}+\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)aB}{\sqrt{a^2+b^2}\,b}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}+\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b}{\sqrt{a^2+b^2}\,b}\right.}{\sqrt{a^2+b^2}\,b}$

```
[In] int((A+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] B/b*ln(tanh(1/2*x)+1)-B/b*ln(tanh(1/2*x)-1)-2*(-A*b+B*a)/b/(a^2+b^2)^(1/2)*
arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(51) = 102.

Time = 0.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.67

$$\int\frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{(Ba-Ab)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^2b+b^3}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((B*a - A*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x)/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.11 (sec) , antiderivative size = 309, normalized size of antiderivative = 5.62

$$\int \frac{A + B\sinh(x)}{a + b\sinh(x)}\,dx = \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right) + Bx\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx}{b} \\ \frac{Ax+B\cosh(x)}{a} \\ \frac{2iA}{b\tanh\left(\frac{x}{2}\right)-ib} + \frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{iBx}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{2B}{b\tanh\left(\frac{x}{2}\right)-ib} \\ -\frac{2iA}{b\tanh\left(\frac{x}{2}\right)+ib} + \frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib} + \frac{iBx}{b\tanh\left(\frac{x}{2}\right)+ib} - \frac{2B}{b\tanh\left(\frac{x}{2}\right)+ib} \\ -\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}} + \frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}} + \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} \end{cases}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), ((A*log(tanh
(x/2)) + B*x)/b, Eq(a, 0)), ((A*x + B*cosh(x))/a, Eq(b, 0)), (2*I*A/(b*tanh
(x/2) - I*b) + B*x*tanh(x/2)/(b*tanh(x/2) - I*b) - I*B*x/(b*tanh(x/2) - I*b
) - 2*B/(b*tanh(x/2) - I*b), Eq(a, -I*b)), (-2*I*A/(b*tanh(x/2) + I*b) + B*
x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*B*x/(b*tanh(x/2) + I*b) - 2*B/(b*tanh(x
/2) + I*b), Eq(a, I*b)), (-A*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqr
t(a**2 + b**2) + A*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b
**2) + B*a*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2))
 - B*a*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) + B
*x/b, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

$$\int \frac{A + B\sinh(x)}{a + b\sinh(x)}\,dx = -B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{x}{b}\right) + \frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^
(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{Bx}{b} - \frac{(Ba-Ab)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a - A*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 269, normalized size of antiderivative = 4.89

$$\int \frac{A+B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{B\,x}{b} - \frac{2\,\mathrm{atan}\left(\frac{b^2\,\mathrm{e}^x\,\sqrt{-a^2\,b^2-b^4}\,\left(\frac{2\,\left(A\,b\,\sqrt{-a^2\,b^2-b^4}-B\,a\,\sqrt{-a^2\,b^2-b^4}\right)}{b^4\,\sqrt{-a^2\,b^2-b^4}\,\sqrt{(A\,b-B\,a)^2}}+\frac{2\,a^2\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{b^2\,\sqrt{-b^2\,\left(a^2+b^2\right)}\,\sqrt{-a^2\,b^2-b^4}\,(A\,b-B\,a)}\right)}{2} - \frac{a\,b\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2 \ldots}}{\sqrt{-b^2\,(a^2+b^2)}\,(A\,b-B \ldots}\right.}{\sqrt{-a^2\,b^2-b^4}}$$

```
[In] int((A + B*sinh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*x)/b - (2*atan((b^2*exp(x)*(- b^4 - a^2*b^2)^(1/2)*((2*(A*b*(- b^4 - a^2
*b^2)^(1/2) - B*a*(- b^4 - a^2*b^2)^(1/2)))/(b^4*(- b^4 - a^2*b^2)^(1/2)*((
A*b - B*a)^2)^(1/2)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^2*(
-b^2*(a^2 + b^2))^(1/2)*(- b^4 - a^2*b^2)^(1/2)*(A*b - B*a))))/2 - (a*b*(A^
2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((-b^2*(a^2 + b^2))^(1/2)*(A*b - B*a)))
*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(- b^4 - a^2*b^2)^(1/2)
```

3.130 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx=-\frac{2(aA+bB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{(Ab-aB)\cosh(x)}{\left(a^2+b^2\right)(a+b\sinh(x))}$$

```
[Out] -2*(A*a+B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-(A*
b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2833, 12, 2739, 632, 212}

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx=-\frac{2(aA+bB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{\cosh(x)(Ab-aB)}{\left(a^2+b^2\right)(a+b\sinh(x))}$$

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*(a*A + b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/
2) - ((A*b - a*B)*Cosh[x])/((a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}-\frac{\int \frac{-aA-bB}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}+\frac{(aA+bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}+\frac{(2(aA+bB))\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}-\frac{(4(aA+bB))\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2(aA+bB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{(Ab-aB)\cosh(x)}{(a^2+b^2)(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2(aA+bB)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{(-Ab+aB)\cosh(x)}{a+b\sinh(x)}}{a^2+b^2}$$

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^2,x]
[Out] ((2*(a*A + b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + ((-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x]))/(a^2 + b^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\left(-\frac{b(Ab-aB)\tanh\left(\frac{x}{2}\right)}{a\left(a^2+b^2\right)}-\frac{Ab-aB}{a^2+b^2}\right)}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{2(Aa+Bb)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}$
risch	$\frac{2(Ab-aB)(\mathrm{e}^x a-b)}{b(a^2+b^2)(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^x a-b)}+\frac{\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)Aa}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)Bb}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(\mathrm{e}^x+\frac{a\left(a^2+\right.}{}\right.}{}$

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*(-b*(A*b-B*a)/a/(a^2+b^2)*tanh(1/2*x)-(A*b-B*a)/(a^2+b^2))/(tanh(1/2*x)^
2*a-2*b*tanh(1/2*x)-a)+2*(A*a+B*b)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/
2*x)-2*b)/(a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(69) = 138.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 6.00

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx =$$

$$-\frac{2\,Ba^3b-2\,Aa^2b^2+2\,Bab^3-2\,Ab^4-\left(Aab^2+Bb^3-(Aab^2+Bb^3)\cosh(x)^2-(Aab^2+Bb^3)\sinh(x)^2-\right.}{a^4b^2+2}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*B*a^3*b - 2*A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4 - (A*a*b^2 + B*b^3 - (A*a*b
^2 + B*b^3)*cosh(x)^2 - (A*a*b^2 + B*b^3)*sinh(x)^2 - 2*(A*a^2*b + B*a*b^2)
*cosh(x) - 2*(A*a^2*b + B*a*b^2 + (A*a*b^2 + B*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 2*(B*a^4 - A*a^3*b + B*a^2*b^2 - A*a*b^3)*cosh(x) - 2*(B*a^4 - A*a
^3*b + B*a^2*b^2 - A*a*b^3)*sinh(x))/(a^4*b^2 + 2*a^2*b^4 + b^6 - (a^4*b^2
+ 2*a^2*b^4 + b^6)*cosh(x)^2 - (a^4*b^2 + 2*a^2*b^4 + b^6)*sinh(x)^2 - 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*cosh(x) - 2*(a^5*b + 2*a^3*b^3 + a*b^5 + (a^4*b^2
 + 2*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(69) = 138$.

Time = 0.29 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.09

$$\begin{aligned}\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx &= A\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(ae^{(-x)}+b\right)}{a^2b+b^3+2\,(a^3+ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}}\right)\\ &\quad + B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(a^2e^{(-x)}+ab\right)}{a^2b^2+b^4+2\,(a^3b+ab^3)e^{(-x)}-(a^2b^2+b^4)e^{(-2\,x)}}\right)\end{aligned}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] A*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))
/(a^2 + b^2)^(3/2) - 2*(a*e^(-x) + b)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x)
 - (a^2*b + b^3)*e^(-2*x))) + B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a^2*e^(-x) + a*b)/(a^
2*b^2 + b^4 + 2*(a^3*b + a*b^3)*e^(-x) - (a^2*b^2 + b^4)*e^(-2*x)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.61

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{(Aa+Bb)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\,(Ba^2e^x - Aabe^x - Bab + Ab^2)}{(a^2b+b^3)(be^{(2\,x)}+2\,ae^x-b)}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (A*a + B*b)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a +
2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(B*a^2*e^x - A*a*b*e^x - B*a*b +
A*b^2)/((a^2*b + b^3)*(b*e^(2*x) + 2*a*e^x - b))
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.01

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\ln\left(\frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,(a^2+b^2)^{3/2}} - \frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3}\right)\,(A\,a+B\,b)}{(a^2+b^2)^{3/2}} - \frac{\ln\left(-\frac{2\,e^x\,(A\,a+B\,b)}{a^2\,b+b^3} - \frac{2\,(b-a\,e^x)\,(A\,a+B\,b)}{b\,(a^2+b^2)^{3/2}}\right)\,(A\,a+B\,b)}{(a^2+b^2)^{3/2}} - \frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,(a^2\,b+b^3)} + \frac{2\,e^x\,(B\,a^2\,b^2-A\,a\,b^3)}{b^2\,(a^2\,b+b^3)}}{2\,a\,e^x-b+b\,e^{2\,x}}$$

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^2,x)
```

```
[Out] (log((2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)^(3/2)) - (2*exp(x)*(A*a
+ B*b))/(a^2*b + b^3))*(A*a + B*b))/(a^2 + b^2)^(3/2) - (log(- (2*exp(x)*(A
*a + B*b))/(a^2*b + b^3) - (2*(b - a*exp(x))*(A*a + B*b))/(b*(a^2 + b^2)^(3
/2)))*(A*a + B*b))/(a^2 + b^2)^(3/2) - ((2*(A*b^3 - B*a*b^2))/(b*(a^2*b + b
^3)) + (2*exp(x)*(B*a^2*b^2 - A*a*b^3))/(b^2*(a^2*b + b^3)))/(2*a*exp(x) -
b + b*exp(2*x))
```

3.131 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3} dx = -\frac{(2a^2A-Ab^2+3abB)\,\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{(Ab-aB)\cosh(x)}{2(a^2+b^2)(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2(a^2+b^2)^2(a+b\sinh(x))}$$

```
[Out] -(2*A*a^2-A*b^2+3*B*a*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(5/2)-1/2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^2-1/2*(3*A*a*b-B*a^2
+2*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2833, 12, 2739, 632, 212}

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3} dx = -\frac{(2a^2A+3abB-Ab^2)\,\mathrm{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{\cosh(x)\left(a^2(-B)+3aAb+2b^2B\right)}{2(a^2+b^2)^2(a+b\sinh(x))} - \frac{\cosh(x)(Ab-aB)}{2(a^2+b^2)(a+b\sinh(x))^2}$$

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

```
[Out] -(((2*a^2*A - A*b^2 + 3*a*b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/
(a^2 + b^2)^(5/2)) - ((A*b - a*B)*Cosh[x])/(2*(a^2 + b^2)*(a + b*Sinh[x])^2
) - ((3*a*A*b - a^2*B + 2*b^2*B)*Cosh[x])/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(Ab-aB)\cosh(x)}{2(a^2+b^2)(a+b\sinh(x))^2} - \frac{\int \frac{-2(aA+bB)+(Ab-aB)\sinh(x)}{(a+b\sinh(x))^2}\,dx}{2(a^2+b^2)} \\
&= -\frac{(Ab-aB)\cosh(x)}{2(a^2+b^2)(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2(a^2+b^2)^2(a+b\sinh(x))} + \frac{\int \frac{2a^2A-Ab^2+3abB}{a+b\sinh(x)}\,dx}{2(a^2+b^2)^2} \\
&= -\frac{(Ab-aB)\cosh(x)}{2(a^2+b^2)(a+b\sinh(x))^2} - \frac{(3aAb-a^2B+2b^2B)\cosh(x)}{2(a^2+b^2)^2(a+b\sinh(x))} \\
&\quad + \frac{(2a^2A-Ab^2+3abB)\int \frac{1}{a+b\sinh(x)}\,dx}{2(a^2+b^2)^2}
\end{aligned}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\left(2a^2A-Ab^2+3abB\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2+b^2\right)^2}$$

$$= -\frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} - \frac{\left(2\left(2a^2A-Ab^2+3abB\right)\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{\left(a^2+b^2\right)^2}$$

$$= -\frac{\left(2a^2A-Ab^2+3abB\right)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{(Ab-aB)\cosh(x)}{2\left(a^2+b^2\right)(a+b\sinh(x))^2} - \frac{\left(3aAb-a^2B+2b^2B\right)\cosh(x)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = \frac{\frac{2\left(2a^2A-Ab^2+3abB\right)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{\left(a^2+b^2\right)(-Ab+aB)\cosh(x)}{(a+b\sinh(x))^2} + \frac{\left(-3aAb+a^2B-2b^2B\right)\cosh(x)}{a+b\sinh(x)}}{2\left(a^2+b^2\right)^2}$$

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^3,x]
```

```
[Out] ((2*(2*a^2*A - A*b^2 + 3*a*b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])
/Sqrt[-a^2 - b^2] + ((a^2 + b^2)*(-(A*b) + a*B)*Cosh[x])/(a + b*Sinh[x])^2
+ ((-3*a*A*b + a^2*B - 2*b^2*B)*Cosh[x])/(a + b*Sinh[x]))/(2*(a^2 + b^2)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(118) = 236.

Time = 0.66 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.45

method	result
default	$-\frac{2\left(-\frac{b\left(5A\,a^{2}b+2A\,b^{3}-3a^{3}B\right)\tanh\left(\frac{x}{2}\right)^{3}}{2a\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{\left(4A\,a^{4}b-7a^{2}A\,b^{3}-2A\,b^{5}-2B\,a^{5}+5a^{3}B\,b^{2}-2Ba\,b^{4}\right)\tanh\left(\frac{x}{2}\right)^{2}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)a^{2}}+\frac{b\left(11A\,a^{2}b+2A\,b^{3}-5a^{3}B+4Ba\,b^{2}\right)\mathrm{t}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)a}}{\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)^{2}}$
risch	$\frac{2A\,a^{2}b^{2}\mathrm{e}^{3x}-A\,b^{4}\mathrm{e}^{3x}+3Ba\,b^{3}\mathrm{e}^{3x}+6A\,a^{3}b\,\mathrm{e}^{2x}-3Aa\,b^{3}\mathrm{e}^{2x}-2B\,a^{4}\mathrm{e}^{2x}+5B\,a^{2}b^{2}\mathrm{e}^{2x}-2B\,b^{4}\mathrm{e}^{2x}-10A\,a^{2}b^{2}\mathrm{e}^{x}-A\,b^{4}\mathrm{e}^{x}+4B\,a^{3}b\,\mathrm{e}^{x}-5Ba\,b^{3}}{b(a^{2}+b^{2})^{2}(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}a-b)^{2}}$

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*b*(5*A*a^2*b+2*A*b^3-3*B*a^3)/a/(a^4+2*a^2*b^2+b^4)*tanh(1/2*x)^3-
1/2*(4*A*a^4*b-7*A*a^2*b^3-2*A*b^5-2*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4+2*a^
2*b^2+b^4)/a^2*tanh(1/2*x)^2+1/2*b*(11*A*a^2*b+2*A*b^3-5*B*a^3+4*B*a*b^2)/(
a^4+2*a^2*b^2+b^4)/a*tanh(1/2*x)+1/2*(4*A*a^2*b+A*b^3-2*B*a^3+B*a*b^2)/(a^4
+2*a^2*b^2+b^4))/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)^2+(2*A*a^2-A*b^2+3*B*a
*b)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1614 vs. $2(119) = 238$.

Time = 0.36 (sec) , antiderivative size = 1614, normalized size of antiderivative = 12.61

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*a^4*b^2 - 6*A*a^3*b^3 - 2*B*a^2*b^4 - 6*A*a*b^5 - 4*B*b^6 - 2*(2*
A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*cosh(x)^3 - 2*(2*A
*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*B*a*b^5 - A*b^6)*sinh(x)^3 + 2*(2*B*
a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B
*b^6)*cosh(x)^2 + 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3*b^3 - 3*B*
a^2*b^4 + 3*A*a*b^5 + 2*B*b^6 - 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 +
3*B*a*b^5 - A*b^6)*cosh(x))*sinh(x)^2 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5 +
(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*
b^5)*sinh(x)^4 + 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^3 + 4*(2*A
*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 + (2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x
))*sinh(x)^3 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
)*cosh(x)^2 + 2*(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5
+ 3*(2*A*a^2*b^3 + 3*B*a*b^4 - A*b^5)*cosh(x)^2 + 6*(2*A*a^3*b^2 + 3*B*a^2*
b^3 - A*a*b^4)*cosh(x))*sinh(x)^2 - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)
*cosh(x) - 4*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4 - (2*A*a^2*b^3 + 3*B*a*b^
4 - A*b^5)*cosh(x)^3 - 3*(2*A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cosh(x)^2 -
```

```
(4*A*a^4*b + 6*B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*cosh(x))*sinh(x
))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a
^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b
*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*a^2*b^4 - 5
*B*a*b^5 - A*b^6)*cosh(x) - 2*(4*B*a^5*b - 10*A*a^4*b^2 - B*a^3*b^3 - 11*A*
a^2*b^4 - 5*B*a*b^5 - A*b^6 + 3*(2*A*a^4*b^2 + 3*B*a^3*b^3 + A*a^2*b^4 + 3*
B*a*b^5 - A*b^6)*cosh(x)^2 - 2*(2*B*a^6 - 6*A*a^5*b - 3*B*a^4*b^2 - 3*A*a^3
*b^3 - 3*B*a^2*b^4 + 3*A*a*b^5 + 2*B*b^6)*cosh(x))*sinh(x))/(a^6*b^3 + 3*a^
4*b^5 + 3*a^2*b^7 + b^9 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^4
 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*sinh(x)^4 + 4*(a^7*b^2 + 3*a^5*b
^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^3 + 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*
b^8 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x))*sinh(x)^3 + 2*(2*a^8
*b + 5*a^6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x)^2 + 2*(2*a^8*b + 5*a^6*
b^3 + 3*a^4*b^5 - a^2*b^7 - b^9 + 3*(a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)
*cosh(x)^2 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2
 - 4*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x) - 4*(a^7*b^2 + 3*a^5
*b^4 + 3*a^3*b^6 + a*b^8 - (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9)*cosh(x)^
3 - 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*cosh(x)^2 - (2*a^8*b + 5*a^
6*b^3 + 3*a^4*b^5 - a^2*b^7 - b^9)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(119) = 238$.

Time = 0.29 (sec) , antiderivative size = 537, normalized size of antiderivative = 4.20

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^3}\,dx$$
$$= \frac{1}{2}\left(\frac{3\,ab\log\left(\frac{be^{(-x)} - a - \sqrt{a^2 + b^2}}{be^{(-x)} - a + \sqrt{a^2 + b^2}}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2 + b^2}} + \frac{2\,(3\,ab^3e^{(-3\,x)} + a^2b^2 - 2\,b^4 + (4\,a^3b - 5\,ab}{a^4b^3 + 2\,a^2b^5 + b^7 + 4\,(a^5b^2 + 2\,a^3b^4 + ab^6)e^{(-x)} + 2\,(2\,a^6b + 3\,a^4b^3 - b^7}\right.$$
$$+ \frac{1}{2}\,A\left(\frac{(2\,a^2 - b^2)\log\left(\frac{be^{(-x)} - a - \sqrt{a^2 + b^2}}{be^{(-x)} - a + \sqrt{a^2 + b^2}}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2 + b^2}} - \frac{2\,(3\,ab^2 + (10\,a^2b + b^3)e^{(-x)} +}{a^4b^2 + 2\,a^2b^4 + b^6 + 4\,(a^5b + 2\,a^3b^3 + ab^5)e^{(-x)} + 2\,(2\,a^6 + 3\,a}\right.$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 +
b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^3*e^(-3*x) + a^
2*b^2 - 2*b^4 + (4*a^3*b - 5*a*b^3)*e^(-x) + (2*a^4 - 5*a^2*b^2 + 2*b^4)*e^
(-2*x))/(a^4*b^3 + 2*a^2*b^5 + b^7 + 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*e^(-x)
 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*e^(-2*x) - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)
*e^(-3*x) + (a^4*b^3 + 2*a^2*b^5 + b^7)*e^(-4*x)))*B + 1/2*A*((2*a^2 - b^2)
*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a
^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a*b^2 + (10*a^2*b + b^3)*e^(-
x) + 3*(2*a^3 - a*b^2)*e^(-2*x) - (2*a^2*b - b^3)*e^(-3*x))/(a^4*b^2 + 2*a^
2*b^4 + b^6 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-x) + 2*(2*a^6 + 3*a^4*b^2 -
 b^6)*e^(-2*x) - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*e^(-3*x) + (a^4*b^2 + 2*a^2*
b^4 + b^6)*e^(-4*x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(119) = 238$.

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.18

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx = -\frac{(2\,Aa^2+3\,Bab-Ab^2)\log\left(\frac{\left|-2\,be^x-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|-2\,be^x-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\,Aa^2b^2e^{(3\,x)}+3\,Bab^3e^{(3\,x)}-Ab^4e^{(3\,x)}-2\,Ba^4e^{(2\,x)}+6\,Aa^3be^{(2\,x)}+5\,Ba^2b^2e^{(2\,x)}-3\,Aab^3e^{(2\,x)}-2\,Bb^4e^{(}}{(a^4b+2\,a^2b^3+b^5)(be^{(2\,x)}+2\,ae^{x}}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*A*a^2 + 3*B*a*b - A*b^2)*log(abs(-2*b*e^x - 2*a - 2*sqrt(a^2 + b^2)
)/abs(-2*b*e^x - 2*a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^
2 + b^2)) + (2*A*a^2*b^2*e^(3*x) + 3*B*a*b^3*e^(3*x) - A*b^4*e^(3*x) - 2*B*
a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) + 5*B*a^2*b^2*e^(2*x) - 3*A*a*b^3*e^(2*x) -
 2*B*b^4*e^(2*x) + 4*B*a^3*b*e^x - 10*A*a^2*b^2*e^x - 5*B*a*b^3*e^x - A*b^4
*e^x - B*a^2*b^2 + 3*A*a*b^3 + 2*B*b^4)/((a^4*b + 2*a^2*b^3 + b^5)*(b*e^(2*
x) + 2*a*e^x - b)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^3}\,dx=\int \frac{A+B\sinh(x)}{(a+b\sinh{(x)})^3}\,dx$$

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^3,x)
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^3, x)
```

3.132 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 187

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = -\frac{(2a^3A-3aAb^2+4a^2bB-b^3B)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2)}} - \frac{(Ab-aB)\cosh(x)}{3\,(a^2+b^2)\,(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6\,(a^2+b^2)^2\,(a+b\sinh(x))^2} - \frac{(11a^2Ab-4Ab^3-2a^3B+13ab^2B)\cosh(x)}{6\,(a^2+b^2)^3\,(a+b\sinh(x))}$$

```
[Out] -(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1
/2))/(a^2+b^2)^(7/2)-1/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^3-1/6*(5
*A*a*b-2*B*a^2+3*B*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^2-1/6*(11*A*a^2*b
-4*A*b^3-2*B*a^3+13*B*a*b^2)*cosh(x)/(a^2+b^2)^3/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2833, 12, 2739, 632, 212}

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = -\frac{\cosh(x)\left(-2a^2B+5aAb+3b^2B\right)}{6\,(a^2+b^2)^2\,(a+b\sinh(x))^2} - \frac{\cosh(x)(Ab-aB)}{3\,(a^2+b^2)\,(a+b\sinh(x))^3} - \frac{(2a^3A+4a^2bB-3aAb^2-b^3B)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{\cosh(x)\left(-2a^3B+11a^2Ab+13ab^2B-4Ab^3\right)}{6\,(a^2+b^2)^3\,(a+b\sinh(x))}$$

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt
[a^2 + b^2]])/(a^2 + b^2)^(7/2)) - ((A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a
+ b*Sinh[x])^3) - ((5*a*A*b - 2*a^2*B + 3*b^2*B)*Cosh[x])/(6*(a^2 + b^2)^2*
(a + b*Sinh[x])^2) - ((11*a^2*A*b - 4*A*b^3 - 2*a^3*B + 13*a*b^2*B)*Cosh[x]
)/(6*(a^2 + b^2)^3*(a + b*Sinh[x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^3}-\frac{\int \frac{-3(aA+bB)+2(Ab-aB)\sinh(x)}{(a+b\sinh(x))^3}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6(a^2+b^2)^2(a+b\sinh(x))^2} + \frac{\int \frac{2(3a^2A-2Ab^2+5abB)-(5aAb-2a^2B+3b^2B)\sinh(x)}{(a+b\sinh(x))^2}\,dx}{6(a^2+b^2)^2}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6(a^2+b^2)^2(a+b\sinh(x))^2} - \frac{(11a^2Ab-4Ab^3-2a^3B+13ab^2B)\cosh(x)}{6(a^2+b^2)^3(a+b\sinh(x))} - \frac{\int -\frac{3(2a^3A-3aAb^2+4a^2bB-b^3B)}{a+b\sinh(x)}\,dx}{6(a^2+b^2)^3}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6(a^2+b^2)^2(a+b\sinh(x))^2} - \frac{(11a^2Ab-4Ab^3-2a^3B+13ab^2B)\cosh(x)}{6(a^2+b^2)^3(a+b\sinh(x))} + \frac{(2a^3A-3aAb^2+4a^2bB-b^3B)\int \frac{1}{a+b\sinh(x)}\,dx}{2(a^2+b^2)^3}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6(a^2+b^2)^2(a+b\sinh(x))^2} - \frac{(11a^2Ab-4Ab^3-2a^3B+13ab^2B)\cosh(x)}{6(a^2+b^2)^3(a+b\sinh(x))} + \frac{(2a^3A-3aAb^2+4a^2bB-b^3B)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= -\frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6(a^2+b^2)^2(a+b\sinh(x))^2} - \frac{(11a^2Ab-4Ab^3-2a^3B+13ab^2B)\cosh(x)}{6(a^2+b^2)^3(a+b\sinh(x))} - \frac{(2(2a^3A-3aAb^2+4a^2bB-b^3B))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= -\frac{(2a^3A-3aAb^2+4a^2bB-b^3B)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^3} - \frac{(5aAb-2a^2B+3b^2B)\cosh(x)}{6(a^2+b^2)^2(a+b\sinh(x))^2} - \frac{(11a^2Ab-4Ab^3-2a^3B+13ab^2B)\cosh(x)}{6(a^2+b^2)^3(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx$$
$$=\frac{\frac{6(2a^3A-3aAb^2+4a^2bB-b^3B)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+\frac{2(a^2+b^2)^2(-Ab+aB)\cosh(x)}{(a+b\sinh(x))^3}+\frac{(a^2+b^2)(-5aAb+2a^2B-3b^2B)\cosh(x)}{(a+b\sinh(x))^2}+\frac{(-11\ldots}{\ldots}}{6\left(a^2+b^2\right)^3}$$

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^4,x]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*b^2 + 4*a^2*b*B - b^3*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt
[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (2*(a^2 + b^2)^2*(-(A*b) + a*B)*Cosh[x])/
(a + b*Sinh[x])^3 + ((a^2 + b^2)*(-5*a*A*b + 2*a^2*B - 3*b^2*B)*Cosh[x])/(a
 + b*Sinh[x])^2 + ((-11*a^2*A*b + 4*A*b^3 + 2*a^3*B - 13*a*b^2*B)*Cosh[x])/
(a + b*Sinh[x]))/(6*(a^2 + b^2)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs. $2(175) = 350$.

Time = 0.83 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.39

method	result
default	$-\frac{2\left(-\frac{b\left(9A\,a^4b+6a^2A\,b^3+2A\,b^5-4B\,a^5+a^3B\,b^2\right)\tanh\left(\frac{x}{2}\right)^5}{2a\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{\left(6A\,a^6b-27A\,a^4b^3-12A\,a^2b^5-4A\,b^7-2B\,a^7+14B\,a^5b^2-11B\,a^3b^4-2Ba\,b^6\right)\tan\ldots}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)a^2}\ldots}{\ldots}$
risch	$\frac{-8B\,a^6e^{3x}-12A\,b^6e^{2x}-13Ba\,b^5-3B\,b^6e^{5x}-11A\,a^2b^4+2B\,a^3b^3+3B\,b^6e^{x}-78B\,a^2b^4e^{3x}-102A\,a^4b^2e^{2x}+36A\,a^2b^4e^{2x}+24B\,a^5b\,e^{2x}\ldots}{\ldots}$

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*b*(9*A*a^4*b+6*A*a^2*b^3+2*A*b^5-4*B*a^5+B*a^3*b^2)/a/(a^6+3*a^4*b
^2+3*a^2*b^4+b^6)*tanh(1/2*x)^5-1/2*(6*A*a^6*b-27*A*a^4*b^3-12*A*a^2*b^5-4*
A*b^7-2*B*a^7+14*B*a^5*b^2-11*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)/a^2*tanh(1/2*x)^4+1/3/a^3*b*(54*A*a^6*b-21*A*a^4*b^3-4*A*a^2*b^5-4*A*
b^7-18*B*a^7+42*B*a^5*b^2-17*B*a^3*b^4-2*B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6)*tanh(1/2*x)^3+1/a^2*(6*A*a^6*b-20*A*a^4*b^3-3*A*a^2*b^5-2*A*b^7-2*B*a^
7+10*B*a^5*b^2-14*B*a^3*b^4-B*a*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2
*x)^2-1/2/a*b*(27*A*a^4*b+4*A*a^2*b^3+2*A*b^5-8*B*a^5+19*B*a^3*b^2+2*B*a*b^
4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tanh(1/2*x)-1/6*(18*A*a^4*b+5*A*a^2*b^3+2*
A*b^5-6*B*a^5+10*B*a^3*b^2+B*a*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6))/(tanh(1/
```

```
2*x)^2*a-2*b*tanh(1/2*x)-a)^3+(2*A*a^3-3*A*a*b^2+4*B*a^2*b-B*b^3)/(a^6+3*a^
4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2
+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3870 vs. $2(177) = 354$.

Time = 0.42 (sec) , antiderivative size = 3870, normalized size of antiderivative = 20.70

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*B*a^5*b^3 - 22*A*a^4*b^4 - 22*B*a^3*b^5 - 14*A*a^2*b^6 - 26*B*a*b^7
 + 8*A*b^8 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a
*b^7 - B*b^8)*cosh(x)^5 + 6*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^
2*b^6 - 3*A*a*b^7 - B*b^8)*sinh(x)^5 + 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^
4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^4 + 30*(2*A*a^6*b^2 +
4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7 + (2*A*a^5*b^
3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*cosh(x))*sin
h(x)^4 - 4*(4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^
4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x)^3 - 4*(4*B*a^8 - 22*A
*a^7*b - 28*B*a^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^
2*b^6 - 12*A*a*b^7 - 15*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^
6 - 3*A*a*b^7 - B*b^8)*cosh(x)^2 - 30*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^
4 + 3*B*a^3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x))*sinh(x)^3 + 12*(4*B*a^7*b
 - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6
+ 4*B*a*b^7 - 2*A*b^8)*cosh(x)^2 + 12*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*
b^3 - 11*A*a^4*b^4 - 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8 + 5*(
2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*b^7 - B*b^8)*co
sh(x)^3 + 15*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^3*b^5 - 3*A*a^2
*b^6 - B*a*b^7)*cosh(x)^2 - (4*B*a^8 - 22*A*a^7*b - 28*B*a^6*b^2 + 19*A*a^5
*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*b^7)*cosh(x))*sin
h(x)^2 + 3*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7 - (2*A*a^3*b^4 +
4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^6 - (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3
*A*a*b^6 - B*b^7)*sinh(x)^6 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 -
B*a*b^6)*cosh(x)^5 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 +
 (2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x))*sinh(x)^5 - 3*(8*
A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*
cosh(x)^4 - 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*
A*a*b^6 + B*b^7 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)
^2 + 10*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x))*sinh(x
)^4 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^
5 + 3*B*a*b^6)*cosh(x)^3 - 4*(4*A*a^6*b + 8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B
```

*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6 + 5*(2*A*a^3*b^4 + 4*B*a^2*b^5 - 3*A*a*b
^6 - B*b^7)*cosh(x)^3 + 15*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b
^6)*cosh(x)^2 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5
+ 3*A*a*b^6 + B*b^7)*cosh(x))*sinh(x)^3 + 3*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 1
4*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 + 3*(8*A*a^5*b^2 +
16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7 - 5*(2*A*a^3
*b^4 + 4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^4 - 20*(2*A*a^4*b^3 + 4*B*a
^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(x)^3 - 6*(8*A*a^5*b^2 + 16*B*a^4*b^3 -
14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^2 - 4*(4*A*a^6*b +
8*B*a^5*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(
x))*sinh(x)^2 - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6)*cosh(
x) - 6*(2*A*a^4*b^3 + 4*B*a^3*b^4 - 3*A*a^2*b^5 - B*a*b^6 + (2*A*a^3*b^4 +
4*B*a^2*b^5 - 3*A*a*b^6 - B*b^7)*cosh(x)^5 + 5*(2*A*a^4*b^3 + 4*B*a^3*b^4 -
3*A*a^2*b^5 - B*a*b^6)*cosh(x)^4 + 2*(8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^
3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7)*cosh(x)^3 + 2*(4*A*a^6*b + 8*B*a^5
*b^2 - 12*A*a^4*b^3 - 14*B*a^3*b^4 + 9*A*a^2*b^5 + 3*B*a*b^6)*cosh(x)^2 - (
8*A*a^5*b^2 + 16*B*a^4*b^3 - 14*A*a^3*b^4 - 8*B*a^2*b^5 + 3*A*a*b^6 + B*b^7
)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*
a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^
2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
2*(b*cosh(x) + a)*sinh(x) - b)) - 6*(4*B*a^6*b^2 - 20*A*a^5*b^3 - 18*B*a^4*
b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^7 - B*b^8)*cosh(x) - 6*(4*B*a^6
*b^2 - 20*A*a^5*b^3 - 18*B*a^4*b^4 - 15*A*a^3*b^5 - 23*B*a^2*b^6 + 5*A*a*b^
7 - B*b^8 - 5*(2*A*a^5*b^3 + 4*B*a^4*b^4 - A*a^3*b^5 + 3*B*a^2*b^6 - 3*A*a*
b^7 - B*b^8)*cosh(x)^4 - 20*(2*A*a^6*b^2 + 4*B*a^5*b^3 - A*a^4*b^4 + 3*B*a^
3*b^5 - 3*A*a^2*b^6 - B*a*b^7)*cosh(x)^3 + 2*(4*B*a^8 - 22*A*a^7*b - 28*B*a
^6*b^2 + 19*A*a^5*b^3 + 7*B*a^4*b^4 + 29*A*a^3*b^5 + 39*B*a^2*b^6 - 12*A*a*
b^7)*cosh(x)^2 - 4*(4*B*a^7*b - 17*A*a^6*b^2 - 13*B*a^5*b^3 - 11*A*a^4*b^4
- 13*B*a^3*b^5 + 4*A*a^2*b^6 + 4*B*a*b^7 - 2*A*b^8)*cosh(x))*sinh(x))/(a^8*
b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12 - (a^8*b^4 + 4*a^6*b^6 + 6*
a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^6 - (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 +
4*a^2*b^10 + b^12)*sinh(x)^6 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*
b^9 + a*b^11)*cosh(x)^5 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
a*b^11 + (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x))*sin
h(x)^5 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(
x)^4 - 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12 + 5*(a^8
*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^2 + 10*(a^9*b^3 +
4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x))*sinh(x)^4 - 4*(2*a^11
*b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11)*cosh(x)^3 - 4*(2*a^11*
b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11 + 5*(a^8*b^4 + 4*a^6*b^6
+ 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^3 + 15*(a^9*b^3 + 4*a^7*b^5 + 6*a
^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x)^2 + 3*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^
6*b^6 + 10*a^4*b^8 - b^12)*cosh(x))*sinh(x)^3 + 3*(4*a^10*b^2 + 15*a^8*b^4
+ 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x)^2 + 3*(4*a^10*b^2 + 15*a^8*b^4 +
20*a^6*b^6 + 10*a^4*b^8 - b^12 - 5*(a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2

```
*b^10 + b^12)*cosh(x)^4 - 20*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
 a*b^11)*cosh(x)^3 - 6*(4*a^10*b^2 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 -
 b^12)*cosh(x)^2 - 4*(2*a^11*b + 5*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*
b^11)*cosh(x))*sinh(x)^2 - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 +
 a*b^11)*cosh(x) - 6*(a^9*b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11
+ (a^8*b^4 + 4*a^6*b^6 + 6*a^4*b^8 + 4*a^2*b^10 + b^12)*cosh(x)^5 + 5*(a^9*
b^3 + 4*a^7*b^5 + 6*a^5*b^7 + 4*a^3*b^9 + a*b^11)*cosh(x)^4 + 2*(4*a^10*b^2
 + 15*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x)^3 + 2*(2*a^11*b + 5
*a^9*b^3 - 10*a^5*b^7 - 10*a^3*b^9 - 3*a*b^11)*cosh(x)^2 - (4*a^10*b^2 + 15
*a^8*b^4 + 20*a^6*b^6 + 10*a^4*b^8 - b^12)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 982 vs. $2(177) = 354$.

Time = 0.31 (sec) , antiderivative size = 982, normalized size of antiderivative = 5.25

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*a^2 - 3*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a
 + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2))
- 2*(11*a^2*b^3 - 4*b^5 + 15*(4*a^3*b^2 - a*b^4)*e^(-x) + 6*(17*a^4*b - 6*a
^2*b^3 + 2*b^5)*e^(-2*x) + 2*(22*a^5 - 41*a^3*b^2 + 12*a*b^4)*e^(-3*x) - 15
*(2*a^4*b - 3*a^2*b^3)*e^(-4*x) + 3*(2*a^3*b^2 - 3*a*b^4)*e^(-5*x))/(a^6*b^
3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + 6*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^
8)*e^(-x) + 3*(4*a^8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-2*x) +
 4*(2*a^9 + 3*a^7*b^2 - 3*a^5*b^4 - 7*a^3*b^6 - 3*a*b^8)*e^(-3*x) - 3*(4*a^
8*b + 11*a^6*b^3 + 9*a^4*b^5 + a^2*b^7 - b^9)*e^(-4*x) + 6*(a^7*b^2 + 3*a^5
*b^4 + 3*a^3*b^6 + a*b^8)*e^(-5*x) - (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9
)*e^(-6*x)))*A + 1/6*B*(3*(4*a^2*b - b^3)*log((b*e^(-x) - a - sqrt(a^2 + b^
2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*
sqrt(a^2 + b^2)) + 2*(2*a^3*b^3 - 13*a*b^5 + 3*(4*a^4*b^2 - 22*a^2*b^4 - b^
```

```
6)*e^(-x) + 6*(4*a^5*b - 17*a^3*b^3 + 4*a*b^5)*e^(-2*x) + 2*(4*a^6 - 32*a^4
*b^2 + 39*a^2*b^4)*e^(-3*x) + 15*(4*a^3*b^3 - a*b^5)*e^(-4*x) - 3*(4*a^2*b^
4 - b^6)*e^(-5*x))/(a^6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10 + 6*(a^7*b^3 + 3
*a^5*b^5 + 3*a^3*b^7 + a*b^9)*e^(-x) + 3*(4*a^8*b^2 + 11*a^6*b^4 + 9*a^4*b^
6 + a^2*b^8 - b^10)*e^(-2*x) + 4*(2*a^9*b + 3*a^7*b^3 - 3*a^5*b^5 - 7*a^3*b
^7 - 3*a*b^9)*e^(-3*x) - 3*(4*a^8*b^2 + 11*a^6*b^4 + 9*a^4*b^6 + a^2*b^8 -
b^10)*e^(-4*x) + 6*(a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*e^(-5*x) - (a^
6*b^4 + 3*a^4*b^6 + 3*a^2*b^8 + b^10)*e^(-6*x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(177) = 354$.

Time = 0.29 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.55

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^4}\,dx = \frac{(2\,Aa^3 + 4\,Ba^2b - 3\,Aab^2 - Bb^3)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\sqrt{a^2+b^2}} + \frac{6\,Aa^3b^3e^{(5\,x)} + 12\,Ba^2b^4e^{(5\,x)} - 9\,Aab^5e^{(5\,x)} - 3\,Bb^6e^{(5\,x)} + 30\,Aa^4b^2e^{(4\,x)} + 60\,Ba^3b^3e^{(4\,x)} - 45\,Aa^2b^4e^{(4}}{}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*A*a^3 + 4*B*a^2*b - 3*A*a*b^2 - B*b^3)*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 1/3*(6*A*a^3*b^3*e^(5*x) + 12*B*a^2*b^4*
e^(5*x) - 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x) + 30*A*a^4*b^2*e^(4*x) + 60*B
*a^3*b^3*e^(4*x) - 45*A*a^2*b^4*e^(4*x) - 15*B*a*b^5*e^(4*x) - 8*B*a^6*e^(3
*x) + 44*A*a^5*b*e^(3*x) + 64*B*a^4*b^2*e^(3*x) - 82*A*a^3*b^3*e^(3*x) - 78
*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) + 24*B*a^5*b*e^(2*x) - 102*A*a^4*b^
2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a^2*b^4*e^(2*x) + 24*B*a*b^5*e^(2*
x) - 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x + 60*A*a^3*b^3*e^x + 66*B*a^2*b^4*
e^x - 15*A*a*b^5*e^x + 3*B*b^6*e^x + 2*B*a^3*b^3 - 11*A*a^2*b^4 - 13*B*a*b^
5 + 4*A*b^6)/((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*(b*e^(2*x) + 2*a*e^x -
b)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^4}\,dx = \int \frac{A+B\sinh(x)}{(a+b\sinh{(x)})^4}\,dx$$

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^4,x)
```

```
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^4, x)
```

3.133 $\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}+\frac{2(a^2-b^2)\,B\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}$$

```
[Out] B*x/b+2*(a^2-b^2)*B*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a/b/(a^2+b^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2814, 2739, 632, 212}

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{2B(a^2-b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}+\frac{Bx}{b}$$

```
[In] Int[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b + (2*(a^2 - b^2)*B*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a*b
*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{Bx}{b} - \frac{\left(i\left(-iaB + \frac{ib^2B}{a}\right)\right)\int \frac{1}{a+b\sinh(x)}\,dx}{b}\\ &= \frac{Bx}{b} - \frac{\left(2i\left(-iaB + \frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b} + \frac{\left(4i\left(-iaB + \frac{ib^2B}{a}\right)\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b}\\ &= \frac{Bx}{b} + \frac{2(a^2-b^2)\,B\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{ab\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\frac{bB}{a} + B\sinh(x)}{a+b\sinh(x)}\,dx = \frac{B\left(ax - \frac{2(a^2-b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}\right)}{ab}$$

```
[In] Integrate[((b*B)/a + B*Sinh[x])/(a + b*Sinh[x]),x]
[Out] (B*(a*x - (2*(a^2 - b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-
a^2 - b^2]))/(a*b)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

method	result
default	$\frac{2B\left(-\frac{(a^2-b^2)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2b}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}\right)}{a}$
risch	$\frac{Bx}{b}+\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)aB}{\sqrt{a^2+b^2}\,b}-\frac{Bb\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,a}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)aB}{\sqrt{a^2+b^2}\,b}+\frac{Bb\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2+b^2}}{\sqrt{a^2+}}\right.}{\sqrt{a^2+b^2}\,a}$

[In] int((b*B/a+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)

[Out] 2*B/a*(-(a^2-b^2)/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+1/2*a/b*ln(tanh(1/2*x)+1)-1/2*a/b*ln(tanh(1/2*x)-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(56) = 112$.

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.57

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = -\frac{(Ba^2-Bb^2)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^3b+ab^3}$$

[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")

[Out] -((B*a^2 - B*b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - (B*a^3 + B*a*b^2)*x)/(a^3*b + a*b^3)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.51 (sec) , antiderivative size = 258, normalized size of antiderivative = 4.30

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = \begin{cases} \text{NaN} \\ \frac{B\cosh(x)}{a} \\ \frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{iBx}{b\tanh\left(\frac{x}{2}\right)-ib} - \frac{4B}{b\tanh\left(\frac{x}{2}\right)-ib} \\ \frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib} + \frac{iBx}{b\tanh\left(\frac{x}{2}\right)+ib} - \frac{4B}{b\tanh\left(\frac{x}{2}\right)+ib} \\ \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} - \frac{Ba\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{b\sqrt{a^2+b^2}} + \frac{Bx}{b} - \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a\sqrt{a^2+b^2}} + \frac{Bb\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\right.}{a\sqrt{a^2+b^2}} \end{cases}$$

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*cosh(x)/a, Eq(b, 0)), (B*x*tanh(x/
2)/(b*tanh(x/2) - I*b) - I*B*x/(b*tanh(x/2) - I*b) - 4*B/(b*tanh(x/2) - I*b
), Eq(a, -I*b)), (B*x*tanh(x/2)/(b*tanh(x/2) + I*b) + I*B*x/(b*tanh(x/2) +
I*b) - 4*B/(b*tanh(x/2) + I*b), Eq(a, I*b)), (B*a*log(tanh(x/2) - b/a - sqr
t(a**2 + b**2)/a)/(b*sqrt(a**2 + b**2)) - B*a*log(tanh(x/2) - b/a + sqrt(a*
*2 + b**2)/a)/(b*sqrt(a**2 + b**2)) + B*x/b - B*b*log(tanh(x/2) - b/a - sqr
t(a**2 + b**2)/a)/(a*sqrt(a**2 + b**2)) + B*b*log(tanh(x/2) - b/a + sqrt(a*
*2 + b**2)/a)/(a*sqrt(a**2 + b**2)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(56) = 112.

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.13

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{x}{b}\right) + \frac{Bb\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a}$$

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + B*b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}-\frac{(Ba^2-Bb^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}ab}$$

```
[In] integrate((b*B/a+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] B*x/b - (B*a^2 - B*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*
e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a*b)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 331, normalized size of antiderivative = 5.52

$$\int \frac{\frac{bB}{a}+B\sinh(x)}{a+b\sinh(x)}\,dx$$

$$=\frac{2\,\mathrm{atan}\left(\frac{a\,b^2\,\mathrm{e}^x\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(\frac{2\,\left(B\,a^2\,\sqrt{-a^4\,b^2-a^2\,b^4}-B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\right)}{a^2\,b^4\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\sqrt{B^2\,\left(a^2-b^2\right)^2}}+\frac{2\,a^2\,\sqrt{B^2\,a^4-2\,B^2\,a^2\,b^2+B^2\,b^4}}{B\,b^2\,\sqrt{-a^4\,b^2-a^2\,b^4}\,\left(a^2-b^2\right)\,\sqrt{-a^2\,b^2\,\left(a^2+b^2\right)}}\right)}{2}-\frac{a^2\,b\,\sqrt{B^2\,a \cdots}}{B\,(a^2-b^2 \cdots}\right.}{\sqrt{-a^4\,b^2-a^2\,b^4}}$$
$$+\frac{B\,x}{b}$$

```
[In] int((B*sinh(x) + (B*b)/a)/(a + b*sinh(x)),x)
```

```
[Out] (2*atan((a*b^2*exp(x)*(- a^2*b^4 - a^4*b^2)^(1/2)*((2*(B*a^2*(- a^2*b^4 - a
^4*b^2)^(1/2) - B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)))/(a^2*b^4*(- a^2*b^4 - a
^4*b^2)^(1/2)*(B^2*(a^2 - b^2)^2)^(1/2)) + (2*a^2*(B^2*a^4 + B^2*b^4 - 2*B^
2*a^2*b^2)^(1/2))/(B*b^2*(- a^2*b^4 - a^4*b^2)^(1/2)*(a^2 - b^2)*(-a^2*b^2*
(a^2 + b^2))^(1/2))))/2 - (a^2*b*(B^2*a^4 + B^2*b^4 - 2*B^2*a^2*b^2)^(1/2))
/(B*(a^2 - b^2)*(-a^2*b^2*(a^2 + b^2))^(1/2)))*(B^2*a^4 + B^2*b^4 - 2*B^2*a
^2*b^2)^(1/2))/(- a^2*b^4 - a^4*b^2)^(1/2) + (B*x)/b
```

3.134 $\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 6

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

```
[Out] B*x/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

```
[In] Int[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{B\int 1\,dx}{b} \\ &= \frac{Bx}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

```
[In] Integrate[((a*B)/b + B*Sinh[x])/(a + b*Sinh[x]),x]
[Out] (B*x)/b
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{Bx}{b}$	7
risch	$\frac{Bx}{b}$	7

```
[In] int((a*B/b+B*sinh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] B*x/b
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b}+B\sinh(x)}{a+b\sinh(x)}\,dx=\frac{Bx}{b}$$

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="fricas")
[Out] B*x/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{Bx}{b}$$

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x)
[Out] B*x/b
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 21.33

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = -B\left(\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b} - \frac{x}{b}\right) + \frac{Ba\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b}$$

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -B*(a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*b) - x/b) + B*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*
e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{Bx}{b}$$

```
[In] integrate((a*B/b+B*sinh(x))/(a+b*sinh(x)),x, algorithm="giac")
[Out] B*x/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\sinh(x)}{a + b\sinh(x)}\,dx = \frac{B\,x}{b}$$

```
[In] int((B*sinh(x) + (B*a)/b)/(a + b*sinh(x)),x)
[Out] (B*x)/b
```

3.135 $\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = -\frac{\cosh(x)}{b+a\sinh(x)}$$

```
[Out] -cosh(x)/(b+a*sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2833, 8}

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = -\frac{\cosh(x)}{a\sinh(x)+b}$$

```
[In] Int[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
```

```
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\cosh(x)}{b+a\sinh(x)} - \frac{\int 0\,dx}{a^2+b^2}\\ &= -\frac{\cosh(x)}{b+a\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = -\frac{\cosh(x)}{b+a\sinh(x)}$$

```
[In] Integrate[(a - b*Sinh[x])/(b + a*Sinh[x])^2,x]
[Out] -(Cosh[x]/(b + a*Sinh[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

method	result	size
parallelrisch	$\frac{-a\sinh(x)-b(\cosh(x)+1)}{b(b+a\sinh(x))}$	26
risch	$-\frac{2(-e^{x}b+a)}{a(e^{2x}a+2e^{x}b-a)}$	30
default	$-\frac{2\left(\frac{a\tanh\left(\frac{x}{2}\right)}{2b}+\frac{1}{2}\right)}{-\frac{\tanh\left(\frac{x}{2}\right)^{2}b}{2}+a\tanh\left(\frac{x}{2}\right)+\frac{b}{2}}$	36

```
[In] int((a-b*sinh(x))/(b+a*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] (-a*sinh(x)-b*(cosh(x)+1))/b/(b+a*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(12) = 24$.

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = \frac{2\,(b\cosh(x)+b\sinh(x)-a)}{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,(a^2\cosh(x)+ab)\sinh(x)}$$

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="fricas")
[Out] 2*(b*cosh(x) + b*sinh(x) - a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x
) - a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 19.17

$$\int \frac{a-b\sinh(x)}{(b+a\sinh(x))^2}\,dx = -b\left(\frac{a\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\,(b^2e^{(-x)}+ab)}{a^4+a^2b^2+2\,(a^3b+ab^3)e^{(-x)}-(a^4+a^2b^2)e^{(-2\,x)}}\right) + a\left(\frac{b\log\left(\frac{ae^{(-x)}-b-\sqrt{a^2+b^2}}{ae^{(-x)}-b+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,(be^{(-x)}+a)}{a^3+ab^2+2\,(a^2b+b^3)e^{(-x)}-(a^3+ab^2)e^{(-2\,x)}}\right)$$

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="maxima")
[Out] -b*(a*log((a*e^(-x) - b - sqrt(a^2 + b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2))
)/(a^2 + b^2)^(3/2) + 2*(b^2*e^(-x) + a*b)/(a^4 + a^2*b^2 + 2*(a^3*b + a*b^
3)*e^(-x) - (a^4 + a^2*b^2)*e^(-2*x))) + a*(b*log((a*e^(-x) - b - sqrt(a^2
+ b^2))/(a*e^(-x) - b + sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^(-x) +
 a)/(a^3 + a*b^2 + 2*(a^2*b + b^3)*e^(-x) - (a^3 + a*b^2)*e^(-2*x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{a - b\sinh(x)}{(b + a\sinh(x))^2}\,dx = \frac{2\,(be^x - a)}{(ae^{(2\,x)} + 2\,be^x - a)a}$$

```
[In] integrate((a-b*sinh(x))/(b+a*sinh(x))^2,x, algorithm="giac")
[Out] 2*(b*e^x - a)/((a*e^(2*x) + 2*b*e^x - a)*a)
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int \frac{a - b\sinh(x)}{(b + a\sinh(x))^2}\,dx = \frac{\frac{2\,e^x\,(a^3\,b + a\,b^3)}{a\,(a^3 + a\,b^2)} - 2}{2\,b\,e^x - a + a\,e^{2\,x}}$$

```
[In] int((a - b*sinh(x))/(b + a*sinh(x))^2,x)
[Out] ((2*exp(x)*(a*b^3 + a^3*b))/(a*(a*b^2 + a^3)) - 2)/(2*b*exp(x) - a + a*exp(
2*x))
```

3.136 $\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = -x+\frac{4x}{\sqrt{5}}-\frac{8\operatorname{arctanh}\left(\frac{\cosh(x)}{2+\sqrt{5}+\sinh(x)}\right)}{\sqrt{5}}$$

```
[Out] -x+4/5*x*5^(1/2)-8/5*arctanh(cosh(x)/(2+sinh(x)+5^(1/2)))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2814, 2736}

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = -\frac{8\operatorname{arctanh}\left(\frac{\cosh(x)}{\sinh(x)+\sqrt{5}+2}\right)}{\sqrt{5}}+\frac{4x}{\sqrt{5}}-x$$

```
[In] Int[(2 - Sinh[x])/(2 + Sinh[x]),x]
```

```
[Out] -x + (4*x)/Sqrt[5] - (8*ArcTanh[Cosh[x]/(2 + Sqrt[5] + Sinh[x])])/Sqrt[5]
```

Rule 2736

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\text{integral} = -x + 4\int \frac{1}{2+\sinh(x)}\,dx$$
$$= -x + \frac{4x}{\sqrt{5}} - \frac{8\text{arctanh}\left(\frac{\cosh(x)}{2+\sqrt{5}+\sinh(x)}\right)}{\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = -x - \frac{8\text{arctanh}\left(\frac{1-2\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Integrate[(2 - Sinh[x])/(2 + Sinh[x]),x]
[Out] -x - (8*ArcTanh[(1 - 2*Tanh[x/2])/Sqrt[5]])/Sqrt[5]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
risch	$-x+\frac{4\sqrt{5}\,\ln\left(e^{x}+2-\sqrt{5}\right)}{5}-\frac{4\sqrt{5}\,\ln\left(e^{x}+2+\sqrt{5}\right)}{5}$	33
default	$\frac{8\sqrt{5}\,\text{arctanh}\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-1\right)\sqrt{5}}{5}\right)}{5}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	37

```
[In] int((2-sinh(x))/(2+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -x+4/5*5^(1/2)*ln(exp(x)+2-5^(1/2))-4/5*5^(1/2)*ln(exp(x)+2+5^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4}{5}\sqrt{5}\log\left(-\frac{(2\sqrt{5}-5)\cosh(x)-2(\sqrt{5}-2)\sinh(x)+\sqrt{5}-2}{\sinh(x)+2}\right)-x$$

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(5)*log(-((2*sqrt(5) - 5)*cosh(x) - 2*(sqrt(5) - 2)*sinh(x) + sqrt(
5) - 2)/(sinh(x) + 2)) - x
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = -x+\frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{5}-\frac{4\sqrt{5}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)}{5}$$

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x)
```

```
[Out] -x + 4*sqrt(5)*log(tanh(x/2) - 1/2 + sqrt(5)/2)/5 - 4*sqrt(5)*log(tanh(x/2)
 - sqrt(5)/2 - 1/2)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-e^{(-x)}+2}{\sqrt{5}+e^{(-x)}-2}\right)-x$$

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="maxima")
```

```
[Out] 4/5*sqrt(5)*log(-(sqrt(5) - e^(-x) + 2)/(sqrt(5) + e^(-x) - 2)) - x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4}{5}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+2e^x+4\right|}{2\left(\sqrt{5}+e^x+2\right)}\right)-x$$

```
[In] integrate((2-sinh(x))/(2+sinh(x)),x, algorithm="giac")
[Out] 4/5*sqrt(5)*log(1/2*abs(-2*sqrt(5) + 2*e^x + 4)/(sqrt(5) + e^x + 2)) - x
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{2-\sinh(x)}{2+\sinh(x)}\,dx = \frac{4\sqrt{5}\,\ln\left(-8\,\mathrm{e}^x-\frac{4\sqrt{5}\,(4\,\mathrm{e}^x-2)}{5}\right)}{5}-x-\frac{4\sqrt{5}\,\ln\left(\frac{4\sqrt{5}\,(4\,\mathrm{e}^x-2)}{5}-8\,\mathrm{e}^x\right)}{5}$$

```
[In] int(-(sinh(x) - 2)/(sinh(x) + 2),x)
[Out] (4*5^(1/2)*log(- 8*exp(x) - (4*5^(1/2)*(4*exp(x) - 2))/5))/5 - x - (4*5^(1/
2)*log((4*5^(1/2)*(4*exp(x) - 2))/5 - 8*exp(x)))/5
```

3.137 $\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 136

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\frac{2iBE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2i(Ab-aB)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] 2*I*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*P
i+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/((a+b*sinh(x))/
(a-I*b))^(1/2)+2*I*(A*b-B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I
*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))
/(a-I*b))^(1/2)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2831, 2742, 2740, 2734, 2732}

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx=\frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}+\frac{2iB\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[In] Int[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
```

[Out] ((2*I)*B*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b *Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*(A*b - a*B)*EllipticF[Pi/4 - (I/ 2)*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{B\int \sqrt{a+b\sinh(x)}\,dx}{b}+\frac{(i(-iAb+iaB))\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b}\\
&= \frac{\left(B\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{\left(i(-iAb+iaB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{b\sqrt{a+b\sinh(x)}}
\end{aligned}
$$

$$=\frac{2iBE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\sqrt{\frac{a+b\sinh(x)}{a-ib}}}+\frac{2i(Ab-aB)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

$$\int\frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx$$
$$=\frac{2\left((ia+b)BE\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)+i(Ab-aB)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[In] Integrate[(A + B*Sinh[x])/Sqrt[a + b*Sinh[x]],x]
[Out] (2*((I*a + b)*B*EllipticE[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)] + I*(A*b
- a*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)])*Sqrt[(a + b*Sinh[
x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]])
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.96

method	result
default	$-\frac{2(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\left(iB\operatorname{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)b-iB\operatorname{EllipticE}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\right.\right.}{b^{2}\cosh(x)\sqrt{a+b\sinh(x)}}$
parts	$-\frac{2A(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\operatorname{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)}{b\cosh(x)\sqrt{a+b\sinh(x)}}+\frac{2B(ib-a)\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh}{ib+}}}{}$
risch	$\frac{B\left(b\,e^{2x}+2\,e^{x}a-b\right)\sqrt{2}\,e^{-x}}{b\sqrt{\left(b\,e^{2x}+2\,e^{x}a-b\right)e^{-x}}}+\frac{\left(\frac{4A\left(a+\sqrt{a^{2}+b^{2}}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^{2}+b^{2}}}{b}\right)b}{a+\sqrt{a^{2}+b^{2}}}}\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^{2}+b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}+b^{2}}}{b}-\frac{-a+\sqrt{a^{2}+b^{2}}}{b}}}\sqrt{-\frac{e^{x}b}{a+\sqrt{a^{2}+b^{2}}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(e^{x}+\frac{a}{}\right.}{a+}}\right.}{b\sqrt{e^{3x}b+2\,e^{2x}a-e^{x}b}}\right.}{}$

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(I*b-a)*(-(a+b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I
+sinh(x))*b/(I*b-a))^(1/2)*(I*B*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-
(I*b-a)/(I*b+a))^(1/2))*b-I*B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I
*b-a)/(I*b+a))^(1/2))*b+A*EllipticF((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a
)/(I*b+a))^(1/2))*b-B*EllipticE((-(a+b*sinh(x))/(I*b-a))^(1/2),(-(I*b-a)/(I
*b+a))^(1/2))*a)/b^2/cosh(x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.35

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\left(3\sqrt{2}Bb^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2+3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3+9\,ab^2)}{27\,b^3},\frac{3\,b}{\cdots}\right.\right.\right.}{\cdots}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*sqrt(2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8
*a^3 + 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*
a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + sqrt(2)*(2*
B*a - 3*A*b)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*
a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*sqrt(b*sin
h(x) + a)*B*b)/b^2
```

Sympy [F]

$$\int \frac{A+B\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{A+B\sinh{(x)}}{\sqrt{a+b\sinh{(x)}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral((A + B*sinh(x))/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh(x)}}\,dx = \int \frac{B\sinh\left(x\right) + A}{\sqrt{b\sinh\left(x\right) + a}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh(x)}}\,dx = \int \frac{B\sinh\left(x\right) + A}{\sqrt{b\sinh\left(x\right) + a}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh(x)}}\,dx = \int \frac{A + B\sinh(x)}{\sqrt{a + b\sinh\left(x\right)}}\,dx$$

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(1/2),x)
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(1/2), x)
```

3.138 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 176

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{2i(Ab-aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2iB\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

```
[Out] -2*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(1/2)+2*I*(A*b-B*a)*(sin(1/4*P
i+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/
2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/(a^2+b^2)/((a+b*sinh(x))/(a-I*b
))^(1/2)+2*I*B*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(
cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b*sinh(x))/(a-I*b))^(1/2
)/b/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = -\frac{2\cosh(x)(Ab-aB)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{2i(Ab-aB)\sqrt{a+b\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{b(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2iB\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{b\sqrt{a+b\sinh(x)}}$$

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/((a^2 + b^2)*Sqrt[a + b*Sinh[x]]) + ((2*I)*(A*b -
a*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[a + b*Sinh[x]])/(b*(a^
2 + b^2)*Sqrt[(a + b*Sinh[x])/(a - I*b)]) + ((2*I)*B*EllipticF[Pi/4 - (I/2)
*x, (2*b)/(I*a + b)]*Sqrt[(a + b*Sinh[x])/(a - I*b)])/(b*Sqrt[a + b*Sinh[x]
])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} - \frac{2\int \frac{\frac{1}{2}(-aA-bB)-\frac{1}{2}(Ab-aB)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{a^2+b^2}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{B\int \frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{b} + \frac{(Ab-aB)\int \sqrt{a+b\sinh(x)}\,dx}{b\,(a^2+b^2)}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{\left((Ab-aB)\sqrt{a+b\sinh(x)}\right)\int \sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{b\,(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{\left(B\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int \frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{b\sqrt{a+b\sinh(x)}}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{(a^2+b^2)\sqrt{a+b\sinh(x)}} + \frac{2i(Ab-aB)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{b\,(a^2+b^2)\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + \frac{2iB\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{b\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = \frac{2b(-Ab+aB)\cosh(x) + \frac{2i(Ab-aB)E\left(\frac{1}{4}(\pi-2ix)\middle|-\frac{2ib}{a-ib}\right)(a+b\sinh(x))}{\sqrt{\frac{a+b\sinh(x)}{a-ib}}} + 2i(a^2+b^2)\,B\,\text{Elli}}{b\,(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(3/2),x]
```

```
[Out] (2*b*(-(A*b) + a*B)*Cosh[x] + ((2*I)*(A*b - a*B)*EllipticE[(Pi - (2*I)*x)/4
, ((-2*I)*b)/(a - I*b)]*(a + b*Sinh[x]))/Sqrt[(a + b*Sinh[x])/(a - I*b)] +
(2*I)*(a^2 + b^2)*B*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b)/(a - I*b)]*Sqrt[
(a + b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(210) = 420.

Time = 3.63 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.94

method	result
default	$\frac{\sqrt{\cosh(x)^2(a+b\sinh(x))}\left(\frac{2B\left(\frac{a}{b}-i\right)\sqrt{\frac{-a-b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-a-b\sinh(x)}{ib-a}},\sqrt{\frac{-ib+a}{ib+a}}\right)}{b\sqrt{\cosh(x)^2(a+b\sinh(x))}}+\frac{(Ab-aB)\left(-\frac{}{(a^2+}\right.}{}\right.}{}$
parts	$\frac{2A\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{a+b\sinh(x)}{ib-a}},\sqrt{-\frac{ib-a}{ib+a}}\right)a^2+\sqrt{-\frac{a+b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x)}{ib-a}}\right.}{}$

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*(2*B/b*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/
2)*((I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(
a+b*sinh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+
a))^(1/2))+(A*b-B*a)/b*(-2*b*cosh(x)^2/(a^2+b^2)/(cosh(x)^2*(a+b*sinh(x)))^
(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)
*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+2*b/(a^2
+b^2)*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b+a))^(1/2)*
((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((-a/b-I)*Ell
ipticE(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I*EllipticF(
((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))))/cosh(x)/(a+b*sin
h(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.60

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx =$$
$$-\frac{2\left(\left(\sqrt{2}(2\,Ba^2b+Aab^2+3\,Bb^3)\cosh(x)^2+\sqrt{2}(2\,Ba^2b+Aab^2+3\,Bb^3)\sinh(x)^2+2\sqrt{2}(2\,Ba^3+Aa^2b+\right.\right.}{}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*((sqrt(2)*(2*B*a^2*b + A*a*b^2 + 3*B*b^3)*cosh(x)^2 + sqrt(2)*(2*B*a^2
*b + A*a*b^2 + 3*B*b^3)*sinh(x)^2 + 2*sqrt(2)*(2*B*a^3 + A*a^2*b + 3*B*a*b^
2)*cosh(x) + 2*(sqrt(2)*(2*B*a^2*b + A*a*b^2 + 3*B*b^3)*cosh(x) + sqrt(2)*(
2*B*a^3 + A*a^2*b + 3*B*a*b^2))*sinh(x) - sqrt(2)*(2*B*a^2*b + A*a*b^2 + 3*
B*b^3))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 +
 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(B*a*b
^2 - A*b^3)*cosh(x)^2 + sqrt(2)*(B*a*b^2 - A*b^3)*sinh(x)^2 + 2*sqrt(2)*(B*
a^2*b - A*a*b^2)*cosh(x) + 2*(sqrt(2)*(B*a*b^2 - A*b^3)*cosh(x) + sqrt(2)*(
B*a^2*b - A*a*b^2))*sinh(x) - sqrt(2)*(B*a*b^2 - A*b^3))*sqrt(b)*weierstras
sZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierstrassPInv
erse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b*cosh(x)
 + 3*b*sinh(x) + 2*a)/b)) + 6*((B*a*b^2 - A*b^3)*cosh(x)^2 + (B*a*b^2 - A*b
^3)*sinh(x)^2 + (B*a^2*b - A*a*b^2)*cosh(x) + (B*a^2*b - A*a*b^2 + 2*(B*a*b
^2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b*sinh(x) + a))/(a^2*b^3 + b^5 - (a^2*b^
3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^3*b^2 + a*b^4)*cosh(x
) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(3/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx = \int \frac{B\sinh(x)+A}{(b\sinh(x)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\,dx = \int \frac{B\sinh(x) + A}{(b\sinh(x) + a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(3/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\,dx = \int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{3/2}}\,dx$$

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(3/2),x)
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(3/2), x)
```

3.139 $\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 251

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{2(4aAb-a^2B+3b^2B)\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{2i(4aAb-a^2B+3b^2B)\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(Ab-aB)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3b(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

```
[Out] -2/3*(A*b-B*a)*cosh(x)/(a^2+b^2)/(a+b*sinh(x))^(3/2)-2/3*(4*A*a*b-B*a^2+3*B
*b^2)*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))^(1/2)+2/3*I*(4*A*a*b-B*a^2+3*B*b^2)
*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2
*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*(a+b*sinh(x))^(1/2)/b/(a^2+b^2)^2/((a+b*si
nh(x))/(a-I*b))^(1/2)-2/3*I*(A*b-B*a)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4
*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2)*(b/(I*a+b))^(1/2))*((a+b
*sinh(x))/(a-I*b))^(1/2)/b/(a^2+b^2)/(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2833, 2831, 2742, 2740, 2734, 2732}

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = -\frac{2\cosh(x)\left(a^2(-B)+4aAb+3b^2B\right)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}} - \frac{2\cosh(x)(Ab-aB)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}} - \frac{2i(Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}} + \frac{2i\left(a^2(-B)+4aAb+3b^2B\right)\sqrt{a+b\sinh(x)}\,E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)}{3b\left(a^2+b^2\right)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}$$

```
[In] Int[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cosh[x])/(3*(a^2 + b^2)*(a + b*Sinh[x])^(3/2)) - (2*(4*a*A*
b - a^2*B + 3*b^2*B)*Cosh[x])/(3*(a^2 + b^2)^2*Sqrt[a + b*Sinh[x]]) + (((2*
I)/3)*(4*a*A*b - a^2*B + 3*b^2*B)*EllipticE[Pi/4 - (I/2)*x, (2*b)/(I*a + b)
]*Sqrt[a + b*Sinh[x]])/(b*(a^2 + b^2)^2*Sqrt[(a + b*Sinh[x])/(a - I*b)]) -
(((2*I)/3)*(A*b - a*B)*EllipticF[Pi/4 - (I/2)*x, (2*b)/(I*a + b)]*Sqrt[(a +
 b*Sinh[x])/(a - I*b)])/(b*(a^2 + b^2)*Sqrt[a + b*Sinh[x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
```

```
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2831

```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(b*c - a*d)/b, Int[1/Sqrt[a + b*Sin[e + f*x]
], x], x] + Dist[d/b, Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2\int\frac{-\frac{3}{2}(aA+bB)+\frac{1}{2}(Ab-aB)\sinh(x)}{(a+b\sinh(x))^{3/2}}\,dx}{3\left(a^2+b^2\right)}\\
&= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}\\
&\quad+\frac{4\int\frac{\frac{1}{4}\left(3a^2A-Ab^2+4abB\right)+\frac{1}{4}\left(4aAb-a^2B+3b^2B\right)\sinh(x)}{\sqrt{a+b\sinh(x)}}\,dx}{3\left(a^2+b^2\right)^2}\\
&= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}\\
&\quad-\frac{(Ab-aB)\int\frac{1}{\sqrt{a+b\sinh(x)}}\,dx}{3b\left(a^2+b^2\right)}+\frac{\left(4aAb-a^2B+3b^2B\right)\int\sqrt{a+b\sinh(x)}\,dx}{3b\left(a^2+b^2\right)^2}\\
&= -\frac{2(Ab-aB)\cosh(x)}{3\left(a^2+b^2\right)(a+b\sinh(x))^{3/2}}-\frac{2\left(4aAb-a^2B+3b^2B\right)\cosh(x)}{3\left(a^2+b^2\right)^2\sqrt{a+b\sinh(x)}}\\
&\quad+\frac{\left(\left(4aAb-a^2B+3b^2B\right)\sqrt{a+b\sinh(x)}\right)\int\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}\,dx}{3b\left(a^2+b^2\right)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}}\\
&\quad-\frac{\left((Ab-aB)\sqrt{\frac{a+b\sinh(x)}{a-ib}}\right)\int\frac{1}{\sqrt{\frac{a}{a-ib}+\frac{b\sinh(x)}{a-ib}}}\,dx}{3b\left(a^2+b^2\right)\sqrt{a+b\sinh(x)}}
\end{aligned}$$

$$= -\frac{2(Ab-aB)\cosh(x)}{3(a^2+b^2)(a+b\sinh(x))^{3/2}} - \frac{2(4aAb-a^2B+3b^2B)\cosh(x)}{3(a^2+b^2)^2\sqrt{a+b\sinh(x)}} + \frac{2i(4aAb-a^2B+3b^2B)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\frac{2b}{ia+b}\right)\sqrt{a+b\sinh(x)}}{3b(a^2+b^2)^2\sqrt{\frac{a+b\sinh(x)}{a-ib}}} - \frac{2i(Ab-aB)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},\frac{2b}{ia+b}\right)\sqrt{\frac{a+b\sinh(x)}{a-ib}}}{3b(a^2+b^2)\sqrt{a+b\sinh(x)}}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.94

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \frac{2i\left(\left(b(3a^2A-Ab^2+4abB)\operatorname{EllipticF}\left(\frac{1}{4}(\pi-2ix),-\frac{2ib}{a-ib}\right)+(4aAb-a^2B+3b^2B)\right.\right.}{}$$

```
[In] Integrate[(A + B*Sinh[x])/(a + b*Sinh[x])^(5/2),x]
```

```
[Out] (((2*I)/3)*((b*(3*a^2*A - A*b^2 + 4*a*b*B)*EllipticF[(Pi - (2*I)*x)/4, ((-2
*I)*b)/(a - I*b)] + (4*a*A*b - a^2*B + 3*b^2*B)*((a - I*b)*EllipticE[(Pi -
(2*I)*x)/4, ((-2*I)*b)/(a - I*b)] - a*EllipticF[(Pi - (2*I)*x)/4, ((-2*I)*b
)/(a - I*b)]))*(a + b*Sinh[x])*Sqrt[(a + b*Sinh[x])/(a - I*b)] + I*b*Cosh[x
]*(-((a^2 + b^2)*(-(A*b) + a*B)) - (-4*a*A*b + a^2*B - 3*b^2*B)*(a + b*Sinh
[x]))))/(b*(a^2 + b^2)^2*(a + b*Sinh[x])^(3/2))
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(275) = 550.

Time = 4.20 (sec) , antiderivative size = 806, normalized size of antiderivative = 3.21

method	result
default	$\frac{\sqrt{\cosh(x)^2(a+b\sinh(x))}\left(B\left(-\frac{2b\cosh(x)^2}{(a^2+b^2)\sqrt{\cosh(x)^2(a+b\sinh(x))}}+\frac{2a\left(\frac{a}{b}-i\right)\sqrt{\frac{-a-b\sinh(x)}{ib-a}}\sqrt{\frac{(i-\sinh(x))b}{ib+a}}\sqrt{\frac{(i+\sinh(x))b}{ib-a}}\operatorname{EllipticF}\left(\sqrt{\frac{-a-b}{ib}}\right.}{(a^2+b^2)\sqrt{\cosh(x)^2(a+b\sinh(x))}}\right.\right.}{}$
parts	Expression too large to display

```
[In] int((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (cosh(x)^2*(a+b*sinh(x)))^(1/2)*(B/b*(-2*b*cosh(x)^2/(a^2+b^2)/(cosh(x)^2*(
a+b*sinh(x)))^(1/2)+2*a/(a^2+b^2)*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((
I-sinh(x))*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*s
inh(x)))^(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(
1/2))+2*b/(a^2+b^2)*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(
I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)
*((-a/b-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)
)+I*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))))+(A*
b-B*a)/b*(-2/3/b/(a^2+b^2)*(cosh(x)^2*(a+b*sinh(x)))^(1/2)/(sinh(x)+a/b)^2-
8/3*b*cosh(x)^2/(a^2+b^2)^2*a/(cosh(x)^2*(a+b*sinh(x)))^(1/2)+2*(3*a^2-b^2)
/(3*a^4+6*a^2*b^2+3*b^4)*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x)
)*b/(I*b+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^
(1/2)*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+8/3
*a*b/(a^2+b^2)^2*(a/b-I)*((-a-b*sinh(x))/(I*b-a))^(1/2)*((I-sinh(x))*b/(I*b
+a))^(1/2)*((I+sinh(x))*b/(I*b-a))^(1/2)/(cosh(x)^2*(a+b*sinh(x)))^(1/2)*((
-a/b-I)*EllipticE(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2))+I
*EllipticF(((-a-b*sinh(x))/(I*b-a))^(1/2),((a-I*b)/(I*b+a))^(1/2)))))/cosh(
x)/(a+b*sinh(x))^(1/2)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 2167, normalized size of antiderivative = 8.63

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*((sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^4 + s
qrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*sinh(x)^4 + 4*sqrt(2
)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x)^3 + 4*(sqrt(2)*
(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x) + sqrt(2)*(2*B*a^4*
b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4))*sinh(x)^3 + 2*sqrt(2)*(4*B*a^5 +
2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^2 + 2
*(3*sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^2 + 6*s
qrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x) + sqrt(2)*
(4*B*a^5 + 2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5))*s
inh(x)^2 - 4*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh
(x) + 4*(sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5)*cosh(x)^3
+ 3*sqrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4)*cosh(x)^2 + s
qrt(2)*(4*B*a^5 + 2*A*a^4*b + 10*B*a^3*b^2 - 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*
b^5)*cosh(x) - sqrt(2)*(2*B*a^4*b + A*a^3*b^2 + 6*B*a^2*b^3 - 3*A*a*b^4))*s
inh(x) + sqrt(2)*(2*B*a^3*b^2 + A*a^2*b^3 + 6*B*a*b^4 - 3*A*b^5))*sqrt(b)*w
eierstrassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/
```

```
3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3*(sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4
- 3*B*b^5)*cosh(x)^4 + sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*sinh(x)^4
+ 4*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4)*cosh(x)^3 + 4*(sqrt(2)*(B
*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh(x) + sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3
- 3*B*a*b^4))*sinh(x)^3 + 2*sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3
+ 4*A*a*b^4 + 3*B*b^5)*cosh(x)^2 + 2*(3*sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*
B*b^5)*cosh(x)^2 + 6*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4)*cosh(x)
+ sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5))*si
nh(x)^2 - 4*sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 - 3*B*a*b^4)*cosh(x) + 4*(sqrt
(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh(x)^3 + 3*sqrt(2)*(B*a^3*b^2 - 4*
A*a^2*b^3 - 3*B*a*b^4)*cosh(x)^2 + sqrt(2)*(2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a
^2*b^3 + 4*A*a*b^4 + 3*B*b^5)*cosh(x) - sqrt(2)*(B*a^3*b^2 - 4*A*a^2*b^3 -
3*B*a*b^4))*sinh(x) + sqrt(2)*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5))*sqrt(b)*we
ierstrassZeta(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, weierst
rassPInverse(4/3*(4*a^2 + 3*b^2)/b^2, -8/27*(8*a^3 + 9*a*b^2)/b^3, 1/3*(3*b
*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 6*((B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*co
sh(x)^4 + (B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*sinh(x)^4 + (4*B*a^3*b^2 - 13*A
*a^2*b^3 - 8*B*a*b^4 - A*b^5)*cosh(x)^3 + (4*B*a^3*b^2 - 13*A*a^2*b^3 - 8*B
*a*b^4 - A*b^5 + 4*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh(x))*sinh(x)^3 + (
2*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5)*cosh(x)^2 + (2
*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5 + 6*(B*a^2*b^3 -
 4*A*a*b^4 - 3*B*b^5)*cosh(x)^2 + 3*(4*B*a^3*b^2 - 13*A*a^2*b^3 - 8*B*a*b^4
 - A*b^5)*cosh(x))*sinh(x)^2 + (3*A*a^2*b^3 + 4*B*a*b^4 - A*b^5)*cosh(x) +
(3*A*a^2*b^3 + 4*B*a*b^4 - A*b^5 + 4*(B*a^2*b^3 - 4*A*a*b^4 - 3*B*b^5)*cosh
(x)^3 + 3*(4*B*a^3*b^2 - 13*A*a^2*b^3 - 8*B*a*b^4 - A*b^5)*cosh(x)^2 + 2*(2
*B*a^4*b - 8*A*a^3*b^2 - 7*B*a^2*b^3 + 4*A*a*b^4 + 3*B*b^5)*cosh(x))*sinh(x
))*sqrt(b*sinh(x) + a))/(a^4*b^4 + 2*a^2*b^6 + b^8 + (a^4*b^4 + 2*a^2*b^6 +
 b^8)*cosh(x)^4 + (a^4*b^4 + 2*a^2*b^6 + b^8)*sinh(x)^4 + 4*(a^5*b^3 + 2*a^
3*b^5 + a*b^7)*cosh(x)^3 + 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7 + (a^4*b^4 + 2*a^
2*b^6 + b^8)*cosh(x))*sinh(x)^3 + 2*(2*a^6*b^2 + 3*a^4*b^4 - b^8)*cosh(x)^2
 + 2*(2*a^6*b^2 + 3*a^4*b^4 - b^8 + 3*(a^4*b^4 + 2*a^2*b^6 + b^8)*cosh(x)^2
 + 6*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x))*sinh(x)^2 - 4*(a^5*b^3 + 2*a^3*
b^5 + a*b^7)*cosh(x) - 4*(a^5*b^3 + 2*a^3*b^5 + a*b^7 - (a^4*b^4 + 2*a^2*b^
6 + b^8)*cosh(x)^3 - 3*(a^5*b^3 + 2*a^3*b^5 + a*b^7)*cosh(x)^2 - (2*a^6*b^2
 + 3*a^4*b^4 - b^8)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\sinh(x)}{(a + b\sinh(x))^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{B\sinh{(x)}+A}{(b\sinh{(x)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="maxima")
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{B\sinh{(x)}+A}{(b\sinh{(x)}+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((A+B*sinh(x))/(a+b*sinh(x))^(5/2),x, algorithm="giac")
[Out] integrate((B*sinh(x) + A)/(b*sinh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sinh(x)}{(a+b\sinh(x))^{5/2}}\,dx = \int \frac{A+B\sinh(x)}{(a+b\sinh{(x)})^{5/2}}\,dx$$

```
[In] int((A + B*sinh(x))/(a + b*sinh(x))^(5/2),x)
[Out] int((A + B*sinh(x))/(a + b*sinh(x))^(5/2), x)
```

3.140 $\int \left(a \sinh^2(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \left(a \sinh^2(x)\right)^{5/2} dx = \frac{8}{15}a^2 \coth(x)\sqrt{a \sinh^2(x)} - \frac{4}{15}a \coth(x) \left(a \sinh^2(x)\right)^{3/2} + \frac{1}{5}\coth(x) \left(a \sinh^2(x)\right)^{5/2}$$

```
[Out] -4/15*a*coth(x)*(a*sinh(x)^2)^(3/2)+1/5*coth(x)*(a*sinh(x)^2)^(5/2)+8/15*a^
2*coth(x)*(a*sinh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3282, 3286, 2718}

$$\int \left(a \sinh^2(x)\right)^{5/2} dx = \frac{8}{15}a^2 \coth(x)\sqrt{a \sinh^2(x)} + \frac{1}{5}\coth(x) \left(a \sinh^2(x)\right)^{5/2} - \frac{4}{15}a \coth(x) \left(a \sinh^2(x)\right)^{3/2}$$

```
[In] Int[(a*Sinh[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Coth[x]*Sqrt[a*Sinh[x]^2])/15 - (4*a*Coth[x]*(a*Sinh[x]^2)^(3/2))/15
 + (Coth[x]*(a*Sinh[x]^2)^(5/2))/5
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}-\frac{1}{5}(4a)\int\left(a\sinh^2(x)\right)^{3/2}\,dx\\
&=-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\right)\int\sqrt{a\sinh^2(x)}\,dx\\
&=-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}\\
&\quad+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\text{csch}(x)\sqrt{a\sinh^2(x)}\right)\int\sinh(x)\,dx\\
&=\frac{8}{15}a^2\coth(x)\sqrt{a\sinh^2(x)}-\frac{4}{15}a\coth(x)\left(a\sinh^2(x)\right)^{3/2}+\frac{1}{5}\coth(x)\left(a\sinh^2(x)\right)^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int\left(a\sinh^2(x)\right)^{5/2}\,dx=\frac{1}{240}a^2(150\cosh(x)-25\cosh(3x)+3\cosh(5x))\text{csch}(x)\sqrt{a\sinh^2(x)}$$

```
[In] Integrate[(a*Sinh[x]^2)^(5/2),x]
[Out] (a^2*(150*Cosh[x] - 25*Cosh[3*x] + 3*Cosh[5*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/
240
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$\frac{a^{3} \sinh(x) \cosh(x) \left(3 \sinh(x)^{4}-4 \sinh(x)^{2}+8\right)}{15 \sqrt{a \sinh(x)^{2}}}$
risch	$\frac{a^{2} \mathrm{e}^{6x} \sqrt{a (\mathrm{e}^{2x}-1)^{2} \mathrm{e}^{-2x}}}{160\, \mathrm{e}^{2x}-160}-\frac{5 a^{2} \mathrm{e}^{4x} \sqrt{a (\mathrm{e}^{2x}-1)^{2} \mathrm{e}^{-2x}}}{96 (\mathrm{e}^{2x}-1)}+\frac{5 a^{2} \mathrm{e}^{2x} \sqrt{a (\mathrm{e}^{2x}-1)^{2} \mathrm{e}^{-2x}}}{16 (\mathrm{e}^{2x}-1)}+\frac{5 \sqrt{a (\mathrm{e}^{2x}-1)^{2} \mathrm{e}^{-2x}}\, a^{2}}{16 (\mathrm{e}^{2x}-1)}-\frac{5 a^{2} \mathrm{e}^{-2x} \sqrt{a (\mathrm{e}^{2x}-1}}{96 (\mathrm{e}^{2x}-1)}$

```
[In] int((a*sinh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/15*a^3*sinh(x)*cosh(x)*(3*sinh(x)^4-4*sinh(x)^2+8)/(a*sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(41) = 82$.

Time = 0.28 (sec) , antiderivative size = 511, normalized size of antiderivative = 9.64

$$\int \left(a \sinh^2(x)\right)^{5/2} \, dx = \frac{\left(30 \, a^{2} \cosh\left(x\right) e^{x} \sinh\left(x\right)^{9} + 3 \, a^{2} e^{x} \sinh\left(x\right)^{10} + 5 \, \left(27 \, a^{2} \cosh\left(x\right)^{2} - 5 \, a^{2}\right) e^{x} \sinh\left(x\right)^{8}}{}$$

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cosh
(x)^2 - 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 - 5*a^2*cosh(x))*e^x*sin
h(x)^7 + 10*(63*a^2*cosh(x)^4 - 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6 +
4*(189*a^2*cosh(x)^5 - 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)^5 +
 10*(63*a^2*cosh(x)^6 - 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 + 15*a^2)*e^x
*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 - 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^3 + 15
*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 - 140*a^2*cosh(x)^6 + 450
*a^2*cosh(x)^4 + 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a^2*cosh(
x)^9 - 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 + 60*a^2*cosh(x)^3 - 5*a^2*cosh(
x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 - 25*a^2*cosh(x)^8 + 150*a^2*cosh(x)^6
+ 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 + 3*a^2)*e^x)*sqrt(a*e^(4*x) - 2*a*e
^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) - 1)*sinh(x)^5 - cosh(x)^5
 + 5*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(2*x) - cosh(x
)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) - cosh(x)^3)*sinh(x)^2 + 5*(cosh(x)^
4*e^(2*x) - cosh(x)^4)*sinh(x))
```

Sympy [F]

$$\int \left(a\sinh^2(x)\right)^{5/2}\,dx=\int \left(a\sinh^2{(x)}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sinh(x)**2)**(5/2),x)
[Out] Integral((a*sinh(x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a\sinh^2(x)\right)^{5/2}\,dx=-\frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)}+\frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)}-\frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)}+\frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)}-\frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)}-\frac{5}{16}\,a^{\frac{5}{2}}e^{x}$$

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="maxima")
[Out] -1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) + 5/96*
a^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) - 5/16*a^(5/2)*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(41) = 82.

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.26

$$\int \left(a\sinh^2(x)\right)^{5/2}\,dx=\frac{1}{480}\left(3\,a^2e^{(5\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)-25\,a^2e^{(3\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)+150\,a^2e^{x}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right.$$

```
[In] integrate((a*sinh(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/480*(3*a^2*e^(5*x)*sgn(e^(3*x) - e^x) - 25*a^2*e^(3*x)*sgn(e^(3*x) - e^x)
 + 150*a^2*e^x*sgn(e^(3*x) - e^x) + (150*a^2*e^(4*x)*sgn(e^(3*x) - e^x) - 2
5*a^2*e^(2*x)*sgn(e^(3*x) - e^x) + 3*a^2*sgn(e^(3*x) - e^x))*e^(-5*x))*sqrt
(a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^2(x)\right)^{5/2} \, dx = \int \left(a\sinh(x)^2\right)^{5/2} dx$$

```
[In] int((a*sinh(x)^2)^(5/2),x)
[Out] int((a*sinh(x)^2)^(5/2), x)
```

3.141 $\int \left(a\sinh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx=-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}+\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}$$

```
[Out] 1/3*coth(x)*(a*sinh(x)^2)^(3/2)-2/3*a*coth(x)*(a*sinh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3282, 3286, 2718}

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx=\frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}-\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)}$$

```
[In] Int[(a*Sinh[x]^2)^(3/2),x]
[Out] (-2*a*Coth[x]*Sqrt[a*Sinh[x]^2])/3 + (Coth[x]*(a*Sinh[x]^2)^(3/2))/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && Gt
```

```
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2} - \frac{1}{3}(2a)\int \sqrt{a\sinh^2(x)}\,dx\\ &= \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2} - \frac{1}{3}\left(2a\text{csch}(x)\sqrt{a\sinh^2(x)}\right)\int \sinh(x)\,dx\\ &= -\frac{2}{3}a\coth(x)\sqrt{a\sinh^2(x)} + \frac{1}{3}\coth(x)\left(a\sinh^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = \frac{1}{12}a(-9\cosh(x)+\cosh(3x))\text{csch}(x)\sqrt{a\sinh^2(x)}$$

```
[In] Integrate[(a*Sinh[x]^2)^(3/2),x]
[Out] (a*(-9*Cosh[x] + Cosh[3*x])*Csch[x]*Sqrt[a*Sinh[x]^2])/12
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{a^2\sinh(x)\cosh(x)\left(\sinh(x)^2-2\right)}{3\sqrt{a\sinh(x)^2}}$	24
risch	$\frac{a\,e^{4x}\sqrt{a(e^{2x}-1)^2e^{-2x}}}{24\,e^{2x}-24} - \frac{3a\,e^{2x}\sqrt{a(e^{2x}-1)^2e^{-2x}}}{8(e^{2x}-1)} - \frac{3\sqrt{a(e^{2x}-1)^2e^{-2x}}\,a}{8(e^{2x}-1)} + \frac{a\,e^{-2x}\sqrt{a(e^{2x}-1)^2e^{-2x}}}{24\,e^{2x}-24}$	122

```
[In] int((a*sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/3*a^2*sinh(x)*cosh(x)*(sinh(x)^2-2)/(a*sinh(x)^2)^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(26) = 52$.

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 6.65

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = \frac{\left(6\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)^{5} + ae^{x}\sinh\left(x\right)^{6} + 3\left(5\,a\cosh\left(x\right)^{2} - 3\,a\right)e^{x}\sinh\left(x\right)^{4} + 4\left(5\,a\right.}{}$$

[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="fricas")

[Out] 1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*cosh(x)^4 - 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 - 6*a*cosh(x)^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 - 9*a*cosh(x)^4 - 9*a*cosh(x)^2 + a)*e^x)*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x) + (e^(2*x) - 1)*sinh(x)^3 - cosh(x)^3 + 3*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^2 + 3*(cosh(x)^2*e^(2*x) - cosh(x)^2)*sinh(x))

Sympy [F]

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = \int \left(a\sinh^{2}\left(x\right)\right)^{\frac{3}{2}}\,dx$$

[In] integrate((a*sinh(x)**2)**(3/2),x)

[Out] Integral((a*sinh(x)**2)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a\sinh^2(x)\right)^{3/2}\,dx = -\frac{1}{24}\,a^{\frac{3}{2}}e^{\left(3\,x\right)} + \frac{3}{8}\,a^{\frac{3}{2}}e^{\left(-x\right)} - \frac{1}{24}\,a^{\frac{3}{2}}e^{\left(-3\,x\right)} + \frac{3}{8}\,a^{\frac{3}{2}}e^{x}$$

[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="maxima")

[Out] -1/24*a^(3/2)*e^(3*x) + 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^(3/2)*e^x

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \left(a\sinh^2(x)\right)^{3/2} dx =$$
$$-\frac{1}{24}\left(\left(9\,e^{(2\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)-\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right)e^{(-3\,x)}-e^{(3\,x)}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)+9\,e^{x}\mathrm{sgn}\left(e^{(3\,x)}-e^{x}\right)\right)a^{\frac{3}{2}}$$

```
[In] integrate((a*sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/24*((9*e^(2*x)*sgn(e^(3*x) - e^x) - sgn(e^(3*x) - e^x))*e^(-3*x) - e^(3*
x)*sgn(e^(3*x) - e^x) + 9*e^x*sgn(e^(3*x) - e^x))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^2(x)\right)^{3/2} dx = \int \left(a\sinh(x)^2\right)^{3/2} dx$$

```
[In] int((a*sinh(x)^2)^(3/2),x)
[Out] int((a*sinh(x)^2)^(3/2), x)
```

3.142 $\int \sqrt{a \sinh^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \sqrt{a \sinh^2(x)}\,dx = \coth(x)\sqrt{a \sinh^2(x)}$$

```
[Out] coth(x)*(a*sinh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3286, 2718}

$$\int \sqrt{a \sinh^2(x)}\,dx = \coth(x)\sqrt{a \sinh^2(x)}$$

```
[In] Int[Sqrt[a*Sinh[x]^2],x]
```

```
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\operatorname{csch}(x)\sqrt{a\sinh^2(x)}\right)\int \sinh(x)\,dx\\ &= \coth(x)\sqrt{a\sinh^2(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a\sinh^2(x)}\,dx = \coth(x)\sqrt{a\sinh^2(x)}$$

```
[In] Integrate[Sqrt[a*Sinh[x]^2],x]
[Out] Coth[x]*Sqrt[a*Sinh[x]^2]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{a\sinh(x)\cosh(x)}{\sqrt{a\sinh(x)^2}}$	15
risch	$\frac{\sqrt{a(e^{2x}-1)^2 e^{-2x}}\,e^{2x}}{2\,e^{2x}-2}+\frac{\sqrt{a(e^{2x}-1)^2 e^{-2x}}}{2\,e^{2x}-2}$	58

```
[In] int((a*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(a*sinh(x)^2)^(1/2)*a*sinh(x)*cosh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 5.46

$$\int \sqrt{a\sinh^2(x)}\,dx = \frac{\left(2\cosh(x)\,e^x\sinh(x) + e^x\sinh(x)^2 + \left(\cosh(x)^2 + 1\right)e^x\right)\sqrt{ae^{(4x)} - 2\,ae^{(2x)} + a}e^{(-x)}}{2\left(\cosh(x)\,e^{(2x)} + \left(e^{(2x)} - 1\right)\sinh(x) - \cosh(x)\right)}$$

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 + 1)*e^x)*sqrt(a*e^
(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) - 1)*sinh(x) -
cosh(x))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{a\sinh^2(x)}\,dx = \frac{\sqrt{a\sinh^2(x)}\cosh(x)}{\sinh(x)}$$

```
[In] integrate((a*sinh(x)**2)**(1/2),x)
[Out] sqrt(a*sinh(x)**2)*cosh(x)/sinh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sqrt{a\sinh^2(x)}\,dx = -\frac{1}{2}\sqrt{a}e^{(-x)} - \frac{1}{2}\sqrt{a}e^x$$

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(a)*e^(-x) - 1/2*sqrt(a)*e^x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.62

$$\int \sqrt{a\sinh^2(x)}\,dx = \frac{1}{2}\left(e^{(-x)}\mathrm{sgn}\left(e^{(3x)} - e^{x}\right) + e^{x}\mathrm{sgn}\left(e^{(3x)} - e^{x}\right)\right)\sqrt{a}$$

```
[In] integrate((a*sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(e^(-x)*sgn(e^(3*x) - e^x) + e^x*sgn(e^(3*x) - e^x))*sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \sqrt{a\sinh^2(x)}\,dx = \sqrt{a}\coth(x)\sqrt{\left(\frac{e^{-x}}{2} - \frac{e^{x}}{2}\right)^2}$$

```
[In] int((a*sinh(x)^2)^(1/2),x)
[Out] a^(1/2)*coth(x)*((exp(-x)/2 - exp(x)/2)^2)^(1/2)
```

3.143 $\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 17

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))\sinh(x)}{\sqrt{a\sinh^2(x)}}$$

```
[Out] -arctanh(cosh(x))*sinh(x)/(a*sinh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3286, 3855}

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = -\frac{\sinh(x)\operatorname{arctanh}(\cosh(x))}{\sqrt{a\sinh^2(x)}}$$

```
[In] Int[1/Sqrt[a*Sinh[x]^2],x]
```

```
[Out] -((ArcTanh[Cosh[x]]*Sinh[x])/Sqrt[a*Sinh[x]^2])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{\sqrt{a\sinh^2(x)}}$$
$$= -\frac{\operatorname{arctanh}(\cosh(x))\sinh(x)}{\sqrt{a\sinh^2(x)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \frac{\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)}{\sqrt{a\sinh^2(x)}}$$

```
[In] Integrate[1/Sqrt[a*Sinh[x]^2],x]
[Out] ((-Log[Cosh[x/2]] + Log[Sinh[x/2]])*Sinh[x])/Sqrt[a*Sinh[x]^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(15) = 30.

Time = 0.91 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

method	result	size
default	$-\frac{\sinh(x)\sqrt{a\cosh(x)^2}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cosh(x)^2}+2a}{\sinh(x)}\right)}{\sqrt{a}\,\cosh(x)\sqrt{a\sinh(x)^2}}$	49
risch	$-\frac{e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}+1\right)}{\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}+\frac{e^{-x}\left(e^{2x}-1\right)\ln\left(e^{x}-1\right)}{\sqrt{a\left(e^{2x}-1\right)^2 e^{-2x}}}$	67

```
[In] int(1/(a*sinh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -sinh(x)*(a*cosh(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2)+a)/s
inh(x))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.47

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \left[\frac{\sqrt{ae^{(4\,x)}-2\,ae^{(2\,x)}+a}\log\left(\frac{\cosh(x)+\sinh(x)-1}{\cosh(x)+\sinh(x)+1}\right)}{ae^{(2\,x)}-a}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{(4\,x)}-2\,ae^{(2\,x)}+a}\sqrt{-a}}{a\cosh(x)e^{(2\,x)}-a\cosh(x)+\left(ae^{(2\,x)}-a\right)\sinh(x)}\right)}{a}\right]$$

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*log((cosh(x) + sinh(x) - 1)/(cosh(x) + s
inh(x) + 1))/(a*e^(2*x) - a), 2*sqrt(-a)*arctan(sqrt(a*e^(4*x) - 2*a*e^(2*x
) + a)*sqrt(-a)/(a*cosh(x)*e^(2*x) - a*cosh(x) + (a*e^(2*x) - a)*sinh(x)))/
a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sinh(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \frac{\log\left(e^{(-x)}+1\right)}{\sqrt{a}} - \frac{\log\left(e^{(-x)}-1\right)}{\sqrt{a}}$$

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(e^(-x) + 1)/sqrt(a) - log(e^(-x) - 1)/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = 0$$

```
[In] integrate(1/(a*sinh(x)^2)^(1/2),x, algorithm="giac")
[Out] 0
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\,\sinh\left(x\right)^2}}\,dx$$

```
[In] int(1/(a*sinh(x)^2)^(1/2),x)
[Out] int(1/(a*sinh(x)^2)^(1/2), x)
```

3.144 $\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx=-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}+\frac{\operatorname{arctanh}(\cosh(x))\sinh(x)}{2a\sqrt{a\sinh^2(x)}}$$

```
[Out] -1/2*coth(x)/a/(a*sinh(x)^2)^(1/2)+1/2*arctanh(cosh(x))*sinh(x)/a/(a*sinh(x
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3283, 3286, 3855}

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx=\frac{\sinh(x)\operatorname{arctanh}(\cosh(x))}{2a\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}}$$

```
[In] Int[(a*Sinh[x]^2)^(-3/2),x]
```

```
[Out] -1/2*Coth[x]/(a*Sqrt[a*Sinh[x]^2]) + (ArcTanh[Cosh[x]]*Sinh[x])/(2*a*Sqrt[a
*Sinh[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] && !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\int \frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{2a}\\ &= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} - \frac{\sinh(x)\int \operatorname{csch}(x)\,dx}{2a\sqrt{a\sinh^2(x)}}\\ &= -\frac{\coth(x)}{2a\sqrt{a\sinh^2(x)}} + \frac{\operatorname{arctanh}(\cosh(x))\sinh(x)}{2a\sqrt{a\sinh^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{\left(\operatorname{csch}^2\left(\frac{x}{2}\right) - 4\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right) + \operatorname{sech}^2\left(\frac{x}{2}\right)\right)\sinh^3(x)}{8\left(a\sinh^2(x)\right)^{3/2}}$$

```
[In] Integrate[(a*Sinh[x]^2)^(-3/2),x]
```

```
[Out] -1/8*((Csch[x/2]^2 - 4*Log[Cosh[x/2]] + 4*Log[Sinh[x/2]] + Sech[x/2]^2)*Sin
h[x]^3)/(a*Sinh[x]^2)^(3/2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(34) = 68.

Time = 0.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

method	result	size
default	$-\frac{\sqrt{a\cosh(x)^2}\left(-\ln\left(\frac{2\sqrt{a}\,\sqrt{a\cosh(x)^2}+2a}{\sinh(x)}\right)a\sinh(x)^2+\sqrt{a}\,\sqrt{a\cosh(x)^2}\right)}{2a^{\frac{5}{2}}\sinh(x)\cosh(x)\sqrt{a\sinh(x)^2}}$	71
risch	$-\frac{1+e^{2x}}{a(e^{2x}-1)\sqrt{a(e^{2x}-1)^2 e^{-2x}}}+\frac{(e^{2x}-1)e^{-x}\ln(e^x+1)}{2a\sqrt{a(e^{2x}-1)^2 e^{-2x}}}-\frac{(e^{2x}-1)e^{-x}\ln(e^x-1)}{2a\sqrt{a(e^{2x}-1)^2 e^{-2x}}}$	109

```
[In] int(1/(a*sinh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(5/2)/sinh(x)*(a*cosh(x)^2)^(1/2)*(-ln(2*(a^(1/2)*(a*cosh(x)^2)^(1/2
)+a)/sinh(x))*a*sinh(x)^2+a^(1/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)^2
)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(34) = 68.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 7.79

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = \frac{\left(6\cosh\left(x\right)e^{x}\sinh\left(x\right)^{2}+2\,e^{x}\sinh\left(x\right)^{3}+2\left(3\cosh\left(x\right)^{2}+1\right)e^{x}\sinh\left(x\right)+2\left(\cosh\left(x\right.\right.\right.}{2\left(a^{2}\cosh\left(x\right)^{4}-\left(a^{2}e^{\left(2x\right)}-a^{2}\right)\sinh\left(x\right)^{4}-2\,a^{2}\cosh\left(x\right)^{2}-4\left(a^{2}\cosh\left(x\right)e^{\left(2x\right)}-\right.\right.}$$

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(x)*e^x*sinh(x)^2 + 2*e^x*sinh(x)^3 + 2*(3*cosh(x)^2 + 1)*e^x*si
nh(x) + 2*(cosh(x)^3 + cosh(x))*e^x - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x
)^4 + 2*(3*cosh(x)^2 - 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 - cosh(x))*e^x*sinh(
x) + (cosh(x)^4 - 2*cosh(x)^2 + 1)*e^x)*log((cosh(x) + sinh(x) + 1)/(cosh(x
) + sinh(x) - 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4
- (a^2*e^(2*x) - a^2)*sinh(x)^4 - 2*a^2*cosh(x)^2 - 4*(a^2*cosh(x)*e^(2*x)
- a^2*cosh(x))*sinh(x)^3 + 2*(3*a^2*cosh(x)^2 - a^2 - (3*a^2*cosh(x)^2 - a^
2)*e^(2*x))*sinh(x)^2 + a^2 - (a^2*cosh(x)^4 - 2*a^2*cosh(x)^2 + a^2)*e^(2*
x) + 4*(a^2*cosh(x)^3 - a^2*cosh(x) - (a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x)
)*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**2)**(3/2),x)
[Out] Integral((a*sinh(x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{e^{(-x)} + e^{(-3x)}}{2a^{\frac{3}{2}}e^{(-2x)} - a^{\frac{3}{2}}e^{(-4x)} - a^{\frac{3}{2}}} - \frac{\log\left(e^{(-x)} + 1\right)}{2a^{\frac{3}{2}}} + \frac{\log\left(e^{(-x)} - 1\right)}{2a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="maxima")
[Out] -(e^(-x) + e^(-3*x))/(2*a^(3/2)*e^(-2*x) - a^(3/2)*e^(-4*x) - a^(3/2)) - 1/
2*log(e^(-x) + 1)/a^(3/2) + 1/2*log(e^(-x) - 1)/a^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = -\frac{e^{(-x)} + e^{x}}{\left(\left(e^{(-x)} + e^{x}\right)^2 - 4\right)a^{\frac{3}{2}}\mathrm{sgn}\left(e^{(3x)} - e^{x}\right)}$$

```
[In] integrate(1/(a*sinh(x)^2)^(3/2),x, algorithm="giac")
[Out] -(e^(-x) + e^x)/(((e^(-x) + e^x)^2 - 4)*a^(3/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh{(x)}^2\right)^{3/2}}\,dx$$

```
[In] int(1/(a*sinh(x)^2)^(3/2),x)
[Out] int(1/(a*sinh(x)^2)^(3/2), x)
```

3.145 $\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx=-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}+\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\text{arctanh}(\cosh(x))\sinh(x)}{8a^2\sqrt{a\sinh^2(x)}}$$

```
[Out] -1/4*coth(x)/a/(a*sinh(x)^2)^(3/2)+3/8*coth(x)/a^2/(a*sinh(x)^2)^(1/2)-3/8*
arctanh(cosh(x))*sinh(x)/a^2/(a*sinh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3283, 3286, 3855}

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx=-\frac{3\sinh(x)\text{arctanh}(\cosh(x))}{8a^2\sqrt{a\sinh^2(x)}}+\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}$$

```
[In] Int[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -1/4*Coth[x]/(a*(a*Sinh[x]^2)^(3/2)) + (3*Coth[x])/(8*a^2*Sqrt[a*Sinh[x]^2]
) - (3*ArcTanh[Cosh[x]]*Sinh[x])/(8*a^2*Sqrt[a*Sinh[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] && !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}-\frac{3\int\frac{1}{\left(a\sinh^2(x)\right)^{3/2}}\,dx}{4a}\\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}+\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}+\frac{3\int\frac{1}{\sqrt{a\sinh^2(x)}}\,dx}{8a^2}\\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}+\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}+\frac{(3\sinh(x))\int\operatorname{csch}(x)\,dx}{8a^2\sqrt{a\sinh^2(x)}}\\
&= -\frac{\coth(x)}{4a\left(a\sinh^2(x)\right)^{3/2}}+\frac{3\coth(x)}{8a^2\sqrt{a\sinh^2(x)}}-\frac{3\operatorname{arctanh}(\cosh(x))\sinh(x)}{8a^2\sqrt{a\sinh^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int\frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx =$$

$$-\frac{\operatorname{csch}(x)\left(-6\operatorname{csch}^2\left(\frac{x}{2}\right)+\operatorname{csch}^4\left(\frac{x}{2}\right)+24\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)-6\operatorname{sech}^2\left(\frac{x}{2}\right)-\operatorname{sech}^4\left(\frac{x}{2}\right)\right)\sqrt{a\sinh}}{64a^3}$$

```
[In] Integrate[(a*Sinh[x]^2)^(-5/2),x]
```

```
[Out] -1/64*(Csch[x]*(-6*Csch[x/2]^2 + Csch[x/2]^4 + 24*(Log[Cosh[x/2]] - Log[Sin
h[x/2]]) - 6*Sech[x/2]^2 - Sech[x/2]^4)*Sqrt[a*Sinh[x]^2])/a^3
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

method	result	size
default	$\frac{\sqrt{a \cosh \left(x \right)^{2}}\, \left(-3 \ln \left(\frac{2 \sqrt{a}\, \sqrt{a \cosh \left(x \right)^{2}}+2 a}{\sinh \left(x \right)}\right) a \sinh \left(x \right)^{4}+3 \sinh \left(x \right)^{2} \sqrt{a}\, \sqrt{a \cosh \left(x \right)^{2}}-2 \sqrt{a}\, \sqrt{a \cosh \left(x \right)^{2}}\right)}{8 a^{\frac{7}{2}} \sinh \left(x \right)^{3} \cosh \left(x \right) \sqrt{a \sinh \left(x \right)^{2}}}$	89
risch	$\frac{3 \,{\mathrm e}^{6 x}-11 \,{\mathrm e}^{4 x}-11 \,{\mathrm e}^{2 x}+3}{4 a^{2} \left({\mathrm e}^{2 x}-1\right)^{3} \sqrt{a \left({\mathrm e}^{2 x}-1\right)^{2} {\mathrm e}^{-2 x}}}+\frac{3 \left({\mathrm e}^{2 x}-1\right) {\mathrm e}^{-x} \ln \left({\mathrm e}^{x}-1\right)}{8 a^{2} \sqrt{a \left({\mathrm e}^{2 x}-1\right)^{2} {\mathrm e}^{-2 x}}}-\frac{3 \left({\mathrm e}^{2 x}-1\right) {\mathrm e}^{-x} \ln \left({\mathrm e}^{x}+1\right)}{8 a^{2} \sqrt{a \left({\mathrm e}^{2 x}-1\right)^{2} {\mathrm e}^{-2 x}}}$	123

```
[In] int(1/(a*sinh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(7/2)/sinh(x)^3*(a*cosh(x)^2)^(1/2)*(-3*ln(2*(a^(1/2)*(a*cosh(x)^2)^(
1/2)+a)/sinh(x))*a*sinh(x)^4+3*sinh(x)^2*a^(1/2)*(a*cosh(x)^2)^(1/2)-2*a^(1
/2)*(a*cosh(x)^2)^(1/2))/cosh(x)/(a*sinh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. 2(49) = 98.

Time = 0.31 (sec) , antiderivative size = 875, normalized size of antiderivative = 14.34

$$\int \frac{1}{\left(a \sinh^2(x)\right)^{5/2}}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(42*cosh(x)*e^x*sinh(x)^6 + 6*e^x*sinh(x)^7 + 2*(63*cosh(x)^2 - 11)*e^
x*sinh(x)^5 + 10*(21*cosh(x)^3 - 11*cosh(x))*e^x*sinh(x)^4 + 2*(105*cosh(x)
^4 - 110*cosh(x)^2 - 11)*e^x*sinh(x)^3 + 2*(63*cosh(x)^5 - 110*cosh(x)^3 -
33*cosh(x))*e^x*sinh(x)^2 + 2*(21*cosh(x)^6 - 55*cosh(x)^4 - 33*cosh(x)^2 +
 3)*e^x*sinh(x) + 2*(3*cosh(x)^7 - 11*cosh(x)^5 - 11*cosh(x)^3 + 3*cosh(x))
*e^x + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*e^x
*sinh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(x)^4 -
30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x)
)*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*e^x*sinh
(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*e^x*sinh(x) + (
cosh(x)^8 - 4*cosh(x)^6 + 6*cosh(x)^4 - 4*cosh(x)^2 + 1)*e^x)*log((cosh(x)
+ sinh(x) - 1)/(cosh(x) + sinh(x) + 1)))*sqrt(a*e^(4*x) - 2*a*e^(2*x) + a)*
e^(-x)/(a^3*cosh(x)^8 - 4*a^3*cosh(x)^6 - (a^3*e^(2*x) - a^3)*sinh(x)^8 - 8
*(a^3*cosh(x)*e^(2*x) - a^3*cosh(x))*sinh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3
*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*co
sh(x)^3 - 3*a^3*cosh(x) - (7*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x))*sinh(x
)^5 - 4*a^3*cosh(x)^2 + 2*(35*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3 - (3
5*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*c
```

```
osh(x)^5 - 10*a^3*cosh(x)^3 + 3*a^3*cosh(x) - (7*a^3*cosh(x)^5 - 10*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 - 15*
a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 - a^3 - (7*a^3*cosh(x)^6 - 15*a^3*cosh(x)^4
 + 9*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^2 - (a^3*cosh(x)^8 - 4*a^3*cosh(
x)^6 + 6*a^3*cosh(x)^4 - 4*a^3*cosh(x)^2 + a^3)*e^(2*x) + 8*(a^3*cosh(x)^7
- 3*a^3*cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x) - (a^3*cosh(x)^7 - 3*a^3*
cosh(x)^5 + 3*a^3*cosh(x)^3 - a^3*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**2)**(5/2),x)
[Out] Integral((a*sinh(x)**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.57

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \frac{3\,e^{(-x)} - 11\,e^{(-3\,x)} - 11\,e^{(-5\,x)} + 3\,e^{(-7\,x)}}{4\left(4\,a^{\frac{5}{2}}e^{(-2\,x)} - 6\,a^{\frac{5}{2}}e^{(-4\,x)} + 4\,a^{\frac{5}{2}}e^{(-6\,x)} - a^{\frac{5}{2}}e^{(-8\,x)} - a^{\frac{5}{2}}\right)} + \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a^{\frac{5}{2}}} - \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="maxima")
[Out] 1/4*(3*e^(-x) - 11*e^(-3*x) - 11*e^(-5*x) + 3*e^(-7*x))/(4*a^(5/2)*e^(-2*x)
 - 6*a^(5/2)*e^(-4*x) + 4*a^(5/2)*e^(-6*x) - a^(5/2)*e^(-8*x) - a^(5/2)) +
3/8*log(e^(-x) + 1)/a^(5/2) - 3/8*log(e^(-x) - 1)/a^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \frac{3\left(e^{(-x)} + e^{x}\right)^3 - 20\,e^{(-x)} - 20\,e^{x}}{4\left(\left(e^{(-x)} + e^{x}\right)^2 - 4\right)^2 a^{\frac{5}{2}}\mathrm{sgn}\left(e^{(3\,x)} - e^{x}\right)}$$

```
[In] integrate(1/(a*sinh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(e^(-x) + e^x)^3 - 20*e^(-x) - 20*e^x)/(((e^(-x) + e^x)^2 - 4)^2*a^(
5/2)*sgn(e^(3*x) - e^x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^2\right)^{5/2}}\,dx$$

```
[In] int(1/(a*sinh(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*sinh(x)^2)^(5/2), x)
```

3.146 $\int \left(a\sinh^3(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\begin{aligned}\int \left(a\sinh^3(x)\right)^{5/2}\,dx = {} & -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} \\ & + \frac{26}{77}ia^2\operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)} \\ & + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} \\ & + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)}\end{aligned}$$

```
[Out] -26/77*a^2*coth(x)*(a*sinh(x)^3)^(1/2)+78/385*a^2*cosh(x)*sinh(x)*(a*sinh(x
)^3)^(1/2)-26/165*a^2*cosh(x)*sinh(x)^3*(a*sinh(x)^3)^(1/2)+2/15*a^2*cosh(x
)*sinh(x)^5*(a*sinh(x)^3)^(1/2)+26/77*I*a^2*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^
2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh
(x))^(1/2)*(a*sinh(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{3286, 2715, 2721, 2720\}$

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx = -\frac{26}{165}a^2\sinh^3(x)\cosh(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\sinh(x)\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^3(x)} - \frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{26}{77}ia^2\sqrt{i\sinh(x)}\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{a\sinh^3(x)}$$

```
[In] Int[(a*Sinh[x]^3)^(5/2),x]
```

```
[Out] (-26*a^2*Coth[x]*Sqrt[a*Sinh[x]^3])/77 + ((26*I)/77)*a^2*Csch[x]^2*Elliptic
F[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3] + (78*a^2*Cosh[x]*Si
nh[x]*Sqrt[a*Sinh[x]^3])/385 - (26*a^2*Cosh[x]*Sinh[x]^3*Sqrt[a*Sinh[x]^3])
/165 + (2*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]^3])/15
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\left(a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{15}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}$$

$$= \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} - \frac{\left(13a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{11}{2}}(x)\,dx}{15\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} + \frac{\left(39a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{7}{2}}(x)\,dx}{55\sinh^{\frac{3}{2}}(x)}$$

$$= \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} - \frac{\left(39a^2\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{3}{2}}(x)\,dx}{77\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} + \frac{\left(13a^2\sqrt{a\sinh^3(x)}\right)\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{77\sinh^{\frac{3}{2}}(x)}$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)} + \frac{1}{77}\left(13a^2\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx$$

$$= -\frac{26}{77}a^2\coth(x)\sqrt{a\sinh^3(x)} + \frac{26}{77}ia^2\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)} + \frac{78}{385}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^3(x)} - \frac{26}{165}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^3(x)} + \frac{2}{15}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^3(x)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.50

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx = \frac{a^2\text{csch}(x)\left(-15465\cosh(x)+3657\cosh(3x)-749\cosh(5x)+77\cosh(7x)-\frac{12480\,\text{Elli}}{\sqrt{}}\right.}{36960}$$

```
[In] Integrate[(a*Sinh[x]^3)^(5/2),x]
```

```
[Out] (a^2*Csch[x]*(-15465*Cosh[x] + 3657*Cosh[3*x] - 749*Cosh[5*x] + 77*Cosh[7*x
] - (12480*EllipticF[(Pi - (2*I)*x)/4, 2])/Sqrt[I*Sinh[x]])*Sqrt[a*Sinh[x]^
3])/36960
```

Maple [F]

$$\int \left(a\sinh\left(x\right)^{3}\right)^{\frac{5}{2}}dx$$

```
[In] int((a*sinh(x)^3)^(5/2),x)
```

```
[Out] int((a*sinh(x)^3)^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 823, normalized size of antiderivative = 6.10

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/73920*(24960*(sqrt(2)*a^2*cosh(x)^7 + 7*sqrt(2)*a^2*cosh(x)^6*sinh(x) + 2
1*sqrt(2)*a^2*cosh(x)^5*sinh(x)^2 + 35*sqrt(2)*a^2*cosh(x)^4*sinh(x)^3 + 35
*sqrt(2)*a^2*cosh(x)^3*sinh(x)^4 + 21*sqrt(2)*a^2*cosh(x)^2*sinh(x)^5 + 7*s
qrt(2)*a^2*cosh(x)*sinh(x)^6 + sqrt(2)*a^2*sinh(x)^7)*sqrt(a)*weierstrassPI
nverse(4, 0, cosh(x) + sinh(x)) + (77*a^2*cosh(x)^14 + 1078*a^2*cosh(x)*sin
h(x)^13 + 77*a^2*sinh(x)^14 - 749*a^2*cosh(x)^12 + 7*(1001*a^2*cosh(x)^2 -
107*a^2)*sinh(x)^12 + 3657*a^2*cosh(x)^10 + 28*(1001*a^2*cosh(x)^3 - 321*a^
2*cosh(x))*sinh(x)^11 + (77077*a^2*cosh(x)^4 - 49434*a^2*cosh(x)^2 + 3657*a
^2)*sinh(x)^10 - 15465*a^2*cosh(x)^8 + 2*(77077*a^2*cosh(x)^5 - 82390*a^2*c
osh(x)^3 + 18285*a^2*cosh(x))*sinh(x)^9 + 3*(77077*a^2*cosh(x)^6 - 123585*a
^2*cosh(x)^4 + 54855*a^2*cosh(x)^2 - 5155*a^2)*sinh(x)^8 - 15465*a^2*cosh(x
)^6 + 24*(11011*a^2*cosh(x)^7 - 24717*a^2*cosh(x)^5 + 18285*a^2*cosh(x)^3 -
 5155*a^2*cosh(x))*sinh(x)^7 + 3*(77077*a^2*cosh(x)^8 - 230692*a^2*cosh(x)^
```

```
6 + 255990*a^2*cosh(x)^4 - 144340*a^2*cosh(x)^2 - 5155*a^2)*sinh(x)^6 + 365
7*a^2*cosh(x)^4 + 2*(77077*a^2*cosh(x)^9 - 296604*a^2*cosh(x)^7 + 460782*a^
2*cosh(x)^5 - 433020*a^2*cosh(x)^3 - 46395*a^2*cosh(x))*sinh(x)^5 + (77077*
a^2*cosh(x)^10 - 370755*a^2*cosh(x)^8 + 767970*a^2*cosh(x)^6 - 1082550*a^2*
cosh(x)^4 - 231975*a^2*cosh(x)^2 + 3657*a^2)*sinh(x)^4 - 749*a^2*cosh(x)^2
+ 4*(7007*a^2*cosh(x)^11 - 41195*a^2*cosh(x)^9 + 109710*a^2*cosh(x)^7 - 216
510*a^2*cosh(x)^5 - 77325*a^2*cosh(x)^3 + 3657*a^2*cosh(x))*sinh(x)^3 + (70
07*a^2*cosh(x)^12 - 49434*a^2*cosh(x)^10 + 164565*a^2*cosh(x)^8 - 433020*a^
2*cosh(x)^6 - 231975*a^2*cosh(x)^4 + 21942*a^2*cosh(x)^2 - 749*a^2)*sinh(x)
^2 + 77*a^2 + 2*(539*a^2*cosh(x)^13 - 4494*a^2*cosh(x)^11 + 18285*a^2*cosh(
x)^9 - 61860*a^2*cosh(x)^7 - 46395*a^2*cosh(x)^5 + 7314*a^2*cosh(x)^3 - 749
*a^2*cosh(x))*sinh(x))*sqrt(a*sinh(x)))/(cosh(x)^7 + 7*cosh(x)^6*sinh(x) +
21*cosh(x)^5*sinh(x)^2 + 35*cosh(x)^4*sinh(x)^3 + 35*cosh(x)^3*sinh(x)^4 +
21*cosh(x)^2*sinh(x)^5 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7)
```

Sympy [F]

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx=\int \left(a\sinh^3{(x)}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sinh(x)**3)**(5/2),x)
[Out] Integral((a*sinh(x)**3)**(5/2), x)
```

Maxima [F]

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx=\int \left(a\sinh{(x)}^3\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx=\int \left(a\sinh{(x)}^3\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sinh(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^3(x)\right)^{5/2}\,dx = \int \left(a\sinh(x)^3\right)^{5/2}\,dx$$

```
[In] int((a*sinh(x)^3)^(5/2),x)
[Out] int((a*sinh(x)^3)^(5/2), x)
```

3.147 $\int \left(a\sinh^3(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 83

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{14iacsch(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)}$$

```
[Out] -14/45*a*cosh(x)*(a*sinh(x)^3)^(1/2)+2/9*a*cosh(x)*sinh(x)^2*(a*sinh(x)^3)^
(1/2)+14/15*I*a*csch(x)*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*E
llipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*(a*sinh(x)^3)^(1/2)/(I*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3286, 2715, 2721, 2719}

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\sinh^2(x)\cosh(x)\sqrt{a\sinh^3(x)} + \frac{14iacsch(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}}$$

```
[In] Int[(a*Sinh[x]^3)^(3/2),x]
```

```
[Out] (-14*a*Cosh[x]*Sqrt[a*Sinh[x]^3])/45 + (((14*I)/15)*a*Csch[x]*EllipticE[Pi/
4 - (I/2)*x, 2]*Sqrt[a*Sinh[x]^3])/Sqrt[I*Sinh[x]] + (2*a*Cosh[x]*Sinh[x]^2
*Sqrt[a*Sinh[x]^3])/9
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{9}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)}\\
&= \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} - \frac{\left(7a\sqrt{a\sinh^3(x)}\right)\int \sinh^{\frac{5}{2}}(x)\,dx}{9\sinh^{\frac{3}{2}}(x)}\\
&= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)}\\
&\quad + \frac{\left(7a\sqrt{a\sinh^3(x)}\right)\int \sqrt{\sinh(x)}\,dx}{15\sinh^{\frac{3}{2}}(x)}
\end{aligned}
$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)} + \frac{\left(7a\text{csch}(x)\sqrt{a\sinh^3(x)}\right)\int\sqrt{i\sinh(x)}\,dx}{15\sqrt{i\sinh(x)}}$$

$$= -\frac{14}{45}a\cosh(x)\sqrt{a\sinh^3(x)} + \frac{14ia\text{csch}(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{a\sinh^3(x)}}{15\sqrt{i\sinh(x)}} + \frac{2}{9}a\cosh(x)\sinh^2(x)\sqrt{a\sinh^3(x)}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = \frac{1}{180}a\text{csch}(x)\sqrt{a\sinh^3(x)}\left(168\text{csch}(x)E\left(\frac{1}{4}(\pi-2ix)\middle|2\right)\sqrt{i\sinh(x)} - 38\sinh(2x) + 5\sinh(4x)\right)$$

```
[In] Integrate[(a*Sinh[x]^3)^(3/2),x]
[Out] (a*Csch[x]*Sqrt[a*Sinh[x]^3]*(168*Csch[x]*EllipticE[(Pi - (2*I)*x)/4, 2]*Sq
rt[I*Sinh[x]] - 38*Sinh[2*x] + 5*Sinh[4*x]))/180
```

Maple [F]

$$\int \left(a\sinh(x)^3\right)^{\frac{3}{2}} dx$$

```
[In] int((a*sinh(x)^3)^(3/2),x)
[Out] int((a*sinh(x)^3)^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.82

$$\int \left(a\sinh^3(x)\right)^{3/2} dx = -\frac{336\left(\sqrt{2}a\cosh(x)^4 + 4\sqrt{2}a\cosh(x)^3\sinh(x) + 6\sqrt{2}a\cosh(x)^2\sinh(x)^2 + 4\sqrt{2}a\cosh(x)\sinh(x)^3 + \sqrt{2}\right.}{}$$

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/360*(336*(sqrt(2)*a*cosh(x)^4 + 4*sqrt(2)*a*cosh(x)^3*sinh(x) + 6*sqrt(2
)*a*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*a*cosh(x)*sinh(x)^3 + sqrt(2)*a*sinh(x)
^4)*sqrt(a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + sinh(
x))) - (5*a*cosh(x)^8 + 40*a*cosh(x)*sinh(x)^7 + 5*a*sinh(x)^8 - 38*a*cosh(
x)^6 + 2*(70*a*cosh(x)^2 - 19*a)*sinh(x)^6 + 4*(70*a*cosh(x)^3 - 57*a*cosh(
x))*sinh(x)^5 - 336*a*cosh(x)^4 + 2*(175*a*cosh(x)^4 - 285*a*cosh(x)^2 - 16
8*a)*sinh(x)^4 + 8*(35*a*cosh(x)^5 - 95*a*cosh(x)^3 - 168*a*cosh(x))*sinh(x
)^3 + 38*a*cosh(x)^2 + 2*(70*a*cosh(x)^6 - 285*a*cosh(x)^4 - 1008*a*cosh(x)
^2 + 19*a)*sinh(x)^2 + 4*(10*a*cosh(x)^7 - 57*a*cosh(x)^5 - 336*a*cosh(x)^3
 + 19*a*cosh(x))*sinh(x) - 5*a)*sqrt(a*sinh(x)))/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F]

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx = \int \left(a\sinh^3\left(x\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sinh(x)**3)**(3/2),x)
[Out] Integral((a*sinh(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx = \int \left(a\sinh\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx = \int \left(a\sinh\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sinh(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^3(x)\right)^{3/2}\,dx=\int \left(a\sinh(x)^3\right)^{3/2}\,dx$$

```
[In] int((a*sinh(x)^3)^(3/2),x)
[Out] int((a*sinh(x)^3)^(3/2), x)
```

3.148 $\int \sqrt{a \sinh^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \sqrt{a \sinh^3(x)}\,dx = \frac{2}{3}\coth(x)\sqrt{a \sinh^3(x)} - \frac{2}{3} i \operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i \sinh(x)}\sqrt{a \sinh^3(x)}$$

```
[Out] 2/3*coth(x)*(a*sinh(x)^3)^(1/2)-2/3*I*csch(x)^2*(sin(1/4*Pi+1/2*I*x)^2)^(1/
2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(1/4*Pi+1/2*I*x),2^(1/2))*(I*sinh(x))^(
1/2)*(a*sinh(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3286, 2715, 2721, 2720}

$$\int \sqrt{a \sinh^3(x)}\,dx = \frac{2}{3}\coth(x)\sqrt{a \sinh^3(x)} - \frac{2}{3} i \sqrt{i \sinh(x)}\operatorname{csch}^2(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{a \sinh^3(x)}$$

```
[In] Int[Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (2*Coth[x]*Sqrt[a*Sinh[x]^3])/3 - ((2*I)/3)*Csch[x]^2*EllipticF[Pi/4 - (I/2
)*x, 2]*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a\sinh^3(x)}\int \sinh^{\frac{3}{2}}(x)\,dx}{\sinh^{\frac{3}{2}}(x)} \\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{\sqrt{a\sinh^3(x)}\int \frac{1}{\sqrt{\sinh(x)}}\,dx}{3\sinh^{\frac{3}{2}}(x)} \\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{1}{3}\left(\operatorname{csch}^2(x)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}\right)\int \frac{1}{\sqrt{i\sinh(x)}}\,dx \\
&= \frac{2}{3}\coth(x)\sqrt{a\sinh^3(x)} - \frac{2}{3}i\operatorname{csch}^2(x)\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sqrt{a \sinh^3(x)}\, dx = \frac{2}{3}\sqrt{a \sinh^3(x)}\Big(\coth(x) - \sqrt{2}\text{csch}^2(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2x) + \sinh(2x)\right)\sqrt{-\sinh(x)(\cosh(x)+\sinh(x))}\Big)$$

```
[In] Integrate[Sqrt[a*Sinh[x]^3],x]
[Out] (2*Sqrt[a*Sinh[x]^3]*(Coth[x] - Sqrt[2]*Csch[x]^2*Hypergeometric2F1[1/4, 1/
2, 5/4, Cosh[2*x] + Sinh[2*x]]*Sqrt[-(Sinh[x]*(Cosh[x] + Sinh[x]))]))/3
```

Maple [F]

$$\int \sqrt{a \sinh{(x)}^3}dx$$

```
[In] int((a*sinh(x)^3)^(1/2),x)
[Out] int((a*sinh(x)^3)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \sqrt{a \sinh^3(x)}\, dx = -\frac{2\left(\sqrt{2}\cosh{(x)} + \sqrt{2}\sinh{(x)}\right)\sqrt{a}\text{weierstrassPInverse}(4,0,\cosh{(x)}+\sinh{(x)}) - \left(\cosh{(x)}^2 + 2\cosh{(x)}\right.}{3\left(\cosh{(x)}+\sinh{(x)}\right)}$$

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="fricas")
[Out] -1/3*(2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(a)*weierstrassPInverse(4,
0, cosh(x) + sinh(x)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqr
t(a*sinh(x)))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \sqrt{a\sinh^3(x)}\,dx = \int \sqrt{a\sinh^3{(x)}}\,dx$$

```
[In] integrate((a*sinh(x)**3)**(1/2),x)
[Out] Integral(sqrt(a*sinh(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\sinh^3(x)}\,dx = \int \sqrt{a\sinh{(x)}^3}\,dx$$

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\sinh^3(x)}\,dx = \int \sqrt{a\sinh{(x)}^3}\,dx$$

```
[In] integrate((a*sinh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sinh(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\sinh^3(x)}\,dx = \int \sqrt{a\sinh{(x)}^3}\,dx$$

```
[In] int((a*sinh(x)^3)^(1/2),x)
[Out] int((a*sinh(x)^3)^(1/2), x)
```

3.149 $\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx = -\frac{2 \cosh(x) \sinh(x)}{\sqrt{a \sinh^3(x)}} + \frac{2iE\left(\frac{\pi}{4} - \frac{ix}{2} \middle| 2\right) \sinh^2(x)}{\sqrt{i \sinh(x)} \sqrt{a \sinh^3(x)}}$$

```
[Out] -2*cosh(x)*sinh(x)/(a*sinh(x)^3)^(1/2)+2*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/si
n(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2/(I*sinh(
x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3286, 2716, 2721, 2719}

$$\int \frac{1}{\sqrt{a \sinh^3(x)}} \, dx = -\frac{2 \sinh(x) \cosh(x)}{\sqrt{a \sinh^3(x)}} + \frac{2i \sinh^2(x) E\left(\frac{\pi}{4} - \frac{ix}{2} \middle| 2\right)}{\sqrt{i \sinh(x)} \sqrt{a \sinh^3(x)}}$$

```
[In] Int[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^3] + ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2
]*Sinh[x]^2)/(Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{\sinh^{\frac{3}{2}}(x)\int \sqrt{\sinh(x)}\,dx}{\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{\sinh^2(x)\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}\\ &= -\frac{2\cosh(x)\sinh(x)}{\sqrt{a\sinh^3(x)}}+\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|\,2\right)\sinh^2(x)}{\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = -\frac{2\left(\cosh(x) - E\left(\frac{1}{4}(\pi - 2ix)\middle|\,2\right)\sqrt{i\sinh(x)}\right)\sinh(x)}{\sqrt{a\sinh^3(x)}}$$

```
[In] Integrate[1/Sqrt[a*Sinh[x]^3],x]
```

```
[Out] (-2*(Cosh[x] - EllipticE[(Pi - (2*I)*x)/4, 2]*Sqrt[I*Sinh[x]])*Sinh[x])/Sqr
t[a*Sinh[x]^3]
```

Maple [F]

$$\int \frac{1}{\sqrt{a\sinh\left(x\right)^{3}}}dx$$

```
[In] int(1/(a*sinh(x)^3)^(1/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh\left(x\right)^{2} + 2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right) + \sqrt{2}\sinh\left(x\right)^{2} - \sqrt{2}\right)\sqrt{a}\mathrm{weierstrassZeta}(4, 0, \mathrm{weierstrassPInv}}{a\cosh\left(x\right)^{2} + 2\,a\cosh\left(x\right)\sinh\left(x\right)}$$

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sq
rt(2))*sqrt(a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x) + si
nh(x))) + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a*sinh(x)))/(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh^3{(x)}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*sinh(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh{(x)}^3}}\,dx$$

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh{(x)}^3}}\,dx$$

```
[In] integrate(1/(a*sinh(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*sinh(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\sinh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\sinh{(x)}^3}}\,dx$$

```
[In] int(1/(a*sinh(x)^3)^(1/2),x)
[Out] int(1/(a*sinh(x)^3)^(1/2), x)
```

3.150 $\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}(x)}{7a\sqrt{a\sinh^3(x)}} + \frac{10i\,\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sinh(x)}{21a\sqrt{a\sinh^3(x)}}$$

```
[Out] 10/21*cosh(x)/a/(a*sinh(x)^3)^(1/2)-2/7*coth(x)*csch(x)/a/(a*sinh(x)^3)^(1/
2)+10/21*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/sin(1/4*Pi+1/2*I*x)*EllipticF(cos(
1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)*(I*sinh(x))^(1/2)/a/(a*sinh(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3286, 2716, 2721, 2720}

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}} + \frac{10i\sqrt{i\sinh(x)}\sinh(x)\operatorname{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)}{21a\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}(x)}{7a\sqrt{a\sinh^3(x)}}$$

```
[In] Int[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (10*Cosh[x])/(21*a*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x])/(7*a*Sqrt[a*Sin
h[x]^3]) + (((10*I)/21)*EllipticF[Pi/4 - (I/2)*x, 2]*Sqrt[I*Sinh[x]]*Sinh[x
])/(a*Sqrt[a*Sinh[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sinh^{\frac{3}{2}}(x)\int\frac{1}{\sinh^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sinh^3(x)}}\\
&= -\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}-\frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int\frac{1}{\sinh^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\sinh^3(x)}}\\
&= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}+\frac{\left(5\sinh^{\frac{3}{2}}(x)\right)\int\frac{1}{\sqrt{\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}\\
&= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}+\frac{\left(5\sqrt{i\sinh(x)}\sinh(x)\right)\int\frac{1}{\sqrt{i\sinh(x)}}\,dx}{21a\sqrt{a\sinh^3(x)}}\\
&= \frac{10\cosh(x)}{21a\sqrt{a\sinh^3(x)}}-\frac{2\coth(x)\text{csch}(x)}{7a\sqrt{a\sinh^3(x)}}+\frac{10i\,\text{EllipticF}\left(\frac{\pi}{4}-\frac{ix}{2},2\right)\sqrt{i\sinh(x)}\sinh(x)}{21a\sqrt{a\sinh^3(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \frac{2\left(5\cosh(x)-3\coth(x)\text{csch}(x)+5\,\text{EllipticF}\left(\frac{1}{4}(\pi-2ix),2\right)(i\sinh(x))^{3/2}\right)}{21a\sqrt{a\sinh^3(x)}}$$

```
[In] Integrate[(a*Sinh[x]^3)^(-3/2),x]
```

```
[Out] (2*(5*Cosh[x] - 3*Coth[x]*Csch[x] + 5*EllipticF[(Pi - (2*I)*x)/4, 2]*(I*Sin
h[x])^(3/2)))/(21*a*Sqrt[a*Sinh[x]^3])
```

Maple [F]

$$\int \frac{1}{\left(a\sinh\left(x\right)^{3}\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a*sinh(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sinh(x)^3)^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 639, normalized size of antiderivative = 7.34

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(5*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^
8 + 4*(7*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^6 - 4*sqrt(2)*cosh(x)^6 + 8*(
7*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^
4 - 30*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^4 + 6*sqrt(2)*cosh(x)^4 + 8*(
7*sqrt(2)*cosh(x)^5 - 10*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*sinh(x)^3 +
 4*(7*sqrt(2)*cosh(x)^6 - 15*sqrt(2)*cosh(x)^4 + 9*sqrt(2)*cosh(x)^2 - sqrt
(2))*sinh(x)^2 - 4*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 3*sqrt(2)*cos
h(x)^5 + 3*sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*sqrt(a)*
weierstrassPInverse(4, 0, cosh(x) + sinh(x)) + 2*(5*cosh(x)^7 + 35*cosh(x)*
sinh(x)^6 + 5*sinh(x)^7 + (105*cosh(x)^2 - 17)*sinh(x)^5 - 17*cosh(x)^5 + 5
*(35*cosh(x)^3 - 17*cosh(x))*sinh(x)^4 + (175*cosh(x)^4 - 170*cosh(x)^2 - 1
7)*sinh(x)^3 - 17*cosh(x)^3 + (105*cosh(x)^5 - 170*cosh(x)^3 - 51*cosh(x))*
sinh(x)^2 + (35*cosh(x)^6 - 85*cosh(x)^4 - 51*cosh(x)^2 + 5)*sinh(x) + 5*co
```

```
sh(x))*sqrt(a*sinh(x)))/(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh
(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(
x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4
- 30*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^
5 - 10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a
^2*cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 -
3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a \sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a \sinh^{3}{\left(x \right)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**3)**(3/2),x)
[Out] Integral((a*sinh(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a \sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a \sinh\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a \sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a \sinh\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{3/2}}\,dx$$

```
[In] int(1/(a*sinh(x)^3)^(3/2),x)
[Out] int(1/(a*sinh(x)^3)^(3/2), x)
```

3.151 $\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 135

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{154iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

```
[Out] -154/585*coth(x)/a^2/(a*sinh(x)^3)^(1/2)+22/117*coth(x)*csch(x)^2/a^2/(a*si
nh(x)^3)^(1/2)-2/13*coth(x)*csch(x)^4/a^2/(a*sinh(x)^3)^(1/2)+154/195*cosh(
x)*sinh(x)/a^2/(a*sinh(x)^3)^(1/2)-154/195*I*(sin(1/4*Pi+1/2*I*x)^2)^(1/2)/
sin(1/4*Pi+1/2*I*x)*EllipticE(cos(1/4*Pi+1/2*I*x),2^(1/2))*sinh(x)^2/a^2/(I
*sinh(x))^(1/2)/(a*sinh(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3286, 2716, 2721, 2719}

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \frac{154\sinh(x)\cosh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} - \frac{154i\sinh^2(x)E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\operatorname{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\operatorname{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}}$$

```
[In] Int[(a*Sinh[x]^3)^(-5/2),x]
```

```
[Out] (-154*Coth[x])/(585*a^2*Sqrt[a*Sinh[x]^3]) + (22*Coth[x]*Csch[x]^2)/(117*a^
2*Sqrt[a*Sinh[x]^3]) - (2*Coth[x]*Csch[x]^4)/(13*a^2*Sqrt[a*Sinh[x]^3]) + (
154*Cosh[x]*Sinh[x])/(195*a^2*Sqrt[a*Sinh[x]^3]) - (((154*I)/195)*EllipticE
[Pi/4 - (I/2)*x, 2]*Sinh[x]^2)/(a^2*Sqrt[I*Sinh[x]]*Sqrt[a*Sinh[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\sinh^{\frac{3}{2}}(x)\int \frac{1}{\sinh^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(11\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\sinh^3(x)}}$$

$$= \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int \frac{1}{\sinh^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} - \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int\frac{1}{\sinh^{\frac{3}{2}}(x)}dx}{195a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{\left(77\sinh^{\frac{3}{2}}(x)\right)\int\sqrt{\sinh(x)}dx}{195a^2\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{\left(77\sinh^2(x)\right)\int\sqrt{i\sinh(x)}dx}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

$$= -\frac{154\coth(x)}{585a^2\sqrt{a\sinh^3(x)}} + \frac{22\coth(x)\text{csch}^2(x)}{117a^2\sqrt{a\sinh^3(x)}} - \frac{2\coth(x)\text{csch}^4(x)}{13a^2\sqrt{a\sinh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\sinh^3(x)}} - \frac{154iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sinh^2(x)}{195a^2\sqrt{i\sinh(x)}\sqrt{a\sinh^3(x)}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.51

$$\int\frac{1}{\left(a\sinh^3(x)\right)^{5/2}}dx = \frac{-2\coth(x)\left(77-55\text{csch}^2(x)+45\text{csch}^4(x)\right)+462iE\left(\frac{1}{4}(\pi-2ix)\middle|2\right)(i\sinh(x))^{3/2}+}{585a^2\sqrt{a\sinh^3(x)}}$$

```
[In] Integrate[(a*Sinh[x]^3)^(-5/2),x]
```

```
[Out] (-2*Coth[x]*(77 - 55*Csch[x]^2 + 45*Csch[x]^4) + (462*I)*EllipticE[(Pi - (2
*I)*x)/4, 2]*(I*Sinh[x])^(3/2) + 462*Cosh[x]*Sinh[x])/(585*a^2*Sqrt[a*Sinh[
x]^3])
```

Maple [F]

$$\int \frac{1}{\left(a \sinh \left(x \right)^{3}\right)^{\frac{5}{2}}}d x$$

[In] int(1/(a*sinh(x)^3)^(5/2),x)

[Out] int(1/(a*sinh(x)^3)^(5/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1676, normalized size of antiderivative = 12.41

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx=\text{Too large to display}$$

[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="fricas")

[Out] 2/585*(231*(sqrt(2)*cosh(x)^14 + 14*sqrt(2)*cosh(x)*sinh(x)^13 + sqrt(2)*si
nh(x)^14 + 7*(13*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^12 - 7*sqrt(2)*cosh(x
)^12 + 28*(13*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^11 + 7*(143*sq
rt(2)*cosh(x)^4 - 66*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x)^10 + 21*sqrt(2)
cosh(x)^10 + 14(143*sqrt(2)*cosh(x)^5 - 110*sqrt(2)*cosh(x)^3 + 15*sqrt(2
)*cosh(x))*sinh(x)^9 + 7*(429*sqrt(2)*cosh(x)^6 - 495*sqrt(2)*cosh(x)^4 + 1
35*sqrt(2)*cosh(x)^2 - 5*sqrt(2))*sinh(x)^8 - 35*sqrt(2)*cosh(x)^8 + 8*(429
*sqrt(2)*cosh(x)^7 - 693*sqrt(2)*cosh(x)^5 + 315*sqrt(2)*cosh(x)^3 - 35*sqr
t(2)*cosh(x))*sinh(x)^7 + 7*(429*sqrt(2)*cosh(x)^8 - 924*sqrt(2)*cosh(x)^6
+ 630*sqrt(2)*cosh(x)^4 - 140*sqrt(2)*cosh(x)^2 + 5*sqrt(2))*sinh(x)^6 + 35
*sqrt(2)*cosh(x)^6 + 14*(143*sqrt(2)*cosh(x)^9 - 396*sqrt(2)*cosh(x)^7 + 37
8*sqrt(2)*cosh(x)^5 - 140*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x))*sinh(x)^5
 + 7*(143*sqrt(2)*cosh(x)^10 - 495*sqrt(2)*cosh(x)^8 + 630*sqrt(2)*cosh(x)^
6 - 350*sqrt(2)*cosh(x)^4 + 75*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^4 - 2
1*sqrt(2)*cosh(x)^4 + 28*(13*sqrt(2)*cosh(x)^11 - 55*sqrt(2)*cosh(x)^9 + 90
*sqrt(2)*cosh(x)^7 - 70*sqrt(2)*cosh(x)^5 + 25*sqrt(2)*cosh(x)^3 - 3*sqrt(2
)*cosh(x))*sinh(x)^3 + 7*(13*sqrt(2)*cosh(x)^12 - 66*sqrt(2)*cosh(x)^10 + 1
35*sqrt(2)*cosh(x)^8 - 140*sqrt(2)*cosh(x)^6 + 75*sqrt(2)*cosh(x)^4 - 18*sq
rt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 7*sqrt(2)*cosh(x)^2 + 14*(sqrt(2)*co
sh(x)^13 - 6*sqrt(2)*cosh(x)^11 + 15*sqrt(2)*cosh(x)^9 - 20*sqrt(2)*cosh(x)
^7 + 15*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x)
- sqrt(2))*sqrt(a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(x)
+ sinh(x))) + 2*(231*cosh(x)^14 + 3234*cosh(x)*sinh(x)^13 + 231*sinh(x)^14
+ 77*(273*cosh(x)^2 - 20)*sinh(x)^12 - 1540*cosh(x)^12 + 924*(91*cosh(x)^3
- 20*cosh(x))*sinh(x)^11 + 11*(21021*cosh(x)^4 - 9240*cosh(x)^2 + 397)*sinh
(x)^10 + 4367*cosh(x)^10 + 22*(21021*cosh(x)^5 - 15400*cosh(x)^3 + 1985*cos

```
h(x))*sinh(x)^9 + (693693*cosh(x)^6 - 762300*cosh(x)^4 + 196515*cosh(x)^2 -
 6808)*sinh(x)^8 - 6808*cosh(x)^8 + 8*(99099*cosh(x)^7 - 152460*cosh(x)^5 +
 65505*cosh(x)^3 - 6808*cosh(x))*sinh(x)^7 + (693693*cosh(x)^8 - 1422960*co
sh(x)^6 + 917070*cosh(x)^4 - 190624*cosh(x)^2 + 1277)*sinh(x)^6 + 1277*cosh
(x)^6 + 2*(231231*cosh(x)^9 - 609840*cosh(x)^7 + 550242*cosh(x)^5 - 190624*
cosh(x)^3 + 3831*cosh(x))*sinh(x)^5 + (231231*cosh(x)^10 - 762300*cosh(x)^8
 + 917070*cosh(x)^6 - 476560*cosh(x)^4 + 19155*cosh(x)^2 - 484)*sinh(x)^4 -
 484*cosh(x)^4 + 4*(21021*cosh(x)^11 - 84700*cosh(x)^9 + 131010*cosh(x)^7 -
 95312*cosh(x)^5 + 6385*cosh(x)^3 - 484*cosh(x))*sinh(x)^3 + (21021*cosh(x)
^12 - 101640*cosh(x)^10 + 196515*cosh(x)^8 - 190624*cosh(x)^6 + 19155*cosh(
x)^4 - 2904*cosh(x)^2 + 77)*sinh(x)^2 + 77*cosh(x)^2 + 2*(1617*cosh(x)^13 -
 9240*cosh(x)^11 + 21835*cosh(x)^9 - 27232*cosh(x)^7 + 3831*cosh(x)^5 - 968
*cosh(x)^3 + 77*cosh(x))*sinh(x))*sqrt(a*sinh(x)))/(a^3*cosh(x)^14 + 14*a^3
*cosh(x)*sinh(x)^13 + a^3*sinh(x)^14 - 7*a^3*cosh(x)^12 + 21*a^3*cosh(x)^10
 + 7*(13*a^3*cosh(x)^2 - a^3)*sinh(x)^12 + 28*(13*a^3*cosh(x)^3 - 3*a^3*cos
h(x))*sinh(x)^11 - 35*a^3*cosh(x)^8 + 7*(143*a^3*cosh(x)^4 - 66*a^3*cosh(x)
^2 + 3*a^3)*sinh(x)^10 + 14*(143*a^3*cosh(x)^5 - 110*a^3*cosh(x)^3 + 15*a^3
*cosh(x))*sinh(x)^9 + 35*a^3*cosh(x)^6 + 7*(429*a^3*cosh(x)^6 - 495*a^3*cos
h(x)^4 + 135*a^3*cosh(x)^2 - 5*a^3)*sinh(x)^8 + 8*(429*a^3*cosh(x)^7 - 693*
a^3*cosh(x)^5 + 315*a^3*cosh(x)^3 - 35*a^3*cosh(x))*sinh(x)^7 - 21*a^3*cosh
(x)^4 + 7*(429*a^3*cosh(x)^8 - 924*a^3*cosh(x)^6 + 630*a^3*cosh(x)^4 - 140*
a^3*cosh(x)^2 + 5*a^3)*sinh(x)^6 + 14*(143*a^3*cosh(x)^9 - 396*a^3*cosh(x)^
7 + 378*a^3*cosh(x)^5 - 140*a^3*cosh(x)^3 + 15*a^3*cosh(x))*sinh(x)^5 + 7*a
^3*cosh(x)^2 + 7*(143*a^3*cosh(x)^10 - 495*a^3*cosh(x)^8 + 630*a^3*cosh(x)^
6 - 350*a^3*cosh(x)^4 + 75*a^3*cosh(x)^2 - 3*a^3)*sinh(x)^4 + 28*(13*a^3*co
sh(x)^11 - 55*a^3*cosh(x)^9 + 90*a^3*cosh(x)^7 - 70*a^3*cosh(x)^5 + 25*a^3*
cosh(x)^3 - 3*a^3*cosh(x))*sinh(x)^3 - a^3 + 7*(13*a^3*cosh(x)^12 - 66*a^3*
cosh(x)^10 + 135*a^3*cosh(x)^8 - 140*a^3*cosh(x)^6 + 75*a^3*cosh(x)^4 - 18*
a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 14*(a^3*cosh(x)^13 - 6*a^3*cosh(x)^11 + 15
*a^3*cosh(x)^9 - 20*a^3*cosh(x)^7 + 15*a^3*cosh(x)^5 - 6*a^3*cosh(x)^3 + a^
3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh^3{(x)}\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**3)**(5/2),x)
[Out] Integral((a*sinh(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sinh(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\sinh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh\left(x\right)^3\right)^{5/2}}\,dx$$

```
[In] int(1/(a*sinh(x)^3)^(5/2),x)
[Out] int(1/(a*sinh(x)^3)^(5/2), x)
```

3.152 $\int \left(a\sinh^4(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx=\frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)}-\frac{63}{256}a^2x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}-\frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)}+\frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)}-\frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)}+\frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)}$$

```
[Out] 63/256*a^2*coth(x)*(a*sinh(x)^4)^(1/2)-63/256*a^2*x*csch(x)^2*(a*sinh(x)^4)
^(1/2)-21/128*a^2*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+21/160*a^2*cosh(x)*si
nh(x)^3*(a*sinh(x)^4)^(1/2)-9/80*a^2*cosh(x)*sinh(x)^5*(a*sinh(x)^4)^(1/2)+
1/10*a^2*cosh(x)*sinh(x)^7*(a*sinh(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3286, 2715, 8}

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx=-\frac{21}{128}a^2\sinh(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{1}{10}a^2\sinh^7(x)\cosh(x)\sqrt{a\sinh^4(x)}-\frac{9}{80}a^2\sinh^5(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{21}{160}a^2\sinh^3(x)\cosh(x)\sqrt{a\sinh^4(x)}+\frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)}-\frac{63}{256}a^2x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}$$

```
[In] Int[(a*Sinh[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*Coth[x]*Sqrt[a*Sinh[x]^4])/256 - (63*a^2*x*Csch[x]^2*Sqrt[a*Sinh[x]
^4])/256 - (21*a^2*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/128 + (21*a^2*Cosh[x]
*Sinh[x]^3*Sqrt[a*Sinh[x]^4])/160 - (9*a^2*Cosh[x]*Sinh[x]^5*Sqrt[a*Sinh[x]
^4])/80 + (a^2*Cosh[x]*Sinh[x]^7*Sqrt[a*Sinh[x]^4])/10
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^{10}(x)\,dx\\
&= \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} - \frac{1}{10}\left(9a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^8(x)\,dx\\
&= -\frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)}\\
&\quad + \frac{1}{80}\left(63a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^6(x)\,dx\\
&= \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)}\\
&\quad + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} - \frac{1}{32}\left(21a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^4(x)\,dx\\
&= -\frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)}\\
&\quad - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)}\\
&\quad + \frac{1}{128}\left(63a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^2(x)\,dx
\end{aligned}$$

$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)} - \frac{1}{256}\left(63a^2\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int 1\,dx$$

$$= \frac{63}{256}a^2\coth(x)\sqrt{a\sinh^4(x)} - \frac{63}{256}a^2x\text{csch}^2(x)\sqrt{a\sinh^4(x)} - \frac{21}{128}a^2\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{21}{160}a^2\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{9}{80}a^2\cosh(x)\sinh^5(x)\sqrt{a\sinh^4(x)} + \frac{1}{10}a^2\cosh(x)\sinh^7(x)\sqrt{a\sinh^4(x)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.40

$$\int \left(a\sinh^4(x)\right)^{5/2}\,dx = \frac{a\text{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}(-2520x + 2100\sinh(2x) - 600\sinh(4x) + 150\sinh(6x) -}{10240}$$

```
[In] Integrate[(a*Sinh[x]^4)^(5/2),x]
```

```
[Out] (a*Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-2520*x + 2100*Sinh[2*x] - 600*Sinh[4*x]
+ 150*Sinh[6*x] - 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

Maple [A] (verified)

Time = 8.39 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

method	result
default	$\frac{a^{\frac{3}{2}}(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(8\sqrt{a}\sqrt{a\sinh(2x)^2}\sinh(2x)^4-50\sqrt{a}\sqrt{a\sinh(2x)^2}\cosh(2x)\sinh(2x)^2+160\sqrt{a}\right.}{2560\sinh(2x)\sqrt{(-1+c}}$
risch	$-\frac{63a^2e^{2x}\sqrt{a(e^{2x}-1)^4e^{-4x}}\,x}{256(e^{2x}-1)^2}+\frac{a^2e^{12x}\sqrt{a(e^{2x}-1)^4e^{-4x}}}{10240(e^{2x}-1)^2}-\frac{5a^2e^{10x}\sqrt{a(e^{2x}-1)^4e^{-4x}}}{4096(e^{2x}-1)^2}+\frac{15a^2e^{8x}\sqrt{a(e^{2x}-1)^4e^{-4x}}}{2048(e^{2x}-1)^2}-\frac{15a^2e^{}}{}$

```
[In] int((a*sinh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2560*a^(3/2)*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*(8*a^(
1/2)*(a*sinh(2*x)^2)^(1/2)*sinh(2*x)^4-50*a^(1/2)*(a*sinh(2*x)^2)^(1/2)*cos
h(2*x)*sinh(2*x)^2+160*a^(1/2)*(a*sinh(2*x)^2)^(1/2)*sinh(2*x)^2-325*cosh(2
*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-315*ln(
cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/((-1+cosh(2*x))^2*a)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. 2(108) = 216.

Time = 0.31 (sec) , antiderivative size = 1597, normalized size of antiderivative = 12.10

$$\int \left(a\sinh^4(x)\right)^{5/2} dx = \text{Too large to display}$$

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="fricas")
[Out] 1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*(
76*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 - 15*a^
2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 - 255*a^2*cosh(x)^2 +
 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 - 425*a^2*cosh(x)^3 + 5
0*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 - 1275*a^2*cosh(
x)^4 + 300*a^2*cosh(x)^2 - 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2*cosh(
x)^7 - 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sin
h(x)^13 + 60*(4199*a^2*cosh(x)^8 - 7735*a^2*cosh(x)^6 + 4550*a^2*cosh(x)^4
- 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*cosh(x)^9 -
 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 - 2730*a^2*cosh(x)^3 + 315*a^2*cos
h(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 - 546975*a^2*cosh(x)^8
+ 600600*a^2*cosh(x)^6 - 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(x)^2 - 2520*
a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 - 60775*a^2*cosh(x)^9
+ 85800*a^2*cosh(x)^7 - 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(x)^3 - 2520*a^
2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 - 36465*a^2*cosh(x
)^10 + 64350*a^2*cosh(x)^8 - 60060*a^2*cosh(x)^6 + 34650*a^2*cosh(x)^4 - 75
60*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*a^2*cosh(x)^13 -
3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 - 8580*a^2*cosh(x)^7 + 6930*a^2*co
sh(x)^5 - 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)*sinh(x)^7 + 60*(12
92*a^2*cosh(x)^14 - 7735*a^2*cosh(x)^12 + 20020*a^2*cosh(x)^10 - 30030*a^2*
cosh(x)^8 + 32340*a^2*cosh(x)^6 - 17640*a^2*x*cosh(x)^4 - 980*a^2*cosh(x)^2
 + 10*a^2)*e^(2*x)*sinh(x)^6 + 24*(1292*a^2*cosh(x)^15 - 8925*a^2*cosh(x)^1
3 + 27300*a^2*cosh(x)^11 - 50050*a^2*cosh(x)^9 + 69300*a^2*cosh(x)^7 - 5292
0*a^2*x*cosh(x)^5 - 4900*a^2*cosh(x)^3 + 150*a^2*cosh(x))*e^(2*x)*sinh(x)^5
 + 30*(323*a^2*cosh(x)^16 - 2550*a^2*cosh(x)^14 + 9100*a^2*cosh(x)^12 - 200
20*a^2*cosh(x)^10 + 34650*a^2*cosh(x)^8 - 35280*a^2*x*cosh(x)^6 - 4900*a^2*
cosh(x)^4 + 300*a^2*cosh(x)^2 - 5*a^2)*e^(2*x)*sinh(x)^4 + 120*(19*a^2*cosh
(x)^17 - 170*a^2*cosh(x)^15 + 700*a^2*cosh(x)^13 - 1820*a^2*cosh(x)^11 + 38
50*a^2*cosh(x)^9 - 5040*a^2*x*cosh(x)^7 - 980*a^2*cosh(x)^5 + 100*a^2*cosh(
x)^3 - 5*a^2*cosh(x))*e^(2*x)*sinh(x)^3 + 5*(76*a^2*cosh(x)^18 - 765*a^2*co
sh(x)^16 + 3600*a^2*cosh(x)^14 - 10920*a^2*cosh(x)^12 + 27720*a^2*cosh(x)^1
0 - 45360*a^2*x*cosh(x)^8 - 11760*a^2*cosh(x)^6 + 1800*a^2*cosh(x)^4 - 180*
a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^2 + 10*(4*a^2*cosh(x)^19 - 45*a^2*co
sh(x)^17 + 240*a^2*cosh(x)^15 - 840*a^2*cosh(x)^13 + 2520*a^2*cosh(x)^11 -
5040*a^2*x*cosh(x)^9 - 1680*a^2*cosh(x)^7 + 360*a^2*cosh(x)^5 - 60*a^2*cosh
```

```
(x)^3 + 5*a^2*cosh(x))*e^(2*x)*sinh(x) + (2*a^2*cosh(x)^20 - 25*a^2*cosh(x)
^18 + 150*a^2*cosh(x)^16 - 600*a^2*cosh(x)^14 + 2100*a^2*cosh(x)^12 - 5040*
a^2*x*cosh(x)^10 - 2100*a^2*cosh(x)^8 + 600*a^2*cosh(x)^6 - 150*a^2*cosh(x)
^4 + 25*a^2*cosh(x)^2 - 2*a^2)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*
e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^10*e^(4*x) - 2*cosh(x)^10*e^(2
*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^10 + cosh(x)^10 + 10*(cosh(x)*e^(4*
x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 + 45*(cosh(x)^2*e^(4*x) - 2*cos
h(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^8 + 120*(cosh(x)^3*e^(4*x) - 2*cosh(x)^
3*e^(2*x) + cosh(x)^3)*sinh(x)^7 + 210*(cosh(x)^4*e^(4*x) - 2*cosh(x)^4*e^(
2*x) + cosh(x)^4)*sinh(x)^6 + 252*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5*e^(2*x)
+ cosh(x)^5)*sinh(x)^5 + 210*(cosh(x)^6*e^(4*x) - 2*cosh(x)^6*e^(2*x) + cos
h(x)^6)*sinh(x)^4 + 120*(cosh(x)^7*e^(4*x) - 2*cosh(x)^7*e^(2*x) + cosh(x)^
7)*sinh(x)^3 + 45*(cosh(x)^8*e^(4*x) - 2*cosh(x)^8*e^(2*x) + cosh(x)^8)*sin
h(x)^2 + 10*(cosh(x)^9*e^(4*x) - 2*cosh(x)^9*e^(2*x) + cosh(x)^9)*sinh(x))
```

Sympy [F]

$$\int \left(a\sinh^4(x)\right)^{5/2}\, dx = \int \left(a\sinh^4\left(x\right)\right)^{\frac{5}{2}}\, dx$$

```
[In] integrate((a*sinh(x)**4)**(5/2),x)
[Out] Integral((a*sinh(x)**4)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \left(a\sinh^4(x)\right)^{5/2}\, dx = -\frac{63}{256}\, a^{\frac{5}{2}}x - \frac{1}{20480}\left(25\, a^{\frac{5}{2}}e^{(-2\,x)} - 150\, a^{\frac{5}{2}}e^{(-4\,x)} + 600\, a^{\frac{5}{2}}e^{(-6\,x)} - 2100\, a^{\frac{5}{2}}e^{(-8\,x)} + 2100\, a^{\frac{5}{2}}e^{(-12\,x)} - 600\, a^{\frac{5}{2}}e^{(-14\,x)} + 15$$

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="maxima")
[Out] -63/256*a^(5/2)*x - 1/20480*(25*a^(5/2)*e^(-2*x) - 150*a^(5/2)*e^(-4*x) + 6
00*a^(5/2)*e^(-6*x) - 2100*a^(5/2)*e^(-8*x) + 2100*a^(5/2)*e^(-12*x) - 600*
a^(5/2)*e^(-14*x) + 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) + 2*a^(5/2
)*e^(-20*x) - 2*a^(5/2))*e^(10*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \left(a \sinh^4(x)\right)^{5/2} dx =$$
$$-\frac{1}{20480} \left(5040 a^2 x - 2 a^2 e^{(10 x)} + 25 a^2 e^{(8 x)} - 150 a^2 e^{(6 x)} + 600 a^2 e^{(4 x)} - 2100 a^2 e^{(2 x)} - \left(5754 a^2 e^{(10 x)} - 210\right.\right.$$

```
[In] integrate((a*sinh(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*(5040*a^2*x - 2*a^2*e^(10*x) + 25*a^2*e^(8*x) - 150*a^2*e^(6*x) +
600*a^2*e^(4*x) - 2100*a^2*e^(2*x) - (5754*a^2*e^(10*x) - 2100*a^2*e^(8*x)
+ 600*a^2*e^(6*x) - 150*a^2*e^(4*x) + 25*a^2*e^(2*x) - 2*a^2)*e^(-10*x))*sq
rt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a \sinh^4(x)\right)^{5/2} dx = \int \left(a \sinh(x)^4\right)^{5/2} dx$$

```
[In] int((a*sinh(x)^4)^(5/2),x)
```

```
[Out] int((a*sinh(x)^4)^(5/2), x)
```

3.153 $\int \left(a \sinh^4(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \left(a \sinh^4(x)\right)^{3/2} dx = \frac{5}{16} a \coth(x) \sqrt{a \sinh^4(x)} - \frac{5}{16} a x \operatorname{csch}^2(x) \sqrt{a \sinh^4(x)} - \frac{5}{24} a \cosh(x) \sinh(x) \sqrt{a \sinh^4(x)} + \frac{1}{6} a \cosh(x) \sinh^3(x) \sqrt{a \sinh^4(x)}$$

```
[Out] 5/16*a*coth(x)*(a*sinh(x)^4)^(1/2)-5/16*a*x*csch(x)^2*(a*sinh(x)^4)^(1/2)-5
/24*a*cosh(x)*sinh(x)*(a*sinh(x)^4)^(1/2)+1/6*a*cosh(x)*sinh(x)^3*(a*sinh(x
)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3286, 2715, 8}

$$\int \left(a \sinh^4(x)\right)^{3/2} dx = -\frac{5}{24} a \sinh(x) \cosh(x) \sqrt{a \sinh^4(x)} + \frac{1}{6} a \sinh^3(x) \cosh(x) \sqrt{a \sinh^4(x)} + \frac{5}{16} a \coth(x) \sqrt{a \sinh^4(x)} - \frac{5}{16} a x \operatorname{csch}^2(x) \sqrt{a \sinh^4(x)}$$

```
[In] Int[(a*Sinh[x]^4)^(3/2),x]
```

```
[Out] (5*a*Coth[x]*Sqrt[a*Sinh[x]^4])/16 - (5*a*x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/16
 - (5*a*Cosh[x]*Sinh[x]*Sqrt[a*Sinh[x]^4])/24 + (a*Cosh[x]*Sinh[x]^3*Sqrt[a
*Sinh[x]^4])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^6(x)\,dx \\
&= \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{1}{6}\left(5a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^4(x)\,dx \\
&= -\frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} \\
&\quad + \frac{1}{8}\left(5a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^2(x)\,dx \\
&= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} \\
&\quad + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)} - \frac{1}{16}\left(5a\text{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int 1\,dx \\
&= \frac{5}{16}a\coth(x)\sqrt{a\sinh^4(x)} - \frac{5}{16}ax\text{csch}^2(x)\sqrt{a\sinh^4(x)} \\
&\quad - \frac{5}{24}a\cosh(x)\sinh(x)\sqrt{a\sinh^4(x)} + \frac{1}{6}a\cosh(x)\sinh^3(x)\sqrt{a\sinh^4(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \frac{1}{192}\operatorname{csch}^6(x)\left(a\sinh^4(x)\right)^{3/2}\left(-60x+45\sinh(2x)-9\sinh(4x)+\sinh(6x)\right)$$

```
[In] Integrate[(a*Sinh[x]^4)^(3/2),x]
[Out] (Csch[x]^6*(a*Sinh[x]^4)^(3/2)*(-60*x + 45*Sinh[2*x] - 9*Sinh[4*x] + Sinh[6
*x]))/192
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{a}\,(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(2\sqrt{a}\,\sqrt{a\sinh(2x)^2}\,\sinh(2x)^2-9\cosh(2x)\sqrt{a\sinh(2x)^2}\,\sqrt{a}+24\sqrt{a\sinh(2x)^2}\,\sqrt{}\right.}{96\sinh(2x)\sqrt{(-1+\cosh(2x))^2 a}}$
risch	$-\frac{5a\,e^{2x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}\,x}{16(e^{2x}-1)^2}+\frac{a\,e^{8x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{384(e^{2x}-1)^2}-\frac{3a\,e^{6x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{128(e^{2x}-1)^2}+\frac{15a\,e^{4x}\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{128(e^{2x}-1)^2}-\frac{15\sqrt{a(e^{2x}-}}{128(e^{2x}}$

```
[In] int((a*sinh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/96*a^(1/2)*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*(2*a^(1/
2)*(a*sinh(2*x)^2)^(1/2)*sinh(2*x)^2-9*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1
/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)-15*ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2
)^(1/2))*a)/sinh(2*x)/((-1+cosh(2*x))^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. 2(62) = 124.

Time = 0.30 (sec) , antiderivative size = 659, normalized size of antiderivative = 8.45

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="fricas")
[Out] 1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*cos
h(x)^2 - 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 - 9*a*cosh(x))*e^(2*x
)*sinh(x)^9 + 45*(11*a*cosh(x)^4 - 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8 + 7
2*(11*a*cosh(x)^5 - 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6*(15
4*a*cosh(x)^6 - 315*a*cosh(x)^4 + 210*a*cosh(x)^2 - 20*a*x)*e^(2*x)*sinh(x)
```

```
^6 + 36*(22*a*cosh(x)^7 - 63*a*cosh(x)^5 + 70*a*cosh(x)^3 - 20*a*x*cosh(x))
*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 - 42*a*cosh(x)^6 + 70*a*cosh(x)^4 -
 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 - 54*a*cosh(x
)^7 + 126*a*cosh(x)^5 - 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*sinh(x)^3
+ 3*(22*a*cosh(x)^10 - 135*a*cosh(x)^8 + 420*a*cosh(x)^6 - 600*a*x*cosh(x)^
4 - 90*a*cosh(x)^2 + 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11 - 15*a*cosh
(x)^9 + 60*a*cosh(x)^7 - 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 + 3*a*cosh(x))*
e^(2*x)*sinh(x) + (a*cosh(x)^12 - 9*a*cosh(x)^10 + 45*a*cosh(x)^8 - 120*a*x
*cosh(x)^6 - 45*a*cosh(x)^4 + 9*a*cosh(x)^2 - a)*e^(2*x))*sqrt(a*e^(8*x) -
4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^6*e^(4*x) -
2*cosh(x)^6*e^(2*x) + (e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(
cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 + 15*(cosh(x)^2*e^
(4*x) - 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(cosh(x)^3*e^(4*x)
- 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(x)^4*e^(4*x) - 2*co
sh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^5*e^(4*x) - 2*cosh(x)^5
*e^(2*x) + cosh(x)^5)*sinh(x))
```

Sympy [F]

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx=\int \left(a\sinh^4{(x)}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sinh(x)**4)**(3/2),x)
[Out] Integral((a*sinh(x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx=-\frac{5}{16}\,a^{\frac{3}{2}}x-\frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)}-45\,a^{\frac{3}{2}}e^{(-4\,x)}+45\,a^{\frac{3}{2}}e^{(-8\,x)}-9\,a^{\frac{3}{2}}e^{(-10\,x)}+a^{\frac{3}{2}}e^{(-12\,x)}-a^{\frac{3}{2}}\right)e^{(6\,x)}$$

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="maxima")
[Out] -5/16*a^(3/2)*x - 1/384*(9*a^(3/2)*e^(-2*x) - 45*a^(3/2)*e^(-4*x) + 45*a^(3
/2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) + a^(3/2)*e^(-12*x) - a^(3/2))*e^(6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \frac{1}{384}\left(\left(110\,e^{(6\,x)} - 45\,e^{(4\,x)} + 9\,e^{(2\,x)} - 1\right)e^{(-6\,x)} - 120\,x + e^{(6\,x)} - 9\,e^{(4\,x)} + 45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

```
[In] integrate((a*sinh(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*((110*e^(6*x) - 45*e^(4*x) + 9*e^(2*x) - 1)*e^(-6*x) - 120*x + e^(6*x
) - 9*e^(4*x) + 45*e^(2*x))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\sinh^4(x)\right)^{3/2}\,dx = \int \left(a\sinh(x)^4\right)^{3/2}\,dx$$

```
[In] int((a*sinh(x)^4)^(3/2),x)
```

```
[Out] int((a*sinh(x)^4)^(3/2), x)
```

3.154 $\int \sqrt{a \sinh^4(x)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{a \sinh^4(x)} \, dx = \frac{1}{2} \coth(x) \sqrt{a \sinh^4(x)} - \frac{1}{2} x \operatorname{csch}^2(x) \sqrt{a \sinh^4(x)}$$

```
[Out] 1/2*coth(x)*(a*sinh(x)^4)^(1/2)-1/2*x*csch(x)^2*(a*sinh(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3286, 2715, 8}

$$\int \sqrt{a \sinh^4(x)} \, dx = \frac{1}{2} \coth(x) \sqrt{a \sinh^4(x)} - \frac{1}{2} x \operatorname{csch}^2(x) \sqrt{a \sinh^4(x)}$$

```
[In] Int[Sqrt[a*Sinh[x]^4],x]
```

```
[Out] (Coth[x]*Sqrt[a*Sinh[x]^4])/2 - (x*Csch[x]^2*Sqrt[a*Sinh[x]^4])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int \sinh^2(x)\,dx\\ &= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}\left(\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\right)\int 1\,dx\\ &= \frac{1}{2}\coth(x)\sqrt{a\sinh^4(x)}-\frac{1}{2}x\operatorname{csch}^2(x)\sqrt{a\sinh^4(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{a\sinh^4(x)}\,dx=\frac{1}{2}\left(\coth(x)-x\operatorname{csch}^2(x)\right)\sqrt{a\sinh^4(x)}$$

```
[In] Integrate[Sqrt[a*Sinh[x]^4],x]
[Out] ((Coth[x] - x*Csch[x]^2)*Sqrt[a*Sinh[x]^4])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(28) = 56.

Time = 1.83 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{(-1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(\sqrt{a\sinh(2x)^2}\,\sqrt{a}-\ln\left(\cosh(2x)\sqrt{a}+\sqrt{a\sinh(2x)^2}\right)a\right)}{4\sqrt{a}\,\sinh(2x)\sqrt{(-1+\cosh(2x))^2 a}}$	84
risch	$-\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}\,e^{2x}x}{2(e^{2x}-1)^2}+\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}\,e^{4x}}{8(e^{2x}-1)^2}-\frac{\sqrt{a(e^{2x}-1)^4 e^{-4x}}}{8(e^{2x}-1)^2}$	89

```
[In] int((a*sinh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*((a*sinh(2*x)^2)^
(1/2)*a^(1/2)-ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2)^(1/2))*a)/a^(1/2)/sinh(2
*x)/((-1+cosh(2*x))^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.00

$$\int \sqrt{a\sinh^4(x)}\,dx$$
$$= \frac{\left(4\cosh\left(x\right)e^{(2x)}\sinh\left(x\right)^3 + e^{(2x)}\sinh\left(x\right)^4 + 2\left(3\cosh\left(x\right)^2 - 2x\right)e^{(2x)}\sinh\left(x\right)^2 + 4\left(\cosh\left(x\right)^3 - 2x\cosh\left(\right.\right.\right.}{8\left(\cosh\left(x\right)^2 e^{(4x)} - 2\cosh\left(x\right)^2 e^{(2x)} + \left(e^{(4x)} - 2e^{(2x)} + 1\right)\sinh\left(x\right)^2 + \right.}$$

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 - 2*x
)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 - 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh(x
)^4 - 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x
) - 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) - 2*cosh(x)^2*e^(2*x) + (e
^(4*x) - 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) - 2*cosh
(x)*e^(2*x) + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \sqrt{a\sinh^4(x)}\,dx = \int \sqrt{a\sinh^4\left(x\right)}\,dx$$

```
[In] integrate((a*sinh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sinh(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{a\sinh^4(x)}\,dx = -\frac{1}{8}\left(\sqrt{a}e^{(-4x)} - \sqrt{a}\right)e^{(2x)} - \frac{1}{2}\sqrt{a}x$$

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) - 1/2*sqrt(a)*x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{a \sinh^4(x)}\, dx = \frac{1}{8} \left((2 e^{(2x)} - 1) e^{(-2x)} - 4x + e^{(2x)} \right) \sqrt{a}$$

```
[In] integrate((a*sinh(x)^4)^(1/2),x, algorithm="giac")
[Out] 1/8*((2*e^(2*x) - 1)*e^(-2*x) - 4*x + e^(2*x))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a \sinh^4(x)}\, dx = \int \sqrt{a \sinh(x)^4}\, dx$$

```
[In] int((a*sinh(x)^4)^(1/2),x)
[Out] int((a*sinh(x)^4)^(1/2), x)
```

3.155 $\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = -\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}$$

```
[Out] -cosh(x)*sinh(x)/(a*sinh(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3286, 3852, 8}

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = -\frac{\sinh(x)\cosh(x)}{\sqrt{a\sinh^4(x)}}$$

```
[In] Int[1/Sqrt[a*Sinh[x]^4],x]
[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sinh^2(x)\int \operatorname{csch}^2(x)\,dx}{\sqrt{a\sinh^4(x)}}\\ &= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}\left(\int 1\,dx, x, -i\coth(x)\right)}{\sqrt{a\sinh^4(x)}}\\ &= -\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = -\frac{\cosh(x)\sinh(x)}{\sqrt{a\sinh^4(x)}}$$

```
[In] Integrate[1/Sqrt[a*Sinh[x]^4],x]
[Out] -((Cosh[x]*Sinh[x])/Sqrt[a*Sinh[x]^4])
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
risch	$-\frac{2\,{\mathrm e}^{-2x}\left({\mathrm e}^{2x}-1\right)}{\sqrt{a\left({\mathrm e}^{2x}-1\right)^{4}{\mathrm e}^{-4x}}}$	29
default	$-\frac{\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}\,\sqrt{a\sinh\left(2x\right)^{2}}}{a\sinh\left(2x\right)\sqrt{\left(-1+\cosh\left(2x\right)\right)^{2}a}}$	50

```
[In] int(1/(a*sinh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/(a*(exp(2*x)-1)^4*exp(-4*x))^(1/2)*exp(-2*x)*(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(14) = 28.

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 7.62

$$\int \frac{1}{\sqrt{a \sinh^4(x)}}\, dx =$$
$$-\frac{2\sqrt{ae^{(8x)} - 4\,ae^{(6x)} + 6\,ae^{(4x)} - 4\,ae^{(2x)} + a}}{a\cosh{(x)}^2 + (ae^{(4x)} - 2\,ae^{(2x)} + a)\sinh{(x)}^2 + \left(a\cosh{(x)}^2 - a\right)e^{(4x)} - 2\left(a\cosh{(x)}^2 - a\right)e^{(2x)} + 2\,(a\,c}$$

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(a*cosh(x)
^2 + (a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 - a)*e^(4*x) -
2*(a*cosh(x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) - 2*a*cosh(x)*e^(2*x) +
a*cosh(x))*sinh(x) - a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a \sinh^4(x)}}\, dx = \int \frac{1}{\sqrt{a \sinh^4{(x)}}}\, dx$$

```
[In] integrate(1/(a*sinh(x)**4)**(1/2),x)
[Out] Integral(1/sqrt(a*sinh(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{a \sinh^4(x)}}\, dx = \frac{2}{\sqrt{a}e^{(-2x)} - \sqrt{a}}$$

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="maxima")
[Out] 2/(sqrt(a)*e^(-2*x) - sqrt(a))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = -\frac{2}{\sqrt{a}(e^{(2x)}-1)}$$

```
[In] integrate(1/(a*sinh(x)^4)^(1/2),x, algorithm="giac")
[Out] -2/(sqrt(a)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{a\sinh^4(x)}}\,dx = \frac{\mathrm{e}^{-x}\sqrt{a\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^4}}{a\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)^3}$$

```
[In] int(1/(a*sinh(x)^4)^(1/2),x)
[Out] (exp(-x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 - exp(x)/2)^3)
```

3.156 $\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx=\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}-\frac{\cosh(x)\sinh(x)}{a\sqrt{a\sinh^4(x)}}$$

```
[Out] 2/3*cosh(x)^2*coth(x)/a/(a*sinh(x)^4)^(1/2)-1/5*cosh(x)^2*coth(x)^3/a/(a*si
nh(x)^4)^(1/2)-cosh(x)*sinh(x)/a/(a*sinh(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3286, 3852}

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx=-\frac{\sinh(x)\cosh(x)}{a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}+\frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}$$

```
[In] Int[(a*Sinh[x]^4)^(-3/2),x]
```

```
[Out] (2*Cosh[x]^2*Coth[x])/(3*a*Sqrt[a*Sinh[x]^4]) - (Cosh[x]^2*Coth[x]^3)/(5*a*
Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a*Sqrt[a*Sinh[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sinh^2(x)\int \operatorname{csch}^6(x)\,dx}{a\sqrt{a\sinh^4(x)}}\\ &= -\frac{\left(i\sinh^2(x)\right)\operatorname{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,-i\coth(x)\right)}{a\sqrt{a\sinh^4(x)}}\\ &= \frac{2\cosh^2(x)\coth(x)}{3a\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^3(x)}{5a\sqrt{a\sinh^4(x)}}-\frac{\cosh(x)\sinh(x)}{a\sqrt{a\sinh^4(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = -\frac{\cosh(x)\left(8-4\operatorname{csch}^2(x)+3\operatorname{csch}^4(x)\right)\sinh^5(x)}{15\left(a\sinh^4(x)\right)^{3/2}}$$

```
[In] Integrate[(a*Sinh[x]^4)^(-3/2),x]
[Out] -1/15*(Cosh[x]*(8 - 4*Csch[x]^2 + 3*Csch[x]^4)*Sinh[x]^5)/(a*Sinh[x]^4)^(3/
2)
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

method	result	size
risch	$-\frac{16\,{\rm e}^{-2x}\left(10\,{\rm e}^{4x}-5\,{\rm e}^{2x}+1\right)}{15a\left({\rm e}^{2x}-1\right)^{3}\sqrt{a\left({\rm e}^{2x}-1\right)^{4}{\rm e}^{-4x}}}$	48
default	$-\frac{4\left(2\cosh\left(2x\right)^{2}-6\cosh\left(2x\right)+7\right)\sqrt{a\sinh\left(2x\right)^{2}}\,\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}}{15a^{2}\left(-1+\cosh\left(2x\right)\right)^{2}\sinh\left(2x\right)\sqrt{\left(-1+\cosh\left(2x\right)\right)^{2}a}}$	74

```
[In] int(1/(a*sinh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15/a/(exp(2*x)-1)^3*exp(-2*x)/(a*(exp(2*x)-1)^4*exp(-4*x))^(1/2)*(10*ex
p(4*x)-5*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 1163, normalized size of antiderivative = 17.10

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x)
^2 - 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (1
0*cosh(x)^4 - 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e^(6*x) + 6*a*
e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) - 2*a^2*
e^(2*x) + a^2)*sinh(x)^10 - 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x) - 2*a
^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 - a^2 + (9
*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(9*a^2*cosh(x)^2 - a^2)*e^(2*x))*sinh(x)^
8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^3
 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x
)^7 + 10*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^4 - 1
4*a^2*cosh(x)^2 + a^2)*e^(4*x) - 2*(21*a^2*cosh(x)^4 - 14*a^2*cosh(x)^2 + a
^2)*e^(2*x))*sinh(x)^6 - 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5 - 70*a^2*co
sh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*co
sh(x))*e^(4*x) - 2*(63*a^2*cosh(x)^5 - 70*a^2*cosh(x)^3 + 15*a^2*cosh(x))*e
^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x
)^2 - a^2 + (21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(4*x) - 2*(21*a^2*cosh(x)^6 - 35*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 - a^2)*
e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^
5 + 5*a^2*cosh(x)^3 - a^2*cosh(x) + (3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5 + 5*
a^2*cosh(x)^3 - a^2*cosh(x))*e^(4*x) - 2*(3*a^2*cosh(x)^7 - 7*a^2*cosh(x)^5
 + 5*a^2*cosh(x)^3 - a^2*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(9*a^2*cosh(x)^8 -
 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2 + (9*a^2*cosh
(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh(x)^2 + a^2)*e^(4*
x) - 2*(9*a^2*cosh(x)^8 - 28*a^2*cosh(x)^6 + 30*a^2*cosh(x)^4 - 12*a^2*cosh
(x)^2 + a^2)*e^(2*x))*sinh(x)^2 - a^2 + (a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 +
 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*a^2*cosh(x)^2 - a^2)*e^(4*x) - 2*(
a^2*cosh(x)^10 - 5*a^2*cosh(x)^8 + 10*a^2*cosh(x)^6 - 10*a^2*cosh(x)^4 + 5*
a^2*cosh(x)^2 - a^2)*e^(2*x) + 10*(a^2*cosh(x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*
cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^9 - 4*a^2*cosh(x)^
7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) - 2*(a^2*cosh(
```

```
x)^9 - 4*a^2*cosh(x)^7 + 6*a^2*cosh(x)^5 - 4*a^2*cosh(x)^3 + a^2*cosh(x))*e
^(2*x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\sinh^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**4)**(3/2),x)
[Out] Integral((a*sinh(x)**4)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(58) = 116.

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.51

$$\begin{aligned}\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = & -\frac{16\,e^{(-2\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)} \\ & + \frac{32\,e^{(-4\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)} \\ & + \frac{16}{15\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} - 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} - 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}\right)}\end{aligned}$$

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="maxima")
[Out] -16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6
*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 32/3*e^(-4*x)/(5*
a^(3/2)*e^(-2*x) - 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^
(-8*x) + a^(3/2)*e^(-10*x) - a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) - 10*a^(3
/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) - 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x)
 - a^(3/2))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = -\frac{16\left(10\,e^{(4\,x)} - 5\,e^{(2\,x)} + 1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)} - 1\right)^5}$$

```
[In] integrate(1/(a*sinh(x)^4)^(3/2),x, algorithm="giac")
[Out] -16/15*(10*e^(4*x) - 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{3/2}}\,dx = -\frac{64\,\mathrm{e}^{2\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}-\frac{\mathrm{e}^{x}}{2}\right)}^4}\,\left(10\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}+1\right)}{15\,a^2\,{\left(\mathrm{e}^{2\,x}-1\right)}^7}$$

```
[In] int(1/(a*sinh(x)^4)^(3/2),x)
[Out] -(64*exp(2*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2)*(10*exp(4*x) - 5*exp(2*x)
+ 1))/(15*a^2*(exp(2*x) - 1)^7)
```

3.157 $\int \frac{1}{(a\sinh^4(x))^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 118

$$\int \frac{1}{(a\sinh^4(x))^{5/2}}\,dx = \frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}} - \frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}} + \frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\sinh^4(x)}}$$

```
[Out] 4/3*cosh(x)^2*coth(x)/a^2/(a*sinh(x)^4)^(1/2)-6/5*cosh(x)^2*coth(x)^3/a^2/(
a*sinh(x)^4)^(1/2)+4/7*cosh(x)^2*coth(x)^5/a^2/(a*sinh(x)^4)^(1/2)-1/9*cosh
(x)^2*coth(x)^7/a^2/(a*sinh(x)^4)^(1/2)-cosh(x)*sinh(x)/a^2/(a*sinh(x)^4)^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3286, 3852}

$$\int \frac{1}{(a\sinh^4(x))^{5/2}}\,dx = -\frac{\sinh(x)\cosh(x)}{a^2\sqrt{a\sinh^4(x)}} - \frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}} + \frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}} - \frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}} + \frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}$$

```
[In] Int[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] (4*Cosh[x]^2*Coth[x])/(3*a^2*Sqrt[a*Sinh[x]^4]) - (6*Cosh[x]^2*Coth[x]^3)/(
5*a^2*Sqrt[a*Sinh[x]^4]) + (4*Cosh[x]^2*Coth[x]^5)/(7*a^2*Sqrt[a*Sinh[x]^4]
) - (Cosh[x]^2*Coth[x]^7)/(9*a^2*Sqrt[a*Sinh[x]^4]) - (Cosh[x]*Sinh[x])/(a^
2*Sqrt[a*Sinh[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sinh^2(x)\int \operatorname{csch}^{10}(x)\,dx}{a^2\sqrt{a\sinh^4(x)}}\\
&= -\frac{(i\sinh^2(x))\,\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-i\coth(x)\right)}{a^2\sqrt{a\sinh^4(x)}}\\
&= \frac{4\cosh^2(x)\coth(x)}{3a^2\sqrt{a\sinh^4(x)}}-\frac{6\cosh^2(x)\coth^3(x)}{5a^2\sqrt{a\sinh^4(x)}}\\
&\quad+\frac{4\cosh^2(x)\coth^5(x)}{7a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh^2(x)\coth^7(x)}{9a^2\sqrt{a\sinh^4(x)}}-\frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\sinh^4(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = -\frac{\cosh(x)\left(128-64\text{csch}^2(x)+48\text{csch}^4(x)-40\text{csch}^6(x)+35\text{csch}^8(x)\right)\sinh(x)}{315a^2\sqrt{a\sinh^4(x)}}$$

```
[In] Integrate[(a*Sinh[x]^4)^(-5/2),x]
```

```
[Out] -1/315*(Cosh[x]*(128 - 64*Csch[x]^2 + 48*Csch[x]^4 - 40*Csch[x]^6 + 35*Csch
[x]^8)*Sinh[x])/(a^2*Sqrt[a*Sinh[x]^4])
```

Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256\,\mathrm{e}^{-2x}\left(126\,\mathrm{e}^{8x}-84\,\mathrm{e}^{6x}+36\,\mathrm{e}^{4x}-9\,\mathrm{e}^{2x}+1\right)}{315a^{2}\left(\mathrm{e}^{2x}-1\right)^{7}\sqrt{a\left(\mathrm{e}^{2x}-1\right)^{4}\mathrm{e}^{-4x}}}$	60
default	$-\frac{16\left(8\cosh(2x)^{4}-40\cosh(2x)^{3}+84\cosh(2x)^{2}-100\cosh(2x)+83\right)\sqrt{a\sinh(2x)^{2}}\,\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}}{315a^{3}(-1+\cosh(2x))^{4}\sinh(2x)\sqrt{(-1+\cosh(2x))^{2}a}}$	90

```
[In] int(1/(a*sinh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -256/315/a^2/(exp(2*x)-1)^7*exp(-2*x)/(a*(exp(2*x)-1)^4*exp(-4*x))^(1/2)*(1
26*exp(8*x)-84*exp(6*x)+36*exp(4*x)-9*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3093 vs. 2(100) = 200.

Time = 0.35 (sec) , antiderivative size = 3093, normalized size of antiderivative = 26.21

$$\int \frac{1}{\left(a\sinh^{4}(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(1008*cosh(x)*e^(2*x)*sinh(x)^7 + 126*e^(2*x)*sinh(x)^8 + 84*(42*c
osh(x)^2 - 1)*e^(2*x)*sinh(x)^6 + 504*(14*cosh(x)^3 - cosh(x))*e^(2*x)*sinh
(x)^5 + 36*(245*cosh(x)^4 - 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(147*c
osh(x)^5 - 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(392*cosh(x)^6 -
 140*cosh(x)^4 + 24*cosh(x)^2 - 1)*e^(2*x)*sinh(x)^2 + 18*(56*cosh(x)^7 - 2
8*cosh(x)^5 + 8*cosh(x)^3 - cosh(x))*e^(2*x)*sinh(x) + (126*cosh(x)^8 - 84*
cosh(x)^6 + 36*cosh(x)^4 - 9*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) - 4*a*e
^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)*e^(-2*x)/(a^3*cosh(x)^18 - 9*a^3*co
sh(x)^16 + (a^3*e^(4*x) - 2*a^3*e^(2*x) + a^3)*sinh(x)^18 + 18*(a^3*cosh(x)
*e^(4*x) - 2*a^3*cosh(x)*e^(2*x) + a^3*cosh(x))*sinh(x)^17 + 36*a^3*cosh(x)
^14 + 9*(17*a^3*cosh(x)^2 - a^3 + (17*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(17*
a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^16 + 48*(17*a^3*cosh(x)^3 - 3*a^3*cos
h(x) + (17*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^3 - 3
*a^3*cosh(x))*e^(2*x))*sinh(x)^15 - 84*a^3*cosh(x)^12 + 36*(85*a^3*cosh(x)^
```

4 - 30*a^3*cosh(x)^2 + a^3 + (85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^
(4*x) - 2*(85*a^3*cosh(x)^4 - 30*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^14 +
 504*(17*a^3*cosh(x)^5 - 10*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^5
 - 10*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^5 - 10*a^3*c
osh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^13 + 126*a^3*cosh(x)^10 + 84*(221*
a^3*cosh(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3 + (221*a^3*cosh(
x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2*(221*a^3*cos
h(x)^6 - 195*a^3*cosh(x)^4 + 39*a^3*cosh(x)^2 - a^3)*e^(2*x))*sinh(x)^12 +
144*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh(
x) + (221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^7 - 273*a^3*cosh(x)^5 + 91*a^3*cosh(x)^3
- 7*a^3*cosh(x))*e^(2*x))*sinh(x)^11 - 126*a^3*cosh(x)^8 + 18*(2431*a^3*cos
h(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^
3 + (2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 + 2002*a^3*cosh(x)^4 - 308*a^3
*cosh(x)^2 + 7*a^3)*e^(4*x) - 2*(2431*a^3*cosh(x)^8 - 4004*a^3*cosh(x)^6 +
2002*a^3*cosh(x)^4 - 308*a^3*cosh(x)^2 + 7*a^3)*e^(2*x))*sinh(x)^10 + 4*(12
155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*co
sh(x)^3 + 315*a^3*cosh(x) + (12155*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18
018*a^3*cosh(x)^5 - 4620*a^3*cosh(x)^3 + 315*a^3*cosh(x))*e^(4*x) - 2*(1215
5*a^3*cosh(x)^9 - 25740*a^3*cosh(x)^7 + 18018*a^3*cosh(x)^5 - 4620*a^3*cosh
(x)^3 + 315*a^3*cosh(x))*e^(2*x))*sinh(x)^9 + 84*a^3*cosh(x)^6 + 18*(2431*a
^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^
4 + 315*a^3*cosh(x)^2 - 7*a^3 + (2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 +
 6006*a^3*cosh(x)^6 - 2310*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(4*
x) - 2*(2431*a^3*cosh(x)^10 - 6435*a^3*cosh(x)^8 + 6006*a^3*cosh(x)^6 - 231
0*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2 - 7*a^3)*e^(2*x))*sinh(x)^8 + 144*(221*
a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5
+ 105*a^3*cosh(x)^3 - 7*a^3*cosh(x) + (221*a^3*cosh(x)^11 - 715*a^3*cosh(x)
^9 + 858*a^3*cosh(x)^7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh
(x))*e^(4*x) - 2*(221*a^3*cosh(x)^11 - 715*a^3*cosh(x)^9 + 858*a^3*cosh(x)^
7 - 462*a^3*cosh(x)^5 + 105*a^3*cosh(x)^3 - 7*a^3*cosh(x))*e^(2*x))*sinh(x)
^7 - 36*a^3*cosh(x)^4 + 84*(221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*
a^3*cosh(x)^8 - 924*a^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 +
a^3 + (221*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a
^3*cosh(x)^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(221
*a^3*cosh(x)^12 - 858*a^3*cosh(x)^10 + 1287*a^3*cosh(x)^8 - 924*a^3*cosh(x)
^6 + 315*a^3*cosh(x)^4 - 42*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 504*(
17*a^3*cosh(x)^13 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)
^7 + 63*a^3*cosh(x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x) + (17*a^3*cosh(x)^13
 - 78*a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(
x)^5 - 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x)^13 - 78*
a^3*cosh(x)^11 + 143*a^3*cosh(x)^9 - 132*a^3*cosh(x)^7 + 63*a^3*cosh(x)^5 -
 14*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 9*a^3*cosh(x)^2 + 36*
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3 +

```
(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 - 1155*a^3*co
sh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)^2 - a^3)*e
^(4*x) - 2*(85*a^3*cosh(x)^14 - 455*a^3*cosh(x)^12 + 1001*a^3*cosh(x)^10 -
1155*a^3*cosh(x)^8 + 735*a^3*cosh(x)^6 - 245*a^3*cosh(x)^4 + 35*a^3*cosh(x)
^2 - a^3)*e^(2*x))*sinh(x)^4 + 48*(17*a^3*cosh(x)^15 - 105*a^3*cosh(x)^13 +
 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 - 147*a^3*cosh(
x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x) + (17*a^3*cosh(x)^15 - 105*a^3*cosh
(x)^13 + 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^3*cosh(x)^7 - 147*a
^3*cosh(x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(4*x) - 2*(17*a^3*cosh(x
)^15 - 105*a^3*cosh(x)^13 + 273*a^3*cosh(x)^11 - 385*a^3*cosh(x)^9 + 315*a^
3*cosh(x)^7 - 147*a^3*cosh(x)^5 + 35*a^3*cosh(x)^3 - 3*a^3*cosh(x))*e^(2*x)
)*sinh(x)^3 - a^3 + 9*(17*a^3*cosh(x)^16 - 120*a^3*cosh(x)^14 + 364*a^3*cos
h(x)^12 - 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 - 392*a^3*cosh(x)^6 + 140*
a^3*cosh(x)^4 - 24*a^3*cosh(x)^2 + a^3 + (17*a^3*cosh(x)^16 - 120*a^3*cosh(
x)^14 + 364*a^3*cosh(x)^12 - 616*a^3*cosh(x)^10 + 630*a^3*cosh(x)^8 - 392*a
^3*cosh(x)^6 + 140*a^3*cosh(x)^4 - 24*a^3*cosh(x)^2 + a^3)*e^(4*x) - 2*(17*
a^3*cosh(x)^16 - 120*a^3*cosh(x)^14 + 364*a^3*cosh(x)^12 - 616*a^3*cosh(x)^
10 + 630*a^3*cosh(x)^8 - 392*a^3*cosh(x)^6 + 140*a^3*cosh(x)^4 - 24*a^3*cos
h(x)^2 + a^3)*e^(2*x))*sinh(x)^2 + (a^3*cosh(x)^18 - 9*a^3*cosh(x)^16 + 36*
a^3*cosh(x)^14 - 84*a^3*cosh(x)^12 + 126*a^3*cosh(x)^10 - 126*a^3*cosh(x)^8
 + 84*a^3*cosh(x)^6 - 36*a^3*cosh(x)^4 + 9*a^3*cosh(x)^2 - a^3)*e^(4*x) - 2
*(a^3*cosh(x)^18 - 9*a^3*cosh(x)^16 + 36*a^3*cosh(x)^14 - 84*a^3*cosh(x)^12
 + 126*a^3*cosh(x)^10 - 126*a^3*cosh(x)^8 + 84*a^3*cosh(x)^6 - 36*a^3*cosh(
x)^4 + 9*a^3*cosh(x)^2 - a^3)*e^(2*x) + 18*(a^3*cosh(x)^17 - 8*a^3*cosh(x)^
15 + 28*a^3*cosh(x)^13 - 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)^9 - 56*a^3*cosh
(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3*cosh(x)^3 + a^3*cosh(x) + (a^3*cosh(x)^17
- 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 - 56*a^3*cosh(x)^11 + 70*a^3*cosh(x)
^9 - 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3*cosh(x)^3 + a^3*cosh(x))*e
^(4*x) - 2*(a^3*cosh(x)^17 - 8*a^3*cosh(x)^15 + 28*a^3*cosh(x)^13 - 56*a^3*
cosh(x)^11 + 70*a^3*cosh(x)^9 - 56*a^3*cosh(x)^7 + 28*a^3*cosh(x)^5 - 8*a^3
*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\sinh^4\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sinh(x)**4)**(5/2),x)
[Out] Integral((a*sinh(x)**4)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(100) = 200.

Time = 0.29 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.96

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx =$$

$$-\frac{256\,e^{(-2\,x)}}{35\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} - 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} - 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} - 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} - \cdots\right.}$$

$$+\frac{1024\,e^{(-4\,x)}}{35\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} - 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} - 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} - 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} - \cdots\right.}$$

$$-\frac{1024\,e^{(-6\,x)}}{15\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} - 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} - 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} - 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} - \cdots\right.}$$

$$+\frac{512\,e^{(-8\,x)}}{5\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} - 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} - 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} - 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} - 9\cdots\right.}$$

$$+\frac{256}{315\left(9\,a^{\frac{5}{2}}e^{(-2\,x)} - 36\,a^{\frac{5}{2}}e^{(-4\,x)} + 84\,a^{\frac{5}{2}}e^{(-6\,x)} - 126\,a^{\frac{5}{2}}e^{(-8\,x)} + 126\,a^{\frac{5}{2}}e^{(-10\,x)} - 84\,a^{\frac{5}{2}}e^{(-12\,x)} + 36\,a^{\frac{5}{2}}e^{(-14\,x)} - \cdots\right.}$$

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(
-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-12*x)
 + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(5/2)
) + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)
*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-1
2*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^(
5/2)) - 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(
5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e
^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) -
 a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a
^(5/2)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)
*e^(-12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
 - a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) - 36*a^(5/2)*e^(-4*x) + 84*a^(5/2
)*e^(-6*x) - 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) - 84*a^(5/2)*e^(-
12*x) + 36*a^(5/2)*e^(-14*x) - 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) - a^
(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx = -\frac{256\left(126\,e^{(8\,x)} - 84\,e^{(6\,x)} + 36\,e^{(4\,x)} - 9\,e^{(2\,x)} + 1\right)}{315\,a^{\frac{5}{2}}\left(e^{(2\,x)} - 1\right)^9}$$

```
[In] integrate(1/(a*sinh(x)^4)^(5/2),x, algorithm="giac")
[Out] -256/315*(126*e^(8*x) - 84*e^(6*x) + 36*e^(4*x) - 9*e^(2*x) + 1)/(a^(5/2)*(
e^(2*x) - 1)^9)
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.17

$$\begin{aligned}\int \frac{1}{\left(a\sinh^4(x)\right)^{5/2}}\,dx &= -\frac{2048\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{5\,a^3\left(e^{2\,x}-1\right)^5\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)} \\ &\quad - \frac{4096\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{3\,a^3\left(e^{2\,x}-1\right)^6\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)} - \frac{12288\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{7\,a^3\left(e^{2\,x}-1\right)^7\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)} \\ &\quad - \frac{1024\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{a^3\left(e^{2\,x}-1\right)^8\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)} - \frac{2048\,e^{4\,x}\sqrt{a\left(\frac{e^{-x}}{2}-\frac{e^{x}}{2}\right)^4}}{9\,a^3\left(e^{2\,x}-1\right)^9\left(e^{2\,x}-2\,e^{4\,x}+e^{6\,x}\right)}\end{aligned}$$

```
[In] int(1/(a*sinh(x)^4)^(5/2),x)
[Out] - (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(5*a^3*(exp(2*x) - 1)^
5*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (4096*exp(4*x)*(a*(exp(-x)/2 - exp(
x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) - 1)^6*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
 - (12288*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x) - 1
)^7*(exp(2*x) - 2*exp(4*x) + exp(6*x))) - (1024*exp(4*x)*(a*(exp(-x)/2 - ex
p(x)/2)^4)^(1/2))/(a^3*(exp(2*x) - 1)^8*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
 - (2048*exp(4*x)*(a*(exp(-x)/2 - exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*x) - 1)
^9*(exp(2*x) - 2*exp(4*x) + exp(6*x)))
```

3.158 $\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = -\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{5}{16}i\cosh(x)\sinh(x) -\frac{5}{24}i\cosh^3(x)\sinh(x)-\frac{1}{6}i\cosh^5(x)\sinh(x)$$

```
[Out] -5/16*I*x+1/7*cosh(x)^7-5/16*I*cosh(x)*sinh(x)-5/24*I*cosh(x)^3*sinh(x)-1/6
*I*cosh(x)^5*sinh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2761, 2715, 8}

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = -\frac{5ix}{16}+\frac{\cosh^7(x)}{7}-\frac{1}{6}i\sinh(x)\cosh^5(x) -\frac{5}{24}i\sinh(x)\cosh^3(x)-\frac{5}{16}i\sinh(x)\cosh(x)$$

```
[In] Int[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*x + Cosh[x]^7/7 - ((5*I)/16)*Cosh[x]*Sinh[x] - ((5*I)/24)*Cosh[
x]^3*Sinh[x] - (I/6)*Cosh[x]^5*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^7(x)}{7} - i\int \cosh^6(x)\,dx \\
&= \frac{\cosh^7(x)}{7} - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{6}i\int \cosh^4(x)\,dx \\
&= \frac{\cosh^7(x)}{7} - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{8}i\int \cosh^2(x)\,dx \\
&= \frac{\cosh^7(x)}{7} - \frac{5}{16}i\cosh(x)\sinh(x) - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x) - \frac{5}{16}i\int 1\,dx \\
&= -\frac{5ix}{16} + \frac{\cosh^7(x)}{7} - \frac{5}{16}i\cosh(x)\sinh(x) - \frac{5}{24}i\cosh^3(x)\sinh(x) - \frac{1}{6}i\cosh^5(x)\sinh(x)
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 219 vs. $2(50) = 100$.

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.38

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = \frac{\cosh^9(x)\left(6i\left(35\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}+8\sqrt{1+i\sinh(x)}\right)+279\sqrt{1+i\sinh(x)}\sinh(x)-\right.}{}$$

```
[In] Integrate[Cosh[x]^8/(I + Sinh[x]),x]
```

```
[Out] (Cosh[x]^9*((6*I)*(35*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x
]] + 8*Sqrt[1 + I*Sinh[x]]) + 279*Sqrt[1 + I*Sinh[x]]*Sinh[x] - (87*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^2 + 326*Sqrt[1 + I*Sinh[x]]*Sinh[x]^3 - (38*I)*Sqrt
[1 + I*Sinh[x]]*Sinh[x]^4 + 200*Sqrt[1 + I*Sinh[x]]*Sinh[x]^5 - (8*I)*Sqrt[
1 + I*Sinh[x]]*Sinh[x]^6 + 48*Sqrt[1 + I*Sinh[x]]*Sinh[x]^7))/(336*Sqrt[1 +
 I*Sinh[x]]*(-I + Sinh[x])^4*(I + Sinh[x])^5)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(36) = 72.

Time = 0.48 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.72

$$\frac{-\frac{1}{2}+\frac{i}{6}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{6}}+\frac{-\frac{11}{16}-\frac{19i}{16}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{9}{8}-\frac{7i}{6}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{-\frac{5}{4}+i}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{-\frac{1}{2}-\frac{i}{6}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{6}}+\frac{\frac{5}{16}-\frac{11i}{16}}{\tanh\left(\frac{x}{2}\right)+1}-$$

```
[In] int(cosh(x)^8/(I+sinh(x)),x)
```

```
[Out] (-1/2+1/6*I)/(tanh(1/2*x)+1)^6-(11/16+19/16*I)/(tanh(1/2*x)-1)^2-(9/8+7/6*I
)/(tanh(1/2*x)-1)^3+(-5/4+I)/(tanh(1/2*x)+1)^4-(1/2+1/6*I)/(tanh(1/2*x)-1)^
6+(5/16-11/16*I)/(tanh(1/2*x)+1)-5/16*I*ln(tanh(1/2*x)+1)+1/7/(tanh(1/2*x)+
1)^7-(5/4+I)/(tanh(1/2*x)-1)^4+(9/8-7/6*I)/(tanh(1/2*x)+1)^3-(1+1/2*I)/(tan
h(1/2*x)-1)^5+5/16*I*ln(tanh(1/2*x)-1)-(5/16+11/16*I)/(tanh(1/2*x)-1)+(1-1/
2*I)/(tanh(1/2*x)+1)^5+(-11/16+19/16*I)/(tanh(1/2*x)+1)^2-1/7/(tanh(1/2*x)-
1)^7
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(32) = 64.

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^{8}(x)}{i+\sinh(x)}\,dx$$
$$=\frac{1}{2688}\left(-840i\,xe^{(7\,x)}+3\,e^{(14\,x)}-7i\,e^{(13\,x)}+21\,e^{(12\,x)}-63i\,e^{(11\,x)}+63\,e^{(10\,x)}-315i\,e^{(9\,x)}+105\,e^{(8\,x)}+105\,e^{(\right.$$

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2688*(-840*I*x*e^(7*x) + 3*e^(14*x) - 7*I*e^(13*x) + 21*e^(12*x) - 63*I*e
^(11*x) + 63*e^(10*x) - 315*I*e^(9*x) + 105*e^(8*x) + 105*e^(6*x) + 315*I*e
^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x) + 7*I*e^x + 3)*e^(-7*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(49) = 98$.

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.48

$$\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx = -\frac{5ix}{16} + \frac{e^{7x}}{896} - \frac{ie^{6x}}{384} + \frac{e^{5x}}{128} - \frac{3ie^{4x}}{128} + \frac{3e^{3x}}{128} - \frac{15ie^{2x}}{128} + \frac{5e^{x}}{128} + \frac{5e^{-x}}{128} + \frac{15ie^{-2x}}{128} + \frac{3e^{-3x}}{128} + \frac{3ie^{-4x}}{128} + \frac{e^{-5x}}{128} + \frac{ie^{-6x}}{384} + \frac{e^{-7x}}{896}$$

```
[In] integrate(cosh(x)**8/(I+sinh(x)),x)
```

```
[Out] -5*I*x/16 + exp(7*x)/896 - I*exp(6*x)/384 + exp(5*x)/128 - 3*I*exp(4*x)/128
 + 3*exp(3*x)/128 - 15*I*exp(2*x)/128 + 5*exp(x)/128 + 5*exp(-x)/128 + 15*I
*exp(-2*x)/128 + 3*exp(-3*x)/128 + 3*I*exp(-4*x)/128 + exp(-5*x)/128 + I*ex
p(-6*x)/384 + exp(-7*x)/896
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx &= -\frac{1}{2688}\left(7i\,e^{(-x)} - 21\,e^{(-2\,x)} + 63i\,e^{(-3\,x)} - 63\,e^{(-4\,x)} + 315i\,e^{(-5\,x)} - 105\,e^{(-6\,x)} - 3\right)e^{(7\,x)} \\ &\quad - \frac{5}{16}i\,x + \frac{5}{128}\,e^{(-x)} + \frac{15}{128}i\,e^{(-2\,x)} + \frac{3}{128}\,e^{(-3\,x)} \\ &\quad + \frac{3}{128}i\,e^{(-4\,x)} + \frac{1}{128}\,e^{(-5\,x)} + \frac{1}{384}i\,e^{(-6\,x)} + \frac{1}{896}\,e^{(-7\,x)}\end{aligned}$$

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/2688*(7*I*e^(-x) - 21*e^(-2*x) + 63*I*e^(-3*x) - 63*e^(-4*x) + 315*I*e^(
-5*x) - 105*e^(-6*x) - 3)*e^(7*x) - 5/16*I*x + 5/128*e^(-x) + 15/128*I*e^(-
2*x) + 3/128*e^(-3*x) + 3/128*I*e^(-4*x) + 1/128*e^(-5*x) + 1/384*I*e^(-6*x
) + 1/896*e^(-7*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.72

$$\begin{aligned}&\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx\\&=\frac{1}{2688}\left(105\,e^{(6\,x)}+315i\,e^{(5\,x)}+63\,e^{(4\,x)}+63i\,e^{(3\,x)}+21\,e^{(2\,x)}+7i\,e^{x}+3\right)e^{(-7\,x)}-\frac{5}{16}i\,x\\&+\frac{1}{896}\,e^{(7\,x)}-\frac{1}{384}i\,e^{(6\,x)}+\frac{1}{128}\,e^{(5\,x)}-\frac{3}{128}i\,e^{(4\,x)}+\frac{3}{128}\,e^{(3\,x)}-\frac{15}{128}i\,e^{(2\,x)}+\frac{5}{128}\,e^{x}\end{aligned}$$

```
[In] integrate(cosh(x)^8/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2688*(105*e^(6*x) + 315*I*e^(5*x) + 63*e^(4*x) + 63*I*e^(3*x) + 21*e^(2*x
) + 7*I*e^x + 3)*e^(-7*x) - 5/16*I*x + 1/896*e^(7*x) - 1/384*I*e^(6*x) + 1/
128*e^(5*x) - 3/128*I*e^(4*x) + 3/128*e^(3*x) - 15/128*I*e^(2*x) + 5/128*e^
x
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.86

$$\begin{aligned}\int \frac{\cosh^8(x)}{i+\sinh(x)}\,dx=&\frac{5\,\mathrm{e}^{-x}}{128}+\frac{3\,\mathrm{e}^{-3\,x}}{128}+\frac{3\,\mathrm{e}^{3\,x}}{128}+\frac{\mathrm{e}^{-5\,x}}{128}+\frac{\mathrm{e}^{5\,x}}{128}+\frac{\mathrm{e}^{-7\,x}}{896}+\frac{\mathrm{e}^{7\,x}}{896}+\frac{5\,\mathrm{e}^{x}}{128}\\&-\frac{x\,5\mathrm{i}}{16}+\frac{\mathrm{e}^{-2\,x}\,15\mathrm{i}}{128}-\frac{\mathrm{e}^{2\,x}\,15\mathrm{i}}{128}+\frac{\mathrm{e}^{-4\,x}\,3\mathrm{i}}{128}-\frac{\mathrm{e}^{4\,x}\,3\mathrm{i}}{128}+\frac{\mathrm{e}^{-6\,x}\,1\mathrm{i}}{384}-\frac{\mathrm{e}^{6\,x}\,1\mathrm{i}}{384}\end{aligned}$$

```
[In] int(cosh(x)^8/(sinh(x) + 1i),x)
```

```
[Out] (5*exp(-x))/128 - (x*5i)/16 + (exp(-2*x)*15i)/128 - (exp(2*x)*15i)/128 + (3
*exp(-3*x))/128 + (3*exp(3*x))/128 + (exp(-4*x)*3i)/128 - (exp(4*x)*3i)/128
 + exp(-5*x)/128 + exp(5*x)/128 + (exp(-6*x)*1i)/384 - (exp(6*x)*1i)/384 +
exp(-7*x)/896 + exp(7*x)/896 + (5*exp(x))/128
```

3.159 $\int \frac{\cosh^7(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\cosh^7(x)}{i+\sinh(x)} dx = -(i-\sinh(x))^4 - \frac{4}{5}i(i-\sinh(x))^5 + \frac{1}{6}(i-\sinh(x))^6$$

```
[Out] -(I-sinh(x))^4-4/5*I*(I-sinh(x))^5+1/6*(I-sinh(x))^6
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 45}

$$\int \frac{\cosh^7(x)}{i+\sinh(x)} dx = \frac{1}{6}(-\sinh(x)+i)^6 - \frac{4}{5}i(-\sinh(x)+i)^5 - (-\sinh(x)+i)^4$$

```
[In] Int[Cosh[x]^7/(I + Sinh[x]),x]
```

```
[Out] -(I - Sinh[x])^4 - ((4*I)/5)*(I - Sinh[x])^5 + (I - Sinh[x])^6/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int (i-x)^3(i+x)^2\,dx, x, \sinh(x)\right)\\ &= -\text{Subst}\left(\int \left(-4(i-x)^3 - 4i(i-x)^4 + (i-x)^5\right)\,dx, x, \sinh(x)\right)\\ &= -(i-\sinh(x))^4 - \frac{4}{5}i(i-\sinh(x))^5 + \frac{1}{6}(i-\sinh(x))^6\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = \frac{1}{30}\sinh(x)\left(-30i + 15\sinh(x) - 20i\sinh^2(x) + 15\sinh^3(x) - 6i\sinh^4(x) + 5\sinh^5(x)\right)$$

```
[In] Integrate[Cosh[x]^7/(I + Sinh[x]),x]
[Out] (Sinh[x]*(-30*I + 15*Sinh[x] - (20*I)*Sinh[x]^2 + 15*Sinh[x]^3 - (6*I)*Sinh
[x]^4 + 5*Sinh[x]^5))/30
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$-i\sinh(x) + \frac{\sinh(x)^6}{6} - \frac{i\sinh(x)^5}{5} + \frac{\sinh(x)^4}{2} - \frac{2i\sinh(x)^3}{3} + \frac{\sinh(x)^2}{2}$$

```
[In] int(cosh(x)^7/(I+sinh(x)),x)
[Out] -I*sinh(x)+1/6*sinh(x)^6-1/5*I*sinh(x)^5+1/2*sinh(x)^4-2/3*I*sinh(x)^3+1/2*
sinh(x)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.67

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx$$
$$= \frac{1}{1920}\left(5\,e^{(12\,x)} - 12i\,e^{(11\,x)} + 30\,e^{(10\,x)} - 100i\,e^{(9\,x)} + 75\,e^{(8\,x)} - 600i\,e^{(7\,x)} + 600i\,e^{(5\,x)} + 75\,e^{(4\,x)} + 100i\,e^{(3\right.$$

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*e^(12*x) - 12*I*e^(11*x) + 30*e^(10*x) - 100*I*e^(9*x) + 75*e^(8*
x) - 600*I*e^(7*x) + 600*I*e^(5*x) + 75*e^(4*x) + 100*I*e^(3*x) + 30*e^(2*x
) + 12*I*e^x + 5)*e^(-6*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(26) = 52.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.33

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = \frac{e^{6x}}{384} - \frac{ie^{5x}}{160} + \frac{e^{4x}}{64} - \frac{5ie^{3x}}{96} + \frac{5e^{2x}}{128} - \frac{5ie^{x}}{16} + \frac{5ie^{-x}}{16} + \frac{5e^{-2x}}{128} + \frac{5ie^{-3x}}{96} + \frac{e^{-4x}}{64} + \frac{ie^{-5x}}{160} + \frac{e^{-6x}}{384}$$

```
[In] integrate(cosh(x)**7/(I+sinh(x)),x)
```

```
[Out] exp(6*x)/384 - I*exp(5*x)/160 + exp(4*x)/64 - 5*I*exp(3*x)/96 + 5*exp(2*x)/
128 - 5*I*exp(x)/16 + 5*I*exp(-x)/16 + 5*exp(-2*x)/128 + 5*I*exp(-3*x)/96 +
 exp(-4*x)/64 + I*exp(-5*x)/160 + exp(-6*x)/384
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.74

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx$$
$$= -\frac{1}{1920}\left(12i\,e^{(-x)} - 30\,e^{(-2\,x)} + 100i\,e^{(-3\,x)} - 75\,e^{(-4\,x)} + 600i\,e^{(-5\,x)} - 5\right)e^{(6\,x)} + \frac{5}{16}i\,e^{(-x)} + \frac{5}{128}\,e^{(-2\,x)} + \frac{5}{96}i\,e^{(-3\,x)} + \frac{1}{64}\,e^{(-4\,x)} + \frac{1}{160}i\,e^{(-5\,x)} + \frac{1}{384}\,e^{(-6\,x)}$$

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/1920*(12*I*e^(-x) - 30*e^(-2*x) + 100*I*e^(-3*x) - 75*e^(-4*x) + 600*I*e
^(-5*x) - 5)*e^(6*x) + 5/16*I*e^(-x) + 5/128*e^(-2*x) + 5/96*I*e^(-3*x) + 1
/64*e^(-4*x) + 1/160*I*e^(-5*x) + 1/384*e^(-6*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = -\frac{1}{1920}\left(-600i\,e^{(5\,x)} - 75\,e^{(4\,x)} - 100i\,e^{(3\,x)} - 30\,e^{(2\,x)} - 12i\,e^{x} - 5\right)e^{(-6\,x)} + \frac{1}{384}\,e^{(6\,x)} - \frac{1}{160}i\,e^{(5\,x)} + \frac{1}{64}\,e^{(4\,x)} - \frac{5}{96}i\,e^{(3\,x)} + \frac{5}{128}\,e^{(2\,x)} - \frac{5}{16}i\,e^{x}$$

```
[In] integrate(cosh(x)^7/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/1920*(-600*I*e^(5*x) - 75*e^(4*x) - 100*I*e^(3*x) - 30*e^(2*x) - 12*I*e^
x - 5)*e^(-6*x) + 1/384*e^(6*x) - 1/160*I*e^(5*x) + 1/64*e^(4*x) - 5/96*I*e
^(3*x) + 5/128*e^(2*x) - 5/16*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^7(x)}{i+\sinh(x)}\,dx = \frac{\mathrm{e}^{-x}\,5\mathrm{i}}{16} + \frac{5\,\mathrm{e}^{-2\,x}}{128} + \frac{5\,\mathrm{e}^{2\,x}}{128} + \frac{\mathrm{e}^{-3\,x}\,5\mathrm{i}}{96} - \frac{\mathrm{e}^{3\,x}\,5\mathrm{i}}{96} + \frac{\mathrm{e}^{-4\,x}}{64} + \frac{\mathrm{e}^{4\,x}}{64} + \frac{\mathrm{e}^{-5\,x}\,1\mathrm{i}}{160} - \frac{\mathrm{e}^{5\,x}\,1\mathrm{i}}{160} + \frac{\mathrm{e}^{-6\,x}}{384} + \frac{\mathrm{e}^{6\,x}}{384} - \frac{\mathrm{e}^{x}\,5\mathrm{i}}{16}$$

```
[In] int(cosh(x)^7/(sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*5i)/16 + (5*exp(-2*x))/128 + (5*exp(2*x))/128 + (exp(-3*x)*5i)/96
- (exp(3*x)*5i)/96 + exp(-4*x)/64 + exp(4*x)/64 + (exp(-5*x)*1i)/160 - (exp
(5*x)*1i)/160 + exp(-6*x)/384 + exp(6*x)/384 - (exp(x)*5i)/16
```

3.160 $\int \frac{\cosh^6(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\cosh^6(x)}{i+\sinh(x)} dx = -\frac{3ix}{8} + \frac{\cosh^5(x)}{5} - \frac{3}{8} i \cosh(x) \sinh(x) - \frac{1}{4} i \cosh^3(x) \sinh(x)$$

```
[Out] -3/8*I*x+1/5*cosh(x)^5-3/8*I*cosh(x)*sinh(x)-1/4*I*cosh(x)^3*sinh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2761, 2715, 8}

$$\int \frac{\cosh^6(x)}{i+\sinh(x)} dx = -\frac{3ix}{8} + \frac{\cosh^5(x)}{5} - \frac{1}{4} i \sinh(x) \cosh^3(x) - \frac{3}{8} i \sinh(x) \cosh(x)$$

```
[In] Int[Cosh[x]^6/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/8)*x + Cosh[x]^5/5 - ((3*I)/8)*Cosh[x]*Sinh[x] - (I/4)*Cosh[x]^3*Si
nh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\cosh^5(x)}{5} - i\int \cosh^4(x)\,dx\\ &= \frac{\cosh^5(x)}{5} - \frac{1}{4}i\cosh^3(x)\sinh(x) - \frac{3}{4}i\int \cosh^2(x)\,dx\\ &= \frac{\cosh^5(x)}{5} - \frac{3}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x) - \frac{3}{8}i\int 1\,dx\\ &= -\frac{3ix}{8} + \frac{\cosh^5(x)}{5} - \frac{3}{8}i\cosh(x)\sinh(x) - \frac{1}{4}i\cosh^3(x)\sinh(x)\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 131 vs. 2(38) = 76.

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.45

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = -\frac{i\cosh^7(x)\left(8i + \frac{30i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}} + 33\sinh(x) - 9i\sinh^2(x) + 26\sinh^3(x) - 2i\sinh^4(x) + 8\right.}{40\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)^6}$$

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x]),x]
```

```
[Out] ((-1/40*I)*Cosh[x]^7*(8*I + ((30*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqr
t[1 - I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] + 33*Sinh[x] - (9*I)*Sinh[x]^2 + 26*S
inh[x]^3 - (2*I)*Sinh[x]^4 + 8*Sinh[x]^5))/((Cosh[x/2] - I*Sinh[x/2])^8*(Co
sh[x/2] + I*Sinh[x/2])^6)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(27) = 54.

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.63

$$\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{-\frac{1}{2}-\frac{i}{4}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{-\frac{3}{8}-\frac{5i}{8}}{\tanh\left(\frac{x}{2}\right)-1}+\frac{-\frac{5}{8}-\frac{7i}{8}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{3}{4}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{5\left(\tanh\left(\frac{x}{2}\right)-\right.}$$

```
[In] int(cosh(x)^6/(I+sinh(x)),x)
```

```
[Out] 3/8*I*ln(tanh(1/2*x)-1)-(1/2+1/4*I)/(tanh(1/2*x)-1)^4-(3/8+5/8*I)/(tanh(1/2
*x)-1)-(5/8+7/8*I)/(tanh(1/2*x)-1)^2-(3/4+1/2*I)/(tanh(1/2*x)-1)^3-1/5/(tan
h(1/2*x)-1)^5-3/8*I*ln(tanh(1/2*x)+1)+(-1/2+1/4*I)/(tanh(1/2*x)+1)^4+(3/4-1
/2*I)/(tanh(1/2*x)+1)^3+(3/8-5/8*I)/(tanh(1/2*x)+1)+(-5/8+7/8*I)/(tanh(1/2*
x)+1)^2+1/5/(tanh(1/2*x)+1)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(24) = 48.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int\frac{\cosh^{6}(x)}{i+\sinh(x)}\,dx$$
$$=\frac{1}{320}\left(-120i\,xe^{(5\,x)}+2\,e^{(10\,x)}-5i\,e^{(9\,x)}+10\,e^{(8\,x)}-40i\,e^{(7\,x)}+20\,e^{(6\,x)}+20\,e^{(4\,x)}+40i\,e^{(3\,x)}+10\,e^{(2\,x)}+5\right.$$

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/320*(-120*I*x*e^(5*x) + 2*e^(10*x) - 5*I*e^(9*x) + 10*e^(8*x) - 40*I*e^(7
*x) + 20*e^(6*x) + 20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^
(-5*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int\frac{\cosh^{6}(x)}{i+\sinh(x)}\,dx=-\frac{3ix}{8}+\frac{e^{5x}}{160}-\frac{ie^{4x}}{64}+\frac{e^{3x}}{32}-\frac{ie^{2x}}{8}+\frac{e^{x}}{16}+\frac{e^{-x}}{16}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{32}+\frac{ie^{-4x}}{64}+\frac{e^{-5x}}{160}$$

```
[In] integrate(cosh(x)**6/(I+sinh(x)),x)
```

```
[Out] -3*I*x/8 + exp(5*x)/160 - I*exp(4*x)/64 + exp(3*x)/32 - I*exp(2*x)/8 + exp(
x)/16 + exp(-x)/16 + I*exp(-2*x)/8 + exp(-3*x)/32 + I*exp(-4*x)/64 + exp(-5
*x)/160
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(24) = 48$.

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = -\frac{1}{320}\left(5i\,e^{(-x)} - 10\,e^{(-2\,x)} + 40i\,e^{(-3\,x)} - 20\,e^{(-4\,x)} - 2\right)e^{(5\,x)} - \frac{3}{8}i\,x + \frac{1}{16}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)} + \frac{1}{32}\,e^{(-3\,x)} + \frac{1}{64}i\,e^{(-4\,x)} + \frac{1}{160}\,e^{(-5\,x)}$$

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/320*(5*I*e^(-x) - 10*e^(-2*x) + 40*I*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)
 - 3/8*I*x + 1/16*e^(-x) + 1/8*I*e^(-2*x) + 1/32*e^(-3*x) + 1/64*I*e^(-4*x)
 + 1/160*e^(-5*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = \frac{1}{320}\left(20\,e^{(4\,x)} + 40i\,e^{(3\,x)} + 10\,e^{(2\,x)} + 5i\,e^{x} + 2\right)e^{(-5\,x)} - \frac{3}{8}i\,x + \frac{1}{160}\,e^{(5\,x)} - \frac{1}{64}i\,e^{(4\,x)} + \frac{1}{32}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{16}\,e^{x}$$

```
[In] integrate(cosh(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/320*(20*e^(4*x) + 40*I*e^(3*x) + 10*e^(2*x) + 5*I*e^x + 2)*e^(-5*x) - 3/8
*I*x + 1/160*e^(5*x) - 1/64*I*e^(4*x) + 1/32*e^(3*x) - 1/8*I*e^(2*x) + 1/16
*e^x
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{\cosh^6(x)}{i+\sinh(x)}\,dx = \frac{e^{-x}}{16}+\frac{e^{-3x}}{32}+\frac{e^{3x}}{32}+\frac{e^{-5x}}{160}+\frac{e^{5x}}{160}+\frac{e^{x}}{16} - \frac{x\,3\mathrm{i}}{8}+\frac{e^{-2x}\,1\mathrm{i}}{8}-\frac{e^{2x}\,1\mathrm{i}}{8}+\frac{e^{-4x}\,1\mathrm{i}}{64}-\frac{e^{4x}\,1\mathrm{i}}{64}$$

```
[In] int(cosh(x)^6/(sinh(x) + 1i),x)
```

```
[Out] exp(-x)/16 - (x*3i)/8 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/32 +
 exp(3*x)/32 + (exp(-4*x)*1i)/64 - (exp(4*x)*1i)/64 + exp(-5*x)/160 + exp(5
*x)/160 + exp(x)/16
```

3.161 $\int \frac{\cosh^5(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\cosh^5(x)}{i+\sinh(x)} dx = -i\sinh(x) + \frac{\sinh^2(x)}{2} - \frac{1}{3}i\sinh^3(x) + \frac{\sinh^4(x)}{4}$$

```
[Out] -I*sinh(x)+1/2*sinh(x)^2-1/3*I*sinh(x)^3+1/4*sinh(x)^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 45}

$$\int \frac{\cosh^5(x)}{i+\sinh(x)} dx = \frac{\sinh^4(x)}{4} - \frac{1}{3}i\sinh^3(x) + \frac{\sinh^2(x)}{2} - i\sinh(x)$$

```
[In] Int[Cosh[x]^5/(I + Sinh[x]),x]
```

```
[Out] (-I)*Sinh[x] + Sinh[x]^2/2 - (I/3)*Sinh[x]^3 + Sinh[x]^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int (i-x)^2(i+x)\,dx, x, \sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-i+x-ix^2+x^3\right)\,dx, x, \sinh(x)\right)\\ &= -i\sinh(x)+\frac{\sinh^2(x)}{2}-\frac{1}{3}i\sinh^3(x)+\frac{\sinh^4(x)}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \frac{1}{12}\sinh(x)\left(-12i+6\sinh(x)-4i\sinh^2(x)+3\sinh^3(x)\right)$$

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x]),x]
[Out] (Sinh[x]*(-12*I + 6*Sinh[x] - (4*I)*Sinh[x]^2 + 3*Sinh[x]^3))/12
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$-i\sinh(x)+\frac{\sinh(x)^2}{2}-\frac{i\sinh(x)^3}{3}+\frac{\sinh(x)^4}{4}$$

```
[In] int(cosh(x)^5/(I+sinh(x)),x)
[Out] -I*sinh(x)+1/2*sinh(x)^2-1/3*I*sinh(x)^3+1/4*sinh(x)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \frac{1}{192}\left(3e^{(8x)}-8ie^{(7x)}+12e^{(6x)}-72ie^{(5x)}+72ie^{(3x)}+12e^{(2x)}+8ie^{x}+3\right)e^{(-4x)}$$

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*e^(8*x) - 8*I*e^(7*x) + 12*e^(6*x) - 72*I*e^(5*x) + 72*I*e^(3*x) +
 12*e^(2*x) + 8*I*e^x + 3)*e^(-4*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = \frac{e^{4x}}{64} - \frac{ie^{3x}}{24} + \frac{e^{2x}}{16} - \frac{3ie^{x}}{8} + \frac{3ie^{-x}}{8} + \frac{e^{-2x}}{16} + \frac{ie^{-3x}}{24} + \frac{e^{-4x}}{64}$$

```
[In] integrate(cosh(x)**5/(I+sinh(x)),x)
```

```
[Out] exp(4*x)/64 - I*exp(3*x)/24 + exp(2*x)/16 - 3*I*exp(x)/8 + 3*I*exp(-x)/8 +
exp(-2*x)/16 + I*exp(-3*x)/24 + exp(-4*x)/64
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(23) = 46.

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = -\frac{1}{192}\left(8i\,e^{(-x)} - 12\,e^{(-2\,x)} + 72i\,e^{(-3\,x)} - 3\right)e^{(4\,x)} + \frac{3}{8}i\,e^{(-x)} + \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{24}i\,e^{(-3\,x)} + \frac{1}{64}\,e^{(-4\,x)}$$

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*I*e^(-x) - 12*e^(-2*x) + 72*I*e^(-3*x) - 3)*e^(4*x) + 3/8*I*e^(-x
) + 1/16*e^(-2*x) + 1/24*I*e^(-3*x) + 1/64*e^(-4*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

$$\int \frac{\cosh^5(x)}{i+\sinh(x)}\,dx = -\frac{1}{192}\left(-72i\,e^{(3\,x)} - 12\,e^{(2\,x)} - 8i\,e^{x} - 3\right)e^{(-4\,x)} + \frac{1}{64}\,e^{(4\,x)} - \frac{1}{24}i\,e^{(3\,x)} + \frac{1}{16}\,e^{(2\,x)} - \frac{3}{8}i\,e^{x}$$

```
[In] integrate(cosh(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/192*(-72*I*e^(3*x) - 12*e^(2*x) - 8*I*e^x - 3)*e^(-4*x) + 1/64*e^(4*x) -
 1/24*I*e^(3*x) + 1/16*e^(2*x) - 3/8*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\cosh^5(x)}{i+\sinh(x)} \, dx = \frac{e^{-x} \, 3i}{8} + \frac{e^{-2x}}{16} + \frac{e^{2x}}{16} + \frac{e^{-3x} \, 1i}{24} - \frac{e^{3x} \, 1i}{24} + \frac{e^{-4x}}{64} + \frac{e^{4x}}{64} - \frac{e^{x} \, 3i}{8}$$

```
[In] int(cosh(x)^5/(sinh(x) + 1i),x)
```

```
[Out] (exp(-x)*3i)/8 + exp(-2*x)/16 + exp(2*x)/16 + (exp(-3*x)*1i)/24 - (exp(3*x)
*1i)/24 + exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*3i)/8
```

3.162 $\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx=-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\cosh(x)\sinh(x)$$

```
[Out] -1/2*I*x+1/3*cosh(x)^3-1/2*I*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2761, 2715, 8}

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx=-\frac{ix}{2}+\frac{\cosh^3(x)}{3}-\frac{1}{2}i\sinh(x)\cosh(x)$$

```
[In] Int[Cosh[x]^4/(I + Sinh[x]),x]
```

```
[Out] (-1/2*I)*x + Cosh[x]^3/3 - (I/2)*Cosh[x]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\cosh^3(x)}{3} - i\int \cosh^2(x)\,dx\\ &= \frac{\cosh^3(x)}{3} - \frac{1}{2}i\cosh(x)\sinh(x) - \frac{1}{2}i\int 1\,dx\\ &= -\frac{ix}{2} + \frac{\cosh^3(x)}{3} - \frac{1}{2}i\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 93 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.58

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = \frac{\cosh^5(x)\left(2i + \frac{6i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}} + 5\sinh(x) - i\sinh^2(x) + 2\sinh^3(x)\right)}{6(-i+\sinh(x))^2(i+\sinh(x))^3}$$

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x]),x]
[Out] (Cosh[x]^5*(2*I + ((6*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sin
h[x]])/Sqrt[1 + I*Sinh[x]] + 5*Sinh[x] - I*Sinh[x]^2 + 2*Sinh[x]^3))/(6*(-I
 + Sinh[x])^2*(I + Sinh[x])^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(18) = 36$.

Time = 210.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{ix}{2}+\frac{{\mathrm e}^{3x}}{24}-\frac{i{\mathrm e}^{2x}}{8}+\frac{{\mathrm e}^{x}}{8}+\frac{{\mathrm e}^{-x}}{8}+\frac{i{\mathrm e}^{-2x}}{8}+\frac{{\mathrm e}^{-3x}}{24}$
default	$-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{\frac{1}{2}-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)+1}+\frac{-\frac{1}{2}+\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{-\frac{1}{2}-\frac{i}{2}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{1}{2}-\frac{i}{2}}{\tanh\left(\frac{x}{2}\right)-1}$

```
[In] int(cosh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*x+1/24*exp(x)^3-1/8*I*exp(x)^2+1/8*exp(x)+1/8/exp(x)+1/8*I/exp(x)^2+
1/24/exp(x)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = \frac{1}{24}\left(-12i\,xe^{(3\,x)}+e^{(6\,x)}-3i\,e^{(5\,x)}+3\,e^{(4\,x)}+3\,e^{(2\,x)}+3i\,e^{x}+1\right)e^{(-3\,x)}$$

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(-12*I*x*e^(3*x) + e^(6*x) - 3*I*e^(5*x) + 3*e^(4*x) + 3*e^(2*x) + 3*I
*e^x + 1)*e^(-3*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx = -\frac{ix}{2}+\frac{e^{3x}}{24}-\frac{ie^{2x}}{8}+\frac{e^{x}}{8}+\frac{e^{-x}}{8}+\frac{ie^{-2x}}{8}+\frac{e^{-3x}}{24}$$

```
[In] integrate(cosh(x)**4/(I+sinh(x)),x)
```

```
[Out] -I*x/2 + exp(3*x)/24 - I*exp(2*x)/8 + exp(x)/8 + exp(-x)/8 + I*exp(-2*x)/8
+ exp(-3*x)/24
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^4(x)}{i + \sinh(x)}\,dx = -\frac{1}{24}\left(3i\,e^{(-x)} - 3\,e^{(-2\,x)} - 1\right)e^{(3\,x)} - \frac{1}{2}i\,x + \frac{1}{8}\,e^{(-x)} + \frac{1}{8}i\,e^{(-2\,x)} + \frac{1}{24}\,e^{(-3\,x)}$$

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*I*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x) - 1/2*I*x + 1/8*e^(-x) + 1/8*I*
e^(-2*x) + 1/24*e^(-3*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{\cosh^4(x)}{i + \sinh(x)}\,dx = \frac{1}{24}\left(3\,e^{(2\,x)} + 3i\,e^{x} + 1\right)e^{(-3\,x)} - \frac{1}{2}i\,x + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{8}i\,e^{(2\,x)} + \frac{1}{8}\,e^{x}$$

```
[In] integrate(cosh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) + 3*I*e^x + 1)*e^(-3*x) - 1/2*I*x + 1/24*e^(3*x) - 1/8*I*e^
(2*x) + 1/8*e^x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^4(x)}{i + \sinh(x)}\,dx = \frac{\mathrm{e}^{-x}}{8} + \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} + \frac{\mathrm{e}^{x}}{8} - \frac{x\,1\mathrm{i}}{2} + \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{8} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{8}$$

```
[In] int(cosh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] exp(-x)/8 - (x*1i)/2 + (exp(-2*x)*1i)/8 - (exp(2*x)*1i)/8 + exp(-3*x)/24 +
exp(3*x)/24 + exp(x)/8
```

3.163 $\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = -i\sinh(x)+\frac{\sinh^2(x)}{2}$$

```
[Out] -I*sinh(x)+1/2*sinh(x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2746}

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{\sinh^2(x)}{2}-i\sinh(x)$$

```
[In] Int[Cosh[x]^3/(I + Sinh[x]),x]
[Out] (-I)*Sinh[x] + Sinh[x]^2/2
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int (i-x)\,dx,x,\sinh(x)\right) \\ &= -i\sinh(x)+\frac{\sinh^2(x)}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx=\frac{1}{2}\sinh(x)(-2i+\sinh(x))$$

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x]),x]
[Out] (Sinh[x]*(-2*I + Sinh[x]))/2
```

Maple [A] (verified)

Time = 16.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$-i\sinh\left(x\right)+\frac{\sinh\left(x\right)^{2}}{2}$	13
default	$-i\sinh\left(x\right)+\frac{\sinh\left(x\right)^{2}}{2}$	13
risch	$\frac{{\mathrm e}^{2x}}{8}-\frac{i{\mathrm e}^{x}}{2}+\frac{i{\mathrm e}^{-x}}{2}+\frac{{\mathrm e}^{-2x}}{8}$	26

```
[In] int(cosh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -I*sinh(x)+1/2*sinh(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx=\frac{1}{8}\left(e^{(4x)}-4i\,e^{(3x)}+4i\,e^{x}+1\right)e^{(-2x)}$$

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/8*(e^(4*x) - 4*I*e^(3*x) + 4*I*e^x + 1)*e^(-2*x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{e^{2x}}{8} - \frac{ie^{x}}{2} + \frac{ie^{-x}}{2} + \frac{e^{-2x}}{8}$$

```
[In] integrate(cosh(x)**3/(I+sinh(x)),x)
[Out] exp(2*x)/8 - I*exp(x)/2 + I*exp(-x)/2 + exp(-2*x)/8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(11) = 22.

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = \frac{1}{8}\left(-4i\,e^{(-x)}+1\right)e^{(2\,x)} + \frac{1}{2}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="maxima")
[Out] 1/8*(-4*I*e^(-x) + 1)*e^(2*x) + 1/2*I*e^(-x) + 1/8*e^(-2*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx = -\frac{1}{8}\left(-4i\,e^{x}-1\right)e^{(-2\,x)} + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{2}i\,e^{x}$$

```
[In] integrate(cosh(x)^3/(I+sinh(x)),x, algorithm="giac")
[Out] -1/8*(-4*I*e^x - 1)*e^(-2*x) + 1/8*e^(2*x) - 1/2*I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\cosh^3(x)}{i+\sinh(x)}\,dx=\frac{\mathrm{e}^{-2\,x}\,\left(\mathrm{e}^{4\,x}+1\right)}{8}-\frac{\mathrm{e}^{-2\,x}\,\left(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^{x}\right)\,1\mathrm{i}}{8}$$

```
[In] int(cosh(x)^3/(sinh(x) + 1i),x)
[Out] (exp(-2*x)*(exp(4*x) + 1))/8 - (exp(-2*x)*(4*exp(3*x) - 4*exp(x))*1i)/8
```

3.164 $\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = -ix+\cosh(x)$$

```
[Out] -I*x+cosh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2761, 8}

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = \cosh(x)-ix$$

```
[In] Int[Cosh[x]^2/(I + Sinh[x]),x]
```

```
[Out] (-I)*x + Cosh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(x) - i\int 1\,dx \\ &= -ix + \cosh(x) \end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = \cosh(x) + 2\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{\cosh^2(x)}\operatorname{sech}(x)$$

```
[In] Integrate[Cosh[x]^2/(I + Sinh[x]),x]
[Out] Cosh[x] + 2*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 7.89 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

method	result	size
risch	$-ix + \frac{e^x}{2} + \frac{e^{-x}}{2}$	16
default	$i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - \frac{1}{\tanh\left(\frac{x}{2}\right)-1} - i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{1}{\tanh\left(\frac{x}{2}\right)+1}$	40

```
[In] int(cosh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -I*x+1/2*exp(x)+1/2*exp(-x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\left(-2i\,xe^{x} + e^{(2\,x)} + 1\right)e^{(-x)}$$

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/2*(-2*I*x*e^x + e^(2*x) + 1)*e^(-x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$.

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = -ix + \frac{e^{x}}{2} + \frac{e^{-x}}{2}$$

```
[In] integrate(cosh(x)**2/(I+sinh(x)),x)
[Out] -I*x + exp(x)/2 + exp(-x)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx = -i\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="maxima")
[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^2(x)}{i + \sinh(x)}\, dx = -i\, x + \frac{1}{2}\, e^{(-x)} + \frac{1}{2}\, e^{x}$$

```
[In] integrate(cosh(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] -I*x + 1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(x)}{i + \sinh(x)}\, dx = \cosh(x) - x\, 1\mathrm{i}$$

```
[In] int(cosh(x)^2/(sinh(x) + 1i),x)
[Out] cosh(x) - x*1i
```

3.165 $\int \frac{\cosh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 7

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = \log(i+\sinh(x))$$

```
[Out] ln(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2746, 31}

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = \log(\sinh(x)+i)$$

```
[In] Int[Cosh[x]/(I + Sinh[x]),x]
```

```
[Out] Log[I + Sinh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right) \\ &= \log(i+\sinh(x)) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = \log(i+\sinh(x))$$

```
[In] Integrate[Cosh[x]/(I + Sinh[x]),x]
[Out] Log[I + Sinh[x]]
```

Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln(i+\sinh(x))$	7
default	$\ln(i+\sinh(x))$	7
risch	$-x+2\ln(e^{x}+i)$	13

```
[In] int(cosh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] ln(I+sinh(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = -x+2\,\log\left(e^{x}+i\right)$$

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] -x + 2*log(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = -x + 2\log\left(e^{x}+i\right)$$

```
[In] integrate(cosh(x)/(I+sinh(x)),x)
[Out] -x + 2*log(exp(x) + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = \log\left(\sinh\left(x\right)+i\right)$$

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] log(sinh(x) + I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11 vs. 2(5) = 10.

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = -x + 2\,\log\left(e^{x}+i\right)$$

```
[In] integrate(cosh(x)/(I+sinh(x)),x, algorithm="giac")
[Out] -x + 2*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(x)}{i+\sinh(x)}\,dx = \ln\left(\cosh(x)\right) - \mathrm{atan}(\sinh(x))\ 1\mathrm{i}$$

```
[In] int(cosh(x)/(sinh(x) + 1i),x)
[Out] log(cosh(x)) - atan(sinh(x))*1i
```

3.166 $\int \frac{\text{sech}(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{\text{sech}(x)}{i+\sinh(x)}\,dx = -\frac{1}{2}i\arctan(\sinh(x)) - \frac{i}{2(i+\sinh(x))}$$

```
[Out] -1/2*I*arctan(sinh(x))-1/2*I/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2746, 46, 209}

$$\int \frac{\text{sech}(x)}{i+\sinh(x)}\,dx = -\frac{1}{2}i\arctan(\sinh(x)) - \frac{i}{2(\sinh(x)+i)}$$

```
[In] Int[Sech[x]/(I + Sinh[x]),x]
```

```
[Out] (-1/2*I)*ArcTan[Sinh[x]] - (I/2)/(I + Sinh[x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{(i-x)(i+x)^2}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{i}{2(i+x)^2}+\frac{i}{2\,(1+x^2)}\right)\,dx, x, \sinh(x)\right)\\
&= -\frac{i}{2(i+\sinh(x))}-\frac{1}{2}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{2}i\arctan(\sinh(x))-\frac{i}{2(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\text{sech}(x)}{i+\sinh(x)}\,dx = -\frac{1}{2}i\left(\arctan(\sinh(x))+\frac{1}{i+\sinh(x)}\right)$$

```
[In] Integrate[Sech[x]/(I + Sinh[x]),x]
[Out] (-1/2*I)*(ArcTan[Sinh[x]] + (I + Sinh[x])^(-1))
```

Maple [A] (verified)

Time = 8.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

method	result	size
risch	$-\frac{i e^{x}}{\left(e^{x}+i\right)^{2}}-\frac{\ln\left(e^{x}-i\right)}{2}+\frac{\ln\left(e^{x}+i\right)}{2}$	30
default	$\frac{i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{2}-\frac{\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{2}$	43

```
[In] int(sech(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -I*exp(x)/(exp(x)+I)^2-1/2*ln(exp(x)-I)+1/2*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(14) = 28.

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \frac{\left(e^{(2\,x)} + 2i\,e^{x} - 1\right)\log\left(e^{x} + i\right) - \left(e^{(2\,x)} + 2i\,e^{x} - 1\right)\log\left(e^{x} - i\right) - 2i\,e^{x}}{2\left(e^{(2\,x)} + 2i\,e^{x} - 1\right)}$$

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] 1/2*((e^(2*x) + 2*I*e^x - 1)*log(e^x + I) - (e^(2*x) + 2*I*e^x - 1)*log(e^x
 - I) - 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{\sinh(x) + i}\,dx$$

```
[In] integrate(sech(x)/(I+sinh(x)),x)
[Out] Integral(sech(x)/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(14) = 28.

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \frac{2i\,e^{(-x)}}{-4i\,e^{(-x)} + 2\,e^{(-2\,x)} - 2} - \frac{1}{2}\log\left(e^{(-x)} + i\right) + \frac{1}{2}\log\left(e^{(-x)} - i\right)$$

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] 2*I*e^(-x)/(-4*I*e^(-x) + 2*e^(-2*x) - 2) - 1/2*log(e^(-x) + I) + 1/2*log(e
^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = -\frac{e^{(-x)}-e^x-6i}{4\left(e^{(-x)}-e^x-2i\right)}+\frac{1}{4}\log\left(-e^{(-x)}+e^x+2i\right)-\frac{1}{4}\log\left(-e^{(-x)}+e^x-2i\right)$$

```
[In] integrate(sech(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/4*(e^(-x) - e^x - 6*I)/(e^(-x) - e^x - 2*I) + 1/4*log(-e^(-x) + e^x + 2*
I) - 1/4*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{\operatorname{sech}(x)}{i+\sinh(x)}\,dx = \frac{\ln\left(-1+\mathrm{e}^x\,1\mathrm{i}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)}{2}-\frac{1}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}}-\frac{1\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

```
[In] int(1/(cosh(x)*(sinh(x) + 1i)),x)
```

```
[Out] log(exp(x)*1i - 1)/2 - log(exp(x)*1i + 1)/2 - 1/(exp(2*x) + exp(x)*2i - 1)
- 1i/(exp(x) + 1i)
```

3.167 $\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{i\operatorname{sech}(x)}{3(i+\sinh(x))} - \frac{2}{3}i\tanh(x)$$

```
[Out] -1/3*I*sech(x)/(I+sinh(x))-2/3*I*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2751, 3852, 8}

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{2}{3}i\tanh(x) - \frac{i\operatorname{sech}(x)}{3(\sinh(x)+i)}$$

```
[In] Int[Sech[x]^2/(I + Sinh[x]),x]
```

```
[Out] ((-1/3*I)*Sech[x])/(I + Sinh[x]) - ((2*I)/3)*Tanh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
```

```
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{i\text{sech}(x)}{3(i+\sinh(x))} - \frac{2}{3}i\int \text{sech}^2(x)\,dx \\
&= -\frac{i\text{sech}(x)}{3(i+\sinh(x))} + \frac{2}{3}\text{Subst}(\int 1\,dx, x, -i\tanh(x)) \\
&= -\frac{i\text{sech}(x)}{3(i+\sinh(x))} - \frac{2}{3}i\tanh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$
\int \frac{\text{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{1}{3}i\left(\frac{\text{sech}(x)}{i+\sinh(x)} + 2\tanh(x)\right)
$$

```
[In] Integrate[Sech[x]^2/(I + Sinh[x]),x]
[Out] (-1/3*I)*(Sech[x]/(I + Sinh[x]) + 2*Tanh[x])
```

Maple [A] (verified)

Time = 27.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{4(2\,e^{x}+i)}{3(e^{x}+i)^{3}(e^{x}-i)}$	24
default	$-\frac{i}{2(-i+\tanh(\frac{x}{2}))} - \frac{1}{(\tanh(\frac{x}{2})+i)^{2}} + \frac{2i}{3(\tanh(\frac{x}{2})+i)^{3}} - \frac{3i}{2(\tanh(\frac{x}{2})+i)}$	49

```
[In] int(sech(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -4/3*(2*exp(x)+I)/(exp(x)+I)^3/(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{4\,(2\,e^{x}+i)}{3\,(e^{(4\,x)}+2i\,e^{(3\,x)}+2i\,e^{x}-1)}$$

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] -4/3*(2*e^x + I)/(e^(4*x) + 2*I*e^(3*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\sinh(x)+i}\,dx$$

```
[In] integrate(sech(x)**2/(I+sinh(x)),x)
[Out] Integral(sech(x)**2/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(15) = 30.

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{8\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}+\frac{4i}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="maxima")
[Out] -8*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 4*I/(-6*I*e^(-x)
- 6*I*e^(-3*x) + 3*e^(-4*x) - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = \frac{1}{2\left(e^{x}-i\right)} - \frac{3\,e^{(2\,x)}+12i\,e^{x}-5}{6\left(e^{x}+i\right)^{3}}$$

```
[In] integrate(sech(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] 1/2/(e^x - I) - 1/6*(3*e^(2*x) + 12*I*e^x - 5)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{\operatorname{sech}^2(x)}{i+\sinh(x)}\,dx = -\frac{8\,e^{x}}{3\left(e^{2\,x}+1\right)^{3}} - \frac{8\,e^{x}\left(e^{2\,x}-1\right)}{3\left(e^{2\,x}+1\right)^{3}} + \frac{e^{2\,x}\,16i}{3\left(e^{2\,x}+1\right)^{3}} - \frac{\left(e^{2\,x}-1\right)\,4i}{3\left(e^{2\,x}+1\right)^{3}}$$

```
[In] int(1/(cosh(x)^2*(sinh(x) + 1i)),x)
[Out] (exp(2*x)*16i)/(3*(exp(2*x) + 1)^3) - (8*exp(x))/(3*(exp(2*x) + 1)^3) - ((e
xp(2*x) - 1)*4i)/(3*(exp(2*x) + 1)^3) - (8*exp(x)*(exp(2*x) - 1))/(3*(exp(2
*x) + 1)^3)
```

3.168 $\int \frac{\text{sech}^3(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{\text{sech}^3(x)}{i+\sinh(x)} \, dx = -\frac{3}{8}i \arctan(\sinh(x)) + \frac{i}{8(i-\sinh(x))} + \frac{1}{8(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

```
[Out] -3/8*I*arctan(sinh(x))+1/8*I/(I-sinh(x))+1/8/(I+sinh(x))^2-1/4*I/(I+sinh(x)
)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 46, 209}

$$\int \frac{\text{sech}^3(x)}{i+\sinh(x)} \, dx = -\frac{3}{8}i \arctan(\sinh(x)) + \frac{i}{8(-\sinh(x)+i)} - \frac{i}{4(\sinh(x)+i)} + \frac{1}{8(\sinh(x)+i)^2}$$

```
[In] Int[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-3*I)/8)*ArcTan[Sinh[x]] + (I/8)/(I - Sinh[x]) + 1/(8*(I + Sinh[x])^2) -
(I/4)/(I + Sinh[x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^3}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{i}{8(-i+x)^2}-\frac{1}{4(i+x)^3}+\frac{i}{4(i+x)^2}-\frac{3i}{8\left(1+x^2\right)}\right)\,dx, x, \sinh(x)\right)\\
&= \frac{i}{8(i-\sinh(x))}+\frac{1}{8(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}-\frac{3}{8}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{3}{8}i\arctan(\sinh(x))+\frac{i}{8(i-\sinh(x))}+\frac{1}{8(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = -\frac{i\operatorname{sech}^2(x)\left(2+3i\arctan(\sinh(x))+3(i+\arctan(\sinh(x)))\sinh(x)+(3+3i\arctan(\sinh(x)))\sinh^2(x)+\right.}{8(i+\sinh(x))}$$

```
[In] Integrate[Sech[x]^3/(I + Sinh[x]),x]
```

```
[Out] ((-1/8*I)*Sech[x]^2*(2 + (3*I)*ArcTan[Sinh[x]] + 3*(I + ArcTan[Sinh[x]])*Si
nh[x] + (3 + (3*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 3*ArcTan[Sinh[x]]*Sinh[x]^3
))/(I + Sinh[x])
```

Maple [A] (verified)

Time = 177.48 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{i\mathrm{e}^{x}\left(6i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}-6i\mathrm{e}^{x}+2\,\mathrm{e}^{2x}+3\right)}{4\left(\mathrm{e}^{x}+i\right)^{4}\left(\mathrm{e}^{x}-i\right)^{2}}-\frac{3\ln\left(\mathrm{e}^{x}-i\right)}{8}+\frac{3\ln\left(\mathrm{e}^{x}+i\right)}{8}$
default	$-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{i}{\tanh\left(\frac{x}{2}\right)+i}-\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8}+\frac{i}{-4i+4\tanh\left(\frac{x}{2}\right)}-\frac{1}{4(-i+\tanh$

```
[In] int(sech(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*exp(x)*(6*I*exp(x)^3+3*exp(x)^4-6*I*exp(x)+2*exp(x)^2+3)/(exp(x)+I)^
4/(exp(x)-I)^2-3/8*ln(exp(x)-I)+3/8*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(30) = 60.

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.75

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \frac{3\left(e^{(6x)}+2i\,e^{(5x)}+e^{(4x)}+4i\,e^{(3x)}-e^{(2x)}+2i\,e^{x}-1\right)\log\left(e^{x}+i\right)-3\left(e^{(6x)}+2i\,e^{(5x)}+e^{(4x)}+4i\,e^{(3x)}\cdots\right.}{8\left(e^{(6x)}+2i\,e^{(5x)}+e^{(4x)}+4i\,e^{(3x)}-e^{(2\cdots}\right.}$$

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*x) + 2*I*e^x -
 1)*log(e^x + I) - 3*(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*
x) + 2*I*e^x - 1)*log(e^x - I) - 6*I*e^(5*x) + 12*e^(4*x) - 4*I*e^(3*x) - 1
2*e^(2*x) - 6*I*e^x)/(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*
x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{\sinh(x)+i}\,dx$$

```
[In] integrate(sech(x)**3/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**3/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(30) = 60.

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \frac{8\left(3i\,e^{(-x)}-6\,e^{(-2\,x)}+2i\,e^{(-3\,x)}+6\,e^{(-4\,x)}+3i\,e^{(-5\,x)}\right)}{-64i\,e^{(-x)}-32\,e^{(-2\,x)}-128i\,e^{(-3\,x)}+32\,e^{(-4\,x)}-64i\,e^{(-5\,x)}+32\,e^{(-6\,x)}-32} - \frac{3}{8}\,\log\left(e^{(-x)}+i\right)+\frac{3}{8}\,\log\left(e^{(-x)}-i\right)$$

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 8*(3*I*e^(-x) - 6*e^(-2*x) + 2*I*e^(-3*x) + 6*e^(-4*x) + 3*I*e^(-5*x))/(-64
*I*e^(-x) - 32*e^(-2*x) - 128*I*e^(-3*x) + 32*e^(-4*x) - 64*I*e^(-5*x) + 32
*e^(-6*x) - 32) - 3/8*log(e^(-x) + I) + 3/8*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\operatorname{sech}^3(x)}{i+\sinh(x)}\,dx = \frac{3\,e^{(-x)}-3\,e^{x}+10i}{16\left(e^{(-x)}-e^{x}+2i\right)} - \frac{9\left(e^{(-x)}-e^{x}\right)^2-52i\,e^{(-x)}+52i\,e^{x}-84}{32\left(e^{(-x)}-e^{x}-2i\right)^2} + \frac{3}{16}\,\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{3}{16}\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

```
[In] integrate(sech(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*e^(-x) - 3*e^x + 10*I)/(e^(-x) - e^x + 2*I) - 1/32*(9*(e^(-x) - e^x
)^2 - 52*I*e^(-x) + 52*I*e^x - 84)/(e^(-x) - e^x - 2*I)^2 + 3/16*log(-e^(-x
) + e^x + 2*I) - 3/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.21

$$\int \frac{\mathrm{sech}^3(x)}{i+\sinh(x)}\,dx = \frac{3\,\ln\left(-\frac{3}{4}+\frac{\mathrm{e}^x\,3\mathrm{i}}{4}\right)}{8}-\frac{3\,\ln\left(\frac{3}{4}+\frac{\mathrm{e}^x\,3\mathrm{i}}{4}\right)}{8}-\frac{1}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}\right)}-\frac{1}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^x-\mathrm{i}\right)}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^x+1\mathrm{i}\right)}-\frac{1\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i}}$$

```
[In] int(1/(cosh(x)^3*(sinh(x) + 1i)),x)
```

```
[Out] (3*log((exp(x)*3i)/4 - 3/4))/8 - (3*log((exp(x)*3i)/4 + 3/4))/8 - 1/(2*(exp
(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - 1/(4*(exp(x)*2i - exp(
2*x) + 1)) - 1i/(4*(exp(x) - 1i)) - 1i/(2*(exp(x) + 1i)) - 1i/(exp(2*x)*3i
+ exp(3*x) - 3*exp(x) - 1i)
```

3.169 $\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{i\operatorname{sech}^3(x)}{5(i+\sinh(x))} - \frac{4}{5}i\tanh(x) + \frac{4}{15}i\tanh^3(x)$$

```
[Out] -1/5*I*sech(x)^3/(I+sinh(x))-4/5*I*tanh(x)+4/15*I*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2751, 3852}

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = \frac{4}{15}i\tanh^3(x) - \frac{4}{5}i\tanh(x) - \frac{i\operatorname{sech}^3(x)}{5(\sinh(x)+i)}$$

```
[In] Int[Sech[x]^4/(I + Sinh[x]),x]
```

```
[Out] ((-1/5*I)*Sech[x]^3)/(I + Sinh[x]) - ((4*I)/5)*Tanh[x] + ((4*I)/15)*Tanh[x]
^3
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{sech}^3(x)}{5(i+\sinh(x))}-\frac{4}{5}i\int \text{sech}^4(x)\,dx\\ &= -\frac{i\text{sech}^3(x)}{5(i+\sinh(x))}+\frac{4}{5}\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\tanh(x)\right)\\ &= -\frac{i\text{sech}^3(x)}{5(i+\sinh(x))}-\frac{4}{5}i\tanh(x)+\frac{4}{15}i\tanh^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\text{sech}^4(x)}{i+\sinh(x)}\,dx=-\frac{1}{15}i\left(\frac{3\text{sech}^3(x)}{i+\sinh(x)}+12\text{sech}^2(x)\tanh(x)+8\tanh^3(x)\right)$$

```
[In] Integrate[Sech[x]^4/(I + Sinh[x]),x]
[Out] (-1/15*I)*((3*Sech[x]^3)/(I + Sinh[x]) + 12*Sech[x]^2*Tanh[x] + 8*Tanh[x]^3
)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(27) = 54.

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.51

$$\frac{i}{6\left(-i+\tanh\left(\frac{x}{2}\right)\right)^3}-\frac{5i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)}+\frac{1}{4\left(-i+\tanh\left(\frac{x}{2}\right)\right)^2}-\frac{2i}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^5}+\frac{5i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^3}-\frac{}{8\left(\text{ta}\right.}$$

```
[In] int(sech(x)^4/(I+sinh(x)),x)
[Out] 1/6*I/(-I+tanh(1/2*x))^3-5/8*I/(-I+tanh(1/2*x))+1/4/(-I+tanh(1/2*x))^2-2/5*
I/(tanh(1/2*x)+I)^5+5/3*I/(tanh(1/2*x)+I)^3-11/8*I/(tanh(1/2*x)+I)+1/(tanh(
1/2*x)+I)^4-3/2/(tanh(1/2*x)+I)^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = -\frac{16\left(6\,e^{(3\,x)}+2i\,e^{(2\,x)}+2\,e^{x}+i\right)}{15\left(e^{(8\,x)}+2i\,e^{(7\,x)}+2\,e^{(6\,x)}+6i\,e^{(5\,x)}+6i\,e^{(3\,x)}-2\,e^{(2\,x)}+2i\,e^{x}-1\right)}$$

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="fricas")
[Out] -16/15*(6*e^(3*x) + 2*I*e^(2*x) + 2*e^x + I)/(e^(8*x) + 2*I*e^(7*x) + 2*e^(
6*x) + 6*I*e^(5*x) + 6*I*e^(3*x) - 2*e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{\sinh(x)+i}\,dx$$

```
[In] integrate(sech(x)**4/(I+sinh(x)),x)
[Out] Integral(sech(x)**4/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(23) = 46$.

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.54

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx &= -\frac{32\,e^{(-x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad+\frac{32i\,e^{(-2\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad-\frac{96\,e^{(-3\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad+\frac{16i}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\end{aligned}$$

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] -32*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 32*I*e^(-2*x)/(-30*I*e^(-x)
 - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x
) + 15*e^(-8*x) - 15) - 96*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 16
*I/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x
) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = \frac{9\,e^{(2\,x)}-24i\,e^{x}-11}{24\,(e^{x}-i)^{3}}-\frac{45\,e^{(4\,x)}+240i\,e^{(3\,x)}-490\,e^{(2\,x)}-320i\,e^{x}+73}{120\,(e^{x}+i)^{5}}$$

```
[In] integrate(sech(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(9*e^(2*x) - 24*I*e^x - 11)/(e^x - I)^3 - 1/120*(45*e^(4*x) + 240*I*e^
(3*x) - 490*e^(2*x) - 320*I*e^x + 73)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 231, normalized size of antiderivative = 6.24

$$\begin{aligned}\int \frac{\operatorname{sech}^4(x)}{i+\sinh(x)}\,dx = &-\frac{1}{6\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{\frac{3\,\mathrm{e}^{x}}{40}+\frac{1}{8}\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}\\&-\frac{\frac{3\,\mathrm{e}^{2\,x}}{40}-\frac{5}{24}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}+\frac{1\mathrm{i}}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{3}{8\,\left(\mathrm{e}^{x}-\mathrm{i}\right)}\\&-\frac{3}{40\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}-\frac{\frac{\mathrm{e}^{2\,x}\,3\mathrm{i}}{8}+\frac{3\,\mathrm{e}^{3\,x}}{40}-\frac{5\,\mathrm{e}^{x}}{8}-\frac{1}{8}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}}\\&-\frac{\frac{3\,\mathrm{e}^{4\,x}}{40}-\frac{5\,\mathrm{e}^{2\,x}}{4}+\frac{3}{40}+\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}}{\mathrm{e}^{5\,x}-10\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}\,5\mathrm{i}-\mathrm{e}^{2\,x}\,10\mathrm{i}+5\,\mathrm{e}^{x}+1\mathrm{i}}\end{aligned}$$

```
[In] int(1/(cosh(x)^4*(sinh(x) + 1i)),x)
```

```
[Out] 1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((3*exp(x))/40 + 1i/8)/(exp(2*x) + exp(
x)*2i - 1) - ((3*exp(2*x))/40 + (exp(x)*1i)/4 - 5/24)/(exp(2*x)*3i + exp(3*
x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 3/(8
*(exp(x) - 1i)) - 3/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (3*exp(3*x))/40
 - (5*exp(x))/8 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i +
1) - ((exp(3*x)*1i)/2 - (5*exp(2*x))/4 + (3*exp(4*x))/40 - (exp(x)*1i)/2 +
3/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i)
```

3.170 $\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx = -\frac{5}{16}i\arctan(\sinh(x)) - \frac{1}{32(i-\sinh(x))^2} + \frac{i}{8(i-\sinh(x))} + \frac{i}{24(i+\sinh(x))^3} + \frac{3}{32(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))}$$

```
[Out] -5/16*I*arctan(sinh(x))-1/32/(I-sinh(x))^2+1/8*I/(I-sinh(x))+1/24*I/(I+sinh
(x))^3+3/32/(I+sinh(x))^2-3/16*I/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 46, 209}

$$\int \frac{\text{sech}^5(x)}{i+\sinh(x)}\,dx = -\frac{5}{16}i\arctan(\sinh(x)) + \frac{i}{8(-\sinh(x)+i)} - \frac{3i}{16(\sinh(x)+i)} - \frac{1}{32(-\sinh(x)+i)^2} + \frac{3}{32(\sinh(x)+i)^2} + \frac{i}{24(\sinh(x)+i)^3}$$

```
[In] Int[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-5*I)/16)*ArcTan[Sinh[x]] - 1/(32*(I - Sinh[x])^2) + (I/8)/(I - Sinh[x])
+ (I/24)/(I + Sinh[x])^3 + 3/(32*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x]
)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{(i-x)^3(i+x)^4}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{16(-i+x)^3}-\frac{i}{8(-i+x)^2}+\frac{i}{8(i+x)^4}+\frac{3}{16(i+x)^3}-\frac{3i}{16(i+x)^2}\right.\right.\\
&\qquad\left.\left.+\frac{5i}{16\left(1+x^2\right)}\right)dx, x, \sinh(x)\right)\\
&= -\frac{1}{32(i-\sinh(x))^2}+\frac{i}{8(i-\sinh(x))}+\frac{i}{24(i+\sinh(x))^3}+\frac{3}{32(i+\sinh(x))^2}\\
&\quad -\frac{3i}{16(i+\sinh(x))}-\frac{5}{16}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{5}{16}i\arctan(\sinh(x))-\frac{1}{32(i-\sinh(x))^2}+\frac{i}{8(i-\sinh(x))}\\
&\quad +\frac{i}{24(i+\sinh(x))^3}+\frac{3}{32(i+\sinh(x))^2}-\frac{3i}{16(i+\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = -\frac{i\operatorname{sech}^4(x)\left(8+15i\arctan(\sinh(x))+5(5i+3\arctan(\sinh(x)))\sinh(x)+5(5+6i\arctan(\sinh(x)))\sinh^2\right.}{48(i-}$$

```
[In] Integrate[Sech[x]^5/(I + Sinh[x]),x]
```

```
[Out] ((-1/48*I)*Sech[x]^4*(8 + (15*I)*ArcTan[Sinh[x]] + 5*(5*I + 3*ArcTan[Sinh[x
]])*Sinh[x] + 5*(5 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^2 + 15*(I + 2*ArcTan[Si
nh[x]])*Sinh[x]^3 + 15*(1 + I*ArcTan[Sinh[x]])*Sinh[x]^4 + 15*ArcTan[Sinh[x
]]*Sinh[x]^5))/(I + Sinh[x])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 136 vs. 2(59) = 118.

Time = 2.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.71

$$\frac{3i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)}-\frac{i}{4\left(-i+\tanh\left(\frac{x}{2}\right)\right)^3}+\frac{1}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)^4}-\frac{1}{2\left(-i+\tanh\left(\frac{x}{2}\right)\right)^2}-\frac{5\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{16}+$$

```
[In] int(sech(x)^5/(I+sinh(x)),x)
```

```
[Out] 3/8*I/(-I+tanh(1/2*x))-1/4*I/(-I+tanh(1/2*x))^3+1/8/(-I+tanh(1/2*x))^4-1/2/
(-I+tanh(1/2*x))^2-5/16*ln(-I+tanh(1/2*x))+I/(tanh(1/2*x)+I)^5+I/(tanh(1/2*
x)+I)-25/12*I/(tanh(1/2*x)+I)^3+1/3/(tanh(1/2*x)+I)^6-15/8/(tanh(1/2*x)+I)^
4+15/8/(tanh(1/2*x)+I)^2+5/16*ln(tanh(1/2*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(46) = 92.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.06

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \frac{15\left(e^{(10x)}+2i\,e^{(9x)}+3\,e^{(8x)}+8i\,e^{(7x)}+2\,e^{(6x)}+12i\,e^{(5x)}-2\,e^{(4x)}+8i\,e^{(3x)}-3\,e^{(2x)}+2i\,e^{x}-1\right)\log\left(e^{x}\right.}{}$$

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/48*(15*(e^(10*x) + 2*I*e^(9*x) + 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 12
*I*e^(5*x) - 2*e^(4*x) + 8*I*e^(3*x) - 3*e^(2*x) + 2*I*e^x - 1)*log(e^x + I
) - 15*(e^(10*x) + 2*I*e^(9*x) + 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 12*I
*e^(5*x) - 2*e^(4*x) + 8*I*e^(3*x) - 3*e^(2*x) + 2*I*e^x - 1)*log(e^x - I)
- 30*I*e^(9*x) + 60*e^(8*x) - 80*I*e^(7*x) + 220*e^(6*x) - 36*I*e^(5*x) - 2
20*e^(4*x) - 80*I*e^(3*x) - 60*e^(2*x) - 30*I*e^x)/(e^(10*x) + 2*I*e^(9*x)
+ 3*e^(8*x) + 8*I*e^(7*x) + 2*e^(6*x) + 12*I*e^(5*x) - 2*e^(4*x) + 8*I*e^(3
*x) - 3*e^(2*x) + 2*I*e^x - 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \int \frac{\operatorname{sech}^5{(x)}}{\sinh{(x)}+i}\,dx$$

```
[In] integrate(sech(x)**5/(I+sinh(x)),x)
```

```
[Out] Integral(sech(x)**5/(sinh(x) + I), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(46) = 92$.

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \frac{32\left(15i\,e^{(-x)} - 30\,e^{(-2\,x)} + 40i\,e^{(-3\,x)} - 110\,e^{(-4\,x)} + 18i\,e^{(-5\,x)} + 110\,e^{(-6\,x)} + 40i\,e^{(-7\,x)}\right.}{-1536i\,e^{(-x)} - 2304\,e^{(-2\,x)} - 6144i\,e^{(-3\,x)} - 1536\,e^{(-4\,x)} - 9216i\,e^{(-5\,x)} + 1536\,e^{(-6\,x)} - 6144i\,e^{(-7\,x)} + 230} - \frac{5}{16}\,\log\left(e^{(-x)} + i\right) + \frac{5}{16}\,\log\left(e^{(-x)} - i\right)$$

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 32*(15*I*e^(-x) - 30*e^(-2*x) + 40*I*e^(-3*x) - 110*e^(-4*x) + 18*I*e^(-5*x
) + 110*e^(-6*x) + 40*I*e^(-7*x) + 30*e^(-8*x) + 15*I*e^(-9*x))/(-1536*I*e^
(-x) - 2304*e^(-2*x) - 6144*I*e^(-3*x) - 1536*e^(-4*x) - 9216*I*e^(-5*x) +
1536*e^(-6*x) - 6144*I*e^(-7*x) + 2304*e^(-8*x) - 1536*I*e^(-9*x) + 768*e^(
-10*x) - 768) - 5/16*log(e^(-x) + I) + 5/16*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \frac{15\left(e^{(-x)}-e^x\right)^2+76i\,e^{(-x)}-76i\,e^x-100}{64\left(e^{(-x)}-e^x+2i\right)^2} - \frac{55\left(e^{(-x)}-e^x\right)^3-402i\left(e^{(-x)}-e^x\right)^2-1020\,e^{(-x)}+1020\,e^x+936i}{192\left(e^{(-x)}-e^x-2i\right)^3} + \frac{5}{32}\log\left(-e^{(-x)}+e^x+2i\right) - \frac{5}{32}\log\left(-e^{(-x)}+e^x-2i\right)$$

```
[In] integrate(sech(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/64*(15*(e^(-x) - e^x)^2 + 76*I*e^(-x) - 76*I*e^x - 100)/(e^(-x) - e^x + 2
*I)^2 - 1/192*(55*(e^(-x) - e^x)^3 - 402*I*(e^(-x) - e^x)^2 - 1020*e^(-x) +
 1020*e^x + 936*I)/(e^(-x) - e^x - 2*I)^3 + 5/32*log(-e^(-x) + e^x + 2*I) -
 5/32*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.11

$$\int \frac{\operatorname{sech}^5(x)}{i+\sinh(x)}\,dx = \frac{5\ln\left(-\frac{5}{8}+\frac{e^x\,5i}{8}\right)}{16} - \frac{5\ln\left(\frac{5}{8}+\frac{e^x\,5i}{8}\right)}{16} - \frac{1i}{e^{5x}-10\,e^{3x}+e^{4x}\,5i-e^{2x}\,10i+5\,e^x+1i} + \frac{1i}{4\left(e^{2x}\,3i-e^{3x}+3\,e^x-i\right)} + \frac{1}{8\left(e^{4x}-6\,e^{2x}+1-e^{3x}\,4i+e^x\,4i\right)} + \frac{5}{8\left(e^{4x}-6\,e^{2x}+1+e^{3x}\,4i-e^x\,4i\right)} - \frac{1}{8\left(1-e^{2x}+e^x\,2i\right)} - \frac{1i}{4\left(e^x-i\right)} - \frac{3i}{8\left(e^x+1i\right)} - \frac{1}{3\left(15\,e^{2x}-15\,e^{4x}+e^{6x}-1-e^{3x}\,20i+e^{5x}\,6i+e^x\,6i\right)} - \frac{5i}{12\left(e^{2x}\,3i+e^{3x}-3\,e^x-i\right)}$$

```
[In] int(1/(cosh(x)^5*(sinh(x) + 1i)),x)
```

```
[Out] (5*log((exp(x)*5i)/8 - 5/8))/16 - (5*log((exp(x)*5i)/8 + 5/8))/16 - 1i/(exp
(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) + 1i/(4*(
exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) + 1/(8*(exp(4*x) - exp(3*x)*4i - 6
*exp(2*x) + exp(x)*4i + 1)) + 5/(8*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - e
xp(x)*4i + 1)) - 1/(8*(exp(x)*2i - exp(2*x) + 1)) - 1i/(4*(exp(x) - 1i)) -
```

```
3i/(8*(exp(x) + 1i)) - 1/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp
(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 5i/(12*(exp(2*x)*3i + exp(3*x) - 3*
exp(x) - 1i))
```

3.171 $\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} dx = -\frac{5x}{8} - \frac{5}{12} i \cosh^3(x) - \frac{5}{8} \cosh(x) \sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))}$$

```
[Out] -5/8*x-5/12*I*cosh(x)^3-5/8*cosh(x)*sinh(x)+1/4*cosh(x)^5/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2758, 2761, 2715, 8}

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2} dx = -\frac{5x}{8} - \frac{5}{12} i \cosh^3(x) + \frac{\cosh^5(x)}{4(\sinh(x)+i)} - \frac{5}{8} \sinh(x) \cosh(x)$$

```
[In] Int[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-5*x)/8 - ((5*I)/12)*Cosh[x]^3 - (5*Cosh[x]*Sinh[x])/8 + Cosh[x]^5/(4*(I +
 Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5}{4}i\int \frac{\cosh^4(x)}{i+\sinh(x)}\,dx \\
&= -\frac{5}{12}i\cosh^3(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5}{4}\int \cosh^2(x)\,dx \\
&= -\frac{5}{12}i\cosh^3(x) - \frac{5}{8}\cosh(x)\sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))} - \frac{5\int 1\,dx}{8} \\
&= -\frac{5x}{8} - \frac{5}{12}i\cosh^3(x) - \frac{5}{8}\cosh(x)\sinh(x) + \frac{\cosh^5(x)}{4(i+\sinh(x))}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 121 vs. 2(40) = 80.

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.02

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{i\cosh^7(x)\left(16+\frac{30\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}-25i\sinh(x)+7\sinh^2(x)-10i\sinh^3(x)+6\sinh^4(x)\right)}{24\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^8\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^6}$$

```
[In] Integrate[Cosh[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/24*I)*Cosh[x]^7*(16 + (30*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 -
 I*Sinh[x]])/Sqrt[1 + I*Sinh[x]] - (25*I)*Sinh[x] + 7*Sinh[x]^2 - (10*I)*Si
nh[x]^3 + 6*Sinh[x]^4))/((Cosh[x/2] - I*Sinh[x/2])^8*(Cosh[x/2] + I*Sinh[x/
2])^6)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(30) = 60$.

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.80

$$\frac{\frac{1}{2}+\frac{2i}{3}}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{-\frac{1}{8}+i}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{-\frac{3}{8}+i}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}+\frac{\frac{1}{2}-\frac{2i}{3}}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}$$

```
[In] int(cosh(x)^6/(I+sinh(x))^2,x)
```

```
[Out] (1/2+2/3*I)/(tanh(1/2*x)-1)^3+(-1/8+I)/(tanh(1/2*x)-1)^2+(-3/8+I)/(tanh(1/2
*x)-1)+1/4/(tanh(1/2*x)-1)^4+5/8*ln(tanh(1/2*x)-1)+(1/2-2/3*I)/(tanh(1/2*x)
+1)^3+(1/8+I)/(tanh(1/2*x)+1)^2-(3/8+I)/(tanh(1/2*x)+1)-1/4/(tanh(1/2*x)+1)
^4-5/8*ln(tanh(1/2*x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{192}\left(120\,xe^{(4\,x)}-3\,e^{(8\,x)}+16i\,e^{(7\,x)}+24\,e^{(6\,x)}+48i\,e^{(5\,x)}+48i\,e^{(3\,x)}-24\,e^{(2\,x)}+16i\,e^{x}+3\right)e^{(-4\,x)}$$

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/192*(120*x*e^(4*x) - 3*e^(8*x) + 16*I*e^(7*x) + 24*e^(6*x) + 48*I*e^(5*x
) + 48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{5x}{8} + \frac{e^{4x}}{64} - \frac{ie^{3x}}{12} - \frac{e^{2x}}{8} - \frac{ie^{x}}{4} - \frac{ie^{-x}}{4} + \frac{e^{-2x}}{8} - \frac{ie^{-3x}}{12} - \frac{e^{-4x}}{64}$$

```
[In] integrate(cosh(x)**6/(I+sinh(x))**2,x)
```

```
[Out] -5*x/8 + exp(4*x)/64 - I*exp(3*x)/12 - exp(2*x)/8 - I*exp(x)/4 - I*exp(-x)/
4 + exp(-2*x)/8 - I*exp(-3*x)/12 - exp(-4*x)/64
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{192}\left(16i\,e^{(-x)} + 24\,e^{(-2\,x)} + 48i\,e^{(-3\,x)} - 3\right)e^{(4\,x)} - \frac{5}{8}\,x - \frac{1}{4}i\,e^{(-x)} + \frac{1}{8}\,e^{(-2\,x)} - \frac{1}{12}i\,e^{(-3\,x)} - \frac{1}{64}\,e^{(-4\,x)}$$

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/192*(16*I*e^(-x) + 24*e^(-2*x) + 48*I*e^(-3*x) - 3)*e^(4*x) - 5/8*x - 1/
4*I*e^(-x) + 1/8*e^(-2*x) - 1/12*I*e^(-3*x) - 1/64*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{192}\left(48i\,e^{(3\,x)} - 24\,e^{(2\,x)} + 16i\,e^{x} + 3\right)e^{(-4\,x)} - \frac{5}{8}\,x + \frac{1}{64}\,e^{(4\,x)} - \frac{1}{12}i\,e^{(3\,x)} - \frac{1}{8}\,e^{(2\,x)} - \frac{1}{4}i\,e^{x}$$

```
[In] integrate(cosh(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/192*(48*I*e^(3*x) - 24*e^(2*x) + 16*I*e^x + 3)*e^(-4*x) - 5/8*x + 1/64*e
^(4*x) - 1/12*I*e^(3*x) - 1/8*e^(2*x) - 1/4*I*e^x
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.35

$$\int \frac{\cosh^6(x)}{(i+\sinh(x))^2}\,dx = \frac{\mathrm{e}^{-2\,x}}{8} - \frac{\mathrm{e}^{-x}\,1\mathrm{i}}{4} - \frac{5\,x}{8} - \frac{\mathrm{e}^{2\,x}}{8} - \frac{\mathrm{e}^{-3\,x}\,1\mathrm{i}}{12} - \frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{12} - \frac{\mathrm{e}^{-4\,x}}{64} + \frac{\mathrm{e}^{4\,x}}{64} - \frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}$$

```
[In] int(cosh(x)^6/(sinh(x) + 1i)^2,x)
```

```
[Out] exp(-2*x)/8 - (exp(-x)*1i)/4 - (5*x)/8 - exp(2*x)/8 - (exp(-3*x)*1i)/12 - (
exp(3*x)*1i)/12 - exp(-4*x)/64 + exp(4*x)/64 - (exp(x)*1i)/4
```

3.172 $\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx = -\frac{1}{3}(i-\sinh(x))^3$$

```
[Out] -1/3*(I-sinh(x))^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2746, 32}

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2} dx = -\frac{1}{3}(-\sinh(x)+i)^3$$

```
[In] Int[Cosh[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -1/3*(I - Sinh[x])^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int (i-x)^2\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{3}(i-\sinh(x))^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{6}(-7+\cosh(2x)-6i\sinh(x))\sinh(x)$$

```
[In] Integrate[Cosh[x]^5/(I + Sinh[x])^2,x]
[Out] ((-7 + Cosh[2*x] - (6*I)*Sinh[x])*Sinh[x])/6
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$-\frac{(i-\sinh(x))^3}{3}$$

```
[In] int(cosh(x)^5/(I+sinh(x))^2,x)
[Out] -1/3*(I-sinh(x))^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(8) = 16.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\left(e^{(6x)}-6i\,e^{(5x)}-15\,e^{(4x)}+15\,e^{(2x)}-6i\,e^{x}-1\right)e^{(-3x)}$$

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
[Out] 1/24*(e^(6*x) - 6*I*e^(5*x) - 15*e^(4*x) + 15*e^(2*x) - 6*I*e^x - 1)*e^(-3*
x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(8) = 16.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.14

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{3x}}{24} - \frac{ie^{2x}}{4} - \frac{5e^x}{8} + \frac{5e^{-x}}{8} - \frac{ie^{-2x}}{4} - \frac{e^{-3x}}{24}$$

```
[In] integrate(cosh(x)**5/(I+sinh(x))**2,x)
[Out] exp(3*x)/24 - I*exp(2*x)/4 - 5*exp(x)/8 + 5*exp(-x)/8 - I*exp(-2*x)/4 - exp
(-3*x)/24
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(8) = 16.

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{24}\left(6i\,e^{(-x)} + 15\,e^{(-2\,x)} - 1\right)e^{(3\,x)} + \frac{5}{8}\,e^{(-x)} - \frac{1}{4}i\,e^{(-2\,x)} - \frac{1}{24}\,e^{(-3\,x)}$$

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -1/24*(6*I*e^(-x) + 15*e^(-2*x) - 1)*e^(3*x) + 5/8*e^(-x) - 1/4*I*e^(-2*x)
- 1/24*e^(-3*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{24}\left(15\,e^{(2\,x)} - 6i\,e^x - 1\right)e^{(-3\,x)} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{4}i\,e^{(2\,x)} - \frac{5}{8}\,e^x$$

```
[In] integrate(cosh(x)^5/(I+sinh(x))^2,x, algorithm="giac")
[Out] 1/24*(15*e^(2*x) - 6*I*e^x - 1)*e^(-3*x) + 1/24*e^(3*x) - 1/4*I*e^(2*x) - 5
/8*e^x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.64

$$\int \frac{\cosh^5(x)}{(i+\sinh(x))^2}\,dx = \frac{5\,\mathrm{e}^{-x}}{8} - \frac{\mathrm{e}^{-3\,x}}{24} + \frac{\mathrm{e}^{3\,x}}{24} - \frac{5\,\mathrm{e}^{x}}{8} - \frac{\mathrm{e}^{-2\,x}\,1\mathrm{i}}{4} - \frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{4}$$

```
[In] int(cosh(x)^5/(sinh(x) + 1i)^2,x)
```

```
[Out] (5*exp(-x))/8 - (exp(-2*x)*1i)/4 - (exp(2*x)*1i)/4 - exp(-3*x)/24 + exp(3*x
)/24 - (5*exp(x))/8
```

3.173 $\int \frac{\cosh^4(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2} dx = -\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))}$$

```
[Out] -3/2*x-3/2*I*cosh(x)+1/2*cosh(x)^3/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2758, 2761, 8}

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2} dx = -\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(\sinh(x)+i)}$$

```
[In] Int[Cosh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-3*x)/2 - ((3*I)/2)*Cosh[x] + Cosh[x]^3/(2*(I + Sinh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2758

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(a*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f
, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] ||
```

```
EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && Int
egersQ[2*m, 2*p]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3}{2}i\int \frac{\cosh^2(x)}{i+\sinh(x)}\,dx \\
&= -\frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))} - \frac{3\int 1\,dx}{2} \\
&= -\frac{3x}{2} - \frac{3}{2}i\cosh(x) + \frac{\cosh^3(x)}{2(i+\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$
\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -3i\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{\cosh^2(x)}\operatorname{sech}(x) + \frac{1}{2}\cosh(x)(-4i+\sinh(x))
$$

```
[In] Integrate[Cosh[x]^4/(I + Sinh[x])^2,x]
[Out] (-3*I)*ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[Cosh[x]^2]*Sech[x] + (Cosh[
x]*(-4*I + Sinh[x]))/2
```

Maple [A] (verified)

Time = 89.74 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{3x}{2}+\frac{e^{2x}}{8}-ie^{x}-ie^{-x}-\frac{e^{-2x}}{8}$	29
default	$\frac{\frac{1}{2}+2i}{\tanh(\frac{x}{2})-1}+\frac{1}{2(\tanh(\frac{x}{2})-1)^2}+\frac{3\ln(\tanh(\frac{x}{2})-1)}{2}+\frac{\frac{1}{2}-2i}{\tanh(\frac{x}{2})+1}-\frac{1}{2(\tanh(\frac{x}{2})+1)^2}-\frac{3\ln(\tanh(\frac{x}{2})+1)}{2}$	64

```
[In] int(cosh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -3/2*x+1/8*exp(x)^2-I*exp(x)-I/exp(x)-1/8/exp(x)^2
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{8}\left(12\,xe^{(2\,x)} - e^{(4\,x)} + 8i\,e^{(3\,x)} + 8i\,e^{x} + 1\right)e^{(-2\,x)}$$

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
[Out] -1/8*(12*x*e^(2*x) - e^(4*x) + 8*I*e^(3*x) + 8*I*e^x + 1)*e^(-2*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{3x}{2} + \frac{e^{2x}}{8} - ie^{x} - ie^{-x} - \frac{e^{-2x}}{8}$$

```
[In] integrate(cosh(x)**4/(I+sinh(x))**2,x)
[Out] -3*x/2 + exp(2*x)/8 - I*exp(x) - I*exp(-x) - exp(-2*x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{8}\left(8i\,e^{(-x)} - 1\right)e^{(2\,x)} - \frac{3}{2}\,x - i\,e^{(-x)} - \frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -1/8*(8*I*e^(-x) - 1)*e^(2*x) - 3/2*x - I*e^(-x) - 1/8*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{8}\,(8i\,e^x+1)e^{(-2\,x)} - \frac{3}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - i\,e^x$$

```
[In] integrate(cosh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
[Out] -1/8*(8*I*e^x + 1)*e^(-2*x) - 3/2*x + 1/8*e^(2*x) - I*e^x
```

Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{2\,x}}{8} - e^{-x}\,1i - \frac{e^{-2\,x}}{8} - \frac{3\,x}{2} - e^x\,1i$$

```
[In] int(cosh(x)^4/(sinh(x) + 1i)^2,x)
[Out] exp(2*x)/8 - exp(-x)*1i - exp(-2*x)/8 - (3*x)/2 - exp(x)*1i
```

3.174 $\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\log(i+\sinh(x))+\sinh(x)$$

```
[Out] -2*I*ln(I+sinh(x))+sinh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 45}

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = \sinh(x)-2i\log(\sinh(x)+i)$$

```
[In] Int[Cosh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{i-x}{i+x}\,dx, x, \sinh(x)\right)\\ &= -\text{Subst}\left(\int \left(-1+\frac{2i}{i+x}\right)\,dx, x, \sinh(x)\right)\\ &= -2i\log(i+\sinh(x))+\sinh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\log(i+\sinh(x))+\sinh(x)$$

```
[In] Integrate[Cosh[x]^3/(I + Sinh[x])^2,x]
[Out] (-2*I)*Log[I + Sinh[x]] + Sinh[x]
```

Maple [A] (verified)

Time = 25.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

method	result	size
risch	$2ix+\frac{e^{x}}{2}-\frac{e^{-x}}{2}-4i\ln\left(e^{x}+i\right)$	25
default	$-4i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+2i\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+2i\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}$	53

```
[In] int(cosh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] 2*I*x+1/2*exp(x)-1/2*exp(-x)-4*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{2}\left(4i\,xe^{x} - 8i\,e^{x}\log\left(e^{x}+i\right) + e^{(2\,x)} - 1\right)e^{(-x)}$$

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
[Out] 1/2*(4*I*x*e^x - 8*I*e^x*log(e^x + I) + e^(2*x) - 1)*e^(-x)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = 2ix + \frac{e^{x}}{2} - 4i\log\left(e^{x}+i\right) - \frac{e^{-x}}{2}$$

```
[In] integrate(cosh(x)**3/(I+sinh(x))**2,x)
[Out] 2*I*x + exp(x)/2 - 4*I*log(exp(x) + I) - exp(-x)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = -2i\,x - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4i\,\log\left(e^{(-x)} - i\right)$$

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = 2i\,x - \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4i\,\log\left(e^{x}+i\right)$$

```
[In] integrate(cosh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
[Out] 2*I*x - 1/2*e^(-x) + 1/2*e^x - 4*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{x}}{2} - \frac{e^{-x}}{2} + x\,2\mathrm{i} - \ln\left(e^{x}+1\mathrm{i}\right)\,4\mathrm{i}$$

```
[In] int(cosh(x)^3/(sinh(x) + 1i)^2,x)
[Out] x*2i - exp(-x)/2 - log(exp(x) + 1i)*4i + exp(x)/2
```

3.175 $\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x - \frac{2\cosh(x)}{i+\sinh(x)}$$

```
[Out] x-2*cosh(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2759, 8}

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x - \frac{2\cosh(x)}{\sinh(x)+i}$$

```
[In] Int[Cosh[x]^2/(I + Sinh[x])^2,x]
[Out] x - (2*Cosh[x])/(I + Sinh[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2759

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Dist[g^2*((p - 1)/(b^2*(2*m + p + 1
))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
```

```
NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{2\cosh(x)}{i+\sinh(x)} + \int 1\,dx \\ &= x - \frac{2\cosh(x)}{i+\sinh(x)} \end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 69 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.93

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2\cosh^3(x)\left(-1 - \frac{\arcsin\left(\frac{\sqrt{1-i\sinh(x)}}{\sqrt{2}}\right)\sqrt{1-i\sinh(x)}}{\sqrt{1+i\sinh(x)}}\right)}{(-i+\sinh(x))(i+\sinh(x))^2}$$

```
[In] Integrate[Cosh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*Cosh[x]^3*(-1 - (ArcSin[Sqrt[1 - I*Sinh[x]]/Sqrt[2]]*Sqrt[1 - I*Sinh[x]]
)/Sqrt[1 + I*Sinh[x]]))/((-I + Sinh[x])*(I + Sinh[x])^2)
```

Maple [A] (verified)

Time = 13.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
risch	$x+\frac{4i}{e^{x}+i}$	13
default	$-\frac{4}{\tanh\left(\frac{x}{2}\right)+i}-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	29

```
[In] int(cosh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+4*I/(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{xe^x + i\,x + 4i}{e^x + i}$$

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
[Out] (x*e^x + I*x + 4*I)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4i}{e^x + i}$$

```
[In] integrate(cosh(x)**2/(I+sinh(x))**2,x)
[Out] x + 4*I/(exp(x) + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4i}{e^{(-x)} - i}$$

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
[Out] x + 4*I/(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4i}{e^x + i}$$

```
[In] integrate(cosh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
[Out] x + 4*I/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(x)}{(i+\sinh(x))^2}\,dx = x + \frac{4\mathrm{i}}{\mathrm{e}^x + 1\mathrm{i}}$$

```
[In] int(cosh(x)^2/(sinh(x) + 1i)^2,x)
[Out] x + 4i/(exp(x) + 1i)
```

3.176 $\int \frac{\cosh(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2} \, dx = -\frac{1}{i+\sinh(x)}$$

```
[Out] -1/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2746, 32}

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2} \, dx = -\frac{1}{\sinh(x)+i}$$

```
[In] Int[Cosh[x]/(I + Sinh[x])^2,x]
[Out] -(I + Sinh[x])^(-1)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{(i+x)^2}\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{i+\sinh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{i+\sinh(x)}$$

```
[In] Integrate[Cosh[x]/(I + Sinh[x])^2,x]
[Out] -(I + Sinh[x])^(-1)
```

Maple [A] (verified)

Time = 10.89 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{1}{i+\sinh(x)}$	10
default	$-\frac{1}{i+\sinh(x)}$	10
risch	$-\frac{2\,e^{x}}{(e^{x}+i)^{2}}$	12

```
[In] int(cosh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/(I+sinh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,e^x}{e^{(2\,x)}+2i\,e^x-1}$$

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="fricas")
[Out] -2*e^x/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2e^x}{e^{2x}+2ie^x-1}$$

```
[In] integrate(cosh(x)/(I+sinh(x))**2,x)
[Out] -2*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{\sinh(x)+i}$$

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -1/(sinh(x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,e^x}{(e^x+i)^2}$$

```
[In] integrate(cosh(x)/(I+sinh(x))^2,x, algorithm="giac")
[Out] -2*e^x/(e^x + I)^2
```

Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cosh(x)}{(i+\sinh(x))^2}\,dx = -\frac{1\mathrm{i}}{-1+\sinh(x)\;1\mathrm{i}}$$

```
[In] int(cosh(x)/(sinh(x) + 1i)^2,x)
[Out] -1i/(sinh(x)*1i - 1)
```

3.177 $\int \frac{\text{sech}(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{\text{sech}(x)}{(i+\sinh(x))^2} dx = -\frac{1}{4}\arctan(\sinh(x)) - \frac{i}{4(i+\sinh(x))^2} - \frac{1}{4(i+\sinh(x))}$$

```
[Out] -1/4*arctan(sinh(x))-1/4*I/(I+sinh(x))^2-1/4/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2746, 46, 209}

$$\int \frac{\text{sech}(x)}{(i+\sinh(x))^2} dx = -\frac{1}{4}\arctan(\sinh(x)) - \frac{1}{4(\sinh(x)+i)} - \frac{i}{4(\sinh(x)+i)^2}$$

```
[In] Int[Sech[x]/(I + Sinh[x])^2,x]
```

```
[Out] -1/4*ArcTan[Sinh[x]] - (I/4)/(I + Sinh[x])^2 - 1/(4*(I + Sinh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{(i-x)(i+x)^3}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{i}{2(i+x)^3} - \frac{1}{4(i+x)^2} + \frac{1}{4\,(1+x^2)}\right)dx, x, \sinh(x)\right)\\
&= -\frac{i}{4(i+\sinh(x))^2} - \frac{1}{4(i+\sinh(x))} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{4}\arctan(\sinh(x)) - \frac{i}{4(i+\sinh(x))^2} - \frac{1}{4(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{4}\left(-\arctan(\sinh(x)) - \frac{2i+\sinh(x)}{(i+\sinh(x))^2}\right)$$

```
[In] Integrate[Sech[x]/(I + Sinh[x])^2,x]
[Out] (-ArcTan[Sinh[x]] - (2*I + Sinh[x])/(I + Sinh[x])^2)/4
```

Maple [A] (verified)

Time = 24.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{-e^{x}+4ie^{2x}+e^{3x}}{2(e^{x}+i)^{4}}+\frac{i\ln(e^{x}-i)}{4}-\frac{i\ln(e^{x}+i)}{4}$	45
default	$\frac{i\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{4}+\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{4}-\frac{5i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}-\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}$	70

```
[In] int(sech(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-exp(x)+4*I*exp(x)^2+exp(x)^3)/(exp(x)+I)^4+1/4*I*ln(exp(x)-I)-1/4*I*
ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(22) = 44.

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.03

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \frac{\left(-i\,e^{(4\,x)}+4\,e^{(3\,x)}+6i\,e^{(2\,x)}-4\,e^{x}-i\right)\log\left(e^{x}+i\right)+\left(i\,e^{(4\,x)}-4\,e^{(3\,x)}-6i\,e^{(2\,x)}+4\,e^{x}+i\right)\log\left(e^{x}-i\right)-}{4\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((-I*e^(4*x) + 4*e^(3*x) + 6*I*e^(2*x) - 4*e^x - I)*log(e^x + I) + (I*e
^(4*x) - 4*e^(3*x) - 6*I*e^(2*x) + 4*e^x + I)*log(e^x - I) - 2*e^(3*x) - 8*
I*e^(2*x) + 2*e^x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}(x)}{(\sinh(x)+i)^2}\,dx$$

```
[In] integrate(sech(x)/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(22) = 44.

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{sech}(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(e^{(-x)}+4i\,e^{(-2\,x)}-e^{(-3\,x)}\right)}{16i\,e^{(-x)}-24\,e^{(-2\,x)}-16i\,e^{(-3\,x)}+4\,e^{(-4\,x)}+4} - \frac{1}{4}i\,\log\left(i\,e^{(-x)}+1\right)+\frac{1}{4}i\,\log\left(i\,e^{(-x)}-1\right)$$

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(e^(-x) + 4*I*e^(-2*x) - e^(-3*x))/(16*I*e^(-x) - 24*e^(-2*x) - 16*I*e^(
-3*x) + 4*e^(-4*x) + 4) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \frac{\mathrm{sech}(x)}{(i+\sinh(x))^2}\,dx = \frac{3i\left(e^{(-x)}-e^x\right)^2+20\,e^{(-x)}-20\,e^x-44i}{16\left(e^{(-x)}-e^x-2i\right)^2} - \frac{1}{8}i\,\log\left(-e^{(-x)}+e^x+2i\right)+\frac{1}{8}i\,\log\left(-e^{(-x)}+e^x-2i\right)$$

```
[In] integrate(sech(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(3*I*(e^(-x) - e^x)^2 + 20*e^(-x) - 20*e^x - 44*I)/(e^(-x) - e^x - 2*I
)^2 - 1/8*I*log(-e^(-x) + e^x + 2*I) + 1/8*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.53

$$\int \frac{\mathrm{sech}(x)}{(i+\sinh(x))^2}\,dx = -\frac{\mathrm{atan}(e^x)}{2}-\frac{1i}{2\,\left(e^{2\,x}-1+e^x\,2i\right)}+\frac{1i}{e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^x\,4i} - \frac{1}{2\,\left(e^x+1i\right)}-\frac{2}{e^{2\,x}\,3i+e^{3\,x}-3\,e^x-i}$$

```
[In] int(1/(cosh(x)*(sinh(x) + 1i)^2),x)
```

```
[Out] 1i/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) - 1i/(2*(exp(2*x)
+ exp(x)*2i - 1)) - atan(exp(x))/2 - 1/(2*(exp(x) + 1i)) - 2/(exp(2*x)*3i +
 exp(3*x) - 3*exp(x) - 1i)
```

3.178 $\int \frac{\text{sech}^2(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\text{sech}^2(x)}{(i+\sinh(x))^2} dx = -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2} - \frac{\text{sech}(x)}{5(i+\sinh(x))} - \frac{2\tanh(x)}{5}$$

```
[Out] -1/5*I*sech(x)/(I+sinh(x))^2-1/5*sech(x)/(I+sinh(x))-2/5*tanh(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2751, 3852, 8}

$$\int \frac{\text{sech}^2(x)}{(i+\sinh(x))^2} dx = -\frac{2\tanh(x)}{5} - \frac{\text{sech}(x)}{5(\sinh(x)+i)} - \frac{i\text{sech}(x)}{5(\sinh(x)+i)^2}$$

```
[In] Int[Sech[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/5*I)*Sech[x])/(I + Sinh[x])^2 - Sech[x]/(5*(I + Sinh[x])) - (2*Tanh[x]
)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
```

```
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2} - \frac{3}{5}i\int \frac{\text{sech}^2(x)}{i+\sinh(x)}\,dx \\ &= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2} - \frac{\text{sech}(x)}{5(i+\sinh(x))} - \frac{2}{5}\int \text{sech}^2(x)\,dx \\ &= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2} - \frac{\text{sech}(x)}{5(i+\sinh(x))} - \frac{2}{5}i\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right) \\ &= -\frac{i\text{sech}(x)}{5(i+\sinh(x))^2} - \frac{\text{sech}(x)}{5(i+\sinh(x))} - \frac{2\tanh(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\text{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{\text{sech}(x)(4i\cosh(2x) - 5\sinh(x) + \sinh(3x))}{10(i+\sinh(x))^2}$$

```
[In] Integrate[Sech[x]^2/(I + Sinh[x])^2,x]
[Out] -1/10*(Sech[x]*((4*I)*Cosh[2*x] - 5*Sinh[x] + Sinh[3*x]))/(I + Sinh[x])^2
```

Maple [A] (verified)

Time = 49.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{4(5\,\mathrm{e}^{2x}+4i\mathrm{e}^{x}-1)}{5(\mathrm{e}^{x}-i)(\mathrm{e}^{x}+i)^{5}}$	30
default	$-\frac{1}{4(-i+\tanh(\frac{x}{2}))} - \frac{2i}{(\tanh(\frac{x}{2})+i)^{4}} + \frac{5i}{2(\tanh(\frac{x}{2})+i)^{2}} - \frac{4}{5(\tanh(\frac{x}{2})+i)^{5}} + \frac{3}{(\tanh(\frac{x}{2})+i)^{3}} - \frac{7}{4(\tanh(\frac{x}{2})+i)}$	70

```
[In] int(sech(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -4/5*(5*exp(2*x)+4*I*exp(x)-1)/(exp(x)-I)/(exp(x)+I)^5
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{4\left(5\,e^{(2\,x)}+4i\,e^{x}-1\right)}{5\left(e^{(6\,x)}+4i\,e^{(5\,x)}-5\,e^{(4\,x)}-5\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
[Out] -4/5*(5*e^(2*x) + 4*I*e^x - 1)/(e^(6*x) + 4*I*e^(5*x) - 5*e^(4*x) - 5*e^(2*
x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^2(x)}{(\sinh(x)+i)^2}\,dx$$

```
[In] integrate(sech(x)**2/(I+sinh(x))**2,x)
[Out] Integral(sech(x)**2/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.16

$$\begin{aligned}\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = &-\frac{16i\,e^{(-x)}}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+5}\\&+\frac{20\,e^{(-2\,x)}}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+5}\\&-\frac{4}{20i\,e^{(-x)}-25\,e^{(-2\,x)}-25\,e^{(-4\,x)}-20i\,e^{(-5\,x)}+5\,e^{(-6\,x)}+5}\end{aligned}$$

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -16*I*e^(-x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*e^(-5*x) + 5*e
^(-6*x) + 5) + 20*e^(-2*x)/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 20*I*
e^(-5*x) + 5*e^(-6*x) + 5) - 4/(20*I*e^(-x) - 25*e^(-2*x) - 25*e^(-4*x) - 2
0*I*e^(-5*x) + 5*e^(-6*x) + 5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{i}{4\,(e^x-i)} - \frac{-5i\,e^{(4\,x)}+30\,e^{(3\,x)}+80i\,e^{(2\,x)}-50\,e^x-11i}{20\,(e^x+i)^5}$$

```
[In] integrate(sech(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*I/(e^x - I) - 1/20*(-5*I*e^(4*x) + 30*e^(3*x) + 80*I*e^(2*x) - 50*e^x
- 11*I)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{sech}^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{16\,\mathrm{e}^x\,(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^x)}{5\,(\mathrm{e}^{2\,x}+1)^5} - \frac{\left(4\,\mathrm{e}^{2\,x}-\frac{4}{5}\right)\,(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1)}{(\mathrm{e}^{2\,x}+1)^5} - \frac{\mathrm{e}^x\,(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1)\,16\mathrm{i}}{5\,(\mathrm{e}^{2\,x}+1)^5} + \frac{(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^x)\,\left(4\,\mathrm{e}^{2\,x}-\frac{4}{5}\right)\,1\mathrm{i}}{(\mathrm{e}^{2\,x}+1)^5}$$

```
[In] int(1/(cosh(x)^2*(sinh(x) + 1i)^2),x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*(4*exp(2*x) - 4/5)*1i)/(exp(2*x) + 1)^5 - (exp(x)*
(exp(4*x) - 6*exp(2*x) + 1)*16i)/(5*(exp(2*x) + 1)^5) - (16*exp(x)*(4*exp(3
*x) - 4*exp(x)))/(5*(exp(2*x) + 1)^5) - ((4*exp(2*x) - 4/5)*(exp(4*x) - 6*e
xp(2*x) + 1))/(exp(2*x) + 1)^5
```

3.179 $\int \frac{\text{sech}^3(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\text{sech}^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{1}{4}\arctan(\sinh(x)) + \frac{1}{16(i-\sinh(x))} + \frac{1}{12(i+\sinh(x))^3} - \frac{i}{8(i+\sinh(x))^2} - \frac{3}{16(i+\sinh(x))}$$

```
[Out] -1/4*arctan(sinh(x))+1/16/(I-sinh(x))+1/12/(I+sinh(x))^3-1/8*I/(I+sinh(x))^
2-3/16/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2746, 46, 209}

$$\int \frac{\text{sech}^3(x)}{(i+\sinh(x))^2} \, dx = -\frac{1}{4}\arctan(\sinh(x)) + \frac{1}{16(-\sinh(x)+i)} - \frac{3}{16(\sinh(x)+i)} - \frac{i}{8(\sinh(x)+i)^2} + \frac{1}{12(\sinh(x)+i)^3}$$

```
[In] Int[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -1/4*ArcTan[Sinh[x]] + 1/(16*(I - Sinh[x])) + 1/(12*(I + Sinh[x])^3) - (I/8
)/(I + Sinh[x])^2 - 3/(16*(I + Sinh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(i-x)^2(i+x)^4}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{16(-i+x)^2}-\frac{1}{4(i+x)^4}+\frac{i}{4(i+x)^3}+\frac{3}{16(i+x)^2}-\frac{1}{4\,(1+x^2)}\right)\,dx, x, \sinh(x)\right)\\
&= \frac{1}{16(i-\sinh(x))}+\frac{1}{12(i+\sinh(x))^3}-\frac{i}{8(i+\sinh(x))^2}\\
&\quad -\frac{3}{16(i+\sinh(x))}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{4}\arctan(\sinh(x))+\frac{1}{16(i-\sinh(x))}\\
&\quad +\frac{1}{12(i+\sinh(x))^3}-\frac{i}{8(i+\sinh(x))^2}-\frac{3}{16(i+\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$
\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{\operatorname{sech}^2(x)\,(4i-3\arctan(\sinh(x))+(-1+6i\arctan(\sinh(x)))\sinh(x)+6i\sinh^2(x)+(3+6i\arctan(\sinh}{12(i+\sinh(x))^2}
$$

```
[In] Integrate[Sech[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] -1/12*(Sech[x]^2*(4*I - 3*ArcTan[Sinh[x]] + (-1 + (6*I)*ArcTan[Sinh[x]])*Si
nh[x] + (6*I)*Sinh[x]^2 + (3 + (6*I)*ArcTan[Sinh[x]])*Sinh[x]^3 + 3*ArcTan[
Sinh[x]]*Sinh[x]^4))/(I + Sinh[x])^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(45) = 90.

Time = 0.62 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\frac{i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{i\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{4}+\frac{1}{-8i+8\tanh\left(\frac{x}{2}\right)}+\frac{7i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{i\ln(}{}$$

```
[In] int(sech(x)^3/(I+sinh(x))^2,x)
```

```
[Out] 1/8*I/(-I+tanh(1/2*x))^2+1/4*I*ln(-I+tanh(1/2*x))+1/8/(-I+tanh(1/2*x))+7/2*
I/(tanh(1/2*x)+I)^4-2/3*I/(tanh(1/2*x)+I)^6-1/4*I*ln(tanh(1/2*x)+I)-23/8*I/
(tanh(1/2*x)+I)^2+2/(tanh(1/2*x)+I)^5-11/3/(tanh(1/2*x)+I)^3+11/8/(tanh(1/2
*x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(38) = 76.

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.35

$$\int\frac{\operatorname{sech}^{3}(x)}{(i+\sinh(x))^{2}}\,dx = -\frac{3\left(i\,e^{(8x)}-4\,e^{(7x)}-4i\,e^{(6x)}-4\,e^{(5x)}-10i\,e^{(4x)}+4\,e^{(3x)}-4i\,e^{(2x)}+4\,e^{x}+i\right)\log\left(e^{x}+i\right)+3\left(-i\,e^{(8x}\right.}{12\left(e^{(8x)}+4i\,e\right.}$$

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/12*(3*(I*e^(8*x) - 4*e^(7*x) - 4*I*e^(6*x) - 4*e^(5*x) - 10*I*e^(4*x) +
4*e^(3*x) - 4*I*e^(2*x) + 4*e^x + I)*log(e^x + I) + 3*(-I*e^(8*x) + 4*e^(7*
x) + 4*I*e^(6*x) + 4*e^(5*x) + 10*I*e^(4*x) - 4*e^(3*x) + 4*I*e^(2*x) - 4*e
^x - I)*log(e^x - I) + 6*e^(7*x) + 24*I*e^(6*x) - 26*e^(5*x) + 16*I*e^(4*x)
 + 26*e^(3*x) + 24*I*e^(2*x) - 6*e^x)/(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) +
4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^3(x)}{(\sinh(x)+i)^2}\,dx$$

```
[In] integrate(sech(x)**3/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**3/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(38) = 76.

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.00

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{8\left(3\,e^{(-x)}+12i\,e^{(-2\,x)}-13\,e^{(-3\,x)}+8i\,e^{(-4\,x)}+13\,e^{(-5\,x)}+12i\,e^{(-6\,x)}-3\,e^{(-7\,x)}\right)}{192i\,e^{(-x)}-192\,e^{(-2\,x)}+192i\,e^{(-3\,x)}-480\,e^{(-4\,x)}-192i\,e^{(-5\,x)}-192\,e^{(-6\,x)}-192i\,e^{(-7\,x)}+48\,e^{(-8\,x)}+4} - \frac{1}{4}i\,\log\left(i\,e^{(-x)}+1\right)+\frac{1}{4}i\,\log\left(i\,e^{(-x)}-1\right)$$

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -8*(3*e^(-x) + 12*I*e^(-2*x) - 13*e^(-3*x) + 8*I*e^(-4*x) + 13*e^(-5*x) + 1
2*I*e^(-6*x) - 3*e^(-7*x))/(192*I*e^(-x) - 192*e^(-2*x) + 192*I*e^(-3*x) -
480*e^(-4*x) - 192*I*e^(-5*x) - 192*e^(-6*x) - 192*I*e^(-7*x) + 48*e^(-8*x)
 + 48) - 1/4*I*log(I*e^(-x) + 1) + 1/4*I*log(I*e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.75

$$\begin{aligned}\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx &= \frac{-i\,e^{(-x)}+i\,e^{x}+3}{8\left(e^{(-x)}-e^{x}+2i\right)} \\ &\quad+\frac{11i\left(e^{(-x)}-e^{x}\right)^3+84\left(e^{(-x)}-e^{x}\right)^2-228i\,e^{(-x)}+228i\,e^{x}-240}{48\left(e^{(-x)}-e^{x}-2i\right)^3} \\ &\quad-\frac{1}{8}i\,\log\left(-e^{(-x)}+e^{x}+2i\right)+\frac{1}{8}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)\end{aligned}$$

```
[In] integrate(sech(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/8*(-I*e^(-x) + I*e^x + 3)/(e^(-x) - e^x + 2*I) + 1/48*(11*I*(e^(-x) - e^x
)^3 + 84*(e^(-x) - e^x)^2 - 228*I*e^(-x) + 228*I*e^x - 240)/(e^(-x) - e^x -
 2*I)^3 - 1/8*I*log(-e^(-x) + e^x + 2*I) + 1/8*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.30

$$\int \frac{\operatorname{sech}^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{\operatorname{atan}(e^x)}{2} - \frac{2}{e^{5x} - 10\,e^{3x} + e^{4x}\,5i - e^{2x}\,10i + 5\,e^x + 1i} - \frac{1i}{8\,\left(e^{2x} - 1 + e^x\,2i\right)} - \frac{3i}{2\,\left(e^{4x} - 6\,e^{2x} + 1 + e^{3x}\,4i - e^x\,4i\right)} + \frac{1i}{8\,\left(1 - e^{2x} + e^x\,2i\right)} - \frac{1}{8\,\left(e^x - i\right)} - \frac{3}{8\,\left(e^x + 1i\right)} + \frac{2i}{3\,\left(15\,e^{2x} - 15\,e^{4x} + e^{6x} - 1 - e^{3x}\,20i + e^{5x}\,6i + e^x\,6i\right)} - \frac{1}{3\,\left(e^{2x}\,3i + e^{3x} - 3\,e^x - i\right)}$$

```
[In] int(1/(cosh(x)^3*(sinh(x) + 1i)^2),x)
```

```
[Out] 1i/(8*(exp(x)*2i - exp(2*x) + 1)) - 2/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)
*10i + exp(5*x) + 5*exp(x) + 1i) - 1i/(8*(exp(2*x) + exp(x)*2i - 1)) - 3i/(
2*(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) - atan(exp(x))/2 -
 1/(8*(exp(x) - 1i)) - 3/(8*(exp(x) + 1i)) + 2i/(3*(15*exp(2*x) - exp(3*x)*
20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)*6i - 1)) - 1/(3*(exp(2*
x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

3.180 $\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{i\operatorname{sech}^3(x)}{7(i+\sinh(x))^2}-\frac{\operatorname{sech}^3(x)}{7(i+\sinh(x))}-\frac{4\tanh(x)}{7}+\frac{4\tanh^3(x)}{21}$$

```
[Out] -1/7*I*sech(x)^3/(I+sinh(x))^2-1/7*sech(x)^3/(I+sinh(x))-4/7*tanh(x)+4/21*t
anh(x)^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2751, 3852}

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = \frac{4\tanh^3(x)}{21}-\frac{4\tanh(x)}{7}-\frac{\operatorname{sech}^3(x)}{7(\sinh(x)+i)}-\frac{i\operatorname{sech}^3(x)}{7(\sinh(x)+i)^2}$$

```
[In] Int[Sech[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((-1/7*I)*Sech[x]^3)/(I + Sinh[x])^2 - Sech[x]^3/(7*(I + Sinh[x])) - (4*Tan
h[x])/7 + (4*Tanh[x]^3)/21
```

Rule 2751

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Dist[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x],
 x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplif
```

```
y[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{5}{7}i\int \frac{\text{sech}^4(x)}{i+\sinh(x)}\,dx \\ &= -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\text{sech}^3(x)}{7(i+\sinh(x))} - \frac{4}{7}\int \text{sech}^4(x)\,dx \\ &= -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\text{sech}^3(x)}{7(i+\sinh(x))} - \frac{4}{7}i\text{Subst}\left(\int (1+x^2)\,dx, x, -i\tanh(x)\right) \\ &= -\frac{i\text{sech}^3(x)}{7(i+\sinh(x))^2} - \frac{\text{sech}^3(x)}{7(i+\sinh(x))} - \frac{4\tanh(x)}{7} + \frac{4\tanh^3(x)}{21}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{\text{sech}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{\text{sech}^3(x)(8i\cosh(2x)+4i\cosh(4x)-14\sinh(x)-3\sinh(3x)+\sinh(5x))}{42(i+\sinh(x))^2}$$

```
[In] Integrate[Sech[x]^4/(I + Sinh[x])^2,x]
[Out] -1/42*(Sech[x]^3*((8*I)*Cosh[2*x] + (4*I)*Cosh[4*x] - 14*Sinh[x] - 3*Sinh[3
*x] + Sinh[5*x]))/(I + Sinh[x])^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(38) = 76.

Time = 0.41 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.37

$$-\frac{i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)^2}+\frac{1}{12\left(-i+\tanh\left(\frac{x}{2}\right)\right)^3}-\frac{3}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)}+\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^6}-\frac{5i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^4}+\frac{\cdots}{8(\text{ta}\cdots}$$

```
[In] int(sech(x)^4/(I+sinh(x))^2,x)
```

```
[Out] -1/8*I/(-I+tanh(1/2*x))^2+1/12/(-I+tanh(1/2*x))^3-3/8/(-I+tanh(1/2*x))+2*I/
(tanh(1/2*x)+I)^6-5*I/(tanh(1/2*x)+I)^4+23/8*I/(tanh(1/2*x)+I)^2+4/7/(tanh(
1/2*x)+I)^7-4/(tanh(1/2*x)+I)^5+55/12/(tanh(1/2*x)+I)^3-13/8/(tanh(1/2*x)+I
)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(35) = 70$.

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{16\left(14\,e^{(4\,x)}+8i\,e^{(3\,x)}+3\,e^{(2\,x)}+4i\,e^{x}-1\right)}{21\,\left(e^{(10\,x)}+4i\,e^{(9\,x)}-3\,e^{(8\,x)}+8i\,e^{(7\,x)}-14\,e^{(6\,x)}-14\,e^{(4\,x)}-8i\,e^{(3\,x)}-3\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -16/21*(14*e^(4*x) + 8*I*e^(3*x) + 3*e^(2*x) + 4*I*e^x - 1)/(e^(10*x) + 4*I
*e^(9*x) - 3*e^(8*x) + 8*I*e^(7*x) - 14*e^(6*x) - 14*e^(4*x) - 8*I*e^(3*x)
- 3*e^(2*x) - 4*I*e^x + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^4(x)}{(\sinh(x)+i)^2}\,dx$$

```
[In] integrate(sech(x)**4/(I+sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**4/(sinh(x) + I)**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(35) = 70$.

Time = 0.21 (sec) , antiderivative size = 317, normalized size of antiderivative = 6.47

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx =$$

$$-\frac{64i\,e^{(-x)}}{84i\,e^{(-x)} - 63\,e^{(-2\,x)} + 168i\,e^{(-3\,x)} - 294\,e^{(-4\,x)} - 294\,e^{(-6\,x)} - 168i\,e^{(-7\,x)} - 63\,e^{(-8\,x)} - 84i\,e^{(-9\,x)} + 21}$$
$$+\frac{48\,e^{(-2\,x)}}{84i\,e^{(-x)} - 63\,e^{(-2\,x)} + 168i\,e^{(-3\,x)} - 294\,e^{(-4\,x)} - 294\,e^{(-6\,x)} - 168i\,e^{(-7\,x)} - 63\,e^{(-8\,x)} - 84i\,e^{(-9\,x)} + 21}$$
$$-\frac{128i\,e^{(-3\,x)}}{84i\,e^{(-x)} - 63\,e^{(-2\,x)} + 168i\,e^{(-3\,x)} - 294\,e^{(-4\,x)} - 294\,e^{(-6\,x)} - 168i\,e^{(-7\,x)} - 63\,e^{(-8\,x)} - 84i\,e^{(-9\,x)} + 21}$$
$$+\frac{224\,e^{(-4\,x)}}{84i\,e^{(-x)} - 63\,e^{(-2\,x)} + 168i\,e^{(-3\,x)} - 294\,e^{(-4\,x)} - 294\,e^{(-6\,x)} - 168i\,e^{(-7\,x)} - 63\,e^{(-8\,x)} - 84i\,e^{(-9\,x)} + 21}$$
$$-\frac{16}{84i\,e^{(-x)} - 63\,e^{(-2\,x)} + 168i\,e^{(-3\,x)} - 294\,e^{(-4\,x)} - 294\,e^{(-6\,x)} - 168i\,e^{(-7\,x)} - 63\,e^{(-8\,x)} - 84i\,e^{(-9\,x)} + 21}$$

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
[Out] -64*I*e^(-x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*x) - 2
94*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-10*x) +
 21) + 48*e^(-2*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 294*e^(-4*
x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) + 21*e^(-1
0*x) + 21) - 128*I*e^(-3*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x) - 2
94*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*x) +
 21*e^(-10*x) + 21) + 224*e^(-4*x)/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3
*x) - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(
-9*x) + 21*e^(-10*x) + 21) - 16/(84*I*e^(-x) - 63*e^(-2*x) + 168*I*e^(-3*x)
 - 294*e^(-4*x) - 294*e^(-6*x) - 168*I*e^(-7*x) - 63*e^(-8*x) - 84*I*e^(-9*
x) + 21*e^(-10*x) + 21)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{6i\,e^{(2\,x)} + 15\,e^{x} - 7i}{24\,(e^{x} - i)^{3}} - \frac{-42i\,e^{(6\,x)} + 315\,e^{(5\,x)} + 1015i\,e^{(4\,x)} - 1750\,e^{(3\,x)} - 1344i\,e^{(2\,x)} + 511\,e^{x} + 79i}{168\,(e^{x} + i)^{7}}$$

```
[In] integrate(sech(x)^4/(I+sinh(x))^2,x, algorithm="giac")
[Out] -1/24*(6*I*e^(2*x) + 15*e^x - 7*I)/(e^x - I)^3 - 1/168*(-42*I*e^(6*x) + 315
*e^(5*x) + 1015*I*e^(4*x) - 1750*e^(3*x) - 1344*I*e^(2*x) + 511*e^x + 79*I)
/(e^x + I)^7
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \frac{\operatorname{sech}^4(x)}{(i+\sinh(x))^2}\,dx = \frac{\left(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^{x}\right)\,\left(\frac{16\,\mathrm{e}^{2\,x}}{7}+\frac{32\,\mathrm{e}^{4\,x}}{3}-\frac{16}{21}\right)\,1\mathrm{i}}{{\left(\mathrm{e}^{2\,x}+1\right)}^7} - \frac{\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1\right)\,\left(\frac{16\,\mathrm{e}^{2\,x}}{7}+\frac{32\,\mathrm{e}^{4\,x}}{3}-\frac{16}{21}\right)}{{\left(\mathrm{e}^{2\,x}+1\right)}^7} - \frac{\left(4\,\mathrm{e}^{3\,x}-4\,\mathrm{e}^{x}\right)\,\left(\frac{128\,\mathrm{e}^{3\,x}}{21}+\frac{64\,\mathrm{e}^{x}}{21}\right)}{{\left(\mathrm{e}^{2\,x}+1\right)}^7} - \frac{\left(\frac{128\,\mathrm{e}^{3\,x}}{21}+\frac{64\,\mathrm{e}^{x}}{21}\right)\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1\right)\,1\mathrm{i}}{{\left(\mathrm{e}^{2\,x}+1\right)}^7}$$

```
[In] int(1/(cosh(x)^4*(sinh(x) + 1i)^2),x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*((16*exp(2*x))/7 + (32*exp(4*x))/3 - 16/21)*1i)/(e
xp(2*x) + 1)^7 - ((exp(4*x) - 6*exp(2*x) + 1)*((16*exp(2*x))/7 + (32*exp(4*
x))/3 - 16/21))/(exp(2*x) + 1)^7 - ((4*exp(3*x) - 4*exp(x))*((128*exp(3*x))
/21 + (64*exp(x))/21))/(exp(2*x) + 1)^7 - (((128*exp(3*x))/21 + (64*exp(x))
/21)*(exp(4*x) - 6*exp(2*x) + 1)*1i)/(exp(2*x) + 1)^7
```

3.181 $\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = i\log(i-\sinh(x)) + \frac{2i}{1+i\sinh(x)}$$

```
[Out] I*ln(I-sinh(x))+2*I/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2746, 45}

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = \frac{2i}{1+i\sinh(x)} + i\log(-\sinh(x)+i)$$

```
[In] Int[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
```

```
[Out] I*Log[I - Sinh[x]] + (2*I)/(1 + I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, i\sinh(x)\right)\right)\\
&= -\left(i\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, i\sinh(x)\right)\right)\\
&= i\log(i-\sinh(x))+\frac{2i}{1+i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = \frac{\cosh^4(x)(2+\log(i-\sinh(x))+i\log(i-\sinh(x))\sinh(x))}{(-i+\sinh(x))^3(i+\sinh(x))^2}$$

```
[In] Integrate[Cosh[x]^3/(1 + I*Sinh[x])^3,x]
[Out] (Cosh[x]^4*(2 + Log[I - Sinh[x]] + I*Log[I - Sinh[x]]*Sinh[x]))/((-I + Sinh
[x])^3*(I + Sinh[x])^2)
```

Maple [A] (verified)

Time = 60.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{i\ln\left(\sinh(x)^2+1\right)}{2}-\arctan\left(\sinh\left(x\right)\right)+\frac{2}{\sinh(x)-i}$	26
default	$\frac{i\ln\left(\sinh(x)^2+1\right)}{2}-\arctan\left(\sinh\left(x\right)\right)+\frac{2}{\sinh(x)-i}$	26
risch	$-ix+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-i)^2}+2i\ln\left(\mathrm{e}^{x}-i\right)$	26

```
[In] int(cosh(x)^3/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] 1/2*I*ln(sinh(x)^2+1)-arctan(sinh(x))+2/(sinh(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(20) = 40.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = \frac{-i\,xe^{(2\,x)} - 2\,(x-2)e^x - 2\left(-i\,e^{(2\,x)} - 2\,e^x + i\right)\log\left(e^x - i\right) + i\,x}{e^{(2\,x)} - 2i\,e^x - 1}$$

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (-I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(-I*e^(2*x) - 2*e^x + I)*log(e^x - I) + I
*x)/(e^(2*x) - 2*I*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -ix + 2i\log\left(e^x - i\right) + \frac{4e^x}{e^{2x} - 2ie^x - 1}$$

```
[In] integrate(cosh(x)**3/(1+I*sinh(x))**3,x)
```

```
[Out] -I*x + 2*I*log(exp(x) - I) + 4*exp(x)/(exp(2*x) - 2*I*exp(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = i\,x - \frac{4\,e^{(-x)}}{2i\,e^{(-x)} + e^{(-2\,x)} - 1} + 2i\,\log\left(e^{(-x)} + i\right)$$

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] I*x - 4*e^(-x)/(2*I*e^(-x) + e^(-2*x) - 1) + 2*I*log(e^(-x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -i\,x + \frac{4\,e^x}{(e^x-i)^2} + 2i\,\log\left(e^x-i\right)$$

```
[In] integrate(cosh(x)^3/(1+I*sinh(x))^3,x, algorithm="giac")
[Out] -I*x + 4*e^x/(e^x - I)^2 + 2*I*log(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{\cosh^3(x)}{(1+i\sinh(x))^3}\,dx = -x\,1\mathrm{i} + \ln\left(\mathrm{e}^x - \mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x-\mathrm{i}}$$

```
[In] int(cosh(x)^3/(sinh(x)*1i + 1)^3,x)
[Out] log(exp(x) - 1i)*2i - x*1i - 4i/(exp(x)*2i - exp(2*x) + 1) + 4/(exp(x) - 1i
)
```

3.182 $\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3} dx = \frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

```
[Out] 1/3*I*cosh(x)^3/(1+I*sinh(x))^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2750}

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3} dx = \frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

```
[In] Int[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
```

```
[Out] ((I/3)*Cosh[x]^3)/(1 + I*Sinh[x])^3
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{i\cosh^3(x)}{3(1+i\sinh(x))^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{\cosh^3(x)}{3(-i+\sinh(x))^3}$$

```
[In] Integrate[Cosh[x]^2/(1 + I*Sinh[x])^3,x]
[Out] -1/3*Cosh[x]^3/(-I + Sinh[x])^3
```

Maple [A] (verified)

Time = 57.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{2i(3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{x}-i)^3}$	19
default	$\frac{4i}{\left(-i+\tanh\left(\frac{x}{2}\right)\right)^2}-\frac{8}{3\left(-i+\tanh\left(\frac{x}{2}\right)\right)^3}+\frac{2}{-i+\tanh\left(\frac{x}{2}\right)}$	36

```
[In] int(cosh(x)^2/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] -2/3*I*(3*exp(2*x)-1)/(exp(x)-I)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2\left(3i\,e^{(2\,x)}-i\right)}{3\left(e^{(3\,x)}-3i\,e^{(2\,x)}-3\,e^{x}+i\right)}$$

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="fricas")
[Out] -2/3*(3*I*e^(2*x) - I)/(e^(3*x) - 3*I*e^(2*x) - 3*e^x + I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{-6ie^{2x}+2i}{3e^{3x}-9ie^{2x}-9e^{x}+3i}$$

```
[In] integrate(cosh(x)**2/(1+I*sinh(x))**3,x)
[Out] (-6*I*exp(2*x) + 2*I)/(3*exp(3*x) - 9*I*exp(2*x) - 9*exp(x) + 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = \frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3} - \frac{2}{-9i\,e^{(-x)}-9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}+3}$$

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="maxima")
[Out] 6*e^(-2*x)/(-9*I*e^(-x) - 9*e^(-2*x) + 3*I*e^(-3*x) + 3) - 2/(-9*I*e^(-x) -
 9*e^(-2*x) + 3*I*e^(-3*x) + 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2\left(3i\,e^{(2\,x)}-i\right)}{3\left(e^{x}-i\right)^3}$$

```
[In] integrate(cosh(x)^2/(1+I*sinh(x))^3,x, algorithm="giac")
[Out] -2/3*(3*I*e^(2*x) - I)/(e^x - I)^3
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^2(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2\,e^{2\,x}-\frac{2}{3}}{\left(1+e^{x}\,1\mathrm{i}\right)^3}$$

```
[In] int(cosh(x)^2/(sinh(x)*1i + 1)^3,x)
```

```
[Out] -(2*exp(2*x) - 2/3)/(exp(x)*1i + 1)^3
```

3.183 $\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = \frac{i}{2(1+i\sinh(x))^2}$$

```
[Out] 1/2*I/(1+I*sinh(x))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 32}

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = \frac{i}{2(1+i\sinh(x))^2}$$

```
[In] Int[Cosh[x]/(1 + I*Sinh[x])^3,x]
[Out] (I/2)/(1 + I*Sinh[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\text{integral} = -\left(i\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, i\sinh(x)\right)\right)$$
$$= \frac{i}{2(1+i\sinh(x))^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{i}{2(-i+\sinh(x))^2}$$

```
[In] Integrate[Cosh[x]/(1 + I*Sinh[x])^3,x]
[Out] (-1/2*I)/(-I + Sinh[x])^2
```

Maple [A] (verified)

Time = 58.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{i}{2(1+i\sinh(x))^2}$	13
default	$\frac{i}{2(1+i\sinh(x))^2}$	13
risch	$-\frac{2i\mathrm{e}^{2x}}{(\mathrm{e}^{x}-i)^4}$	15

```
[In] int(cosh(x)/(1+I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] 1/2*I/(1+I*sinh(x))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2i\,e^{(2\,x)}}{e^{(4\,x)}-4i\,e^{(3\,x)}-6\,e^{(2\,x)}+4i\,e^{x}+1}$$

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="fricas")
[Out] -2*I*e^(2*x)/(e^(4*x) - 4*I*e^(3*x) - 6*e^(2*x) + 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2ie^{2x}}{e^{4x}-4ie^{3x}-6e^{2x}+4ie^{x}+1}$$

```
[In] integrate(cosh(x)/(1+I*sinh(x))**3,x)
[Out] -2*I*exp(2*x)/(exp(4*x) - 4*I*exp(3*x) - 6*exp(2*x) + 4*I*exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = \frac{i}{2\,(i\,\sinh\left(x\right)+1)^2}$$

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="maxima")
[Out] 1/2*I/(I*sinh(x) + 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{2i\,e^{(2\,x)}}{(e^x-i)^4}$$

```
[In] integrate(cosh(x)/(1+I*sinh(x))^3,x, algorithm="giac")
[Out] -2*I*e^(2*x)/(e^x - I)^4
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(1+i\sinh(x))^3}\,dx = -\frac{e^{2\,x}\,2i}{(1+e^x\,1i)^4}$$

```
[In] int(cosh(x)/(sinh(x)*1i + 1)^3,x)
[Out] -(exp(2*x)*2i)/(exp(x)*1i + 1)^4
```

3.184 $\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = -i\log(i+\sinh(x)) - \frac{2i}{1-i\sinh(x)}$$

```
[Out] -I*ln(I+sinh(x))-2*I/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2746, 45}

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = -\frac{2i}{1-i\sinh(x)} - i\log(\sinh(x)+i)$$

```
[In] Int[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
```

```
[Out] (-I)*Log[I + Sinh[x]] - (2*I)/(1 - I*Sinh[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
```

```
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= i\text{Subst}\left(\int \frac{1-x}{(1+x)^2}\,dx, x, -i\sinh(x)\right)\\
&= i\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{2}{(1+x)^2}\right)\,dx, x, -i\sinh(x)\right)\\
&= -i\log(i+\sinh(x)) - \frac{2i}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = \frac{\cosh^4(x)(2+\log(i+\sinh(x)) - i\log(i+\sinh(x))\sinh(x))}{(-i+\sinh(x))^2(i+\sinh(x))^3}$$

```
[In] Integrate[Cosh[x]^3/(1 - I*Sinh[x])^3,x]
[Out] (Cosh[x]^4*(2 + Log[I + Sinh[x]] - I*Log[I + Sinh[x]]*Sinh[x]))/((-I + Sinh
[x])^2*(I + Sinh[x])^3)
```

Maple [A] (verified)

Time = 59.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{2}{i+\sinh(x)} - \frac{i\ln\left(\sinh(x)^2+1\right)}{2} - \arctan\left(\sinh\left(x\right)\right)$	26
default	$\frac{2}{i+\sinh(x)} - \frac{i\ln\left(\sinh(x)^2+1\right)}{2} - \arctan\left(\sinh\left(x\right)\right)$	26
risch	$ix + \frac{4\,e^x}{(e^x+i)^2} - 2i\ln\left(e^x+i\right)$	26

```
[In] int(cosh(x)^3/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] 2/(I+sinh(x))-1/2*I*ln(sinh(x)^2+1)-arctan(sinh(x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = \frac{i\,xe^{(2\,x)} - 2\,(x-2)e^x - 2\,\left(i\,e^{(2\,x)} - 2\,e^x - i\right)\log\left(e^x + i\right) - i\,x}{e^{(2\,x)} + 2i\,e^x - 1}$$

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="fricas")
```

```
[Out] (I*x*e^(2*x) - 2*(x - 2)*e^x - 2*(I*e^(2*x) - 2*e^x - I)*log(e^x + I) - I*x
)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = ix - 2i\log\left(e^x + i\right) + \frac{4e^x}{e^{2x} + 2ie^x - 1}$$

```
[In] integrate(cosh(x)**3/(1-I*sinh(x))**3,x)
```

```
[Out] I*x - 2*I*log(exp(x) + I) + 4*exp(x)/(exp(2*x) + 2*I*exp(x) - 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = -i\,x - \frac{4\,e^{(-x)}}{-2i\,e^{(-x)} + e^{(-2\,x)} - 1} - 2i\,\log\left(e^{(-x)} - i\right)$$

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="maxima")
```

```
[Out] -I*x - 4*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - 2*I*log(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = i\,x + \frac{4\,e^x}{(e^x+i)^2} - 2i\,\log\left(e^x+i\right)$$

```
[In] integrate(cosh(x)^3/(1-I*sinh(x))^3,x, algorithm="giac")
[Out] I*x + 4*e^x/(e^x + I)^2 - 2*I*log(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(x)}{(1-i\sinh(x))^3}\,dx = x\,1\mathrm{i} - \ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,2\mathrm{i} - \frac{4\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}} + \frac{4}{\mathrm{e}^x+1\mathrm{i}}$$

```
[In] int(-cosh(x)^3/(sinh(x)*1i - 1)^3,x)
[Out] x*1i - log(exp(x) + 1i)*2i - 4i/(exp(2*x) + exp(x)*2i - 1) + 4/(exp(x) + 1i
)
```

3.185 $\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

```
[Out] -1/3*I*cosh(x)^3/(1-I*sinh(x))^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2750}

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

```
[In] Int[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
```

```
[Out] ((-1/3*I)*Cosh[x]^3)/(1 - I*Sinh[x])^3
```

Rule 2750

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\frac{i\cosh^3(x)}{3(1-i\sinh(x))^3}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{\cosh^3(x)}{3(i+\sinh(x))^3}$$

```
[In] Integrate[Cosh[x]^2/(1 - I*Sinh[x])^3,x]
[Out] -1/3*Cosh[x]^3/(I + Sinh[x])^3
```

Maple [A] (verified)

Time = 57.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{2i(3\,\mathrm{e}^{2x}-1)}{3(\mathrm{e}^{x}+i)^{3}}$	19
default	$\frac{2}{\tanh(\frac{x}{2})+i}-\frac{4i}{(\tanh(\frac{x}{2})+i)^{2}}-\frac{8}{3(\tanh(\frac{x}{2})+i)^{3}}$	36

```
[In] int(cosh(x)^2/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] 2/3*I*(3*exp(2*x)-1)/(exp(x)+I)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{2\left(-3i\,e^{(2\,x)}+i\right)}{3\left(e^{(3\,x)}+3i\,e^{(2\,x)}-3\,e^{x}-i\right)}$$

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="fricas")
[Out] -2/3*(-3*I*e^(2*x) + I)/(e^(3*x) + 3*I*e^(2*x) - 3*e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = \frac{6ie^{2x}-2i}{3e^{3x}+9ie^{2x}-9e^x-3i}$$

```
[In] integrate(cosh(x)**2/(1-I*sinh(x))**3,x)
[Out] (6*I*exp(2*x) - 2*I)/(3*exp(3*x) + 9*I*exp(2*x) - 9*exp(x) - 3*I)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(14) = 28.

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{6\,e^{(-2\,x)}}{-9i\,e^{(-x)}+9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}-3} + \frac{2}{-9i\,e^{(-x)}+9\,e^{(-2\,x)}+3i\,e^{(-3\,x)}-3}$$

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="maxima")
[Out] -6*e^(-2*x)/(-9*I*e^(-x) + 9*e^(-2*x) + 3*I*e^(-3*x) - 3) + 2/(-9*I*e^(-x)
+ 9*e^(-2*x) + 3*I*e^(-3*x) - 3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^2(x)}{(1-i\sinh(x))^3}\,dx = -\frac{2\left(-3i\,e^{(2\,x)}+i\right)}{3\,(e^x+i)^3}$$

```
[In] integrate(cosh(x)^2/(1-I*sinh(x))^3,x, algorithm="giac")
[Out] -2/3*(-3*I*e^(2*x) + I)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^2(x)}{(1 - i\sinh(x))^3}\,dx = \frac{2\,(3\,\mathrm{e}^{2\,x} - 1)}{3\,{(-1 + \mathrm{e}^{x}\,1\mathrm{i})}^3}$$

```
[In] int(-cosh(x)^2/(sinh(x)*1i - 1)^3,x)
[Out] (2*(3*exp(2*x) - 1))/(3*(exp(x)*1i - 1)^3)
```

3.186 $\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i}{2(1-i\sinh(x))^2}$$

```
[Out] -1/2*I/(1-I*sinh(x))^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2746, 32}

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = -\frac{i}{2(1-i\sinh(x))^2}$$

```
[In] Int[Cosh[x]/(1 - I*Sinh[x])^3,x]
[Out] (-1/2*I)/(1 - I*Sinh[x])^2
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
```

```
])
```

Rubi steps

$$\text{integral} = i\text{Subst}\left(\int \frac{1}{(1+x)^3}\,dx, x, -i\sinh(x)\right)$$
$$= -\frac{i}{2(1-i\sinh(x))^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(x)}{(1-i\sinh(x))^3}\,dx = \frac{i}{2(i+\sinh(x))^2}$$

```
[In] Integrate[Cosh[x]/(1 - I*Sinh[x])^3,x]
[Out] (I/2)/(I + Sinh[x])^2
```

Maple [A] (verified)

Time = 58.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{i}{2(1-i\sinh(x))^2}$	13
default	$-\frac{i}{2(1-i\sinh(x))^2}$	13
risch	$\frac{2i\,\mathrm{e}^{2x}}{(\mathrm{e}^{x}+i)^4}$	15

```
[In] int(cosh(x)/(1-I*sinh(x))^3,x,method=_RETURNVERBOSE)
[Out] -1/2*I/(1-I*sinh(x))^2
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\cosh(x)}{(1 - i\sinh(x))^3}\,dx = \frac{2i\,e^{(2\,x)}}{e^{(4\,x)} + 4i\,e^{(3\,x)} - 6\,e^{(2\,x)} - 4i\,e^{x} + 1}$$

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="fricas")
[Out] 2*I*e^(2*x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{\cosh(x)}{(1 - i\sinh(x))^3}\,dx = \frac{2ie^{2x}}{e^{4x} + 4ie^{3x} - 6e^{2x} - 4ie^{x} + 1}$$

```
[In] integrate(cosh(x)/(1-I*sinh(x))**3,x)
[Out] 2*I*exp(2*x)/(exp(4*x) + 4*I*exp(3*x) - 6*exp(2*x) - 4*I*exp(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(x)}{(1 - i\sinh(x))^3}\,dx = -\frac{i}{2\,(-i\,\sinh\,(x) + 1)^2}$$

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="maxima")
[Out] -1/2*I/(-I*sinh(x) + 1)^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cosh(x)}{(1 - i\sinh(x))^3}\,dx = \frac{2i\,e^{(2\,x)}}{(e^x + i)^4}$$

```
[In] integrate(cosh(x)/(1-I*sinh(x))^3,x, algorithm="giac")
[Out] 2*I*e^(2*x)/(e^x + I)^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(1 - i\sinh(x))^3}\,dx = \frac{\mathrm{e}^{2\,x}\,2\mathrm{i}}{(-1 + \mathrm{e}^x\,1\mathrm{i})^4}$$

```
[In] int(-cosh(x)/(sinh(x)*1i - 1)^3,x)
[Out] (exp(2*x)*2i)/(exp(x)*1i - 1)^4
```

3.187 $\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 138

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx=\frac{(a^2+b^2)^3\log(a+b\sinh(x))}{b^7}-\frac{a(a^4+3a^2b^2+3b^4)\sinh(x)}{b^6}+\frac{(a^4+3a^2b^2+3b^4)\sinh^2(x)}{2b^5}-\frac{a(a^2+3b^2)\sinh^3(x)}{3b^4}+\frac{(a^2+3b^2)\sinh^4(x)}{4b^3}-\frac{a\sinh^5(x)}{5b^2}+\frac{\sinh^6(x)}{6b}$$

```
[Out] (a^2+b^2)^3*ln(a+b*sinh(x))/b^7-a*(a^4+3*a^2*b^2+3*b^4)*sinh(x)/b^6+1/2*(a^
4+3*a^2*b^2+3*b^4)*sinh(x)^2/b^5-1/3*a*(a^2+3*b^2)*sinh(x)^3/b^4+1/4*(a^2+3
*b^2)*sinh(x)^4/b^3-1/5*a*sinh(x)^5/b^2+1/6*sinh(x)^6/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2747, 711}

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx=\frac{(a^2+b^2)^3\log(a+b\sinh(x))}{b^7}-\frac{a(a^2+3b^2)\sinh^3(x)}{3b^4}+\frac{(a^2+3b^2)\sinh^4(x)}{4b^3}-\frac{a(a^4+3a^2b^2+3b^4)\sinh(x)}{b^6}+\frac{(a^4+3a^2b^2+3b^4)\sinh^2(x)}{2b^5}-\frac{a\sinh^5(x)}{5b^2}+\frac{\sinh^6(x)}{6b}$$

```
[In] Int[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^3*Log[a + b*Sinh[x]])/b^7 - (a*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[
x])/b^6 + ((a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x]^2)/(2*b^5) - (a*(a^2 + 3*b^2)*
Sinh[x]^3)/(3*b^4) + ((a^2 + 3*b^2)*Sinh[x]^4)/(4*b^3) - (a*Sinh[x]^5)/(5*b
^2) + Sinh[x]^6/(6*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(-b^2-x^2)^3}{a+x}\,dx, x, b\sinh(x)\right)}{b^7}\\
&= -\frac{\text{Subst}\left(\int \left(a^5\left(1+\frac{3b^2(a^2+b^2)}{a^4}\right) - (a^4+3a^2b^2+3b^4)\,x + a(a^2+3b^2)\,x^2 - (a^2+3b^2)\,x^3 + ax^4 - x^5\right.\right.}{b^7}\\
&= \frac{(a^2+b^2)^3\log(a+b\sinh(x))}{b^7} - \frac{a(a^4+3a^2b^2+3b^4)\sinh(x)}{b^6}\\
&\quad + \frac{(a^4+3a^2b^2+3b^4)\sinh^2(x)}{2b^5} - \frac{a(a^2+3b^2)\sinh^3(x)}{3b^4}\\
&\quad + \frac{(a^2+3b^2)\sinh^4(x)}{4b^3} - \frac{a\sinh^5(x)}{5b^2} + \frac{\sinh^6(x)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{15b^4(a^2+b^2)\cosh^4(x) + 10b^6\cosh^6(x) + 60(a^2+b^2)^3\log(a+b\sinh(x)) - 60ab(a^4+3a^2b^2+3b^4)\sinh(x)}{60b^7}$$

```
[In] Integrate[Cosh[x]^7/(a + b*Sinh[x]),x]
```

```
[Out] (15*b^4*(a^2 + b^2)*Cosh[x]^4 + 10*b^6*Cosh[x]^6 + 60*(a^2 + b^2)^3*Log[a +
 b*Sinh[x]] - 60*a*b*(a^4 + 3*a^2*b^2 + 3*b^4)*Sinh[x] + 30*b^2*(a^2 + b^2)
^2*Sinh[x]^2 - 20*a*b^3*(a^2 + 3*b^2)*Sinh[x]^3 - 12*a*b^5*Sinh[x]^5)/(60*b
^7)
```

Maple [A] (verified)

Time = 128.98 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{-\frac{\sinh(x)^6 b^5}{6}+\frac{a\sinh(x)^5 b^4}{5}-\frac{b\left(a^2b^2+3b^4\right)\sinh(x)^4}{4}+\frac{a\left(a^2b^2+3b^4\right)\sinh(x)^3}{3}-\frac{\left(a^4+3a^2b^2+3b^4\right)\sinh(x)^2 b}{2}+a(a^4+3a^2b^2+3b}{b^6}$
default	$-\frac{-\frac{\sinh(x)^6 b^5}{6}+\frac{a\sinh(x)^5 b^4}{5}-\frac{b\left(a^2b^2+3b^4\right)\sinh(x)^4}{4}+\frac{a\left(a^2b^2+3b^4\right)\sinh(x)^3}{3}-\frac{\left(a^4+3a^2b^2+3b^4\right)\sinh(x)^2 b}{2}+a(a^4+3a^2b^2+3b}{b^6}$
risch	$-\frac{19a\,\mathrm{e}^{x}}{16b^2}-\frac{3x\,a^2}{b^3}-\frac{x}{b}+\frac{29\,\mathrm{e}^{-2x}}{128b}+\frac{\mathrm{e}^{-4x}}{32b}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{b}+\frac{\mathrm{e}^{4x}}{32b}+\frac{29\,\mathrm{e}^{2x}}{128b}+\frac{\mathrm{e}^{-6x}}{384b}+\frac{\mathrm{e}^{6x}}{384b}+\frac{\mathrm{e}^{-2x}a}{8b^5}$

```
[In] int(cosh(x)^7/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/6*sinh(x)^6*b^5+1/5*a*sinh(x)^5*b^4-1/4*b*(a^2*b^2+3*b^4)*sinh(x
)^4+1/3*a*(a^2*b^2+3*b^4)*sinh(x)^3-1/2*(a^4+3*a^2*b^2+3*b^4)*sinh(x)^2*b+a
*(a^4+3*a^2*b^2+3*b^4)*sinh(x))+(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/b^7*ln(a+b*si
nh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2105 vs. 2(128) = 256.

Time = 0.32 (sec) , antiderivative size = 2105, normalized size of antiderivative = 15.25

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*b
^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^10 + 6*(55*b^
6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 + 10*b^6)*sinh(x)^10 - 20*(4*a^3
*b^3 + 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 - 4*a
^3*b^3 - 9*a*b^5 + 15*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^4*b^2
 + 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5*cosh(
x)^3 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 + 2*b^6)*cosh(x)^2 -
12*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*x*cosh(x)^6 - 120*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^7 +
 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b - 22*a^3*b^3 - 19*a*b
^5 + 30*(a^2*b^4 + 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + (
16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 + 12*a*b^5*cosh(x) + 1
2*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 + 2*b^6)*cosh(x)^
4 - 140*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*
b^6)*cosh(x)^2 - 160*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 70*(8*a^5*b +
22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 + 120*(8*a^5*b + 22*a^3*b
```

^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b^5*cosh(x)^6 + 40
*a^5*b + 110*a^3*b^3 + 95*a*b^5 + 315*(a^2*b^4 + 2*b^6)*cosh(x)^5 - 105*(4*
a^3*b^3 + 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x
)^3 - 480*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x) - 105*(8*a^5*b + 22
*a^3*b^3 + 19*a*b^5)*cosh(x)^2)*sinh(x)^5 + 15*(16*a^4*b^2 + 40*a^2*b^4 + 2
9*b^6)*cosh(x)^4 + 15*(165*b^6*cosh(x)^8 - 264*a*b^5*cosh(x)^7 + 420*(a^2*b
^4 + 2*b^6)*cosh(x)^6 + 16*a^4*b^2 + 40*a^2*b^4 + 29*b^6 - 168*(4*a^3*b^3 +
 9*a*b^5)*cosh(x)^5 + 70*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^4 - 192
0*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^2 - 280*(8*a^5*b + 22*a^3*b
^3 + 19*a*b^5)*cosh(x)^3 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x))*si
nh(x)^4 + 20*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^3 + 20*(55*b^6*cosh(x)^9 - 99*a*
b^5*cosh(x)^8 + 180*(a^2*b^4 + 2*b^6)*cosh(x)^7 - 84*(4*a^3*b^3 + 9*a*b^5)*
cosh(x)^6 + 4*a^3*b^3 + 9*a*b^5 + 42*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cos
h(x)^5 - 1920*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^3 - 210*(8*a^5*
b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^4 + 60*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)
cosh(x)^2 + 3(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^3 + 30*(
a^2*b^4 + 2*b^6)*cosh(x)^2 + 30*(11*b^6*cosh(x)^10 - 22*a*b^5*cosh(x)^9 + 4
5*(a^2*b^4 + 2*b^6)*cosh(x)^8 - 24*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^7 + 14*(16
*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^6 + a^2*b^4 + 2*b^6 - 960*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^4 - 84*(8*a^5*b + 22*a^3*b^3 + 19*a*b^
5)*cosh(x)^5 + 40*(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^3 + 3*(16*a^4*b
^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^2 + 2*(4*a^3*b^3 + 9*a*b^5)*cosh(x))*sinh
(x)^2 + 1920*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5*sinh(x) + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^4*sinh(x)^2 + 20*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(
x)^3*sinh(x)^3 + 15*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2*sinh(x)^4
 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*sinh(x)^6)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)))
 + 12*(5*b^6*cosh(x)^11 - 11*a*b^5*cosh(x)^10 + 25*(a^2*b^4 + 2*b^6)*cosh(x
)^9 - 15*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^8 + 10*(16*a^4*b^2 + 40*a^2*b^4 + 29
*b^6)*cosh(x)^7 - 960*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x*cosh(x)^5 - 70*
(8*a^5*b + 22*a^3*b^3 + 19*a*b^5)*cosh(x)^6 + a*b^5 + 50*(8*a^5*b + 22*a^3*
b^3 + 19*a*b^5)*cosh(x)^4 + 5*(16*a^4*b^2 + 40*a^2*b^4 + 29*b^6)*cosh(x)^3
+ 5*(4*a^3*b^3 + 9*a*b^5)*cosh(x)^2 + 5*(a^2*b^4 + 2*b^6)*cosh(x))*sinh(x))
/(b^7*cosh(x)^6 + 6*b^7*cosh(x)^5*sinh(x) + 15*b^7*cosh(x)^4*sinh(x)^2 + 20
*b^7*cosh(x)^3*sinh(x)^3 + 15*b^7*cosh(x)^2*sinh(x)^4 + 6*b^7*cosh(x)*sinh(
x)^5 + b^7*sinh(x)^6)

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**7/(a+b*sinh(x)),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(128) = 256$.

Time = 0.22 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.23

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = -\frac{(12\,ab^4e^{(-x)} - 5\,b^5 - 30\,(a^2b^3 + 2\,b^5)e^{(-2\,x)} + 20\,(4\,a^3b^2 + 9\,ab^4)e^{(-3\,x)} - 15\,(16\,a^4b + 40\,a^2b^3 + 29\,b^5)e^{(-}}{1920\,b^6} + \frac{12\,ab^4e^{(-5\,x)} + 5\,b^5e^{(-6\,x)} + 120\,(8\,a^5 + 22\,a^3b^2 + 19\,ab^4)e^{(-x)} + 15\,(16\,a^4b + 40\,a^2b^3 + 29\,b^5)e^{(-2\,x)} + 20}{1920\,b^6} + \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)x}{b^7} + \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b^7}$$

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 + 2*b^5)*e^(-2*x) + 20*(4*a^
3*b^2 + 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-4*x) +
120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 + 1/1920*(12*a*b^
4*e^(-5*x) + 5*b^5*e^(-6*x) + 120*(8*a^5 + 22*a^3*b^2 + 19*a*b^4)*e^(-x) +
15*(16*a^4*b + 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 + 9*a*b^4)*e^(
-3*x) + 30*(a^2*b^3 + 2*b^5)*e^(-4*x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*x/b^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(-2*a*e^(-x) + b*e^(-2
*x) - b)/b^7
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.84

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \frac{5\,b^5\left(e^{(-x)} - e^x\right)^6 + 12\,ab^4\left(e^{(-x)} - e^x\right)^5 + 30\,a^2b^3\left(e^{(-x)} - e^x\right)^4 + 90\,b^5\left(e^{(-x)} - e^x\right)^4 + 80\,a^3b^2\left(e^{(-x)} - e^x\right)^3}{} + \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{b^7}$$

```
[In] integrate(cosh(x)^7/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/1920*(5*b^5*(e^(-x) - e^x)^6 + 12*a*b^4*(e^(-x) - e^x)^5 + 30*a^2*b^3*(e^
(-x) - e^x)^4 + 90*b^5*(e^(-x) - e^x)^4 + 80*a^3*b^2*(e^(-x) - e^x)^3 + 240
*a*b^4*(e^(-x) - e^x)^3 + 240*a^4*b*(e^(-x) - e^x)^2 + 720*a^2*b^3*(e^(-x)
- e^x)^2 + 720*b^5*(e^(-x) - e^x)^2 + 960*a^5*(e^(-x) - e^x) + 2880*a^3*b^2
*(e^(-x) - e^x) + 2880*a*b^4*(e^(-x) - e^x))/b^6 + (a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^7
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.08

$$\int \frac{\cosh^7(x)}{a+b\sinh(x)}\,dx = \frac{e^{-6x}}{384\,b}+\frac{e^{6x}}{384\,b}+\frac{e^{-x}\,(8\,a^5+22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6}+\frac{e^{-3x}\,(4\,a^3+9\,a\,b^2)}{96\,b^4} - \frac{e^{3x}\,(4\,a^3+9\,a\,b^2)}{96\,b^4}+\frac{e^{-4x}\,(a^2+2\,b^2)}{64\,b^3}+\frac{e^{4x}\,(a^2+2\,b^2)}{64\,b^3}+\frac{a\,e^{-5x}}{160\,b^2} - \frac{a\,e^{5x}}{160\,b^2}-\frac{x\,(a^2+b^2)^3}{b^7}+\frac{e^{-2x}\,(16\,a^4+40\,a^2\,b^2+29\,b^4)}{128\,b^5} + \frac{e^{2x}\,(16\,a^4+40\,a^2\,b^2+29\,b^4)}{128\,b^5}-\frac{e^{x}\,(8\,a^5+22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6} + \frac{\ln\left(2\,a\,e^{x}-b+b\,e^{2x}\right)\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}{b^7}$$

```
[In] int(cosh(x)^7/(a + b*sinh(x)),x)
```

```
[Out] exp(-6*x)/(384*b) + exp(6*x)/(384*b) + (exp(-x)*(19*a*b^4 + 8*a^5 + 22*a^3*
b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 + 4*a^3))/(96*b^4) - (exp(3*x)*(9*a*b^
2 + 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 + 2*b^2))/(64*b^3) + (exp(4*x)*(a^2
+ 2*b^2))/(64*b^3) + (a*exp(-5*x))/(160*b^2) - (a*exp(5*x))/(160*b^2) - (x*
(a^2 + b^2)^3)/b^7 + (exp(-2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b^5) +
 (exp(2*x)*(16*a^4 + 29*b^4 + 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*b^4 +
8*a^5 + 22*a^3*b^2))/(16*b^6) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2))/b^7
```

3.188 $\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 145

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6}-\frac{2(a^2+b^2)^{5/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh^5(x)}{5b}+\frac{\cosh^3(x)\left(4(a^2+b^2)-3ab\sinh(x)\right)}{12b^3}+\frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5}$$

```
[Out] -1/8*a*(8*a^4+20*a^2*b^2+15*b^4)*x/b^6-2*(a^2+b^2)^(5/2)*arctanh((b-a*tanh(
1/2*x))/(a^2+b^2)^(1/2))/b^6+1/5*cosh(x)^5/b+1/12*cosh(x)^3*(4*a^2+4*b^2-3*
a*b*sinh(x))/b^3+1/8*cosh(x)*(8*(a^2+b^2)^2-a*b*(4*a^2+7*b^2)*sinh(x))/b^5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2774, 2944, 2814, 2739, 632, 212}

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = -\frac{2(a^2+b^2)^{5/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6}+\frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5}+\frac{\cosh^3(x)\left(4(a^2+b^2)-3ab\sinh(x)\right)}{12b^3}-\frac{ax(8a^4+20a^2b^2+15b^4)}{8b^6}+\frac{\cosh^5(x)}{5b}$$

```
[In] Int[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] -1/8*(a*(8*a^4 + 20*a^2*b^2 + 15*b^4)*x)/b^6 - (2*(a^2 + b^2)^(5/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/b^6 + Cosh[x]^5/(5*b) + (Cosh[x]^3*(4*
(a^2 + b^2) - 3*a*b*Sinh[x]))/(12*b^3) + (Cosh[x]*(8*(a^2 + b^2)^2 - a*b*(4
*a^2 + 7*b^2)*Sinh[x]))/(8*b^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
```

```
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{\cosh^5(x)}{5b} + \frac{i\int \frac{\cosh^4(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3} - \frac{i\int \frac{\cosh^2(x)(ib(a^2+4b^2)-ia(4a^2+7b^2)\sinh(x))}{a+b\sinh(x)}\,dx}{4b^3}$$

$$= \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3} + \frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5} + \frac{i\int \frac{-ib(4a^4+9a^2b^2+8b^4)+ia(8a^4+20a^2b^2+15b^4)\sinh(x)}{a+b\sinh(x)}\,dx}{8b^5}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3} + \frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5} + \frac{(a^2+b^2)^3\int \frac{1}{a+b\sinh(x)}\,dx}{b^6}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3} + \frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5} + \frac{\left(2(a^2+b^2)^3\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^6}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3} + \frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5} - \frac{\left(4(a^2+b^2)^3\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^6}$$

$$= -\frac{a(8a^4+20a^2b^2+15b^4)\,x}{8b^6} - \frac{2(a^2+b^2)^{5/2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^6} + \frac{\cosh^5(x)}{5b} + \frac{\cosh^3(x)\,(4(a^2+b^2)-3ab\sinh(x))}{12b^3} + \frac{\cosh(x)\left(8(a^2+b^2)^2-ab(4a^2+7b^2)\sinh(x)\right)}{8b^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.68 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.19

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\cosh(x)\Bigg(8(15a^4+35a^2b^2+23b^4)-15ab(4a^2+9b^2)\sinh(x)+8b^2(5a^2+11b^2)\sinh^2(x)-30ab^3\sinh^3(x)\cdots}{\cdots}$$

```
[In] Integrate[Cosh[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(8*(15*a^4 + 35*a^2*b^2 + 23*b^4) - 15*a*b*(4*a^2 + 9*b^2)*Sinh[x]
 + 8*b^2*(5*a^2 + 11*b^2)*Sinh[x]^2 - 30*a*b^3*Sinh[x]^3 + 24*b^4*Sinh[x]^4
 - (30*(-1)^(3/4)*Sqrt[b]*(8*a^4 - (4*I)*a^3*b + 16*a^2*b^2 - (7*I)*a*b^3 +
 8*b^4)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b
))])/Sqrt[b]])/(Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))]) - (240*(a^2 + b^2)^2*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/(a - I*b
))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]])/(Sqrt[-((b*(-I + Sinh[x]))/(a +
 I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]) + (240*(a - I*b)^(5/2)*(a + I
*b)^(3/2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqr
t[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])])/(Sqrt[-((b*(-I + Sinh[x
]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(120*b^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(132) = 264.

Time = 57.61 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.37

method	result
risch	$-\frac{a^5x}{b^6}-\frac{5a^3x}{2b^4}-\frac{15ax}{8b^2}+\frac{e^{5x}}{160b}-\frac{a\,e^{4x}}{64b^2}+\frac{e^{3x}a^2}{24b^3}+\frac{7\,e^{3x}}{96b}-\frac{a^3e^{2x}}{8b^4}-\frac{a\,e^{2x}}{4b^2}+\frac{e^{x}a^4}{2b^5}+\frac{9\,e^{x}a^2}{8b^3}+\frac{11\,e^{x}}{16b}+\frac{e^{-x}a^4}{2b^5}+\cdots$
default	$-\frac{2(-a^6-3a^4b^2-3a^2b^4-b^6)\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^6\sqrt{a^2+b^2}}-\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)-1\right)^5}-\frac{2b+a}{4b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}-\frac{4a^2+6ab+13b^2}{12b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\cdots$

```
[In] int(cosh(x)^6/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a^5*x/b^6-5/2*a^3*x/b^4-15/8*a*x/b^2+1/160/b*exp(x)^5-1/64*a/b^2*exp(x)^4+
1/24/b^3*exp(x)^3*a^2+7/96/b*exp(x)^3-1/8*a^3/b^4*exp(x)^2-1/4*a/b^2*exp(x)
```

```
^2+1/2/b^5*exp(x)*a^4+9/8/b^3*exp(x)*a^2+11/16/b*exp(x)+1/2/b^5/exp(x)*a^4+
9/8/b^3/exp(x)*a^2+11/16/b/exp(x)+1/8*a^3/b^4/exp(x)^2+1/4*a/b^2/exp(x)^2+1
/24/b^3/exp(x)^3*a^2+7/96/b/exp(x)^3+1/64*a/b^2/exp(x)^4+1/160/b/exp(x)^5+(
a^2+b^2)^(5/2)/b^6*ln(exp(x)-((a^2+b^2)^(5/2)-a^5-2*a^3*b^2-a*b^4)/b/(a^4+2
*a^2*b^2+b^4))-(a^2+b^2)^(5/2)/b^6*ln(exp(x)+((a^2+b^2)^(5/2)+a^5+2*a^3*b^2
+a*b^4)/b/(a^4+2*a^2*b^2+b^4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. $2(133) = 266$.

Time = 0.32 (sec) , antiderivative size = 1486, normalized size of antiderivative = 10.25

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b^5
*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 + 7*b^5)*cosh(x)^8 + 5*(54*b^5*
cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 + 14*b^5)*sinh(x)^8 - 120*(a^3*b^2
 + 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*a^3*b
^2 - 12*a*b^4 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b + 54*a^2*b^3 + 3
3*b^5 + 14*(4*a^2*b^3 + 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 + 2*a*b^4)*cosh(x))*
sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*cosh(x)^4 +
 280*(4*a^2*b^3 + 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 + 2*a*b^4)*cosh(x)^2 - 6
0*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*c
osh(x))*sinh(x)^5 + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^4 +
10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 + 48*a^4*b + 108*a^2*b^3 + 66*b
^5 + 70*(4*a^2*b^3 + 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 -
 60*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b + 18*a^2*b^3 +
11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 + 2*a*b^4)*cosh(x)^3 + 20*(36*b
^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 + 7*b^5)*cosh(x)^5 + 6*a^
3*b^2 + 12*a*b^4 - 210*(a^3*b^2 + 2*a*b^4)*cosh(x)^4 - 60*(8*a^5 + 20*a^3*b
^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 +
 12*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x)^3 + 10*(4*a^2*b^3 + 7*
b^5)*cosh(x)^2 + 10*(27*b^5*cosh(x)^8 - 54*a*b^4*cosh(x)^7 + 28*(4*a^2*b^3
+ 7*b^5)*cosh(x)^6 - 252*(a^3*b^2 + 2*a*b^4)*cosh(x)^5 + 4*a^2*b^3 + 7*b^5
- 120*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^3 + 90*(8*a^4*b + 18*a^2*b^
3 + 11*b^5)*cosh(x)^4 + 36*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^2 + 36*(
a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x)^2 + 960*((a^4 + 2*a^2*b^2 + b^4)*cosh(x
)^5 + 5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4*sinh(x) + 10*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)^3*sinh(x)^2 + 10*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^3 +
```

```
5*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^4 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^5)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) + 5*(12*b^5*cosh(x)^9 - 27*a*b^4*cosh(x)^8 + 16*(4*a^2*b^
3 + 7*b^5)*cosh(x)^7 - 168*(a^3*b^2 + 2*a*b^4)*cosh(x)^6 - 120*(8*a^5 + 20*
a^3*b^2 + 15*a*b^4)*x*cosh(x)^4 + 72*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x
)^5 + 3*a*b^4 + 48*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*cosh(x)^3 + 72*(a^3*b^2
+ 2*a*b^4)*cosh(x)^2 + 4*(4*a^2*b^3 + 7*b^5)*cosh(x))*sinh(x))/(b^6*cosh(x)
^5 + 5*b^6*cosh(x)^4*sinh(x) + 10*b^6*cosh(x)^3*sinh(x)^2 + 10*b^6*cosh(x)^
2*sinh(x)^3 + 5*b^6*cosh(x)*sinh(x)^4 + b^6*sinh(x)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**6/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(133) = 266.

Time = 0.30 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.95

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = -\frac{\left(15\,ab^3e^{(-x)} - 6\,b^4 - 10\,(4\,a^2b^2 + 7\,b^4)e^{(-2\,x)} + 120\,(a^3b + 2\,ab^3)e^{(-3\,x)} - 60\,(8\,a^4 + 18\,a^2b^2 + 11\,b^4)e^{(-4\,x)} \cdots}{960\,b^5} + \frac{15\,ab^3e^{(-4\,x)} + 6\,b^4e^{(-5\,x)} + 60\,(8\,a^4 + 18\,a^2b^2 + 11\,b^4)e^{(-x)} + 120\,(a^3b + 2\,ab^3)e^{(-2\,x)} + 10\,(4\,a^2b^2 + 7\,b^4)e \cdots}{960\,b^5} - \frac{(8\,a^5 + 20\,a^3b^2 + 15\,ab^4)x}{8\,b^6} + \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^6}$$

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/960*(15*a*b^3*e^(-x) - 6*b^4 - 10*(4*a^2*b^2 + 7*b^4)*e^(-2*x) + 120*(a^
3*b + 2*a*b^3)*e^(-3*x) - 60*(8*a^4 + 18*a^2*b^2 + 11*b^4)*e^(-4*x))*e^(5*x
)/b^5 + 1/960*(15*a*b^3*e^(-4*x) + 6*b^4*e^(-5*x) + 60*(8*a^4 + 18*a^2*b^2
+ 11*b^4)*e^(-x) + 120*(a^3*b + 2*a*b^3)*e^(-2*x) + 10*(4*a^2*b^2 + 7*b^4)*
e^(-3*x))/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + (a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(133) = 266$.

Time = 0.28 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.99

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} + 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} - 240\,ab^3e^{(2\,x)} + 480\,a^4e^x + 1080\,a^2b^2e^x + \cdots}{960\,b^5}$$
$$- \frac{(8\,a^5 + 20\,a^3b^2 + 15\,ab^4)x}{8\,b^6}$$
$$+ \frac{(15\,ab^4e^x + 6\,b^5 + 60\,(8\,a^4b + 18\,a^2b^3 + 11\,b^5)e^{(4\,x)} + 120\,(a^3b^2 + 2\,ab^4)e^{(3\,x)} + 10\,(4\,a^2b^3 + 7\,b^5)e^{(2\,x)})e^{(\cdots}}{960\,b^6}$$
$$+ \frac{(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^6}$$

```
[In] integrate(cosh(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) + 70*b^4*e^(3*
x) - 120*a^3*b*e^(2*x) - 240*a*b^3*e^(2*x) + 480*a^4*e^x + 1080*a^2*b^2*e^x
 + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*(15
*a*b^4*e^x + 6*b^5 + 60*(8*a^4*b + 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(a^3*
b^2 + 2*a*b^4)*e^(3*x) + 10*(4*a^2*b^3 + 7*b^5)*e^(2*x))*e^(-5*x)/b^6 + (a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))
/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6)
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.08

$$\int \frac{\cosh^6(x)}{a+b\sinh(x)}\,dx = \frac{e^{-5\,x}}{160\,b} + \frac{e^{5\,x}}{160\,b} - \frac{\ln\left(-\frac{2\,e^x\,{\left(a^2+b^2\right)}^3}{b^7} - \frac{2\,\left(b-a\,e^x\right)\,{\left(a^2+b^2\right)}^{5/2}}{b^7}\right)\,{\left(a^2+b^2\right)}^{5/2}}{b^6}$$
$$+ \frac{\ln\left(\frac{2\,\left(b-a\,e^x\right)\,{\left(a^2+b^2\right)}^{5/2}}{b^7} - \frac{2\,e^x\,{\left(a^2+b^2\right)}^3}{b^7}\right)\,{\left(a^2+b^2\right)}^{5/2}}{b^6}$$
$$- \frac{x\,\left(8\,a^5+20\,a^3\,b^2+15\,a\,b^4\right)}{8\,b^6} + \frac{e^x\,\left(8\,a^4+18\,a^2\,b^2+11\,b^4\right)}{16\,b^5}$$
$$+ \frac{a\,e^{-4\,x}}{64\,b^2} - \frac{a\,e^{4\,x}}{64\,b^2} + \frac{e^{-x}\,\left(8\,a^4+18\,a^2\,b^2+11\,b^4\right)}{16\,b^5} + \frac{e^{-3\,x}\,\left(4\,a^2+7\,b^2\right)}{96\,b^3}$$
$$+ \frac{e^{3\,x}\,\left(4\,a^2+7\,b^2\right)}{96\,b^3} + \frac{e^{-2\,x}\,\left(a^3+2\,a\,b^2\right)}{8\,b^4} - \frac{e^{2\,x}\,\left(a^3+2\,a\,b^2\right)}{8\,b^4}$$

```
[In] int(cosh(x)^6/(a + b*sinh(x)),x)
```

```
[Out] exp(-5*x)/(160*b) + exp(5*x)/(160*b) - (log(- (2*exp(x)*(a^2 + b^2)^3)/b^7
- (2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7)*(a^2 + b^2)^(5/2))/b^6 + (log((
2*(b - a*exp(x))*(a^2 + b^2)^(5/2))/b^7 - (2*exp(x)*(a^2 + b^2)^3)/b^7)*(a^
2 + b^2)^(5/2))/b^6 - (x*(15*a*b^4 + 8*a^5 + 20*a^3*b^2))/(8*b^6) + (exp(x)
*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x))/(64*b^2) - (a*exp(
4*x))/(64*b^2) + (exp(-x)*(8*a^4 + 11*b^4 + 18*a^2*b^2))/(16*b^5) + (exp(-3
*x)*(4*a^2 + 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 + 7*b^2))/(96*b^3) + (exp(
-2*x)*(2*a*b^2 + a^3))/(8*b^4) - (exp(2*x)*(2*a*b^2 + a^3))/(8*b^4)
```

3.189 $\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5} - \frac{a(a^2+2b^2)\sinh(x)}{b^4} + \frac{(a^2+2b^2)\sinh^2(x)}{2b^3} - \frac{a\sinh^3(x)}{3b^2} + \frac{\sinh^4(x)}{4b}$$

```
[Out] (a^2+b^2)^2*ln(a+b*sinh(x))/b^5-a*(a^2+2*b^2)*sinh(x)/b^4+1/2*(a^2+2*b^2)*s
inh(x)^2/b^3-1/3*a*sinh(x)^3/b^2+1/4*sinh(x)^4/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2747, 711}

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5} - \frac{a(a^2+2b^2)\sinh(x)}{b^4} + \frac{(a^2+2b^2)\sinh^2(x)}{2b^3} - \frac{a\sinh^3(x)}{3b^2} + \frac{\sinh^4(x)}{4b}$$

```
[In] Int[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)^2*Log[a + b*Sinh[x]])/b^5 - (a*(a^2 + 2*b^2)*Sinh[x])/b^4 + ((
a^2 + 2*b^2)*Sinh[x]^2)/(2*b^3) - (a*Sinh[x]^3)/(3*b^2) + Sinh[x]^4/(4*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{(-b^2-x^2)^2}{a+x}\,dx, x, b\sinh(x)\right)}{b^5}$$

$$= \frac{\text{Subst}\left(\int \left(-a(a^2+2b^2)+(a^2+2b^2)\,x-ax^2+x^3+\frac{(a^2+b^2)^2}{a+x}\right)\,dx, x, b\sinh(x)\right)}{b^5}$$

$$= \frac{(a^2+b^2)^2\log(a+b\sinh(x))}{b^5} - \frac{a(a^2+2b^2)\sinh(x)}{b^4} + \frac{(a^2+2b^2)\sinh^2(x)}{2b^3} - \frac{a\sinh^3(x)}{3b^2} + \frac{\sinh^4(x)}{4b}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{3b^4\cosh^4(x)+12(a^2+b^2)^2\log(a+b\sinh(x))-12ab(a^2+2b^2)\sinh(x)+6b^2(a^2+b^2)\sinh^2(x)-4ab^3\sinh}{12b^5}$$

```
[In] Integrate[Cosh[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (3*b^4*Cosh[x]^4 + 12*(a^2 + b^2)^2*Log[a + b*Sinh[x]] - 12*a*b*(a^2 + 2*b^
2)*Sinh[x] + 6*b^2*(a^2 + b^2)*Sinh[x]^2 - 4*a*b^3*Sinh[x]^3)/(12*b^5)
```

Maple [A] (verified)

Time = 23.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{-\frac{\sinh(x)^4 b^3}{4}+\frac{a\,b^2\sinh(x)^3}{3}-\frac{\left(a^2+2b^2\right)\sinh(x)^2 b}{2}+a\left(a^2+2b^2\right)\sinh(x)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\sinh(x))}{b^5}$
default	$-\frac{-\frac{\sinh(x)^4 b^3}{4}+\frac{a\,b^2\sinh(x)^3}{3}-\frac{\left(a^2+2b^2\right)\sinh(x)^2 b}{2}+a\left(a^2+2b^2\right)\sinh(x)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\sinh(x))}{b^5}$
risch	$-\frac{x\,a^4}{b^5}-\frac{2x\,a^2}{b^3}-\frac{x}{b}+\frac{e^{4x}}{64b}-\frac{a\,e^{3x}}{24b^2}+\frac{e^{2x}a^2}{8b^3}+\frac{3\,e^{2x}}{16b}-\frac{a^3e^{x}}{2b^4}-\frac{7a\,e^{x}}{8b^2}+\frac{a^3e^{-x}}{2b^4}+\frac{7a\,e^{-x}}{8b^2}+\frac{e^{-2x}a^2}{8b^3}+$

```
[In] int(cosh(x)^5/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/4*sinh(x)^4*b^3+1/3*a*b^2*sinh(x)^3-1/2*(a^2+2*b^2)*sinh(x)^2*b+
a*(a^2+2*b^2)*sinh(x))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(75) = 150.

Time = 0.30 (sec) , antiderivative size = 865, normalized size of antiderivative = 10.68

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*cos
h(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^6 + 4*(21*b^4*cosh
(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 + 9*b^4)*sinh(x)^6 - 192*(a^4 + 2*a^2*
b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b + 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*cosh(
x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b - 7*a*b^3 + 3*(2*a^2*b^2 + 3*b^4)*cosh(x
))*sinh(x)^5 + 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cosh(x)^3
 + 90*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 - 96*(a^4 + 2*a^2*b^2 + b^4)*x - 60*(4*
a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 + 24*(4*a^3*b + 7*a*b^3)*cosh(x
)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 + 12*a^3*b + 21*a*b^3 + 30*(
2*a^2*b^2 + 3*b^4)*cosh(x)^3 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x) - 30*(4
*a^3*b + 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 + 3*b^4)*cosh(x)^2 +
 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 + 3*b^4)*cosh(x)^
4 + 2*a^2*b^2 + 3*b^4 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^2 - 20*(4*a^3*
b + 7*a*b^3)*cosh(x)^3 + 6*(4*a^3*b + 7*a*b^3)*cosh(x))*sinh(x)^2 + 192*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3*sinh(
x) + 6*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4 + 2*a^2*b^2 + b
^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^4)*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3*cosh(x)^6 + 9*(2*
```

```
a^2*b^2 + 3*b^4)*cosh(x)^5 - 96*(a^4 + 2*a^2*b^2 + b^4)*x*cosh(x)^3 - 15*(4
*a^3*b + 7*a*b^3)*cosh(x)^4 + a*b^3 + 9*(4*a^3*b + 7*a*b^3)*cosh(x)^2 + 3*(
2*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(
x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(75) = 150$.

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.22

$$\begin{aligned}\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx &= -\frac{\left(8\,ab^2e^{(-x)} - 3\,b^3 - 12\,(2\,a^2b + 3\,b^3)e^{(-2\,x)} + 24\,(4\,a^3 + 7\,ab^2)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4} \\ &\quad + \frac{8\,ab^2e^{(-3\,x)} + 3\,b^3e^{(-4\,x)} + 24\,(4\,a^3 + 7\,ab^2)e^{(-x)} + 12\,(2\,a^2b + 3\,b^3)e^{(-2\,x)}}{192\,b^4} \\ &\quad + \frac{(a^4 + 2\,a^2b^2 + b^4)x}{b^5} + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b^5}\end{aligned}$$

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b + 3*b^3)*e^(-2*x) + 24*(4*a^3
+ 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 + 1/192*(8*a*b^2*e^(-3*x) + 3*b^3*e^(-4*x)
 + 24*(4*a^3 + 7*a*b^2)*e^(-x) + 12*(2*a^2*b + 3*b^3)*e^(-2*x))/b^4 + (a^4
+ 2*a^2*b^2 + b^4)*x/b^5 + (a^4 + 2*a^2*b^2 + b^4)*log(-2*a*e^(-x) + b*e^(-
2*x) - b)/b^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.72

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{3\,b^3\left(e^{(-x)}-e^x\right)^4+8\,ab^2\left(e^{(-x)}-e^x\right)^3+24\,a^2b\left(e^{(-x)}-e^x\right)^2+48\,b^3\left(e^{(-x)}-e^x\right)^2+96\,a^3\left(e^{(-x)}-e^x\right)+19}{192\,b^4} + \frac{\left(a^4+2\,a^2b^2+b^4\right)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^5}$$

```
[In] integrate(cosh(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/192*(3*b^3*(e^(-x) - e^x)^4 + 8*a*b^2*(e^(-x) - e^x)^3 + 24*a^2*b*(e^(-x)
 - e^x)^2 + 48*b^3*(e^(-x) - e^x)^2 + 96*a^3*(e^(-x) - e^x) + 192*a*b^2*(e^
(-x) - e^x))/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(-b*(e^(-x) - e^x) + 2*a)
)/b^5
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.09

$$\int \frac{\cosh^5(x)}{a+b\sinh(x)}\,dx = \frac{\mathrm{e}^{-4\,x}}{64\,b}+\frac{\mathrm{e}^{4\,x}}{64\,b}+\frac{\ln\left(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{b^5} + \frac{\mathrm{e}^{-x}\,\left(4\,a^3+7\,a\,b^2\right)}{8\,b^4}+\frac{a\,\mathrm{e}^{-3\,x}}{24\,b^2}-\frac{a\,\mathrm{e}^{3\,x}}{24\,b^2}-\frac{x\,{\left(a^2+b^2\right)}^2}{b^5} + \frac{\mathrm{e}^{-2\,x}\,\left(2\,a^2+3\,b^2\right)}{16\,b^3}+\frac{\mathrm{e}^{2\,x}\,\left(2\,a^2+3\,b^2\right)}{16\,b^3}-\frac{\mathrm{e}^x\,\left(4\,a^3+7\,a\,b^2\right)}{8\,b^4}$$

```
[In] int(cosh(x)^5/(a + b*sinh(x)),x)
```

```
[Out] exp(-4*x)/(64*b) + exp(4*x)/(64*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^4
 + b^4 + 2*a^2*b^2))/b^5 + (exp(-x)*(7*a*b^2 + 4*a^3))/(8*b^4) + (a*exp(-3*
x))/(24*b^2) - (a*exp(3*x))/(24*b^2) - (x*(a^2 + b^2)^2)/b^5 + (exp(-2*x)*(
2*a^2 + 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 + 3*b^2))/(16*b^3) - (exp(x)*(7
*a*b^2 + 4*a^3))/(8*b^4)
```

3.190 $\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a(2a^2+3b^2)\,x}{2b^4} - \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3}$$

```
[Out] -1/2*a*(2*a^2+3*b^2)*x/b^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*tanh(1/2*x))/(a^2
+b^2)^(1/2))/b^4+1/3*cosh(x)^3/b+1/2*cosh(x)*(2*a^2+2*b^2-a*b*sinh(x))/b^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2774, 2944, 2814, 2739, 632, 212}

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = -\frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} - \frac{ax(2a^2+3b^2)}{2b^4} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} + \frac{\cosh^3(x)}{3b}$$

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(a*(2*a^2 + 3*b^2)*x)/b^4 - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b - a*Tanh[x
/2])/Sqrt[a^2 + b^2]])/b^4 + Cosh[x]^3/(3*b) + (Cosh[x]*(2*(a^2 + b^2) - a*
b*Sinh[x]))/(2*b^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{\cosh^3(x)}{3b} + \frac{i \int \frac{\cosh^2(x)(-ib+ia\sinh(x))}{a+b\sinh(x)}\,dx}{b}$$

$$= \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} - \frac{i\int \frac{ib(a^2+2b^2)-ia(2a^2+3b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} + \frac{(a^2+b^2)^2\int \frac{1}{a+b\sinh(x)}\,dx}{b^4}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} + \frac{\left(2(a^2+b^2)^2\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^4}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3} - \frac{\left(4(a^2+b^2)^2\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^4}$$

$$= -\frac{a(2a^2+3b^2)\,x}{2b^4} - \frac{2(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} + \frac{\cosh^3(x)}{3b} + \frac{\cosh(x)\left(2(a^2+b^2)-ab\sinh(x)\right)}{2b^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.76 (sec) , antiderivative size = 553, normalized size of antiderivative = 5.70

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{\cosh^3(x)\left(-12\sqrt{a-ib}\sqrt{a+ib}(a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)} + 12(a-ib)^2(a+ib)\text{arct}\right.}{}$$

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]^3*(-12*Sqrt[a - I*b]*Sqrt[a + I*b]*(a^2 + b^2)*ArcTanh[Sqrt[-((b*(
I + Sinh[x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I
*Sinh[x]] + 12*(a - I*b)^2*(a + I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I +
Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]
```

```
)]*Sqrt[1 + I*Sinh[x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))
]*((3 - 3*I)*Sqrt[2]*Sqrt[b]*(2*a^2 - I*a*b + 2*b^2)*ArcSin[((1/2 + I/2)*Sq
rt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*Sqrt[a - I*b
]*(3*a^2 + 4*b^2)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]
- 3*a*Sqrt[a - I*b]*b*Sqrt[1 + I*Sinh[x]]*Sinh[x]*Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))] + 2*Sqrt[a - I*b]*b^2*Sqrt[1 + I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(
I + Sinh[x]))/(a - I*b))])))/(6*(a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sqrt[1 +
I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b)))^(3/2)*(-((b*(I + Sinh[x]))/(a
- I*b)))^(3/2))
```

Maple [A] (verified)

Time = 8.01 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{a^{3} x}{b^{4}}-\frac{3 a x}{2 b^{2}}+\frac{{\mathrm e}^{3 x}}{24 b}-\frac{a \,{\mathrm e}^{2 x}}{8 b^{2}}+\frac{{\mathrm e}^{x} a^{2}}{2 b^{3}}+\frac{5 \,{\mathrm e}^{x}}{8 b}+\frac{{\mathrm e}^{-x} a^{2}}{2 b^{3}}+\frac{5 \,{\mathrm e}^{-x}}{8 b}+\frac{a \,{\mathrm e}^{-2 x}}{8 b^{2}}+\frac{{\mathrm e}^{-3 x}}{24 b}+\frac{\left(a^{2}+b^{2}\right)^{\frac{3}{2}} \ln \left({\mathrm e}^{x}-\frac{-a+\sqrt{a^{2}+b^{2}}}{b}\right)}{b^{4}}$
default	$-\frac{2 \left(-a^{4}-2 a^{2} b^{2}-b^{4}\right) \operatorname{arctanh}\left(\frac{2 a \tanh \left(\frac{x}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{b^{4} \sqrt{a^{2}+b^{2}}}-\frac{1}{3 b \left(\tanh \left(\frac{x}{2}\right)-1\right)^{3}}-\frac{a +b}{2 b^{2} \left(\tanh \left(\frac{x}{2}\right)-1\right)^{2}}-\frac{2 a^{2}+a b +3 b^{2}}{2 b^{3} \left(\tanh \left(\frac{x}{2}\right)-1\right)}+\frac{a \left(2 a^{2}+3 b^{2}\right)}{}$

```
[In] int(cosh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*x/b^4-3/2*a*x/b^2+1/24/b*exp(x)^3-1/8*a/b^2*exp(x)^2+1/2/b^3*exp(x)*a^
2+5/8/b*exp(x)+1/2/b^3/exp(x)*a^2+5/8/b/exp(x)+1/8*a/b^2/exp(x)^2+1/24/b/ex
p(x)^3+(a^2+b^2)^(3/2)/b^4*ln(exp(x)-(-a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(3/2
)/b^4*ln(exp(x)+(a+(a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(87) = 174.

Time = 0.31 (sec) , antiderivative size = 569, normalized size of antiderivative = 5.87

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{b^{3} \cosh\left(x\right)^{6} + b^{3} \sinh\left(x\right)^{6} - 3\, a b^{2} \cosh\left(x\right)^{5} + 3\, {\left(2\, b^{3} \cosh\left(x\right) - a b^{2}\right)} \sinh\left(x\right)^{5} - 12\, {\left(2\, a^{3} + 3\, a b^{2}\right)} x \cosh\left(x\right)}{}$$

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x)
- a*b^2)*sinh(x)^5 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b + 5*b^3)
*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b + 5*b^3)*sinh(x
)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(2*a^3
```

```
 + 3*a*b^2)*x + 6*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x)^3 + b^3 + 3*(4*a^2*b +
 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 + 4*a^2*b + 5*b
^3 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b + 5*b^3)*cosh(x)^2)*sinh(x
)^2 + 24*((a^2 + b^2)*cosh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2*sinh(x) + 3*(a^2
+ b^2)*cosh(x)*sinh(x)^2 + (a^2 + b^2)*sinh(x)^3)*sqrt(a^2 + b^2)*log((b^2*
cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) +
a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2
+ b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 3*(2*b^3*co
sh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 + 3*a*b^2)*x*cosh(x)^2 + 4*(4*a^2*b
 + 5*b^3)*cosh(x)^3 + a*b^2 + 2*(4*a^2*b + 5*b^3)*cosh(x))*sinh(x))/(b^4*co
sh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.75

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = -\frac{\left(3\,abe^{(-x)}-b^2-3\,(4\,a^2+5\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,b^3} + \frac{3\,abe^{(-2\,x)}+b^2e^{(-3\,x)}+3\,(4\,a^2+5\,b^2)e^{(-x)}}{24\,b^3} - \frac{(2\,a^3+3\,ab^2)x}{2\,b^4} + \frac{(a^4+2\,a^2b^2+b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^4}$$

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - b^2 - 3*(4*a^2 + 5*b^2)*e^(-2*x))*e^(3*x)/b^3 + 1/24*
(3*a*b*e^(-2*x) + b^2*e^(-3*x) + 3*(4*a^2 + 5*b^2)*e^(-x))/b^3 - 1/2*(2*a^3
 + 3*a*b^2)*x/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - sqrt(a^2 +
b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.73

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{b^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^2e^x + 15\,b^2e^x}{24\,b^3} - \frac{(2\,a^3 + 3\,ab^2)x}{2\,b^4} + \frac{\left(3\,ab^2e^x + b^3 + 3\,(4\,a^2b + 5\,b^3)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4} + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(\frac{\left|2\,be^x + 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x + 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}$$

```
[In] integrate(cosh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 15*b^2*e^x)/b^3 - 1/2*(2*a
^3 + 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x + b^3 + 3*(4*a^2*b + 5*b^3)*e^(2*x)
)*e^(-3*x)/b^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2
 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4)
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.06

$$\int \frac{\cosh^4(x)}{a+b\sinh(x)}\,dx = \frac{\mathrm{e}^{-3\,x}}{24\,b} + \frac{\mathrm{e}^{3\,x}}{24\,b} - \frac{\ln\left(-\frac{2\,\mathrm{e}^x\,{(a^2+b^2)}^2}{b^5} - \frac{2\,(b-a\,\mathrm{e}^x)\,{(a^2+b^2)}^{3/2}}{b^5}\right)\,{(a^2+b^2)}^{3/2}}{b^4} + \frac{\ln\left(\frac{2\,(b-a\,\mathrm{e}^x)\,{(a^2+b^2)}^{3/2}}{b^5} - \frac{2\,\mathrm{e}^x\,{(a^2+b^2)}^2}{b^5}\right)\,{(a^2+b^2)}^{3/2}}{b^4} - \frac{x\,(2\,a^3+3\,a\,b^2)}{2\,b^4} + \frac{\mathrm{e}^x\,(4\,a^2+5\,b^2)}{8\,b^3} + \frac{a\,\mathrm{e}^{-2\,x}}{8\,b^2} - \frac{a\,\mathrm{e}^{2\,x}}{8\,b^2} + \frac{\mathrm{e}^{-x}\,(4\,a^2+5\,b^2)}{8\,b^3}$$

```
[In] int(cosh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] exp(-3*x)/(24*b) + exp(3*x)/(24*b) - (log(- (2*exp(x)*(a^2 + b^2)^2)/b^5 -
(2*(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5)*(a^2 + b^2)^(3/2))/b^4 + (log((2*
(b - a*exp(x))*(a^2 + b^2)^(3/2))/b^5 - (2*exp(x)*(a^2 + b^2)^2)/b^5)*(a^2
+ b^2)^(3/2))/b^4 - (x*(3*a*b^2 + 2*a^3))/(2*b^4) + (exp(x)*(4*a^2 + 5*b^2)
)/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) + (exp(-x)*(4*a^2
+ 5*b^2))/(8*b^3)
```

3.191 $\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx=\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

```
[Out] (a^2+b^2)*ln(a+b*sinh(x))/b^3-a*sinh(x)/b^2+1/2*sinh(x)^2/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2747, 711}

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx=\frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}$$

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Sinh[x]])/b^3 - (a*Sinh[x])/b^2 + Sinh[x]^2/(2*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{a+x}\,dx, x, b\sinh(x)\right)}{b^3}\\
&= -\frac{\text{Subst}\left(\int \left(a-x+\frac{-a^2-b^2}{a+x}\right)\,dx, x, b\sinh(x)\right)}{b^3}\\
&= \frac{(a^2+b^2)\log(a+b\sinh(x))}{b^3}-\frac{a\sinh(x)}{b^2}+\frac{\sinh^2(x)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx = -\frac{-((a^2+b^2)\log(a+b\sinh(x)))+ab\sinh(x)-\frac{1}{2}b^2\sinh^2(x)}{b^3}$$

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x]),x]
[Out] -((-((a^2 + b^2)*Log[a + b*Sinh[x]]) + a*b*Sinh[x] - (b^2*Sinh[x]^2)/2)/b^3
)
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{-\frac{b\sinh(x)^2}{2}+a\sinh(x)}{b^2}+\frac{(a^2+b^2)\ln(a+b\sinh(x))}{b^3}$	37
default	$-\frac{-\frac{b\sinh(x)^2}{2}+a\sinh(x)}{b^2}+\frac{(a^2+b^2)\ln(a+b\sinh(x))}{b^3}$	37
risch	$-\frac{x\,a^2}{b^3}-\frac{x}{b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}+\frac{a\,e^{-x}}{2b^2}+\frac{e^{-2x}}{8b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)a^2}{b^3}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}$	94

```
[In] int(cosh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] -1/b^2*(-1/2*b*sinh(x)^2+a*sinh(x))+(a^2+b^2)*ln(a+b*sinh(x))/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 5.82

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{b^2\cosh{(x)}^4 + b^2\sinh{(x)}^4 - 4\,ab\cosh{(x)}^3 - 8\,(a^2 + b^2)x\cosh{(x)}^2 + 4\,(b^2\cosh{(x)} - ab)\sinh{(x)}^3 + 4\,ab\cos}{}$$

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 + b^2)*x*cosh
(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*b^2*cosh(x)^
2 - 6*a*b*cosh(x) - 4*(a^2 + b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 + b^2)*cosh(
x)^2 + 2*(a^2 + b^2)*cosh(x)*sinh(x) + (a^2 + b^2)*sinh(x)^2)*log(2*(b*sinh
(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4*(a^2
 + b^2)*x*cosh(x) + a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) +
b^3*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^3(x)}{a + b\sinh(x)}\,dx = -\frac{\left(4\,ae^{(-x)} - b\right)e^{(2\,x)}}{8\,b^2} + \frac{4\,ae^{(-x)} + be^{(-2\,x)}}{8\,b^2} + \frac{(a^2 + b^2)x}{b^3} + \frac{(a^2 + b^2)\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b^3}$$

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 + 1/8*(4*a*e^(-x) + b*e^(-2*x))/b^2 + (a^
2 + b^2)*x/b^3 + (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx = \frac{b\left(e^{(-x)}-e^x\right)^2+4\,a\left(e^{(-x)}-e^x\right)}{8\,b^2}+\frac{(a^2+b^2)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{b^3}$$

```
[In] integrate(cosh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/8*(b*(e^(-x) - e^x)^2 + 4*a*(e^(-x) - e^x))/b^2 + (a^2 + b^2)*log(abs(-b*
(e^(-x) - e^x) + 2*a))/b^3
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int \frac{\cosh^3(x)}{a+b\sinh(x)}\,dx = \frac{e^{-2\,x}}{8\,b}+\frac{e^{2\,x}}{8\,b}+\frac{\ln\left(2\,a\,e^x-b+b\,e^{2\,x}\right)\,\left(a^2+b^2\right)}{b^3}-\frac{a\,e^x}{2\,b^2}-\frac{x\,\left(a^2+b^2\right)}{b^3}+\frac{a\,e^{-x}}{2\,b^2}$$

```
[In] int(cosh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(2*a*exp(x) - b + b*exp(2*x))*(a^2 +
 b^2))/b^3 - (a*exp(x))/(2*b^2) - (x*(a^2 + b^2))/b^3 + (a*exp(-x))/(2*b^2)
```

3.192 $\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx=-\frac{ax}{b^2}-\frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}+\frac{\cosh(x)}{b}$$

```
[Out] -a*x/b^2+cosh(x)/b-2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(
1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2774, 2814, 2739, 632, 212}

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx=-\frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2}-\frac{ax}{b^2}+\frac{\cosh(x)}{b}$$

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] -((a*x)/b^2) - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]
])/b^2 + Cosh[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2774

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Dist[g^2*((p - 1)/(b*(m + p))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh(x)}{b} + \frac{i \int \frac{-ib+ia\sinh(x)}{a+b\sinh(x)}\,dx}{b} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} + \frac{(a^2+b^2)\int \frac{1}{a+b\sinh(x)}\,dx}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} + \frac{(2(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{ax}{b^2} + \frac{\cosh(x)}{b} - \frac{(4(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2} \\
&= -\frac{ax}{b^2} - \frac{2\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2} + \frac{\cosh(x)}{b}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 396, normalized size of antiderivative = 7.33

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = \frac{\cosh(x)\left(-2\sqrt{a-ib}\sqrt{a+ib}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}+2(a-ib)\operatorname{arctanh}\left(\frac{\sqrt{a-ib}\sqrt{-\frac{b(i+\sinh(x}{a-ib}}}{\sqrt{a+ib}\sqrt{-\frac{b(-i+\sinh(}{a+ib}}}\right.\right.}{\sqrt{a-ib}\sqrt{a+i}}$$

[In] Integrate[Cosh[x]^2/(a + b*Sinh[x]),x]

[Out] (Cosh[x]*(-2*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]] + 2*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1 + I*Sinh[x]] + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*(-2*(-1)^(3/4)*Sqrt[b]*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + Sqrt[a - I*b]*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(Sqrt[a - I*b]*Sqrt[a + I*b]*b*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.72

method	result	size
risch	$-\frac{ax}{b^2}+\frac{e^x}{2b}+\frac{e^{-x}}{2b}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^x-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^x+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{b^2}$	93
default	$-\frac{2(-a^2-b^2)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}$	100

[In] int(cosh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)

[Out] -a*x/b^2+1/2/b*exp(x)+1/2/b/exp(x)+(a^2+b^2)^(1/2)/b^2*ln(exp(x)-(-a+(a^2+b^2)^(1/2))/b)-(a^2+b^2)^(1/2)/b^2*ln(exp(x)+(a+(a^2+b^2)^(1/2))/b)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(50) = 100$.

Time = 0.32 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.17

$$\int \frac{\cosh^2(x)}{a + b\sinh(x)}\,dx =$$
$$-\frac{2\,ax\cosh(x) - b\cosh(x)^2 - b\sinh(x)^2 - 2\sqrt{a^2+b^2}(\cosh(x)+\sinh(x))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cos\cdots}{b\cosh(x\cdots}\right.}{2\left(b^2\cosh(x)+b^2\,s\cdots\right.}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 + b^2)*(cosh(x
) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b
^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(
x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(
x) - b)) + 2*(a*x - b*cosh(x))*sinh(x) - b)/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(46) = 92$.

Time = 67.57 (sec) , antiderivative size = 377, normalized size of antiderivative = 6.98

$$\int \frac{\cosh^2(x)}{a + b\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(\frac{\log(\tanh(\frac{x}{2}))\tanh^2(\frac{x}{2})}{\tanh^2(\frac{x}{2})-1} - \frac{\log(\tanh(\frac{x}{2}))}{\tanh^2(\frac{x}{2})-1} - \frac{2}{\tanh^2(\frac{x}{2})-1}\right) \\ \frac{\frac{\log(\tanh(\frac{x}{2}))\tanh^2(\frac{x}{2})}{\tanh^2(\frac{x}{2})-1} - \frac{\log(\tanh(\frac{x}{2}))}{\tanh^2(\frac{x}{2})-1} - \frac{2}{\tanh^2(\frac{x}{2})-1}}{b} \\ \frac{-\frac{x\sinh^2(x)}{2} + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2}}{a} \\ -\frac{ax\tanh^2(\frac{x}{2})}{b^2\tanh^2(\frac{x}{2})-b^2} + \frac{ax}{b^2\tanh^2(\frac{x}{2})-b^2} - \frac{2b}{b^2\tanh^2(\frac{x}{2})-b^2} - \frac{\sqrt{a^2+b^2}\log\left(\tanh(\frac{x}{2}) - \frac{b}{a} - \frac{\sqrt{a^2+b^2}}{a}\right)\tanh^2(\frac{x}{2})}{b^2\tanh^2(\frac{x}{2})-b^2} + \frac{\sqrt{a^2+b^2}\log\left(\tanh(\frac{x}{2}\cdots\right.}{b^2\tanh^2(\cdots} \end{cases}$$

```
[In] integrate(cosh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(log(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x
/2))/(tanh(x/2)**2 - 1) - 2/(tanh(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), ((lo
g(tanh(x/2))*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(tanh(x/2))/(tanh(x/2)**2
 - 1) - 2/(tanh(x/2)**2 - 1))/b, Eq(a, 0)), ((-x*sinh(x)**2/2 + x*cosh(x)**
2/2 + sinh(x)*cosh(x)/2)/a, Eq(b, 0)), (-a*x*tanh(x/2)**2/(b**2*tanh(x/2)**
2 - b**2) + a*x/(b**2*tanh(x/2)**2 - b**2) - 2*b/(b**2*tanh(x/2)**2 - b**2)
```

```
 - sqrt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)*tanh(x/2)**
2/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/2) - b/a - sqrt
(a**2 + b**2)/a)/(b**2*tanh(x/2)**2 - b**2) + sqrt(a**2 + b**2)*log(tanh(x/
2) - b/a + sqrt(a**2 + b**2)/a)*tanh(x/2)**2/(b**2*tanh(x/2)**2 - b**2) - s
qrt(a**2 + b**2)*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/(b**2*tanh(x/2)
**2 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = -\frac{ax}{b^2}+\frac{e^{(-x)}}{2\,b}+\frac{e^x}{2\,b}+\frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^2}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log((b*e^(-x) - a - s
qrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = -\frac{ax}{b^2}+\frac{e^{(-x)}}{2\,b}+\frac{e^x}{2\,b}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
[Out] -a*x/b^2 + 1/2*e^(-x)/b + 1/2*e^x/b + sqrt(a^2 + b^2)*log(abs(2*b*e^x + 2*a
 - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/b^2
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^2(x)}{a+b\sinh(x)}\,dx = \frac{\mathrm{e}^{-x}}{2\,b}+\frac{\mathrm{e}^x}{2\,b}-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^4}}{b^2\,\sqrt{a^2+b^2}}+\frac{\mathrm{e}^x\,\sqrt{-b^4}}{b\,\sqrt{a^2+b^2}}\right)\,\sqrt{a^2+b^2}}{\sqrt{-b^4}}-\frac{a\,x}{b^2}$$

```
[In] int(cosh(x)^2/(a + b*sinh(x)),x)
[Out] exp(-x)/(2*b) + exp(x)/(2*b) - (2*atan((a*(-b^4)^(1/2))/(b^2*(a^2 + b^2)^(1
/2)) + (exp(x)*(-b^4)^(1/2))/(b*(a^2 + b^2)^(1/2))))*(a^2 + b^2)^(1/2))/(-b^
4)^(1/2) - (a*x)/b^2
```

3.193 $\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\frac{\log(a+b\sinh(x))}{b}$$

```
[Out] ln(a+b*sinh(x))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2747, 31}

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\frac{\log(a+b\sinh(x))}{b}$$

```
[In] Int[Cosh[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[a + b*Sinh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{b}\\ &= \frac{\log(a+b\sinh(x))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\log(a+b\sinh(x))}{b}$$

```
[In] Integrate[Cosh[x]/(a + b*Sinh[x]),x]
[Out] Log[a + b*Sinh[x]]/b
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\sinh(x))}{b}$	12
default	$\frac{\ln(a+b\sinh(x))}{b}$	12
risch	$-\frac{x}{b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{b}$	27

```
[In] int(cosh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*sinh(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = -\frac{x-\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="fricas")
[Out] -(x - log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{b}+\sinh(x)\right)}{b} & \text{for } b\neq 0\\ \frac{\sinh(x)}{a} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x)
[Out] Piecewise((log(a/b + sinh(x))/b, Ne(b, 0)), (sinh(x)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\log\left(b\sinh(x)+a\right)}{b}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="maxima")
[Out] log(b*sinh(x) + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx = \frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{b}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x)),x, algorithm="giac")
[Out] log(abs(-b*(e^(-x) - e^x) + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx=\frac{\ln\left(a+b\sinh(x)\right)}{b}$$

```
[In] int(cosh(x)/(a + b*sinh(x)),x)
```

```
[Out] log(a + b*sinh(x))/b
```

3.194 $\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx=\frac{a\arctan(\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}+\frac{b\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] a*arctan(sinh(x))/(a^2+b^2)-b*ln(cosh(x))/(a^2+b^2)+b*ln(a+b*sinh(x))/(a^2+
b^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {2747, 720, 31, 649, 210, 266}

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx=\frac{a\arctan(\sinh(x))}{a^2+b^2}+\frac{b\log(a+b\sinh(x))}{a^2+b^2}-\frac{b\log(\cosh(x))}{a^2+b^2}$$

```
[In] Int[Sech[x]/(a + b*Sinh[x]),x]
```

```
[Out] (a*ArcTan[Sinh[x]])/(a^2 + b^2) - (b*Log[Cosh[x]])/(a^2 + b^2) + (b*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\right)\\
&= \frac{b\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{a^2+b^2} + \frac{b\text{Subst}\left(\int \frac{-a+x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{b\log(a+b\sinh(x))}{a^2+b^2} + \frac{b\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2} - \frac{(ab)\text{Subst}\left(\int \frac{1}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{a\arctan(\sinh(x))}{a^2+b^2} - \frac{b\log(\cosh(x))}{a^2+b^2} + \frac{b\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 99 vs. 2(48) = 96.

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = -\frac{b\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh\right.\right.}{2\sqrt{-b^2}\,(a^2+b^2)}$$

```
[In] Integrate[Sech[x]/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a
 + b*Sinh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/(Sqrt[-b^2]*
(a^2 + b^2))
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a^2+b^2}+\frac{-b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$	64
risch	$\frac{i\ln(\mathrm{e}^x+i)a}{a^2+b^2}-\frac{\ln(\mathrm{e}^x+i)b}{a^2+b^2}-\frac{i\ln(\mathrm{e}^x-i)a}{a^2+b^2}-\frac{\ln(\mathrm{e}^x-i)b}{a^2+b^2}+\frac{b\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^x}{b}-1\right)}{a^2+b^2}$	102

```
[In] int(sech(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b/(a^2+b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a^2+b^2)*(-1/2*b*ln(1+
tanh(1/2*x)^2)+a*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{2\,a\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+b\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-b\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*arctan(cosh(x) + sinh(x)) + b*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) - b*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{sech}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

```
[In] integrate(sech(x)/(a+b*sinh(x)),x)
[Out] Integral(sech(x)/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = -\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2} + \frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2} - \frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -2*a*arctan(e^(-x))/(a^2 + b^2) + b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) - b*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^2b+b^3} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)a}{2\left(a^2+b^2\right)} - \frac{b\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\left(a^2+b^2\right)}$$

```
[In] integrate(sech(x)/(a+b*sinh(x)),x, algorithm="giac")
[Out] b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*a/(a^2 + b^2) - 1/2*b*log((e^(-x) - e^x)^2 + 4)/(a
^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.94

$$\int \frac{\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{b\,\ln\left(4\,b^3\,\mathrm{e}^{2\,x}-a^2\,b-4\,b^3+2\,a^3\,\mathrm{e}^{x}+8\,a\,b^2\,\mathrm{e}^{x}+a^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2+b^2} - \frac{\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)}{b+a\,1\mathrm{i}} - \frac{\ln\left(1+\mathrm{e}^{x}\,1\mathrm{i}\right)\,1\mathrm{i}}{a+b\,1\mathrm{i}}$$

```
[In] int(1/(cosh(x)*(a + b*sinh(x))),x)
```

```
[Out] (b*log(4*b^3*exp(2*x) - a^2*b - 4*b^3 + 2*a^3*exp(x) + 8*a*b^2*exp(x) + a^2
*b*exp(2*x)))/(a^2 + b^2) - log(exp(x) + 1i)/(a*1i + b) - (log(exp(x)*1i +
1)*1i)/(a + b*1i)
```

3.195 $\int \frac{\text{sech}^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\text{sech}^2(x)}{a+b\sinh(x)}\,dx=-\frac{2b^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{\text{sech}(x)(b+a\sinh(x))}{a^2+b^2}$$

```
[Out] -2*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+sech(x)*(
b+a*sinh(x))/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2775, 12, 2739, 632, 212}

$$\int \frac{\text{sech}^2(x)}{a+b\sinh(x)}\,dx=\frac{\text{sech}(x)(a\sinh(x)+b)}{a^2+b^2}-\frac{2b^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[In] Int[Sech[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (Se
ch[x]*(b + a*Sinh[x]))/(a^2 + b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{\int \frac{b^2}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{b^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}+\frac{(2b^2)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= \frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}-\frac{(4b^2)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\\
&= -\frac{2b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{\operatorname{sech}(x)(b+a\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2b^2\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+b\operatorname{sech}(x)+a\tanh(x)}{a^2+b^2}$$

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x]),x]
[Out] ((2*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + b*Se
ch[x] + a*Tanh[x])/(a^2 + b^2)
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{2b^2\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(-a\tanh\left(\frac{x}{2}\right)-b\right)}{(a^2+b^2)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}$	71
risch	$-\frac{2(-e^x b+a)}{(1+e^{2x})(a^2+b^2)}+\frac{b^2\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{b^2\ln\left(e^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	145

```
[In] int(sech(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-2/
(a^2+b^2)*(-a*tanh(1/2*x)-b)/(1+tanh(1/2*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(55) = 110$.

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 4.39

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = -\frac{2\,a^3+2\,ab^2-\left(b^2\cosh\left(x\right)^2+2\,b^2\cosh\left(x\right)\sinh\left(x\right)+b^2\sinh\left(x\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\cdots}{b\,c\cdots}\right.}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)^2+2\,(a^4\cdots}$$

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
[Out] -(2*a^3 + 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
+ b^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
```

```
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
 + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^
4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^
2 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(sech(x)**2/(a+b*sinh(x)),x)
[Out] Integral(sech(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(be^{(-x)}+a\right)}{a^2+b^2+(a^2+b^2)e^{(-2x)}}$$

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
[Out] b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(a^2 + b^2)^(3/2) + 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = \frac{b^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\,(be^x-a)}{(a^2+b^2)(e^{(2x)}+1)}$$

```
[In] integrate(sech(x)^2/(a+b*sinh(x)),x, algorithm="giac")
[Out] b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 321, normalized size of antiderivative = 5.44

$$\int \frac{\operatorname{sech}^2(x)}{a+b\sinh(x)}\,dx = -\frac{\frac{2a}{a^2+b^2}-\frac{2b\,e^x}{a^2+b^2}}{e^{2x}+1} - \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2}{\sqrt{b^4}\,(a^2+b^2)^2}+\frac{2a\left(a^3\sqrt{b^4}+a\,b^2\sqrt{b^4}\right)}{b^4\sqrt{-(a^2+b^2)^3}\,(a^2+b^2)\sqrt{-a^6-3a^4b^2-3a^2b^4-b^6}}\right)-\frac{2a\left(b^3\sqrt{b^4}+a^2\,b\sqrt{b^4}\right)}{b^4\sqrt{-(a^2+b^2)^3}\,(a^2+b^2)\sqrt{-a^6-3a^4b^2-3a^2\cdots}}\cdots}{\sqrt{-a^6-3a^4b^2-3a^2b^4-b^6}}$$

```
[In] int(1/(cosh(x)^2*(a + b*sinh(x))),x)
```

```
[Out] - ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan((
exp(x)*(2/((b^4)^(1/2)*(a^2 + b^2)^2) + (2*a*(a^3*(b^4)^(1/2) + a*b^2*(b^4)
^(1/2)))/(b^4*(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 -
 3*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) + a^2*b*(b^4)^(1/2)))/(b^4*(-(a
^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))
)*((b^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^6 - b^
6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(- a^6 - b^6 - 3*a^2*b^4
 - 3*a^4*b^2)^(1/2)
```

3.196 $\int \frac{\text{sech}^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\text{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{a(a^2+3b^2)\arctan(\sinh(x))}{2\left(a^2+b^2\right)^2} - \frac{b^3\log(\cosh(x))}{\left(a^2+b^2\right)^2} + \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)}$$

```
[Out] 1/2*a*(a^2+3*b^2)*arctan(sinh(x))/(a^2+b^2)^2-b^3*ln(cosh(x))/(a^2+b^2)^2+b
^3*ln(a+b*sinh(x))/(a^2+b^2)^2+1/2*sech(x)^2*(b+a*sinh(x))/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2747, 755, 815, 649, 209, 266}

$$\int \frac{\text{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{a(a^2+3b^2)\arctan(\sinh(x))}{2\left(a^2+b^2\right)^2} + \frac{\text{sech}^2(x)(a\sinh(x)+b)}{2\left(a^2+b^2\right)} + \frac{b^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^2} - \frac{b^3\log(\cosh(x))}{\left(a^2+b^2\right)^2}$$

```
[In] Int[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (a*(a^2 + 3*b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) - (b^3*Log[Cosh[x]])/(a
^2 + b^2)^2 + (b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(b + a*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = b^3\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)} - \frac{b\text{Subst}\left(\int \frac{a^2+2b^2+ax}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}$$

$$= \frac{\operatorname{sech}^2(x)(b + a\sinh(x))}{2(a^2 + b^2)} - \frac{b\text{Subst}\left(\int \left(-\frac{2b^2}{(a^2+b^2)(a+x)} + \frac{-a^3-3ab^2+2b^2x}{(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(x)\right)}{2(a^2 + b^2)}$$

$$= \frac{b^3\log(a + b\sinh(x))}{(a^2 + b^2)^2} + \frac{\operatorname{sech}^2(x)(b + a\sinh(x))}{2(a^2 + b^2)} - \frac{b\text{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2} dx, x, b\sinh(x)\right)}{2(a^2 + b^2)^2}$$

$$= \frac{b^3\log(a + b\sinh(x))}{(a^2 + b^2)^2} + \frac{\operatorname{sech}^2(x)(b + a\sinh(x))}{2(a^2 + b^2)} - \frac{b^3\text{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b\sinh(x)\right)}{(a^2 + b^2)^2} + \frac{(ab(a^2 + 3b^2))\,\text{Subst}\left(\int \frac{1}{b^2+x^2} dx, x, b\sinh(x)\right)}{2(a^2 + b^2)^2}$$

$$= \frac{a(a^2 + 3b^2)\arctan(\sinh(x))}{2(a^2 + b^2)^2} - \frac{b^3\log(\cosh(x))}{(a^2 + b^2)^2} + \frac{b^3\log(a + b\sinh(x))}{(a^2 + b^2)^2} + \frac{\operatorname{sech}^2(x)(b + a\sinh(x))}{2(a^2 + b^2)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{sech}^3(x)}{a + b\sinh(x)} dx = \frac{2a(a^2 + 3b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + 2b^3(-\log(\cosh(x)) + \log(a + b\sinh(x))) + b(a^2 + b^2)\operatorname{sech}^2(x) + a(a^2 +}{2(a^2 + b^2)^2}$$

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*(a^2 + 3*b^2)*ArcTan[Tanh[x/2]] + 2*b^3*(-Log[Cosh[x]] + Log[a + b*Sin
h[x]]) + b*(a^2 + b^2)*Sech[x]^2 + a*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 +
 b^2)^2)
```

Maple [A] (verified)

Time = 17.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.85

method	result
default	$\frac{b^3\ln\left(\tanh\left(\frac{x}{2}\right)^2 a - 2b\tanh\left(\frac{x}{2}\right) - a\right)}{a^4+2a^2b^2+b^4} + \frac{\frac{2\left(\left(-\frac{1}{2}a^3-\frac{1}{2}ab^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^2b-b^3\right)\tanh\left(\frac{x}{2}\right)^2+\left(\frac{1}{2}a^3+\frac{1}{2}ab^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2} - b^3\ln\left(1+\tanh\left(\frac{x}{2}\right.\right.}{a^4+2a^2b^2+b^4}$
risch	$\frac{e^x\left(e^{2x}a+2e^xb-a\right)}{\left(1+e^{2x}\right)^2(a^2+b^2)} + \frac{i\ln(e^x+i)a^3}{2a^4+4a^2b^2+2b^4} + \frac{3i\ln(e^x+i)ab^2}{2(a^4+2a^2b^2+b^4)} - \frac{\ln(e^x+i)b^3}{a^4+2a^2b^2+b^4} - \frac{i\ln(e^x-i)a^3}{2(a^4+2a^2b^2+b^4)} - \frac{3i\ln(e^x-i)ab^2}{2(a^4+2a^2b^2+b^4)} - \frac{\ln}{a^4+}$

```
[In] int(sech(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b^3/(a^4+2*a^2*b^2+b^4)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a^4+2*a^2*
b^2+b^4)*(((-1/2*a^3-1/2*a*b^2)*tanh(1/2*x)^3+(-a^2*b-b^3)*tanh(1/2*x)^2+(1
/2*a^3+1/2*a*b^2)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2-1/2*b^3*ln(1+tanh(1/2*x)
^2)+1/2*(a^3+3*a*b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. 2(83) = 166.

Time = 0.31 (sec) , antiderivative size = 652, normalized size of antiderivative = 7.49

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{(a^3+ab^2)\cosh(x)^3+(a^3+ab^2)\sinh(x)^3+2\,(a^2b+b^3)\cosh(x)^2+(2\,a^2b+2\,b^3+3\,(a^3+ab^2)\cosh(x))\,\mathrm{si}}{}$$

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a*b^2)*cosh(x)^3 + (a^3 + a*b^2)*sinh(x)^3 + 2*(a^2*b + b^3)*cosh(x
)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(x))*sinh(x)^2 + ((a^3 + 3*a*b
^2)*cosh(x)^4 + 4*(a^3 + 3*a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 3*a*b^2)*sinh(
x)^4 + a^3 + 3*a*b^2 + 2*(a^3 + 3*a*b^2)*cosh(x)^2 + 2*(a^3 + 3*a*b^2 + 3*(
a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a*b^2)*cosh(x)^3 + (a^3 +
 3*a*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^3 + a*b^2)*cosh(
x) + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(
x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*c
osh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (b^3*cosh(x)^
4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*
b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))*l
og(2*cosh(x)/(cosh(x) - sinh(x))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(x)^
2 - 4*(a^2*b + b^3)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(
x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4
 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a
^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(sech(x)**3/(a+b*sinh(x)),x)
[Out] Integral(sech(x)**3/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{b^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4} - \frac{b^3\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{(a^3+3\,ab^2)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4} + \frac{ae^{(-x)}+2\,be^{(-2\,x)}-ae^{(-3\,x)}}{a^2+b^2+2\,(a^2+b^2)e^{(-2\,x)}+(a^2+b^2)e^{(-4\,x)}}$$

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
[Out] b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(e^(
-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^3 + 3*a*b^2)*arctan(e^(-x))/(a^4 +
2*a^2*b^2 + b^4) + (a*e^(-x) + 2*b*e^(-2*x) - a*e^(-3*x))/(a^2 + b^2 + 2*(a
^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.46

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{b^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^4+2\,a^2b^2+b^4)} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(a^3+3\,ab^2)}{4\,(a^4+2\,a^2b^2+b^4)} + \frac{b^3\left(e^{(-x)}-e^x\right)^2-2\,a^3\left(e^{(-x)}-e^x\right)-2\,ab^2\left(e^{(-x)}-e^x\right)+4\,a^2b+8\,b^3}{2\,(a^4+2\,a^2b^2+b^4)\left(\left(e^{(-x)}-e^x\right)^2+4\right)}$$

```
[In] integrate(sech(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*b^3*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/2*(
e^(2*x) - 1)*e^(-x)))*(a^3 + 3*a*b^2)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(b^3*(e
^(-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 4*a^2*b +
8*b^3)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.34

$$\int \frac{\operatorname{sech}^3(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,(a^2\,b+b^3)}{(a^2+b^2)^2}+\frac{e^x\,(a^3+a\,b^2)}{(a^2+b^2)^2}}{e^{2\,x}+1} - \frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\,e^x}{a^2+b^2}}{2\,e^{2\,x}+e^{4\,x}+1} - \frac{\ln\left(1+e^x\,1i\right)\,\left(a+b\,2i\right)}{2\,\left(-a^2\,1i+2\,a\,b+b^2\,1i\right)} + \frac{b^3\,\ln\left(16\,b^7\,e^{2\,x}-a^6\,b-16\,b^7-9\,a^2\,b^5-6\,a^4\,b^3+2\,a^7\,e^x+9\,a^2\,b^5\,e^{2\,x}+6\,a^4\,b^3\,e^{2\,x}+32\,a\,b^6\,e^x+a^6\,b\,e^{2\,x}\cdots\right.}{a^4+2\,a^2\,b^2+b^4} - \frac{\ln\left(e^x+1i\right)\,\left(2\,b+a\,1i\right)}{2\,\left(-a^2+a\,b\,2i+b^2\right)}$$

```
[In] int(1/(cosh(x)^3*(a + b*sinh(x))),x)
```

```
[Out] ((2*(a^2*b + b^3))/(a^2 + b^2)^2 + (exp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(e
xp(2*x) + 1) - ((2*b)/(a^2 + b^2) + (2*a*exp(x))/(a^2 + b^2))/(2*exp(2*x) +
 exp(4*x) + 1) - (log(exp(x)*1i + 1)*(a + b*2i))/(2*(2*a*b - a^2*1i + b^2*1
i)) + (b^3*log(16*b^7*exp(2*x) - a^6*b - 16*b^7 - 9*a^2*b^5 - 6*a^4*b^3 + 2
*a^7*exp(x) + 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a*b^6*exp(x) + a
^6*b*exp(2*x) + 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^4 + b^4 + 2*a^2*
b^2) - (log(exp(x) + 1i)*(a*1i + 2*b))/(2*(a*b*2i - a^2 + b^2))
```

3.197 $\int \frac{\text{sech}^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\text{sech}^4(x)}{a+b\sinh(x)}\,dx = -\frac{2b^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{\text{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\text{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2}$$

```
[Out] -2*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/3*sech(
x)^3*(b+a*sinh(x))/(a^2+b^2)+1/3*sech(x)*(3*b^3+a*(2*a^2+5*b^2)*sinh(x))/(a
^2+b^2)^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2775, 2945, 12, 2739, 632, 212}

$$\int \frac{\text{sech}^4(x)}{a+b\sinh(x)}\,dx = -\frac{2b^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{\text{sech}^3(x)(a\sinh(x)+b)}{3(a^2+b^2)} + \frac{\text{sech}(x)\left(a(2a^2+5b^2)\sinh(x)+3b^3\right)}{3(a^2+b^2)^2}$$

```
[In] Int[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (Se
ch[x]^3*(b + a*Sinh[x]))/(3*(a^2 + b^2)) + (Sech[x]*(3*b^3 + a*(2*a^2 + 5*b
^2)*Sinh[x]))/(3*(a^2 + b^2)^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{\text{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} - \frac{\int \frac{\text{sech}^2(x)(-2a^2-3b^2-2ab\sinh(x))}{a+b\sinh(x)}\,dx}{3(a^2+b^2)}$$

$$= \frac{\text{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\text{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\int \frac{3b^4}{a+b\sinh(x)}\,dx}{3(a^2+b^2)^2}$$

$$= \frac{\text{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\text{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{b^4\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}$$

$$= \frac{\text{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\text{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}$$

$$= \frac{\text{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\text{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2} - \frac{(4b^4)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}$$

$$= -\frac{2b^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{\text{sech}^3(x)(b+a\sinh(x))}{3(a^2+b^2)} + \frac{\text{sech}(x)\left(3b^3+a(2a^2+5b^2)\sinh(x)\right)}{3(a^2+b^2)^2}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{\text{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{\frac{6b^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 3b^3\text{sech}(x) + (a^2+b^2)\,\text{sech}^3(x)(b+a\sinh(x)) + a(2a^2+5b^2)\tanh(x)}{3(a^2+b^2)^2}$$

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 3*b^
3*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) + a*(2*a^2 + 5*b^2)*Tanh[
x])/(3*(a^2 + b^2)^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(90) = 180$.

Time = 47.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.82

method	result
default	$\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(\left(-a^3-2a\,b^2\right)\tanh\left(\frac{x}{2}\right)^5+\left(-a^2b-2b^3\right)\tanh\left(\frac{x}{2}\right)^4+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\tanh\left(\frac{x}{2}\right)^3-2b^3\tanh\left(\frac{x}{2}\right)^2+(-a}{(a^4+2a^2b^2+b^4)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}$
risch	$-\frac{2\left(-3b^3 e^{5x}+3\,e^{4x}a\,b^2-4a^2b\,e^{3x}-10\,e^{3x}b^3+6a^3e^{2x}+12a\,e^{2x}b^2-3b^3e^x+2a^3+5a\,b^2\right)}{3(a^4+2a^2b^2+b^4)(1+e^{2x})^3} + \frac{b^4\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-3a^4b^2-3a^2b^4-b^6}{b\left(a^2+b^2\right)^{\frac{5}{2}}}\right.}{\left(a^2+b^2\right)^{\frac{5}{2}}}$

```
[In] int(sech(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)
/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tanh(1/2*x)^5+(-a^2
*b-2*b^3)*tanh(1/2*x)^4+(-2/3*a^3-8/3*a*b^2)*tanh(1/2*x)^3-2*b^3*tanh(1/2*x
)^2+(-a^3-2*a*b^2)*tanh(1/2*x)-1/3*a^2*b-4/3*b^3)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1142 vs. $2(92) = 184$.

Time = 0.29 (sec) , antiderivative size = 1142, normalized size of antiderivative = 11.42

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*b^3 + b^5)*cosh(x)^5 + 6*(a^2*b^3 + b^5)*sinh(x)^5 - 4*a^5 - 14
*a^3*b^2 - 10*a*b^4 - 6*(a^3*b^2 + a*b^4)*cosh(x)^4 - 6*(a^3*b^2 + a*b^4 -
5*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh
(x)^3 + 4*(2*a^4*b + 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 + b^5)*cosh(x)^2 - 6*(
a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x
)^2 - 12*(a^5 + 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 + b^5)*cosh(x)^3 + 3*(a^3*
b^2 + a*b^4)*cosh(x)^2 - (2*a^4*b + 7*a^2*b^3 + 5*b^5)*cosh(x))*sinh(x)^2 +
 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 + 3*b^4*cosh(x)
^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*sinh(x)^4 + b^4 + 4*(5*b^4
*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^
2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh
(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
```

```
a)*sinh(x) - b)) + 6*(a^2*b^3 + b^5)*cosh(x) + 6*(a^2*b^3 + b^5 + 5*(a^2*b^
3 + b^5)*cosh(x)^4 - 4*(a^3*b^2 + a*b^4)*cosh(x)^3 + 2*(2*a^4*b + 7*a^2*b^3
 + 5*b^5)*cosh(x)^2 - 4*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(x))*sinh(x))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sinh(x)^6 +
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*c
osh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x)^3 + 3*
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 6*(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{sech}^4{(x)}}{a+b\sinh{(x)}}\,dx$$

```
[In] integrate(sech(x)**4/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(92) = 184$.

Time = 0.29 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.30

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx=\frac{b^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(3\,b^3e^{(-x)}+3\,ab^2e^{(-4\,x)}+3\,b^3e^{(-5\,x)}+2\,a^3+5\,ab^2+6\,(a^3+2\,ab^2)e^{(-2\,x)}+2\,(2\,a^2b+5\,b^3)e^{(-3\,x)}\right)}{3\,(a^4+2\,a^2b^2+b^4+3\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+3\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+(a^4+2\,a^2b^2+b^4)e^{(-6\,x)})}$$

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(-x) + 3*a*b^2*e^(
-4*x) + 3*b^3*e^(-5*x) + 2*a^3 + 5*a*b^2 + 6*(a^3 + 2*a*b^2)*e^(-2*x) + 2*(
2*a^2*b + 5*b^3)*e^(-3*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^
4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 + 2*a^2*b^2 + b^4)*
e^(-6*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{b^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,b^3e^{(5\,x)}-3\,ab^2e^{(4\,x)}+4\,a^2be^{(3\,x)}+10\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}-12\,ab^2e^{(2\,x)}+3\,b^3e^x-2\,a^3-5\,ab^2\right)}{3\,(a^4+2\,a^2b^2+b^4)(e^{(2\,x)}+1)^3}$$

```
[In] integrate(sech(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3*(3*b^3*e^(5*x)
- 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) + 10*b^3*e^(3*x) - 6*a^3*e^(2*x) - 12*a
*b^2*e^(2*x) + 3*b^3*e^x - 2*a^3 - 5*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(e^(2*
x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 634, normalized size of antiderivative = 6.34

$$\int \frac{\operatorname{sech}^4(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,b^3\,\mathrm{e}^x}{(a^2+b^2)^2}-\frac{2\,a\,b^2}{(a^2+b^2)^2}}{\mathrm{e}^{2\,x}+1} - \frac{\frac{4\,(a^3+a\,b^2)}{(a^2+b^2)^2}-\frac{8\,\mathrm{e}^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + \frac{\frac{8\,a}{3\,(a^2+b^2)}-\frac{8\,b\,\mathrm{e}^x}{3\,(a^2+b^2)}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(\mathrm{e}^x\left(\frac{2\,b^2}{\sqrt{b^8}\,(a^2+b^2)^2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,a\left(a^5\,\sqrt{b^8}+2\,a^3\,b^2\,\sqrt{b^8}+a\,b^4\,\sqrt{b^8}\right)}{b^6\,\sqrt{-(a^2+b^2)^5}\,(a^4+2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}\right)-\right.\right.}{}$$

```
[In] int(1/(cosh(x)^4*(a + b*sinh(x))),x)
```

```
[Out] ((2*b^3*exp(x))/(a^2 + b^2)^2 - (2*a*b^2)/(a^2 + b^2)^2)/(exp(2*x) + 1) - (
(4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b + b^3))/(3*(a^2 + b^2)^2
))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2)) - (8*b*exp(x))/(3*(
a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (2*atan((exp(x)*((2
*b^2)/((b^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(a^5*(b^8)
^(1/2) + 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 + b^2)^5)^
(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*
a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*a*(b^5*(b^8)^(1/2) + 2*a^2*b^3*(b^8)^(1/2
) + a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)
*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*
((b^5*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/
2))/2 + (a^4*b*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8
```

```
*b^2)^(1/2))/2 + a^2*b^3*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b
^4 - 5*a^8*b^2)^(1/2)))*(b^8)^(1/2))/(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^
6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

3.198 $\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 135

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{a(3a^4+10a^2b^2+15b^4)\arctan(\sinh(x))}{8(a^2+b^2)^3} - \frac{b^5\log(\cosh(x))}{(a^2+b^2)^3} + \frac{b^5\log(a+b\sinh(x))}{(a^2+b^2)^3} + \frac{\operatorname{sech}^4(x)(b+a\sinh(x))}{4(a^2+b^2)} + \frac{\operatorname{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8(a^2+b^2)^2}$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*arctan(sinh(x))/(a^2+b^2)^3-b^5*ln(cosh(x))
/(a^2+b^2)^3+b^5*ln(a+b*sinh(x))/(a^2+b^2)^3+1/4*sech(x)^4*(b+a*sinh(x))/(a
^2+b^2)+1/8*sech(x)^2*(4*b^3+a*(3*a^2+7*b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2747, 755, 837, 815, 649, 209, 266}

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{\operatorname{sech}^4(x)(a\sinh(x)+b)}{4(a^2+b^2)} + \frac{b^5\log(a+b\sinh(x))}{(a^2+b^2)^3} - \frac{b^5\log(\cosh(x))}{(a^2+b^2)^3} + \frac{\operatorname{sech}^2(x)\left(a(3a^2+7b^2)\sinh(x)+4b^3\right)}{8(a^2+b^2)^2} + \frac{a(3a^4+10a^2b^2+15b^4)\arctan(\sinh(x))}{8(a^2+b^2)^3}$$

```
[In] Int[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*ArcTan[Sinh[x]])/(8*(a^2 + b^2)^3) - (b^5*
Log[Cosh[x]])/(a^2 + b^2)^3 + (b^5*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (Sec
h[x]^4*(b + a*Sinh[x]))/(4*(a^2 + b^2)) + (Sech[x]^2*(4*b^3 + a*(3*a^2 + 7*
b^2)*Sinh[x]))/(8*(a^2 + b^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(b^5\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)^3}\,dx, x, b\sinh(x)\right)\right)\\
&= \frac{\text{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)}+\frac{b^3\text{Subst}\left(\int \frac{3a^2+4b^2+3ax}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)}{4\left(a^2+b^2\right)}\\
&= \frac{\text{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)}+\frac{\text{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{-3a^4-7a^2b^2-8b^4-a(3a^2+7b^2)x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{8\left(a^2+b^2\right)^2}\\
&= \frac{\text{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)}+\frac{\text{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}\\
&\quad+\frac{b\text{Subst}\left(\int \left(\frac{8b^4}{(a^2+b^2)(a+x)}+\frac{3a^5+10a^3b^2+15ab^4-8b^4x}{(a^2+b^2)(b^2+x^2)}\right)dx, x, b\sinh(x)\right)}{8\left(a^2+b^2\right)^2}\\
&= \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{\text{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)}\\
&\quad+\frac{\text{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{3a^5+10a^3b^2+15ab^4-8b^4x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{8\left(a^2+b^2\right)^3}\\
&= \frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}+\frac{\text{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)}\\
&\quad+\frac{\text{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}-\frac{b^5\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^3}\\
&\quad+\frac{\left(ab(3a^4+10a^2b^2+15b^4)\right)\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{8\left(a^2+b^2\right)^3}\\
&= \frac{a(3a^4+10a^2b^2+15b^4)\arctan(\sinh(x))}{8\left(a^2+b^2\right)^3}-\frac{b^5\log(\cosh(x))}{\left(a^2+b^2\right)^3}+\frac{b^5\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}\\
&\quad+\frac{\text{sech}^4(x)(b+a\sinh(x))}{4\left(a^2+b^2\right)}+\frac{\text{sech}^2(x)\left(4b^3+a(3a^2+7b^2)\sinh(x)\right)}{8\left(a^2+b^2\right)^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 284 vs. $2(135) = 270$.

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{-\left(\left(8b^6+3a^5\sqrt{-b^2}+15ab^4\sqrt{-b^2}-10a^3(-b^2)^{3/2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)\right)+16b^6\log(a+b\sinh(x))-}{}$$

```
[In] Integrate[Sech[x]^5/(a + b*Sinh[x]),x]
```

```
[Out] (-((8*b^6 + 3*a^5*Sqrt[-b^2] + 15*a*b^4*Sqrt[-b^2] - 10*a^3*(-b^2)^(3/2))*L
og[Sqrt[-b^2] - b*Sinh[x]]) + 16*b^6*Log[a + b*Sinh[x]] - 8*b^6*Log[Sqrt[-b
^2] + b*Sinh[x]] + 3*a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Sinh[x]] + 15*a*b^4*
Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Sinh[x]] - 10*a^3*(-b^2)^(3/2)*Log[Sqrt[-b^2]
 + b*Sinh[x]] + 8*b^4*(a^2 + b^2)*Sech[x]^2 + 4*b^2*(a^2 + b^2)^2*Sech[x]^4
 + 2*a*b*(3*a^4 + 10*a^2*b^2 + 7*b^4)*Sech[x]*Tanh[x] + 4*a*b*(a^2 + b^2)^2
*Sech[x]^3*Tanh[x])/(16*b*(a^2 + b^2)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(129) = 258$.

Time = 92.65 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.32

method	result
default	$\frac{b^5\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2\left(\left(-\frac{5}{8}a^5-\frac{7}{4}a^3b^2-\frac{9}{8}ab^4\right)\tanh\left(\frac{x}{2}\right)^7+\left(-a^4b-3a^2b^3-2b^5\right)\tanh\left(\frac{x}{2}\right)^6+\left(\frac{3}{8}a^5+\frac{1}{4}a^3b^2-\frac{1}{8}ab^4\right)\tanh\left(\right.}{}$
risch	$\frac{\left(3a^3e^{6x}+7e^{6x}ab^2+8b^3e^{5x}+11a^3e^{4x}+15e^{4x}ab^2+16a^2be^{3x}+32e^{3x}b^3-11a^3e^{2x}-15ae^{2x}b^2+8b^3e^x-3a^3-7ab^2\right)e^x}{4(a^4+2a^2b^2+b^4)(1+e^{2x})^4}+\frac{3i\ln(e}{8(a^6+3a^4b^2}$

```
[In] int(sech(x)^5/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b^5/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(
a^6+3*a^4*b^2+3*a^2*b^4+b^6)*(((-5/8*a^5-7/4*a^3*b^2-9/8*a*b^4)*tanh(1/2*x)
^7+(-a^4*b-3*a^2*b^3-2*b^5)*tanh(1/2*x)^6+(3/8*a^5+1/4*a^3*b^2-1/8*a*b^4)*t
anh(1/2*x)^5+(-2*a^2*b^3-2*b^5)*tanh(1/2*x)^4+(-3/8*a^5-1/4*a^3*b^2+1/8*a*b
^4)*tanh(1/2*x)^3+(-a^4*b-3*a^2*b^3-2*b^5)*tanh(1/2*x)^2+(5/8*a^5+7/4*a^3*b
^2+9/8*a*b^4)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^4-1/2*b^5*ln(1+tanh(1/2*x)^2)+
1/8*(3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2707 vs. $2(129) = 258$.

Time = 0.37 (sec) , antiderivative size = 2707, normalized size of antiderivative = 20.05

$$\int \frac{\operatorname{sech}^5(x)}{a + b\sinh(x)}\, dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^7 + (3*a^5 + 10*a^3*b^2 + 7*a*b
^4)*sinh(x)^7 + 8*(a^2*b^3 + b^5)*cosh(x)^6 + (8*a^2*b^3 + 8*b^5 + 7*(3*a^5
 + 10*a^3*b^2 + 7*a*b^4)*cosh(x))*sinh(x)^6 + (11*a^5 + 26*a^3*b^2 + 15*a*b
^4)*cosh(x)^5 + (11*a^5 + 26*a^3*b^2 + 15*a*b^4 + 21*(3*a^5 + 10*a^3*b^2 +
7*a*b^4)*cosh(x)^2 + 48*(a^2*b^3 + b^5)*cosh(x))*sinh(x)^5 + 16*(a^4*b + 3*
a^2*b^3 + 2*b^5)*cosh(x)^4 + (16*a^4*b + 48*a^2*b^3 + 32*b^5 + 35*(3*a^5 +
10*a^3*b^2 + 7*a*b^4)*cosh(x)^3 + 120*(a^2*b^3 + b^5)*cosh(x)^2 + 5*(11*a^5
 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^4 - (11*a^5 + 26*a^3*b^2 + 15*a*
b^4)*cosh(x)^3 - (11*a^5 + 26*a^3*b^2 + 15*a*b^4 - 35*(3*a^5 + 10*a^3*b^2 +
 7*a*b^4)*cosh(x)^4 - 160*(a^2*b^3 + b^5)*cosh(x)^3 - 10*(11*a^5 + 26*a^3*b
^2 + 15*a*b^4)*cosh(x)^2 - 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^
3 + 8*(a^2*b^3 + b^5)*cosh(x)^2 + (21*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x
)^5 + 8*a^2*b^3 + 8*b^5 + 120*(a^2*b^3 + b^5)*cosh(x)^4 + 10*(11*a^5 + 26*a
^3*b^2 + 15*a*b^4)*cosh(x)^3 + 96*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^2 - 3
*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^2 + ((3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^8 + 8*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^
7 + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(3*a^5 + 10*a^3*b^2 + 15*
a*b^4)*cosh(x)^6 + 4*(3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 7*(3*a^5 + 10*a^3*b^2
 + 15*a*b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*co
sh(x)^3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 3*a^5 + 10
*a^3*b^2 + 15*a*b^4 + 6*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(9*a^
5 + 30*a^3*b^2 + 45*a*b^4 + 35*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^4 +
30*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(3*a^5 + 10*
a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 10*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
3 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(3*a^5 + 10*a^
3*b^2 + 15*a*b^4)*cosh(x)^2 + 4*(7*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^
6 + 3*a^5 + 10*a^3*b^2 + 15*a*b^4 + 15*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^4 + 9*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((3*a^5
+ 10*a^3*b^2 + 15*a*b^4)*cosh(x)^7 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh
(x)^5 + 3*(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (3*a^5 + 10*a^3*b^2 +
 15*a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (3*a^5 + 10*a^3*b^
2 + 7*a*b^4)*cosh(x) + 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sin
h(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cos
h(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b
^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*c
```

```
osh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6
 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 +
 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*(b*sinh(x)
 + a)/(cosh(x) - sinh(x))) - 4*(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b
^5*sinh(x)^8 + 4*b^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b
^5*cosh(x)^2 + b^5)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)
^5 + b^5 + 2*(35*b^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7
*b^5*cosh(x)^5 + 10*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cos
h(x)^6 + 15*b^5*cosh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(
x)^7 + 3*b^5*cosh(x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*log(2*cosh
(x)/(cosh(x) - sinh(x))) + (7*(3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cosh(x)^6 + 48
*(a^2*b^3 + b^5)*cosh(x)^5 - 3*a^5 - 10*a^3*b^2 - 7*a*b^4 + 5*(11*a^5 + 26*
a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 64*(a^4*b + 3*a^2*b^3 + 2*b^5)*cosh(x)^3 -
3*(11*a^5 + 26*a^3*b^2 + 15*a*b^4)*cosh(x)^2 + 16*(a^2*b^3 + b^5)*cosh(x))*
sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^8 + 8*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^7 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*sinh(x)^8 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 4*(a^6 + 3*
a^4*b^2 + 3*a^2*b^4 + b^6 + 7*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2
)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 8*(7*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*
sinh(x)^5 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 2*(3*a^6 + 9*
a^4*b^2 + 9*a^2*b^4 + 3*b^6 + 35*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
)^4 + 30*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh(x
)^3 + 4*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 + 4*(7*(a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 15*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 9*(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*cosh(x)^2)*sinh(x)^2 + 8*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh
(x)^7 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 3*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x
))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{a + b\sinh(x)}\,dx = \int \frac{\operatorname{sech}^5(x)}{a + b\sinh(x)}\,dx$$

```
[In] integrate(sech(x)**5/(a+b*sinh(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(129) = 258.

Time = 0.30 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.56

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{b^5\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{b^5\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(3\,a^5+10\,a^3b^2+15\,ab^4)\arctan\left(e^{(-x)}\right)}{4\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{8\,b^3e^{(-2\,x)}+8\,b^3e^{(-6\,x)}+(3\,a^3+7\,ab^2)e^{(-x)}+(11\,a^3+15\,ab^2)e^{(-3\,x)}+16\,(a^2b+2\,b^3)e^{(-4\,x)}-(11\,a^3+15}{4\,(a^4+2\,a^2b^2+b^4+4\,(a^4+2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4+2\,a^2b^2+b^4)e^{(-4\,x)}+4\,(a^4+2\,a^2b^2+b^4)e^{(-6}}$$

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b^5*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 b^5*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^5 + 1
0*a^3*b^2 + 15*a*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 + 7*a*b^2)*e^(-x) + (11*a^3 +
 15*a*b^2)*e^(-3*x) + 16*(a^2*b + 2*b^3)*e^(-4*x) - (11*a^3 + 15*a*b^2)*e^(
-5*x) - (3*a^3 + 7*a*b^2)*e^(-7*x))/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2
*b^2 + b^4)*e^(-2*x) + 6*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + 4*(a^4 + 2*a^2*
b^2 + b^4)*e^(-6*x) + (a^4 + 2*a^2*b^2 + b^4)*e^(-8*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(129) = 258.

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.73

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{b^6\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{b^5\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(3\,a^5+10\,a^3b^2+15\,ab^4)}{16\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{3\,b^5\left(e^{(-x)}-e^x\right)^4-3\,a^5\left(e^{(-x)}-e^x\right)^3-10\,a^3b^2\left(e^{(-x)}-e^x\right)^3-7\,ab^4\left(e^{(-x)}-e^x\right)^3+8\,a^2b^3\left(e^{(-x)}-e^x\right)^2+}{4\,(a^6+3\,a^4b^2+3\,a^2b^4}$$

```
[In] integrate(sech(x)^5/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)
 - 1/2*b^5*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
1/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^5 + 10*a^3*b^2 + 15*a*b
^4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(3*b^5*(e^(-x) - e^x)^4 - 3*a
^5*(e^(-x) - e^x)^3 - 10*a^3*b^2*(e^(-x) - e^x)^3 - 7*a*b^4*(e^(-x) - e^x)^
```

```
3 + 8*a^2*b^3*(e^(-x) - e^x)^2 + 32*b^5*(e^(-x) - e^x)^2 - 20*a^5*(e^(-x) -
 e^x) - 56*a^3*b^2*(e^(-x) - e^x) - 36*a*b^4*(e^(-x) - e^x) + 16*a^4*b + 64
*a^2*b^3 + 96*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*((e^(-x) - e^x)^2 +
 4)^2)
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 548, normalized size of antiderivative = 4.06

$$\int \frac{\operatorname{sech}^5(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,(2\,a^2\,b+b^3)}{(a^2+b^2)^2}-\frac{e^x\,(3\,a\,b^2-a^3)}{2\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1}-\frac{\frac{8\,(a^2\,b+b^3)}{(a^2+b^2)^2}+\frac{6\,e^x\,(a^3+a\,b^2)}{(a^2+b^2)^2}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} + \frac{\frac{4\,b}{a^2+b^2}+\frac{4\,a\,e^x}{a^2+b^2}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}+\frac{\frac{2\,(a^2\,b^3+b^5)}{(a^2+b^2)^3}+\frac{e^x\,(3\,a^5+10\,a^3\,b^2+7\,a\,b^4)}{4\,(a^2+b^2)^3}}{e^{2\,x}+1} + \frac{b^5\,\ln(256\,b^{11}\,e^{2\,x}-9\,a^{10}\,b-256\,b^{11}-225\,a^2\,b^9-300\,a^4\,b^7-190\,a^6\,b^5-60\,a^8\,b^3+18\,a^{11}\,e^x+225\,a^2\,b^9}{} - \frac{\ln(1+e^x\,1i)\,(-a^2\,3i+9\,a\,b+b^2\,8i)}{8\,(-a^3-a^2\,b\,3i+3\,a\,b^2+b^3\,1i)}-\frac{\ln(e^x+1i)\,(-3\,a^2+a\,b\,9i+8\,b^2)}{8\,(-a^3\,1i-3\,a^2\,b+a\,b^2\,3i+b^3)}$$

```
[In] int(1/(cosh(x)^5*(a + b*sinh(x))),x)
```

```
[Out] ((2*(2*a^2*b + b^3))/(a^2 + b^2)^2 - (exp(x)*(3*a*b^2 - a^3))/(2*(a^2 + b^2
)^2))/(2*exp(2*x) + exp(4*x) + 1) - ((8*(a^2*b + b^3))/(a^2 + b^2)^2 + (6*e
xp(x)*(a*b^2 + a^3))/(a^2 + b^2)^2)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1
) + ((4*b)/(a^2 + b^2) + (4*a*exp(x))/(a^2 + b^2))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) + ((2*(b^5 + a^2*b^3))/(a^2 + b^2)^3 + (exp(x
)*(7*a*b^4 + 3*a^5 + 10*a^3*b^2))/(4*(a^2 + b^2)^3))/(exp(2*x) + 1) + (b^5*
log(256*b^11*exp(2*x) - 9*a^10*b - 256*b^11 - 225*a^2*b^9 - 300*a^4*b^7 - 1
90*a^6*b^5 - 60*a^8*b^3 + 18*a^11*exp(x) + 225*a^2*b^9*exp(2*x) + 300*a^4*b
^7*exp(2*x) + 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b^10*exp(x
) + 9*a^10*b*exp(2*x) + 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) + 380*a^7*b
^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (log
(exp(x)*1i + 1)*(9*a*b - a^2*3i + b^2*8i))/(8*(3*a*b^2 - a^2*b*3i - a^3 + b
^3*1i)) - (log(exp(x) + 1i)*(a*b*9i - 3*a^2 + 8*b^2))/(8*(a*b^2*3i - 3*a^2*
b - a^3*1i + b^3))
```

3.199 $\int \frac{\text{sech}^6(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 146

$$\int \frac{\text{sech}^6(x)}{a+b\sinh(x)}\,dx = -\frac{2b^6\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{\text{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)}+\frac{\text{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2}+\frac{\text{sech}(x)\left(15b^5+a(8a^4+26a^2b^2+33b^4)\sinh(x)\right)}{15(a^2+b^2)^3}$$

```
[Out] -2*b^6*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+1/5*sech(
x)^5*(b+a*sinh(x))/(a^2+b^2)+1/15*sech(x)^3*(5*b^3+a*(4*a^2+9*b^2)*sinh(x))
/(a^2+b^2)^2+1/15*sech(x)*(15*b^5+a*(8*a^4+26*a^2*b^2+33*b^4)*sinh(x))/(a^2
+b^2)^3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2775, 2945, 12, 2739, 632, 212}

$$\int \frac{\text{sech}^6(x)}{a+b\sinh(x)}\,dx = -\frac{2b^6\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{\text{sech}^5(x)(a\sinh(x)+b)}{5(a^2+b^2)}+\frac{\text{sech}^3(x)\left(a(4a^2+9b^2)\sinh(x)+5b^3\right)}{15(a^2+b^2)^2}+\frac{\text{sech}(x)\left(a(8a^4+26a^2b^2+33b^4)\sinh(x)+15b^5\right)}{15(a^2+b^2)^3}$$

```
[In] Int[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] (-2*b^6*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (Se
ch[x]^5*(b + a*Sinh[x]))/(5*(a^2 + b^2)) + (Sech[x]^3*(5*b^3 + a*(4*a^2 + 9
*b^2)*Sinh[x]))/(15*(a^2 + b^2)^2) + (Sech[x]*(15*b^5 + a*(8*a^4 + 26*a^2*b
^2 + 33*b^4)*Sinh[x]))/(15*(a^2 + b^2)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2775

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
```

```
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} - \frac{\int \frac{\operatorname{sech}^4(x)(-4a^2-5b^2-4ab\sinh(x))}{a+b\sinh(x)}\,dx}{5(a^2+b^2)}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\int \frac{\operatorname{sech}^2(x)(8a^4+18a^2b^2+15b^4+2ab(4a^2+9b^2)\sinh(x))}{a+b\sinh(x)}\,dx}{15(a^2+b^2)^2}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8a^4+26a^2b^2+33b^4)\sinh(x)\right)}{15(a^2+b^2)^3} - \frac{\int -\frac{15b^6}{a+b\sinh(x)}\,dx}{15(a^2+b^2)^3}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8a^4+26a^2b^2+33b^4)\sinh(x)\right)}{15(a^2+b^2)^3} + \frac{b^6\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8a^4+26a^2b^2+33b^4)\sinh(x)\right)}{15(a^2+b^2)^3} + \frac{(2b^6)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\left(5b^3+a(4a^2+9b^2)\sinh(x)\right)}{15(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\left(15b^5+a(8a^4+26a^2b^2+33b^4)\sinh(x)\right)}{15(a^2+b^2)^3} - \frac{(4b^6)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= -\frac{2b^6 \operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{\operatorname{sech}^5(x)(b+a\sinh(x))}{5\,(a^2+b^2)} + \frac{\operatorname{sech}^3(x)\,(5b^3+a(4a^2+9b^2)\sinh(x))}{15\,(a^2+b^2)^2} + \frac{\operatorname{sech}(x)\,(15b^5+a(8a^4+26a^2b^2+33b^4)\sinh(x))}{15\,(a^2+b^2)^3}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{\frac{30b^6\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 15b^5\operatorname{sech}(x) + 3(a^2+b^2)^2\operatorname{sech}^5(x)(b+a\sinh(x)) + (a^2+b^2)\operatorname{sech}^3(x)\,(5b^3+a(4a}{15\,(a^2+b^2)^3}$$

```
[In] Integrate[Sech[x]^6/(a + b*Sinh[x]),x]
```

```
[Out] ((30*b^6*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 15*
b^5*Sech[x] + 3*(a^2 + b^2)^2*Sech[x]^5*(b + a*Sinh[x]) + (a^2 + b^2)*Sech[
x]^3*(5*b^3 + a*(4*a^2 + 9*b^2)*Sinh[x]) + a*(8*a^4 + 26*a^2*b^2 + 33*b^4)*
Tanh[x])/(15*(a^2 + b^2)^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(134) = 268.

Time = 174.74 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.40

method	result
default	$\frac{2b^6 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(\left(-a^5-3a^3b^2-3a\,b^4\right)\tanh\left(\frac{x}{2}\right)^9+\left(-a^4b-3a^2b^3-3b^5\right)\tanh\left(\frac{x}{2}\right)^8+\left(-\frac{4}{3}a^5-\frac{16}{3}a^3b^2-8a\,b^4\right)\tan\right.}{}$
risch	$-\frac{2\left(-15b^5 e^{9x}+15a\,b^4 e^{8x}-20a^2b^3 e^{7x}-80b^5 e^{7x}+30a^3b^2 e^{6x}+90a\,b^4 e^{6x}-48a^4b\,e^{5x}-136a^2b^3 e^{5x}-178\,e^{5x}b^5+80a^5 e^{4x}+230a^3b^2 e^{4x}+\right.}{15(a^6+3a^4b^2+3a^2b^4+b^6)(1+e}$

```
[In] int(sech(x)^6/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^6/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1
/2*x)-2*b)/(a^2+b^2)^(1/2))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^5-3*a^3*b^
2-3*a*b^4)*tanh(1/2*x)^9+(-a^4*b-3*a^2*b^3-3*b^5)*tanh(1/2*x)^8+(-4/3*a^5-1
```

```
6/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^7+(-2*a^2*b^3-6*b^5)*tanh(1/2*x)^6+(-58/15
*a^5-166/15*a^3*b^2-66/5*a*b^4)*tanh(1/2*x)^5+(-2*a^4*b-16/3*a^2*b^3-28/3*b
^5)*tanh(1/2*x)^4+(-4/3*a^5-16/3*a^3*b^2-8*a*b^4)*tanh(1/2*x)^3+(-2/3*a^2*b
^3-14/3*b^5)*tanh(1/2*x)^2+(-a^5-3*a^3*b^2-3*a*b^4)*tanh(1/2*x)-1/5*a^4*b-1
1/15*a^2*b^3-23/15*b^5)/(1+tanh(1/2*x)^2)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3175 vs. $2(136) = 272$.

Time = 0.31 (sec) , antiderivative size = 3175, normalized size of antiderivative = 21.75

$$\int \frac{\operatorname{sech}^6(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/15*(30*(a^2*b^5 + b^7)*cosh(x)^9 + 30*(a^2*b^5 + b^7)*sinh(x)^9 - 30*(a^3
*b^4 + a*b^6)*cosh(x)^8 - 30*(a^3*b^4 + a*b^6 - 9*(a^2*b^5 + b^7)*cosh(x))*
sinh(x)^8 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^7 + 40*(a^4*b^3 + 5*a^
2*b^5 + 4*b^7 + 27*(a^2*b^5 + b^7)*cosh(x)^2 - 6*(a^3*b^4 + a*b^6)*cosh(x))
*sinh(x)^7 - 16*a^7 - 68*a^5*b^2 - 118*a^3*b^4 - 66*a*b^6 - 60*(a^5*b^2 + 4
*a^3*b^4 + 3*a*b^6)*cosh(x)^6 - 20*(3*a^5*b^2 + 12*a^3*b^4 + 9*a*b^6 - 126*
(a^2*b^5 + b^7)*cosh(x)^3 + 42*(a^3*b^4 + a*b^6)*cosh(x)^2 - 14*(a^4*b^3 +
5*a^2*b^5 + 4*b^7)*cosh(x))*sinh(x)^6 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*
b^5 + 89*b^7)*cosh(x)^5 + 4*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7 +
 945*(a^2*b^5 + b^7)*cosh(x)^4 - 420*(a^3*b^4 + a*b^6)*cosh(x)^3 + 210*(a^4
*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 90*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*co
sh(x))*sinh(x)^5 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^
4 - 20*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6 - 189*(a^2*b^5 + b^7)*co
sh(x)^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^4 - 70*(a^4*b^3 + 5*a^2*b^5 + 4*b^7
)*cosh(x)^3 + 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^2 - (24*a^6*b + 92
*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x))*sinh(x)^4 + 40*(a^4*b^3 + 5*a^2*b
^5 + 4*b^7)*cosh(x)^3 + 40*(a^4*b^3 + 5*a^2*b^5 + 4*b^7 + 63*(a^2*b^5 + b^7
)*cosh(x)^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^5 + 35*(a^4*b^3 + 5*a^2*b^5 + 4*
b^7)*cosh(x)^4 - 30*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^3 + (24*a^6*b +
 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^2 - 2*(8*a^7 + 31*a^5*b^2 + 47*
a^3*b^4 + 24*a*b^6)*cosh(x))*sinh(x)^3 - 20*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^
4 + 15*a*b^6)*cosh(x)^2 + 20*(54*(a^2*b^5 + b^7)*cosh(x)^7 - 4*a^7 - 17*a^5
*b^2 - 28*a^3*b^4 - 15*a*b^6 - 42*(a^3*b^4 + a*b^6)*cosh(x)^6 + 42*(a^4*b^3
 + 5*a^2*b^5 + 4*b^7)*cosh(x)^5 - 45*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x
)^4 + 2*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^3 - 6*(8*a^7
 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^2 + 6*(a^4*b^3 + 5*a^2*b^5 +
 4*b^7)*cosh(x))*sinh(x)^2 + 15*(b^6*cosh(x)^10 + 10*b^6*cosh(x)*sinh(x)^9
+ b^6*sinh(x)^10 + 5*b^6*cosh(x)^8 + 10*b^6*cosh(x)^6 + 10*b^6*cosh(x)^4 +
```

5*(9*b^6*cosh(x)^2 + b^6)*sinh(x)^8 + 5*b^6*cosh(x)^2 + 40*(3*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^7 + 10*(21*b^6*cosh(x)^4 + 14*b^6*cosh(x)^2 + b^6)*sinh(x)^6 + b^6 + 4*(63*b^6*cosh(x)^5 + 70*b^6*cosh(x)^3 + 15*b^6*cosh(x))*sinh(x)^5 + 10*(21*b^6*cosh(x)^6 + 35*b^6*cosh(x)^4 + 15*b^6*cosh(x)^2 + b^6)*sinh(x)^4 + 40*(3*b^6*cosh(x)^7 + 7*b^6*cosh(x)^5 + 5*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x)^3 + 5*(9*b^6*cosh(x)^8 + 28*b^6*cosh(x)^6 + 30*b^6*cosh(x)^4 + 12*b^6*cosh(x)^2 + b^6)*sinh(x)^2 + 10*(b^6*cosh(x)^9 + 4*b^6*cosh(x)^7 + 6*b^6*cosh(x)^5 + 4*b^6*cosh(x)^3 + b^6*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 30*(a^2*b^5 + b^7)*cosh(x) + 10*(27*(a^2*b^5 + b^7)*cosh(x)^8 - 24*(a^3*b^4 + a*b^6)*cosh(x)^7 + 3*a^2*b^5 + 3*b^7 + 28*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^6 - 36*(a^5*b^2 + 4*a^3*b^4 + 3*a*b^6)*cosh(x)^5 + 2*(24*a^6*b + 92*a^4*b^3 + 157*a^2*b^5 + 89*b^7)*cosh(x)^4 - 8*(8*a^7 + 31*a^5*b^2 + 47*a^3*b^4 + 24*a*b^6)*cosh(x)^3 + 12*(a^4*b^3 + 5*a^2*b^5 + 4*b^7)*cosh(x)^2 - 4*(4*a^7 + 17*a^5*b^2 + 28*a^3*b^4 + 15*a*b^6)*cosh(x))*sinh(x))/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^10 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)*sinh(x)^9 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*sinh(x)^10 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^8 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 9*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^8 + a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 40*(3*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x))*sinh(x)^7 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^6 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 14*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 70*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + 15*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x))*sinh(x)^5 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 10*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 21*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^6 + 35*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 15*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^7 + 7*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x))*sinh(x)^3 + 5*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2 + 5*(9*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^8 + a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8 + 28*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^6 + 30*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^4 + 12*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^2)*sinh(x)^2 + 10*((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^9 + 4*(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^7 + 6*(a^8 + 4*a^6*b^

```
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*cosh(x)^5 + 4*(a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8)*cosh(x)^3 + (a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 +
b^8)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx=\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(sech(x)**6/(a+b*sinh(x)),x)
[Out] Integral(sech(x)**6/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(136) = 272.

Time = 0.30 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx=\frac{b^6\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{2\left(15\,b^5e^{(-x)}+15\,ab^4e^{(-8\,x)}+15\,b^5e^{(-9\,x)}+8\,a^5+26\,a^3b^2+33\,ab^4+10\,(4\,a^5+13\,a^3b^2+15\,ab^4)e^{(-2\,x)}+\cdots\right.}{15\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6+5\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)e^{(-2\,x)}+10\,(a^6+3\,a^4b^2+3\,\cdots}$$

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="maxima")
[Out] b^6*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(15*b^5*e^(-x)
 + 15*a*b^4*e^(-8*x) + 15*b^5*e^(-9*x) + 8*a^5 + 26*a^3*b^2 + 33*a*b^4 + 10
*(4*a^5 + 13*a^3*b^2 + 15*a*b^4)*e^(-2*x) + 20*(a^2*b^3 + 4*b^5)*e^(-3*x) +
 10*(8*a^5 + 23*a^3*b^2 + 24*a*b^4)*e^(-4*x) + 2*(24*a^4*b + 68*a^2*b^3 + 8
9*b^5)*e^(-5*x) + 30*(a^3*b^2 + 3*a*b^4)*e^(-6*x) + 20*(a^2*b^3 + 4*b^5)*e^
(-7*x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*e^(-2*x) + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-4*x) + 10*(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-6*x) + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*e^(-8*x) + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*e^(-10*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(136) = 272$.

Time = 0.28 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.21

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{b^6\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,b^5e^{(9\,x)}-15\,ab^4e^{(8\,x)}+20\,a^2b^3e^{(7\,x)}+80\,b^5e^{(7\,x)}-30\,a^3b^2e^{(6\,x)}-90\,ab^4e^{(6\,x)}+48\,a^4be^{(5\,x)}+136\,a^2\cdots\right.}{\cdots}$$

```
[In] integrate(sech(x)^6/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/15*(1
5*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2*b^3*e^(7*x) + 80*b^5*e^(7*x) - 30
*a^3*b^2*e^(6*x) - 90*a*b^4*e^(6*x) + 48*a^4*b*e^(5*x) + 136*a^2*b^3*e^(5*x
) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) - 230*a^3*b^2*e^(4*x) - 240*a*b^4*e^(4
*x) + 20*a^2*b^3*e^(3*x) + 80*b^5*e^(3*x) - 40*a^5*e^(2*x) - 130*a^3*b^2*e^
(2*x) - 150*a*b^4*e^(2*x) + 15*b^5*e^x - 8*a^5 - 26*a^3*b^2 - 33*a*b^4)/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 1010, normalized size of antiderivative = 6.92

$$\int \frac{\operatorname{sech}^6(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,b^5\,e^x}{(a^2+b^2)^3}-\frac{2\,a\,b^4}{(a^2+b^2)^3}}{e^{2\,x}+1} - \frac{\frac{8\,(4\,a^3+3\,a\,b^2)}{3\,(a^2+b^2)^2}-\frac{8\,e^x\,(12\,a^2\,b+7\,b^3)}{15\,(a^2+b^2)^2}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{\frac{4\,(a^3\,b^2+a\,b^4)}{(a^2+b^2)^3}-\frac{8\,e^x\,(a^2\,b^3+b^5)}{3\,(a^2+b^2)^3}}{2\,e^{2\,x}+e^{4\,x}+1} - \frac{\frac{32\,a}{5\,(a^2+b^2)}-\frac{32\,b\,e^x}{5\,(a^2+b^2)}}{5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1} + \frac{\frac{16\,(a^3+a\,b^2)}{(a^2+b^2)^2}-\frac{64\,e^x\,(a^2\,b+b^3)}{5\,(a^2+b^2)^2}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,b^4}{\sqrt{b^{12}}\,(a^2+b^2)^3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{2\,a\left(a^7\,\sqrt{b^{12}}+3\,a^3\,b^4\,\sqrt{b^{12}}+3\,a^5\,b^2\,\sqrt{b^{12}}+a\,b^6\cdots\right.}{b^8\,\sqrt{-(a^2+b^2)^7}\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)\,\sqrt{-a^{14}-7\,a^{12}\,b^2-21\,a^{10}\,b^4-35\,a^8\,b\cdots}}\right.\right.\right.}{\cdots}$$

```
[In] int(1/(cosh(x)^6*(a + b*sinh(x))),x)
```

```
[Out] ((2*b^5*exp(x))/(a^2 + b^2)^3 - (2*a*b^4)/(a^2 + b^2)^3)/(exp(2*x) + 1) - (
(8*(3*a*b^2 + 4*a^3))/(3*(a^2 + b^2)^2) - (8*exp(x)*(12*a^2*b + 7*b^3))/(15
*(a^2 + b^2)^2))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a*b^4 + a^
3*b^2))/(a^2 + b^2)^3 - (8*exp(x)*(b^5 + a^2*b^3))/(3*(a^2 + b^2)^3))/(2*ex
```

```
p(2*x) + exp(4*x) + 1) - ((32*a)/(5*(a^2 + b^2)) - (32*b*exp(x))/(5*(a^2 +
b^2)))/(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1
) + ((16*(a*b^2 + a^3))/(a^2 + b^2)^2 - (64*exp(x)*(a^2*b + b^3))/(5*(a^2 +
 b^2)^2))/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) - (2*atan((
exp(x)*((2*b^4)/((b^12)^(1/2)*(a^2 + b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) + 3*a^5*b^2*(b^12)^
(1/2) + a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 + b^2)^7)^(1/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 -
35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^12)^(1/2) + 3
*a^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) + a^6*b*(b^12)^(1/2)))/(b^8*
(-(a^2 + b^2)^7)^(1/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(- a^14 - b^14 -
 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*
b^2)^(1/2)))*((b^7*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 -
 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (3*a^2*b^5*(- a^14 - b^1
4 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^
12*b^2)^(1/2))/2 + (3*a^4*b^3*(- a^14 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 3
5*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2))/2 + (a^6*b*(- a^1
4 - b^14 - 7*a^2*b^12 - 21*a^4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4
 - 7*a^12*b^2)^(1/2))/2))*(b^12)^(1/2))/(- a^14 - b^14 - 7*a^2*b^12 - 21*a^
4*b^10 - 35*a^6*b^8 - 35*a^8*b^6 - 21*a^10*b^4 - 7*a^12*b^2)^(1/2)
```

3.200 $\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{3(2a^2+b^2)\,x}{2b^4} + \frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

```
[Out] 3/2*(2*a^2+b^2)*x/b^4-3/2*cosh(x)*(2*a-b*sinh(x))/b^3-cosh(x)^3/b/(a+b*sinh
(x))+6*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^4
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2772, 2944, 2814, 2739, 632, 212}

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4} + \frac{3x(2a^2+b^2)}{2b^4} - \frac{3\cosh(x)(2a-b\sinh(x))}{2b^3} - \frac{\cosh^3(x)}{b(a+b\sinh(x))}$$

```
[In] Int[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (3*(2*a^2 + b^2)*x)/(2*b^4) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2]
)/Sqrt[a^2 + b^2]])/b^4 - (3*Cosh[x]*(2*a - b*Sinh[x]))/(2*b^3) - Cosh[x]^3
/(b*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh^3(x)}{b(a+b\sinh(x))}+\frac{3\int\frac{\cosh^2(x)\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= -\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}+\frac{(3i)\int\frac{iab-i(2a^2+b^2)\sinh(x)}{a+b\sinh(x)}\,dx}{2b^3}\\
&= \frac{3(2a^2+b^2)\,x}{2b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}-\frac{(3a(a^2+b^2))\int\frac{1}{a+b\sinh(x)}\,dx}{b^4}\\
&= \frac{3(2a^2+b^2)\,x}{2b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}\\
&\quad-\frac{(6a(a^2+b^2))\,\text{Subst}\left(\int\frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^4}\\
&= \frac{3(2a^2+b^2)\,x}{2b^4}-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}\\
&\quad+\frac{(12a(a^2+b^2))\,\text{Subst}\left(\int\frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^4}\\
&= \frac{3(2a^2+b^2)\,x}{2b^4}+\frac{6a\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^4}\\
&\quad-\frac{3\cosh(x)(2a-b\sinh(x))}{2b^3}-\frac{\cosh^3(x)}{b(a+b\sinh(x))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.60 (sec) , antiderivative size = 660, normalized size of antiderivative = 7.02

$$\int\frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$=\frac{\cosh^3(x)\left(12a\sqrt{a-ib}(a+ib)^{3/2}\text{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}(a+b\sinh(x))-12a(a^2+b^2)\,a\right.}{}$$

```
[In] Integrate[Cosh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (Cosh[x]^3*(12*a*Sqrt[a - I*b]*(a + I*b)^(3/2)*ArcTanh[Sqrt[-((b*(I + Sinh[
x]))/(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]
*(a + b*Sinh[x]) - 12*a*(a^2 + b^2)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + S
```

```
inh[x]))/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])
]*Sqrt[1 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[
x]))/(a + I*b))]*(6*(-1)^(3/4)*a*Sqrt[b]*(2*a^2 + I*a*b + b^2)*ArcSin[((1/2
 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 6*(-
1)^(3/4)*b^(3/2)*(2*a^2 + I*a*b + b^2)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sq
rt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]]*Sinh[x] - 2*Sqrt[a - I*b]*(3*a
^3 + (3*I)*a^2*b + a*b^2 + I*b^3)*Sqrt[1 + I*Sinh[x]]*Sqrt[-((b*(I + Sinh[x
]))/(a - I*b))] - 3*a*Sqrt[a - I*b]*(a + I*b)*b*Sqrt[1 + I*Sinh[x]]*Sinh[x]
*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))] + Sqrt[a - I*b]*(a + I*b)*b^2*Sqrt[1
+ I*Sinh[x]]*Sinh[x]^2*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])))/(2*(a - I*b)
^(3/2)*(a + I*b)^(5/2)*b*Sqrt[1 + I*Sinh[x]]*(-((b*(-I + Sinh[x]))/(a + I*b
)))^(3/2)*(-((b*(I + Sinh[x]))/(a - I*b)))^(3/2)*(a + b*Sinh[x]))
```

Maple [A] (verified)

Time = 17.49 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.71

method	result
risch	$\frac{3x\,a^2}{b^4}+\frac{3x}{2b^2}+\frac{e^{2x}}{8b^2}-\frac{a\,e^{x}}{b^3}-\frac{a\,e^{-x}}{b^3}-\frac{e^{-2x}}{8b^2}+\frac{2(a^2+b^2)(e^{x}a-b)}{b^4(b\,e^{2x}+2\,e^{x}a-b)}+\frac{3\sqrt{a^2+b^2}\,a\ln\left(e^{x}+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{b^4}-\frac{3\sqrt{a^2+b^2}\,a\ln\left(e^{x}-\right.}{b^4}$
default	$\frac{\frac{2\left(\frac{b^2\left(a^2+b^2\right)\tanh\left(\frac{x}{2}\right)}{a}+b\left(a^2+b^2\right)\right)}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-6a\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^4}+\frac{1}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-b-4a}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-6a^2-3}{}$

```
[In] int(cosh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3*x/b^4*a^2+3/2/b^2*x+1/8/b^2*exp(x)^2-1/b^3*a*exp(x)-1/b^3*a/exp(x)-1/8/b^
2/exp(x)^2+2*(a^2+b^2)*(exp(x)*a-b)/b^4/(b*exp(x)^2+2*exp(x)*a-b)+3*(a^2+b^
2)^(1/2)*a/b^4*ln(exp(x)+(a+(a^2+b^2)^(1/2))/b)-3*(a^2+b^2)^(1/2)*a/b^4*ln(
exp(x)-(-a+(a^2+b^2)^(1/2))/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 833 vs. $2(85) = 170$.

Time = 0.31 (sec) , antiderivative size = 833, normalized size of antiderivative = 8.86

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 6*a*b^2*cosh(x)^5 + 6*(b^3*cosh(x) - a
*b^2)*sinh(x)^5 - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^4 + (15*b
^3*cosh(x)^2 - 30*a*b^2*cosh(x) - 16*a^2*b - b^3 + 12*(2*a^2*b + b^3)*x)*si
```

```
nh(x)^4 + 6*a*b^2*cosh(x) + 8*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(
x)^3 + 4*(5*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 + 4*a^3 + 4*a*b^2 + 6*(2*a^3
 + a*b^2)*x - (16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x)^3 +
b^3 - (32*a^2*b + 17*b^3 + 12*(2*a^2*b + b^3)*x)*cosh(x)^2 + (15*b^3*cosh(x
)^4 - 60*a*b^2*cosh(x)^3 - 32*a^2*b - 17*b^3 - 6*(16*a^2*b + b^3 - 12*(2*a^
2*b + b^3)*x)*cosh(x)^2 - 12*(2*a^2*b + b^3)*x + 24*(2*a^3 + 2*a*b^2 + 3*(2
*a^3 + a*b^2)*x)*cosh(x))*sinh(x)^2 + 24*(a*b*cosh(x)^4 + a*b*sinh(x)^4 + 2
*a^2*cosh(x)^3 - a*b*cosh(x)^2 + 2*(2*a*b*cosh(x) + a^2)*sinh(x)^3 + (6*a*b
*cosh(x)^2 + 6*a^2*cosh(x) - a*b)*sinh(x)^2 + 2*(2*a*b*cosh(x)^3 + 3*a^2*co
sh(x)^2 - a*b*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*si
nh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*
a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(3*b^3*cosh(x)^5 - 15*a*b^2
*cosh(x)^4 - 2*(16*a^2*b + b^3 - 12*(2*a^2*b + b^3)*x)*cosh(x)^3 + 3*a*b^2
+ 12*(2*a^3 + 2*a*b^2 + 3*(2*a^3 + a*b^2)*x)*cosh(x)^2 - (32*a^2*b + 17*b^3
 + 12*(2*a^2*b + b^3)*x)*cosh(x))*sinh(x))/(b^5*cosh(x)^4 + b^5*sinh(x)^4 +
 2*a*b^4*cosh(x)^3 - b^5*cosh(x)^2 + 2*(2*b^5*cosh(x) + a*b^4)*sinh(x)^3 +
(6*b^5*cosh(x)^2 + 6*a*b^4*cosh(x) - b^5)*sinh(x)^2 + 2*(2*b^5*cosh(x)^3 +
3*a*b^4*cosh(x)^2 - b^5*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**4/(a+b*sinh(x))**2,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{6\,ab^2e^{(-x)} - b^3 + (32\,a^2b + 17\,b^3)e^{(-2\,x)} + 8\,(2\,a^3 + ab^2)e^{(-3\,x)}}{8\,(b^5e^{(-2\,x)} + 2\,ab^4e^{(-3\,x)} - b^5e^{(-4\,x)})} - \frac{3\,\sqrt{a^2+b^2}a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{b^4} - \frac{8\,ae^{(-x)} + be^{(-2\,x)}}{8\,b^3} + \frac{3\,(2\,a^2+b^2)x}{2\,b^4}$$

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(6*a*b^2*e^(-x) - b^3 + (32*a^2*b + 17*b^3)*e^(-2*x) + 8*(2*a^3 + a*b^
2)*e^(-3*x))/(b^5*e^(-2*x) + 2*a*b^4*e^(-3*x) - b^5*e^(-4*x)) - 3*sqrt(a^2
+ b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^
2)))/b^4 - 1/8*(8*a*e^(-x) + b*e^(-2*x))/b^3 + 3/2*(2*a^2 + b^2)*x/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.89

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{3\,(2\,a^2+b^2)x}{2\,b^4} - \frac{3\,(a^3+ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4} + \frac{b^2e^{(2\,x)}-8\,abe^x}{8\,b^4} + \frac{\left(6\,ab^2e^x+b^3+8\,(2\,a^3+ab^2)e^{(3\,x)}-(32\,a^2b+17\,b^3)e^{(2\,x)}\right)e^{(-2\,x)}}{8\,(be^{(2\,x)}+2\,ae^x-b)b^4}$$

```
[In] integrate(cosh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*(2*a^2 + b^2)*x/b^4 - 3*(a^3 + a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^
2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 1
/8*(b^2*e^(2*x) - 8*a*b*e^x)/b^4 + 1/8*(6*a*b^2*e^x + b^3 + 8*(2*a^3 + a*b^
2)*e^(3*x) - (32*a^2*b + 17*b^3)*e^(2*x))*e^(-2*x)/((b*e^(2*x) + 2*a*e^x -
b)*b^4)
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.72

$$\int \frac{\cosh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{e^{2\,x}}{8\,b^2} - \frac{e^{-2\,x}}{8\,b^2} - \frac{\frac{2\,(a^4\,b^2+2\,a^2\,b^4+b^6)}{b^4\,(a^2\,b+b^3)} - \frac{2\,e^x\,(a^5\,b^2+2\,a^3\,b^4+a\,b^6)}{b^5\,(a^2\,b+b^3)}}{2\,a\,e^x-b+b\,e^{2\,x}} + \frac{x\,(6\,a^2+3\,b^2)}{2\,b^4} - \frac{a\,e^x}{b^3} - \frac{a\,e^{-x}}{b^3} - \frac{3\,a\,\ln\left(\frac{6\,a\,e^x\,(a^2+b^2)}{b^5} - \frac{6\,a\,(b-a\,e^x)\,\sqrt{a^2+b^2}}{b^5}\right)\sqrt{a^2+b^2}}{b^4} + \frac{3\,a\,\ln\left(\frac{6\,a\,(b-a\,e^x)\,\sqrt{a^2+b^2}}{b^5} + \frac{6\,a\,e^x\,(a^2+b^2)}{b^5}\right)\sqrt{a^2+b^2}}{b^4}$$

```
[In] int(cosh(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] exp(2*x)/(8*b^2) - exp(-2*x)/(8*b^2) - ((2*(b^6 + 2*a^2*b^4 + a^4*b^2))/(b^
4*(a^2*b + b^3)) - (2*exp(x)*(a*b^6 + 2*a^3*b^4 + a^5*b^2))/(b^5*(a^2*b + b
^3)))/(2*a*exp(x) - b + b*exp(2*x)) + (x*(6*a^2 + 3*b^2))/(2*b^4) - (a*exp(
```

```
x))/b^3 - (a*exp(-x))/b^3 - (3*a*log((6*a*exp(x)*(a^2 + b^2))/b^5 - (6*a*(b
 - a*exp(x))*(a^2 + b^2)^(1/2))/b^5)*(a^2 + b^2)^(1/2))/b^4 + (3*a*log((6*a
*(b - a*exp(x))*(a^2 + b^2)^(1/2))/b^5 + (6*a*exp(x)*(a^2 + b^2))/b^5)*(a^2
 + b^2)^(1/2))/b^4
```

3.201 $\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2} dx = -\frac{2a\log(a+b\sinh(x))}{b^3} + \frac{\sinh(x)}{b^2} - \frac{a^2+b^2}{b^3(a+b\sinh(x))}$$

```
[Out] -2*a*ln(a+b*sinh(x))/b^3+sinh(x)/b^2+(-a^2-b^2)/b^3/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2747, 711}

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2} dx = -\frac{a^2+b^2}{b^3(a+b\sinh(x))} - \frac{2a\log(a+b\sinh(x))}{b^3} + \frac{\sinh(x)}{b^2}$$

```
[In] Int[Cosh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Sinh[x]])/b^3 + Sinh[x]/b^2 - (a^2 + b^2)/(b^3*(a + b*Sinh[
x]))
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{(a+x)^2}\,dx, x, b\sinh(x)\right)}{b^3}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{-a^2-b^2}{(a+x)^2}+\frac{2a}{a+x}\right)\,dx, x, b\sinh(x)\right)}{b^3}\\ &= -\frac{2a\log(a+b\sinh(x))}{b^3}+\frac{\sinh(x)}{b^2}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a\log(a+b\sinh(x))-b\sinh(x)+\frac{a^2+b^2}{a+b\sinh(x)}}{b^3}$$

```
[In] Integrate[Cosh[x]^3/(a + b*Sinh[x])^2,x]
[Out] -((2*a*Log[a + b*Sinh[x]] - b*Sinh[x] + (a^2 + b^2)/(a + b*Sinh[x]))/b^3)
```

Maple [A] (verified)

Time = 6.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{\sinh(x)}{b^2}-\frac{2a\ln(a+b\sinh(x))}{b^3}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}$	41
default	$\frac{\sinh(x)}{b^2}-\frac{2a\ln(a+b\sinh(x))}{b^3}-\frac{a^2+b^2}{b^3(a+b\sinh(x))}$	41
risch	$\frac{2ax}{b^3}+\frac{e^x}{2b^2}-\frac{e^{-x}}{2b^2}-\frac{2(a^2+b^2)e^x}{b^3(b\,e^{2x}+2\,e^x a-b)}-\frac{2a\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{b^3}$	77

```
[In] int(cosh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] sinh(x)/b^2-2*a*ln(a+b*sinh(x))/b^3-(a^2+b^2)/b^3/(a+b*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(40) = 80.

Time = 0.29 (sec) , antiderivative size = 370, normalized size of antiderivative = 9.25

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 + 2\,(2\,abx + ab)\cosh(x)^3 + 2\,(2\,abx + 2\,b^2\cosh(x) + ab)\sinh(x)^3 + 2\,(4\,a^2x - 2}{}$$

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 2*(2*a*b*x + a*b)*cosh(x)^3 + 2*(2*a*b
*x + 2*b^2*cosh(x) + a*b)*sinh(x)^3 + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x)^2
 + 2*(3*b^2*cosh(x)^2 + 4*a^2*x - 2*a^2 - 3*b^2 + 3*(2*a*b*x + a*b)*cosh(x)
)*sinh(x)^2 + b^2 - 2*(2*a*b*x + a*b)*cosh(x) - 4*(a*b*cosh(x)^3 + a*b*sinh
(x)^3 + 2*a^2*cosh(x)^2 - a*b*cosh(x) + (3*a*b*cosh(x) + 2*a^2)*sinh(x)^2 +
 (3*a*b*cosh(x)^2 + 4*a^2*cosh(x) - a*b)*sinh(x))*log(2*(b*sinh(x) + a)/(co
sh(x) - sinh(x))) + 2*(2*b^2*cosh(x)^3 - 2*a*b*x + 3*(2*a*b*x + a*b)*cosh(x
)^2 - a*b + 2*(4*a^2*x - 2*a^2 - 3*b^2)*cosh(x))*sinh(x))/(b^4*cosh(x)^3 +
b^4*sinh(x)^3 + 2*a*b^3*cosh(x)^2 - b^4*cosh(x) + (3*b^4*cosh(x) + 2*a*b^3)
*sinh(x)^2 + (3*b^4*cosh(x)^2 + 4*a*b^3*cosh(x) - b^4)*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(39) = 78.

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.32

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \begin{cases} -\frac{2a^2\log\left(\frac{a}{b}+\sinh(x)\right)}{ab^3+b^4\sinh(x)} - \frac{2a^2}{ab^3+b^4\sinh(x)} - \frac{2ab\log\left(\frac{a}{b}+\sinh(x)\right)\sinh(x)}{ab^3+b^4\sinh(x)} + \frac{2b^2\sinh^2(x)}{ab^3+b^4\sinh(x)} - \frac{b^2\cosh^2(x)}{ab^3+b^4\sinh(x)} & \text{for } b\neq 0 \\ \frac{-\frac{2\sinh^3(x)}{3}+\sinh(x)\cosh^2(x)}{a^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((-2*a**2*log(a/b + sinh(x))/(a*b**3 + b**4*sinh(x)) - 2*a**2/(a*b
**3 + b**4*sinh(x)) - 2*a*b*log(a/b + sinh(x))*sinh(x)/(a*b**3 + b**4*sinh(
x)) + 2*b**2*sinh(x)**2/(a*b**3 + b**4*sinh(x)) - b**2*cosh(x)**2/(a*b**3 +
 b**4*sinh(x)), Ne(b, 0)), ((-2*sinh(x)**3/3 + sinh(x)*cosh(x)**2)/a**2, Tr
ue))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(40) = 80$.

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.55

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,abe^{(-x)} + b^2 - (4\,a^2 + 5\,b^2)e^{(-2\,x)}}{2\,(b^4e^{(-x)} + 2\,ab^3e^{(-2\,x)} - b^4e^{(-3\,x)})} - \frac{2\,ax}{b^3} - \frac{e^{(-x)}}{2\,b^2} - \frac{2\,a\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{b^3}$$

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b*e^(-x) + b^2 - (4*a^2 + 5*b^2)*e^(-2*x))/(b^4*e^(-x) + 2*a*b^3*e
^(-2*x) - b^4*e^(-3*x)) - 2*a*x/b^3 - 1/2*e^(-x)/b^2 - 2*a*log(-2*a*e^(-x)
+ b*e^(-2*x) - b)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = -\frac{e^{(-x)} - e^x}{2\,b^2} - \frac{2\,a\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{b^3} + \frac{2\,\left(ab\left(e^{(-x)} - e^x\right) - a^2 + b^2\right)}{\left(b\left(e^{(-x)} - e^x\right) - 2\,a\right)b^3}$$

```
[In] integrate(cosh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/b^2 - 2*a*log(abs(-b*(e^(-x) - e^x) + 2*a))/b^3 + 2*(a*
b*(e^(-x) - e^x) - a^2 + b^2)/((b*(e^(-x) - e^x) - 2*a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{\cosh(x)^2}{b} - \frac{2\sinh(x)^3}{a} + \frac{2\cosh(x)^2\sinh(x)}{a} + \frac{2\,a\sinh(x)}{b^2}}{a + b\sinh\left(x\right)} - \frac{2\,a\,\ln\left(a + b\sinh(x)\right)}{b^3}$$

```
[In] int(cosh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] (cosh(x)^2/b - (2*sinh(x)^3)/a + (2*cosh(x)^2*sinh(x))/a + (2*a*sinh(x))/b^
2)/(a + b*sinh(x)) - (2*a*log(a + b*sinh(x)))/b^3
```

3.202 $\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{x}{b^2} + \frac{2a\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\cosh(x)}{b(a+b\sinh(x))}$$

```
[Out] x/b^2-cosh(x)/b/(a+b*sinh(x))+2*a*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/b^2/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2772, 2814, 2739, 632, 212}

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2a\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{\cosh(x)}{b(a+b\sinh(x))} + \frac{x}{b^2}$$

```
[In] Int[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] x/b^2 + (2*a*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]) - Cosh[x]/(b*(a + b*Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2772

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Dist[g^2*((p - 1)/(b*(m + 1))), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{\int \frac{\sinh(x)}{a+b\sinh(x)}\,dx}{b}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{a\int \frac{1}{a+b\sinh(x)}\,dx}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}-\frac{(2a)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}-\frac{\cosh(x)}{b(a+b\sinh(x))}+\frac{(4a)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{x}{b^2}+\frac{2a\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{\cosh(x)}{b(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 502, normalized size of antiderivative = 8.10

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\cosh(x)\left(2a\sqrt{a-ib}\sqrt{a+ib}\,\text{arctanh}\left(\frac{\sqrt{-\frac{b(i+\sinh(x))}{a-ib}}}{\sqrt{-\frac{b(-i+\sinh(x))}{a+ib}}}\right)\sqrt{1+i\sinh(x)}(a+b\sinh(x))-2a(a-ib)\text{arctanh}\left(\right.\right.}{}$$

```
[In] Integrate[Cosh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (Cosh[x]*(2*a*Sqrt[a - I*b]*Sqrt[a + I*b]*ArcTanh[Sqrt[-((b*(I + Sinh[x]))/
(a - I*b))]/Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]]*Sqrt[1 + I*Sinh[x]]*(a +
 b*Sinh[x]) - 2*a*(a - I*b)*ArcTanh[(Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))
/(a - I*b))])/(Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))])]*Sqrt[1
 + I*Sinh[x]]*(a + b*Sinh[x]) + Sqrt[a + I*b]*Sqrt[-((b*(-I + Sinh[x]))/(a
+ I*b))]*(2*(-1)^(1/4)*a*Sqrt[b]*(I*a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b
]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b))])/Sqrt[b]] + 2*(-1)^(1/4)*b^(3/2)*(I*
a + b)*ArcSin[((1/2 + I/2)*Sqrt[a - I*b]*Sqrt[-((b*(I + Sinh[x]))/(a - I*b)
)])/Sqrt[b]]*Sinh[x] - Sqrt[a - I*b]*(a^2 + b^2)*Sqrt[1 + I*Sinh[x]]*Sqrt[-
((b*(I + Sinh[x]))/(a - I*b))])))/((a - I*b)^(3/2)*(a + I*b)^(3/2)*b*Sqrt[1
 + I*Sinh[x]]*Sqrt[-((b*(-I + Sinh[x]))/(a + I*b))]*Sqrt[-((b*(I + Sinh[x])
)/(a - I*b))]*(a + b*Sinh[x]))
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

method	result	size
default	$\frac{\frac{2\left(\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}+b\right)}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{2a\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b^2}$	101
risch	$\frac{x}{b^2}+\frac{2\,e^x a-2b}{b^2\left(b\,e^{2x}+2\,e^x a-b\right)}+\frac{a\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{a\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)}{\sqrt{a^2+b^2}\,b^2}$	140

```
[In] int(cosh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*((1/a*b^2*tanh(1/2*x)+b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-a/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/b^2*ln(tanh
(1/2*x)-1)+1/b^2*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(58) = 116.

Time = 0.31 (sec) , antiderivative size = 362, normalized size of antiderivative = 5.84

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^2b+b^3)x\cosh(x)^2+(a^2b+b^3)x\sinh(x)^2-2a^2b-2b^3+\left(ab\cosh(x)^2+ab\sinh(x)^2+2a^2\cosh(x\right.}{}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*x*cosh(x)^2 + (a^2*b + b^3)*x*sinh(x)^2 - 2*a^2*b - 2*b^3 +
 (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^
2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) - b)) - (a^2*b + b^3)*x + 2*(a^3 + a*b^2 + (a^3 + a*b^2)*
x)*cosh(x) + 2*(a^3 + a*b^2 + (a^2*b + b^3)*x*cosh(x) + (a^3 + a*b^2)*x)*si
nh(x))/(a^2*b^3 + b^5 - (a^2*b^3 + b^5)*cosh(x)^2 - (a^2*b^3 + b^5)*sinh(x)
^2 - 2*(a^3*b^2 + a*b^4)*cosh(x) - 2*(a^3*b^2 + a*b^4 + (a^2*b^3 + b^5)*cos
h(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Timed out}$$

```
[In] integrate(cosh(x)**2/(a+b*sinh(x))**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\left(ae^{(-x)}+b\right)}{2\,ab^2e^{(-x)}-b^3e^{(-2\,x)}+b^3}-\frac{a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}b^2}+\frac{x}{b^2}$$

```
[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(a*e^(-x) + b)/(2*a*b^2*e^(-x) - b^3*e^(-2*x) + b^3) - a*log((b*e^(-x) -
 a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^
2) + x/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{a\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2} + \frac{x}{b^2} + \frac{2\,(ae^x-b)}{(be^{(2\,x)}+2\,ae^x-b)b^2}$$

[In] integrate(cosh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")

[Out] -a*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + x/b^2 + 2*(a*e^x - b)/((b*e^(2*x) + 2*a*e^x - b)*b^2)

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

$$\int \frac{\cosh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{x}{b^2} - \frac{\frac{2}{b}-\frac{2\,a\,e^x}{b^2}}{2\,a\,e^x-b+b\,e^{2\,x}} - \frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}-\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}} + \frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}+\frac{2\,a\,(b-a\,e^x)}{b^3\,\sqrt{a^2+b^2}}\right)}{b^2\,\sqrt{a^2+b^2}}$$

[In] int(cosh(x)^2/(a + b*sinh(x))^2,x)

[Out] x/b^2 - (2/b - (2*a*exp(x))/b^2)/(2*a*exp(x) - b + b*exp(2*x)) - (a*log((2*a*exp(x))/b^3 - (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2)) + (a*log((2*a*exp(x))/b^3 + (2*a*(b - a*exp(x)))/(b^3*(a^2 + b^2)^(1/2))))/(b^2*(a^2 + b^2)^(1/2))

3.203 $\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{1}{b(a+b\sinh(x))}$$

```
[Out] -1/b/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2747, 32}

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{1}{b(a+b\sinh(x))}$$

```
[In] Int[Cosh[x]/(a + b*Sinh[x])^2,x]
[Out] -(1/(b*(a + b*Sinh[x])))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, b\sinh(x)\right)}{b}\\ &= -\frac{1}{b(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{1}{b(a+b\sinh(x))}$$

```
[In] Integrate[Cosh[x]/(a + b*Sinh[x])^2,x]
[Out] -(1/(b*(a + b*Sinh[x])))
```

Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{1}{b(a+b\sinh(x))}$	14
default	$-\frac{1}{b(a+b\sinh(x))}$	14
risch	$-\frac{2\,e^{x}}{b(b\,e^{2x}+2\,e^{x}a-b)}$	25

```
[In] int(cosh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/b/(a+b*sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(13) = 26.

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.92

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\,(\cosh(x)+\sinh(x))}{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*(cosh(x) + sinh(x))/(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) - b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = \begin{cases} -\frac{1}{ab+b^2\sinh(x)} & \text{for } b\neq 0 \\ \frac{\sinh(x)}{a^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Piecewise((-1/(a*b + b**2*sinh(x)), Ne(b, 0)), (sinh(x)/a**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{1}{(b\sinh(x)+a)b}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -1/((b*sinh(x) + a)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = \frac{2}{(b(e^{(-x)}-e^x)-2\,a)b}$$

```
[In] integrate(cosh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2/((b*(e^(-x) - e^x) - 2*a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\cosh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\sinh(x)}{a\ (a+b\sinh{(x)})}$$

```
[In] int(cosh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] sinh(x)/(a*(a + b*sinh(x)))
```

3.204 $\int \frac{\mathrm{sech}(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 79

$$\int \frac{\mathrm{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^2} - \frac{2ab\log(\cosh(x))}{(a^2+b^2)^2} + \frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2} - \frac{b}{(a^2+b^2)\,(a+b\sinh(x))}$$

```
[Out] (a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^2-2*a*b*ln(cosh(x))/(a^2+b^2)^2+2*a*b*l
n(a+b*sinh(x))/(a^2+b^2)^2-b/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {2747, 724, 815, 649, 209, 266}

$$\int \frac{\mathrm{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^2} - \frac{b}{(a^2+b^2)\,(a+b\sinh(x))} + \frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2} - \frac{2ab\log(\cosh(x))}{(a^2+b^2)^2}$$

```
[In] Int[Sech[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^2 - (2*a*b*Log[Cosh[x]])/(a^2 + b
^2)^2 + (2*a*b*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 - b/((a^2 + b^2)*(a + b*Si
nh[x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 724

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[e*((d
 + e*x)^(m + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + Dist[c/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*((d - e*x)/(a + c*x^2)), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\right)\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{b\text{Subst}\left(\int \frac{a-x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{b}{\left(a^2+b^2\right)(a+b\sinh(x))}-\frac{b\text{Subst}\left(\int \left(-\frac{2a}{\left(a^2+b^2\right)(a+x)}+\frac{-a^2+b^2+2ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)\,dx, x, b\sinh(x)\right)}{a^2+b^2}
\end{aligned}
$$

$$=\frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2}-\frac{b}{(a^2+b^2)(a+b\sinh(x))}-\frac{b\text{Subst}\left(\int\frac{-a^2+b^2+2ax}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{(a^2+b^2)^2}$$

$$=\frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2}-\frac{b}{(a^2+b^2)(a+b\sinh(x))}-\frac{(2ab)\text{Subst}\left(\int\frac{x}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{(a^2+b^2)^2}+\frac{(b(a^2-b^2))\,\text{Subst}\left(\int\frac{1}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{(a^2+b^2)^2}$$

$$=\frac{(a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^2}-\frac{2ab\log(\cosh(x))}{(a^2+b^2)^2}+\frac{2ab\log(a+b\sinh(x))}{(a^2+b^2)^2}-\frac{b}{(a^2+b^2)(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.53

$$\int\frac{\text{sech}(x)}{(a+b\sinh(x))^2}\,dx=-\frac{b\left(\left(2a+\frac{-a^2+b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-4a\log(a+b\sinh(x))+\left(2a+\frac{a^2-b^2}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\sinh(x\right.\right.}{2\left(a^2+b^2\right)^2}$$

```
[In] Integrate[Sech[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*(b*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a*
Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh
[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x])))/(a^2 + b^2)^2
```

Maple [A] (verified)

Time = 13.92 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

method	result
default	$\frac{2b\left(-\frac{b(a^2+b^2)\tanh(\frac{x}{2})}{a\left(\tanh(\frac{x}{2})^2a-2b\tanh(\frac{x}{2})-a\right)}+a\ln\left(\tanh(\frac{x}{2})^2a-2b\tanh(\frac{x}{2})-a\right)\right)}{(a^2+b^2)^2}+\frac{-2ab\ln\left(1+\tanh(\frac{x}{2})^2\right)+2(a^2-b^2)\arctan(\tanh(\frac{x}{2}))}{a^4+2a^2b^2+b^4}$
risch	$-\frac{2b\,e^{x}}{(a^2+b^2)(b\,e^{2x}+2\,e^{x}a-b)}-\frac{i\ln(e^{x}-i)a^2}{a^4+2a^2b^2+b^4}+\frac{i\ln(e^{x}-i)b^2}{a^4+2a^2b^2+b^4}-\frac{2\ln(e^{x}-i)ab}{a^4+2a^2b^2+b^4}+\frac{i\ln(e^{x}+i)a^2}{a^4+2a^2b^2+b^4}-\frac{i\ln(e^{x}+i)b^2}{a^4+2a^2b^2+b^4}-\frac{2\ln(e}{a^4+2a}$

```
[In] int(sech(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b/(a^2+b^2)^2*(-b*(a^2+b^2)/a*tanh(1/2*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x
)-a)+a*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a))+2/(a^4+2*a^2*b^2+b^4)*(-a*b*l
n(1+tanh(1/2*x)^2)+(a^2-b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(79) = 158$.

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.85

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{2\left(\left(a^2b-b^3-(a^2b-b^3)\cosh(x)^2-(a^2b-b^3)\sinh(x)^2-2\left(a^3-ab^2\right)\cosh(x)-2\left(a^3-ab^2+(a^2b-b^3)\right.\right.\right.}{}$$

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^
3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*arcta
n(cosh(x) + sinh(x)) + (a^2*b + b^3)*cosh(x) - (a*b^2*cosh(x)^2 + a*b^2*sin
h(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2
*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2
+ 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2*cosh(x) + a^2*b)*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + (a^2*b + b^3)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 -
(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 -
 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b
+ 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(sech(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,ab\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4} - \frac{2\,ab\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{2\,(a^2-b^2)\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4} - \frac{2\,be^{(-x)}}{a^2b+b^3+2\,(a^3+ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}}$$

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) - 2*a*b*log
(e^(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a^2 - b^2)*arctan(e^(-x))/(a^4
+ 2*a^2*b^2 + b^4) - 2*b*e^(-x)/(a^2*b + b^3 + 2*(a^3 + a*b^2)*e^(-x) - (a^
2*b + b^3)*e^(-2*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(79) = 158.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \frac{\operatorname{sech}(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,ab^2\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{ab\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)(a^2-b^2)}{2\,(a^4+2\,a^2b^2+b^4)} - \frac{2\left(ab^2\left(e^{(-x)}-e^x\right)-3\,a^2b-b^3\right)}{(a^4+2\,a^2b^2+b^4)(b(e^{(-x)}-e^x)-2\,a)}$$

```
[In] integrate(sech(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*b^2*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*l
og((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(pi + 2*arctan(1/2*(
e^(2*x) - 1)*e^(-x)))*(a^2 - b^2)/(a^4 + 2*a^2*b^2 + b^4) - 2*(a*b^2*(e^(-x
) - e^x) - 3*a^2*b - b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x) - 2*a)
)
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.35

$$\int \frac{\mathrm{sech}(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{2\,a\,b\,\ln\left(b^5\,\mathrm{e}^{2\,x}-a^4\,b-b^5-14\,a^2\,b^3+2\,a^5\,\mathrm{e}^x+14\,a^2\,b^3\,\mathrm{e}^{2\,x}+2\,a\,b^4\,\mathrm{e}^x+a^4\,b\,\mathrm{e}^{2\,x}+28\,a^3\,b^2\,\mathrm{e}^x\right)}{a^4+2\,a^2\,b^2+b^4}$$
$$-\frac{2\,b^2\,\mathrm{e}^x}{\left(a^2\,b+b^3\right)\,\left(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}\right)}-\frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)}{-a^2\,1\mathrm{i}+2\,a\,b+b^2\,1\mathrm{i}}-\frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{-a^2+a\,b\,2\mathrm{i}+b^2}$$

```
[In] int(1/(cosh(x)*(a + b*sinh(x))^2),x)
```

```
[Out] (2*a*b*log(b^5*exp(2*x) - a^4*b - b^5 - 14*a^2*b^3 + 2*a^5*exp(x) + 14*a^2*
b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) + 28*a^3*b^2*exp(x)))/(a^4 +
 b^4 + 2*a^2*b^2) - (log(exp(x) + 1i)*1i)/(a*b*2i - a^2 + b^2) - (2*b^2*exp
(x))/((a^2*b + b^3)*(2*a*exp(x) - b + b*exp(2*x))) - log(exp(x)*1i + 1)/(2*
a*b - a^2*1i + b^2*1i)
```

3.205 $\int \frac{\text{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \frac{\text{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{6ab^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\text{sech}(x)}{(a^2+b^2)\,(a+b\sinh(x))} + \frac{\text{sech}(x)\,(3ab+(a^2-2b^2)\sinh(x))}{(a^2+b^2)^2}$$

```
[Out] -6*a*b^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-b*sech(
x)/(a^2+b^2)/(a+b*sinh(x))+sech(x)*(3*a*b+(a^2-2*b^2)*sinh(x))/(a^2+b^2)^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2773, 2945, 12, 2739, 632, 212}

$$\int \frac{\text{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{6ab^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\text{sech}(x)}{(a^2+b^2)\,(a+b\sinh(x))} + \frac{\text{sech}(x)\,((a^2-2b^2)\sinh(x)+3ab)}{(a^2+b^2)^2}$$

```
[In] Int[Sech[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-6*a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
b*Sech[x])/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]*(3*a*b + (a^2 - 2*b^2)*
Sinh[x]))/(a^2 + b^2)^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} - \frac{\int \frac{\text{sech}^2(x)(-a+2b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{\int \frac{3ab^2}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}$$

$$= -\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(3ab^2)\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}$$

$$= -\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(6ab^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}$$

$$= -\frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2} - \frac{(12ab^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}$$

$$= -\frac{6ab^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\text{sech}(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}(x)\left(3ab+(a^2-2b^2)\sinh(x)\right)}{(a^2+b^2)^2}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{\text{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{6ab^2\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 2ab\text{sech}(x) - \frac{b^3\cosh(x)}{a+b\sinh(x)} + a^2\tanh(x) - b^2\tanh(x)}{(a^2+b^2)^2}$$

```
[In] Integrate[Sech[x]^2/(a + b*Sinh[x])^2,x]
[Out] ((6*a*b^2*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 2*
a*b*Sech[x] - (b^3*Cosh[x])/(a + b*Sinh[x]) + a^2*Tanh[x] - b^2*Tanh[x])/(a
^2 + b^2)^2
```

Maple [A] (verified)

Time = 50.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.48

method	result
default	$-\frac{2b^2\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{3a\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2}-\frac{2\left(\left(-a^2+b^2\right)\tanh\left(\frac{x}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}$
risch	$-\frac{2\left(-3a\,b^2e^{3x}-3a^2b\,e^{2x}+2a^3e^x-a\,b^2e^x-a^2b+2b^3\right)}{\left(b\,e^{2x}+2\,e^x a-b\right)\left(1+e^{2x}\right)\left(a^4+2a^2b^2+b^4\right)}+\frac{3b^2a\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-3a^4b^2-3a^2b^4-b^6}{b\left(a^2+b^2\right)^{\frac{5}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{5}{2}}}-\frac{3b^2a\ln\left(e^x+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}}{}\right.}{\left(a^2+\right.}$

[In] int(sech(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] -2/(a^2+b^2)^2*b^2*((-1/a*b^2*tanh(1/2*x)-b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(1+tanh(1/2*x)^2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. 2(89) = 178.

Time = 0.30 (sec) , antiderivative size = 802, normalized size of antiderivative = 8.62

$$\int\frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\,a^4b-2\,a^2b^3-4\,b^5+6\,(a^3b^2+ab^4)\cosh{(x)}^3+6\,(a^3b^2+ab^4)\sinh{(x)}^3+6\,(a^4b+a^2b^3)\cosh{(x)}^2+6\,(a}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7-(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cosh}$$

[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")

[Out] -(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 6*(a^3*b^2 + a*b^4)*cosh(x)^3 + 6*(a^3*b^2 + a*b^4)*sinh(x)^3 + 6*(a^4*b + a^2*b^3)*cosh(x)^2 + 6*(a^4*b + a^2*b^3 + 3*(a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^2 + 3*(a*b^3*cosh(x)^4 + a*b^3*sinh(x)^4 + 2*a^2*b^2*cosh(x)^3 + 2*a^2*b^2*cosh(x) - a*b^3 + 2*(2*a*b^3*cosh(x) + a^2*b^2)*sinh(x)^3 + 6*(a*b^3*cosh(x)^2 + a^2*b^2*cosh(x))*sinh(x)^2 + 2*(2*a*b^3*cosh(x)^3 + 3*a^2*b^2*cosh(x)^2 + a^2*b^2)*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2*(2*a^5 + a^3*b^2 - a*b^4)*cosh(x) - 2*(2*a^5 + a^3*b^2 - a*b^4 - 9*(a^3*b^2 + a*b^4)*cosh(x)^2 - 6*(a^4*b + a^2*b^3)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (

```
a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7 + 3*a^5*b^2 + 3*a^3
*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*((a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x
))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) - 2*(a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2)*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(sech(x)**2/(a+b*sinh(x))**2,x)
[Out] Integral(sech(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(89) = 178$.

Time = 0.29 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.31

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{3\,ab^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,a^2be^{(-2\,x)}-3\,ab^2e^{(-3\,x)}+a^2b-2\,b^3+(2\,a^3-ab^2)e^{(-x)}\right)}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+2\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}-(a^4b+2\,a^2b^3+b^5)e^{(-4\,x)}}$$

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] 3*a*b^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2
)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a^2*b*e^(-2*x) - 3*a*b
^2*e^(-3*x) + a^2*b - 2*b^3 + (2*a^3 - a*b^2)*e^(-x))/(a^4*b + 2*a^2*b^3 +
b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-
3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{3\,ab^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3\,ab^2e^{(3\,x)}+3\,a^2be^{(2\,x)}-2\,a^3e^x+ab^2e^x+a^2b-2\,b^3\right)}{(a^4+2\,a^2b^2+b^4)(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b)}$$

```
[In] integrate(sech(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 3*a*b^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(3*a*b^2*e^(3
*x) + 3*a^2*b*e^(2*x) - 2*a^3*e^x + a*b^2*e^x + a^2*b - 2*b^3)/((a^4 + 2*a^
2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b))
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{sech}^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{6\,a^4\,b^4\,e^{2\,x}}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)} - \frac{2\,(2\,a^2\,b^6-a^4\,b^4)}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)} + \frac{6\,a^3\,b^5\,e^{3\,x}}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)} + \frac{2\,a\,e^x\,(a^2\,b^6-2\,a^4\,b^4)}{b\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}}{2\,a\,e^x-b+2\,a\,e^{3\,x}+b\,e^{4\,x}} - \frac{3\,a\,b^2\,\ln\left(-\frac{6\,a\,b\,e^x}{(a^2+b^2)^2}-\frac{6\,a\,b\,(b-a\,e^x)}{(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}} + \frac{3\,a\,b^2\,\ln\left(\frac{6\,a\,b\,(b-a\,e^x)}{(a^2+b^2)^{5/2}}-\frac{6\,a\,b\,e^x}{(a^2+b^2)^2}\right)}{(a^2+b^2)^{5/2}}$$

```
[In] int(1/(cosh(x)^2*(a + b*sinh(x))^2),x)
```

```
[Out] ((6*a^4*b^4*exp(2*x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*(2*a^2*b^6 - a
^4*b^4))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^3*b^5*exp(3*x))/((a*b^2 +
 a^3)*(a*b^5 + a^3*b^3)) + (2*a*exp(x)*(a^2*b^6 - 2*a^4*b^4))/(b*(a*b^2 + a
^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*exp(3*x) + b*exp(4*x)) - (3*a
*b^2*log(- (6*a*b*exp(x))/(a^2 + b^2)^2 - (6*a*b*(b - a*exp(x)))/(a^2 + b^2
)^(5/2)))/(a^2 + b^2)^(5/2) + (3*a*b^2*log((6*a*b*(b - a*exp(x)))/(a^2 + b^
2)^(5/2) - (6*a*b*exp(x))/(a^2 + b^2)^2))/(a^2 + b^2)^(5/2)
```

3.206 $\int \frac{\text{sech}^3(x)}{(a+b\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 136

$$\int \frac{\text{sech}^3(x)}{(a+b\sinh(x))^2} dx = \frac{(a^4+6a^2b^2-3b^4)\arctan(\sinh(x))}{2(a^2+b^2)^3} - \frac{4ab^3\log(\cosh(x))}{(a^2+b^2)^3} + \frac{4ab^3\log(a+b\sinh(x))}{(a^2+b^2)^3} + \frac{b(a^2-3b^2)}{2(a^2+b^2)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)(b+a\sinh(x))}{2(a^2+b^2)(a+b\sinh(x))}$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(sinh(x))/(a^2+b^2)^3-4*a*b^3*ln(cosh(x))/(
a^2+b^2)^3+4*a*b^3*ln(a+b*sinh(x))/(a^2+b^2)^3+1/2*b*(a^2-3*b^2)/(a^2+b^2)^
2/(a+b*sinh(x))+1/2*sech(x)^2*(b+a*sinh(x))/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2747, 755, 815, 649, 209, 266}

$$\int \frac{\text{sech}^3(x)}{(a+b\sinh(x))^2} dx = \frac{b(a^2-3b^2)}{2(a^2+b^2)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)(a\sinh(x)+b)}{2(a^2+b^2)(a+b\sinh(x))} + \frac{4ab^3\log(a+b\sinh(x))}{(a^2+b^2)^3} - \frac{4ab^3\log(\cosh(x))}{(a^2+b^2)^3} + \frac{(a^4+6a^2b^2-3b^4)\arctan(\sinh(x))}{2(a^2+b^2)^3}$$

```
[In] Int[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^3) - (4*a*b^3*Lo
g[Cosh[x]])/(a^2 + b^2)^3 + (4*a*b^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^3 + (b
*(a^2 - 3*b^2))/(2*(a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(b + a*Sinh[
x]))/(2*(a^2 + b^2)*(a + b*Sinh[x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= b^3\text{Subst}\left(\int \frac{1}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)\\
&= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\text{Subst}\left(\int \frac{a^2+3b^2+2ax}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))}\\
&\quad - \frac{b\text{Subst}\left(\int \left(\frac{a^2-3b^2}{\left(a^2+b^2\right)(a+x)^2} - \frac{8ab^2}{\left(a^2+b^2\right)^2(a+x)} + \frac{-a^4-6a^2b^2+3b^4+8ab^2x}{\left(a^2+b^2\right)^2\left(b^2+x^2\right)}\right)\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)}\\
&= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}\\
&\quad + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{b\text{Subst}\left(\int \frac{-a^4-6a^2b^2+3b^4+8ab^2x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)^3}\\
&= \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))}\\
&\quad + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))} - \frac{\left(4ab^3\right)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^3}\\
&\quad + \frac{\left(b\left(a^4+6a^2b^2-3b^4\right)\right)\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{2\left(a^2+b^2\right)^3}\\
&= \frac{\left(a^4+6a^2b^2-3b^4\right)\arctan(\sinh(x))}{2\left(a^2+b^2\right)^3} - \frac{4ab^3\log(\cosh(x))}{\left(a^2+b^2\right)^3} + \frac{4ab^3\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3}\\
&\quad + \frac{b\left(a^2-3b^2\right)}{2\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)(b+a\sinh(x))}{2\left(a^2+b^2\right)(a+b\sinh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.91

$$
\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx =
$$

$$
-\frac{-\frac{2\operatorname{sech}^2(x)(b+a\sinh(x))}{a+b\sinh(x)} + \frac{b\left(\frac{2a\left(a^2+b^2\right)\left(\left(-a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}-b\sinh(x)\right)-2\sqrt{-b^2}\log(a+b\sinh(x))+\left(a+\sqrt{-b^2}\right)\log\left(\sqrt{-b^2}+b\sinh(x)\right)\right)}{\sqrt{-b^2}}+(-\cdots\right.}{\cdots}}{4\left(a^2+b^2\right)\cdots}
$$

```
[In] Integrate[Sech[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/4*((-2*Sech[x]^2*(b + a*Sinh[x]))/(a + b*Sinh[x]) + (b*((2*a*(a^2 + b^2)
*((-a + Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 2*Sqrt[-b^2]*Log[a + b*Si
nh[x]] + (a + Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sinh[x]]))/Sqrt[-b^2] + (-a^2
+ 3*b^2)*((2*a + (-a^2 + b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Sinh[x]] - 4*a
*Log[a + b*Sinh[x]] + (2*a + (a^2 - b^2)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Sin
h[x]] + (2*(a^2 + b^2))/(a + b*Sinh[x]))))/(a^2 + b^2)^2)/(a^2 + b^2)
```

Maple [A] (verified)

Time = 95.75 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.55

method	result
default	$\frac{2b^3\left(-\frac{b\left(a^2+b^2\right)\tanh\left(\frac{x}{2}\right)}{a\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}+2a\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)\right)}{\left(a^2+b^2\right)^3}+\frac{2\left(\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tanh\left(\frac{x}{2}\right)^3+\left(-2a^3b-2b^3a\right)\tanh\left(\frac{x}{2}\right)^2+\left(\right.}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}$
risch	$\frac{\left(a^2b\,\mathrm{e}^{4x}-3\,\mathrm{e}^{4x}b^3+2a^3\mathrm{e}^{3x}+2a\,b^2\mathrm{e}^{3x}+6a^2b\,\mathrm{e}^{2x}-2b^3\mathrm{e}^{2x}-2a^3\mathrm{e}^{x}-2a\,b^2\mathrm{e}^{x}+a^2b-3b^3\right)\mathrm{e}^{x}}{\left(a^4+2a^2b^2+b^4\right)\left(1+\mathrm{e}^{2x}\right)^2\left(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}a-b\right)}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)a^4}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}-\frac{3i\ln\left(\mathrm{e}^{x}-i\right)}{a^6+3a^4b^2+3a}$

```
[In] int(sech(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^3/(a^2+b^2)^3*(-b*(a^2+b^2)/a*tanh(1/2*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/2
*x)-a)+2*a*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a))+2/(a^6+3*a^4*b^2+3*a^2*b^
4+b^6)*(((-1/2*a^4+1/2*b^4)*tanh(1/2*x)^3+(-2*a^3*b-2*a*b^3)*tanh(1/2*x)^2+
(1/2*a^4-1/2*b^4)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2-2*b^3*a*ln(1+tanh(1/2*x)
^2)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(130) = 260$.

Time = 0.33 (sec) , antiderivative size = 2615, normalized size of antiderivative = 19.23

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^5 + (a^4*b - 2*a^2*b^3 - 3*b^5)*sinh(
x)^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^4 + (2*a^5 + 4*a^3*b^2 + 2*a*b^4
 + 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^4 + 2*(3*a^4*b + 2*a^2*b^
3 - b^5)*cosh(x)^3 + 2*(3*a^4*b + 2*a^2*b^3 - b^5 + 5*(a^4*b - 2*a^2*b^3 -
3*b^5)*cosh(x)^2 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 2*(a^5
+ 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 - 5*(a^4*b - 2*
a^2*b^3 - 3*b^5)*cosh(x)^3 - 6*(a^5 + 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 3*(3*a
^4*b + 2*a^2*b^3 - b^5)*cosh(x))*sinh(x)^2 + ((a^4*b + 6*a^2*b^3 - 3*b^5)*c
```

osh(x)^6 + (a^4*b + 6*a^2*b^3 - 3*b^5)*sinh(x)^6 + 2*(a^5 + 6*a^3*b^2 - 3*a
*b^4)*cosh(x)^5 + 2*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 3*(a^4*b + 6*a^2*b^3 - 3*b
^5)*cosh(x))*sinh(x)^5 - a^4*b - 6*a^2*b^3 + 3*b^5 + (a^4*b + 6*a^2*b^3 - 3
*b^5)*cosh(x)^4 + (a^4*b + 6*a^2*b^3 - 3*b^5 + 15*(a^4*b + 6*a^2*b^3 - 3*b^
5)*cosh(x)^2 + 10*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^4 + 4*(a^5 +
 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 + 4*(a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^4*b +
 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^2 + (
a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - (a^4*b + 6*a^2*b^3 - 3*b^5)
*cosh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5 - 15*(a^4*b + 6*a^2*b^3 - 3*b^5)*co
sh(x)^4 - 20*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^3 - 6*(a^4*b + 6*a^2*b^3 -
 3*b^5)*cosh(x)^2 - 12*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x))*sinh(x)^2 + 2*(
a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x) + 2*(3*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(
x)^5 + a^5 + 6*a^3*b^2 - 3*a*b^4 + 5*(a^5 + 6*a^3*b^2 - 3*a*b^4)*cosh(x)^4
+ 2*(a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x)^3 + 6*(a^5 + 6*a^3*b^2 - 3*a*b^4)*c
osh(x)^2 - (a^4*b + 6*a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))*arctan(cosh(x) + s
inh(x)) + (a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x) + 4*(a*b^4*cosh(x)^6 + a*b^4*
sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3 - a
*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh(x)^
5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4 + 4
*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*sinh(x
)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 + 12*a
^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*cosh(x
)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3)*si
nh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - 4*(a*b^4*cosh(x)^6 + a*
b^4*sinh(x)^6 + 2*a^2*b^3*cosh(x)^5 + a*b^4*cosh(x)^4 + 4*a^2*b^3*cosh(x)^3
 - a*b^4*cosh(x)^2 + 2*a^2*b^3*cosh(x) + 2*(3*a*b^4*cosh(x) + a^2*b^3)*sinh
(x)^5 - a*b^4 + (15*a*b^4*cosh(x)^2 + 10*a^2*b^3*cosh(x) + a*b^4)*sinh(x)^4
 + 4*(5*a*b^4*cosh(x)^3 + 5*a^2*b^3*cosh(x)^2 + a*b^4*cosh(x) + a^2*b^3)*si
nh(x)^3 + (15*a*b^4*cosh(x)^4 + 20*a^2*b^3*cosh(x)^3 + 6*a*b^4*cosh(x)^2 +
12*a^2*b^3*cosh(x) - a*b^4)*sinh(x)^2 + 2*(3*a*b^4*cosh(x)^5 + 5*a^2*b^3*co
sh(x)^4 + 2*a*b^4*cosh(x)^3 + 6*a^2*b^3*cosh(x)^2 - a*b^4*cosh(x) + a^2*b^3
)*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + (a^4*b - 2*a^2*b^3 - 3*b^5
+ 5*(a^4*b - 2*a^2*b^3 - 3*b^5)*cosh(x)^4 + 8*(a^5 + 2*a^3*b^2 + a*b^4)*cos
h(x)^3 + 6*(3*a^4*b + 2*a^2*b^3 - b^5)*cosh(x)^2 - 4*(a^5 + 2*a^3*b^2 + a*b
^4)*cosh(x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4
*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*s
inh(x)^6 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a
^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x
))*sinh(x)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3
*a^4*b^3 + 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x)^2 + 10*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7
 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*
a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b
^7)*cosh(x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 +

```
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7)*cosh(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
4 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2
+ 3*a^3*b^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(sech(x)**3/(a+b*sinh(x))**2,x)
[Out] Integral(sech(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(130) = 260$.

Time = 0.30 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.76

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{4\,ab^3\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,ab^3\log\left(e^{(-2\,x)}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(a^4+6\,a^2b^2-3\,b^4)\arctan\left(e^{(-x)}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^2b-3\,b^3)e^{(-x)}+2\,(a^3+ab^2)e^{(-2\,x)}+2\,(3\,a^2b-b^3)e^{(-3\,x)}-}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+(a^4b+2\,a^2b^3+b^5)e^{(-2\,x)}+4\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}-}$$

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] 4*a*b^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) - 4*a*b^3*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 +
 6*a^2*b^2 - 3*b^4)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + ((
a^2*b - 3*b^3)*e^(-x) + 2*(a^3 + a*b^2)*e^(-2*x) + 2*(3*a^2*b - b^3)*e^(-3*
x) - 2*(a^3 + a*b^2)*e^(-4*x) + (a^2*b - 3*b^3)*e^(-5*x))/(a^4*b + 2*a^2*b^
3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^
(-2*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e
^(-4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*
e^(-6*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(130) = 260.

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{4\,ab^4\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{2\,ab^3\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)\left(a^4+6\,a^2b^2-3\,b^4\right)}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)} - \frac{a^2b\left(e^{(-x)}-e^x\right)^2-3\,b^3\left(e^{(-x)}-e^x\right)^2-2\,a^3\left(e^{(-x)}-e^x\right)-2\,ab^2\left(e^{(-x)}-e^x\right)+8\,a^2b-8\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\left(e^{(-x)}-e^x\right)^3-2\,a\left(e^{(-x)}-e^x\right)^2+4\,b\left(e^{(-x)}-e^x\right)-8\,a\right)}$$

```
[In] integrate(sech(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 4*a*b^4*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 2*a*b^3*log((e^(-x) - e^x)^2 + 4)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(a^4 + 6*a^2*b^2 - 3*b^4)
/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^2*b*(e^(-x) - e^x)^2 - 3*b^3*(e^(
-x) - e^x)^2 - 2*a^3*(e^(-x) - e^x) - 2*a*b^2*(e^(-x) - e^x) + 8*a^2*b - 8*
b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-x) - e^x)^2 +
4*b*(e^(-x) - e^x) - 8*a))
```

Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.82

$$\int \frac{\operatorname{sech}^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{4\,\left(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)^2}+\frac{\mathrm{e}^x\,\left(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8\right)}{\left(a^2+b^2\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)^2}}{\mathrm{e}^{2\,x}+1} - \frac{\frac{4\,a\,b}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,\mathrm{e}^x\,\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + \frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,\left(a-b\,3\mathrm{i}\right)}{2\,\left(-a^3\,1\mathrm{i}-3\,a^2\,b+a\,b^2\,3\mathrm{i}+b^3\right)} + \frac{\ln\left(1+\mathrm{e}^x\,1\mathrm{i}\right)\,\left(-3\,b+a\,1\mathrm{i}\right)}{2\,\left(-a^3-a^2\,b\,3\mathrm{i}+3\,a\,b^2+b^3\,1\mathrm{i}\right)} + \frac{4\,a\,b^3\,\ln\left(9\,b^9\,\mathrm{e}^{2\,x}-a^8\,b-9\,b^9-220\,a^2\,b^7-30\,a^4\,b^5-12\,a^6\,b^3+2\,a^9\,\mathrm{e}^x+220\,a^2\,b^7\,\mathrm{e}^{2\,x}+30\,a^4\,b^5\,\mathrm{e}^{2\,x}+\right.}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} - \frac{2\,\mathrm{e}^x\,\left(a^4\,b^6+2\,a^2\,b^8+b^{10}\right)}{b^2\,\left(a^2\,b+b^3\right)\,\left(a^2+b^2\right)\,\left(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}$$

```
[In] int(1/(cosh(x)^3*(a + b*sinh(x))^2),x)
```

```
[Out] ((4*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(a^4 + b^4 + 2*a^
2*b^2)^2) + (exp(x)*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^4
+ b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((4*a*b)/(a^4 + b^4 + 2*a^2*b^2) +
```

```
(2*exp(x)*(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x) + 1)
 + (log(exp(x) + 1i)*(a - b*3i))/(2*(a*b^2*3i - 3*a^2*b - a^3*1i + b^3)) +
(log(exp(x)*1i + 1)*(a*1i - 3*b))/(2*(3*a*b^2 - a^2*b*3i - a^3 + b^3*1i)) +
 (4*a*b^3*log(9*b^9*exp(2*x) - a^8*b - 9*b^9 - 220*a^2*b^7 - 30*a^4*b^5 - 1
2*a^6*b^3 + 2*a^9*exp(x) + 220*a^2*b^7*exp(2*x) + 30*a^4*b^5*exp(2*x) + 12*
a^6*b^3*exp(2*x) + 18*a*b^8*exp(x) + a^8*b*exp(2*x) + 440*a^3*b^6*exp(x) +
60*a^5*b^4*exp(x) + 24*a^7*b^2*exp(x)))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
 - (2*exp(x)*(b^10 + 2*a^2*b^8 + a^4*b^6))/(b^2*(a^2*b + b^3)*(a^2 + b^2)*(
2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))
```

3.207 $\int \frac{\text{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 144

$$\int \frac{\text{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{10ab^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))} + \frac{\text{sech}^3(x)\,(5ab+(a^2-4b^2)\sinh(x))}{3\,(a^2+b^2)^2} + \frac{\text{sech}(x)\,(15ab^3+(2a^4+9a^2b^2-8b^4)\sinh(x))}{3\,(a^2+b^2)^3}$$

```
[Out] -10*a*b^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-b*sech
(x)^3/(a^2+b^2)/(a+b*sinh(x))+1/3*sech(x)^3*(5*a*b+(a^2-4*b^2)*sinh(x))/(a^
2+b^2)^2+1/3*sech(x)*(15*a*b^3+(2*a^4+9*a^2*b^2-8*b^4)*sinh(x))/(a^2+b^2)^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2773, 2945, 12, 2739, 632, 212}

$$\int \frac{\text{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{10ab^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))} + \frac{\text{sech}^3(x)\,((a^2-4b^2)\sinh(x)+5ab)}{3\,(a^2+b^2)^2} + \frac{\text{sech}(x)\,((2a^4+9a^2b^2-8b^4)\sinh(x)+15ab^3)}{3\,(a^2+b^2)^3}$$

```
[In] Int[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-10*a*b^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) -
(b*Sech[x]^3)/((a^2 + b^2)*(a + b*Sinh[x])) + (Sech[x]^3*(5*a*b + (a^2 - 4*
b^2)*Sinh[x]))/(3*(a^2 + b^2)^2) + (Sech[x]*(15*a*b^3 + (2*a^4 + 9*a^2*b^2
- 8*b^4)*Sinh[x]))/(3*(a^2 + b^2)^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
```

```
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rubi steps

$$\text{integral} = -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} - \frac{\int \frac{\text{sech}^4(x)(-a+4b\sinh(x))}{a+b\sinh(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\int \frac{\text{sech}^2(x)\left(a(2a^2+7b^2)+2b(a^2-4b^2)\sinh(x)\right)}{a+b\sinh(x)}\,dx}{3(a^2+b^2)^2}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+(2a^4+9a^2b^2-8b^4)\sinh(x)\right)}{3(a^2+b^2)^3} - \frac{\int -\frac{15ab^4}{a+b\sinh(x)}\,dx}{3(a^2+b^2)^3}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+(2a^4+9a^2b^2-8b^4)\sinh(x)\right)}{3(a^2+b^2)^3} + \frac{(5ab^4)\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+(2a^4+9a^2b^2-8b^4)\sinh(x)\right)}{3(a^2+b^2)^3} + \frac{(10ab^4)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= -\frac{b\text{sech}^3(x)}{(a^2+b^2)(a+b\sinh(x))} + \frac{\text{sech}^3(x)\left(5ab+(a^2-4b^2)\sinh(x)\right)}{3(a^2+b^2)^2} + \frac{\text{sech}(x)\left(15ab^3+(2a^4+9a^2b^2-8b^4)\sinh(x)\right)}{3(a^2+b^2)^3} - \frac{(20ab^4)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= -\frac{10ab^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{b\text{sech}^3(x)}{(a^2+b^2)\,(a+b\sinh(x))} + \frac{\text{sech}^3(x)\,(5ab+(a^2-4b^2)\sinh(x))}{3\,(a^2+b^2)^2} + \frac{\text{sech}(x)\,(15ab^3+(2a^4+9a^2b^2-8b^4)\sinh(x))}{3\,(a^2+b^2)^3}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95

$$\int \frac{\text{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{30ab^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + 12ab^3\text{sech}(x) - \frac{3b^5\cosh(x)}{a+b\sinh(x)} + (a^2+b^2)\,\text{sech}^3(x)\,(2ab+(a^2-b^2)\sinh(x)) + (2a^4+9\cdots}{3\,(a^2+b^2)^3}$$

```
[In] Integrate[Sech[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((30*a*b^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + 1
2*a*b^3*Sech[x] - (3*b^5*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2)*Sech[x]^3*(
2*a*b + (a^2 - b^2)*Sinh[x]) + (2*a^4 + 9*a^2*b^2 - 5*b^4)*Tanh[x])/(3*(a^2
 + b^2)^3)
```

Maple [A] (verified)

Time = 179.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.85

method	result
default	$-\frac{2b^4\left(\frac{-\frac{b^2\tanh\left(\frac{x}{2}\right)}{a}-b}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{5a\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)} - \frac{2\left(\left(-a^4-3a^2b^2+2b^4\right)\tanh\left(\frac{x}{2}\right)^5+\left(-2a^3b-6b^3a\right)\tanh\left(\frac{x}{2}\right)^4+(-\cdots}{(a^2+\cdots}$
risch	$-\frac{2(-15a\,b^4e^{7x}-15a^2b^3e^{6x}+10a^3b^2e^{5x}-35a\,b^4e^{5x}-10a^4b\,e^{4x}-55a^2b^3e^{4x}+12a^5e^{3x}+44a^3b^2e^{3x}-13a\,b^4e^{3x}-4a^4b\,e^{2x}-33a^2b^3e^{2x}+1\cdots}{3(a^6+3a^4b^2+3a^2b^4+b^6)(1+e^{2x})^3(b\,e^{2x}+2\,e^xa-b)}$

```
[In] int(sech(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-1/a*b^2*tanh(1/2*x)-b)/(tanh(1/2*x)
^2*a-2*b*tanh(1/2*x)-a)-5*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*
b)/(a^2+b^2)^(1/2)))-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^4-3*a^2*b^2+2*b^4
```

```
)*tanh(1/2*x)^5+(-2*a^3*b-6*a*b^3)*tanh(1/2*x)^4+(-2/3*a^4-6*a^2*b^2+8/3*b^
4)*tanh(1/2*x)^3-8*tanh(1/2*x)^2*a*b^3+(-a^4-3*a^2*b^2+2*b^4)*tanh(1/2*x)-2
/3*a^3*b-14/3*b^3*a)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3044 vs. $2(136) = 272$.

Time = 0.32 (sec) , antiderivative size = 3044, normalized size of antiderivative = 21.14

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(30*(a^3*b^4 + a*b^6)*cosh(x)^7 + 30*(a^3*b^4 + a*b^6)*sinh(x)^7 + 4*a
^6*b + 22*a^4*b^3 + 2*a^2*b^5 - 16*b^7 + 30*(a^4*b^3 + a^2*b^5)*cosh(x)^6 +
 30*(a^4*b^3 + a^2*b^5 + 7*(a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^6 - 10*(2*a^5
*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^5 - 10*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6
 - 63*(a^3*b^4 + a*b^6)*cosh(x)^2 - 18*(a^4*b^3 + a^2*b^5)*cosh(x))*sinh(x)
^5 + 10*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^4 + 10*(2*a^6*b + 13*a^
4*b^3 + 11*a^2*b^5 + 105*(a^3*b^4 + a*b^6)*cosh(x)^3 + 45*(a^4*b^3 + a^2*b^
5)*cosh(x)^2 - 5*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x))*sinh(x)^4 - 2*(
12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 - 13*a*b^6)*cosh(x)^3 - 2*(12*a^7 + 56*a^5
*b^2 + 31*a^3*b^4 - 13*a*b^6 - 525*(a^3*b^4 + a*b^6)*cosh(x)^4 - 300*(a^4*b
^3 + a^2*b^5)*cosh(x)^3 + 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^2 -
20*(2*a^6*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x))*sinh(x)^3 + 2*(4*a^6*b + 37
*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*cosh(x)^2 + 2*(4*a^6*b + 37*a^4*b^3 + 17*a^
2*b^5 - 16*b^7 + 315*(a^3*b^4 + a*b^6)*cosh(x)^5 + 225*(a^4*b^3 + a^2*b^5)*
cosh(x)^4 - 50*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^3 + 30*(2*a^6*b +
13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^2 - 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b^4 -
13*a*b^6)*cosh(x))*sinh(x)^2 + 15*(a*b^5*cosh(x)^8 + a*b^5*sinh(x)^8 + 2*a^
2*b^4*cosh(x)^7 + 2*a*b^5*cosh(x)^6 + 6*a^2*b^4*cosh(x)^5 + 6*a^2*b^4*cosh(
x)^3 - 2*a*b^5*cosh(x)^2 + 2*(4*a*b^5*cosh(x) + a^2*b^4)*sinh(x)^7 + 2*a^2*
b^4*cosh(x) + 2*(14*a*b^5*cosh(x)^2 + 7*a^2*b^4*cosh(x) + a*b^5)*sinh(x)^6
- a*b^5 + 2*(28*a*b^5*cosh(x)^3 + 21*a^2*b^4*cosh(x)^2 + 6*a*b^5*cosh(x) +
3*a^2*b^4)*sinh(x)^5 + 10*(7*a*b^5*cosh(x)^4 + 7*a^2*b^4*cosh(x)^3 + 3*a*b^
5*cosh(x)^2 + 3*a^2*b^4*cosh(x))*sinh(x)^4 + 2*(28*a*b^5*cosh(x)^5 + 35*a^2
*b^4*cosh(x)^4 + 20*a*b^5*cosh(x)^3 + 30*a^2*b^4*cosh(x)^2 + 3*a^2*b^4)*sin
h(x)^3 + 2*(14*a*b^5*cosh(x)^6 + 21*a^2*b^4*cosh(x)^5 + 15*a*b^5*cosh(x)^4
+ 30*a^2*b^4*cosh(x)^3 + 9*a^2*b^4*cosh(x) - a*b^5)*sinh(x)^2 + 2*(4*a*b^5*
cosh(x)^7 + 7*a^2*b^4*cosh(x)^6 + 6*a*b^5*cosh(x)^5 + 15*a^2*b^4*cosh(x)^4
+ 9*a^2*b^4*cosh(x)^2 - 2*a*b^5*cosh(x) + a^2*b^4)*sinh(x))*sqrt(a^2 + b^2)
*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
```

cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) - 2
*(4*a^7 + 22*a^5*b^2 + 17*a^3*b^4 - a*b^6)*cosh(x) - 2*(4*a^7 + 22*a^5*b^2
+ 17*a^3*b^4 - a*b^6 - 105*(a^3*b^4 + a*b^6)*cosh(x)^6 - 90*(a^4*b^3 + a^2*
b^5)*cosh(x)^5 + 25*(2*a^5*b^2 - 5*a^3*b^4 - 7*a*b^6)*cosh(x)^4 - 20*(2*a^6
*b + 13*a^4*b^3 + 11*a^2*b^5)*cosh(x)^3 + 3*(12*a^7 + 56*a^5*b^2 + 31*a^3*b
^4 - 13*a*b^6)*cosh(x)^2 - 2*(4*a^6*b + 37*a^4*b^3 + 17*a^2*b^5 - 16*b^7)*c
osh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 - (a^8*b
+ 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^8 - (a^8*b + 4*a^6*b^3 +
6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 +
a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(
x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^6 - 2*(a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 + 14*(a^8*b + 4*a^6*b^3 + 6*
a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a
^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 +
3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 +
21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 + 6*(a^8*b
+ 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(x)^5 - 10*(7*(a^8*
b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 + 7*(a^9 + 4*a^7*b^2
+ 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(a^8*b + 4*a^6*b^3 + 6*a^4*
b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b
^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
+ a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*a^5*b^4 + 12*a^3*b^6 + 3*a
b^8 + 28(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^5 + 35*
(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 + 20*(a^8*b + 4
*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3 + 30*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2)*sinh(x)^3 + 2*(a^8*b + 4*a^6*b^3 +
6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5
+ 4*a^2*b^7 + b^9 - 14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*co
sh(x)^6 - 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 -
15*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^4 - 30*(a^9 +
4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 9*(a^9 + 4*a^7*b^2 +
6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^2 - 2*(a^9 + 4*a^7*b^2 + 6
*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x) - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(
x)^7 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^6 + 6*(a
^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^5 + 15*(a^9 + 4*a^7
*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 + 9*(a^9 + 4*a^7*b^2 + 6*a^
5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4
*a^2*b^7 + b^9)*cosh(x))*sinh(x))

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(sech(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(sech(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(136) = 272$.

Time = 0.33 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.40

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{5\,ab^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,a^2b^3e^{(-6\,x)}-15\,ab^4e^{(-7\,x)}+2\,a^4b+9\,a^2b^3-8\,b^5+(4\,a\right.}{3\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-x)}+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-2\,x)}}$$

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2
)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2/3*(15*a^2*b^3
*e^(-6*x) - 15*a*b^4*e^(-7*x) + 2*a^4*b + 9*a^2*b^3 - 8*b^5 + (4*a^5 + 18*a
^3*b^2 - a*b^4)*e^(-x) + (4*a^4*b + 33*a^2*b^3 - 16*b^5)*e^(-2*x) + (12*a^5
 + 44*a^3*b^2 - 13*a*b^4)*e^(-3*x) + 5*(2*a^4*b + 11*a^2*b^3)*e^(-4*x) + 5*
(2*a^3*b^2 - 7*a*b^4)*e^(-5*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 + 2*(a
^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b
^5 + b^7)*e^(-2*x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*x) + 6*(
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b^3 + 3*a^
2*b^5 + b^7)*e^(-6*x) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-7*x) -
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-8*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(136) = 272$.

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.99

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{5\,ab^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\,(ab^4e^x-b^5)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(be^{(2\,x)}+2\,ae^x-b)}$$
$$+ \frac{2\left(12\,ab^3e^{(5\,x)}-9\,a^2b^2e^{(4\,x)}+3\,b^4e^{(4\,x)}+8\,a^3be^{(3\,x)}+32\,ab^3e^{(3\,x)}-6\,a^4e^{(2\,x)}-18\,a^2b^2e^{(2\,x)}+12\,b^4e^{(2\,x)}+\right.}{3\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(e^{(2\,x)}+1)^3}$$

```
[In] integrate(sech(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 5*a*b^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sq
rt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(
a*b^4*e^x - b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x) + 2*a*e^x
- b)) + 2/3*(12*a*b^3*e^(5*x) - 9*a^2*b^2*e^(4*x) + 3*b^4*e^(4*x) + 8*a^3*b
*e^(3*x) + 32*a*b^3*e^(3*x) - 6*a^4*e^(2*x) - 18*a^2*b^2*e^(2*x) + 12*b^4*e
^(2*x) + 12*a*b^3*e^x - 2*a^4 - 9*a^2*b^2 + 5*b^4)/((a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 476, normalized size of antiderivative = 3.31

$$\int \frac{\operatorname{sech}^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{8\,(a^2-b^2)}{3\,(a^4+2\,a^2\,b^2+b^4)}-\frac{16\,a\,b\,\mathrm{e}^x}{3\,(a^4+2\,a^2\,b^2+b^4)}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1} - \frac{\frac{4\,(a^6+a^4\,b^2-a^2\,b^4-b^6)}{(a^4+2\,a^2\,b^2+b^4)^2}-\frac{16\,\mathrm{e}^x\,(a^5\,b+2\,a^3\,b^3+a\,b^5)}{3\,(a^4+2\,a^2\,b^2+b^4)^2}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} - \frac{\frac{2\,(3\,a^4\,b^2+2\,a^2\,b^4-b^6)}{(a^4+2\,a^2\,b^2+b^4)^2}-\frac{8\,\mathrm{e}^x\,(a^3\,b^3+a\,b^5)}{(a^4+2\,a^2\,b^2+b^4)^2}}{\mathrm{e}^{2\,x}+1} - \frac{\frac{2\,(a^2\,b^9+b^{11})}{b^3\,(a^2\,b+b^3)\,(a^2+b^2)^3}-\frac{2\,\mathrm{e}^x\,(a^3\,b^9+a\,b^{11})}{b^4\,(a^2\,b+b^3)\,(a^2+b^2)^3}}{2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x}} - \frac{5\,a\,b^4\,\ln\left(-\frac{10\,a\,b^3\,(b-a\,\mathrm{e}^x)}{(a^2+b^2)^{7/2}}-\frac{10\,a\,b^3\,\mathrm{e}^x}{(a^2+b^2)^3}\right)}{(a^2+b^2)^{7/2}} + \frac{5\,a\,b^4\,\ln\left(\frac{10\,a\,b^3\,(b-a\,\mathrm{e}^x)}{(a^2+b^2)^{7/2}}-\frac{10\,a\,b^3\,\mathrm{e}^x}{(a^2+b^2)^3}\right)}{(a^2+b^2)^{7/2}}$$

```
[In] int(1/(cosh(x)^4*(a + b*sinh(x))^2),x)
```

```
[Out] ((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b^
4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^6
- a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^5*b
 + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) -
 ((2*(2*a^2*b^4 - b^6 + 3*a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (8*exp(x)*(
a*b^5 + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*x) + 1) - ((2*(b^11 + a
^2*b^9))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x)*(a*b^11 + a^3*b^9))/
(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b + b*exp(2*x)) - (5*a*b^4
*log(- (10*a*b^3*(b - a*exp(x)))/(a^2 + b^2)^(7/2) - (10*a*b^3*exp(x))/(a^2
 + b^2)^3))/(a^2 + b^2)^(7/2) + (5*a*b^4*log((10*a*b^3*(b - a*exp(x)))/(a^2
 + b^2)^(7/2) - (10*a*b^3*exp(x))/(a^2 + b^2)^3))/(a^2 + b^2)^(7/2)
```

3.208 $\int \frac{\tanh^4(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\tanh^4(x)}{i+\sinh(x)} dx = -\operatorname{sech}(x) + \frac{2\operatorname{sech}^3(x)}{3} - \frac{\operatorname{sech}^5(x)}{5} - \frac{1}{5}i\tanh^5(x)$$

```
[Out] -sech(x)+2/3*sech(x)^3-1/5*sech(x)^5-1/5*I*tanh(x)^5
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2686, 200}

$$\int \frac{\tanh^4(x)}{i+\sinh(x)} dx = -\frac{1}{5}i\tanh^5(x) - \frac{1}{5}\operatorname{sech}^5(x) + \frac{2\operatorname{sech}^3(x)}{3} - \operatorname{sech}(x)$$

```
[In] Int[Tanh[x]^4/(I + Sinh[x]),x]
```

```
[Out] -Sech[x] + (2*Sech[x]^3)/3 - Sech[x]^5/5 - (I/5)*Tanh[x]^5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \operatorname{sech}^2(x)\tanh^4(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^5(x)\,dx\\
&= -\text{Subst}\left(\int x^4\,dx, x, i\tanh(x)\right)-\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \operatorname{sech}(x)\right)\\
&= -\frac{1}{5}i\tanh^5(x)-\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \operatorname{sech}(x)\right)\\
&= -\operatorname{sech}(x)+\frac{2\operatorname{sech}^3(x)}{3}-\frac{\operatorname{sech}^5(x)}{5}-\frac{1}{5}i\tanh^5(x)
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 96 vs. $2(31) = 62$.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.10

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = -\frac{200-534\cosh(x)+288\cosh(2x)-178\cosh(3x)+24\cosh(4x)+64i\sinh(x)+178i\sinh(2x)-192i\,\text{s}}{960\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)^3}$$

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x]),x]
```

```
[Out] -1/960*(200 - 534*Cosh[x] + 288*Cosh[2*x] - 178*Cosh[3*x] + 24*Cosh[4*x] +
(64*I)*Sinh[x] + (178*I)*Sinh[2*x] - (192*I)*Sinh[3*x] + (89*I)*Sinh[4*x])/
((Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

Time = 13.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{2(25ie^{4x}+5\,e^{5x}+21ie^{2x}+13\,e^{3x}+15ie^{6x}+15\,e^{7x}-9\,e^{x}+3i)}{15(e^{x}+i)^{5}(e^{x}-i)^{3}}$
default	$\frac{3i}{8(-i+\tanh(\frac{x}{2}))}+\frac{i}{6(-i+\tanh(\frac{x}{2}))^{3}}+\frac{1}{4(-i+\tanh(\frac{x}{2}))^{2}}+\frac{i}{3(\tanh(\frac{x}{2})+i)^{3}}-\frac{2i}{5(\tanh(\frac{x}{2})+i)^{5}}-\frac{3i}{8(\tanh(\frac{x}{2})+i)}+\frac{1}{(\tanh(\frac{x}{2}$

```
[In] int(tanh(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(25*I*exp(x)^4+5*exp(x)^5+21*I*exp(x)^2+13*exp(x)^3+15*I*exp(x)^6+15*
exp(x)^7-9*exp(x)+3*I)/(exp(x)+I)^5/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.77

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = -\frac{2\left(15\,e^{(7\,x)}+15i\,e^{(6\,x)}+5\,e^{(5\,x)}+25i\,e^{(4\,x)}+13\,e^{(3\,x)}+21i\,e^{(2\,x)}-9\,e^{x}+3i\right)}{15\left(e^{(8\,x)}+2i\,e^{(7\,x)}+2\,e^{(6\,x)}+6i\,e^{(5\,x)}+6i\,e^{(3\,x)}-2\,e^{(2\,x)}+2i\,e^{x}-1\right)}$$

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -2/15*(15*e^(7*x) + 15*I*e^(6*x) + 5*e^(5*x) + 25*I*e^(4*x) + 13*e^(3*x) +
21*I*e^(2*x) - 9*e^x + 3*I)/(e^(8*x) + 2*I*e^(7*x) + 2*e^(6*x) + 6*I*e^(5*x
) + 6*I*e^(3*x) - 2*e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(26) = 52.

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.45

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = \frac{-30e^{7x}-30ie^{6x}-10e^{5x}-50ie^{4x}-26e^{3x}-42ie^{2x}+18e^{x}-6i}{15e^{8x}+30ie^{7x}+30e^{6x}+90ie^{5x}+90ie^{3x}-30e^{2x}+30ie^{x}-15}$$

```
[In] integrate(tanh(x)**4/(I+sinh(x)),x)
```

```
[Out] (-30*exp(7*x) - 30*I*exp(6*x) - 10*exp(5*x) - 50*I*exp(4*x) - 26*exp(3*x) -
 42*I*exp(2*x) + 18*exp(x) - 6*I)/(15*exp(8*x) + 30*I*exp(7*x) + 30*exp(6*x
) + 90*I*exp(5*x) + 90*I*exp(3*x) - 30*exp(2*x) + 30*I*exp(x) - 15)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(23) = 46.

Time = 0.23 (sec) , antiderivative size = 413, normalized size of antiderivative = 13.32

$$\begin{aligned}\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx
&= \frac{18\,e^{(-x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad+\frac{42i\,e^{(-2\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad-\frac{26\,e^{(-3\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad+\frac{50i\,e^{(-4\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad-\frac{10\,e^{(-5\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad+\frac{30i\,e^{(-6\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad-\frac{30\,e^{(-7\,x)}}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\\
&\quad+\frac{6i}{-30i\,e^{(-x)}-30\,e^{(-2\,x)}-90i\,e^{(-3\,x)}-90i\,e^{(-5\,x)}+30\,e^{(-6\,x)}-30i\,e^{(-7\,x)}+15\,e^{(-8\,x)}-15}\end{aligned}$$

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 18*e^(-x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 42*I*e^(-2*x)/(-30*I*e^(-x)
```

```
- 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
 + 15*e^(-8*x) - 15) - 26*e^(-3*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3
*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 50*
I*e^(-4*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30
*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 10*e^(-5*x)/(-30*I*e^(-x) -
 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x)
+ 15*e^(-8*x) - 15) + 30*I*e^(-6*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-
3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) - 30
*e^(-7*x)/(-30*I*e^(-x) - 30*e^(-2*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*
e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-8*x) - 15) + 6*I/(-30*I*e^(-x) - 30*e^(-2
*x) - 90*I*e^(-3*x) - 90*I*e^(-5*x) + 30*e^(-6*x) - 30*I*e^(-7*x) + 15*e^(-
8*x) - 15)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = -\frac{15\,e^{(2\,x)}-24i\,e^{x}-13}{24\,(e^{x}-i)^{3}}-\frac{165\,e^{(4\,x)}+480i\,e^{(3\,x)}-650\,e^{(2\,x)}-400i\,e^{x}+113}{120\,(e^{x}+i)^{5}}$$

```
[In] integrate(tanh(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/24*(15*e^(2*x) - 24*I*e^x - 13)/(e^x - I)^3 - 1/120*(165*e^(4*x) + 480*I
*e^(3*x) - 650*e^(2*x) - 400*I*e^x + 113)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 231, normalized size of antiderivative = 7.45

$$\begin{aligned}\int \frac{\tanh^4(x)}{i+\sinh(x)}\,dx = & -\frac{1}{6\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{\frac{11\,\mathrm{e}^{x}}{40}+\frac{1}{8}\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}} \\ & -\frac{\frac{11\,\mathrm{e}^{2\,x}}{40}-\frac{17}{120}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{4}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}+\frac{1\mathrm{i}}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)}-\frac{5}{8\,\left(\mathrm{e}^{x}-\mathrm{i}\right)} \\ & -\frac{11}{40\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}-\frac{\frac{\mathrm{e}^{2\,x}\,3\mathrm{i}}{8}+\frac{11\,\mathrm{e}^{3\,x}}{40}-\frac{17\,\mathrm{e}^{x}}{40}-\frac{1}{8}\mathrm{i}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}} \\ & -\frac{\frac{11\,\mathrm{e}^{4\,x}}{40}-\frac{17\,\mathrm{e}^{2\,x}}{20}+\frac{11}{40}+\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}}{\mathrm{e}^{5\,x}-10\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}\,5\mathrm{i}-\mathrm{e}^{2\,x}\,10\mathrm{i}+5\,\mathrm{e}^{x}+1\mathrm{i}}\end{aligned}$$

```
[In] int(tanh(x)^4/(sinh(x) + 1i),x)
```

```
[Out] 1i/(4*(exp(x)*2i - exp(2*x) + 1)) - ((11*exp(x))/40 + 1i/8)/(exp(2*x) + exp
(x)*2i - 1) - ((11*exp(2*x))/40 + (exp(x)*1i)/4 - 17/120)/(exp(2*x)*3i + ex
p(3*x) - 3*exp(x) - 1i) - 1/(6*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) -
5/(8*(exp(x) - 1i)) - 11/(40*(exp(x) + 1i)) - ((exp(2*x)*3i)/8 + (11*exp(3*
x))/40 - (17*exp(x))/40 - 1i/8)/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(
x)*4i + 1) - ((exp(3*x)*1i)/2 - (17*exp(2*x))/20 + (11*exp(4*x))/40 - (exp(
x)*1i)/2 + 11/40)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*
exp(x) + 1i)
```

3.209 $\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{3}{8}\arctan(\sinh(x)) - \frac{3}{8}\mathrm{sech}(x)\tanh(x) - \frac{1}{4}\mathrm{sech}(x)\tanh^3(x) - \frac{1}{4}i\tanh^4(x)$$

```
[Out] 3/8*arctan(sinh(x))-3/8*sech(x)*tanh(x)-1/4*sech(x)*tanh(x)^3-1/4*I*tanh(x)
^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2785, 2687, 30, 2691, 3855}

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{3}{8}\arctan(\sinh(x)) - \frac{1}{4}i\tanh^4(x) - \frac{1}{4}\tanh^3(x)\mathrm{sech}(x) - \frac{3}{8}\tanh(x)\mathrm{sech}(x)$$

```
[In] Int[Tanh[x]^3/(I + Sinh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 - (3*Sech[x]*Tanh[x])/8 - (Sech[x]*Tanh[x]^3)/4 - (I/
4)*Tanh[x]^4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \operatorname{sech}^2(x)\tanh^3(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^4(x)\,dx\\
&= -\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-i\text{Subst}\left(\int x^3\,dx,x,i\tanh(x)\right)+\frac{3}{4}\int \operatorname{sech}(x)\tanh^2(x)\,dx\\
&= -\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)+\frac{3}{8}\int \operatorname{sech}(x)\,dx\\
&= \frac{3}{8}\arctan(\sinh(x))-\frac{3}{8}\operatorname{sech}(x)\tanh(x)-\frac{1}{4}\operatorname{sech}(x)\tanh^3(x)-\frac{1}{4}i\tanh^4(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx=\frac{1}{8}\left(3\arctan(\sinh(x))-\frac{2+i\sinh(x)+5\sinh^2(x)}{(-i+\sinh(x))(i+\sinh(x))^2}\right)$$

```
[In] Integrate[Tanh[x]^3/(I + Sinh[x]),x]
[Out] (3*ArcTan[Sinh[x]] - (2 + I*Sinh[x] + 5*Sinh[x]^2)/((-I + Sinh[x])*(I + Sin
h[x])^2))/8
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(27) = 54.

Time = 9.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{2i e^{4x}-2 e^{3x}-2i e^{2x}+5 e^{5x}+5 e^{x}}{4(e^{x}+i)^{4}(e^{x}-i)^{2}}-\frac{3i\ln(e^{x}-i)}{8}+\frac{3i\ln(e^{x}+i)}{8}$
default	$-\frac{3i\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)}{8}+\frac{i}{4\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{-4i+4\tanh\left(\frac{x}{2}\right)}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}+\frac{3i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)}{8}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}+\frac{}{2\cdots}$

```
[In] int(tanh(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*I*exp(x)^4-2*exp(x)^3-2*I*exp(x)^2+5*exp(x)^5+5*exp(x))/(exp(x)+I)^
4/(exp(x)-I)^2-3/8*I*ln(exp(x)-I)+3/8*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(26) = 52.

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 4.19

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = -\frac{3\left(-i\,e^{(6\,x)}+2\,e^{(5\,x)}-i\,e^{(4\,x)}+4\,e^{(3\,x)}+i\,e^{(2\,x)}+2\,e^{x}+i\right)\log\left(e^{x}+i\right)+3\left(i\,e^{(6\,x)}-2\,e^{(5\,x)}+i\,e^{(4\,x)}-4\,e\cdots\right.}{8\left(e^{(6\,x)}+2i\,e^{(5\,x)}+e^{(4\,x)}+4i\,e^{(3\,x)}-e\cdots\right.}$$

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(3*(-I*e^(6*x) + 2*e^(5*x) - I*e^(4*x) + 4*e^(3*x) + I*e^(2*x) + 2*e^x
 + I)*log(e^x + I) + 3*(I*e^(6*x) - 2*e^(5*x) + I*e^(4*x) - 4*e^(3*x) - I*e
^(2*x) - 2*e^x - I)*log(e^x - I) + 10*e^(5*x) + 4*I*e^(4*x) - 4*e^(3*x) - 4
*I*e^(2*x) + 10*e^x)/(e^(6*x) + 2*I*e^(5*x) + e^(4*x) + 4*I*e^(3*x) - e^(2*
x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(36) = 72.

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.75

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{-5e^{5x}-2ie^{4x}+2e^{3x}+2ie^{2x}-5e^{x}}{4e^{6x}+8ie^{5x}+4e^{4x}+16ie^{3x}-4e^{2x}+8ie^{x}-4} + \operatorname{RootSum}\left(64z^2+9,\left(i\mapsto i\log\left(\frac{8i}{3}+e^{x}\right)\right)\right)$$

```
[In] integrate(tanh(x)**3/(I+sinh(x)),x)
```

```
[Out] (-5*exp(5*x) - 2*I*exp(4*x) + 2*exp(3*x) + 2*I*exp(2*x) - 5*exp(x))/(4*exp(
6*x) + 8*I*exp(5*x) + 4*exp(4*x) + 16*I*exp(3*x) - 4*exp(2*x) + 8*I*exp(x)
- 4) + RootSum(64*_z**2 + 9, Lambda(_i, _i*log(8*_i/3 + exp(x))))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(26) = 52.

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.64

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{5\,e^{(-x)}+2i\,e^{(-2\,x)}-2\,e^{(-3\,x)}-2i\,e^{(-4\,x)}+5\,e^{(-5\,x)}}{-8i\,e^{(-x)}-4\,e^{(-2\,x)}-16i\,e^{(-3\,x)}+4\,e^{(-4\,x)}-8i\,e^{(-5\,x)}+4\,e^{(-6\,x)}-4} + \frac{3}{8}i\,\log\left(i\,e^{(-x)}+1\right) - \frac{3}{8}i\,\log\left(i\,e^{(-x)}-1\right)$$

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] (5*e^(-x) + 2*I*e^(-2*x) - 2*e^(-3*x) - 2*I*e^(-4*x) + 5*e^(-5*x))/(-8*I*e^
(-x) - 4*e^(-2*x) - 16*I*e^(-3*x) + 4*e^(-4*x) - 8*I*e^(-5*x) + 4*e^(-6*x)
- 4) + 3/8*I*log(I*e^(-x) + 1) - 3/8*I*log(I*e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{3i\,e^{(-x)}-3i\,e^{x}-2}{16\left(e^{(-x)}-e^{x}+2i\right)} - \frac{9i\left(e^{(-x)}-e^{x}\right)^2+4\,e^{(-x)}-4\,e^{x}+12i}{32\left(e^{(-x)}-e^{x}-2i\right)^2} + \frac{3}{16}i\,\log\left(-e^{(-x)}+e^{x}+2i\right) - \frac{3}{16}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

```
[In] integrate(tanh(x)^3/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/16*(3*I*e^(-x) - 3*I*e^x - 2)/(e^(-x) - e^x + 2*I) - 1/32*(9*I*(e^(-x) -
e^x)^2 + 4*e^(-x) - 4*e^x + 12*I)/(e^(-x) - e^x - 2*I)^2 + 3/16*I*log(-e^(-
x) + e^x + 2*I) - 3/16*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.14

$$\int \frac{\tanh^3(x)}{i+\sinh(x)}\,dx = \frac{3\,\mathrm{atan}(\mathrm{e}^x)}{4}+\frac{3\mathrm{i}}{2\,\left(\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{1\mathrm{i}}{2\,\left(\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^x\,4\mathrm{i}\right)}+\frac{1\mathrm{i}}{4\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^x\,2\mathrm{i}\right)}-\frac{1}{4\,\left(\mathrm{e}^x-\mathrm{i}\right)}-\frac{1}{\mathrm{e}^x+1\mathrm{i}}+\frac{1}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^x-\mathrm{i}}$$

```
[In] int(tanh(x)^3/(sinh(x) + 1i),x)
```

```
[Out] (3*atan(exp(x)))/4 + 3i/(2*(exp(2*x) + exp(x)*2i - 1)) - 1i/(2*(exp(3*x)*4i
 - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)) + 1i/(4*(exp(x)*2i - exp(2*x) +
1)) - 1/(4*(exp(x) - 1i)) - 1/(exp(x) + 1i) + 1/(exp(2*x)*3i + exp(3*x) - 3
*exp(x) - 1i)
```

3.210 $\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\operatorname{sech}(x)+\frac{\operatorname{sech}^3(x)}{3}-\frac{1}{3}i\tanh^3(x)$$

```
[Out] -sech(x)+1/3*sech(x)^3-1/3*I*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2785, 2687, 30, 2686}

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\frac{1}{3}i\tanh^3(x)+\frac{\operatorname{sech}^3(x)}{3}-\operatorname{sech}(x)$$

```
[In] Int[Tanh[x]^2/(I + Sinh[x]),x]
```

```
[Out] -Sech[x] + Sech[x]^3/3 - (I/3)*Tanh[x]^3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\int \operatorname{sech}^2(x)\tanh^2(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^3(x)\,dx\\ &= \operatorname{Subst}\left(\int x^2\,dx, x, i\tanh(x)\right)+\operatorname{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \operatorname{sech}(x)\right)\\ &= -\operatorname{sech}(x)+\frac{\operatorname{sech}^3(x)}{3}-\frac{1}{3}i\tanh^3(x)\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 67 vs. 2(23) = 46.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx=\frac{-3-\cosh(2x)+\cosh(x)(5-5i\sinh(x))+4i\sinh(x)}{6\left(\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)\right)^3\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)}$$

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x]),x]
[Out] (-3 - Cosh[2*x] + Cosh[x]*(5 - (5*I)*Sinh[x]) + (4*I)*Sinh[x])/(6*(Cosh[x/2
] - I*Sinh[x/2])^3*(Cosh[x/2] + I*Sinh[x/2]))
```

Maple [A] (verified)

Time = 5.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

method	result	size
risch	$-\frac{2\left(3i\,{\mathrm e}^{2x}+3\,{\mathrm e}^{3x}+i-{\mathrm e}^{x}\right)}{3\left({\mathrm e}^{x}+i\right)^{3}\left({\mathrm e}^{x}-i\right)}$	37
default	$\frac{i}{-2i+2\tanh\left(\frac{x}{2}\right)}-\frac{2i}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}$	47

```
[In] int(tanh(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -2/3*(3*I*exp(x)^2+3*exp(x)^3+I-exp(x))/(exp(x)+I)^3/(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx=-\frac{2\left(3\,e^{(3\,x)}+3i\,e^{(2\,x)}-e^{x}+i\right)}{3\left(e^{(4\,x)}+2i\,e^{(3\,x)}+2i\,e^{x}-1\right)}$$

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] -2/3*(3*e^(3*x) + 3*I*e^(2*x) - e^x + I)/(e^(4*x) + 2*I*e^(3*x) + 2*I*e^x -
 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx=\frac{-6e^{3x}-6ie^{2x}+2e^{x}-2i}{3e^{4x}+6ie^{3x}+6ie^{x}-3}$$

```
[In] integrate(tanh(x)**2/(I+sinh(x)),x)
[Out] (-6*exp(3*x) - 6*I*exp(2*x) + 2*exp(x) - 2*I)/(3*exp(4*x) + 6*I*exp(3*x) +
6*I*exp(x) - 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(17) = 34.

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.74

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = \frac{2\,e^{(-x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3} + \frac{6i\,e^{(-2\,x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3} - \frac{6\,e^{(-3\,x)}}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3} + \frac{2i}{-6i\,e^{(-x)}-6i\,e^{(-3\,x)}+3\,e^{(-4\,x)}-3}$$

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) + 6*I*e^(-2*x)/(-6*I
*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3) - 6*e^(-3*x)/(-6*I*e^(-x) - 6*I*e^
(-3*x) + 3*e^(-4*x) - 3) + 2*I/(-6*I*e^(-x) - 6*I*e^(-3*x) + 3*e^(-4*x) - 3
)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\frac{1}{2\,(e^x-i)} - \frac{9\,e^{(2\,x)}+12i\,e^x-7}{6\,(e^x+i)^3}$$

```
[In] integrate(tanh(x)^2/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/2/(e^x - I) - 1/6*(9*e^(2*x) + 12*I*e^x - 7)/(e^x + I)^3
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.48

$$\int \frac{\tanh^2(x)}{i+\sinh(x)}\,dx = -\frac{\frac{e^x}{2}+\frac{1}{6}i}{e^{2\,x}-1+e^x\,2i} - \frac{\frac{e^{2\,x}}{2}-\frac{1}{2}+\frac{e^x\,1i}{3}}{e^{2\,x}\,3i+e^{3\,x}-3\,e^x-i} - \frac{1}{2\,\left(e^x-i\right)} - \frac{1}{2\,\left(e^x+1i\right)}$$

```
[In] int(tanh(x)^2/(sinh(x) + 1i),x)
```

```
[Out] - (exp(x)/2 + 1i/6)/(exp(2*x) + exp(x)*2i - 1) - (exp(2*x)/2 + (exp(x)*1i)/
3 - 1/2)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) - 1/(2*(exp(x) - 1i)) - 1
/(2*(exp(x) + 1i))
```

3.211 $\int \frac{\tanh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\arctan(\sinh(x)) + \frac{1}{2}i\mathrm{sech}^2(x) - \frac{1}{2}\mathrm{sech}(x)\tanh(x)$$

```
[Out] 1/2*arctan(sinh(x))+1/2*I*sech(x)^2-1/2*sech(x)*tanh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2785, 2686, 30, 2691, 3855}

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\arctan(\sinh(x)) + \frac{1}{2}i\mathrm{sech}^2(x) - \frac{1}{2}\tanh(x)\mathrm{sech}(x)$$

```
[In] Int[Tanh[x]/(I + Sinh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/2 + (I/2)*Sech[x]^2 - (Sech[x]*Tanh[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\int \operatorname{sech}^2(x)\tanh(x)\,dx\right)+\int \operatorname{sech}(x)\tanh^2(x)\,dx\\ &= -\frac{1}{2}\operatorname{sech}(x)\tanh(x)+i\text{Subst}\left(\int x\,dx, x, \operatorname{sech}(x)\right)+\frac{1}{2}\int \operatorname{sech}(x)\,dx\\ &= \frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}i\operatorname{sech}^2(x)-\frac{1}{2}\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{1}{2}\arctan(\sinh(x)) - \frac{1}{2(i+\sinh(x))}$$

```
[In] Integrate[Tanh[x]/(I + Sinh[x]),x]
[Out] ArcTan[Sinh[x]]/2 - 1/(2*(I + Sinh[x]))
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result	size
risch	$-\frac{e^{x}}{\left(e^{x}+i\right)^{2}}+\frac{i \ln \left(e^{x}+i\right)}{2}-\frac{i \ln \left(e^{x}-i\right)}{2}$	31
default	$-\frac{i \ln \left(-i+\tanh \left(\frac{x}{2}\right)\right)}{2}-\frac{i}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{2}}+\frac{i \ln \left(\tanh \left(\frac{x}{2}\right)+i\right)}{2}+\frac{1}{\tanh \left(\frac{x}{2}\right)+i}$	45

```
[In] int(tanh(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(exp(x)+I)^2*exp(x)+1/2*I*ln(exp(x)+I)-1/2*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(18) = 36.

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{\left(i\,e^{(2\,x)} - 2\,e^{x} - i\right)\log\left(e^{x} + i\right) + \left(-i\,e^{(2\,x)} + 2\,e^{x} + i\right)\log\left(e^{x} - i\right) - 2\,e^{x}}{2\left(e^{(2\,x)} + 2i\,e^{x} - 1\right)}$$

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*((I*e^(2*x) - 2*e^x - I)*log(e^x + I) + (-I*e^(2*x) + 2*e^x + I)*log(e^
x - I) - 2*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \operatorname{RootSum}\left(4z^2 + 1, \left(i \mapsto i\log\left(2i + e^{x}\right)\right)\right) - \frac{e^{x}}{e^{2x} + 2ie^{x} - 1}$$

```
[In] integrate(tanh(x)/(I+sinh(x)),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x)))) - exp(x)/(exp(2*x)
+ 2*I*exp(x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(18) = 36.

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}+\frac{1}{2}i\,\log\left(i\,e^{(-x)}+1\right)-\frac{1}{2}i\,\log\left(i\,e^{(-x)}-1\right)$$

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) + 1/2*I*log(I*e^(-x) + 1) - 1/2*I*log(I
*e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(18) = 36.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \frac{-i\,e^{(-x)}+i\,e^{x}+2}{4\left(e^{(-x)}-e^{x}-2i\right)}+\frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}+2i\right)-\frac{1}{4}i\,\log\left(-e^{(-x)}+e^{x}-2i\right)$$

```
[In] integrate(tanh(x)/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(-I*e^(-x) + I*e^x + 2)/(e^(-x) - e^x - 2*I) + 1/4*I*log(-e^(-x) + e^x
+ 2*I) - 1/4*I*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{\tanh(x)}{i+\sinh(x)}\,dx = \mathrm{atan}(\mathrm{e}^{x})+\frac{1\mathrm{i}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}}-\frac{1}{\mathrm{e}^{x}+1\mathrm{i}}$$

```
[In] int(tanh(x)/(sinh(x) + 1i),x)
```

```
[Out] atan(exp(x)) + 1i/(exp(2*x) + exp(x)*2i - 1) - 1/(exp(x) + 1i)
```

3.212 $\int \frac{\coth(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\log(\sinh(x)) + i\log(i+\sinh(x))$$

```
[Out] -I*ln(sinh(x))+I*ln(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2786, 36, 29, 31}

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = i\log(\sinh(x)+i) - i\log(\sinh(x))$$

```
[In] Int[Coth[x]/(I + Sinh[x]),x]
[Out] (-I)*Log[Sinh[x]] + I*Log[I + Sinh[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x(i+x)}\,dx, x, \sinh(x)\right)\\
&= -\left(i\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh(x)\right)\right) + i\text{Subst}\left(\int \frac{1}{i+x}\,dx, x, \sinh(x)\right)\\
&= -i\log(\sinh(x)) + i\log(i+\sinh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = i(-\log(\sinh(x)) + \log(i+\sinh(x)))$$

```
[In] Integrate[Coth[x]/(I + Sinh[x]),x]
[Out] I*(-Log[Sinh[x]] + Log[I + Sinh[x]])
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
default	$2i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	21
risch	$2i\ln\left(e^{x}+i\right)-i\ln\left(e^{2x}-1\right)$	21

```
[In] int(coth(x)/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] 2*I*ln(tanh(1/2*x)+I)-I*ln(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\,\log\left(e^{(2\,x)}-1\right)+2i\,\log\left(e^{x}+i\right)$$

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="fricas")
[Out] -I*log(e^(2*x) - 1) + 2*I*log(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = 2i\log\left(e^{x}+i\right)-i\log\left(e^{2x}-1\right)$$

```
[In] integrate(coth(x)/(I+sinh(x)),x)
[Out] 2*I*log(exp(x) + I) - I*log(exp(2*x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\,\log\left(e^{(-x)}+1\right)+2i\,\log\left(e^{(-x)}-i\right)-i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="maxima")
[Out] -I*log(e^(-x) + 1) + 2*I*log(e^(-x) - I) - I*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = -i\,\log\left(e^{x}+1\right)+2i\,\log\left(e^{x}+i\right)-i\,\log\left(|e^{x}-1|\right)$$

```
[In] integrate(coth(x)/(I+sinh(x)),x, algorithm="giac")
[Out] -I*log(e^x + 1) + 2*I*log(e^x + I) - I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\coth(x)}{i+\sinh(x)}\,dx = \ln\left(-36\,\mathrm{e}^{x}-36\mathrm{i}\right)\,2\mathrm{i}-\ln\left(3-3\,\mathrm{e}^{2\,x}\right)\,1\mathrm{i}$$

```
[In] int(coth(x)/(sinh(x) + 1i),x)
[Out] log(- 36*exp(x) - 36i)*2i - log(3 - 3*exp(2*x))*1i
```

3.213 $\int \frac{\coth^2(x)}{i+\sinh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\coth^2(x)}{i+\sinh(x)} dx = -\operatorname{arctanh}(\cosh(x)) + i\coth(x)$$

```
[Out] -arctanh(cosh(x))+I*coth(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2785, 3852, 8, 3855}

$$\int \frac{\coth^2(x)}{i+\sinh(x)} dx = -\operatorname{arctanh}(\cosh(x)) + i\coth(x)$$

```
[In] Int[Coth[x]^2/(I + Sinh[x]),x]
[Out] -ArcTanh[Cosh[x]] + I*Coth[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\int \operatorname{csch}^2(x)\,dx\right)+\int \operatorname{csch}(x)\,dx\\ &= -\operatorname{arctanh}(\cosh(x))-\operatorname{Subst}\left(\int 1\,dx,x,-i\coth(x)\right)\\ &= -\operatorname{arctanh}(\cosh(x))+i\coth(x)\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx=\frac{1}{2}i\coth\left(\frac{x}{2}\right)-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{1}{2}i\tanh\left(\frac{x}{2}\right)$$

```
[In] Integrate[Coth[x]^2/(I + Sinh[x]),x]
[Out] (I/2)*Coth[x/2] - Log[Cosh[x/2]] + Log[Sinh[x/2]] + (I/2)*Tanh[x/2]
```

Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{i\tanh\left(\frac{x}{2}\right)}{2}+\frac{i}{2\tanh\left(\frac{x}{2}\right)}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	23
risch	$\frac{2i}{e^{2x}-1}+\ln\left(e^{x}-1\right)-\ln\left(e^{x}+1\right)$	25

```
[In] int(coth(x)^2/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*I*tanh(1/2*x)+1/2*I/tanh(1/2*x)+ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \frac{\coth^2(x)}{i + \sinh(x)}\,dx = -\frac{\left(e^{(2x)} - 1\right)\log\left(e^x + 1\right) - \left(e^{(2x)} - 1\right)\log\left(e^x - 1\right) - 2i}{e^{(2x)} - 1}$$

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="fricas")
[Out] -((e^(2*x) - 1)*log(e^x + 1) - (e^(2*x) - 1)*log(e^x - 1) - 2*I)/(e^(2*x) -
 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\coth^2(x)}{i + \sinh(x)}\,dx = \log\left(e^x - 1\right) - \log\left(e^x + 1\right) + \frac{2i}{e^{2x} - 1}$$

```
[In] integrate(coth(x)**2/(I+sinh(x)),x)
[Out] log(exp(x) - 1) - log(exp(x) + 1) + 2*I/(exp(2*x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{\coth^2(x)}{i + \sinh(x)}\,dx = -\frac{2i}{e^{(-2x)} - 1} - \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="maxima")
[Out] -2*I/(e^(-2*x) - 1) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = \frac{2i}{e^{(2x)}-1} - \log\left(e^x+1\right) + \log\left(|e^x-1|\right)$$

```
[In] integrate(coth(x)^2/(I+sinh(x)),x, algorithm="giac")
[Out] 2*I/(e^(2*x) - 1) - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{\coth^2(x)}{i+\sinh(x)}\,dx = \ln\left(2-2\,\mathrm{e}^x\right) - \ln\left(-2\,\mathrm{e}^x-2\right) + \frac{2\mathrm{i}}{\mathrm{e}^{2\,x}-1}$$

```
[In] int(coth(x)^2/(sinh(x) + 1i),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + 2i/(exp(2*x) - 1)
```

3.214 $\int \frac{\coth^3(x)}{i+\sinh(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{\coth^3(x)}{i+\sinh(x)} \, dx = -\operatorname{csch}(x) + \frac{1}{2} i \operatorname{csch}^2(x)$$

```
[Out] -csch(x)+1/2*I*csch(x)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2785, 2686, 30, 8}

$$\int \frac{\coth^3(x)}{i+\sinh(x)} \, dx = -\operatorname{csch}(x) + \frac{1}{2} i \operatorname{csch}^2(x)$$

```
[In] Int[Coth[x]^3/(I + Sinh[x]),x]
```

```
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\int \coth(x)\operatorname{csch}^2(x)\,dx\right)+\int \coth(x)\operatorname{csch}(x)\,dx\\ &= -(i\text{Subst}(\int 1\,dx,x,-i\operatorname{csch}(x)))-i\text{Subst}(\int x\,dx,x,-i\operatorname{csch}(x))\\ &= -\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx=-\operatorname{csch}(x)+\frac{1}{2}i\operatorname{csch}^2(x)$$

```
[In] Integrate[Coth[x]^3/(I + Sinh[x]),x]
[Out] -Csch[x] + (I/2)*Csch[x]^2
```

Maple [A] (verified)

Time = 9.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

method	result	size
risch	$-\frac{2\,\mathrm{e}^{x}\left(-i\,\mathrm{e}^{x}+\mathrm{e}^{2x}-1\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}}$	24
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}+\frac{i\tanh\left(\frac{x}{2}\right)^{2}}{8}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}+\frac{i}{8\tanh\left(\frac{x}{2}\right)^{2}}$	34

```
[In] int(coth(x)^3/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

[Out] -2*exp(x)*(-I*exp(x)+exp(2*x)-1)/(exp(2*x)-1)^2

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = -\frac{2\left(e^{(3x)} - i\,e^{(2x)} - e^{x}\right)}{e^{(4x)} - 2\,e^{(2x)} + 1}$$

[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="fricas")

[Out] -2*(e^(3*x) - I*e^(2*x) - e^x)/(e^(4*x) - 2*e^(2*x) + 1)

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(10) = 20.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = \frac{-2e^{3x} + 2ie^{2x} + 2e^{x}}{e^{4x} - 2e^{2x} + 1}$$

[In] integrate(coth(x)**3/(I+sinh(x)),x)

[Out] (-2*exp(3*x) + 2*I*exp(2*x) + 2*exp(x))/(exp(4*x) - 2*exp(2*x) + 1)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(11) = 22.

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.47

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = \frac{2\,e^{(-x)}}{2\,e^{(-2x)} - e^{(-4x)} - 1} - \frac{2i\,e^{(-2x)}}{2\,e^{(-2x)} - e^{(-4x)} - 1} - \frac{2\,e^{(-3x)}}{2\,e^{(-2x)} - e^{(-4x)} - 1}$$

[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="maxima")

[Out] 2*e^(-x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*I*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2*e^(-3*x)/(2*e^(-2*x) - e^(-4*x) - 1)

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = \frac{2\left(e^{(-x)}-e^{x}+i\right)}{\left(e^{(-x)}-e^{x}\right)^2}$$

```
[In] integrate(coth(x)^3/(I+sinh(x)),x, algorithm="giac")
[Out] 2*(e^(-x) - e^x + I)/(e^(-x) - e^x)^2
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{\coth^3(x)}{i+\sinh(x)}\,dx = \frac{2\,\mathrm{e}^{x}\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,1\mathrm{i}\right)}{\left(\mathrm{e}^{2\,x}-1\right)^2}$$

```
[In] int(coth(x)^3/(sinh(x) + 1i),x)
[Out] (2*exp(x)*(exp(x)*1i - exp(2*x) + 1))/(exp(2*x) - 1)^2
```

3.215 $\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = -\frac{1}{2}\operatorname{arctanh}(\cosh(x)) + \frac{1}{3}i\coth^3(x) - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

```
[Out] -1/2*arctanh(cosh(x))+1/3*I*coth(x)^3-1/2*coth(x)*csch(x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2785, 2687, 30, 2691, 3855}

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = -\frac{1}{2}\operatorname{arctanh}(\cosh(x)) + \frac{1}{3}i\coth^3(x) - \frac{1}{2}\coth(x)\operatorname{csch}(x)$$

```
[In] Int[Coth[x]^4/(I + Sinh[x]),x]
[Out] -1/2*ArcTanh[Cosh[x]] + (I/3)*Coth[x]^3 - (Coth[x]*Csch[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
```

```
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \coth^2(x)\operatorname{csch}^2(x)\,dx\right)+\int \coth^2(x)\operatorname{csch}(x)\,dx\\
&= -\frac{1}{2}\coth(x)\operatorname{csch}(x)+\frac{1}{2}\int \operatorname{csch}(x)\,dx-\operatorname{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)\\
&= -\frac{1}{2}\operatorname{arctanh}(\cosh(x))+\frac{1}{3}i\coth^3(x)-\frac{1}{2}\coth(x)\operatorname{csch}(x)
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 111 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.27

$$\begin{aligned}
\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx &= \frac{1}{6}i\coth\left(\frac{x}{2}\right)-\frac{1}{8}\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{24}i\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)\\
&\quad-\frac{1}{2}\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sinh\left(\frac{x}{2}\right)\right)-\frac{1}{8}\operatorname{sech}^2\left(\frac{x}{2}\right)\\
&\quad+\frac{1}{6}i\tanh\left(\frac{x}{2}\right)-\frac{1}{24}i\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)
\end{aligned}$$

```
[In] Integrate[Coth[x]^4/(I + Sinh[x]),x]
```

```
[Out] (I/6)*Coth[x/2] - Csch[x/2]^2/8 + (I/24)*Coth[x/2]*Csch[x/2]^2 - Log[Cosh[x
/2]]/2 + Log[Sinh[x/2]]/2 - Sech[x/2]^2/8 + (I/6)*Tanh[x/2] - (I/24)*Sech[x
/2]^2*Tanh[x/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(19) = 38$.

Time = 13.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

method	result	size
risch	$-\frac{-6i\,\mathrm{e}^{4x}+3\,\mathrm{e}^{5x}-2i-3\,\mathrm{e}^{x}}{3(\mathrm{e}^{2x}-1)^{3}}+\frac{\ln(\mathrm{e}^{x}-1)}{2}-\frac{\ln(\mathrm{e}^{x}+1)}{2}$	46
default	$\frac{i\tanh(\frac{x}{2})}{8}+\frac{i\tanh(\frac{x}{2})^{3}}{24}+\frac{\tanh(\frac{x}{2})^{2}}{8}+\frac{i}{8\tanh(\frac{x}{2})}-\frac{1}{8\tanh(\frac{x}{2})^{2}}+\frac{i}{24\tanh(\frac{x}{2})^{3}}+\frac{\ln(\tanh(\frac{x}{2}))}{2}$	59

```
[In] int(coth(x)^4/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-6*I*exp(x)^4+3*exp(x)^5-2*I-3*exp(x))/(exp(x)^2-1)^3+1/2*ln(exp(x)-1
)-1/2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = -\frac{3\left(e^{(6x)}-3e^{(4x)}+3e^{(2x)}-1\right)\log\left(e^{x}+1\right)-3\left(e^{(6x)}-3e^{(4x)}+3e^{(2x)}-1\right)\log\left(e^{x}-1\right)+6e^{(5x)}-12}{6\left(e^{(6x)}-3e^{(4x)}+3e^{(2x)}-1\right)}$$

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/6*(3*(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)*log(e^x + 1) - 3*(e^(6*x) - 3
*e^(4*x) + 3*e^(2*x) - 1)*log(e^x - 1) + 6*e^(5*x) - 12*I*e^(4*x) - 6*e^x -
 4*I)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = \frac{-3e^{5x}+6ie^{4x}+3e^{x}+2i}{3e^{6x}-9e^{4x}+9e^{2x}-3} + \frac{\log(e^x-1)}{2} - \frac{\log(e^x+1)}{2}$$

```
[In] integrate(coth(x)**4/(I+sinh(x)),x)
```

```
[Out] (-3*exp(5*x) + 6*I*exp(4*x) + 3*exp(x) + 2*I)/(3*exp(6*x) - 9*exp(4*x) + 9*
exp(2*x) - 3) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(18) = 36.

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = \frac{3\,e^{(-x)}-6i\,e^{(-4\,x)}-3\,e^{(-5\,x)}-2i}{3\,(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1)} - \frac{1}{2}\log\left(e^{(-x)}+1\right) + \frac{1}{2}\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*e^(-x) - 6*I*e^(-4*x) - 3*e^(-5*x) - 2*I)/(3*e^(-2*x) - 3*e^(-4*x) +
 e^(-6*x) - 1) - 1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = -\frac{3\,e^{(5\,x)}-6i\,e^{(4\,x)}-3\,e^{x}-2i}{3\,\left(e^{(2\,x)}-1\right)^3} - \frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(|e^x-1|\right)$$

```
[In] integrate(coth(x)^4/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/3*(3*e^(5*x) - 6*I*e^(4*x) - 3*e^x - 2*I)/(e^(2*x) - 1)^3 - 1/2*log(e^x
+ 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.85

$$\int \frac{\coth^4(x)}{i+\sinh(x)}\,dx = \frac{\ln(1-e^x)}{2} - \frac{\ln(e^x+1)}{2} - \frac{e^x}{e^{2x}-1} - \frac{2\,e^x}{(e^{2x}-1)^2} + \frac{2i}{e^{2x}-1} + \frac{4i}{(e^{2x}-1)^2} + \frac{8i}{3\,(e^{2x}-1)^3}$$

```
[In] int(coth(x)^4/(sinh(x) + 1i),x)
```

```
[Out] log(1 - exp(x))/2 - log(exp(x) + 1)/2 - exp(x)/(exp(2*x) - 1) - (2*exp(x))/
(exp(2*x) - 1)^2 + 2i/(exp(2*x) - 1) + 4i/(exp(2*x) - 1)^2 + 8i/(3*(exp(2*x
) - 1)^3)
```

3.216 $\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{1}{4}i\coth^4(x) - \operatorname{csch}(x) - \frac{\operatorname{csch}^3(x)}{3}$$

```
[Out] 1/4*I*coth(x)^4-csch(x)-1/3*csch(x)^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2785, 2687, 30, 2686}

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{1}{4}i\coth^4(x) - \frac{\operatorname{csch}^3(x)}{3} - \operatorname{csch}(x)$$

```
[In] Int[Coth[x]^5/(I + Sinh[x]),x]
```

```
[Out] (I/4)*Coth[x]^4 - Csch[x] - Csch[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \coth^3(x)\text{csch}^2(x)\,dx\right)+\int \coth^3(x)\text{csch}(x)\,dx\\
&= i\text{Subst}\left(\int x^3\,dx, x, i\coth(x)\right)+i\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, -i\text{csch}(x)\right)\\
&= \frac{1}{4}i\coth^4(x)-\text{csch}(x)-\frac{\text{csch}^3(x)}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = -\text{csch}(x)+\frac{1}{2}i\text{csch}^2(x)-\frac{\text{csch}^3(x)}{3}+\frac{1}{4}i\text{csch}^4(x)$$

```
[In] Integrate[Coth[x]^5/(I + Sinh[x]),x]
[Out] -Csch[x] + (I/2)*Csch[x]^2 - Csch[x]^3/3 + (I/4)*Csch[x]^4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(18) = 36.

Time = 17.70 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.96

method	result	size
risch	$-\frac{2 \mathrm{e}^{x} \left(-3 i \mathrm{e}^{5 x}+3 \mathrm{e}^{6 x}-5 \mathrm{e}^{4 x}-3 i \mathrm{e}^{x}+5 \mathrm{e}^{2 x}-3\right)}{3 \left(\mathrm{e}^{2 x}-1\right)^{4}}$	45
default	$\frac{3 \tanh \left(\frac{x}{2}\right)}{8}+\frac{i \tanh \left(\frac{x}{2}\right)^{4}}{64}+\frac{\tanh \left(\frac{x}{2}\right)^{3}}{24}+\frac{i \tanh \left(\frac{x}{2}\right)^{2}}{16}+\frac{i}{64 \tanh \left(\frac{x}{2}\right)^{4}}-\frac{3}{8 \tanh \left(\frac{x}{2}\right)}+\frac{i}{16 \tanh \left(\frac{x}{2}\right)^{2}}-\frac{1}{24 \tanh \left(\frac{x}{2}\right)^{3}}$	68

```
[In] int(coth(x)^5/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*exp(x)*(-3*I*exp(x)^5+3*exp(x)^6-5*exp(x)^4-3*I*exp(x)+5*exp(x)^2-3)/(
exp(x)^2-1)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\, dx = -\frac{2 \left(3 e^{(7 x)} - 3i e^{(6 x)} - 5 e^{(5 x)} + 5 e^{(3 x)} - 3i e^{(2 x)} - 3 e^{x}\right)}{3 \left(e^{(8 x)} - 4 e^{(6 x)} + 6 e^{(4 x)} - 4 e^{(2 x)} + 1\right)}$$

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -2/3*(3*e^(7*x) - 3*I*e^(6*x) - 5*e^(5*x) + 5*e^(3*x) - 3*I*e^(2*x) - 3*e^x
)/(e^(8*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.04

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\, dx = \frac{-6e^{7x} + 6ie^{6x} + 10e^{5x} - 10e^{3x} + 6ie^{2x} + 6e^{x}}{3e^{8x} - 12e^{6x} + 18e^{4x} - 12e^{2x} + 3}$$

```
[In] integrate(coth(x)**5/(I+sinh(x)),x)
```

```
[Out] (-6*exp(7*x) + 6*I*exp(6*x) + 10*exp(5*x) - 10*exp(3*x) + 6*I*exp(2*x) + 6*
exp(x))/(3*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.91

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{2\,e^{(-x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} - \frac{2i\,e^{(-2\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} - \frac{10\,e^{(-3\,x)}}{3\,(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1)} + \frac{10\,e^{(-5\,x)}}{3\,(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1)} - \frac{2i\,e^{(-6\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} - \frac{2\,e^{(-7\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1}$$

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-2*
x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 10/3*e^(-3*x)/(4
*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 10/3*e^(-5*x)/(4*e^(-
2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*I*e^(-6*x)/(4*e^(-2*x) -
 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 2*e^(-7*x)/(4*e^(-2*x) - 6*e^(-4
*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{2\left(3\left(e^{(-x)} - e^{x}\right)^3 + 3i\left(e^{(-x)} - e^{x}\right)^2 + 4\,e^{(-x)} - 4\,e^{x} + 6i\right)}{3\left(e^{(-x)} - e^{x}\right)^4}$$

```
[In] integrate(coth(x)^5/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] 2/3*(3*(e^(-x) - e^x)^3 + 3*I*(e^(-x) - e^x)^2 + 4*e^(-x) - 4*e^x + 6*I)/(e
^(-x) - e^x)^4
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{\coth^5(x)}{i+\sinh(x)}\,dx = \frac{2\,\mathrm{e}^{x}\,(5\,\mathrm{e}^{4\,x}-5\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{6\,x}+3+\mathrm{e}^{5\,x}\,3\mathrm{i}+\mathrm{e}^{x}\,3\mathrm{i})}{3\,(\mathrm{e}^{2\,x}-1)^4}$$

```
[In] int(coth(x)^5/(sinh(x) + 1i),x)
```

```
[Out] (2*exp(x)*(5*exp(4*x) - 5*exp(2*x) + exp(5*x)*3i - 3*exp(6*x) + exp(x)*3i +
 3))/(3*(exp(2*x) - 1)^4)
```

3.217 $\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = -\frac{3}{8}\operatorname{arctanh}(\cosh(x))+\frac{1}{5}i\coth^5(x)-\frac{3}{8}\coth(x)\operatorname{csch}(x)-\frac{1}{4}\coth^3(x)\operatorname{csch}(x)$$

```
[Out] -3/8*arctanh(cosh(x))+1/5*I*coth(x)^5-3/8*coth(x)*csch(x)-1/4*coth(x)^3*csc
h(x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2785, 2687, 30, 2691, 3855}

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = -\frac{3}{8}\operatorname{arctanh}(\cosh(x))+\frac{1}{5}i\coth^5(x)-\frac{1}{4}\coth^3(x)\operatorname{csch}(x)-\frac{3}{8}\coth(x)\operatorname{csch}(x)$$

```
[In] Int[Coth[x]^6/(I + Sinh[x]),x]
```

```
[Out] (-3*ArcTanh[Cosh[x]])/8 + (I/5)*Coth[x]^5 - (3*Coth[x]*Csch[x])/8 - (Coth[x
]^3*Csch[x])/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i \int \coth^4(x)\operatorname{csch}^2(x)\,dx\right) + \int \coth^4(x)\operatorname{csch}(x)\,dx \\
&= -\frac{1}{4}\coth^3(x)\operatorname{csch}(x) + \frac{3}{4}\int \coth^2(x)\operatorname{csch}(x)\,dx + \operatorname{Subst}\left(\int x^4\,dx, x, i\coth(x)\right) \\
&= \frac{1}{5} i\coth^5(x) - \frac{3}{8}\coth(x)\operatorname{csch}(x) - \frac{1}{4}\coth^3(x)\operatorname{csch}(x) + \frac{3}{8}\int \operatorname{csch}(x)\,dx \\
&= -\frac{3}{8}\operatorname{arctanh}(\cosh(x)) + \frac{1}{5} i\coth^5(x) - \frac{3}{8}\coth(x)\operatorname{csch}(x) - \frac{1}{4}\coth^3(x)\operatorname{csch}(x)
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 175 vs. $2(36) = 72$.

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.86

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \frac{1}{10}i\coth\left(\frac{x}{2}\right) - \frac{5}{32}\text{csch}^2\left(\frac{x}{2}\right) + \frac{7}{160}i\coth\left(\frac{x}{2}\right)\text{csch}^2\left(\frac{x}{2}\right) - \frac{1}{64}\text{csch}^4\left(\frac{x}{2}\right) + \frac{1}{160}i\coth\left(\frac{x}{2}\right)\text{csch}^4\left(\frac{x}{2}\right) - \frac{3}{8}\log\left(\cosh\left(\frac{x}{2}\right)\right) + \frac{3}{8}\log\left(\sinh\left(\frac{x}{2}\right)\right) - \frac{5}{32}\text{sech}^2\left(\frac{x}{2}\right) + \frac{1}{64}\text{sech}^4\left(\frac{x}{2}\right) + \frac{1}{10}i\tanh\left(\frac{x}{2}\right) - \frac{7}{160}i\text{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right) + \frac{1}{160}i\text{sech}^4\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

```
[In] Integrate[Coth[x]^6/(I + Sinh[x]),x]
```

```
[Out] (I/10)*Coth[x/2] - (5*Csch[x/2]^2)/32 + ((7*I)/160)*Coth[x/2]*Csch[x/2]^2 -
 Csch[x/2]^4/64 + (I/160)*Coth[x/2]*Csch[x/2]^4 - (3*Log[Cosh[x/2]])/8 + (3
*Log[Sinh[x/2]])/8 - (5*Sech[x/2]^2)/32 + Sech[x/2]^4/64 + (I/10)*Tanh[x/2]
 - ((7*I)/160)*Sech[x/2]^2*Tanh[x/2] + (I/160)*Sech[x/2]^4*Tanh[x/2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(27) = 54.

Time = 25.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{-40i\,e^{8x}+25\,e^{9x}-10\,e^{7x}-80i\,e^{4x}+10\,e^{3x}-8i-25\,e^{x}}{20(e^{2x}-1)^5}+\frac{3\ln(e^{x}-1)}{8}-\frac{3\ln(e^{x}+1)}{8}$
default	$\frac{i\tanh(\frac{x}{2})}{16}+\frac{i\tanh(\frac{x}{2})^5}{160}+\frac{\tanh(\frac{x}{2})^4}{64}+\frac{i\tanh(\frac{x}{2})^3}{32}+\frac{\tanh(\frac{x}{2})^2}{8}-\frac{1}{64\tanh(\frac{x}{2})^4}+\frac{i}{160\tanh(\frac{x}{2})^5}+\frac{i}{16\tanh(\frac{x}{2})}-$ 8 tan

```
[In] int(coth(x)^6/(I+sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(-40*I*exp(x)^8+25*exp(x)^9-10*exp(x)^7-80*I*exp(x)^4+10*exp(x)^3-8*I
-25*exp(x))/(exp(x)^2-1)^5+3/8*ln(exp(x)-1)-3/8*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.00

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = -\frac{15\left(e^{(10\,x)} - 5\,e^{(8\,x)} + 10\,e^{(6\,x)} - 10\,e^{(4\,x)} + 5\,e^{(2\,x)} - 1\right)\log\left(e^{x}+1\right) - 15\left(e^{(10\,x)} - 5\,e^{(8\,x)} + 10\,e^{(6\,x)} - 10\,e}{40\left(e^{(10\,x)} - 5\,e^{(8\,x)} + 10\,e^{(6\,x)}\right.}$$

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/40*(15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x) - 1)*
log(e^x + 1) - 15*(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*
x) - 1)*log(e^x - 1) + 50*e^(9*x) - 80*I*e^(8*x) - 20*e^(7*x) - 160*I*e^(4*
x) + 20*e^(3*x) - 50*e^x - 16*I)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^
(4*x) + 5*e^(2*x) - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(36) = 72.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.78

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \frac{3\log\left(e^{x}-1\right)}{8} - \frac{3\log\left(e^{x}+1\right)}{8} + \frac{-25e^{9x} + 40ie^{8x} + 10e^{7x} + 80ie^{4x} - 10e^{3x} + 25e^{x} + 8i}{20e^{10x} - 100e^{8x} + 200e^{6x} - 200e^{4x} + 100e^{2x} - 20}$$

```
[In] integrate(coth(x)**6/(I+sinh(x)),x)
```

```
[Out] 3*log(exp(x) - 1)/8 - 3*log(exp(x) + 1)/8 + (-25*exp(9*x) + 40*I*exp(8*x) +
 10*exp(7*x) + 80*I*exp(4*x) - 10*exp(3*x) + 25*exp(x) + 8*I)/(20*exp(10*x)
 - 100*exp(8*x) + 200*exp(6*x) - 200*exp(4*x) + 100*exp(2*x) - 20)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.53

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \frac{25\,e^{(-x)} - 10\,e^{(-3\,x)} - 80i\,e^{(-4\,x)} + 10\,e^{(-7\,x)} - 40i\,e^{(-8\,x)} - 25\,e^{(-9\,x)} - 8i}{20\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)} - \frac{3}{8}\log\left(e^{(-x)}+1\right) + \frac{3}{8}\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/20*(25*e^(-x) - 10*e^(-3*x) - 80*I*e^(-4*x) + 10*e^(-7*x) - 40*I*e^(-8*x)
 - 25*e^(-9*x) - 8*I)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
+ e^(-10*x) - 1) - 3/8*log(e^(-x) + 1) + 3/8*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.72

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = -\frac{25\,e^{(9\,x)} - 40i\,e^{(8\,x)} - 10\,e^{(7\,x)} - 80i\,e^{(4\,x)} + 10\,e^{(3\,x)} - 25\,e^{x} - 8i}{20\,(e^{(2\,x)}-1)^5} - \frac{3}{8}\log\left(e^{x}+1\right) + \frac{3}{8}\log\left(|e^{x}-1|\right)$$

```
[In] integrate(coth(x)^6/(I+sinh(x)),x, algorithm="giac")
```

```
[Out] -1/20*(25*e^(9*x) - 40*I*e^(8*x) - 10*e^(7*x) - 80*I*e^(4*x) + 10*e^(3*x) -
 25*e^x - 8*I)/(e^(2*x) - 1)^5 - 3/8*log(e^x + 1) + 3/8*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.44

$$\int \frac{\coth^6(x)}{i+\sinh(x)}\,dx = \frac{3\,\ln\left(\frac{3}{4}-\frac{3\,e^{x}}{4}\right)}{8} - \frac{3\,\ln\left(\frac{3\,e^{x}}{4}+\frac{3}{4}\right)}{8} - \frac{5\,e^{x}}{4\,(e^{2\,x}-1)} - \frac{9\,e^{x}}{2\,(e^{2\,x}-1)^2} - \frac{6\,e^{x}}{(e^{2\,x}-1)^3} - \frac{4\,e^{x}}{(e^{2\,x}-1)^4} + \frac{2i}{e^{2\,x}-1} + \frac{8i}{(e^{2\,x}-1)^2} + \frac{16i}{(e^{2\,x}-1)^3} + \frac{16i}{(e^{2\,x}-1)^4} + \frac{32i}{5\,(e^{2\,x}-1)^5}$$

```
[In] int(coth(x)^6/(sinh(x) + 1i),x)
```

```
[Out] (3*log(3/4 - (3*exp(x))/4))/8 - (3*log((3*exp(x))/4 + 3/4))/8 - (5*exp(x))/
(4*(exp(2*x) - 1)) - (9*exp(x))/(2*(exp(2*x) - 1)^2) - (6*exp(x))/(exp(2*x)
 - 1)^3 - (4*exp(x))/(exp(2*x) - 1)^4 + 2i/(exp(2*x) - 1) + 8i/(exp(2*x) -
1)^2 + 16i/(exp(2*x) - 1)^3 + 16i/(exp(2*x) - 1)^4 + 32i/(5*(exp(2*x) - 1)^
5)
```

3.218 $\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{3}i\operatorname{sech}^3(x) - \frac{4}{5}i\operatorname{sech}^5(x) + \frac{2}{7}i\operatorname{sech}^7(x) - \frac{\tanh^5(x)}{5} + \frac{2\tanh^7(x)}{7}$$

```
[Out] 2/3*I*sech(x)^3-4/5*I*sech(x)^5+2/7*I*sech(x)^7-1/5*tanh(x)^5+2/7*tanh(x)^7
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2790, 2687, 14, 2686, 276, 30}

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{2\tanh^7(x)}{7} - \frac{\tanh^5(x)}{5} + \frac{2}{7}i\operatorname{sech}^7(x) - \frac{4}{5}i\operatorname{sech}^5(x) + \frac{2}{3}i\operatorname{sech}^3(x)$$

```
[In] Int[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((4*I)/5)*Sech[x]^5 + ((2*I)/7)*Sech[x]^7 - Tanh[x]^5
/5 + (2*Tanh[x]^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\operatorname{sech}^4(x)\tanh^4(x) - 2i\operatorname{sech}^3(x)\tanh^5(x) + \operatorname{sech}^2(x)\tanh^6(x)\right)\,dx \\
&= -\left(2i\int \operatorname{sech}^3(x)\tanh^5(x)\,dx\right) - \int \operatorname{sech}^4(x)\tanh^4(x)\,dx + \int \operatorname{sech}^2(x)\tanh^6(x)\,dx \\
&= i\text{Subst}\left(\int x^6\,dx, x, i\tanh(x)\right) + i\text{Subst}\left(\int x^4\left(1+x^2\right)\,dx, x, i\tanh(x)\right) \\
&\quad + 2i\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \operatorname{sech}(x)\right) \\
&= \frac{\tanh^7(x)}{7} + i\text{Subst}\left(\int \left(x^4+x^6\right)\,dx, x, i\tanh(x)\right) \\
&\quad + 2i\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \operatorname{sech}(x)\right)
\end{aligned}$$

$$= \frac{2}{3}i\text{sech}^3(x) - \frac{4}{5}i\text{sech}^5(x) + \frac{2}{7}i\text{sech}^7(x) - \frac{\tanh^5(x)}{5} + \frac{2\tanh^7(x)}{7}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 112 vs. 2(47) = 94.

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.38

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{-672i + 1442i\cosh(x) - 1664i\cosh(2x) + 309i\cosh(3x) + 288i\cosh(4x) - 103i\cosh(5x) + 1232\sinh(\ldots}{13440\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^7\left(\cosh\left(\frac{x}{2}\right) + i\sin\ldots}$$

```
[In] Integrate[Tanh[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] -1/13440*(-672*I + (1442*I)*Cosh[x] - (1664*I)*Cosh[2*x] + (309*I)*Cosh[3*x
] + (288*I)*Cosh[4*x] - (103*I)*Cosh[5*x] + 1232*Sinh[x] + 824*Sinh[2*x] -
1896*Sinh[3*x] + 412*Sinh[4*x] + 72*Sinh[5*x])/((Cosh[x/2] - I*Sinh[x/2])^7
*(Cosh[x/2] + I*Sinh[x/2])^3)
```

Maple [A] (verified)

Time = 28.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.47

method	result
risch	$-\frac{2\left(68i\,e^{3x}-132\,e^{2x}-36i\,e^{x}+14\,e^{4x}+9+84i\,e^{5x}-140\,e^{6x}+140i\,e^{7x}+105\,e^{8x}\right)}{105\left(e^{x}+i\right)^{7}\left(e^{x}-i\right)^{3}}$
default	$-\frac{i}{8\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{1}{12\left(-i+\tanh\left(\frac{x}{2}\right)\right)^{3}}+\frac{1}{-8i+8\tanh\left(\frac{x}{2}\right)}+\frac{2i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{6}}-\frac{i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i}{8\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{\ldots}{7(\text{ta}\ldots}$

```
[In] int(tanh(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/105*(68*I*exp(x)^3-132*exp(x)^2-36*I*exp(x)+14*exp(x)^4+9+84*I*exp(x)^5-
140*exp(x)^6+140*I*exp(x)^7+105*exp(x)^8)/(exp(x)+I)^7/(exp(x)-I)^3
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(31) = 62.

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.21

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(105\,e^{(8\,x)}+140i\,e^{(7\,x)}-140\,e^{(6\,x)}+84i\,e^{(5\,x)}+14\,e^{(4\,x)}+68i\,e^{(3\,x)}-132\,e^{(2\,x)}-36i\,e^{x}+9\right)}{105\left(e^{(10\,x)}+4i\,e^{(9\,x)}-3\,e^{(8\,x)}+8i\,e^{(7\,x)}-14\,e^{(6\,x)}-14\,e^{(4\,x)}-8i\,e^{(3\,x)}-3\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/105*(105*e^(8*x) + 140*I*e^(7*x) - 140*e^(6*x) + 84*I*e^(5*x) + 14*e^(4*
x) + 68*I*e^(3*x) - 132*e^(2*x) - 36*I*e^x + 9)/(e^(10*x) + 4*I*e^(9*x) - 3
*e^(8*x) + 8*I*e^(7*x) - 14*e^(6*x) - 14*e^(4*x) - 8*I*e^(3*x) - 3*e^(2*x)
- 4*I*e^x + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(44) = 88.

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.72

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \frac{-210e^{8x}-280ie^{7x}+280e^{6x}-168ie^{5x}-28e^{4x}-136ie^{3x}+264e^{2x}+72ie^{x}-18}{105e^{10x}+420ie^{9x}-315e^{8x}+840ie^{7x}-1470e^{6x}-1470e^{4x}-840ie^{3x}-315e^{2x}-420ie^{x}+105}$$

```
[In] integrate(tanh(x)**4/(I+sinh(x))**2,x)
```

```
[Out] (-210*exp(8*x) - 280*I*exp(7*x) + 280*exp(6*x) - 168*I*exp(5*x) - 28*exp(4*
x) - 136*I*exp(3*x) + 264*exp(2*x) + 72*I*exp(x) - 18)/(105*exp(10*x) + 420
*I*exp(9*x) - 315*exp(8*x) + 840*I*exp(7*x) - 1470*exp(6*x) - 1470*exp(4*x)
 - 840*I*exp(3*x) - 315*exp(2*x) - 420*I*exp(x) + 105)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(31) = 62$.

Time = 0.21 (sec) , antiderivative size = 573, normalized size of antiderivative = 12.19

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 72*I*e^(-x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) -
 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-1
0*x) + 105) - 264*e^(-2*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) -
1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9
*x) + 105*e^(-10*x) + 105) - 136*I*e^(-3*x)/(420*I*e^(-x) - 315*e^(-2*x) +
840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8
*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 28*e^(-4*x)/(420*I*e^(-x) - 3
15*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*
x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) - 168*I*e^(-5*x)/
(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*
x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
- 280*e^(-6*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x
) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*e^(-9*x) + 105*e^
(-10*x) + 105) - 280*I*e^(-7*x)/(420*I*e^(-x) - 315*e^(-2*x) + 840*I*e^(-3*
x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^(-8*x) - 420*I*
e^(-9*x) + 105*e^(-10*x) + 105) + 210*e^(-8*x)/(420*I*e^(-x) - 315*e^(-2*x)
 + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 315*e^
(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105) + 18/(420*I*e^(-x) - 315*e^(
-2*x) + 840*I*e^(-3*x) - 1470*e^(-4*x) - 1470*e^(-6*x) - 840*I*e^(-7*x) - 3
15*e^(-8*x) - 420*I*e^(-9*x) + 105*e^(-10*x) + 105)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(31) = 62$.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{-6i\,e^{(2\,x)} - 9\,e^{x} + 5i}{24\,(e^{x} - i)^{3}} - \frac{210i\,e^{(6\,x)} - 105\,e^{(5\,x)} + 175i\,e^{(4\,x)} - 910\,e^{(3\,x)} - 756i\,e^{(2\,x)} + 427\,e^{x} + 31i}{840\,(e^{x} + i)^{7}}$$

```
[In] integrate(tanh(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/24*(-6*I*e^(2*x) - 9*e^x + 5*I)/(e^x - I)^3 - 1/840*(210*I*e^(6*x) - 105
*e^(5*x) + 175*I*e^(4*x) - 910*e^(3*x) - 756*I*e^(2*x) + 427*e^x + 31*I)/(e
^x + I)^7
```

Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 395, normalized size of antiderivative = 8.40

$$
\begin{aligned}
\int \frac{\tanh^4(x)}{(i+\sinh(x))^2}\,dx = & -\frac{\frac{25\,\mathrm{e}^{4\,x}}{168}-\frac{\mathrm{e}^{2\,x}}{4}+\frac{5}{168}+\frac{\mathrm{e}^{3\,x}\,5\mathrm{i}}{42}+\frac{\mathrm{e}^{5\,x}\,1\mathrm{i}}{28}-\frac{\mathrm{e}^{x}\,5\mathrm{i}}{84}}{15\,\mathrm{e}^{2\,x}-15\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1-\mathrm{e}^{3\,x}\,20\mathrm{i}+\mathrm{e}^{5\,x}\,6\mathrm{i}+\mathrm{e}^{x}\,6\mathrm{i}} \\
& +\frac{1\mathrm{i}}{12\,\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}-\mathrm{i}\right)}-\frac{\frac{5}{168}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{28}}{\mathrm{e}^{2\,x}-1+\mathrm{e}^{x}\,2\mathrm{i}} \\
& -\frac{\frac{5\,\mathrm{e}^{5\,x}}{28}-\frac{\mathrm{e}^{3\,x}}{2}+\frac{\mathrm{e}^{4\,x}\,5\mathrm{i}}{28}-\frac{\mathrm{e}^{2\,x}\,5\mathrm{i}}{28}+\frac{\mathrm{e}^{6\,x}\,1\mathrm{i}}{28}+\frac{5\,\mathrm{e}^{x}}{28}-\frac{1}{28}\mathrm{i}}{\mathrm{e}^{2\,x}\,21\mathrm{i}+35\,\mathrm{e}^{3\,x}-\mathrm{e}^{4\,x}\,35\mathrm{i}-21\,\mathrm{e}^{5\,x}+\mathrm{e}^{6\,x}\,7\mathrm{i}+\mathrm{e}^{7\,x}-7\,\mathrm{e}^{x}-\mathrm{i}} \\
& -\frac{\frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{28}+\frac{5\,\mathrm{e}^{x}}{84}+\frac{1}{84}\mathrm{i}}{\mathrm{e}^{2\,x}\,3\mathrm{i}+\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}-\mathrm{i}}+\frac{1}{8\,\left(1-\mathrm{e}^{2\,x}+\mathrm{e}^{x}\,2\mathrm{i}\right)}+\frac{1\mathrm{i}}{4\,\left(\mathrm{e}^{x}-\mathrm{i}\right)} \\
& -\frac{1\mathrm{i}}{28\,\left(\mathrm{e}^{x}+1\mathrm{i}\right)}-\frac{\frac{5\,\mathrm{e}^{2\,x}}{56}-\frac{1}{40}+\frac{\mathrm{e}^{3\,x}\,1\mathrm{i}}{28}+\frac{\mathrm{e}^{x}\,1\mathrm{i}}{28}}{\mathrm{e}^{4\,x}-6\,\mathrm{e}^{2\,x}+1+\mathrm{e}^{3\,x}\,4\mathrm{i}-\mathrm{e}^{x}\,4\mathrm{i}} \\
& -\frac{\frac{\mathrm{e}^{2\,x}\,1\mathrm{i}}{14}+\frac{5\,\mathrm{e}^{3\,x}}{42}+\frac{\mathrm{e}^{4\,x}\,1\mathrm{i}}{28}-\frac{\mathrm{e}^{x}}{10}-\frac{1}{84}\mathrm{i}}{\mathrm{e}^{5\,x}-10\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}\,5\mathrm{i}-\mathrm{e}^{2\,x}\,10\mathrm{i}+5\,\mathrm{e}^{x}+1\mathrm{i}}
\end{aligned}
$$

```
[In] int(tanh(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] 1i/(12*(exp(2*x)*3i - exp(3*x) + 3*exp(x) - 1i)) - ((exp(3*x)*5i)/42 - exp(
2*x)/4 + (25*exp(4*x))/168 + (exp(5*x)*1i)/28 - (exp(x)*5i)/84 + 5/168)/(15
*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(6*x) + exp(x)*6i
 - 1) - ((exp(x)*1i)/28 + 5/168)/(exp(2*x) + exp(x)*2i - 1) - ((exp(4*x)*5i
)/28 - exp(3*x)/2 - (exp(2*x)*5i)/28 + (5*exp(5*x))/28 + (exp(6*x)*1i)/28 +
 (5*exp(x))/28 - 1i/28)/(exp(2*x)*21i + 35*exp(3*x) - exp(4*x)*35i - 21*exp
(5*x) + exp(6*x)*7i + exp(7*x) - 7*exp(x) - 1i) - ((exp(2*x)*1i)/28 + (5*ex
p(x))/84 + 1i/84)/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i) + 1/(8*(exp(x)*2
i - exp(2*x) + 1)) + 1i/(4*(exp(x) - 1i)) - 1i/(28*(exp(x) + 1i)) - ((5*exp
(2*x))/56 + (exp(3*x)*1i)/28 + (exp(x)*1i)/28 - 1/40)/(exp(3*x)*4i - 6*exp(
2*x) + exp(4*x) - exp(x)*4i + 1) - ((exp(2*x)*1i)/14 + (5*exp(3*x))/42 + (e
xp(4*x)*1i)/28 - exp(x)/10 - 1i/84)/(exp(4*x)*5i - 10*exp(3*x) - exp(2*x)*1
0i + exp(5*x) + 5*exp(x) + 1i)
```

3.219 $\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} dx = -\frac{1}{8} i \arctan(\sinh(x)) - \frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))}$$

```
[Out] -1/8*I*arctan(sinh(x))-1/16*I/(I-sinh(x))+1/12*I/(I+sinh(x))^3-1/4/(I+sinh(
x))^2-3/16*I/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2786, 90, 209}

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2} dx = -\frac{1}{8} i \arctan(\sinh(x)) - \frac{i}{16(-\sinh(x)+i)} - \frac{3i}{16(\sinh(x)+i)} - \frac{1}{4(\sinh(x)+i)^2} + \frac{i}{12(\sinh(x)+i)^3}$$

```
[In] Int[Tanh[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (-1/8*I)*ArcTan[Sinh[x]] - (I/16)/(I - Sinh[x]) + (I/12)/(I + Sinh[x])^3 -
1/(4*(I + Sinh[x])^2) - ((3*I)/16)/(I + Sinh[x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^3}{(i-x)^2(i+x)^4}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{i}{16(-i+x)^2} - \frac{i}{4(i+x)^4} + \frac{1}{2(i+x)^3} + \frac{3i}{16(i+x)^2} - \frac{i}{8(1+x^2)}\right) dx, x, \sinh(x)\right)\\
&= -\frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))} - \frac{1}{8}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{8}i\arctan(\sinh(x)) - \frac{i}{16(i-\sinh(x))} + \frac{i}{12(i+\sinh(x))^3} - \frac{1}{4(i+\sinh(x))^2} - \frac{3i}{16(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{48}i\left(-6\arctan(\sinh(x)) - \frac{2\left(2i+7\sinh(x)-6i\sinh^2(x)+3\sinh^3(x)\right)}{(-i+\sinh(x))(i+\sinh(x))^3}\right)$$

[In] Integrate[Tanh[x]^3/(I + Sinh[x])^2,x]

[Out] (I/48)*(-6*ArcTan[Sinh[x]] - (2*(2*I + 7*Sinh[x] - (6*I)*Sinh[x]^2 + 3*Sinh[x]^3))/((-I + Sinh[x])*(I + Sinh[x])^3))

Maple [A] (verified)

Time = 20.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{i e^{x} \left(-12 i e^{5 x}+3 e^{6 x}+40 i e^{3 x}+19 e^{4 x}-12 i e^{x}-19 e^{2 x}-3\right)}{12 \left(e^{x}-i\right)^{2} \left(e^{x}+i\right)^{6}}+\frac{\ln \left(e^{x}+i\right)}{8}-\frac{\ln \left(e^{x}-i\right)}{8}$
default	$-\frac{i}{8\left(-i+\tanh \left(\frac{x}{2}\right)\right)}+\frac{1}{8\left(-i+\tanh \left(\frac{x}{2}\right)\right)^{2}}-\frac{\ln \left(-i+\tanh \left(\frac{x}{2}\right)\right)}{8}+\frac{2 i}{\left(\tanh \left(\frac{x}{2}\right)+i\right)^{5}}-\frac{2 i}{3\left(\tanh \left(\frac{x}{2}\right)+i\right)^{3}}-\frac{i}{8\left(\tanh \left(\frac{x}{2}\right)+i\right)}+\frac{}{3(\tan}$

[In] int(tanh(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] -1/12*I*exp(x)*(-12*I*exp(x)^5+3*exp(x)^6+40*I*exp(x)^3+19*exp(x)^4-12*I*exp(x)-19*exp(x)^2-3)/(exp(x)-I)^2/(exp(x)+I)^6+1/8*ln(exp(x)+I)-1/8*ln(exp(x)-I)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.98

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{3\left(e^{(8x)}+4i\,e^{(7x)}-4\,e^{(6x)}+4i\,e^{(5x)}-10\,e^{(4x)}-4i\,e^{(3x)}-4\,e^{(2x)}-4i\,e^{x}+1\right)\log\left(e^{x}+i\right)-3\left(e^{(8x)}+4i\right.}{24\left(e^{(8x)}+4i\,e^{(7x)}\right.}$$

[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="fricas")

[Out] 1/24*(3*(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - 3*(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)*log(e^x - I) - 6*I*e^(7*x) - 24*e^(6*x) - 38*I*e^(5*x) + 80*e^(4*x) + 38*I*e^(3*x) - 24*e^(2*x) + 6*I*e^x)/(e^(8*x) + 4*I*e^(7*x) - 4*e^(6*x) + 4*I*e^(5*x) - 10*e^(4*x) - 4*I*e^(3*x) - 4*e^(2*x) - 4*I*e^x + 1)

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.95

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{-3ie^{7x}-12e^{6x}-19ie^{5x}+40e^{4x}+19ie^{3x}-12e^{2x}+3ie^{x}}{12e^{8x}+48ie^{7x}-48e^{6x}+48ie^{5x}-120e^{4x}-48ie^{3x}-48e^{2x}-48ie^{x}+12} - \frac{\log(e^x-i)}{8}+\frac{\log(e^x+i)}{8}$$

```
[In] integrate(tanh(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (-3*I*exp(7*x) - 12*exp(6*x) - 19*I*exp(5*x) + 40*exp(4*x) + 19*I*exp(3*x)
- 12*exp(2*x) + 3*I*exp(x))/(12*exp(8*x) + 48*I*exp(7*x) - 48*exp(6*x) + 48
*I*exp(5*x) - 120*exp(4*x) - 48*I*exp(3*x) - 48*exp(2*x) - 48*I*exp(x) + 12
) - log(exp(x) - I)/8 + log(exp(x) + I)/8
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(38) = 76.

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.74

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{-3i\,e^{(-x)}-12\,e^{(-2x)}-19i\,e^{(-3x)}+40\,e^{(-4x)}+19i\,e^{(-5x)}-12\,e^{(-6x)}+3i\,e^{(-7x)}}{48i\,e^{(-x)}-48\,e^{(-2x)}+48i\,e^{(-3x)}-120\,e^{(-4x)}-48i\,e^{(-5x)}-48\,e^{(-6x)}-48i\,e^{(-7x)}+12\,e^{(-8x)}+12} - \frac{1}{8}\log\left(e^{(-x)}+i\right)+\frac{1}{8}\log\left(e^{(-x)}-i\right)$$

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (-3*I*e^(-x) - 12*e^(-2*x) - 19*I*e^(-3*x) + 40*e^(-4*x) + 19*I*e^(-5*x) -
12*e^(-6*x) + 3*I*e^(-7*x))/(48*I*e^(-x) - 48*e^(-2*x) + 48*I*e^(-3*x) - 12
0*e^(-4*x) - 48*I*e^(-5*x) - 48*e^(-6*x) - 48*I*e^(-7*x) + 12*e^(-8*x) + 12
) - 1/8*log(e^(-x) + I) + 1/8*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(38) = 76.

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.55

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{e^{(-x)}-e^x}{16\left(e^{(-x)}-e^x+2i\right)} - \frac{11\left(e^{(-x)}-e^x\right)^3-102i\left(e^{(-x)}-e^x\right)^2-180\,e^{(-x)}+180\,e^x+104i}{96\left(e^{(-x)}-e^x-2i\right)^3} + \frac{1}{16}\log\left(-e^{(-x)}+e^x+2i\right) - \frac{1}{16}\log\left(-e^{(-x)}+e^x-2i\right)$$

```
[In] integrate(tanh(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/16*(e^(-x) - e^x)/(e^(-x) - e^x + 2*I) - 1/96*(11*(e^(-x) - e^x)^3 - 102*
I*(e^(-x) - e^x)^2 - 180*e^(-x) + 180*e^x + 104*I)/(e^(-x) - e^x - 2*I)^3 +
 1/16*log(-e^(-x) + e^x + 2*I) - 1/16*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.17

$$\int \frac{\tanh^3(x)}{(i+\sinh(x))^2}\,dx = \frac{\ln\left(-\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8} - \frac{\ln\left(\frac{1}{4}+\frac{e^x\,1i}{4}\right)}{8} - \frac{2i}{e^{5x}-10\,e^{3x}+e^{4x}\,5i-e^{2x}\,10i+5\,e^x+1i} - \frac{11}{8\,\left(e^{2x}-1+e^x\,2i\right)} + \frac{3}{e^{4x}-6\,e^{2x}+1+e^{3x}\,4i-e^x\,4i} + \frac{1}{8\,\left(1-e^{2x}+e^x\,2i\right)} + \frac{1i}{8\,\left(e^x-i\right)} - \frac{3i}{8\,\left(e^x+1i\right)} - \frac{2}{3\,\left(15\,e^{2x}-15\,e^{4x}+e^{6x}-1-e^{3x}\,20i+e^{5x}\,6i+e^x\,6i\right)} + \frac{8i}{3\,\left(e^{2x}\,3i+e^{3x}-3\,e^x-i\right)}$$

```
[In] int(tanh(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] log((exp(x)*1i)/4 - 1/4)/8 - log((exp(x)*1i)/4 + 1/4)/8 - 2i/(exp(4*x)*5i -
 10*exp(3*x) - exp(2*x)*10i + exp(5*x) + 5*exp(x) + 1i) - 11/(8*(exp(2*x) +
 exp(x)*2i - 1)) + 3/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1)
+ 1/(8*(exp(x)*2i - exp(2*x) + 1)) + 1i/(8*(exp(x) - 1i)) - 3i/(8*(exp(x) +
 1i)) - 2/(3*(15*exp(2*x) - exp(3*x)*20i - 15*exp(4*x) + exp(5*x)*6i + exp(
6*x) + exp(x)*6i - 1)) + 8i/(3*(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i))
```

3.220 $\int \frac{\tanh^2(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2} dx = \frac{2}{3}i\text{sech}^3(x) - \frac{2}{5}i\text{sech}^5(x) - \frac{\tanh^3(x)}{3} + \frac{2\tanh^5(x)}{5}$$

```
[Out] 2/3*I*sech(x)^3-2/5*I*sech(x)^5-1/3*tanh(x)^3+2/5*tanh(x)^5
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2790, 2687, 14, 2686, 30}

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2} dx = \frac{2\tanh^5(x)}{5} - \frac{\tanh^3(x)}{3} - \frac{2}{5}i\text{sech}^5(x) + \frac{2}{3}i\text{sech}^3(x)$$

```
[In] Int[Tanh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] ((2*I)/3)*Sech[x]^3 - ((2*I)/5)*Sech[x]^5 - Tanh[x]^3/3 + (2*Tanh[x]^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2790

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \left(\operatorname{sech}^4(x)\tanh^2(x)+2i\operatorname{sech}^3(x)\tanh^3(x)-\operatorname{sech}^2(x)\tanh^4(x)\right)\,dx\\
&= -\left(2i\int \operatorname{sech}^3(x)\tanh^3(x)\,dx\right)-\int \operatorname{sech}^4(x)\tanh^2(x)\,dx+\int \operatorname{sech}^2(x)\tanh^4(x)\,dx\\
&= -\left(i\text{Subst}\left(\int x^4\,dx,x,i\tanh(x)\right)\right)-i\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\tanh(x)\right)\\
&\quad -2i\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\operatorname{sech}(x)\right)\\
&= \frac{\tanh^5(x)}{5}-i\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\tanh(x)\right)-2i\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\operatorname{sech}(x)\right)\\
&= \frac{2}{3}i\operatorname{sech}^3(x)-\frac{2}{5}i\operatorname{sech}^5(x)-\frac{\tanh^3(x)}{3}+\frac{2\tanh^5(x)}{5}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 84 vs. 2(37) = 74.

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.27

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{80i - 55i\cosh(x) - 16i\cosh(2x) + 11i\cosh(3x) + 140\sinh(x) - 44\sinh(2x) - 4\sinh(3x)}{240\left(\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)\right)^5\left(\cosh\left(\frac{x}{2}\right) + i\sinh\left(\frac{x}{2}\right)\right)}$$

```
[In] Integrate[Tanh[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (80*I - (55*I)*Cosh[x] - (16*I)*Cosh[2*x] + (11*I)*Cosh[3*x] + 140*Sinh[x]
- 44*Sinh[2*x] - 4*Sinh[3*x])/(240*(Cosh[x/2] - I*Sinh[x/2])^5*(Cosh[x/2] +
 I*Sinh[x/2]))
```

Maple [A] (verified)

Time = 13.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{2\left(-20\,{\mathrm e}^{2 x}-4 i {\mathrm e}^{x}+1+20 i {\mathrm e}^{3 x}+15\,{\mathrm e}^{4 x}\right)}{15\left({\mathrm e}^{x}-i\right)\left({\mathrm e}^{x}+i\right)^{5}}$	43
default	$\frac{1}{-4 i+4 \tanh\left(\frac{x}{2}\right)}+\frac{2 i}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^{4}}-\frac{i}{2\left(\tanh\left(\frac{x}{2}\right)+i\right)^{2}}+\frac{4}{5\left(\tanh\left(\frac{x}{2}\right)+i\right)^{5}}-\frac{5}{3\left(\tanh\left(\frac{x}{2}\right)+i\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+i\right)}$	70

```
[In] int(tanh(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(-20*exp(x)^2-4*I*exp(x)+1+20*I*exp(x)^3+15*exp(x)^4)/(exp(x)-I)/(exp
(x)+I)^5
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(15\,e^{(4\,x)} + 20i\,e^{(3\,x)} - 20\,e^{(2\,x)} - 4i\,e^{x} + 1\right)}{15\left(e^{(6\,x)} + 4i\,e^{(5\,x)} - 5\,e^{(4\,x)} - 5\,e^{(2\,x)} - 4i\,e^{x} + 1\right)}$$

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/15*(15*e^(4*x) + 20*I*e^(3*x) - 20*e^(2*x) - 4*I*e^x + 1)/(e^(6*x) + 4*I
*e^(5*x) - 5*e^(4*x) - 5*e^(2*x) - 4*I*e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.78

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{-30e^{4x} - 40ie^{3x} + 40e^{2x} + 8ie^{x} - 2}{15e^{6x} + 60ie^{5x} - 75e^{4x} - 75e^{2x} - 60ie^{x} + 15}$$

```
[In] integrate(tanh(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (-30*exp(4*x) - 40*I*exp(3*x) + 40*exp(2*x) + 8*I*exp(x) - 2)/(15*exp(6*x)
+ 60*I*exp(5*x) - 75*exp(4*x) - 75*exp(2*x) - 60*I*exp(x) + 15)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(25) = 50.

Time = 0.22 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.32

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{8i\,e^{(-x)}}{60i\,e^{(-x)} - 75\,e^{(-2\,x)} - 75\,e^{(-4\,x)} - 60i\,e^{(-5\,x)} + 15\,e^{(-6\,x)} + 15} - \frac{40\,e^{(-2\,x)}}{60i\,e^{(-x)} - 75\,e^{(-2\,x)} - 75\,e^{(-4\,x)} - 60i\,e^{(-5\,x)} + 15\,e^{(-6\,x)} + 15} - \frac{40i\,e^{(-3\,x)}}{60i\,e^{(-x)} - 75\,e^{(-2\,x)} - 75\,e^{(-4\,x)} - 60i\,e^{(-5\,x)} + 15\,e^{(-6\,x)} + 15} + \frac{30\,e^{(-4\,x)}}{60i\,e^{(-x)} - 75\,e^{(-2\,x)} - 75\,e^{(-4\,x)} - 60i\,e^{(-5\,x)} + 15\,e^{(-6\,x)} + 15} + \frac{2}{60i\,e^{(-x)} - 75\,e^{(-2\,x)} - 75\,e^{(-4\,x)} - 60i\,e^{(-5\,x)} + 15\,e^{(-6\,x)} + 15}$$

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 8*I*e^(-x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^
(-6*x) + 15) - 40*e^(-2*x)/(60*I*e^(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*
e^(-5*x) + 15*e^(-6*x) + 15) - 40*I*e^(-3*x)/(60*I*e^(-x) - 75*e^(-2*x) - 7
5*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 30*e^(-4*x)/(60*I*e^(-x) -
 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15) + 2/(60*I*e^
(-x) - 75*e^(-2*x) - 75*e^(-4*x) - 60*I*e^(-5*x) + 15*e^(-6*x) + 15)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{i}{4\,(e^x-i)} - \frac{15i\,e^{(4\,x)} + 30\,e^{(3\,x)} + 40i\,e^{(2\,x)} - 50\,e^x - 7i}{60\,(e^x+i)^5}$$

```
[In] integrate(tanh(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*I/(e^x - I) - 1/60*(15*I*e^(4*x) + 30*e^(3*x) + 40*I*e^(2*x) - 50*e^x -
 7*I)/(e^x + I)^5
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.76

$$\int \frac{\tanh^2(x)}{(i+\sinh(x))^2}\,dx = \frac{(4\,e^{3\,x}-4\,e^x)\left(2\,e^{4\,x}-\frac{8\,e^{2\,x}}{3}+\frac{2}{15}\right)1i}{(e^{2\,x}+1)^5} - \frac{(e^{4\,x}-6\,e^{2\,x}+1)\left(2\,e^{4\,x}-\frac{8\,e^{2\,x}}{3}+\frac{2}{15}\right)}{(e^{2\,x}+1)^5} - \frac{(4\,e^{3\,x}-4\,e^x)\left(\frac{8\,e^{3\,x}}{3}-\frac{8\,e^x}{15}\right)}{(e^{2\,x}+1)^5} - \frac{\left(\frac{8\,e^{3\,x}}{3}-\frac{8\,e^x}{15}\right)(e^{4\,x}-6\,e^{2\,x}+1)\,1i}{(e^{2\,x}+1)^5}$$

```
[In] int(tanh(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] ((4*exp(3*x) - 4*exp(x))*(2*exp(4*x) - (8*exp(2*x))/3 + 2/15)*1i)/(exp(2*x)
 + 1)^5 - ((exp(4*x) - 6*exp(2*x) + 1)*(2*exp(4*x) - (8*exp(2*x))/3 + 2/15)
)/(exp(2*x) + 1)^5 - ((4*exp(3*x) - 4*exp(x))*((8*exp(3*x))/3 - (8*exp(x))/
15))/(exp(2*x) + 1)^5 - (((8*exp(3*x))/3 - (8*exp(x))/15)*(exp(4*x) - 6*exp
(2*x) + 1)*1i)/(exp(2*x) + 1)^5
```

3.221 $\int \frac{\tanh(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2} dx = -\frac{1}{4}i\arctan(\sinh(x)) - \frac{1}{4(i+\sinh(x))^2} - \frac{i}{4(i+\sinh(x))}$$

```
[Out] -1/4*I*arctan(sinh(x))-1/4/(I+sinh(x))^2-1/4*I/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2786, 78, 209}

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2} dx = -\frac{1}{4}i\arctan(\sinh(x)) - \frac{i}{4(\sinh(x)+i)} - \frac{1}{4(\sinh(x)+i)^2}$$

```
[In] Int[Tanh[x]/(I + Sinh[x])^2,x]
```

```
[Out] (-1/4*I)*ArcTan[Sinh[x]] - 1/(4*(I + Sinh[x])^2) - (I/4)/(I + Sinh[x])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x}{(i-x)(i+x)^3}\,dx, x, \sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{2(i+x)^3}-\frac{i}{4(i+x)^2}+\frac{i}{4\,(1+x^2)}\right)dx, x, \sinh(x)\right)\\
&= -\frac{1}{4(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}-\frac{1}{4}i\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(x)\right)\\
&= -\frac{1}{4}i\arctan(\sinh(x))-\frac{1}{4(i+\sinh(x))^2}-\frac{i}{4(i+\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = -\frac{i(\sinh(x)+\arctan(\sinh(x))(i+\sinh(x))^2)}{4(i+\sinh(x))^2}$$

```
[In] Integrate[Tanh[x]/(I + Sinh[x])^2,x]
[Out] ((-1/4*I)*(Sinh[x] + ArcTan[Sinh[x]]*(I + Sinh[x])^2))/(I + Sinh[x])^2
```

Maple [A] (verified)

Time = 9.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{i(e^{2x}-1)e^{x}}{2(e^{x}+i)^{4}}+\frac{\ln(e^{x}+i)}{4}-\frac{\ln(e^{x}-i)}{4}$	36
default	$-\frac{\ln(-i+\tanh(\frac{x}{2}))}{4}+\frac{2i}{(\tanh(\frac{x}{2})+i)^{3}}-\frac{i}{2(\tanh(\frac{x}{2})+i)}+\frac{1}{(\tanh(\frac{x}{2})+i)^{4}}-\frac{3}{2(\tanh(\frac{x}{2})+i)^{2}}+\frac{\ln(\tanh(\frac{x}{2})+i)}{4}$	66

```
[In] int(tanh(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/2*I*(exp(2*x)-1)*exp(x)/(exp(x)+I)^4+1/4*ln(exp(x)+I)-1/4*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.61

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)\log\left(e^{x}+i\right)-\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)\log\left(e^{x}-i\right)-2}{4\left(e^{(4\,x)}+4i\,e^{(3\,x)}-6\,e^{(2\,x)}-4i\,e^{x}+1\right)}$$

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="fricas")
[Out] 1/4*((e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x + I) - (e^(4
*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)*log(e^x - I) - 2*I*e^(3*x) + 2
*I*e^x)/(e^(4*x) + 4*I*e^(3*x) - 6*e^(2*x) - 4*I*e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{-ie^{3x}+ie^{x}}{2e^{4x}+8ie^{3x}-12e^{2x}-8ie^{x}+2}-\frac{\log\left(e^{x}-i\right)}{4}+\frac{\log\left(e^{x}+i\right)}{4}$$

```
[In] integrate(tanh(x)/(I+sinh(x))**2,x)
[Out] (-I*exp(3*x) + I*exp(x))/(2*exp(4*x) + 8*I*exp(3*x) - 12*exp(2*x) - 8*I*exp
(x) + 2) - log(exp(x) - I)/4 + log(exp(x) + I)/4
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{-i\,e^{(-x)}+i\,e^{(-3\,x)}}{8i\,e^{(-x)}-12\,e^{(-2\,x)}-8i\,e^{(-3\,x)}+2\,e^{(-4\,x)}+2} - \frac{1}{4}\log\left(e^{(-x)}+i\right)+\frac{1}{4}\log\left(e^{(-x)}-i\right)$$

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="maxima")
[Out] (-I*e^(-x) + I*e^(-3*x))/(8*I*e^(-x) - 12*e^(-2*x) - 8*I*e^(-3*x) + 2*e^(-4
*x) + 2) - 1/4*log(e^(-x) + I) + 1/4*log(e^(-x) - I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(22) = 44.

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = -\frac{3\left(e^{(-x)}-e^x\right)^2-20i\,e^{(-x)}+20i\,e^x-12}{16\left(e^{(-x)}-e^x-2i\right)^2} + \frac{1}{8}\log\left(-e^{(-x)}+e^x+2i\right)-\frac{1}{8}\log\left(-e^{(-x)}+e^x-2i\right)$$

```
[In] integrate(tanh(x)/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/16*(3*(e^(-x) - e^x)^2 - 20*I*e^(-x) + 20*I*e^x - 12)/(e^(-x) - e^x - 2*
I)^2 + 1/8*log(-e^(-x) + e^x + 2*I) - 1/8*log(-e^(-x) + e^x - 2*I)
```

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.75

$$\int \frac{\tanh(x)}{(i+\sinh(x))^2}\,dx = \frac{\ln\left(-\frac{1}{2}+\frac{e^x\,1i}{2}\right)}{4}-\frac{\ln\left(\frac{1}{2}+\frac{e^x\,1i}{2}\right)}{4}-\frac{3}{2\,\left(e^{2\,x}-1+e^x\,2i\right)} + \frac{1}{e^{4\,x}-6\,e^{2\,x}+1+e^{3\,x}\,4i-e^x\,4i} - \frac{1i}{2\,\left(e^x+1i\right)}+\frac{2i}{e^{2\,x}\,3i+e^{3\,x}-3\,e^x-i}$$

```
[In] int(tanh(x)/(sinh(x) + 1i)^2,x)
```

```
[Out] log((exp(x)*1i)/2 - 1/2)/4 - log((exp(x)*1i)/2 + 1/2)/4 - 3/(2*(exp(2*x) +
exp(x)*2i - 1)) + 1/(exp(3*x)*4i - 6*exp(2*x) + exp(4*x) - exp(x)*4i + 1) -
 1i/(2*(exp(x) + 1i)) + 2i/(exp(2*x)*3i + exp(3*x) - 3*exp(x) - 1i)
```

3.222 $\int \frac{\coth(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{\coth(x)}{(i+\sinh(x))^2} \, dx = -\log(\sinh(x)) + \log(i+\sinh(x)) - \frac{i}{i+\sinh(x)}$$

```
[Out] -ln(sinh(x))+ln(I+sinh(x))-I/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2786, 46}

$$\int \frac{\coth(x)}{(i+\sinh(x))^2} \, dx = -\frac{i}{\sinh(x)+i} - \log(\sinh(x)) + \log(\sinh(x)+i)$$

```
[In] Int[Coth[x]/(I + Sinh[x])^2,x]
```

```
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
```

```
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{x(i+x)^2}\,dx, x, \sinh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{x}+\frac{i}{(i+x)^2}+\frac{1}{i+x}\right)\,dx, x, \sinh(x)\right)\\
&= -\log(\sinh(x))+\log(i+\sinh(x))-\frac{i}{i+\sinh(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = -\log(\sinh(x))+\log(i+\sinh(x))-\frac{i}{i+\sinh(x)}$$

```
[In] Integrate[Coth[x]/(I + Sinh[x])^2,x]
[Out] -Log[Sinh[x]] + Log[I + Sinh[x]] - I/(I + Sinh[x])
```

Maple [A] (verified)

Time = 8.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

method	result	size
risch	$-\frac{2i\,e^{x}}{(e^{x}+i)^2}-\ln\left(e^{2x}-1\right)+2\ln\left(e^{x}+i\right)$	31
default	$\frac{2i}{\tanh\left(\frac{x}{2}\right)+i}+\frac{2}{\left(\tanh\left(\frac{x}{2}\right)+i\right)^2}+2\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	42

```
[In] int(coth(x)/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2*I/(exp(x)+I)^2*exp(x)-ln(exp(2*x)-1)+2*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(19) = 38.

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = -\frac{\left(e^{(2\,x)}+2i\,e^{x}-1\right)\log\left(e^{(2\,x)}-1\right)-2\left(e^{(2\,x)}+2i\,e^{x}-1\right)\log\left(e^{x}+i\right)+2i\,e^{x}}{e^{(2\,x)}+2i\,e^{x}-1}$$

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -((e^(2*x) + 2*I*e^x - 1)*log(e^(2*x) - 1) - 2*(e^(2*x) + 2*I*e^x - 1)*log(
e^x + I) + 2*I*e^x)/(e^(2*x) + 2*I*e^x - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = 2\log\left(e^{x}+i\right)-\log\left(e^{2x}-1\right)-\frac{2ie^{x}}{e^{2x}+2ie^{x}-1}$$

```
[In] integrate(coth(x)/(I+sinh(x))**2,x)
```

```
[Out] 2*log(exp(x) + I) - log(exp(2*x) - 1) - 2*I*exp(x)/(exp(2*x) + 2*I*exp(x) -
 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = \frac{2i\,e^{(-x)}}{-2i\,e^{(-x)}+e^{(-2\,x)}-1}-\log\left(e^{(-x)}+1\right) + 2\,\log\left(e^{(-x)}-i\right)-\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*I*e^(-x)/(-2*I*e^(-x) + e^(-2*x) - 1) - log(e^(-x) + 1) + 2*log(e^(-x) -
I) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = -\frac{2i\,e^x}{(e^x+i)^2} - \log\left(e^x+1\right) + 2\,\log\left(e^x+i\right) - \log\left(|e^x-1|\right)$$

```
[In] integrate(coth(x)/(I+sinh(x))^2,x, algorithm="giac")
[Out] -2*I*e^x/(e^x + I)^2 - log(e^x + 1) + 2*log(e^x + I) - log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{\coth(x)}{(i+\sinh(x))^2}\,dx = 2\,\ln\left(36\,\mathrm{e}^x+36\mathrm{i}\right) - \ln\left(\mathrm{e}^{2\,x}\,3\mathrm{i}-3\mathrm{i}\right) - \frac{2}{\mathrm{e}^{2\,x}-1+\mathrm{e}^x\,2\mathrm{i}} - \frac{2\mathrm{i}}{\mathrm{e}^x+1\mathrm{i}}$$

```
[In] int(coth(x)/(sinh(x) + 1i)^2,x)
[Out] 2*log(36*exp(x) + 36i) - log(exp(2*x)*3i - 3i) - 2/(exp(2*x) + exp(x)*2i -
1) - 2i/(exp(x) + 1i)
```

3.223 $\int \frac{\coth^2(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2} dx = 2i\text{arctanh}(\cosh(x)) + \coth(x) + \frac{2i\coth(x)}{i-\text{csch}(x)}$$

```
[Out] 2*I*arctanh(cosh(x))+3*coth(x)-2*I*coth(x)/(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2788, 3855, 3852, 8, 3862}

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2} dx = 2i\text{arctanh}(\cosh(x)) + \coth(x) + \frac{2i\coth(x)}{-\text{csch}(x)+i}$$

```
[In] Int[Coth[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*ArcTanh[Cosh[x]] + Coth[x] + ((2*I)*Coth[x])/(I - Csch[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
```

```
p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(2 - 2i\text{csch}(x) - \text{csch}^2(x) + \frac{2i}{-i+\text{csch}(x)}\right) dx \\
&= 2x - 2i\int \text{csch}(x)\,dx + 2i\int \frac{1}{-i+\text{csch}(x)}\,dx - \int \text{csch}^2(x)\,dx \\
&= 2x + 2i\text{arctanh}(\cosh(x)) + \frac{2i\coth(x)}{i-\text{csch}(x)} + i\text{Subst}\left(\int 1\,dx, x, -i\coth(x)\right) + 2i\int i\,dx \\
&= 2i\text{arctanh}(\cosh(x)) + \coth(x) + \frac{2i\coth(x)}{i-\text{csch}(x)}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \frac{1}{2}\left(\coth\left(\frac{x}{2}\right) + 4i\log\left(\cosh\left(\frac{x}{2}\right)\right) - 4i\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{8\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right) - i\sinh\left(\frac{x}{2}\right)} + \tanh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Coth[x]^2/(I + Sinh[x])^2,x]
```

```
[Out] (Coth[x/2] + (4*I)*Log[Cosh[x/2]] - (4*I)*Log[Sinh[x/2]] + (8*Sinh[x/2])/(C
osh[x/2] - I*Sinh[x/2]) + Tanh[x/2])/2
```

Maple [A] (verified)

Time = 14.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2}+\frac{4}{\tanh\left(\frac{x}{2}\right)+i}-2i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2\tanh\left(\frac{x}{2}\right)}$	35
risch	$-\frac{2i\left(i\,\mathrm{e}^{x}+2\,\mathrm{e}^{2x}-3\right)}{\left(\mathrm{e}^{2x}-1\right)\left(\mathrm{e}^{x}+i\right)}-2i\ln\left(\mathrm{e}^{x}-1\right)+2i\ln\left(\mathrm{e}^{x}+1\right)$	49

```
[In] int(coth(x)^2/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*tanh(1/2*x)+4/(tanh(1/2*x)+I)-2*I*ln(tanh(1/2*x))+1/2/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(20) = 40.

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(\left(-i\,e^{(3\,x)}+e^{(2\,x)}+i\,e^{x}-1\right)\log\left(e^{x}+1\right)+\left(i\,e^{(3\,x)}-e^{(2\,x)}-i\,e^{x}+1\right)\log\left(e^{x}-1\right)+2i\,e^{(2\,x)}-e^{x}-3\right.}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i}$$

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2*((-I*e^(3*x) + e^(2*x) + I*e^x - 1)*log(e^x + 1) + (I*e^(3*x) - e^(2*x)
- I*e^x + 1)*log(e^x - 1) + 2*I*e^(2*x) - e^x - 3*I)/(e^(3*x) + I*e^(2*x) -
 e^x - I)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(24) = 48.

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \frac{-4ie^{2x}+2e^{x}+6i}{e^{3x}+ie^{2x}-e^{x}-i}+2\,\mathrm{RootSum}\left(z^2+1,\left(i\mapsto i\log\left(-ii+e^{x}\right)\right)\right)$$

```
[In] integrate(coth(x)**2/(I+sinh(x))**2,x)
```

```
[Out] (-4*I*exp(2*x) + 2*exp(x) + 6*I)/(exp(3*x) + I*exp(2*x) - exp(x) - I) + 2*R
ootSum(_z**2 + 1, Lambda(_i, _i*log(-_i*I + exp(x))))
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(20) = 40.

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = \frac{2\left(e^{(-x)}+2i\,e^{(-2\,x)}-3i\right)}{e^{(-x)}+i\,e^{(-2\,x)}-e^{(-3\,x)}-i}+2i\,\log\left(e^{(-x)}+1\right)-2i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + 2*I*e^(-2*x) - 3*I)/(e^(-x) + I*e^(-2*x) - e^(-3*x) - I) + 2*I*
log(e^(-x) + 1) - 2*I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(2i\,e^{(2\,x)}-e^{x}-3i\right)}{e^{(3\,x)}+i\,e^{(2\,x)}-e^{x}-i}+2i\,\log\left(e^{x}+1\right)-2i\,\log\left(|e^{x}-1|\right)$$

```
[In] integrate(coth(x)^2/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(2*I*e^(2*x) - e^x - 3*I)/(e^(3*x) + I*e^(2*x) - e^x - I) + 2*I*log(e^x
+ 1) - 2*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{\coth^2(x)}{(i+\sinh(x))^2}\,dx = -\ln\left(\mathrm{e}^{x}\,4\mathrm{i}-4\mathrm{i}\right)\,2\mathrm{i}+\ln\left(\mathrm{e}^{x}\,4\mathrm{i}+4\mathrm{i}\right)\,2\mathrm{i}+\frac{2\,\mathrm{e}^{x}-\mathrm{e}^{2\,x}\,4\mathrm{i}+6\mathrm{i}}{\mathrm{e}^{2\,x}\,1\mathrm{i}+\mathrm{e}^{3\,x}-\mathrm{e}^{x}-\mathrm{i}}$$

```
[In] int(coth(x)^2/(sinh(x) + 1i)^2,x)
```

```
[Out] log(exp(x)*4i + 4i)*2i - log(exp(x)*4i - 4i)*2i + (2*exp(x) - exp(2*x)*4i +
 6i)/(exp(2*x)*1i + exp(3*x) - exp(x) - 1i)
```

3.224 $\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx = 2i\text{csch}(x) + \frac{\text{csch}^2(x)}{2} + 2\log(\sinh(x)) - 2\log(i+\sinh(x))$$

```
[Out] 2*I*csch(x)+1/2*csch(x)^2+2*ln(sinh(x))-2*ln(I+sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2786, 78}

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2} dx = \frac{\text{csch}^2(x)}{2} + 2i\text{csch}(x) + 2\log(\sinh(x)) - 2\log(\sinh(x)+i)$$

```
[In] Int[Coth[x]^3/(I + Sinh[x])^2,x]
```

```
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{i-x}{x^3(i+x)}\,dx, x, \sinh(x)\right)\\ &= -\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{2i}{x^2}-\frac{2}{x}+\frac{2}{i+x}\right)dx, x, \sinh(x)\right)\\ &= 2i\text{csch}(x)+\frac{\text{csch}^2(x)}{2}+2\log(\sinh(x))-2\log(i+\sinh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = 2i\text{csch}(x)+\frac{\text{csch}^2(x)}{2}+2\log(\sinh(x))-2\log(i+\sinh(x))$$

```
[In] Integrate[Coth[x]^3/(I + Sinh[x])^2,x]
[Out] (2*I)*Csch[x] + Csch[x]^2/2 + 2*Log[Sinh[x]] - 2*Log[I + Sinh[x]]
```

Maple [A] (verified)

Time = 21.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

method	result	size
risch	$\frac{2i e^{x}\left(2\,e^{2x}-2-i e^{x}\right)}{\left(e^{2x}-1\right)^{2}}+2\ln\left(e^{2x}-1\right)-4\ln\left(e^{x}+i\right)$	45
default	$-i\tanh\left(\frac{x}{2}\right)+\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8}-4\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+\frac{i}{\tanh\left(\frac{x}{2}\right)}+\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2}}+2\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	51

```
[In] int(coth(x)^3/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] 2*I*exp(x)*(2*exp(2*x)-2-I*exp(x))/(exp(2*x)-1)^2+2*ln(exp(2*x)-1)-4*ln(exp
(x)+I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(23) = 46.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.41

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = \frac{2\left(\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^{(2x)}-1\right)-2\left(e^{(4x)}-2e^{(2x)}+1\right)\log\left(e^{x}+i\right)+2ie^{(3x)}+e^{(2x)}-2ie^{x}\right)}{e^{(4x)}-2e^{(2x)}+1}$$

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] 2*((e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) - 2*(e^(4*x) - 2*e^(2*x) + 1)
*log(e^x + I) + 2*I*e^(3*x) + e^(2*x) - 2*I*e^x)/(e^(4*x) - 2*e^(2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = \frac{4ie^{3x}+2e^{2x}-4ie^{x}}{e^{4x}-2e^{2x}+1}-4\log\left(e^{x}+i\right)+2\log\left(e^{2x}-1\right)$$

```
[In] integrate(coth(x)**3/(I+sinh(x))**2,x)
```

```
[Out] (4*I*exp(3*x) + 2*exp(2*x) - 4*I*exp(x))/(exp(4*x) - 2*exp(2*x) + 1) - 4*lo
g(exp(x) + I) + 2*log(exp(2*x) - 1)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(23) = 46.

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.17

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(2ie^{(-x)}+e^{(-2x)}-2ie^{(-3x)}\right)}{2e^{(-2x)}-e^{(-4x)}-1}+2\log\left(e^{(-x)}+1\right)-4\log\left(e^{(-x)}-i\right)+2\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(2*I*e^(-x) + e^(-2*x) - 2*I*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 2*l
og(e^(-x) + 1) - 4*log(e^(-x) - I) + 2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.86

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(-2i\,e^{(3x)} - e^{(2x)} + 2i\,e^{x}\right)}{\left(e^{x}+1\right)^2\left(e^{x}-1\right)^2} + 2\,\log\left(e^{x}+1\right) - 4\,\log\left(e^{x}+i\right) + 2\,\log\left(\left|e^{x}-1\right|\right)$$

```
[In] integrate(coth(x)^3/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2*(-2*I*e^(3*x) - e^(2*x) + 2*I*e^x)/((e^x + 1)^2*(e^x - 1)^2) + 2*log(e^x
 + 1) - 4*log(e^x + I) + 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.93

$$\int \frac{\coth^3(x)}{(i+\sinh(x))^2}\,dx = \frac{2}{e^{4x} - 2\,e^{2x} + 1} + 2\,\ln\left(-e^{2x}\,6i + 6i\right) - 4\,\ln\left(144\,e^{x} + 144i\right) + \frac{2 + e^{x}\,4i}{e^{2x} - 1}$$

```
[In] int(coth(x)^3/(sinh(x) + 1i)^2,x)
```

```
[Out] 2*log(6i - exp(2*x)*6i) - 4*log(144*exp(x) + 144i) + 2/(exp(4*x) - 2*exp(2*
x) + 1) + (exp(x)*4i + 2)/(exp(2*x) - 1)
```

3.225 $\int \frac{\coth^4(x)}{(i+\sinh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2} \, dx = -i\text{arctanh}(\cosh(x)) - 2\coth(x) + \frac{\coth^3(x)}{3} + i\coth(x)\text{csch}(x)$$

```
[Out] -I*arctanh(cosh(x))-2*coth(x)+1/3*coth(x)^3+I*coth(x)*csch(x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2787, 2836, 3852, 8, 3853, 3855}

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2} \, dx = -i\text{arctanh}(\cosh(x)) + \frac{\coth^3(x)}{3} - 2\coth(x) + i\coth(x)\text{csch}(x)$$

```
[In] Int[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-I)*ArcTanh[Cosh[x]] - 2*Coth[x] + Coth[x]^3/3 + I*Coth[x]*Csch[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2787

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Dist[a^p, Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x], x
] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[
p, 2*m]
```

Rule 2836

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \operatorname{csch}^4(x)(i-\sinh(x))^2\,dx \\
&= \int \left(\operatorname{csch}^2(x)-2i\operatorname{csch}^3(x)-\operatorname{csch}^4(x)\right)\,dx \\
&= -\left(2i\int \operatorname{csch}^3(x)\,dx\right)+\int \operatorname{csch}^2(x)\,dx-\int \operatorname{csch}^4(x)\,dx \\
&= i\coth(x)\operatorname{csch}(x)+i\int \operatorname{csch}(x)\,dx-i\text{Subst}\left(\int 1\,dx,x,-i\coth(x)\right) \\
&\quad -i\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\coth(x)\right) \\
&= -i\text{arctanh}(\cosh(x))-2\coth(x)+\frac{\coth^3(x)}{3}+i\coth(x)\operatorname{csch}(x)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 107 vs. 2(28) = 56.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.82

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{5}{6}\coth\left(\frac{x}{2}\right)+\frac{1}{4}i\text{csch}^2\left(\frac{x}{2}\right)+\frac{1}{24}\coth\left(\frac{x}{2}\right)\text{csch}^2\left(\frac{x}{2}\right) - i\log\left(\cosh\left(\frac{x}{2}\right)\right)+i\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{1}{4}i\text{sech}^2\left(\frac{x}{2}\right) - \frac{5}{6}\tanh\left(\frac{x}{2}\right)-\frac{1}{24}\text{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

```
[In] Integrate[Coth[x]^4/(I + Sinh[x])^2,x]
```

```
[Out] (-5*Coth[x/2])/6 + (I/4)*Csch[x/2]^2 + (Coth[x/2]*Csch[x/2]^2)/24 - I*Log[C
osh[x/2]] + I*Log[Sinh[x/2]] + (I/4)*Sech[x/2]^2 - (5*Tanh[x/2])/6 - (Sech[
x/2]^2*Tanh[x/2])/24
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(24) = 48.

Time = 31.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

method	result	size
risch	$\frac{2i\left(3i\,\mathrm{e}^{4x}+3\,\mathrm{e}^{5x}-12i\,\mathrm{e}^{2x}+5i-3\,\mathrm{e}^{x}\right)}{3\left(\mathrm{e}^{2x}-1\right)^{3}}+i\ln\left(\mathrm{e}^{x}-1\right)-i\ln\left(\mathrm{e}^{x}+1\right)$	56
default	$-\frac{7\tanh\left(\frac{x}{2}\right)}{8}+\frac{\tanh\left(\frac{x}{2}\right)^{3}}{24}-\frac{i\tanh\left(\frac{x}{2}\right)^{2}}{4}-\frac{7}{8\tanh\left(\frac{x}{2}\right)}+\frac{i}{4\tanh\left(\frac{x}{2}\right)^{2}}+\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}+i\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	58

```
[In] int(coth(x)^4/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*(3*I*exp(x)^4+3*exp(x)^5-12*I*exp(x)^2+5*I-3*exp(x))/(exp(x)^2-1)^3+I
*ln(exp(x)-1)-I*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(22) = 44.

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.57

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{3\left(i\,e^{(6\,x)}-3i\,e^{(4\,x)}+3i\,e^{(2\,x)}-i\right)\log\left(e^x+1\right)+3\left(-i\,e^{(6\,x)}+3i\,e^{(4\,x)}-3i\,e^{(2\,x)}+i\right)\log\left(e^x-1\right)-6i\,e^{(5\,x)}}{3\left(e^{(6\,x)}-3\,e^{(4\,x)}+3\,e^{(2\,x)}-1\right)}$$

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*(I*e^(6*x) - 3*I*e^(4*x) + 3*I*e^(2*x) - I)*log(e^x + 1) + 3*(-I*e^
(6*x) + 3*I*e^(4*x) - 3*I*e^(2*x) + I)*log(e^x - 1) - 6*I*e^(5*x) + 6*e^(4*
x) - 24*e^(2*x) + 6*I*e^x + 10)/(e^(6*x) - 3*e^(4*x) + 3*e^(2*x) - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = \operatorname{RootSum}\left(z^2+1,\left(i\mapsto i\log\left(ii+e^x\right)\right)\right) + \frac{6ie^{5x}-6e^{4x}+24e^{2x}-6ie^x-10}{3e^{6x}-9e^{4x}+9e^{2x}-3}$$

```
[In] integrate(coth(x)**4/(I+sinh(x))**2,x)
```

```
[Out] RootSum(_z**2 + 1, Lambda(_i, _i*log(_i*I + exp(x)))) + (6*I*exp(5*x) - 6*e
xp(4*x) + 24*exp(2*x) - 6*I*exp(x) - 10)/(3*exp(6*x) - 9*exp(4*x) + 9*exp(2
*x) - 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(22) = 44.

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3i\,e^{(-x)}+12\,e^{(-2\,x)}-3\,e^{(-4\,x)}-3i\,e^{(-5\,x)}-5\right)}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)} - i\,\log\left(e^{(-x)}+1\right)+i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(3*I*e^(-x) + 12*e^(-2*x) - 3*e^(-4*x) - 3*I*e^(-5*x) - 5)/(3*e^(-2*x)
 - 3*e^(-4*x) + e^(-6*x) - 1) - I*log(e^(-x) + 1) + I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(-3i\,e^{(5x)}+3\,e^{(4x)}-12\,e^{(2x)}+3i\,e^{x}+5\right)}{3\left(e^{(2x)}-1\right)^3} - i\,\log\left(e^{x}+1\right)+i\,\log\left(\left|e^{x}-1\right|\right)$$

```
[In] integrate(coth(x)^4/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(-3*I*e^(5*x) + 3*e^(4*x) - 12*e^(2*x) + 3*I*e^x + 5)/(e^(2*x) - 1)^3
- I*log(e^x + 1) + I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.96

$$\int \frac{\coth^4(x)}{(i+\sinh(x))^2}\,dx = -\ln\left(-e^{x}\,2i-2i\right)\,1i+\ln\left(-e^{x}\,2i+2i\right)\,1i - \frac{\frac{2\,e^{4x}}{3}-4\,e^{2x}+\frac{2}{3}-\frac{e^{3x}\,8i}{3}+\frac{e^{x}\,8i}{3}}{3\,e^{2x}-3\,e^{4x}+e^{6x}-1}+\frac{\frac{4}{3}+\frac{e^{x}\,4i}{3}}{e^{4x}-2\,e^{2x}+1}+\frac{-\frac{4}{3}+e^{x}\,2i}{e^{2x}-1}$$

```
[In] int(coth(x)^4/(sinh(x) + 1i)^2,x)
```

```
[Out] log(2i - exp(x)*2i)*1i - log(- exp(x)*2i - 2i)*1i - ((2*exp(4*x))/3 - (exp(
3*x)*8i)/3 - 4*exp(2*x) + (exp(x)*8i)/3 + 2/3)/(3*exp(2*x) - 3*exp(4*x) + e
xp(6*x) - 1) + ((exp(x)*4i)/3 + 4/3)/(exp(4*x) - 2*exp(2*x) + 1) + (exp(x)*
2i - 4/3)/(exp(2*x) - 1)
```

3.226 $\int \frac{\coth^5(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2} dx = -\frac{1}{2}\operatorname{csch}^2(x) + \frac{2}{3}i\operatorname{csch}^3(x) + \frac{\operatorname{csch}^4(x)}{4}$$

```
[Out] -1/2*csch(x)^2+2/3*I*csch(x)^3+1/4*csch(x)^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2786, 45}

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2} dx = \frac{\operatorname{csch}^4(x)}{4} + \frac{2}{3}i\operatorname{csch}^3(x) - \frac{\operatorname{csch}^2(x)}{2}$$

```
[In] Int[Coth[x]^5/(I + Sinh[x])^2,x]
```

```
[Out] -1/2*Csch[x]^2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
```

```
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(i-x)^2}{x^5}\,dx, x, \sinh(x)\right)\\ &= \text{Subst}\left(\int \left(-\frac{1}{x^5}-\frac{2i}{x^4}+\frac{1}{x^3}\right)\,dx, x, \sinh(x)\right)\\ &= -\frac{1}{2}\text{csch}^2(x)+\frac{2}{3}i\text{csch}^3(x)+\frac{\text{csch}^4(x)}{4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{1}{2}\text{csch}^2(x)+\frac{2}{3}i\text{csch}^3(x)+\frac{\text{csch}^4(x)}{4}$$

```
[In] Integrate[Coth[x]^5/(I + Sinh[x])^2,x]
[Out] -1/2*Csch[x]^2 + ((2*I)/3)*Csch[x]^3 + Csch[x]^4/4
```

Maple [A] (verified)

Time = 43.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

method	result	size
risch	$-\frac{2\,\mathrm{e}^{2x}\left(-8i\mathrm{e}^{3x}+3\,\mathrm{e}^{4x}+8i\mathrm{e}^{x}-12\,\mathrm{e}^{2x}+3\right)}{3\left(\mathrm{e}^{2x}-1\right)^{4}}$	41
default	$\frac{i\tanh(\frac{x}{2})}{4}+\frac{\tanh(\frac{x}{2})^4}{64}-\frac{i\tanh(\frac{x}{2})^3}{12}-\frac{3\tanh(\frac{x}{2})^2}{16}+\frac{1}{64\tanh(\frac{x}{2})^4}-\frac{i}{4\tanh(\frac{x}{2})}-\frac{3}{16\tanh(\frac{x}{2})^2}+\frac{i}{12\tanh(\frac{x}{2})^3}$	68

```
[In] int(coth(x)^5/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -2/3*exp(x)^2*(-8*I*exp(x)^3+3*exp(x)^4+8*I*exp(x)-12*exp(x)^2+3)/(exp(x)^2
-1)^4
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\,e^{(6\,x)} - 8i\,e^{(5\,x)} - 12\,e^{(4\,x)} + 8i\,e^{(3\,x)} + 3\,e^{(2\,x)}\right)}{3\left(e^{(8\,x)} - 4\,e^{(6\,x)} + 6\,e^{(4\,x)} - 4\,e^{(2\,x)} + 1\right)}$$

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*e^(6*x) - 8*I*e^(5*x) - 12*e^(4*x) + 8*I*e^(3*x) + 3*e^(2*x))/(e^(8
*x) - 4*e^(6*x) + 6*e^(4*x) - 4*e^(2*x) + 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(22) = 44.

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = \frac{-6e^{6x} + 16ie^{5x} + 24e^{4x} - 16ie^{3x} - 6e^{2x}}{3e^{8x} - 12e^{6x} + 18e^{4x} - 12e^{2x} + 3}$$

```
[In] integrate(coth(x)**5/(I+sinh(x))**2,x)
```

```
[Out] (-6*exp(6*x) + 16*I*exp(5*x) + 24*exp(4*x) - 16*I*exp(3*x) - 6*exp(2*x))/(3
*exp(8*x) - 12*exp(6*x) + 18*exp(4*x) - 12*exp(2*x) + 3)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(19) = 38.

Time = 0.21 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.33

$$\begin{aligned}\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx &= \frac{2\,e^{(-2\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} \\ &\quad - \frac{16i\,e^{(-3\,x)}}{3\left(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1\right)} \\ &\quad - \frac{8\,e^{(-4\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} \\ &\quad + \frac{16i\,e^{(-5\,x)}}{3\left(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1\right)} \\ &\quad + \frac{2\,e^{(-6\,x)}}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1}\end{aligned}$$

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-2*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 16/3*I*e
^(-3*x)/(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) - 8*e^(-4*x)/
(4*e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 16/3*I*e^(-5*x)/(4*
e^(-2*x) - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 2*e^(-6*x)/(4*e^(-2*x)
 - 6*e^(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\left(3\left(e^{(-x)}-e^{x}\right)^2+8i\,e^{(-x)}-8i\,e^{x}-6\right)}{3\left(e^{(-x)}-e^{x}\right)^4}$$

```
[In] integrate(coth(x)^5/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*(e^(-x) - e^x)^2 + 8*I*e^(-x) - 8*I*e^x - 6)/(e^(-x) - e^x)^4
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\coth^5(x)}{(i+\sinh(x))^2}\,dx = -\frac{2\,\mathrm{e}^{2\,x}\,\left(3\,\mathrm{e}^{4\,x}-12\,\mathrm{e}^{2\,x}+3-\mathrm{e}^{3\,x}\,8\mathrm{i}+\mathrm{e}^{x}\,8\mathrm{i}\right)}{3\,{\left(\mathrm{e}^{2\,x}-1\right)}^4}$$

```
[In] int(coth(x)^5/(sinh(x) + 1i)^2,x)
```

```
[Out] -(2*exp(2*x)*(3*exp(4*x) - exp(3*x)*8i - 12*exp(2*x) + exp(x)*8i + 3))/(3*(
exp(2*x) - 1)^4)
```

3.227 $\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx = -\frac{1}{4}i\text{arctanh}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\text{csch}(x) + \frac{1}{2}i\coth(x)\text{csch}^3(x)$$

```
[Out] -1/4*I*arctanh(cosh(x))-2/3*coth(x)^3+1/5*coth(x)^5+1/4*I*coth(x)*csch(x)+1
/2*I*coth(x)*csch(x)^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2788, 3852, 8, 3853, 3855}

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2} dx = -\frac{1}{4}i\text{arctanh}(\cosh(x)) + \frac{\coth^5(x)}{5} - \frac{2\coth^3(x)}{3} + \frac{1}{2}i\coth(x)\text{csch}^3(x) + \frac{1}{4}i\coth(x)\text{csch}(x)$$

```
[In] Int[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-1/4*I)*ArcTanh[Cosh[x]] - (2*Coth[x]^3)/3 + Coth[x]^5/5 + (I/4)*Coth[x]*C
sch[x] + (I/2)*Coth[x]*Csch[x]^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2788

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
+ f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\text{csch}^2(x) - 2i\text{csch}^3(x) - 2i\text{csch}^5(x) - \text{csch}^6(x)\right)\,dx \\
&= -\left(2i\int \text{csch}^3(x)\,dx\right) - 2i\int \text{csch}^5(x)\,dx + \int \text{csch}^2(x)\,dx - \int \text{csch}^6(x)\,dx \\
&= i\coth(x)\text{csch}(x) + \frac{1}{2}i\coth(x)\text{csch}^3(x) \\
&\quad + i\int \text{csch}(x)\,dx - i\text{Subst}(\int 1\,dx, x, -i\coth(x)) \\
&\quad + i\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\coth(x)\right) + \frac{3}{2}i\int \text{csch}^3(x)\,dx \\
&= -i\text{arctanh}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} \\
&\quad + \frac{1}{4}i\coth(x)\text{csch}(x) + \frac{1}{2}i\coth(x)\text{csch}^3(x) - \frac{3}{4}i\int \text{csch}(x)\,dx \\
&= -\frac{1}{4}i\text{arctanh}(\cosh(x)) - \frac{2\coth^3(x)}{3} + \frac{\coth^5(x)}{5} + \frac{1}{4}i\coth(x)\text{csch}(x) + \frac{1}{2}i\coth(x)\text{csch}^3(x)
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 175 vs. 2(48) = 96.

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.65

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{7}{30}\coth\left(\frac{x}{2}\right)+\frac{1}{16}i\operatorname{csch}^2\left(\frac{x}{2}\right)-\frac{19}{480}\coth\left(\frac{x}{2}\right)\operatorname{csch}^2\left(\frac{x}{2}\right)+\frac{1}{32}i\operatorname{csch}^4\left(\frac{x}{2}\right)+\frac{1}{160}\coth\left(\frac{x}{2}\right)\operatorname{csch}^4\left(\frac{x}{2}\right)-\frac{1}{4}i\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{4}i\log\left(\sinh\left(\frac{x}{2}\right)\right)+\frac{1}{16}i\operatorname{sech}^2\left(\frac{x}{2}\right)-\frac{1}{32}i\operatorname{sech}^4\left(\frac{x}{2}\right)-\frac{7}{30}\tanh\left(\frac{x}{2}\right)+\frac{19}{480}\operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)+\frac{1}{160}\operatorname{sech}^4\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)$$

```
[In] Integrate[Coth[x]^6/(I + Sinh[x])^2,x]
```

```
[Out] (-7*Coth[x/2])/30 + (I/16)*Csch[x/2]^2 - (19*Coth[x/2]*Csch[x/2]^2)/480 + (
I/32)*Csch[x/2]^4 + (Coth[x/2]*Csch[x/2]^4)/160 - (I/4)*Log[Cosh[x/2]] + (I
/4)*Log[Sinh[x/2]] + (I/16)*Sech[x/2]^2 - (I/32)*Sech[x/2]^4 - (7*Tanh[x/2]
)/30 + (19*Sech[x/2]^2*Tanh[x/2])/480 + (Sech[x/2]^4*Tanh[x/2])/160
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 61.78 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

method	result
default	$-\frac{3\tanh\left(\frac{x}{2}\right)}{16}+\frac{\tanh\left(\frac{x}{2}\right)^5}{160}-\frac{i\tanh\left(\frac{x}{2}\right)^4}{32}-\frac{5\tanh\left(\frac{x}{2}\right)^3}{96}+\frac{i}{32\tanh\left(\frac{x}{2}\right)^4}+\frac{1}{160\tanh\left(\frac{x}{2}\right)^5}-\frac{3}{16\tanh\left(\frac{x}{2}\right)}-\frac{5}{96\tanh\left(\frac{x}{2}\right)^3}+$
risch	$\frac{i\left(60ie^{8x}+15\,e^{9x}-240ie^{6x}+90\,e^{7x}+40ie^{4x}-80ie^{2x}-90\,e^{3x}+28i-15\,e^{x}\right)}{30\left(e^{2x}-1\right)^5}-\frac{i\ln\left(e^x+1\right)}{4}+\frac{i\ln\left(e^x-1\right)}{4}$

```
[In] int(coth(x)^6/(I+sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*tanh(1/2*x)+1/160*tanh(1/2*x)^5-1/32*I*tanh(1/2*x)^4-5/96*tanh(1/2*x)
^3+1/32*I/tanh(1/2*x)^4+1/160/tanh(1/2*x)^5-3/16/tanh(1/2*x)-5/96/tanh(1/2*
x)^3+1/4*I*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.33

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{15\left(i\,e^{(10\,x)} - 5i\,e^{(8\,x)} + 10i\,e^{(6\,x)} - 10i\,e^{(4\,x)} + 5i\,e^{(2\,x)} - i\right)\log\left(e^{x}+1\right) + 15\left(-i\,e^{(10\,x)} + 5i\,e^{(8\,x)} - 10i\,e^{\cdots}\right.}{60\left(e^{(10\,x)}\cdots\right.}$$

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/60*(15*(I*e^(10*x) - 5*I*e^(8*x) + 10*I*e^(6*x) - 10*I*e^(4*x) + 5*I*e^(
2*x) - I)*log(e^x + 1) + 15*(-I*e^(10*x) + 5*I*e^(8*x) - 10*I*e^(6*x) + 10*
I*e^(4*x) - 5*I*e^(2*x) + I)*log(e^x - 1) - 30*I*e^(9*x) + 120*e^(8*x) - 18
0*I*e^(7*x) - 480*e^(6*x) + 80*e^(4*x) + 180*I*e^(3*x) - 160*e^(2*x) + 30*I
*e^x + 56)/(e^(10*x) - 5*e^(8*x) + 10*e^(6*x) - 10*e^(4*x) + 5*e^(2*x) - 1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(44) = 88$.

Time = 0.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.38

$$\begin{aligned}\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx &= \operatorname{RootSum}\left(16z^2+1, \left(i \mapsto i\log\left(4ii+e^{x}\right)\right)\right) \\ &\quad + \frac{15ie^{9x} - 60e^{8x} + 90ie^{7x} + 240e^{6x} - 40e^{4x} - 90ie^{3x} + 80e^{2x} - 15ie^{x} - 28}{30e^{10x} - 150e^{8x} + 300e^{6x} - 300e^{4x} + 150e^{2x} - 30}\end{aligned}$$

```
[In] integrate(coth(x)**6/(I+sinh(x))**2,x)
```

```
[Out] RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i*I + exp(x)))) + (15*I*exp(9*x)
 - 60*exp(8*x) + 90*I*exp(7*x) + 240*exp(6*x) - 40*exp(4*x) - 90*I*exp(3*x)
 + 80*exp(2*x) - 15*I*exp(x) - 28)/(30*exp(10*x) - 150*exp(8*x) + 300*exp(6
*x) - 300*exp(4*x) + 150*exp(2*x) - 30)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(32) = 64.

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx$$
$$= \frac{-15i\,e^{(-x)} - 80\,e^{(-2\,x)} - 90i\,e^{(-3\,x)} + 40\,e^{(-4\,x)} - 240\,e^{(-6\,x)} + 90i\,e^{(-7\,x)} + 60\,e^{(-8\,x)} + 15i\,e^{(-9\,x)} + 28}{30\,(5\,e^{(-2\,x)} - 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} - 5\,e^{(-8\,x)} + e^{(-10\,x)} - 1)}$$
$$- \frac{1}{4}i\,\log\left(e^{(-x)} + 1\right) + \frac{1}{4}i\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(-15*I*e^(-x) - 80*e^(-2*x) - 90*I*e^(-3*x) + 40*e^(-4*x) - 240*e^(-6*
x) + 90*I*e^(-7*x) + 60*e^(-8*x) + 15*I*e^(-9*x) + 28)/(5*e^(-2*x) - 10*e^(
-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*x) - 1) - 1/4*I*log(e^(-x) + 1) +
 1/4*I*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx =$$
$$-\frac{-15i\,e^{(9\,x)} + 60\,e^{(8\,x)} - 90i\,e^{(7\,x)} - 240\,e^{(6\,x)} + 40\,e^{(4\,x)} + 90i\,e^{(3\,x)} - 80\,e^{(2\,x)} + 15i\,e^{x} + 28}{30\,\left(e^{(2\,x)} - 1\right)^5}$$
$$- \frac{1}{4}i\,\log\left(e^{x} + 1\right) + \frac{1}{4}i\,\log\left(|e^{x} - 1|\right)$$

```
[In] integrate(coth(x)^6/(I+sinh(x))^2,x, algorithm="giac")
```

```
[Out] -1/30*(-15*I*e^(9*x) + 60*e^(8*x) - 90*I*e^(7*x) - 240*e^(6*x) + 40*e^(4*x)
 + 90*I*e^(3*x) - 80*e^(2*x) + 15*I*e^x + 28)/(e^(2*x) - 1)^5 - 1/4*I*log(e
^x + 1) + 1/4*I*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.12

$$\int \frac{\coth^6(x)}{(i+\sinh(x))^2}\,dx = -\frac{80\,\mathrm{e}^{4\,x}-160\,\mathrm{e}^{2\,x}-480\,\mathrm{e}^{6\,x}+120\,\mathrm{e}^{8\,x}+56-\ln\left(-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}-\frac{1}{2}\mathrm{i}\right)\,15\mathrm{i}+\ln\left(-\frac{\mathrm{e}^{x}\,1\mathrm{i}}{2}+\frac{1}{2}\mathrm{i}\right)\,15\mathrm{i}+\mathrm{e}^{3\,x}\,180\mathrm{i}-\mathrm{e}^{7\,x}$$

```
[In] int(coth(x)^6/(sinh(x) + 1i)^2,x)
```

```
[Out] -(log(1i/2 - (exp(x)*1i)/2)*15i - log(- (exp(x)*1i)/2 - 1i/2)*15i - 160*exp
(2*x) + exp(3*x)*180i + 80*exp(4*x) - 480*exp(6*x) - exp(7*x)*180i + 120*ex
p(8*x) - exp(9*x)*30i + exp(x)*30i + log(- (exp(x)*1i)/2 - 1i/2)*exp(2*x)*7
5i - log(1i/2 - (exp(x)*1i)/2)*exp(2*x)*75i - log(- (exp(x)*1i)/2 - 1i/2)*e
xp(4*x)*150i + log(1i/2 - (exp(x)*1i)/2)*exp(4*x)*150i + log(- (exp(x)*1i)/
2 - 1i/2)*exp(6*x)*150i - log(1i/2 - (exp(x)*1i)/2)*exp(6*x)*150i - log(- (
exp(x)*1i)/2 - 1i/2)*exp(8*x)*75i + log(1i/2 - (exp(x)*1i)/2)*exp(8*x)*75i
+ log(- (exp(x)*1i)/2 - 1i/2)*exp(10*x)*15i - log(1i/2 - (exp(x)*1i)/2)*exp
(10*x)*15i + 56)/(60*(exp(2*x) - 1)^5)
```

3.228 $\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = -\frac{2a^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{b\operatorname{sech}(x)}{a^2+b^2} + \frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)} - \frac{a^3\tanh(x)}{(a^2+b^2)^2} - \frac{a\tanh^3(x)}{3(a^2+b^2)}$$

```
[Out] -2*a^4*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-a^2*b*sec
h(x)/(a^2+b^2)^2-b*sech(x)/(a^2+b^2)+1/3*b*sech(x)^3/(a^2+b^2)-a^3*tanh(x)/
(a^2+b^2)^2-1/3*a*tanh(x)^3/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2806, 2687, 30, 2686, 3852, 8, 2739, 632, 212}

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = -\frac{a\tanh^3(x)}{3(a^2+b^2)} + \frac{b\operatorname{sech}^3(x)}{3(a^2+b^2)} - \frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{b\operatorname{sech}(x)}{a^2+b^2} - \frac{2a^4\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{a^3\tanh(x)}{(a^2+b^2)^2}$$

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^4*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (a^
2*b*Sech[x])/(a^2 + b^2)^2 - (b*Sech[x])/(a^2 + b^2) + (b*Sech[x]^3)/(3*(a^
2 + b^2)) - (a^3*Tanh[x])/(a^2 + b^2)^2 - (a*Tanh[x]^3)/(3*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
```

```
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a\int \operatorname{sech}^2(x)\tanh^2(x)\,dx}{a^2+b^2}+\frac{a^2\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx}{a^2+b^2}+\frac{b\int \operatorname{sech}(x)\tanh^3(x)\,dx}{a^2+b^2}\\
&= -\frac{a^3\int \operatorname{sech}^2(x)\,dx}{(a^2+b^2)^2}+\frac{a^4\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^2}+\frac{(a^2b)\int \operatorname{sech}(x)\tanh(x)\,dx}{(a^2+b^2)^2}\\
&\quad -\frac{(ia)\text{Subst}\left(\int x^2\,dx, x, i\tanh(x)\right)}{a^2+b^2}+\frac{b\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \operatorname{sech}(x)\right)}{a^2+b^2}\\
&= -\frac{b\operatorname{sech}(x)}{a^2+b^2}+\frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}-\frac{(ia^3)\,\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)}{(a^2+b^2)^2}\\
&\quad +\frac{(2a^4)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}-\frac{(a^2b)\,\text{Subst}\left(\int 1\,dx, x, \operatorname{sech}(x)\right)}{(a^2+b^2)^2}\\
&= -\frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}+\frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a^3\tanh(x)}{(a^2+b^2)^2}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}\\
&\quad -\frac{(4a^4)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}\\
&= -\frac{2a^4\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{a^2b\operatorname{sech}(x)}{(a^2+b^2)^2}-\frac{b\operatorname{sech}(x)}{a^2+b^2}+\frac{b\operatorname{sech}^3(x)}{3\left(a^2+b^2\right)}-\frac{a^3\tanh(x)}{(a^2+b^2)^2}-\frac{a\tanh^3(x)}{3\left(a^2+b^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$$

$$=\frac{\frac{6a^4\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-3b(2a^2+b^2)\operatorname{sech}(x)+(a^2+b^2)\operatorname{sech}^3(x)(b+a\sinh(x))-a(4a^2+b^2)\tanh(x)}{3\left(a^2+b^2\right)^2}$$

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] ((6*a^4*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 3*b*
(2*a^2 + b^2)*Sech[x] + (a^2 + b^2)*Sech[x]^3*(b + a*Sinh[x]) - a*(4*a^2 +
b^2)*Tanh[x])/(3*(a^2 + b^2)^2)
```

Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.38

method	result
default	$\frac{32a^4 \operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(16a^4+32a^2b^2+16b^4)\sqrt{a^2+b^2}}+\frac{-2a^3\tanh\left(\frac{x}{2}\right)^5-2a^2b\tanh\left(\frac{x}{2}\right)^4+2\left(-\frac{10}{3}a^3-\frac{4}{3}ab^2\right)\tanh\left(\frac{x}{2}\right)^3+2\left(-4a^2b-2b^3\right)\tanh\left(\frac{x}{2}\right)^2-2\tan}{(a^4+2a^2b^2+b^4)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}$
risch	$\frac{-4a^2b\,e^{5x}-2b^3e^{5x}+4a^3e^{4x}+2\,e^{4x}a\,b^2-\frac{16a^2b\,e^{3x}}{3}-\frac{4\,e^{3x}b^3}{3}+4a^3e^{2x}-4\,e^{x}a^2b-2b^3e^{x}+\frac{8a^3}{3}+\frac{2a\,b^2}{3}}{(a^4+2a^2b^2+b^4)(1+e^{2x})^3}+\frac{a^4\ln\left(e^{x}+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}a-a^6-3a}{b\left(a^2+b^2\right.}\right.}{(a^2+b^2)^{\frac{5}{2}}}$

```
[In] int(tanh(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 32*a^4/(16*a^4+32*a^2*b^2+16*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2
*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(-a^3*tanh(1/2*x)^5-a^2*b*t
anh(1/2*x)^4+(-10/3*a^3-4/3*a*b^2)*tanh(1/2*x)^3+(-4*a^2*b-2*b^3)*tanh(1/2*
x)^2-tanh(1/2*x)*a^3-5/3*a^2*b-2/3*b^3)/(1+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1199 vs. $2(116) = 232$.

Time = 0.29 (sec) , antiderivative size = 1199, normalized size of antiderivative = 9.67

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b + 3*a^2*b^3 + b^
5)*sinh(x)^5 - 8*a^5 - 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4)
*cosh(x)^4 - 6*(2*a^5 + 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*c
osh(x))*sinh(x)^4 + 4*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^3 + 4*(4*a^4*b +
5*a^2*b^3 + b^5 + 15*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^2 - 6*(2*a^5 + 3*a
^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 + a^3*b^2)*cosh(x)^2 - 12*(a^5
 + a^3*b^2 - 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^3 + 3*(2*a^5 + 3*a^3*b^2
 + a*b^4)*cosh(x)^2 - (4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^2 - 3*(a
^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 +
3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh
```

```
(x)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a
^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))*
sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
+ b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*si
nh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*si
nh(x) - b)) + 6*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*a^4*b + 3*a^2*b^
3 + b^5 + 5*(2*a^4*b + 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a^5 + 3*a^3*b^2 +
a*b^4)*cosh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2 - 4*(a^5 + a^3*b
^2)*cosh(x))*sinh(x))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^6 + 6*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)*sinh(x)^5 + (a^6 + 3*a^4*b^2 + 3*
a^2*b^4 + b^6)*sinh(x)^6 + a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 3*(a^6 + 3*a
^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^4 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
+ 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 +
3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*cosh(x))*sinh(x)^3 + 3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2 +
3*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
)*cosh(x)^4 + 6*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^2)*sinh(x)^2 +
6*((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x)^5 + 2*(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6)*cosh(x)^3 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*cosh(x))*sinh
(x))
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*sinh(x)),x)
[Out] Integral(tanh(x)**4/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(116) = 232.

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.94

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \frac{a^4\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(6a^3e^{(-2x)}+4a^3+ab^2+3(2a^2b+b^3)e^{(-x)}+2(4a^2b+b^3)e^{(-3x)}+3(2a^3+ab^2)e^{(-4x)}+3(2a^2b+b\right.}{3\left(a^4+2a^2b^2+b^4+3(a^4+2a^2b^2+b^4)e^{(-2x)}+3(a^4+2a^2b^2+b^4)e^{(-4x)}+(a^4+2a^2b^2+b^4)e^{(-6\right.}}$$

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^4*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^3*e^(-2*x) + 4*a^3 + a
*b^2 + 3*(2*a^2*b + b^3)*e^(-x) + 2*(4*a^2*b + b^3)*e^(-3*x) + 3*(2*a^3 + a
*b^2)*e^(-4*x) + 3*(2*a^2*b + b^3)*e^(-5*x))/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^
4 + 2*a^2*b^2 + b^4)*e^(-2*x) + 3*(a^4 + 2*a^2*b^2 + b^4)*e^(-4*x) + (a^4 +
 2*a^2*b^2 + b^4)*e^(-6*x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.59

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \frac{a^4\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(6\,a^2be^{(5\,x)}+3\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}-3\,ab^2e^{(4\,x)}+8\,a^2be^{(3\,x)}+2\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}+6\,a^2be^x+3\,b^3e^x-\right.}{3\,(a^4+2\,a^2b^2+b^4)(e^{(2\,x)}+1)^3}$$

```
[In] integrate(tanh(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^4*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3*(6*a^2*b*e^(5*x
) + 3*b^3*e^(5*x) - 6*a^3*e^(4*x) - 3*a*b^2*e^(4*x) + 8*a^2*b*e^(3*x) + 2*b
^3*e^(3*x) - 6*a^3*e^(2*x) + 6*a^2*b*e^x + 3*b^3*e^x - 4*a^3 - a*b^2)/((a^4
 + 2*a^2*b^2 + b^4)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 654, normalized size of antiderivative = 5.27

$$\int \frac{\tanh^4(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,a\,(2\,a^2+b^2)}{(a^2+b^2)^2}-\frac{2\,b\,e^x\,(2\,a^2+b^2)}{(a^2+b^2)^2}}{e^{2\,x}+1} - \frac{\frac{4\,(a^3+a\,b^2)}{(a^2+b^2)^2}-\frac{8\,e^x\,(a^2\,b+b^3)}{3\,(a^2+b^2)^2}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{\frac{8\,a}{3\,(a^2+b^2)}-\frac{8\,b\,e^x}{3\,(a^2+b^2)}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1} - \frac{2\,\mathrm{atan}\left(\left(\left(e^x\left(\frac{2\,a^4}{b^2\,\sqrt{a^8}\,(a^2+b^2)^2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\left(a^5\,\sqrt{a^8}+2\,a^3\,b^2\,\sqrt{a^8}+a\,b^4\,\sqrt{a^8}\right)}{a^3\,b^2\,\sqrt{-(a^2+b^2)^5}\,(a^4+2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}-5\,a^8\,b^2-10\,a^6\,b^4-10\,a^4\,b^6-5\,a^2\,b^8-b^{10}}}\right.\right.\right.\right.}{}$$

```
[In] int(tanh(x)^4/(a + b*sinh(x)),x)
```

```
[Out] ((2*a*(2*a^2 + b^2))/(a^2 + b^2)^2 - (2*b*exp(x)*(2*a^2 + b^2))/(a^2 + b^2)
^2)/(exp(2*x) + 1) - ((4*(a*b^2 + a^3))/(a^2 + b^2)^2 - (8*exp(x)*(a^2*b +
b^3))/(3*(a^2 + b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) + ((8*a)/(3*(a^2 + b^2
```

```
)) - (8*b*exp(x))/(3*(a^2 + b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
 - (2*atan((exp(x)*((2*a^4)/(b^2*(a^8)^(1/2)*(a^2 + b^2)^2*(a^4 + b^4 + 2*a
^2*b^2)) + (2*(a^5*(a^8)^(1/2) + 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^(1/2))
)/(a^3*b^2*(-(a^2 + b^2)^5)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 -
5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))) - (2*(b^5*(a^8)^(1
/2) + 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 + b^2)^5)
^(1/2)*(a^4 + b^4 + 2*a^2*b^2)*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10
*a^6*b^4 - 5*a^8*b^2)^(1/2)))*((b^5*(- a^10 - b^10 - 5*a^2*b^8 - 10*a^4*b^6
 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + (a^4*b*(- a^10 - b^10 - 5*a^2*b^8 - 1
0*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/2 + a^2*b^3*(- a^10 - b^10 - 5*a
^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)))*(a^8)^(1/2))/(- a^10
- b^10 - 5*a^2*b^8 - 10*a^4*b^6 - 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

3.229 $\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx=\frac{b(3a^2+b^2)\arctan(\sinh(x))}{2(a^2+b^2)^2}+\frac{a^3\log(\cosh(x))}{(a^2+b^2)^2}-\frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2(a^2+b^2)}$$

```
[Out] 1/2*b*(3*a^2+b^2)*arctan(sinh(x))/(a^2+b^2)^2+a^3*ln(cosh(x))/(a^2+b^2)^2-a
^3*ln(a+b*sinh(x))/(a^2+b^2)^2+1/2*sech(x)^2*(a-b*sinh(x))/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2800, 1661, 815, 649, 209, 266}

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx=\frac{b(3a^2+b^2)\arctan(\sinh(x))}{2(a^2+b^2)^2}+\frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2(a^2+b^2)}-\frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{a^3\log(\cosh(x))}{(a^2+b^2)^2}$$

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*(3*a^2 + b^2)*ArcTan[Sinh[x]])/(2*(a^2 + b^2)^2) + (a^3*Log[Cosh[x]])/(a
^2 + b^2)^2 - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (Sech[x]^2*(a - b*Si
nh[x]))/(2*(a^2 + b^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^3}{(a+x)\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2\left(a^2+b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2+b^2}+\frac{b^2\left(2a^2+b^2\right)x}{a^2+b^2}}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2b^2}$$

$$= \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2(a^2+b^2)} - \frac{\operatorname{Subst}\left(\int\left(\frac{2a^3b^2}{(a^2+b^2)^2(a+x)} - \frac{b^2(3a^2b^2+b^4+2a^3x)}{(a^2+b^2)^2(b^2+x^2)}\right)dx, x, b\sinh(x)\right)}{2b^2}$$

$$= -\frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2(a^2+b^2)} + \frac{\operatorname{Subst}\left(\int\frac{3a^2b^2+b^4+2a^3x}{b^2+x^2}dx, x, b\sinh(x)\right)}{2(a^2+b^2)^2}$$

$$= -\frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2(a^2+b^2)} + \frac{a^3\operatorname{Subst}\left(\int\frac{x}{b^2+x^2}dx, x, b\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(b^2(3a^2+b^2))\operatorname{Subst}\left(\int\frac{1}{b^2+x^2}dx, x, b\sinh(x)\right)}{2(a^2+b^2)^2}$$

$$= \frac{b(3a^2+b^2)\arctan(\sinh(x))}{2(a^2+b^2)^2} + \frac{a^3\log(\cosh(x))}{(a^2+b^2)^2} - \frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{\operatorname{sech}^2(x)(a-b\sinh(x))}{2(a^2+b^2)}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.74

$$\int\frac{\tanh^3(x)}{a+b\sinh(x)}dx = -\frac{b\arctan(\sinh(x))}{2(a^2+b^2)} + \frac{(a^3-i(2a^2b+b^3))\log(i-\sinh(x))}{2(a^2+b^2)^2} + \frac{(a^3+i(2a^2b+b^3))\log(i+\sinh(x))}{2(a^2+b^2)^2} - \frac{a^3\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{a\operatorname{sech}^2(x)}{2(a^2+b^2)} - \frac{b\operatorname{sech}(x)\tanh(x)}{2(a^2+b^2)}$$

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] -1/2*(b*ArcTan[Sinh[x]])/(a^2 + b^2) + ((a^3 - I*(2*a^2*b + b^3))*Log[I - S
inh[x]])/(2*(a^2 + b^2)^2) + ((a^3 + I*(2*a^2*b + b^3))*Log[I + Sinh[x]])/(
2*(a^2 + b^2)^2) - (a^3*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + (a*Sech[x]^2)/(
2*(a^2 + b^2)) - (b*Sech[x]*Tanh[x])/(2*(a^2 + b^2))
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.89

method	result
default	$-\frac{8a^3\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{8a^4+16a^2b^2+8b^4}+\frac{\frac{2\left(\left(\frac{1}{2}a^2b+\frac{1}{2}b^3\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^3-a\,b^2\right)\tanh\left(\frac{x}{2}\right)^2+\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+a^3\ln\left(1+\tanh\left(\right.\right.}{a^4+2a^2b^2+b^4}$
risch	$\frac{e^x\left(-b\,e^{2x}+2\,e^x a+b\right)}{\left(1+e^{2x}\right)^2\left(a^2+b^2\right)}+\frac{3i\ln\left(e^x+i\right)a^2b}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{i\ln\left(e^x+i\right)b^3}{2a^4+4a^2b^2+2b^4}+\frac{\ln\left(e^x+i\right)a^3}{a^4+2a^2b^2+b^4}-\frac{3i\ln\left(e^x-i\right)a^2b}{2\left(a^4+2a^2b^2+b^4\right)}-\frac{i\ln\left(e^x-i\right)b^3}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{\ln}{a^4+}$

```
[In] int(tanh(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -8*a^3/(8*a^4+16*a^2*b^2+8*b^4)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a^
4+2*a^2*b^2+b^4)*(((1/2*a^2*b+1/2*b^3)*tanh(1/2*x)^3+(-a^3-a*b^2)*tanh(1/2*
x)^2+(-1/2*a^2*b-1/2*b^3)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+1/2*a^3*ln(1+tan
h(1/2*x)^2)+1/2*(3*a^2*b+b^3)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(85) = 170.

Time = 0.30 (sec) , antiderivative size = 655, normalized size of antiderivative = 7.44

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = -\frac{(a^2b+b^3)\cosh(x)^3+(a^2b+b^3)\sinh(x)^3-2(a^3+ab^2)\cosh(x)^2-(2a^3+2ab^2-3(a^2b+b^3)\cosh(x)}{}$$

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((a^2*b + b^3)*cosh(x)^3 + (a^2*b + b^3)*sinh(x)^3 - 2*(a^3 + a*b^2)*cosh(
x)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(x))*sinh(x)^2 - ((3*a^2*b +
b^3)*cosh(x)^4 + 4*(3*a^2*b + b^3)*cosh(x)*sinh(x)^3 + (3*a^2*b + b^3)*sinh
(x)^4 + 3*a^2*b + b^3 + 2*(3*a^2*b + b^3)*cosh(x)^2 + 2*(3*a^2*b + b^3 + 3*
(3*a^2*b + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((3*a^2*b + b^3)*cosh(x)^3 + (3*a^
2*b + b^3)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b + b^3)*cosh
(x) + (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh
(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*
cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^3*cosh(x)
^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3
*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*
log(2*cosh(x)/(cosh(x) - sinh(x))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(x)
^2 + 4*(a^3 + a*b^2)*cosh(x))*sinh(x))/((a^4 + 2*a^2*b^2 + b^4)*cosh(x)^4 +
 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh
```

```
(x)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^
4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)
)
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx = \int \frac{\tanh^3{(x)}}{a + b\sinh{(x)}}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*sinh(x)),x)
[Out] Integral(tanh(x)**3/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\begin{aligned}\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx = &-\frac{a^3\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^4 + 2\,a^2b^2 + b^4} \\ &+ \frac{a^3\log\left(e^{(-2\,x)} + 1\right)}{a^4 + 2\,a^2b^2 + b^4} - \frac{(3\,a^2b + b^3)\arctan\left(e^{(-x)}\right)}{a^4 + 2\,a^2b^2 + b^4} \\ &- \frac{be^{(-x)} - 2\,ae^{(-2\,x)} - be^{(-3\,x)}}{a^2 + b^2 + 2\,(a^2 + b^2)e^{(-2\,x)} + (a^2 + b^2)e^{(-4\,x)}}\end{aligned}$$

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -a^3*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + a^3*log(e^
(-2*x) + 1)/(a^4 + 2*a^2*b^2 + b^4) - (3*a^2*b + b^3)*arctan(e^(-x))/(a^4 +
 2*a^2*b^2 + b^4) - (b*e^(-x) - 2*a*e^(-2*x) - b*e^(-3*x))/(a^2 + b^2 + 2*(
a^2 + b^2)*e^(-2*x) + (a^2 + b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.40

$$\begin{aligned}\int \frac{\tanh^3(x)}{a + b\sinh(x)}\,dx = &-\frac{a^3b\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^4b + 2\,a^2b^3 + b^5} + \frac{a^3\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\,(a^4 + 2\,a^2b^2 + b^4)} \\ &+ \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)(3\,a^2b + b^3)}{4\,(a^4 + 2\,a^2b^2 + b^4)} \\ &- \frac{a^3\left(e^{(-x)} - e^x\right)^2 - 2\,a^2b\left(e^{(-x)} - e^x\right) - 2\,b^3\left(e^{(-x)} - e^x\right) - 4\,ab^2}{2\,(a^4 + 2\,a^2b^2 + b^4)\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}\end{aligned}$$

```
[In] integrate(tanh(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^
3*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) + 1/4*(pi + 2*arctan(1/
2*(e^(2*x) - 1)*e^(-x)))*(3*a^2*b + b^3)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^3
*(e^(-x) - e^x)^2 - 2*a^2*b*(e^(-x) - e^x) - 2*b^3*(e^(-x) - e^x) - 4*a*b^2
)/((a^4 + 2*a^2*b^2 + b^4)*((e^(-x) - e^x)^2 + 4))
```

Mupad [B] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.31

$$\int \frac{\tanh^3(x)}{a+b\sinh(x)}\,dx = \frac{\frac{2\,(a^3+a\,b^2)}{(a^2+b^2)^2}-\frac{e^x\,(a^2\,b+b^3)}{(a^2+b^2)^2}}{e^{2\,x}+1}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,e^x}{a^2+b^2}}{2\,e^{2\,x}+e^{4\,x}+1}+\frac{\ln\left(1+e^x\,1i\right)\,\left(2\,a+b\,1i\right)}{2\,\left(a^2+a\,b\,2i-b^2\right)} -\frac{a^3\,\ln\left(b^7\,e^{2\,x}-16\,a^6\,b-b^7-6\,a^2\,b^5-9\,a^4\,b^3+32\,a^7\,e^x+6\,a^2\,b^5\,e^{2\,x}+9\,a^4\,b^3\,e^{2\,x}+2\,a\,b^6\,e^x+16\,a^6\,b\,e^{2\,x}\right.}{a^4+2\,a^2\,b^2+b^4} +\frac{\ln\left(e^x+1i\right)\,\left(b+a\,2i\right)}{2\,\left(a^2\,1i+2\,a\,b-b^2\,1i\right)}$$

```
[In] int(tanh(x)^3/(a + b*sinh(x)),x)
```

```
[Out] ((2*(a*b^2 + a^3))/(a^2 + b^2)^2 - (exp(x)*(a^2*b + b^3))/(a^2 + b^2)^2)/(e
xp(2*x) + 1) - ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(2*exp(2*x) +
 exp(4*x) + 1) + (log(exp(x)*1i + 1)*(2*a + b*1i))/(2*(a*b*2i + a^2 - b^2))
 - (a^3*log(b^7*exp(2*x) - 16*a^6*b - b^7 - 6*a^2*b^5 - 9*a^4*b^3 + 32*a^7*
exp(x) + 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) + 16*a^6*
b*exp(2*x) + 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 + 2*a^2*b^2
) + (log(exp(x) + 1i)*(a*2i + b))/(2*(2*a*b + a^2*1i - b^2*1i))
```

3.230 $\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx=-\frac{2a^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\text{sech}(x)}{a^2+b^2}-\frac{a\tanh(x)}{a^2+b^2}$$

```
[Out] -2*a^2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-b*sech(x)
/(a^2+b^2)-a*tanh(x)/(a^2+b^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2806, 3852, 8, 2686, 2739, 632, 212}

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx=-\frac{2a^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\tanh(x)}{a^2+b^2}-\frac{b\text{sech}(x)}{a^2+b^2}$$

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-2*a^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*
Sech[x])/(a^2 + b^2) - (a*Tanh[x])/(a^2 + b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2806

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[a/(a^2 - b^2), Int[(g*Tan[e + f*x])^p/Sin[e + f*x]^
2, x], x] + (-Dist[b*(g/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 1)/Cos[e +
f*x], x], x] - Dist[a^2*(g^2/(a^2 - b^2)), Int[(g*Tan[e + f*x])^(p - 2)/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0
] && IntegersQ[2*p] && GtQ[p, 1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = -\frac{a\int \operatorname{sech}^2(x)\,dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^2+b^2}+\frac{b\int \operatorname{sech}(x)\tanh(x)\,dx}{a^2+b^2}$$

$$= -\frac{(ia)\text{Subst}(\int 1\,dx, x, -i\tanh(x))}{a^2+b^2} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2} - \frac{b\text{Subst}(\int 1\,dx, x, \text{sech}(x))}{a^2+b^2}$$

$$= -\frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2} - \frac{(4a^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$= -\frac{2a^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b\text{sech}(x)}{a^2+b^2} - \frac{a\tanh(x)}{a^2+b^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \frac{-b\text{sech}(x) + a\left(\frac{2a\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - \tanh(x)\right)}{a^2+b^2}$$

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (-(b*Sech[x]) + a*((2*a*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] - Tanh[x]))/(a^2 + b^2)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{8a^2\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(4a^2+4b^2)\sqrt{a^2+b^2}} + \frac{-2a\tanh\left(\frac{x}{2}\right)-2b}{(a^2+b^2)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}$	84
risch	$\frac{-2\,\mathrm{e}^x b+2a}{(1+\mathrm{e}^{2x})(a^2+b^2)} + \frac{a^2\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}-a^4-2a^2b^2-b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{a^2\ln\left(\mathrm{e}^x+\frac{a\left(a^2+b^2\right)^{\frac{3}{2}}+a^4+2a^2b^2+b^4}{b\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	145

```
[In] int(tanh(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+
b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*x)-b)/(1+tanh(1/2*x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(65) = 130$.

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.72

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \frac{2\,a^3+2\,ab^2+\left(a^2\cosh\left(x\right)^2+2\,a^2\cosh\left(x\right)\sinh\left(x\right)+a^2\sinh\left(x\right)^2+a^2\right)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,a}{b\cos}\right.}{a^4+2\,a^2b^2+b^4+\left(a^4+2\,a^2b^2+b^4\right)\cosh\left(x\right)^2+2\left(a^4+2\right.}$$

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 +
 a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
 a)*sinh(x) - b)) - 2*(a^2*b + b^3)*cosh(x) - 2*(a^2*b + b^3)*sinh(x))/(a^4
 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(a^4 + 2*a^2*b^2
 + b^4)*cosh(x)*sinh(x) + (a^4 + 2*a^2*b^2 + b^4)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \int \frac{\tanh^2\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx = \frac{a^2\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}} - \frac{2\left(be^{(-x)}+a\right)}{a^2+b^2+(a^2+b^2)e^{(-2\,x)}}$$

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] a^2*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/
(a^2 + b^2)^(3/2) - 2*(b*e^(-x) + a)/(a^2 + b^2 + (a^2 + b^2)*e^(-2*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx=\frac{a^2\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\,(be^x-a)}{(a^2+b^2)(e^{(2\,x)}+1)}$$

```
[In] integrate(tanh(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a
^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*e^x - a)/((a^2 + b^2)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.78

$$\int \frac{\tanh^2(x)}{a+b\sinh(x)}\,dx=\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,e^x}{a^2+b^2}}{e^{2\,x}+1}-\frac{2\,\mathrm{atan}\left(\left(\frac{b^3\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}+\frac{a^2\,b\,\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}{2}\right)\left(e^x\left(\frac{2\,a^2}{b^2\,\sqrt{a^4}\,(a^2+b^2)^2}+\frac{2\left(a^3\,\sqrt{a^4}+a\right.}{a\,b^2\,\sqrt{-(a^2+b^2)^3}\,(a^2+b^2)\,\sqrt{-}}\right.\right.\right.}{\sqrt{-a^6-3\,a^4\,b^2-3\,a^2\,b^4-b^6}}$$

```
[In] int(tanh(x)^2/(a + b*sinh(x)),x)
```

```
[Out] ((2*a)/(a^2 + b^2) - (2*b*exp(x))/(a^2 + b^2))/(exp(2*x) + 1) - (2*atan(((b
^3*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/2 + (a^2*b*(- a^6 - b^6 - 3
*a^2*b^4 - 3*a^4*b^2)^(1/2))/2)*(exp(x)*((2*a^2)/(b^2*(a^4)^(1/2)*(a^2 + b^
2)^2) + (2*(a^3*(a^4)^(1/2) + a*b^2*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^3)^(
1/2)*(a^2 + b^2)*(- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))) - (2*(b^3*(a
^4)^(1/2) + a^2*b*(a^4)^(1/2)))/(a*b^2*(-(a^2 + b^2)^3)^(1/2)*(a^2 + b^2)*(
- a^6 - b^6 - 3*a^2*b^4 - 3*a^4*b^2)^(1/2))))*(a^4)^(1/2))/(- a^6 - b^6 - 3
*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

3.231 $\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 48

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{b\arctan(\sinh(x))}{a^2+b^2} + \frac{a\log(\cosh(x))}{a^2+b^2} - \frac{a\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] b*arctan(sinh(x))/(a^2+b^2)+a*ln(cosh(x))/(a^2+b^2)-a*ln(a+b*sinh(x))/(a^2+
b^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2800, 815, 649, 209, 266}

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{b\arctan(\sinh(x))}{a^2+b^2} - \frac{a\log(a+b\sinh(x))}{a^2+b^2} + \frac{a\log(\cosh(x))}{a^2+b^2}$$

```
[In] Int[Tanh[x]/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTan[Sinh[x]])/(a^2 + b^2) + (a*Log[Cosh[x]])/(a^2 + b^2) - (a*Log[a +
 b*Sinh[x]])/(a^2 + b^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{a}{\left(a^2+b^2\right)(a+x)}+\frac{-b^2-ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2}-\frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{a\log(a+b\sinh(x))}{a^2+b^2}+\frac{a\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}+\frac{b^2\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{b\arctan(\sinh(x))}{a^2+b^2}+\frac{a\log(\cosh(x))}{a^2+b^2}-\frac{a\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{(a-ib)\log(i-\sinh(x))+(a+ib)\log(i+\sinh(x))-2a\log(a+b\sinh(x))}{2(a^2+b^2)}$$

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x]),x]
```

```
[Out] ((a - I*b)*Log[I - Sinh[x]] + (a + I*b)*Log[I + Sinh[x]] - 2*a*Log[a + b*Si
nh[x]])/(2*(a^2 + b^2))
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{2a\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2a^2+2b^2}+\frac{2a\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+4b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{2a^2+2b^2}$	73
risch	$\frac{i\ln(e^x+i)b}{a^2+b^2}+\frac{\ln(e^x+i)a}{a^2+b^2}-\frac{i\ln(e^x-i)b}{a^2+b^2}+\frac{\ln(e^x-i)a}{a^2+b^2}-\frac{a\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{a^2+b^2}$	101

```
[In] int(tanh(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/(2*a^2+2*b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+4/(2*a^2+2*b^2)*(1
/2*a*ln(1+tanh(1/2*x)^2)+b*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{2\,b\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-a\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+a\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2+b^2}$$

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*b*arctan(cosh(x) + sinh(x)) - a*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + a*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2 + b^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \int \frac{\tanh(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(tanh(x)/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = -\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2} - \frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2} + \frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}$$

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(e^(-x))/(a^2 + b^2) - a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^2
+ b^2) + a*log(e^(-2*x) + 1)/(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.85

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = -\frac{ab\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^2b+b^3} + \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)b}{2\,(a^2+b^2)} + \frac{a\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^2+b^2)}$$

```
[In] integrate(tanh(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^2*b + b^3) + 1/2*(pi + 2*arctan(1
/2*(e^(2*x) - 1)*e^(-x)))*b/(a^2 + b^2) + 1/2*a*log((e^(-x) - e^x)^2 + 4)/(
a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx = \frac{\ln\left(e^x+1i\right)}{a-b\,1i} - \frac{a\,\ln\left(b^3\,e^{2\,x}-4\,a^2\,b-b^3+8\,a^3\,e^x+2\,a\,b^2\,e^x+4\,a^2\,b\,e^{2\,x}\right)}{a^2+b^2} + \frac{\ln\left(1+e^x\,1i\right)\,1i}{-b+a\,1i}$$

```
[In] int(tanh(x)/(a + b*sinh(x)),x)
```

```
[Out] (log(exp(x)*1i + 1)*1i)/(a*1i - b) + log(exp(x) + 1i)/(a - b*1i) - (a*log(b
^3*exp(2*x) - 4*a^2*b - b^3 + 8*a^3*exp(x) + 2*a*b^2*exp(x) + 4*a^2*b*exp(2
*x)))/(a^2 + b^2)
```

3.232 $\int \frac{\coth(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = \frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}$$

```
[Out] ln(sinh(x))/a-ln(a+b*sinh(x))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2800, 36, 29, 31}

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = \frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}$$

```
[In] Int[Coth[x]/(a + b*Sinh[x]),x]
```

```
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sinh(x)\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sinh(x)\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{a}\\ &= \frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = \frac{\log(\sinh(x))}{a} - \frac{\log(a+b\sinh(x))}{a}$$

```
[In] Integrate[Coth[x]/(a + b*Sinh[x]),x]
[Out] Log[Sinh[x]]/a - Log[a + b*Sinh[x]]/a
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{\ln(\mathrm{e}^{2x}-1)}{a}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a}$	33
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	36

```
[In] int(coth(x)/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*ln(exp(2*x)-1)-1/a*ln(exp(2*x)+2*a/b*exp(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\coth(x)}{a + b\sinh(x)}\,dx = -\frac{\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right) - \log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -(log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(x) - sin
h(x))))/a
```

Sympy [F]

$$\int \frac{\coth(x)}{a + b\sinh(x)}\,dx = \int \frac{\coth(x)}{a + b\sinh(x)}\,dx$$

```
[In] integrate(coth(x)/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\coth(x)}{a + b\sinh(x)}\,dx = -\frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a} + \frac{\log\left(e^{(-x)} + 1\right)}{a} + \frac{\log\left(e^{(-x)} - 1\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -log(-2*a*e^(-x) + b*e^(-2*x) - b)/a + log(e^(-x) + 1)/a + log(e^(-x) - 1)/
a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = -\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a + log(abs(-e^(-x) + e^x))/a
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 195, normalized size of antiderivative = 9.75

$$\int \frac{\coth(x)}{a+b\sinh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\sqrt{-a^2}+b\,\mathrm{e}^{x}\,\sqrt{-a^2}-2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^2}-b\,\mathrm{e}^{3\,x}\,\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}} - \frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,\sqrt{-a^2}+4\,a^2\,b^3\,\sqrt{-a^2}\right)\,\left(\frac{1}{8\,a\,b\,{\left(a^2+b^2\right)}^2}-\mathrm{e}^{x}\,\left(\frac{1}{16\,b^2\,{\left(a^2+b^2\right)}^2}-\frac{{\left(a^2+2\,b^2\right)}^2}{16\,a^4\,b^2\,{\left(a^2+b^2\right)}^2}\right)+\frac{a^2+2\,b^2}{8\,a^3\,b\,{\left(a^2+b^2\right)}^2}\right)\right)}{\sqrt{-a^2}}$$

```
[In] int(coth(x)/(a + b*sinh(x)),x)
```

```
[Out] (2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) - 2*a*exp(2*x)*(-a^2)^(1/2)
 - b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^(1
/2) + 4*a^2*b^3*(-a^2)^(1/2))*(1/(8*a*b*(a^2 + b^2)^2) - exp(x)*(1/(16*b^2*
(a^2 + b^2)^2) - (a^2 + 2*b^2)^2/(16*a^4*b^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2
)/(8*a^3*b*(a^2 + b^2)^2))))/(-a^2)^(1/2)
```

3.233 $\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx=\frac{b\text{arctanh}(\cosh(x))}{a^2}-\frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\coth(x)}{a}$$

```
[Out] b*arctanh(cosh(x))/a^2-coth(x)/a-2*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2
))*(a^2+b^2)^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 212}

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx=-\frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\text{arctanh}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[In] Int[Coth[x]^2/(a + b*Sinh[x]),x]
```

```
[Out] (b*ArcTanh[Cosh[x]])/a^2 - (2*Sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/a^2 - Coth[x]/a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\text{csch}^2(x)\left(1+\sinh^2(x)\right)}{a+b\sinh(x)}\,dx\\
&= -\frac{\coth(x)}{a}+\frac{i\int \frac{\text{csch}(x)(ib-ia\sinh(x))}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{\coth(x)}{a}-\frac{b\int \text{csch}(x)\,dx}{a^2}+\frac{(a^2+b^2)\int \frac{1}{a+b\sinh(x)}\,dx}{a^2}\\
&= \frac{b\text{arctanh}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}+\frac{(2(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\text{arctanh}(\cosh(x))}{a^2}-\frac{\coth(x)}{a}-\frac{(4(a^2+b^2))\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= \frac{b\text{arctanh}(\cosh(x))}{a^2}-\frac{2\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\coth(x)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

$$
\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = -\frac{4\sqrt{-a^2-b^2}\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\coth\left(\frac{x}{2}\right)-2b\log\left(\cosh\left(\frac{x}{2}\right)\right)+2b\log\left(\sinh\left(\frac{x}{2}\right)\right)+a\tanh\left(\frac{x}{2}\right)}{2a^2}
$$

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x]),x]
[Out] -1/2*(4*Sqrt[-a^2 - b^2]*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Cot
h[x/2] - 2*b*Log[Cosh[x/2]] + 2*b*Log[Sinh[x/2]] + a*Tanh[x/2])/a^2
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a}-\frac{(-4a^2-4b^2)\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{1}{2a\tanh\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$	81
risch	$-\frac{2}{a(e^{2x}-1)}+\frac{b\ln(e^x+1)}{a^2}-\frac{b\ln(e^x-1)}{a^2}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^x-\frac{-a+\sqrt{a^2+b^2}}{b}\right)}{a^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^x+\frac{a+\sqrt{a^2+b^2}}{b}\right)}{a^2}$	104

```
[In] int(coth(x)^2/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*tanh(1/2*x)-1/2/a^2*(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/2/a/tanh(1/2*x)-1/a^2*b*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(52) = 104$.

Time = 0.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.07

$$\int \frac{\coth^2(x)}{a + b\sinh(x)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1\right)\log\left(\frac{b^2\cosh(x)^2 + b^2\sinh(x)^2 + 2ab\cosh(x) + 2a^2 + b^2 + 2(b^2\cosh(}{b\cosh(x)^2 + b\sinh(x)^2 + 2a\cosh(x) +}\right.}{}$$

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log((b^2*c
osh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a
*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 +
 b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + (b*cosh(x)^2
 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) + 1) - (b*c
osh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(cosh(x) + sinh(x) - 1
) - 2*a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a + b\sinh(x)}\,dx = \int \frac{\coth^2(x)}{a + b\sinh(x)}\,dx$$

```
[In] integrate(coth(x)**2/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**2/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.73

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \frac{b\log\left(e^{(-x)}+1\right)}{a^2} - \frac{b\log\left(e^{(-x)}-1\right)}{a^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{a^2} + \frac{2}{ae^{(-2x)}-a}$$

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] b*log(e^(-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + sqrt(a^2 + b^2)*log((b*e^(-
x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/a^2 + 2/(a*e^(-
2*x) - a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \frac{b\log\left(e^x+1\right)}{a^2} - \frac{b\log\left(|e^x-1|\right)}{a^2} + \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{a^2} - \frac{2}{a(e^{(2x)}-1)}$$

```
[In] integrate(coth(x)^2/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] b*log(e^x + 1)/a^2 - b*log(abs(e^x - 1))/a^2 + sqrt(a^2 + b^2)*log(abs(2*b*
e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/a^2
- 2/(a*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 304, normalized size of antiderivative = 5.43

$$\int \frac{\coth^2(x)}{a+b\sinh(x)}\,dx = \frac{2}{a-a\,\mathrm{e}^{2\,x}} - \frac{b\,\ln\left(32\,a^2+32\,b^2-32\,a^2\,\mathrm{e}^x-32\,b^2\,\mathrm{e}^x\right)}{a^2} + \frac{b\,\ln\left(32\,a^2+32\,b^2+32\,a^2\,\mathrm{e}^x+32\,b^2\,\mathrm{e}^x\right)}{a^2} + \frac{\ln\left(128\,a^4\,\mathrm{e}^x-64\,a\,b^3-64\,a^3\,b-32\,b^3\,\sqrt{a^2+b^2}+32\,b^4\,\mathrm{e}^x+128\,a^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x-64\,a^2\,b\right.}{a^2} - \frac{\ln\left(32\,b^3\,\sqrt{a^2+b^2}-64\,a\,b^3-64\,a^3\,b+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2+b^2}+160\,a^2\,b^2\,\mathrm{e}^x+64\,a^2\,b\right.}{a^2}$$

```
[In] int(coth(x)^2/(a + b*sinh(x)),x)

[Out] 2/(a - a*exp(2*x)) - (b*log(32*a^2 + 32*b^2 - 32*a^2*exp(x) - 32*b^2*exp(x)
))/a^2 + (b*log(32*a^2 + 32*b^2 + 32*a^2*exp(x) + 32*b^2*exp(x)))/a^2 + (lo
g(128*a^4*exp(x) - 64*a*b^3 - 64*a^3*b - 32*b^3*(a^2 + b^2)^(1/2) + 32*b^4*
exp(x) + 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x) - 64*a^2*b*(
a^2 + b^2)^(1/2) + 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^
2 - (log(32*b^3*(a^2 + b^2)^(1/2) - 64*a*b^3 - 64*a^3*b + 128*a^4*exp(x) +
32*b^4*exp(x) - 128*a^3*exp(x)*(a^2 + b^2)^(1/2) + 160*a^2*b^2*exp(x) + 64*
a^2*b*(a^2 + b^2)^(1/2) - 96*a*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1
/2))/a^2
```

3.234 $\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{b\operatorname{csch}(x)}{a^2} - \frac{\operatorname{csch}^2(x)}{2a} + \frac{(a^2+b^2)\log(\sinh(x))}{a^3} - \frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}$$

```
[Out] b*csch(x)/a^2-1/2*csch(x)^2/a+(a^2+b^2)*ln(sinh(x))/a^3-(a^2+b^2)*ln(a+b*si
nh(x))/a^3
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2800, 908}

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{b\operatorname{csch}(x)}{a^2} + \frac{(a^2+b^2)\log(\sinh(x))}{a^3} - \frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3} - \frac{\operatorname{csch}^2(x)}{2a}$$

```
[In] Int[Coth[x]^3/(a + b*Sinh[x]),x]
```

```
[Out] (b*Csch[x])/a^2 - Csch[x]^2/(2*a) + ((a^2 + b^2)*Log[Sinh[x]])/a^3 - ((a^2
+ b^2)*Log[a + b*Sinh[x]])/a^3
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)}\,dx, x, b\sinh(x)\right)\\ &= -\text{Subst}\left(\int \left(-\frac{b^2}{ax^3}+\frac{b^2}{a^2x^2}+\frac{-a^2-b^2}{a^3x}+\frac{a^2+b^2}{a^3(a+x)}\right)dx, x, b\sinh(x)\right)\\ &= \frac{b\text{csch}(x)}{a^2}-\frac{\text{csch}^2(x)}{2a}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}-\frac{(a^2+b^2)\log(a+b\sinh(x))}{a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{2ab\text{csch}(x)-a^2\text{csch}^2(x)+2(a^2+b^2)\left(\log(\sinh(x))-\log(a+b\sinh(x))\right)}{2a^3}$$

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x]),x]
[Out] (2*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + b^2)*(Log[Sinh[x]] - Log[a + b*Si
nh[x]]))/(2*a^3)
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.88

method	result
risch	$-\frac{2\,e^{x}\left(-b\,e^{2x}+e^{x}a+b\right)}{\left(e^{2x}-1\right)^{2}a^{2}}+\frac{\ln\left(e^{2x}-1\right)}{a}+\frac{\ln\left(e^{2x}-1\right)b^{2}}{a^{3}}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)}{a}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}-1\right)b^{2}}{a^{3}}$
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2}a}{2}+2b\tanh\left(\frac{x}{2}\right)}{4a^{2}}+\frac{\left(-4a^{2}-4b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{4a^{3}}-\frac{1}{8a\tanh\left(\frac{x}{2}\right)^{2}}+\frac{\left(4a^{2}+4b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^{3}}+\frac{b}{2a^{2}\tan}$

```
[In] int(coth(x)^3/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*exp(x)*(-b*exp(2*x)+exp(x)*a+b)/(exp(2*x)-1)^2/a^2+1/a*ln(exp(2*x)-1)+1/
a^3*ln(exp(2*x)-1)*b^2-1/a*ln(exp(2*x)+2*a/b*exp(x)-1)-1/a^3*ln(exp(2*x)+2*
a/b*exp(x)-1)*b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(50) = 100$.

Time = 0.29 (sec) , antiderivative size = 427, normalized size of antiderivative = 8.21

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{2\,ab\cosh\left(x\right)^3+2\,ab\sinh\left(x\right)^3-2\,a^2\cosh\left(x\right)^2-2\,ab\cosh\left(x\right)+2\left(3\,ab\cosh\left(x\right)-a^2\right)\sinh\left(x\right)^2-\left(\left(a^2+b^2\right.\right.}{}$$

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 - 2*a*b*cosh(x) + 2*(3
*a*b*cosh(x) - a^2)*sinh(x)^2 - ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh
(x)*sinh(x)^3 + (a^2 + b^2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2
 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(
x)^3 - (a^2 + b^2)*cosh(x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(
x))) + ((a^2 + b^2)*cosh(x)^4 + 4*(a^2 + b^2)*cosh(x)*sinh(x)^3 + (a^2 + b^
2)*sinh(x)^4 - 2*(a^2 + b^2)*cosh(x)^2 + 2*(3*(a^2 + b^2)*cosh(x)^2 - a^2 -
 b^2)*sinh(x)^2 + a^2 + b^2 + 4*((a^2 + b^2)*cosh(x)^3 - (a^2 + b^2)*cosh(x
))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)^2 - 2*a^2
*cosh(x) - a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sin
h(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 + 4*(a
^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx=\int \frac{\coth^3\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*sinh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(50) = 100$.

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.23

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = -\frac{2\left(be^{(-x)}-ae^{(-2x)}-be^{(-3x)}\right)}{2a^2e^{(-2x)}-a^2e^{(-4x)}-a^2} - \frac{(a^2+b^2)\log\left(-2ae^{(-x)}+be^{(-2x)}-b\right)}{a^3} + \frac{(a^2+b^2)\log\left(e^{(-x)}+1\right)}{a^3} + \frac{(a^2+b^2)\log\left(e^{(-x)}-1\right)}{a^3}$$

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -2*(b*e^(-x) - a*e^(-2*x) - b*e^(-3*x))/(2*a^2*e^(-2*x) - a^2*e^(-4*x) - a^
2) - (a^2 + b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^3 + (a^2 + b^2)*log(e^
(-x) + 1)/a^3 + (a^2 + b^2)*log(e^(-x) - 1)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.40

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{(a^2+b^2)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^3} - \frac{(a^2b+b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2a\right|\right)}{a^3b} - \frac{3a^2\left(e^{(-x)}-e^x\right)^2+3b^2\left(e^{(-x)}-e^x\right)^2+4ab\left(e^{(-x)}-e^x\right)+4a^2}{2a^3\left(e^{(-x)}-e^x\right)^2}$$

```
[In] integrate(coth(x)^3/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(-e^(-x) + e^x))/a^3 - (a^2*b + b^3)*log(abs(-b*(e^(-x)
- e^x) + 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) - e^x)^2 + 3*b^2*(e^(-x) - e^x)
^2 + 4*a*b*(e^(-x) - e^x) + 4*a^2)/(a^3*(e^(-x) - e^x)^2)
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 1163, normalized size of antiderivative = 22.37

$$\int \frac{\coth^3(x)}{a+b\sinh(x)}\,dx = \frac{\left(2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{-a^6}\,\sqrt{a^4+2\,a^2\,b^2+b^4}+2\,b^2\,\sqrt{-a^6}\,\sqrt{a^4+2\,a^2\,b^2+b^4}}{2\,a^3\,\left(a^2+b^2\right)^2}+\frac{\left(a^7+a^5\,b^2\right)\,\sqrt{-a^6}}{2\,a^6\,\sqrt{\left(a^2+b^2\right)^2}\,\left(a^2+b^2\right)}-\frac{a^6\,b^2\,\mathrm{e}^x\,\sqrt{-a^6}\,\left(\frac{8\,\left(a^4+2\,a^2\,b^2+b^4\right)}{a^8\,b\,\left(a^2+b^2\right)^2}-\frac{4\,(2\cdots}{\cdots}\right.}{\cdots}\right.\right.}{\cdots}$$

$$-\frac{2}{a\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)}-\frac{\frac{2}{a}-\frac{2\,b\,\mathrm{e}^x}{a^2}}{\mathrm{e}^{2\,x}-1}$$

```
[In] int(coth(x)^3/(a + b*sinh(x)),x)
```

```
[Out] ((2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/
2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(2*a^3*(a^2 + b^2)^2) + ((a^7 + a^5*b^2)*
(-a^6)^(1/2))/(2*a^6*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (a^6*b^2*exp(x)*(
-a^6)^(1/2)*((8*(a^4 + b^4 + 2*a^2*b^2))/(a^8*b*(a^2 + b^2)^2) - (4*(2*a^6*
b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2)^(1/
2)*(a^2 + b^2)) + (2*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b
^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^6)^(1/2)*
(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^
(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*(-a^6)^(1/2)*(a^2 + b^2)^2)
))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)) - (a^6*b^2*exp(2*x)*(-a^6)^(1/2)*((4*(
a^2 + 2*b^2)*(a^4 + b^4 + 2*a^2*b^2))/(a^9*b^2*(a^2 + b^2)^2) + (4*(a^2*(-a
^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 + 2
*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^9*b^2*(-a^6)^(1/2)*(a^2
+ b^2)^2) + (2*(2*a^6*b + 2*a^4*b^3)*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^11*b
^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) + (4*(a^7 + a^5*b^2)*(a^4 + b^4 + 2*a
^2*b^2)^(1/2))/(a^12*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2))))/(8*(a^4 + b^4
 + 2*a^2*b^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 + a^5*b^2)*(a^4 + b^4 + 2
*a^2*b^2)^(1/2))/(a^11*b^3*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2
*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 + 2*a^2*b^2)^(1/2) + 2*b^2*(-a^6)^(1/2)*
(a^4 + b^4 + 2*a^2*b^2)^(1/2))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(a^10*b^3*(-a
^6)^(1/2)*(a^2 + b^2)^2))*(-a^6)^(1/2))/(8*(a^4 + b^4 + 2*a^2*b^2)^(1/2)))
- 2*atan((4*a^6*b*(-a^6)^(1/2)*(a^2 + b^2)^2 + 4*a^4*b^3*(-a^6)^(1/2)*(a^2
+ b^2)^2)*(1/(8*a^5*b*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3) - exp(x)*(1/(16*
a^4*b^2*((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3) - (a^2 + 2*b^2)^2/(16*a^8*b^2*
((a^2 + b^2)^2)^(1/2)*(a^2 + b^2)^3)) + (a^2 + 2*b^2)/(8*a^7*b*((a^2 + b^2)
^2)^(1/2)*(a^2 + b^2)^3))))*(a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-a^6)^(1/2) - 2
/(a*(exp(4*x) - 2*exp(2*x) + 1)) - (2/a - (2*b*exp(x))/a^2)/(exp(2*x) - 1)
```

3.235 $\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{b(3a^2+2b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} - \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4} - \frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

```
[Out] 1/2*b*(3*a^2+2*b^2)*arctanh(cosh(x))/a^4-2*(a^2+b^2)^(3/2)*arctanh((b-a*tan
h(1/2*x))/(a^2+b^2)^(1/2))/a^4-1/3*(4*a^2+3*b^2)*coth(x)/a^3+1/2*b*coth(x)*
csch(x)/a^2-1/3*coth(x)*csch(x)^2/a
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2804, 3134, 3080, 3855, 2739, 632, 212}

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{b\coth(x)\operatorname{csch}(x)}{2a^2} + \frac{b(3a^2+2b^2)\operatorname{arctanh}(\cosh(x))}{2a^4} - \frac{2(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4} - \frac{(4a^2+3b^2)\coth(x)}{3a^3} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a}$$

```
[In] Int[Coth[x]^4/(a + b*Sinh[x]),x]
```

```
[Out] (b*(3*a^2 + 2*b^2)*ArcTanh[Cosh[x]])/(2*a^4) - (2*(a^2 + b^2)^(3/2)*ArcTanh
[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^4 - ((4*a^2 + 3*b^2)*Coth[x])/(3*a^3
) + (b*Coth[x]*Csch[x])/(2*a^2) - (Coth[x]*Csch[x]^2)/(3*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2804

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(6*a^2), Int[((a + b*Sin[e + f*x])^m/Sin[e + f*
x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x] - (6*a^2 - b^2*
m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[b*(m - 2)*Cos[e + f*x]*((a + b
*Sin[e + f*x])^(m + 1)/(6*a^2*f*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} \\
&\quad + \frac{\int \frac{\text{csch}^2(x)\left(2(4a^2+3b^2)-ab\sinh(x)+3(2a^2+b^2)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{6a^2} \\
&= -\frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} \\
&\quad + \frac{i\int \frac{\text{csch}(x)\left(3ib(3a^2+2b^2)-3ia(2a^2+b^2)\sinh(x)\right)}{a+b\sinh(x)}\,dx}{6a^3} \\
&= -\frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a} \\
&\quad + \frac{(a^2+b^2)^2\int \frac{1}{a+b\sinh(x)}\,dx}{a^4} - \frac{(b(3a^2+2b^2))\int \text{csch}(x)\,dx}{2a^4} \\
&= \frac{b(3a^2+2b^2)\,\text{arctanh}(\cosh(x))}{2a^4} - \frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} \\
&\quad - \frac{\coth(x)\text{csch}^2(x)}{3a} + \frac{\left(2(a^2+b^2)^2\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^4} \\
&= \frac{b(3a^2+2b^2)\,\text{arctanh}(\cosh(x))}{2a^4} - \frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} \\
&\quad - \frac{\coth(x)\text{csch}^2(x)}{3a} - \frac{\left(4(a^2+b^2)^2\right)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^4} \\
&= \frac{b(3a^2+2b^2)\,\text{arctanh}(\cosh(x))}{2a^4} - \frac{2(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^4} \\
&\quad - \frac{(4a^2+3b^2)\coth(x)}{3a^3} + \frac{b\coth(x)\text{csch}(x)}{2a^2} - \frac{\coth(x)\text{csch}^2(x)}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.82

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx = \frac{48(-a^2-b^2)^{3/2}\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)-4a(4a^2+3b^2)\coth\left(\frac{x}{2}\right)+3a^2b\,\mathrm{csch}^2\left(\frac{x}{2}\right)+12b(3a^2+2b^2)\log\left(\cosh\right.}{}$$

[In] Integrate[Coth[x]^4/(a + b*Sinh[x]),x]

[Out] (48*(-a^2 - b^2)^(3/2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] - 4*a*(4*a^2 + 3*b^2)*Coth[x/2] + 3*a^2*b*Csch[x/2]^2 + 12*b*(3*a^2 + 2*b^2)*Log[Cosh[x/2]] - 12*b*(3*a^2 + 2*b^2)*Log[Sinh[x/2]] + 3*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x])/2 - 4*a*(4*a^2 + 3*b^2)*Tanh[x/2])/(24*a^4)

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\frac{a^2\tanh\left(\frac{x}{2}\right)^3}{3}+ab\tanh\left(\frac{x}{2}\right)^2+5a^2\tanh\left(\frac{x}{2}\right)+4b^2\tanh\left(\frac{x}{2}\right)}{8a^3}-\frac{(-16a^4-32a^2b^2-16b^4)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{8a^4\sqrt{a^2+b^2}}-\frac{1}{24\tanh\left(\frac{x}{2}\right)^3 a}$
risch	$-\frac{-3ab\,e^{5x}+12\,e^{4x}a^2+6b^2e^{4x}-12\,e^{2x}a^2-12\,e^{2x}b^2+3b\,e^{x}a+8a^2+6b^2}{3(e^{2x}-1)^3a^3}+\frac{3b\ln(e^x+1)}{2a^2}+\frac{b^3\ln(e^x+1)}{a^4}+\frac{(a^2+b^2)^{\frac{3}{2}}\ln\left(e^x-\frac{(a^2+b}{b}\right.}{a^4}$

[In] int(coth(x)^4/(a+b*sinh(x)),x,method=_RETURNVERBOSE)

[Out] -1/8/a^3*(1/3*a^2*tanh(1/2*x)^3+a*b*tanh(1/2*x)^2+5*a^2*tanh(1/2*x)+4*b^2*tanh(1/2*x))-1/8/a^4*(-16*a^4-32*a^2*b^2-16*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/24/tanh(1/2*x)^3/a-1/8*(5*a^2+4*b^2)/a^3/tanh(1/2*x)+1/8*b/tanh(1/2*x)^2/a^2-1/2/a^4*b*(3*a^2+2*b^2)*ln(tanh(1/2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(96) = 192$.

Time = 0.34 (sec) , antiderivative size = 1303, normalized size of antiderivative = 12.06

$$\int \frac{\coth^4(x)}{a + b\sinh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*a^2*b*cosh(x)^5 + 6*a^2*b*sinh(x)^5 - 12*(2*a^3 + a*b^2)*cosh(x)^4 +
 6*(5*a^2*b*cosh(x) - 4*a^3 - 2*a*b^2)*sinh(x)^4 - 6*a^2*b*cosh(x) + 12*(5*
a^2*b*cosh(x)^2 - 4*(2*a^3 + a*b^2)*cosh(x))*sinh(x)^3 - 16*a^3 - 12*a*b^2
+ 24*(a^3 + a*b^2)*cosh(x)^2 + 12*(5*a^2*b*cosh(x)^3 + 2*a^3 + 2*a*b^2 - 6*
(2*a^3 + a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((a^2 + b^2)*cosh(x)^6 + 6*(a^2 +
b^2)*cosh(x)*sinh(x)^5 + (a^2 + b^2)*sinh(x)^6 - 3*(a^2 + b^2)*cosh(x)^4 +
3*(5*(a^2 + b^2)*cosh(x)^2 - a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 + b^2)*cosh(x
)^3 - 3*(a^2 + b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 + b^2)*cosh(x)^2 + 3*(5*(a^
2 + b^2)*cosh(x)^4 - 6*(a^2 + b^2)*cosh(x)^2 + a^2 + b^2)*sinh(x)^2 - a^2 -
 b^2 + 6*((a^2 + b^2)*cosh(x)^5 - 2*(a^2 + b^2)*cosh(x)^3 + (a^2 + b^2)*cos
h(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b
*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) - b)) + 3*((3*a^2*b + 2*b^3)*cosh(x)^6 + 6*(3*a^2*b +
2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh(x)^6 - 3*(3*a^2*b + 2*b^3
)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^4
 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b + 2*b^3)*cosh(x))*sinh(x)^
3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)^2 + 3*(5*(3*a^2*b + 2*b^3
)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^3)*cosh(x)^2)*sinh(x)^2 +
6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b^3)*cosh(x)^3 + (3*a^2*b +
 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b + 2*b^3)
*cosh(x)^6 + 6*(3*a^2*b + 2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b + 2*b^3)*sinh
(x)^6 - 3*(3*a^2*b + 2*b^3)*cosh(x)^4 - 3*(3*a^2*b + 2*b^3 - 5*(3*a^2*b + 2
*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(3*a^2*b + 2*b^3)*cosh(x)^3 - 3*(3*a^2*b
+ 2*b^3)*cosh(x))*sinh(x)^3 - 3*a^2*b - 2*b^3 + 3*(3*a^2*b + 2*b^3)*cosh(x)
^2 + 3*(5*(3*a^2*b + 2*b^3)*cosh(x)^4 + 3*a^2*b + 2*b^3 - 6*(3*a^2*b + 2*b^
3)*cosh(x)^2)*sinh(x)^2 + 6*((3*a^2*b + 2*b^3)*cosh(x)^5 - 2*(3*a^2*b + 2*b
^3)*cosh(x)^3 + (3*a^2*b + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) -
 1) + 6*(5*a^2*b*cosh(x)^4 - 8*(2*a^3 + a*b^2)*cosh(x)^3 - a^2*b + 8*(a^3 +
 a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*si
nh(x)^6 - 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sin
h(x)^4 - a^4 + 4*(5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cos
h(x)^4 - 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x
)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a + b\sinh(x)}\,dx = \int \frac{\coth^4(x)}{a + b\sinh(x)}\,dx$$

```
[In] integrate(coth(x)**4/(a+b*sinh(x)),x)
[Out] Integral(coth(x)**4/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(96) = 192$.

Time = 0.32 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.96

$$\begin{aligned}\int \frac{\coth^4(x)}{a + b\sinh(x)}\,dx &= -\frac{3\,abe^{(-x)} - 3\,abe^{(-5\,x)} - 8\,a^2 - 6\,b^2 + 12\,(a^2 + b^2)e^{(-2\,x)} - 6\,(2\,a^2 + b^2)e^{(-4\,x)}}{3\,(3\,a^3e^{(-2\,x)} - 3\,a^3e^{(-4\,x)} + a^3e^{(-6\,x)} - a^3)}\\&\quad + \frac{(3\,a^2b + 2\,b^3)\log\left(e^{(-x)} + 1\right)}{2\,a^4} - \frac{(3\,a^2b + 2\,b^3)\log\left(e^{(-x)} - 1\right)}{2\,a^4}\\&\quad + \frac{(a^4 + 2\,a^2b^2 + b^4)\log\left(\frac{be^{(-x)} - a - \sqrt{a^2+b^2}}{be^{(-x)} - a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2 + b^2}a^4}\end{aligned}$$

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -1/3*(3*a*b*e^(-x) - 3*a*b*e^(-5*x) - 8*a^2 - 6*b^2 + 12*(a^2 + b^2)*e^(-2*
x) - 6*(2*a^2 + b^2)*e^(-4*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(-6
*x) - a^3) + 1/2*(3*a^2*b + 2*b^3)*log(e^(-x) + 1)/a^4 - 1/2*(3*a^2*b + 2*b
^3)*log(e^(-x) - 1)/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log((b*e^(-x) - a - sqrt(
a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(96) = 192$.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.80

$$
\begin{aligned}
&\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx \\
&= \frac{(3a^2b+2b^3)\log(e^x+1)}{2a^4} - \frac{(3a^2b+2b^3)\log(|e^x-1|)}{2a^4} \\
&\quad + \frac{(a^4+2a^2b^2+b^4)\log\left(\frac{\left|2be^x+2a-2\sqrt{a^2+b^2}\right|}{\left|2be^x+2a+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^4} \\
&\quad + \frac{3abe^{(5x)}-12a^2e^{(4x)}-6b^2e^{(4x)}+12a^2e^{(2x)}+12b^2e^{(2x)}-3abe^x-8a^2-6b^2}{3a^3(e^{(2x)}-1)^3}
\end{aligned}
$$

```
[In] integrate(coth(x)^4/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(3*a^2*b + 2*b^3)*log(e^x + 1)/a^4 - 1/2*(3*a^2*b + 2*b^3)*log(abs(e^x
- 1))/a^4 + (a^4 + 2*a^2*b^2 + b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^
2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/3*(3*
a*b*e^(5*x) - 12*a^2*e^(4*x) - 6*b^2*e^(4*x) + 12*a^2*e^(2*x) + 12*b^2*e^(2
*x) - 3*a*b*e^x - 8*a^2 - 6*b^2)/(a^3*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 778, normalized size of antiderivative = 7.20

$$\int \frac{\coth^4(x)}{a+b\sinh(x)}\,dx$$

$$= \frac{\ln\left(-\frac{8\left(-30\,e^x a^9+18\,a^8 b-101\,e^x a^7 b^2+60\,a^6 b^3-126\,e^x a^5 b^4+74\,a^4 b^5-69\,e^x a^3 b^6+40\,a^2 b^7-14\,e^x a b^8+8\,b^9\right)}{a^9 b^3}-\frac{\sqrt{(a^2+b^2)^3}\left(\frac{8\left(4\,a^8-36 \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$-\frac{\frac{2\left(2\,a^2+b^2\right)}{a^3}-\frac{b\,e^x}{a^2}}{e^{2x}-1}-\frac{\frac{4}{a}-\frac{2\,b\,e^x}{a^2}}{e^{4x}-2\,e^{2x}+1}$$

$$-\frac{\ln\left(\frac{\sqrt{(a^2+b^2)^3}\left(\frac{8\left(4\,a^8-36\,e^x a^7 b+34\,a^6 b^2-75\,e^x a^5 b^3+57\,a^4 b^4-52\,e^x a^3 b^5+36\,a^2 b^6-12\,e^x a b^7+8\,b^8\right)}{a^6 b^4}+\frac{\left(\frac{16\left(-8\,e^x a^5+4\,a^4 b-15\,e^x a^3 b^2+8\,a^2 b \cdots\right.}{a\,b^5}\cdots\right.}{\cdots}\right.}{a^4}\cdots\right.}{\cdots}$$

$$-\frac{8}{3\,a\,\left(3\,e^{2x}-3\,e^{4x}+e^{6x}-1\right)}-\frac{\ln\left(e^x-1\right)\,\left(3\,a^2\,b+2\,b^3\right)}{2\,a^4}+\frac{\ln\left(e^x+1\right)\,\left(3\,a^2\,b+2\,b^3\right)}{2\,a^4}$$

```
[In] int(coth(x)^4/(a + b*sinh(x)),x)
```

```
[Out] (log(- (8*(18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 30*a^9
*exp(x) - 14*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) - 101*a^
7*b^2*exp(x)))/(a^9*b^3) - (((a^2 + b^2)^3)^(1/2)*((8*(4*a^8 + 8*b^8 + 36*a
^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*exp(x) - 36*a^7*b*exp(x) - 52*a
^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^4) - (((16*(4*a^4*b + 4*b^5 + 8*
a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) - 15*a^3*b^2*exp(x)))/(a*b^5) + (32
*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(
x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a^4))/a^4)*((a^2 + b^2)^3)^(1/2))/a^
4 - ((2*(2*a^2 + b^2))/a^3 - (b*exp(x))/a^2)/(exp(2*x) - 1) - (4/a - (2*b*e
xp(x))/a^2)/(exp(4*x) - 2*exp(2*x) + 1) - (log((((a^2 + b^2)^3)^(1/2)*((8*(
4*a^8 + 8*b^8 + 36*a^2*b^6 + 57*a^4*b^4 + 34*a^6*b^2 - 12*a*b^7*exp(x) - 36
*a^7*b*exp(x) - 52*a^3*b^5*exp(x) - 75*a^5*b^3*exp(x)))/(a^6*b^4) + (((16*(
4*a^4*b + 4*b^5 + 8*a^2*b^3 - 8*a^5*exp(x) - 7*a*b^4*exp(x) - 15*a^3*b^2*ex
```

```
p(x)))/(a*b^5) - (32*((a^2 + b^2)^3)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp
(x) - 3*a^3*b^2*exp(x)))/(a^4*b^5))*((a^2 + b^2)^3)^(1/2))/a^4))/a^4 - (8*(
18*a^8*b + 8*b^9 + 40*a^2*b^7 + 74*a^4*b^5 + 60*a^6*b^3 - 30*a^9*exp(x) - 1
4*a*b^8*exp(x) - 69*a^3*b^6*exp(x) - 126*a^5*b^4*exp(x) - 101*a^7*b^2*exp(x
)))/(a^9*b^3))*((a^2 + b^2)^3)^(1/2))/a^4 - 8/(3*a*(3*exp(2*x) - 3*exp(4*x)
 + exp(6*x) - 1)) - (log(exp(x) - 1)*(3*a^2*b + 2*b^3))/(2*a^4) + (log(exp(
x) + 1)*(3*a^2*b + 2*b^3))/(2*a^4)
```

3.236 $\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 224

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a^5\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{8a^3b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3} - \frac{(a^2-b^2)\tanh^3(x)}{3(a^2+b^2)^2}$$

```
[Out] -2*a^5*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)+8*a^3*b^2
*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-4*a^3*b*sech(x)
/(a^2+b^2)^3+2/3*a*b*sech(x)^3/(a^2+b^2)^2-a^4*b*cosh(x)/(a^2+b^2)^3/(a+b*s
inh(x))+(a^2-b^2)*tanh(x)/(a^2+b^2)^2-(2*a^4-3*a^2*b^2-b^4)*tanh(x)/(a^2+b^
2)^3-1/3*(a^2-b^2)*tanh(x)^3/(a^2+b^2)^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used

$= \{2810, 2743, 12, 2739, 632, 212, 2748, 3852, 8\}$

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{(a^2-b^2)\tanh^3(x)}{3\,(a^2+b^2)^2} + \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} + \frac{2ab\text{sech}^3(x)}{3\,(a^2+b^2)^2} - \frac{2a^5\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3\,(a+b\sinh(x))} - \frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3} + \frac{8a^3b^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\text{sech}(x)}{(a^2+b^2)^3}$$

```
[In] Int[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^5*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) + (8*
a^3*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) - (4*
a^3*b*Sech[x])/(a^2 + b^2)^3 + (2*a*b*Sech[x]^3)/(3*(a^2 + b^2)^2) - (a^4*b
*Cosh[x])/((a^2 + b^2)^3*(a + b*Sinh[x])) + ((a^2 - b^2)*Tanh[x])/(a^2 + b^
2)^2 - ((2*a^4 - 3*a^2*b^2 - b^4)*Tanh[x])/(a^2 + b^2)^3 - ((a^2 - b^2)*Tan
h[x]^3)/(3*(a^2 + b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{a^4}{(a^2+b^2)^2 (a+b\sinh(x))^2} - \frac{4a^3b^2}{(a^2+b^2)^3 (a+b\sinh(x))} + \frac{\operatorname{sech}^4(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)}{(a^2+b^2)^2} + \frac{\operatorname{sech}^2(x)\left(-2a^4\left(1-\frac{3a^2b^2+b^4}{2a^4}\right)+4a^3b\sinh(x)\right)}{(a^2+b^2)^3} \right) dx$$

$$= \frac{\int \operatorname{sech}^2(x) \left(-2a^4\left(1-\frac{3a^2b^2+b^4}{2a^4}\right)+4a^3b\sinh(x)\right)\,dx}{(a^2+b^2)^3} - \frac{(4a^3b^2)\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3} + \frac{\int \operatorname{sech}^4(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)\,dx}{(a^2+b^2)^2} + \frac{a^4\int \frac{1}{(a+b\sinh(x))^2}\,dx}{(a^2+b^2)^2}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{a^4\int \frac{a}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3} - \frac{(8a^3b^2)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3} + \frac{(a^2-b^2)\int \operatorname{sech}^4(x)\,dx}{(a^2+b^2)^2} - \frac{(2a^4-3a^2b^2-b^4)\int \operatorname{sech}^2(x)\,dx}{(a^2+b^2)^3}$$

$$= -\frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{a^5\int \frac{1}{a+b\sinh(x)}\,dx}{(a^2+b^2)^3} + \frac{(16a^3b^2)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3} + \frac{(i(a^2-b^2))\operatorname{Subst}\left(\int (1+x^2)\,dx, x, -i\tanh(x)\right)}{(a^2+b^2)^2} - \frac{(i(2a^4-3a^2b^2-b^4))\operatorname{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)}{(a^2+b^2)^3}$$

$$= \frac{8a^3b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3} - \frac{(a^2-b^2)\tanh^3(x)}{3(a^2+b^2)^2} + \frac{(2a^5)\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= \frac{8a^3b^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\operatorname{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\operatorname{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3} - \frac{(a^2-b^2)\tanh^3(x)}{3(a^2+b^2)^2} - \frac{(4a^5)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^3}$$

$$= -\frac{2a^5\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{8a^3b^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{4a^3b\text{sech}(x)}{(a^2+b^2)^3} + \frac{2ab\text{sech}^3(x)}{3(a^2+b^2)^2} - \frac{a^4b\cosh(x)}{(a^2+b^2)^3(a+b\sinh(x))} + \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{(2a^4-3a^2b^2-b^4)\tanh(x)}{(a^2+b^2)^3} - \frac{(a^2-b^2)\tanh^3(x)}{3(a^2+b^2)^2}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.64

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{6a^3(a^2-4b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 12a^3b\text{sech}(x) - \frac{3a^4b\cosh(x)}{a+b\sinh(x)} + (a^2+b^2)\text{sech}^3(x)\left(2ab+(a^2-b^2)\sinh(x)\right) + (}{3(a^2+b^2)^3}$$

```
[In] Integrate[Tanh[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] ((6*a^3*(a^2 - 4*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2
 - b^2] - 12*a^3*b*Sech[x] - (3*a^4*b*Cosh[x])/(a + b*Sinh[x]) + (a^2 + b^2
)*Sech[x]^3*(2*a*b + (a^2 - b^2)*Sinh[x]) + (-4*a^4 + 9*a^2*b^2 + b^4)*Tanh
[x])/(3*(a^2 + b^2)^3)
```

Maple [A] (verified)

Time = 2.70 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.09

method	result
default	$-\frac{2a^3\left(\frac{-b^2\tanh\left(\frac{x}{2}\right)-ab}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{(a^2-4b^2)\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)} + \frac{2(-a^4+3a^2b^2)\tanh\left(\frac{x}{2}\right)^5+2(-2a^3b+2b^3a)\tanh\left(\frac{x}{2}\right)^4}{}$
risch	$\frac{2a^5e^{7x}-8a^3b^2e^{7x}-14a^4b\,e^{6x}-6a^2b^3e^{6x}-2b^5e^{6x}+14a^5e^{5x}-\frac{44a^3b^2e^{5x}}{3}+\frac{4a\,b^4e^{5x}}{3}-\frac{82a^4b\,e^{4x}}{3}+\frac{14a^2b^3e^{4x}}{3}+2b^5e^{4x}+14a^5e^{3x}-\frac{64a^3b^2e}{3}}{(a^6+3a^4b^2+3a^2b^4+b^6)(1+e^{2x})^3(b\,e^{2x}+2\,e}$

```
[In] int(tanh(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^3/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2*tanh(1/2*x)-a*b)/(tanh(1/2*x)^2
*a-2*b*tanh(1/2*x)-a)-(a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2
*x)-2*b)/(a^2+b^2)^(1/2)))+2/(a^2+b^2)^3*((-a^4+3*a^2*b^2)*tanh(1/2*x)^5+(-
```

```
2*a^3*b+2*a*b^3)*tanh(1/2*x)^4+(-10/3*a^4+6*a^2*b^2+4/3*b^4)*tanh(1/2*x)^3-
8*tanh(1/2*x)^2*a^3*b+(-a^4+3*a^2*b^2)*tanh(1/2*x)-10/3*a^3*b+2/3*b^3*a)/(1
+tanh(1/2*x)^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3534 vs. $2(212) = 424$.

Time = 0.32 (sec) , antiderivative size = 3534, normalized size of antiderivative = 15.78

$$\int \frac{\tanh^4(x)}{(a + b\sinh(x))^2}\, dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^7 + 6*(a^7 - 3*a^5*b^2 - 4*a^
3*b^4)*sinh(x)^7 - 14*a^6*b + 4*a^4*b^3 + 20*a^2*b^5 + 2*b^7 - 6*(7*a^6*b +
 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^6 - 6*(7*a^6*b + 10*a^4*b^3 + 4*a^2*
b^5 + b^7 - 7*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x))*sinh(x)^6 + 2*(21*a^7
- a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^5 + 2*(21*a^7 - a^5*b^2 - 20*a^3*
b^4 + 2*a*b^6 + 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^2 - 18*(7*a^6*b +
10*a^4*b^3 + 4*a^2*b^5 + b^7)*cosh(x))*sinh(x)^5 - 2*(41*a^6*b + 34*a^4*b^3
 - 10*a^2*b^5 - 3*b^7)*cosh(x)^4 - 2*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 -
3*b^7 - 105*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^3 + 45*(7*a^6*b + 10*a^4*
b^3 + 4*a^2*b^5 + b^7)*cosh(x)^2 - 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b
^6)*cosh(x))*sinh(x)^4 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cos
h(x)^3 + 2*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6 + 105*(a^7 - 3*a^5*b
^2 - 4*a^3*b^4)*cosh(x)^4 - 60*(7*a^6*b + 10*a^4*b^3 + 4*a^2*b^5 + b^7)*cos
h(x)^3 + 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)*cosh(x)^2 - 4*(41*a^6
*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x))*sinh(x)^3 - 2*(35*a^6*b + 26
*a^4*b^3 - 8*a^2*b^5 + b^7)*cosh(x)^2 - 2*(35*a^6*b + 26*a^4*b^3 - 8*a^2*b^
5 + b^7 - 63*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^5 + 45*(7*a^6*b + 10*a^4
*b^3 + 4*a^2*b^5 + b^7)*cosh(x)^4 - 10*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a
*b^6)*cosh(x)^3 + 6*(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^2
- 3*(21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x))*sinh(x)^2 - 3*((a
^5*b - 4*a^3*b^3)*cosh(x)^8 + (a^5*b - 4*a^3*b^3)*sinh(x)^8 + 2*(a^6 - 4*a^
4*b^2)*cosh(x)^7 + 2*(a^6 - 4*a^4*b^2 + 4*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh
(x)^7 + 2*(a^5*b - 4*a^3*b^3)*cosh(x)^6 + 2*(a^5*b - 4*a^3*b^3 + 14*(a^5*b
- 4*a^3*b^3)*cosh(x)^2 + 7*(a^6 - 4*a^4*b^2)*cosh(x))*sinh(x)^6 - a^5*b + 4
*a^3*b^3 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 2*(3*a^6 - 12*a^4*b^2 + 28*(a^5*
b - 4*a^3*b^3)*cosh(x)^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^2 + 6*(a^5*b - 4*a^
3*b^3)*cosh(x))*sinh(x)^5 + 10*(7*(a^5*b - 4*a^3*b^3)*cosh(x)^4 + 7*(a^6 -
4*a^4*b^2)*cosh(x)^3 + 3*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 3*(a^6 - 4*a^4*b^2
)*cosh(x))*sinh(x)^4 + 6*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 2*(3*a^6 - 12*a^4*b^
2 + 28*(a^5*b - 4*a^3*b^3)*cosh(x)^5 + 35*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 20*
```

(a^5*b - 4*a^3*b^3)*cosh(x)^3 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^2)*sinh(x)^3 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x)^2 + 2*(14*(a^5*b - 4*a^3*b^3)*cosh(x)^6 - a^
5*b + 4*a^3*b^3 + 21*(a^6 - 4*a^4*b^2)*cosh(x)^5 + 15*(a^5*b - 4*a^3*b^3)*c
osh(x)^4 + 30*(a^6 - 4*a^4*b^2)*cosh(x)^3 + 9*(a^6 - 4*a^4*b^2)*cosh(x))*si
nh(x)^2 + 2*(a^6 - 4*a^4*b^2)*cosh(x) + 2*(4*(a^5*b - 4*a^3*b^3)*cosh(x)^7
 + 7*(a^6 - 4*a^4*b^2)*cosh(x)^6 + a^6 - 4*a^4*b^2 + 6*(a^5*b - 4*a^3*b^3)*c
osh(x)^5 + 15*(a^6 - 4*a^4*b^2)*cosh(x)^4 + 9*(a^6 - 4*a^4*b^2)*cosh(x)^2 -
 2*(a^5*b - 4*a^3*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2
 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)) + 2*(11*a^7 + 5*a^5*b^
2 - 8*a^3*b^4 - 2*a*b^6)*cosh(x) + 2*(11*a^7 + 5*a^5*b^2 - 8*a^3*b^4 - 2*a*
b^6 + 21*(a^7 - 3*a^5*b^2 - 4*a^3*b^4)*cosh(x)^6 - 18*(7*a^6*b + 10*a^4*b^3
 + 4*a^2*b^5 + b^7)*cosh(x)^5 + 5*(21*a^7 - a^5*b^2 - 20*a^3*b^4 + 2*a*b^6)
cosh(x)^4 - 4(41*a^6*b + 34*a^4*b^3 - 10*a^2*b^5 - 3*b^7)*cosh(x)^3 + 3*(
21*a^7 - 11*a^5*b^2 - 40*a^3*b^4 - 8*a*b^6)*cosh(x)^2 - 2*(35*a^6*b + 26*a^
4*b^3 - 8*a^2*b^5 + b^7)*cosh(x))*sinh(x))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 +
 4*a^2*b^7 + b^9 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
)^8 - (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sinh(x)^8 - 2*(a^9
 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^7 - 2*(a^9 + 4*a^7*b^2
 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
*b^7 + b^9)*cosh(x))*sinh(x)^7 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b
^7 + b^9)*cosh(x)^6 - 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 +
14*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 7*(a^9 + 4
*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^6 - 6*(a^9 + 4*a
^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^5 - 2*(3*a^9 + 12*a^7*b^2 +
 18*a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*
a^2*b^7 + b^9)*cosh(x)^3 + 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*
b^8)*cosh(x)^2 + 6*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x
))*sinh(x)^5 - 10*(7*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh
(x)^4 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 + 3*(
a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 3*(a^9 + 4*a^7
*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^4 - 6*(a^9 + 4*a^7*b
^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^3 - 2*(3*a^9 + 12*a^7*b^2 + 18*
a^5*b^4 + 12*a^3*b^6 + 3*a*b^8 + 28*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*
b^7 + b^9)*cosh(x)^5 + 35*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)
cosh(x)^4 + 20(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^3
 + 30*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2)*sinh(x)^
3 + 2*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^2 + 2*(a^8*
b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9 - 14*(a^8*b + 4*a^6*b^3 + 6*a^4
*b^5 + 4*a^2*b^7 + b^9)*cosh(x)^6 - 21*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3
*b^6 + a*b^8)*cosh(x)^5 - 15*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b
^9)*cosh(x)^4 - 30*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x
)^3 - 9*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x))*sinh(x)^
2 - 2*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x) - 2*(a^9 +

```
4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8 + 4*(a^8*b + 4*a^6*b^3 + 6*a^4*b^
5 + 4*a^2*b^7 + b^9)*cosh(x)^7 + 7*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6
 + a*b^8)*cosh(x)^6 + 6*(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*c
osh(x)^5 + 15*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^4 +
 9*(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cosh(x)^2 - 2*(a^8*b +
 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\tanh^4{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(212) = 424.

Time = 0.35 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.33

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-4\,b^2)a^3\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(7\,a^4b-9\,a^2b^3-b^5+(11\,a^5-6\,a^3b^2-2\,ab^4)e^{(-x)}+(35\,a^4b-9\,a^2b^3+b^5)\right.}{3\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)e^{(-x)}+2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)e^{(-2\,x)}+\right.}$$

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] (a^2 - 4*b^2)*a^3*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt
(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2/3*(
7*a^4*b - 9*a^2*b^3 - b^5 + (11*a^5 - 6*a^3*b^2 - 2*a*b^4)*e^(-x) + (35*a^4
*b - 9*a^2*b^3 + b^5)*e^(-2*x) + (21*a^5 - 32*a^3*b^2 - 8*a*b^4)*e^(-3*x) +
 (41*a^4*b - 7*a^2*b^3 - 3*b^5)*e^(-4*x) + (21*a^5 - 22*a^3*b^2 + 2*a*b^4)*
e^(-5*x) + 3*(7*a^4*b + 3*a^2*b^3 + b^5)*e^(-6*x) + 3*(a^5 - 4*a^3*b^2)*e^(
-7*x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*e^(-x) + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-2*x) + 6*(a
^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-3*x) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*e^(-5*x) - 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*e^(-6*x) + 2
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*e^(-7*x) - (a^6*b + 3*a^4*b^3 + 3*a^
2*b^5 + b^7)*e^(-8*x))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.30

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^5-4a^3b^2)\log\left(\frac{|2be^x+2a-2\sqrt{a^2+b^2}|}{|2be^x+2a+2\sqrt{a^2+b^2}|}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2(a^5e^x-a^4b)}{(a^6+3a^4b^2+3a^2b^4+b^6)(be^{(2x)}+2ae^x-b)} - \frac{2\left(12a^3be^{(5x)}-6a^4e^{(4x)}+9a^2b^2e^{(4x)}+3b^4e^{(4x)}+16a^3be^{(3x)}-8ab^3e^{(3x)}-6a^4e^{(2x)}+18a^2b^2e^{(2x)}+\right.}{3(a^6+3a^4b^2+3a^2b^4+b^6)(e^{(2x)}+1)^3}$$

```
[In] integrate(tanh(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^5 - 4*a^3*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x +
2*a + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b
^2)) + 2*(a^5*e^x - a^4*b)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*e^(2*x)
+ 2*a*e^x - b)) - 2/3*(12*a^3*b*e^(5*x) - 6*a^4*e^(4*x) + 9*a^2*b^2*e^(4*x)
 + 3*b^4*e^(4*x) + 16*a^3*b*e^(3*x) - 8*a*b^3*e^(3*x) - 6*a^4*e^(2*x) + 18*
a^2*b^2*e^(2*x) + 12*a^3*b*e^x - 4*a^4 + 9*a^2*b^2 + b^4)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.42

$$\int \frac{\tanh^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{8(a^2-b^2)}{3(a^4+2a^2b^2+b^4)}-\frac{16ab\,e^x}{3(a^4+2a^2b^2+b^4)}}{3e^{2x}+3e^{4x}+e^{6x}+1} - \frac{\frac{4(a^6+a^4b^2-a^2b^4-b^6)}{(a^4+2a^2b^2+b^4)^2}-\frac{16e^x(a^5b+2a^3b^3+ab^5)}{3(a^4+2a^2b^2+b^4)^2}}{2e^{2x}+e^{4x}+1} - \frac{\frac{2(a^6b^5+a^4b^7)}{b^3(a^2b+b^3)(a^2+b^2)^3}-\frac{2e^x(a^7b^5+a^5b^7)}{b^4(a^2b+b^3)(a^2+b^2)^3}}{2a\,e^x-b+b\,e^{2x}} - \frac{\frac{2(-2a^6+a^4b^2+4a^2b^4+b^6)}{(a^4+2a^2b^2+b^4)^2}+\frac{8e^x(a^5b+a^3b^3)}{(a^4+2a^2b^2+b^4)^2}}{e^{2x}+1} - \frac{\ln\left(-\frac{2e^x(a^5-4a^3b^2)}{b(a^2+b^2)^3}-\frac{2(a^5-4a^3b^2)(b-a\,e^x)}{b(a^2+b^2)^{7/2}}\right)(a^5-4a^3b^2)}{(a^2+b^2)^{7/2}} + \frac{\ln\left(\frac{2(a^5-4a^3b^2)(b-a\,e^x)}{b(a^2+b^2)^{7/2}}-\frac{2e^x(a^5-4a^3b^2)}{b(a^2+b^2)^3}\right)(a^5-4a^3b^2)}{(a^2+b^2)^{7/2}}$$

```
[In] int(tanh(x)^4/(a + b*sinh(x))^2,x)

[Out] ((8*(a^2 - b^2))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b*exp(x))/(3*(a^4 + b^
4 + 2*a^2*b^2)))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - ((4*(a^6 - b^6
- a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^2 - (16*exp(x)*(a*b^5 + a^5*b
 + 2*a^3*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)^2))/(2*exp(2*x) + exp(4*x) + 1) -
 ((2*(a^4*b^7 + a^6*b^5))/(b^3*(a^2*b + b^3)*(a^2 + b^2)^3) - (2*exp(x)*(a^
5*b^7 + a^7*b^5))/(b^4*(a^2*b + b^3)*(a^2 + b^2)^3))/(2*a*exp(x) - b + b*ex
p(2*x)) - ((2*(b^6 - 2*a^6 + 4*a^2*b^4 + a^4*b^2))/(a^4 + b^4 + 2*a^2*b^2)^
2 + (8*exp(x)*(a^5*b + a^3*b^3))/(a^4 + b^4 + 2*a^2*b^2)^2)/(exp(2*x) + 1)
- (log(- (2*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3) - (2*(a^5 - 4*a^3*b
^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(7/2)))*(a^5 - 4*a^3*b^2))/(a^2 + b^2)^(
7/2) + (log((2*(a^5 - 4*a^3*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(7/2)) - (2
*exp(x)*(a^5 - 4*a^3*b^2))/(b*(a^2 + b^2)^3))*(a^5 - 4*a^3*b^2))/(a^2 + b^2
)^(7/2)
```

3.237 $\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 135

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2} dx = \frac{ab(3a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^3} + \frac{a^2(a^2-3b^2)\log(\cosh(x))}{(a^2+b^2)^3} - \frac{a^2(a^2-3b^2)\log(a+b\sinh(x))}{(a^2+b^2)^3} + \frac{a^3}{(a^2+b^2)^2(a+b\sinh(x))} + \frac{\operatorname{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2(a^2+b^2)^2}$$

```
[Out] a*b*(3*a^2-b^2)*arctan(sinh(x))/(a^2+b^2)^3+a^2*(a^2-3*b^2)*ln(cosh(x))/(a^
2+b^2)^3-a^2*(a^2-3*b^2)*ln(a+b*sinh(x))/(a^2+b^2)^3+a^3/(a^2+b^2)^2/(a+b*s
inh(x))+1/2*sech(x)^2*(a^2-b^2-2*a*b*sinh(x))/(a^2+b^2)^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2800, 1661, 1643, 649, 209, 266}

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2} dx = \frac{ab(3a^2-b^2)\arctan(\sinh(x))}{(a^2+b^2)^3} - \frac{a^2(a^2-3b^2)\log(a+b\sinh(x))}{(a^2+b^2)^3} + \frac{a^2(a^2-3b^2)\log(\cosh(x))}{(a^2+b^2)^3} + \frac{\operatorname{sech}^2(x)\left(a^2-2ab\sinh(x)-b^2\right)}{2(a^2+b^2)^2} + \frac{a^3}{(a^2+b^2)^2(a+b\sinh(x))}$$

```
[In] Int[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*b*(3*a^2 - b^2)*ArcTan[Sinh[x]])/(a^2 + b^2)^3 + (a^2*(a^2 - 3*b^2)*Log[
Cosh[x]])/(a^2 + b^2)^3 - (a^2*(a^2 - 3*b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2
)^3 + a^3/((a^2 + b^2)^2*(a + b*Sinh[x])) + (Sech[x]^2*(a^2 - b^2 - 2*a*b*S
inh[x]))/(2*(a^2 + b^2)^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{x^3}{(a+x)^2\left(-b^2-x^2\right)^2}\,dx, x, b\sinh(x)\right)$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^3b^4}{\left(a^2+b^2\right)^2}+\frac{2a^2b^2x}{a^2+b^2}-\frac{2ab^4x^2}{\left(a^2+b^2\right)^2}}{(a+x)^2\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right)}{2b^2}$$

$$= \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{\left(a^2+b^2\right)^2(a+x)^2}+\frac{2a^2b^2\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3(a+x)}+\frac{2ab^2\left(-b^2\left(3a^2-b^2\right)-a\left(a^2-3b^2\right)x\right)}{\left(a^2+b^2\right)^3\left(b^2+x^2\right)}\right)dx, x, b\sinh(x)\right)}{2b^2}$$

$$= -\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} - \frac{a\text{Subst}\left(\int \frac{-b^2\left(3a^2-b^2\right)-a\left(a^2-3b^2\right)x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^3}$$

$$= -\frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2} + \frac{\left(a^2\left(a^2-3b^2\right)\right)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^3} + \frac{\left(ab^2\left(3a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{\left(a^2+b^2\right)^3}$$

$$= \frac{ab\left(3a^2-b^2\right)\arctan(\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^2\left(a^2-3b^2\right)\log(\cosh(x))}{\left(a^2+b^2\right)^3} - \frac{a^2\left(a^2-3b^2\right)\log(a+b\sinh(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{\left(a^2+b^2\right)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2-b^2-2ab\sinh(x)\right)}{2\left(a^2+b^2\right)^2}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{-2ab(a^2+b^2)\arctan(\sinh(x))+a^2(a-ib)(a-3ib)\log(i-\sinh(x))+a^2(a+ib)(a+3ib)\log(i+\sinh(x))}{2\,(a^2+b^2}$$

```
[In] Integrate[Tanh[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a*b*(a^2 + b^2)*ArcTan[Sinh[x]] + a^2*(a - I*b)*(a - (3*I)*b)*Log[I - S
inh[x]] + a^2*(a + I*b)*(a + (3*I)*b)*Log[I + Sinh[x]] - 2*a^2*(a^2 - 3*b^2
)*Log[a + b*Sinh[x]] + (a^4 - b^4)*Sech[x]^2 + (2*a^3*(a^2 + b^2))/(a + b*S
inh[x]) - 2*a*b*(a^2 + b^2)*Sech[x]*Tanh[x])/(2*(a^2 + b^2)^3)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2a^2\left(\frac{\left(-a^2b-b^3\right)\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{\left(a^2-3b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{2\left(\left(a^3b+b^3a\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^4+b^4\right)\tanh\left(\frac{x}{2}\right)^2+\left(-a^3b\right.\right.}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}$
risch	$\frac{2\,e^x\left(a^3e^{4x}-e^{4x}a\,b^2-a^2b\,e^{3x}-e^{3x}b^3+4a^3e^{2x}+e^xa^2b+b^3e^x+a^3-a\,b^2\right)}{(a^4+2a^2b^2+b^4)(1+e^{2x})^2(b\,e^{2x}+2\,e^xa-b)}-\frac{a^4\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{3a^2\ln\left(e^{2x}+\frac{2a\,e^x}{b}-1\right)b^2}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{3}{a^6+}$

```
[In] int(tanh(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-a^2*b-b^3)*tanh(1/2*x)/(tanh(1/2*x)
^2*a-2*b*tanh(1/2*x)-a)+1/2*(a^2-3*b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-
a))+2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*(((a^3*b+a*b^3)*tanh(1/2*x)^3+(-a^4+b^4
)*tanh(1/2*x)^2+(-a^3*b-a*b^3)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+a*(1/2*(a^3
-3*a*b^2)*ln(1+tanh(1/2*x)^2)+(3*a^2*b-b^3)*arctan(tanh(1/2*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2850 vs. $2(133) = 266$.

Time = 0.35 (sec) , antiderivative size = 2850, normalized size of antiderivative = 21.11

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(2*(a^5 - a*b^4)*cosh(x)^5 + 2*(a^5 - a*b^4)*sinh(x)^5 - 2*(a^4*b + 2*a^2*
b^3 + b^5)*cosh(x)^4 - 2*(a^4*b + 2*a^2*b^3 + b^5 - 5*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^4 + 8*(a^5 + a^3*b^2)*cosh(x)^3 + 4*(2*a^5 + 2*a^3*b^2 + 5*(a^5 -
 a*b^4)*cosh(x)^2 - 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 + 2*(a^4
*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5 + 10*(a^5 - a*
b^4)*cosh(x)^3 - 6*(a^4*b + 2*a^2*b^3 + b^5)*cosh(x)^2 + 12*(a^5 + a^3*b^2)
*cosh(x))*sinh(x)^2 + 2*((3*a^3*b^2 - a*b^4)*cosh(x)^6 + (3*a^3*b^2 - a*b^4
)*sinh(x)^6 + 2*(3*a^4*b - a^2*b^3)*cosh(x)^5 + 2*(3*a^4*b - a^2*b^3 + 3*(3
*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^5 - 3*a^3*b^2 + a*b^4 + (3*a^3*b^2 - a*b
^4)*cosh(x)^4 + (3*a^3*b^2 - a*b^4 + 15*(3*a^3*b^2 - a*b^4)*cosh(x)^2 + 10*
(3*a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^4*b - a^2*b^3)*cosh(x)^3 +
4*(3*a^4*b - a^2*b^3 + 5*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 5*(3*a^4*b - a^2*b
^3)*cosh(x)^2 + (3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - (3*a^3*b^2 - a*b^4
)*cosh(x)^2 - (3*a^3*b^2 - a*b^4 - 15*(3*a^3*b^2 - a*b^4)*cosh(x)^4 - 20*(3
*a^4*b - a^2*b^3)*cosh(x)^3 - 6*(3*a^3*b^2 - a*b^4)*cosh(x)^2 - 12*(3*a^4*b
 - a^2*b^3)*cosh(x))*sinh(x)^2 + 2*(3*a^4*b - a^2*b^3)*cosh(x) + 2*(3*(3*a^
3*b^2 - a*b^4)*cosh(x)^5 + 3*a^4*b - a^2*b^3 + 5*(3*a^4*b - a^2*b^3)*cosh(x
)^4 + 2*(3*a^3*b^2 - a*b^4)*cosh(x)^3 + 6*(3*a^4*b - a^2*b^3)*cosh(x)^2 - (
3*a^3*b^2 - a*b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^5 - a
*b^4)*cosh(x) - ((a^4*b - 3*a^2*b^3)*cosh(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x
)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2
*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^
4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3
*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b
^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4
*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b
 - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cosh(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh
(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 - 12*(a^5 - 3*a^3*b^2)*cosh(x))*sin
h(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 +
 a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*cosh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*co
sh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 - (a^4*b - 3*a^2*b^3)*cosh(x))*sinh
(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^4*b - 3*a^2*b^3)*cosh
(x)^6 + (a^4*b - 3*a^2*b^3)*sinh(x)^6 + 2*(a^5 - 3*a^3*b^2)*cosh(x)^5 + 2*(
a^5 - 3*a^3*b^2 + 3*(a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^5 - a^4*b + 3*a^2*
b^3 + (a^4*b - 3*a^2*b^3)*cosh(x)^4 + (a^4*b - 3*a^2*b^3 + 15*(a^4*b - 3*a^
```

```
2*b^3)*cosh(x)^2 + 10*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^4 + 4*(a^5 - 3*a^3
*b^2)*cosh(x)^3 + 4*(a^5 - 3*a^3*b^2 + 5*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 5*
(a^5 - 3*a^3*b^2)*cosh(x)^2 + (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x)^3 - (a^4
*b - 3*a^2*b^3)*cosh(x)^2 - (a^4*b - 3*a^2*b^3 - 15*(a^4*b - 3*a^2*b^3)*cos
h(x)^4 - 20*(a^5 - 3*a^3*b^2)*cosh(x)^3 - 6*(a^4*b - 3*a^2*b^3)*cosh(x)^2 -
 12*(a^5 - 3*a^3*b^2)*cosh(x))*sinh(x)^2 + 2*(a^5 - 3*a^3*b^2)*cosh(x) + 2*
(3*(a^4*b - 3*a^2*b^3)*cosh(x)^5 + a^5 - 3*a^3*b^2 + 5*(a^5 - 3*a^3*b^2)*co
sh(x)^4 + 2*(a^4*b - 3*a^2*b^3)*cosh(x)^3 + 6*(a^5 - 3*a^3*b^2)*cosh(x)^2 -
 (a^4*b - 3*a^2*b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 2*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*cosh(x)^4 - 4*(a^4*b + 2*a^2*b^3 + b^5)*c
osh(x)^3 + 12*(a^5 + a^3*b^2)*cosh(x)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(
x))*sinh(x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^6 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^6
- 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^5 - 2*(a^7 + 3*a^5*b^2 +
3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x
)^5 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3
+ 3*a^2*b^5 + b^7 + 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + 10
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x))*sinh(x)^4 - 4*(a^7 + 3*a^5*
b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 4*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6
 + 5*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 5*(a^7 + 3*a^5*b^2 +
 3*a^3*b^4 + a*b^6)*cosh(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(
x))*sinh(x)^3 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^6*b +
3*a^4*b^3 + 3*a^2*b^5 + b^7 - 15*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh
(x)^4 - 20*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 6*(a^6*b + 3*a
^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 - 12*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b
^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x) -
2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 3*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^5 + 5*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^4 + 2*(a^
6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^3 + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*cosh(x)^2 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh
(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\tanh^3{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(133) = 266$.

Time = 0.34 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.78

$$\int \frac{\tanh^3(x)}{(a + b\sinh(x))^2}\,dx = -\frac{2\,(3\,a^3b - ab^3)\arctan\left(e^{(-x)}\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} - \frac{(a^4 - 3\,a^2b^2)\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{(a^4 - 3\,a^2b^2)\log\left(e^{(-2\,x)} + 1\right)}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{2\,\left(4\,a^3e^{(-3\,x)} + (a^3 - ab^2)e^{(-x)} - (a^2b + b^3)e^{(-2\,x)} + (a\right.}{a^4b + 2\,a^2b^3 + b^5 + 2\,(a^5 + 2\,a^3b^2 + ab^4)e^{(-x)} + (a^4b + 2\,a^2b^3 + b^5)e^{(-2\,x)} + 4\,(a^5 + 2\,a^3b^2 + ab^4)e^{(-3\,x)} -}$$

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(3*a^3*b - a*b^3)*arctan(e^(-x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (
a^4 - 3*a^2*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6) + (a^4 - 3*a^2*b^2)*log(e^(-2*x) + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*
b^4 + b^6) + 2*(4*a^3*e^(-3*x) + (a^3 - a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2
*x) + (a^2*b + b^3)*e^(-4*x) + (a^3 - a*b^2)*e^(-5*x))/(a^4*b + 2*a^2*b^3 +
 b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + (a^4*b + 2*a^2*b^3 + b^5)*e^(-2
*x) + 4*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-
4*x) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-5*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(
-6*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. $2(133) = 266$.

Time = 0.30 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.27

$$\int \frac{\tanh^3(x)}{(a + b\sinh(x))^2}\,dx = \frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)(3\,a^3b - ab^3)}{2\,(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)} + \frac{(a^4 - 3\,a^2b^2)\log\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)}{2\,(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6)} - \frac{(a^4b - 3\,a^2b^3)\log\left(\left|-b\left(e^{(-x)} - e^x\right) + 2\,a\right|\right)}{a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7} - \frac{2\,\left(a^3\left(e^{(-x)} - e^x\right)^2 - ab^2\left(e^{(-x)} - e^x\right)^2 + a^2b\left(e^{(-x)} - e^x\right) + b^3\left(e^{(-x)} - e^x\right) + 6\,a^3 - 2\,ab^2\right)}{(a^4 + 2\,a^2b^2 + b^4)\left(b\left(e^{(-x)} - e^x\right)^3 - 2\,a\left(e^{(-x)} - e^x\right)^2 + 4\,b\left(e^{(-x)} - e^x\right) - 8\,a\right)}$$

```
[In] integrate(tanh(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*(3*a^3*b - a*b^3)/(a^6 + 3*a^
4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b^2)*log((e^(-x) - e^x)^2 + 4)/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2*b^3)*log(abs(-b*(e^(-x
```

```
) - e^x) + 2*a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 2*(a^3*(e^(-x) - e
^x)^2 - a*b^2*(e^(-x) - e^x)^2 + a^2*b*(e^(-x) - e^x) + b^3*(e^(-x) - e^x)
+ 6*a^3 - 2*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(-x) - e^x)^3 - 2*a*(e^(-
x) - e^x)^2 + 4*b*(e^(-x) - e^x) - 8*a))
```

Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 501, normalized size of antiderivative = 3.71

$$\int \frac{\tanh^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2\,(a^8+2\,a^6\,b^2-2\,a^2\,b^6-b^8)}{(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)^2}-\frac{2\,\mathrm{e}^x\,(a^7\,b+3\,a^5\,b^3+3\,a^3\,b^5+a\,b^7)}{(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)^2}}{\mathrm{e}^{2\,x}+1} - \frac{\frac{2\,(a^2-b^2)}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a\,b\,\mathrm{e}^x}{a^4+2\,a^2\,b^2+b^4}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} - \frac{a\,\ln(\mathrm{e}^x+1\mathrm{i})}{-a^3+a^2\,b\,3\mathrm{i}+3\,a\,b^2-b^3\,1\mathrm{i}} - \frac{\ln(15\,a^6\,b^3-a^2\,b^7-30\,a^4\,b^5-4\,a^8\,b+8\,a^9\,\mathrm{e}^x+a^2\,b^7\,\mathrm{e}^{2\,x}+30\,a^4\,b^5\,\mathrm{e}^{2\,x}-15\,a^6\,b^3\,\mathrm{e}^{2\,x}+4\,a^8\,b\,\mathrm{e}^{2\,x}+2\,a^3\,b}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6} + \frac{2\,\mathrm{e}^x\,(a^7\,b^2+2\,a^5\,b^4+a^3\,b^6)}{b\,(a^2\,b+b^3)\,(a^2+b^2)\,(2\,a\,\mathrm{e}^x-b+b\,\mathrm{e}^{2\,x})\,(a^4+2\,a^2\,b^2+b^4)} - \frac{a\,\ln(1+\mathrm{e}^x\,1\mathrm{i})\,1\mathrm{i}}{-a^3\,1\mathrm{i}+3\,a^2\,b+a\,b^2\,3\mathrm{i}-b^3}$$

```
[In] int(tanh(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*(a^8 - b^8 - 2*a^2*b^6 + 2*a^6*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^
2)^2) - (2*exp(x)*(a*b^7 + a^7*b + 3*a^3*b^5 + 3*a^5*b^3))/((a^2 + b^2)*(a^
4 + b^4 + 2*a^2*b^2)^2))/(exp(2*x) + 1) - ((2*(a^2 - b^2))/(a^4 + b^4 + 2*a
^2*b^2) - (4*a*b*exp(x))/(a^4 + b^4 + 2*a^2*b^2))/(2*exp(2*x) + exp(4*x) +
1) - (a*log(exp(x) + 1i))/(3*a*b^2 + a^2*b*3i - a^3 - b^3*1i) - (log(15*a^6
*b^3 - a^2*b^7 - 30*a^4*b^5 - 4*a^8*b + 8*a^9*exp(x) + a^2*b^7*exp(2*x) + 3
0*a^4*b^5*exp(2*x) - 15*a^6*b^3*exp(2*x) + 4*a^8*b*exp(2*x) + 2*a^3*b^6*exp
(x) + 60*a^5*b^4*exp(x) - 30*a^7*b^2*exp(x))*(a^4 - 3*a^2*b^2))/(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2) - (a*log(exp(x)*1i + 1)*1i)/(a*b^2*3i + 3*a^2*b -
a^3*1i - b^3) + (2*exp(x)*(a^3*b^6 + 2*a^5*b^4 + a^7*b^2))/(b*(a^2*b + b^3)
*(a^2 + b^2)*(2*a*exp(x) - b + b*exp(2*x))*(a^4 + b^4 + 2*a^2*b^2))
```

3.238 $\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 144

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2a^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{4ab^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))} - \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2}$$

```
[Out] -2*a^3*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+4*a*b^2*a
rctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-2*a*b*sech(x)/(a^
2+b^2)^2-a^2*b*cosh(x)/(a^2+b^2)^2/(a+b*sinh(x))-(a^2-b^2)*tanh(x)/(a^2+b^2
)^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {2810, 2743, 12, 2739, 632, 212, 2748, 3852, 8}

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{4ab^2\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))} - \frac{2a^3\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

```
[In] Int[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (-2*a^3*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (4*
a*b^2*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (2*a*
```

```
b*Sech[x])/(a^2 + b^2)^2 - (a^2*b*Cosh[x])/((a^2 + b^2)^2*(a + b*Sinh[x]))
- ((a^2 - b^2)*Tanh[x])/(a^2 + b^2)^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\text{integral} = -\int \left(-\frac{a^2}{(a^2+b^2)(a+b\sinh(x))^2} + \frac{2ab^2}{(a^2+b^2)^2(a+b\sinh(x))} + \frac{\text{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right)}{(a^2+b^2)^2} \right) dx$$

$$= -\frac{\int \text{sech}^2(x)\left(a^2\left(1-\frac{b^2}{a^2}\right)-2ab\sinh(x)\right) dx}{(a^2+b^2)^2} - \frac{(2ab^2)\int \frac{1}{a+b\sinh(x)} dx}{(a^2+b^2)^2} + \frac{a^2\int \frac{1}{(a+b\sinh(x))^2} dx}{a^2+b^2}$$

$$= -\frac{2ab\text{sech}(x)}{(a^2+b^2)^2} - \frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))} + \frac{a^2\int \frac{a}{a+b\sinh(x)} dx}{(a^2+b^2)^2} - \frac{(4ab^2)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2} dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2} - \frac{(a^2-b^2)\int \text{sech}^2(x)\,dx}{(a^2+b^2)^2}$$

$$= -\frac{2ab\text{sech}(x)}{(a^2+b^2)^2} - \frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))} + \frac{a^3\int \frac{1}{a+b\sinh(x)} dx}{(a^2+b^2)^2} + \frac{(8ab^2)\,\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2} dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2} - \frac{(i(a^2-b^2))\,\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)}{(a^2+b^2)^2}$$

$$= \frac{4ab^2\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{2ab\text{sech}(x)}{(a^2+b^2)^2} - \frac{a^2b\cosh(x)}{(a^2+b^2)^2(a+b\sinh(x))} - \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{a+2bx-ax^2} dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}$$

$$= \frac{4ab^2 \operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{a^2 b\cosh(x)}{(a^2+b^2)^2 (a+b\sinh(x))} - \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2} - \frac{(4a^3)\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2}$$

$$= -\frac{2a^3 \operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{4ab^2 \operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{2ab\operatorname{sech}(x)}{(a^2+b^2)^2} - \frac{a^2 b\cosh(x)}{(a^2+b^2)^2 (a+b\sinh(x))} - \frac{(a^2-b^2)\tanh(x)}{(a^2+b^2)^2}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.69

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{2a(a^2-2b^2)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 2ab\operatorname{sech}(x) - \frac{a^2 b\cosh(x)}{a+b\sinh(x)} + (-a^2+b^2)\tanh(x)}{(a^2+b^2)^2}$$

```
[In] Integrate[Tanh[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] ((2*a*(a^2 - 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 -
 b^2] - 2*a*b*Sech[x] - (a^2*b*Cosh[x])/(a + b*Sinh[x]) + (-a^2 + b^2)*Tanh
[x])/(a^2 + b^2)^2
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

method	result
default	$-\frac{2a\left(\frac{-b^2\tanh\left(\frac{x}{2}\right)-ab}{\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}+\frac{2\left(-a^2+b^2\right)\tanh\left(\frac{x}{2}\right)-4ab}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}$
risch	$\frac{2a^3 e^{3x}-4a\,b^2 e^{3x}-8a^2 b\,e^{2x}-2b^3 e^{2x}+6a^3 e^{x}-4a^2 b+2b^3}{\left(b\,e^{2x}+2\,e^{x} a-b\right)\left(1+e^{2x}\right)\left(a^4+2a^2b^2+b^4\right)}+\frac{a^3\ln\left(e^{x}+\frac{\left(a^2+b^2\right)^{\frac{5}{2}} a-a^6-3a^4b^2-3a^2b^4-b^6}{b\left(a^2+b^2\right)^{\frac{5}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{5}{2}}}-\frac{2b^2 a\ln\left(e^{x}+\frac{\left(a^2+b^2\right)^{\frac{5}{2}}}{}\right.}{(a^2}$

```
[In] int(tanh(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/(a^2+b^2)^2*((-b^2*tanh(1/2*x)-a*b)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a
)-(a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(
1/2)))+2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tanh(1/2*x)-2*a*b)/(1+tanh(1/2*x)^
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(136) = 272$.

Time = 0.29 (sec) , antiderivative size = 900, normalized size of antiderivative = 6.25

$$\int \frac{\tanh^2(x)}{(a + b\sinh(x))^2} \, dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^4*b + 2*a^2*b^3 - 2*b^5 - 2*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x)^3 - 2*(a
^5 - a^3*b^2 - 2*a*b^4)*sinh(x)^3 + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x)^2
 + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - 3*(a^5 - a^3*b^2 - 2*a*b^4)*cosh(x))*sinh
(x)^2 + ((a^3*b - 2*a*b^3)*cosh(x)^4 + (a^3*b - 2*a*b^3)*sinh(x)^4 - a^3*b
+ 2*a*b^3 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 2*(a^4 - 2*a^2*b^2 + 2*(a^3*b -
 2*a*b^3)*cosh(x))*sinh(x)^3 + 6*((a^3*b - 2*a*b^3)*cosh(x)^2 + (a^4 - 2*a^
2*b^2)*cosh(x))*sinh(x)^2 + 2*(a^4 - 2*a^2*b^2)*cosh(x) + 2*(a^4 - 2*a^2*b^
2 + 2*(a^3*b - 2*a*b^3)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x))
*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*s
inh(x) - b)) - 6*(a^5 + a^3*b^2)*cosh(x) - 2*(3*a^5 + 3*a^3*b^2 + 3*(a^5 -
a^3*b^2 - 2*a*b^4)*cosh(x)^2 - 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cosh(x))*sinh(
x))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7)*cosh(x)^4 - (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sinh(x)^4 - 2*(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^3 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 - 6*(
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cosh(x)^2 + (a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*cosh(x))*sinh(x)^2 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*c
osh(x) - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cosh(x)^
2)*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*sinh(x))**2,x)
[Out] Integral(tanh(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.55

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2-2\,b^2)a\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^3e^{(-x)}+2\,a^2b-b^3+(4\,a^2b+b^3)e^{(-2\,x)}+(a^3-2\,ab^2)e^{(-3\,x)}\right)}{a^4b+2\,a^2b^3+b^5+2\,(a^5+2\,a^3b^2+ab^4)e^{(-x)}+2\,(a^5+2\,a^3b^2+ab^4)e^{(-3\,x)}-(a^4b+2\,a^2b^3+b^5)e^{(-4\,x)}}$$

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] (a^2 - 2*b^2)*a*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a
^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*e^(-x) + 2
*a^2*b - b^3 + (4*a^2*b + b^3)*e^(-2*x) + (a^3 - 2*a*b^2)*e^(-3*x))/(a^4*b
+ 2*a^2*b^3 + b^5 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*e^(-x) + 2*(a^5 + 2*a^3*b^2
 + a*b^4)*e^(-3*x) - (a^4*b + 2*a^2*b^3 + b^5)*e^(-4*x))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^3-2\,ab^2)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(a^3e^{(3\,x)}-2\,ab^2e^{(3\,x)}-4\,a^2be^{(2\,x)}-b^3e^{(2\,x)}+3\,a^3e^x-2\,a^2b+b^3\right)}{(a^4+2\,a^2b^2+b^4)(be^{(4\,x)}+2\,ae^{(3\,x)}+2\,ae^x-b)}$$

```
[In] integrate(tanh(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
[Out] (a^3 - 2*a*b^2)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*
a + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(a^3*
e^(3*x) - 2*a*b^2*e^(3*x) - 4*a^2*b*e^(2*x) - b^3*e^(2*x) + 3*a^3*e^x - 2*a
^2*b + b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*e^(4*x) + 2*a*e^(3*x) + 2*a*e^x - b
))
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.62

$$\int \frac{\tanh^2(x)}{(a+b\sinh(x))^2}\,dx$$
$$= \frac{\frac{2\,(a^2\,b^9-2\,a^4\,b^7)}{b^3\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}-\frac{2\,e^{2\,x}\,(4\,a^4\,b^7+a^2\,b^9)}{b^3\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}+\frac{6\,a^5\,b^3\,e^x}{(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}-\frac{2\,a\,e^{3\,x}\,(2\,a^2\,b^9-a^4\,b^7)}{b^4\,(a^3+a\,b^2)\,(a^3\,b^3+a\,b^5)}}{2\,a\,e^x-b+2\,a\,e^{3\,x}+b\,e^{4\,x}}$$
$$-\frac{a\,\ln\left(\frac{2\,e^x\,(2\,a\,b^2-a^3)}{b\,(a^2+b^2)^2}-\frac{2\,a\,(a^2-2\,b^2)\,(b-a\,e^x)}{b\,(a^2+b^2)^{5/2}}\right)\,(a^2-2\,b^2)}{(a^2+b^2)^{5/2}}$$
$$+\frac{a\,\ln\left(\frac{2\,e^x\,(2\,a\,b^2-a^3)}{b\,(a^2+b^2)^2}+\frac{2\,a\,(a^2-2\,b^2)\,(b-a\,e^x)}{b\,(a^2+b^2)^{5/2}}\right)\,(a^2-2\,b^2)}{(a^2+b^2)^{5/2}}$$

```
[In] int(tanh(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] ((2*(a^2*b^9 - 2*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*exp(2
*x)*(a^2*b^9 + 4*a^4*b^7))/(b^3*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)) + (6*a^5*b
^3*exp(x))/((a*b^2 + a^3)*(a*b^5 + a^3*b^3)) - (2*a*exp(3*x)*(2*a^2*b^9 - a
^4*b^7))/(b^4*(a*b^2 + a^3)*(a*b^5 + a^3*b^3)))/(2*a*exp(x) - b + 2*a*exp(3
*x) + b*exp(4*x)) - (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2) - (
2*a*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2))/(a^
2 + b^2)^(5/2) + (a*log((2*exp(x)*(2*a*b^2 - a^3))/(b*(a^2 + b^2)^2) + (2*a
*(a^2 - 2*b^2)*(b - a*exp(x)))/(b*(a^2 + b^2)^(5/2)))*(a^2 - 2*b^2))/(a^2 +
 b^2)^(5/2)
```

3.239 $\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 85

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{2ab\arctan(\sinh(x))}{(a^2+b^2)^2} + \frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2} - \frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{a}{(a^2+b^2)(a+b\sinh(x))}$$

```
[Out] 2*a*b*arctan(sinh(x))/(a^2+b^2)^2+(a^2-b^2)*ln(cosh(x))/(a^2+b^2)^2-(a^2-b^
2)*ln(a+b*sinh(x))/(a^2+b^2)^2+a/(a^2+b^2)/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {2800, 815, 649, 209, 266}

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{2ab\arctan(\sinh(x))}{(a^2+b^2)^2} + \frac{a}{(a^2+b^2)(a+b\sinh(x))} - \frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2}$$

```
[In] Int[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*a*b*ArcTan[Sinh[x]])/(a^2 + b^2)^2 + ((a^2 - b^2)*Log[Cosh[x]])/(a^2 + b
^2)^2 - ((a^2 - b^2)*Log[a + b*Sinh[x]])/(a^2 + b^2)^2 + a/((a^2 + b^2)*(a
+ b*Sinh[x]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x}{(a+x)^2\,(-b^2-x^2)}\,dx, x, b\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{a}{(a^2+b^2)\,(a+x)^2}+\frac{a^2-b^2}{(a^2+b^2)^2\,(a+x)}+\frac{-2ab^2-(a^2-b^2)\,x}{(a^2+b^2)^2\,(b^2+x^2)}\right)\,dx, x, b\sinh(x)\right)\\
&= -\frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)\,(a+b\sinh(x))}-\frac{\text{Subst}\left(\int \frac{-2ab^2-(a^2-b^2)x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{(a^2+b^2)^2}
\end{aligned}$$

$$= -\frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{a}{(a^2+b^2)(a+b\sinh(x))} + \frac{(2ab^2)\,\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{(a^2+b^2)^2} + \frac{(a^2-b^2)\,\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx, x, b\sinh(x)\right)}{(a^2+b^2)^2}$$

$$= \frac{2ab\arctan(\sinh(x))}{(a^2+b^2)^2} + \frac{(a^2-b^2)\log(\cosh(x))}{(a^2+b^2)^2} - \frac{(a^2-b^2)\log(a+b\sinh(x))}{(a^2+b^2)^2} + \frac{a}{(a^2+b^2)(a+b\sinh(x))}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.72

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{a((a-ib)^2\log(i-\sinh(x)) + (a+ib)^2\log(i+\sinh(x)) + 2(a^2+b^2+(-a^2+b^2)\log(a+b\sinh(x)))) + b(\ldots}{2(a^2+b^2)^2(a+b\,\mathrm{s}\ldots}$$

```
[In] Integrate[Tanh[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] (a*((a - I*b)^2*Log[I - Sinh[x]] + (a + I*b)^2*Log[I + Sinh[x]] + 2*(a^2 +
b^2 + (-a^2 + b^2)*Log[a + b*Sinh[x]])) + b*((a - I*b)^2*Log[I - Sinh[x]] +
 (a + I*b)^2*Log[I + Sinh[x]] + 2*(-a^2 + b^2)*Log[a + b*Sinh[x]])*Sinh[x])
/(2*(a^2 + b^2)^2*(a + b*Sinh[x]))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

method	result
default	$-\frac{2\left(\frac{(-a^2b-b^3)\tanh(\frac{x}{2})}{\tanh(\frac{x}{2})^2a-2b\tanh(\frac{x}{2})-a}+\frac{(a^2-b^2)\ln\left(\tanh(\frac{x}{2})^2a-2b\tanh(\frac{x}{2})-a\right)}{2}\right)}{(a^2+b^2)^2}+\frac{2(a^2-b^2)\ln\left(1+\tanh(\frac{x}{2})^2\right)+8ab\arctan(\tanh(\frac{x}{2}))}{2a^4+4a^2b^2+2b^4}$
risch	$\frac{2a\,e^x}{(a^2+b^2)(b\,e^{2x}+2\,e^x a-b)}+\frac{2i\ln(e^x+i)ab}{a^4+2a^2b^2+b^4}+\frac{\ln(e^x+i)a^2}{a^4+2a^2b^2+b^4}-\frac{\ln(e^x+i)b^2}{a^4+2a^2b^2+b^4}-\frac{2i\ln(e^x-i)ab}{a^4+2a^2b^2+b^4}+\frac{\ln(e^x-i)a^2}{a^4+2a^2b^2+b^4}-\frac{\ln(e^x-i)b\ldots}{a^4+2a^2b^2+\ldots}$

```
[In] int(tanh(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a^2+b^2)^2*((-a^2*b-b^3)*tanh(1/2*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a
)+1/2*(a^2-b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a))+4/(2*a^4+4*a^2*b^2+2
*b^4)*(1/2*(a^2-b^2)*ln(1+tanh(1/2*x)^2)+2*a*b*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.98

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{4\left(ab^2\cosh\left(x\right)^2+ab^2\sinh\left(x\right)^2+2\,a^2b\cosh\left(x\right)-ab^2+2\left(ab^2\cosh\left(x\right)+a^2b\right)\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)\right.}{}$$

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] -(4*(a*b^2*cosh(x)^2 + a*b^2*sinh(x)^2 + 2*a^2*b*cosh(x) - a*b^2 + 2*(a*b^2
*cosh(x) + a^2*b)*sinh(x))*arctan(cosh(x) + sinh(x)) + 2*(a^3 + a*b^2)*cosh
(x) + (a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*
(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2 + (a^2*b - b^3)*cosh(x))*sinh(x))*lo
g(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) - (a^2*b - b^3 - (a^2*b - b^3)*cos
h(x)^2 - (a^2*b - b^3)*sinh(x)^2 - 2*(a^3 - a*b^2)*cosh(x) - 2*(a^3 - a*b^2
 + (a^2*b - b^3)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(
a^3 + a*b^2)*sinh(x))/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b + 2*a^2*b^3 + b^5)*
cosh(x)^2 - (a^4*b + 2*a^2*b^3 + b^5)*sinh(x)^2 - 2*(a^5 + 2*a^3*b^2 + a*b^
4)*cosh(x) - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(x)
)*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\tanh\left(x\right)}{\left(a+b\sinh\left(x\right)\right)^2}\,dx$$

```
[In] integrate(tanh(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(tanh(x)/(a + b*sinh(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.82

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = -\frac{4\,ab\arctan\left(e^{(-x)}\right)}{a^4+2\,a^2b^2+b^4} + \frac{2\,ae^{(-x)}}{a^2b+b^3+2\,(a^3+ab^2)e^{(-x)}-(a^2b+b^3)e^{(-2\,x)}} - \frac{(a^2-b^2)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^2-b^2)\log\left(e^{(-2\,x)}+1\right)}{a^4+2\,a^2b^2+b^4}$$

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] -4*a*b*arctan(e^(-x))/(a^4 + 2*a^2*b^2 + b^4) + 2*a*e^(-x)/(a^2*b + b^3 + 2
*(a^3 + a*b^2)*e^(-x) - (a^2*b + b^3)*e^(-2*x)) - (a^2 - b^2)*log(-2*a*e^(-
x) + b*e^(-2*x) - b)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*log(e^(-2*x) + 1
)/(a^4 + 2*a^2*b^2 + b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.34

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)ab}{a^4+2\,a^2b^2+b^4} + \frac{(a^2-b^2)\log\left(\left(e^{(-x)}-e^x\right)^2+4\right)}{2\,(a^4+2\,a^2b^2+b^4)} - \frac{(a^2b-b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b+2\,a^2b^3+b^5} + \frac{a^2b\left(e^{(-x)}-e^x\right)-b^3\left(e^{(-x)}-e^x\right)-4\,a^3}{(a^4+2\,a^2b^2+b^4)\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)}$$

```
[In] integrate(tanh(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a*b/(a^4 + 2*a^2*b^2 + b^4) + 1/2
*(a^2 - b^2)*log((e^(-x) - e^x)^2 + 4)/(a^4 + 2*a^2*b^2 + b^4) - (a^2*b - b
^3)*log(abs(-b*(e^(-x) - e^x) + 2*a))/(a^4*b + 2*a^2*b^3 + b^5) + (a^2*b*(e
^(-x) - e^x) - b^3*(e^(-x) - e^x) - 4*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*(e^(
-x) - e^x) - 2*a))
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.24

$$\int \frac{\tanh(x)}{(a+b\sinh(x))^2}\,dx = \frac{\ln\left(1+e^x\,1i\right)}{a^2+a\,b\,2i-b^2} - \frac{\ln\left(b^5\,e^{2\,x}-a^4\,b-b^5+a^2\,b^3+2\,a^5\,e^x-a^2\,b^3\,e^{2\,x}+2\,a\,b^4\,e^x+a^4\,b\,e^{2\,x}-2\,a^3\,b^2\,e^x\right)\,\left(a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4} + \frac{2\,a\,b\,e^x}{\left(a^2\,b+b^3\right)\,\left(2\,a\,e^x-b+b\,e^{2\,x}\right)} + \frac{\ln\left(e^x+1i\right)\,1i}{a^2\,1i+2\,a\,b-b^2\,1i}$$

```
[In] int(tanh(x)/(a + b*sinh(x))^2,x)
```

```
[Out] log(exp(x)*1i + 1)/(a*b*2i + a^2 - b^2) + (log(exp(x) + 1i)*1i)/(2*a*b + a^
2*1i - b^2*1i) - (log(b^5*exp(2*x) - a^4*b - b^5 + a^2*b^3 + 2*a^5*exp(x) -
 a^2*b^3*exp(2*x) + 2*a*b^4*exp(x) + a^4*b*exp(2*x) - 2*a^3*b^2*exp(x))*(a^
2 - b^2))/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b*exp(x))/((a^2*b + b^3)*(2*a*exp(
x) - b + b*exp(2*x)))
```

3.240 $\int \frac{\coth(x)}{(a+b\sinh(x))^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2} dx = \frac{\log(\sinh(x))}{a^2} - \frac{\log(a+b\sinh(x))}{a^2} + \frac{1}{a(a+b\sinh(x))}$$

```
[Out] ln(sinh(x))/a^2-ln(a+b*sinh(x))/a^2+1/a/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2800, 46}

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2} dx = -\frac{\log(a+b\sinh(x))}{a^2} + \frac{\log(\sinh(x))}{a^2} + \frac{1}{a(a+b\sinh(x))}$$

```
[In] Int[Coth[x]/(a + b*Sinh[x])^2,x]
```

```
[Out] Log[Sinh[x]]/a^2 - Log[a + b*Sinh[x]]/a^2 + 1/(a*(a + b*Sinh[x]))
```

<u>Rule 46</u>

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

<u>Rule 2800</u>

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x(a+x)^2}\,dx, x, b\sinh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{1}{a(a+x)^2}-\frac{1}{a^2(a+x)}\right)dx, x, b\sinh(x)\right)\\ &= \frac{\log(\sinh(x))}{a^2}-\frac{\log(a+b\sinh(x))}{a^2}+\frac{1}{a(a+b\sinh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{\log(\sinh(x))-\log(a+b\sinh(x))+\frac{a}{a+b\sinh(x)}}{a^2}$$

```
[In] Integrate[Coth[x]/(a + b*Sinh[x])^2,x]
[Out] (Log[Sinh[x]] - Log[a + b*Sinh[x]] + a/(a + b*Sinh[x]))/a^2
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

method	result	size
risch	$\frac{2\,\mathrm{e}^{x}}{a(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}a-b)}+\frac{\ln(\mathrm{e}^{2x}-1)}{a^2}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a^2}$	57
default	$-\frac{2\left(-\frac{b\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{a^2}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2}$	67

```
[In] int(coth(x)/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] 2/a*exp(x)/(b*exp(2*x)+2*exp(x)*a-b)+1/a^2*ln(exp(2*x)-1)-1/a^2*ln(exp(2*x)
+2*a/b*exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(32) = 64$.

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.94

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,a\cosh\left(x\right) - \left(b\cosh\left(x\right)^2 + b\sinh\left(x\right)^2 + 2\,a\cosh\left(x\right) + 2\,\left(b\cosh\left(x\right) + a\right)\sinh\left(x\right) - b\right)\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{a^2b\cosh\left(x\right)^2 + a^2b\sinh\left(x\right)^2 + 2\,a^3\cosh}$$

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a*cosh(x) - (b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)
*sinh(x) - b)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + (b*cosh(x)^2 + b
*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)*log(2*sinh(x)/(co
sh(x) - sinh(x))) + 2*a*sinh(x))/(a^2*b*cosh(x)^2 + a^2*b*sinh(x)^2 + 2*a^3
*cosh(x) - a^2*b + 2*(a^2*b*cosh(x) + a^3)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\coth\left(x\right)}{\left(a+b\sinh\left(x\right)\right)^2}\,dx$$

```
[In] integrate(coth(x)/(a+b*sinh(x))**2,x)
```

```
[Out] Integral(coth(x)/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,e^{(-x)}}{2\,a^2e^{(-x)} - abe^{(-2\,x)} + ab} - \frac{\log\left(-2\,ae^{(-x)} + be^{(-2\,x)} - b\right)}{a^2} + \frac{\log\left(e^{(-x)} + 1\right)}{a^2} + \frac{\log\left(e^{(-x)} - 1\right)}{a^2}$$

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(2*a^2*e^(-x) - a*b*e^(-2*x) + a*b) - log(-2*a*e^(-x) + b*e^(-2*x)
 - b)/a^2 + log(e^(-x) + 1)/a^2 + log(e^(-x) - 1)/a^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(32) = 64$.

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.34

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)+2\,a\right|\right)}{a^2} + \frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a^2} + \frac{b\left(e^{(-x)}-e^{x}\right)-4\,a}{\left(b\left(e^{(-x)}-e^{x}\right)-2\,a\right)a^2}$$

```
[In] integrate(coth(x)/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] -log(abs(-b*(e^(-x) - e^x) + 2*a))/a^2 + log(abs(-e^(-x) + e^x))/a^2 + (b*(
e^(-x) - e^x) - 4*a)/((b*(e^(-x) - e^x) - 2*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 240, normalized size of antiderivative = 7.50

$$\int \frac{\coth(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^4}+b\,\mathrm{e}^{x}\,\sqrt{-a^4}-2\,a\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}-b\,\mathrm{e}^{3\,x}\,\sqrt{-a^4}}{a^3}\right)-2\,\mathrm{atan}\left(\left(4\,a^5\,b\,\sqrt{-a^4}+4\,a^3\,b^3\,\sqrt{-a^4}\right)\,\left(\frac{1}{8\,a^3\,b\,\left(a^2+b^2\right)^2}-\mathrm{e}^{x}\,\left(\right.\right.\right.}{\sqrt{-a^4}} + \frac{2\,b^3\,\mathrm{e}^{x}\,\left(a^2+b^2\right)}{a\,\left(a^2\,b^3+b^5\right)\,\left(2\,a\,\mathrm{e}^{x}-b+b\,\mathrm{e}^{2\,x}\right)}$$

```
[In] int(coth(x)/(a + b*sinh(x))^2,x)
```

```
[Out] (2*atan((a*(-a^4)^(1/2) + b*exp(x)*(-a^4)^(1/2) - 2*a*exp(2*x)*(-a^4)^(1/2)
 - b*exp(3*x)*(-a^4)^(1/2))/a^3) - 2*atan((4*a^5*b*(-a^4)^(1/2) + 4*a^3*b^3
*(-a^4)^(1/2))*(1/(8*a^3*b*(a^2 + b^2)^2) - exp(x)*(1/(16*a^2*b^2*(a^2 + b^
2)^2) - (a^2 + 2*b^2)^2/(16*a^6*b^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^5*
b*(a^2 + b^2)^2))))/(-a^4)^(1/2) + (2*b^3*exp(x)*(a^2 + b^2))/(a*(b^5 + a^2
*b^3)*(2*a*exp(x) - b + b*exp(2*x)))
```

3.241 $\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b\text{arctanh}(\cosh(x))}{a^3} - \frac{2(a^2+2b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))}$$

```
[Out] 2*b*arctanh(cosh(x))/a^3-2*coth(x)/a^2+coth(x)/a/(a+b*sinh(x))-2*(a^2+2*b^2
)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/a^3/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2802, 3135, 3080, 3855, 2739, 632, 212}

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2b\text{arctanh}(\cosh(x))}{a^3} - \frac{2\coth(x)}{a^2} - \frac{2(a^2+2b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} + \frac{\coth(x)}{a(a+b\sinh(x))}$$

```
[In] Int[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*ArcTanh[Cosh[x]])/a^3 - (2*(a^2 + 2*b^2)*ArcTanh[(b - a*Tanh[x/2])/Sqr
t[a^2 + b^2]])/(a^3*Sqrt[a^2 + b^2]) - (2*Coth[x])/a^2 + Coth[x]/(a*(a + b*
Sinh[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2802

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3135

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[
e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n +
 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*
(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d
, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 -
 d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
!(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\text{csch}^2(x)\left(1+\sinh^2(x)\right)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{\int \frac{\text{csch}^2(x)\left(2\left(a^2+b^2\right)+\left(a^2+b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{a\left(a^2+b^2\right)}$$

$$= -\frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{i\int \frac{\text{csch}(x)\left(2ib\left(a^2+b^2\right)-ia\left(a^2+b^2\right)\sinh(x)\right)}{a+b\sinh(x)}\,dx}{a^2\left(a^2+b^2\right)}$$

$$= -\frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} - \frac{(2b)\int \text{csch}(x)\,dx}{a^3} + \frac{\left(a^2+2b^2\right)\int \frac{1}{a+b\sinh(x)}\,dx}{a^3}$$

$$= \frac{2b\text{arctanh}(\cosh(x))}{a^3} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} + \frac{\left(2\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3}$$

$$= \frac{2b\text{arctanh}(\cosh(x))}{a^3} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))} - \frac{\left(4\left(a^2+2b^2\right)\right)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^3}$$

$$= \frac{2b\text{arctanh}(\cosh(x))}{a^3} - \frac{2\left(a^2+2b^2\right)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}} - \frac{2\coth(x)}{a^2} + \frac{\coth(x)}{a(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.40

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = -\frac{-\frac{4\left(a^2+2b^2\right)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + a\coth\left(\frac{x}{2}\right) - 4b\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4b\log\left(\sinh\left(\frac{x}{2}\right)\right) + \frac{2ab\cosh(x)}{a+b\sinh(x)} + a\tanh\left(\frac{x}{2}\right)}{2a^3}$$

```
[In] Integrate[Coth[x]^2/(a + b*Sinh[x])^2,x]
```

```
[Out] -1/2*((-4*(a^2 + 2*b^2)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a
^2 - b^2] + a*Coth[x/2] - 4*b*Log[Cosh[x/2]] + 4*b*Log[Sinh[x/2]] + (2*a*b*
Cosh[x])/(a + b*Sinh[x]) + a*Tanh[x/2])/a^3
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.48

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2a^{2}}-\frac{2\left(\frac{-b^{2}\tanh\left(\frac{x}{2}\right)-ab}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^{2}+2b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}\right)}{a^{3}}-\frac{1}{2a^{2}\tanh\left(\frac{x}{2}\right)}-\frac{2b\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{3}}$
risch	$\frac{2a\,e^{3x}-4b\,e^{2x}-6\,e^{x}a+4b}{\left(e^{2x}-1\right)a^{2}\left(b\,e^{2x}+2\,e^{x}a-b\right)}+\frac{2b\ln\left(e^{x}+1\right)}{a^{3}}-\frac{2b\ln\left(e^{x}-1\right)}{a^{3}}+\frac{\ln\left(e^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)}{\sqrt{a^{2}+b^{2}}\,a}+\frac{2\ln\left(e^{x}+\frac{a\sqrt{a^{2}+b^{2}}-a^{2}-b^{2}}{\sqrt{a^{2}+b^{2}}\,b}\right)b^{2}}{\sqrt{a^{2}+b^{2}}\,a^{3}}-$

```
[In] int(coth(x)^2/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*tanh(1/2*x)-2/a^3*((-b^2*tanh(1/2*x)-a*b)/(tanh(1/2*x)^2*a-2*b*tan
h(1/2*x)-a)-(a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(
a^2+b^2)^(1/2)))-1/2/a^2/tanh(1/2*x)-2/a^3*b*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1257 vs. $2(76) = 152$.

Time = 0.32 (sec) , antiderivative size = 1257, normalized size of antiderivative = 15.71

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (4*a^3*b + 4*a*b^3 + 2*(a^4 + a^2*b^2)*cosh(x)^3 + 2*(a^4 + a^2*b^2)*sinh(x
)^3 - 4*(a^3*b + a*b^3)*cosh(x)^2 - 2*(2*a^3*b + 2*a*b^3 - 3*(a^4 + a^2*b^2
)*cosh(x))*sinh(x)^2 + ((a^2*b + 2*b^3)*cosh(x)^4 + (a^2*b + 2*b^3)*sinh(x)
^4 + 2*(a^3 + 2*a*b^2)*cosh(x)^3 + 2*(a^3 + 2*a*b^2 + 2*(a^2*b + 2*b^3)*cos
h(x))*sinh(x)^3 + a^2*b + 2*b^3 - 2*(a^2*b + 2*b^3)*cosh(x)^2 - 2*(a^2*b +
2*b^3 - 3*(a^2*b + 2*b^3)*cosh(x)^2 - 3*(a^3 + 2*a*b^2)*cosh(x))*sinh(x)^2
- 2*(a^3 + 2*a*b^2)*cosh(x) + 2*(2*(a^2*b + 2*b^3)*cosh(x)^3 - a^3 - 2*a*b^
2 + 3*(a^3 + 2*a*b^2)*cosh(x)^2 - 2*(a^2*b + 2*b^3)*cosh(x))*sinh(x))*sqrt(
a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x)
 + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x)
 - b)) - 6*(a^4 + a^2*b^2)*cosh(x) + 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2*b^
2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^3*b
 + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh(x)
^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*cos
h(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*b^2
 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh(x)
```

```
)*sinh(x))*log(cosh(x) + sinh(x) + 1) - 2*((a^2*b^2 + b^4)*cosh(x)^4 + (a^2
*b^2 + b^4)*sinh(x)^4 + a^2*b^2 + b^4 + 2*(a^3*b + a*b^3)*cosh(x)^3 + 2*(a^
3*b + a*b^3 + 2*(a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 - 2*(a^2*b^2 + b^4)*cosh
(x)^2 - 2*(a^2*b^2 + b^4 - 3*(a^2*b^2 + b^4)*cosh(x)^2 - 3*(a^3*b + a*b^3)*
cosh(x))*sinh(x)^2 - 2*(a^3*b + a*b^3)*cosh(x) - 2*(a^3*b + a*b^3 - 2*(a^2*
b^2 + b^4)*cosh(x)^3 - 3*(a^3*b + a*b^3)*cosh(x)^2 + 2*(a^2*b^2 + b^4)*cosh
(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) - 2*(3*a^4 + 3*a^2*b^2 - 3*(a^4 +
a^2*b^2)*cosh(x)^2 + 4*(a^3*b + a*b^3)*cosh(x))*sinh(x))/(a^5*b + a^3*b^3 +
 (a^5*b + a^3*b^3)*cosh(x)^4 + (a^5*b + a^3*b^3)*sinh(x)^4 + 2*(a^6 + a^4*b
^2)*cosh(x)^3 + 2*(a^6 + a^4*b^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x)^3 -
 2*(a^5*b + a^3*b^3)*cosh(x)^2 - 2*(a^5*b + a^3*b^3 - 3*(a^5*b + a^3*b^3)*c
osh(x)^2 - 3*(a^6 + a^4*b^2)*cosh(x))*sinh(x)^2 - 2*(a^6 + a^4*b^2)*cosh(x)
 - 2*(a^6 + a^4*b^2 - 2*(a^5*b + a^3*b^3)*cosh(x)^3 - 3*(a^6 + a^4*b^2)*cos
h(x)^2 + 2*(a^5*b + a^3*b^3)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\coth^2{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

```
[In] integrate(coth(x)**2/(a+b*sinh(x))**2,x)
[Out] Integral(coth(x)**2/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(76) = 152$.

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.06

$$\begin{aligned}\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx &= -\frac{2\left(3\,ae^{(-x)}-2\,be^{(-2\,x)}-ae^{(-3\,x)}+2\,b\right)}{2\,a^3e^{(-x)}-2\,a^2be^{(-2\,x)}-2\,a^3e^{(-3\,x)}+a^2be^{(-4\,x)}+a^2b}\\ &\quad+\frac{2\,b\log\left(e^{(-x)}+1\right)}{a^3}-\frac{2\,b\log\left(e^{(-x)}-1\right)}{a^3}\\ &\quad+\frac{(a^2+2\,b^2)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^3}\end{aligned}$$

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] -2*(3*a*e^(-x) - 2*b*e^(-2*x) - a*e^(-3*x) + 2*b)/(2*a^3*e^(-x) - 2*a^2*b*e
^(-2*x) - 2*a^3*e^(-3*x) + a^2*b*e^(-4*x) + a^2*b) + 2*b*log(e^(-x) + 1)/a^
3 - 2*b*log(e^(-x) - 1)/a^3 + (a^2 + 2*b^2)*log((b*e^(-x) - a - sqrt(a^2 +
b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.85

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{2\,b\log\left(e^x+1\right)}{a^3} - \frac{2\,b\log\left(\left|e^x-1\right|\right)}{a^3} + \frac{\left(a^2+2\,b^2\right)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^3} + \frac{2\left(ae^{(3\,x)}-2\,be^{(2\,x)}-3\,ae^x+2\,b\right)}{\left(be^{(4\,x)}+2\,ae^{(3\,x)}-2\,be^{(2\,x)}-2\,ae^x+b\right)a^2}$$

```
[In] integrate(coth(x)^2/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(e^x + 1)/a^3 - 2*b*log(abs(e^x - 1))/a^3 + (a^2 + 2*b^2)*log(abs(2*
b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(s
qrt(a^2 + b^2)*a^3) + 2*(a*e^(3*x) - 2*b*e^(2*x) - 3*a*e^x + 2*b)/((b*e^(4*
x) + 2*a*e^(3*x) - 2*b*e^(2*x) - 2*a*e^x + b)*a^2)
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 897, normalized size of antiderivative = 11.21

$$\int \frac{\coth^2(x)}{(a+b\sinh(x))^2}\,dx = \frac{\frac{4\left(25\,a^8\,b^8+65\,a^6\,b^{10}+56\,a^4\,b^{12}+16\,a^2\,b^{14}\right)}{a^4\,b^4\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)} - \frac{6\,\mathrm{e}^x\left(25\,a^9\,b^8+65\,a^7\,b^{10}+56\,a^5\,b^{12}+16\,a^3\,b^{14}\right)}{a^4\,b^5\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)} - \frac{4\,\mathrm{e}^{2\,x}\left(25\,a^8\,b^8+65\,a^6\,b^{10}+56\,a^4\,b^{12}+16\,a^2\,b^{14}\right.}{a^4\,b^4\left(25\,a^6\,b^3+65\,a^4\,b^5+56\,a^2\,b^7+16\,b^9\right)}}{b-2\,a\,\mathrm{e}^x+2\,a\,\mathrm{e}^{3\,x}-2\,b\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{4\,x}} - \frac{2\,b\,\ln\left(64\,\mathrm{e}^x-64\right)}{a^3} + \frac{2\,b\,\ln\left(64\,\mathrm{e}^x+64\right)}{a^3} - \frac{\ln\left(\frac{\left(a^2+2\,b^2\right)\left(\frac{32\left(a^4-16\,\mathrm{e}^x\,a^3\,b+12\,a^2\,b^2-12\,\mathrm{e}^x\,a\,b^3+8\,b^4\right)}{a^4\,b^4}+\frac{\left(a^2+2\,b^2\right)\left(\frac{32\left(-4\,\mathrm{e}^x\,a^3+2\,a^2\,b-7\,\mathrm{e}^x\,a\,b^2+4\,b^3\right)}{b^5}-\frac{32\left(a^2+2\,b^2\right)\sqrt{a^2+b^2}\left(-4\,\mathrm{e}^x\,a^5+3\cdots\right.}{b^5\left(a^5+a^3\,b\cdots\right.}\right.}{a^5+a^3\,b^2}\right.}{a^5+a^3\,b^2}\right.}{a^5+a^3\,b^2} + \frac{\ln\left(-\frac{\left(a^2+2\,b^2\right)\left(\frac{32\left(a^4-16\,\mathrm{e}^x\,a^3\,b+12\,a^2\,b^2-12\,\mathrm{e}^x\,a\,b^3+8\,b^4\right)}{a^4\,b^4}-\frac{\left(a^2+2\,b^2\right)\left(\frac{32\left(-4\,\mathrm{e}^x\,a^3+2\,a^2\,b-7\,\mathrm{e}^x\,a\,b^2+4\,b^3\right)}{b^5}+\frac{32\left(a^2+2\,b^2\right)\sqrt{a^2+b^2}\left(-4\,\mathrm{e}^x\,a^5\cdots\right.}{b^5\left(a^5+a\cdots\right.}\right.}{a^5+a^3\,b^2}\right.}{a^5+a^3\,b^2}\right.}{a^5+a^3\,b^2}$$

```
[In] int(coth(x)^2/(a + b*sinh(x))^2,x)
```

```
[Out] ((4*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^10 + 25*a^8*b^8))/(a^4*b^4*(16*b^
9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) - (6*exp(x)*(16*a^3*b^14 + 56*a^
5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a^4*b^5*(16*b^9 + 56*a^2*b^7 + 65*a^4*
b^5 + 25*a^6*b^3)) - (4*exp(2*x)*(16*a^2*b^14 + 56*a^4*b^12 + 65*a^6*b^10 +
 25*a^8*b^8))/(a^4*b^4*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)) + (
2*exp(3*x)*(16*a^3*b^14 + 56*a^5*b^12 + 65*a^7*b^10 + 25*a^9*b^8))/(a^4*b^5
*(16*b^9 + 56*a^2*b^7 + 65*a^4*b^5 + 25*a^6*b^3)))/(b - 2*a*exp(x) + 2*a*ex
p(3*x) - 2*b*exp(2*x) + b*exp(4*x)) - (2*b*log(64*exp(x) - 64))/a^3 + (2*b*
log(64*exp(x) + 64))/a^3 - (log(((a^2 + 2*b^2)*((32*(a^4 + 8*b^4 + 12*a^2*b
^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))/(a^4*b^4) + ((a^2 + 2*b^2)*((32*(2
*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2*exp(x)))/b^5 - (32*(a^2 + 2*b^2)*(a
^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*a^3*b^2*exp(x)))/(b
^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2))*(a^2 + b^2)^(1/2))
/(a^5 + a^3*b^2) - (64*(a^2 + 2*b^2)*(4*b - 7*a*exp(x)))/(a^6*b^3))*(a^2 +
2*b^2)*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) + (log(- ((a^2 + 2*b^2)*((32*(a^4
 + 8*b^4 + 12*a^2*b^2 - 12*a*b^3*exp(x) - 16*a^3*b*exp(x)))/(a^4*b^4) - ((a
^2 + 2*b^2)*((32*(2*a^2*b + 4*b^3 - 4*a^3*exp(x) - 7*a*b^2*exp(x)))/b^5 + (
32*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*(3*a^4*b + 2*a^2*b^3 - 4*a^5*exp(x) - 3*
a^3*b^2*exp(x)))/(b^5*(a^5 + a^3*b^2)))*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2))
*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2) - (64*(a^2 + 2*b^2)*(4*b - 7*a*exp(x)))
/(a^6*b^3))*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2))/(a^5 + a^3*b^2)
```

3.242 $\int \frac{\coth^3(x)}{(a+b\sinh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2} dx = \frac{2b\text{csch}(x)}{a^3} - \frac{\text{csch}^2(x)}{2a^2} + \frac{(a^2+3b^2)\log(\sinh(x))}{a^4} - \frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4} + \frac{a^2+b^2}{a^3(a+b\sinh(x))}$$

```
[Out] 2*b*csch(x)/a^3-1/2*csch(x)^2/a^2+(a^2+3*b^2)*ln(sinh(x))/a^4-(a^2+3*b^2)*l
n(a+b*sinh(x))/a^4+(a^2+b^2)/a^3/(a+b*sinh(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2800, 908}

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2} dx = \frac{2b\text{csch}(x)}{a^3} - \frac{\text{csch}^2(x)}{2a^2} + \frac{(a^2+3b^2)\log(\sinh(x))}{a^4} - \frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4} + \frac{a^2+b^2}{a^3(a+b\sinh(x))}$$

```
[In] Int[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (2*b*Csch[x])/a^3 - Csch[x]^2/(2*a^2) + ((a^2 + 3*b^2)*Log[Sinh[x]])/a^4 -
((a^2 + 3*b^2)*Log[a + b*Sinh[x]])/a^4 + (a^2 + b^2)/(a^3*(a + b*Sinh[x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)^2}\,dx, x, b\sinh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{b^2}{a^2x^3}+\frac{2b^2}{a^3x^2}+\frac{-a^2-3b^2}{a^4x}+\frac{a^2+b^2}{a^3(a+x)^2}+\frac{a^2+3b^2}{a^4(a+x)}\right)\,dx, x, b\sinh(x)\right)\\
&= \frac{2b\text{csch}(x)}{a^3}-\frac{\text{csch}^2(x)}{2a^2}+\frac{(a^2+3b^2)\log(\sinh(x))}{a^4}\\
&\quad -\frac{(a^2+3b^2)\log(a+b\sinh(x))}{a^4}+\frac{a^2+b^2}{a^3(a+b\sinh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{4ab\text{csch}(x)-a^2\text{csch}^2(x)+2(a^2+3b^2)\log(\sinh(x))-2(a^2+3b^2)\log(a+b\sinh(x))+\frac{2a(a^2+b^2)}{a+b\sinh(x)}}{2a^4}$$

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x])^2,x]
```

```
[Out] (4*a*b*Csch[x] - a^2*Csch[x]^2 + 2*(a^2 + 3*b^2)*Log[Sinh[x]] - 2*(a^2 + 3*
b^2)*Log[a + b*Sinh[x]] + (2*a*(a^2 + b^2))/(a + b*Sinh[x]))/(2*a^4)
```

Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.87

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2} a}{2}+4b\tanh\left(\frac{x}{2}\right)}{4a^{3}}-\frac{2\left(\frac{\left(-a^{2}b-b^{3}\right)\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2} a-2b\tanh\left(\frac{x}{2}\right)-a}+\frac{\left(a^{2}+3b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2} a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{2}\right)}{a^{4}}-\frac{1}{8\tanh\left(\frac{x}{2}\right)^{2} a^{2}}+\frac{(4a^{2}$
risch	$\frac{2\,\mathrm{e}^{x}\left(\mathrm{e}^{4x}a^{2}+3b^{2}\mathrm{e}^{4x}+3ab\,\mathrm{e}^{3x}-4\,\mathrm{e}^{2x}a^{2}-6\,\mathrm{e}^{2x}b^{2}-3b\,\mathrm{e}^{x}a+a^{2}+3b^{2}\right)}{\left(\mathrm{e}^{2x}-1\right)^{2}a^{3}\left(b\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}a-b\right)}+\frac{\ln\left(\mathrm{e}^{2x}-1\right)}{a^{2}}+\frac{3\ln\left(\mathrm{e}^{2x}-1\right)b^{2}}{a^{4}}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}-1\right)}{a^{2}}-\frac{3\ln(\mathrm{e}$

```
[In] int(coth(x)^3/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/a^3*(1/2*tanh(1/2*x)^2*a+4*b*tanh(1/2*x))-2/a^4*((-a^2*b-b^3)*tanh(1/2
*x)/(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+1/2*(a^2+3*b^2)*ln(tanh(1/2*x)^2*a-
2*b*tanh(1/2*x)-a))-1/8/tanh(1/2*x)^2/a^2+1/4/a^4*(4*a^2+12*b^2)*ln(tanh(1/
2*x))+b/tanh(1/2*x)/a^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1463 vs. 2(74) = 148.

Time = 0.30 (sec) , antiderivative size = 1463, normalized size of antiderivative = 19.25

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="fricas")
```

```
[Out] (6*a^2*b*cosh(x)^4 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2)*sinh(x
)^5 - 6*a^2*b*cosh(x)^2 + 2*(3*a^2*b + 5*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4
 - 4*(2*a^3 + 3*a*b^2)*cosh(x)^3 + 4*(6*a^2*b*cosh(x) - 2*a^3 - 3*a*b^2 + 5
*(a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x)^3 + 2*(18*a^2*b*cosh(x)^2 + 10*(a^3 + 3
*a*b^2)*cosh(x)^3 - 3*a^2*b - 6*(2*a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a
^3 + 3*a*b^2)*cosh(x) - ((a^2*b + 3*b^3)*cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x
)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*co
sh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2
*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*
a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*cosh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3
+ 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^3 - a^2*b - 3*b^3
 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2*b + 3*b^3)*cosh(x)^4 + 20*(a^3 +
3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a
^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b
+ 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x
)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*cosh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(
x))*sinh(x))*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x))) + ((a^2*b + 3*b^3)*
```

```
cosh(x)^6 + (a^2*b + 3*b^3)*sinh(x)^6 + 2*(a^3 + 3*a*b^2)*cosh(x)^5 + 2*(a^
3 + 3*a*b^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x)^5 - 3*(a^2*b + 3*b^3)*cosh
(x)^4 - (3*a^2*b + 9*b^3 - 15*(a^2*b + 3*b^3)*cosh(x)^2 - 10*(a^3 + 3*a*b^2
)*cosh(x))*sinh(x)^4 - 4*(a^3 + 3*a*b^2)*cosh(x)^3 + 4*(5*(a^2*b + 3*b^3)*c
osh(x)^3 - a^3 - 3*a*b^2 + 5*(a^3 + 3*a*b^2)*cosh(x)^2 - 3*(a^2*b + 3*b^3)*
cosh(x))*sinh(x)^3 - a^2*b - 3*b^3 + 3*(a^2*b + 3*b^3)*cosh(x)^2 + (15*(a^2
*b + 3*b^3)*cosh(x)^4 + 20*(a^3 + 3*a*b^2)*cosh(x)^3 + 3*a^2*b + 9*b^3 - 18
*(a^2*b + 3*b^3)*cosh(x)^2 - 12*(a^3 + 3*a*b^2)*cosh(x))*sinh(x)^2 + 2*(a^3
 + 3*a*b^2)*cosh(x) + 2*(3*(a^2*b + 3*b^3)*cosh(x)^5 + 5*(a^3 + 3*a*b^2)*co
sh(x)^4 - 6*(a^2*b + 3*b^3)*cosh(x)^3 + a^3 + 3*a*b^2 - 6*(a^3 + 3*a*b^2)*c
osh(x)^2 + 3*(a^2*b + 3*b^3)*cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sin
h(x))) + 2*(12*a^2*b*cosh(x)^3 + 5*(a^3 + 3*a*b^2)*cosh(x)^4 - 6*a^2*b*cosh
(x) + a^3 + 3*a*b^2 - 6*(2*a^3 + 3*a*b^2)*cosh(x)^2)*sinh(x))/(a^4*b*cosh(x
)^6 + a^4*b*sinh(x)^6 + 2*a^5*cosh(x)^5 - 3*a^4*b*cosh(x)^4 - 4*a^5*cosh(x)
^3 + 3*a^4*b*cosh(x)^2 + 2*a^5*cosh(x) + 2*(3*a^4*b*cosh(x) + a^5)*sinh(x)^
5 - a^4*b + (15*a^4*b*cosh(x)^2 + 10*a^5*cosh(x) - 3*a^4*b)*sinh(x)^4 + 4*(
5*a^4*b*cosh(x)^3 + 5*a^5*cosh(x)^2 - 3*a^4*b*cosh(x) - a^5)*sinh(x)^3 + (1
5*a^4*b*cosh(x)^4 + 20*a^5*cosh(x)^3 - 18*a^4*b*cosh(x)^2 - 12*a^5*cosh(x)
+ 3*a^4*b)*sinh(x)^2 + 2*(3*a^4*b*cosh(x)^5 + 5*a^5*cosh(x)^4 - 6*a^4*b*cos
h(x)^3 - 6*a^5*cosh(x)^2 + 3*a^4*b*cosh(x) + a^5)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx=\int \frac{\coth^3{(x)}}{(a+b\sinh{(x)})^2}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*sinh(x))**2,x)
[Out] Integral(coth(x)**3/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(74) = 148$.

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.66

$$\begin{aligned}\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx &= \frac{2\left(3\,abe^{(-2\,x)}-3\,abe^{(-4\,x)}+(a^2+3\,b^2)e^{(-x)}-2\,(2\,a^2+3\,b^2)e^{(-3\,x)}+(a^2+3\,b^2)e^{(-5\,x)}\right)}{2\,a^4e^{(-x)}-3\,a^3be^{(-2\,x)}-4\,a^4e^{(-3\,x)}+3\,a^3be^{(-4\,x)}+2\,a^4e^{(-5\,x)}-a^3be^{(-6\,x)}+a^3b}\\&\quad-\frac{(a^2+3\,b^2)\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^4}\\&\quad+\frac{(a^2+3\,b^2)\log\left(e^{(-x)}+1\right)}{a^4}+\frac{(a^2+3\,b^2)\log\left(e^{(-x)}-1\right)}{a^4}\end{aligned}$$

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(3*a*b*e^(-2*x) - 3*a*b*e^(-4*x) + (a^2 + 3*b^2)*e^(-x) - 2*(2*a^2 + 3*b^
2)*e^(-3*x) + (a^2 + 3*b^2)*e^(-5*x))/(2*a^4*e^(-x) - 3*a^3*b*e^(-2*x) - 4*
a^4*e^(-3*x) + 3*a^3*b*e^(-4*x) + 2*a^4*e^(-5*x) - a^3*b*e^(-6*x) + a^3*b)
- (a^2 + 3*b^2)*log(-2*a*e^(-x) + b*e^(-2*x) - b)/a^4 + (a^2 + 3*b^2)*log(e
^(-x) + 1)/a^4 + (a^2 + 3*b^2)*log(e^(-x) - 1)/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.50

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \frac{(a^2+3\,b^2)\log\left(\left|-e^{(-x)}+e^x\right|\right)}{a^4} - \frac{(a^2b+3\,b^3)\log\left(\left|-b\left(e^{(-x)}-e^x\right)+2\,a\right|\right)}{a^4b} + \frac{a^2b\left(e^{(-x)}-e^x\right)+3\,b^3\left(e^{(-x)}-e^x\right)-4\,a^3-8\,ab^2}{\left(b\left(e^{(-x)}-e^x\right)-2\,a\right)a^4} - \frac{3\,a^2\left(e^{(-x)}-e^x\right)^2+9\,b^2\left(e^{(-x)}-e^x\right)^2+8\,ab\left(e^{(-x)}-e^x\right)+4\,a^2}{2\,a^4\left(e^{(-x)}-e^x\right)^2}$$

```
[In] integrate(coth(x)^3/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (a^2 + 3*b^2)*log(abs(-e^(-x) + e^x))/a^4 - (a^2*b + 3*b^3)*log(abs(-b*(e^(
-x) - e^x) + 2*a))/(a^4*b) + (a^2*b*(e^(-x) - e^x) + 3*b^3*(e^(-x) - e^x) -
 4*a^3 - 8*a*b^2)/((b*(e^(-x) - e^x) - 2*a)*a^4) - 1/2*(3*a^2*(e^(-x) - e^x
)^2 + 9*b^2*(e^(-x) - e^x)^2 + 8*a*b*(e^(-x) - e^x) + 4*a^2)/(a^4*(e^(-x) -
 e^x)^2)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 1375, normalized size of antiderivative = 18.09

$$\int \frac{\coth^3(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] int(coth(x)^3/(a + b*sinh(x))^2,x)
```

```
[Out] (2*exp(x)*(a*b^7 + 2*a^3*b^5 + a^5*b^3))/(a^4*(b^5 + a^2*b^3)*(2*a*exp(x) -
 b + b*exp(2*x))) - 2/(a^2*(exp(4*x) - 2*exp(2*x) + 1)) - ((2*atan((4*a^9*b
*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2) + 12*a^5*b^5*((a^2 + 3*b^2)^2)^(1/2)*
(-a^8)^(1/2) + 16*a^7*b^3*((a^2 + 3*b^2)^2)^(1/2)*(-a^8)^(1/2))*(exp(x)*((a
^2 + 2*b^2)^2/(16*a^10*b^2*(a^4 + 3*b^4 + 4*a^2*b^2)^2) - 1/(16*a^6*b^2*(a^
```

```
2 + 3*b^2)^2*(a^2 + b^2)^2)) + (a^2 + 2*b^2)/(8*a^9*b*(a^4 + 3*b^4 + 4*a^2*
b^2)^2) + 1/(8*a^7*b*(a^2 + 3*b^2)^2*(a^2 + b^2)^2))) - 2*atan((a^2*(-a^8)^
(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6
*a^2*b^2)^(1/2))/(2*a^4*(a^4 + 3*b^4 + 4*a^2*b^2)) + ((a^8 + 3*a^6*b^2)*(-a
^8)^(1/2))/(2*a^8*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) - (a^8*b^2*exp(2*x)*
(-a^8)^(1/2)*((4*(a^2 + 2*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2))/(a^12*b^2*(a^4 +
3*b^4 + 4*a^2*b^2)) + (4*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2)
+ 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 + 9*b^4 + 6*a^2*
b^2)^(1/2))/(a^12*b^2*(-a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)) + (2*(2*a^7*b
 + 6*a^5*b^3)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2)^
(1/2)*(a^2 + b^2)) + (4*(a^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/
(a^16*b^2*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2))))/(8*(a^4 + 9*b^4 + 6*a^2*b^
2)^(1/2)) + (a^8*b^2*exp(3*x)*((2*(a^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^
2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)
*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^
4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^13*b^3*(-
a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)))*(-a^8)^(1/2))/(8*(a^4 + 9*b^4 + 6*a^
2*b^2)^(1/2)) - (a^8*b^2*exp(x)*(-a^8)^(1/2)*((8*(a^4 + 9*b^4 + 6*a^2*b^2))
/(a^11*b*(a^4 + 3*b^4 + 4*a^2*b^2)) - (4*(2*a^7*b + 6*a^5*b^3)*(a^4 + 9*b^4
 + 6*a^2*b^2)^(1/2))/(a^16*b^2*((a^2 + 3*b^2)^2)^(1/2)*(a^2 + b^2)) + (2*(a
^8 + 3*a^6*b^2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/(a^15*b^3*((a^2 + 3*b^2)^2
)^(1/2)*(a^2 + b^2)) - (2*(a^2 + 2*b^2)*(a^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*
a^2*b^2)^(1/2) + 2*b^2*(-a^8)^(1/2)*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))*(a^4 +
 9*b^4 + 6*a^2*b^2)^(1/2))/(a^13*b^3*(-a^8)^(1/2)*(a^4 + 3*b^4 + 4*a^2*b^2)
)))/(8*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))))*(a^4 + 9*b^4 + 6*a^2*b^2)^(1/2))/
(-a^8)^(1/2) - (2/a^2 - (4*b*exp(x))/a^3)/(exp(2*x) - 1)
```

3.243 $\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\begin{aligned}\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = {} & \frac{b(3a^2+4b^2)\operatorname{arctanh}(\cosh(x))}{a^5} \\ & - \frac{2\sqrt{a^2+b^2}(a^2+4b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} \\ & - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} \\ & - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}\end{aligned}$$

```
[Out] b*(3*a^2+4*b^2)*arctanh(cosh(x))/a^5-1/3*(7*a^2+12*b^2)*coth(x)/a^4+(a^2+2*
b^2)*coth(x)*csch(x)/a^3/b-1/3*(3+4*b^2/a^2)*coth(x)*csch(x)/b/(a+b*sinh(x)
)-1/3*coth(x)*csch(x)^2/a/(a+b*sinh(x))-2*(a^2+4*b^2)*arctanh((b-a*tanh(1/2
*x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^5
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used

$= \{2803, 3134, 3080, 3855, 2739, 632, 212\}$

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{\left(\frac{4b^2}{a^2}+3\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))} + \frac{b(3a^2+4b^2)\,\text{arctanh}(\cosh(x))}{a^5} - \frac{2\sqrt{a^2+b^2}(a^2+4b^2)\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\text{csch}(x)}{a^3b} - \frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))}$$

```
[In] Int[Coth[x]^4/(a + b*Sinh[x])^2,x]
```

```
[Out] (b*(3*a^2 + 4*b^2)*ArcTanh[Cosh[x]])/a^5 - (2*Sqrt[a^2 + b^2]*(a^2 + 4*b^2)
*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/a^5 - ((7*a^2 + 12*b^2)*Coth[x
])/(3*a^4) + ((a^2 + 2*b^2)*Coth[x]*Csch[x])/(a^3*b) - ((3 + (4*b^2)/a^2)*C
oth[x]*Csch[x])/(3*b*(a + b*Sinh[x])) - (Coth[x]*Csch[x]^2)/(3*a*(a + b*Sin
h[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2803

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Dist[1/(3*a^2*b*(m + 1)), Int[((a + b*Sin[e + f*x])^(m
 + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e
+ f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x] - Simp[(3*a^2 +
 b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(3*a^2*b*f*(m + 1)
*Sin[e + f*x]^2)), x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && Lt
```

```
Q[m, -1] && IntegerQ[2*m]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))} \\
&\quad - \frac{\int \frac{\text{csch}^3(x)\left(6\left(a^2+2b^2\right)-ab\sinh(x)+\left(3a^2+8b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{3a^2b} \\
&= \frac{\left(a^2+2b^2\right)\coth(x)\text{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\text{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\text{csch}^2(x)}{3a(a+b\sinh(x))} \\
&\quad - \frac{i\int \frac{\text{csch}^2(x)\left(2ib\left(7a^2+12b^2\right)-4iab^2\sinh(x)+6ib\left(a^2+2b^2\right)\sinh^2(x)\right)}{a+b\sinh(x)}\,dx}{6a^3b}
\end{aligned}$$

$$= -\frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} + \frac{\int \frac{\operatorname{csch}(x)(-6b^2(3a^2+4b^2)+6ab(a^2+2b^2)\sinh(x))}{a+b\sinh(x)}\,dx}{6a^4b}$$

$$= -\frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} + \frac{((a^2+b^2)(a^2+4b^2))\int \frac{1}{a+b\sinh(x)}\,dx}{a^5} - \frac{(b(3a^2+4b^2))\int \operatorname{csch}(x)\,dx}{a^5}$$

$$= \frac{b(3a^2+4b^2)\operatorname{arctanh}(\cosh(x))}{a^5} - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} + \frac{(2(a^2+b^2)(a^2+4b^2))\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^5}$$

$$= \frac{b(3a^2+4b^2)\operatorname{arctanh}(\cosh(x))}{a^5} - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))} - \frac{(4(a^2+b^2)(a^2+4b^2))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a^5}$$

$$= \frac{b(3a^2+4b^2)\operatorname{arctanh}(\cosh(x))}{a^5} - \frac{2\sqrt{a^2+b^2}(a^2+4b^2)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a^5} - \frac{(7a^2+12b^2)\coth(x)}{3a^4} + \frac{(a^2+2b^2)\coth(x)\operatorname{csch}(x)}{a^3b} - \frac{\left(3+\frac{4b^2}{a^2}\right)\coth(x)\operatorname{csch}(x)}{3b(a+b\sinh(x))} - \frac{\coth(x)\operatorname{csch}^2(x)}{3a(a+b\sinh(x))}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.48

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$$

$$= \frac{\frac{48(a^4+5a^2b^2+4b^4)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4a(4a^2+9b^2)\coth\left(\frac{x}{2}\right) + 6a^2b\operatorname{csch}^2\left(\frac{x}{2}\right) + 24b(3a^2+4b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right) \cdots}{\cdots}$$

[In] Integrate[Coth[x]^4/(a + b*Sinh[x])^2,x]

[Out] ((48*(a^4 + 5*a^2*b^2 + 4*b^4)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/
Sqrt[-a^2 - b^2] - 4*a*(4*a^2 + 9*b^2)*Coth[x/2] + 6*a^2*b*Csch[x/2]^2 + 24
b(3*a^2 + 4*b^2)*Log[Cosh[x/2]] - 24*b*(3*a^2 + 4*b^2)*Log[Sinh[x/2]] + 6
*a^2*b*Sech[x/2]^2 + 8*a^3*Csch[x]^3*Sinh[x/2]^4 - (a^3*Csch[x/2]^4*Sinh[x]
)/2 - (24*a*b*(a^2 + b^2)*Cosh[x])/(a + b*Sinh[x]) - 4*a*(4*a^2 + 9*b^2)*Ta
nh[x/2])/(24*a^5)

Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.39

method	result
default	$-\frac{\frac{a^{2} \tanh\left(\frac{x}{2}\right)^{3}}{3}+2ab\tanh\left(\frac{x}{2}\right)^{2}+5a^{2}\tanh\left(\frac{x}{2}\right)+12b^{2}\tanh\left(\frac{x}{2}\right)}{8a^{4}}-\frac{2\left(\frac{-b^{2}\left(a^{2}+b^{2}\right)\tanh\left(\frac{x}{2}\right)-\left(a^{2}+b^{2}\right)ab}{\tanh\left(\frac{x}{2}\right)^{2}a-2b\tanh\left(\frac{x}{2}\right)-a}-\frac{\left(a^{4}+5a^{2}b^{2}+4b^{4}\right)\operatorname{arctanh}\left(\frac{2a\,t\cdots}{\cdots}\right.}{\sqrt{a^{2}+b^{2}}}\right.}{a^{5}}$
risch	$\frac{2a^{3}e^{7x}+4a\,b^{2}e^{7x}-2a^{2}b\,e^{6x}-8b^{3}e^{6x}-14a^{3}e^{5x}-20a\,b^{2}e^{5x}+14a^{2}b\,e^{4x}+24\,e^{4x}b^{3}+14a^{3}e^{3x}+28a\,b^{2}e^{3x}-\frac{50a^{2}b\,e^{2x}}{3}-24b^{3}e^{2x}-\frac{22a^{3}e^{x}}{3}-\cdots}{\left(e^{2x}-1\right)^{3}a^{4}\left(b\,e^{2x}+2\,e^{x}a-b\right)}$

[In] int(coth(x)^4/(a+b*sinh(x))^2,x,method=_RETURNVERBOSE)

[Out] -1/8/a^4*(1/3*a^2*tanh(1/2*x)^3+2*a*b*tanh(1/2*x)^2+5*a^2*tanh(1/2*x)+12*b^
2*tanh(1/2*x))-2/a^5*((-b^2*(a^2+b^2)*tanh(1/2*x)-(a^2+b^2)*a*b)/(tanh(1/2*
x)^2*a-2*b*tanh(1/2*x)-a)-(a^4+5*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2
*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/24/a^2/tanh(1/2*x)^3-1/8*(5*a^2+
12*b^2)/a^4/tanh(1/2*x)+1/4/a^3*b/tanh(1/2*x)^2-1/a^5*b*(3*a^2+4*b^2)*ln(ta
nh(1/2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3648 vs. 2(149) = 298.

Time = 0.35 (sec) , antiderivative size = 3648, normalized size of antiderivative = 22.94

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="fricas")

[Out] 1/3*(6*(a^4 + 2*a^2*b^2)*cosh(x)^7 + 6*(a^4 + 2*a^2*b^2)*sinh(x)^7 - 6*(a^3
*b + 4*a*b^3)*cosh(x)^6 - 6*(a^3*b + 4*a*b^3 - 7*(a^4 + 2*a^2*b^2)*cosh(x))
sinh(x)^6 - 6(7*a^4 + 10*a^2*b^2)*cosh(x)^5 - 6*(7*a^4 + 10*a^2*b^2 - 21*
(a^4 + 2*a^2*b^2)*cosh(x)^2 + 6*(a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^5 + 6*(7
*a^3*b + 12*a*b^3)*cosh(x)^4 + 6*(7*a^3*b + 12*a*b^3 + 35*(a^4 + 2*a^2*b^2)

cosh(x)^3 - 15(a^3*b + 4*a*b^3)*cosh(x)^2 - 5*(7*a^4 + 10*a^2*b^2)*cosh(x
))*sinh(x)^4 + 14*a^3*b + 24*a*b^3 + 42*(a^4 + 2*a^2*b^2)*cosh(x)^3 + 6*(35
*(a^4 + 2*a^2*b^2)*cosh(x)^4 + 7*a^4 + 14*a^2*b^2 - 20*(a^3*b + 4*a*b^3)*co
sh(x)^3 - 10*(7*a^4 + 10*a^2*b^2)*cosh(x)^2 + 4*(7*a^3*b + 12*a*b^3)*cosh(x
))*sinh(x)^3 - 2*(25*a^3*b + 36*a*b^3)*cosh(x)^2 + 2*(63*(a^4 + 2*a^2*b^2)*
cosh(x)^5 - 45*(a^3*b + 4*a*b^3)*cosh(x)^4 - 25*a^3*b - 36*a*b^3 - 30*(7*a^
4 + 10*a^2*b^2)*cosh(x)^3 + 18*(7*a^3*b + 12*a*b^3)*cosh(x)^2 + 63*(a^4 + 2
*a^2*b^2)*cosh(x))*sinh(x)^2 + 3*((a^2*b + 4*b^3)*cosh(x)^8 + (a^2*b + 4*b^
3)*sinh(x)^8 + 2*(a^3 + 4*a*b^2)*cosh(x)^7 + 2*(a^3 + 4*a*b^2 + 4*(a^2*b +
4*b^3)*cosh(x))*sinh(x)^7 - 4*(a^2*b + 4*b^3)*cosh(x)^6 - 2*(2*a^2*b + 8*b^
3 - 14*(a^2*b + 4*b^3)*cosh(x)^2 - 7*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^6 - 6
*(a^3 + 4*a*b^2)*cosh(x)^5 + 2*(28*(a^2*b + 4*b^3)*cosh(x)^3 - 3*a^3 - 12*a
b^2 + 21(a^3 + 4*a*b^2)*cosh(x)^2 - 12*(a^2*b + 4*b^3)*cosh(x))*sinh(x)^5
 + 6*(a^2*b + 4*b^3)*cosh(x)^4 + 2*(35*(a^2*b + 4*b^3)*cosh(x)^4 + 35*(a^3
+ 4*a*b^2)*cosh(x)^3 + 3*a^2*b + 12*b^3 - 30*(a^2*b + 4*b^3)*cosh(x)^2 - 15
*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^4 + 6*(a^3 + 4*a*b^2)*cosh(x)^3 + 2*(28*(
a^2*b + 4*b^3)*cosh(x)^5 + 35*(a^3 + 4*a*b^2)*cosh(x)^4 - 40*(a^2*b + 4*b^3
)*cosh(x)^3 + 3*a^3 + 12*a*b^2 - 30*(a^3 + 4*a*b^2)*cosh(x)^2 + 12*(a^2*b +
 4*b^3)*cosh(x))*sinh(x)^3 + a^2*b + 4*b^3 - 4*(a^2*b + 4*b^3)*cosh(x)^2 +
2*(14*(a^2*b + 4*b^3)*cosh(x)^6 + 21*(a^3 + 4*a*b^2)*cosh(x)^5 - 30*(a^2*b
+ 4*b^3)*cosh(x)^4 - 30*(a^3 + 4*a*b^2)*cosh(x)^3 - 2*a^2*b - 8*b^3 + 18*(a
^2*b + 4*b^3)*cosh(x)^2 + 9*(a^3 + 4*a*b^2)*cosh(x))*sinh(x)^2 - 2*(a^3 + 4
*a*b^2)*cosh(x) + 2*(4*(a^2*b + 4*b^3)*cosh(x)^7 + 7*(a^3 + 4*a*b^2)*cosh(x
)^6 - 12*(a^2*b + 4*b^3)*cosh(x)^5 - 15*(a^3 + 4*a*b^2)*cosh(x)^4 + 12*(a^2
*b + 4*b^3)*cosh(x)^3 - a^3 - 4*a*b^2 + 9*(a^3 + 4*a*b^2)*cosh(x)^2 - 4*(a^
2*b + 4*b^3)*cosh(x))*sinh(x))*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sin
h(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sq
rt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a
cosh(x) + 2(b*cosh(x) + a)*sinh(x) - b)) - 2*(11*a^4 + 18*a^2*b^2)*cosh(x
) + 3*((3*a^2*b^2 + 4*b^4)*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3
*a^3*b + 4*a*b^3)*cosh(x)^7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*
cosh(x))*sinh(x)^7 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4
 - 14*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x
)^6 - 6*(3*a^3*b + 4*a*b^3)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b
^2 + 4*b^4)*cosh(x)^3 - 21*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 +
4*b^4)*cosh(x))*sinh(x)^5 + 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*
b^2 + 4*b^4)*cosh(x)^4 + 9*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x
)^3 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*si
nh(x)^4 + 3*a^2*b^2 + 4*b^4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^
2*b^2 + 4*b^4)*cosh(x)^5 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*
a*b^3 - 40*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^2
 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^3 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x
)^2 + 2*(14*(3*a^2*b^2 + 4*b^4)*cosh(x)^6 + 21*(3*a^3*b + 4*a*b^3)*cosh(x)^
5 - 30*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 - 6*a^2*b^2 - 8*b^4 - 30*(3*a^3*b + 4*
a*b^3)*cosh(x)^3 + 18*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 + 9*(3*a^3*b + 4*a*b^3)

```
*cosh(x))*sinh(x)^2 - 2*(3*a^3*b + 4*a*b^3)*cosh(x) + 2*(4*(3*a^2*b^2 + 4*b
^4)*cosh(x)^7 + 7*(3*a^3*b + 4*a*b^3)*cosh(x)^6 - 12*(3*a^2*b^2 + 4*b^4)*co
sh(x)^5 - 15*(3*a^3*b + 4*a*b^3)*cosh(x)^4 - 3*a^3*b - 4*a*b^3 + 12*(3*a^2*
b^2 + 4*b^4)*cosh(x)^3 + 9*(3*a^3*b + 4*a*b^3)*cosh(x)^2 - 4*(3*a^2*b^2 + 4
*b^4)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) + 1) - 3*((3*a^2*b^2 + 4*b^4)
*cosh(x)^8 + (3*a^2*b^2 + 4*b^4)*sinh(x)^8 + 2*(3*a^3*b + 4*a*b^3)*cosh(x)^
7 + 2*(3*a^3*b + 4*a*b^3 + 4*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^7 - 4*(3*
a^2*b^2 + 4*b^4)*cosh(x)^6 - 2*(6*a^2*b^2 + 8*b^4 - 14*(3*a^2*b^2 + 4*b^4)*
cosh(x)^2 - 7*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^6 - 6*(3*a^3*b + 4*a*b^3
)*cosh(x)^5 - 2*(9*a^3*b + 12*a*b^3 - 28*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 - 21
*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x)^5
 + 6*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 2*(35*(3*a^2*b^2 + 4*b^4)*cosh(x)^4 + 9
*a^2*b^2 + 12*b^4 + 35*(3*a^3*b + 4*a*b^3)*cosh(x)^3 - 30*(3*a^2*b^2 + 4*b^
4)*cosh(x)^2 - 15*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^4 + 3*a^2*b^2 + 4*b^
4 + 6*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 2*(28*(3*a^2*b^2 + 4*b^4)*cosh(x)^5 +
 35*(3*a^3*b + 4*a*b^3)*cosh(x)^4 + 9*a^3*b + 12*a*b^3 - 40*(3*a^2*b^2 + 4*
b^4)*cosh(x)^3 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^2 + 12*(3*a^2*b^2 + 4*b^4)*
cosh(x))*sinh(x)^3 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x)^2 + 2*(14*(3*a^2*b^2 + 4
*b^4)*cosh(x)^6 + 21*(3*a^3*b + 4*a*b^3)*cosh(x)^5 - 30*(3*a^2*b^2 + 4*b^4)
*cosh(x)^4 - 6*a^2*b^2 - 8*b^4 - 30*(3*a^3*b + 4*a*b^3)*cosh(x)^3 + 18*(3*a
^2*b^2 + 4*b^4)*cosh(x)^2 + 9*(3*a^3*b + 4*a*b^3)*cosh(x))*sinh(x)^2 - 2*(3
*a^3*b + 4*a*b^3)*cosh(x) + 2*(4*(3*a^2*b^2 + 4*b^4)*cosh(x)^7 + 7*(3*a^3*b
 + 4*a*b^3)*cosh(x)^6 - 12*(3*a^2*b^2 + 4*b^4)*cosh(x)^5 - 15*(3*a^3*b + 4*
a*b^3)*cosh(x)^4 - 3*a^3*b - 4*a*b^3 + 12*(3*a^2*b^2 + 4*b^4)*cosh(x)^3 + 9
*(3*a^3*b + 4*a*b^3)*cosh(x)^2 - 4*(3*a^2*b^2 + 4*b^4)*cosh(x))*sinh(x))*lo
g(cosh(x) + sinh(x) - 1) + 2*(21*(a^4 + 2*a^2*b^2)*cosh(x)^6 - 18*(a^3*b +
4*a*b^3)*cosh(x)^5 - 15*(7*a^4 + 10*a^2*b^2)*cosh(x)^4 - 11*a^4 - 18*a^2*b^
2 + 12*(7*a^3*b + 12*a*b^3)*cosh(x)^3 + 63*(a^4 + 2*a^2*b^2)*cosh(x)^2 - 2*
(25*a^3*b + 36*a*b^3)*cosh(x))*sinh(x))/(a^5*b*cosh(x)^8 + a^5*b*sinh(x)^8
+ 2*a^6*cosh(x)^7 - 4*a^5*b*cosh(x)^6 - 6*a^6*cosh(x)^5 + 6*a^5*b*cosh(x)^4
 + 6*a^6*cosh(x)^3 - 4*a^5*b*cosh(x)^2 + 2*(4*a^5*b*cosh(x) + a^6)*sinh(x)^
7 - 2*a^6*cosh(x) + 2*(14*a^5*b*cosh(x)^2 + 7*a^6*cosh(x) - 2*a^5*b)*sinh(x
)^6 + a^5*b + 2*(28*a^5*b*cosh(x)^3 + 21*a^6*cosh(x)^2 - 12*a^5*b*cosh(x) -
 3*a^6)*sinh(x)^5 + 2*(35*a^5*b*cosh(x)^4 + 35*a^6*cosh(x)^3 - 30*a^5*b*cos
h(x)^2 - 15*a^6*cosh(x) + 3*a^5*b)*sinh(x)^4 + 2*(28*a^5*b*cosh(x)^5 + 35*a
^6*cosh(x)^4 - 40*a^5*b*cosh(x)^3 - 30*a^6*cosh(x)^2 + 12*a^5*b*cosh(x) + 3
*a^6)*sinh(x)^3 + 2*(14*a^5*b*cosh(x)^6 + 21*a^6*cosh(x)^5 - 30*a^5*b*cosh(
x)^4 - 30*a^6*cosh(x)^3 + 18*a^5*b*cosh(x)^2 + 9*a^6*cosh(x) - 2*a^5*b)*sin
h(x)^2 + 2*(4*a^5*b*cosh(x)^7 + 7*a^6*cosh(x)^6 - 12*a^5*b*cosh(x)^5 - 15*a
^6*cosh(x)^4 + 12*a^5*b*cosh(x)^3 + 9*a^6*cosh(x)^2 - 4*a^5*b*cosh(x) - a^6
)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx$$

```
[In] integrate(coth(x)**4/(a+b*sinh(x))**2,x)
[Out] Integral(coth(x)**4/(a + b*sinh(x))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(149) = 298.

Time = 0.29 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.13

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = -\frac{2\left(7a^2b+12b^3+(11a^3+18ab^2)e^{(-x)}-(25a^2b+36b^3)e^{(-2x)}-21(a^3+2ab^2)e^{(-3x)}+3(7a^2b+12b^3\right.}{3\left(2a^5e^{(-x)}-4a^4be^{(-2x)}-6a^5e^{(-3x)}+6a^4be^{(-4x)}+6a^5e^{(-5x)}-4\right.} + \frac{(3a^2b+4b^3)\log\left(e^{(-x)}+1\right)}{a^5} - \frac{(3a^2b+4b^3)\log\left(e^{(-x)}-1\right)}{a^5} + \frac{(a^4+5a^2b^2+4b^4)\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^5}$$

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="maxima")
[Out] -2/3*(7*a^2*b + 12*b^3 + (11*a^3 + 18*a*b^2)*e^(-x) - (25*a^2*b + 36*b^3)*e
^(-2*x) - 21*(a^3 + 2*a*b^2)*e^(-3*x) + 3*(7*a^2*b + 12*b^3)*e^(-4*x) + 3*(
7*a^3 + 10*a*b^2)*e^(-5*x) - 3*(a^2*b + 4*b^3)*e^(-6*x) - 3*(a^3 + 2*a*b^2)
*e^(-7*x))/(2*a^5*e^(-x) - 4*a^4*b*e^(-2*x) - 6*a^5*e^(-3*x) + 6*a^4*b*e^(-
4*x) + 6*a^5*e^(-5*x) - 4*a^4*b*e^(-6*x) - 2*a^5*e^(-7*x) + a^4*b*e^(-8*x)
+ a^4*b) + (3*a^2*b + 4*b^3)*log(e^(-x) + 1)/a^5 - (3*a^2*b + 4*b^3)*log(e^
(-x) - 1)/a^5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log((b*e^(-x) - a - sqrt(a^2 + b^
2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.52

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \frac{(3\,a^2b+4\,b^3)\log\left(e^x+1\right)}{a^5} - \frac{(3\,a^2b+4\,b^3)\log\left(\left|e^x-1\right|\right)}{a^5} + \frac{(a^4+5\,a^2b^2+4\,b^4)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^5} + \frac{2\,(a^3e^x+ab^2e^x-a^2b-b^3)}{(be^{(2\,x)}+2\,ae^x-b)a^4} + \frac{2\,\left(3\,abe^{(5\,x)}-6\,a^2e^{(4\,x)}-9\,b^2e^{(4\,x)}+6\,a^2e^{(2\,x)}+18\,b^2e^{(2\,x)}-3\,abe^x-4\,a^2-9\,b^2\right)}{3\,a^4\left(e^{(2\,x)}-1\right)^3}$$

```
[In] integrate(coth(x)^4/(a+b*sinh(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^2*b + 4*b^3)*log(e^x + 1)/a^5 - (3*a^2*b + 4*b^3)*log(abs(e^x - 1))/a^
5 + (a^4 + 5*a^2*b^2 + 4*b^4)*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/ab
s(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^5) + 2*(a^3*e^x +
a*b^2*e^x - a^2*b - b^3)/((b*e^(2*x) + 2*a*e^x - b)*a^4) + 2/3*(3*a*b*e^(5*
x) - 6*a^2*e^(4*x) - 9*b^2*e^(4*x) + 6*a^2*e^(2*x) + 18*b^2*e^(2*x) - 3*a*b
*e^x - 4*a^2 - 9*b^2)/(a^4*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 1450, normalized size of antiderivative = 9.12

$$\int \frac{\coth^4(x)}{(a+b\sinh(x))^2}\,dx = \text{Too large to display}$$

```
[In] int(coth(x)^4/(a + b*sinh(x))^2,x)
```

```
[Out] (3*b*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*exp(x) + 128*b^4*exp(x) +
224*a^2*b^2*exp(x)))/a^3 - 4/(a^2*exp(2*x) - a^2) - (6*b^2)/(a^4*exp(2*x) -
 a^4) - 8/(3*(3*a^2*exp(2*x) - 3*a^2*exp(4*x) + a^2*exp(6*x) - a^2)) - (4*a
^3*b^7)/(a^5*b^7*exp(2*x) - a^7*b^5 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^
6*exp(x) + 2*a^8*b^4*exp(x)) - (2*a^5*b^5)/(a^5*b^7*exp(2*x) - a^7*b^5 - a^
5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^6*exp(x) + 2*a^8*b^4*exp(x)) - (3*b*log(
96*a^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 224*a^2*b
^2*exp(x)))/a^3 - 4/(a^2*exp(4*x) - 2*a^2*exp(2*x) + a^2) - (4*b^3*log(96*a
^4 + 128*b^4 + 224*a^2*b^2 - 96*a^4*exp(x) - 128*b^4*exp(x) - 224*a^2*b^2*e
xp(x)))/a^5 + (4*b^3*log(96*a^4 + 128*b^4 + 224*a^2*b^2 + 96*a^4*exp(x) + 1
28*b^4*exp(x) + 224*a^2*b^2*exp(x)))/a^5 + (log(128*a^6*exp(x) - 256*a*b^5
```

```
- 64*a^5*b - 320*a^3*b^3 - 128*b^5*(a^2 + b^2)^(1/2) + 128*b^6*exp(x) - 288
*a^2*b^3*(a^2 + b^2)^(1/2) + 128*a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4
*exp(x) + 672*a^4*b^2*exp(x) - 64*a^4*b*(a^2 + b^2)^(1/2) + 384*a*b^4*exp(x
)*(a^2 + b^2)^(1/2) + 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/
2))/a^3 - (log(128*b^5*(a^2 + b^2)^(1/2) - 256*a*b^5 - 64*a^5*b - 320*a^3*b
^3 + 128*a^6*exp(x) + 128*b^6*exp(x) + 288*a^2*b^3*(a^2 + b^2)^(1/2) - 128*
a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672*a^4*b^2*exp(x) + 64
*a^4*b*(a^2 + b^2)^(1/2) - 384*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 608*a^3*b^2
*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^3 - (2*a*b^9)/(a^5*b^7*exp(
2*x) - a^7*b^5 - a^5*b^7 + a^7*b^5*exp(2*x) + 2*a^6*b^6*exp(x) + 2*a^8*b^4*
exp(x)) + (4*b*exp(x))/(a^3*exp(4*x) - 2*a^3*exp(2*x) + a^3) + (2*b*exp(x))
/(a^3*exp(2*x) - a^3) + (4*b^2*log(128*a^6*exp(x) - 256*a*b^5 - 64*a^5*b -
320*a^3*b^3 - 128*b^5*(a^2 + b^2)^(1/2) + 128*b^6*exp(x) - 288*a^2*b^3*(a^2
 + b^2)^(1/2) + 128*a^5*exp(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672
*a^4*b^2*exp(x) - 64*a^4*b*(a^2 + b^2)^(1/2) + 384*a*b^4*exp(x)*(a^2 + b^2)
^(1/2) + 608*a^3*b^2*exp(x)*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^5 - (4*
b^2*log(128*b^5*(a^2 + b^2)^(1/2) - 256*a*b^5 - 64*a^5*b - 320*a^3*b^3 + 12
8*a^6*exp(x) + 128*b^6*exp(x) + 288*a^2*b^3*(a^2 + b^2)^(1/2) - 128*a^5*exp
(x)*(a^2 + b^2)^(1/2) + 672*a^2*b^4*exp(x) + 672*a^4*b^2*exp(x) + 64*a^4*b*
(a^2 + b^2)^(1/2) - 384*a*b^4*exp(x)*(a^2 + b^2)^(1/2) - 608*a^3*b^2*exp(x)
*(a^2 + b^2)^(1/2))*(a^2 + b^2)^(1/2))/a^5 + (2*a^2*b^9*exp(x))/(a^5*b^8*ex
p(2*x) - a^7*b^6 - a^5*b^8 + a^7*b^6*exp(2*x) + 2*a^6*b^7*exp(x) + 2*a^8*b^
5*exp(x)) + (4*a^4*b^7*exp(x))/(a^5*b^8*exp(2*x) - a^7*b^6 - a^5*b^8 + a^7*
b^6*exp(2*x) + 2*a^6*b^7*exp(x) + 2*a^8*b^5*exp(x)) + (2*a^6*b^5*exp(x))/(a
^5*b^8*exp(2*x) - a^7*b^6 - a^5*b^8 + a^7*b^6*exp(2*x) + 2*a^6*b^7*exp(x) +
 2*a^8*b^5*exp(x))
```

3.244 $\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = -2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\sinh(x)}$$

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))*a^(1/2)+2*(a+b*sinh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2800, 52, 65, 213}

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = 2\sqrt{a+b\sinh(x)}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)$$

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]],x]
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\sinh(x)\right)\\
&= 2\sqrt{a+b\sinh(x)} + a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)\\
&= 2\sqrt{a+b\sinh(x)} + (2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)\\
&= -2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right) + 2\sqrt{a+b\sinh(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$
\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = -2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right) + 2\sqrt{a+b\sinh(x)}
$$

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]],x]
[Out] -2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

method	result	size
default	$\text{`int/indef0`}\left(\frac{\frac{a}{\sinh(x)}+b}{\sqrt{a+b\sinh(x)}}, \sinh\left(x\right)\right)$	21

```
[In] int(coth(x)*(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] `int/indef0`((1/sinh(x)*a+b)/(a+b*sinh(x))^(1/2),sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(29) = 58.

Time = 0.45 (sec) , antiderivative size = 356, normalized size of antiderivative = 9.62

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx$$
$$= \left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^2\cosh\left(x\right)^4 + b^2\sinh\left(x\right)^4 + 16\,ab\cosh\left(x\right)^3 + 4\left(b^2\cosh\left(x\right) + 4\,ab\right)\sinh\left(x\right)^3 - 16\,ab\cosh\left(x\right)\right.\right.$$
$$+2\sqrt{b\sinh\left(x\right)+a}, \sqrt{-a}\arctan\left(\frac{4\sqrt{b\sinh\left(x\right)+a}\sqrt{-a}(\cosh\left(x\right)+\sinh\left(x\right))}{b\cosh\left(x\right)^2 + b\sinh\left(x\right)^2 + 4\,a\cosh\left(x\right) + 2\left(b\cosh\left(x\right)+2\,a\right)\sinh\left(x\right) - \right.}\right.$$
$$\left. + 2\sqrt{b\sinh\left(x\right)+a}\right]$$

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="fricas")
[Out] [1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^
2*cosh(x) + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2
+ 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh
(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cos
h(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(
a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cos
h(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^
2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*
sqrt(b*sinh(x) + a), sqrt(-a)*arctan(4*sqrt(b*sinh(x) + a)*sqrt(-a)*(cosh(x
) + sinh(x))/(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2*a)
*sinh(x) - b)) + 2*sqrt(b*sinh(x) + a)]
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \sqrt{a+b\sinh{(x)}}\coth{(x)}\,dx$$

```
[In] integrate(coth(x)*(a+b*sinh(x))**(1/2),x)
[Out] Integral(sqrt(a + b*sinh(x))*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \sqrt{b\sinh{(x)}+a}\coth{(x)}\,dx$$

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

Giac [F]

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \sqrt{b\sinh{(x)}+a}\coth{(x)}\,dx$$

```
[In] integrate(coth(x)*(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sinh(x) + a)*coth(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\sinh(x)}\,dx = \int \coth(x)\,\sqrt{a+b\sinh{(x)}}\,dx$$

```
[In] int(coth(x)*(a + b*sinh(x))^(1/2),x)
[Out] int(coth(x)*(a + b*sinh(x))^(1/2), x)
```

3.245 $\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((a+b*sinh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2800, 65, 213}

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\sinh(x)\right)\\ &= 2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\sinh(x)}\right)\\ &= -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\sinh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]]/Sqrt[a]])/Sqrt[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

method	result	size
default	‘int/indef0‘ $\left(\frac{1}{\sinh(x)\sqrt{a+b\sinh(x)}}, \sinh\left(x\right)\right)$	17

```
[In] int(coth(x)/(a+b*sinh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ‘int/indef0‘(1/sinh(x)/(a+b*sinh(x))^(1/2),sinh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(18) = 36.

Time = 0.33 (sec) , antiderivative size = 370, normalized size of antiderivative = 15.42

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \left[\frac{\log\left(\frac{b^2\cosh(x)^4+b^2\sinh(x)^4+16\,ab\cosh(x)^3+4\,(b^2\cosh(x)+4\,ab)\sinh(x)^3-16\,ab\cosh(x)+2\,(16\,a^2-b^2)\cosh(x)^2+2\,\left(3\,b^2\cosh(x)^2+24\,ab\,c\right.}{\cosh(x}\right.}{} \right.$$

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x)
 + 4*a*b)*sinh(x)^3 - 16*a*b*cosh(x) + 2*(16*a^2 - b^2)*cosh(x)^2 + 2*(3*b^
2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 - b^2)*sinh(x)^2 - 8*(b*cosh(x)^3 + b
*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 - b*cosh(x) + (3
*b*cosh(x)^2 + 8*a*cosh(x) - b)*sinh(x))*sqrt(b*sinh(x) + a)*sqrt(a) + b^2
+ 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 - 4*a*b + (16*a^2 - b^2)*cosh(x))*sin
h(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*si
nh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1))/sqrt(a), sqrt
(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2
*a)*sinh(x) - b)*sqrt(b*sinh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^
2 + 2*a^2*cosh(x) - a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx$$

```
[In] integrate(coth(x)/(a+b*sinh(x))**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\coth\left(x\right)}{\sqrt{b\sinh\left(x\right)+a}}\,dx$$

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

Giac [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\coth\left(x\right)}{\sqrt{b\sinh\left(x\right)+a}}\,dx$$

```
[In] integrate(coth(x)/(a+b*sinh(x))^(1/2),x, algorithm="giac")
[Out] integrate(coth(x)/sqrt(b*sinh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+b\sinh\left(x\right)}}\,dx$$

```
[In] int(coth(x)/(a + b*sinh(x))^(1/2),x)
[Out] int(coth(x)/(a + b*sinh(x))^(1/2), x)
```

3.246 $\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx=-\frac{2A\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(a+b\sinh(x))}{b}$$

```
[Out] B*ln(a+b*sinh(x))/b-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4486, 2739, 632, 212, 2747, 31}

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx=\frac{B\log(a+b\sinh(x))}{b}-\frac{2A\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] Int[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\cosh(x)}{a+b\sinh(x)}\right)dx\\
&= A\int \frac{1}{a+b\sinh(x)}\,dx+B\int \frac{\cosh(x)}{a+b\sinh(x)}\,dx\\
&= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,b\sinh(x)\right)}{b}\\
&= \frac{B\log(a+b\sinh(x))}{b}-(4A)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\\
&= -\frac{2A\,\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(a+b\sinh(x))}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{A + B\cosh(x)}{a + b\sinh(x)}\,dx = \frac{2A\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{B\log(a + b\sinh(x))}{b}$$

```
[In] Integrate[(A + B*Cosh[x])/(a + b*Sinh[x]),x]
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*Log[
a + b*Sinh[x]])/b
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result
parts	$\frac{2A\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\ln(a+b\sinh(x))}{b}$
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)+\frac{2Ab\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}$
risch	$\frac{Bx}{b}-\frac{2xB\,a^2b}{a^2b^2+b^4}-\frac{2xB\,b^3}{a^2b^2+b^4}+\frac{\ln\left(e^x+\frac{Aab-\sqrt{A^2a^2b^2+A^2b^4}}{A\,b^2}\right)B\,a^2}{(a^2+b^2)b}+\frac{b\ln\left(e^x+\frac{Aab-\sqrt{A^2a^2b^2+A^2b^4}}{A\,b^2}\right)B}{a^2+b^2}+\frac{\ln\left(e^x+\frac{Aab-\sqrt{A^2a}}{A\,b}\right.}{}$

```
[In] int((A+B*cosh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 2*A/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+B*ln
(a+b*sinh(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(47) = 94.

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.33

$$\int \frac{A + B\cosh(x)}{a + b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(B}{a^2b+b^3}$$

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*x + (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a
)/(cosh(x) - sinh(x))))/(a^2*b + b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.13 (sec) , antiderivative size = 517, normalized size of antiderivative = 10.14

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)\right) \\ \frac{A\log\left(\tanh\left(\frac{x}{2}\right)\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)+B\log\left(\tanh\left(\frac{x}{2}\right)\right)}{b} \\ \frac{Ax+B\sinh(x)}{a} \\ \frac{2iA}{b\tanh\left(\frac{x}{2}\right)-ib}+\frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib}-\frac{iBx}{b\tanh\left(\frac{x}{2}\right)-ib}-\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)-ib}+\frac{2iB\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b\tanh\left(\frac{x}{2}\right)-ib}+\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)-i\right)\tanh}{b\tanh\left(\frac{x}{2}\right)-ib} \\ -\frac{2iA}{b\tanh\left(\frac{x}{2}\right)+ib}+\frac{Bx\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib}+\frac{iBx}{b\tanh\left(\frac{x}{2}\right)+ib}-\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)\tanh\left(\frac{x}{2}\right)}{b\tanh\left(\frac{x}{2}\right)+ib}-\frac{2iB\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b\tanh\left(\frac{x}{2}\right)+ib}+\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+i\right)\tan}{b\tanh\left(\frac{x}{2}\right)+ib} \\ -\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{A\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{Bx}{b}-\frac{2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)}{b}+\frac{B\log\left(\tanh\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{b} - \end{cases}$$

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x)
```

```
[Out] Piecewise((zoo*(A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log(tan
h(x/2))), Eq(a, 0) & Eq(b, 0)), ((A*log(tanh(x/2)) + B*x - 2*B*log(tanh(x/2
) + 1) + B*log(tanh(x/2)))/b, Eq(a, 0)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (
2*I*A/(b*tanh(x/2) - I*b) + B*x*tanh(x/2)/(b*tanh(x/2) - I*b) - I*B*x/(b*ta
nh(x/2) - I*b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) - I*b) + 2*I
*B*log(tanh(x/2) + 1)/(b*tanh(x/2) - I*b) + 2*B*log(tanh(x/2) - I)*tanh(x/2
)/(b*tanh(x/2) - I*b) - 2*I*B*log(tanh(x/2) - I)/(b*tanh(x/2) - I*b), Eq(a,
 -I*b)), (-2*I*A/(b*tanh(x/2) + I*b) + B*x*tanh(x/2)/(b*tanh(x/2) + I*b) +
I*B*x/(b*tanh(x/2) + I*b) - 2*B*log(tanh(x/2) + 1)*tanh(x/2)/(b*tanh(x/2) +
 I*b) - 2*I*B*log(tanh(x/2) + 1)/(b*tanh(x/2) + I*b) + 2*B*log(tanh(x/2) +
I)*tanh(x/2)/(b*tanh(x/2) + I*b) + 2*I*B*log(tanh(x/2) + I)/(b*tanh(x/2) +
I*b), Eq(a, I*b)), (-A*log(tanh(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2
 + b**2) + A*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2) +
 B*x/b - 2*B*log(tanh(x/2) + 1)/b + B*log(tanh(x/2) - b/a - sqrt(a**2 + b**
2)/a)/b + B*log(tanh(x/2) - b/a + sqrt(a**2 + b**2)/a)/b, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx = \frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{B\log\left(b\sinh\left(x\right)+a\right)}{b}$$

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sq
rt(a^2 + b^2) + B*log(b*sinh(x) + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.71

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx = \frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} - \frac{Bx}{b} + \frac{B\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{b}$$

```
[In] integrate((A+B*cosh(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) - B*x/b + B*log(abs(b*e^(2*x) + 2*a*e^x - b))/b
```

Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.88

$$\int \frac{A+B\cosh(x)}{a+b\sinh(x)}\,dx = \frac{B\,b^3\,\ln\left(8\,A^2\,a\,\mathrm{e}^{x}-4\,A^2\,b+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2\,b^2+b^4} - \frac{B\,x}{b} - \frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^{x}\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b+A\,b^3\right)\,\sqrt{A^2}}+\frac{A^2\,a\,b\,\sqrt{-a^2-b^2}}{\left(A\,a^2\,b+A\,b^3\right)\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{-a^2-b^2}} + \frac{B\,a^2\,b\,\ln\left(8\,A^2\,a\,\mathrm{e}^{x}-4\,A^2\,b+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2\,b^2+b^4}$$

```
[In] int((A + B*cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*b^3*log(8*A^2*a*exp(x) - 4*A^2*b + 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2) -
(B*x)/b - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)*(
A^2)^(1/2)) + (A^2*a*b*(- a^2 - b^2)^(1/2))/((A*b^3 + A*a^2*b)*(A^2)^(1/2))
)*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) + (B*a^2*b*log(8*A^2*a*exp(x) - 4*A^2*b
+ 4*A^2*b*exp(2*x)))/(b^4 + a^2*b^2)
```

3.247 $\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = B\log(i+\sinh(x)) - \frac{A\cosh(x)}{1-i\sinh(x)}$$

```
[Out] B*ln(I+sinh(x))-A*cosh(x)/(1-I*sinh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4486, 2727, 2746, 31}

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = B\log(\sinh(x)+i) - \frac{A\cosh(x)}{1-i\sinh(x)}$$

```
[In] Int[(A + B*Cosh[x])/(I + Sinh[x]),x]
```

```
[Out] B*Log[I + Sinh[x]] - (A*Cosh[x])/(1 - I*Sinh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{iA}{-1+i\sinh(x)}+\frac{iB\cosh(x)}{-1+i\sinh(x)}\right)dx\\
&= (iA)\int \frac{1}{-1+i\sinh(x)}\,dx+(iB)\int \frac{\cosh(x)}{-1+i\sinh(x)}\,dx\\
&= -\frac{A\cosh(x)}{1-i\sinh(x)}+B\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,i\sinh(x)\right)\\
&= B\log(i+\sinh(x))-\frac{A\cosh(x)}{1-i\sinh(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = -2iB\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+B\log(\cosh(x))-\frac{2iA\sinh\left(\frac{x}{2}\right)}{\cosh\left(\frac{x}{2}\right)-i\sinh\left(\frac{x}{2}\right)}$$

```
[In] Integrate[(A + B*Cosh[x])/(I + Sinh[x]),x]
[Out] (-2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]] - ((2*I)*A*Sinh[x/2])/(Cosh[x/2
] - I*Sinh[x/2])
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
parts	$-\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}+B\ln\left(i+\sinh\left(x\right)\right)$	23
risch	$-Bx-\frac{2A}{\mathrm{e}^{x}+i}+2B\ln\left(\mathrm{e}^{x}+i\right)$	25
default	$2B\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\frac{2iA}{\tanh\left(\frac{x}{2}\right)+i}-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	46

```
[In] int((A+B*cosh(x))/(I+sinh(x)),x,method=_RETURNVERBOSE)
[Out] -2*I*A/(tanh(1/2*x)+I)+B*ln(I+sinh(x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx=-\frac{Bxe^{x}+i\,Bx-2\left(Be^{x}+i\,B\right)\log\left(e^{x}+i\right)+2\,A}{e^{x}+i}$$

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="fricas")
[Out] -(B*x*e^x + I*B*x - 2*(B*e^x + I*B)*log(e^x + I) + 2*A)/(e^x + I)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx=-\frac{2A}{e^{x}+i}-Bx+2B\log\left(e^{x}+i\right)$$

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x)
[Out] -2*A/(exp(x) + I) - B*x + 2*B*log(exp(x) + I)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = B\log\left(\sinh\left(x\right)+i\right) - \frac{2\,A}{e^{(-x)}-i}$$

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="maxima")
[Out] B*log(sinh(x) + I) - 2*A/(e^(-x) - I)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = -Bx + 2\,B\log\left(e^{x}+i\right) - \frac{2\,A}{e^{x}+i}$$

```
[In] integrate((A+B*cosh(x))/(I+sinh(x)),x, algorithm="giac")
[Out] -B*x + 2*B*log(e^x + I) - 2*A/(e^x + I)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{A+B\cosh(x)}{i+\sinh(x)}\,dx = -B\,x - \frac{2\,A}{\mathrm{e}^{x}+1\mathrm{i}} + 2\,B\,\ln\left(\mathrm{e}^{x}+1\mathrm{i}\right)$$

```
[In] int((A + B*cosh(x))/(sinh(x) + 1i),x)
[Out] 2*B*log(exp(x) + 1i) - (2*A)/(exp(x) + 1i) - B*x
```

3.248 $\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = -B\log(i-\sinh(x)) + \frac{A\cosh(x)}{1+i\sinh(x)}$$

```
[Out] -B*ln(I-sinh(x))+A*cosh(x)/(1+I*sinh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4486, 2727, 2746, 31}

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = \frac{A\cosh(x)}{1+i\sinh(x)} - B\log(-\sinh(x)+i)$$

```
[In] Int[(A + B*Cosh[x])/(I - Sinh[x]),x]
[Out] -(B*Log[I - Sinh[x]]) + (A*Cosh[x])/(1 + I*Sinh[x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{iA}{1+i\sinh(x)} - \frac{iB\cosh(x)}{1+i\sinh(x)}\right) dx\\ &= -\left((iA)\int \frac{1}{1+i\sinh(x)}\,dx\right) - (iB)\int \frac{\cosh(x)}{1+i\sinh(x)}\,dx\\ &= \frac{A\cosh(x)}{1+i\sinh(x)} - B\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, i\sinh(x)\right)\\ &= -B\log(i-\sinh(x)) + \frac{A\cosh(x)}{1+i\sinh(x)}\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 81 vs. 2(27) = 54.

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.00

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = -\frac{\left(\cosh\left(\frac{x}{2}\right)+i\sinh\left(\frac{x}{2}\right)\right)\left(B\cosh\left(\frac{x}{2}\right)\left(2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-i\log(\cosh(x))\right)+\left(2A+2iB\arctan\left(\tanh\left(\right.\right.\right.\right.}{-i+\sinh(x)}$$

```
[In] Integrate[(A + B*Cosh[x])/(I - Sinh[x]),x]
```

```
[Out] -(((Cosh[x/2] + I*Sinh[x/2])*(B*Cosh[x/2]*(2*ArcTan[Tanh[x/2]] - I*Log[Cosh
[x]]) + (2*A + (2*I)*B*ArcTan[Tanh[x/2]] + B*Log[Cosh[x]])*Sinh[x/2]))/(-I
+ Sinh[x]))
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
risch	$Bx+\frac{2A}{\mathrm{e}^{x}-i}-2B\ln\left(\mathrm{e}^{x}-i\right)$	24
parts	$-\frac{2iA}{-i+\tanh\left(\frac{x}{2}\right)}+B\left(-\frac{\ln\left(\sinh\left(x\right)^{2}+1\right)}{2}-i\arctan\left(\sinh\left(x\right)\right)\right)$	33
default	$-\frac{2iA}{-i+\tanh\left(\frac{x}{2}\right)}-2B\ln\left(-i+\tanh\left(\frac{x}{2}\right)\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	44

```
[In] int((A+B*cosh(x))/(I-sinh(x)),x,method=_RETURNVERBOSE)
[Out] B*x+2*A/(exp(x)-I)-2*B*ln(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx=\frac{Bxe^{x}-i\,Bx-2\,(Be^{x}-i\,B)\log\left(e^{x}-i\right)+2\,A}{e^{x}-i}$$

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="fricas")
[Out] (B*x*e^x - I*B*x - 2*(B*e^x - I*B)*log(e^x - I) + 2*A)/(e^x - I)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx=\frac{2A}{e^{x}-i}+Bx-2B\log\left(e^{x}-i\right)$$

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x)
[Out] 2*A/(exp(x) - I) + B*x - 2*B*log(exp(x) - I)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = -B\log\left(\sinh\left(x\right)-i\right)+\frac{2\,A}{e^{(-x)}+i}$$

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="maxima")
[Out] -B*log(sinh(x) - I) + 2*A/(e^(-x) + I)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = Bx-2\,B\log\left(e^{x}-i\right)+\frac{2\,A}{e^{x}-i}$$

```
[In] integrate((A+B*cosh(x))/(I-sinh(x)),x, algorithm="giac")
[Out] B*x - 2*B*log(e^x - I) + 2*A/(e^x - I)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{A+B\cosh(x)}{i-\sinh(x)}\,dx = B\,x+\frac{2\,A}{\mathrm{e}^{x}-\mathrm{i}}-2\,B\,\ln\left(\mathrm{e}^{x}-\mathrm{i}\right)$$

```
[In] int(-(A + B*cosh(x))/(sinh(x) - 1i),x)
[Out] B*x + (2*A)/(exp(x) - 1i) - 2*B*log(exp(x) - 1i)
```

3.249 $\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \frac{bB\arctan(\sinh(x))}{a^2+b^2} - \frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{aB\log(\cosh(x))}{a^2+b^2} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] b*B*arctan(sinh(x))/(a^2+b^2)+a*B*ln(cosh(x))/(a^2+b^2)-a*B*ln(a+b*sinh(x))
/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4486, 2739, 632, 212, 2800, 815, 649, 209, 266}

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = -\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{bB\arctan(\sinh(x))}{a^2+b^2} - \frac{aB\log(a+b\sinh(x))}{a^2+b^2} + \frac{aB\log(\cosh(x))}{a^2+b^2}$$

```
[In] Int[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (b*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] + (a*B*Log[Cosh[x]])/(a^2 + b^2) - (a*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\tanh(x)}{a+b\sinh(x)}\right)dx\\
&= A\int \frac{1}{a+b\sinh(x)}\,dx+B\int \frac{\tanh(x)}{a+b\sinh(x)}\,dx\\
&= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\\
&\quad -B\text{Subst}\left(\int \frac{x}{(a+x)\left(-b^2-x^2\right)}\,dx,x,b\sinh(x)\right)\\
&= -\left((4A)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)\right)\\
&\quad -B\text{Subst}\left(\int \left(\frac{a}{\left(a^2+b^2\right)(a+x)}+\frac{-b^2-ax}{\left(a^2+b^2\right)\left(b^2+x^2\right)}\right)dx,x,b\sinh(x)\right)\\
&= -\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}-\frac{B\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{a^2+b^2}\\
&= -\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}\\
&\quad +\frac{(aB)\text{Subst}\left(\int \frac{x}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{a^2+b^2}+\frac{\left(b^2B\right)\text{Subst}\left(\int \frac{1}{b^2+x^2}\,dx,x,b\sinh(x)\right)}{a^2+b^2}\\
&= \frac{bB\arctan(\sinh(x))}{a^2+b^2}-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{aB\log(\cosh(x))}{a^2+b^2}-\frac{aB\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.67

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx$$
$$=\frac{\cosh(x)\left(2b\sqrt{-a^2-b^2}B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+2A\left(a^2+b^2\right)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)+a\sqrt{-a^2-b^2}B(\log(\cosh(x}{(a-ib)(a+ib)\sqrt{-a^2-b^2}(A\cosh(x)+B\sinh(x))}$$

```
[In] Integrate[(A + B*Tanh[x])/(a + b*Sinh[x]),x]
```

```
[Out] (Cosh[x]*(2*b*Sqrt[-a^2 - b^2]*B*ArcTan[Tanh[x/2]] + 2*A*(a^2 + b^2)*ArcTan
[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]] + a*Sqrt[-a^2 - b^2]*B*(Log[Cosh[x]] -
 Log[a + b*Sinh[x]]))*(A + B*Tanh[x]))/((a - I*b)*(a + I*b)*Sqrt[-a^2 - b^2
]*(A*Cosh[x] + B*Sinh[x]))
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

method	result
default	$\frac{-Ba\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-a^2A-A\,b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a^2+b^2}+\frac{2B\left(\frac{a\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{2}+b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2+b^2}$
risch	$-\frac{2xBa}{a^2+b^2}-\frac{2x\,a^3B}{-a^4-2a^2b^2-b^4}-\frac{2xBa\,b^2}{-a^4-2a^2b^2-b^4}+\frac{iB\ln(\mathrm{e}^x+i)b}{a^2+b^2}+\frac{B\ln(\mathrm{e}^x+i)a}{a^2+b^2}-\frac{iB\ln(\mathrm{e}^x-i)b}{a^2+b^2}+\frac{B\ln(\mathrm{e}^x-i)a}{a^2+b^2}-\frac{\ln\left(\mathrm{e}^x+\right.}{}$

```
[In] int((A+B*tanh(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)*(-1/2*B*a*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)-(-A*a^2-A*b^2)/
(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+2*B/(a^
2+b^2)*(1/2*a*ln(1+tanh(1/2*x)^2)+b*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(85) = 170.

Time = 1.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx$$
$$=\frac{2\,Bb\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-Ba\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)+Ba\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}A\log\left(\frac{b^2\,\mathrm{c}}{}\right.}{a^2+b^2}$$

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*b*arctan(cosh(x) + sinh(x)) - B*a*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) + B*a*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

Sympy [F]

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx=\int \frac{A+B\tanh{(x)}}{a+b\sinh{(x)}}\,dx$$

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x)
[Out] Integral((A + B*tanh(x))/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{2\,b\arctan\left(e^{(-x)}\right)}{a^2+b^2}+\frac{a\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}-\frac{a\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right) + \frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="maxima")
[Out] -B*(2*b*arctan(e^(-x))/(a^2 + b^2) + a*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a
^2 + b^2) - a*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt(a
^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx = \frac{2\,Bb\arctan\left(e^x\right)}{a^2+b^2}+\frac{Ba\log\left(e^{(2\,x)}+1\right)}{a^2+b^2} - \frac{Ba\log\left(\left|be^{(2\,x)}+2\,ae^x-b\right|\right)}{a^2+b^2}+\frac{A\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate((A+B*tanh(x))/(a+b*sinh(x)),x, algorithm="giac")
[Out] 2*B*b*arctan(e^x)/(a^2 + b^2) + B*a*log(e^(2*x) + 1)/(a^2 + b^2) - B*a*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 914, normalized size of antiderivative = 10.27

$$\int \frac{A+B\tanh(x)}{a+b\sinh(x)}\,dx =$$

$$-\frac{\ln\left(\frac{32\,B\,\left(A^2\,a\,b+\mathrm{e}^x\,A^2\,b^2-4\,\mathrm{e}^x\,A\,B\,a^2+2\,A\,B\,a\,b-\mathrm{e}^x\,A\,B\,b^2-4\,\mathrm{e}^x\,B^2\,a^2+B^2\,a\,b\right)}{b^5}-\frac{\left(\frac{32\,\left(-A^2\,a^2\,b-2\,\mathrm{e}^x\,A^2\,a\,b^2+A^2\,b^3+8\,\mathrm{e}^x\,A\,B\,a^3-4\,A\,B\,a\right.}{}\right.}{}\right.}{}$$

$$-\frac{\ln\left(\frac{32\,B\,\left(A^2\,a\,b+\mathrm{e}^x\,A^2\,b^2-4\,\mathrm{e}^x\,A\,B\,a^2+2\,A\,B\,a\,b-\mathrm{e}^x\,A\,B\,b^2-4\,\mathrm{e}^x\,B^2\,a^2+B^2\,a\,b\right)}{b^5}-\frac{\left(\frac{32\,\left(-A^2\,a^2\,b-2\,\mathrm{e}^x\,A^2\,a\,b^2+A^2\,b^3+8\,\mathrm{e}^x\,A\,B\,a^3-4\,A\,B\,a\right.}{}\right.}{}\right.}{}$$

$$+\frac{B\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)}{a-b\,1\mathrm{i}}+\frac{B\,\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}}{-b+a\,1\mathrm{i}}$$

```
[In] int((A + B*tanh(x))/(a + b*sinh(x)),x)
[Out] (B*log(exp(x) + 1i))/(a - b*1i) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*ex
p(x) + A^2*a*b + B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/
b^5 - (((32*(A^2*b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp(x)
 - 5*B^2*a*b^2*exp(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*exp(x)
 + 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*b^2)*
(a*b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) + a^
3*b^3*(128*A - 256*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2*b^4*ex
p(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A - 3*B) + 96*A*a^2*b*((a^2 + b^2)^3)^(
1/2) - 128*A*a^3*exp(x)*((a^2 + b^2)^3)^(1/2) - 32*A*a*b^2*exp(x)*((a^2 + b
^2)^3)^(1/2)))/(b^5*(a^2 + b^2)^3))*(B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*
b^2))/(a^2 + b^2)^2)*(B*a^3 - A*((a^2 + b^2)^3)^(1/2) + B*a*b^2))/(a^4 + b^
4 + 2*a^2*b^2) - (log((32*B*(A^2*b^2*exp(x) - 4*B^2*a^2*exp(x) + A^2*a*b +
B^2*a*b - 4*A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5 - (((32*(A^2*
b^3 + B^2*b^3 - A^2*a^2*b - 3*B^2*a^2*b + 4*B^2*a^3*exp(x) - 5*B^2*a*b^2*ex
p(x) - 4*A*B*a^2*b + 8*A*B*a^3*exp(x) - 2*A^2*a*b^2*exp(x) + 2*A*B*a*b^2*ex
p(x)))/b^5 - ((A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2)*(a*b^5*(64*A - 12
8*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) + a^3*b^3*(128*A - 25
6*B) - 128*exp(x)*(A - 2*B)*(a^2 + b^2)^3 + 192*a^2*b^4*exp(x)*(A - 3*B) +
96*a^4*b^2*exp(x)*(A - 3*B) - 96*A*a^2*b*((a^2 + b^2)^3)^(1/2) + 128*A*a^3*
exp(x)*((a^2 + b^2)^3)^(1/2) + 32*A*a*b^2*exp(x)*((a^2 + b^2)^3)^(1/2)))/(b
^5*(a^2 + b^2)^3))*(A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2))/(a^2 + b^2)
^2)*(A*((a^2 + b^2)^3)^(1/2) + B*a^3 + B*a*b^2))/(a^4 + b^4 + 2*a^2*b^2) +
(B*log(exp(x) - 1i)*1i)/(a*1i - b)
```

3.250 $\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx=-\frac{2A\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\log(\sinh(x))}{a}-\frac{B\log(a+b\sinh(x))}{a}$$

```
[Out] B*ln(sinh(x))/a-B*ln(a+b*sinh(x))/a-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)
^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4486, 2739, 632, 212, 2800, 36, 29, 31}

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx=-\frac{2A\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{B\log(a+b\sinh(x))}{a}+\frac{B\log(\sinh(x))}{a}$$

```
[In] Int[(A + B*Coth[x])/(a + b*Sinh[x]),x]
```

```
[Out] (-2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (B*Log[
Sinh[x]])/a - (B*Log[a + b*Sinh[x]])/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\coth(x)}{a+b\sinh(x)}\right)\,dx \\ &= A\int \frac{1}{a+b\sinh(x)}\,dx+B\int \frac{\coth(x)}{a+b\sinh(x)}\,dx \end{aligned}$$

$$= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) + B\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\sinh(x)\right)$$

$$= -\left((4A)\text{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) + \frac{B\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\sinh(x)\right)}{a} - \frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{a}$$

$$= -\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{B\log(\sinh(x))}{a} - \frac{B\log(a+b\sinh(x))}{a}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{2A\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + \frac{B(\log(\sinh(x))-\log(a+b\sinh(x)))}{a}$$

```
[In] Integrate[(A + B*Coth[x])/(a + b*Sinh[x]),x]
[Out] (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + (B*(Log
[Sinh[x]] - Log[a + b*Sinh[x]]))/a
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result
parts	$\frac{2A\ \text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a} - \frac{B\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a}$
default	$\frac{-B\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-2b\tanh\left(\frac{x}{2}\right)-a\right)+\frac{2Aa\ \text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a} + \frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$
risch	$-\frac{2xB}{a} - \frac{2x\,a^3 B}{-a^4-a^2b^2} - \frac{2xBa\,b^2}{-a^4-a^2b^2} + \frac{B\ln\left(e^{2x}-1\right)}{a} - \frac{a\ln\left(e^x+\frac{a^2A-\sqrt{A^2a^4+A^2a^2b^2}}{Aab}\right)B}{a^2+b^2} - \frac{\ln\left(e^x+\frac{a^2A-\sqrt{A^2a^4+A^2a^2b^2}}{Aab}\right)B\,b^2}{(a^2+b^2)a}$

```
[In] int((A+B*coth(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] 2*A/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+B/a*
ln(tanh(1/2*x))-B/a*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(56) = 112.

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.05

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{\sqrt{a^2+b^2}Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right)-(B}{a^3+ab^2}$$

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) +
 b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) +
a)*sinh(x) - b)) - (B*a^2 + B*b^2)*log(2*(b*sinh(x) + a)/(cosh(x) - sinh(x)
)) + (B*a^2 + B*b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^3 + a*b^2)
```

Sympy [F]

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \int \frac{A+B\coth{(x)}}{a+b\sinh{(x)}}\,dx$$

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*coth(x))/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.77

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a}-\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right) + \frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(log(-2*a*e^(-x) + b*e^(-2*x) - b)/a - log(e^(-x) + 1)/a - log(e^(-x) -
1)/a) + A*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b
^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}} + \frac{B\log\left(e^{x}+1\right)}{a} - \frac{B\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a} + \frac{B\log\left(\left|e^{x}-1\right|\right)}{a}$$

```
[In] integrate((A+B*coth(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] A*log(abs(2*b*e^x + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2
 + b^2)))/sqrt(a^2 + b^2) + B*log(e^x + 1)/a - B*log(abs(b*e^(2*x) + 2*a*e^
x - b))/a + B*log(abs(e^x - 1))/a
```

Mupad [B] (verification not implemented)

Time = 12.32 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{A+B\coth(x)}{a+b\sinh(x)}\,dx = \frac{B\,\ln\left(16\,B^2\,a^2+16\,B^2\,b^2-16\,B^2\,a^2\,\mathrm{e}^{2\,x}-16\,B^2\,b^2\,\mathrm{e}^{2\,x}\right)}{a} - \frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^{x}\,\sqrt{-a^2-b^2}+A^2\,a\,b\,\sqrt{-a^2-b^2}}{A\,b\,\sqrt{A^2}\,\left(a^2+b^2\right)}\right)\,\sqrt{A^2}}{\sqrt{-a^2-b^2}} - \frac{B\,\ln\left(32\,B^2\,a\,\mathrm{e}^{x}-16\,B^2\,b+16\,B^2\,b\,\mathrm{e}^{2\,x}\right)}{a}$$

```
[In] int((A + B*coth(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*log(16*B^2*a^2 + 16*B^2*b^2 - 16*B^2*a^2*exp(2*x) - 16*B^2*b^2*exp(2*x))
)/a - (2*atan((A^2*b^2*exp(x)*(- a^2 - b^2)^(1/2) + A^2*a*b*(- a^2 - b^2)^(
1/2))/(A*b*(A^2)^(1/2)*(a^2 + b^2)))*(A^2)^(1/2))/(- a^2 - b^2)^(1/2) - (B*
log(32*B^2*a*exp(x) - 16*B^2*b + 16*B^2*b*exp(2*x)))/a
```

3.251 $\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{aB\arctan(\sinh(x))}{a^2+b^2} - \frac{2A\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \frac{bB\log(\cosh(x))}{a^2+b^2} + \frac{bB\log(a+b\sinh(x))}{a^2+b^2}$$

```
[Out] a*B*arctan(sinh(x))/(a^2+b^2)-b*B*ln(cosh(x))/(a^2+b^2)+b*B*ln(a+b*sinh(x))
/(a^2+b^2)-2*A*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {4311, 4486, 2739, 632, 212, 2747, 720, 31, 649, 210, 266}

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = -\frac{2A\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{aB\arctan(\sinh(x))}{a^2+b^2} + \frac{bB\log(a+b\sinh(x))}{a^2+b^2} - \frac{bB\log(\cosh(x))}{a^2+b^2}$$

```
[In] Int[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

```
[Out] (a*B*ArcTan[Sinh[x]])/(a^2 + b^2) - (2*A*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a^2
 + b^2]])/Sqrt[a^2 + b^2] - (b*B*Log[Cosh[x]])/(a^2 + b^2) + (b*B*Log[a + b
*Sinh[x]])/(a^2 + b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4311

```
Int[(u_)*((A_) + (B_.)*sec[(a_.) + (b_.)*(x_)]), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Cos[a + b*x])/Cos[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(B+A\cosh(x))\text{sech}(x)}{a+b\sinh(x)}\,dx \\
&= \int \left(\frac{A}{a+b\sinh(x)}+\frac{B\text{sech}(x)}{a+b\sinh(x)}\right)dx \\
&= A\int \frac{1}{a+b\sinh(x)}\,dx + B\int \frac{\text{sech}(x)}{a+b\sinh(x)}\,dx \\
&= (2A)\text{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right) \\
&\quad -(bB)\text{Subst}\left(\int \frac{1}{(a+x)\left(-b^2-x^2\right)}\,dx, x, b\sinh(x)\right) \\
&= -\left((4A)\text{Subst}\left(\int \frac{1}{4\left(a^2+b^2\right)-x^2}\,dx, x, 2b-2a\tanh\left(\frac{x}{2}\right)\right)\right) \\
&\quad +\frac{(bB)\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\sinh(x)\right)}{a^2+b^2}+\frac{(bB)\text{Subst}\left(\int \frac{-a+x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2} \\
&= -\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2} \\
&\quad +\frac{(bB)\text{Subst}\left(\int \frac{x}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2}-\frac{(abB)\text{Subst}\left(\int \frac{1}{-b^2-x^2}\,dx, x, b\sinh(x)\right)}{a^2+b^2} \\
&= \frac{aB\arctan(\sinh(x))}{a^2+b^2}-\frac{2A\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{bB\log(\cosh(x))}{a^2+b^2}+\frac{bB\log(a+b\sinh(x))}{a^2+b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx=\frac{2aB\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{2A\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}-\frac{bB(\log(\cosh(x))-\log(a+b\sinh(x)))}{a^2+b^2}$$

```
[In] Integrate[(A + B*Sech[x])/(a + b*Sinh[x]),x]
```

```
[Out] (2*a*B*ArcTan[Tanh[x/2]])/(a^2 + b^2) + (2*A*ArcTan[(b - a*Tanh[x/2])/Sqrt[
-a^2 - b^2]])/Sqrt[-a^2 - b^2] - (b*B*(Log[Cosh[x]] - Log[a + b*Sinh[x]]))/
(a^2 + b^2)
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.15

method	result
parts	$\frac{2A\,\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+B\left(\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)}{a^2+b^2}+\frac{-b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^2+b^2}\right)$
default	$\frac{bB\ln\left(\tanh\left(\frac{x}{2}\right)^2a-2b\tanh\left(\frac{x}{2}\right)-a\right)-\frac{2\left(-a^2A-A\,b^2\right)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a^2+b^2}+\frac{2B\left(-\frac{b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{2}+a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2+b^2}$
risch	$\frac{2xbB}{a^2+b^2}+\frac{2xB\,a^2b}{-a^4-2a^2b^2-b^4}+\frac{2xB\,b^3}{-a^4-2a^2b^2-b^4}+\frac{iB\ln(\mathrm{e}^x+i)a}{a^2+b^2}-\frac{B\ln(\mathrm{e}^x+i)b}{a^2+b^2}-\frac{iB\ln(\mathrm{e}^x-i)a}{a^2+b^2}-\frac{B\ln(\mathrm{e}^x-i)b}{a^2+b^2}+\frac{\ln\left(\mathrm{e}^x+\frac{Aa-}{}\right.}{}$

```
[In] int((A+B*sech(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*A/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^2)^(1/2))+B*(b
/(a^2+b^2)*ln(tanh(1/2*x)^2*a-2*b*tanh(1/2*x)-a)+2/(a^2+b^2)*(-1/2*b*ln(1+t
anh(1/2*x)^2)+a*arctan(tanh(1/2*x))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(85) = 170.

Time = 1.89 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

$$\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx$$
$$= \frac{2\,Ba\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+Bb\log\left(\frac{2\,(b\sinh(x)+a)}{\cosh(x)-\sinh(x)}\right)-Bb\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2+b^2}A\log\left(\frac{b^2 c\cdots}{\cdots}\right.}{a^2+b^2}$$

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] (2*B*a*arctan(cosh(x) + sinh(x)) + B*b*log(2*(b*sinh(x) + a)/(cosh(x) - sin
h(x))) - B*b*log(2*cosh(x)/(cosh(x) - sinh(x))) + sqrt(a^2 + b^2)*A*log((b^
2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^
2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) - b)))/(a^2 + b^2
)
```

Sympy [F]

$$\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx = \int \frac{A+B\,\text{sech}\left(x\right)}{a+b\sinh\left(x\right)}\,dx$$

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*sech(x))/(a + b*sinh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.40

$$\int \frac{A+B\text{sech}(x)}{a+b\sinh(x)}\,dx$$
$$= -B\left(\frac{2\,a\arctan\left(e^{(-x)}\right)}{a^2+b^2}-\frac{b\log\left(-2\,ae^{(-x)}+be^{(-2\,x)}-b\right)}{a^2+b^2}+\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2+b^2}\right)$$
$$+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(2*a*arctan(e^(-x))/(a^2 + b^2) - b*log(-2*a*e^(-x) + b*e^(-2*x) - b)/(a
^2 + b^2) + b*log(e^(-2*x) + 1)/(a^2 + b^2)) + A*log((b*e^(-x) - a - sqrt(a
^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{2\,Ba\arctan\left(e^{x}\right)}{a^{2}+b^{2}} - \frac{Bb\log\left(e^{(2\,x)}+1\right)}{a^{2}+b^{2}} + \frac{Bb\log\left(\left|be^{(2\,x)}+2\,ae^{x}-b\right|\right)}{a^{2}+b^{2}} + \frac{A\log\left(\frac{\left|2\,be^{x}+2\,a-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,be^{x}+2\,a+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}}$$

```
[In] integrate((A+B*sech(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(e^x)/(a^2 + b^2) - B*b*log(e^(2*x) + 1)/(a^2 + b^2) + B*b*log(
abs(b*e^(2*x) + 2*a*e^x - b))/(a^2 + b^2) + A*log(abs(2*b*e^x + 2*a - 2*sqr
t(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 11.67 (sec) , antiderivative size = 864, normalized size of antiderivative = 9.71

$$\int \frac{A+B\operatorname{sech}(x)}{a+b\sinh(x)}\,dx = \frac{\ln\left(\frac{\left(A\,\sqrt{{\left(a^2+b^2\right)}^3}+B\,b^3+B\,a^2\,b\right)\,\left(b^3\,\left(32\,A^2-96\,{\mathrm{e}}^x\,A\,B+64\,B^2\right)-128\,A^2\,{\mathrm{e}}^x\,\sqrt{{\left(a^2+b^2\right)}^3}-a\,b^2\,\left(96\,{\mathrm{e}}^x\,A^2-192\,A\,B+128\,{\mathrm{e}}^x\,B^2\right)-128\,a^3\,{\mathrm{e}}^x\,\left(A^2\right.\right.}{}\right)}{} - \frac{B\,\ln\left({\mathrm{e}}^x+1{}\mathrm{i}\right)}{b+a\,1{}\mathrm{i}} + \frac{\ln\left(-\frac{32\,B\,\left({\mathrm{e}}^x\,A^2\,a\,b-A^2\,b^2+4\,{\mathrm{e}}^x\,A\,B\,a^2-2\,A\,B\,a\,b+{\mathrm{e}}^x\,A\,B\,b^2-4\,{\mathrm{e}}^x\,B^2\,a\,b+2\,B^2\,b^2\right)}{b^5}-\frac{\left(B\,b^3-A\,\sqrt{{\left(a^2+b^2\right)}^3}+B\,a^2\,b\right)\,\left(a\,b^2\,\left(96\,{\mathrm{e}}^x\,A^2-1\right.\right.}{}\right)}{} - \frac{B\,\ln\left({\mathrm{e}}^x-\mathrm{i}\right)\,1{}\mathrm{i}}{a+b\,1{}\mathrm{i}}$$

```
[In] int((A + B/cosh(x))/(a + b*sinh(x)),x)
```

```
[Out] (log(((A*((a^2 + b^2)^3)^(1/2) + B*b^3 + B*a^2*b)*(b^3*(32*A^2 + 64*B^2 - 9
6*A*B*exp(x)) - 128*A^2*exp(x)*((a^2 + b^2)^3)^(1/2) - a*b^2*(96*A^2*exp(x)
 + 128*B^2*exp(x) - 192*A*B) - 128*a^3*exp(x)*(A^2 + B^2) + a^2*b*(64*A^2 +
 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6*(2*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(
```

```
1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^2*
b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^3*b^3*(4*A - 19*B*e
xp(x)))/((a^2 + b^2)^3)^(1/2) + (64*A*a*b^5*(A - 4*B*exp(x)))/((a^2 + b^2)^
3)^(1/2) + (32*A*a^5*b*(2*A - 11*B*exp(x)))/((a^2 + b^2)^3)^(1/2)))/(b^5*(a
^2 + b^2)^2) - (32*B*(2*B^2*b^2 - A^2*b^2 + 4*A*B*a^2*exp(x) + A*B*b^2*exp(
x) + A^2*a*b*exp(x) - 4*B^2*a*b*exp(x) - 2*A*B*a*b))/b^5)*(A*((a^2 + b^2)^3
)^(1/2) + B*b^3 + B*a^2*b))/(a^4 + b^4 + 2*a^2*b^2) - (B*log(exp(x) + 1i))/
(a*1i + b) - (B*log(exp(x) - 1i)*1i)/(a + b*1i) + (log(- (32*B*(2*B^2*b^2 -
 A^2*b^2 + 4*A*B*a^2*exp(x) + A*B*b^2*exp(x) + A^2*a*b*exp(x) - 4*B^2*a*b*e
xp(x) - 2*A*B*a*b))/b^5 - ((B*b^3 - A*((a^2 + b^2)^3)^(1/2) + B*a^2*b)*(a*b
^2*(96*A^2*exp(x) + 128*B^2*exp(x) - 192*A*B) - 128*A^2*exp(x)*((a^2 + b^2)
^3)^(1/2) - b^3*(32*A^2 + 64*B^2 - 96*A*B*exp(x)) + 128*a^3*exp(x)*(A^2 + B
^2) - a^2*b*(64*A^2 + 64*B^2 - 384*A*B*exp(x)) + (32*A*b^6*(2*B + 3*A*exp(x
)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^4*b^2*(5*B + 3*A*exp(x)))/((a^2 + b^2)^
3)^(1/2) + (32*A*a^2*b^4*(7*B + 6*A*exp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*
a^3*b^3*(4*A - 19*B*exp(x)))/((a^2 + b^2)^3)^(1/2) + (64*A*a*b^5*(A - 4*B*e
xp(x)))/((a^2 + b^2)^3)^(1/2) + (32*A*a^5*b*(2*A - 11*B*exp(x)))/((a^2 + b^
2)^3)^(1/2)))/(b^5*(a^2 + b^2)^2))*(B*b^3 - A*((a^2 + b^2)^3)^(1/2) + B*a^2
*b))/(a^4 + b^4 + 2*a^2*b^2)
```

3.252 $\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx=-\frac{B\text{arctanh}(\cosh(x))}{a}-\frac{2(aA-bB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] -B*arctanh(cosh(x))/a-2*(A*a-B*b)*arctanh((b-a*tanh(1/2*x))/(a^2+b^2)^(1/2)
)/a/(a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2907, 3080, 3855, 2739, 632, 212}

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx=-\frac{2(aA-bB)\text{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}-\frac{B\text{arctanh}(\cosh(x))}{a}$$

```
[In] Int[(A + B*Csch[x])/(a + b*Sinh[x]),x]
```

```
[Out] -((B*ArcTanh[Cosh[x]])/a) - (2*(a*A - b*B)*ArcTanh[(b - a*Tanh[x/2])/Sqrt[a
^2 + b^2]])/(a*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2907

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

Rule 3080

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i\int \frac{\operatorname{csch}(x)(iB+iA\sinh(x))}{a+b\sinh(x)}\,dx\right)\\
&= \frac{B\int \operatorname{csch}(x)\,dx}{a}+\frac{(aA-bB)\int \frac{1}{a+b\sinh(x)}\,dx}{a}\\
&= -\frac{B\operatorname{arctanh}(\cosh(x))}{a}+\frac{(2(aA-bB))\operatorname{Subst}\left(\int \frac{1}{a+2bx-ax^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{B\operatorname{arctanh}(\cosh(x))}{a}-\frac{(4(aA-bB))\operatorname{Subst}\left(\int \frac{1}{4(a^2+b^2)-x^2}\,dx,x,2b-2a\tanh\left(\frac{x}{2}\right)\right)}{a}\\
&= -\frac{B\operatorname{arctanh}(\cosh(x))}{a}-\frac{2(aA-bB)\operatorname{arctanh}\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx=\frac{\frac{2(aA-bB)\arctan\left(\frac{b-a\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}+B\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{a}$$

```
[In] Integrate[(A + B*Csch[x])/(a + b*Sinh[x]),x]
[Out] ((2*(a*A - b*B)*ArcTan[(b - a*Tanh[x/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2
] + B*(-Log[Cosh[x/2]] + Log[Sinh[x/2]]))/a
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

method	result
default	$-\frac{(-2Aa+2Bb)\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$
parts	$\frac{2A\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2Bb\,\text{arctanh}\left(\frac{2a\tanh\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$
risch	$\frac{B\ln(e^x-1)}{a}-\frac{B\ln(e^x+1)}{a}+\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}-\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}-a^2-b^2}{\sqrt{a^2+b^2}\,b}\right)Bb}{\sqrt{a^2+b^2}\,a}-\frac{\ln\left(e^x+\frac{a\sqrt{a^2+b^2}+a^2+b^2}{\sqrt{a^2+b^2}\,b}\right)A}{\sqrt{a^2+b^2}}+$

```
[In] int((A+B*csch(x))/(a+b*sinh(x)),x,method=_RETURNVERBOSE)
[Out] -(-2*A*a+2*B*b)/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*x)-2*b)/(a^2+b^
2)^(1/2))+B/a*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(54) = 108$.

Time = 0.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.97

$$\int \frac{A+B\text{csch}(x)}{a+b\sinh(x)}\,dx = -\frac{(Aa-Bb)\sqrt{a^2+b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2+b^2+2\,\left(b^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{a^2+b^2}(b\cosh(x)+b\sinh(x)+}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)-b}\right.}{a^3+ab^2}$$

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="fricas")
```

```
[Out] -((A*a - B*b)*sqrt(a^2 + b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*co
sh(x) + 2*a^2 + b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 + b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) - b)) + (B*a^2 + B*b^2)*log(cosh(x) + sinh(x) + 1) - (B
*a^2 + B*b^2)*log(cosh(x) + sinh(x) - 1))/(a^3 + a*b^2)
```

Sympy [F]

$$\int \frac{A+B\mathrm{csch}(x)}{a+b\sinh(x)}\,dx = \int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx$$

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x)
```

```
[Out] Integral((A + B*csch(x))/(a + b*sinh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(54) = 108.

Time = 0.33 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

$$\int \frac{A+B\mathrm{csch}(x)}{a+b\sinh(x)}\,dx = -B\left(\frac{b\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(e^{(-x)}+1\right)}{a}-\frac{\log\left(e^{(-x)}-1\right)}{a}\right)+\frac{A\log\left(\frac{be^{(-x)}-a-\sqrt{a^2+b^2}}{be^{(-x)}-a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="maxima")
```

```
[Out] -B*(b*log((b*e^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2))
)/(sqrt(a^2 + b^2)*a) + log(e^(-x) + 1)/a - log(e^(-x) - 1)/a) + A*log((b*e
^(-x) - a - sqrt(a^2 + b^2))/(b*e^(-x) - a + sqrt(a^2 + b^2)))/sqrt(a^2 + b
^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{A+B\mathrm{csch}(x)}{a+b\sinh(x)}\,dx = -\frac{B\log\left(e^x+1\right)}{a}+\frac{B\log\left(\left|e^x-1\right|\right)}{a}+\frac{(Aa-Bb)\log\left(\frac{\left|2\,be^x+2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,be^x+2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}$$

```
[In] integrate((A+B*csch(x))/(a+b*sinh(x)),x, algorithm="giac")
```

```
[Out] -B*log(e^x + 1)/a + B*log(abs(e^x - 1))/a + (A*a - B*b)*log(abs(2*b*e^x + 2
*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^x + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*a)
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 539, normalized size of antiderivative = 9.29

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\sinh(x)}\,dx = \frac{B\ \ln\left(\mathrm{e}^x-1\right)}{a} - \frac{B\ \ln\left(\mathrm{e}^x+1\right)}{a} - \frac{\ln\left(\frac{(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,\mathrm{e}^x\,B^2\,a^3+2\,B^2\,a^2\,b-3\,\mathrm{e}^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}-\frac{(A\,a-B\,b)\left(\frac{32\,a^2\left(2\,B\,b^2+4\,A\,a^2\,\mathrm{e}^x+A\,b^2\,\mathrm{e}^x-2\,A\,a\,b-3\,B\,a\,b\,\mathrm{e}^x\right)}{b^5}\cdots}{a\,\sqrt{a^2+b^2}}\right)}{a\,\sqrt{a^2+b^2}}\cdots\right)\cdots}{a^3+a\,b^2\cdots} + \frac{\ln\left(\frac{32\,B\,(A\,a-B\,b)\,(A\,b\,\mathrm{e}^x-2\,B\,b+4\,B\,a\,\mathrm{e}^x)}{b^5}-\frac{(A\,a-B\,b)\left(\frac{32\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,\mathrm{e}^x\,B^2\,a^3+2\,B^2\,a^2\,b-3\,\mathrm{e}^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}+\frac{(A\,a-B\,b)\left(\frac{32\,a\cdots}{\cdots}\right)}{\cdots}\right)}{a\sqrt{\cdots}}\right)\cdots}{a^3+a\,b^2\cdots}$$

```
[In] int((A + B/sinh(x))/(a + b*sinh(x)),x)
```

```
[Out] (B*log(exp(x) - 1))/a - (B*log(exp(x) + 1))/a - (log(((A*a - B*b)*((32*(2*B
^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*a*b^2*exp(x) -
2*A*B*a*b^2))/b^5 - ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^2*exp(x) + A*b^2
*exp(x) - 2*A*a*b - 3*B*a*b*exp(x)))/b^5 + (32*a*(A*a - B*b)*(3*a^2*b + 2*b
^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(a*(a^2 + b^
2)^(1/2))))/(a*(a^2 + b^2)^(1/2)) + (32*B*(A*a - B*b)*(A*b*exp(x) - 2*B*b +
 4*B*a*exp(x)))/b^5)*(A*a - B*b)*(a^2 + b^2)^(1/2))/(a*b^2 + a^3) + (log((3
2*B*(A*a - B*b)*(A*b*exp(x) - 2*B*b + 4*B*a*exp(x)))/b^5 - ((A*a - B*b)*((3
2*(2*B^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b - 4*B^2*a^3*exp(x) - 3*B^2*a*b^2*exp
(x) - 2*A*B*a*b^2))/b^5 + ((A*a - B*b)*((32*a^2*(2*B*b^2 + 4*A*a^2*exp(x) +
 A*b^2*exp(x) - 2*A*a*b - 3*B*a*b*exp(x)))/b^5 - (32*a*(A*a - B*b)*(3*a^2*b
 + 2*b^3 - 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a^2 + b^2)^(1/2))))/(a*(a^
2 + b^2)^(1/2))))/(a*(a^2 + b^2)^(1/2)))*(A*a - B*b)*(a^2 + b^2)^(1/2))/(a*
b^2 + a^3)
```

3.253 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 81

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx=\frac{Cx}{c}-\frac{2(Ac-aC)\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}e}+\frac{B\log(a+c\sinh(d+ex))}{ce}$$

```
[Out] C*x/c+B*ln(a+c*sinh(e*x+d))/c/e-2*(A*c-C*a)*arctanh((c-a*tanh(1/2*e*x+1/2*d
))/(a^2+c^2)^(1/2))/c/e/(a^2+c^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4461, 2814, 2739, 632, 210, 2747, 31}

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx=-\frac{2(Ac-aC)\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{ce\sqrt{a^2+c^2}}+\frac{B\log(a+c\sinh(d+ex))}{ce}+\frac{Cx}{c}$$

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
```

```
[Out] (C*x)/c - (2*(A*c - a*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]]
)/(c*Sqrt[a^2 + c^2]*e) + (B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \frac{\cosh(d+ex)}{a+c\sinh(d+ex)}\,dx+\int \frac{A+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx\\
&= \frac{Cx}{c}-\frac{(i(iAc-iaC))\int \frac{1}{a+c\sinh(d+ex)}\,dx}{c}+\frac{B\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,c\sinh(d+ex)\right)}{ce}\\
&= \frac{Cx}{c}+\frac{B\log(a+c\sinh(d+ex))}{ce}-\frac{(2i(Ac-aC))\text{Subst}\left(\int \frac{1}{a-2icx+ax^2}\,dx,x,\tan\left(\frac{1}{2}(id+iex)\right)\right)}{ce}\\
&= \frac{Cx}{c}+\frac{B\log(a+c\sinh(d+ex))}{ce}\\
&\quad+\frac{(4i(Ac-aC))\text{Subst}\left(\int \frac{1}{-4(a^2+c^2)-x^2}\,dx,x,-2ic+2a\tan\left(\frac{1}{2}(id+iex)\right)\right)}{ce}\\
&= \frac{Cx}{c}-\frac{2(Ac-aC)\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}e}+\frac{B\log(a+c\sinh(d+ex))}{ce}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$
\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx
= \frac{C(d+ex)+\frac{2(Ac-aC)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}+B\log(a+c\sinh(d+ex))}{ce}
$$

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x]),x]
[Out] (C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 -
c^2]])/Sqrt[-a^2 - c^2] + B*Log[a + c*Sinh[d + e*x]])/(c*e)
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

method	result
parts	$\frac{-\frac{2(-Ac+Ca)\operatorname{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{c\sqrt{a^2+c^2}}+\frac{C\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{c}-\frac{C\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{c}}{e}+\frac{B\ln(a+c\sinh(ex+d))}{ce}$
derivativedivides	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{c}+\frac{(-B+C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{c}+\frac{B\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)-\frac{2(-Ac+Ca)\operatorname{arctanh}}{\sqrt{}}}{c}}{e}$
default	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{c}+\frac{(-B+C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{c}+\frac{B\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)-\frac{2(-Ac+Ca)\operatorname{arctanh}}{\sqrt{}}}{c}}{e}$
risch	$\frac{xB}{c}+\frac{Cx}{c}-\frac{2B\,a^2c\,e^2x}{a^2c^2e^2+c^4e^2}-\frac{2B\,c^3e^2x}{a^2c^2e^2+c^4e^2}-\frac{2B\,a^2cde}{a^2c^2e^2+c^4e^2}-\frac{2B\,c^3de}{a^2c^2e^2+c^4e^2}+\frac{\ln\left(e^{ex+d}+\frac{Aac-a^2C-\sqrt{A^2a^2c^2+A}}{}\right.}{}$

[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x,method=_RETURNVERBOSE)

[Out] 1/e*(-2*(-A*c+C*a)/c/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2))+C/c*ln(tanh(1/2*e*x+1/2*d)+1)-C/c*ln(tanh(1/2*e*x+1/2*d)-1))+B*ln(a+c*sinh(e*x+d))/c/e

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 3.07

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx =$$
$$-\frac{((B-C)a^2+(B-C)c^2)ex+(Ca-Ac)\sqrt{a^2+c^2}\log\left(\frac{c^2\cosh(ex+d)^2+c^2\sinh(ex+d)^2+2\,ac\cosh(ex+d)+2\,a^2+c^2+2\,(c}{c\cosh(ex+d)^2+c\sinh(ex+d)^2+2\,a\,c}\right.}{(a^2c+c^3}$$

[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="fricas")

[Out] -(((B - C)*a^2 + (B - C)*c^2)*e*x + (C*a - A*c)*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e*x + d) + a)*sinh(e*x + d) - c)) - (B*a^2 + B*c^2)*log(2*(c*sinh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*c + c^3)*e)

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 18.12 (sec) , antiderivative size = 1318, normalized size of antiderivative = 16.27

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{a + c\sinh(d+ex)}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x)
```

```
[Out] Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/sinh(d), Eq(a, 0) & Eq(c, 0) &
 Eq(e, 0)), ((A*log(tanh(d/2 + e*x/2))/e + B*x - 2*B*log(tanh(d/2 + e*x/2)
+ 1)/e + B*log(tanh(d/2 + e*x/2))/e + C*x)/c, Eq(a, 0)), (2*I*A/(c*e*tanh(d
/2 + e*x/2) - I*c*e) + B*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c
*e) - I*B*e*x/(c*e*tanh(d/2 + e*x/2) - I*c*e) - 2*B*log(tanh(d/2 + e*x/2) +
 1)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c*e) + 2*I*B*log(tanh(d/2
+ e*x/2) + 1)/(c*e*tanh(d/2 + e*x/2) - I*c*e) + 2*B*log(tanh(d/2 + e*x/2) -
 I)*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) - I*c*e) - 2*I*B*log(tanh(d/2
+ e*x/2) - I)/(c*e*tanh(d/2 + e*x/2) - I*c*e) + C*e*x*tanh(d/2 + e*x/2)/(c*
e*tanh(d/2 + e*x/2) - I*c*e) - I*C*e*x/(c*e*tanh(d/2 + e*x/2) - I*c*e) - 2*
C/(c*e*tanh(d/2 + e*x/2) - I*c*e), Eq(a, -I*c)), (-2*I*A/(c*e*tanh(d/2 + e*
x/2) + I*c*e) + B*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + I
*B*e*x/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*B*log(tanh(d/2 + e*x/2) + 1)*tan
h(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*I*B*log(tanh(d/2 + e*x/2
) + 1)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*B*log(tanh(d/2 + e*x/2) + I)*tan
h(d/2 + e*x/2)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + 2*I*B*log(tanh(d/2 + e*x/2
) + I)/(c*e*tanh(d/2 + e*x/2) + I*c*e) + C*e*x*tanh(d/2 + e*x/2)/(c*e*tanh(
d/2 + e*x/2) + I*c*e) + I*C*e*x/(c*e*tanh(d/2 + e*x/2) + I*c*e) - 2*C/(c*e*
tanh(d/2 + e*x/2) + I*c*e), Eq(a, I*c)), ((A*x + B*sinh(d + e*x)/e + C*cosh
(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cosh(d) + C*sinh(d))/(a + c*sinh(d)),
Eq(e, 0)), (-A*c*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2
+ c**2)/a)/(a**2*c*e + c**3*e) + A*c*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2
) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + B*a**2*e*x/(a**2*c*e +
 c**3*e) - 2*B*a**2*log(tanh(d/2 + e*x/2) + 1)/(a**2*c*e + c**3*e) + B*a**2
*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e) + B
*a**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c**2)/a)/(a**2*c*e + c**3*e
) + B*c**2*e*x/(a**2*c*e + c**3*e) - 2*B*c**2*log(tanh(d/2 + e*x/2) + 1)/(a
**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/
a)/(a**2*c*e + c**3*e) + B*c**2*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c
**2)/a)/(a**2*c*e + c**3*e) + C*a**2*e*x/(a**2*c*e + c**3*e) + C*a*sqrt(a**
2 + c**2)*log(tanh(d/2 + e*x/2) - c/a - sqrt(a**2 + c**2)/a)/(a**2*c*e + c*
*3*e) - C*a*sqrt(a**2 + c**2)*log(tanh(d/2 + e*x/2) - c/a + sqrt(a**2 + c**
2)/a)/(a**2*c*e + c**3*e) + C*c**2*e*x/(a**2*c*e + c**3*e), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(78) = 156$.

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.17

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{a + c\sinh(d+ex)}\,dx = -C\left(\frac{a\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}ce} - \frac{ex+d}{ce}\right) + \frac{A\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{\sqrt{a^2+c^2}e} + \frac{B\log\left(c\sinh\left(ex+d\right)+a\right)}{ce}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="m
axima")
```

```
[Out] -C*(a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt
(a^2 + c^2)))/(sqrt(a^2 + c^2)*c*e) - (e*x + d)/(c*e)) + A*log((c*e^(-e*x -
 d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 + c^2)))/(sqrt(a^
2 + c^2)*e) + B*log(c*sinh(e*x + d) + a)/(c*e)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{a + c\sinh(d+ex)}\,dx = -\frac{\frac{(ex+d)(B-C)}{c} - \frac{B\log\left(\left|ce^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}-c\right|\right)}{c} + \frac{(Ca-Ac)\log\left(\frac{\left|2\,ce^{(ex+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(ex+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\sqrt{a^2+c^2}c}}{e}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d)),x, algorithm="g
iac")
```

```
[Out] -((e*x + d)*(B - C)/c - B*log(abs(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c))
/c + (C*a - A*c)*log(abs(2*c*e^(e*x + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c
*e^(e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/(sqrt(a^2 + c^2)*c))/e
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 656, normalized size of antiderivative = 8.10

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+c\sinh(d+ex)}\,dx = \frac{C\,x}{c}-\frac{B\,x}{c}$$
$$-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c+A\,e\,a^2\,c^2-C\,e\,a\,c^3+A\,e\,c^4}-\frac{a^2\,c^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}\,\sqrt{A^2\,c^2-2\,A\,C\,a\,c+C^2\,a^2}}{-C\,e\,a^3\,c^4+A\,e\,a^2\,c^5-C\,e\,a\,c^6+A\,e\,c^7}+\frac{A\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{-a^2\,c^2\,e}}{c\,e\,\sqrt{A^2\,c^2-2\,A\,C\,a}}\right.}{\sqrt{-a^2\,c^2\,e^2-c^4\,e^2}}$$
$$+\frac{B\,c^3\,e\,\ln\left(8\,A\,C\,a\,c^2-4\,C^2\,a^2\,c-4\,A^2\,c^3+8\,C^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,A^2\,c^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,c^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,C^2\,a^2\,c\right.}{a^2\,c^2\,e^2+c^4\,e^2}$$
$$+\frac{B\,a^2\,c\,e\,\ln\left(8\,A\,C\,a\,c^2-4\,C^2\,a^2\,c-4\,A^2\,c^3+8\,C^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,A^2\,c^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,c^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,C^2\,a^2\right.}{a^2\,c^2\,e^2+c^4\,e^2}$$

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x)),x)
```

```
[Out] (C*x)/c - (B*x)/c - (2*atan((a*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 + C
^2*a^2 - 2*A*C*a*c)^(1/2))/(A*c^4*e - C*a*c^3*e - C*a^3*c*e + A*a^2*c^2*e)
- (a^2*c^2*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)*(A^2*c^2 + C^2*a
^2 - 2*A*C*a*c)^(1/2))/(A*c^7*e - C*a*c^6*e + A*a^2*c^5*e - C*a^3*c^4*e) +
(A*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2))/(c*e*(A^2*c^2 + C^2*a^2
 - 2*A*C*a*c)^(1/2)) - (C*a*exp(e*x)*exp(d)*(- c^4*e^2 - a^2*c^2*e^2)^(1/2)
)/(c^2*e*(A^2*c^2 + C^2*a^2 - 2*A*C*a*c)^(1/2)))*(A^2*c^2 + C^2*a^2 - 2*A*C
*a*c)^(1/2))/(- c^4*e^2 - a^2*c^2*e^2)^(1/2) + (B*c^3*e*log(8*A*C*a*c^2 - 4
*C^2*a^2*c - 4*A^2*c^3 + 8*C^2*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp(2*d)*exp
(2*e*x) + 8*A^2*a*c^2*exp(e*x)*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(2*e*x) - 1
6*A*C*a^2*c*exp(e*x)*exp(d) - 8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/(c^4*e^2 +
a^2*c^2*e^2) + (B*a^2*c*e*log(8*A*C*a*c^2 - 4*C^2*a^2*c - 4*A^2*c^3 + 8*C^2
*a^3*exp(e*x)*exp(d) + 4*A^2*c^3*exp(2*d)*exp(2*e*x) + 8*A^2*a*c^2*exp(e*x)
*exp(d) + 4*C^2*a^2*c*exp(2*d)*exp(2*e*x) - 16*A*C*a^2*c*exp(e*x)*exp(d) -
8*A*C*a*c^2*exp(2*d)*exp(2*e*x)))/(c^4*e^2 + a^2*c^2*e^2)
```

3.254 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 113

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = -\frac{2(aA+cC)\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{3/2}\,e} - \frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))}$$

```
[Out] -2*(A*a+C*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))/(a^2+c^2)^(
3/2)/e-B/c/e/(a+c*sinh(e*x+d))-(A*c-C*a)*cosh(e*x+d)/(a^2+c^2)/e/(a+c*sinh(
e*x+d))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = -\frac{2(aA+cC)\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\,(a^2+c^2)^{3/2}} - \frac{(Ac-aC)\cosh(d+ex)}{e\,(a^2+c^2)\,(a+c\sinh(d+ex))} - \frac{B}{ce(a+c\sinh(d+ex))}$$

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x]
```

```
[Out] (-2*(a*A + c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 + c^2]])/((a^2 +
 c^2)^(3/2)*e) - B/(c*e*(a + c*Sinh[d + e*x])) - ((A*c - a*C)*Cosh[d + e*x]
)/((a^2 + c^2)*e*(a + c*Sinh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
```

```
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\text{integral} = B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx + \int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$

$$= -\frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} - \frac{\int \frac{-aA-cC}{a+c\sinh(d+ex)}\,dx}{a^2+c^2} + \frac{B\text{Subst}\left(\int \frac{1}{(a+x)^2}\,dx, x, c\sinh(d+ex)\right)}{ce}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} + \frac{(aA+cC)\int \frac{1}{a+c\sinh(d+ex)}\,dx}{a^2+c^2}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} - \frac{(2i(aA+cC))\text{Subst}\left(\int \frac{1}{a-2icx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(id+iex)\right)\right)}{(a^2+c^2)\,e}$$

$$= -\frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))} + \frac{(4i(aA+cC))\text{Subst}\left(\int \frac{1}{-4(a^2+c^2)-x^2}\,dx, x, -2ic+2a\tan\left(\frac{1}{2}(id+iex)\right)\right)}{(a^2+c^2)\,e}$$

$$= -\frac{2(aA+cC)\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{3/2}\,e} - \frac{B}{ce(a+c\sinh(d+ex))} - \frac{(Ac-aC)\cosh(d+ex)}{(a^2+c^2)\,e(a+c\sinh(d+ex))}$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = \frac{\frac{2(aA+cC)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}}-\frac{B(a^2+c^2)+c(Ac-aC)\cosh(d+ex)}{c(a+c\sinh(d+ex))}}{(a^2+c^2)\,e}$$

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^2,x
]
```

```
[Out] ((2*(a*A + c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2 - c^2]])/Sqrt[-a
^2 - c^2] - (B*(a^2 + c^2) + c*(A*c - a*C)*Cosh[d + e*x])/(c*(a + c*Sinh[d
+ e*x])))/((a^2 + c^2)*e)
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(A\,c^2-B\,a^2-B\,c^2-Cac\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^2+c^2\right)}-\frac{Ac-Ca}{a^2+c^2}\right)}{a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a}+\frac{2(Aa+cC)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}}}{e}$
default	$\frac{-\frac{2\left(-\frac{\left(A\,c^2-B\,a^2-B\,c^2-Cac\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^2+c^2\right)}-\frac{Ac-Ca}{a^2+c^2}\right)}{a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a}+\frac{2(Aa+cC)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}}}{e}$
parts	$\frac{-\frac{2\left(-\frac{c(Ac-Ca)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^2+c^2\right)}-\frac{Ac-Ca}{a^2+c^2}\right)}{a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a}+\frac{2(Aa+cC)\,\mathrm{arctanh}\left(\frac{2a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-2c}{2\sqrt{a^2+c^2}}\right)}{\left(a^2+c^2\right)^{\frac{3}{2}}}}{e}-\frac{B}{ce(a+c\sinh(ex+d))}$
risch	$\frac{2Aac\,\mathrm{e}^{ex+d}-2B\,a^2\mathrm{e}^{ex+d}-2B\,c^2\mathrm{e}^{ex+d}-2C\,a^2\mathrm{e}^{ex+d}-2A\,c^2+2Cac}{ce(a^2+c^2)\left(c\,\mathrm{e}^{2ex+2d}+2a\,\mathrm{e}^{ex+d}-c\right)}+\frac{\ln\left(\mathrm{e}^{ex+d}+\frac{\left(a^2+c^2\right)^{\frac{3}{2}}a-a^4-2a^2c^2-c^4}{c\left(a^2+c^2\right)^{\frac{3}{2}}}\right)Aa}{(a^2+c^2)^{\frac{3}{2}}e}+$

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-(A*c^2-B*a^2-B*c^2-C*a*c)/a/(a^2+c^2)*tanh(1/2*e*x+1/2*d)-(A*c-C*
a)/(a^2+c^2))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*e*x+1/2*d)-a)+2*(A*a+C*
c)/(a^2+c^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(109) = 218.

Time = 0.27 (sec) , antiderivative size = 570, normalized size of antiderivative = 5.04

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx$$
$$= \frac{2\,Ca^3c - 2\,Aa^2c^2 + 2\,Cac^3 - 2\,Ac^4 - \left(Aac^2 + Cc^3 - (Aac^2 + Cc^3)\cosh\left(ex+d\right)^2 - (Aac^2 + Cc^3)\sinh\left(ex\right.\right.}{}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"fricas")
```

```
[Out] (2*C*a^3*c - 2*A*a^2*c^2 + 2*C*a*c^3 - 2*A*c^4 - (A*a*c^2 + C*c^3 - (A*a*c^
2 + C*c^3)*cosh(e*x + d)^2 - (A*a*c^2 + C*c^3)*sinh(e*x + d)^2 - 2*(A*a^2*c
 + C*a*c^2)*cosh(e*x + d) - 2*(A*a^2*c + C*a*c^2 + (A*a*c^2 + C*c^3)*cosh(e
*x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh
(e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*
c)*sinh(e*x + d) - 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a
))/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e
*x + d) + a)*sinh(e*x + d) - c)) - 2*((B + C)*a^4 - A*a^3*c + (2*B + C)*a^2
*c^2 - A*a*c^3 + B*c^4)*cosh(e*x + d) - 2*((B + C)*a^4 - A*a^3*c + (2*B + C
)*a^2*c^2 - A*a*c^3 + B*c^4)*sinh(e*x + d))/((a^4*c^2 + 2*a^2*c^4 + c^6)*e*
cosh(e*x + d)^2 + (a^4*c^2 + 2*a^2*c^4 + c^6)*e*sinh(e*x + d)^2 + 2*(a^5*c
+ 2*a^3*c^3 + a*c^5)*e*cosh(e*x + d) - (a^4*c^2 + 2*a^2*c^4 + c^6)*e + 2*((
a^4*c^2 + 2*a^2*c^4 + c^6)*e*cosh(e*x + d) + (a^5*c + 2*a^3*c^3 + a*c^5)*e)
*sinh(e*x + d))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(109) = 218$.

Time = 0.35 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.00

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^2}\,dx$$
$$= A\left(\frac{a\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{\frac{3}{2}}e} - \frac{2\left(ae^{(-ex-d)}+c\right)}{(a^2c+c^3+2\,(a^3+ac^2)e^{(-ex-d)}-(a^2c+c^3)e^{(-2\,ex-2\,d)})e}\right)$$
$$+C\left(\frac{c\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{\frac{3}{2}}e} + \frac{2\left(a^2e^{(-ex-d)}+ac\right)}{(a^2c^2+c^4+2\,(a^3c+ac^3)e^{(-ex-d)}-(a^2c^2+c^4)e^{(-2\,ex-2\,d)})e}\right)$$
$$-\frac{2\,Be^{(-ex-d)}}{(2\,ace^{(-ex-d)}-c^2e^{(-2\,ex-2\,d)}+c^2)e}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"maxima")
```

```
[Out] A*(a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(
a^2 + c^2)))/((a^2 + c^2)^(3/2)*e) - 2*(a*e^(-e*x - d) + c)/((a^2*c + c^3 +
 2*(a^3 + a*c^2)*e^(-e*x - d) - (a^2*c + c^3)*e^(-2*e*x - 2*d))*e)) + C*(c*
log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 +
 c^2)))/((a^2 + c^2)^(3/2)*e) + 2*(a^2*e^(-e*x - d) + a*c)/((a^2*c^2 + c^4
+ 2*(a^3*c + a*c^3)*e^(-e*x - d) - (a^2*c^2 + c^4)*e^(-2*e*x - 2*d))*e)) -
2*B*e^(-e*x - d)/((2*a*c*e^(-e*x - d) - c^2*e^(-2*e*x - 2*d) + c^2)*e)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.50

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^2}\,dx$$
$$= \frac{\frac{(Aa+Cc)\log\left(\frac{\left|2\,ce^{(ex+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(ex+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{(a^2+c^2)^{\frac{3}{2}}} - \frac{2\,(Ba^2e^{(ex+d)}+Ca^2e^{(ex+d)}-Aace^{(ex+d)}+Bc^2e^{(ex+d)}-Cac+Ac^2)}{(a^2c+c^3)\left(ce^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}-c\right)}}{e}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^2,x, algorithm=
"giac")
```

```
[Out] ((A*a + C*c)*log(abs(2*c*e^(e*x + d) + 2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(
e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/(a^2 + c^2)^(3/2) - 2*(B*a^2*e^(e*x +
d) + C*a^2*e^(e*x + d) - A*a*c*e^(e*x + d) + B*c^2*e^(e*x + d) - C*a*c + A*
c^2)/((a^2*c + c^3)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c)))/e
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.47

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx = \frac{\ln\left(\frac{2\,(A\,a+C\,c)\,(c-a\,\mathrm{e}^{d+e\,x})}{c\,(a^2+c^2)^{3/2}}-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a+C\,c)}{c\,(a^2+c^2)}\right)\,(A\,a+C\,c)}{e\,\left(a^2+c^2\right)^{3/2}} - \frac{\ln\left(-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a+C\,c)}{c\,(a^2+c^2)}-\frac{2\,(A\,a+C\,c)\,(c-a\,\mathrm{e}^{d+e\,x})}{c\,(a^2+c^2)^{3/2}}\right)\,(A\,a+C\,c)}{e\,\left(a^2+c^2\right)^{3/2}} - \frac{\frac{2\,(A\,c^3-C\,a\,c^2)}{c\,e\,(a^2\,c+c^3)}+\frac{2\,\mathrm{e}^{d+e\,x}\,(B\,c^4+B\,a^2\,c^2+C\,a^2\,c^2-A\,a\,c^3)}{c^2\,e\,(a^2\,c+c^3)}}{2\,a\,\mathrm{e}^{d+e\,x}-c+c\,\mathrm{e}^{2\,d+2\,e\,x}}$$

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^2,x)
```

```
[Out] (log((2*(A*a + C*c)*(c - a*exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)) - (2*exp(d
+ e*x)*(A*a + C*c))/(c*(a^2 + c^2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)) - (
log(- (2*exp(d + e*x)*(A*a + C*c))/(c*(a^2 + c^2)) - (2*(A*a + C*c)*(c - a*
exp(d + e*x)))/(c*(a^2 + c^2)^(3/2)))*(A*a + C*c))/(e*(a^2 + c^2)^(3/2)) -
((2*(A*c^3 - C*a*c^2))/(c*e*(a^2*c + c^3)) + (2*exp(d + e*x)*(B*c^4 + B*a^2
*c^2 + C*a^2*c^2 - A*a*c^3))/(c^2*e*(a^2*c + c^3)))/(2*a*exp(d + e*x) - c +
 c*exp(2*d + 2*e*x))
```

3.255 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 180

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = -\frac{(2a^2A-Ac^2+3acC)\operatorname{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{5/2}\,e}-\frac{B}{2ce(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{2\,(a^2+c^2)\,e(a+c\sinh(d+ex))^2}-\frac{(3aAc-a^2C+2c^2C)\cosh(d+ex)}{2\,(a^2+c^2)^2\,e(a+c\sinh(d+ex))}$$

```
[Out] -(2*A*a^2-A*c^2+3*C*a*c)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2+c^2)^(1/2))
/(a^2+c^2)^(5/2)/e-1/2*B/c/e/(a+c*sinh(e*x+d))^2-1/2*(A*c-C*a)*cosh(e*x+d)/
(a^2+c^2)/e/(a+c*sinh(e*x+d))^2-1/2*(3*A*a*c-C*a^2+2*C*c^2)*cosh(e*x+d)/(a^
2+c^2)^2/e/(a+c*sinh(e*x+d))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {4461, 2833, 12, 2739, 632, 210, 2747, 32}

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx$$
$$= -\frac{(2a^2A+3acC-Ac^2)\,\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\,(a^2+c^2)^{5/2}}$$
$$-\frac{(a^2(-C)+3aAc+2c^2C)\cosh(d+ex)}{2e\,(a^2+c^2)^2\,(a+c\sinh(d+ex))}$$
$$-\frac{(Ac-aC)\cosh(d+ex)}{2e\,(a^2+c^2)\,(a+c\sinh(d+ex))^2}-\frac{B}{2ce(a+c\sinh(d+ex))^2}$$

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x]
```

```
[Out] -(((2*a^2*A - A*c^2 + 3*a*c*C)*ArcTanh[(c - a*Tanh[(d + e*x)/2])/Sqrt[a^2 +
 c^2]])/((a^2 + c^2)^(5/2)*e)) - B/(2*c*e*(a + c*Sinh[d + e*x])^2) - ((A*c
- a*C)*Cosh[d + e*x])/(2*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^2) - ((3*a*A*c
 - a^2*C + 2*c^2*C)*Cosh[d + e*x])/(2*(a^2 + c^2)^2*e*(a + c*Sinh[d + e*x])
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx+\int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx \\
&= -\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2}-\frac{\int \frac{-2(aA+cC)+(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx}{2\left(a^2+c^2\right)} \\
&\quad+\frac{B\text{Subst}\left(\int \frac{1}{(a+x)^3}\,dx,x,c\sinh(d+ex)\right)}{ce} \\
&= -\frac{B}{2ce(a+c\sinh(d+ex))^2}-\frac{(Ac-aC)\cosh(d+ex)}{2\left(a^2+c^2\right)e(a+c\sinh(d+ex))^2} \\
&\quad-\frac{\left(3aAc-a^2C+2c^2C\right)\cosh(d+ex)}{2\left(a^2+c^2\right)^2e(a+c\sinh(d+ex))}+\frac{\int \frac{2a^2A-Ac^2+3acC}{a+c\sinh(d+ex)}\,dx}{2\left(a^2+c^2\right)^2}
\end{aligned}
$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2(a^2+c^2)e(a+c\sinh(d+ex))^2} - \frac{(3aAc-a^2C+2c^2C)\cosh(d+ex)}{2(a^2+c^2)^2e(a+c\sinh(d+ex))} + \frac{(2a^2A-Ac^2+3acC)\int\frac{1}{a+c\sinh(d+ex)}dx}{2(a^2+c^2)^2}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2(a^2+c^2)e(a+c\sinh(d+ex))^2} - \frac{(3aAc-a^2C+2c^2C)\cosh(d+ex)}{2(a^2+c^2)^2e(a+c\sinh(d+ex))} - \frac{(i(2a^2A-Ac^2+3acC))\,\mathrm{Subst}\left(\int\frac{1}{a-2icx+ax^2}dx, x, \tan\left(\frac{1}{2}(id+iex)\right)\right)}{(a^2+c^2)^2e}$$

$$= -\frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2(a^2+c^2)e(a+c\sinh(d+ex))^2} - \frac{(3aAc-a^2C+2c^2C)\cosh(d+ex)}{2(a^2+c^2)^2e(a+c\sinh(d+ex))} + \frac{(2i(2a^2A-Ac^2+3acC))\,\mathrm{Subst}\left(\int\frac{1}{-4(a^2+c^2)-x^2}dx, x, -2ic+2a\tan\left(\frac{1}{2}(id+iex)\right)\right)}{(a^2+c^2)^2e}$$

$$= -\frac{(2a^2A-Ac^2+3acC)\,\mathrm{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{5/2}e} - \frac{B}{2ce(a+c\sinh(d+ex))^2} - \frac{(Ac-aC)\cosh(d+ex)}{2(a^2+c^2)e(a+c\sinh(d+ex))^2} - \frac{(3aAc-a^2C+2c^2C)\cosh(d+ex)}{2(a^2+c^2)^2e(a+c\sinh(d+ex))}$$

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.94

$$\int\frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}dx = \frac{\frac{2(2a^2A-Ac^2+3acC)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{(a^2+c^2)(B(a^2+c^2)+c(Ac-aC)\cosh(d+ex))}{c(a+c\sinh(d+ex))^2} + \frac{(-3aAc+a^2C-2c^2C)\cosh(d+ex)}{a+c\sinh(d+ex)}}{2(a^2+c^2)^2e}$$

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^3,x
]
```

```
[Out] ((2*(2*a^2*A - A*c^2 + 3*a*c*C)*ArcTan[(c - a*Tanh[(d + e*x)/2])/Sqrt[-a^2
- c^2]])/Sqrt[-a^2 - c^2] - ((a^2 + c^2)*(B*(a^2 + c^2) + c*(A*c - a*C)*Cos
h[d + e*x]))/(c*(a + c*Sinh[d + e*x])^2) + ((-3*a*A*c + a^2*C - 2*c^2*C)*Co
sh[d + e*x])/(a + c*Sinh[d + e*x]))/(2*(a^2 + c^2)^2*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(169) = 338$.

Time = 15.39 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.06

method	result
parts	$-\frac{2\left(-\frac{c\left(5Aa^2c+2Ac^3-3Ca^3\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{2a\left(a^4+2a^2c^2+c^4\right)}-\frac{\left(4Aa^4c-7Aa^2c^3-2Ac^5-2Ca^5+5Ca^3c^2-2Cac^4\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{2\left(a^4+2a^2c^2+c^4\right)a^2}+\frac{c\left(11Aa^2c\cdots\right.}{\cdots}\right.}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-2c\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-a\right)^2} \cdots \over e$
derivativedivides	$-\frac{2\left(-\frac{\left(5Ac^2a^2+2Ac^4-2Ba^4-4Ba^2c^2-2Bc^4-3Ca^3c\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{2a\left(a^4+2a^2c^2+c^4\right)}-\frac{\left(4Aa^4c-7Aa^2c^3-2Ac^5+2Ba^4c+4Ba^2c^3+2Bc^5-2C\cdots\right.}{2\left(a^4+2a^2c^2+c^4\right)\cdots}\right.}{\left(a\tanh\left(\frac{ex}{2}\cdots\right.\right.}$
default	$-\frac{2\left(-\frac{\left(5Ac^2a^2+2Ac^4-2Ba^4-4Ba^2c^2-2Bc^4-3Ca^3c\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{2a\left(a^4+2a^2c^2+c^4\right)}-\frac{\left(4Aa^4c-7Aa^2c^3-2Ac^5+2Ba^4c+4Ba^2c^3+2Bc^5-2C\cdots\right.}{2\left(a^4+2a^2c^2+c^4\right)\cdots}\right.}{\left(a\tanh\left(\frac{ex}{2}\cdots\right.\right.}$
risch	$\frac{2Aa^2c^2e^{3ex+3d}-Ac^4e^{3ex+3d}+3Cac^3e^{3ex+3d}+6Aa^3ce^{2ex+2d}-3Aac^3e^{2ex+2d}-2Ba^4e^{2ex+2d}-4Ba^2c^2e^{2ex+2d}-2B\cdots}{ce(a^2+\cdots}$

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-1/2*c*(5*A*a^2*c+2*A*c^3-3*C*a^3)/a/(a^4+2*a^2*c^2+c^4)*tanh(1/2*
e*x+1/2*d)^3-1/2*(4*A*a^4*c-7*A*a^2*c^3-2*A*c^5-2*C*a^5+5*C*a^3*c^2-2*C*a*c
^4)/(a^4+2*a^2*c^2+c^4)/a^2*tanh(1/2*e*x+1/2*d)^2+1/2*c*(11*A*a^2*c+2*A*c^3
-5*C*a^3+4*C*a*c^2)/(a^4+2*a^2*c^2+c^4)/a*tanh(1/2*e*x+1/2*d)+1/2*(4*A*a^2*
c+A*c^3-2*C*a^3+C*a*c^2)/(a^4+2*a^2*c^2+c^4))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*
tanh(1/2*e*x+1/2*d)-a)^2+(2*A*a^2-A*c^2+3*C*a*c)/(a^4+2*a^2*c^2+c^4)/(a^2+c
^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^(1/2)))-1/2*B
/c/e/(a+c*sinh(e*x+d))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1880 vs. $2(170) = 340$.

Time = 0.30 (sec) , antiderivative size = 1880, normalized size of antiderivative = 10.44

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"fricas")
```

```
[Out] -1/2*(2*C*a^4*c^2 - 6*A*a^3*c^3 - 2*C*a^2*c^4 - 6*A*a*c^5 - 4*C*c^6 - 2*(2*
A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d)^3 -
2*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*sinh(e*x + d)
^3 + 2*(2*(B + C)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(
2*B - C)*a^2*c^4 + 3*A*a*c^5 + 2*(B + C)*c^6)*cosh(e*x + d)^2 + 2*(2*(B + C
)*a^6 - 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4
 + 3*A*a*c^5 + 2*(B + C)*c^6 - 3*(2*A*a^4*c^2 + 3*C*a^3*c^3 + A*a^2*c^4 + 3
*C*a*c^5 - A*c^6)*cosh(e*x + d))*sinh(e*x + d)^2 + (2*A*a^2*c^3 + 3*C*a*c^4
 - A*c^5 + (2*A*a^2*c^3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d)^4 + (2*A*a^2*c^3
 + 3*C*a*c^4 - A*c^5)*sinh(e*x + d)^4 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*
c^4)*cosh(e*x + d)^3 + 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 + (2*A*a^2*c^
3 + 3*C*a*c^4 - A*c^5)*cosh(e*x + d))*sinh(e*x + d)^3 + 2*(4*A*a^4*c + 6*C*
a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cosh(e*x + d)^2 + 2*(4*A*a^4*c +
 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 + 3*(2*A*a^2*c^3 + 3*C*a*c^4
 - A*c^5)*cosh(e*x + d)^2 + 6*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*
x + d))*sinh(e*x + d)^2 - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*x
+ d) - 4*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4 - (2*A*a^2*c^3 + 3*C*a*c^4 -
A*c^5)*cosh(e*x + d)^3 - 3*(2*A*a^3*c^2 + 3*C*a^2*c^3 - A*a*c^4)*cosh(e*x +
 d)^2 - (4*A*a^4*c + 6*C*a^3*c^2 - 4*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cosh(e*
x + d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh(
e*x + d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*c
)*sinh(e*x + d) + 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a)
)/(c*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e*
x + d) + a)*sinh(e*x + d) - c)) - 2*(4*C*a^5*c - 10*A*a^4*c^2 - C*a^3*c^3 -
 11*A*a^2*c^4 - 5*C*a*c^5 - A*c^6)*cosh(e*x + d) - 2*(4*C*a^5*c - 10*A*a^4*
c^2 - C*a^3*c^3 - 11*A*a^2*c^4 - 5*C*a*c^5 - A*c^6 + 3*(2*A*a^4*c^2 + 3*C*a
^3*c^3 + A*a^2*c^4 + 3*C*a*c^5 - A*c^6)*cosh(e*x + d)^2 - 2*(2*(B + C)*a^6
- 6*A*a^5*c + 3*(2*B - C)*a^4*c^2 - 3*A*a^3*c^3 + 3*(2*B - C)*a^2*c^4 + 3*A
*a*c^5 + 2*(B + C)*c^6)*cosh(e*x + d))*sinh(e*x + d))/((a^6*c^3 + 3*a^4*c^5
 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d)^4 + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 +
c^9)*e*sinh(e*x + d)^4 + 4*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e*cosh
(e*x + d)^3 + 2*(2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*e*cosh(e*
x + d)^2 + 4*((a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d) + (a^
7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e)*sinh(e*x + d)^3 - 4*(a^7*c^2 + 3*
a^5*c^4 + 3*a^3*c^6 + a*c^8)*e*cosh(e*x + d) + 2*(3*(a^6*c^3 + 3*a^4*c^5 +
3*a^2*c^7 + c^9)*e*cosh(e*x + d)^2 + 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a
*c^8)*e*cosh(e*x + d) + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 - a^2*c^7 - c^9)*e
)*sinh(e*x + d)^2 + (a^6*c^3 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e + 4*((a^6*c^3
 + 3*a^4*c^5 + 3*a^2*c^7 + c^9)*e*cosh(e*x + d)^3 + 3*(a^7*c^2 + 3*a^5*c^4
+ 3*a^3*c^6 + a*c^8)*e*cosh(e*x + d)^2 + (2*a^8*c + 5*a^6*c^3 + 3*a^4*c^5 -
 a^2*c^7 - c^9)*e*cosh(e*x + d) - (a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)
*e)*sinh(e*x + d))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^3}\,dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**3,x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 726 vs. $2(170) = 340$.

Time = 0.34 (sec) , antiderivative size = 726, normalized size of antiderivative = 4.03

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^3}\,dx$$
$$= \frac{1}{2}C\left(\frac{3\,ac\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^4+2\,a^2c^2+c^4)\sqrt{a^2+c^2}e} + \frac{2\,(3\,ac^3e^{(-3\,ex-3\,d)}+a^2c^2-2\,c^4+(\cdots}{(a^4c^3+2\,a^2c^5+c^7+4\,(a^5c^2+2\,a^3c^4+ac^6)e^{(-ex-d)}+2\,(2\,a^6c+3\,a\cdots}\right.$$
$$+\frac{1}{2}A\left(\frac{(2\,a^2-c^2)\log\left(\frac{ce^{(-ex-d)}-a-\sqrt{a^2+c^2}}{ce^{(-ex-d)}-a+\sqrt{a^2+c^2}}\right)}{(a^4+2\,a^2c^2+c^4)\sqrt{a^2+c^2}e} - \frac{2\,(3\,ac^2+(10\,a^2c+c^3)\cdots}{(a^4c^2+2\,a^2c^4+c^6+4\,(a^5c+2\,a^3c^3+ac^5)e^{(-ex-d)}+2\,(2\,a\cdots}\right.$$
$$-\frac{2\,Be^{(-2\,ex-2\,d)}}{(4\,ac^2e^{(-ex-d)}-4\,ac^2e^{(-3\,ex-3\,d)}+c^3e^{(-4\,ex-4\,d)}+c^3+2\,(2\,a^2c-c^3)e^{(-2\,ex-2\,d)})e}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"maxima")
[Out] 1/2*C*(3*a*c*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a
 + sqrt(a^2 + c^2)))/((a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2)*e) + 2*(3*a*c
^3*e^(-3*e*x - 3*d) + a^2*c^2 - 2*c^4 + (4*a^3*c - 5*a*c^3)*e^(-e*x - d) +
(2*a^4 - 5*a^2*c^2 + 2*c^4)*e^(-2*e*x - 2*d))/((a^4*c^3 + 2*a^2*c^5 + c^7 +
 4*(a^5*c^2 + 2*a^3*c^4 + a*c^6)*e^(-e*x - d) + 2*(2*a^6*c + 3*a^4*c^3 - c^
7)*e^(-2*e*x - 2*d) - 4*(a^5*c^2 + 2*a^3*c^4 + a*c^6)*e^(-3*e*x - 3*d) + (a
^4*c^3 + 2*a^2*c^5 + c^7)*e^(-4*e*x - 4*d))*e)) + 1/2*A*((2*a^2 - c^2)*log(
(c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 + c^2
)))/((a^4 + 2*a^2*c^2 + c^4)*sqrt(a^2 + c^2)*e) - 2*(3*a*c^2 + (10*a^2*c +
c^3)*e^(-e*x - d) + 3*(2*a^3 - a*c^2)*e^(-2*e*x - 2*d) - (2*a^2*c - c^3)*e^
(-3*e*x - 3*d))/((a^4*c^2 + 2*a^2*c^4 + c^6 + 4*(a^5*c + 2*a^3*c^3 + a*c^5)
*e^(-e*x - d) + 2*(2*a^6 + 3*a^4*c^2 - c^6)*e^(-2*e*x - 2*d) - 4*(a^5*c + 2
*a^3*c^3 + a*c^5)*e^(-3*e*x - 3*d) + (a^4*c^2 + 2*a^2*c^4 + c^6)*e^(-4*e*x
- 4*d))*e)) - 2*B*e^(-2*e*x - 2*d)/((4*a*c^2*e^(-e*x - d) - 4*a*c^2*e^(-3*e
*x - 3*d) + c^3*e^(-4*e*x - 4*d) + c^3 + 2*(2*a^2*c - c^3)*e^(-2*e*x - 2*d)
)*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(170) = 340.

Time = 0.33 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.25

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx =$$

$$-\frac{\frac{(2\,Aa^2+3\,Cac-Ac^2)\log\left(\frac{\left|-2\,ce^{(ex+d)}-2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|-2\,ce^{(ex+d)}-2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{(a^4+2\,a^2c^2+c^4)\sqrt{a^2+c^2}} - \frac{2\,(2\,Aa^2c^2e^{(3\,ex+3\,d)}+3\,Cac^3e^{(3\,ex+3\,d)}-Ac^4e^{(3\,ex+3\,d)}-2\,Ba^4e^{(2\,ex+2\,d)}-2\,Ca^4}{}}{}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^3,x, algorithm=
"giac")
```

```
[Out] -1/2*((2*A*a^2 + 3*C*a*c - A*c^2)*log(abs(-2*c*e^(e*x + d) - 2*a - 2*sqrt(a
^2 + c^2))/abs(-2*c*e^(e*x + d) - 2*a + 2*sqrt(a^2 + c^2)))/((a^4 + 2*a^2*c
^2 + c^4)*sqrt(a^2 + c^2)) - 2*(2*A*a^2*c^2*e^(3*e*x + 3*d) + 3*C*a*c^3*e^(
3*e*x + 3*d) - A*c^4*e^(3*e*x + 3*d) - 2*B*a^4*e^(2*e*x + 2*d) - 2*C*a^4*e^
(2*e*x + 2*d) + 6*A*a^3*c*e^(2*e*x + 2*d) - 4*B*a^2*c^2*e^(2*e*x + 2*d) + 5
*C*a^2*c^2*e^(2*e*x + 2*d) - 3*A*a*c^3*e^(2*e*x + 2*d) - 2*B*c^4*e^(2*e*x +
 2*d) - 2*C*c^4*e^(2*e*x + 2*d) + 4*C*a^3*c*e^(e*x + d) - 10*A*a^2*c^2*e^(e
*x + d) - 5*C*a*c^3*e^(e*x + d) - A*c^4*e^(e*x + d) - C*a^2*c^2 + 3*A*a*c^3
 + 2*C*c^4)/((a^4*c + 2*a^2*c^3 + c^5)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d)
 - c)^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx = \int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{{\left(a+c\sinh\left(d+e\,x\right)\right)}^3}\,dx$$

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3,x)
```

```
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^3, x)
```

3.256 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 250

$$\begin{aligned}
\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx
&= -\frac{(2a^3A-3aAc^2+4a^2cC-c^3C)\,\text{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{7/2}\,e}-\frac{B}{3ce(a+c\sinh(d+ex))^3} \\
&\quad-\frac{(Ac-aC)\cosh(d+ex)}{3\,(a^2+c^2)\,e(a+c\sinh(d+ex))^3}-\frac{(5aAc-2a^2C+3c^2C)\cosh(d+ex)}{6\,(a^2+c^2)^2\,e(a+c\sinh(d+ex))^2} \\
&\quad-\frac{(11a^2Ac-4Ac^3-2a^3C+13ac^2C)\cosh(d+ex)}{6\,(a^2+c^2)^3\,e(a+c\sinh(d+ex))}
\end{aligned}$$

```
[Out] -(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)*arctanh((c-a*tanh(1/2*e*x+1/2*d))/(a^2
+c^2)^(1/2))/(a^2+c^2)^(7/2)/e-1/3*B/c/e/(a+c*sinh(e*x+d))^3-1/3*(A*c-C*a)*
cosh(e*x+d)/(a^2+c^2)/e/(a+c*sinh(e*x+d))^3-1/6*(5*A*a*c-2*C*a^2+3*C*c^2)*c
osh(e*x+d)/(a^2+c^2)^2/e/(a+c*sinh(e*x+d))^2-1/6*(11*A*a^2*c-4*A*c^3-2*C*a^
3+13*C*a*c^2)*cosh(e*x+d)/(a^2+c^2)^3/e/(a+c*sinh(e*x+d))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used

$= \{4461, 2833, 12, 2739, 632, 210, 2747, 32\}$

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a + c\sinh(d+ex))^4}\,dx$$
$$= -\frac{(-2a^2C + 5aAc + 3c^2C)\cosh(d+ex)}{6e\left(a^2+c^2\right)^2(a+c\sinh(d+ex))^2} - \frac{(Ac - aC)\cosh(d+ex)}{3e\left(a^2+c^2\right)(a+c\sinh(d+ex))^3}$$
$$- \frac{(2a^3A + 4a^2cC - 3aAc^2 - c^3C)\operatorname{arctanh}\left(\frac{c - a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{e\left(a^2+c^2\right)^{7/2}}$$
$$- \frac{(-2a^3C + 11a^2Ac + 13ac^2C - 4Ac^3)\cosh(d+ex)}{6e\left(a^2+c^2\right)^3(a+c\sinh(d+ex))} - \frac{B}{3ce(a+c\sinh(d+ex))^3}$$

```
[In] Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x]
```

```
[Out] -(((2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTanh[(c - a*Tanh[(d + e*x)/
2])/Sqrt[a^2 + c^2]])/((a^2 + c^2)^(7/2)*e)) - B/(3*c*e*(a + c*Sinh[d + e*x
])^3) - ((A*c - a*C)*Cosh[d + e*x])/(3*(a^2 + c^2)*e*(a + c*Sinh[d + e*x])^
3) - ((5*a*A*c - 2*a^2*C + 3*c^2*C)*Cosh[d + e*x])/(6*(a^2 + c^2)^2*e*(a +
c*Sinh[d + e*x])^2) - ((11*a^2*A*c - 4*A*c^3 - 2*a^3*C + 13*a*c^2*C)*Cosh[d
 + e*x])/(6*(a^2 + c^2)^3*e*(a + c*Sinh[d + e*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2833

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4461

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= B\int \frac{\cosh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx+\int \frac{A+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx\\
&= -\frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3}-\frac{\int \frac{-3(aA+cC)+2(Ac-aC)\sinh(d+ex)}{(a+c\sinh(d+ex))^3}\,dx}{3\left(a^2+c^2\right)}\\
&\quad+\frac{B\text{Subst}\left(\int \frac{1}{(a+x)^4}\,dx,x,c\sinh(d+ex)\right)}{ce}\\
&= -\frac{B}{3ce(a+c\sinh(d+ex))^3}-\frac{(Ac-aC)\cosh(d+ex)}{3\left(a^2+c^2\right)e(a+c\sinh(d+ex))^3}\\
&\quad-\frac{\left(5aAc-2a^2C+3c^2C\right)\cosh(d+ex)}{6\left(a^2+c^2\right)^2e(a+c\sinh(d+ex))^2}\\
&\quad+\frac{\int \frac{2\left(3a^2A-2Ac^2+5acC\right)-\left(5aAc-2a^2C+3c^2C\right)\sinh(d+ex)}{(a+c\sinh(d+ex))^2}\,dx}{6\left(a^2+c^2\right)^2}
\end{aligned}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))^3} - \frac{(5aAc-2a^2C+3c^2C)\cosh(d+ex)}{6(a^2+c^2)^2e(a+c\sinh(d+ex))^2} - \frac{(11a^2Ac-4Ac^3-2a^3C+13ac^2C)\cosh(d+ex)}{6(a^2+c^2)^3e(a+c\sinh(d+ex))} - \frac{\int -\frac{3(2a^3A-3aAc^2+4a^2cC-c^3C)}{a+c\sinh(d+ex)}\,dx}{6(a^2+c^2)^3}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))^3} - \frac{(5aAc-2a^2C+3c^2C)\cosh(d+ex)}{6(a^2+c^2)^2e(a+c\sinh(d+ex))^2} - \frac{(11a^2Ac-4Ac^3-2a^3C+13ac^2C)\cosh(d+ex)}{6(a^2+c^2)^3e(a+c\sinh(d+ex))} + \frac{(2a^3A-3aAc^2+4a^2cC-c^3C)\int \frac{1}{a+c\sinh(d+ex)}\,dx}{2(a^2+c^2)^3}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))^3} - \frac{(5aAc-2a^2C+3c^2C)\cosh(d+ex)}{6(a^2+c^2)^2e(a+c\sinh(d+ex))^2} - \frac{(11a^2Ac-4Ac^3-2a^3C+13ac^2C)\cosh(d+ex)}{6(a^2+c^2)^3e(a+c\sinh(d+ex))} - \frac{(i(2a^3A-3aAc^2+4a^2cC-c^3C))\,\mathrm{Subst}\left(\int \frac{1}{a-2icx+ax^2}\,dx, x, \tan\left(\frac{1}{2}(id+iex)\right)\right)}{(a^2+c^2)^3e}$$

$$= -\frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))^3} - \frac{(5aAc-2a^2C+3c^2C)\cosh(d+ex)}{6(a^2+c^2)^2e(a+c\sinh(d+ex))^2} - \frac{(11a^2Ac-4Ac^3-2a^3C+13ac^2C)\cosh(d+ex)}{6(a^2+c^2)^3e(a+c\sinh(d+ex))} + \frac{(2i(2a^3A-3aAc^2+4a^2cC-c^3C))\,\mathrm{Subst}\left(\int \frac{1}{-4(a^2+c^2)-x^2}\,dx, x, -2ic+2a\tan\left(\frac{1}{2}(id+iex)\right)\right)}{(a^2+c^2)^3e}$$

$$= -\frac{(2a^3A - 3aAc^2 + 4a^2cC - c^3C)\operatorname{arctanh}\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2+c^2}}\right)}{(a^2+c^2)^{7/2}e} - \frac{B}{3ce(a+c\sinh(d+ex))^3} - \frac{(Ac-aC)\cosh(d+ex)}{3(a^2+c^2)e(a+c\sinh(d+ex))^3} - \frac{(5aAc-2a^2C+3c^2C)\cosh(d+ex)}{6(a^2+c^2)^2e(a+c\sinh(d+ex))^2} - \frac{(11a^2Ac-4Ac^3-2a^3C+13ac^2C)\cosh(d+ex)}{6(a^2+c^2)^3e(a+c\sinh(d+ex))}$$

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx = \frac{\frac{6(2a^3A-3aAc^2+4a^2cC-c^3C)\arctan\left(\frac{c-a\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2-c^2}}\right)}{\sqrt{-a^2-c^2}} - \frac{2(a^2+c^2)^2(B(a^2+c^2)+c(Ac-aC)\cosh(d+ex))}{c(a+c\sinh(d+ex))^3} + \frac{(a^2+c^2)(-5aAc+2a^2C-3}{(a+c\sinh(d+}}{6(a^2+c^2)^3e}$$

```
[In] Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + c*Sinh[d + e*x])^4,x
]
```

```
[Out] ((6*(2*a^3*A - 3*a*A*c^2 + 4*a^2*c*C - c^3*C)*ArcTan[(c - a*Tanh[(d + e*x)/
2])/Sqrt[-a^2 - c^2]])/Sqrt[-a^2 - c^2] - (2*(a^2 + c^2)^2*(B*(a^2 + c^2) +
 c*(A*c - a*C)*Cosh[d + e*x]))/(c*(a + c*Sinh[d + e*x])^3) + ((a^2 + c^2)*(
-5*a*A*c + 2*a^2*C - 3*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d + e*x])^2 + ((-1
1*a^2*A*c + 4*A*c^3 + 2*a^3*C - 13*a*c^2*C)*Cosh[d + e*x])/(a + c*Sinh[d +
e*x]))/(6*(a^2 + c^2)^3*e)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. 2(237) = 474.

Time = 75.69 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.80

method	result
parts	$-\frac{2\left(-\frac{c\left(9A\,a^4c+6A\,a^2c^3+2A\,c^5-4C\,a^5+C\,a^3c^2\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a\left(a^6+3a^4c^2+3a^2c^4+c^6\right)}-\frac{\left(6A\,a^6c-27A\,a^4c^3-12A\,a^2c^5-4A\,c^7-2C\,a^7+14C\,a^5c^2-11C\,a^3c\right.}{2\left(a^6+3a^4c^2+3a^2c^4+c^6\right)a^2}\right.}{}$
derivativedivides	$-\frac{2\left(-\frac{\left(9A\,a^4c^2+6A\,a^2c^4+2A\,c^6-2B\,a^6-6c^2B\,a^4-6c^4B\,a^2-2c^6B-4C\,a^5c+C\,a^3c^3\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a\left(a^6+3a^4c^2+3a^2c^4+c^6\right)}-\frac{\left(6A\,a^6c-27A\,a^4c^3-12A\,a^2c^5\right.}{}\right.}{}$
default	$-\frac{2\left(-\frac{\left(9A\,a^4c^2+6A\,a^2c^4+2A\,c^6-2B\,a^6-6c^2B\,a^4-6c^4B\,a^2-2c^6B-4C\,a^5c+C\,a^3c^3\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a\left(a^6+3a^4c^2+3a^2c^4+c^6\right)}-\frac{\left(6A\,a^6c-27A\,a^4c^3-12A\,a^2c^5\right.}{}\right.}{}$
risch	Expression too large to display

```
[In] int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/e*(-2*(-1/2*c*(9*A*a^4*c+6*A*a^2*c^3+2*A*c^5-4*C*a^5+C*a^3*c^2)/a/(a^6+3*
a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^5-1/2*(6*A*a^6*c-27*A*a^4*c^3-12
*A*a^2*c^5-4*A*c^7-2*C*a^7+14*C*a^5*c^2-11*C*a^3*c^4-2*C*a*c^6)/(a^6+3*a^4*
c^2+3*a^2*c^4+c^6)/a^2*tanh(1/2*e*x+1/2*d)^4+1/3/a^3*c*(54*A*a^6*c-21*A*a^4
*c^3-4*A*a^2*c^5-4*A*c^7-18*C*a^7+42*C*a^5*c^2-17*C*a^3*c^4-2*C*a*c^6)/(a^6
+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^3+1/a^2*(6*A*a^6*c-20*A*a^4*c
^3-3*A*a^2*c^5-2*A*c^7-2*C*a^7+10*C*a^5*c^2-14*C*a^3*c^4-C*a*c^6)/(a^6+3*a^
4*c^2+3*a^2*c^4+c^6)*tanh(1/2*e*x+1/2*d)^2-1/2/a*c*(27*A*a^4*c+4*A*a^2*c^3+
2*A*c^5-8*C*a^5+19*C*a^3*c^2+2*C*a*c^4)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6)*tanh(
1/2*e*x+1/2*d)-1/6*(18*A*a^4*c+5*A*a^2*c^3+2*A*c^5-6*C*a^5+10*C*a^3*c^2+C*a
*c^4)/(a^6+3*a^4*c^2+3*a^2*c^4+c^6))/(a*tanh(1/2*e*x+1/2*d)^2-2*c*tanh(1/2*
e*x+1/2*d)-a)^3+(2*A*a^3-3*A*a*c^2+4*C*a^2*c-C*c^3)/(a^6+3*a^4*c^2+3*a^2*c^
4+c^6)/(a^2+c^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*e*x+1/2*d)-2*c)/(a^2+c^2)^
(1/2)))-1/3*B/c/e/(a+c*sinh(e*x+d))^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4350 vs. $2(239) = 478$.

Time = 0.40 (sec) , antiderivative size = 4350, normalized size of antiderivative = 17.40

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4}\,dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"fricas")
```

```
[Out] 1/6*(4*C*a^5*c^3 - 22*A*a^4*c^4 - 22*C*a^3*c^5 - 14*A*a^2*c^6 - 26*C*a*c^7
+ 8*A*c^8 + 6*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*
c^7 - C*c^8)*cosh(e*x + d)^5 + 6*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3
*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*sinh(e*x + d)^5 + 30*(2*A*a^6*c^2 + 4*C*a^5
*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(e*x + d)^4 + 3
0*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*
c^7 + (2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*
c^8)*cosh(e*x + d))*sinh(e*x + d)^4 - 4*(4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*
B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16
*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d)^3 - 4*(4*(B + C)*a
^8 - 22*A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c
^4 + 29*A*a^3*c^5 + (16*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8 - 15*(2*A*
a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(e
*x + d)^2 - 30*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a
^2*c^6 - C*a*c^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 12*(4*C*a^7*c - 17*A*a^6
*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6 + 4*C*a*c^7
 - 2*A*c^8)*cosh(e*x + d)^2 + 12*(4*C*a^7*c - 17*A*a^6*c^2 - 13*C*a^5*c^3 -
 11*A*a^4*c^4 - 13*C*a^3*c^5 + 4*A*a^2*c^6 + 4*C*a*c^7 - 2*A*c^8 + 5*(2*A*a
^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*a*c^7 - C*c^8)*cosh(e*
x + d)^3 + 15*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4 + 3*C*a^3*c^5 - 3*A*a^
2*c^6 - C*a*c^7)*cosh(e*x + d)^2 - (4*(B + C)*a^8 - 22*A*a^7*c + 4*(4*B - 7
*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 + 29*A*a^3*c^5 + (16*B +
39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d))*sinh(e*x + d)^2 + 3*(2
*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7 - (2*A*a^3*c^4 + 4*C*a^2*c^5 -
 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^6 - (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^
6 - C*c^7)*sinh(e*x + d)^6 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C
*a*c^6)*cosh(e*x + d)^5 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*
c^6 + (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d))*sinh(e
*x + d)^5 - 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*
A*a*c^6 + C*c^7)*cosh(e*x + d)^4 - 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3
*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7 + 5*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A
*a*c^6 - C*c^7)*cosh(e*x + d)^2 + 10*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c
^5 - C*a*c^6)*cosh(e*x + d))*sinh(e*x + d)^4 - 4*(4*A*a^6*c + 8*C*a^5*c^2 -
 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(e*x + d)^3 - 4
*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C
*a*c^6 + 5*(2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^3
+ 15*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d)^2 +
3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*
c^7)*cosh(e*x + d))*sinh(e*x + d)^3 + 3*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*
a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x + d)^2 + 3*(8*A*a^5*c^2
 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7 - 5*(2*A*a
^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh(e*x + d)^4 - 20*(2*A*a^4*c^3
 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d)^3 - 6*(8*A*a^5*c^2 +
16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x + d
)^2 - 4*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3 - 14*C*a^3*c^4 + 9*A*a^2*c^
```

5 + 3*C*a*c^6)*cosh(e*x + d))*sinh(e*x + d)^2 - 6*(2*A*a^4*c^3 + 4*C*a^3*c^
4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*x + d) - 6*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3
*A*a^2*c^5 - C*a*c^6 + (2*A*a^3*c^4 + 4*C*a^2*c^5 - 3*A*a*c^6 - C*c^7)*cosh
(e*x + d)^5 + 5*(2*A*a^4*c^3 + 4*C*a^3*c^4 - 3*A*a^2*c^5 - C*a*c^6)*cosh(e*
x + d)^4 + 2*(8*A*a^5*c^2 + 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A
*a*c^6 + C*c^7)*cosh(e*x + d)^3 + 2*(4*A*a^6*c + 8*C*a^5*c^2 - 12*A*a^4*c^3
 - 14*C*a^3*c^4 + 9*A*a^2*c^5 + 3*C*a*c^6)*cosh(e*x + d)^2 - (8*A*a^5*c^2 +
 16*C*a^4*c^3 - 14*A*a^3*c^4 - 8*C*a^2*c^5 + 3*A*a*c^6 + C*c^7)*cosh(e*x +
d))*sinh(e*x + d))*sqrt(a^2 + c^2)*log((c^2*cosh(e*x + d)^2 + c^2*sinh(e*x
+ d)^2 + 2*a*c*cosh(e*x + d) + 2*a^2 + c^2 + 2*(c^2*cosh(e*x + d) + a*c)*si
nh(e*x + d) + 2*sqrt(a^2 + c^2)*(c*cosh(e*x + d) + c*sinh(e*x + d) + a))/(c
*cosh(e*x + d)^2 + c*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*(c*cosh(e*x +
d) + a)*sinh(e*x + d) - c)) - 6*(4*C*a^6*c^2 - 20*A*a^5*c^3 - 18*C*a^4*c^4
- 15*A*a^3*c^5 - 23*C*a^2*c^6 + 5*A*a*c^7 - C*c^8)*cosh(e*x + d) - 6*(4*C*a
^6*c^2 - 20*A*a^5*c^3 - 18*C*a^4*c^4 - 15*A*a^3*c^5 - 23*C*a^2*c^6 + 5*A*a*
c^7 - C*c^8 - 5*(2*A*a^5*c^3 + 4*C*a^4*c^4 - A*a^3*c^5 + 3*C*a^2*c^6 - 3*A*
a*c^7 - C*c^8)*cosh(e*x + d)^4 - 20*(2*A*a^6*c^2 + 4*C*a^5*c^3 - A*a^4*c^4
+ 3*C*a^3*c^5 - 3*A*a^2*c^6 - C*a*c^7)*cosh(e*x + d)^3 + 2*(4*(B + C)*a^8 -
 22*A*a^7*c + 4*(4*B - 7*C)*a^6*c^2 + 19*A*a^5*c^3 + (24*B + 7*C)*a^4*c^4 +
 29*A*a^3*c^5 + (16*B + 39*C)*a^2*c^6 - 12*A*a*c^7 + 4*B*c^8)*cosh(e*x + d)
^2 - 4*(4*C*a^7*c - 17*A*a^6*c^2 - 13*C*a^5*c^3 - 11*A*a^4*c^4 - 13*C*a^3*c
^5 + 4*A*a^2*c^6 + 4*C*a*c^7 - 2*A*c^8)*cosh(e*x + d))*sinh(e*x + d))/((a^8
*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^6 + (a^8*
c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*sinh(e*x + d)^6 + 6*(a^9
*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^5 + 3*(4
*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^4
+ 6*((a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)
+ (a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e)*sinh(e*x + d)^5
 + 4*(2*a^11*c + 5*a^9*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x
 + d)^3 + 3*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh
(e*x + d)^2 + 10*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*c
osh(e*x + d) + (4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e
)*sinh(e*x + d)^4 - 3*(4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 -
c^12)*e*cosh(e*x + d)^2 + 4*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^1
0 + c^12)*e*cosh(e*x + d)^3 + 15*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c
^9 + a*c^11)*e*cosh(e*x + d)^2 + 3*(4*a^10*c^2 + 15*a^8*c^4 + 20*a^6*c^6 +
10*a^4*c^8 - c^12)*e*cosh(e*x + d) + (2*a^11*c + 5*a^9*c^3 - 10*a^5*c^7 - 1
0*a^3*c^9 - 3*a*c^11)*e)*sinh(e*x + d)^3 + 6*(a^9*c^3 + 4*a^7*c^5 + 6*a^5*c
^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d) + 3*(5*(a^8*c^4 + 4*a^6*c^6 + 6*a^
4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^4 + 20*(a^9*c^3 + 4*a^7*c^5 + 6*
a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^3 + 6*(4*a^10*c^2 + 15*a^8*c^
4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^2 + 4*(2*a^11*c + 5*a^9
*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x + d) - (4*a^10*c^2 +
15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e)*sinh(e*x + d)^2 - (a^8*c^4
+ 4*a^6*c^6 + 6*a^4*c^8 + 4*a^2*c^10 + c^12)*e + 6*((a^8*c^4 + 4*a^6*c^6 +

```
6*a^4*c^8 + 4*a^2*c^10 + c^12)*e*cosh(e*x + d)^5 + 5*(a^9*c^3 + 4*a^7*c^5 +
 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e*cosh(e*x + d)^4 + 2*(4*a^10*c^2 + 15*a^8
*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d)^3 + 2*(2*a^11*c + 5*
a^9*c^3 - 10*a^5*c^7 - 10*a^3*c^9 - 3*a*c^11)*e*cosh(e*x + d)^2 - (4*a^10*c
^2 + 15*a^8*c^4 + 20*a^6*c^6 + 10*a^4*c^8 - c^12)*e*cosh(e*x + d) + (a^9*c^
3 + 4*a^7*c^5 + 6*a^5*c^7 + 4*a^3*c^9 + a*c^11)*e)*sinh(e*x + d))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4} \, dx = \text{Timed out}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))**4,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1263 vs. $2(239) = 478$.

Time $= 0.34$ (sec) , antiderivative size $= 1263$, normalized size of antiderivative $= 5.05$

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + c\sinh(d + ex))^4} \, dx = \text{Too large to display}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"maxima")
```

```
[Out] 1/6*A*(3*(2*a^2 - 3*c^2)*a*log((c*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^
(-e*x - d) - a + sqrt(a^2 + c^2)))/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqr
t(a^2 + c^2)*e) - 2*(11*a^2*c^3 - 4*c^5 + 15*(4*a^3*c^2 - a*c^4)*e^(-e*x -
d) + 6*(17*a^4*c - 6*a^2*c^3 + 2*c^5)*e^(-2*e*x - 2*d) + 2*(22*a^5 - 41*a^3
*c^2 + 12*a*c^4)*e^(-3*e*x - 3*d) - 15*(2*a^4*c - 3*a^2*c^3)*e^(-4*e*x - 4*
d) + 3*(2*a^3*c^2 - 3*a*c^4)*e^(-5*e*x - 5*d))/((a^6*c^3 + 3*a^4*c^5 + 3*a^
2*c^7 + c^9 + 6*(a^7*c^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-e*x - d) + 3*
(4*a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-2*e*x - 2*d) + 4*(2*
a^9 + 3*a^7*c^2 - 3*a^5*c^4 - 7*a^3*c^6 - 3*a*c^8)*e^(-3*e*x - 3*d) - 3*(4*
a^8*c + 11*a^6*c^3 + 9*a^4*c^5 + a^2*c^7 - c^9)*e^(-4*e*x - 4*d) + 6*(a^7*c
^2 + 3*a^5*c^4 + 3*a^3*c^6 + a*c^8)*e^(-5*e*x - 5*d) - (a^6*c^3 + 3*a^4*c^5
 + 3*a^2*c^7 + c^9)*e^(-6*e*x - 6*d))*e)) + 1/6*C*(3*(4*a^2*c - c^3)*log((c
*e^(-e*x - d) - a - sqrt(a^2 + c^2))/(c*e^(-e*x - d) - a + sqrt(a^2 + c^2))
)/((a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)*e) + 2*(2*a^3*c^3 -
13*a*c^5 + 3*(4*a^4*c^2 - 22*a^2*c^4 - c^6)*e^(-e*x - d) + 6*(4*a^5*c - 17*
a^3*c^3 + 4*a*c^5)*e^(-2*e*x - 2*d) + 2*(4*a^6 - 32*a^4*c^2 + 39*a^2*c^4)*e
```

```
^(-3*e*x - 3*d) + 15*(4*a^3*c^3 - a*c^5)*e^(-4*e*x - 4*d) - 3*(4*a^2*c^4 -
c^6)*e^(-5*e*x - 5*d))/((a^6*c^4 + 3*a^4*c^6 + 3*a^2*c^8 + c^10 + 6*(a^7*c^
3 + 3*a^5*c^5 + 3*a^3*c^7 + a*c^9)*e^(-e*x - d) + 3*(4*a^8*c^2 + 11*a^6*c^4
 + 9*a^4*c^6 + a^2*c^8 - c^10)*e^(-2*e*x - 2*d) + 4*(2*a^9*c + 3*a^7*c^3 -
3*a^5*c^5 - 7*a^3*c^7 - 3*a*c^9)*e^(-3*e*x - 3*d) - 3*(4*a^8*c^2 + 11*a^6*c
^4 + 9*a^4*c^6 + a^2*c^8 - c^10)*e^(-4*e*x - 4*d) + 6*(a^7*c^3 + 3*a^5*c^5
+ 3*a^3*c^7 + a*c^9)*e^(-5*e*x - 5*d) - (a^6*c^4 + 3*a^4*c^6 + 3*a^2*c^8 +
c^10)*e^(-6*e*x - 6*d))*e)) - 8/3*B*e^(-3*e*x - 3*d)/((6*a*c^3*e^(-e*x - d)
 + 6*a*c^3*e^(-5*e*x - 5*d) - c^4*e^(-6*e*x - 6*d) + c^4 + 3*(4*a^2*c^2 - c
^4)*e^(-2*e*x - 2*d) + 4*(2*a^3*c - 3*a*c^3)*e^(-3*e*x - 3*d) - 3*(4*a^2*c^
2 - c^4)*e^(-4*e*x - 4*d))*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(239) = 478$.

Time = 0.35 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.74

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a + c\sinh(d+ex))^4}\,dx$$

$$= \frac{\frac{3\left(2\,Aa^3+4\,Ca^2c-3\,Aac^2-Cc^3\right)\log\left(\frac{\left|2\,ce^{(ex+d)}+2\,a-2\,\sqrt{a^2+c^2}\right|}{\left|2\,ce^{(ex+d)}+2\,a+2\,\sqrt{a^2+c^2}\right|}\right)}{\left(a^6+3\,a^4c^2+3\,a^2c^4+c^6\right)\sqrt{a^2+c^2}} + \frac{2\left(6\,Aa^3c^3e^{(5\,ex+5\,d)}+12\,Ca^2c^4e^{(5\,ex+5\,d)}-9\,Aac^5e^{(5\,ex+5\,d)}-3\,Cc^6e^{(5\,ex+}\right.}{}}{}$$

```
[In] integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+c*sinh(e*x+d))^4,x, algorithm=
"giac")
```

```
[Out] 1/6*(3*(2*A*a^3 + 4*C*a^2*c - 3*A*a*c^2 - C*c^3)*log(abs(2*c*e^(e*x + d) +
2*a - 2*sqrt(a^2 + c^2))/abs(2*c*e^(e*x + d) + 2*a + 2*sqrt(a^2 + c^2)))/((
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6)*sqrt(a^2 + c^2)) + 2*(6*A*a^3*c^3*e^(5*e
*x + 5*d) + 12*C*a^2*c^4*e^(5*e*x + 5*d) - 9*A*a*c^5*e^(5*e*x + 5*d) - 3*C*
c^6*e^(5*e*x + 5*d) + 30*A*a^4*c^2*e^(4*e*x + 4*d) + 60*C*a^3*c^3*e^(4*e*x
+ 4*d) - 45*A*a^2*c^4*e^(4*e*x + 4*d) - 15*C*a*c^5*e^(4*e*x + 4*d) - 8*B*a^
6*e^(3*e*x + 3*d) - 8*C*a^6*e^(3*e*x + 3*d) + 44*A*a^5*c*e^(3*e*x + 3*d) -
24*B*a^4*c^2*e^(3*e*x + 3*d) + 64*C*a^4*c^2*e^(3*e*x + 3*d) - 82*A*a^3*c^3*
e^(3*e*x + 3*d) - 24*B*a^2*c^4*e^(3*e*x + 3*d) - 78*C*a^2*c^4*e^(3*e*x + 3*
d) + 24*A*a*c^5*e^(3*e*x + 3*d) - 8*B*c^6*e^(3*e*x + 3*d) + 24*C*a^5*c*e^(2
*e*x + 2*d) - 102*A*a^4*c^2*e^(2*e*x + 2*d) - 102*C*a^3*c^3*e^(2*e*x + 2*d)
 + 36*A*a^2*c^4*e^(2*e*x + 2*d) + 24*C*a*c^5*e^(2*e*x + 2*d) - 12*A*c^6*e^(
2*e*x + 2*d) - 12*C*a^4*c^2*e^(e*x + d) + 60*A*a^3*c^3*e^(e*x + d) + 66*C*a
^2*c^4*e^(e*x + d) - 15*A*a*c^5*e^(e*x + d) + 3*C*c^6*e^(e*x + d) + 2*C*a^3
*c^3 - 11*A*a^2*c^4 - 13*C*a*c^5 + 4*A*c^6)/((a^6*c + 3*a^4*c^3 + 3*a^2*c^5
 + c^7)*(c*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) - c)^3))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+c\sinh(d+ex))^4}\,dx=\int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+c\sinh\left(d+e\,x\right)\right)^4}\,dx$$

```
[In] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4,x)
[Out] int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + c*sinh(d + e*x))^4, x)
```

3.257 $\int \frac{x^3}{a+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 439

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{8\sqrt{a}\sqrt{a-b}} - \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{8\sqrt{a}\sqrt{a-b}}$$

```
[Out] 1/2*x^3*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-
1/2*x^3*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+
3/4*x^2*polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^
(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/
(a-b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1
/2)/(a-b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/
a^(1/2)/(a-b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2))
)/a^(1/2)/(a-b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2
)))/a^(1/2)/(a-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5748, 3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx = \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{8\sqrt{a}\sqrt{a-b}} - \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{8\sqrt{a}\sqrt{a-b}} + \frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}$$

```
[In] Int[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^3*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^3*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (3*x^2*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sq
rt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x^2*PolyLog[2, -((b*E^(2*x))
/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a - b]) - (3*x*PolyLo
g[3, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))])/(4*Sqrt[a]*Sqrt[a -
 b]) + (3*x*PolyLog[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))])/(
4*Sqrt[a]*Sqrt[a - b]) + (3*PolyLog[4, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[
a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b]) - (3*PolyLog[4, -((b*E^(2*x))/(2*a +
 2*Sqrt[a]*Sqrt[a - b] - b))])/(8*Sqrt[a]*Sqrt[a - b])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5748

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\int \frac{x^3}{2a-b+b\cosh(2x)}\,dx$$

$$= 4\int \frac{e^{2x}x^3}{b+2(2a-b)e^{2x}+be^{4x}}\,dx$$

$$= \frac{(2b)\int \frac{e^{2x}x^3}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^3}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{3\int x^2\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}} + \frac{3\int x^2\log\left(1+\frac{2be^{2x}}{4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3\int x\,\text{PolyLog}\left(2,-\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}} + \frac{3\int x\,\text{PolyLog}\left(2,-\frac{2be^{2x}}{4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3x\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3\int \text{PolyLog}\left(3,-\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{4\sqrt{a}\sqrt{a-b}} - \frac{3\int \text{PolyLog}\left(3,-\frac{2be^{2x}}{4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{4\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3 \log\left(1 + \frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3 \log\left(1 + \frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2 \operatorname{PolyLog}\left(2, -\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2 \operatorname{PolyLog}\left(2, -\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x \operatorname{PolyLog}\left(3, -\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3x \operatorname{PolyLog}\left(3, -\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, -\frac{bx}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{x} dx, x, e^{2x}\right)}{8\sqrt{a}\sqrt{a-b}} + \frac{3\text{Subst}\left(\int \frac{\operatorname{PolyLog}\left(3, \frac{bx}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)}{x} dx, x, e^{2x}\right)}{8\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^3 \log\left(1 + \frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^3 \log\left(1 + \frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{3x^2 \operatorname{PolyLog}\left(2, -\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x^2 \operatorname{PolyLog}\left(2, -\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{3x \operatorname{PolyLog}\left(3, -\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3x \operatorname{PolyLog}\left(3, -\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{3 \operatorname{PolyLog}\left(4, -\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{8\sqrt{a}\sqrt{a-b}} - \frac{3 \operatorname{PolyLog}\left(4, -\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{8\sqrt{a}\sqrt{a-b}}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.36

$$\int \frac{x^3}{a + b\sinh^2(x)} dx = \frac{-x^3 \log\left(1 - \frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right) - x^3 \log\left(1 + \frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right) + x^3 \log\left(1 - \frac{e^x}{\sqrt{\frac{-2a+2\sqrt{a(a-b)}+b}{b}}}\right) + x^3 \log\left(1\right.}{}$$

```
[In] Integrate[x^3/(a + b*Sinh[x]^2),x]
```

```
[Out] (-(x^3*Log[1 - E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]]) - x^3*Log[1 + E
^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + x^3*Log[1 - E^x/Sqrt[(-2*a + 2
*Sqrt[a*(a - b)] + b)/b]] + x^3*Log[1 + E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)]
+ b)/b]] - 3*x^2*PolyLog[2, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])]
```

```
- 3*x^2*PolyLog[2, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + 3*x^2*Poly
Log[2, -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b])] + 3*x^2*PolyLog[2, E^
x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]] + 6*x*PolyLog[3, -(E^x/Sqrt[(-2*a
 - 2*Sqrt[a*(a - b)] + b)/b])] + 6*x*PolyLog[3, E^x/Sqrt[(-2*a - 2*Sqrt[a*(
a - b)] + b)/b]] - 6*x*PolyLog[3, -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)
/b])] - 6*x*PolyLog[3, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]] - 6*Poly
Log[4, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])] - 6*PolyLog[4, E^x/Sq
rt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + 6*PolyLog[4, -(E^x/Sqrt[(-2*a + 2*S
qrt[a*(a - b)] + b)/b])] + 6*PolyLog[4, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)]
+ b)/b]])/(2*Sqrt[a*(a - b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(351) = 702$.

Time = 1.74 (sec) , antiderivative size = 919, normalized size of antiderivative = 2.09

method	result	size
risch	Expression too large to display	919

```
[In] int(x^3/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x^
3+1/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a
)^(1/2)-2*a+b))*a*x^3-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b
*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b*x^3-1/2/(-2*((a-b)*a)^(1/2)-2*a+b)*
x^4-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*x^4*a+1/4/((a-b)*a)^(1/2
)/(-2*((a-b)*a)^(1/2)-2*a+b)*b*x^4+3/2/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2
,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x^2+3/2/((a-b)*a)^(1/2)/(-2*((a-b)*
a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a*x^2-3/4/
((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*
a)^(1/2)-2*a+b))*b*x^2-3/2/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/
(-2*((a-b)*a)^(1/2)-2*a+b))*x-3/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b
)*polylog(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a*x+3/4/((a-b)*a)^(1/2)/
(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))
*b*x+3/4/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(4,b*exp(2*x)/(-2*((a-b)*a)^(1/2
)-2*a+b))+3/4/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(4,b*exp(2*
x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a-3/8/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*
a+b)*polylog(4,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b+1/2/((a-b)*a)^(1/2)
*x^3*ln(1-b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))-1/4/((a-b)*a)^(1/2)*x^4+3/4
/((a-b)*a)^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))-3/4/((
a-b)*a)^(1/2)*x*polylog(3,b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))+3/8/((a-b)*
a)^(1/2)*polylog(4,b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1655 vs. $2(339) = 678$.

Time = 0.30 (sec) , antiderivative size = 1655, normalized size of antiderivative = 3.77

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x^3*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^3*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^3*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 3*b*x^2*sqrt((a^2 - a*b)
/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sin
h(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)
 + b)/b + 1) + 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2
*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2
*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*
b)/b^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*s
inh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b
) + b)/b + 1) - 3*b*x^2*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (
2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2
*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 6*b*x*sqrt((a^2 - a*b)
/b^2)*polylog(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*
sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)
/b)/b) - 6*b*x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a
- b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*
sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*x*sqrt((a^2 - a*b)/b^2)*polylo
g(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqr
t((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 6*b*
x*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
+ 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*
b)/b^2) - 2*a + b)/b)/b) + 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*
cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^
2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) + 6*b*sqrt((a^2 - a*b
)/b^2)*polylog(4, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a -
b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4, ((2*a - b)*cosh(x) + (2*a -
```

```
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sq
rt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) - 6*b*sqrt((a^2 - a*b)/b^2)*polylog(4,
 -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((
a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b))/(a^2 - a
*b)
```

Sympy [F]

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx = \int \frac{x^3}{a + b\sinh^2(x)}\,dx$$

```
[In] integrate(x**3/(a+b*sinh(x)**2),x)
[Out] Integral(x**3/(a + b*sinh(x)**2), x)
```

Maxima [F]

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx = \int \frac{x^3}{b\sinh(x)^2 + a}\,dx$$

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="maxima")
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x^3}{a + b\sinh^2(x)}\,dx = \int \frac{x^3}{b\sinh(x)^2 + a}\,dx$$

```
[In] integrate(x^3/(a+b*sinh(x)^2),x, algorithm="giac")
[Out] integrate(x^3/(b*sinh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\sinh^2(x)}\,dx=\int \frac{x^3}{b\sinh{(x)}^2+a}\,dx$$

```
[In] int(x^3/(a + b*sinh(x)^2),x)
[Out] int(x^3/(a + b*sinh(x)^2), x)
```

3.258 $\int \frac{x^2}{a+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 327

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}$$

```
[Out] 1/2*x^2*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-
1/2*x^2*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+
1/2*x*polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1
/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b
)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a
-b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/
(a-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{5748, 3401, 2296, 2221, 2611, 2320, 6724\}$

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx = \frac{x\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}$$

```
[In] Int[x^2/(a + b*Sinh[x]^2),x]
```

```
[Out] (x^2*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqr
t[a - b]) - (x^2*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2
*Sqrt[a]*Sqrt[a - b]) + (x*PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a
 - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - (x*PolyLog[2, -((b*E^(2*x))/(2*a +
2*Sqrt[a]*Sqrt[a - b] - b))])/(2*Sqrt[a]*Sqrt[a - b]) - PolyLog[3, -((b*E^(
2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b]) + PolyLog
[3, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b
])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5748

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{x^2}{2a-b+b\cosh(2x)}\,dx \\
&= 4\int \frac{e^{2x}x^2}{b+2(2a-b)e^{2x}+be^{4x}}\,dx \\
&= \frac{(2b)\int \frac{e^{2x}x^2}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x^2}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} \\
&= \frac{x^2\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} \\
&\quad - \frac{\int x\log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{\sqrt{a}\sqrt{a-b}} + \frac{\int x\log\left(1+\frac{2be^{2x}}{4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{\sqrt{a}\sqrt{a-b}}
\end{aligned}
$$

$$= \frac{x^2 \log \left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2 \log \left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\int \operatorname{PolyLog}\left(2,-\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right) dx}{2\sqrt{a}\sqrt{a-b}} + \frac{\int \operatorname{PolyLog}\left(2,-\frac{2be^{2x}}{4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right) dx}{2\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^2 \log \left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2 \log \left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,-\frac{bx}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{x} dx, x, e^{2x}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2,\frac{bx}{-2a+2\sqrt{a}\sqrt{a-b}+b}\right)}{x} dx, x, e^{2x}\right)}{4\sqrt{a}\sqrt{a-b}}$$

$$= \frac{x^2 \log \left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x^2 \log \left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{a+b\sinh^2(x)} dx = \frac{-x^2 \log \left(1-\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right) - x^2 \log \left(1+\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right) + x^2 \log \left(1-\frac{e^x}{\sqrt{\frac{-2a+2\sqrt{a(a-b)}+b}{b}}}\right) + x^2 \log \left(1\right.}{}$$

```
[In] Integrate[x^2/(a + b*Sinh[x]^2),x]
[Out] (-(x^2*Log[1 - E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]]) - x^2*Log[1 + E
^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + x^2*Log[1 - E^x/Sqrt[(-2*a + 2
```

```
*Sqrt[a*(a - b)] + b)/b]] + x^2*Log[1 + E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)]
+ b)/b]] - 2*x*PolyLog[2, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])] -
2*x*PolyLog[2, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + 2*x*PolyLog[2,
 -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b])] + 2*x*PolyLog[2, E^x/Sqrt[(
-2*a + 2*Sqrt[a*(a - b)] + b)/b]] + 2*PolyLog[3, -(E^x/Sqrt[(-2*a - 2*Sqrt[
a*(a - b)] + b)/b])] + 2*PolyLog[3, E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)
/b]] - 2*PolyLog[3, -(E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b])] - 2*Poly
Log[3, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]])/(2*Sqrt[a*(a - b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(261) = 522$.

Time = 0.13 (sec) , antiderivative size = 710, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{2x^3}{3\left(-2\sqrt{(a-b)a}-2a+b\right)}+\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{(a-b)a}-2a+b}\right)x^2}{-2\sqrt{(a-b)a}-2a+b}+\frac{\operatorname{polylog}\left(2,\frac{b\,e^{2x}}{-2\sqrt{(a-b)a}-2a+b}\right)x}{-2\sqrt{(a-b)a}-2a+b}-\frac{\operatorname{polylog}\left(3,\frac{b\,e^{2x}}{-2\sqrt{(a-b)a}-2a+b}\right)}{2\left(-2\sqrt{(a-b)a}-2a+b\right)}$

```
[In] int(x^2/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(-2*((a-b)*a)^(1/2)-2*a+b)*x^3+1/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp
(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x^2+1/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(
2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x-1/2/(-2*((a-b)*a)^(1/2)-2*a+b)*p
olylog(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))-2/3/((a-b)*a)^(1/2)/(-2*((a
-b)*a)^(1/2)-2*a+b)*x^3*a+1/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1
-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a*x^2+1/((a-b)*a)^(1/2)/(-2*((a-b)*
a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a*x-1/2/((
a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*((a-b)*a)
^(1/2)-2*a+b))*a+1/3/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*b*x^3-1/2/(
(a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/
2)-2*a+b))*b*x^2-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b
*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b*x+1/4/((a-b)*a)^(1/2)/(-2*((a-b)*a)
^(1/2)-2*a+b)*polylog(3,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b-1/3/((a-b)
*a)^(1/2)*x^3+1/2/((a-b)*a)^(1/2)*x^2*ln(1-b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*
a+b))+1/2/((a-b)*a)^(1/2)*x*polylog(2,b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))
-1/4/((a-b)*a)^(1/2)*polylog(3,b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(252) = 504$.

Time = 0.30 (sec) , antiderivative size = 1247, normalized size of antiderivative = 3.81

$$\int \frac{x^2}{a + b\sinh^2(x)}\, dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="fricas")
[Out] -1/2*(b*x^2*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(
x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
- a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a
 - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x^2*sq
rt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2
*a + b)/b) + b)/b) - b*x^2*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) +
 (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(
(2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + 2*b*x*sqrt((a^2 - a*b)/b
^2)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(
x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) +
 b)/b + 1) + 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a -
 b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*s
qrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2
)*dilog(-(((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x)
)*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)
/b + 1) - 2*b*x*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)
*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt(
(a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b + 1) - 2*b*sqrt((a^2 - a*b)/b^2)*poly
log(3, ((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*s
qrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b)/b) - 2
*b*sqrt((a^2 - a*b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b)/b) + 2*b*sqrt((a^2 - a*b)/b^2)*polylog(3, ((2*a - b
)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b)/b) + 2*b*sqrt((a^2 - a*
b)/b^2)*polylog(3, -((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
 b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
b)/b)/b))/(a^2 - a*b)
```

Sympy [F]

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx=\int \frac{x^2}{a+b\sinh^2{(x)}}\,dx$$

```
[In] integrate(x**2/(a+b*sinh(x)**2),x)
[Out] Integral(x**2/(a + b*sinh(x)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx=\int \frac{x^2}{b\sinh{(x)}^2+a}\,dx$$

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="maxima")
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx=\int \frac{x^2}{b\sinh{(x)}^2+a}\,dx$$

```
[In] integrate(x^2/(a+b*sinh(x)^2),x, algorithm="giac")
[Out] integrate(x^2/(b*sinh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\sinh^2(x)}\,dx=\int \frac{x^2}{b\sinh{(x)}^2+a}\,dx$$

```
[In] int(x^2/(a + b*sinh(x)^2),x)
[Out] int(x^2/(a + b*sinh(x)^2), x)
```

3.259 $\int \frac{x}{a+b\sinh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 215

$$\int \frac{x}{a+b\sinh^2(x)}\,dx = \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} + \frac{\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}$$

```
[Out] 1/2*x*ln(1+b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-1/
2*x*ln(1+b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)+1/4*
polylog(2,-b*exp(2*x)/(2*a-b-2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)-1/
4*polylog(2,-b*exp(2*x)/(2*a-b+2*a^(1/2)*(a-b)^(1/2)))/a^(1/2)/(a-b)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5748, 3401, 2296, 2221, 2317, 2438}

$$\int \frac{x}{a+b\sinh^2(x)}\,dx = \frac{\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a-b}\sqrt{a}-b}\right)}{4\sqrt{a}\sqrt{a-b}} + \frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a-b}+2a-b}+1\right)}{2\sqrt{a}\sqrt{a-b}}$$

```
[In] Int[x/(a + b*Sinh[x]^2),x]
```

```
[Out] (x*Log[1 + (b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqrt[a]*Sqrt[
a - b]) - (x*Log[1 + (b*E^(2*x))/(2*a + 2*Sqrt[a]*Sqrt[a - b] - b)])/(2*Sqr
```

```
t[a]*Sqrt[a - b]) + PolyLog[2, -((b*E^(2*x))/(2*a - 2*Sqrt[a]*Sqrt[a - b] -
 b))]/(4*Sqrt[a]*Sqrt[a - b]) - PolyLog[2, -((b*E^(2*x))/(2*a + 2*Sqrt[a]*S
qrt[a - b] - b))]/(4*Sqrt[a]*Sqrt[a - b])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5748

```
Int[(x_)^(m_.)*((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]^2)^(n_), x_Symbol] :>
 Dist[1/2^n, Int[x^m*(2*a - b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1] |
| (EqQ[m, 1] && EqQ[n, -2]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{x}{2a-b+b\cosh(2x)}\,dx \\
&= 4\int \frac{e^{2x}x}{b+2(2a-b)e^{2x}+be^{4x}}\,dx \\
&= \frac{(2b)\int \frac{e^{2x}x}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} - \frac{(2b)\int \frac{e^{2x}x}{4\sqrt{a}\sqrt{a-b}+2(2a-b)+2be^{2x}}\,dx}{\sqrt{a}\sqrt{a-b}} \\
&= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} \\
&\quad - \frac{\int \log\left(1+\frac{2be^{2x}}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}} + \frac{\int \log\left(1+\frac{2be^{2x}}{4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)dx}{2\sqrt{a}\sqrt{a-b}} \\
&= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{-4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)}{x}\,dx, x, e^{2x}\right)}{4\sqrt{a}\sqrt{a-b}} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2bx}{4\sqrt{a}\sqrt{a-b}+2(2a-b)}\right)}{x}\,dx, x, e^{2x}\right)}{4\sqrt{a}\sqrt{a-b}} \\
&= \frac{x\log\left(1+\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{2\sqrt{a}\sqrt{a-b}} \\
&\quad + \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}} - \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a}\sqrt{a-b}-b}\right)}{4\sqrt{a}\sqrt{a-b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.37

$$\int \frac{x}{a+b\sinh^2(x)}\,dx$$

$$= \frac{-x\log\left(1-\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right) - x\log\left(1+\frac{e^x}{\sqrt{\frac{-2a-2\sqrt{a(a-b)}+b}{b}}}\right) + x\log\left(1-\frac{e^x}{\sqrt{\frac{-2a+2\sqrt{a(a-b)}+b}{b}}}\right) + x\log\left(1+\cdots\right.}{\cdots}$$

```
[In] Integrate[x/(a + b*Sinh[x]^2),x]
```

```
[Out] (-(x*Log[1 - E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]]) - x*Log[1 + E^x/S
qrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + x*Log[1 - E^x/Sqrt[(-2*a + 2*Sqrt[
a*(a - b)] + b)/b]] + x*Log[1 + E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)] + b)/b]]
 - PolyLog[2, -(E^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b])] - PolyLog[2, E
^x/Sqrt[(-2*a - 2*Sqrt[a*(a - b)] + b)/b]] + PolyLog[2, -(E^x/Sqrt[(-2*a +
2*Sqrt[a*(a - b)] + b)/b])] + PolyLog[2, E^x/Sqrt[(-2*a + 2*Sqrt[a*(a - b)]
 + b)/b]])/(2*Sqrt[a*(a - b)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. $2(171) = 342$.

Time = 0.11 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.35

method	result
risch	$\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{(a-b)a}-2a+b}\right)x}{-2\sqrt{(a-b)a}-2a+b}+\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{(a-b)a}-2a+b}\right)ax}{\sqrt{(a-b)a}\left(-2\sqrt{(a-b)a}-2a+b\right)}-\frac{\ln\left(1-\frac{b\,\mathrm{e}^{2x}}{-2\sqrt{(a-b)a}-2a+b}\right)bx}{2\sqrt{(a-b)a}\left(-2\sqrt{(a-b)a}-2a+b\right)}-\frac{x^2}{-2\sqrt{(a-b)a}-2a+b}-$

```
[In] int(x/(a+b*sinh(x)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*x+
1/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp(2*x)/(-2*((a-b)*a)^
(1/2)-2*a+b))*a*x-1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*ln(1-b*exp
(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b*x-1/(-2*((a-b)*a)^(1/2)-2*a+b)*x^2-1/((
a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b)*x^2*a+1/2/((a-b)*a)^(1/2)/(-2*((a-
b)*a)^(1/2)-2*a+b)*b*x^2+1/2/(-2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x
)/(-2*((a-b)*a)^(1/2)-2*a+b))+1/2/((a-b)*a)^(1/2)/(-2*((a-b)*a)^(1/2)-2*a+b
)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*a-1/4/((a-b)*a)^(1/2)/(-
2*((a-b)*a)^(1/2)-2*a+b)*polylog(2,b*exp(2*x)/(-2*((a-b)*a)^(1/2)-2*a+b))*b
+1/2/((a-b)*a)^(1/2)*x*ln(1-b*exp(2*x)/(2*((a-b)*a)^(1/2)-2*a+b))-1/2/((a-b
)*a)^(1/2)*x^2+1/4/((a-b)*a)^(1/2)*polylog(2,b*exp(2*x)/(2*((a-b)*a)^(1/2)-
2*a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(165) = 330$.

Time = 0.32 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.89

$$\int \frac{x}{a+b\sinh^2(x)}\,dx =$$

$$-\frac{bx\sqrt{\frac{a^2-ab}{b^2}}\log\left(\frac{\left((2a-b)\cosh(x)+(2a-b)\sinh(x)-2\left(b\cosh(x)+b\sinh(x)\right)\sqrt{\frac{a^2-ab}{b^2}}\right)\sqrt{-\frac{2b\sqrt{\frac{a^2-ab}{b^2}}+2a-b}{b}}+b}{b}\right)+bx\sqrt{\frac{a^2-ab}{b^2}}\log}{}$$

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(b*x*sqrt((a^2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x)
 - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 -
a*b)/b^2) + 2*a - b)/b) + b)/b) + b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b
)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) - b)/b) - b*x*sqrt((a^
2 - a*b)/b^2)*log((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) +
b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b
)/b) + b)/b) - b*x*sqrt((a^2 - a*b)/b^2)*log(-(((2*a - b)*cosh(x) + (2*a -
b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqr
t((a^2 - a*b)/b^2) - 2*a + b)/b) - b)/b) + b*sqrt((a^2 - a*b)/b^2)*dilog(-(
((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^
2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/b^2) + 2*a - b)/b) + b)/b + 1) +
 b*sqrt((a^2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) - 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 - a*b)/
b^2) + 2*a - b)/b) - b)/b + 1) - b*sqrt((a^2 - a*b)/b^2)*dilog(-(((2*a - b)
*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 - a*b)/b
^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a + b)/b) + b)/b + 1) - b*sqrt((a^
2 - a*b)/b^2)*dilog((((2*a - b)*cosh(x) + (2*a - b)*sinh(x) + 2*(b*cosh(x)
+ b*sinh(x))*sqrt((a^2 - a*b)/b^2))*sqrt((2*b*sqrt((a^2 - a*b)/b^2) - 2*a +
 b)/b) - b)/b + 1))/(a^2 - a*b)
```

Sympy [F]

$$\int \frac{x}{a+b\sinh^2(x)}\,dx = \int \frac{x}{a+b\sinh^2{(x)}}\,dx$$

```
[In] integrate(x/(a+b*sinh(x)**2),x)
```

```
[Out] Integral(x/(a + b*sinh(x)**2), x)
```

Maxima [F]

$$\int \frac{x}{a+b\sinh^2(x)}\,dx = \int \frac{x}{b\sinh{(x)}^2+a}\,dx$$

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x}{a + b\sinh^2(x)}\,dx = \int \frac{x}{b\sinh{(x)}^2 + a}\,dx$$

```
[In] integrate(x/(a+b*sinh(x)^2),x, algorithm="giac")
[Out] integrate(x/(b*sinh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + b\sinh^2(x)}\,dx = \int \frac{x}{b\sinh{(x)}^2 + a}\,dx$$

```
[In] int(x/(a + b*sinh(x)^2),x)
[Out] int(x/(a + b*sinh(x)^2), x)
```

3.260 $\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = -\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx) -\frac{9}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

```
[Out] -9/4*Chi(b*x)*cosh(a)+1/4*Chi(3*b*x)*cosh(3*a)-9/4*Shi(b*x)*sinh(a)+1/4*Shi
(3*b*x)*sinh(3*a)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6874, 3384, 3379, 3382, 5556}

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = -\frac{9}{4}\cosh(a)\text{Chi}(bx)+\frac{1}{4}\cosh(3a)\text{Chi}(3bx) -\frac{9}{4}\sinh(a)\text{Shi}(bx)+\frac{1}{4}\sinh(3a)\text{Shi}(3bx)$$

```
[In] Int[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
```

```
[Out] (-9*Cosh[a]*CoshIntegral[b*x])/4 + (Cosh[3*a]*CoshIntegral[3*b*x])/4 - (9*S
inh[a]*SinhIntegral[b*x])/4 + (Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{2\cosh(a+bx)}{x}+\frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\right)dx \\
&= -\left(2\int \frac{\cosh(a+bx)}{x}\,dx\right)+\int \frac{\cosh(a+bx)\sinh^2(a+bx)}{x}\,dx \\
&= -\left((2\cosh(a))\int \frac{\cosh(bx)}{x}\,dx\right)-(2\sinh(a))\int \frac{\sinh(bx)}{x}\,dx \\
&\quad +\int \left(-\frac{\cosh(a+bx)}{4x}+\frac{\cosh(3a+3bx)}{4x}\right)dx \\
&= -2\cosh(a)\text{Chi}(bx)-2\sinh(a)\text{Shi}(bx)-\frac{1}{4}\int \frac{\cosh(a+bx)}{x}\,dx+\frac{1}{4}\int \frac{\cosh(3a+3bx)}{x}\,dx \\
&= -2\cosh(a)\text{Chi}(bx)-2\sinh(a)\text{Shi}(bx) \\
&\quad -\frac{1}{4}\cosh(a)\int \frac{\cosh(bx)}{x}\,dx+\frac{1}{4}\cosh(3a)\int \frac{\cosh(3bx)}{x}\,dx \\
&\quad -\frac{1}{4}\sinh(a)\int \frac{\sinh(bx)}{x}\,dx+\frac{1}{4}\sinh(3a)\int \frac{\sinh(3bx)}{x}\,dx
\end{aligned}$$

$$= -\frac{9}{4}\cosh(a)\mathrm{Chi}(bx) + \frac{1}{4}\cosh(3a)\mathrm{Chi}(3bx) - \frac{9}{4}\sinh(a)\mathrm{Shi}(bx) + \frac{1}{4}\sinh(3a)\mathrm{Shi}(3bx)$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \frac{1}{4}(-9\cosh(a)\mathrm{Chi}(bx) + \cosh(3a)\mathrm{Chi}(3bx) - 9\sinh(a)\mathrm{Shi}(bx) + \sinh(3a)\mathrm{Shi}(3bx))$$

```
[In] Integrate[(Cosh[a + b*x]*(-2 + Sinh[a + b*x]^2))/x,x]
[Out] (-9*Cosh[a]*CoshIntegral[b*x] + Cosh[3*a]*CoshIntegral[3*b*x] - 9*Sinh[a]*S
inhIntegral[b*x] + Sinh[3*a]*SinhIntegral[3*b*x])/4
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{e^{-3a}\,\mathrm{Ei}_1(3bx)}{8}+\frac{9\,e^{-a}\,\mathrm{Ei}_1(bx)}{8}+\frac{9\,e^{a}\,\mathrm{Ei}_1(-bx)}{8}-\frac{e^{3a}\,\mathrm{Ei}_1(-3bx)}{8}$	47

```
[In] int(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x,method=_RETURNVERBOSE)
[Out] -1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)+9/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \frac{1}{8}\left(\mathrm{Ei}(3\,bx) + \mathrm{Ei}(-3\,bx)\right)\cosh(3\,a) - \frac{9}{8}\left(\mathrm{Ei}(bx) + \mathrm{Ei}(-bx)\right)\cosh(a) + \frac{1}{8}\left(\mathrm{Ei}(3\,bx) - \mathrm{Ei}(-3\,bx)\right)\sinh(3\,a) - \frac{9}{8}\left(\mathrm{Ei}(bx) - \mathrm{Ei}(-bx)\right)\sinh(a)$$

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="fricas")
[Out] 1/8*(Ei(3*b*x) + Ei(-3*b*x))*cosh(3*a) - 9/8*(Ei(b*x) + Ei(-b*x))*cosh(a) +
 1/8*(Ei(3*b*x) - Ei(-3*b*x))*sinh(3*a) - 9/8*(Ei(b*x) - Ei(-b*x))*sinh(a)
```

Sympy [F]

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \int \frac{\left(\sinh^2\left(a+bx\right)-2\right)\cosh\left(a+bx\right)}{x}\,dx$$

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)**2)/x,x)
[Out] Integral((sinh(a + b*x)**2 - 2)*cosh(a + b*x)/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="maxima")
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} + \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} - \frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

```
[In] integrate(cosh(b*x+a)*(-2+sinh(b*x+a)^2)/x,x, algorithm="giac")
[Out] 1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) + 1/8*Ei(-3*b*x)*e^(-3*a) - 9/8
*Ei(b*x)*e^a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)\left(-2+\sinh^2(a+bx)\right)}{x}\,dx = \int \frac{\cosh(a+b\,x)\,\left(\sinh(a+b\,x)^2-2\right)}{x}\,dx$$

```
[In] int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x,x)
[Out] int((cosh(a + b*x)*(sinh(a + b*x)^2 - 2))/x, x)
```

3.261 $\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{3\operatorname{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\operatorname{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

```
[Out] 3/4*Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Shi(3*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6813, 3393, 3379}

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{3\operatorname{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a} - \frac{\operatorname{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4a}$$

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/(4*a) - SinhIntegral[(3*Sqrt[
1 - a*x])/Sqrt[1 + a*x]]/(4*a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{i\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x}-\frac{i\sinh(3x)}{4x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}+\frac{3\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}\\
&= \frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\frac{3\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)-\text{Shi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

```
[Out] (3*SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - SinhIntegral[(3*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/(4*a)
```

Maple [F]

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{-a^2x^2+1}dx$$

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

```
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.262 $\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} + \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

```
[Out] -1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6813, 3393, 3382}

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a} - \frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2a}$$

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a + Log[Sqrt[1 - a*x]/Sq
rt[1 + a*x]]/(2*a)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sinh^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2x}-\frac{\cosh(2x)}{2x}\right)\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\
&= \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}-\frac{\text{Subst}\left(\int \frac{\cosh(2x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}\\
&= -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}+\frac{\log(1-ax)}{4a}-\frac{\log(1+ax)}{4a}$$

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a + Log[1 - a*x]/(4*a) -
 Log[1 + a*x]/(4*a)
```

Maple [F]

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{-a^2x^2+1}dx$$

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
[Out] -1/4*log(a*x + 1)/a + 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1
)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sqr
t(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

```
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

```
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.263 $\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 26

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[Out] -Shi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6813, 3379}

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

```
[In] Int[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 6813

```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Dist[2*e*(g/(C*(e*f - d
```

```
*g))), Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]], x
] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\\ &= -\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{Shi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

```
[In] Integrate[Sinh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(SinhIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Maple [F]

$$\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

```
[In] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
[Out] int(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
[Out] integral(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
[Out] -Integral(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
[Out] -integrate(sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

```
[In] integrate(sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
[Out] integrate(-sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

```
[In] int(-sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
[Out] -int(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.264 $\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)}, x\right)$$

```
[Out] Unintegrable(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$$

```
[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] -(Defer[Subst][Defer[Int][Csch[x]/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi steps

$$\text{integral} = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}(x)}{x} \, dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [N/A]

Not integrable

Time = 12.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

```
[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.88 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(-a^2x^2+1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}dx$$

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [N/A]

Not integrable

Time = 10.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{a^2x^2\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

```
[Out] -Integral(1/(a**2*x**2*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sinh(sqrt(-a*x
+ 1)/sqrt(a*x + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

```
[Out] -integrate(1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Mupad [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

```
[In] int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
[Out] -int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.265 $\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

```
[Out] Unintegrable(csch((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

```
[In] Int[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

```
[Out] -(Defer[Subst][Defer[Int][Csch[x]^2/x, x], x, Sqrt[1 - a*x]/Sqrt[1 + a*x]]/
a)
```

Rubi steps

$$\text{integral} = -\frac{\operatorname{Subst}\left(\int \frac{\operatorname{csch}^2(x)}{x}\,dx, x, \frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

Mathematica [N/A]

Not integrable

Time = 44.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

```
[In] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
[Out] Integrate[Csch[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(-a^2x^2+1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}dx$$

```
[In] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
[Out] int(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{1}{\left(a^2x^2-1\right)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
[Out] integral(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Sympy [N/A]

Not integrable

Time = 34.71 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\int \frac{1}{a^2x^2\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\sinh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

```
[In] integrate(1/(-a**2*x**2+1)/sinh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
[Out] -Integral(1/(a**2*x**2*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - sinh(sqrt(-a
*x + 1)/sqrt(a*x + 1))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 5.19

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{1}{(a^2x^2-1)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
[Out] 2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - sqrt
(-a*x + 1)*a) - integrate(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sq
rt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x) + integrate
(sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x + 1)*e^(sqrt(-a*x + 1)/sqrt(a*x + 1
)) - (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{1}{(a^2x^2-1)\sinh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

```
[In] integrate(1/(-a^2*x^2+1)/sinh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
[Out] integrate(-1/((a^2*x^2 - 1)*sinh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\int \frac{1}{\sinh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

```
[In] int(-1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
[Out] -int(1/(sinh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.266 $\int \sinh(a + b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \sinh(a + b\log(cx^n))\,dx = -\frac{bnx\cosh(a + b\log(cx^n))}{1 - b^2n^2} + \frac{x\sinh(a + b\log(cx^n))}{1 - b^2n^2}$$

```
[Out] -b*n*x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5628}

$$\int \sinh(a + b\log(cx^n))\,dx = \frac{x\sinh(a + b\log(cx^n))}{1 - b^2n^2} - \frac{bnx\cosh(a + b\log(cx^n))}{1 - b^2n^2}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2)) + (x*Sinh[a + b*Log[c*x^n]]
)/(1 - b^2*n^2)
```

Rule 5628

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Cosh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{bnx\cosh(a + b\log(cx^n))}{1 - b^2n^2} + \frac{x\sinh(a + b\log(cx^n))}{1 - b^2n^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \sinh(a+b\log(cx^n))\,dx=\frac{x(bn\cosh(a+b\log(cx^n))-\sinh(a+b\log(cx^n)))}{-1+b^2n^2}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]],x]
[Out] (x*(b*n*Cosh[a + b*Log[c*x^n]] - Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{x(bn\cosh(a+b\ln(c\,x^n))-\sinh(a+b\ln(c\,x^n)))}{b^2n^2-1}$	42

```
[In] int(sinh(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
[Out] x*(b*n*cosh(a+b*ln(c*x^n))-sinh(a+b*ln(c*x^n)))/(b^2*n^2-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \sinh(a+b\log(cx^n))\,dx=\frac{bnx\cosh(bn\log(x)+b\log(c)+a)-x\sinh(bn\log(x)+b\log(c)+a)}{b^2n^2-1}$$

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a) - x*sinh(b*n*log(x) + b*log(c) + a))
/(b^2*n^2 - 1)
```

Sympy [F]

$$\int \sinh \left(a+b \log \left(c x^{n}\right)\right) d x=\begin{cases}\int \sinh \left(a-\frac{\log \left(c x^{n}\right)}{n}\right) d x & \text{for } b=-\frac{1}{n} \\ \int \sinh \left(a+\frac{\log \left(c x^{n}\right)}{n}\right) d x & \text{for } b=\frac{1}{n} \\ \frac{b n x \cosh \left(a+b \log \left(c x^{n}\right)\right)}{b^{2} n^{2}-1}-\frac{x \sinh \left(a+b \log \left(c x^{n}\right)\right)}{b^{2} n^{2}-1} & \text{otherwise}\end{cases}$$

```
[In] integrate(sinh(a+b*ln(c*x**n)),x)
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(si
nh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*cosh(a + b*log(c*x**n))/(b**
2*n**2 - 1) - x*sinh(a + b*log(c*x**n))/(b**2*n**2 - 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \sinh \left(a+b \log \left(c x^{n}\right)\right) d x=\frac{c^{b} x e^{\left(b \log \left(x^{n}\right)+a\right)}}{2(b n+1)}+\frac{x e^{\left(-b \log \left(x^{n}\right)-a\right)}}{2\left(b c^{b} n-c^{b}\right)}$$

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] 1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) + 1/2*x*e^(-b*log(x^n) - a)/(b*c^b*n
 - c^b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \sinh \left(a+b \log \left(c x^{n}\right)\right) d x=\frac{c^{b} x x^{b n} e^{a}}{2(b n+1)}+\frac{x e^{(-a)}}{2(b n-1) c^{b} x^{b n}}$$

```
[In] integrate(sinh(a+b*log(c*x^n)),x, algorithm="giac")
[Out] 1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) + 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sinh\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,b\,n-2\right)}+\frac{x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,b\,n+2}$$

```
[In] int(sinh(a + b*log(c*x^n)),x)
[Out] (x*exp(-a))/((c*x^n)^b*(2*b*n - 2)) + (x*exp(a)*(c*x^n)^b)/(2*b*n + 2)
```

3.267 $\int \sinh^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \sinh^2(a+b\log(cx^n))\,dx=\frac{2b^2n^2x}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}$$

```
[Out] 2*b^2*n^2*x/(-4*b^2*n^2+1)-2*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/
(-4*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5630, 8}

$$\int \sinh^2(a+b\log(cx^n))\,dx=\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{2b^2n^2x}{1-4b^2n^2}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x)/(1 - 4*b^2*n^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*
Log[c*x^n]])/(1 - 4*b^2*n^2) + (x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5630

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (-Dist[b
^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Sinh[d*(a + b*Log[c*x^n])
]^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a +
b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2} \\ &\quad+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}+\frac{(2b^2n^2)\int 1\,dx}{1-4b^2n^2} \\ &= \frac{2b^2n^2x}{1-4b^2n^2}-\frac{2bnx\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-4b^2n^2}+\frac{x\sinh^2(a+b\log(cx^n))}{1-4b^2n^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

$$\begin{aligned}&\int \sinh^2(a+b\log(cx^n))\,dx \\ &= -\frac{x(-1+4b^2n^2+\cosh(2(a+b\log(cx^n)))-2bn\sinh(2(a+b\log(cx^n))))}{-2+8b^2n^2}\end{aligned}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2,x]
[Out] -((x*(-1 + 4*b^2*n^2 + Cosh[2*(a + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a + b*Log
[c*x^n])]))/(-2 + 8*b^2*n^2))
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.66

method	result	size
parallelrisch	$-\frac{x\left(4b^2n^2-2bn\sinh(2b\ln(c\,x^n)+2a)+\cosh(2b\ln(c\,x^n)+2a)-1\right)}{8b^2n^2-2}$	58

```
[In] int(sinh(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -x*(4*b^2*n^2-2*b*n*sinh(2*b*ln(c*x^n)+2*a)+cosh(2*b*ln(c*x^n)+2*a)-1)/(8*b
^2*n^2-2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

$$\int \sinh^2(a+b\log(cx^n))\,dx$$
$$=\frac{4\,bnx\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)-x\cosh(bn\log(x)+b\log(c)+a)^2-}{2\,(4\,b^2n^2-1)}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) + a)
^2 - (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

Sympy [F]

$$\int \sinh^2(a+b\log(cx^n))\,dx$$
$$=\begin{cases}\int \sinh^2\left(a-\frac{\log(cx^n)}{2n}\right)dx \\ \int \sinh^2\left(a+\frac{\log(cx^n)}{2n}\right)dx \\ \frac{2b^2n^2x\sinh^2(a+b\log(cx^n))}{4b^2n^2-1}-\frac{2b^2n^2x\cosh^2(a+b\log(cx^n))}{4b^2n^2-1}+\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2n^2-1}-\frac{x\sinh^2(a+b\log(cx^n))}{4b^2n^2-1}\end{cases}$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))), (
Integral(sinh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (2*b**2*n**2*
x*sinh(a + b*log(c*x**n))**2/(4*b**2*n**2 - 1) - 2*b**2*n**2*x*cosh(a + b*l
og(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*log(c*x**n))*cosh(a +
 b*log(c*x**n))/(4*b**2*n**2 - 1) - x*sinh(a + b*log(c*x**n))**2/(4*b**2*n*
*2 - 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+2\,a\right)}}{4\left(2\,bn+1\right)}-\frac{1}{2}\,x-\frac{xe^{\left(-2\,a\right)}}{4\left(2\,bc^{2\,b}n-c^{2\,b}\right)\left(x^n\right)^{2\,b}}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) - 1/2*x - 1/4*x*e^(-2*a)/(
(2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.92

$$\begin{aligned}\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx=&\frac{bc^{2\,b}nxx^{2\,bn}e^{\left(2\,a\right)}}{2\left(4\,b^2n^2-1\right)}-\frac{2\,b^2n^2x}{4\,b^2n^2-1}\\&-\frac{c^{2\,b}xx^{2\,bn}e^{\left(2\,a\right)}}{4\left(4\,b^2n^2-1\right)}-\frac{bnxe^{\left(-2\,a\right)}}{2\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}\\&+\frac{x}{2\left(4\,b^2n^2-1\right)}-\frac{xe^{\left(-2\,a\right)}}{4\left(4\,b^2n^2-1\right)c^{2\,b}x^{2\,bn}}\end{aligned}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 2*b^2*n^2*x/(4*b^2*n^
2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-2*
a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{x\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}-\frac{x\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)}-\frac{x}{2}$$

```
[In] int(sinh(a + b*log(c*x^n))^2,x)
[Out] (x*exp(2*a)*(c*x^n)^(2*b))/(8*b*n + 4) - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*
n - 4)) - x/2
```

3.268 $\int \sinh^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \sinh^3(a+b\log(cx^n))\,dx = -\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4}+\frac{6b^2n^2x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} - \frac{3bnx\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{1-9b^2n^2} + \frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}$$

```
[Out] -6*b^3*n^3*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+6*b^2*n^2*x*sinh(
a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)-3*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+
b*ln(c*x^n))^2/(-9*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^3/(-9*b^2*n^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5630, 5628}

$$\int \sinh^3(a+b\log(cx^n))\,dx = \frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2} - \frac{3bnx\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-9b^2n^2} + \frac{6b^2n^2x\sinh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} - \frac{6b^3n^3x\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x*Cosh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) + (6*b^2
*n^2*x*Sinh[a + b*Log[c*x^n]])/(1 - 10*b^2*n^2 + 9*b^4*n^4) - (3*b*n*x*Cosh
```

```
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/(1 - 9*b^2*n^2) + (x*Sinh[a +
b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2)
```

Rule 5628

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Cosh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Rule 5630

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (-Dist[b
^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Sinh[d*(a + b*Log[c*x^n])
]^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a +
b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3bnx\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{1-9b^2n^2} \\
&\quad + \frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2} + \frac{(6b^2n^2)\int\sinh(a+b\log(cx^n))\,dx}{1-9b^2n^2} \\
&= -\frac{6b^3n^3x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} + \frac{6b^2n^2x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} \\
&\quad - \frac{3bnx\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{1-9b^2n^2} + \frac{x\sinh^3(a+b\log(cx^n))}{1-9b^2n^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$
\int \sinh^3(a+b\log(cx^n))\,dx = \frac{x(-3bn(-1+9b^2n^2)\cosh(a+b\log(cx^n))+3bn(-1+b^2n^2)\cosh(3(a+b\log(cx^n)))-2(1-13b^2n^2+(}{4-40b^2n^2+36b^4n^4}
$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(-3*b*n*(-1 + 9*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + 3*b*n*(-1 + b^2*n^2)*C
osh[3*(a + b*Log[c*x^n])] - 2*(1 - 13*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a +
b*Log[c*x^n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

Maple [A] (verified)

Time = 5.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{x\left(-27b^3n^3\cosh(a+b\ln(c\,x^n))+3b^3n^3\cosh(3b\ln(c\,x^n)+3a)+27b^2n^2\sinh(a+b\ln(c\,x^n))-b^2n^2\sinh(3b\ln(c\,x^n)+3a)+3bn\cosh(a\right.}{36b^4n^4-40b^2n^2+4}$

[In] int(sinh(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)

[Out] x*(-27*b^3*n^3*cosh(a+b*ln(c*x^n))+3*b^3*n^3*cosh(3*b*ln(c*x^n)+3*a)+27*b^2*n^2*sinh(a+b*ln(c*x^n))-b^2*n^2*sinh(3*b*ln(c*x^n)+3*a)+3*b*n*cosh(a+b*ln(c*x^n))-3*b*n*cosh(3*b*ln(c*x^n)+3*a)-3*sinh(a+b*ln(c*x^n))+sinh(3*b*ln(c*x^n)+3*a))/(36*b^4*n^4-40*b^2*n^2+4)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.34

$$\int \sinh^3\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{3\,(b^3n^3-bn)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+9\,(b^3n^3-bn)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\right.}{}$$

[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")

[Out] 1/4*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - (b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^3 - 3*(9*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) + a) - 3*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (9*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*b^4*n^4 - 10*b^2*n^2 + 1)

Sympy [F]

$$\int \sinh^3(a + b\log(cx^n))\,dx$$

$$= \begin{cases} \int \sinh^3\left(a - \frac{\log(cx^n)}{n}\right) dx \\ \int \sinh^3\left(a - \frac{\log(cx^n)}{3n}\right) dx \\ \int \sinh^3\left(a + \frac{\log(cx^n)}{3n}\right) dx \\ \int \sinh^3\left(a + \frac{\log(cx^n)}{n}\right) dx \\ \frac{9b^3n^3x\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} - \frac{6b^3n^3x\cosh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} - \frac{7b^2n^2x\sinh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} + \frac{6b^2n^2x\sinh(a+b\log}{9b^4n} \end{cases}$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integral
(sinh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(sinh(a + l
og(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(sinh(a + log(c*x**n)/n
)**3, x), Eq(b, 1/n)), (9*b**3*n**3*x*sinh(a + b*log(c*x**n))**2*cosh(a + b
*log(c*x**n))/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 6*b**3*n**3*x*cosh(a + b*l
og(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) - 7*b**2*n**2*x*sinh(a + b*
log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sinh(a + b
*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*n**2 + 1) -
 3*b*n*x*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 -
10*b**2*n**2 + 1) + x*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n**
2 + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.77

$$\int \sinh^3(a + b\log(cx^n))\,dx = \frac{c^{3b}xe^{(3b\log(x^n)+3a)}}{8(3bn+1)} - \frac{3c^bxe^{(b\log(x^n)+a)}}{8(bn+1)} - \frac{3xe^{(-b\log(x^n)-a)}}{8(bc^bn-c^b)} + \frac{xe^{(-3b\log(x^n)-3a)}}{8(3bc^{3b}n-c^{3b})}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) - 3/8*c^b*x*e^(b*log(x^n)
+ a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) + 1/8*x*e^(-3*b*
log(x^n) - 3*a)/(3*b*c^(3*b)*n - c^(3*b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(150) = 300$.

Time = 0.30 (sec) , antiderivative size = 665, normalized size of antiderivative = 4.46

$$\int \sinh^3(a+b\log(cx^n))\,dx = \frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^3c^bn^3xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{27\,b^2c^bn^2xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{3\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^3n^3xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} + \frac{3\,b^3n^3xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} + \frac{3\,bc^bnxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^2n^2xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} + \frac{b^2n^2xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} - \frac{3\,c^bxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{3\,bnxe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} - \frac{3\,bnxe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} + \frac{3\,xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} - \frac{xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8
*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)*n
^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/8*b^2*c^b*n^2*x*x^
(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a
)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) + 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 +
1)*c^(3*b)*x^(3*b*n)) + 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 +
 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b
^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) + 1/8*b^2*n^2*x*
e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^(b*
n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n^4 - 10*b^2
*n^2 + 1)*c^b*x^(b*n)) - 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c
^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n))
 - 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \sinh^3(a+b\log(cx^n))\,dx = \frac{x\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(24\,b\,n-8\right)}-\frac{3\,x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(8\,b\,n-8\right)}+\frac{x\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{24\,b\,n+8}-\frac{3\,x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{8\,b\,n+8}$$

```
[In] int(sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] (x*exp(-3*a))/((c*x^n)^(3*b)*(24*b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b*
n - 8)) + (x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (3*x*exp(a)*(c*x^n)^b)/
(8*b*n + 8)
```

3.269 $\int \sinh^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 191

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4} - \frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} - \frac{4bnx\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{1-16b^2n^2} + \frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}$$

```
[Out] 24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-24*b^3*n^3*x*cosh(a+b*ln(c*x^n))*sin
h(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)+12*b^2*n^2*x*sinh(a+b*ln(c*x^n))
^2/(64*b^4*n^4-20*b^2*n^2+1)-4*b*n*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n)
)^3/(-16*b^2*n^2+1)+x*sinh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used

= {5630, 8}

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2} - \frac{4bnx\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-16b^2n^2} + \frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} - \frac{24b^3n^3x\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} + \frac{24b^4n^4x}{64b^4n^4-20b^2n^2+1}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (24*b^3*n^3*x*Cosh[a + b*Log
[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 20*b^2*n^2 + 64*b^4*n^4) + (12*b^2*n^
2*x*Sinh[a + b*Log[c*x^n]]^2)/(1 - 20*b^2*n^2 + 64*b^4*n^4) - (4*b*n*x*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(1 - 16*b^2*n^2) + (x*Sinh[a +
 b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5630

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Sinh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (-Dist[b
^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1)), Int[Sinh[d*(a + b*Log[c*x^n])
]^(p - 2), x], x] + Simp[b*d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(a +
b*Log[c*x^n])]^(p - 1)/(b^2*d^2*n^2*p^2 - 1)), x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{4bnx\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{1-16b^2n^2} + \frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2} + \frac{(12b^2n^2)\int\sinh^2(a+b\log(cx^n))\,dx}{1-16b^2n^2}$$

$$= -\frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} - \frac{4bnx\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{1-16b^2n^2} + \frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2} + \frac{(24b^4n^4)\int 1\,dx}{1-20b^2n^2+64b^4n^4}$$

$$= \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4} - \frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} + \frac{12b^2n^2x\sinh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4} - \frac{4bnx\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{1-16b^2n^2} + \frac{x\sinh^4(a+b\log(cx^n))}{1-16b^2n^2}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{x(3-60b^2n^2+192b^4n^4+(-4+64b^2n^2)\cosh(2(a+b\log(cx^n)))+(1-4b^2n^2)\cosh(4(a+b\log(cx^n)))+}{8}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (-4 + 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] + 8*b*n*Sinh[2*(a + b*Log
[c*x^n])] - 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*Lo
g[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2 + 6
4*b^4*n^4))
```

Maple [A] (verified)

Time = 14.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69

method	result
parallelrisch	$-\frac{128x\left(\left(-\frac{1}{8}b^3n^3+\frac{1}{32}bn\right)\sinh(4b\ln(cx^n)+4a)+\left(\frac{b^2n^2}{32}-\frac{1}{128}\right)\cosh(4b\ln(cx^n)+4a)+\left(bn-\frac{1}{4}\right)\left(-\frac{3b^2n^2}{2}+bn\sinh(2b\ln(cx^n)+2a\right.\right.}{512b^4n^4-160b^2n^2+8}$

```
[In] int(sinh(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -128*x*((-1/8*b^3*n^3+1/32*b*n)*sinh(4*b*ln(c*x^n)+4*a)+(1/32*b^2*n^2-1/128
)*cosh(4*b*ln(c*x^n)+4*a)+(b*n-1/4)*(-3/2*b^2*n^2+b*n*sinh(2*b*ln(c*x^n)+2*
a)-1/2*cosh(2*b*ln(c*x^n)+2*a)+3/8)*(b*n+1/4))/(512*b^4*n^4-160*b^2*n^2+8)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.54

$$\int \sinh^4(a+b\log(cx^n))\,dx = -\frac{(4\,b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^4-16\,(4\,b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)\sinh(}{}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] -1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3 -
 b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 +
 (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 - 4*(16*b^2*n^2 - 1)*x
*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*log(x)
 + b*log(c) + a)^2 - 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c) + a)^
2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^3 - (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*log(c) +
a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

Sympy [F]

$$\int \sinh^4(a+b\log(cx^n))\,dx = \begin{cases} \int \sinh^4\left(a-\frac{\log(cx^n)}{2n}\right)dx \\ \int \sinh^4\left(a-\frac{\log(cx^n)}{4n}\right)dx \\ \int \sinh^4\left(a+\frac{\log(cx^n)}{4n}\right)dx \\ \int \sinh^4\left(a+\frac{\log(cx^n)}{2n}\right)dx \\ \frac{24b^4n^4x\sinh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} - \frac{48b^4n^4x\sinh^2(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} + \frac{24b^4n^4x\cosh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} + \frac{40b^3n^3x\sinh^3(}{} \end{cases}$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Piecewise((Integral(sinh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))), (
Integral(sinh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(si
```

```
nh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(sinh(a + log(c
*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*log(c*x**
n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a + b*log(c*
x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 24
*b**4*n**4*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) +
 40*b**3*n**3*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64*b**4
*n**4 - 20*b**2*n**2 + 1) - 24*b**3*n**3*x*sinh(a + b*log(c*x**n))*cosh(a +
 b*log(c*x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 16*b**2*n**2*x*sinh(
a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**2*n**2*x*si
nh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2
*n**2 + 1) - 4*b*n*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64
*b**4*n**4 - 20*b**2*n**2 + 1) + x*sinh(a + b*log(c*x**n))**4/(64*b**4*n**4
 - 20*b**2*n**2 + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

$$\int \sinh^4\left(a+b\log\left(cx^n\right)\right)\,dx=\frac{c^{4\,b}xe^{\left(4\,b\log\left(x^n\right)+4\,a\right)}}{16\,\left(4\,bn+1\right)}-\frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+2\,a\right)}}{4\,\left(2\,bn+1\right)}+\frac{3}{8}\,x+\frac{xe^{\left(-2\,b\log\left(x^n\right)-2\,a\right)}}{4\,\left(2\,bc^{2\,b}n-c^{2\,b}\right)}-\frac{xe^{\left(-4\,a\right)}}{16\,\left(4\,bc^{4\,b}n-c^{4\,b}\right)\left(x^n\right)^{4\,b}}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) - 1/4*c^(2*b)*x*e^(2*b*lo
g(x^n) + 2*a)/(2*b*n + 1) + 3/8*x + 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^(2
*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(192) = 384.

Time = 0.32 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.07

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{b^3c^{4\,b}n^3xx^{4\,bn}e^{(4\,a)}}{64\,b^4n^4-20\,b^2n^2+1} - \frac{8\,b^3c^{2\,b}n^3xx^{2\,bn}e^{(2\,a)}}{64\,b^4n^4-20\,b^2n^2+1} + \frac{24\,b^4n^4x}{64\,b^4n^4-20\,b^2n^2+1} - \frac{b^2c^{4\,b}n^2xx^{4\,bn}e^{(4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)} + \frac{4\,b^2c^{2\,b}n^2xx^{2\,bn}e^{(2\,a)}}{64\,b^4n^4-20\,b^2n^2+1} - \frac{bc^{4\,b}nxx^{4\,bn}e^{(4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)} + \frac{bc^{2\,b}nxx^{2\,bn}e^{(2\,a)}}{2\,(64\,b^4n^4-20\,b^2n^2+1)} + \frac{8\,b^3n^3xe^{(-2\,a)}}{(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}} - \frac{b^3n^3xe^{(-4\,a)}}{(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}} - \frac{15\,b^2n^2x}{2\,(64\,b^4n^4-20\,b^2n^2+1)} + \frac{c^{4\,b}xx^{4\,bn}e^{(4\,a)}}{16\,(64\,b^4n^4-20\,b^2n^2+1)} - \frac{c^{2\,b}xx^{2\,bn}e^{(2\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)} + \frac{4\,b^2n^2xe^{(-2\,a)}}{(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}} - \frac{b^2n^2xe^{(-4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}} - \frac{bnxe^{(-2\,a)}}{2\,(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}} + \frac{bnxe^{(-4\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}} + \frac{3\,x}{8\,(64\,b^4n^4-20\,b^2n^2+1)} - \frac{xe^{(-2\,a)}}{4\,(64\,b^4n^4-20\,b^2n^2+1)c^{2\,b}x^{2\,bn}} + \frac{xe^{(-4\,a)}}{16\,(64\,b^4n^4-20\,b^2n^2+1)c^{4\,b}x^{4\,bn}}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 8*b^3*c
^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4*x
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)/(6
4*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64*b^4
*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 -
20*b^2*n^2 + 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*
n^2 + 1) + 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2
*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n
)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x*x^(4*b*n
)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(
```

```
64*b^4*n^4 - 20*b^2*n^2 + 1) + 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n
^2 + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 +
 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*
c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*x*e^(-2*a)/(
(64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e^(-4*a)/((64*b^4
*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \sinh^4(a+b\log(cx^n))\,dx = \frac{3\,x}{8}+\frac{x\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(8\,b\,n-4\right)}-\frac{x\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{8\,b\,n+4}-\frac{x\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(64\,b\,n-16\right)}+\frac{x\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{64\,b\,n+16}$$

```
[In] int(sinh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x)/8 + (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) - (x*exp(2*a)*(c*x^n)^(
2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(4*
a)*(c*x^n)^(4*b))/(64*b*n + 16)
```

3.270 $\int x^m \sinh(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int x^m \sinh(a+b\log(cx^n))\,dx = -\frac{bnx^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2} + \frac{(1+m)x^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

```
[Out] -b*n*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*l
n(c*x^n))/((1+m)^2-b^2*n^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5638}

$$\int x^m \sinh(a+b\log(cx^n))\,dx = \frac{(m+1)x^{m+1}\sinh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)} - \frac{bnx^{m+1}\cosh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}$$

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]],x]
```

```
[Out] -((b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)) + ((1 + m)*
x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m - b*n)*(1 + m + b*n))
```

Rule 5638

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Cosh[d*(a + b*Log
```

```
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-b^2n^2}+\frac{(1+m)x^{1+m}\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m-bn)(1+m+bn)}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^m\sinh\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x^{1+m}\left(-bn\cosh\left(a+b\log\left(cx^n\right)\right)+(1+m)\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{(1+m-bn)(1+m+bn)}$$

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]],x]
[Out] (x^(1 + m)*(-(b*n*Cosh[a + b*Log[c*x^n]]) + (1 + m)*Sinh[a + b*Log[c*x^n]])
)/((1 + m - b*n)*(1 + m + b*n))
```

Maple [F]

$$\int x^m\sinh\left(a+b\ln\left(c\,x^n\right)\right)dx$$

```
[In] int(x^m*sinh(a+b*ln(c*x^n)),x)
[Out] int(x^m*sinh(a+b*ln(c*x^n)),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.34

$$\int x^m\sinh\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\cosh\left(m\log\left(x\right)\right)+bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(m\log\left(x\right)\right)-}{b^2n^2-m^2-2\,m-}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] (b*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + b*n*x*cosh(b*n*log(
x) + b*log(c) + a)*sinh(m*log(x)) - ((m + 1)*x*cosh(m*log(x)) + (m + 1)*x*s
inh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m - 1)
```

Sympy [F]

$$\int x^m \sinh(a+b\log(cx^n))\,dx$$
$$= \begin{cases} \log(x)\sinh(a) & \text{for } b=0 \wedge m=-1 \\ -\int x^m \sinh\left(-a+\frac{m\log(cx^n)}{n}+\frac{\log(cx^n)}{n}\right)dx & \text{for } b=-\frac{m+1}{n} \\ \int x^m \sinh\left(a+\frac{m\log(cx^n)}{n}+\frac{\log(cx^n)}{n}\right)dx & \text{for } b=\frac{m+1}{n} \\ \frac{bnxx^m\cosh(a+b\log(cx^n))}{b^2n^2-m^2-2m-1}-\frac{mxx^m\sinh(a+b\log(cx^n))}{b^2n^2-m^2-2m-1}-\frac{xx^m\sinh(a+b\log(cx^n))}{b^2n^2-m^2-2m-1} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n)),x)
```

```
[Out] Piecewise((log(x)*sinh(a), Eq(b, 0) & Eq(m, -1)), (-Integral(x**m*sinh(-a +
 m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, -(m + 1)/n)), (Integral(x**m*s
inh(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*x**
m*cosh(a + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*sinh(a +
b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*sinh(a + b*log(c*x**n)
)/(b**2*n**2 - m**2 - 2*m - 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int x^m \sinh(a+b\log(cx^n))\,dx = \frac{c^b x e^{(b\log(x^n)+m\log(x)+a)}}{2(bn+m+1)}+\frac{x e^{(-b\log(x^n)+m\log(x)-a)}}{2(bc^bn-c^b(m+1))}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="maxima")
```

```
[Out] 1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) + 1/2*x*e^(-b*log(x^n
) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(75) = 150$.

Time = 0.29 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.22

$$\int x^m \sinh(a+b\log(cx^n))\,dx = \frac{bc^b nxx^{bn}x^m e^a}{2(b^2n^2-m^2-2m-1)} - \frac{c^b mxx^{bn}x^m e^a}{2(b^2n^2-m^2-2m-1)} - \frac{c^b xx^{bn}x^m e^a}{2(b^2n^2-m^2-2m-1)} + \frac{bnxx^m e^{(-a)}}{2(b^2n^2-m^2-2m-1)c^b x^{bn}} + \frac{mxx^m e^{(-a)}}{2(b^2n^2-m^2-2m-1)c^b x^{bn}} + \frac{xx^m e^{(-a)}}{2(b^2n^2-m^2-2m-1)c^b x^{bn}}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n)),x, algorithm="giac")
```

```
[Out] 1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b*
n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2 -
 m^2 - 2*m - 1) + 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*
n)) + 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) + 1/2*x*x^
m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int x^m \sinh(a+b\log(cx^n))\,dx = \frac{x\,x^m\,\mathrm{e}^a\,(c\,x^n)^b}{2\,m+2\,b\,n+2} - \frac{x\,x^m\,\mathrm{e}^{-a}}{(c\,x^n)^b\,(2\,m-2\,b\,n+2)}$$

```
[In] int(x^m*sinh(a + b*log(c*x^n)),x)
```

```
[Out] (x*x^m*exp(a)*(c*x^n)^b)/(2*m + 2*b*n + 2) - (x*x^m*exp(-a))/((c*x^n)^b*(2*
m - 2*b*n + 2))
```

3.271 $\int x^m \sinh^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx = \frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} - \frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} + \frac{(1+m)x^{1+m}\sinh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}$$

```
[Out] 2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^n
))*sinh(a+b*ln(c*x^n))/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n)
)^2/((1+m)^2-4*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5640, 30}

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx = \frac{(m+1)x^{m+1}\sinh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1} - \frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2} + \frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) - (2*b*n*x^(1 + m)*
Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2) + ((
1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/(1 + 2*m + m^2 - 4*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5640

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (-Dist[b^2*d^2*n^2*p*((p - 1)/(b
^2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2
), x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(
a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-4b^2n^2} \\ &\quad+\frac{(1+m)x^{1+m}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{1+2m+m^2-4b^2n^2}+\frac{\left(2b^2n^2\right)\int x^m\,dx}{(1+m)^2-4b^2n^2} \\ &= \frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)}-\frac{2bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-4b^2n^2} \\ &\quad+\frac{(1+m)x^{1+m}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{1+2m+m^2-4b^2n^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int x^m\sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{x^{1+m}\left(-1-2m-m^2+4b^2n^2+(1+m)^2\cosh\left(2\left(a+b\log\left(cx^n\right)\right)\right)-2b(1+m)n\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)\right)}{2(1+m)(1+m-2bn)(1+m+2bn)}$$

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x^(1 + m)*(-1 - 2*m - m^2 + 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n]
)] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)*
(1 + m + 2*b*n))
```

Maple [F]

$$\int x^{m}\sinh \left(a+b\ln \left(c\,x^{n}\right)\right)^{2}dx$$

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^2,x)
[Out] int(x^m*sinh(a+b*ln(c*x^n))^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.07

$$\int x^m \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{(m^2+2\,m+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2\cosh\left(m\log\left(x\right)\right)+(4\,b^2n^2-m^2-2\,m-1)x\cosh\left(m\log\right.}{}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] 1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (
4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*log
(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2
- 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*m +
b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*
log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (4*b^2
*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2 + 3*m^2
 + 3*m + 1)
```

Sympy [F]

$$\int x^m \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\begin{cases}\log\left(x\right)\sinh^2\left(a\right)\\ \int x^m\sinh^2\left(-a+\frac{m\log\left(cx^n\right)}{2n}+\frac{\log\left(cx^n\right)}{2n}\right)dx\\ \int x^m\sinh^2\left(a+\frac{m\log\left(cx^n\right)}{2n}+\frac{\log\left(cx^n\right)}{2n}\right)dx\\ \int\frac{\sinh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx\\ \frac{2b^2n^2xx^m\sinh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}-\frac{2b^2n^2xx^m\cosh^2\left(a+b\log\left(cx^n\right)\right)}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}+\frac{2bmnxx^m\sinh\left(a+b\log\left(cx^n\right)\right)\cosh\left(a+b\log\left(cx^n\right)\right)}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1}+\frac{2bnxx}{}\end{cases}$$

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**2,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*sinh(-a
 + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq(b, -(m + 1)/(2*n))),
 (Integral(x**m*sinh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), E
q(b, (m + 1)/(2*n))), (Integral(sinh(a + b*log(c*x**n))**2/x, x), Eq(m, -1)
), (2*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n
**2 - m**3 - 3*m**2 - 3*m - 1) - 2*b**2*n**2*x*x**m*cosh(a + b*log(c*x**n))
**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*m*n*x*x**
m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n
**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x**m*sinh(a + b*log(c*x**n))*cosh(
a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1)
- m**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**
3 - 3*m**2 - 3*m - 1) - 2*m*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**
2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*sinh(a + b*log(c*x**n))
**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int x^m \sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx = \frac{c^{2\,b}xe^{\left(2\,b\log\left(x^n\right)+m\log\left(x\right)+2\,a\right)}}{4\,\left(2\,bn+m+1\right)} - \frac{xe^{\left(-2\,b\log\left(x^n\right)+m\log\left(x\right)-2\,a\right)}}{4\,\left(2\,bc^{2\,b}n-c^{2\,b}(m+1)\right)} - \frac{x^{m+1}}{2\,\left(m+1\right)}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(
-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/2*x^(
m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 758 vs. $2(127) = 254$.

Time = 0.29 (sec) , antiderivative size = 758, normalized size of antiderivative = 6.32

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx = \frac{bc^{2b}mnxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{c^{2b}m^2xx^{2bn}x^me^{(2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} + \frac{bc^{2b}nxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{2b^2n^2xx^m}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{c^{2b}mxx^{2bn}x^me^{(2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{c^{2b}xx^{2bn}x^me^{(2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} + \frac{m^2xx^m}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{bmnxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} + \frac{mxx^m}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} - \frac{m^2xx^me^{(-2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} - \frac{bnxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} + \frac{xx^m}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)} - \frac{mxx^me^{(-2a)}}{2(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}} - \frac{xx^me^{(-2a)}}{4(4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1)c^{2b}x^{2bn}}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4
*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)
/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 2*b^2*n^2*x*x^m/(4*b^2
*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n)*x^m
*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*x*
```

```
x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1
/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*b*m*n*
x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x
^(2*b*n)) + m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4
*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2
*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3
*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) + 1/2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m
^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^
3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)/((4*b^2*m*n^2
+ 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.62

$$\int x^m \sinh^2(a+b\log(cx^n))\,dx=\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}-\frac{x\,x^m}{2\,m+2}+\frac{x\,x^m\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}$$

```
[In] int(x^m*sinh(a + b*log(c*x^n))^2,x)
```

```
[Out] (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) - (x*x^m)/(2*m + 2) + (
x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)
```

3.272 $\int x^m \sinh^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 203

$$\begin{aligned}\int x^m \sinh^3(a+b\log(cx^n))\,dx = & -\frac{6b^3n^3x^{1+m}\cosh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)} \\ & + \frac{6b^2(1+m)n^2x^{1+m}\sinh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)((1+m)^2-b^2n^2)} \\ & - \frac{3bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} \\ & + \frac{(1+m)x^{1+m}\sinh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2}\end{aligned}$$

```
[Out] -6*b^3*n^3*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^2
)+6*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-
b^2*n^2)-3*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^2/((1+m)^2-9
*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used

$= \{5640, 5638\}$

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx = \frac{(m+1)x^{m+1}\sinh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1} + \frac{6b^2(m+1)n^2x^{m+1}\sinh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)((m+1)^2-9b^2n^2)} - \frac{3bnx^{m+1}\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-9b^2n^2} - \frac{6b^3n^3x^{m+1}\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2(m+1)^2n^2+(m+1)^4}$$

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (-6*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m)^4 - 10*b^2*(1 + m)^2
*n^2 + 9*b^4*n^4) + (6*b^2*(1 + m)*n^2*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((
1 + m - b*n)*(1 + m + b*n)*((1 + m)^2 - 9*b^2*n^2)) - (3*b*n*x^(1 + m)*Cosh
[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^2)/((1 + m)^2 - 9*b^2*n^2) + ((1
+ m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]^3)/(1 + 2*m + m^2 - 9*b^2*n^2)
```

Rule 5638

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Cosh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Rule 5640

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (-Dist[b^2*d^2*n^2*p*((p - 1)/(b
^2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2
), x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(
a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Rubi steps

$$\text{integral} = -\frac{3bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} + \frac{(1+m)x^{1+m}\sinh^3(a+b\log(cx^n))}{1+2m+m^2-9b^2n^2} + \frac{(6b^2n^2)\int x^m\sinh(a+b\log(cx^n))\,dx}{(1+m)^2-9b^2n^2}$$

$$= -\frac{6b^3n^3x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^4-10b^2(1+m)^2n^2+9b^4n^4} + \frac{6b^2(1+m)n^2x^{1+m}\sinh(a+b\log(cx^n))}{(1+m-bn)(1+m+bn)((1+m)^2-9b^2n^2)} - \frac{3bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^2(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} + \frac{(1+m)x^{1+m}\sinh^3(a+b\log(cx^n))}{1+2m+m^2-9b^2n^2}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.44

$$\int x^m\sinh^3(a+b\log(cx^n))\,dx$$
$$= \frac{1}{4}x^{1+m}\left(-\frac{3\cosh(bn\log(x))(-bn\cosh(a-bn\log(x)+b\log(cx^n))+(1+m)\sinh(a-bn\log(x)+b\log(cx^n)))}{(1+m-bn)(1+m+bn)} - \frac{3\sinh(bn\log(x))((1+m)\cosh(a-bn\log(x)+b\log(cx^n))-bn\sinh(a-bn\log(x)+b\log(cx^n)))}{(1+m-bn)(1+m+bn)} + \frac{\cosh(3bn\log(x))(-3bn\cosh(3(a-bn\log(x)+b\log(cx^n)))+(1+m)\sinh(3(a-bn\log(x)+b\log(cx^n))))}{(1+m-3bn)(1+m+3bn)} + \frac{\sinh(3bn\log(x))((1+m)\cosh(3(a-bn\log(x)+b\log(cx^n)))-3bn\sinh(3(a-bn\log(x)+b\log(cx^n))))}{(1+m-3bn)(1+m+3bn)}\right)$$

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x^(1 + m)*((-3*Cosh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]]
) + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b
*n)) - (3*Sinh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] - b
*n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) + (C
osh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)
*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3*b*n)
) + (Sinh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] -
3*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3
*b*n))))/4
```

Maple [F]

$$\int x^m \sinh\left(a+b\ln\left(c\,x^n\right)\right)^3 dx$$

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
[Out] int(x^m*sinh(a+b*ln(c*x^n))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(214) = 428$.

Time = 0.27 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.88

$$\int x^m \sinh^3\left(a+b\log\left(cx^n\right)\right) dx$$
$$= \frac{3\left(b^3n^3-(bm^2+2\,bm+b)n\right)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3\cosh\left(m\log\left(x\right)\right)-3\left(9\,b^3n^3-(bm^2+2\,bm\right.}{}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="fricas")
[Out] 1/4*(3*(b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^
3*cosh(m*log(x)) - 3*(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x)
+ b*log(c) + a)*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m +
1)*x*cosh(m*log(x)) + (m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*sinh(m*
log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 9*((b^3*n^3 - (b*m^2 + 2*b*m +
 b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n
*log(x) + b*log(c) + a)^2 + 3*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*
x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) - (m^3 - 9*(b^2*m + b^2)
*n^2 + 3*m^2 + 3*m + 1)*x*cosh(m*log(x)) + ((m^3 - (b^2*m + b^2)*n^2 + 3*m^
2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 - (m^3 - 9*(b^2*m + b^2)*n
^2 + 3*m^2 + 3*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) +
3*((b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 -
(9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a))*sinh
(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6
*m^2 + 4*m + 1)
```

Sympy [F]

$$\int x^m \sinh^3\left(a+b\log\left(cx^n\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**3,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**3, Eq(b, 0) & Eq(m, -1)), (-Integral(x**m*sinh(-
a + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*n))**3, x), Eq(b, (-m - 1)/(3*n)))
, (-Integral(x**m*sinh(-a + m*log(c*x**n)/n + log(c*x**n)/n)**3, x), Eq(b,
(-m - 1)/n)), (Integral(x**m*sinh(a + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*
n))**3, x), Eq(b, (m + 1)/(3*n))), (Integral(x**m*sinh(a + m*log(c*x**n)/n
+ log(c*x**n)/n)**3, x), Eq(b, (m + 1)/n)), (9*b**3*n**3*x*x**m*sinh(a + b*
log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*m**2*n**2 -
20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 6*b**3*
n**3*x*x**m*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 2
0*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 7*b**2*m
*n**2*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 -
20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 6*b**2*
m*n**2*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n*
*4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*
m**2 + 4*m + 1) - 7*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**
4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m
**2 + 4*m + 1) + 6*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(
c*x**n))**2/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**
2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 3*b*m**2*n*x*x**m*sinh(a + b*log(c*
x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**
2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 6*b*m*n*x*x**
m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2
*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m +
 1) - 3*b*n*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b*
*4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3
 + 6*m**2 + 4*m + 1) + m**3*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4
- 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**
2 + 4*m + 1) + 3*m**2*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b
**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*
m + 1) + 3*m*x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*
n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) +
x*x**m*sinh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**
2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx = \frac{c^{3\,b}xe^{(3\,b\log(x^n)+m\log(x)+3\,a)}}{8\,(3\,bn+m+1)} - \frac{3\,c^b xe^{(b\log(x^n)+m\log(x)+a)}}{8\,(bn+m+1)} - \frac{3\,xe^{(-b\log(x^n)+m\log(x)-a)}}{8\,(bc^bn-c^b(m+1))} + \frac{xe^{(-3\,b\log(x^n)+m\log(x)-3\,a)}}{8\,(3\,bc^{3\,b}n-c^{3\,b}(m+1))}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) - 3/8*c^b*x
*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*log
(x) - a)/(b*c^b*n - c^b*(m + 1)) + 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3*a)
/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. 2(214) = 428.

Time = 0.35 (sec) , antiderivative size = 3225, normalized size of antiderivative = 15.89

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

```
[Out] 3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^3*c^b*n^
3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x^m*e
^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^
3 + 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10
*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) -
 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b
)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*m^2*n*x*x^(b*n)*x^m
*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1) + 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2
*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1/8
*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^(3*b)*m*n*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
```

```
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*c^b*m^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
+ 1) + 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2
*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^(3*b)*m^2*x*x^
(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*x^m*e^(3*a)
/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*
m^2 + 4*m + 1) - 27/8*b^3*n^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 3/
8*b^3*n^3*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m^2*x*
x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n
^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*c^b*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 -
10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)
 + 3/8*c^(3*b)*m*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b
^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*m*n^2*x*x
^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 +
4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) + 1/8*b^2*m*n^2*x*x^m*e^(-3*a)/((9*b^
4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 +
4*m + 1)*c^(3*b)*x^(3*b*n)) - 9/8*c^b*m*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 1
/8*c^(3*b)*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n
^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*b*m^2*n*x*x^m*e^(-a)
/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6
*m^2 + 4*m + 1)*c^b*x^(b*n)) - 27/8*b^2*n^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b
^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b
*x^(b*n)) - 3/8*b*m^2*n*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^
2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) +
1/8*b^2*n^2*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^
4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*c^b*x*x^
(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2
 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*m^3*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2
*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*
n)) + 3/4*b*m*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 +
m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 1/8*m^3*x*x^m*e^
(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m
^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/4*b*m*n*x*x^m*e^(-3*a)/((9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4
*m + 1)*c^(3*b)*x^(3*b*n)) + 9/8*m^2*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*
n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n
)) + 3/8*b*n*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4
 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/8*m^2*x*x^m*e^(-3
*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3
+ 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) - 3/8*b*n*x*x^m*e^(-3*a)/((9*b^4*n^4
- 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m +
```

```
1)*c^(3*b)*x^(3*b*n)) + 9/8*m*x*x^m*e^(-a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 2
0*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^b*x^(b*n)) - 3/
8*m*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b
^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*x^m*e^(-a)/((9
*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2
 + 4*m + 1)*c^b*x^(b*n)) - 1/8*x*x^m*e^(-3*a)/((9*b^4*n^4 - 10*b^2*m^2*n^2
- 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1)*c^(3*b)*x^(3*b
*n))
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.58

$$\int x^m \sinh^3(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m\,\mathrm{e}^{-a}}{\left(c\,x^n\right)^b\,\left(8\,m-8\,b\,n+8\right)}-\frac{x\,x^m\,\mathrm{e}^{-3\,a}}{\left(c\,x^n\right)^{3\,b}\,\left(8\,m-24\,b\,n+8\right)}+\frac{x\,x^m\,\mathrm{e}^{3\,a}\,\left(c\,x^n\right)^{3\,b}}{8\,m+24\,b\,n+8}-\frac{3\,x\,x^m\,\mathrm{e}^{a}\,\left(c\,x^n\right)^b}{8\,m+8\,b\,n+8}$$

```
[In] int(x^m*sinh(a + b*log(c*x^n))^3,x)
```

```
[Out] (3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) - (x*x^m*exp(-3*a))/((c*x^n
)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*n
+ 8) - (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

3.273 $\int x^m \sinh^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 266

$$\begin{aligned}\int x^m \sinh^4(a+b\log(cx^n))\,dx &= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad - \frac{24b^3n^3x^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad + \frac{12b^2(1+m)n^2x^{1+m}\sinh^2(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad - \frac{4bnx^{1+m}\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} \\ &\quad + \frac{(1+m)x^{1+m}\sinh^4(a+b\log(cx^n))}{(1+m)^2-16b^2n^2}\end{aligned}$$

```
[Out] 24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-24*b^3*n^
3*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((1+
m)^2-4*b^2*n^2)+12*b^2*(1+m)*n^2*x^(1+m)*sinh(a+b*ln(c*x^n))^2/((1+m)^2-16*
b^2*n^2)/((1+m)^2-4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(
c*x^n))^3/((1+m)^2-16*b^2*n^2)+(1+m)*x^(1+m)*sinh(a+b*ln(c*x^n))^4/((1+m)^2
-16*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5640, 30}

$$\int x^m \sinh^4(a+b\log(cx^n))\, dx = \frac{(m+1)x^{m+1}\sinh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1} + \frac{12b^2(m+1)n^2x^{m+1}\sinh^2(a+b\log(cx^n))}{((m+1)^2-16b^2n^2)(-4b^2n^2+m^2+2m+1)} - \frac{4bnx^{m+1}\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-16b^2n^2} - \frac{24b^3n^3x^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{64b^4n^4-20b^2(m+1)^2n^2+(m+1)^4} + \frac{24b^4n^4x^{m+1}}{(m+1)((m+1)^2-16b^2n^2)((m+1)^2-4b^2n^2)}$$

```
[In] Int[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (24*b^4*n^4*x^(1 + m))/((1 + m)*((1 + m)^2 - 16*b^2*n^2)*((1 + m)^2 - 4*b^2
*n^2)) - (24*b^3*n^3*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]
])/((1 + m)^4 - 20*b^2*(1 + m)^2*n^2 + 64*b^4*n^4) + (12*b^2*(1 + m)*n^2*x^
(1 + m)*Sinh[a + b*Log[c*x^n]]^2)/(((1 + m)^2 - 16*b^2*n^2)*(1 + 2*m + m^2
- 4*b^2*n^2)) - (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^
n]]^3)/((1 + m)^2 - 16*b^2*n^2) + ((1 + m)*x^(1 + m)*Sinh[a + b*Log[c*x^n]]
^4)/(1 + 2*m + m^2 - 16*b^2*n^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5640

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Sinh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (-Dist[b^2*d^2*n^2*p*((p - 1)/(b
^2*d^2*n^2*p^2 - (m + 1)^2)), Int[(e*x)^m*Sinh[d*(a + b*Log[c*x^n])]^(p - 2
), x], x] + Simp[b*d*n*p*(e*x)^(m + 1)*Cosh[d*(a + b*Log[c*x^n])]*(Sinh[d*(
a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{4bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-16b^2n^2} \\
&\quad+\frac{(1+m)x^{1+m}\sinh^4\left(a+b\log\left(cx^n\right)\right)}{1+2m+m^2-16b^2n^2}+\frac{\left(12b^2n^2\right)\int x^m\sinh^2\left(a+b\log\left(cx^n\right)\right)\,dx}{(1+m)^2-16b^2n^2} \\
&= -\frac{24b^3n^3x^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} \\
&\quad+\frac{12b^2(1+m)n^2x^{1+m}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{\left((1+m)^2-16b^2n^2\right)\left(1+2m+m^2-4b^2n^2\right)} \\
&\quad-\frac{4bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-16b^2n^2} \\
&\quad+\frac{(1+m)x^{1+m}\sinh^4\left(a+b\log\left(cx^n\right)\right)}{1+2m+m^2-16b^2n^2}+\frac{\left(24b^4n^4\right)\int x^m\,dx}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} \\
&= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^4-20b^2(1+m)^2n^2+64b^4n^4\right)} \\
&\quad-\frac{24b^3n^3x^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{(1+m)^4-20b^2(1+m)^2n^2+64b^4n^4} \\
&\quad+\frac{12b^2(1+m)n^2x^{1+m}\sinh^2\left(a+b\log\left(cx^n\right)\right)}{\left((1+m)^2-16b^2n^2\right)\left(1+2m+m^2-4b^2n^2\right)} \\
&\quad-\frac{4bnx^{1+m}\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^3\left(a+b\log\left(cx^n\right)\right)}{(1+m)^2-16b^2n^2} \\
&\quad+\frac{(1+m)x^{1+m}\sinh^4\left(a+b\log\left(cx^n\right)\right)}{1+2m+m^2-16b^2n^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.17

$$
\begin{aligned}
\int x^m\sinh^4\left(a+b\log\left(cx^n\right)\right)\,dx &= \frac{1}{8}x^{1+m}\left(\frac{3}{1+m}\right. \\
&\quad-\frac{4\sinh(2bn\log(x))\left(-2bn\cosh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sinh\left(2(a-bn\log(x)+b\log\left(c\right.\right.\right.}{(1+m-2bn)(1+m+2bn)} \\
&\quad-\frac{4\cosh(2bn\log(x))\left((1+m)\cosh\left(2(a-bn\log(x)+b\log\left(cx^n\right))\right)-2bn\sinh\left(2(a-bn\log(x)+b\log\left(cx\right.\right.\right.}{(1+m-2bn)(1+m+2bn)} \\
&\quad+\frac{\sinh(4bn\log(x))\left(-4bn\cosh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)+(1+m)\sinh\left(4(a-bn\log(x)+b\log\left(cx\right.\right.\right.}{(1+m-4bn)(1+m+4bn)} \\
&\quad+\frac{\cosh(4bn\log(x))\left((1+m)\cosh\left(4(a-bn\log(x)+b\log\left(cx^n\right))\right)-4bn\sinh\left(4(a-bn\log(x)+b\log\left(cx^n\right.\right.\right.}{(1+m-4bn)(1+m+4bn)}
\end{aligned}
$$

```
[In] Integrate[x^m*Sinh[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (x^(1 + m)*(3/(1 + m) - (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[x
] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1
+ m - 2*b*n)*(1 + m + 2*b*n)) - (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a -
b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]
))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cosh[4*(
a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*Log[c*
x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1 + m)*C
osh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[x] + b*L
og[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

Maple [F]

$$\int x^m \sinh\left(a+b\ln\left(c\,x^n\right)\right)^4 dx$$

```
[In] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

```
[Out] int(x^m*sinh(a+b*ln(c*x^n))^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(283) = 566$.

Time = 0.32 (sec) , antiderivative size = 1125, normalized size of antiderivative = 4.23

$$\int x^m \sinh^4\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

```
[Out] 1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*co
sh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) - 4*(m^4 + 4*m^3 - 16*(b^2*m
^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a
)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^
2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*
n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^4
+ 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*lo
g(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m
^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b
*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^4 + 4*m^3 -
 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2*cosh(m*log(x)) - 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)
*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 - 4*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 -
2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sin
```

```
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b^3*m + b^3)*n^3 -
 (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3*cosh(
m*log(x)) - (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^3 - (16*(b^3*m
 + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^4 + 4*m^3 - 4*
(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(
c) + a)^4 - 4*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m
 + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 2
0*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x)*sinh(m*log(x)))/(m^5
+ 64*(b^4*m + b^4)*n^4 + 5*m^4 + 10*m^3 - 20*(b^2*m^3 + 3*b^2*m^2 + 3*b^2*m
 + b^2)*n^2 + 10*m^2 + 5*m + 1)
```

Sympy [F]

$$\int x^m \sinh^4\left(a + b\log\left(cx^n\right)\right) dx = \text{Too large to display}$$

```
[In] integrate(x**m*sinh(a+b*ln(c*x**n))**4,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**4, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*sinh(-a
 + m*log(c*x**n)/(4*n) + log(c*x**n)/(4*n))**4, x), Eq(b, (-m - 1)/(4*n))),
 (Integral(x**m*sinh(-a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**4, x),
Eq(b, (-m - 1)/(2*n))), (Integral(x**m*sinh(a + m*log(c*x**n)/(4*n) + log(c
*x**n)/(4*n))**4, x), Eq(b, (m + 1)/(4*n))), (Integral(x**m*sinh(a + m*log(
c*x**n)/(2*n) + log(c*x**n)/(2*n))**4, x), Eq(b, (m + 1)/(2*n))), (Integral
(sinh(a + b*log(c*x**n))**4/x, x), Eq(m, -1)), (24*b**4*n**4*x*x**m*sinh(a
+ b*log(c*x**n))**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60
*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 +
 10*m**2 + 5*m + 1) - 48*b**4*n**4*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a
 + b*log(c*x**n))**2/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 6
0*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3
+ 10*m**2 + 5*m + 1) + 24*b**4*n**4*x*x**m*cosh(a + b*log(c*x**n))**4/(64*b
**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2
*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) + 40*
b**3*m*n**3*x*x**m*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64*b
**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2
*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) - 24*
b**3*m*n**3*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**3/(64*b
**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2
*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) + 40*
b**3*n**3*x*x**m*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64*b**
4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m
```

```
*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) - 24*b*
*3*n**3*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**3/(64*b**4*
m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n
**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) - 16*b**2
*m**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**4/(64*b**4*m*n**4 + 64*b**4*n**4
 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 +
m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) + 12*b**2*m**2*n**2*x*x**m*sin
h(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*m*n**4 + 64*b**
4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n
**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) - 32*b**2*m*n**2*x*x**m*
sinh(a + b*log(c*x**n))**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n*
*2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10
*m**3 + 10*m**2 + 5*m + 1) + 24*b**2*m*n**2*x*x**m*sinh(a + b*log(c*x**n))*
*2*cosh(a + b*log(c*x**n))**2/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3
*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 +
 10*m**3 + 10*m**2 + 5*m + 1) - 16*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))
**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2
- 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m +
 1) + 12*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n)
)**2/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2
 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m
+ 1) - 4*b*m**3*n*x*x**m*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))
/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 6
0*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1)
 - 12*b*m**2*n*x*x**m*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(6
4*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b
**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) -
12*b*m*n*x*x**m*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64*b**4
*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*
n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) - 4*b*n*
x*x**m*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/(64*b**4*m*n**4 +
 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20
*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) + m**4*x*x**m*sin
h(a + b*log(c*x**n))**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2
- 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m*
*3 + 10*m**2 + 5*m + 1) + 4*m**3*x*x**m*sinh(a + b*log(c*x**n))**4/(64*b**4
*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*
n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1) + 6*m**2
*x*x**m*sinh(a + b*log(c*x**n))**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2
*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m
**4 + 10*m**3 + 10*m**2 + 5*m + 1) + 4*m*x*x**m*sinh(a + b*log(c*x**n))**4/
(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**2*m**3*n**2 - 60*b**2*m**2*n**2 - 60
*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*m**4 + 10*m**3 + 10*m**2 + 5*m + 1)
+ x*x**m*sinh(a + b*log(c*x**n))**4/(64*b**4*m*n**4 + 64*b**4*n**4 - 20*b**
2*m**3*n**2 - 60*b**2*m**2*n**2 - 60*b**2*m*n**2 - 20*b**2*n**2 + m**5 + 5*
```

```
m**4 + 10*m**3 + 10*m**2 + 5*m + 1), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.61

$$\int x^m \sinh^4(a + b\log(cx^n))\,dx = \frac{c^{4b}xe^{(4b\log(x^n)+m\log(x)+4a)}}{16(4bn+m+1)} - \frac{c^{2b}xe^{(2b\log(x^n)+m\log(x)+2a)}}{4(2bn+m+1)} + \frac{xe^{(-2b\log(x^n)+m\log(x)-2a)}}{4(2bc^{2b}n - c^{2b}(m+1))} - \frac{xe^{(-4b\log(x^n)+m\log(x)-4a)}}{16(4bc^{4b}n - c^{4b}(m+1))} + \frac{3x^{m+1}}{8(m+1)}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

```
[Out] 1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) - 1/4*c^(2
*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) + 1/4*x*e^(-2*b*log
(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4*b*
log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m + 1
)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6884 vs. 2(283) = 566.

Time = 0.42 (sec) , antiderivative size = 6884, normalized size of antiderivative = 25.88

$$\int x^m \sinh^4(a + b\log(cx^n))\,dx = \text{Too large to display}$$

```
[In] integrate(x^m*sinh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

```
[Out] b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b
^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*
m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b
^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^
2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*m^2*n
^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n
^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*
n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 8*b^3*c^(2
```

```
*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2
 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m
^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*
m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4
+ 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(4*b)*m*n^2*x*x^(4*b
*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^
2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) +
1/2*b*c^(2*b)*m^3*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20
*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) + 8*b^2*c^(2*b)*m*n^2*x*x^(2*b*n)*x^m*e^(2*a)/(64
*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*
n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*m^4*x*
x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m +
 1) - 3/4*b*c^(4*b)*m^2*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^
4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n
^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*c^(2*b)*m
^4*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60
*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 +
5*m + 1) + 3/2*b*c^(2*b)*m^2*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b
^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*
b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*x^m*e^
(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 -
60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*m
^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 +
 m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4
*c^(4*b)*m^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^
3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 +
 10*m^2 + 5*m + 1) - 3/4*b*c^(4*b)*m*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^
4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m
^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - c^(2*b)*m^3*x*x^(2*b*n)*x^m*
e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5
 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/2*b*c^
(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n
^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) + 8*b^3*m*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 -
20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - b^3*m*n^3*x*x^m*e^(-4*a)/(
(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2
*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n))
 - 15*b^2*m*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*
```

m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 3/8*c^(4*b)*m^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 4*b^2*m^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 8*b^3*n^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*m^2*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - b^3*n^3*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/8*m^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 15/2*b^2*n^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/4*c^(4*b)*m*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - c^(2*b)*m*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b*m^3*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 8*b^2*m*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*m^3*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/2*b^2*m*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/2*m^3*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) + 1/16*c^(4*b)*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*c^(2*b)*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*m^4*x*x^m*e^(-

2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 3/2*b*m^2*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 4*b^2*n^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*m^4*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/4*b*m^2*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*b^2*n^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 9/4*m^2*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - m^3*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 3/2*b*m*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*m^3*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/4*b*m*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/2*m*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 3/2*m^2*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 3/8*m^2*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 1/4*b*n*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - m*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/4*m*x*x^m*e^(-4*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*n)) - 1/4*x*x^m*e^(-2*a)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b

```
^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*x^m*e^(-4*a
)/((64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*
b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)*c^(4*b)*x^(4*b*
n))
```

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.51

$$\int x^m \sinh^4(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m}{8\,m+8}-\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}-\frac{x\,x^m\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}+\frac{x\,x^m\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(16\,m-64\,b\,n+16\right)}+\frac{x\,x^m\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{16\,m+64\,b\,n+16}$$

```
[In] int(x^m*sinh(a + b*log(c*x^n))^4,x)
```

```
[Out] (3*x*x^m)/(8*m + 8) - (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) -
 (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c*x
^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m + 64
*b*n + 16)
```

3.274 $\int \frac{\sinh(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sinh(a+b\log(cx^n))}{x}\,dx=\frac{\cosh(a+b\log(cx^n))}{bn}$$

```
[Out] cosh(a+b*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2718}

$$\int \frac{\sinh(a+b\log(cx^n))}{x}\,dx=\frac{\cosh(a+b\log(cx^n))}{bn}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]/x,x]
```

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\text{Subst}\left(\int \sinh(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\&=\frac{\cosh(a+b\log(cx^n))}{bn}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. $2(18) = 36$.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\,dx = \frac{\cosh(a)\cosh(b\log(cx^n))}{bn} + \frac{\sinh(a)\sinh(b\log(cx^n))}{bn}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]/x,x]
[Out] (Cosh[a]*Cosh[b*Log[c*x^n]])/(b*n) + (Sinh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\cosh(a+b\ln(c\,x^n))}{bn}$	19
default	$\frac{\cosh(a+b\ln(c\,x^n))}{bn}$	19
parallelrisch	$\frac{\cosh(2b\ln(\sqrt{c\,x^n})+a)+1}{bn}$	24

```
[In] int(sinh(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] cosh(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\,dx = \frac{\cosh(bn\log(x) + b\log(c) + a)}{bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] cosh(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\sinh(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\sinh(a + b\log(c)) & \text{for } n = 0 \\ \frac{\cosh(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b*ln(c*x**n))/x,x)
[Out] Piecewise((log(x)*sinh(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*sinh(
a + b*log(c)), Eq(n, 0)), (cosh(a + b*log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\,dx = \frac{\cosh(b\log(cx^n) + a)}{bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] cosh(b*log(c*x^n) + a)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.22

$$\int \frac{\sinh(a + b\log(cx^n))}{x}\,dx = \frac{\left(c^{2b}x^{bn}e^{(2a)} + \frac{1}{x^{bn}}\right)e^{(-a)}}{2\,bc^bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] 1/2*(c^(2*b)*x^(b*n)*e^(2*a) + 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

Mupad [B] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh \left(a+b\log \left(cx^n\right)\right)}{x}\,dx=\frac{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

[In] `int(sinh(a + b*log(c*x^n))/x,x)`

[Out] `cosh(a + b*log(c*x^n))/(b*n)`

3.275 $\int \frac{\sinh^2(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x} dx = -\frac{\log(x)}{2} + \frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn}$$

```
[Out] -1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2715, 8}

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x} dx = \frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2bn} - \frac{\log(x)}{2}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] -1/2*Log[x] + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sinh^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn} - \frac{\text{Subst}\left(\int 1\,dx, x, \log\left(cx^n\right)\right)}{2n}\\ &= -\frac{\log(x)}{2} + \frac{\cosh\left(a+b\log\left(cx^n\right)\right)\sinh\left(a+b\log\left(cx^n\right)\right)}{2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{-2\left(a+b\log\left(cx^n\right)\right)+\sinh\left(2\left(a+b\log\left(cx^n\right)\right)\right)}{4bn}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^2/x,x]
[Out] (-2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{-2\ln(x)bn+\sinh(2b\ln(c\,x^n)+2a)}{4bn}$	30
derivativedivides	$\frac{\frac{\cosh(a+b\ln(c\,x^n))\sinh(a+b\ln(c\,x^n))}{2}-\frac{b\ln(c\,x^n)}{2}-\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cosh(a+b\ln(c\,x^n))\sinh(a+b\ln(c\,x^n))}{2}-\frac{b\ln(c\,x^n)}{2}-\frac{a}{2}}{nb}$	45

```
[In] int(sinh(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
[Out] 1/4*(-2*ln(x)*b*n+sinh(2*b*ln(c*x^n)+2*a))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = -\frac{bn\log(x)-\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)}{2\,bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -1/2*(b*n*log(x) - cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(
c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} - \frac{1}{2}\log(x)$$

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) -
1/2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(35) = 70$.

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.08

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = -\frac{\left(4\,bc^{2\,b}ne^{(2\,a)}\log(x) - c^{4\,b}x^{2\,bn}e^{(4\,a)} - \frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
[Out] -1/8*(4*b*c^(2*b)*n*e^(2*a)*log(x) - c^(4*b)*x^(2*b*n)*e^(4*a) - (2*c^(2*b)
*x^(2*b*n)*e^(2*a) - 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^2(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(2\,a+2\,b\,\ln(c\,x^n))}{4\,b\,n} - \frac{\ln(x^n)}{2\,n}$$

```
[In] int(sinh(a + b*log(c*x^n))^2/x,x)
[Out] sinh(2*a + 2*b*log(c*x^n))/(4*b*n) - log(x^n)/(2*n)
```

3.276 $\int \frac{\sinh^3(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x} \, dx = -\frac{\cosh(a+b\log(cx^n))}{bn}+\frac{\cosh^3(a+b\log(cx^n))}{3bn}$$

```
[Out] -cosh(a+b*ln(c*x^n))/b/n+1/3*cosh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2713}

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x} \, dx = \frac{\cosh^3(a+b\log(cx^n))}{3bn}-\frac{\cosh(a+b\log(cx^n))}{bn}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -(Cosh[a + b*Log[c*x^n]]/(b*n)) + Cosh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh^3(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\
&= -\frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn} + \frac{\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$
\int \frac{\sinh^3\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = -\frac{3\cosh\left(a+b\log\left(cx^n\right)\right)}{4bn} + \frac{\cosh\left(3\left(a+b\log\left(cx^n\right)\right)\right)}{12bn}
$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^3/x,x]
[Out] (-3*Cosh[a + b*Log[c*x^n]])/(4*b*n) + Cosh[3*(a + b*Log[c*x^n])]/(12*b*n)
```

Maple [A] (verified)

Time = 6.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\left(-\frac{2}{3}+\frac{\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{3}\right)\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$	36
default	$\frac{\left(-\frac{2}{3}+\frac{\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{3}\right)\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$	36
parallelrisch	$\frac{-8+\cosh\left(3b\ln\left(c\,x^{n}\right)+3a\right)-9\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)}{12bn}$	38

```
[In] int(sinh(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b*(-2/3+1/3*sinh(a+b*ln(c*x^n))^2)*cosh(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \frac{\cosh(bn\log(x)+b\log(c)+a)^3+3\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^2-9\cosh(bn\log(x)+b\log(c)+a)}{12\,bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
[Out] 1/12*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^2 - 9*cosh(b*n*log(x) + b*log(c) + a))/(b*
n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(32) = 64.

Time = 1.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\sinh^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\sinh^3(a+b\log(c)) & \text{for } n=0 \\ \frac{\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{bn} - \frac{2\cosh^3(a+b\log(cx^n))}{3bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**3/x,x)
[Out] Piecewise((log(x)*sinh(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*si
nh(a + b*log(c))**3, Eq(n, 0)), (sinh(a + b*log(c*x**n))**2*cosh(a + b*log(
c*x**n))/(b*n) - 2*cosh(a + b*log(c*x**n))**3/(3*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(41) = 82.

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \frac{e^{(3\,b\log(cx^n)+3\,a)}}{24\,bn} - \frac{3\,e^{(b\log(cx^n)+a)}}{8\,bn} - \frac{3\,e^{(-b\log(cx^n)-a)}}{8\,bn} + \frac{e^{(-3\,b\log(cx^n)-3\,a)}}{24\,bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) - 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8*
e^(-b*log(c*x^n) - a)/(b*n) + 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = \frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)} - 9\,c^{4\,b}x^{bn}e^{(4\,a)} - \frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}-1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] 1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) - 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x^
(2*b*n)*e^(2*a) - 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^3(a+b\log(cx^n))}{x}\,dx = -\frac{3\cosh(a+b\,\ln(c\,x^n)) - \cosh(a+b\,\ln(c\,x^n))^3}{3\,b\,n}$$

```
[In] int(sinh(a + b*log(c*x^n))^3/x,x)
```

```
[Out] -(3*cosh(a + b*log(c*x^n)) - cosh(a + b*log(c*x^n))^3)/(3*b*n)
```

3.277 $\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{3\log(x)}{8} - \frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn} + \frac{\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{4bn}$$

```
[Out] 3/8*ln(x)-3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(c
*x^n))*sinh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2715, 8}

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{\sinh^3(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4bn} - \frac{3\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{8bn} + \frac{3\log(x)}{8}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (3*Log[x])/8 - (3*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(8*b*n) +
(Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]]^3)/(4*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh^4(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{4bn} - \frac{3\text{Subst}\left(\int \sinh^2(a+bx)\,dx, x, \log(cx^n)\right)}{4n} \\
&= -\frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn} \\
&\quad + \frac{\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{4bn} + \frac{3\text{Subst}\left(\int 1\,dx, x, \log(cx^n)\right)}{8n} \\
&= \frac{3\log(x)}{8} - \frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn} \\
&\quad + \frac{\cosh(a+b\log(cx^n))\sinh^3(a+b\log(cx^n))}{4bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$
\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{12(a+b\log(cx^n)) - 8\sinh(2(a+b\log(cx^n))) + \sinh(4(a+b\log(cx^n)))}{32bn}
$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] (12*(a + b*Log[c*x^n]) - 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[c
*x^n])])/(32*b*n)
```

Maple [A] (verified)

Time = 21.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{12\ln(x)bn-8\sinh(2b\ln(c\,x^{n})+2a)+\sinh(4b\ln(c\,x^{n})+4a)}{32bn}$	46
derivativedivides	$\frac{\left(\frac{\sinh(a+b\ln(c\,x^{n}))^{3}}{4}-\frac{3\sinh(a+b\ln(c\,x^{n}))}{8}\right)\cosh(a+b\ln(c\,x^{n}))+\frac{3b\ln(c\,x^{n})}{8}+\frac{3a}{8}}{nb}$	62
default	$\frac{\left(\frac{\sinh(a+b\ln(c\,x^{n}))^{3}}{4}-\frac{3\sinh(a+b\ln(c\,x^{n}))}{8}\right)\cosh(a+b\ln(c\,x^{n}))+\frac{3b\ln(c\,x^{n})}{8}+\frac{3a}{8}}{nb}$	62

```
[In] int(sinh(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
[Out] 1/32*(12*ln(x)*b*n-8*sinh(2*b*ln(c*x^n)+2*a)+sinh(4*b*ln(c*x^n)+4*a))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^3+3\,bn\log(x)+\left(\cosh(bn\log(x)+b\log(c)\right.}{8\,bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
[Out] 1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(x) + b*log
(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**4/x,x)
[Out] Integral(sinh(a + b*log(c*x**n))**4/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{e^{(4\,b\log(cx^n)+4\,a)}}{64\,bn} - \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} + \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} - \frac{e^{(-4\,b\log(cx^n)-4\,a)}}{64\,bn} + \frac{3}{8}\log(x)$$

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) - 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) +
1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n)
+ 3/8*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{\left(24\,bc^{4\,b}ne^{(4\,a)}\log(x) + c^{8\,b}x^{4\,bn}e^{(8\,a)} - 8\,c^{6\,b}x^{2\,bn}e^{(6\,a)} - \frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 8\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 1/64*(24*b*c^(4*b)*n*e^(4*a)*log(x) + c^(8*b)*x^(4*b*n)*e^(8*a) - 8*c^(6*b)
*x^(2*b*n)*e^(6*a) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) - 8*c^(2*b)*x^(2*b*n)*e^
(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(a+b\log(cx^n))}{x}\,dx = \frac{3\,\ln(x^n)}{8\,n} - \frac{\frac{\sinh(2\,a+2\,b\,\ln(c\,x^n))}{4} - \frac{\sinh(4\,a+4\,b\,\ln(c\,x^n))}{32}}{b\,n}$$

```
[In] int(sinh(a + b*log(c*x^n))^4/x,x)
```

```
[Out] (3*log(x^n))/(8*n) - (sinh(2*a + 2*b*log(c*x^n))/4 - sinh(4*a + 4*b*log(c*x
^n))/32)/(b*n)
```

3.278 $\int \frac{\sinh^5(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x} dx = \frac{\cosh(a+b\log(cx^n))}{bn} - \frac{2\cosh^3(a+b\log(cx^n))}{3bn} + \frac{\cosh^5(a+b\log(cx^n))}{5bn}$$

```
[Out] cosh(a+b*ln(c*x^n))/b/n-2/3*cosh(a+b*ln(c*x^n))^3/b/n+1/5*cosh(a+b*ln(c*x^n
))^5/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2713}

$$\int \frac{\sinh^5(a+b\log(cx^n))}{x} dx = \frac{\cosh^5(a+b\log(cx^n))}{5bn} - \frac{2\cosh^3(a+b\log(cx^n))}{3bn} + \frac{\cosh(a+b\log(cx^n))}{bn}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Cosh[a + b*Log[c*x^n]]/(b*n) - (2*Cosh[a + b*Log[c*x^n]]^3)/(3*b*n) + Cosh[
a + b*Log[c*x^n]]^5/(5*b*n)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh^5(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \cosh\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\
&= \frac{\cosh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{2\cosh^3\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\cosh^5\left(a+b\log\left(cx^n\right)\right)}{5bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{5\cosh\left(a+b\log\left(cx^n\right)\right)}{8bn} - \frac{5\cosh\left(3\left(a+b\log\left(cx^n\right)\right)\right)}{48bn} + \frac{\cosh\left(5\left(a+b\log\left(cx^n\right)\right)\right)}{80bn}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (5*Cosh[a + b*Log[c*x^n]])/(8*b*n) - (5*Cosh[3*(a + b*Log[c*x^n])])/(48*b*n
) + Cosh[5*(a + b*Log[c*x^n])]/(80*b*n)
```

Maple [A] (verified)

Time = 74.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\sinh\left(a+b\ln\left(c\,x^n\right)\right)^4}{5}-\frac{4\sinh\left(a+b\ln\left(c\,x^n\right)\right)^2}{15}\right)\cosh\left(a+b\ln\left(c\,x^n\right)\right)}{nb}$	51
default	$\frac{\left(\frac{8}{15}+\frac{\sinh\left(a+b\ln\left(c\,x^n\right)\right)^4}{5}-\frac{4\sinh\left(a+b\ln\left(c\,x^n\right)\right)^2}{15}\right)\cosh\left(a+b\ln\left(c\,x^n\right)\right)}{nb}$	51
parallelrisch	$\frac{128-25\cosh\left(3b\ln\left(c\,x^n\right)+3a\right)+150\cosh\left(a+b\ln\left(c\,x^n\right)\right)+3\cosh\left(5b\ln\left(c\,x^n\right)+5a\right)}{240bn}$	56

```
[In] int(sinh(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(8/15+1/5*sinh(a+b*ln(c*x^n))^4-4/15*sinh(a+b*ln(c*x^n))^2)*cosh(a+b*
ln(c*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^5(a + b\log(cx^n))}{x}\,dx$$
$$= \frac{3\cosh(bn\log(x) + b\log(c) + a)^5 + 15\cosh(bn\log(x) + b\log(c) + a)\sinh(bn\log(x) + b\log(c) + a)^4 - 2$$

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/240*(3*cosh(b*n*log(x) + b*log(c) + a)^5 + 15*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a)^4 - 25*cosh(b*n*log(x) + b*log(c) + a)
^3 + 15*(2*cosh(b*n*log(x) + b*log(c) + a)^3 - 5*cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + 150*cosh(b*n*log(x) + b*log(c) +
 a))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 7.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\sinh^5(a + b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \log(x)\sinh^5(a) & \text{for } b = 0 \wedge (b = \\ \log(x)\sinh^5(a + b\log(c)) & \text{for } n = 0 \\ \frac{\sinh^4(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{bn} - \frac{4\sinh^2(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{3bn} + \frac{8\cosh^5(a+b\log(cx^n))}{15bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((log(x)*sinh(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*si
nh(a + b*log(c))**5, Eq(n, 0)), (sinh(a + b*log(c*x**n))**4*cosh(a + b*log(
c*x**n))/(b*n) - 4*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**3/(3
*b*n) + 8*cosh(a + b*log(c*x**n))**5/(15*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.

Time = 0.23 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{e^{\left(5\,b\log\left(cx^n\right)+5\,a\right)}}{160\,bn} - \frac{5\,e^{\left(3\,b\log\left(cx^n\right)+3\,a\right)}}{96\,bn} + \frac{5\,e^{\left(b\log\left(cx^n\right)+a\right)}}{16\,bn} + \frac{5\,e^{\left(-b\log\left(cx^n\right)-a\right)}}{16\,bn} - \frac{5\,e^{\left(-3\,b\log\left(cx^n\right)-3\,a\right)}}{96\,bn} + \frac{e^{\left(-5\,b\log\left(cx^n\right)-5\,a\right)}}{160\,bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) - 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
+ 5/16*e^(b*log(c*x^n) + a)/(b*n) + 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/96
*e^(-3*b*log(c*x^n) - 3*a)/(b*n) + 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)} - 25\,c^{8\,b}x^{3\,bn}e^{(8\,a)} + 150\,c^{6\,b}x^{bn}e^{(6\,a)} + \frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 25\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) - 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*c
^(6*b)*x^(b*n)*e^(6*a) + (150*c^(4*b)*x^(4*b*n)*e^(4*a) - 25*c^(2*b)*x^(2*b
*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^5\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\frac{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^5}{5} - \frac{2\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^3}{3} + \cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}{b\,n}$$

```
[In] int(sinh(a + b*log(c*x^n))^5/x,x)
```

```
[Out] (cosh(a + b*log(c*x^n)) - (2*cosh(a + b*log(c*x^n))^3)/3 + cosh(a + b*log(c
*x^n))^5/5)/(b*n)
```

3.279 $\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{5bn\sqrt{i\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\log(cx^n))\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{5bn}$$

```
[Out] 2/5*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))^(3/2)/b/n-6/5*I*(sin(1/2*I*a+1/
4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Elli
pticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c*x^n))^(1
/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2715, 2721, 2719}

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sinh^{\frac{3}{2}}(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{5bn} + \frac{6i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{5bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((6*I)/5)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sinh[a + b*Log[c*x^n]]^(3/2))/(5*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh^{\frac{5}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn} - \frac{3\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{5n} \\
&= \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn} \\
&\quad - \frac{\left(3\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{5n\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}} \\
&= \frac{6iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{5bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}} \\
&\quad + \frac{2\cosh\left(a+b\log\left(cx^n\right)\right)\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{5bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-6E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}+\sinh(a+b\log(cx^n))\sinh(2(a+b\log(cx$$
$$5bn\sqrt{\sinh(a+b\log(cx^n))}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (-6*EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]] + Sinh[a + b*Log[c*x^n]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n*Sq
rt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A] (verified)

Time = 6.45 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.05

method	result
derivativedivides	$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(}{}}{n\cosh(a+b\ln}$
default	$\frac{-\frac{6\sqrt{1-i\sinh(a+b\ln(cx^n))}\sqrt{2}\sqrt{1+i\sinh(a+b\ln(cx^n))}\sqrt{i\sinh(a+b\ln(cx^n))}\operatorname{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{5}+\frac{3\sqrt{1-i\sinh(}{}}{n\cosh(a+b\ln}$

```
[In] int(sinh(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-6/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))
^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1
/2),1/2*2^(1/2))+3/5*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*
ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(
c*x^n)))^(1/2),1/2*2^(1/2))+2/5*cosh(a+b*ln(c*x^n))^4-2/5*cosh(a+b*ln(c*x^n
))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.98

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{12\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^2+2\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)\right.}{}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/10*(12*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log
(x) + b*log(c) + a)^2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh
(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + (cosh(b*n
*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(
x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 6*(cosh(b*n*log(
x) + b*log(c) + a)^2 + 2)*sinh(b*n*log(x) + b*log(c) + a)^2 + 12*cosh(b*n*l
og(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + 6*cosh(b*n
*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) - 1)*sqrt(sinh(b*n
*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*co
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*
n*log(x) + b*log(c) + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{5}{2}}(a + b\log(cx^n))}{x} \, dx = \int \frac{\sinh(b\log(cx^n) + a)^{\frac{5}{2}}}{x} \, dx$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
[Out] integrate(sinh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{5}{2}}(a + b\log(cx^n))}{x} \, dx = \int \frac{\sinh(a + b\ln(c\,x^n))^{5/2}}{x} \, dx$$

```
[In] int(sinh(a + b*log(c*x^n))^(5/2)/x,x)
[Out] int(sinh(a + b*log(c*x^n))^(5/2)/x, x)
```

3.280 $\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn}$$

```
[Out] -2/3*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1
/2*I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*
(I*sinh(a+b*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2/3*cosh(a+b*ln
(c*x^n))*sinh(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2715, 2721, 2720}

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\sinh(a+b\log(cx^n))}\cosh(a+b\log(cx^n))}{3bn} + \frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[In] Int[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (((2*I)/3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*
Log[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) + (2*Cosh[a + b*Log[c*x^n]
]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(3*b*n)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \sinh^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn} - \frac{\sqrt{i\sinh(a+b\log(cx^n))}\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n\sqrt{\sinh(a+b\log(cx^n))}}$$

$$= \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right), 2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{2\cosh(a+b\log(cx^n))\sqrt{\sinh(a+b\log(cx^n))}}{3bn}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{-2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+b\log(cx^n)))+\sinh(2(a+b\log(cx^n)))\right)\sqrt{1-\cosh(2(a+b}}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[In] Integrate[Sinh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] (-2*Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a
 + b*Log[c*x^n])]]*Sqrt[1 - Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[
c*x^n])]] + Sinh[2*(a + b*Log[c*x^n])]])/(3*b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]
)
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\sinh(a+b}{}}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b}$
default	$\frac{-\frac{i\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2\sinh(a+b}{}}{n\cosh(a+b\ln(cx^n))\sqrt{\sinh(a+b\ln(cx^n))}\,b}$

```
[In] int(sinh(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-1/3*I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)
))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n)))^
(1/2),1/2*2^(1/2))+2/3*sinh(a+b*ln(c*x^n))*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*
ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.54

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)+\sqrt{2}\sinh(bn\log(x)+b\log(c)+a)\right)\text{weierstrassPInverse}(4,0,\cos$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
+ b*log(c) + a))*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a)
+ sinh(b*n*log(x) + b*log(c) + a)) - (cosh(b*n*log(x) + b*log(c) + a)^2 + 2
*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n
*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)))/(b*n*
cosh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sinh(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\sinh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\sinh(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\sinh\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
[Out] integrate(sinh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx=\int \frac{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}{x}\,dx$$

```
[In] int(sinh(a + b*log(c*x^n))^(3/2)/x,x)
[Out] int(sinh(a + b*log(c*x^n))^(3/2)/x, x)
```

3.281 $\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx = -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sin
h(a+b*ln(c*x^n))^(1/2)/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 2719}

$$\int \frac{\sqrt{\sinh(a+b\log(cx^n))}}{x}\,dx = -\frac{2i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[In] Int[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*EllipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c
*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\ &= \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}\\ &= -\frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \frac{2E\left(\frac{1}{2}\left(\frac{\pi}{2}-i\left(a+b\log\left(cx^n\right)\right)\right)\middle|2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}$$

```
[In] Integrate[Sqrt[Sinh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] (2*EllipticE[(Pi/2 - I*(a + b*Log[c*x^n]))/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.03

method	result
derivativedivides	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\right.\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$
default	$\frac{\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(c\right.\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$

```
[In] int(sinh(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*(2*EllipticE((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))
)/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\sinh (a+b\log (cx^n))}}{x}\,dx =$$
$$-\frac{2\left(\sqrt{2}\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cosh (bn\log (x)+b\log (c)+a)+\sinh (bn\log (x)+\right.}{bn}$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*(sqrt(2)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cosh(b*n*log(x)
 + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + sqrt(sinh(b*n*log(x)
 + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\sinh (a+b\log (cx^n))}}{x}\,dx = \int \frac{\sqrt{\sinh (a+b\log (cx^n))}}{x}\,dx$$

```
[In] integrate(sinh(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sinh(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\sinh (a+b\log (cx^n))}}{x}\,dx = \int \frac{\sqrt{\sinh (b\log (cx^n)+a)}}{x}\,dx$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

```
[In] integrate(sinh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(sinh(b*log(c*x^n) + a))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}}{x}\,dx$$

```
[In] int(sinh(a + b*log(c*x^n))^(1/2)/x,x)
[Out] int(sinh(a + b*log(c*x^n))^(1/2)/x, x)
```

3.282 $\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[Out] 2*I*(sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*
I*b*ln(c*x^n))*EllipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*(I*
sinh(a+b*ln(c*x^n)))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2721, 2720}

$$\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx = -\frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[In] Int[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log
[c*x^n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\sqrt{i\sinh(a+b\log(cx^n))}\,\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n\sqrt{\sinh(a+b\log(cx^n))}} \\ &= -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{bn\sqrt{\sinh(a+b\log(cx^n))}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{\sinh(a+b\log(cx^n))}}\,dx = -\frac{2\,\text{EllipticF}\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n)),2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[In] Integrate[1/(x*Sqrt[Sinh[a + b*Log[c*x^n]]]),x]
```

```
[Out] (-2*EllipticF[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n])/4, 2]*Sqrt[Sinh[a + b*Lo
g[c*x^n]]])/(b*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.67

method	result
derivativedivides	$\frac{i\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(c\,x^n))}\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$
default	$\frac{i\sqrt{-i(\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{2}\,\sqrt{-i(-\sinh(a+b\ln(c\,x^n))+i)}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\operatorname{EllipticF}\left(\sqrt{-i(\sinh(a+b\ln(c\,x^n))}\right.}{n\cosh(a+b\ln(c\,x^n))\sqrt{\sinh(a+b\ln(c\,x^n))}\,b}$

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/n*(-I*(sinh(a+b*ln(c*x^n))+I))^(1/2)*2^(1/2)*(-I*(-sinh(a+b*ln(c*x^n))+I)
)^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((-I*(sinh(a+b*ln(c*x^n))+I)
)^(1/2),1/2*2^(1/2))/cosh(a+b*ln(c*x^n))/sinh(a+b*ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(
b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(sinh(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\sinh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(x*sqrt(sinh(b*log(c*x^n) + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\,\sqrt{\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)}}\,dx$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(1/2)),x)
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(1/2)), x)
```

3.283 $\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))} dx = -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[Out] -2*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(1/2)+2*I*(sin(1/2*I*a+1/4*P
i+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ellipti
cE(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*sinh(a+b*ln(c*x^n))^(1/2)
/b/n/(I*sinh(a+b*ln(c*x^n)))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2716, 2721, 2719}

$$\int \frac{1}{x \sinh^{\frac{3}{2}}(a+b\log(cx^n))} dx = -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} - \frac{2i\sqrt{\sinh(a+b\log(cx^n))}E\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right)\middle|2\right)}{bn\sqrt{i\sinh(a+b\log(cx^n))}}$$

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]]) - ((2*I)*Ell
ipticE[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[Sinh[a + b*Log[c*x^n]]])/(b
*n*Sqrt[I*Sinh[a + b*Log[c*x^n]]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sinh^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} \\
&\quad + \frac{\sqrt{\sinh(a+b\log(cx^n))}\text{Subst}\left(\int \sqrt{i\sinh(a+bx)}\,dx, x, \log(cx^n)\right)}{n\sqrt{i\sinh(a+b\log(cx^n))}} \\
&= -\frac{2\cosh(a+b\log(cx^n))}{bn\sqrt{\sinh(a+b\log(cx^n))}} - \frac{2iE\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right)\middle|2\right)\sqrt{\sinh(a+b\log(cx^n))}}{bn\sqrt{i\sinh(a+b\log(cx^n))}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\left(\cosh(a+b\log(cx^n)) - E\left(\frac{1}{4}(-2ia+\pi-2ib\log(cx^n))\middle|2\right)\sqrt{i\sinh(a+b\log(cx^n))}\right)}{bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] - EllipticE[((-2*I)*a + Pi - (2*I)*b*Log[c*x^n]
)/4, 2]*Sqrt[I*Sinh[a + b*Log[c*x^n]]]))/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.98

method	result
derivativedivides	$\frac{2\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)-}{n\cosh(a+b}$
default	$\frac{2\sqrt{1-i\sinh(a+b\ln(cx^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(cx^n))}\,\sqrt{i\sinh(a+b\ln(cx^n))}\,\text{EllipticE}\left(\sqrt{1-i\sinh(a+b\ln(cx^n))},\frac{\sqrt{2}}{2}\right)-}{n\cosh(a+b}$

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^n)))^(1
/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticE((1-I*sinh(a+b*ln(c*x^n)))^(1/2)
,1/2*2^(1/2))-(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)*(1+I*sinh(a+b*ln(c*x^
n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1-I*sinh(a+b*ln(c*x^n))
)^(1/2),1/2*2^(1/2))-2*cosh(a+b*ln(c*x^n))^2)/cosh(a+b*ln(c*x^n))/sinh(a+b*
ln(c*x^n))^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.30

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^2+2\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)\right.\right.}{}$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) +
 b*log(c) + a)^2 - sqrt(2))*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
 cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + 2*(c
osh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b
*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2)*sqrt(sinh(b*
n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b
*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F]

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(1/(x*sinh(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sinh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sinh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sinh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sinh\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(3/2)),x)
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(3/2)), x)
```

3.284 $\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 111

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))} dx = -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log(cx^n)\right),2\right)\sqrt{i\sinh(a+b\log(cx^n))}}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[Out] -2/3*cosh(a+b*ln(c*x^n))/b/n/sinh(a+b*ln(c*x^n))^(3/2)-2/3*I*(sin(1/2*I*a+1
/4*Pi+1/2*I*b*ln(c*x^n))^2)^(1/2)/sin(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n))*Ell
ipticF(cos(1/2*I*a+1/4*Pi+1/2*I*b*ln(c*x^n)),2^(1/2))*(I*sinh(a+b*ln(c*x^n)
))^(1/2)/b/n/sinh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2716, 2721, 2720}

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))} dx = -\frac{2\cosh(a+b\log(cx^n))}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{2i\sqrt{i\sinh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}\left(ia+ib\log(cx^n)-\frac{\pi}{2}\right),2\right)}{3bn\sqrt{\sinh(a+b\log(cx^n))}}$$

```
[In] Int[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*Cosh[a + b*Log[c*x^n]])/(3*b*n*Sinh[a + b*Log[c*x^n]]^(3/2)) + (((2*I)/
3)*EllipticF[(I*a - Pi/2 + I*b*Log[c*x^n])/2, 2]*Sqrt[I*Sinh[a + b*Log[c*x^
n]]])/(b*n*Sqrt[Sinh[a + b*Log[c*x^n]]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sinh^{\frac{5}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{3n}\\
&= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}-\frac{\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{i\sinh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{3n\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}\\
&= -\frac{2\cosh\left(a+b\log\left(cx^n\right)\right)}{3bn\sinh^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\\
&\quad+\frac{2i\,\text{EllipticF}\left(\frac{1}{2}\left(ia-\frac{\pi}{2}+ib\log\left(cx^n\right)\right),2\right)\sqrt{i\sinh\left(a+b\log\left(cx^n\right)\right)}}{3bn\sqrt{\sinh\left(a+b\log\left(cx^n\right)\right)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\Big(\cosh(a+b\log(cx^n)) + \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\cosh(2(a+b\log(cx^n))) + \sinh(2(a+b\log(cx^n\ldots}{3bn\sinh^{\frac{3}{2}}(a+b\log(cx\ldots}$$

```
[In] Integrate[1/(x*Sinh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2*(Cosh[a + b*Log[c*x^n]] + Hypergeometric2F1[1/4, 1/2, 5/4, Cosh[2*(a +
b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]*Sinh[a + b*Log[c*x^n]]*Sqrt[1
- Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Sinh[a
+ b*Log[c*x^n]]^(3/2))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)\ldots}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh(a+b\ln(c\,x^n))b}$
default	$-\frac{i\sqrt{1-i\sinh(a+b\ln(c\,x^n))}\,\sqrt{2}\,\sqrt{1+i\sinh(a+b\ln(c\,x^n))}\,\sqrt{i\sinh(a+b\ln(c\,x^n))}\,\text{EllipticF}\left(\sqrt{1-i\sinh(a+b\ln(c\,x^n))},\frac{\sqrt{2}}{2}\right)\ldots}{3n\sinh(a+b\ln(c\,x^n))^{\frac{3}{2}}\cosh(a+b\ln(c\,x^n))b}$

```
[In] int(1/x/sinh(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/n/sinh(a+b*ln(c*x^n))^(3/2)*(I*(1-I*sinh(a+b*ln(c*x^n)))^(1/2)*2^(1/2)
*(1+I*sinh(a+b*ln(c*x^n)))^(1/2)*(I*sinh(a+b*ln(c*x^n)))^(1/2)*EllipticF((1
-I*sinh(a+b*ln(c*x^n)))^(1/2),1/2*2^(1/2))*sinh(a+b*ln(c*x^n))+2*cosh(a+b*l
n(c*x^n))^2)/cosh(a+b*ln(c*x^n))/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 504, normalized size of antiderivative = 4.54

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{2\left(\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^4+4\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(\right.\right.}{}$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*sqrt(2)*cosh(b*n*log(x
) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*sinh(b*n*log(
x) + b*log(c) + a)^4 + 2*(3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 - sqr
t(2))*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*sqrt(2)*cosh(b*n*log(x) + b*log
(c) + a)^2 + 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 - sqrt(2)*cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2))*weiers
trassPInverse(4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*l
og(c) + a)) + 2*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*
log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c)
+ a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(
c) + a) + cosh(b*n*log(x) + b*log(c) + a))*sqrt(sinh(b*n*log(x) + b*log(c)
+ a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(
c) + a)^4 - 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log
(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b
*n*cosh(b*n*log(x) + b*log(c) + a)^3 - b*n*cosh(b*n*log(x) + b*log(c) + a))
*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x \sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/sinh(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sinh(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sinh(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/sinh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] integrate(1/(x*sinh(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sinh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\sinh(a+b\,\ln(c\,x^n))^{5/2}}\,dx$$

```
[In] int(1/(x*sinh(a + b*log(c*x^n))^(5/2)),x)
[Out] int(1/(x*sinh(a + b*log(c*x^n))^(5/2)), x)
```

3.285 $\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 209

$$\begin{aligned}\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx = &-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\\ &-\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2}\\ &+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}\\ &-\frac{5e^{-3a}x(cx^n)^{-6/n}\csc^{-1}\left(e^a(cx^n)^{2/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}\end{aligned}$$

```
[Out] -1/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)-5/4*x*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(2*
a)/((c*x^n)^(4/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)))^2+5/12*x*sinh(a+2*ln(c*x^
n)/n)^(5/2)/(1-1/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsc(exp(a)*(c*x^n)^(2/n
))*sinh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1-1/exp(2*a)/((c*x
^n)^(4/n)))^(5/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {5636, 5644, 360, 356, 352, 248, 283, 222}

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = -\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2} - \frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) + \frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)} - \frac{5e^{-3a}x(cx^n)^{-6/n}\csc^{-1}\left(e^a(cx^n)^{2/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

```
[Out] -1/4*(x*Sinh[a + (2*Log[c*x^n])/n]^(5/2)) - (5*x*Sinh[a + (2*Log[c*x^n])/n]
^(5/2))/(4*E^(2*a)*(c*x^n)^(4/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^2) + (5*x*
Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(12*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))) - (5
*x*ArcCsc[E^a*(c*x^n)^(2/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/(4*E^(3*a)*(
c*x^n)^(6/n)*(1 - 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 356

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Dist[b*n*(p/(m + 1)), Int[x^(m + n)*(a + b*x^n
)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && GtQ
[p, 0]
```

Rule 360

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(x)}{n}\right)\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1-e^{-2a}x^{-4/n}\right)^{5/2}\,dx,x,cx^n\right)}{n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}\\
&= -\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\\
&\quad+\frac{\left(5x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{6}{n}}\left(1-e^{-2a}x^{-4/n}\right)^{3/2}\,dx,x,cx^n\right)}{2n\left(1-e^{-2a}\left(cx^n\right)^{-4/n}\right)^{5/2}}
\end{aligned}$$

$$=-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}-\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sqrt{1-e^{-2a}x^{-4/n}}\,dx,x,cx^n\right)}{2n\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$=-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}-\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int\sqrt{1-\frac{e^{-2a}}{x^2}}\,dx,x,(cx^n)^{2/n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$=-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}+\frac{\left(5e^{-2a}x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int\frac{\sqrt{1-e^{-2a}x^2}}{x^2}\,dx,x,(cx^n)^{-2/n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$=-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)-\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}-\frac{\left(5e^{-4a}x(cx^n)^{-6/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-e^{-2a}x^2}}\,dx,x,(cx^n)^{-2/n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

$$=-\frac{1}{4}x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)-\frac{5e^{-2a}x(cx^n)^{-4/n}\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1-e^{-2a}(cx^n)^{-4/n}\right)}-\frac{5e^{-3a}x(cx^n)^{-6/n}\arcsin\left(e^{-a}(cx^n)^{-2/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.41

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$$
$$=\frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(-1+e^{2a}(cx^n)^{4/n}\right)\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},1-e^{2a}(cx^n)^{4/n}\right)\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)$$

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(5/2),x]
[Out] (E^(2*a)*x*(c*x^n)^(4/n)*(-1 + E^(2*a)*(c*x^n)^(4/n))*Hypergeometric2F1[2,
7/2, 9/2, 1 - E^(2*a)*(c*x^n)^(4/n)]*Sinh[a + (2*Log[c*x^n])/n]^(5/2))/14
```

Maple [F]

$$\int \sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{5}{2}}dx$$

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
[Out] int(sinh(a+2*ln(c*x^n)/n)^(5/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.78

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$$
$$=\frac{\left(15\sqrt{2}x^3\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2(an+2\log(c))}{n}\right)}-1}{x^2}}\right)e^{\left(\frac{3(an+2\log(c))}{2n}\right)}+2\sqrt{\frac{1}{2}}\left(2x^8e^{\left(\frac{4(an+2\log(c))}{n}\right)}-14x^4e^{\left(\frac{2(a}{}\right.}\right.\right.}{96x^3}$$

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
[Out] 1/96*(15*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log
(c))/n) - 1)/x^2))*e^(3/2*(a*n + 2*log(c))/n) + 2*sqrt(1/2)*(2*x^8*e^(4*(a*
n + 2*log(c))/n) - 14*x^4*e^(2*(a*n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n
 + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c
))/n)/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)\,dx=\text{Timed out}$$

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)\,dx=\int \sinh\left(a+\frac{2\,\log\left(cx^{n}\right)}{n}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.97

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)\,dx=\frac{1}{48}\,\sqrt{2}\sqrt{c^{\frac{6}{n}}x^{6}e^{(3\,a)}-c^{\frac{2}{n}}x^{2}e^{a}}c^{\frac{2}{n}}x^{3}e^{a}+\frac{\sqrt{2}\left(15\,c^{\frac{8}{n}}\arctan\left(\sqrt{c^{\frac{4}{n}}x^{4}e^{(3\,a)}-e^{a}}e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(\frac{9}{2}\,a\right)}-14\,\sqrt{c^{\frac{4}{n}}x^{4}e^{(3\,a)}-e^{a}}c^{\frac{8}{n}}e^{(4\,a)}-\frac{3\,\sqrt{c^{\frac{4}{n}}x^{4}e^{(3\,a)}-e^{a}}c^{\frac{8}{n}}e^{(2\,a)}}{c^{\frac{4}{n}}x^{4}}\right)}{96\,c^{\frac{8}{n}}c^{\left(\frac{1}{n}\right)}\mathrm{sgn}\left(x\right)}$$

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
[Out] 1/48*sqrt(2)*sqrt(c^(6/n)*x^6*e^(3*a) - c^(2/n)*x^2*e^a)*c^(2/n)*x^3*e^a +
1/96*sqrt(2)*(15*c^(8/n)*arctan(sqrt(c^(4/n)*x^4*e^(3*a) - e^a)*e^(-1/2*a))
*e^(9/2*a) - 14*sqrt(c^(4/n)*x^4*e^(3*a) - e^a)*c^(8/n)*e^(4*a) - 3*sqrt(c^
(4/n)*x^4*e^(3*a) - e^a)*c^(8/n)*e^(2*a)/(c^(4/n)*x^4))*e^(-5*a)/(c^(8/n)*c
^(1/n)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx=\int \sinh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{5/2}\,dx$$

```
[In] int(sinh(a + (2*log(c*x^n))/n)^(5/2),x)
[Out] int(sinh(a + (2*log(c*x^n))/n)^(5/2), x)
```

3.286 $\int \sqrt{\sinh \left(a+\frac{2 \log(c x^n)}{n}\right)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \sqrt{\sinh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} \, dx = \frac{1}{2} x \sqrt{\sinh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} + \frac{e^{-a} x (c x^n)^{-2/n} \csc^{-1}\left(e^a (c x^n)^{2/n}\right) \sqrt{\sinh \left(a+\frac{2 \log(c x^n)}{n}\right)}}{2 \sqrt{1-e^{-2 a} \left(c x^n\right)^{-4/n}}}$$

```
[Out] 1/2*x*sinh(a+2*ln(c*x^n)/n)^(1/2)+1/2*x*arccsc(exp(a)*(c*x^n)^(2/n))*sinh(a
+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1-1/exp(2*a)/((c*x^n)^(4/n)))
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5636, 5644, 352, 248, 283, 222}

$$\int \sqrt{\sinh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} \, dx = \frac{1}{2} x \sqrt{\sinh \left(a+\frac{2 \log \left(c x^n\right)}{n}\right)} + \frac{e^{-a} x (c x^n)^{-2/n} \csc^{-1}\left(e^a (c x^n)^{2/n}\right) \sqrt{\sinh \left(a+\frac{2 \log(c x^n)}{n}\right)}}{2 \sqrt{1-e^{-2 a} \left(c x^n\right)^{-4/n}}}$$

```
[In] Int[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
```

```
[Out] (x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2 + (x*ArcCsc[E^a*(c*x^n)^(2/n)]*Sqrt[
Sinh[a + (2*Log[c*x^n])/n]])/(2*E^a*(c*x^n)^(2/n)*Sqrt[1 - 1/(E^(2*a)*(c*x^
n)^(4/n))])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right) \text{Subst}\left(\int x^{-1+\frac{1}{n}}\sqrt{\sinh\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$=\frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\sqrt{1-e^{-2a}x^{-4/n}}\,dx,x,cx^n\right)}{n\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$=\frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int\sqrt{1-\frac{e^{-2a}}{x^2}}\,dx,x,\left(cx^n\right)^{2/n}\right)}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$=-\frac{\left(x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int\frac{\sqrt{1-e^{-2a}x^2}}{x^2}\,dx,x,\left(cx^n\right)^{-2/n}\right)}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$=\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}+\frac{\left(e^{-2a}x(cx^n)^{-2/n}\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-e^{-2a}x^2}}\,dx,x,\left(cx^n\right)^{-2/n}\right)}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

$$=\frac{1}{2}x\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}+\frac{e^{-a}x(cx^n)^{-2/n}\arcsin\left(e^{-a}(cx^n)^{-2/n}\right)\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1-e^{-2a}\left(cx^n\right)^{-4/n}}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\int\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

$$=\frac{1}{2}x\left(1-\frac{\arctan\left(\sqrt{-1+e^{2a}\left(cx^n\right)^{4/n}}\right)}{\sqrt{-1+e^{2a}\left(cx^n\right)^{4/n}}}\right)\sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}$$

```
[In] Integrate[Sqrt[Sinh[a + (2*Log[c*x^n])/n]],x]
[Out] (x*(1 - ArcTan[Sqrt[-1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[-1 + E^(2*a)*(c*x^n)^
(4/n)])*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])/2
```

Maple [F]

$$\int \sqrt{\sinh\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)}dx$$

```
[In] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

```
[Out] int(sinh(a+2*ln(c*x^n)/n)^(1/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14

$$\int \sqrt{\sinh\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx = \frac{1}{4}\left(2\sqrt{\frac{1}{2}}x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^{2}}}e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^{2}}}\right)e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}\right.$$

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(1/2*(a*n
 + 2*log(c))/n) - sqrt(2)*arctan(sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n +
2*log(c))/n) - 1)/x^2))*e^(1/2*(a*n + 2*log(c))/n))*e^(-(a*n + 2*log(c))/n)
```

Sympy [F]

$$\int \sqrt{\sinh\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx = \int \sqrt{\sinh\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx$$

```
[In] integrate(sinh(a+2*ln(c*x**n)/n)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(a + 2*log(c*x**n)/n)), x)
```

Maxima [F]

$$\int \sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx = \int \sqrt{\sinh\left(a+\frac{2\,\log\left(cx^n\right)}{n}\right)}\,dx$$

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sinh(a + 2*log(c*x^n)/n)), x)
```

Giac [F(-1)]

Timed out.

$$\int \sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(sinh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sinh\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx = \int \sqrt{\sinh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)}\,dx$$

```
[In] int(sinh(a + (2*log(c*x^n))/n)^(1/2),x)
[Out] int(sinh(a + (2*log(c*x^n))/n)^(1/2), x)
```

3.287 $\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 43

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/2*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5636, 5644, 270}

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] -1/2*(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/Sinh[a + (2*Log[c*x^n])/n]^(3/2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{2/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{3/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{3/2}}\,dx, x, cx^n\right)}{n\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{2\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.42

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{-\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)}{x\sqrt{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

```
[Out] (-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n]
)/n])/(x*Sqrt[Sinh[a + (2*Log[c*x^n])/n]])
```

Maple [F]

$$\int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{3}{2}}}dx$$

[In] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)

[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(3/2),x)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{2\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}$$

[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")

[Out] -2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) - 1)

Sympy [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log{(cx^n)}}{n}\right)}\,dx$$

[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(3/2),x)

[Out] Integral(sinh(a + 2*log(c*x**n)/n)**(-3/2), x)

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\sinh\left(a+\frac{2\,\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")

[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-3/2), x)

Giac [A] (verification not implemented)

none

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{\sqrt{2}}{\sqrt{c^{\frac{4}{n}}e^{(3a)}-\frac{e^a}{x^4}}c^{\left(\frac{1}{n}\right)}x^2\operatorname{sgn}(x)}$$

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
[Out] -sqrt(2)/(sqrt(c^(4/n)*e^(3*a) - e^a/x^4)*c^(1/n)*x^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{3/2}}\,dx$$

```
[In] int(1/sinh(a + (2*log(c*x^n))/n)^(3/2),x)
[Out] int(1/sinh(a + (2*log(c*x^n))/n)^(3/2), x)
```

3.288 $\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}+\frac{e^{-2a}x(cx^n)^{-4/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[Out] -1/6*x*(1-1/exp(2*a)/((c*x^n)^(4/n)))/sinh(a+2*ln(c*x^n)/n)^(7/2)+1/15*x*(1
-1/exp(2*a)/((c*x^n)^(4/n)))/exp(2*a)/((c*x^n)^(4/n))/sinh(a+2*ln(c*x^n)/n)
^(7/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5636, 5644, 277, 270}

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\frac{e^{-2a}x(cx^n)^{-4/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}-\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[In] Int[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] -1/6*(x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/Sinh[a + (2*Log[c*x^n])/n]^(7/2) +
 (x*(1 - 1/(E^(2*a)*(c*x^n)^(4/n))))/(15*E^(2*a)*(c*x^n)^(4/n)*Sinh[a + (2*
Log[c*x^n])/n]^(7/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 5636

```
Int[Sinh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sinh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5644

```
Int[((e_.)*(x_))^(m_.)*Sinh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_), x_Symbol]
:> Dist[Sinh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p
), Int[(e*x)^m*x^(b*d*p)*(1 - 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[{
a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(x)}{n}\right)}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{6}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{7/2}}\,dx, x, cx^n\right)}{n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{\left(2e^{-2a}x(cx^n)^{6/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)^{7/2}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{10}{n}}}{\left(1-e^{-2a}x^{-4/n}\right)^{7/2}}\,dx, x, cx^n\right)}{3n\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$= -\frac{x\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{6\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} + \frac{e^{-2a}x(cx^n)^{-4/n}\left(1-e^{-2a}(cx^n)^{-4/n}\right)}{15\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$$
$$=\frac{\left((-2+5x^4)\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+(2+5x^4)\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)\right)\left(-\cosh\left(2a-4\right.\right.}{15x^5\sinh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

```
[In] Integrate[Sinh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

```
[Out] (((-2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (2 + 5*x^4)*Sinh[a -
 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n] +
Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Sinh[a + (2*Log[c*x^n])/n
]^(5/2))
```

Maple [F]

$$\int \frac{1}{\sinh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{7}{2}}}dx$$

```
[In] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

```
[Out] int(1/sinh(a+2*ln(c*x^n)/n)^(7/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$$
$$=-\frac{8\sqrt{\frac{1}{2}}\left(5\,x^5e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,x\right)\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1}{x^2}}e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}-3\,x^8e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-1\right)}$$

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

```
[Out] -8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) - 2*x)*sqrt((x^4*e^(2*(a*n
+ 2*log(c))/n) - 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*lo
g(c))/n) - 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/n)
- 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/sinh(a+2*ln(c*x**n)/n)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\sinh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
[Out] integrate(sinh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{4\sqrt{2}c^{\frac{7}{n}}\left(\frac{5\,e^{a}}{c^{\frac{4}{n}}\operatorname{sgn}(x)}-\frac{2\,e^{(-a)}}{c^{\frac{8}{n}}x^{4}\operatorname{sgn}(x)}\right)e^{(3\,a)}}{15\left(c^{\frac{4}{n}}e^{(3\,a)}-\frac{e^{a}}{x^{4}}\right)^{\frac{5}{2}}x^{6}}$$

```
[In] integrate(1/sinh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
[Out] -4/15*sqrt(2)*c^(7/n)*(5*e^a/(c^(4/n)*sgn(x)) - 2*e^(-a)/(c^(8/n)*x^4*sgn(x
)))*e^(3*a)/((c^(4/n)*e^(3*a) - e^a/x^4)^(5/2)*x^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sinh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\sinh\left(a+\frac{2\,\ln(c\,x^n)}{n}\right)^{7/2}}\,dx$$

```
[In] int(1/sinh(a + (2*log(c*x^n))/n)^(7/2),x)
[Out] int(1/sinh(a + (2*log(c*x^n))/n)^(7/2), x)
```

3.289 $\int \sinh\left(\frac{a}{c+dx}\right) dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sinh\left(\frac{a}{c+dx}\right) dx = -\frac{a\mathrm{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}$$

```
[Out] -a*Chi(a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5418, 5410, 3378, 3382}

$$\int \sinh\left(\frac{a}{c+dx}\right) dx = \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\mathrm{Chi}\left(\frac{a}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[a/(c + d*x)],x]
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sinh\left(\frac{a}{x}\right)\,dx, x, c+dx\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{\sinh(ax)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\ &= \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{\cosh(ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d}\\ &= -\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx = -\frac{a\text{Chi}\left(\frac{a}{c+dx}\right)}{d} + \frac{(c+dx)\sinh\left(\frac{a}{c+dx}\right)}{d}$$

```
[In] Integrate[Sinh[a/(c + d*x)],x]
[Out] -((a*CoshIntegral[a/(c + d*x)])/d) + ((c + d*x)*Sinh[a/(c + d*x)])/d
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{a\left(-\frac{(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{a}+\operatorname{Chi}\left(\frac{a}{dx+c}\right)\right)}{d}$	38
default	$-\frac{a\left(-\frac{(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{a}+\operatorname{Chi}\left(\frac{a}{dx+c}\right)\right)}{d}$	38
risch	$-\frac{e^{-\frac{a}{dx+c}}x}{2}-\frac{e^{-\frac{a}{dx+c}}c}{2d}+\frac{a\operatorname{Ei}_1\left(\frac{a}{dx+c}\right)}{2d}+\frac{e^{\frac{a}{dx+c}}x}{2}+\frac{e^{\frac{a}{dx+c}}c}{2d}+\frac{a\operatorname{Ei}_1\left(-\frac{a}{dx+c}\right)}{2d}$	99

```
[In] int(sinh(1/(d*x+c)*a),x,method=_RETURNVERBOSE)
[Out] -1/d*a*(-(d*x+c)/a*sinh(1/(d*x+c)*a)+Chi(1/(d*x+c)*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx=-\frac{a\mathrm{Ei}\left(\frac{a}{dx+c}\right)+a\mathrm{Ei}\left(-\frac{a}{dx+c}\right)-2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{2\,d}$$

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(a*Ei(a/(d*x + c)) + a*Ei(-a/(d*x + c)) - 2*(d*x + c)*sinh(a/(d*x + c)
))/d
```

Sympy [F]

$$\int \sinh\left(\frac{a}{c+dx}\right)\,dx=\int \sinh\left(\frac{a}{c+dx}\right)\,dx$$

```
[In] integrate(sinh(a/(d*x+c)),x)
[Out] Integral(sinh(a/(c + d*x)), x)
```

Maxima [F]

$$\int \sinh\left(\frac{a}{c+dx}\right) dx = \int \sinh\left(\frac{a}{dx+c}\right) dx$$

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*a*d
*integrate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/2*x*e^(a/(d
*x + c)) - 1/2*x*e^(-a/(d*x + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

$$\int \sinh\left(\frac{a}{c+dx}\right) dx = -\frac{\left(\frac{a^3 \text{Ei}\left(\frac{a}{dx+c}\right)}{dx+c} - a^2 e^{\left(\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2 d} - \frac{\left(\frac{a^3 \text{Ei}\left(-\frac{a}{dx+c}\right)}{dx+c} + a^2 e^{\left(-\frac{a}{dx+c}\right)}\right)(dx+c)}{2\,a^2 d}$$

```
[In] integrate(sinh(a/(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a^3*Ei(a/(d*x + c))/(d*x + c) - a^2*e^(a/(d*x + c)))*(d*x + c)/(a^2*d
) - 1/2*(a^3*Ei(-a/(d*x + c))/(d*x + c) + a^2*e^(-a/(d*x + c)))*(d*x + c)/(
a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(\frac{a}{c+dx}\right) dx = \int \sinh\left(\frac{a}{c+d\,x}\right) dx$$

```
[In] int(sinh(a/(c + d*x)),x)
```

```
[Out] int(sinh(a/(c + d*x)), x)
```

3.290 $\int \sinh^2\left(\frac{a}{c+dx}\right) dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \sinh^2\left(\frac{a}{c+dx}\right) dx = \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

```
[Out] -a*Shi(2*a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5418, 5410, 3394, 12, 3379}

$$\int \sinh^2\left(\frac{a}{c+dx}\right) dx = \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[a/(c + d*x)]^2,x]
```

```
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2)/d - (a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh^2\left(\frac{a}{x}\right)\,dx, x, c+dx\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh^2(ax)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} + \frac{(2ia)\text{Subst}\left(\int \frac{i\sinh(2ax)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{\sinh(2ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)}{d} - \frac{a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx=\frac{(c+dx)\sinh^2\left(\frac{a}{c+dx}\right)-a\text{Shi}\left(\frac{2a}{c+dx}\right)}{d}$$

```
[In] Integrate[Sinh[a/(c + d*x)]^2,x]
[Out] ((c + d*x)*Sinh[a/(c + d*x)]^2 - a*SinhIntegral[(2*a)/(c + d*x)])/d
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

method	result	size
derivativedivides	$-\frac{a\left(\frac{dx+c}{2a}-\frac{(dx+c)\cosh\left(\frac{2a}{dx+c}\right)}{2a}+\text{Shi}\left(\frac{2a}{dx+c}\right)\right)}{d}$	50
default	$-\frac{a\left(\frac{dx+c}{2a}-\frac{(dx+c)\cosh\left(\frac{2a}{dx+c}\right)}{2a}+\text{Shi}\left(\frac{2a}{dx+c}\right)\right)}{d}$	50
risch	$-\frac{x}{2}+\frac{e^{-\frac{2a}{dx+c}}x}{4}+\frac{e^{-\frac{2a}{dx+c}}c}{4d}-\frac{a\,\text{Ei}_1\left(\frac{2a}{dx+c}\right)}{2d}+\frac{e^{\frac{2a}{dx+c}}x}{4}+\frac{e^{\frac{2a}{dx+c}}c}{4d}+\frac{a\,\text{Ei}_1\left(-\frac{2a}{dx+c}\right)}{2d}$	103

```
[In] int(sinh(1/(d*x+c)*a)^2,x,method=_RETURNVERBOSE)
[Out] -1/d*a*(1/2*(d*x+c)/a-1/2*(d*x+c)/a*cosh(2/(d*x+c)*a)+Shi(2/(d*x+c)*a))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.87

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx = \frac{(dx+c)\cosh\left(\frac{a}{dx+c}\right)^2+(dx+c)\sinh\left(\frac{a}{dx+c}\right)^2-dx-a\text{Ei}\left(\frac{2a}{dx+c}\right)+a\text{Ei}\left(-\frac{2a}{dx+c}\right)}{2d}$$

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="fricas")
[Out] 1/2*((d*x + c)*cosh(a/(d*x + c))^2 + (d*x + c)*sinh(a/(d*x + c))^2 - d*x -
a*Ei(2*a/(d*x + c)) + a*Ei(-2*a/(d*x + c)))/d
```

Sympy [F]

$$\int \sinh^2\left(\frac{a}{c+dx}\right) dx = \int \sinh^2\left(\frac{a}{c+dx}\right) dx$$

```
[In] integrate(sinh(a/(d*x+c))**2,x)
```

```
[Out] Integral(sinh(a/(c + d*x))**2, x)
```

Maxima [F]

$$\int \sinh^2\left(\frac{a}{c+dx}\right) dx = \int \sinh\left(\frac{a}{dx+c}\right)^2 dx$$

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*d*integrate(x*e^(2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*a
*d*integrate(x*e^(-2*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*x*e^(
2*a/(d*x + c)) + 1/4*x*e^(-2*a/(d*x + c)) - 1/2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(39) = 78$.

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.49

$$\int \sinh^2\left(\frac{a}{c+dx}\right) dx = -\frac{\left(\frac{2a^3 \operatorname{Ei}\left(\frac{2a}{dx+c}\right)}{dx+c} - \frac{2a^3 \operatorname{Ei}\left(-\frac{2a}{dx+c}\right)}{dx+c} - a^2 e^{\left(\frac{2a}{dx+c}\right)} - a^2 e^{\left(-\frac{2a}{dx+c}\right)} + 2a^2\right)(dx+c)}{4a^2 d}$$

```
[In] integrate(sinh(a/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*a^3*Ei(2*a/(d*x + c))/(d*x + c) - 2*a^3*Ei(-2*a/(d*x + c))/(d*x + c
) - a^2*e^(2*a/(d*x + c)) - a^2*e^(-2*a/(d*x + c)) + 2*a^2)*(d*x + c)/(a^2*
d)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(\frac{a}{c+dx}\right)\,dx = \int \sinh\left(\frac{a}{c+d\,x}\right)^2\,dx$$

```
[In] int(sinh(a/(c + d*x))^2,x)
[Out] int(sinh(a/(c + d*x))^2, x)
```

3.291 $\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx=\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d}-\frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d}+\frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

```
[Out] 3/4*a*Chi(a/(d*x+c))/d-3/4*a*Chi(3*a/(d*x+c))/d+(d*x+c)*sinh(a/(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5418, 5410, 3394, 3382}

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx=\frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d}-\frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d}+\frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[a/(c + d*x)]^3,x]
```

```
[Out] (3*a*CoshIntegral[a/(c + d*x)])/(4*d) - (3*a*CoshIntegral[(3*a)/(c + d*x)])
/(4*d) + ((c + d*x)*Sinh[a/(c + d*x)]^3)/d
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
```

```
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5410

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol] :> -Subs
t[Int[(a + b*Sinh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5418

```
Int[((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(u_)^(n_)])^(p_.), x_Symbol] :> Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Sinh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sinh^3\left(\frac{a}{x}\right)\,dx, x, c+dx\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{\sinh^3(ax)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\text{Subst}\left(\int \left(\frac{\cosh(ax)}{4x} - \frac{\cosh(3ax)}{4x}\right)\,dx, x, \frac{1}{c+dx}\right)}{d}\\
&= \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d} + \frac{(3a)\text{Subst}\left(\int \frac{\cosh(ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d} - \frac{(3a)\text{Subst}\left(\int \frac{\cosh(3ax)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d}\\
&= \frac{3a\text{Chi}\left(\frac{a}{c+dx}\right)}{4d} - \frac{3a\text{Chi}\left(\frac{3a}{c+dx}\right)}{4d} + \frac{(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = \frac{3a\text{Chi}\left(\frac{a}{c+dx}\right) - 3a\text{Chi}\left(\frac{3a}{c+dx}\right) + 4(c+dx)\sinh^3\left(\frac{a}{c+dx}\right)}{4d}$$

```
[In] Integrate[Sinh[a/(c + d*x)]^3,x]
[Out] (3*a*CoshIntegral[a/(c + d*x)] - 3*a*CoshIntegral[(3*a)/(c + d*x)] + 4*(c +
 d*x)*Sinh[a/(c + d*x)]^3)/(4*d)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{a\left(\frac{3(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{4a}-\frac{3\,\mathrm{Chi}\left(\frac{a}{dx+c}\right)}{4}-\frac{(dx+c)\sinh\left(\frac{3a}{dx+c}\right)}{4a}+\frac{3\,\mathrm{Chi}\left(\frac{3a}{dx+c}\right)}{4}\right)}{d}$
default	$-\frac{a\left(\frac{3(dx+c)\sinh\left(\frac{a}{dx+c}\right)}{4a}-\frac{3\,\mathrm{Chi}\left(\frac{a}{dx+c}\right)}{4}-\frac{(dx+c)\sinh\left(\frac{3a}{dx+c}\right)}{4a}+\frac{3\,\mathrm{Chi}\left(\frac{3a}{dx+c}\right)}{4}\right)}{d}$
risch	$-\frac{\mathrm{e}^{-\frac{3a}{dx+c}}x}{8}-\frac{\mathrm{e}^{-\frac{3a}{dx+c}}c}{8d}+\frac{3a\,\mathrm{Ei}_1\left(\frac{3a}{dx+c}\right)}{8d}+\frac{3\,\mathrm{e}^{-\frac{a}{dx+c}}x}{8}+\frac{3\,\mathrm{e}^{-\frac{a}{dx+c}}c}{8d}-\frac{3a\,\mathrm{Ei}_1\left(\frac{a}{dx+c}\right)}{8d}+\frac{\mathrm{e}^{\frac{3a}{dx+c}}x}{8}+\frac{\mathrm{e}^{\frac{3a}{dx+c}}}{8d}$

```
[In] int(sinh(1/(d*x+c)*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*a*(3/4*(d*x+c)/a*sinh(1/(d*x+c)*a)-3/4*Chi(1/(d*x+c)*a)-1/4*(d*x+c)/a*
sinh(3/(d*x+c)*a)+3/4*Chi(3/(d*x+c)*a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(55) = 110.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.00

$$\int \sinh^3\left(\frac{a}{c+dx}\right)dx$$
$$=\frac{2\,(dx+c)\sinh\left(\frac{a}{dx+c}\right)^3-3\,a\mathrm{Ei}\left(\frac{3\,a}{dx+c}\right)+3\,a\mathrm{Ei}\left(\frac{a}{dx+c}\right)+3\,a\mathrm{Ei}\left(-\frac{a}{dx+c}\right)-3\,a\mathrm{Ei}\left(-\frac{3\,a}{dx+c}\right)+6\left((dx+c)\cosh\left(\right.\right.}{8\,d}$$

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(d*x + c)*sinh(a/(d*x + c))^3 - 3*a*Ei(3*a/(d*x + c)) + 3*a*Ei(a/(d*
x + c)) + 3*a*Ei(-a/(d*x + c)) - 3*a*Ei(-3*a/(d*x + c)) + 6*((d*x + c)*cosh
(a/(d*x + c))^2 - d*x - c)*sinh(a/(d*x + c)))/d
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a}{c+dx}\right)dx=\text{Timed out}$$

```
[In] integrate(sinh(a/(d*x+c))**3,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = \int \sinh\left(\frac{a}{dx+c}\right)^3\,dx$$

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 3/8*a*d*integrate(x*e^(3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a
*d*integrate(x*e^(a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*a*d*inte
grate(x*e^(-a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 3/8*a*d*integrate(
x*e^(-3*a/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/8*x*e^(3*a/(d*x + c)
) - 3/8*x*e^(a/(d*x + c)) + 3/8*x*e^(-a/(d*x + c)) - 1/8*x*e^(-3*a/(d*x + c
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(55) = 110$.

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.83

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = -\frac{\left(\frac{3\,a^3\mathrm{Ei}\left(\frac{3\,a}{dx+c}\right)}{dx+c} - \frac{3\,a^3\mathrm{Ei}\left(\frac{a}{dx+c}\right)}{dx+c} - \frac{3\,a^3\mathrm{Ei}\left(-\frac{a}{dx+c}\right)}{dx+c} + \frac{3\,a^3\mathrm{Ei}\left(-\frac{3\,a}{dx+c}\right)}{dx+c} - a^2e^{\left(\frac{3\,a}{dx+c}\right)} + 3\,a^2e^{\left(\frac{a}{dx+c}\right)} - 3\,a^2e^{\left(-\frac{a}{dx+c}\right)} + a^2e}{8\,a^2d}$$

```
[In] integrate(sinh(a/(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^3*Ei(3*a/(d*x + c))/(d*x + c) - 3*a^3*Ei(a/(d*x + c))/(d*x + c) -
 3*a^3*Ei(-a/(d*x + c))/(d*x + c) + 3*a^3*Ei(-3*a/(d*x + c))/(d*x + c) - a^
2*e^(3*a/(d*x + c)) + 3*a^2*e^(a/(d*x + c)) - 3*a^2*e^(-a/(d*x + c)) + a^2*
e^(-3*a/(d*x + c)))*(d*x + c)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a}{c+dx}\right)\,dx = \int {\sinh\left(\frac{a}{c+d\,x}\right)}^3\,dx$$

```
[In] int(sinh(a/(c + d*x))^3,x)
```

```
[Out] int(sinh(a/(c + d*x))^3, x)
```

3.292 $\int \sinh\left(\frac{bx}{c+dx}\right) dx$

Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \sinh\left(\frac{bx}{c+dx}\right) dx = \frac{bc\cosh\left(\frac{b}{d}\right)\operatorname{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\sinh\left(\frac{b}{d}\right)\operatorname{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}$$

```
[Out] b*c*Chi(b*c/d/(d*x+c))*cosh(b/d)/d^2-b*c*Shi(b*c/d/(d*x+c))*sinh(b/d)/d^2+(
d*x+c)*sinh(b*x/(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5726, 3378, 3384, 3379, 3382}

$$\int \sinh\left(\frac{bx}{c+dx}\right) dx = \frac{bc\cosh\left(\frac{b}{d}\right)\operatorname{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2} - \frac{bc\sinh\left(\frac{b}{d}\right)\operatorname{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] (b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/d^2 + ((c + d*x)*Sinh[(b*
x)/(c + d*x)])/d - (b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d} \\
&= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} \\
&= \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} + \frac{\left(bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} \\
&\quad - \frac{\left(bc\sinh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}
\end{aligned}
$$

$$= \frac{bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\sinh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc}{d(c+dx)}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int \sinh\left(\frac{bx}{c+dx}\right)dx = \frac{de^{-\frac{bx}{c+dx}}\left(-1+e^{\frac{2bx}{c+dx}}\right)(c+dx) + 2bc\cosh\left(\frac{b}{d}\right)\mathrm{Chi}\left(\frac{bc}{cd+d^2x}\right) - 2bc\sinh\left(\frac{b}{d}\right)\mathrm{Shi}\left(\frac{bc}{cd+d^2x}\right)}{2d^2}$$

```
[In] Integrate[Sinh[(b*x)/(c + d*x)],x]
```

```
[Out] ((d*(-1 + E^((2*b*x)/(c + d*x)))*(c + d*x))/E^((b*x)/(c + d*x)) + 2*b*c*Cos
h[b/d]*CoshIntegral[(b*c)/(c*d + d^2*x)] - 2*b*c*Sinh[b/d]*SinhIntegral[(b*
c)/(c*d + d^2*x)])/(2*d^2)
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.53

method	result	size
risch	$-\frac{e^{-\frac{bx}{dx+c}}(dx+c)}{2d} - \frac{bc\,e^{-\frac{b}{d}}\,\mathrm{Ei}_1\left(-\frac{bc}{d(dx+c)}\right)}{2d^2} + \frac{e^{\frac{bx}{dx+c}}x}{2} + \frac{c\,e^{\frac{bx}{dx+c}}}{2d} - \frac{bc\,e^{\frac{b}{d}}\,\mathrm{Ei}_1\left(\frac{bc}{d(dx+c)}\right)}{2d^2}$	113

```
[In] int(sinh(b*x/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-b*x/(d*x+c))*(d*x+c)-1/2*b*c/d^2*exp(-b/d)*Ei(1,-b*c/d/(d*x+c))
+1/2*exp(b*x/(d*x+c))*x+1/2*c/d*exp(b*x/(d*x+c))-1/2*b*c/d^2*exp(b/d)*Ei(1,
b*c/d/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.42

$$\int \sinh\left(\frac{bx}{c+dx}\right)dx =$$

$$-\frac{bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\sinh\left(\frac{bx}{dx+c}\right)^2 - \left(bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{bx}{dx+c}\right)^2 + bc\mathrm{Ei}\left(\frac{bc}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right) - 2\left(d^2x\right.}{2\left(d^2\cosh\left(\frac{bx}{dx+c}\right)^2\right.}$$

[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="fricas")

[Out] -1/2*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b/d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh(b/d) - 2*(d^2*x + c*d)*sinh(b*x/(d*x + c)) - (b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c))^2 - b*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(b*x/(d*x + c))^2)

Sympy [F]

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{c+dx}\right)\,dx$$

[In] integrate(sinh(b*x/(d*x+c)),x)

[Out] Integral(sinh(b*x/(c + d*x)), x)

Maxima [F]

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{dx+c}\right)\,dx$$

[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="maxima")

[Out] -1/2*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/d) + c^2*e^(b/d)), x) - 1/2*b*c*integrate(x*e^(-b*c/(d^2*x + c*d) + b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/2*(x*e^(b*c/(d^2*x + c*d)) - x*e^(-b*c/(d^2*x + c*d) + 2*b/d))*e^(-b/d)

Giac [F]

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{dx+c}\right)\,dx$$

[In] integrate(sinh(b*x/(d*x+c)),x, algorithm="giac")

[Out] integrate(sinh(b*x/(d*x + c)), x)

Mupad [F(-1)]

Timed out.

$$\int \sinh\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{b\,x}{c+d\,x}\right)\,dx$$

```
[In] int(sinh((b*x)/(c + d*x)),x)
```

```
[Out] int(sinh((b*x)/(c + d*x)), x)
```

3.293 $\int \sinh^2\left(\frac{bx}{c+dx}\right) dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \sinh^2\left(\frac{bx}{c+dx}\right) dx = \frac{bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}$$

```
[Out] -b*c*cosh(2*b/d)*Shi(2*b*c/d/(d*x+c))/d^2+b*c*Chi(2*b*c/d/(d*x+c))*sinh(2*b
/d)/d^2+(d*x+c)*sinh(b*x/(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5726, 3394, 12, 3384, 3379, 3382}

$$\int \sinh^2\left(\frac{bx}{c+dx}\right) dx = \frac{bc\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2} - \frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] (b*c*CoshIntegral[(2*b*c)/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2 + ((c + d*x)*Si
nh[(b*x)/(c + d*x)]^2)/d - (b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(d*(c +
d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{(2ibc)\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} + \frac{(bc)\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(bc\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left(bc\sinh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2bcx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{bc\text{Chi}\left(\frac{2bc}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{bx}{c+dx}\right)}{d} - \frac{bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{d(c+dx)}\right)}{d^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.44

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = \frac{de^{-\frac{2bx}{c+dx}}\left(c\left(1+e^{\frac{4bx}{c+dx}}\right)+d\left(-1+e^{\frac{2bx}{c+dx}}\right)^2 x\right)+4bc\text{Chi}\left(\frac{2bc}{cd+d^2x}\right)\sinh\left(\frac{2b}{d}\right)-4bc\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2bc}{cd+d^2x}\right)}{4d^2}$$

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^2,x]
```

```
[Out] ((d*(c*(1 + E^((4*b*x)/(c + d*x))) + d*(-1 + E^((2*b*x)/(c + d*x)))^2*x))/E
^((2*b*x)/(c + d*x)) + 4*b*c*CoshIntegral[(2*b*c)/(c*d + d^2*x)]*Sinh[(2*b)
/d] - 4*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c)/(c*d + d^2*x)])/(4*d^2)
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.50

method	result	size
risch	$-\frac{x}{2}+\frac{\mathrm{e}^{-\frac{2bx}{dx+c}}(dx+c)}{4d}+\frac{bc\,\mathrm{e}^{-\frac{2b}{d}}\,\mathrm{Ei}_1\left(-\frac{2bc}{d(dx+c)}\right)}{2d^2}+\frac{\mathrm{e}^{\frac{2bx}{dx+c}}x}{4}+\frac{c\,\mathrm{e}^{\frac{2bx}{dx+c}}}{4d}-\frac{bc\,\mathrm{e}^{\frac{2b}{d}}\,\mathrm{Ei}_1\left(\frac{2bc}{d(dx+c)}\right)}{2d^2}$	120

```
[In] int(sinh(b*x/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/4/d*exp(-2*b*x/(d*x+c))*(d*x+c)+1/2*b*c/d^2*exp(-2*b/d)*Ei(1,-2*b*
c/d/(d*x+c))+1/4*exp(2*b*x/(d*x+c))*x+1/4*c/d*exp(2*b*x/(d*x+c))-1/2*b*c/d^
2*exp(2*b/d)*Ei(1,2*b*c/d/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(80) = 160$.

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.46

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = -\frac{d^2x - (d^2x+cd)\cosh\left(\frac{bx}{dx+c}\right)^2 + \left(bc\mathrm{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right) - d^2x - cd\right)\sinh\left(\frac{bx}{dx+c}\right)^2 - \left(bc\mathrm{Ei}\left(-\frac{2\,bc}{d^2x+cd}\right)\right.}{2\left(d^2\right.}$$

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh(b*x/(d*x + c))^2 + (b*c*Ei(-2*b*c/(d^2*x +
 c*d))*cosh(2*b/d) - d^2*x - c*d)*sinh(b*x/(d*x + c))^2 - (b*c*Ei(-2*b*c/(d
^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(2*b*c/(d^2*x + c*d)))*cosh(2*b/
d) - (b*c*Ei(-2*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2 - b*c*Ei(-2*b*c/(d
^2*x + c*d))*sinh(b*x/(d*x + c))^2 + b*c*Ei(2*b*c/(d^2*x + c*d)))*sinh(2*b/
d))/(d^2*cosh(b*x/(d*x + c))^2 - d^2*sinh(b*x/(d*x + c))^2)
```

Sympy [F]

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx$$

```
[In] integrate(sinh(b*x/(d*x+c))**2,x)
```

```
[Out] Integral(sinh(b*x/(c + d*x))**2, x)
```

Maxima [F]

$$\int \sinh^2\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx}{dx+c}\right)^2\,dx$$

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*c*integrate(x*e^(2*b*c/(d^2*x + c*d))/(d^2*x^2*e^(2*b/d) + 2*c*d*x*e^
(2*b/d) + c^2*e^(2*b/d)), x) - 1/2*b*c*integrate(x*e^(-2*b*c/(d^2*x + c*d)
+ 2*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) + 1/4*(x*e^(2*b*c/(d^2*x + c*d)) + x
*e^(-2*b*c/(d^2*x + c*d) + 4*b/d))*e^(-2*b/d) - 1/2*x
```

Giac [F]

$$\int \sinh^2\left(\frac{bx}{c+dx}\right) dx = \int \sinh\left(\frac{bx}{dx+c}\right)^2 dx$$

```
[In] integrate(sinh(b*x/(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sinh(b*x/(d*x + c))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(\frac{bx}{c+dx}\right) dx = \int \sinh\left(\frac{b\,x}{c+d\,x}\right)^2 dx$$

```
[In] int(sinh((b*x)/(c + d*x))^2,x)
[Out] int(sinh((b*x)/(c + d*x))^2, x)
```

3.294 $\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 143

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx = -\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} + \frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} - \frac{3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}$$

```
[Out] -3/4*b*c*Chi(b*c/d/(d*x+c))*cosh(b/d)/d^2+3/4*b*c*Chi(3*b*c/d/(d*x+c))*cosh
(3*b/d)/d^2+3/4*b*c*Shi(b*c/d/(d*x+c))*sinh(b/d)/d^2-3/4*b*c*Shi(3*b*c/d/(d
*x+c))*sinh(3*b/d)/d^2+(d*x+c)*sinh(b*x/(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5726, 3394, 3384, 3379, 3382}

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx = -\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} + \frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2} + \frac{3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} - \frac{3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[(b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(d*(c + d*x))])/(4*d^2) + (3*b*c*Cosh[
(3*b)/d]*CoshIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(b*
x)/(c + d*x)]^3)/d + (3*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(d*(c + d*x))])/(4
*d^2) - (3*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(d*(c + d*x))])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{(3bc)\text{Subst}\left(\int \left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{4x} + \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{4x}\right) dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{(3bc)\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3bc)\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} - \frac{\left(3bc\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{\left(3bc\cosh\left(\frac{3b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{\left(3bc\sinh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{\left(3bc\sinh\left(\frac{3b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3bcx}{d}\right)}{x} dx, x, \frac{1}{c+dx}\right)}{4d^2}$$

$$= -\frac{3bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} + \frac{3bc\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{bx}{c+dx}\right)}{d} + \frac{3bc\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc}{d(c+dx)}\right)}{4d^2} - \frac{3bc\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3bc}{d(c+dx)}\right)}{4d^2}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.62

$$\int \sinh^3\left(\frac{bx}{c+dx}\right) dx = \frac{-cde^{-\frac{3bx}{c+dx}} + 3cde^{-\frac{bx}{c+dx}} - 3cde^{\frac{bx}{c+dx}} + cde^{\frac{3bx}{c+dx}} - d^2e^{-\frac{3bx}{c+dx}}x + d^2e^{\frac{3bx}{c+dx}}x - 6bc\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc}{cd+d^2x}\right) + 6bc\,\text{co}}{8d^2}$$

```
[In] Integrate[Sinh[(b*x)/(c + d*x)]^3,x]
```

```
[Out] (-((c*d)/E^((3*b*x)/(c + d*x))) + (3*c*d)/E^((b*x)/(c + d*x)) - 3*c*d*E^((b
*x)/(c + d*x)) + c*d*E^((3*b*x)/(c + d*x)) - (d^2*x)/E^((3*b*x)/(c + d*x))
+ d^2*E^((3*b*x)/(c + d*x))*x - 6*b*c*Cosh[b/d]*CoshIntegral[(b*c)/(c*d + d
^2*x)] + 6*b*c*Cosh[(3*b)/d]*CoshIntegral[(3*b*c)/(c*d + d^2*x)] - 6*d^2*x*
Sinh[(b*x)/(c + d*x)] + 6*b*c*Sinh[b/d]*SinhIntegral[(b*c)/(c*d + d^2*x)] -
 6*b*c*Sinh[(3*b)/d]*SinhIntegral[(3*b*c)/(c*d + d^2*x)])/(8*d^2)
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.75

method	result
risch	$\frac{3bc\,\mathrm{e}^{\frac{b}{d}}\,\mathrm{Ei}_1\left(\frac{bc}{d(dx+c)}\right)}{8d^2}+\frac{3\,\mathrm{e}^{-\frac{bx}{dx+c}}x}{8}+\frac{3bc\,\mathrm{e}^{-\frac{b}{d}}\,\mathrm{Ei}_1\left(-\frac{bc}{d(dx+c)}\right)}{8d^2}-\frac{3\,\mathrm{e}^{\frac{bx}{dx+c}}x}{8}+\frac{\mathrm{e}^{\frac{3bx}{dx+c}}x}{8}-\frac{3\,\mathrm{e}^{\frac{3b}{d}}\,\mathrm{Ei}_1\left(\frac{3bc}{d(dx+c)}\right)bc}{8d^2}-\frac{\mathrm{e}^{-\frac{3bx}{dx+c}}x}{8}$

```
[In] int(sinh(b*x/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*b*c/d^2*exp(b/d)*Ei(1,b*c/d/(d*x+c))+3/8*exp(-b*x/(d*x+c))*x+3/8*b*c/d^
2*exp(-b/d)*Ei(1,-b*c/d/(d*x+c))-3/8*exp(b*x/(d*x+c))*x+1/8*exp(3*b*x/(d*x+
c))*x-3/8/d^2*exp(3*b/d)*Ei(1,3*b*c/d/(d*x+c))*b*c-1/8*exp(-3*b*x/(d*x+c))*
x-3/8/d^2*exp(-3*b/d)*Ei(1,-3*b*c/d/(d*x+c))*b*c+3/8/d*exp(-b*x/(d*x+c))*c-
3/8*c/d*exp(b*x/(d*x+c))+1/8/d*exp(3*b*x/(d*x+c))*c-1/8/d*exp(-3*b*x/(d*x+c
))*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. $2(135) = 270$.

Time = 0.28 (sec) , antiderivative size = 701, normalized size of antiderivative = 4.90

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)dx$$
$$=\frac{3\left(bc\mathrm{Ei}\left(-\frac{3bc}{d^2x+cd}\right)\cosh\left(\frac{3b}{d}\right)-bc\mathrm{Ei}\left(-\frac{bc}{d^2x+cd}\right)\cosh\left(\frac{b}{d}\right)\right)\sinh\left(\frac{bx}{dx+c}\right)^4+2\left(d^2x+cd\right)\sinh\left(\frac{bx}{dx+c}\right)^3-6\left(bc\mathrm{Ei}\left(-\right.\right.}{}$$

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(3*b/d) - b*c*Ei(-b*c/(d^2*x + c*d
))*cosh(b/d))*sinh(b*x/(d*x + c))^4 + 2*(d^2*x + c*d)*sinh(b*x/(d*x + c))^3
 - 6*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(3*b/d) - b*c*
Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^2*cosh(b/d))*sinh(b*x/(d*x + c))
^2 + 3*(b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x + c))^4 + b*c*Ei(3*b*c/(
d^2*x + c*d)))*cosh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
c))^4 + b*c*Ei(b*c/(d^2*x + c*d)))*cosh(b/d) - 6*(d^2*x - (d^2*x + c*d)*cos
h(b*x/(d*x + c))^2 + c*d)*sinh(b*x/(d*x + c)) + 3*(b*c*Ei(-3*b*c/(d^2*x + c
*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-3*b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-3*b*c/(d^2*x + c*d))*sinh(b*x/(d*x +
 c))^4 - b*c*Ei(3*b*c/(d^2*x + c*d)))*sinh(3*b/d) - 3*(b*c*Ei(-b*c/(d^2*x +
 c*d))*cosh(b*x/(d*x + c))^4 - 2*b*c*Ei(-b*c/(d^2*x + c*d))*cosh(b*x/(d*x +
 c))^2*sinh(b*x/(d*x + c))^2 + b*c*Ei(-b*c/(d^2*x + c*d))*sinh(b*x/(d*x + c
))^4 - b*c*Ei(b*c/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh(b*x/(d*x + c))^4 - 2
*d^2*cosh(b*x/(d*x + c))^2*sinh(b*x/(d*x + c))^2 + d^2*sinh(b*x/(d*x + c))^
4)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx=\text{Timed out}$$

```
[In] integrate(sinh(b*x/(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx=\int \sinh\left(\frac{bx}{dx+c}\right)^3\,dx$$

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="maxima")
[Out] -3/8*b*c*integrate(x*e^(3*b*c/(d^2*x + c*d))/(d^2*x^2*e^(3*b/d) + 2*c*d*x*e
^(3*b/d) + c^2*e^(3*b/d)), x) + 3/8*b*c*integrate(x*e^(b*c/(d^2*x + c*d))/(
d^2*x^2*e^(b/d) + 2*c*d*x*e^(b/d) + c^2*e^(b/d)), x) + 3/8*b*c*integrate(x*
e^(-b*c/(d^2*x + c*d) + b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 3/8*b*c*integr
ate(x*e^(-3*b*c/(d^2*x + c*d) + 3*b/d)/(d^2*x^2 + 2*c*d*x + c^2), x) - 1/8*
(x*e^(3*b*c/(d^2*x + c*d)) - 3*x*e^(b*c/(d^2*x + c*d) + 2*b/d) + 3*x*e^(-b*
c/(d^2*x + c*d) + 4*b/d) - x*e^(-3*b*c/(d^2*x + c*d) + 6*b/d))*e^(-3*b/d)
```

Giac [F]

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx=\int \sinh\left(\frac{bx}{dx+c}\right)^3\,dx$$

```
[In] integrate(sinh(b*x/(d*x+c))^3,x, algorithm="giac")
[Out] integrate(sinh(b*x/(d*x + c))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{b\,x}{c+d\,x}\right)^3\,dx$$

```
[In] int(sinh((b*x)/(c + d*x))^3,x)
[Out] int(sinh((b*x)/(c + d*x))^3, x)
```

3.295 $\int \sinh\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)\,dx = \frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

```
[Out] (-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*cosh(b/d)/d^2-(-a*d+b*c)*Shi((-a*d+b*c
)/d/(d*x+c))*sinh(b/d)/d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5726, 3378, 3384, 3379, 3382}

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)\,dx = \frac{\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} - \frac{\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/d^2 + ((c +
 d*x)*Sinh[(a + b*x)/(c + d*x)])/d - ((b*c - a*d)*Sinh[b/d]*SinhIntegral[(b
*c - a*d)/(d*(c + d*x))])/d^2
```

<u>Rule 3378</u>

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{\left((bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad)\sinh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$=\frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh\left(\frac{a+bx}{c+dx}\right)}{d}-\frac{(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 302 vs. 2(101) = 202.

Time = 0.51 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.99

$$\int\sinh\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{-cde^{-\frac{a+bx}{c+dx}}+cde^{\frac{a+bx}{c+dx}}+2d^2x\cosh\left(\frac{-bc+ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right)+2d^2x\cosh\left(\frac{b}{d}\right)\sinh\left(\frac{-bc+ad}{d(c+dx)}\right)+(bc-ad)\left(\text{Chi}\left(\frac{bc-}{cd+}\right.\right.}{}$$

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)],x]
```

```
[Out] (-((c*d)/E^((a + b*x)/(c + d*x))) + c*d*E^((a + b*x)/(c + d*x)) + 2*d^2*x*C
osh[(-(b*c) + a*d)/(d*(c + d*x))]*Sinh[b/d] + 2*d^2*x*Cosh[b/d]*Sinh[(-(b*c
) + a*d)/(d*(c + d*x))] + (b*c - a*d)*(CoshIntegral[(b*c - a*d)/(c*d + d^2*
x)]*(Cosh[b/d] - Sinh[b/d]) + CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(C
osh[b/d] + Sinh[b/d]) + Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))
] + Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + Cosh[b/d]*SinhIn
tegral[(b*c - a*d)/(c*d + d^2*x)] - Sinh[b/d]*SinhIntegral[(b*c - a*d)/(c*d
 + d^2*x)]))/(2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(101) = 202.

Time = 1.08 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.44

method	result
risch	$-\frac{e^{-\frac{bx+a}{dx+c}}a}{2\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{bx+a}{dx+c}}bc}{2d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{b}{d}}\text{Ei}_1\left(\frac{ad-bc}{d(dx+c)}\right)a}{2d}-\frac{e^{-\frac{b}{d}}\text{Ei}_1\left(\frac{ad-bc}{d(dx+c)}\right)bc}{2d^2}+\frac{d\,e^{\frac{bx+a}{dx+c}}xa}{2ad-2bc}-\frac{e^{\frac{bx+a}{dx+c}}xbc}{2(ad-bc)}+\frac{e^{\frac{bx+}{dx+}}}{2ad-}$

```
[In] int(sinh((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-(b*x+a)/(d*x+c))/(d/(d*x+c)*a-b*c/(d*x+c))*a+1/2/d*exp(-(b*x+a)/(
d*x+c))/(d/(d*x+c)*a-b*c/(d*x+c))*b*c+1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x
+c))*a-1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/2*d*exp((b*x+a)/(d
*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/2*exp((b*x+
a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*
exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d
*x+c))*b*c
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.69

$$\int \sinh\left(\frac{a+bx}{c+dx}\right) dx$$
$$= \frac{\big((bc-ad)\text{Ei}\big(\frac{bc-ad}{d^2x+cd}\big)+(bc-ad)\text{Ei}\big(-\frac{bc-ad}{d^2x+cd}\big)\big)\cosh\left(\frac{b}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)-\big((bc-ad)\text{Ei}\big(\frac{bc-ad}{d^2x+cd}\big)}{2\,d^2}$$

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*cosh(b/d) + 2*(d^2*x + c*d)*sinh((b*x + a)/(d*x + c)) -
((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d
^2*x + c*d)))*sinh(b/d))/d^2
```

Sympy [F]

$$\int \sinh\left(\frac{a+bx}{c+dx}\right) dx = \int \sinh\left(\frac{a+bx}{c+dx}\right) dx$$

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x)
[Out] Integral(sinh((a + b*x)/(c + d*x)), x)
```

Maxima [F]

$$\int \sinh\left(\frac{a+bx}{c+dx}\right) dx = \int \sinh\left(\frac{bx+a}{dx+c}\right) dx$$

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="maxima")
[Out] integrate(sinh((b*x + a)/(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(101) = 202$.

Time = 1.90 (sec) , antiderivative size = 764, normalized size of antiderivative = 7.56

$$\int \sinh\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{\left(b^3c^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}} - 2\,ab^2cd\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}} - \frac{(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\frac{b}{d}}}{dx+c} + a^2bd^2\mathrm{Ei}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right.\right.}{}$$
$$+ \frac{\left(b^3c^2\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)} - 2\,ab^2cd\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)} - \frac{(bx+a)b^2c^2d\mathrm{Ei}\left(\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)e^{\left(-\frac{b}{d}\right)}}{dx+c} + a^2bd^2\mathrm{Ei}\left(\frac{b-}{}\right.\right.}{}$$

```
[In] integrate(sinh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-(
b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x +
 a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x
 + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d
)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*
e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^((b*x
 + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*
d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) + 1/2*(b^3*c^2*Ei((b - (
b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(
-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 2*
(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) -
(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) - a
^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b
*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh \left(\frac{a+bx}{c+dx}\right) \, dx = \int \sinh\left(\frac{a+b\,x}{c+d\,x}\right) \,dx$$

```
[In] int(sinh((a + b*x)/(c + d*x)),x)
[Out] int(sinh((a + b*x)/(c + d*x)), x)
```

3.296 $\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx = \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

```
[Out] -(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/d/(d*x+c))/d^2+(-a*d+b*c)*Chi(2*(-
a*d+b*c)/d/(d*x+c))*sinh(2*b/d)/d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5726, 3394, 12, 3384, 3379, 3382}

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right) dx = \frac{\sinh\left(\frac{2b}{d}\right)(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} - \frac{\cosh\left(\frac{2b}{d}\right)(bc-ad)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d])/d^2
 + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^2)/d - ((b*c - a*d)*Cosh[(2*b)/d]*S
inhIntegral[(2*(b*c - a*d))/(d*(c + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(2i(bc-ad))\text{Subst}\left(\int \frac{i\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(bc-ad)\text{Subst}\left(\int \frac{\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left((bc-ad)\cosh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)\sinh\left(\frac{2b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 475 vs. $2(107) = 214$.

Time = 2.96 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.44

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{cde^{-\frac{2(a+bx)}{c+dx}} + cde^{\frac{2(a+bx)}{c+dx}} - 2d^2x + 2d^2x\cosh\left(\frac{2b}{d}\right)\cosh\left(\frac{2(-bc+ad)}{d(c+dx)}\right) - 2(bc-ad)\text{Chi}\left(\frac{2bc-2ad}{cd+d^2x}\right)\left(\cosh\left(\frac{2b}{d}\right) - \text{s}\right.}{}$$

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^2,x]
```

```
[Out] ((c*d)/E^((2*(a + b*x))/(c + d*x)) + c*d*E^((2*(a + b*x))/(c + d*x)) - 2*d^
2*x + 2*d^2*x*Cosh[(2*b)/d]*Cosh[(2*(-(b*c) + a*d))/(d*(c + d*x))] - 2*(b*c
 - a*d)*CoshIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)]*(Cosh[(2*b)/d] - Sinh[(
2*b)/d]) + 2*(b*c - a*d)*CoshIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*(Co
sh[(2*b)/d] + Sinh[(2*b)/d]) + 2*d^2*x*Sinh[(2*b)/d]*Sinh[(2*(-(b*c) + a*d)
)/(d*(c + d*x))] + 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(
c + d*x))] - 2*a*d*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*
x))] + 2*b*c*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))] -
 2*a*d*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))] - 2*b*c
*Cosh[(2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*a*d*Cosh[(2*
b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*b*c*Sinh[(2*b)/d]*Sin
hIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] - 2*a*d*Sinh[(2*b)/d]*SinhIntegral
[(2*b*c - 2*a*d)/(c*d + d^2*x)])/(4*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(107) = 214.

Time = 7.67 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.35

method	result
risch	$-\frac{x}{2}+\frac{\mathrm{e}^{-\frac{2(bx+a)}{dx+c}}a}{\frac{4da}{dx+c}-\frac{4bc}{dx+c}}-\frac{\mathrm{e}^{-\frac{2(bx+a)}{dx+c}}bc}{4d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{\mathrm{e}^{-\frac{2b}{d}}\operatorname{Ei}_1\left(\frac{2ad-2bc}{(dx+c)d}\right)a}{2d}+\frac{\mathrm{e}^{-\frac{2b}{d}}\operatorname{Ei}_1\left(\frac{2ad-2bc}{(dx+c)d}\right)bc}{2d^2}+\frac{d\,\mathrm{e}^{\frac{2bx+2a}{dx+c}}xa}{4ad-4bc}-\frac{\mathrm{e}^{\frac{2bx+2a}{dx+c}}xbc}{4(ad-bc)}+$

```
[In] int(sinh((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-b*c/(d*x+c))*a-1/4/d*exp(-2
*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-b*c/(d*x+c))*b*c-1/2/d*exp(-2*b/d)*Ei(1,2*(a
*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*b*c+1/
4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*
c)*x*b*c+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+a)/(d*x+
c))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*a-1/2/d^2
*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*b*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(107) = 214.

Time = 0.26 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.46

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx =$$

$$-\frac{d^2x-(d^2x+cd)\cosh\left(\frac{bx+a}{dx+c}\right)^2-\left(d^2x-(bc-ad)\operatorname{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right)+cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2-\left((bc-\right.}{}$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 - (d^2*x - (b*c - a
*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*x
 + c))^2 - ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*
x + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) - ((b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 - (b*
c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2 + (b*
c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x + a)/
(d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

Sympy [F]

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))**2,x)
[Out] Integral(sinh((a + b*x)/(c + d*x))**2, x)
```

Maxima [F]

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx+a}{dx+c}\right)^2\,dx$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
[Out] -1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x)
+ 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(107) = 214$.

Time = 8.41 (sec) , antiderivative size = 749, normalized size of antiderivative = 7.00

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{\left(2\,b^3c^2\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)} - 4\,ab^2cd\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)} - \frac{2\,(bx+a)b^2c^2d\mathrm{Ei}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)e^{\left(\frac{2\,b}{d}\right)}}{dx+c} + \cdots$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
[Out] 1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*d
*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*Ei(
-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(b -
(b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b - (b*
x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*d/(d*x
```

```
+ c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(d*x + c)
)/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)*e^(
-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d*x + c))/d)
*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) + b^2*c^2*d*e^
(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x + c)) + a^2*d^3
*e^(-2*(b*x + a)/(d*x + c)) - 2*b^2*c^2*d + 4*a*b*c*d^2 - 2*a^2*d^3)*(b*c/(
b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(\frac{a+bx}{c+dx}\right)\,dx=\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

```
[In] int(sinh((a + b*x)/(c + d*x))^2,x)
[Out] int(sinh((a + b*x)/(c + d*x))^2, x)
```

3.297 $\int \sinh^3\left(\frac{a+bx}{c+dx}\right) dx$

Optimal result

Integrand size = 16, antiderivative size = 194

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right) dx = -\frac{3(bc-ad)\cosh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)\cosh\left(\frac{3b}{d}\right)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{3(bc-ad)\sinh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\sinh\left(\frac{3b}{d}\right)\text{Shi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}$$

```
[Out] -3/4*(-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*cosh(b/d)/d^2+3/4*(-a*d+b*c)*Chi(
3*(-a*d+b*c)/d/(d*x+c))*cosh(3*b/d)/d^2+3/4*(-a*d+b*c)*Shi((-a*d+b*c)/d/(d*
x+c))*sinh(b/d)/d^2-3/4*(-a*d+b*c)*Shi(3*(-a*d+b*c)/d/(d*x+c))*sinh(3*b/d)/
d^2+(d*x+c)*sinh((b*x+a)/(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used

$= \{5726, 3394, 3384, 3379, 3382\}$

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{3\cosh\left(\frac{b}{d}\right)(bc-ad)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} + \frac{3\cosh\left(\frac{3b}{d}\right)(bc-ad)\text{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{3\sinh\left(\frac{b}{d}\right)(bc-ad)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3\sinh\left(\frac{3b}{d}\right)(bc-ad)\text{Shi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (-3*(b*c - a*d)*Cosh[b/d]*CoshIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2)
+ (3*(b*c - a*d)*Cosh[(3*b)/d]*CoshIntegral[(3*(b*c - a*d))/(d*(c + d*x))])
/(4*d^2) + ((c + d*x)*Sinh[(a + b*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*Sinh[
b/d]*SinhIntegral[(b*c - a*d)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*Sinh
[(3*b)/d]*SinhIntegral[(3*(b*c - a*d))/(d*(c + d*x))])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

```
LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(3(bc-ad))\text{Subst}\left(\int\left(-\frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{4x}+\frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{(3(bc-ad))\text{Subst}\left(\int \frac{\cosh\left(\frac{3b}{d}-\frac{3(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{\cosh\left(\frac{b}{d}-\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{\left(3(bc-ad)\cosh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{\left(3(bc-ad)\cosh\left(\frac{3b}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{3(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} + \frac{\left(3(bc-ad)\sinh\left(\frac{b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{\left(3(bc-ad)\sinh\left(\frac{3b}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{3(bc-ad)x}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2}$$

$$= -\frac{3(bc-ad)\cosh\left(\frac{b}{d}\right)\operatorname{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} + \frac{3(bc-ad)\cosh\left(\frac{3b}{d}\right)\operatorname{Chi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sinh^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{3(bc-ad)\sinh\left(\frac{b}{d}\right)\operatorname{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\sinh\left(\frac{3b}{d}\right)\operatorname{Shi}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 651 vs. $2(194) = 388$.

Time = 5.82 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.36

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = \frac{-cde^{-\frac{3(a+bx)}{c+dx}} + 3cde^{-\frac{a+bx}{c+dx}} - 3cde^{\frac{a+bx}{c+dx}} + cde^{\frac{3(a+bx)}{c+dx}} - 6d^2x\cosh\left(\frac{-bc+ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right) + 2d^2x\cosh\left(\frac{3(-bc+ad)}{d(c+dx)}\right)\operatorname{si}}{}$$

```
[In] Integrate[Sinh[(a + b*x)/(c + d*x)]^3,x]
```

```
[Out] (-((c*d)/E^((3*(a + b*x))/(c + d*x))) + (3*c*d)/E^((a + b*x)/(c + d*x)) - 3
*c*d*E^((a + b*x)/(c + d*x)) + c*d*E^((3*(a + b*x))/(c + d*x)) - 6*d^2*x*Co
sh[(-(b*c) + a*d)/(d*(c + d*x))]*Sinh[b/d] + 2*d^2*x*Cosh[(3*(-(b*c) + a*d)
)/(d*(c + d*x))]*Sinh[(3*b)/d] - 6*d^2*x*Cosh[b/d]*Sinh[(-(b*c) + a*d)/(d*(
c + d*x))] + 2*d^2*x*Cosh[(3*b)/d]*Sinh[(3*(-(b*c) + a*d))/(d*(c + d*x))] +
 3*(b*c - a*d)*(Cosh[(3*b)/d]*CoshIntegral[(3*b*c - 3*a*d)/(c*d + d^2*x)] -
 Cosh[b/d]*CoshIntegral[(b*c - a*d)/(c*d + d^2*x)] + CoshIntegral[(b*c - a*
d)/(c*d + d^2*x)]*Sinh[b/d] - CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(C
osh[b/d] + Sinh[b/d]) - CoshIntegral[(3*b*c - 3*a*d)/(c*d + d^2*x)]*Sinh[(3
*b)/d] + CoshIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))]*(Cosh[(3*b)/d] + Si
nh[(3*b)/d]) - Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - Sinh[
b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + Cosh[(3*b)/d]*SinhIntegra
l[(3*(-(b*c) + a*d))/(d*(c + d*x))] + Sinh[(3*b)/d]*SinhIntegral[(3*(-(b*c)
 + a*d))/(d*(c + d*x))] + Cosh[(3*b)/d]*SinhIntegral[(3*b*c - 3*a*d)/(c*d +
 d^2*x)] - Sinh[(3*b)/d]*SinhIntegral[(3*b*c - 3*a*d)/(c*d + d^2*x)] - Cosh
[b/d]*SinhIntegral[(b*c - a*d)/(c*d + d^2*x)] + Sinh[b/d]*SinhIntegral[(b*c
 - a*d)/(c*d + d^2*x)]))/(8*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(186) = 372$.

Time = 1.89 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.61

method	result
risch	$-\frac{e^{-\frac{3(bx+a)}{dx+c}}a}{8\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{e^{-\frac{3(bx+a)}{dx+c}}bc}{8d\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}+\frac{3\,e^{-\frac{3b}{d}}\,\mathrm{Ei}_1\left(\frac{3ad-3bc}{(dx+c)d}\right)a}{8d}-\frac{3\,e^{-\frac{3b}{d}}\,\mathrm{Ei}_1\left(\frac{3ad-3bc}{(dx+c)d}\right)bc}{8d^2}+\frac{3\,e^{-\frac{bx+a}{dx+c}}a}{8\left(\frac{da}{dx+c}-\frac{bc}{dx+c}\right)}-\frac{3\,e^{-}}{8d\left(\frac{da}{dx+}\right.}$

```
[In] int(sinh((b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*exp(-3*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-b*c/(d*x+c))*a+1/8/d*exp(-3*(b*x+
a)/(d*x+c))/(d/(d*x+c)*a-b*c/(d*x+c))*b*c+3/8/d*exp(-3*b/d)*Ei(1,3*(a*d-b*c
)/d/(d*x+c))*a-3/8/d^2*exp(-3*b/d)*Ei(1,3*(a*d-b*c)/d/(d*x+c))*b*c+3/8*exp(
-(b*x+a)/(d*x+c))/(d/(d*x+c)*a-b*c/(d*x+c))*a-3/8/d*exp(-(b*x+a)/(d*x+c))/(
d/(d*x+c)*a-b*c/(d*x+c))*b*c-3/8/d*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*a+3/
8/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/8*d*exp(3*(b*x+a)/(d*x+c))/
(a*d-b*c)*x*a-1/8*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*x*b*c+1/8*exp(3*(b*x+a)/
(d*x+c))/(a*d-b*c)*c*a-1/8/d*exp(3*(b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b+3/8/d*e
xp(3*b/d)*Ei(1,-3*(a*d-b*c)/d/(d*x+c))*a-3/8/d^2*exp(3*b/d)*Ei(1,-3*(a*d-b*
c)/d/(d*x+c))*b*c-3/8*d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*a+3/8*exp((b*x+a)/
(d*x+c))/(a*d-b*c)*x*b*c-3/8*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c*a+3/8/d*exp((
b*x+a)/(d*x+c))/(a*d-b*c)*c^2*b-3/8/d*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a
+3/8/d^2*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*b*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 717 vs. $2(186) = 372$.

Time = 0.28 (sec) , antiderivative size = 717, normalized size of antiderivative = 3.70

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{6\,(bc-ad)\mathrm{Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{bx+a}{dx+c}\right)^2\cosh\left(\frac{3\,b}{d}\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2-3\,(bc-ad)\mathrm{Ei}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{3\,b}{d}\right)\sinh}{}$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x +
c))^2*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^2 - 3*(b*c - a*d)*Ei(-3*(b*c -
a*d)/(d^2*x + c*d))*cosh(3*b/d)*sinh((b*x + a)/(d*x + c))^4 - 2*(d^2*x + c*
d)*sinh((b*x + a)/(d*x + c))^3 - 3*((b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x +
c*d))*cosh((b*x + a)/(d*x + c))^4 + (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c
*d)))*cosh(3*b/d) + 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a
```

```
*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d) + 6*(d^2*x - (d^2*x + c*d)*co
sh((b*x + a)/(d*x + c))^2 + c*d)*sinh((b*x + a)/(d*x + c)) - 3*((b*c - a*d)
*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^4 - 2*(b*c - a*
d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2*sinh((b*x +
 a)/(d*x + c))^2 + (b*c - a*d)*Ei(-3*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x +
 a)/(d*x + c))^4 - (b*c - a*d)*Ei(3*(b*c - a*d)/(d^2*x + c*d)))*sinh(3*b/d)
 - 3*((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*
d)/(d^2*x + c*d)))*sinh(b/d))/(d^2*cosh((b*x + a)/(d*x + c))^4 - 2*d^2*cosh
((b*x + a)/(d*x + c))^2*sinh((b*x + a)/(d*x + c))^2 + d^2*sinh((b*x + a)/(d
*x + c))^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Timed out}$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sinh\left(\frac{bx+a}{dx+c}\right)^3\,dx$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(sinh((b*x + a)/(d*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1383 vs. $2(186) = 372$.

Time = 9.58 (sec) , antiderivative size = 1383, normalized size of antiderivative = 7.13

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Too large to display}$$

```
[In] integrate(sinh((b*x+a)/(d*x+c))^3,x, algorithm="giac")
[Out] 1/8*(3*b^3*c^2*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 6*a*b^2*c*d
*Ei(-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) - 3*(b*x + a)*b^2*c^2*d*Ei(
-3*(b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) + 3*a^2*b*d^2*Ei(-3*(
```

```
b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d) + 6*(b*x + a)*a*b*c*d^2*Ei(-3*(b -
(b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*(b*x + a)*a^2*d^3*Ei(-3*(
b - (b*x + a)*d/(d*x + c))/d)*e^(3*b/d)/(d*x + c) - 3*b^3*c^2*Ei(-(b - (b*x
 + a)*d/(d*x + c))/d)*e^(b/d) + 6*a*b^2*c*d*Ei(-(b - (b*x + a)*d/(d*x + c))
/d)*e^(b/d) + 3*(b*x + a)*b^2*c^2*d*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b
/d)/(d*x + c) - 3*a^2*b*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 6*
(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) +
3*(b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) -
3*b^3*c^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 6*a*b^2*c*d*Ei((b -
(b*x + a)*d/(d*x + c))/d)*e^(-b/d) + 3*(b*x + a)*b^2*c^2*d*Ei((b - (b*x + a
)*d/(d*x + c))/d)*e^(-b/d)/(d*x + c) - 3*a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x
 + c))/d)*e^(-b/d) - 6*(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d
)*e^(-b/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d
)*e^(-b/d)/(d*x + c) + 3*b^3*c^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*
b/d) - 6*a*b^2*c*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d) - 3*(b*x
+ a)*b^2*c^2*d*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) + 3
*a^2*b*d^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d) + 6*(b*x + a)*a*b
*c*d^2*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) - 3*(b*x +
a)*a^2*d^3*Ei(3*(b - (b*x + a)*d/(d*x + c))/d)*e^(-3*b/d)/(d*x + c) + b^2*c
^2*d*e^(3*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(3*(b*x + a)/(d*x + c)) + a^
2*d^3*e^(3*(b*x + a)/(d*x + c)) - 3*b^2*c^2*d*e^((b*x + a)/(d*x + c)) + 6*a
*b*c*d^2*e^((b*x + a)/(d*x + c)) - 3*a^2*d^3*e^((b*x + a)/(d*x + c)) + 3*b^
2*c^2*d*e^(-(b*x + a)/(d*x + c)) - 6*a*b*c*d^2*e^(-(b*x + a)/(d*x + c)) + 3
*a^2*d^3*e^(-(b*x + a)/(d*x + c)) - b^2*c^2*d*e^(-3*(b*x + a)/(d*x + c)) +
2*a*b*c*d^2*e^(-3*(b*x + a)/(d*x + c)) - a^2*d^3*e^(-3*(b*x + a)/(d*x + c))
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(\frac{a+bx}{c+dx}\right)\,dx=\int \sinh\left(\frac{a+b\,x}{c+d\,x}\right)^3\,dx$$

```
[In] int(sinh((a + b*x)/(c + d*x))^3,x)
[Out] int(sinh((a + b*x)/(c + d*x))^3, x)
```

3.298 $\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right) dx = \frac{(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}$$

```
[Out] (-a*d+b*c)*f*Chi((-a*d+b*c)*f/d/(d*x+c))*cosh(e+b*f/d)/d^2-(-a*d+b*c)*f*Shi
((-a*d+b*c)*f/d/(d*x+c))*sinh(e+b*f/d)/d^2+(d*x+c)*sinh((b*f*x+d*e*x+a*f+c*
e)/(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {5728, 5726, 3378, 3384, 3379, 3382}

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right) dx = \frac{f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} - \frac{f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] ((b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))
])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)])/d - ((b*c
- a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b*c - a*d)*f)/(d*(c + d*x))])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5728

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\text{integral} = \int \sinh\left(\frac{ce+af+(de+bf)x}{c+dx}\right)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{((bc-ad)f)\text{Subst}\left(\int \frac{\cosh\left(\frac{de+bf}{d}-\frac{(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{\left((bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} - \frac{\left((bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2} + \frac{(c+dx)\sinh\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 363 vs. $2(121) = 242$.

Time = 1.59 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.00

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$$
$$= \frac{-cde^{-\frac{ce+af+dex+bfx}{c+dx}} + cde^{\frac{ce+af+dex+bfx}{c+dx}} + 2d^2x\cosh\left(\frac{-bcf+adf}{d(c+dx)}\right)\sinh\left(e+\frac{bf}{d}\right) + 2d^2x\cosh\left(e+\frac{bf}{d}\right)\sinh\left(\frac{-bcf+}{d(c+d}\right.}{}$$

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)],x]
```

```
[Out] (-((c*d)/E^((c*e + a*f + d*e*x + b*f*x)/(c + d*x))) + c*d*E^((c*e + a*f + d
*e*x + b*f*x)/(c + d*x)) + 2*d^2*x*Cosh[(-(b*c*f) + a*d*f)/(d*(c + d*x))]*S
inh[e + (b*f)/d] + 2*d^2*x*Cosh[e + (b*f)/d]*Sinh[(-(b*c*f) + a*d*f)/(d*(c
+ d*x))] + (b*c - a*d)*f*(CoshIntegral[((b*c - a*d)*f)/(d*(c + d*x))]*(Cosh
[e + (b*f)/d] - Sinh[e + (b*f)/d]) + CoshIntegral[(-(b*c*f) + a*d*f)/(d*(c
```

```
+ d*x))]*(Cosh[e + (b*f)/d] + Sinh[e + (b*f)/d]) + Cosh[e + (b*f)/d]*SinhIn
tegral[((b*c - a*d)*f)/(d*(c + d*x))] - Sinh[e + (b*f)/d]*SinhIntegral[((b*
c - a*d)*f)/(d*(c + d*x))] + Cosh[e + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d
*f)/(d*(c + d*x))] + Sinh[e + (b*f)/d]*SinhIntegral[(-(b*c*f) + a*d*f)/(d*(
c + d*x))]))/(2*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(121) = 242$.

Time = 1.66 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.83

method	result
risch	$-\frac{e^{-\frac{bfx+dex+af+ce}{dx+c}} af}{2\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{e^{-\frac{bfx+dex+af+ce}{dx+c}} bcf}{2d\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{e^{-\frac{fb+de}{d}} \operatorname{Ei}_1\left(\frac{adf-bcf}{d(dx+c)}\right) af}{2d}-\frac{e^{-\frac{fb+de}{d}} \operatorname{Ei}_1\left(\frac{adf-bcf}{d(dx+c)}\right) bcf}{2d^2}+\frac{e^{\frac{bfx+dex+a}{dx+c}}}{2d\left(\frac{fa}{dx+c}-\frac{}{(c}\right.}$

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)*a*
f+1/2/d*exp(-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)
*b*c*f+1/2/d*exp(-(b*f+d*e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*a*f-1/2/d^2*
exp(-(b*f+d*e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+1/2/d*exp((b*f*x+d*
e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*a*f-1/2/d^2*exp((b*f*
x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*b*c*f+1/2/d*exp((
b*f+d*e)/d)*Ei(1,-1/d*(a*d*f-b*c*f)/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*a*f-1
/2/d^2*exp((b*f+d*e)/d)*Ei(1,-1/d*(a*d*f-b*c*f)/(d*x+c)-(b*f+d*e)/d-(-b*f-d
*e)/d)*b*c*f
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.67

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx = \frac{\left((bc-ad)f\operatorname{Ei}\left(\frac{(bc-ad)f}{d^2x+cd}\right)+(bc-ad)f\operatorname{Ei}\left(-\frac{(bc-ad)f}{d^2x+cd}\right)\right)\cosh\left(\frac{de+bf}{d}\right)+2\left(d^2x+cd\right)\sinh\left(\frac{ce+af+(de+bf)x}{dx+c}\right)-}{2\,d^2}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-(b*
c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 2*(d^2*x + c*d)*sinh((c*e
+ a*f + (d*e + b*f)*x)/(d*x + c)) - ((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x
+ c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(d^2*x + c*d)))*sinh((d*e + b*f)/
d))/d^2
```

Sympy [F]

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)dx$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x)
[Out] Integral(sinh(e + f*(a + b*x)/(c + d*x)), x)
```

Maxima [F]

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{(bx+a)f}{dx+c}\right)\,dx$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="maxima")
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1624 vs. $2(121) = 242$.

Time = 5.02 (sec) , antiderivative size = 1624, normalized size of antiderivative = 13.42

$$\int \sinh\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\text{Too large to display}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c)),x, algorithm="giac")
[Out] 1/2*(b^2*c^2*d*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x +
c))/d)*e^((d*e + b*f)/d) - 2*a*b*c*d^2*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*
x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + a^2*d^3*e*f^2*Ei(-(d*e +
 b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) + b^3*
c^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*
e + b*f)/d) - 2*a*b^2*c*d*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*
d/(d*x + c))/d)*e^((d*e + b*f)/d) + a^2*b*d^2*f^3*Ei(-(d*e + b*f - (d*e*x +
 b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - (d*e*x + b*f*x + c*
e + a*f)*b^2*c^2*d*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x
+ c))/d)*e^((d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f*x + c*e + a*f)*a*b*c*
d^2*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*
e + b*f)/d)/(d*x + c) - (d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-(d*e +
b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x +
c) + b^2*c^2*d*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 2*a*b*c*d^2*
```

```
f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + a^2*d^3*f^2*e^((d*e*x + b*f
*x + c*e + a*f)/(d*x + c)))*((d*e + b*f)*c/(b*c*f - a*d*f)^2 - (c*e + a*f)*
d/(b*c*f - a*d*f)^2)/(d^3*e + b*d^2*f - (d*e*x + b*f*x + c*e + a*f)*d^3/(d*
x + c)) + 1/2*(b^2*c^2*d*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*
d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 2*a*b*c*d^2*e*f^2*Ei((d*e + b*f - (d*e
*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + a^2*d^3*e*f^2*
Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)
/d) + b^3*c^2*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/
d)*e^(-(d*e + b*f)/d) - 2*a*b^2*c*d*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*
e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + a^2*b*d^2*f^3*Ei((d*e + b*f -
 (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - (d*e*x +
b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f
)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f*x + c*e + a
*f)*a*b*c*d^2*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/
d)*e^(-(d*e + b*f)/d)/(d*x + c) - (d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*E
i((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/
d)/(d*x + c) - b^2*c^2*d*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + 2
*a*b*c*d^2*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - a^2*d^3*f^2*e^(
-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)))*((d*e + b*f)*c/(b*c*f - a*d*f)^2 -
 (c*e + a*f)*d/(b*c*f - a*d*f)^2)/(d^3*e + b*d^2*f - (d*e*x + b*f*x + c*e +
 a*f)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh \left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)\,dx$$

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x)),x)
```

```
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x)), x)
```

3.299 $\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\frac{(bc-ad)f\,\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{(bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}$$

```
[Out] -(-a*d+b*c)*f*cosh(2*e+2*b*f/d)*Shi(2*(-a*d+b*c)*f/d/(d*x+c))/d^2+(-a*d+b*c
)*f*Chi(2*(-a*d+b*c)*f/d/(d*x+c))*sinh(2*e+2*b*f/d)/d^2+(d*x+c)*sinh((b*f*x
+d*e*x+a*f+c*e)/(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5728, 5726, 3394, 12, 3384, 3379, 3382}

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\frac{f(bc-ad)\sinh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}-\frac{f(bc-ad)\cosh\left(2\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sinh^2\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] ((b*c - a*d)*f*CoshIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))]*Sinh[2*(e + (b
*f)/d)])/d^2 + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b*f*x)/(c + d*x)]^2)/d
- ((b*c - a*d)*f*Cosh[2*(e + (b*f)/d)]*SinhIntegral[(2*(b*c - a*d)*f)/(d*(c
 + d*x))])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5728

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\text{integral} = \int \sinh^2\left(\frac{ce+af+(de+bf)x}{c+dx}\right)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh^2\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(2i(bc-ad)f)\text{Subst}\left(\int \frac{i\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)fx}{d}\right)}{2x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{((bc-ad)f)\text{Subst}\left(\int \frac{\sinh\left(2\left(e+\frac{bf}{d}\right)-\frac{2(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{\left((bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2} + \frac{\left((bc-ad)f\sinh\left(2\left(e+\frac{bf}{d}\right)\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(bc-ad)f\,\text{Chi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)\sinh\left(2\left(e+\frac{bf}{d}\right)\right)}{d^2} + \frac{(c+dx)\sinh^2\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(bc-ad)f\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\text{Shi}\left(\frac{2(bc-ad)f}{d(c+dx)}\right)}{d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 572 vs. $2(129) = 258$.

Time = 3.56 (sec) , antiderivative size = 572, normalized size of antiderivative = 4.43

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$$

$$= \frac{cde^{-\frac{2(ce+af+dex+bfx)}{c+dx}} + cde^{\frac{2(ce+af+dex+bfx)}{c+dx}} + 2d^2x\cosh\left(2\left(e+\frac{bf}{d}\right)\right)\cosh\left(\frac{2(-bcf+adf)}{d(c+dx)}\right) + 2d^2x\sinh\left(2\left(e+\frac{bf}{d}\right)\right)\text{s}}{}$$

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^2,x]
```

```
[Out] ((c*d)/E^((2*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)) + c*d*E^((2*(c*e + a*f
 + d*e*x + b*f*x))/(c + d*x)) + 2*d^2*x*Cosh[2*(e + (b*f)/d)]*Cosh[(2*(-(b*
c*f) + a*d*f))/(d*(c + d*x))] + 2*d^2*x*Sinh[2*(e + (b*f)/d)]*Sinh[(2*(-(b*
c*f) + a*d*f))/(d*(c + d*x))] - 2*(d^2*x + (b*c - a*d)*f*CoshIntegral[(2*(b
*c - a*d)*f)/(d*(c + d*x))]*(Cosh[2*(e + (b*f)/d)] - Sinh[2*(e + (b*f)/d)])
 - (b*c - a*d)*f*CoshIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))]*(Cosh[2
*(e + (b*f)/d)] + Sinh[2*(e + (b*f)/d)]) + b*c*f*Cosh[2*(e + (b*f)/d)]*Sinh
Integral[(2*(b*c - a*d)*f)/(d*(c + d*x))] - a*d*f*Cosh[2*(e + (b*f)/d)]*Sin
hIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))] - b*c*f*Sinh[2*(e + (b*f)/d)]*Si
nhIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))] + a*d*f*Sinh[2*(e + (b*f)/d)]*S
inhIntegral[(2*(b*c - a*d)*f)/(d*(c + d*x))] - b*c*f*Cosh[2*(e + (b*f)/d)]*
SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))] + a*d*f*Cosh[2*(e + (b*f
)/d)]*SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))] - b*c*f*Sinh[2*(e
+ (b*f)/d)]*SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))] + a*d*f*Sinh
[2*(e + (b*f)/d)]*SinhIntegral[(2*(-(b*c*f) + a*d*f))/(d*(c + d*x))]))/(4*d
^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(131) = 262$.

Time = 9.18 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.66

method	result
risch	$-\frac{x}{2}+\frac{\mathrm{e}^{-\frac{2(bfx+dex+af+ce)}{dx+c}}af}{\frac{4dfa}{dx+c}-\frac{4bcf}{dx+c}}-\frac{\mathrm{e}^{-\frac{2(bfx+dex+af+ce)}{dx+c}}bcf}{4d\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}-\frac{\mathrm{e}^{-\frac{2(fb+de)}{d}}\mathrm{Ei}_1\left(\frac{2adf-2bcf}{(dx+c)d}\right)af}{2d}+\frac{\mathrm{e}^{-\frac{2(fb+de)}{d}}\mathrm{Ei}_1\left(\frac{2adf-2bcf}{(dx+c)d}\right)}{2d^2}$

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x+1/4*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b
*c*f)*a*f-1/4/d*exp(-2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x
+c)*b*c*f)*b*c*f-1/2/d*exp(-2*(b*f+d*e)/d)*Ei(1,2/d*(a*d*f-b*c*f)/(d*x+c))*
a*f+1/2/d^2*exp(-2*(b*f+d*e)/d)*Ei(1,2/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+1/4/d
*exp(2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*a*f-1
/4/d^2*exp(2*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)
*b*c*f+1/2/d*exp(2*(b*f+d*e)/d)*Ei(1,-2/d*(a*d*f-b*c*f)/(d*x+c)-2*(b*f+d*e)
/d-2*(-b*f-d*e)/d)*a*f-1/2/d^2*exp(2*(b*f+d*e)/d)*Ei(1,-2/d*(a*d*f-b*c*f)/(
d*x+c)-2*(b*f+d*e)/d-2*(-b*f-d*e)/d)*b*c*f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(131) = 262.

Time = 0.28 (sec) , antiderivative size = 477, normalized size of antiderivative = 3.70

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right) dx =$$

$$-\frac{d^2x-(d^2x+cd)\cosh\left(\frac{ce+af+(de+bf)x}{dx+c}\right)^2+\left((bc-ad)f\mathrm{Ei}\left(-\frac{2(bc-ad)f}{d^2x+cd}\right)\cosh\left(\frac{2(de+bf)}{d}\right)-d^2x-cd\right)\sinh}{}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d^2*x - (d^2*x + c*d)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 +
 ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh(2*(d*e + b*f)/d) -
d^2*x - c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 - ((b*c - a*d)*f
*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x +
 c))^2 - (b*c - a*d)*f*Ei(2*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(2*(d*e + b*f
)/d) - ((b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f +
(d*e + b*f)*x)/(d*x + c))^2 - (b*c - a*d)*f*Ei(-2*(b*c - a*d)*f/(d^2*x + c*
d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*Ei(2*(b*c
 - a*d)*f/(d^2*x + c*d)))*sinh(2*(d*e + b*f)/d))/(d^2*cosh((c*e + a*f + (d*
e + b*f)*x)/(d*x + c))^2 - d^2*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^
2)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right) dx = \text{Timed out}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{(bx+a)f}{dx+c}\right)^2\,dx$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*x + 1/4*integrate(e^(2*b*c*f/(d^2*x + c*d) - 2*e - 2*a*f/(d*x + c) - 2
*b*f/d), x) + 1/4*integrate(e^(-2*b*c*f/(d^2*x + c*d) + 2*e + 2*a*f/(d*x +
c) + 2*b*f/d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1596 vs. $2(131) = 262$.

Time = 22.53 (sec) , antiderivative size = 1596, normalized size of antiderivative = 12.37

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\text{Too large to display}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^2*c^2*d*e*f^2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*
x + c))/d)*e^(2*(d*e + b*f)/d) - 4*a*b*c*d^2*e*f^2*Ei(-2*(d*e + b*f - (d*e*
x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d) + 2*a^2*d^3*e*f^
2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e
+ b*f)/d) + 2*b^3*c^2*f^3*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/
(d*x + c))/d)*e^(2*(d*e + b*f)/d) - 4*a*b^2*c*d*f^3*Ei(-2*(d*e + b*f - (d*e
*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d) + 2*a^2*b*d^2*f
^3*Ei(-2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e
 + b*f)/d) - 2*b^2*c^2*d*e*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f
)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) + 4*a*b*c*d^2*e*f^2*Ei(2*(d*e + b*f
- (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) - 2*a^2*
d^3*e*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(
-2*(d*e + b*f)/d) - 2*b^3*c^2*f^3*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e +
a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) + 4*a*b^2*c*d*f^3*Ei(2*(d*e + b*f
 - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d) - 2*a^2
*b*d^2*f^3*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^
(-2*(d*e + b*f)/d) - 2*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(-2*(d*e
 + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d)/(d
*x + c) + 4*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(-2*(d*e + b*f - (d
*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d)/(d*x + c) - 2
*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-2*(d*e + b*f - (d*e*x + b*f*x
+ c*e + a*f)*d/(d*x + c))/d)*e^(2*(d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f
```

```
*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)
*d/(d*x + c))/d)*e^(-2*(d*e + b*f)/d)/(d*x + c) - 4*(d*e*x + b*f*x + c*e +
a*f)*a*b*c*d^2*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c
))/d)*e^(-2*(d*e + b*f)/d)/(d*x + c) + 2*(d*e*x + b*f*x + c*e + a*f)*a^2*d^
3*f^2*Ei(2*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-2*(
d*e + b*f)/d)/(d*x + c) + b^2*c^2*d*f^2*e^(2*(d*e*x + b*f*x + c*e + a*f)/(d
*x + c)) - 2*a*b*c*d^2*f^2*e^(2*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + a^
2*d^3*f^2*e^(2*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + b^2*c^2*d*f^2*e^(-2
*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 2*a*b*c*d^2*f^2*e^(-2*(d*e*x + b*
f*x + c*e + a*f)/(d*x + c)) + a^2*d^3*f^2*e^(-2*(d*e*x + b*f*x + c*e + a*f)
/(d*x + c)) - 2*b^2*c^2*d*f^2 + 4*a*b*c*d^2*f^2 - 2*a^2*d^3*f^2)*((d*e + b*
f)*c/(b*c*f - a*d*f)^2 - (c*e + a*f)*d/(b*c*f - a*d*f)^2)/(d^3*e + b*d^2*f
- (d*e*x + b*f*x + c*e + a*f)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^2\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)^2\,dx$$

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x))^2,x)
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x))^2, x)
```

3.300 $\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=-\frac{3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}+\frac{3(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}-\frac{3(bc-ad)f\sinh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Shi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}$$

```
[Out] -3/4*(-a*d+b*c)*f*Chi((-a*d+b*c)*f/d/(d*x+c))*cosh(e+b*f/d)/d^2+3/4*(-a*d+b
*c)*f*Chi(3*(-a*d+b*c)*f/d/(d*x+c))*cosh(3*e+3*b*f/d)/d^2+3/4*(-a*d+b*c)*f*
Shi((-a*d+b*c)*f/d/(d*x+c))*sinh(e+b*f/d)/d^2-3/4*(-a*d+b*c)*f*Shi(3*(-a*d+
b*c)*f/d/(d*x+c))*sinh(3*e+3*b*f/d)/d^2+(d*x+c)*sinh((b*f*x+d*e*x+a*f+c*e)/
(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5728, 5726, 3394, 3384, 3379, 3382}

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=-\frac{3f(bc-ad)\cosh\left(\frac{bf}{d}+e\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\cosh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3f(bc-ad)\sinh\left(\frac{bf}{d}+e\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}-\frac{3f(bc-ad)\sinh\left(3\left(\frac{bf}{d}+e\right)\right)\text{Shi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{af+bfx+ce+dex}{c+dx}\right)}{d}$$

```
[In] Int[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

```
[Out] (-3*(b*c - a*d)*f*Cosh[e + (b*f)/d]*CoshIntegral[((b*c - a*d)*f)/(d*(c + d*
x))])/(4*d^2) + (3*(b*c - a*d)*f*Cosh[3*(e + (b*f)/d)]*CoshIntegral[(3*(b*c
 - a*d)*f)/(d*(c + d*x))])/(4*d^2) + ((c + d*x)*Sinh[(c*e + a*f + d*e*x + b
*f*x)/(c + d*x)]^3)/d + (3*(b*c - a*d)*f*Sinh[e + (b*f)/d]*SinhIntegral[((b
*c - a*d)*f)/(d*(c + d*x))])/(4*d^2) - (3*(b*c - a*d)*f*Sinh[3*(e + (b*f)/d
)]*SinhIntegral[(3*(b*c - a*d)*f)/(d*(c + d*x))])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 5726

```
Int[Sinh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Dist[-d^(-1), Subst[Int[Sinh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
 x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a
*d, 0]
```

Rule 5728

```
Int[Sinh[u_]^(n_.), x_Symbol] :> With[{lst = QuotientOfLinearsParts[u, x]},
 Int[Sinh[(lst[[1]] + lst[[2]]*x)/(lst[[3]] + lst[[4]]*x)]^n, x]] /; IGtQ[n
, 0] && QuotientOfLinearsQ[u, x]
```

Rubi steps

$$\text{integral} = \int \sinh^3\left(\frac{ce+af+(de+bf)x}{c+dx}\right)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{\sinh^3\left(\frac{de+bf}{d}-\frac{(-d(ce+af)+c(de+bf))x}{d}\right)}{x^2}\,dx, x, \frac{1}{c+dx}\right)}{d}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} - \frac{(3(bc-ad)f)\text{Subst}\left(\int \left(-\frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc-ad)fx}{d}\right)}{4x}+\frac{\cosh\left(e+\frac{bf}{d}-\frac{(bc-ad)fx}{d}\right)}{4x}\right)dx, x, \frac{1}{c+dx}\right)}{d^2}$$

$$= \frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d} + \frac{(3(bc-ad)f)\text{Subst}\left(\int \frac{\cosh\left(3\left(e+\frac{bf}{d}\right)-\frac{3(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2} - \frac{(3(bc-ad)f)\text{Subst}\left(\int \frac{\cosh\left(e+\frac{bf}{d}-\frac{(bc-ad)fx}{d}\right)}{x}\,dx, x, \frac{1}{c+dx}\right)}{4d^2}$$

$$=\frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}-\frac{\left(3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{(bc-ad)fx}{d}\right)}{x}\,dx,x,\frac{1}{c+dx}\right)}{4d^2}+\frac{\left(3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\right)\text{Subst}\left(\int\frac{\cosh\left(\frac{3(bc-ad)fx}{d}\right)}{x}\,dx,x,\frac{1}{c+dx}\right)}{4d^2}+\frac{\left(3(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{(bc-ad)fx}{d}\right)}{x}\,dx,x,\frac{1}{c+dx}\right)}{4d^2}-\frac{\left(3(bc-ad)f\sinh\left(3\left(e+\frac{bf}{d}\right)\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{3(bc-ad)fx}{d}\right)}{x}\,dx,x,\frac{1}{c+dx}\right)}{4d^2}$$

$$=-\frac{3(bc-ad)f\cosh\left(e+\frac{bf}{d}\right)\text{Chi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{3(bc-ad)f\cosh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Chi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}+\frac{(c+dx)\sinh^3\left(\frac{ce+af+dex+bfx}{c+dx}\right)}{d}+\frac{3(bc-ad)f\sinh\left(e+\frac{bf}{d}\right)\text{Shi}\left(\frac{(bc-ad)f}{d(c+dx)}\right)}{4d^2}-\frac{3(bc-ad)f\sinh\left(3\left(e+\frac{bf}{d}\right)\right)\text{Shi}\left(\frac{3(bc-ad)f}{d(c+dx)}\right)}{4d^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 913 vs. $2(226) = 452$.

Time = 6.48 (sec) , antiderivative size = 913, normalized size of antiderivative = 4.04

$$\int\sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)dx=-\frac{ce^{-\frac{3(ce+af+dex+bfx)}{c+dx}}}{8d}+\frac{3ce^{-\frac{ce+af+dex+bfx}{c+dx}}}{8d}-\frac{3ce^{\frac{ce+af+dex+bfx}{c+dx}}}{8d}+\frac{ce^{\frac{3(ce+af+dex+bfx)}{c+dx}}}{8d}-\frac{3}{4}x\cosh\left(\frac{-bcf+adf}{d(c+dx)}\right)\sinh\left(\frac{de+bf}{d}\right)+\frac{1}{4}x\cosh\left(\frac{3(-bcf+adf)}{d(c+dx)}\right)\sinh\left(\frac{3(de+bf)}{d}\right)-\frac{3}{4}x\cosh\left(\frac{de+bf}{d}\right)\sinh\left(\frac{-bcf+adf}{d(c+dx)}\right)+\frac{1}{4}x\cosh\left(\frac{3(de+bf)}{d}\right)\sinh\left(\frac{3(-bcf+adf)}{d(c+dx)}\right)-\frac{3(-bc+ad)f\left(\cosh\left(\frac{3(de+bf)}{d}\right)\text{Chi}\left(\frac{3bcf-3adf}{cd+d^2x}\right)-\cosh\left(\frac{de+bf}{d}\right)\text{Chi}\left(\frac{bcf-adf}{cd+d^2x}\right)-\cosh\left(\frac{de+bf}{d}\right)\text{Chi}\left(\frac{-bcf+adf}{cd+d^2x}\right)\right.}{}$$

```
[In] Integrate[Sinh[e + (f*(a + b*x))/(c + d*x)]^3,x]
```

```
[Out] -1/8*c/(d*E^((3*(c*e + a*f + d*e*x + b*f*x))/(c + d*x))) + (3*c)/(8*d*E^((c
*e + a*f + d*e*x + b*f*x)/(c + d*x))) - (3*c*E^((c*e + a*f + d*e*x + b*f*x)
/(c + d*x)))/(8*d) + (c*E^((3*(c*e + a*f + d*e*x + b*f*x))/(c + d*x)))/(8*d
) - (3*x*Cosh[(-(b*c*f) + a*d*f)/(d*(c + d*x))]*Sinh[(d*e + b*f)/d])/4 + (x
*Cosh[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))]*Sinh[(3*(d*e + b*f))/d])/4 - (3
*x*Cosh[(d*e + b*f)/d]*Sinh[(-(b*c*f) + a*d*f)/(d*(c + d*x))])/4 + (x*Cosh[
(3*(d*e + b*f))/d]*Sinh[(3*(-(b*c*f) + a*d*f))/(d*(c + d*x))])/4 - (3*(-(b*
c) + a*d)*f*(Cosh[(3*(d*e + b*f))/d]*CoshIntegral[(3*b*c*f - 3*a*d*f)/(c*d
+ d^2*x)] - Cosh[(d*e + b*f)/d]*CoshIntegral[(b*c*f - a*d*f)/(c*d + d^2*x)]
 - Cosh[(d*e + b*f)/d]*CoshIntegral[(-(b*c*f) + a*d*f)/(c*d + d^2*x)] + Cos
h[(3*(d*e + b*f))/d]*CoshIntegral[(-3*b*c*f + 3*a*d*f)/(c*d + d^2*x)] + Cos
hIntegral[(b*c*f - a*d*f)/(c*d + d^2*x)]*Sinh[(d*e + b*f)/d] - CoshIntegral
[(-(b*c*f) + a*d*f)/(c*d + d^2*x)]*Sinh[(d*e + b*f)/d] - CoshIntegral[(3*b*
c*f - 3*a*d*f)/(c*d + d^2*x)]*Sinh[(3*(d*e + b*f))/d] + CoshIntegral[(-3*b*
c*f + 3*a*d*f)/(c*d + d^2*x)]*Sinh[(3*(d*e + b*f))/d] + Cosh[(3*(d*e + b*f)
)/d]*SinhIntegral[(3*b*c*f - 3*a*d*f)/(c*d + d^2*x)] - Sinh[(3*(d*e + b*f))
/d]*SinhIntegral[(3*b*c*f - 3*a*d*f)/(c*d + d^2*x)] - Cosh[(d*e + b*f)/d]*S
inhIntegral[(b*c*f - a*d*f)/(c*d + d^2*x)] + Sinh[(d*e + b*f)/d]*SinhIntegr
al[(b*c*f - a*d*f)/(c*d + d^2*x)] - Cosh[(d*e + b*f)/d]*SinhIntegral[(-(b*c
*f) + a*d*f)/(c*d + d^2*x)] - Sinh[(d*e + b*f)/d]*SinhIntegral[(-(b*c*f) +
a*d*f)/(c*d + d^2*x)] + Cosh[(3*(d*e + b*f))/d]*SinhIntegral[(-3*b*c*f + 3*
a*d*f)/(c*d + d^2*x)] + Sinh[(3*(d*e + b*f))/d]*SinhIntegral[(-3*b*c*f + 3*
a*d*f)/(c*d + d^2*x)]))/(8*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(220) = 440$.

Time = 2.50 (sec) , antiderivative size = 930, normalized size of antiderivative = 4.12

method	result
risch	$-\frac{\mathrm{e}^{-\frac{3(bfx+dex+af+ce)}{dx+c}} af}{8\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{\mathrm{e}^{-\frac{3(bfx+dex+af+ce)}{dx+c}} bcf}{8d\left(\frac{dfa}{dx+c}-\frac{bcf}{dx+c}\right)}+\frac{3\,\mathrm{e}^{-\frac{3(fb+de)}{d}}\,\mathrm{Ei}_1\left(\frac{3adf-3bcf}{(dx+c)d}\right)af}{8d}-\frac{3\,\mathrm{e}^{-\frac{3(fb+de)}{d}}\,\mathrm{Ei}_1\left(\frac{3adf-3bcf}{(dx+c)d}\right)bcf}{8d^2}$

```
[In] int(sinh(e+f*(b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)*
a*f+1/8/d*exp(-3*(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*
c*f)*b*c*f+3/8/d*exp(-3*(b*f+d*e)/d)*Ei(1,3/d*(a*d*f-b*c*f)/(d*x+c))*a*f-3/
8/d^2*exp(-3*(b*f+d*e)/d)*Ei(1,3/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+3/8*exp(-(b
*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)*a*f-3/8/d*exp(
-(b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(d*f/(d*x+c)*a-1/(d*x+c)*b*c*f)*b*c*f-3/8/d
*exp(-(b*f+d*e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*a*f+3/8/d^2*exp(-(b*f+d*
e)/d)*Ei(1,1/d*(a*d*f-b*c*f)/(d*x+c))*b*c*f+1/8/d*exp(3*(b*f*x+d*e*x+a*f+c*
```

```
e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*a*f-1/8/d^2*exp(3*(b*f*x+d*e*x+
a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d*x+c)/d*b*c*f)*b*c*f+3/8/d*exp(3*(b*f+d*
e)/d)*Ei(1,-3/d*(a*d*f-b*c*f)/(d*x+c)-3*(b*f+d*e)/d-3*(-b*f-d*e)/d)*a*f-3/8
/d^2*exp(3*(b*f+d*e)/d)*Ei(1,-3/d*(a*d*f-b*c*f)/(d*x+c)-3*(b*f+d*e)/d-3*(-b
*f-d*e)/d)*b*c*f-3/8/d*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-1/(d
*x+c)/d*b*c*f)*a*f+3/8/d^2*exp((b*f*x+d*e*x+a*f+c*e)/(d*x+c))/(f/(d*x+c)*a-
1/(d*x+c)/d*b*c*f)*b*c*f-3/8/d*exp((b*f+d*e)/d)*Ei(1,-1/d*(a*d*f-b*c*f)/(d*
x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*a*f+3/8/d^2*exp((b*f+d*e)/d)*Ei(1,-1/d*(a*d*
f-b*c*f)/(d*x+c)-(b*f+d*e)/d-(-b*f-d*e)/d)*b*c*f
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(220) = 440$.

Time = 0.29 (sec) , antiderivative size = 942, normalized size of antiderivative = 4.17

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx =$$

$$-\frac{6\,(bc-ad)f\operatorname{Ei}\left(-\frac{3\,(bc-ad)f}{d^2x+cd}\right)\cosh\left(\frac{ce+af+(de+bf)x}{dx+c}\right)^2\cosh\left(\frac{3\,(de+bf)}{d}\right)\sinh\left(\frac{ce+af+(de+bf)x}{dx+c}\right)^2-3\,(bc-ad)f}{}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(6*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f +
(d*e + b*f)*x)/(d*x + c))^2*cosh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e +
b*f)*x)/(d*x + c))^2 - 3*(b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*c
osh(3*(d*e + b*f)/d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*(d^2
*x + c*d)*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^3 - 3*((b*c - a*d)*f*
Ei(-3*(b*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x +
c))^4 + (b*c - a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*cosh(3*(d*e + b*f)
/d) + 3*((b*c - a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) + (b*c - a*d)*f*Ei(-
(b*c - a*d)*f/(d^2*x + c*d)))*cosh((d*e + b*f)/d) + 6*(d^2*x - (d^2*x + c*d
)*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + c*d)*sinh((c*e + a*f + (d
*e + b*f)*x)/(d*x + c)) - 3*((b*c - a*d)*f*Ei(-3*(b*c - a*d)*f/(d^2*x + c*d
))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - 2*(b*c - a*d)*f*Ei(-3*(b
*c - a*d)*f/(d^2*x + c*d))*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2*si
nh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2 + (b*c - a*d)*f*Ei(-3*(b*c - a*
d)*f/(d^2*x + c*d))*sinh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^4 - (b*c -
a*d)*f*Ei(3*(b*c - a*d)*f/(d^2*x + c*d)))*sinh(3*(d*e + b*f)/d) - 3*((b*c -
 a*d)*f*Ei((b*c - a*d)*f/(d^2*x + c*d)) - (b*c - a*d)*f*Ei(-(b*c - a*d)*f/(
d^2*x + c*d)))*sinh((d*e + b*f)/d))/(d^2*cosh((c*e + a*f + (d*e + b*f)*x)/(
d*x + c))^4 - 2*d^2*cosh((c*e + a*f + (d*e + b*f)*x)/(d*x + c))^2*sinh((c*e
 + a*f + (d*e + b*f)*x)/(d*x + c))^2 + d^2*sinh((c*e + a*f + (d*e + b*f)*x)
/(d*x + c))^4)
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\text{Timed out}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{(bx+a)f}{dx+c}\right)^3\,dx$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="maxima")
[Out] integrate(sinh(e + (b*x + a)*f/(d*x + c))^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3021 vs. $2(220) = 440$.

Time = 27.95 (sec) , antiderivative size = 3021, normalized size of antiderivative = 13.37

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\text{Too large to display}$$

```
[In] integrate(sinh(e+f*(b*x+a)/(d*x+c))^3,x, algorithm="giac")
[Out] 1/8*(3*b^2*c^2*d*e*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*
x + c))/d)*e^(3*(d*e + b*f)/d) - 6*a*b*c*d^2*e*f^2*Ei(-3*(d*e + b*f - (d*e*
x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d) + 3*a^2*d^3*e*f^
2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e
+ b*f)/d) + 3*b^3*c^2*f^3*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/
(d*x + c))/d)*e^(3*(d*e + b*f)/d) - 6*a*b^2*c*d*f^3*Ei(-3*(d*e + b*f - (d*e
*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d) + 3*a^2*b*d^2*f
^3*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e
 + b*f)/d) - 3*b^2*c^2*d*e*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)
*d/(d*x + c))/d)*e^((d*e + b*f)/d) + 6*a*b*c*d^2*e*f^2*Ei(-(d*e + b*f - (d*
e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*a^2*d^3*e*f^
2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*
f)/d) - 3*b^3*c^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x +
 c))/d)*e^((d*e + b*f)/d) + 6*a*b^2*c*d*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x
```

+ c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*a^2*b*d^2*f^3*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d) - 3*b^2*c^2*d*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + 6*a*b*c*d^2*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 3*a^2*d^3*e*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 3*b^3*c^2*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + 6*a*b^2*c*d*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) - 3*a^2*b*d^2*f^3*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d) + 3*b^2*c^2*d*e*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) - 6*a*b*c*d^2*e*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) + 3*a^2*d^3*e*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) + 3*b^3*c^2*f^3*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) - 6*a*b^2*c*d*f^3*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) + 3*a^2*b*d^2*f^3*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d) - 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d)/(d*x + c) + 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d)/(d*x + c) - 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(3*(d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x + c) - 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(-(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^((d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) - 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) + 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei((d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-(d*e + b*f)/d)/(d*x + c) - 3*(d*e*x + b*f*x + c*e + a*f)*b^2*c^2*d*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d)/(d*x + c) + 6*(d*e*x + b*f*x + c*e + a*f)*a*b*c*d^2*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d)/(d*x + c) - 3*(d*e*x + b*f*x + c*e + a*f)*a^2*d^3*f^2*Ei(3*(d*e + b*f - (d*e*x + b*f*x + c*e + a*f)*d/(d*x + c))/d)*e^(-3*(d*e + b*f)/d)/(d*x + c) + b^2*c^2*d*f^2*e^(3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 2*a*b*c*d^2*f^2*e^(3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + a^2*d^3*f^2*e^(3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 3*b^2*c^2*d*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + 6*a*b*c*d^2*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - 3*a^2*d^3*f^2*e^((d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + 3*b^2*c^2*d*f^2*e^(-(d*e*x + b*f*x + c*e +

```
a*f)/(d*x + c)) - 6*a*b*c*d^2*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c
)) + 3*a^2*d^3*f^2*e^(-(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) - b^2*c^2*d*f
^2*e^(-3*(d*e*x + b*f*x + c*e + a*f)/(d*x + c)) + 2*a*b*c*d^2*f^2*e^(-3*(d*
e*x + b*f*x + c*e + a*f)/(d*x + c)) - a^2*d^3*f^2*e^(-3*(d*e*x + b*f*x + c*
e + a*f)/(d*x + c)))*((d*e + b*f)*c/(b*c*f - a*d*f)^2 - (c*e + a*f)*d/(b*c*
f - a*d*f)^2)/(d^3*e + b*d^2*f - (d*e*x + b*f*x + c*e + a*f)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \sinh^3\left(e+\frac{f(a+bx)}{c+dx}\right)\,dx=\int \sinh\left(e+\frac{f\,(a+b\,x)}{c+d\,x}\right)^3\,dx$$

```
[In] int(sinh(e + (f*(a + b*x))/(c + d*x))^3,x)
[Out] int(sinh(e + (f*(a + b*x))/(c + d*x))^3, x)
```

3.301 $\int e^{a+bx} \sinh^4(a+bx) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int e^{a+bx} \sinh^4(a+bx) \, dx = -\frac{e^{-3a-3bx}}{48b} + \frac{e^{-a-bx}}{4b} + \frac{3e^{a+bx}}{8b} - \frac{e^{3a+3bx}}{12b} + \frac{e^{5a+5bx}}{80b}$$

```
[Out] -1/48*exp(-3*b*x-3*a)/b+1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b-1/12*exp(3*b*x+3
*a)/b+1/80*exp(5*b*x+5*a)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2320, 12, 276}

$$\int e^{a+bx} \sinh^4(a+bx) \, dx = -\frac{e^{-3a-3bx}}{48b} + \frac{e^{-a-bx}}{4b} + \frac{3e^{a+bx}}{8b} - \frac{e^{3a+3bx}}{12b} + \frac{e^{5a+5bx}}{80b}$$

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^4,x]
```

```
[Out] -1/48*E^(-3*a - 3*b*x)/b + E^(-a - b*x)/(4*b) + (3*E^(a + b*x))/(8*b) - E^(
3*a + 3*b*x)/(12*b) + E^(5*a + 5*b*x)/(80*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{16x^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{x^4}\,dx, x, e^{a+bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(6+\frac{1}{x^4}-\frac{4}{x^2}-4x^2+x^4\right)\,dx, x, e^{a+bx}\right)}{16b}\\
&= -\frac{e^{-3a-3bx}}{48b}+\frac{e^{-a-bx}}{4b}+\frac{3e^{a+bx}}{8b}-\frac{e^{3a+3bx}}{12b}+\frac{e^{5a+5bx}}{80b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(-5+60e^{2(a+bx)}+90e^{4(a+bx)}-20e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^4,x]
[Out] (-5 + 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) - 20*E^(6*(a + b*x)) + 3*E^(8
*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	si
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^5}{5}}{b}$	45
default	$\frac{\left(\frac{8}{15}+\frac{\sinh(bx+a)^4}{5}-\frac{4\sinh(bx+a)^2}{15}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^5}{5}}{b}$	45
risch	$-\frac{e^{-3bx-3a}}{48b}+\frac{e^{-bx-a}}{4b}+\frac{3\,e^{bx+a}}{8b}-\frac{e^{3bx+3a}}{12b}+\frac{e^{5bx+5a}}{80b}$	69
parallelrisch	$\frac{e^{bx+a}(-\cosh(4bx+4a)+64\cosh(bx+a)+4\sinh(4bx+4a)-64\sinh(bx+a)-40\sinh(2bx+2a)+20\cosh(2bx+2a)+45)}{120b}$	74

```
[In] int(exp(b*x+a)*sinh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*((8/15+1/5*sinh(b*x+a)^4-4/15*sinh(b*x+a)^2)*cosh(b*x+a)+1/5*sinh(b*x+a
)^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\sinh^4(a+bx)\,dx =$$
$$-\frac{\cosh(bx+a)^4-16\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2-10\right)\sinh(bx+}{120\left(b\cosh(bx+a)-b\sinh(\right.}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 + 2*(3*cosh(b*x + a)^2 - 10)*sinh(b*x + a)^2 - 20*cosh(b*x + a)^2 - 16*(c
osh(b*x + a)^3 - 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) - b*
sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(65) = 130.

Time = 2.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\sinh^4(a+bx)\,dx$$
$$=\begin{cases}\frac{e^ae^{bx}\sinh^4(a+bx)}{5b}+\frac{4e^ae^{bx}\sinh^3(a+bx)\cosh(a+bx)}{5b}-\frac{4e^ae^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}-\frac{8e^ae^{bx}\sinh(a+bx)\cosh^3(a+bx)}{15b}+\frac{8e^ae^{bx}}{} \\ xe^a\sinh^4(a)\end{cases}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**4,x)
```

```
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**4/(5*b) + 4*exp(a)*exp(b*x)*sinh(
a + b*x)**3*cosh(a + b*x)/(5*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a
 + b*x)**2/(5*b) - 8*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/(15*b)
+ 8*exp(a)*exp(b*x)*cosh(a + b*x)**4/(15*b), Ne(b, 0)), (x*exp(a)*sinh(a)**
4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{80\,b} - \frac{e^{(3\,bx+3\,a)}}{12\,b} + \frac{3\,e^{(bx+a)}}{8\,b} + \frac{e^{(-bx-a)}}{4\,b} - \frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/80*e^(5*b*x + 5*a)/b - 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b + 1/4*e
^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{5\left(12\,e^{(2\,bx+2\,a)}-1\right)e^{(-3\,bx-3\,a)} + 3\,e^{(5\,bx+5\,a)} - 20\,e^{(3\,bx+3\,a)} + 90\,e^{(bx+a)}}{240\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/240*(5*(12*e^(2*b*x + 2*a) - 1)*e^(-3*b*x - 3*a) + 3*e^(5*b*x + 5*a) - 20
*e^(3*b*x + 3*a) + 90*e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\sinh^4(a+bx)\,dx = \frac{90\,\mathrm{e}^{a+b\,x} + 60\,\mathrm{e}^{-a-b\,x} - 5\,\mathrm{e}^{-3\,a-3\,b\,x} - 20\,\mathrm{e}^{3\,a+3\,b\,x} + 3\,\mathrm{e}^{5\,a+5\,b\,x}}{240\,b}$$

```
[In] int(exp(a + b*x)*sinh(a + b*x)^4,x)
```

```
[Out] (90*exp(a + b*x) + 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) - 20*exp(3*a +
3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

3.302 $\int e^{a+bx}\sinh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int e^{a+bx}\sinh^3(a+bx)\,dx=\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[Out] 1/16*exp(-2*b*x-2*a)/b-3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 272, 45}

$$\int e^{a+bx}\sinh^3(a+bx)\,dx=\frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}$$

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^3,x]
```

```
[Out] E^(-2*a - 2*b*x)/(16*b) - (3*E^(2*a + 2*b*x))/(16*b) + E^(4*a + 4*b*x)/(32*
b) + (3*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{8x^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x^2)^3}{x^3}\,dx, x, e^{a+bx}\right)}{8b}\\
&= \frac{\text{Subst}\left(\int \frac{(-1+x)^3}{x^2}\,dx, x, e^{2a+2bx}\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, e^{2a+2bx}\right)}{16b}\\
&= \frac{e^{-2a-2bx}}{16b}-\frac{3e^{2a+2bx}}{16b}+\frac{e^{4a+4bx}}{32b}+\frac{3x}{8}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int e^{a+bx}\sinh^3(a+bx)\,dx=\frac{e^{-2(a+bx)}-3e^{2(a+bx)}+\frac{1}{2}e^{4(a+bx)}+6bx}{16b}$$

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^3,x]
[Out] (E^(-2*(a + b*x)) - 3*E^(2*(a + b*x)) + E^(4*(a + b*x))/2 + 6*b*x)/(16*b)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{{\rm e}^{-2bx-2a}}{16b}-\frac{3\,{\rm e}^{2bx+2a}}{16b}+\frac{{\rm e}^{4bx+4a}}{32b}+\frac{3x}{8}$	47
derivativedivides	$\frac{\left(\frac{\sinh(bx+a)^3}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}+\frac{\sinh(bx+a)^4}{4}}{b}$	49
default	$\frac{\left(\frac{\sinh(bx+a)^3}{4}-\frac{3\sinh(bx+a)}{8}\right)\cosh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}+\frac{\sinh(bx+a)^4}{4}}{b}$	49

```
[In] int(exp(b*x+a)*sinh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/16*exp(-2*b*x-2*a)/b-3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\sinh^3(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^3+9\cosh(bx+a)\sinh(bx+a)^2-\sinh(bx+a)^3+6(2bx-1)\cosh(bx+a)-3(4bx+c}{32(b\cosh(bx+a)-b\sinh(bx+a))}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="fricas")
[Out] 1/32*(3*cosh(b*x + a)^3 + 9*cosh(b*x + a)*sinh(b*x + a)^2 - sinh(b*x + a)^3
 + 6*(2*b*x - 1)*cosh(b*x + a) - 3*(4*b*x + cosh(b*x + a)^2 + 2)*sinh(b*x +
 a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(48) = 96.

Time = 0.84 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.63

$$\int e^{a+bx}\sinh^3(a+bx)\,dx$$
$$=\begin{cases}\frac{3xe^ae^{bx}\sinh^3(a+bx)}{8}-\frac{3xe^ae^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8}-\frac{3xe^ae^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8}+\frac{3xe^ae^{bx}\cosh^3(a+bx)}{8}-\frac{3e^ae^{bx}\sinh^3(}{8b}\\ xe^a\sinh^3(a)\end{cases}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**3,x)
```

```
[Out] Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*sin
h(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a +
b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - 3*exp(a)*exp(b*x)*sinh
(a + b*x)**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/b + exp
(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**2/(4*b) - 5*exp(a)*exp(b*x)*cosh(
a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*sinh(a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int e^{a+bx}\sinh^3(a+bx)\,dx = \frac{3\,(bx+a)}{8\,b} + \frac{e^{(4\,bx+4\,a)}}{32\,b} - \frac{3\,e^{(2\,bx+2\,a)}}{16\,b} + \frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b - 3/16*e^(2*b*x + 2*a)/b + 1/16*e^
(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\sinh^3(a+bx)\,dx = \frac{12\,bx - 2\,\left(3\,e^{(2\,bx+2\,a)} - 1\right)e^{(-2\,bx-2\,a)} + 12\,a + e^{(4\,bx+4\,a)} - 6\,e^{(2\,bx+2\,a)}}{32\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) - 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*x
 + 4*a) - 6*e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\sinh^3(a+bx)\,dx = \frac{3\,x}{8} + \frac{\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16} - \frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16} + \frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b}$$

```
[In] int(exp(a + b*x)*sinh(a + b*x)^3,x)
```

```
[Out] (3*x)/8 + (exp(- 2*a - 2*b*x)/16 - (3*exp(2*a + 2*b*x))/16 + exp(4*a + 4*b*
x)/32)/b
```

3.303 $\int e^{a+bx} \sinh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int e^{a+bx} \sinh^2(a+bx)\,dx = -\frac{e^{-a-bx}}{4b} - \frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{12b}$$

```
[Out] -1/4*exp(-b*x-a)/b-1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2320, 12, 276}

$$\int e^{a+bx} \sinh^2(a+bx)\,dx = -\frac{e^{-a-bx}}{4b} - \frac{e^{a+bx}}{2b} + \frac{e^{3a+3bx}}{12b}$$

```
[In] Int[E^(a + b*x)*Sinh[a + b*x]^2,x]
```

```
[Out] -1/4*E^(-a - b*x)/b - E^(a + b*x)/(2*b) + E^(3*a + 3*b*x)/(12*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{4x^2}\,dx, x, e^{a+bx}\right)}{b} \\ &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2}\,dx, x, e^{a+bx}\right)}{4b} \\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, e^{a+bx}\right)}{4b} \\ &= -\frac{e^{-a-bx}}{4b}-\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \frac{e^{-a-bx}\left(-3-6e^{2(a+bx)}+e^{4(a+bx)}\right)}{12b}$$

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x]^2,x]
[Out] (E^(-a - b*x)*(-3 - 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^3}{3}}{b}$	35
default	$\frac{\left(-\frac{2}{3}+\frac{\sinh(bx+a)^2}{3}\right)\cosh(bx+a)+\frac{\sinh(bx+a)^3}{3}}{b}$	35
risch	$-\frac{e^{-bx-a}}{4b}-\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{12b}$	41
parallelrisch	$-\frac{e^{bx+a}(-2\sinh(2bx+2a)-4\sinh(bx+a)+\cosh(2bx+2a)+4\cosh(bx+a)+3)}{6b}$	50

```
[In] int(exp(b*x+a)*sinh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*((-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+1/3*sinh(b*x+a)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = -\frac{\cosh\left(bx+a\right)^2-4\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+3}{6\left(b\cosh\left(bx+a\right)-b\sinh\left(bx+a\right)\right)}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="fricas")
[Out] -1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 3
)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \begin{cases} \frac{e^{a}e^{bx}\sinh^2\left(a+bx\right)}{3b}+\frac{2e^{a}e^{bx}\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{3b}-\frac{2e^{a}e^{bx}\cosh^2\left(a+bx\right)}{3b} & \text{for } b\neq 0 \\ xe^{a}\sinh^2\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)**2,x)
[Out] Piecewise((exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*sinh(
a + b*x)*cosh(a + b*x)/(3*b) - 2*exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b), Ne
(b, 0)), (x*exp(a)*sinh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{12\,b} - \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{4\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="maxima")
[Out] 1/12*e^(3*b*x + 3*a)/b - 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)} - 6\,e^{(bx+a)} - 3\,e^{(-bx-a)}}{12\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a)^2,x, algorithm="giac")
[Out] 1/12*(e^(3*b*x + 3*a) - 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\sinh^2(a+bx)\,dx = -\frac{6\,\mathrm{e}^{a+b\,x} + 3\,\mathrm{e}^{-a-b\,x} - \mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

```
[In] int(exp(a + b*x)*sinh(a + b*x)^2,x)
[Out] -(6*exp(a + b*x) + 3*exp(- a - b*x) - exp(3*a + 3*b*x))/(12*b)
```

3.304 $\int e^{a+bx} \sinh(a+bx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int e^{a+bx} \sinh(a+bx) \, dx = \frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

```
[Out] 1/4*exp(2*b*x+2*a)/b-1/2*x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2320, 12, 14}

$$\int e^{a+bx} \sinh(a+bx) \, dx = \frac{e^{2a+2bx}}{4b} - \frac{x}{2}$$

```
[In] Int[E^(a + b*x)*Sinh[a + b*x],x]
```

```
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx, x, e^{a+bx}\right)}{2b}\\ &= \frac{e^{2a+2bx}}{4b}-\frac{x}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\sinh(a+bx)\,dx = \frac{e^{2a+2bx}}{4b}-\frac{x}{2}$$

```
[In] Integrate[E^(a + b*x)*Sinh[a + b*x],x]
[Out] E^(2*a + 2*b*x)/(4*b) - x/2
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{{\mathrm e}^{2 b x+2 a}}{4 b}-\frac{x}{2}$	19
derivativedivides	$\frac{\frac{\cosh \left(b x+a\right) \sinh \left(b x+a\right)}{2}-\frac{b x}{2}-\frac{a}{2}+\frac{\cosh \left(b x+a\right)^{2}}{2}}{b}$	37
default	$\frac{\frac{\cosh \left(b x+a\right) \sinh \left(b x+a\right)}{2}-\frac{b x}{2}-\frac{a}{2}+\frac{\cosh \left(b x+a\right)^{2}}{2}}{b}$	37

```
[In] int(exp(b*x+a)*sinh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(2*b*x+2*a)/b-1/2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\sinh(a+bx)\,dx = -\frac{(2\,bx-1)\cosh(bx+a)-(2\,bx+1)\sinh(bx+a)}{4\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="fricas")
[Out] -1/4*((2*b*x - 1)*cosh(b*x + a) - (2*b*x + 1)*sinh(b*x + a))/(b*cosh(b*x +
a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int e^{a+bx}\sinh(a+bx)\,dx = \begin{cases} \frac{xe^{a}e^{bx}\sinh(a+bx)}{2} - \frac{xe^{a}e^{bx}\cosh(a+bx)}{2} + \frac{e^{a}e^{bx}\cosh(a+bx)}{2b} & \text{for } b \neq 0 \\ xe^{a}\sinh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x)
[Out] Piecewise((x*exp(a)*exp(b*x)*sinh(a + b*x)/2 - x*exp(a)*exp(b*x)*cosh(a + b
*x)/2 + exp(a)*exp(b*x)*cosh(a + b*x)/(2*b), Ne(b, 0)), (x*exp(a)*sinh(a),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\sinh(a+bx)\,dx = -\frac{1}{2}\,x - \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/2*x - 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\sinh(a+bx)\,dx = -\frac{2\,bx + 2\,a - e^{(2\,bx+2\,a)}}{4\,b}$$

```
[In] integrate(exp(b*x+a)*sinh(b*x+a),x, algorithm="giac")
[Out] -1/4*(2*b*x + 2*a - e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\sinh(a+bx)\,dx = \frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b} - \frac{x}{2}$$

```
[In] int(exp(a + b*x)*sinh(a + b*x),x)
[Out] exp(2*a + 2*b*x)/(4*b) - x/2
```

3.305 $\int e^{a+bx}\operatorname{csch}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 19

$$\int e^{a+bx}\operatorname{csch}(a+bx)\,dx=\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[Out] ln(1-exp(2*b*x+2*a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2320, 12, 266}

$$\int e^{a+bx}\operatorname{csch}(a+bx)\,dx=\frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Int[E^(a + b*x)*Csch[a + b*x],x]
```

```
[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{\log\left(1-e^{2a+2bx}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\text{csch}(a+bx)\,dx = \frac{\log\left(1-e^{2a+2bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x],x]
[Out] Log[1 - E^(2*a + 2*b*x)]/b
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{bx+a+\ln(\sinh(bx+a))}{b}$	17
default	$\frac{bx+a+\ln(\sinh(bx+a))}{b}$	17
risch	$-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	24

```
[In] int(exp(b*x+a)*csch(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(b*x+a+ln(sinh(b*x+a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int e^{a+bx}\text{csch}(a+bx)\,dx=\frac{\log\left(\frac{2\,\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="fricas")
[Out] log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F]

$$\int e^{a+bx}\text{csch}(a+bx)\,dx=e^{a}\int e^{bx}\,\text{csch}\,(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int e^{a+bx}\text{csch}(a+bx)\,dx=\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="maxima")
[Out] log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int e^{a+bx}\text{csch}(a+bx)\,dx=\frac{\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a),x, algorithm="giac")
[Out] (log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^{a+bx}\mathrm{csch}(a+bx)\,dx=\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{b}$$

```
[In] int(exp(a + b*x)/sinh(a + b*x),x)
[Out] log(exp(2*a + 2*b*x) - 1)/b
```

3.306 $\int e^{a+bx}\mathbf{csch}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 42

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx=\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] 2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 294, 212}

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx=\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^2,x]
```

```
[Out] (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{4x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{4\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\text{csch}^2(a+bx)\,dx = \frac{-\frac{2e^{a+bx}}{-1+e^{2(a+bx)}} - 2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^2,x]
[Out] ((-2*E^(a + b*x))/(-1 + E^(2*(a + b*x))) - 2*ArcTanh[E^(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

method	result	size
derivativedivides	$\frac{-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{1}{\sinh(bx+a)}}{b}$	25
default	$\frac{-2\,\operatorname{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{1}{\sinh(bx+a)}}{b}$	25
risch	$-\frac{2\,\mathrm{e}^{bx+a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)}-\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{b}+\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{b}$	53

```
[In] int(exp(b*x+a)*csch(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(-2*arctanh(exp(b*x+a))-1/sinh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.74

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = -\frac{\left(\cosh(bx+a)^2+2\,\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-1\right)\log\left(\cosh(bx+a)+\sinh(bx+a)\right.}{b\cosh(bx+}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="fricas")
[Out] -((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*l
og(cosh(b*x + a) + sinh(b*x + a) + 1) - (cosh(b*x + a)^2 + 2*cosh(b*x + a)*
sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1)
 + 2*cosh(b*x + a) + 2*sinh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a
)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = e^{a}\int e^{bx}\operatorname{csch}^2(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**2,x)
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = -\frac{\log\left(e^{(bx+a)}+1\right)}{b}+\frac{\log\left(e^{(bx+a)}-1\right)}{b}-\frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}-1)}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x + a)/(b*(e^(2*b
*x + 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = -\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1}+\log\left(e^{(bx+a)}+1\right)-\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) + log(e^(b*x + a) + 1) - log(abs(e^(b
*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int e^{a+bx}\operatorname{csch}^2(a+bx)\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(exp(a + b*x)/sinh(a + b*x)^2,x)
```

```
[Out] - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) - (2*exp(a + b*x)
)/(b*(exp(2*a + 2*b*x) - 1))
```

3.307 $\int e^{a+bx}\text{csch}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 31

$$\int e^{a+bx}\text{csch}^3(a+bx)\,dx=-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[Out] -2*exp(4*b*x+4*a)/b/(1-exp(2*b*x+2*a))^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2320, 12, 270}

$$\int e^{a+bx}\text{csch}^3(a+bx)\,dx=-\frac{2e^{4a+4bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^3,x]
```

```
[Out] (-2*E^(4*a + 4*b*x))/(b*(1 - E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{8\text{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}\\ &= -\frac{2e^{4a+4bx}}{b\,(1-e^{2a+2bx})^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\text{csch}^3(a+bx)\,dx = -\frac{2e^{4a+4bx}}{b\,(-1+e^{2a+2bx})^2}$$

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^3,x]
[Out] (-2*E^(4*a + 4*b*x))/(b*(-1 + E^(2*a + 2*b*x))^2)
```

Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\coth(bx+a)-\frac{1}{2\sinh(bx+a)^2}}{b}$	24
default	$\frac{-\coth(bx+a)-\frac{1}{2\sinh(bx+a)^2}}{b}$	24
risch	$-\frac{2(2\,\mathrm{e}^{2bx+2a}-1)}{b(\mathrm{e}^{2bx+2a}-1)^2}$	32
parallelrisch	$\frac{\mathrm{e}^{bx+a}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}{8b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}$	47

```
[In] int(exp(b*x+a)*csch(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-coth(b*x+a)-1/2/sinh(b*x+a)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(27) = 54.

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.84

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = -\frac{2\left(\cosh\left(bx+a\right)+3\,\sinh\left(bx+a\right)\right)}{b\cosh\left(bx+a\right)^3+3\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+b\sinh\left(bx+a\right)^3-b\cosh\left(bx+a\right)+3\left(b\cosh\left(bx+a\right.\right.}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="fricas")
[Out] -2*(cosh(b*x + a) + 3*sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)
*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 - b*cosh(b*x + a) + 3*(b*cosh(b*x + a)
^2 - b)*sinh(b*x + a))
```

Sympy [F]

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = e^{a}\int e^{bx}\operatorname{csch}^3\left(a+bx\right)dx$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**3,x)
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int e^{a+bx}\operatorname{csch}^3(a+bx)\,dx = -\frac{4\,e^{(2\,bx+2\,a)}}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}+\frac{2}{b(e^{(4\,bx+4\,a)}-2\,e^{(2\,bx+2\,a)}+1)}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="maxima")
[Out] -4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1)) + 2/(b*(e^
(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\mathrm{csch}^3(a+bx)\,dx = -\frac{2\left(2\,e^{(2\,bx+2\,a)}-1\right)}{b\left(e^{(2\,bx+2\,a)}-1\right)^2}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^3,x, algorithm="giac")
[Out] -2*(2*e^(2*b*x + 2*a) - 1)/(b*(e^(2*b*x + 2*a) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\mathrm{csch}^3(a+bx)\,dx = -\frac{2\left(2\,\mathrm{e}^{2\,a+2\,b\,x}-1\right)}{b\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^2}$$

```
[In] int(exp(a + b*x)/sinh(a + b*x)^3,x)
[Out] -(2*(2*exp(2*a + 2*b*x) - 1))/(b*(exp(2*a + 2*b*x) - 1)^2)
```

3.308 $\int e^{a+bx} \operatorname{csch}^4(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int e^{a+bx} \operatorname{csch}^4(a+bx)\,dx = \frac{8e^{3a+3bx}}{3b\,(1-e^{2a+2bx})^3} - \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})^2} + \frac{e^{a+bx}}{b\,(1-e^{2a+2bx})} + \frac{\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] 8/3*exp(3*b*x+3*a)/b/(1-exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))
^2+exp(b*x+a)/b/(1-exp(2*b*x+2*a))+arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2320, 12, 294, 205, 212}

$$\int e^{a+bx} \operatorname{csch}^4(a+bx)\,dx = \frac{\operatorname{arctanh}\left(e^{a+bx}\right)}{b} + \frac{e^{a+bx}}{b\,(1-e^{2a+2bx})} - \frac{2e^{a+bx}}{b\,(1-e^{2a+2bx})^2} + \frac{8e^{3a+3bx}}{3b\,(1-e^{2a+2bx})^3}$$

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^4,x]
```

```
[Out] (8*E^(3*a + 3*b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (2*E^(a + b*x))/(b*(1 -
 E^(2*a + 2*b*x))^2) + E^(a + b*x)/(b*(1 - E^(2*a + 2*b*x))) + ArcTanh[E^(a
 + b*x)]/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{16x^4}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{16\text{Subst}\left(\int \frac{x^4}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{8e^{3a+3bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{8\text{Subst}\left(\int \frac{x^2}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{b}
\end{aligned}$$

$$= \frac{8e^{3a+3bx}}{3b(1-e^{2a+2bx})^3} - \frac{2e^{a+bx}}{b(1-e^{2a+2bx})^2} + \frac{2\text{Subst}\left(\int \frac{1}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{8e^{3a+3bx}}{3b(1-e^{2a+2bx})^3} - \frac{2e^{a+bx}}{b(1-e^{2a+2bx})^2} + \frac{e^{a+bx}}{b(1-e^{2a+2bx})} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{8e^{3a+3bx}}{3b(1-e^{2a+2bx})^3} - \frac{2e^{a+bx}}{b(1-e^{2a+2bx})^2} + \frac{e^{a+bx}}{b(1-e^{2a+2bx})} + \frac{\text{arctanh}\left(e^{a+bx}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\text{csch}^4(a+bx)\,dx = \frac{3e^{a+bx} - 8e^{3(a+bx)} - 3e^{5(a+bx)} + 3\left(-1+e^{2(a+bx)}\right)^3 \text{arctanh}\left(e^{a+bx}\right)}{3b\left(-1+e^{2(a+bx)}\right)^3}$$

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^4,x]
[Out] (3*E^(a + b*x) - 8*E^(3*(a + b*x)) - 3*E^(5*(a + b*x)) + 3*(-1 + E^(2*(a + b*x)))^3*ArcTanh[E^(a + b*x)])/(3*b*(-1 + E^(2*(a + b*x)))^3)
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.37

method	result	size
derivativedivides	$\frac{-\frac{\text{csch}(bx+a)\coth(bx+a)}{2}+\text{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{1}{3\sinh(bx+a)^3}}{b}$	37
default	$\frac{-\frac{\text{csch}(bx+a)\coth(bx+a)}{2}+\text{arctanh}\left(\mathrm{e}^{bx+a}\right)-\frac{1}{3\sinh(bx+a)^3}}{b}$	37
risch	$-\frac{\mathrm{e}^{bx+a}\left(3\,\mathrm{e}^{4bx+4a}+8\,\mathrm{e}^{2bx+2a}-3\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^3}+\frac{\ln\left(\mathrm{e}^{bx+a}+1\right)}{2b}-\frac{\ln\left(\mathrm{e}^{bx+a}-1\right)}{2b}$	78

```
[In] int(exp(b*x+a)*csch(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2*csch(b*x+a)*coth(b*x+a)+arctanh(exp(b*x+a))-1/3/sinh(b*x+a)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. 2(87) = 174.

Time = 0.28 (sec) , antiderivative size = 705, normalized size of antiderivative = 6.98

$$\int e^{a+bx}\text{csch}^4(a+bx)\,dx = -\frac{6\cosh(bx+a)^5 + 30\cosh(bx+a)\sinh(bx+a)^4 + 6\sinh(bx+a)^5 + 4\left(15\cosh(bx+a)^2+4\right)\sinh(\cdots}{\cdots}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(6*cosh(b*x + a)^5 + 30*cosh(b*x + a)*sinh(b*x + a)^4 + 6*sinh(b*x + a
)^5 + 4*(15*cosh(b*x + a)^2 + 4)*sinh(b*x + a)^3 + 16*cosh(b*x + a)^3 + 12*
(5*cosh(b*x + a)^3 + 4*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
- 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*
x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 +
 cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) +
 3*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3
*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*
x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh
(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 -
 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) +
sinh(b*x + a) - 1) + 6*(5*cosh(b*x + a)^4 + 8*cosh(b*x + a)^2 - 1)*sinh(b*x
 + a) - 6*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b
)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x +
a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 +
 b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b
*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^4(a+bx)\,dx = e^a\int e^{bx}\,\text{csch}^4\,(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**4,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = \frac{\log\left(e^{(bx+a)}+1\right)}{2b} - \frac{\log\left(e^{(bx+a)}-1\right)}{2b} - \frac{3e^{(5bx+5a)}+8e^{(3bx+3a)}-3e^{(bx+a)}}{3b(e^{(6bx+6a)}-3e^{(4bx+4a)}+3e^{(2bx+2a)}-1)}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="maxima")
[Out] 1/2*log(e^(b*x + a) + 1)/b - 1/2*log(e^(b*x + a) - 1)/b - 1/3*(3*e^(5*b*x +
 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(b*(e^(6*b*x + 6*a) - 3*e^(4*b*x
 + 4*a) + 3*e^(2*b*x + 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = -\frac{\frac{2\left(3e^{(5bx+5a)}+8e^{(3bx+3a)}-3e^{(bx+a)}\right)}{\left(e^{(2bx+2a)}-1\right)^3} - 3\log\left(e^{(bx+a)}+1\right) + 3\log\left(\left|e^{(bx+a)}-1\right|\right)}{6b}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^4,x, algorithm="giac")
[Out] -1/6*(2*(3*e^(5*b*x + 5*a) + 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x +
 2*a) - 1)^3 - 3*log(e^(b*x + a) + 1) + 3*log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

$$\int e^{a+bx}\operatorname{csch}^4(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,e^{a+bx}}{b\,\left(e^{4a+4bx}-2\,e^{2a+2bx}+1\right)} - \frac{8\,e^{3a+3bx}}{3\,b\,\left(3\,e^{2a+2bx}-3\,e^{4a+4bx}+e^{6a+6bx}-1\right)} - \frac{e^{a+bx}}{b\,\left(e^{2a+2bx}-1\right)}$$

```
[In] int(exp(a + b*x)/sinh(a + b*x)^4,x)
[Out] atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b)/(-b^2)^(1/2) - (2*exp(a + b*x))/(b*(
exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(3*
exp(2*a + 2*b*x) - 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - exp(a + b*
x)/(b*(exp(2*a + 2*b*x) - 1))
```

3.309 $\int e^{a+bx} \operatorname{csch}^5(a+bx)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int e^{a+bx} \operatorname{csch}^5(a+bx)\, dx = -\frac{4}{b\left(1-e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[Out] -4/b/(1-exp(2*b*x+2*a))^4+32/3/b/(1-exp(2*b*x+2*a))^3-8/b/(1-exp(2*b*x+2*a)
)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 12, 272, 45}

$$\int e^{a+bx} \operatorname{csch}^5(a+bx)\, dx = -\frac{8}{b\left(1-e^{2a+2bx}\right)^2}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{4}{b\left(1-e^{2a+2bx}\right)^4}$$

```
[In] Int[E^(a + b*x)*Csch[a + b*x]^5,x]
```

```
[Out] -4/(b*(1 - E^(2*a + 2*b*x))^4) + 32/(3*b*(1 - E^(2*a + 2*b*x))^3) - 8/(b*(1
 - E^(2*a + 2*b*x))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{32x^5}{(-1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{32\text{Subst}\left(\int \frac{x^5}{(-1+x^2)^5}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{16\text{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2a+2bx}\right)}{b}\\
&= \frac{16\text{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)\,dx, x, e^{2a+2bx}\right)}{b}\\
&= -\frac{4}{b\left(1-e^{2a+2bx}\right)^4}+\frac{32}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{8}{b\left(1-e^{2a+2bx}\right)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

$$\int e^{a+bx}\text{csch}^5(a+bx)\,dx = -\frac{4\left(1-4e^{2(a+bx)}+6e^{4(a+bx)}\right)}{3b\left(-1+e^{2(a+bx)}\right)^4}$$

```
[In] Integrate[E^(a + b*x)*Csch[a + b*x]^5,x]
[Out] (-4*(1 - 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(-1 + E^(2*(a + b*x))
)^4)
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^{2}}{3}\right)\coth(bx+a)-\frac{1}{4\sinh(bx+a)^{4}}}{b}$	35
default	$\frac{\left(\frac{2}{3}-\frac{\operatorname{csch}(bx+a)^{2}}{3}\right)\coth(bx+a)-\frac{1}{4\sinh(bx+a)^{4}}}{b}$	35
risch	$-\frac{4\left(6\,\mathrm{e}^{4bx+4a}-4\,\mathrm{e}^{2bx+2a}+1\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^{4}}$	43
parallelrisch	$-\frac{\mathrm{e}^{bx+a}\operatorname{sech}\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}\operatorname{csch}\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}\left(4\cosh(bx+a)+4\sinh(bx+a)-\cosh(3bx+3a)-\sinh(3bx+3a)\right)}{192b}$	73

```
[In] int(exp(b*x+a)*csch(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*((2/3-1/3*csch(b*x+a)^2)*coth(b*x+a)-1/4/sinh(b*x+a)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(55) = 110.

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.53

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{6}+6\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+b\sinh\left(bx+a\right)^{6}-4\,b\cosh\left(bx+a\right)^{4}+\left(15\,b\cosh\right.\right.}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="fricas")
[Out] -4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)^
2 - 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x
+ a)^6 - 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 - 4*b)*sinh(b*x + a)^4
 + 4*(5*b*cosh(b*x + a)^3 - 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*cosh(b
*x + a)^2 + (15*b*cosh(b*x + a)^4 - 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b*x +
a)^2 + 2*(3*b*cosh(b*x + a)^5 - 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x + a))*si
nh(b*x + a) - 4*b)
```

Sympy [F]

$$\int e^{a+bx}\text{csch}^5(a+bx)\,dx = e^a\int e^{bx}\,\text{csch}^5\,(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)**5,x)
[Out] exp(a)*Integral(exp(b*x)*csch(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.61

$$\int e^{a+bx}\text{csch}^5(a+bx)\,dx = -\frac{8\,e^{(4\,bx+4\,a)}}{b(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1)} + \frac{16\,e^{(2\,bx+2\,a)}}{3\,b(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1)} - \frac{4}{3\,b(e^{(8\,bx+8\,a)}-4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1)}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="maxima")
[Out] -8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4
*a) - 4*e^(2*b*x + 2*a) + 1)) + 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a) -
4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b*(e
^(8*b*x + 8*a) - 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a)
+ 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\text{csch}^5(a+bx)\,dx = -\frac{4\left(6\,e^{(4\,bx+4\,a)}-4\,e^{(2\,bx+2\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}-1)^4}$$

```
[In] integrate(exp(b*x+a)*csch(b*x+a)^5,x, algorithm="giac")
[Out] -4/3*(6*e^(4*b*x + 4*a) - 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) - 1)^4
)
```

Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\operatorname{csch}^5(a+bx)\,dx = -\frac{4\left(6\,\mathrm{e}^{4\,a+4\,b\,x}-4\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{3\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)^4}$$

```
[In] int(exp(a + b*x)/sinh(a + b*x)^5,x)
[Out] -(4*(6*exp(4*a + 4*b*x) - 4*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) -
 1)^4)
```

3.310 $\int e^x \sinh^2(2x) \, dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \sinh^2(2x) \, dx = -\frac{1}{12}e^{-3x} - \frac{e^x}{2} + \frac{e^{5x}}{20}$$

```
[Out] -1/12/exp(3*x)-1/2*exp(x)+1/20*exp(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2320, 12, 276}

$$\int e^x \sinh^2(2x) \, dx = -\frac{1}{12}e^{-3x} - \frac{e^x}{2} + \frac{e^{5x}}{20}$$

```
[In] Int[E^x*Sinh[2*x]^2,x]
```

```
[Out] -1/12*1/E^(3*x) - E^x/2 + E^(5*x)/20
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^4)^2}{4x^4}\,dx, x, e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x^4)^2}{x^4}\,dx, x, e^x\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^4}+x^4\right)\,dx, x, e^x\right)\\
&= -\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \sinh^2(2x)\,dx = -\frac{1}{12}e^{-3x}-\frac{e^x}{2}+\frac{e^{5x}}{20}$$

```
[In] Integrate[E^x*Sinh[2*x]^2,x]
[Out] -1/12*1/E^(3*x) - E^x/2 + E^(5*x)/20
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^x(15+\cosh(4x)-4\sinh(4x))}{30}$	17
risch	$\frac{e^{5x}}{20}-\frac{e^x}{2}-\frac{e^{-3x}}{12}$	18
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}-\frac{\cosh(x)}{2}-\frac{\cosh(3x)}{12}+\frac{\cosh(5x)}{20}$	34

```
[In] int(exp(x)*sinh(2*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/30*exp(x)*(15+cosh(4*x)-4*sinh(4*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int e^x \sinh^2(2x)\, dx = -\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 15}{30(\cosh(x) - \sinh(x))}$$

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="fricas")
[Out] -1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(x
)*sinh(x)^3 + sinh(x)^4 + 15)/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \sinh^2(2x)\, dx = \frac{7e^x\sinh^2(2x)}{15} + \frac{4e^x\sinh(2x)\cosh(2x)}{15} - \frac{8e^x\cosh^2(2x)}{15}$$

```
[In] integrate(exp(x)*sinh(2*x)**2,x)
[Out] 7*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 - 8*exp(x)*cosh(
2*x)**2/15
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(2x)\, dx = \frac{1}{20}e^{(5x)} - \frac{1}{12}e^{(-3x)} - \frac{1}{2}e^x$$

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="maxima")
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(2x)\,dx = \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{2}\,e^x$$

```
[In] integrate(exp(x)*sinh(2*x)^2,x, algorithm="giac")
[Out] 1/20*e^(5*x) - 1/12*e^(-3*x) - 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(2x)\,dx = \frac{e^{5\,x}}{20} - \frac{e^{-3\,x}}{12} - \frac{e^x}{2}$$

```
[In] int(sinh(2*x)^2*exp(x),x)
[Out] exp(5*x)/20 - exp(-3*x)/12 - exp(x)/2
```

3.311 $\int e^x \sinh(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \sinh(2x)\,dx = \frac{e^{-x}}{2} + \frac{e^{3x}}{6}$$

```
[Out] 1/2/exp(x)+1/6*exp(3*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2320, 12, 14}

$$\int e^x \sinh(2x)\,dx = \frac{e^{-x}}{2} + \frac{e^{3x}}{6}$$

```
[In] Int[E^x*Sinh[2*x],x]
```

```
[Out] 1/(2*E^x) + E^(3*x)/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{-1+x^4}{2x^2}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^4}{x^2}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}+x^2\right)\,dx, x, e^x\right)\\ &= \frac{e^{-x}}{2}+\frac{e^{3x}}{6}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \sinh(2x)\,dx = \frac{1}{6}e^{-x}\left(3+e^{4x}\right)$$

```
[In] Integrate[E^x*Sinh[2*x],x]
[Out] (3 + E^(4*x))/(6*E^x)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{e}^{3x}}{6}+\frac{{e}^{-x}}{2}$	14
parallelrisch	$\frac{{e}^{x} \left(2 \cosh \left(2 x \right)-\sinh \left(2 x \right)\right)}{3}$	18
default	$-\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(3 x \right)}{6}+\frac{\cosh \left(x \right)}{2}+\frac{\cosh \left(3 x \right)}{6}$	22

```
[In] int(exp(x)*sinh(2*x),x,method=_RETURNVERBOSE)
[Out] 1/6*exp(3*x)+1/2*exp(-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int e^x \sinh(2x)\,dx = \frac{2\left(\cosh(x)^2 - \cosh(x)\sinh(x) + \sinh(x)^2\right)}{3\left(\cosh(x) - \sinh(x)\right)}$$

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="fricas")
[Out] 2/3*(cosh(x)^2 - cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \sinh(2x)\,dx = -\frac{e^x \sinh(2x)}{3} + \frac{2e^x \cosh(2x)}{3}$$

```
[In] integrate(exp(x)*sinh(2*x),x)
[Out] -exp(x)*sinh(2*x)/3 + 2*exp(x)*cosh(2*x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(2x)\,dx = \frac{1}{6}\,e^{(3\,x)} + \frac{1}{2}\,e^{(-x)}$$

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="maxima")
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(2x)\,dx = \frac{1}{6}\,e^{(3\,x)} + \frac{1}{2}\,e^{(-x)}$$

```
[In] integrate(exp(x)*sinh(2*x),x, algorithm="giac")
[Out] 1/6*e^(3*x) + 1/2*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \sinh(2x)\,dx = \frac{\mathrm{e}^{-x}\,(\mathrm{e}^{4\,x}+3)}{6}$$

```
[In] int(sinh(2*x)*exp(x),x)
[Out] (exp(-x)*(exp(4*x) + 3))/6
```

3.312 $\int e^x \operatorname{csch}(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 11

$$\int e^x \operatorname{csch}(2x)\,dx = \arctan\left(e^x\right) - \operatorname{arctanh}(e^x)$$

```
[Out] arctan(exp(x))-arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2320, 12, 304, 209, 212}

$$\int e^x \operatorname{csch}(2x)\,dx = \arctan\left(e^x\right) - \operatorname{arctanh}(e^x)$$

```
[In] Int[E^x*Csch[2*x],x]
```

```
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^2}{-1+x^4}\,dx, x, e^x\right)\\
&= 2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, e^x\right)\\
&= -\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \arctan\left(e^x\right) - \operatorname{arctanh}(e^x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^x \operatorname{csch}(2x)\,dx = \arctan\left(e^x\right) - \operatorname{arctanh}(e^x)$$

```
[In] Integrate[E^x*Csch[2*x],x]
[Out] ArcTan[E^x] - ArcTanh[E^x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

method	result	size
risch	$\frac{\ln(e^x-1)}{2}-\frac{\ln(e^x+1)}{2}+\frac{i\ln(e^x+i)}{2}-\frac{i\ln(e^x-i)}{2}$	34

```
[In] int(exp(x)*csch(2*x),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(exp(x)-1)-1/2*ln(exp(x)+1)+1/2*I*ln(exp(x)+I)-1/2*I*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(9) = 18$.

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int e^x \operatorname{csch}(2x)\,dx = \arctan(\cosh(x)+\sinh(x)) - \frac{1}{2}\log(\cosh(x)+\sinh(x)+1) + \frac{1}{2}\log(\cosh(x)+\sinh(x)-1)$$

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="fricas")
[Out] arctan(cosh(x) + sinh(x)) - 1/2*log(cosh(x) + sinh(x) + 1) + 1/2*log(cosh(x
) + sinh(x) - 1)
```

Sympy [F]

$$\int e^x \operatorname{csch}(2x)\,dx = \int e^x \operatorname{csch}(2x)\,dx$$

```
[In] integrate(exp(x)*csch(2*x),x)
[Out] Integral(exp(x)*csch(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int e^x \operatorname{csch}(2x)\,dx = \arctan\left(e^x\right) - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(e^x - 1\right)$$

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="maxima")
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int e^x \operatorname{csch}(2x)\,dx = \arctan\left(e^x\right) - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(|e^x - 1|\right)$$

```
[In] integrate(exp(x)*csch(2*x),x, algorithm="giac")
[Out] arctan(e^x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int e^x \operatorname{csch}(2x)\,dx = \frac{\ln\left(4\,e^x - 4\right)}{2} - \frac{\ln\left(-4\,e^x - 4\right)}{2} - \operatorname{atan}\left(e^{-x}\right)$$

```
[In] int(exp(x)/sinh(2*x),x)
[Out] log(4*exp(x) - 4)/2 - log(- 4*exp(x) - 4)/2 - atan(exp(-x))
```

3.313 $\int e^x \mathrm{csch}^2(2x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 32

$$\int e^x \mathrm{csch}^2(2x)\,dx = \frac{e^x}{1-e^{4x}} - \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2}$$

```
[Out] exp(x)/(1-exp(4*x))-1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2320, 12, 294, 218, 212, 209}

$$\int e^x \mathrm{csch}^2(2x)\,dx = -\frac{1}{2}\arctan(e^x) - \frac{\operatorname{arctanh}(e^x)}{2} + \frac{e^x}{1-e^{4x}}$$

```
[In] Int[E^x*Csch[2*x]^2,x]
[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{4x^4}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= 4\text{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= \frac{e^x}{1-e^{4x}} - \frac{\arctan(e^x)}{2} - \frac{\text{arctanh}(e^x)}{2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int e^x \operatorname{csch}^2(2x)\,dx = \frac{e^x}{1-e^{4x}} - \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2}$$

[In] Integrate[E^x*Csch[2*x]^2,x]

[Out] E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{e^{x}}{e^{4x}-1}-\frac{\ln(e^{x}+1)}{4}+\frac{i\ln(e^{x}-i)}{4}-\frac{i\ln(e^{x}+i)}{4}+\frac{\ln(e^{x}-1)}{4}$	46

[In] int(exp(x)*csch(2*x)^2,x,method=_RETURNVERBOSE)

[Out] -exp(x)/(exp(4*x)-1)-1/4*ln(exp(x)+1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)
+1/4*ln(exp(x)-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(23) = 46.

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.69

$$\int e^x \operatorname{csch}^2(2x)\,dx =$$
$$-\frac{2\left(\cosh(x)^4+4\cosh(x)^3\sinh(x)+6\cosh(x)^2\sinh(x)^2+4\cosh(x)\sinh(x)^3+\sinh(x)^4-1\right)\arctan(\ldots}{\ldots}$$

[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="fricas")

[Out] -1/4*(2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh(x)) + (cosh(x)^4 + 4*cos
h(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 -
1)*log(cosh(x) + sinh(x) + 1) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x
)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x)
- 1) + 4*cosh(x) + 4*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^
2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)

Sympy [F]

$$\int e^x \text{csch}^2(2x)\,dx = \int e^x \operatorname{csch}^2(2x)\,dx$$

```
[In] integrate(exp(x)*csch(2*x)**2,x)
[Out] Integral(exp(x)*csch(2*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int e^x \text{csch}^2(2x)\,dx = -\frac{e^x}{e^{(4x)}-1} - \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(e^x-1)$$

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="maxima")
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int e^x \text{csch}^2(2x)\,dx = -\frac{e^x}{e^{(4x)}-1} - \frac{1}{2}\arctan(e^x) - \frac{1}{4}\log(e^x+1) + \frac{1}{4}\log(|e^x-1|)$$

```
[In] integrate(exp(x)*csch(2*x)^2,x, algorithm="giac")
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) - 1/4*log(e^x + 1) + 1/4*log(abs(e^x -
 1))
```

Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int e^x \text{csch}^2(2x)\,dx = \frac{\ln(1-e^x)}{4} - \frac{\ln(-e^x-1)}{4} - \frac{\operatorname{atan}(e^x)}{2} - \frac{e^x}{e^{4x}-1}$$

```
[In] int(exp(x)/sinh(2*x)^2,x)
[Out] log(1 - exp(x))/4 - log(- exp(x) - 1)/4 - atan(exp(x))/2 - exp(x)/(exp(4*x)
 - 1)
```

3.314 $\int e^x \sinh^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \sinh^2(3x)\,dx = -\frac{1}{20}e^{-5x} - \frac{e^x}{2} + \frac{e^{7x}}{28}$$

```
[Out] -1/20/exp(5*x)-1/2*exp(x)+1/28*exp(7*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2320, 12, 276}

$$\int e^x \sinh^2(3x)\,dx = -\frac{1}{20}e^{-5x} - \frac{e^x}{2} + \frac{e^{7x}}{28}$$

```
[In] Int[E^x*Sinh[3*x]^2,x]
```

```
[Out] -1/20*1/E^(5*x) - E^x/2 + E^(7*x)/28
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(1-x^6)^2}{4x^6}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x^6)^2}{x^6}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^6}+x^6\right)\,dx, x, e^x\right)\\ &= -\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \sinh^2(3x)\,dx = -\frac{1}{20}e^{-5x}-\frac{e^x}{2}+\frac{e^{7x}}{28}$$

```
[In] Integrate[E^x*Sinh[3*x]^2,x]
[Out] -1/20*1/E^(5*x) - E^x/2 + E^(7*x)/28
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x}(35+\cosh(6x)-6\sinh(6x))}{70}$	17
risch	$\frac{e^{7x}}{28}-\frac{e^{x}}{2}-\frac{e^{-5x}}{20}$	18
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(5x)}{20}+\frac{\sinh(7x)}{28}-\frac{\cosh(x)}{2}-\frac{\cosh(5x)}{20}+\frac{\cosh(7x)}{28}$	34

```
[In] int(exp(x)*sinh(3*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/70*exp(x)*(35+cosh(6*x)-6*sinh(6*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int e^x \sinh^2(3x)\,dx = -\frac{\cosh(x)^6 - 36\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 - 120\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 - 36\cosh(x)\sinh(x)^5 + \sinh(x)^6 + 35}{70\,(\cosh(x) - \sinh(x))}$$

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="fricas")
[Out] -1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)^6
 + 35)/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \sinh^2(3x)\,dx = \frac{17e^x\sinh^2(3x)}{35} + \frac{6e^x\sinh(3x)\cosh(3x)}{35} - \frac{18e^x\cosh^2(3x)}{35}$$

```
[In] integrate(exp(x)*sinh(3*x)**2,x)
[Out] 17*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 - 18*exp(x)*cos
h(3*x)**2/35
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(3x)\,dx = \frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} - \frac{1}{2}\,e^x$$

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="maxima")
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(3x)\, dx = \frac{1}{28}\, e^{(7\,x)} - \frac{1}{20}\, e^{(-5\,x)} - \frac{1}{2}\, e^x$$

```
[In] integrate(exp(x)*sinh(3*x)^2,x, algorithm="giac")
[Out] 1/28*e^(7*x) - 1/20*e^(-5*x) - 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(3x)\, dx = \frac{\mathrm{e}^{7\,x}}{28} - \frac{\mathrm{e}^{-5\,x}}{20} - \frac{\mathrm{e}^x}{2}$$

```
[In] int(sinh(3*x)^2*exp(x),x)
[Out] exp(7*x)/28 - exp(-5*x)/20 - exp(x)/2
```

3.315 $\int e^x \sinh(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \sinh(3x)\,dx = \frac{e^{-2x}}{4} + \frac{e^{4x}}{8}$$

```
[Out] 1/4/exp(2*x)+1/8*exp(4*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 12, 14}

$$\int e^x \sinh(3x)\,dx = \frac{e^{-2x}}{4} + \frac{e^{4x}}{8}$$

```
[In] Int[E^x*Sinh[3*x],x]
```

```
[Out] 1/(4*E^(2*x)) + E^(4*x)/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{-1+x^6}{2x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^6}{x^3}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^3}+x^3\right)\,dx, x, e^x\right)\\ &= \frac{e^{-2x}}{4}+\frac{e^{4x}}{8}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \sinh(3x)\,dx = \frac{1}{8}e^{-2x}\left(2+e^{6x}\right)$$

```
[In] Integrate[E^x*Sinh[3*x],x]
[Out] (2 + E^(6*x))/(8*E^(2*x))
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{e^{4x}}{8}+\frac{e^{-2x}}{4}$	14
parallelrisch	$-\frac{e^x(-3\cosh(3x)+\sinh(3x))}{8}$	16
default	$-\frac{\sinh(2x)}{4}+\frac{\sinh(4x)}{8}+\frac{\cosh(2x)}{4}+\frac{\cosh(4x)}{8}$	26

```
[In] int(exp(x)*sinh(3*x),x,method=_RETURNVERBOSE)
[Out] 1/8*exp(4*x)+1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int e^x \sinh(3x)\,dx = \frac{3\,\cosh(x)^3 - 3\,\cosh(x)^2\sinh(x) + 9\,\cosh(x)\sinh(x)^2 - \sinh(x)^3}{8\,(\cosh(x) - \sinh(x))}$$

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="fricas")
[Out] 1/8*(3*cosh(x)^3 - 3*cosh(x)^2*sinh(x) + 9*cosh(x)*sinh(x)^2 - sinh(x)^3)/(
cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \sinh(3x)\,dx = -\frac{e^x\sinh(3x)}{8} + \frac{3e^x\cosh(3x)}{8}$$

```
[In] integrate(exp(x)*sinh(3*x),x)
[Out] -exp(x)*sinh(3*x)/8 + 3*exp(x)*cosh(3*x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(3x)\,dx = \frac{1}{8}\,e^{(4\,x)} + \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="maxima")
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(3x)\,dx = \frac{1}{8}\,e^{(4\,x)} + \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(exp(x)*sinh(3*x),x, algorithm="giac")
[Out] 1/8*e^(4*x) + 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \sinh(3x)\, dx = \frac{e^{-2x}\, (e^{6x} + 2)}{8}$$

```
[In] int(sinh(3*x)*exp(x),x)
[Out] (exp(-2*x)*(exp(6*x) + 2))/8
```

3.316 $\int e^x \mathbf{csch}(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int e^x \text{csch}(3x)\,dx = \frac{\arctan\left(\frac{1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)$$

```
[Out] 1/3*ln(1-exp(2*x))-1/6*ln(1+exp(2*x)+exp(4*x))+1/3*arctan(1/3*(1+2*exp(2*x)
)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {2320, 12, 281, 298, 31, 648, 632, 210, 642}

$$\int e^x \text{csch}(3x)\,dx = \frac{\arctan\left(\frac{2e^{2x}+1}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\log\left(e^{2x}+e^{4x}+1\right)$$

```
[In] Int[E^x*Csch[3*x],x]
```

```
[Out] ArcTan[(1 + 2*E^(2*x))/Sqrt[3]]/Sqrt[3] + Log[1 - E^(2*x)]/3 - Log[1 + E^(2
*x) + E^(4*x)]/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^3}{-1+x^6}\,dx, x, e^x\right)\\
&= 2\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, e^{2x}\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, e^{2x}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^{2x}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, e^{2x}\right)\\
&= \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2e^{2x}\right)\\
&= \frac{\arctan\left(\frac{1+2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log\left(1-e^{2x}\right) - \frac{1}{6}\log\left(1+e^{2x}+e^{4x}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.41

$$
\int e^x \operatorname{csch}(3x)\,dx = -\frac{1}{2}e^{4x}\,\text{Hypergeometric2F1}\left(\frac{2}{3}, 1, \frac{5}{3}, e^{6x}\right)
$$

```
[In] Integrate[E^x*Csch[3*x],x]
[Out] -1/2*(E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, E^(6*x)])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

method	result	size
risch	$-\frac{\ln\left(\mathrm{e}^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(\mathrm{e}^{2x}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(\mathrm{e}^{2x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln(\mathrm{e}^{2x}-1)}{3}$	79

```
[In] int(exp(x)*csch(3*x),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(exp(2*x)+1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)+1/2+1/2*I*3^(1/2))*3^
(1/2)-1/6*ln(exp(2*x)+1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(2*x)+1/2-1/2*I*3^(1/2
))*3^(1/2)+1/3*ln(exp(2*x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int e^x \operatorname{csch}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{3\sqrt{3}\cosh(x)+\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right) - \frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2+1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right) + \frac{1}{3}\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(-1/3*(3*sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - s
inh(x))) - 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 + 1)/(cosh(x)^2 - 2*cosh(x)*s
inh(x) + sinh(x)^2)) + 1/3*log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int e^x \operatorname{csch}(3x)\,dx = \int e^x \operatorname{csch}(3x)\,dx$$

```
[In] integrate(exp(x)*csch(3*x),x)
[Out] Integral(exp(x)*csch(3*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int e^x \operatorname{csch}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) - \frac{1}{6}\log\left(e^{(2x)}+e^x+1\right) - \frac{1}{6}\log\left(e^{(2x)}-e^x+1\right) + \frac{1}{3}\log\left(e^x+1\right) + \frac{1}{3}\log\left(e^x-1\right)$$

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 1/6*log(e^(2*x) + e^x + 1) - 1/6*log(e^(2*x) - e^x + 1) +
 1/3*log(e^x + 1) + 1/3*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int e^x \operatorname{csch}(3x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2e^{(2x)}+1\right)\right) - \frac{1}{6}\log\left(e^{(4x)}+e^{(2x)}+1\right) + \frac{1}{3}\log\left(\left|e^{(2x)}-1\right|\right)$$

```
[In] integrate(exp(x)*csch(3*x),x, algorithm="giac")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) + 1)) - 1/6*log(e^(4*x) + e^(2*x)
 + 1) + 1/3*log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int e^x \operatorname{csch}(3x)\,dx = \frac{\ln\left(8\,\mathrm{e}^{2\,x}-8\right)}{3} + \ln\left(24\,\mathrm{e}^{2\,x}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) - \ln\left(-24\,\mathrm{e}^{2\,x}\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-8\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

```
[In] int(exp(x)/sinh(3*x),x)
[Out] log(8*exp(2*x) - 8)/3 + log(24*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) - 8)*((3^(1/
2)*1i)/6 - 1/6) - log(- 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6) - 8)*((3^(1/2)*1
i)/6 + 1/6)
```

3.317 $\int e^x \mathrm{csch}^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 105

$$\int e^x \mathrm{csch}^2(3x)\,dx = \frac{2e^x}{3(1-e^{6x})} + \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2\mathrm{arctanh}(e^x)}{9} + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right)$$

```
[Out] 2/3*exp(x)/(1-exp(6*x))-2/9*arctanh(exp(x))+1/18*ln(1-exp(x)+exp(2*x))-1/18
*ln(1+exp(x)+exp(2*x))+1/9*arctan(1/3*(1-2*exp(x))*3^(1/2))*3^(1/2)-1/9*arc
tan(1/3*(1+2*exp(x))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2320, 12, 294, 216, 648, 632, 210, 642, 212}

$$\int e^x \mathrm{csch}^2(3x)\,dx = \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2\mathrm{arctanh}(e^x)}{9} + \frac{2e^x}{3(1-e^{6x})} + \frac{1}{18}\log\left(-e^x+e^{2x}+1\right) - \frac{1}{18}\log\left(e^x+e^{2x}+1\right)$$

```
[In] Int[E^x*Csch[3*x]^2,x]
```

```
[Out] (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - ArcTa
n[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x + E^(
2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{4x^6}{(1-x^6)^2}\,dx, x, e^x\right)\\
&= 4\text{Subst}\left(\int \frac{x^6}{(1-x^6)^2}\,dx, x, e^x\right)\\
&= \frac{2e^x}{3(1-e^{6x})} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^6}\,dx, x, e^x\right)\\
&= \frac{2e^x}{3(1-e^{6x})} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right)\\
&\quad - \frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, e^x\right)\\
&= \frac{2e^x}{3(1-e^{6x})} - \frac{2\text{arctanh}(e^x)}{9} + \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{18}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^x\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, e^x\right)\\
&= \frac{2e^x}{3(1-e^{6x})} - \frac{2\text{arctanh}(e^x)}{9} + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right)\\
&\quad + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2e^x\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2e^x\right)\\
&= \frac{2e^x}{3(1-e^{6x})} + \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2\text{arctanh}(e^x)}{9}\\
&\quad + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.32

$$\int e^x \operatorname{csch}^2(3x)\, dx = \frac{2}{3}e^x\left(\frac{1}{1-e^{6x}} - \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},e^{6x}\right)\right)$$

```
[In] Integrate[E^x*Csch[3*x]^2,x]
```

```
[Out] (2*E^x*((1 - E^(6*x))^(-1) - Hypergeometric2F1[1/6, 1, 7/6, E^(6*x)]))/3
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{2\,\mathrm{e}^x}{3(\mathrm{e}^{6x}-1)} + \frac{\ln\left(\mathrm{e}^x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{18} + \frac{i\sqrt{3}\,\ln\left(\mathrm{e}^x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{18} + \frac{\ln\left(\mathrm{e}^x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{18} - \frac{i\sqrt{3}\,\ln\left(\mathrm{e}^x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{18} + \frac{\ln(\mathrm{e}^x-1)}{9} - \frac{\ln(\mathrm{e}^x+1)}{9}$

```
[In] int(exp(x)*csch(3*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*exp(x)/(exp(6*x)-1)+1/18*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/18*I*3^(1/2)*l
n(exp(x)-1/2-1/2*I*3^(1/2))+1/18*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/18*I*3^(1/2
)*ln(exp(x)-1/2+1/2*I*3^(1/2))+1/9*ln(exp(x)-1)-1/9*ln(exp(x)+1)-1/18*ln(ex
p(x)+1/2-1/2*I*3^(1/2))+1/18*I*3^(1/2)*ln(exp(x)+1/2-1/2*I*3^(1/2))-1/18*ln
(exp(x)+1/2+1/2*I*3^(1/2))-1/18*I*3^(1/2)*ln(exp(x)+1/2+1/2*I*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(76) = 152.

Time = 0.28 (sec) , antiderivative size = 560, normalized size of antiderivative = 5.33

$$\int e^x \operatorname{csch}^2(3x)\, dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh
(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sin
h(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(
2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)) + 2*(sqrt(3)*cosh(
x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2 + 20*sq
```

```
rt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(3)*cosh
(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(2/3*sqrt(3)*cosh(x) + 2
/3*sqrt(3)*sinh(x) - 1/3*sqrt(3)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) -
 (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*s
inh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*lo
g((2*cosh(x) - 1)/(cosh(x) - sinh(x))) + 2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x)
 + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4
 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cosh(x) + sinh(x) + 1) - 2*(cos
h(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x
)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cos
h(x) + sinh(x) - 1) + 12*cosh(x) + 12*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sin
h(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(
x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)
```

Sympy [F]

$$\int e^x \operatorname{csch}^2(3x)\,dx = \int e^x \operatorname{csch}^2(3x)\,dx$$

```
[In] integrate(exp(x)*csch(3*x)**2,x)
[Out] Integral(exp(x)*csch(3*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\begin{aligned}\int e^x \operatorname{csch}^2(3x)\,dx = & -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) \\ & - \frac{2e^x}{3\left(e^{(6x)}-1\right)} - \frac{1}{18}\log\left(e^{(2x)}+e^x+1\right) \\ & + \frac{1}{18}\log\left(e^{(2x)}-e^x+1\right) - \frac{1}{9}\log\left(e^x+1\right) + \frac{1}{9}\log\left(e^x-1\right)\end{aligned}$$

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="maxima")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82

$$\int e^x \operatorname{csch}^2(3x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) - \frac{2e^x}{3\left(e^{(6x)}-1\right)} - \frac{1}{18}\log\left(e^{(2x)}+e^x+1\right) + \frac{1}{18}\log\left(e^{(2x)}-e^x+1\right) - \frac{1}{9}\log\left(e^x+1\right) + \frac{1}{9}\log\left(|e^x-1|\right)$$

```
[In] integrate(exp(x)*csch(3*x)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) - 1/18*log(e^(2*x) + e^x + 1) + 1/1
8*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int e^x \operatorname{csch}^2(3x)\,dx = \frac{\ln\left(\frac{2}{3}-\frac{2\,e^x}{3}\right)}{9} - \frac{\ln\left(-\frac{2\,e^x}{3}-\frac{2}{3}\right)}{9} + \frac{\ln\left(\left(\frac{2\,e^x}{3}-\frac{1}{3}\right)^2+\frac{1}{3}\right)}{18} - \frac{\ln\left(\left(\frac{2\,e^x}{3}+\frac{1}{3}\right)^2+\frac{1}{3}\right)}{18} - \frac{2\,e^x}{3\,\left(e^{6\,x}-1\right)} - \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,e^x}{3}-\frac{1}{3}\right)\right)}{9} - \frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,e^x}{3}+\frac{1}{3}\right)\right)}{9}$$

```
[In] int(exp(x)/sinh(3*x)^2,x)
```

```
[Out] log(2/3 - (2*exp(x))/3)/9 - log(- (2*exp(x))/3 - 2/3)/9 + log(((2*exp(x))/3
 - 1/3)^2 + 1/3)/18 - log(((2*exp(x))/3 + 1/3)^2 + 1/3)/18 - (2*exp(x))/(3*
(exp(6*x) - 1)) - (3^(1/2)*atan(3^(1/2)*((2*exp(x))/3 - 1/3)))/9 - (3^(1/2)
*atan(3^(1/2)*((2*exp(x))/3 + 1/3)))/9
```

3.318 $\int e^x \sinh^2(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \sinh^2(4x)\,dx = -\frac{1}{28}e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

```
[Out] -1/28/exp(7*x)-1/2*exp(x)+1/36*exp(9*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2320, 12, 276}

$$\int e^x \sinh^2(4x)\,dx = -\frac{1}{28}e^{-7x} - \frac{e^x}{2} + \frac{e^{9x}}{36}$$

```
[In] Int[E^x*Sinh[4*x]^2,x]
```

```
[Out] -1/28*1/E^(7*x) - E^x/2 + E^(9*x)/36
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(1-x^8)^2}{4x^8}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{(1-x^8)^2}{x^8}\,dx, x, e^x\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(-2+\frac{1}{x^8}+x^8\right)\,dx, x, e^x\right)\\ &= -\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \sinh^2(4x)\,dx = -\frac{1}{28}e^{-7x}-\frac{e^x}{2}+\frac{e^{9x}}{36}$$

```
[In] Integrate[E^x*Sinh[4*x]^2,x]
[Out] -1/28*1/E^(7*x) - E^x/2 + E^(9*x)/36
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^x(\cosh(8x)+63-8\sinh(8x))}{126}$	17
risch	$\frac{e^{9x}}{36}-\frac{e^x}{2}-\frac{e^{-7x}}{28}$	18
default	$-\frac{\sinh(x)}{2}+\frac{\sinh(7x)}{28}+\frac{\sinh(9x)}{36}-\frac{\cosh(x)}{2}-\frac{\cosh(7x)}{28}+\frac{\cosh(9x)}{36}$	34

```
[In] int(exp(x)*sinh(4*x)^2,x,method=_RETURNVERBOSE)
[Out] -1/126*exp(x)*(cosh(8*x)+63-8*sinh(8*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int e^x \sinh^2(4x)\,dx = -\frac{\cosh(x)^8 - 64\cosh(x)^7\sinh(x) + 28\cosh(x)^6\sinh(x)^2 - 448\cosh(x)^5\sinh(x)^3 + 70\cosh(x)^4\sinh(}{126\,(\cosh(x) - \mathrm{s}}$$

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*co
sh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 + 63)/(cosh(x) - sinh(
x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \sinh^2(4x)\,dx = \frac{31e^x\sinh^2(4x)}{63} + \frac{8e^x\sinh(4x)\cosh(4x)}{63} - \frac{32e^x\cosh^2(4x)}{63}$$

```
[In] integrate(exp(x)*sinh(4*x)**2,x)
```

```
[Out] 31*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 - 32*exp(x)*cos
h(4*x)**2/63
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(4x)\,dx = \frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} - \frac{1}{2}\,e^x$$

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="maxima")
```

```
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(4x) \, dx = \frac{1}{36} e^{(9x)} - \frac{1}{28} e^{(-7x)} - \frac{1}{2} e^x$$

```
[In] integrate(exp(x)*sinh(4*x)^2,x, algorithm="giac")
[Out] 1/36*e^(9*x) - 1/28*e^(-7*x) - 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \sinh^2(4x) \, dx = \frac{e^{9x}}{36} - \frac{e^{-7x}}{28} - \frac{e^x}{2}$$

```
[In] int(sinh(4*x)^2*exp(x),x)
[Out] exp(9*x)/36 - exp(-7*x)/28 - exp(x)/2
```

3.319 $\int e^x \sinh(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \sinh(4x)\,dx = \frac{e^{-3x}}{6} + \frac{e^{5x}}{10}$$

```
[Out] 1/6/exp(3*x)+1/10*exp(5*x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 12, 14}

$$\int e^x \sinh(4x)\,dx = \frac{e^{-3x}}{6} + \frac{e^{5x}}{10}$$

```
[In] Int[E^x*Sinh[4*x],x]
```

```
[Out] 1/(6*E^(3*x)) + E^(5*x)/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{-1+x^8}{2x^4}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+x^8}{x^4}\,dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^4}+x^4\right)\,dx, x, e^x\right)\\ &= \frac{e^{-3x}}{6}+\frac{e^{5x}}{10}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^x \sinh(4x)\,dx = \frac{e^{-3x}}{6}+\frac{e^{5x}}{10}$$

```
[In] Integrate[E^x*Sinh[4*x],x]
[Out] 1/(6*E^(3*x)) + E^(5*x)/10
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{e^{5x}}{10}+\frac{e^{-3x}}{6}$	14
parallelrisch	$\frac{e^{x}(4\cosh(4x)-\sinh(4x))}{15}$	18
default	$-\frac{\sinh(3x)}{6}+\frac{\sinh(5x)}{10}+\frac{\cosh(3x)}{6}+\frac{\cosh(5x)}{10}$	26

```
[In] int(exp(x)*sinh(4*x),x,method=_RETURNVERBOSE)
[Out] 1/10*exp(5*x)+1/6*exp(-3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int e^x \sinh(4x)\,dx = \frac{4\left(\cosh(x)^4 - \cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - \cosh(x)\sinh(x)^3 + \sinh(x)^4\right)}{15\left(\cosh(x) - \sinh(x)\right)}$$

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="fricas")
```

```
[Out] 4/15*(cosh(x)^4 - cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - cosh(x)*sinh(
x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \sinh(4x)\,dx = -\frac{e^x\sinh(4x)}{15} + \frac{4e^x\cosh(4x)}{15}$$

```
[In] integrate(exp(x)*sinh(4*x),x)
```

```
[Out] -exp(x)*sinh(4*x)/15 + 4*exp(x)*cosh(4*x)/15
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \sinh(4x)\,dx = \frac{1}{10}\,e^{(5\,x)} + \frac{1}{6}\,e^{(-3\,x)}$$

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^{x}\sinh(4x)\,dx=\frac{1}{10}\,e^{(5\,x)}+\frac{1}{6}\,e^{(-3\,x)}$$

```
[In] integrate(exp(x)*sinh(4*x),x, algorithm="giac")
```

```
[Out] 1/10*e^(5*x) + 1/6*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^{x}\sinh(4x)\,dx=\frac{\mathrm{e}^{-3\,x}}{6}+\frac{\mathrm{e}^{5\,x}}{10}$$

```
[In] int(sinh(4*x)*exp(x),x)
```

```
[Out] exp(-3*x)/6 + exp(5*x)/10
```

3.320 $\int e^x \operatorname{csch}(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 113

$$\int e^x \operatorname{csch}(4x)\,dx = -\frac{1}{2}\arctan(e^x) - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{2} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

```
[Out] -1/2*arctan(exp(x))-1/2*arctanh(exp(x))+1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/
2)+1/4*arctan(1+exp(x)*2^(1/2))*2^(1/2)-1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2
^(1/2)+1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {2320, 12, 307, 217, 1179, 642, 1176, 631, 210, 218, 212, 209}

$$\int e^x \operatorname{csch}(4x)\,dx = -\frac{1}{2}\arctan(e^x) - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{2} - \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}} + \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}$$

```
[In] Int[E^x*Csch[4*x],x]
```

```
[Out] -1/2*ArcTan[E^x] - ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]
*E^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 - Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt
[2]) + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 307

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[x^(m - n/2)/(r
 + s*x^(n/2)), x], x] - Dist[s/(2*b), Int[x^(m - n/2)/(r - s*x^(n/2)), x],
x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && LtQ
[m, n] &&  !GtQ[a/b, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{2x^4}{-1+x^8}\,dx, x, e^x\right)\\
&= 2\text{Subst}\left(\int \frac{x^4}{-1+x^8}\,dx, x, e^x\right)\\
&= -\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) + \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)
\end{aligned}$$

$$= -\frac{1}{2}\arctan(e^x) - \frac{\operatorname{arctanh}(e^x)}{2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}}$$

$$= -\frac{1}{2}\arctan(e^x) - \frac{\operatorname{arctanh}(e^x)}{2} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{2\sqrt{2}}$$

$$= -\frac{1}{2}\arctan(e^x) - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{2} - \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} + \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

$$\int e^x \operatorname{csch}(4x)\,dx = -\frac{2}{5}e^{5x}\,\text{Hypergeometric2F1}\left(\frac{5}{8}, 1, \frac{13}{8}, e^{8x}\right)$$

```
[In] Integrate[E^x*Csch[4*x],x]
[Out] (-2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, E^(8*x)])/5
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.93 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.50

method	result	size
risch	$-\frac{\ln(e^x+1)}{4} + 2\left(\sum_{\_R=\text{RootOf}(4096\_Z^4+1)} \_R\ln\left(e^x+8\_R\right)\right) + \frac{i\ln(e^x-i)}{4} - \frac{i\ln(e^x+i)}{4} + \frac{\ln(e^x-1)}{4}$	56

```
[In] int(exp(x)*csch(4*x),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(exp(x)+1)+2*sum(_R*ln(exp(x)+8*_R),_R=RootOf(4096*_Z^4+1))+1/4*I*ln
(exp(x)-I)-1/4*I*ln(exp(x)+I)+1/4*ln(exp(x)-1)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\begin{aligned}\int e^x \text{csch}(4x)\,dx =& \left(\frac{1}{8}i + \frac{1}{8}\right)\sqrt{2}\log\left((i+1)\sqrt{2} + 2\cosh(x) + 2\sinh(x)\right) \\ & - \left(\frac{1}{8}i - \frac{1}{8}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2} + 2\cosh(x) + 2\sinh(x)\right) \\ & + \left(\frac{1}{8}i - \frac{1}{8}\right)\sqrt{2}\log\left((i-1)\sqrt{2} + 2\cosh(x) + 2\sinh(x)\right) \\ & - \left(\frac{1}{8}i + \frac{1}{8}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2} + 2\cosh(x) + 2\sinh(x)\right) \\ & - \frac{1}{2}\arctan\left(\cosh(x) + \sinh(x)\right) - \frac{1}{4}\log\left(\cosh(x) + \sinh(x) + 1\right) \\ & + \frac{1}{4}\log\left(\cosh(x) + \sinh(x) - 1\right)\end{aligned}$$

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="fricas")
```

```
[Out] (1/8*I + 1/8)*sqrt(2)*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (1/8*I
 - 1/8)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (1/8*I - 1/
8)*sqrt(2)*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (1/8*I + 1/8)*sqr
t(2)*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - 1/2*arctan(cosh(x) + s
inh(x)) - 1/4*log(cosh(x) + sinh(x) + 1) + 1/4*log(cosh(x) + sinh(x) - 1)
```

Sympy [F]

$$\int e^x \text{csch}(4x)\,dx = \int e^x \operatorname{csch}(4x)\,dx$$

```
[In] integrate(exp(x)*csch(4*x),x)
```

```
[Out] Integral(exp(x)*csch(4*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

$$\int e^{x}\mathrm{csch}(4x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right) - \frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(e^{x}-1\right)$$

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int e^{x}\mathrm{csch}(4x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^{x}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^{x}\right)\right) + \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^{x}+e^{(2x)}+1\right)-\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^{x}+e^{(2x)}+1\right) - \frac{1}{2}\arctan\left(e^{x}\right)-\frac{1}{4}\log\left(e^{x}+1\right)+\frac{1}{4}\log\left(|e^{x}-1|\right)$$

```
[In] integrate(exp(x)*csch(4*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/2
*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) -
1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) - 1/4*log(e^x
 + 1) + 1/4*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int e^x\mathrm{csch}(4x)\,dx=\frac{\ln\left(128-128\,\mathrm{e}^x\right)}{4}-\frac{\ln\left(-128\,\mathrm{e}^x-128\right)}{4}-\frac{\mathrm{atan}(\mathrm{e}^x)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(128\,\mathrm{e}^x-64\,\sqrt{2}\right)}{128}\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(128\,\mathrm{e}^x+64\,\sqrt{2}\right)}{128}\right)}{4}-\frac{\sqrt{2}\,\ln\left({\left(128\,\mathrm{e}^x-64\,\sqrt{2}\right)}^2+8192\right)}{8}+\frac{\sqrt{2}\,\ln\left({\left(128\,\mathrm{e}^x+64\,\sqrt{2}\right)}^2+8192\right)}{8}$$

```
[In] int(exp(x)/sinh(4*x),x)
```

```
[Out] log(128 - 128*exp(x))/4 - log(- 128*exp(x) - 128)/4 - atan(exp(x))/2 + (2^(
1/2)*atan((2^(1/2)*(128*exp(x) - 64*2^(1/2)))/128))/4 + (2^(1/2)*atan((2^(1
/2)*(128*exp(x) + 64*2^(1/2)))/128))/4 - (2^(1/2)*log((128*exp(x) - 64*2^(1
/2))^2 + 8192))/8 + (2^(1/2)*log((128*exp(x) + 64*2^(1/2))^2 + 8192))/8
```

3.321 $\int e^x \mathbf{csch}^2(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 131

$$\int e^x \operatorname{csch}^2(4x)\,dx = \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{8} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

```
[Out] 1/2*exp(x)/(1-exp(8*x))-1/8*arctan(exp(x))-1/8*arctanh(exp(x))-1/16*arctan(
-1+exp(x)*2^(1/2))*2^(1/2)-1/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)+1/32*ln(1+
exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {2320, 12, 294, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int e^x \operatorname{csch}^2(4x)\,dx = -\frac{1}{8}\arctan(e^x) + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\arctan\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{8} + \frac{e^x}{2(1-e^{8x})} + \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}} - \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}$$

```
[In] Int[E^x*Csch[4*x]^2,x]
```

```
[Out] E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sqrt[2])
 - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqrt[2]*E
^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{4x^8}{(1-x^8)^2}\,dx, x, e^x\right) \\ &= 4\text{Subst}\left(\int \frac{x^8}{(1-x^8)^2}\,dx, x, e^x\right) \end{aligned}$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} - \frac{\text{arctanh}(e^x)}{8} - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}}$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} - \frac{\text{arctanh}(e^x)}{8} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{8\sqrt{2}}$$

$$= \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\text{arctanh}(e^x)}{8} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.26

$$\int e^x \text{csch}^2(4x)\,dx = \frac{1}{2}e^x\left(\frac{1}{1-e^{8x}} - \text{Hypergeometric2F1}\left(\frac{1}{8}, 1, \frac{9}{8}, e^{8x}\right)\right)$$

```
[In] Integrate[E^x*Csch[4*x]^2,x]
[Out] (E^x*((1 - E^(8*x))^(-1) - Hypergeometric2F1[1/8, 1, 9/8, E^(8*x)]))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.74 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.52

method	result
risch	$-\frac{e^{x}}{2(e^{8x}-1)}+\frac{\ln(e^{x}-1)}{16}+4\left(\sum_{\_R=\mathrm{RootOf}(16777216\_Z^{4}+1)}\_R\ln\left(e^{x}-64\_R\right)\right)-\frac{\ln(e^{x}+1)}{16}+\frac{i\ln(e^{x}-i)}{16}-\frac{i\ln(e$

```
[In] int(exp(x)*csch(4*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(x)/(exp(8*x)-1)+1/16*ln(exp(x)-1)+4*sum(_R*ln(exp(x)-64*_R),_R=Roo
tOf(16777216*_Z^4+1))-1/16*ln(exp(x)+1)+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp(x
)+I)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 855, normalized size of antiderivative = 6.53

$$\int e^{x}\mathrm{csch}^{2}(4x)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(4*(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cos
h(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cos
h(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 - 1)*arctan(cosh(x) + si
nh(x)) - (-(I + 1)*sqrt(2)*cosh(x)^8 - (8*I + 8)*sqrt(2)*cosh(x)^7*sinh(x)
- (28*I + 28)*sqrt(2)*cosh(x)^6*sinh(x)^2 - (56*I + 56)*sqrt(2)*cosh(x)^5*s
inh(x)^3 - (70*I + 70)*sqrt(2)*cosh(x)^4*sinh(x)^4 - (56*I + 56)*sqrt(2)*co
sh(x)^3*sinh(x)^5 - (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I + 8)*sqr
t(2)*cosh(x)*sinh(x)^7 - (I + 1)*sqrt(2)*sinh(x)^8 + (I + 1)*sqrt(2))*log((
I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - ((I - 1)*sqrt(2)*cosh(x)^8 + (8*I
 - 8)*sqrt(2)*cosh(x)^7*sinh(x) + (28*I - 28)*sqrt(2)*cosh(x)^6*sinh(x)^2 +
 (56*I - 56)*sqrt(2)*cosh(x)^5*sinh(x)^3 + (70*I - 70)*sqrt(2)*cosh(x)^4*si
nh(x)^4 + (56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I - 28)*sqrt(2)*cos
h(x)^2*sinh(x)^6 + (8*I - 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I - 1)*sqrt(2)*si
nh(x)^8 - (I - 1)*sqrt(2))*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) -
(-(I - 1)*sqrt(2)*cosh(x)^8 - (8*I - 8)*sqrt(2)*cosh(x)^7*sinh(x) - (28*I -
 28)*sqrt(2)*cosh(x)^6*sinh(x)^2 - (56*I - 56)*sqrt(2)*cosh(x)^5*sinh(x)^3
- (70*I - 70)*sqrt(2)*cosh(x)^4*sinh(x)^4 - (56*I - 56)*sqrt(2)*cosh(x)^3*s
inh(x)^5 - (28*I - 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I - 8)*sqrt(2)*cosh
(x)*sinh(x)^7 - (I - 1)*sqrt(2)*sinh(x)^8 + (I - 1)*sqrt(2))*log((I - 1)*sq
```

```
rt(2) + 2*cosh(x) + 2*sinh(x)) - ((I + 1)*sqrt(2)*cosh(x)^8 + (8*I + 8)*sqr
t(2)*cosh(x)^7*sinh(x) + (28*I + 28)*sqrt(2)*cosh(x)^6*sinh(x)^2 + (56*I +
56)*sqrt(2)*cosh(x)^5*sinh(x)^3 + (70*I + 70)*sqrt(2)*cosh(x)^4*sinh(x)^4 +
 (56*I + 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I + 28)*sqrt(2)*cosh(x)^2*si
nh(x)^6 + (8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*sinh(x)^8 -
 (I + 1)*sqrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + 2*(cosh(x
)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cosh(x)^5*sinh(x)^3
 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6
 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 - 1)*log(cosh(x) + sinh(x) + 1) - 2*(cos
h(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cosh(x)^5*sinh(x
)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x
)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 - 1)*log(cosh(x) + sinh(x) - 1) + 16*
cosh(x) + 16*sinh(x))/(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(
x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(
x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 - 1)
```

Sympy [F]

$$\int e^x \operatorname{csch}^2(4x)\,dx = \int e^x \operatorname{csch}^2(4x)\,dx$$

```
[In] integrate(exp(x)*csch(4*x)**2,x)
[Out] Integral(exp(x)*csch(4*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\begin{aligned}\int e^x \operatorname{csch}^2(4x)\,dx = & -\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) \\ & -\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) \\ & +\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^x}{2\left(e^{(8x)} - 1\right)} \\ & -\frac{1}{8}\arctan\left(e^x\right) - \frac{1}{16}\log\left(e^x + 1\right) + \frac{1}{16}\log\left(e^x - 1\right)\end{aligned}$$

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="maxima")
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int e^x \operatorname{csch}^2(4x)\,dx = -\frac{1}{16}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,e^x\right)\right) - \frac{1}{16}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,e^x\right)\right) - \frac{1}{32}\,\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2\,x)} + 1\right) + \frac{1}{32}\,\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2\,x)} + 1\right) - \frac{e^x}{2\left(e^{(8\,x)} - 1\right)} - \frac{1}{8}\arctan\left(e^x\right) - \frac{1}{16}\log\left(e^x + 1\right) + \frac{1}{16}\log\left(|e^x - 1|\right)$$

```
[In] integrate(exp(x)*csch(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int e^x \operatorname{csch}^2(4x)\,dx = \frac{\ln\left(\frac{1}{2}-\frac{e^x}{2}\right)}{16} - \frac{\ln\left(-\frac{e^x}{2}-\frac{1}{2}\right)}{16} - \frac{\operatorname{atan}(e^x)}{8} - \frac{e^x}{2\left(e^{8\,x}-1\right)} - \frac{\sqrt{2}\operatorname{atan}\left(2\,\sqrt{2}\left(\frac{e^x}{2}-\frac{\sqrt{2}}{4}\right)\right)}{16} - \frac{\sqrt{2}\operatorname{atan}\left(2\,\sqrt{2}\left(\frac{e^x}{2}+\frac{\sqrt{2}}{4}\right)\right)}{16} + \frac{\sqrt{2}\,\ln\left(\left(\frac{e^x}{2}-\frac{\sqrt{2}}{4}\right)^2+\frac{1}{8}\right)}{32} - \frac{\sqrt{2}\,\ln\left(\left(\frac{e^x}{2}+\frac{\sqrt{2}}{4}\right)^2+\frac{1}{8}\right)}{32}$$

```
[In] int(exp(x)/sinh(4*x)^2,x)
```

```
[Out] log(1/2 - exp(x)/2)/16 - log(- exp(x)/2 - 1/2)/16 - atan(exp(x))/8 - exp(x)
/(2*(exp(8*x) - 1)) - (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 - 2^(1/2)/4)))/16 -
 (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 + 2^(1/2)/4)))/16 + (2^(1/2)*log((exp(x)
/2 - 2^(1/2)/4)^2 + 1/8))/32 - (2^(1/2)*log((exp(x)/2 + 2^(1/2)/4)^2 + 1/8)
)/32
```

3.322 $\int F^{c(a+bx)} \sinh^3(d+ex) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 202

$$\int F^{c(a+bx)} \sinh^3(d+ex) \, dx = -\frac{6e^3F^{c(a+bx)}\cosh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sinh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^3(d+ex)}{9e^2-b^2c^2\log^2(F)}$$

```
[Out] -6*e^3*F^(c*(b*x+a))*cosh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F
)^4)+6*b*c*e^2*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^
2+b^4*c^4*ln(F)^4)+3*e*F^(c*(b*x+a))*cosh(e*x+d)*sinh(e*x+d)^2/(9*e^2-b^2*c
^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sinh(e*x+d)^3/(9*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used

$= \{5584, 5582\}$

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = -\frac{bc\log(F)\sinh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)} + \frac{3e\sinh^2(d+ex)\cosh(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)} + \frac{6bce^2\log(F)\sinh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4} - \frac{6e^3\cosh(d+ex)F^{c(a+bx)}}{b^4c^4\log^4(F)-10b^2c^2e^2\log^2(F)+9e^4}$$

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (-6*e^3*F^(c*(a + b*x))*Cosh[d + e*x])/(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b
^4*c^4*Log[F]^4) + (6*b*c*e^2*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x])/(9*e^4
- 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4) + (3*e*F^(c*(a + b*x))*Cosh[d
 + e*x]*Sinh[d + e*x]^2)/(9*e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a + b*x))*
Log[F]*Sinh[d + e*x]^3)/(9*e^2 - b^2*c^2*Log[F]^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^3(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{(6e^2)\int F^{c(a+bx)}\sinh(d+ex)\,dx}{9e^2-b^2c^2\log^2(F)} \\ &= -\frac{6e^3F^{c(a+bx)}\cosh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{6bce^2F^{c(a+bx)}\log(F)\sinh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} \\ &\quad + \frac{3eF^{c(a+bx)}\cosh(d+ex)\sinh^2(d+ex)}{9e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^3(d+ex)}{9e^2-b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(3\cosh(3(d+ex))\left(e^3-b^2c^2e\log^2(F)\right)+3\cosh(d+ex)\left(-9e^3+b^2c^2e\log^2(F)\right)+2bc\log(F)\left(13\right.\right.}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4\right.}$$

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^3,x]
```

```
[Out] (F^(c*(a + b*x))*(3*Cosh[3*(d + e*x)]*(e^3 - b^2*c^2*e*Log[F]^2) + 3*Cosh[d
 + e*x]*(-9*e^3 + b^2*c^2*e*Log[F]^2) + 2*b*c*Log[F]*(13*e^2 - b^2*c^2*Log[
F]^2 + Cosh[2*(d + e*x)]*(-e^2 + b^2*c^2*Log[F]^2))*Sinh[d + e*x]))/(4*(9*e
^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73

method	result
parallelrisch	$-\frac{3\left(\left(\ln(F)^2b^2c^2e-e^3\right)\cosh(3ex+3d)+\frac{\left(-\ln(F)^3b^3c^3+\ln(F)bc\,e^2\right)\sinh(3ex+3d)}{3}+(bc\ln(F)-3e)(bc\ln(F)+3e)(\sinh(ex+d)\ln(F\right.}{4\left(9e^4-10b^2c^2e^2\ln(F)^2+b^4c^4\ln(F)^4\right)}$
risch	$\frac{\left(\ln(F)^3b^3c^3e^{6ex+6d}-3\ln(F)^3b^3c^3e^{4ex+4d}-3\ln(F)^2b^2c^2e\,e^{6ex+6d}+3\ln(F)^3b^3c^3e^{2ex+2d}+3\ln(F)^2b^2c^2e\,e^{4ex+4d}-\ln(F)bc\,e^2\right.}{}$

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*((ln(F)^2*b^2*c^2*e-e^3)*cosh(3*e*x+3*d)+1/3*(-ln(F)^3*b^3*c^3+ln(F)*b
*c*e^2)*sinh(3*e*x+3*d)+(b*c*ln(F)-3*e)*(b*c*ln(F)+3*e)*(sinh(e*x+d)*ln(F)*
b*c-e*cosh(e*x+d)))*F^(c*(b*x+a))/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(
F)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2228 vs. 2(199) = 398.

Time = 0.36 (sec) , antiderivative size = 2228, normalized size of antiderivative = 11.03

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/8*((3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c^3
*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*cosh
(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e*x +
 d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3*c^3)*l
og(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*
b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*c^3*cos
h(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2 - b^3*
c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) + (5*b^3*c
^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^2*e*cos
h(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)
^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3 - 3*(b^2*c^2
*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x + d)^2
+ b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*cosh(e*x + d)^2
+ (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*log(F)^
3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cosh(e*x + d)^2 - b
^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e^2*cosh(e*x + d)^
2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 - 27*b*c*
e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e^2)*log(F) + 6*(3*e
^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) + (b^3*c^
3*cosh(e*x + d)^5 - 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d))*log(
F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 - 2*b^2*c^2*e*cosh(e*x + d)^3 - b^2*c^2
*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 - 18*b*c*e^2*cosh(e*x
 + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*cosh((b*c*x + a*c
)*log(F)) + (3*e^3*cosh(e*x + d)^6 - 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(
F)^3 - 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(
b^3*c^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e
^2*cosh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cos
h(e*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 - b^3
*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 - b^2*c^2*e)*log(F)^
2 - (5*b*c*e^2*cosh(e*x + d)^2 - 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 - 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^2
 - b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 - 27*e^3*cosh(e*x + d) + (
5*b^3*c^3*cosh(e*x + d)^3 - 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2*c^
2*e*cosh(e*x + d)^3 - b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*cosh(e
*x + d)^3 - 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 + 3*e^3 - 3*(
b^2*c^2*e*cosh(e*x + d)^6 - b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cosh(e*x
+ d)^2 + b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 - 54*e^3*cosh(e*x
+ d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 - 6*b^3*c^3*cosh(e*x + d)^2 + b^3*c^3)*
log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 - 6*b^2*c^2*e*cosh(e*x + d
)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 - 54*b*c*e^2*cosh(e*
x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*x + d)^6 -
27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 - b*c*e^2)*log(F) +
 6*(3*e^3*cosh(e*x + d)^5 - 18*e^3*cosh(e*x + d)^3 - 9*e^3*cosh(e*x + d) +
(b^3*c^3*cosh(e*x + d)^5 - 2*b^3*c^3*cosh(e*x + d)^3 + b^3*c^3*cosh(e*x + d
```

```
))*log(F)^3 - (3*b^2*c^2*e*cosh(e*x + d)^5 - 2*b^2*c^2*e*cosh(e*x + d)^3 -
b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (b*c*e^2*cosh(e*x + d)^5 - 18*b*c*e^2*c
osh(e*x + d)^3 + 9*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d))*sinh((b*c*
x + a*c)*log(F)))/(b^4*c^4*cosh(e*x + d)^3*log(F)^4 - 10*b^2*c^2*e^2*cosh(e
*x + d)^3*log(F)^2 + 9*e^4*cosh(e*x + d)^3 + (b^4*c^4*log(F)^4 - 10*b^2*c^2
*e^2*log(F)^2 + 9*e^4)*sinh(e*x + d)^3 + 3*(b^4*c^4*cosh(e*x + d)*log(F)^4
- 10*b^2*c^2*e^2*cosh(e*x + d)*log(F)^2 + 9*e^4*cosh(e*x + d))*sinh(e*x + d
)^2 + 3*(b^4*c^4*cosh(e*x + d)^2*log(F)^4 - 10*b^2*c^2*e^2*cosh(e*x + d)^2*
log(F)^2 + 9*e^4*cosh(e*x + d)^2)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1525 vs. $2(199) = 398$.

Time = 7.77 (sec) , antiderivative size = 1525, normalized size of antiderivative = 7.55

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**3,x)
```

```
[Out] Piecewise((x*sinh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d)**3, Eq(b
, 0) & Eq(e, 0)), (x*sinh(d)**3, Eq(c, 0) & Eq(e, 0)), (-3*F**(a*c + b*c*x)
*x*sinh(b*c*x*log(F) - d)**3/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) - d
)**2*cosh(b*c*x*log(F) - d)/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) - d)
*cosh(b*c*x*log(F) - d)**2/8 - 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)*
*3/8 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b*c*log(F)) - 3*F**(a*
c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/(4*b*c*log(F))
+ 3*F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*log(F)), Eq(e, -b*c*l
og(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**3/8 + 3*F**(a*c + b
*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/8 - 3*F**(a*c
+ b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 - d)**2/8 + F**(a*c
 + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 - 9*F**(a*c + b*c*x)*sinh(b*c*x*l
og(F)/3 - d)**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 - d
)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*c*x*
log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e, -b*c*log(F)/3)), (F**(a*c + b*c*x)*x
*sinh(b*c*x*log(F)/3 + d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 +
 d)**2*cosh(b*c*x*log(F)/3 + d)/8 + 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/
3 + d)*cosh(b*c*x*log(F)/3 + d)**2/8 - F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)
/3 + d)**3/8 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 + d)**3/(8*b*c*log(F))
+ 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/(
b*c*log(F)) - 15*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F
)/3 + d)**2/(4*b*c*log(F)) + 11*F**(a*c + b*c*x)*cosh(b*c*x*log(F)/3 + d)**
3/(8*b*c*log(F)), Eq(e, b*c*log(F)/3)), (3*F**(a*c + b*c*x)*x*sinh(b*c*x*lo
g(F) + d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) + d)**2*cosh(b*c*x*
log(F) + d)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) + d)*cosh(b*c*x*log(
```

```
F) + d)**2/8 + 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) + d)**3/8 - F**(a*c +
 b*c*x)*sinh(b*c*x*log(F) + d)**3/(8*b*c*log(F)) + 3*F**(a*c + b*c*x)*sinh(
b*c*x*log(F) + d)**2*cosh(b*c*x*log(F) + d)/(4*b*c*log(F)) - 3*F**(a*c + b*
c*x)*cosh(b*c*x*log(F) + d)**3/(8*b*c*log(F)), Eq(e, b*c*log(F))), (F**(a*c
 + b*c*x)*b**3*c**3*log(F)**3*sinh(d + e*x)**3/(b**4*c**4*log(F)**4 - 10*b*
*2*c**2*e**2*log(F)**2 + 9*e**4) - 3*F**(a*c + b*c*x)*b**2*c**2*e*log(F)**2
*sinh(d + e*x)**2*cosh(d + e*x)/(b**4*c**4*log(F)**4 - 10*b**2*c**2*e**2*lo
g(F)**2 + 9*e**4) - 7*F**(a*c + b*c*x)*b*c*e**2*log(F)*sinh(d + e*x)**3/(b*
*4*c**4*log(F)**4 - 10*b**2*c**2*e**2*log(F)**2 + 9*e**4) + 6*F**(a*c + b*c
*x)*b*c*e**2*log(F)*sinh(d + e*x)*cosh(d + e*x)**2/(b**4*c**4*log(F)**4 - 1
0*b**2*c**2*e**2*log(F)**2 + 9*e**4) + 9*F**(a*c + b*c*x)*e**3*sinh(d + e*x
)**2*cosh(d + e*x)/(b**4*c**4*log(F)**4 - 10*b**2*c**2*e**2*log(F)**2 + 9*e
**4) - 6*F**(a*c + b*c*x)*e**3*cosh(d + e*x)**3/(b**4*c**4*log(F)**4 - 10*b
**2*c**2*e**2*log(F)**2 + 9*e**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} - \frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log(F)-ee^{d})} - \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) - 3/8*F^(a*c)
*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F)
- e*x)/(b*c*e^d*log(F) - e*e^d) - 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(b*c
*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 1211, normalized size of antiderivative = 6.00

$$\int F^{c(a+bx)} \sinh^3(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(
```

F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) - 3/4*(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) + 3*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d) - 1/4*(2*(b*c*log(abs(F)) - 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d) + I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) - 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - 3*e)*x - 3*d)

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.82

$$\int F^{c(a+bx)}\sinh^3(d+ex)\,dx =$$

$$-\frac{F^{a\,c+b\,c\,x}\,\left(-b^3\,c^3\,\sinh(d+e\,x)^3\,\ln\left(F\right)^3+3\,b^2\,c^2\,e\cosh(d+e\,x)\,\sinh(d+e\,x)^2\,\ln\left(F\right)^2-6\,b\,c\,e^2\cosh(d+e}{b^4\,c^4\ln\left(F\right)^4-}$$

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x)^3,x)
```

```
[Out] -(F^(a*c + b*c*x)*(6*e^3*cosh(d + e*x)^3 - 9*e^3*cosh(d + e*x)*sinh(d + e*x
)^2 - b^3*c^3*sinh(d + e*x)^3*log(F)^3 + 7*b*c*e^2*sinh(d + e*x)^3*log(F) +
 3*b^2*c^2*e*cosh(d + e*x)*sinh(d + e*x)^2*log(F)^2 - 6*b*c*e^2*cosh(d + e*
x)^2*sinh(d + e*x)*log(F)))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(
F)^2)
```

3.323 $\int F^{c(a+bx)} \sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = -\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)} + \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

```
[Out] -2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*co
sh(e*x+d)*sinh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*sinh(
e*x+d)^2/(4*e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5584, 2225}

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx = -\frac{bc\log(F)\sinh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)} + \frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)} - \frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (-2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2) - (b*c
*F^(c*(a + b*x))*Log[F]*Sinh[d + e*x]^2)/(4*e^2 - b^2*c^2*Log[F]^2)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5584

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (-Dist[n*(n - 1)*(e^2/(e^2*n^2 - b^2*c^2*Log[F]^2)), In
t[F^(c*(a + b*x))*Sinh[d + e*x]^(n - 2), x], x] + Simp[e*n*F^(c*(a + b*x))*
Cosh[d + e*x]*(Sinh[d + e*x]^(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} \\ &\quad -\frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}-\frac{(2e^2)\int F^{c(a+bx)}\,dx}{4e^2-b^2c^2\log^2(F)} \\ &= -\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}+\frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)} \\ &\quad -\frac{bcF^{c(a+bx)}\log(F)\sinh^2(d+ex)}{4e^2-b^2c^2\log^2(F)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^2-b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x]^2,x]
```

```
[Out] (F^(c*(a + b*x))*(4*e^2 - b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Log[
F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3*Lo
g[F]^3)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

method	result
parallelrisch	$-\frac{2F^{c(bx+a)}\left(-\frac{b^{2}c^{2}\ln(F)^{2}\cosh(2ex+2d)}{2}+\frac{b^{2}c^{2}\ln(F)^{2}}{2}+ebc\ln(F)\sinh(2ex+2d)-2e^{2}\right)}{2\ln(F)^{3}b^{3}c^{3}-8\ln(F)bc\,e^{2}}$
risch	$\frac{\left(\ln(F)^{2}b^{2}c^{2}\mathrm{e}^{4ex+4d}-2\ln(F)^{2}b^{2}c^{2}\mathrm{e}^{2ex+2d}-2\ln(F)bce\,\mathrm{e}^{4ex+4d}+b^{2}c^{2}\ln(F)^{2}+2ebc\ln(F)+8e^{2}\mathrm{e}^{2ex+2d}\right)\mathrm{e}^{-2ex-2d}F^{c(bx+a)}}{4\ln(F)bc(bc\ln(F)-2e)(bc\ln(F)+2e)}$

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
[Out] -2*F^(c*(b*x+a))*(-1/2*b^2*c^2*ln(F)^2*cosh(2*e*x+2*d)+1/2*b^2*c^2*ln(F)^2+
e*b*c*ln(F)*sinh(2*e*x+2*d)-2*e^2)/(2*ln(F)^3*b^3*c^3-8*ln(F)*b*c*e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. 2(128) = 256.

Time = 0.28 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.33

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \frac{\left(\left(b^2c^2\log(F)^2-2\,bce\log(F)\right)\sinh(ex+d)^4+8\,e^2\cosh(ex+d)^2+4\,\left(b^2c^2\cosh(ex+d)\log(F)^2-2\,bc\right.}{}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="fricas")
[Out] 1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 + 8*e^2*cosh(e*x
+ d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*
sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*b^2*c^2*cosh(e*x + d)^2 + b^
2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x +
 d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)
^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) - 4*e^2*cosh(e*x + d
) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x +
d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(
e*x + d)^4 + 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*
b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 - 2*
b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*lo
g(F) - (3*b^2*c^2*cosh(e*x + d)^2 - b^2*c^2)*log(F)^2 - 4*e^2)*sinh(e*x + d
)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3
*log(F) - 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 - b^2*c^2*cosh(e*x
 + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/(b^3*c^3*cosh(e
*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^3*c^3*log(F)^3 -
 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x + d)*log(F)^3 - 4*
b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(119) = 238$.

Time = 1.07 (sec) , antiderivative size = 709, normalized size of antiderivative = 5.37

$$\int F^{c(a+bx)} \sinh^2(d+ex)\,dx$$

$$= \begin{cases} x\sinh^2(d) \\ \frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ F^{ac}\left(\frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e}\right) \\ \frac{x\sinh^2(d+ex)}{2} - \frac{x\cosh^2(d+ex)}{2} + \frac{\sinh(d+ex)\cosh(d+ex)}{2e} \\ \frac{F^{ac+bcx}x\sinh^2\left(\frac{bcx\log(F)}{2}-d\right)}{4} - \frac{F^{ac+bcx}x\sinh\left(\frac{bcx\log(F)}{2}-d\right)\cosh\left(\frac{bcx\log(F)}{2}-d\right)}{2} + \frac{F^{ac+bcx}x\cosh^2\left(\frac{bcx\log(F)}{2}-d\right)}{4} + \frac{3F^{ac+bcx}\sinh}{} \\ \frac{F^{ac+bcx}x\sinh^2\left(\frac{bcx\log(F)}{2}+d\right)}{4} - \frac{F^{ac+bcx}x\sinh\left(\frac{bcx\log(F)}{2}+d\right)\cosh\left(\frac{bcx\log(F)}{2}+d\right)}{2} + \frac{F^{ac+bcx}x\cosh^2\left(\frac{bcx\log(F)}{2}+d\right)}{4} + \frac{3F^{ac+bcx}\sinh}{} \\ \frac{F^{ac+bcx}b^2c^2\log(F)^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac+bcx}bce\log(F)\sinh(d+ex)\cosh(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} - \frac{2F^{ac+bcx}e^2\sinh^2(d+ex)}{b^3c^3\log(F)^3-4bce^2\log(F)} + \frac{2F^{ac+bcx}e^2\cosh^2(d}{b^3c^3\log(F)^3-4bce^2\log} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d)**2,x)
```

```
[Out] Piecewise((x*sinh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (x*sin
h(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e),
 Eq(F, 1)), (F**(a*c)*(x*sinh(d + e*x)**2/2 - x*cosh(d + e*x)**2/2 + sinh(d
 + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (x*sinh(d + e*x)**2/2 - x*cosh(d +
 e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c + b*c*x
)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2
- d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 -
d)**2/4 + 3*F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*log(F)/2 -
 d)/(2*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 - d)**2/(b*c*log(
F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)**2
/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/2
 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 + d)**2/4 + 3*F**(a*c + b*c*x)*si
nh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F)) - F**(a*c +
b*c*x)*cosh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)), Eq(e, b*c*log(F)/2)), (F**
(a*c + b*c*x)*b**2*c**2*log(F)**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4
*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sinh(d + e*x)*cosh(d +
e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*e**2*si
nh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a*c + b*c*
x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} - \frac{F^{bcx+ac}}{2\,bc\log(F)}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c)
*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) - 1/2*F^(b*c*x
 + a*c)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 890, normalized size of antiderivative = 6.74

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2) -
 (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c
)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(-I*e^(1/2*I*pi*b*c*x*sgn
(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*sgn(
F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*
pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) + 2*I*pi
*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + 1/2*(2
*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
- 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(
abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) +
I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*
I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e) - I*e
^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*
a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) + 16*e))*e^(a*c*l
```

```
og(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*c*log(abs(F)) - 2
*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*
c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2) - (pi*b*c*sgn
(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) +
 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - 2*e)^2))*e^
(a*c*log(abs(F)) + (b*c*log(abs(F)) - 2*e)*x - 2*d) + I*(I*e^(1/2*I*pi*b*c*
x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c
*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn
(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn
(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) - 16*e))*e^(a*c*log(abs(F)) + (b*c*log
(abs(F)) - 2*e)*x - 2*d)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.73

$$\int F^{c(a+bx)}\sinh^2(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,\left(2\,e^2-\frac{b^2\,c^2\,\ln(F)^2}{2}+\frac{b^2\,c^2\,\ln(F)^2\,\cosh(2\,d+2\,e\,x)}{2}-b\,c\,e\,\ln\left(F\right)\,\sinh(2\,d+2\,e\,x)\right)}{b\,c\,\ln\left(F\right)\,\left(4\,e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x)^2,x)
```

```
[Out] -(F^(a*c + b*c*x)*(2*e^2 - (b^2*c^2*log(F)^2)/2 + (b^2*c^2*log(F)^2*cosh(2*
d + 2*e*x))/2 - b*c*e*log(F)*sinh(2*d + 2*e*x)))/(b*c*log(F)*(4*e^2 - b^2*c
^2*log(F)^2))
```

3.324 $\int F^{c(a+bx)} \sinh(d+ex)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int F^{c(a+bx)} \sinh(d+ex)\, dx = \frac{eF^{c(a+bx)} \cosh(d+ex)}{e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

```
[Out] e*F^(c*(b*x+a))*cosh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*ln(F)*s
inh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5582}

$$\int F^{c(a+bx)} \sinh(d+ex)\, dx = \frac{e\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)} - \frac{bc\log(F)\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

```
[In] Int[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (e*F^(c*(a + b*x))*Cosh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2) - (b*c*F^(c*(a +
 b*x))*Log[F]*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = \frac{eF^{c(a+bx)}\cosh(d+ex)}{e^2-b^2c^2\log^2(F)} - \frac{bcF^{c(a+bx)}\log(F)\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx=\frac{F^{c(a+bx)}(e\cosh(d+ex)-bc\log(F)\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

```
[In] Integrate[F^(c*(a + b*x))*Sinh[d + e*x],x]
```

```
[Out] (F^(c*(a + b*x))*(e*Cosh[d + e*x] - b*c*Log[F]*Sinh[d + e*x]))/((e - b*c*Lo
g[F])*(e + b*c*Log[F]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{F^{c(bx+a)}(\sinh(ex+d)\ln(F)bc-e\cosh(ex+d))}{b^2c^2\ln(F)^2-e^2}$	51
risch	$\frac{(\ln(F)bc\,\mathrm{e}^{2ex+2d}-bc\ln(F)-e\,\mathrm{e}^{2ex+2d}-e)\mathrm{e}^{-ex-d}F^{c(bx+a)}}{2(bc\ln(F)-e)(e+bc\ln(F))}$	77

```
[In] int(F^(c*(b*x+a))*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2)*(sinh(e*x+d)*ln(F)*b*c-e*cosh(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(77) = 154.

Time = 0.29 (sec) , antiderivative size = 244, normalized size of antiderivative = 3.25

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = -\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2-bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex+d\right)\right.\right.}{}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x
 + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh
(e*x + d) + e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*log(F
) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 - b*c)*log(F) - 2*(b*c*cosh(e
*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) + e)*sinh((b*c*x + a*c)*log
(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*log(F)
^2 - e^2)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(68) = 136$.

Time = 0.58 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.31

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx$$
$$= \begin{cases} x\sinh{(d)} & \text{fo} \\ F^{ac}x\sinh{(d)} & \text{fo} \\ x\sinh{(d)} & \text{fo} \\ -\frac{F^{ac+bcx}x\sinh{(bcx\log{(F)}-d)}}{2} + \frac{F^{ac+bcx}x\cosh{(bcx\log{(F)}-d)}}{2} + \frac{F^{ac+bcx}\sinh{(bcx\log{(F)}-d)}}{2bc\log{(F)}} - \frac{F^{ac+bcx}\cosh{(bcx\log{(F)}-d)}}{bc\log{(F)}} & \text{fo} \\ \frac{F^{ac+bcx}x\sinh{(bcx\log{(F)}+d)}}{2} - \frac{F^{ac+bcx}x\cosh{(bcx\log{(F)}+d)}}{2} - \frac{F^{ac+bcx}\sinh{(bcx\log{(F)}+d)}}{2bc\log{(F)}} + \frac{F^{ac+bcx}\cosh{(bcx\log{(F)}+d)}}{bc\log{(F)}} & \text{fo} \\ \frac{F^{ac+bcx}bc\log{(F)}\sinh{(d+ex)}}{b^2c^2\log{(F)}^2-e^2} - \frac{F^{ac+bcx}e\cosh{(d+ex)}}{b^2c^2\log{(F)}^2-e^2} & \text{o} \end{cases}$$

```
[In] integrate(F**(c*(b*x+a))*sinh(e*x+d),x)
```

```
[Out] Piecewise((x*sinh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*sinh(d), Eq(b, 0) &
 Eq(e, 0)), (x*sinh(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sinh(b*c
*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)/2 + F**(a*c +
b*c*x)*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)) - F**(a*c + b*c*x)*cosh(b*c*x*
log(F) - d)/(b*c*log(F)), Eq(e, -b*c*log(F))), (F**(a*c + b*c*x)*x*sinh(b*c
*x*log(F) + d)/2 - F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) + d)/2 - F**(a*c +
b*c*x)*sinh(b*c*x*log(F) + d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*cosh(b*c*x*
log(F) + d)/(b*c*log(F)), Eq(e, b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*
sinh(d + e*x)/(b**2*c**2*log(F)**2 - e**2) - F**(a*c + b*c*x)*e*cosh(d + e*
x)/(b**2*c**2*log(F)**2 - e**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}\sinh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log{(F)}+e)} - \frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\,(bce^d\log{(F)}-ee^d)}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) - 1/2*F^(a*c)*e^(b*
c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 598, normalized size of antiderivative = 7.97

$$\int F^{c(a+bx)} \sinh(d+ex)\,dx = \text{Too large to display}$$

```
[In] integrate(F^(c*(b*x+a))*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] (2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a
*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) +
e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2*I*(I
*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi
*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(-1/2*I*
pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*
pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(abs(F)) + (
b*c*log(abs(F)) + e)*x + d) - (2*(b*c*log(abs(F)) - e)*cos(-1/2*pi*b*c*x*sg
n(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*
b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*
b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(
F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F)) + (b*c*log(
abs(F)) - e)*x - d) + 1/2*I*(-I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x +
 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*lo
g(abs(F)) - 2*e) + I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*
a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F))
 - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d)
```

Mupad [B] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)} \sinh(d+ex)\,dx = \frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e+e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)-b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,{\ln\left(F\right)}^2\right)}$$

```
[In] int(F^(c*(a + b*x))*sinh(d + e*x),x)
```

```
[Out] (F^(a*c + b*c*x)*exp(- d - e*x)*(e + e*exp(2*d + 2*e*x) + b*c*log(F) - b*c*
exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

3.325 $\int F^{c(a+bx)}\text{csch}(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 66

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx = -\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)}{e+bc\log(F)}$$

```
[Out] -2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c
*ln(F)/e],exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5601}

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx = -\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{bc\log(F)+e}$$

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x],x]
```

```
[Out] (-2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
```

```
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2e^{d+ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1, \frac{e+bc\log(F)}{2e}, \frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right), e^{2(d+ex)}\right)}{e+bc\log(F)}$$

Mathematica [A] (verified)

Time = 2.62 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(\text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{e}, 1+\frac{bc\log(F)}{e}, -e^{d+ex}\right) - \text{Hypergeometric2F1}\left(1, \frac{bc\log(F)}{e}, 1+\frac{bc\log}{e}\right.\right.}{bc\log(F)}$$

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x],x]
[Out] (F^(c*(a + b*x))*(Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e,
-E^(d + e*x)] - Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, E^
(d + e*x)]))/(b*c*Log[F])
```

Maple [F]

$$\int F^{c(bx+a)}\,\text{csch}\,(ex+d)\,dx$$

```
[In] int(F^(c*(b*x+a))*csch(e*x+d),x)
[Out] int(F^(c*(b*x+a))*csch(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)\,\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{csch}\left(d+ex\right)dx$$

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d),x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="maxima")
```

```
[Out] 4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*d)
*log(F) - e*e^(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2*e*
x) - e), x) - 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) - (b*c*e^(2*
d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Giac [F]

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)}\,dx$$

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x),x)
```

```
[Out] int(F^(c*(a + b*x))/sinh(d + e*x), x)
```

3.326 $\int F^{c(a+bx)}\mathrm{csch}^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int F^{c(a+bx)}\mathrm{csch}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

```
[Out] 4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c*
ln(F)/e],exp(2*e*x+2*d))/(b*c*ln(F)+2*e)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {5601}

$$\int F^{c(a+bx)}\mathrm{csch}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
```

```
[Out] (4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(2
*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
```

```
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.28

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx = -\frac{2F^{c(a+bx)}\left(\left(-1+e^{2d}\right)\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)+\text{csch}(d+ex)\sinh(d)(\cos}{e\left(-1+e^{2d}\right)}$$

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^2,x]
[Out] (-2*F^(c*(a + b*x))*((-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e)
, 1 + (b*c*Log[F])/(2*e), E^(2*(d + e*x))] + Csch[d + e*x]*Sinh[d]*(Cosh[e*
x] - Sinh[e*x])))/(e*(-1 + E^(2*d)))
```

Maple [F]

$$\int F^{c(bx+a)}\,\text{csch}\,(ex+d)^2\,dx$$

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^2\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="fricas")
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx$$

[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**2,x)

[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**2, x)

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^2\,dx$$

[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="maxima")

[Out] 16*F^(a*c)*b*c*e*integrate(-F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) + 8*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d)) *e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2*e ^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F) + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) + 4*(4*F^(a*c)*e + (F^(a*c)*b*c*e^(2*d)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 6* b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2*e^(4*d))*e^(4*e*x) - 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*l og(F) + 8*e^2*e^(2*d))*e^(2*e*x))

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}(ex+d)^2\,dx$$

[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^2,x, algorithm="giac")

[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^2, x)

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^2}\,dx$$

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^2,x)
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^2, x)
```

3.327 $\int F^{c(a+bx)}\mathrm{csch}^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 122

$$\int F^{c(a+bx)}\mathrm{csch}^3(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\mathrm{csch}(d+ex)}{2e}-\frac{bcF^{c(a+bx)}\mathrm{csch}(d+ex)\log(F)}{2e^2}+\frac{e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)(e-bc\log(F))}{e^2}$$

```
[Out] -1/2*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)/e-1/2*b*c*F^(c*(b*x+a))*csch(e*x
+d)*ln(F)/e^2+exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[
3/2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5599, 5601}

$$\int F^{c(a+bx)}\mathrm{csch}^3(d+ex)\,dx = \frac{e^{d+ex}F^{c(a+bx)}(e-bc\log(F))\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),e^{2(d+ex)}\right)}{e^2}-\frac{bc\log(F)\mathrm{csch}(d+ex)F^{c(a+bx)}}{2e^2}-\frac{\coth(d+ex)\mathrm{csch}(d+ex)F^{c(a+bx)}}{2e}$$

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^3,x]
```

```
[Out] -1/2*(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x])/e - (b*c*F^(c*(a + b*x))
*Csch[d + e*x]*Log[F])/(2*e^2) + (E^(d + e*x)*F^(c*(a + b*x))*Hypergeometri
c2F1[1, (e + b*c*Log[F])/(2*e), (3 + (b*c*Log[F])/e)/2, E^(2*(d + e*x))]*(e
 - b*c*Log[F]))/e^2
```

Rule 5599

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(
n - 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a +
 b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F,
a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
 && NeQ[n, 2]
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\text{csch}(d+ex)\log(F)}{2e^2} \\
&\quad - \frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\text{csch}(d+ex)\,dx \\
&= -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}(d+ex)}{2e} - \frac{bcF^{c(a+bx)}\text{csch}(d+ex)\log(F)}{2e^2} \\
&\quad + \frac{e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),e^{2(d+ex)}\right)(e-bc\log(F))}{e^2}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 281 vs. $2(122) = 244$.

Time = 14.38 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.30

$$
\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx
$$

$$
= \frac{F^{c(a+bx)}\left(-e\text{csch}^2\left(\frac{1}{2}(d+ex)\right)-4bc\text{csch}(d)\log(F)+\text{csch}(d)\left(-\frac{4e^2}{bc\log(F)}+4bc\log(F)\right)+\frac{4\left(1-(1+e^d)\text{ Hyperge}\right.}{}\right.}{}
$$

[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^3,x]

[Out] (F^(c*(a + b*x))*(-(e*Csch[(d + e*x)/2]^2) - 4*b*c*Csch[d]*Log[F] + Csch[d]*((-4*e^2)/(b*c*Log[F]) + 4*b*c*Log[F]) + (4*(1 - (1 + E^d)*Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, -E^(d + e*x)])*(e^2 - b^2*c^2*Log[F]^2))/(b*c*(1 + E^d)*Log[F]) + (4*(1 + (-1 + E^d)*Hypergeometric2F1[1, (b*c*Log[F])/e, 1 + (b*c*Log[F])/e, E^(d + e*x)])*(e^2 - b^2*c^2*Log[F]^2))/(b*c*(-1 + E^d)*Log[F]) - e*Sech[(d + e*x)/2]^2 + 2*b*c*Csch[d/2]*Csch[(d + e*x)/2]*Log[F]*Sinh[(e*x)/2] + 2*b*c*Log[F]*Sech[d/2]*Sech[(d + e*x)/2]*Sinh[(e*x)/2]))/(8*e^2)

Maple [F]

$$\int F^{c(bx+a)} \operatorname{csch}(ex+d)^3 \, dx$$

[In] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)

[Out] int(F^(c*(b*x+a))*csch(e*x+d)^3,x)

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{csch}(ex+d)^3 \, dx$$

[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="fricas")

[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^3, x)

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}^3(d+ex)\,dx = \int F^{c(a+bx)} \operatorname{csch}^3(d+ex)\,dx$$

[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**3,x)

[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**3, x)

Maxima [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^3\,dx$$

[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="maxima")

[Out] 48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F) + e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) - 4*(b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x) - 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) + (F^(a*c)*b*c*e^(3*d)*log(F) - 5*F^(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 - (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x))

Giac [F]

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{csch}\,(ex+d)^3\,dx$$

[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^3,x, algorithm="giac")

[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^3, x)

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{csch}^3(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^3}\,dx$$

[In] int(F^(c*(a + b*x))/sinh(d + e*x)^3,x)

[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^3, x)

3.328 $\int F^{c(a+bx)}\mathrm{csch}^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int F^{c(a+bx)}\mathrm{csch}^4(d+ex)\,dx = -\frac{F^{c(a+bx)}\coth(d+ex)\mathrm{csch}^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\mathrm{csch}^2(d+ex)\log(F)}{6e^2} - \frac{2e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2}$$

```
[Out] -1/3*F^(c*(b*x+a))*coth(e*x+d)*csch(e*x+d)^2/e-1/6*b*c*F^(c*(b*x+a))*csch(e
*x+d)^2*ln(F)/e^2-2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*
ln(F)/e],[2+1/2*b*c*ln(F)/e],exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5599, 5601}

$$\int F^{c(a+bx)}\mathrm{csch}^4(d+ex)\,dx = -\frac{2e^{2(d+ex)}F^{c(a+bx)}(2e-bc\log(F))\,\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,e^{2(d+ex)}\right)}{3e^2} - \frac{bc\log(F)\mathrm{csch}^2(d+ex)F^{c(a+bx)}}{6e^2} - \frac{\coth(d+ex)\mathrm{csch}^2(d+ex)F^{c(a+bx)}}{3e}$$

```
[In] Int[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] -1/3*(F^(c*(a + b*x))*Coth[d + e*x]*Csch[d + e*x]^2)/e - (b*c*F^(c*(a + b*x
))*Csch[d + e*x]^2*Log[F])/(6*e^2) - (2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hyp
ergeometric2F1[2, 1 + (b*c*Log[F])/(2*e), 2 + (b*c*Log[F])/(2*e), E^(2*(d +
 e*x))]*(2*e - b*c*Log[F]))/(3*e^2)
```

Rule 5599

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Csch[d + e*x]^(n - 2)/(e^2*(n -
1)*(n - 2))), x] + (-Dist[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(
n - 2)), Int[F^(c*(a + b*x))*Csch[d + e*x]^(n - 2), x], x] - Simp[F^(c*(a +
 b*x))*Csch[d + e*x]^(n - 1)*(Cosh[d + e*x]/(e*(n - 1))), x]) /; FreeQ[{F,
a, b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
 && NeQ[n, 2]
```

Rule 5601

```
Int[Csch[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Simp[(-2)^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hy
pergeometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)),
E^(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\text{csch}^2(d+ex)\log(F)}{6e^2} \\ &\quad -\frac{1}{6}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\text{csch}^2(d+ex)\,dx \\ &= -\frac{F^{c(a+bx)}\coth(d+ex)\text{csch}^2(d+ex)}{3e} - \frac{bcF^{c(a+bx)}\text{csch}^2(d+ex)\log(F)}{6e^2} \\ &\quad -\frac{2e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.24

$$\int F^{c(a+bx)}\text{csch}^4(d+ex)\,dx$$
$$= \frac{F^{c(a+bx)}\left(-e\text{csch}^2(d+ex)(2e\coth(d)+bc\log(F)) - \frac{2\left(1+(-1+e^{2d})\,\text{Hypergeometric2F1}\left(1,\frac{bc\log(F)}{2e},1+\frac{bc\log(F)}{2e},e^{2(d+ex)}\right)\right.}{-1+e^{2d}}\right.}{}$$

```
[In] Integrate[F^(c*(a + b*x))*Csch[d + e*x]^4,x]
```

```
[Out] (F^(c*(a + b*x))*(-(e*Csch[d + e*x]^2*(2*e*Coth[d] + b*c*Log[F])) - (2*(1 +
 (-1 + E^(2*d))*Hypergeometric2F1[1, (b*c*Log[F])/(2*e), 1 + (b*c*Log[F])/(
2*e), E^(2*(d + e*x))])*(-4*e^2 + b^2*c^2*Log[F]^2))/(-1 + E^(2*d)) + 2*e^2
*Csch[d]*Csch[d + e*x]^3*Sinh[e*x] - Csch[d]*Csch[d + e*x]*(4*e^2 - b^2*c^2
*Log[F]^2)*Sinh[e*x]))/(6*e^3)
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{csch}(ex+d)^4\,dx$$

```
[In] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

```
[Out] int(F^(c*(b*x+a))*csch(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}(ex+d)^4\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="fricas")
```

```
[Out] integral(F^(b*c*x + a*c)*csch(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx$$

```
[In] integrate(F**(c*(b*x+a))*csch(e*x+d)**4,x)
```

```
[Out] Integral(F**(c*(a + b*x))*csch(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{csch}(ex+d)^4\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="maxima")
```

```
[Out] 128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(-F^(b
*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 192*
e^3 - (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*b*c
```

```
*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d)*lo
g(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e
^3*e^(8*d))*e^(8*e*x) - 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)
*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 10*(b
^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*
d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) - 5*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b
^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e
^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F^(a*c)*b^2*
c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(a*c)*e^2*e^(
4*d))*e^(4*e*x) + 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*e^2*e^(2*d))*
e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^
2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F
)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*
e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(
F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*
e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x
) - 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e
^2*e^(2*d)*log(F) - 192*e^3*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=\int F^{(bx+a)c}\operatorname{csch}\left(ex+d\right)^4\,dx$$

```
[In] integrate(F^(c*(b*x+a))*csch(e*x+d)^4,x, algorithm="giac")
[Out] integrate(F^((b*x + a)*c)*csch(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{csch}^4(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\sinh\left(d+e\,x\right)^4}\,dx$$

```
[In] int(F^(c*(a + b*x))/sinh(d + e*x)^4,x)
[Out] int(F^(c*(a + b*x))/sinh(d + e*x)^4, x)
```

3.329 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\begin{aligned}\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx &= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc} \\ &- \frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{64bc} \\ &+ \frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{32bc} \\ &- \frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc} \\ &+ \frac{e^{6c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{192bc} - \frac{5}{16}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\end{aligned}$$

```
[Out] 1/128*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(4*c*(b*x+a))-5/64*c
sch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))+5/32*exp(2*c*
(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/128*exp(4*c*(b*x+a
))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/192*exp(6*c*(b*x+a))*csc
h(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-5/16*x*csch(b*c*x+a*c)*(sinh(b*c
*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 2320, 12, 272, 45}

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = \frac{e^{-4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{128bc} - \frac{5e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{64bc} + \frac{5e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{32bc} - \frac{5e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{128bc} + \frac{e^{6c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{192bc} - \frac{5}{16}x\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)$$

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128*b*c*E^(4*c*(a + b*x))) -
 (5*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(64*b*c*E^(2*c*(a + b*x)))
 + (5*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*
c) - (5*E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(128
*b*c) + (E^(6*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(19
2*b*c) - (5*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/16
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh^5(ac+bcx)\,dx\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{32x^5}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{(-1+x^2)^5}{x^5}\,dx, x, e^{c(a+bx)}\right)}{32bc}\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{(-1+x)^5}{x^3}\,dx, x, e^{2c(a+bx)}\right)}{64bc}\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(10-\frac{1}{x^3}+\frac{5}{x^2}-\frac{10}{x}-5x+x^2\right)\,dx, x, e^{2c(a+bx)}\right)}{64bc}\\
&= \frac{e^{-4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc}-\frac{5e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{64bc}\\
&\quad+\frac{5e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{32bc}-\frac{5e^{4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{128bc}\\
&\quad+\frac{e^{6c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{192bc}-\frac{5}{16}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = \frac{\left(\frac{1}{128}e^{-4c(a+bx)}-\frac{5}{64}e^{-2c(a+bx)}+\frac{5}{32}e^{2c(a+bx)}-\frac{5}{128}e^{4c(a+bx)}+\frac{1}{192}e^{6c(a+bx)}-\frac{5bcx}{16}\right)\operatorname{csch}^5(c(a+bx))}{bc}$$

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((1/(128*E^(4*c*(a + b*x))) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b*x
)))/32 - (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 - (5*b*c*x)/16)*
Csch[c*(a + b*x)]^5*(Sinh[c*(a + b*x)]^2)^(5/2))/(b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.35

method	result
default	$\frac{\operatorname{csgn}(\sinh(c(bx+a)))\left(\frac{\sinh(bcx+ac)^6}{6}+\left(\frac{\sinh(bcx+ac)^5}{6}-\frac{5\sinh(bcx+ac)^3}{24}+\frac{5\sinh(bcx+ac)}{16}\right)\cosh(bcx+ac)-\frac{5bcx}{16}-\frac{5ac}{16}\right)}{cb}$
risch	$-\frac{5x\sqrt{\left(e^{2c(bx+a)}-1\right)^2 e^{-2c(bx+a)}}\,e^{c(bx+a)}}{16\left(e^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(e^{2c(bx+a)}-1\right)^2 e^{-2c(bx+a)}}\,e^{7c(bx+a)}}{192cb\left(e^{2c(bx+a)}-1\right)}-\frac{5\sqrt{\left(e^{2c(bx+a)}-1\right)^2 e^{-2c(bx+a)}}\,e^{5c(bx+a)}}{128cb\left(e^{2c(bx+a)}-1\right)}$

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(sinh(c*(b*x+a)))/c/b*(1/6*sinh(b*c*x+a*c)^6+(1/6*sinh(b*c*x+a*c)^5-5/2
4*sinh(b*c*x+a*c)^3+5/16*sinh(b*c*x+a*c))*cosh(b*c*x+a*c)-5/16*b*c*x-5/16*a
*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{5/2}\,dx = \frac{5\cosh(bcx+ac)^5+25\cosh(bcx+ac)\sinh(bcx+ac)^4-\sinh(bcx+ac)^5-5\left(2\cosh(bcx+\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(b*c*x + a*c)^5 + 25*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - s
inh(b*c*x + a*c)^5 - 5*(2*cosh(b*c*x + a*c)^2 - 3)*sinh(b*c*x + a*c)^3 - 45
*cosh(b*c*x + a*c)^3 + 5*(10*cosh(b*c*x + a*c)^3 - 27*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 60*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 5*(cosh(b*c*x + a*
c)^4 - 24*b*c*x - 9*cosh(b*c*x + a*c)^2 - 12)*sinh(b*c*x + a*c))/(b*c*cosh(
b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.36

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx = \frac{\left(2\,e^{(10\,bcx+10\,ac)} - 15\,e^{(8\,bcx+8\,ac)} + 60\,e^{(6\,bcx+6\,ac)} - 30\,e^{(2\,bcx+2\,ac)} + 3\right)e^{(-4\,bcx-4\,ac)}}{384\,bc} - \frac{5\,(bcx+ac)}{16\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
[Out] 1/384*(2*e^(10*b*c*x + 10*a*c) - 15*e^(8*b*c*x + 8*a*c) + 60*e^(6*b*c*x + 6
*a*c) - 30*e^(2*b*c*x + 2*a*c) + 3)*e^(-4*b*c*x - 4*a*c)/(b*c) - 5/16*(b*c*
x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.08

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx = -\frac{120\,bcx\text{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - 3\left(30\,e^{(4\,bcx+4\,ac)}\text{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - 10\,e^{(2\,bcx+2\,ac)}\text{sgn}\left(e^{(bcx+ac)}\right.\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/384*(120*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 3*(30*e^(4*b*c*
x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 10*e^(2*b*c*x + 2*a*c)
*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a*c) - e^(-b*c*x
- a*c)))*e^(-4*b*c*x - 4*a*c) - (2*e^(6*b*c*x + 18*a*c)*sgn(e^(b*c*x + a*c)
 - e^(-b*c*x - a*c)) - 15*e^(4*b*c*x + 16*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*
c*x - a*c)) + 60*e^(2*b*c*x + 14*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c
)))*e^(-12*a*c))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{5/2}\,dx = \int \mathrm{e}^{c\,(a+b\,x)}\,\left(\sinh(a\,c+b\,c\,x)^2\right)^{5/2}\,dx$$

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(5/2), x)
```

3.330 $\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx = \frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc} - \frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc} + \frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{32bc} + \frac{3}{8}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}$$

```
[Out] 1/16*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c/exp(2*c*(b*x+a))-3/16*ex
p(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+1/32*exp(4*c*(
b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c+3/8*x*csch(b*c*x+a*c)
*(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 2320, 12, 272, 45}

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2}\,dx = \frac{e^{-2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc} - \frac{3e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{16bc} + \frac{e^{4c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)}{32bc} + \frac{3}{8}x\sqrt{\sinh^2(ac+bcx)}\operatorname{csch}(ac+bcx)$$

```
[In] Int[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c*E^(2*c*(a + b*x))) -
(3*E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(16*b*c)
+ (E^(4*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(32*b*c)
+ (3*x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh^3(ac+bcx)\,dx$$

$$= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{8x^3}\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$=\frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x^2)^3}{x^3}\,dx,x,e^{c(a+bx)}\right)}{8bc}$$

$$=\frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{(-1+x)^3}{x^2}\,dx,x,e^{2c(a+bx)}\right)}{16bc}$$

$$=\frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx,x,e^{2c(a+bx)}\right)}{16bc}$$

$$=\frac{e^{-2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc}-\frac{3e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{16bc}+\frac{e^{4c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{32bc}+\frac{3}{8}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.47

$$\int e^{c(a+bx)}\sinh^2(ac+bcx)^{3/2}\,dx=\frac{\left(e^{-2c(a+bx)}-3e^{2c(a+bx)}+\frac{1}{2}e^{4c(a+bx)}+6bcx\right)\operatorname{csch}^3(c(a+bx))\sinh^2(c(a+bx))^{3/2}}{16bc}$$

```
[In] Integrate[E^(c*(a + b*x))*(Sinh[a*c + b*c*x]^2)^(3/2),x]
[Out] ((E^(-2*c*(a + b*x)) - 3*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))/2 + 6*b*c*x)
*Csch[c*(a + b*x)]^3*(Sinh[c*(a + b*x)]^2)^(3/2))/(16*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

method	result
default	$\frac{\operatorname{csgn}(\sinh(c(bx+a)))\left(\frac{\sinh(bcx+ac)^4}{4}+\left(\frac{\sinh(bcx+ac)^3}{4}-\frac{3\sinh(bcx+ac)}{8}\right)\cosh(bcx+ac)+\frac{3bcx}{8}+\frac{3ac}{8}\right)}{cb}$
risch	$\frac{3x\sqrt{\left(e^{2c(bx+a)}-1\right)^2e^{-2c(bx+a)}}\,e^{c(bx+a)}}{8\left(e^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(e^{2c(bx+a)}-1\right)^2e^{-2c(bx+a)}}\,e^{5c(bx+a)}}{32cb\left(e^{2c(bx+a)}-1\right)}-\frac{3\sqrt{\left(e^{2c(bx+a)}-1\right)^2e^{-2c(bx+a)}}\,e^{3c(bx+a)}}{16cb\left(e^{2c(bx+a)}-1\right)}+$

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(sinh(c*(b*x+a)))/c/b*(1/4*sinh(b*c*x+a*c)^4+(1/4*sinh(b*c*x+a*c)^3-3/8
*sinh(b*c*x+a*c))*cosh(b*c*x+a*c)+3/8*b*c*x+3/8*a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2} dx = \frac{3 \cosh(bcx+ac)^3 + 9 \cosh(bcx+ac) \sinh(bcx+ac)^2 - \sinh(bcx+ac)^3 + 6(2bcx-1)\cosh \ldots}{32 (bc\cosh(bcx+ac) - bc\sinh(bcx+\ldots}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - sin
h(b*c*x + a*c)^3 + 6*(2*b*c*x - 1)*cosh(b*c*x + a*c) - 3*(4*b*c*x + cosh(b*
c*x + a*c)^2 + 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*
x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2} dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.38

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2} dx = \frac{\left(e^{(6bcx+6ac)} - 6e^{(4bcx+4ac)} + 2\right)e^{(-2bcx-2ac)}}{32bc} + \frac{3(bcx+ac)}{8bc}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(e^(6*b*c*x + 6*a*c) - 6*e^(4*b*c*x + 4*a*c) + 2)*e^(-2*b*c*x - 2*a*c)
/(b*c) + 3/8*(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2} \, dx = \frac{12\,bcx\mathrm{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - 2\left(3\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) - \mathrm{sgn}\left(e^{(bcx+ac)} - e\right.\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 2*(3*e^(2*b*c*x +
2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e^(-
b*c*x - a*c)))*e^(-2*b*c*x - 2*a*c) + (e^(4*b*c*x + 8*a*c)*sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)) - 6*e^(2*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b
*c*x - a*c)))*e^(-4*a*c))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \sinh^2(ac+bcx)^{3/2} \, dx = \int e^{c\,(a+b\,x)}\,\left(\sinh(a\,c+b\,c\,x)^2\right)^{3/2} \, dx$$

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(3/2), x)
```

3.331 $\int e^{c(a+bx)} \sqrt{\sinh^2(ac+bcx)}\, dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int e^{c(a+bx)} \sqrt{\sinh^2(ac+bcx)}\, dx = \frac{e^{2c(a+bx)}\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{4bc} - \frac{1}{2}x\text{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*csch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)/b/c-1/2*x*cs
ch(b*c*x+a*c)*(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 12, 14}

$$\int e^{c(a+bx)} \sqrt{\sinh^2(ac+bcx)}\, dx = \frac{e^{2c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)}{4bc} - \frac{1}{2}x\sqrt{\sinh^2(ac+bcx)}\text{csch}(ac+bcx)$$

```
[In] Int[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/(4*b*c) - (
x*Csch[a*c + b*c*x]*Sqrt[Sinh[a*c + b*c*x]^2])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\int e^{c(a+bx)}\sinh(ac+bcx)\,dx\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{-1+x^2}{2x}\,dx,x,e^{c(a+bx)}\right)}{bc}\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{-1+x^2}{x}\,dx,x,e^{c(a+bx)}\right)}{2bc}\\
&= \frac{\left(\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx,x,e^{c(a+bx)}\right)}{2bc}\\
&= \frac{e^{2c(a+bx)}\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}}{4bc}-\frac{1}{2}x\operatorname{csch}(ac+bcx)\sqrt{\sinh^2(ac+bcx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx=\frac{\left(e^{2c(a+bx)}-2bcx\right)\operatorname{csch}(c(a+bx))\sqrt{\sinh^2(c(a+bx))}}{4bc}$$

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sinh[a*c + b*c*x]^2],x]
[Out] ((E^(2*c*(a + b*x)) - 2*b*c*x)*Csch[c*(a + b*x)]*Sqrt[Sinh[c*(a + b*x)]^2])
/(4*b*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\operatorname{csgn}(\sinh(c(bx+a)))\left(\frac{\cosh(bcx+ac)^2}{2}+\frac{\sinh(bcx+ac)\cosh(bcx+ac)}{2}-\frac{bcx}{2}-\frac{ac}{2}\right)}{cb}$	60
risch	$-\frac{x\sqrt{\left(e^{2c(bx+a)}-1\right)^2e^{-2c(bx+a)}}\,e^{c(bx+a)}}{2\left(e^{2c(bx+a)}-1\right)}+\frac{\sqrt{\left(e^{2c(bx+a)}-1\right)^2e^{-2c(bx+a)}}\,e^{3c(bx+a)}}{4cb\left(e^{2c(bx+a)}-1\right)}$	106

```
[In] int(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] csgn(sinh(c*(b*x+a)))/c/b*(1/2*cosh(b*c*x+a*c)^2+1/2*sinh(b*c*x+a*c)*cosh(b
*c*x+a*c)-1/2*b*c*x-1/2*a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx=-\frac{(2\,bcx-1)\cosh\left(bcx+ac\right)-(2\,bcx+1)\sinh\left(bcx+ac\right)}{4\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] -1/4*((2*b*c*x - 1)*cosh(b*c*x + a*c) - (2*b*c*x + 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(68) = 136$.

Time = 1.57 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.14

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = \begin{cases} 0 & \text{for } a=0 \wedge b=0 \\ x\sqrt{\sinh^2(ac)}e^{ac} & \text{for } b=0 \\ 0 & \text{for } a=-bx \vee c= \\ \frac{x\sqrt{\sinh^2(ac+bcx)}e^{ac}e^{bcx}}{2} - \frac{x\sqrt{\sinh^2(ac+bcx)}e^{ac}e^{bcx}\cosh(ac+bcx)}{2\sinh(ac+bcx)} + \frac{\sqrt{\sinh^2(ac+bcx)}e^{ac}e^{bcx}\cosh(ac+bcx)}{2bc\sinh(ac+bcx)} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (x*sqrt(sinh(a*c)**2)*exp(a*
c), Eq(b, 0)), (0, Eq(c, 0) | Eq(a, -b*x)), (x*sqrt(sinh(a*c + b*c*x)**2)*e
xp(a*c)*exp(b*c*x)/2 - x*sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x)*cos
h(a*c + b*c*x)/(2*sinh(a*c + b*c*x)) + sqrt(sinh(a*c + b*c*x)**2)*exp(a*c)*
exp(b*c*x)*cosh(a*c + b*c*x)/(2*b*c*sinh(a*c + b*c*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = -\frac{bcx+ac}{2\,bc} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*c*x + a*c)/(b*c) + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx = -\frac{1}{2}\,x\mathrm{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right) + \frac{e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)} - e^{(-bcx-ac)}\right)}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 1/4*e^(2*b*c*x + 2*a*c)*sg
n(e^(b*c*x + a*c) - e^(-b*c*x - a*c))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int e^{c(a+bx)}\sqrt{\sinh^2(ac+bcx)}\,dx=-\frac{\left(x\,\mathrm{e}^{a\,c+b\,c\,x}-\frac{\mathrm{e}^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1}$$

```
[In] int(exp(c*(a + b*x))*(sinh(a*c + b*c*x)^2)^(1/2),x)
```

```
[Out] -((x*exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2
- exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) - 1)
```

3.332 $\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 46

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] ln(1-exp(2*c*(b*x+a)))*sinh(b*c*x+a*c)/b/c/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 12, 266}

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
```

```
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[a*c + b*c*x])/(b*c*Sqrt[Sinh[a*c + b*c*x]^
2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\sinh(ac+bcx)\text{Subst}\left(\int \frac{2x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(2\sinh(ac+bcx))\text{Subst}\left(\int \frac{x}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(ac+bcx)}{bc\sqrt{\sinh^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(1-e^{2c(a+bx)}\right)\sinh(c(a+bx))}{bc\sqrt{\sinh^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sinh[a*c + b*c*x]^2],x]
[Out] (Log[1 - E^(2*c*(a + b*x))]*Sinh[c*(a + b*x)])/(b*c*Sqrt[Sinh[c*(a + b*x)]^
2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
default	$\operatorname{csgn}\left(\sinh\left(c(bx+a)\right)\right)\left(x+\frac{\ln\left(\sinh\left(c(bx+a)\right)\right)}{cb}\right)$	29
risch	$\frac{\ln\left(\mathrm{e}^{2bcx}-\mathrm{e}^{-2ac}\right)\left(\mathrm{e}^{2c(bx+a)}-1\right)\mathrm{e}^{-c(bx+a)}}{cb\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}}$	68

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] csgn(sinh(c*(b*x+a)))*(x+1/c/b*ln(sinh(c*(b*x+a))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(\frac{2\sinh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] log(2*sinh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\sinh^2\left(ac+bcx\right)}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sinh(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc}+\frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c) - 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.85

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{(bcx)}+e^{(-ac)}\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\log\left(\left|e^{(bcx)}-e^{(-ac)}\right|\right)\operatorname{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] (log(e^(b*c*x) + e^(-a*c))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + log(ab
s(e^(b*c*x) - e^(-a*c)))*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\sinh^2(ac+bcx)}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\sqrt{{\sinh\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(1/2), x)
```

3.333 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 58

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] -2*exp(4*c*(b*x+a))*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x+
a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6852, 2320, 12, 270}

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (-2*E^(4*c*(a + b*x))*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqr
t[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^3(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\sinh(ac+bcx)\text{Subst}\left(\int \frac{8x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(8\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^3}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= -\frac{2e^{4c(a+bx)}\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{4e^{5c(a+bx)}\sqrt{\sinh^2(c(a+bx))}}{bc\left(-1+e^{2c(a+bx)}\right)^3}$$

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(3/2),x]
[Out] (-4*E^(5*c*(a + b*x))*Sqrt[Sinh[c*(a + b*x)]^2])/(b*c*(-1 + E^(2*c*(a + b*x
)))^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{\operatorname{csgn}(\sinh(c(bx+a)))\left(-\frac{\coth(bcx+ac)^2}{2}-\coth(bcx+ac)\right)}{cb}$	42
risch	$-\frac{2\left(2\,\mathrm{e}^{2c(bx+a)}-1\right)\mathrm{e}^{-c(bx+a)}}{bc\left(\mathrm{e}^{2c(bx+a)}-1\right)\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2\mathrm{e}^{-2c(bx+a)}}}$	69

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] csgn(sinh(c*(b*x+a)))/c/b*(-1/2*coth(b*c*x+a*c)^2-coth(b*c*x+a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.09

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx =$$
$$-\frac{2\left(\cosh\left(bcx+ac\right)+3\,\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3-bc\cosh\left(bcx+ac\right)+3\,(bc$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
[Out] -2*(cosh(b*c*x + a*c) + 3*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 - b*c*co
sh(b*c*x + a*c) + 3*(b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\sinh^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(3/2),x)
[Out] exp(a*c)*Integral(exp(b*c*x)/(sinh(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)} + \frac{2}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) +
1)) + 2/(b*c*(e^(4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.50

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)\right)}{bc(e^{(2\,bcx+2\,ac)}-1)^2}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(2*e^(2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(
b*c*x + a*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}$$

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] -(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) - 1)*((exp(a*c + b*c*x)/2 - ex
p(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) - 1)^3)
```

3.334 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 147

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}} + \frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}} - \frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}}$$

```
[Out] -4*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+32/
3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)-8*si
nh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 2320, 12, 272, 45}

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\sinh^2(ac+bcx)}} + \frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}} - \frac{4\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Sinh[a*c + b*c*x
]^2]) + (32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3*Sqrt[Sinh[a
```

```
*c + b*c*x]^2]) - (8*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt
[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\text{csch}^5(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\ &= \frac{\sinh(ac+bcx)\text{Subst}\left(\int \frac{32x^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\end{aligned}$$

$$= \frac{(32\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(16\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^2}{(-1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= \frac{(16\sinh(ac+bcx))\text{Subst}\left(\int \left(\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{1}{(-1+x)^3}\right)\,dx, x, e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}$$

$$= -\frac{4\sinh(ac+bcx)}{bc\,(1-e^{2c(a+bx)})^4\sqrt{\sinh^2(ac+bcx)}} + \frac{32\sinh(ac+bcx)}{3bc\,(1-e^{2c(a+bx)})^3\sqrt{\sinh^2(ac+bcx)}} - \frac{8\sinh(ac+bcx)}{bc\,(1-e^{2c(a+bx)})^2\sqrt{\sinh^2(ac+bcx)}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.49

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(1-4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\sinh(c(a+bx))}{3bc\,(-1+e^{2c(a+bx)})^4\sqrt{\sinh^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(5/2),x]
[Out] (-4*(1 - 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Sinh[c*(a + b*x)])/(3*b
*c*(-1 + E^(2*c*(a + b*x)))^4*Sqrt[Sinh[c*(a + b*x)]^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\operatorname{csgn}(\sinh(c(bx+a)))\left(-\frac{\coth(bcx+ac)^4}{4}-\frac{\coth(bcx+ac)^3}{3}+\frac{\coth(bcx+ac)^2}{2}+\coth(bcx+ac)\right)}{cb}$	66
risch	$-\frac{4\left(6\,\mathrm{e}^{4c(bx+a)}-4\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{3bc\left(\mathrm{e}^{2c(bx+a)}-1\right)^3\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^2\mathrm{e}^{-2c(bx+a)}}}$	80

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(sinh(c*(b*x+a)))/c/b*(-1/4*coth(b*c*x+a*c)^4-1/3*coth(b*c*x+a*c)^3+1/2
*coth(b*c*x+a*c)^2+coth(b*c*x+a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(130) = 260$.

Time = 0.28 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.14

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^6+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^5+bc\sinh\left(bcx+ac\right)^6-4\,bc\cosh\left(bcx+ac\right)^4\right.}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 - 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 - 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 - 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 - 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 - 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 - 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 - 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.42

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)} + \frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1)) + 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) - 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) - 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.83

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-4\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-a}\right.}{3\,bc(e^{(2\,bcx+2\,ac)}-1)^4}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 4*e^(
2*b*c*x + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + sgn(e^(b*c*x + a
*c) - e^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\,e^{a\,c+b\,c\,x}\,\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}-\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}\,\left(6\,e^{4\,a\,c+4\,b\,c\,x}-4\,e^{2\,a\,c+2\,b\,c\,x}+1\right)}{3\,b\,c\,\left(e^{2\,a\,c+2\,b\,c\,x}-1\right)^5}$$

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2)*
(6*exp(4*a*c + 4*b*c*x) - 4*exp(2*a*c + 2*b*c*x) + 1))/(3*b*c*(exp(2*a*c +
2*b*c*x) - 1)^5)
```

3.335 $\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\, dx$

Optimal result

Integrand size = 25, antiderivative size = 199

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\, dx = &-\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}}\\ &+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}-\frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}\\ &+\frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}\end{aligned}$$

```
[Out] -32/3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^6/(sinh(b*c*x+a*c)^2)^(1/2)+
192/5*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^5/(sinh(b*c*x+a*c)^2)^(1/2)-
48*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(sinh(b*c*x+a*c)^2)^(1/2)+64/
3*sinh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(sinh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{6852, 2320, 12, 272, 45\}$

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = \frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}} - \frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}} + \frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}} - \frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (-32*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^6*Sqrt[Sinh[a*c + b*
c*x]^2]) + (192*Sinh[a*c + b*c*x])/(5*b*c*(1 - E^(2*c*(a + b*x)))^5*Sqrt[Si
nh[a*c + b*c*x]^2]) - (48*Sinh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4
*Sqrt[Sinh[a*c + b*c*x]^2]) + (64*Sinh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c*(a
+ b*x)))^3*Sqrt[Sinh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sinh(ac+bcx)\int e^{c(a+bx)}\operatorname{csch}^7(ac+bcx)\,dx}{\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{\sinh(ac+bcx)\text{Subst}\left(\int \frac{128x^7}{(-1+x^2)^7}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(128\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^7}{(-1+x^2)^7}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(64\sinh(ac+bcx))\text{Subst}\left(\int \frac{x^3}{(-1+x)^7}\,dx,x,e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= \frac{(64\sinh(ac+bcx))\text{Subst}\left(\int \left(\frac{1}{(-1+x)^7}+\frac{3}{(-1+x)^6}+\frac{3}{(-1+x)^5}+\frac{1}{(-1+x)^4}\right)\,dx,x,e^{2c(a+bx)}\right)}{bc\sqrt{\sinh^2(ac+bcx)}}\\
&= -\frac{32\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(ac+bcx)}}+\frac{192\sinh(ac+bcx)}{5bc\left(1-e^{2c(a+bx)}\right)^5\sqrt{\sinh^2(ac+bcx)}}\\
&\quad -\frac{48\sinh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\sinh^2(ac+bcx)}}+\frac{64\sinh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\sinh^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.42

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\left(-1+6e^{2c(a+bx)}-15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\sinh(c(a+bx))}{15bc\left(-1+e^{2c(a+bx)}\right)^6\sqrt{\sinh^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/(Sinh[a*c + b*c*x]^2)^(7/2),x]
[Out] (-16*(-1 + 6*E^(2*c*(a + b*x)) - 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Sinh[c*(a + b*x)])/(15*b*c*(-1 + E^(2*c*(a + b*x)))^6*Sqrt[Sinh[c*(a + b
*x)]^2])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

method	result	size
risch	$-\frac{16\left(20\,\mathrm{e}^{6c(bx+a)}-15\,\mathrm{e}^{4c(bx+a)}+6\,\mathrm{e}^{2c(bx+a)}-1\right)\mathrm{e}^{-c(bx+a)}}{15bc\left(\mathrm{e}^{2c(bx+a)}-1\right)^{5}\sqrt{\left(\mathrm{e}^{2c(bx+a)}-1\right)^{2}\mathrm{e}^{-2c(bx+a)}}}$	91
default	$\frac{\mathrm{csgn}(\sinh(c(bx+a)))\left(-\frac{\coth(bcx+ac)^{6}}{6}-\frac{\coth(bcx+ac)^{5}}{5}+\frac{\coth(bcx+ac)^{4}}{2}+\frac{2\coth(bcx+ac)^{3}}{3}-\frac{\coth(bcx+ac)^{2}}{2}-\coth(bcx+ac)\right)}{cb}$	94

```
[In] int(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/15/b/c*(20*exp(6*c*(b*x+a))-15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))-1)/(
exp(2*c*(b*x+a))-1)^5/((exp(2*c*(b*x+a))-1)^2*exp(-2*c*(b*x+a)))^(1/2)*exp(
-c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(173) = 346.

Time = 0.29 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.97

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^{9}+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{8}+bc\sinh\left(bcx+ac\right)^{9}-6\,bc\cosh\left(bcx+ac\right)^{7}\right.}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(19*cosh(b*c*x + a*c)^3 + 57*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 21*sinh(b*c*x + a*c)^3 + 21*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)
- 9*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 - 6*b*c*cosh(b*c*x + a*c)^7 +
6*(6*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*x
 + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c
*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 - 42*b*c*cosh(b*c*x + a*c)^2 +
5*b*c)*sinh(b*c*x + a*c)^5 - 19*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b*
c*x + a*c)^5 - 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh(
b*c*x + a*c)^4 + 3*(28*b*c*cosh(b*c*x + a*c)^6 - 70*b*c*cosh(b*c*x + a*c)^4
 + 50*b*c*cosh(b*c*x + a*c)^2 - 7*b*c)*sinh(b*c*x + a*c)^3 + 9*b*c*cosh(b*c
*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 - 42*b*c*cosh(b*c*x + a*c)^5 + 50
*b*c*cosh(b*c*x + a*c)^3 - 19*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 +
3*(3*b*c*cosh(b*c*x + a*c)^8 - 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(b*c
*x + a*c)^4 - 21*b*c*cosh(b*c*x + a*c)^2 + 7*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)**2)**(7/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(173) = 346$.

Time = 0.32 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.94

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = -\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)} + \frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)} + \frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}-6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}-20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}-6\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) + 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
- 6*e^(2*b*c*x + 2*a*c) + 1)) + 16/15/(b*c*(e^(12*b*c*x + 12*a*c) - 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) - 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) - 6*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.81

$$\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\left(20\,e^{(6\,bcx+6\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)-15\,e^{(4\,bcx+4\,ac)}\mathrm{sgn}\left(e^{(bcx+ac)}-e^{(-bcx-ac)}\right)+6\,e^{(2\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(\right.}\right.}{15\,bc(e^{(2\,bcx+2\,ac)}-1)^6}$$

```
[In] integrate(exp(c*(b*x+a))/(sinh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - 15
*e^(4*b*c*x + 4*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) + 6*e^(2*b*c*x
 + 2*a*c)*sgn(e^(b*c*x + a*c) - e^(-b*c*x - a*c)) - sgn(e^(b*c*x + a*c) - e
^(-b*c*x - a*c)))/(b*c*(e^(2*b*c*x + 2*a*c) - 1)^6)
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.77

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\sinh^2(ac+bcx)^{7/2}}\,dx &= \frac{128\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}-\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^3}\\ &+\frac{96\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}-\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^4}\\ &+\frac{384\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{5\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}-\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^5}\\ &+\frac{64\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}-\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{3\,b\,c\,\left(\mathrm{e}^{a\,c+b\,c\,x}-\mathrm{e}^{3\,a\,c+3\,b\,c\,x}\right)\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}-1\right)}^6}\end{aligned}$$

```
[In] int(exp(c*(a + b*x))/(sinh(a*c + b*c*x)^2)^(7/2),x)
[Out] (128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c*x)/2)^2)^(
1/2))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x
) - 1)^3) + (96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(- a*c - b*c
*x)/2)^2)^(1/2))/(b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))*(exp(2*a*c
+ 2*b*c*x) - 1)^4) + (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 - exp(-
 a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) - exp(3*a*c + 3*b*c*x))
*(exp(2*a*c + 2*b*c*x) - 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x
)/2 - exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) - exp(3*a*c
+ 3*b*c*x))*(exp(2*a*c + 2*b*c*x) - 1)^6)
```

3.336 $\int e^x \sinh(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int e^x \sinh(a+bx)\,dx = -\frac{be^x\cosh(a+bx)}{1-b^2}+\frac{e^x\sinh(a+bx)}{1-b^2}$$

```
[Out] -b*exp(x)*cosh(b*x+a)/(-b^2+1)+exp(x)*sinh(b*x+a)/(-b^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5582}

$$\int e^x \sinh(a+bx)\,dx = \frac{e^x\sinh(a+bx)}{1-b^2}-\frac{be^x\cosh(a+bx)}{1-b^2}$$

```
[In] Int[E^x*Sinh[a + b*x],x]
[Out] -((b*E^x*Cosh[a + b*x])/(1 - b^2)) + (E^x*Sinh[a + b*x])/(1 - b^2)
```

Rule 5582

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sinh[(d_.) + (e_.)*(x_)], x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{be^x\cosh(a+bx)}{1-b^2}+\frac{e^x\sinh(a+bx)}{1-b^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^x \sinh(a+bx)\,dx = \frac{e^x(b\cosh(a+bx)-\sinh(a+bx))}{-1+b^2}$$

```
[In] Integrate[E^x*Sinh[a + b*x],x]
[Out] (E^x*(b*Cosh[a + b*x] - Sinh[a + b*x]))/(-1 + b^2)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{e^x(b\cosh(bx+a)-\sinh(bx+a))}{b^2-1}$	28
risch	$\frac{e^{bx+a+x}}{2+2b}+\frac{e^{-bx-a+x}}{2b-2}$	33
default	$-\frac{\sinh(x(b-1)+a)}{2(b-1)}+\frac{\sinh((1+b)x+a)}{2+2b}+\frac{\cosh(x(b-1)+a)}{2b-2}+\frac{\cosh((1+b)x+a)}{2+2b}$	62

```
[In] int(exp(x)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] exp(x)/(b^2-1)*(b*cosh(b*x+a)-sinh(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int e^x \sinh(a+bx)\,dx = \frac{b\cosh(bx+a)\cosh(x)+b\cosh(bx+a)\sinh(x)-(\cosh(x)+\sinh(x))\sinh(bx+a)}{b^2-1}$$

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="fricas")
[Out] (b*cosh(b*x + a)*cosh(x) + b*cosh(b*x + a)*sinh(x) - (cosh(x) + sinh(x))*si
nh(b*x + a))/(b^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(31) = 62$.

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int e^x \sinh(a+bx)\,dx = \begin{cases} \frac{xe^x \sinh{(a-x)}}{2} + \frac{xe^x \cosh{(a-x)}}{2} + \frac{e^x \sinh{(a-x)}}{2} & \text{for } b = -1 \\ \frac{xe^x \sinh{(a+x)}}{2} - \frac{xe^x \cosh{(a+x)}}{2} + \frac{e^x \cosh{(a+x)}}{2} & \text{for } b = 1 \\ \frac{be^x \cosh{(a+bx)}}{b^2-1} - \frac{e^x \sinh{(a+bx)}}{b^2-1} & \text{otherwise} \end{cases}$$

```
[In] integrate(exp(x)*sinh(b*x+a),x)
```

```
[Out] Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 + exp(x)*sinh(a
 - x)/2, Eq(b, -1)), (x*exp(x)*sinh(a + x)/2 - x*exp(x)*cosh(a + x)/2 + exp(
x)*cosh(a + x)/2, Eq(b, 1)), (b*exp(x)*cosh(a + b*x)/(b**2 - 1) - exp(x)*si
nh(a + b*x)/(b**2 - 1), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^x \sinh(a+bx)\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(-b>0)', see `assume?` for more deta
ils)Is
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int e^x \sinh(a+bx)\,dx = \frac{e^{(bx+a+x)}}{2(b+1)} + \frac{e^{(-bx-a+x)}}{2(b-1)}$$

```
[In] integrate(exp(x)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*e^(b*x + a + x)/(b + 1) + 1/2*e^(-b*x - a + x)/(b - 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^x \sinh(a+bx)\,dx = \frac{\mathrm{e}^{x-a-b\,x}\,\left(b-\mathrm{e}^{2\,a+2\,b\,x}+b\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{2\,\left(b^2-1\right)}$$

```
[In] int(exp(x)*sinh(a + b*x),x)
```

```
[Out] (exp(x - a - b*x)*(b - exp(2*a + 2*b*x) + b*exp(2*a + 2*b*x) + 1))/(2*(b^2
- 1))
```

3.337 $\int e^x \sinh(a+cx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int e^x \sinh(a+cx^2)\,dx = \frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\text{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*exp(-a+1/4/c)*erf(1/2*(-2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/4*exp(a-1/
4/c)*erfi(1/2*(2*c*x+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5623, 2266, 2236, 2235}

$$\int e^x \sinh(a+cx^2)\,dx = \frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\,\text{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\,\text{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[In] Int[E^x*Sinh[a + c*x^2],x]
```

```
[Out] (E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c]) + (E^(
a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-a+x-cx^2}+\frac{1}{2}e^{a+x+cx^2}\right)\,dx\\
&= -\left(\frac{1}{2}\int e^{-a+x-cx^2}\,dx\right)+\frac{1}{2}\int e^{a+x+cx^2}\,dx\\
&= \frac{1}{2}e^{a-\frac{1}{4c}}\int e^{\frac{(1+2cx)^2}{4c}}\,dx-\frac{1}{2}e^{-a+\frac{1}{4c}}\int e^{-\frac{(1-2cx)^2}{4c}}\,dx\\
&= \frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$
\begin{aligned}
&\int e^x\sinh\left(a+cx^2\right)\,dx\\
&=\frac{e^{-\frac{1}{4}/c}\sqrt{\pi}\left(-e^{\frac{1}{2}/c}\operatorname{erf}\left(\frac{-1+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\operatorname{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}
\end{aligned}
$$

```
[In] Integrate[E^x*Sinh[a + c*x^2],x]
[Out] (Sqrt[Pi]*(-(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]))
 + Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c
)))
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\operatorname{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	72

```
[In] int(exp(x)*sinh(c*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*
Pi^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.21

$$\int e^x \sinh\left(a+cx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{2\,cx-}{2\,\sqrt{c}}\right.}{4\,c}$$

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c))
*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) + sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)/c
) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

Sympy [F]

$$\int e^x \sinh\left(a+cx^2\right)\,dx = \int e^x \sinh\left(a+cx^2\right)dx$$

```
[In] integrate(exp(x)*sinh(c*x**2+a),x)
```

```
[Out] Integral(exp(x)*sinh(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int e^x \sinh\left(a+cx^2\right)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x-\frac{1}{2\sqrt{-c}}\right)e^{\left(a-\frac{1}{4c}\right)}}{4\sqrt{-c}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x-\frac{1}{2\sqrt{c}}\right)e^{\left(-a+\frac{1}{4c}\right)}}{4\sqrt{c}}$$

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="maxima")
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) - 1/4*sq
rt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int e^x \sinh\left(a+cx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{1}{c}\right)\right)e^{\left(\frac{4ac-1}{4c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{1}{c}\right)\right)e^{\left(-\frac{4ac-1}{4c}\right)}}{4\sqrt{c}}$$

```
[In] integrate(exp(x)*sinh(c*x^2+a),x, algorithm="giac")
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c)
 + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt(c
)
```

Mupad [F(-1)]

Timed out.

$$\int e^x \sinh\left(a+cx^2\right)\,dx = \int e^x\,\sinh\left(c\,x^2+a\right)\,dx$$

```
[In] int(exp(x)*sinh(a + c*x^2),x)
[Out] int(exp(x)*sinh(a + c*x^2), x)
```

3.338 $\int e^x \sinh(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int e^x \sinh\left(a+bx+cx^2\right)\,dx=\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\text{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[Out] 1/4*exp(-a+1/4*(1-b)^2/c)*erf(1/2*(-2*c*x-b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)+1/
4*exp(a-1/4*(1+b)^2/c)*erfi(1/2*(2*c*x+b+1)/c^(1/2))*Pi^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5623, 2266, 2236, 2235}

$$\int e^x \sinh\left(a+bx+cx^2\right)\,dx=\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\text{erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\text{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

```
[In] Int[E^x*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/(4*Sqr
t[c]) + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]
)/(4*Sqrt[c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(1+b)x+cx^2}\right)\,dx\\
&= -\left(\frac{1}{2}\int e^{-a+(1-b)x-cx^2}\,dx\right)+\frac{1}{2}\int e^{a+(1+b)x+cx^2}\,dx\\
&= -\left(\frac{1}{2}e^{-a+\frac{(1-b)^2}{4c}}\int e^{-\frac{(1-b-2cx)^2}{4c}}\,dx\right)+\frac{1}{2}e^{a-\frac{(1+b)^2}{4c}}\int e^{\frac{(1+b+2cx)^2}{4c}}\,dx\\
&= \frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int e^x\sinh\left(a+bx+cx^2\right)\,dx$$
$$=\frac{e^{-\frac{(1+b)^2}{4c}}\sqrt{\pi}\left(-e^{\frac{1+b^2}{2c}}\operatorname{erf}\left(\frac{-1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\operatorname{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

```
[In] Integrate[E^x*Sinh[a + b*x + c*x^2],x]
[Out] (Sqrt[Pi]*(-(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a]
 - Sinh[a])) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*S
qrt[c]*E^((1 + b)^2/(4*c)))
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^{2}+2b-1}{4c}}\,\mathrm{erf}\left(\sqrt{c}\,x-\frac{1-b}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^{2}-2b-1}{4c}}\,\mathrm{erf}\left(-\sqrt{-c}\,x+\frac{1+b}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	97

```
[In] int(exp(x)*sinh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)
/c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^(1
/2)*x+1/2*(1+b)/(-c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.28

$$\int e^{x}\sinh\left(a+bx+cx^{2}\right)\,dx=$$
$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^{2}-4\,ac+2\,b+1}{4\,c}\right)+\sinh\left(-\frac{b^{2}-4\,ac+2\,b+1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right)+\sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^{2}-4\,ac-2\,b+}{4\,c}\right.\right.}{4\,c}$$

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*(
b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) + sqrt(pi)*s
qrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c - 2*b
 + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

Sympy [F]

$$\int e^{x}\sinh\left(a+bx+cx^{2}\right)\,dx=\int e^{x}\sinh\left(a+bx+cx^{2}\right)dx$$

```
[In] integrate(exp(x)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x)*sinh(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int e^x \sinh\left(a+bx+cx^2\right)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x-\frac{b+1}{2\sqrt{-c}}\right)e^{\left(a-\frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x+\frac{b-1}{2\sqrt{c}}\right)e^{\left(-a+\frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c)
/sqrt(-c) - 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4*(
b - 1)^2/c)/sqrt(c)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int e^x \sinh\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4ac+2b+1}{4c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4ac-2b+1}{4c}\right)}}{4\sqrt{c}}$$

```
[In] integrate(exp(x)*sinh(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c + 2
*b + 1)/c)/sqrt(-c) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^(1
/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int e^x \sinh\left(a+bx+cx^2\right)\,dx = \int \sinh\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^x\,dx$$

```
[In] int(sinh(a + b*x + c*x^2)*exp(x),x)
```

```
[Out] int(sinh(a + b*x + c*x^2)*exp(x), x)
```

3.339 $\int e^{x^2} \sinh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int e^{x^2} \sinh(a+bx)\,dx = -\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(b+2x)\right)$$

```
[Out] 1/4*exp(-a-1/4*b^2)*erfi(1/2*b-x)*Pi^(1/2)+1/4*exp(a-1/4*b^2)*erfi(1/2*b+x)
*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5623, 2266, 2235}

$$\int e^{x^2} \sinh(a+bx)\,dx = \frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(b+2x)\right)-\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x-b)\right)$$

```
[In] Int[E^x^2*Sinh[a + b*x],x]
```

```
[Out] -1/4*(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2]) + (E^(a - b^2/4)*Sqrt[Pi]
*Erfi[(b + 2*x)/2])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right) dx \\
&= -\left(\frac{1}{2}\int e^{-a-bx+x^2}\,dx\right)+\frac{1}{2}\int e^{a+bx+x^2}\,dx \\
&= -\left(\frac{1}{2}e^{-a-\frac{b^2}{4}}\int e^{\frac{1}{4}(-b+2x)^2}\,dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4}}\int e^{\frac{1}{4}(b+2x)^2}\,dx \\
&= -\frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\,\text{erfi}\left(\frac{1}{2}(b+2x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int e^{x^2}\sinh(a+bx)\,dx=\frac{1}{4}e^{-\frac{b^2}{4}}\sqrt{\pi}\left(\text{erfi}\left(\frac{b}{2}-x\right)(\cosh(a)-\sinh(a))+\text{erfi}\left(\frac{b}{2}+x\right)(\cosh(a)+\sinh(a))\right)$$

```
[In] Integrate[E^x^2*Sinh[a + b*x],x]
[Out] (Sqrt[Pi]*(Erfi[b/2 - x]*(Cosh[a] - Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + Sin
h[a])))/(4*E^(b^2/4))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{i\sqrt{\pi}\,\mathrm{e}^{-a-\frac{b^2}{4}}\operatorname{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4}}\operatorname{erf}\left(ix+\frac{1}{2}ib\right)}{4}$	52

```
[In] int(exp(x^2)*sinh(b*x+a),x,method=_RETURNVERBOSE)
[Out] -1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*
b^2)*erf(I*x+1/2*I*b)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int e^{x^2}\sinh(a+bx)\,dx$$
$$=\frac{1}{4}\sqrt{\pi}\left(\cosh\left(\frac{1}{4}b^2-a\right)\operatorname{erfi}\left(\frac{1}{2}b+x\right)-\cosh\left(\frac{1}{4}b^2+a\right)\operatorname{erfi}\left(-\frac{1}{2}b+x\right)+\operatorname{erfi}\left(-\frac{1}{2}b+x\right)\sinh\left(\frac{1}{4}b^2-\right.\right.$$

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="fricas")
[Out] 1/4*sqrt(pi)*(cosh(1/4*b^2 - a)*erfi(1/2*b + x) - cosh(1/4*b^2 + a)*erfi(-1
/2*b + x) + erfi(-1/2*b + x)*sinh(1/4*b^2 + a) - erfi(1/2*b + x)*sinh(1/4*b
^2 - a))
```

Sympy [F]

$$\int e^{x^2}\sinh(a+bx)\,dx=\int e^{x^2}\sinh\left(a+bx\right)dx$$

```
[In] integrate(exp(x**2)*sinh(b*x+a),x)
[Out] Integral(exp(x**2)*sinh(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2}\sinh(a+bx)\,dx = -\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="maxima")
[Out] -1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(-1
/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2}\sinh(a+bx)\,dx = \frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}-\frac{1}{4}i\sqrt{\pi}\,\mathrm{erf}\left(\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

```
[In] integrate(exp(x^2)*sinh(b*x+a),x, algorithm="giac")
[Out] 1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(1/
2*I*b - I*x)*e^(-1/4*b^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\sinh(a+bx)\,dx = \int \mathrm{e}^{x^2}\,\sinh(a+b\,x)\,dx$$

```
[In] int(exp(x^2)*sinh(a + b*x),x)
[Out] int(exp(x^2)*sinh(a + b*x), x)
```

3.340 $\int e^{x^2} \sinh(a+cx^2)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int e^{x^2} \sinh(a+cx^2)\, dx = -\frac{e^{-a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}} + \frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

```
[Out] -1/4*erfi(x*(1-c)^(1/2))*Pi^(1/2)/exp(a)/(1-c)^(1/2)+1/4*exp(a)*erfi(x*(1+c
)^(1/2))*Pi^(1/2)/(1+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5623, 2235}

$$\int e^{x^2} \sinh(a+cx^2)\, dx = \frac{\sqrt{\pi}e^{a}\mathrm{erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}} - \frac{\sqrt{\pi}e^{-a}\mathrm{erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}}$$

```
[In] Int[E^x^2*Sinh[a + c*x^2],x]
```

```
[Out] -1/4*(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[
Sqrt[1 + c]*x])/(4*Sqrt[1 + c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(-\frac{1}{2}e^{-a+(1-c)x^2} + \frac{1}{2}e^{a+(1+c)x^2}\right) dx \\ &= -\left(\frac{1}{2}\int e^{-a+(1-c)x^2}\, dx\right) + \frac{1}{2}\int e^{a+(1+c)x^2}\, dx \\ &= -\frac{e^{-a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}} + \frac{e^{a}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int e^{x^2}\sinh\left(a+cx^2\right)\, dx = \frac{\sqrt{\pi}\left(-\sqrt{-1+c}(1+c)\mathrm{erf}\left(\sqrt{-1+c}\,x\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\mathrm{erfi}\left(\sqrt{1+c}\,x\right)(\cosh(a)+\sin}{4\left(-1+c^2\right)}$$

```
[In] Integrate[E^x^2*Sinh[a + c*x^2],x]
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]))
+ (-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c
^2))
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erf}\left(\sqrt{c-1}\,x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{e}^{a}\,\mathrm{erf}\left(\sqrt{-c-1}\,x\right)}{4\sqrt{-c-1}}$	48

```
[In] int(exp(x^2)*sinh(c*x^2+a),x,method=_RETURNVERBOSE)
[Out] -1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-
c-1)^(1/2)*erf((-c-1)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\,\mathrm{erf}\left(\sqrt{c-1}x\right)+\sqrt{\pi}((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{\cdots}}{4\left(c^2-1\right)}$$

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
 1)*x) + sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqrt
(-c - 1)*x))/(c^2 - 1)
```

Sympy [F]

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=\int e^{x^2}\sinh\left(a+cx^2\right)dx$$

```
[In] integrate(exp(x**2)*sinh(c*x**2+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt
(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{c-1}x\right)e^{(-a)}}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c-1}x\right)e^{a}}{4\sqrt{-c-1}}$$

```
[In] integrate(exp(x^2)*sinh(c*x^2+a),x, algorithm="giac")
[Out] 1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-sqr
t(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\sinh\left(a+cx^2\right)\,dx=\int e^{x^2}\sinh\left(c\,x^2+a\right)\,dx$$

```
[In] int(exp(x^2)*sinh(a + c*x^2),x)
[Out] int(exp(x^2)*sinh(a + c*x^2), x)
```

3.341 $\int e^{x^2} \sinh \left(a+b x+c x^2\right) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int e^{x^2} \sinh \left(a+b x+c x^2\right) \, dx=\frac{e^{-a-\frac{b^2}{4(1-c)}} \sqrt{\pi } \text{erfi}\left(\frac{b-2 (1-c) x}{2 \sqrt{1-c}}\right)}{4 \sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4 (1+c)}} \sqrt{\pi } \text{erfi}\left(\frac{b+2 (1+c) x}{2 \sqrt{1+c}}\right)}{4 \sqrt{1+c}}$$

```
[Out] 1/4*exp(-a-1/4*b^2/(1-c))*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))*Pi^(1/2)/(1-c
)^(1/2)+1/4*exp(a-1/4*b^2/(1+c))*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2))*Pi^(1/
2)/(1+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {5623, 2266, 2235}

$$\int e^{x^2} \sinh \left(a+b x+c x^2\right) \, dx=\frac{\sqrt{\pi } e^{-a-\frac{b^2}{4 (1-c)}} \text{erfi}\left(\frac{b-2 (1-c) x}{2 \sqrt{1-c}}\right)}{4 \sqrt{1-c}}+\frac{\sqrt{\pi } e^{a-\frac{b^2}{4 (c+1)}} \text{erfi}\left(\frac{b+2 (c+1) x}{2 \sqrt{c+1}}\right)}{4 \sqrt{c+1}}$$

```
[In] Int[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 - c])])
/(4*Sqrt[1 - c]) + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)*x)
/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-a-bx+(1-c)x^2}+\frac{1}{2}e^{a+bx+(1+c)x^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-a-bx+(1-c)x^2}\,dx\right)+\frac{1}{2}\int e^{a+bx+(1+c)x^2}\,dx\\
&= -\left(\frac{1}{2}e^{-a-\frac{b^2}{4(1-c)}}\int e^{\frac{(-b+2(1-c)x)^2}{4(1-c)}}\,dx\right)+\frac{1}{2}e^{a-\frac{b^2}{4(1+c)}}\int e^{\frac{(b+2(1+c)x)^2}{4(1+c)}}\,dx\\
&= \frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)\,dx$$

$$= \frac{e^{-\frac{b^2}{4+4c}}\sqrt{\pi}\left(-\sqrt{-1+c}(1+c)e^{\frac{b^2c}{2(-1+c^2)}}\text{erf}\left(\frac{b+2(-1+c)x}{2\sqrt{-1+c}}\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right.\right.}{4\left(-1+c^2\right)}$$

```
[In] Integrate[E^x^2*Sinh[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[Pi]*(-(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1
 + c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a])) + (-1 + c)*Sqrt[1 + c]*Erfi
[(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(
b^2/(4 + 4*c)))
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^{2}-4a}{4(c-1)}}\,\mathrm{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^{2}+4a}{4+4c}}\,\mathrm{erf}\left(-\sqrt{-c-1}\,x+\frac{b}{2\sqrt{-c-1}}\right)}{4\sqrt{-c-1}}$	105

```
[In] int(exp(x^2)*sinh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x
+1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(1+c))/(-c-1)^(1/2
)*erf(-(-c-1)^(1/2)*x+1/2*b/(-c-1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.43

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\,\mathrm{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)+\sqrt{\pi}\Big((c-1)\cosh}{4\,(c^2-1)}$$

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*si
nh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b)
/sqrt(c - 1)) + sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)) +
(c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*(c
+ 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

Sympy [F]

$$\int e^{x^2}\sinh\left(a+bx+cx^2\right)\,dx = \int e^{x^2}\sinh\left(a+bx+cx^2\right)dx$$

```
[In] integrate(exp(x**2)*sinh(c*x**2+b*x+a),x)
```

```
[Out] Integral(exp(x**2)*sinh(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int e^{x^2} \sinh \left(a+bx+cx^2\right) \, dx = \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-c-1}x - \frac{b}{2\sqrt{-c-1}}\right) e^{\left(a-\frac{b^2}{4\,(c+1)}\right)}}{4\sqrt{-c-1}} - \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{c-1}x + \frac{b}{2\sqrt{c-1}}\right) e^{\left(-a+\frac{b^2}{4\,(c-1)}\right)}}{4\sqrt{c-1}}$$

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="maxima")
[Out] 1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c + 1
))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(-a
 + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int e^{x^2} \sinh \left(a+bx+cx^2\right) \, dx = -\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c-1}\left(2\,x+\frac{b}{c+1}\right)\right) e^{\left(-\frac{b^2-4\,ac-4\,a}{4\,(c+1)}\right)}}{4\sqrt{-c-1}} + \frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2}\sqrt{c-1}\left(2\,x+\frac{b}{c-1}\right)\right) e^{\left(\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)}}{4\sqrt{c-1}}$$

```
[In] integrate(exp(x^2)*sinh(c*x^2+b*x+a),x, algorithm="giac")
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*c
 - 4*a)/(c + 1))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x + b/
(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \sinh \left(a+bx+cx^2\right) \, dx = \int \sinh\left(c\,x^2+b\,x+a\right)\,\mathrm{e}^{x^2} \,d x$$

```
[In] int(sinh(a + b*x + c*x^2)*exp(x^2),x)
[Out] int(sinh(a + b*x + c*x^2)*exp(x^2), x)
```

3.342 $\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=-\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] -1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*
Pi^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f
^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\mathrm{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\mathrm{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Lo
g[f])/(2*Sqrt[f])]) + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*E
rfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx\\
&= \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx-\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,dx\\
&= -\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=\frac{1}{4}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(-e^{\frac{b^2\log^2(f)}{2f}}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2],x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi]*(-(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Co
sh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-f}\,x+\frac{\ln(f)b}{2\sqrt{-f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^2\ln(f)^2-4df}{4f}}}{4\sqrt{-f}}+\frac{\operatorname{erf}\left(-\sqrt{f}\,x+\frac{\ln(f)b}{2\sqrt{f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^2\ln(f)^2-4df}{4f}}}{4\sqrt{f}}$	100

```
[In] int(f^(b*x+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))/(-f)^(1/2)*Pi^(1/2)*f^a*exp(
-1/4*(b^2*ln(f)^2-4*d*f)/f)+1/4*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/2)
*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-4*d*f)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(84) = 168.

Time = 0.29 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.94

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f))\sqrt{-f}}{2\,f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{erf}\left(\right.}{}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*e
rf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) - sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(
f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) - sqrt
```

```
(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqr
t(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=\int f^{a+bx}\sinh\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d),x)
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\,\mathrm{erf}\left(\sqrt{f}x-\frac{b\log(f)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{4\sqrt{-f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))*
e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4af\log(f)-4df}{4f}\right)}}{4\sqrt{f}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4af\log(f)-4df}{4f}\right)}}{4\sqrt{-f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*
a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*lo
g(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\sinh\left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + b*x)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2), x)
```

3.343 $\int f^{a+bx} \sinh^2(d+fx^2) \, dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int f^{a+bx} \sinh^2(d+fx^2) \, dx = \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\text{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right) - \frac{f^{a+bx}}{2b\log(f)}$$

```
[Out] -1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/4*
(4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*b^2*ln(f
)^2/f)*f^(-1/2+a)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2
)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5623, 2225, 2325, 2266, 2236, 2235}

$$\int f^{a+bx} \sinh^2(d+fx^2) \, dx = \frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\text{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\text{erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right) - \frac{f^{a+bx}}{2b\log(f)}$$

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Log
[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)
*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 - f^(a + b*x)/(
2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}f^{a+bx}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx}\,dx-\frac{1}{2}\int f^{a+bx}\,dx\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\int e^{-2d-2fx^2+a\log(f)+bx\log(f)}\,dx+\frac{1}{4}\int e^{2d+2fx^2+a\log(f)+bx\log(f)}\,dx\\
&= -\frac{f^{a+bx}}{2b\log(f)}+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{\frac{(4fx+b\log(f))^2}{8f}}\,dx\\
&\quad+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f}}f^a\right)\int e^{-\frac{(-4fx+b\log(f))^2}{8f}}\,dx
\end{aligned}
$$

$$= \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right) - \frac{f^{a+bx}}{2b\log(f)}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx = \frac{1}{16}f^a\left(-\frac{8f^{bx}}{b\log(f)} + \frac{e^{\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)-\sinh(2d))}{\sqrt{f}} + \frac{e^{-\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)+\sinh(2d))}{\sqrt{f}}\right)$$

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*((-8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4
*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (S
qrt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[2*
d]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{\ln(f)b\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{16\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-2f}\,x+\frac{\ln(f)b}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{8\sqrt{-2f}}-\frac{f^{a}f^{bx}}{2b\ln(f)}$	126

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*ln(f)*b*2^(1/2)/f^(1/2))/f^(1/2)*2^(1/2)*P
i^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-16*d*f)/f)-1/8*erf(-(-2*f)^(1/2)*x+1/2*ln(
f)*b/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/8*(b^2*ln(f)^2-16*d*f)/
f)-1/2*f^a*f^(b*x)/b/ln(f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(114) = 228.

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.88

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)\text{erf}\left(\frac{\sqrt{2}(4\,fx+b\log(f))\sqrt{-f}}{4\,f}\right)\log\left(f\right)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+}{}\right.}{}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2)*
sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*erf(-
1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)
*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2
 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*
(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f*log(f)
 - 16*d*f)/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh((b*x + a)*log(f)))/(b
*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx=\int f^{a+bx}\sinh^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx=\frac{\sqrt{2}\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{8\,f}-2\,d\right)}}{16\sqrt{f}}+\frac{\sqrt{2}\sqrt{\pi}f^a\,\text{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{8\,f}+2\,d\right)}}{16\sqrt{-f}}-\frac{f^{bx+a}}{2\,b\log\left(f\right)}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqrt
(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf(s
qrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/f +
 2*d)/sqrt(-f) - 1/2*f^(b*x + a)/(b*log(f))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.41

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+8\,af\log(f)-16\,df}{8\,f}\right)}}{16\sqrt{f}}$$
$$-\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)}}{16\sqrt{-f}}$$
$$-\left(\frac{2\,b\cos\left(-\frac{1}{2}\pi bx\mathrm{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\mathrm{sgn}(f)+\frac{1}{2}\pi a\right)\log\left(|f|\right)}{4\,b^2\log\left(|f|\right)^2+\left(\pi b\mathrm{sgn}\left(f\right)-\pi b\right)^2}-\frac{\left(\pi b\mathrm{sgn}(f)-\pi b\right)\sin\left(-\frac{1}{2}\pi bx\mathrm{sgn}(f)\right.}{4\,b^2\log\left(|f|\right)^2+\left(\pi b s\right.}\right.$$
$$+i\left(-\frac{i\,e^{\left(\frac{1}{2}i\pi bx\mathrm{sgn}(f)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\mathrm{sgn}(f)-\frac{1}{2}i\pi a\right)}}{2i\pi b\mathrm{sgn}\left(f\right)-2i\pi b+4\,b\log\left(|f|\right)}+\frac{i\,e^{\left(-\frac{1}{2}i\pi bx\mathrm{sgn}(f)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\mathrm{sgn}(f)+\frac{1}{2}i\pi a\right)}}{-2i\pi b\mathrm{sgn}\left(f\right)+2i\pi b+4\,b\log\left(|f|\right)}\right)e^{\left(bx\log(|f|)+a\log(|f|)\right)}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8*
(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)*e
rf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a*f*
log(f) - 16*d*f)/f)/sqrt(-f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1
/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f)
- pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2
*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^
(b*x*log(abs(f)) + a*log(abs(f))) + I*(-I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi
*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*lo
g(abs(f))) + I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) +
 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(ab
s(f)) + a*log(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh^2\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\sinh\left(f\,x^2+d\right)^2\,dx$$

```
[In] int(f^(a + b*x)*sinh(d + f*x^2)^2,x)
[Out] int(f^(a + b*x)*sinh(d + f*x^2)^2, x)
```

3.344 $\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=\frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}e^{3d-\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)$$

```
[Out] -1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*erf(1/6*(6*f*x-b*ln(f))*3^(1/
2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*er
fi(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/16*exp(-d+1/4*b^
2*ln(f)^2/f)*f^(-1/2+a)*erf(1/2*(2*f*x-b*ln(f))/f^(1/2))*Pi^(1/2)-3/16*exp(
d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used

= {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)-\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\operatorname{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)-\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\operatorname{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{b^2\log^2(f)}{12f}}\operatorname{erfi}\left(\frac{b\log(f)+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f
])/(2*Sqrt[f])])/16 - (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[P
i/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E^(d - (b^2*Log[f
]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/16
+ (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x + b*
Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}-\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)\,dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx}\,dx\\
&\quad +\frac{3}{8}\int e^{-d-fx^2}f^{a+bx}\,dx-\frac{3}{8}\int e^{d+fx^2}f^{a+bx}\,dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3fx^2+a\log(f)+bx\log(f)}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2+a\log(f)+bx\log(f)}\,dx\\
&\quad +\frac{3}{8}\int e^{-d-fx^2+a\log(f)+bx\log(f)}\,dx-\frac{3}{8}\int e^{d+fx^2+a\log(f)+bx\log(f)}\,dx\\
&= -\left(\frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{\frac{(2fx+b\log(f))^2}{4f}}\,dx\right)+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{\frac{(6fx+b\log(f))^2}{12f}}\,dx\\
&\quad -\frac{1}{8}\left(e^{-3d+\frac{b^2\log^2(f)}{12f}}f^a\right)\int e^{-\frac{(-6fx+b\log(f))^2}{12f}}\,dx+\frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f}}f^a\right)\int e^{-\frac{(-2fx+b\log(f))^2}{4f}}\,dx\\
&= \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)\\
&\quad -\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\
&\quad -\frac{3}{16}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\\
&\quad +\frac{1}{16}e^{3d-\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.20

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=\frac{1}{16}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\left(-3\sqrt{3}\cosh(d)\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))-3\sqrt{3}\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)-e^{\frac{b^2\log^2(f)}{3f}}\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

```
[In] Integrate[f^(a + b*x)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^(-1/2 + a)*Sqrt[Pi/3]*(-3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqr
t[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqrt
[3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(
2*Sqrt[f])]*(Cosh[d] - Sinh[d]) - 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt
[f])]*Sinh[d] - E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*
Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f*x +
b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f)))
```

Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-3f}\,x+\frac{\ln(f)b}{2\sqrt{-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^2\ln(f)^2-36df}{12f}}}{16\sqrt{-3f}}+\frac{\operatorname{erf}\left(-\sqrt{3}\,\sqrt{f}\,x+\frac{\ln(f)b\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^2\ln(f)^2-36df}{12f}}}{48\sqrt{f}}-\frac{3\operatorname{erf}\left(-\sqrt{f}\,x+\frac{\ln(f)b}{2\sqrt{f}}\right)}{16}$

```
[In] int(f^(b*x+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-(-3*f)^(1/2)*x+1/2*ln(f)*b/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^(1/2)*f
^a*exp(-1/12*(b^2*ln(f)^2-36*d*f)/f)+1/48*erf(-3^(1/2)*f^(1/2)*x+1/6*ln(f)*
b*3^(1/2)/f^(1/2))/f^(1/2)*3^(1/2)*Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-36*d*
f)/f)-3/16*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4
*(b^2*ln(f)^2-4*d*f)/f)+3/16*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))/(-f)
^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(181) = 362$.

Time = 0.32 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.86

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx = -\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}(6\,fx+b\log(f))\sqrt{-f}}{6\,f}\right)-\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+12\,af\log(}{12\,f}\right.}{}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) - sqrt(3)*sqrt(pi
)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*sqr
t(3)*(6*f*x - b*log(f))/sqrt(f)) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3
)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*
d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt
(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) - 9*sqrt(pi)*s
qrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(2*f*x +
b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + 4*a*f*l
og(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(pi)*sqrt(f)
*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4*a*f*log(f)
 - 4*d*f)/f) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f)*s
inh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx = \int f^{a+bx}\sinh^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.84

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx=\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}-\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{12f}-3d\right)}}{48\sqrt{f}}+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{12f}+3d\right)}}{48\sqrt{-f}}-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{16\sqrt{-f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqr
t(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt(3)
*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e^(-1
/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2
*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx = \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{f}} - \frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)}}{48\,\sqrt{-f}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2\,x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{16\,\sqrt{f}} + \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2\,x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{16\,\sqrt{-f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/12*
(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(pi)*
erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 - 12*a
*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*l
og(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) + 3/16*sq
rt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*
log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh^3\left(d+fx^2\right)\,dx = \int f^{a+b\,x}\,{\sinh\left(f\,x^2+d\right)}^3\,dx$$

```
[In] int(f^(a + b*x)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + f*x^2)^3, x)
```

3.345 $\int f^{a+bx} \sinh(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int f^{a+bx} \sinh(d+ex+fx^2)\,dx = -\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right) + \frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)$$

```
[Out] -1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/
2))*Pi^(1/2)+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*
ln(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+bx} \sinh(d+ex+fx^2)\,dx = \frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\operatorname{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right) - \frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\operatorname{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x
 - b*Log[f])/(2*Sqrt[f])]) + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sq
rt[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}\,dx\\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx\right)+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\int e^{\frac{(e+2fx+b\log(f))^2}{4f}}\,dx\\
&= -\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)\\
&\quad+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.08

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{4}e^{-\frac{e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(-e^{\frac{e^2+b^2\log^2(f)}{2f}}\operatorname{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(-(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e
+ 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d])) + Erfi[(e + 2*f*x + b
*Log[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f
)))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-f}\,x+\frac{e+b\ln(f)}{2\sqrt{-f}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{b^2\ln(f)^2+2\ln(f)be-4df+e^2}{4f}}}{4\sqrt{-f}}+\frac{\operatorname{erf}\left(-\sqrt{f}\,x+\frac{b\ln(f)-e}{2\sqrt{f}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{\frac{b^2\ln(f)^2-2\ln(f)be-4df+e^2}{4f}}}{4\sqrt{f}}$	126

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))/(-f)^(1/2)*Pi^(1/2)*f^a*
exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+e^2)/f)+1/4*erf(-f^(1/2)*x+1/2*(b*l
n(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-2*ln(f)*b*e-4*d*
f+e^2)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(90) = 180.

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.20

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df+2\,(be-2\,af)\log(f)}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f)+e)\sqrt{-f}}{2\,f}\right)-\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df-2\,(}{4\,f}\right.}{}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a
*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) - sqrt(pi)*sqrt(f
)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(-1/
2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*lo
g(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*a*f
)*log(f))/f) - sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f))*si
nh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F]

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\sinh\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx=-\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{4\sqrt{-f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
 + 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f
) + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2\,x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,f}\right)}}{4\sqrt{-f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2\,x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,f}\right)}}{4\sqrt{f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log(
f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(-f) + 1/4*sqrt(pi
)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^2 - 2*b*e*l
og(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\,\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2), x)
```

3.346 $\int f^{a+bx} \sinh^2\left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx &= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\\&\quad+\frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{2e+4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\\&\quad-\frac{f^{a+bx}}{2b\log(f)}\end{aligned}$$

```
[Out] -1/2*f^(b*x+a)/b/ln(f)+1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(
1/4*(2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))*2^(1/2)*Pi^(1/2)+1/16*exp(2*d-1/8*
(2*e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2))
*2^(1/2)*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5623, 2225, 2325, 2266, 2236, 2235}

$$\begin{aligned}\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx &= \frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\operatorname{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)\\&\quad+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\operatorname{erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)\\&\quad-\frac{f^{a+bx}}{2b\log(f)}\end{aligned}$$

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*f
*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*f)
)*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])
])/8 - f^(a + b*x)/(2*b*Log[f])
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-\frac{1}{2} f^{a+bx} + \frac{1}{4} e^{-2d-2ex-2fx^2} f^{a+bx} + \frac{1}{4} e^{2d+2ex+2fx^2} f^{a+bx}\right) dx \\ &= \frac{1}{4} \int e^{-2d-2ex-2fx^2} f^{a+bx} \, dx + \frac{1}{4} \int e^{2d+2ex+2fx^2} f^{a+bx} \, dx - \frac{1}{2} \int f^{a+bx} \, dx\end{aligned}$$

$$
\begin{aligned}
&= -\frac{f^{a+bx}}{2b\log(f)} + \frac{1}{4}\int \exp\left(-2d - 2fx^2 + a\log(f) - x(2e - b\log(f))\right)\,dx \\
&\quad + \frac{1}{4}\int \exp\left(2d + 2fx^2 + a\log(f) + x(2e + b\log(f))\right)\,dx \\
&= -\frac{f^{a+bx}}{2b\log(f)} + \frac{1}{4}\left(e^{-2d+\frac{(2e-b\log(f))^2}{8f}} f^a\right)\int e^{-\frac{(-2e-4fx+b\log(f))^2}{8f}}\,dx \\
&\quad + \frac{1}{4}\left(e^{2d-\frac{(2e+b\log(f))^2}{8f}} f^a\right)\int e^{\frac{(2e+4fx+b\log(f))^2}{8f}}\,dx \\
&= \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}} f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) \\
&\quad + \frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}} f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{2e+4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right) - \frac{f^{a+bx}}{2b\log(f)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.37

$$
\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx
= \frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f}} f^{a-\frac{be+f}{2f}}\left(-4\sqrt{2}e^{\frac{4e^2+b^2\log^2(f)}{8f}} f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)} + be^{\frac{4e^2+b^2\log^2(f)}{4f}}\sqrt{\pi}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2d)\right.}{8\sqrt{2}b\log(f)}
$$

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*(-4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/
2 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*e
 + 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d]) +
b*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[
2*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f])
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{(b\ln(f)-2e)\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^a\,\mathrm{e}^{\frac{b^2\ln(f)^2-4\ln(f)be-16df+4e^2}{8f}}}{16\sqrt{f}} - \frac{\operatorname{erf}\left(-\sqrt{-2f}\,x+\frac{2e+b\ln(f)}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^a\,\mathrm{e}^{-\frac{b^2\ln(f)^2+4\ln(f)b}{8f}}}{8\sqrt{-2f}}$

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))/f^(1/2)*2^(
1/2)*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)-1/8*erf
(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f^a*
exp(-1/8*(b^2*ln(f)^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)-1/2*f^a*f^(b*x)/b/ln(f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(126) = 252$.

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.07

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+4e^2-16df+4(be-2af)\log(f)}{8f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4fx+b\log(f)+2e)\sqrt{-f}}{4f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cos}{}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqrt(
-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 -
 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f) + 2
*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f*x +
b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
+ 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*sqrt(2)*
(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f - 4*(b*e - 2*a*f)*log(f))/f) + 8*f*cosh((b*x + a)*log(f)) + 8*f*sinh(
(b*x + a)*log(f)))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx = \int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}(b\log(f)+2e)}{4\sqrt{-f}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^2}{8f}\right)}}{16\sqrt{-f}} + \frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}(b\log(f)-2e)}{4\sqrt{f}}\right)e^{\left(-2d+\frac{(b\log(f)-2e)^2}{8f}\right)}}{16\sqrt{f}} - \frac{f^{bx+a}}{2b\log(f)}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)*s
qrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f))*e
^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) - 1/2*f^(b*x + a)/(b*log(f))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.41

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{-f}\left(4x+\frac{b\log(f)+2e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+4be\log(f)-8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{-f}} - \frac{\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{4}\sqrt{2}\sqrt{f}\left(4x-\frac{b\log(f)-2e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-4be\log(f)+8af\log(f)+4e^2-16df}{8f}\right)}}{16\sqrt{f}} - \left(\frac{2b\cos\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)+\frac{1}{2}\pi bx-\frac{1}{2}\pi a\operatorname{sgn}(f)+\frac{1}{2}\pi a\right)\log(|f|)}{4b^2\log(|f|)^2+(\pi b\operatorname{sgn}(f)-\pi b)^2} - \frac{(\pi b\operatorname{sgn}(f)-\pi b)\sin\left(-\frac{1}{2}\pi bx\operatorname{sgn}(f)\right.}{4b^2\log(|f|)^2+(\pi b s} + i\left(-\frac{ie^{\left(\frac{1}{2}i\pi bx\operatorname{sgn}(f)-\frac{1}{2}i\pi bx+\frac{1}{2}i\pi a\operatorname{sgn}(f)-\frac{1}{2}i\pi a\right)}}{2i\pi b\operatorname{sgn}(f)-2i\pi b+4b\log(|f|)} + \frac{ie^{\left(-\frac{1}{2}i\pi bx\operatorname{sgn}(f)+\frac{1}{2}i\pi bx-\frac{1}{2}i\pi a\operatorname{sgn}(f)+\frac{1}{2}i\pi a\right)}}{-2i\pi b\operatorname{sgn}(f)+2i\pi b+4b\log(|f|)}\right)e^{(bx\log(|f|)+a\log(|f|))}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) + 2*e)/f)
)*e^(-1/8*(b^2*log(f)^2 + 4*b*e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/f)/
```

```
sqrt(-f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(f)
- 2*e)/f))*e^(1/8*(b^2*log(f)^2 - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*
d*f)/f)/sqrt(f) - (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f
) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2) -
(pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f)
+ 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*log(abs(
f)) + a*log(abs(f))) + I*(-I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*
pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(abs(f))) +
I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a)/
(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(f)) + a*log
(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,{\mathrm{sinh}\left(f\,x^2+e\,x+d\right)}^2\,dx$$

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2,x)
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^2, x)
```

3.347 $\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx = \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right) - \frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right) - \frac{3}{16}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right) + \frac{1}{16}e^{3d-\frac{(3e+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)$$

```
[Out] -1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/6*(3*e+6*f*x-b*ln(f
))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+1/48*exp(3*d-1/12*(3*e+b*ln(f))^2/f)*f
^(-1/2+a)*erfi(1/6*(3*e+6*f*x+b*ln(f))*3^(1/2)/f^(1/2))*3^(1/2)*Pi^(1/2)+3/
16*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*erf(1/2*(e+2*f*x-b*ln(f))/f^(1/2)
)*Pi^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*erfi(1/2*(e+2*f*x+b*l
n(f))/f^(1/2))*Pi^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{5623, 2325, 2266, 2236, 2235\}$

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx = \frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right) - \frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\text{erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right) - \frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right) + \frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{(b\log(f)+3e)^2}{12f}}\text{erfi}\left(\frac{b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

```
[In] Int[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b
*Log[f])/(2*Sqrt[f])])/16 - (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 - (3*E
^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b*Log
[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 + a)*S
qrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3(d+ex+fx^2)}f^{a+bx}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3(d+ex+fx^2)\right)f^{a+bx} -\frac{3}{8}\exp\left(4d+4ex+4fx^2-3(d+ex+fx^2)\right)f^{a+bx} +\frac{1}{8}\exp\left(6d+6ex+6fx^2-3(d+ex+fx^2)\right)f^{a+bx}\right)dx\\
&=-\left(\frac{1}{8}\int e^{-3(d+ex+fx^2)}f^{a+bx}\,dx\right)\\
&\quad+\frac{1}{8}\int \exp\left(6d+6ex+6fx^2-3(d+ex+fx^2)\right)f^{a+bx}\,dx\\
&\quad+\frac{3}{8}\int \exp\left(2d+2ex+2fx^2-3(d+ex+fx^2)\right)f^{a+bx}\,dx\\
&\quad-\frac{3}{8}\int \exp\left(4d+4ex+4fx^2-3(d+ex+fx^2)\right)f^{a+bx}\,dx\\
&=-\left(\frac{1}{8}\int \exp\left(-3d-3fx^2+a\log(f)-x(3e-b\log(f))\right)dx\right)\\
&\quad+\frac{1}{8}\int \exp\left(3d+3fx^2+a\log(f)+x(3e+b\log(f))\right)dx\\
&\quad+\frac{3}{8}\int e^{-d-fx^2+a\log(f)-x(e-b\log(f))}\,dx-\frac{3}{8}\int e^{d+fx^2+a\log(f)+x(e+b\log(f))}\,dx\\
&=\frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4f}}f^a\right)\int e^{-\frac{(-e-2fx+b\log(f))^2}{4f}}\,dx\\
&\quad-\frac{1}{8}\left(e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^a\right)\int e^{-\frac{(-3e-6fx+b\log(f))^2}{12f}}\,dx\\
&\quad-\frac{1}{8}\left(3e^{d-\frac{(e+b\log(f))^2}{4f}}f^a\right)\int e^{\frac{(e+2fx+b\log(f))^2}{4f}}\,dx\\
&\quad+\frac{1}{8}\left(e^{3d-\frac{(3e+b\log(f))^2}{12f}}f^a\right)\int e^{\frac{(3e+6fx+b\log(f))^2}{12f}}\,dx\\
&=\frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)\\
&\quad-\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\
&\quad-\frac{3}{16}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\\
&\quad+\frac{1}{16}e^{3d-\frac{(3e+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.38

$$\begin{aligned}
&\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx\\
&=\frac{1}{16}e^{-\frac{3e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\frac{\pi}{3}}\left(-3\sqrt{3}e^{\frac{e^2}{2f}}\cosh(d)\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\right.\\
&\quad+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\
&\quad+3\sqrt{3}e^{\frac{2e^2+b^2\log^2(f)}{2f}}\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))\\
&\quad-3\sqrt{3}e^{\frac{e^2}{2f}}\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)\\
&\quad-e^{\frac{9e^2+2b^2\log^2(f)}{6f}}\text{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))\\
&\quad\left.+e^{\frac{b^2\log^2(f)}{6f}}\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)
\end{aligned}$$

```
[In] Integrate[f^(a + b*x)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(-3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[
(e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Er
fi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 + b^
2*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[
d]) - 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*Sinh
[d] - E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sq
rt[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(3*
e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2 + b^2*
Log[f]^2)/(4*f)))
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\text{erf}\left(-\sqrt{-3f}\,x+\frac{3e+b\ln(f)}{2\sqrt{-3f}}\right)\sqrt{\pi}\,f^a\text{e}^{-\frac{b^2\ln(f)^2+6\ln(f)be-36df+9e^2}{12f}}}{16\sqrt{-3f}}+\frac{\text{erf}\left(-\sqrt{3}\,\sqrt{f}\,x+\frac{(b\ln(f)-3e)\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\,\sqrt{\pi}\,f^a\text{e}^{\frac{b^2\ln(f)^2-6\ln(f)be-}{12f}}}{48\sqrt{f}}$

```
[In] int(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^(
1/2)*f^a*exp(-1/12*(b^2*ln(f)^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)+1/48*erf(-3^(1
/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))/f^(1/2)*3^(1/2)*Pi^(1/2)*f
```

```
^a*exp(1/12*(b^2*ln(f)^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)-3/16*erf(-f^(1/2)*x+1
/2*(b*ln(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-2*ln(f)*b
*e-4*d*f+e^2)/f)+3/16*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))/(-f)^(1
/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+e^2)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(199) = 398$.

Time = 0.30 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.11

$$\int f^{a+bx} \sinh^3\left(d+ex+fx^2\right)\,dx =$$
$$-\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+9e^2-36df+6(be-2af)\log(f)}{12f}\right)\operatorname{erf}\left(\frac{\sqrt{3}(6fx+b\log(f)+3e)\sqrt{-f}}{6f}\right)-\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log\cdots}{\cdots}\right)}{\cdots}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f +
 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt(-
f)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f -
 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/sqrt(
f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sq
rt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*log(f)
)/f) - sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*log(f))/
f) - 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sqrt(pi)*sqr
t(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) + 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
 b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt
(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F]

$$\int f^{a+bx} \sinh^3\left(d+ex+fx^2\right)\,dx = \int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x)*sinh(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}(b\log(f)+3e)}{6\sqrt{-f}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{12f}\right)}}{48\sqrt{-f}}$$
$$+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}$$
$$-\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}(b\log(f)-3e)}{6\sqrt{f}}\right)e^{\left(-3d+\frac{(b\log(f)-3e)^2}{12f}\right)}}{48\sqrt{f}}$$
$$-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{16\sqrt{-f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(pi)
*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*log
(f) - e)^2/f) - 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*sqrt(
3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt(f) -
 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d - 1/4
*(b*log(f) + e)^2/f)/sqrt(-f)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.09

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)+3e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+6be\log(f)-12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{-f}}$$
$$+\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)-3e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-6be\log(f)+12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{f}}$$
$$+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{f}}$$

```
[In] integrate(f^(b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) + 3*e)/f)
)*e^(-1/12*(b^2*log(f)^2 + 6*b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/f
)/sqrt(-f) + 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*log(f
) - 3*e)/f))*e^(1/12*(b^2*log(f)^2 - 6*b*e*log(f) + 12*a*f*log(f) + 9*e^2 -
 36*d*f)/f)/sqrt(f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)
/f))*e^(-1/4*(b^2*log(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/
sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*
(b^2*log(f)^2 - 2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\sinh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,{\mathrm{sinh}\left(f\,x^2+e\,x+d\right)}^3\,dx$$

```
[In] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x)*sinh(d + e*x + f*x^2)^3, x)
```

3.348 $\int f^{a+cx^2} \sinh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int f^{a+cx^2} \sinh(d+ex)\,dx = \frac{e^{-d-\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{d-\frac{e^2}{4c\log(f)}} f^a \sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5623, 2325, 2266, 2235}

$$\int f^{a+cx^2} \sinh(d+ex)\,dx = \frac{\sqrt{\pi} f^a e^{-\frac{e^2}{4c\log(f)}-d}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4c\log(f)}}\,\mathrm{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[c]*
Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f^a*Sq
rt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[L
og[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\
&= -\left(\frac{1}{2}\left(e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx\right)+\frac{1}{2}\left(e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= \frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\sinh(d+ex)\,dx$$
$$=\frac{e^{-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(-\cosh(d)+\sinh(d))+\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[d]
 + Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] +
Sinh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a\,\mathrm{e}^{\frac{4d\ln(f)c-e^2}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a\,\mathrm{e}^{-\frac{4d\ln(f)c+e^2}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}$	117

```
[In] int(f^(c*x^2+a)*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1
/2)*f^a*exp(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)-1/4*erf((-c*ln(f))^(1/2)*x+1/2*e
/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)
/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(101) = 202.

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.63

$$\int f^{a+cx^2}\sinh(d+ex)\,dx =$$
$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)+e)\sqrt{-c\log(f)}}{2\,c\log(f)}\right)}{}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e
^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(
c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-
c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))*
erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sinh(d+ex)\,dx=\int f^{a+cx^2}\sinh{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\sinh(d+ex)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*
e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99

$$\int f^{a+cx^2}\sinh(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*
log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*e
rf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*
d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh(d+ex)\,dx = \int f^{c\,x^2+a}\,\sinh(d+e\,x)\;dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + e*x),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x), x)
```

3.349 $\int f^{a+cx^2} \sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 161

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = -\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-e^2/c/ln(f))*f^a*erfi((-e+c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(
1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*erfi((e+c*x*ln(f))/c^
(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*f^a*erfi(x*c^(1/2)*ln(f
)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5623, 2235, 2325, 2266}

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = -\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{c\log(f)}-2d}\text{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)}}\text{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) - (
E^(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[
Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2} f^{a+cx^2}+\frac{1}{4} e^{-2d-2ex} f^{a+cx^2}+\frac{1}{4} e^{2d+2ex} f^{a+cx^2}\right) dx \\
&= \frac{1}{4}\int e^{-2d-2ex} f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex} f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx \\
&= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d-2ex+a\log(f)+cx^2\log(f)}\,dx \\
&\quad+\frac{1}{4}\int e^{2d+2ex+a\log(f)+cx^2\log(f)}\,dx \\
&= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\left(e^{-2d-\frac{e^2}{c\log(f)}} f^a\right)\int e^{\frac{(-2e+2cx\log(f))^2}{4c\log(f)}}\,dx \\
&\quad+\frac{1}{4}\left(e^{2d-\frac{e^2}{c\log(f)}} f^a\right)\int e^{\frac{(2e+2cx\log(f))^2}{4c\log(f)}}\,dx
\end{aligned}
$$

$$= -\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$
$$+ \frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx$$
$$= \frac{e^{-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\left(-2e^{\frac{e^2}{c\log(f)}}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right) + \operatorname{erfi}\left(\frac{-e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d)) + \operatorname{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(-2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-
e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e +
 c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]*E
^(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a \mathrm{e}^{-\frac{2d\ln(f)c+e^2}{\ln(f)c}}}{8\sqrt{-c\ln(f)}} - \frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a \mathrm{e}^{\frac{2d\ln(f)c-e^2}{\ln(f)c}}}{8\sqrt{-c\ln(f)}} - \frac{f^a\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c\ln\right.}{4\sqrt{-c\ln(f}$

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*erf((-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^
a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)-1/8*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f))^
(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln(f)/c)-1/4*f^a
*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.52

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh(a\log(f))+\sqrt{\pi}\sinh(a\log(f))\right)\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)-\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{ac\log\ldots}{\ldots}\right)\ldots\right.}{\ldots}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f)))*
erf(sqrt(-c*log(f))*x) - sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 + 2*c
*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(f) -
 e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(
-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))) +
 sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf((c*x*lo
g(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \int f^{a+cx^2}\sinh^2\left(d+ex\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(2\,d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(-2\,d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(c
*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sqrt
(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*e
rf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) - e^2
)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x - e/(c
*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f
))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^2(d+ex)\,dx = \int f^{c\,x^2+a}\,\sinh(d+e\,x)^2\,dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + e*x)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x)^2, x)
```

3.350 $\int f^{a+cx^2} \sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 271

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(-e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1
/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*erfi(1
/2*(-3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/1
6*exp(d-1/4*e^2/c/ln(f))*f^a*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*
Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*erfi(1/2*(3*
e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5623, 2325, 2266, 2235}

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = -\frac{3\sqrt{\pi}f^ae^{-\frac{e^2}{4c\log(f)}-d}\text{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^ae^{-\frac{9e^2}{4c\log(f)}-3d}\text{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3\sqrt{\pi}f^ae^{d-\frac{e^2}{4c\log(f)}}\text{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^ae^{3d-\frac{9e^2}{4c\log(f)}}\text{erfi}\left(\frac{2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt[
c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(-3*d - (9*e^2)/(4*c*Log[
f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*
Sqrt[c]*Sqrt[Log[f]]) - (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*
d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*Sqrt[c]
*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}-\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+cx^2}\,dx\\
&\quad+\frac{3}{8}\int e^{-d-ex}f^{a+cx^2}\,dx-\frac{3}{8}\int e^{d+ex}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3ex+a\log(f)+cx^2\log(f)}\,dx\right)+\frac{1}{8}\int e^{3d+3ex+a\log(f)+cx^2\log(f)}\,dx\\
&\quad+\frac{3}{8}\int e^{-d-ex+a\log(f)+cx^2\log(f)}\,dx-\frac{3}{8}\int e^{d+ex+a\log(f)+cx^2\log(f)}\,dx\\
&= -\left(\frac{1}{8}\left(e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-3e+2cx\log(f))^2}{4c\log(f)}}\,dx\right)+\frac{1}{8}\left(e^{3d-\frac{9e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(3e+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&\quad+\frac{1}{8}\left(3e^{-d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(-e+2cx\log(f))^2}{4c\log(f)}}\,dx-\frac{1}{8}\left(3e^{d-\frac{e^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}\\
&\quad-\frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{e^{3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx$$
$$=\frac{e^{-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(-3e^{\frac{2e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\text{si}\right.\right.}{16\sqrt{c}\sqrt{\log}}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c
*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e +
```

```
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e
 + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Erfi[
(-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(-Cosh[3*d] + Sinh[3*d])))/
(16*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\mathrm{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{3d\ln(f)c-\frac{9e^{2}}{4}}{c\ln(f)}}}{16\sqrt{-c\ln(f)}}-\frac{\mathrm{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{3\left(4d\ln(f)c+3e^{2}\right)}{4\ln(f)c}}}{16\sqrt{-c\ln(f)}}+\frac{3\,\mathrm{erf}\left(\sqrt{\ldots}\right.}{\ldots}$

```
[In] int(f^(c*x^2+a)*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-(-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(
1/2)*f^a*exp(3/4*(4*d*ln(f)*c-3*e^2)/ln(f)/c)-1/16*erf((-c*ln(f))^(1/2)*x+3
/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c+
3*e^2)/ln(f)/c)+3/16*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(
f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)+3/16*erf(-(-c*ln
(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(1/4*
(4*d*ln(f)*c-e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(205) = 410.

Time = 0.31 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.58

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = -\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)\right)\mathrm{erf}\left(\frac{(2\,cx\log(f)+3\ldots}{2\,c\log\ldots}\right.}{\ldots}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) - 9
*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f)))
 - 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)/(c
*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(
-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(
f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f))))
```

```
*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(
sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))) + sq
rt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(f))))*erf(1
/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2} \sinh^3(d+ex)\,dx = \int f^{a+cx^2} \sinh^3{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2} \sinh^3(d+ex)\,dx = \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{3e}{2\sqrt{-c\log(f)}}\right) e^{\left(3d - \frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} - \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{e}{2\sqrt{-c\log(f)}}\right) e^{\left(d - \frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} + \frac{3\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x + \frac{e}{2\sqrt{-c\log(f)}}\right) e^{\left(-d - \frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}} - \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x + \frac{3e}{2\sqrt{-c\log(f)}}\right) e^{\left(-3d - \frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d - 9
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*
x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/1
6*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e
^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x +
3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.97

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{3\,e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}} + \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x-\frac{3\,e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*a
*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqr
t(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2
 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f
) - e^2)/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/
(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^3(d+ex)\,dx = \int f^{c\,x^2+a}\,\sinh(d+e\,x)^3\,dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + e*x)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x)^3, x)
```

3.351 $\int f^{a+cx^2} \sinh(d+fx^2)\, dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int f^{a+cx^2} \sinh(d+fx^2)\, dx = -\frac{e^{-d} f^a \sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d} f^a \sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] -1/4*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)+1/4*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5623, 2325, 2236, 2235}

$$\int f^{a+cx^2} \sinh(d+fx^2)\, dx = \frac{\sqrt{\pi} e^{d} f^a \mathrm{erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}} - \frac{\sqrt{\pi} e^{-d} f^a \mathrm{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(E^d*Sqrt[f - c*Log[f]]) + (E
^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int e^{-d+a\log(f)-x^2(f-c\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+a\log(f)+x^2(f+c\log(f))}\,dx\\
&= -\frac{e^{-d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d}f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx=\frac{1}{4}f^a\sqrt{\pi}\left(-\frac{\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)(\cosh(d)-\sinh(d))}{\sqrt{f-c\log(f)}}+\frac{\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)(\cosh(d)+\sinh(d))}{\sqrt{f+c\log(f)}}\right)$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c
*Log[f]]) + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log
[f]]))/4
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{f^{a} {\mathrm e}^{d} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-f}\, x \right)}{4 \sqrt{-c \ln \left(f \right)-f}}-\frac{f^{a} {\mathrm e}^{-d} \sqrt{\pi}\, \operatorname{erf}\left(x \sqrt{f -c \ln \left(f \right)}\right)}{4 \sqrt{f -c \ln \left(f \right)}}$	70

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*f^a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)-1/4*f^
a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(63) = 126.

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.80

$$\int f^{a+cx^2} \sinh \left(d + fx^2\right) \, dx$$
$$= \frac{\left(\sqrt{\pi}(c \log{(f)} + f) \cosh{(a \log{(f)} - d)} + \sqrt{\pi}(c \log{(f)} + f) \sinh{(a \log{(f)} - d)}\right) \sqrt{-c \log{(f)} + f} \operatorname{erf}\left(\sqrt{-c\right.}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f)*
sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) - (sqrt(
pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*log(
f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2 - f^
2)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sinh\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx=-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{4\sqrt{-c\log\left(f\right)+f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^{d}}{4\sqrt{-c\log\left(f\right)-f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1
/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx=-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{(a\log(f)+d)}}{4\sqrt{-c\log\left(f\right)-f}}+\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{(a\log(f)-d)}}{4\sqrt{-c\log\left(f\right)+f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f) -
 f) + 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*log
(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\sinh\left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2),x)
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2), x)
```

3.352 $\int f^{a+cx^2} \sinh^2(d+fx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 128

$$\int f^{a+cx^2}\sinh^2(d+fx^2)\,dx = -\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-2d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}} + \frac{e^{2d}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{2f+c\log(f)}\right)}{8\sqrt{2f+c\log(f)}}$$

```
[Out] -1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*f^a*e
rf(x*(2*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d
)*f^a*erfi(x*(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5623, 2235, 2325, 2236}

$$\int f^{a+cx^2}\sinh^2(d+fx^2)\,dx = \frac{\sqrt{\pi}e^{-2d}f^a\,\mathrm{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}} + \frac{\sqrt{\pi}e^{2d}f^a\,\mathrm{erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}} - \frac{\sqrt{\pi}f^a\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
f^a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]])
+ (E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f
]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= -\frac{f^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int e^{-2d+a\log(f)-x^2(2f-c\log(f))}\,dx\\
&\quad+\frac{1}{4}\int e^{2d+a\log(f)+x^2(2f+c\log(f))}\,dx
\end{aligned}$$

$$= -\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-2d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}} + \frac{e^{2d}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{2f+c\log(f)}\right)}{8\sqrt{2f+c\log(f)}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx$$
$$= \frac{f^a\sqrt{\pi}\left(\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(8f^2-2c^2\log^2(f)\right)+\sqrt{c}\sqrt{\log(f)}\left(\text{erf}\left(x\sqrt{2f-c\log(f)}\right)\sqrt{2f-c\log(f)}(2f\right.\right.}{8\sqrt{c}\sqrt{\log(}}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(8*f^2 - 2*c^2*Log[f]^2) + Sqrt
[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c
*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f - c*L
og[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[
f]]*(-4*f^2 + c^2*Log[f]^2))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{f^a e^{-2d}\sqrt{\pi}\,\text{erf}\left(x\sqrt{2f-c\ln(f)}\right)}{8\sqrt{2f-c\ln(f)}}+\frac{f^a e^{2d}\sqrt{\pi}\,\text{erf}\left(\sqrt{-c\ln(f)-2f}\,x\right)}{8\sqrt{-c\ln(f)-2f}}-\frac{f^a\sqrt{\pi}\,\text{erf}\left(\sqrt{-c\ln(f)}\,x\right)}{4\sqrt{-c\ln(f)}}$	101

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*f^a*exp(-2*d)*Pi^(1/2)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+1
/8*f^a*exp(2*d)*Pi^(1/2)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)-1
/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.98

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx =$$
$$-\frac{\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\cosh\left(a\log\left(f\right)-2\,d\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\sinh\left(a\log\left(f\right)-2\,d\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(p
i)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2*f
)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(f) +
 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) - 2*(sqrt(pi)*(c^
2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(a
*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*c*f^2*l
og(f))
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{(2\,d)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{(-2\,d)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}-\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)}x\right)}{4\,\sqrt{-c\log\left(f\right)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f)
 + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f) +
2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{\left(a\log\left(f\right)+2\,d\right)}}{8\sqrt{-c\log\left(f\right)-2\,f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{\left(a\log\left(f\right)-2\,d\right)}}{8\sqrt{-c\log\left(f\right)+2\,f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/8*s
qrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f) + 2
*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^2\,dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2)^2, x)
```

3.353 $\int f^{a+cx^2} \sinh^3(d+fx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\frac{3e^{-d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{f+c\log(f)}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{3f+c\log(f)}\right)}{16\sqrt{3f+c\log(f)}}$$

```
[Out] 3/16*f^a*erf(x*(f-c*ln(f))^(1/2))*Pi^(1/2)/exp(d)/(f-c*ln(f))^(1/2)-1/16*f^
a*erf(x*(3*f-c*ln(f))^(1/2))*Pi^(1/2)/exp(3*d)/(3*f-c*ln(f))^(1/2)-3/16*exp
(d)*f^a*erfi(x*(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d)*
f^a*erfi(x*(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{5623, 2325, 2236, 2235\}$

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx = \frac{3\sqrt{\pi}e^{-d}f^a\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}} - \frac{\sqrt{\pi}e^{-3d}f^a\operatorname{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}} - \frac{3\sqrt{\pi}e^{d}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}} + \frac{\sqrt{\pi}e^{3d}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+3f}\right)}{16\sqrt{c\log(f)+3f}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) - (f
^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]])
- (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]) +
 (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*Log[f
]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)\,dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+cx^2}\,dx\\
&\quad+\frac{3}{8}\int e^{-d-fx^2}f^{a+cx^2}\,dx-\frac{3}{8}\int e^{d+fx^2}f^{a+cx^2}\,dx\\
&= -\left(\frac{1}{8}\int e^{-3d+a\log(f)-x^2(3f-c\log(f))}\,dx\right)+\frac{1}{8}\int e^{3d+a\log(f)+x^2(3f+c\log(f))}\,dx\\
&\quad+\frac{3}{8}\int e^{-d+a\log(f)-x^2(f-c\log(f))}\,dx-\frac{3}{8}\int e^{d+a\log(f)+x^2(f+c\log(f))}\,dx\\
&= \frac{3e^{-d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d}f^a\sqrt{\pi}\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}\\
&\quad-\frac{3e^{d}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{f+c\log(f)}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d}f^a\sqrt{\pi}\text{erfi}\left(x\sqrt{3f+c\log(f)}\right)}{16\sqrt{3f+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.59

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx$$
$$= \frac{f^a\sqrt{\pi}\left(3\text{erf}\left(x\sqrt{f-c\log(f)}\right)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\sinh}{}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*
f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*Lo
g[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[
f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x*Sqr
t[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) -
Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d]
 + Sinh[3*d])))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

method	result
risch	$\frac{f^{a}\mathrm{e}^{3d}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c\ln(f)-3f}\,x\right)}{16\sqrt{-c\ln(f)-3f}}-\frac{f^{a}\mathrm{e}^{-3d}\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{3f-c\ln(f)}\right)}{16\sqrt{3f-c\ln(f)}}+\frac{3f^{a}\mathrm{e}^{-d}\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{f-c\ln(f)}\right)}{16\sqrt{f-c\ln(f)}}-\frac{3f^{a}\mathrm{e}^{d}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c\ln}\right.}{16\sqrt{-c\ln(f)-}}$

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*f^a*exp(3*d)*Pi^(1/2)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*x)
-1/16*f^a*exp(-3*d)*Pi^(1/2)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))
+3/16*f^a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))-3/16*
f^a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(135) = 270$.

Time = 0.29 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.88

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx$$
$$=\frac{\left(\sqrt{\pi}\left(c^3\log\left(f\right)^3+3\,c^2f\log\left(f\right)^2-cf^2\log\left(f\right)-3\,f^3\right)\cosh\left(a\log\left(f\right)-3\,d\right)+\sqrt{\pi}\left(c^3\log\left(f\right)^3+3\,c^2f\log\left(\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(
f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log(f)
+ 3*f)*x) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9
*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^
2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f
) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9
*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^
2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f
) - f)*x) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f
^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^
2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*erf(sqrt(-c*l
og(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+d)**3,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{(3\,d)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}+\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x\right)e^{(-d)}}{16\,\sqrt{-c\log\left(f\right)+f}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x\right)e^{(-3\,d)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}-\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x\right)e^{d}}{16\,\sqrt{-c\log\left(f\right)-f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*f
) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f)
 - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f) +
 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{\left(a\log\left(f\right)+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}} + \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{\left(a\log\left(f\right)+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{\left(a\log\left(f\right)-d\right)}}{16\,\sqrt{-c\log\left(f\right)+f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+3\,f}x\right)e^{\left(a\log\left(f\right)-3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*log
(f) - 3*f) + 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqr
t(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) -
d)/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^(a*l
og(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^3\left(d+fx^2\right)\,dx = \int f^{c\,x^2+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^3 \,d x$$

```
[In] int(f^(a + c*x^2)*sinh(d + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + f*x^2)^3, x)
```

3.354 $\int f^{a+cx^2} \sinh\left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=-\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] -1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi
(1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\,\mathrm{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\,\mathrm{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[
f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] + (E^(d - e^2/(4*(f + c*Lo
```

```
g[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+cx^2}\right)dx \\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+cx^2}\,dx \\
&= -\left(\frac{1}{2}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx\right)+\frac{1}{2}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,dx \\
&= -\left(\frac{1}{2}\left(e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right) \\
&\quad +\frac{1}{2}\left(e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\right)\int \exp\left(\frac{(e+2x(f+c\log(f)))^2}{4(f+c\log(f))}\right)dx
\end{aligned}$$

$$= -\frac{e^{-d+\frac{e^2}{4f-4c\log(f)}} f^a \sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d-\frac{e^2}{4(f+c\log(f))}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int f^{a+cx^2} \sinh\left(d+ex+fx^2\right) dx$$

$$= \frac{e^{-\frac{e^2}{4(f+c\log(f))}} f^a \sqrt{\pi}\left(-e^{\frac{e^2 f}{2f^2-2c^2\log^2(f)}} \operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}(\cosh(d)-\sinh(d)) + \operatorname{erfi}\left(\frac{e+2fx+2cx\log(f)}{2\sqrt{f+c\log(f)}}\right)\cdots\right)}{4\sqrt{f-c\log(f)}\sqrt{f+c\log(f)}}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*
x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d])) +
 Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]
*(Cosh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sqr
t[f + c*Log[f]])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{\frac{4d\ln(f)c+4df-e^2}{4f+4c\ln(f)}}}{4\sqrt{-c\ln(f)-f}} - \frac{\operatorname{erf}\left(x\sqrt{f-c\ln(f)}+\frac{e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{4d\ln(f)c-4df+e^2}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}$

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2)
*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))-1/4*erf(x*(f-c*l
n(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/
4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(119) = 238$.

Time = 0.31 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.30

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\left(\sqrt{\pi}(c\log\left(f\right)+f)\cosh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\,(cd+af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\,(cd+}{4\,(c\log(f)-f)}\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c
*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*a*c
*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*lo
g(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f)
- f)) - (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f + 4
*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(1/4*(4*
a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c
*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(
f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d),x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(
d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*f^a*erf(sqrt
(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) -
f))/sqrt(-c*log(f) + f)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4
*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f
))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e
/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^
2 + 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+a}\,\mathrm{sinh}\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2),x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2), x)
```

3.355 $\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=-\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+x(2f+c\log(f))}{\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

```
[Out] -1/4*f^a*erfi(x*c^(1/2)*ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(-
2*d+e^2/(2*f-c*ln(f)))*f^a*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^
(1/2)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*erfi((e+x*(2*f
+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{5623, 2235, 2325, 2266, 2236\}$

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\operatorname{erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}} + \frac{\sqrt{\pi}f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\operatorname{erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}} - \frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(Sqrt[c]*Sqrt[Log[f]]) + (
E^(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/S
qrt[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log
[f]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8*
Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}f^{a+cx^2}+\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+cx^2}\right)\,dx\\
&= \frac{1}{4}\int e^{-2d-2ex-2fx^2}f^{a+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex+2fx^2}f^{a+cx^2}\,dx-\frac{1}{2}\int f^{a+cx^2}\,dx\\
&= -\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{1}{4}\int \exp\left(-2d-2ex+a\log(f)-x^2(2f-c\log(f))\right)\,dx\\
&\quad+\frac{1}{4}\int \exp\left(2d+2ex+a\log(f)+x^2(2f+c\log(f))\right)\,dx\\
&= -\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{1}{4}\left(e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\right)\int \exp\left(\frac{(-2e+2x(-2f+c\log(f)))^2}{4(-2f+c\log(f))}\right)\,dx\\
&\quad+\frac{1}{4}\left(e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\right)\int \exp\left(\frac{(2e+2x(2f+c\log(f)))^2}{4(2f+c\log(f))}\right)\,dx\\
&= -\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}\\
&\quad+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+x(2f+c\log(f))}{\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$

$$=\frac{e^{\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\left(2e^{\frac{e^2}{-2f+c\log(f)}}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(4f^2-c^2\log^2(f)\right)-\sqrt{c}\sqrt{\log(f)}\left(\operatorname{erf}\left(\frac{e+2fx-cx\log(f)}{\sqrt{2f-c\log(f)}}\right)\sqrt{2\right.\right.}{}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] (E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(2*E^(e^2/(-2*f + c*Log[f]))*Erfi[Sq
rt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf[(e
 + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f + c*
```

```
Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2))*Erf
i[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqrt[2*f
+ c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f^2 + c^
2*Log[f]^2))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{2f-c\ln(f)}+\frac{e}{\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{2d\ln(f)c-4df+e^{2}}{c\ln(f)-2f}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\, x+\frac{e}{\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{2d\ln(f)c+4df-e^{2}}{2f+c\ln(f)}}}{8\sqrt{-c\ln(f)-2f}}-$

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f))^(1/2)*Pi
^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)-2
*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp((
2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))-1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf
((-c*ln(f))^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(155) = 310$.

Time = 0.28 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.31

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$= \frac{2\left(\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\cosh\left(a\log(f)\right)+\sqrt{\pi}\left(c^2\log(f)^2-4f^2\right)\sinh\left(a\log(f)\right)\right)\sqrt{-c\log(f)}\operatorname{erf}\left(\sqrt{-c\,l\right.}}{}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(
f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) - (sqr
t(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c
*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f)
)*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f) - 2*f)
))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + 2*f)
/(c*log(f) - 2*f)) - (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((a*c*log(
f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^
2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)
*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f) + 2*f*x +
 e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*c*f^2*log(f)
)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log(f)-2f}x-\frac{e}{\sqrt{-c\log(f)-2f}}\right)e^{\left(2d-\frac{e^2}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}} + \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log(f)+2f}x+\frac{e}{\sqrt{-c\log(f)+2f}}\right)e^{\left(-2d-\frac{e^2}{c\log(f)-2f}\right)}}{8\sqrt{-c\log(f)+2f}} - \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^(
2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(
-c*log(f))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}\left(x+\frac{e}{c\log(f)+2\,f}\right)\right)e^{\left(\frac{ac\log(f)^2+2\,cd\log(f)+2\,af\log(f)-e^2+4\,df}{c\log(f)+2\,f}\right)}}{8\sqrt{-c\log\left(f\right)-2\,f}}-\frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}\left(x-\frac{e}{c\log(f)-2\,f}\right)\right)e^{\left(\frac{ac\log(f)^2-2\,cd\log(f)-2\,af\log(f)-e^2+4\,df}{c\log(f)-2\,f}\right)}}{8\sqrt{-c\log\left(f\right)+2\,f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf
(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c*d*
log(f) + 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f
) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*e^((a
*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - 2*f))/
sqrt(-c*log(f) + 2*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\,{\mathrm{sinh}\left(f\,x^2+e\,x+d\right)}^2\,dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

3.356 $\int f^{a+cx^2}\sinh^3(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 300

$$\int f^{a+cx^2}\sinh^3(d+ex+fx^2)\,dx=\frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d-\frac{9e^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

```
[Out] 3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(e+2*x*(f-c*ln(f)))/(f-c*ln(f)
)^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))*f
^a*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(
f))^(1/2)-3/16*exp(d-1/4*e^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+2*x*(f+c*ln(f)))/
(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9/4*e^2/(3*f+c*l
n(f)))*f^a*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(
3*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\frac{3\sqrt{\pi}f^ae^{\frac{e^2}{4f-4c\log(f)}-d}\text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi}f^ae^{\frac{9e^2}{12f-4c\log(f)}-3d}\text{erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^ae^{d-\frac{e^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^ae^{3d-\frac{9e^2}{4(c\log(f)+3f)}}\text{erfi}\left(\frac{2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

```
[In] Int[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f])
)/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (E^(-3*d + (9*e^2)/(12
*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f
 - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) - (3*E^(d - e^2/(4*(f + c*Log[f])
))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*
Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f +
 c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3(d+ex+fx^2)}f^{a+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3(d+ex+fx^2)\right)f^{a+cx^2}\right. \\
&\qquad -\frac{3}{8}\exp\left(4d+4ex+4fx^2-3(d+ex+fx^2)\right)f^{a+cx^2} \\
&\qquad \left.+\frac{1}{8}\exp\left(6d+6ex+6fx^2-3(d+ex+fx^2)\right)f^{a+cx^2}\right)dx \\
&= -\left(\frac{1}{8}\int e^{-3(d+ex+fx^2)}f^{a+cx^2}\,dx\right) \\
&\quad +\frac{1}{8}\int \exp\left(6d+6ex+6fx^2-3(d+ex+fx^2)\right)f^{a+cx^2}\,dx \\
&\quad +\frac{3}{8}\int \exp\left(2d+2ex+2fx^2-3(d+ex+fx^2)\right)f^{a+cx^2}\,dx \\
&\quad -\frac{3}{8}\int \exp\left(4d+4ex+4fx^2-3(d+ex+fx^2)\right)f^{a+cx^2}\,dx \\
&= -\left(\frac{1}{8}\int \exp\left(-3d-3ex+a\log(f)-x^2(3f-c\log(f))\right)dx\right) \\
&\quad +\frac{1}{8}\int \exp\left(3d+3ex+a\log(f)+x^2(3f+c\log(f))\right)dx \\
&\quad +\frac{3}{8}\int e^{-d-ex+a\log(f)-x^2(f-c\log(f))}\,dx-\frac{3}{8}\int e^{d+ex+a\log(f)+x^2(f+c\log(f))}\,dx \\
&= \frac{1}{8}\left(3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx \\
&\quad -\frac{1}{8}\left(e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(-3e+2x(-3f+c\log(f)))^2}{4(-3f+c\log(f))}\right)dx \\
&\quad -\frac{1}{8}\left(3e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\right)\int \exp\left(\frac{(e+2x(f+c\log(f)))^2}{4(f+c\log(f))}\right)dx \\
&\quad +\frac{1}{8}\left(e^{3d-\frac{9e^2}{4(3f+c\log(f))}}f^a\right)\int \exp\left(\frac{(3e+2x(3f+c\log(f)))^2}{4(3f+c\log(f))}\right)dx
\end{aligned}$$

$$= \frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}} - \frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}} - \frac{3e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}} + \frac{e^{3d-\frac{9e^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

Mathematica [A] (verified)

Time = 4.32 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.60

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$$

$$= \frac{e^{-\frac{1}{4}e^2\left(\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}e^2\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(9f^3+\right.}{}$$

```
[In] Integrate[f^(a + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3*
f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*S
qrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*
(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f + c*
Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f])/(2*
Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log
[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(3*f + c
*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f +
 c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) - E^(e^2/(4*(f + c*Log[f]))
)*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f]
)*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*((f + c*Log[
f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f
]^4))
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{\frac{3d\ln(f)c+9df-\frac{9e^2}{4}}{3f+c\ln(f)}}}{16\sqrt{-c\ln(f)-3f}} - \frac{\operatorname{erf}\left(x\sqrt{3f-c\ln(f)}+\frac{3e}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{3\left(4d\ln(f)c-12\right.}{4(c\ln(f)-}}}{16\sqrt{3f-c\ln(f)}}$

```
[In] int(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))/(-c*ln(f)-3*f
)^(1/2)*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))-1/16
*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2)*P
i^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(c*ln(f)-3*f))+3/16*erf(x*(
f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*ex
p(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))+3/16*erf(-(-c*ln(f)-f)^(1/2)*x+
1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f
)*c+4*d*f-e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(253) = 506$.

Time = 0.36 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.83

$$\int f^{a+cx^2} \sinh^3\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(f)
- 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)
*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(
f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e)*sqrt
(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log
(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*
(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)
^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*(c*
d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2*c*x*log(f)
 - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)*(c^3*log(f)
^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^
2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3
- c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 +
 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2
*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)
*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(1/4*(4*a*c*l
og(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(
pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(1/4*(4*a*
c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f) + 3*f)))*sqr
t(-c*log(f) - 3*f)*erf(1/2*(2*c*x*log(f) + 6*f*x + 3*e)*sqrt(-c*log(f) - 3*
f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+a)*sinh(f*x**2+e*x+d)**3,x)
[Out] Integral(f**(a + c*x**2)*sinh(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

$$\begin{aligned}&\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx\\&=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x-\frac{3\,e}{2\,\sqrt{-c\log(f)-3\,f}}\right)e^{\left(3\,d-\frac{9\,e^2}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}\\&-\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{e}{2\,\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}\\&+\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x+\frac{e}{2\,\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}\\&-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x+\frac{3\,e}{2\,\sqrt{-c\log(f)+3\,f}}\right)e^{\left(-3\,d-\frac{9\,e^2}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}\end{aligned}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f)
)*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*
f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^2/(
c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) +
 f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqrt(-c*
log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/sqrt(-c
*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.17

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{3\,e}{c\log(f)+3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)+12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\sqrt{-c\log\left(f\right)-3\,f}}$$
$$+\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{16\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{16\sqrt{-c\log\left(f\right)+f}}$$
$$+\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x-\frac{3\,e}{c\log(f)-3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\sqrt{-c\log\left(f\right)+3\,f}}$$

```
[In] integrate(f^(c*x^2+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f)))
*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 9*e^2 + 36*d*f)/(
c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) +
4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16*sqrt
(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4*a*c*l
og(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - f))/sqrt(-
c*log(f) + f) + 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x - 3*e/(c*
log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f) - 9*
e^2 + 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\sinh\left(f\,x^2+e\,x+d\right)^3\,dx$$

```
[In] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

3.357 $\int f^{a+bx+cx^2} \sinh(d+ex)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int f^{a+bx+cx^2} \sinh(d+ex)\,dx = \frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] -1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(f
))^2/c/ln(f))*f^a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^
(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5623, 2325, 2266, 2235}

$$\int f^{a+bx+cx^2} \sinh(d+ex)\,dx = \frac{\sqrt{\pi} f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\mathrm{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi} f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\mathrm{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f] - 2
*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(d -
(e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x*Log[
f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)\,dx\right)\\
&\quad+\frac{1}{2}\int \exp\left(d+a\log(f)+cx^2\log(f)+x(e+b\log(f))\right)\,dx\\
&= -\left(\frac{1}{2}\left(e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)\,dx\right)\\
&\quad+\frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+b\log(f)+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&= \frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$$
$$=\frac{e^{-\frac{e(e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-e^{\frac{be}{c}}\operatorname{erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x],x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(
2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d])) + Erfi[(e + (b + 2*c*x)*Log[f
])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*
b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\mathrm{e}^{-\frac{2\ln(f)be-4d\ln(f)c+e^2}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}+\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\mathrm{e}^{\frac{2\ln(f)be-}{4\ln}}}{4\sqrt{-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(f))^(
1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(
f)/c)+1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1/2))/(-c*ln(
f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)*b*e-4*d*ln(f)*c-e^2)
/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(121) = 242.

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.72

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx =$$
$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) + e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(p
i)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log
(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log
(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx = \int f^{a+bx+cx^2}\sinh{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d),x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx = \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log{(f)}}x - \frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log{(f)}}} - \frac{\sqrt{\pi}f^a \operatorname{erf}\left(\sqrt{-c\log{(f)}}x - \frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log{(f)}}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f))
)*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(
b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx$$
$$=\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)+e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*e^
(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(
c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (
b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*l
og(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\,\sinh(d+e\,x)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x), x)
```

3.358 $\int f^{a+bx+cx^2} \sinh^2(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\int f^{a+bx+cx^2} \sinh^2(d+ex)\,dx = -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-2*e+b*ln(f)+2*c*x*
ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)+1/8*exp(2*d-1/4*(2
*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(2*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)
*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used

$= \{5623, 2266, 2235, 2325\}$

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = -\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^
a*Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/
(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*S
qrt[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(8*
Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}\right)dx\\
&= \frac{1}{4}\int e^{-2d-2ex}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2ex}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2d+a\log(f)+cx^2\log(f)-x(2e-b\log(f))\right)dx\\
&\quad+\frac{1}{4}\int \exp\left(2d+a\log(f)+cx^2\log(f)+x(2e+b\log(f))\right)dx-\frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx\\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{1}{4}\left(e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-2e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&\quad+\frac{1}{4}\left(e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(2e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$$
$$=\frac{e^{-\frac{e(e+b\log(f))}{c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(-2e^{\frac{e(e+b\log(f))}{c\log(f)}}\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{2be}{c}}\mathrm{erfi}\left(\frac{-2e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^2,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*(-2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b
+ 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*x
)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e + (
b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*S
qrt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-2e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^2}{4c}}\mathrm{e}^{\frac{\ln(f)be-2d\ln(f)c-e^2}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}f^{-\frac{b^2}{4c}}\mathrm{e}^{-\frac{\ln(f)be-2d\,\mathrm{l}}{\ln(f}}}{8\sqrt{-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^(1/2))/(-c*ln(f))
^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((ln(f)*b*e-2*d*ln(f)*c-e^2)/ln(f)/c)
-1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(f))
^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(ln(f)*b*e-2*d*ln(f)*c+e^2)/ln(f)/c
)+1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+
1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(167) = 334.

Time = 0.28 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.57

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)-\sqrt{-c\log(}}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(p
i)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c
) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 4
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*lo
g(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1
/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log(f))) +
sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f)
)/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))/(c*log(f)
)))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \int f^{a+bx+cx^2}\sinh^2\left(d+ex\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+2e}{2\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-2e}{2\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{(b\log(f)-2e)^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(f
)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(
pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^(-2
*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1
/4*b^2/c))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx$$
$$= \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)-4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+2e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)+4be\log(f)+4e^2}{4c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*
a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
 + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8
*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(-1/4*(
b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)/(c*log
(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^2(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\,\sinh(d+e\,x)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^2, x)
```

3.359 $\int f^{a+bx+cx^2} \sinh^3(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 315

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{(3e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[Out] 3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-e+b*ln(f)+2*c*x*ln(f)
)/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-1/4*(3*e-
b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(-3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(
1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f))*f^
a*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln
(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*erfi(1/2*(3*e+b*ln(
f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))*Pi^(1/2)/c^(1/2)/ln(f)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5623, 2325, 2266, 2235}

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = -\frac{3\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi}f^a e^{3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
- 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(
-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f] -
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (3*E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3*d - (
3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*c*x*L
og[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}-\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}\right.\\
&\qquad\left.+\frac{1}{8}e^{3d+3ex}f^{a+bx+cx^2}\right) dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3ex}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3ex}f^{a+bx+cx^2}\,dx\\
&\quad+\frac{3}{8}\int e^{-d-ex}f^{a+bx+cx^2}\,dx-\frac{3}{8}\int e^{d+ex}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{8}\int \exp\left(-3d+a\log(f)+cx^2\log(f)-x(3e-b\log(f))\right)dx\right)\\
&\quad+\frac{1}{8}\int \exp\left(3d+a\log(f)+cx^2\log(f)+x(3e+b\log(f))\right)dx\\
&\quad+\frac{3}{8}\int \exp\left(-d+a\log(f)+cx^2\log(f)-x(e-b\log(f))\right)dx\\
&\quad-\frac{3}{8}\int \exp\left(d+a\log(f)+cx^2\log(f)+x(e+b\log(f))\right)dx\\
&= \frac{1}{8}\left(3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&\quad-\frac{1}{8}\left(e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(-3e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx\\
&\quad-\frac{1}{8}\left(3e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\right)\int e^{\frac{(e+b\log(f)+2cx\log(f))^2}{4c\log(f)}}\,dx\\
&\quad+\frac{1}{8}\left(e^{3d-\frac{(3e+b\log(f))^2}{4c\log(f)}}f^a\right)\int \exp\left(\frac{(3e+b\log(f)+2cx\log(f))^2}{4c\log(f)}\right)dx
\end{aligned}
$$

$$= -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\text{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\text{erfi}\left(\frac{3e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{3e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\text{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{(3e+b\log(f))^2}{4c\log(f)}} f^a\sqrt{\pi}\text{erfi}\left(\frac{3e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx$$
$$= \frac{e^{-\frac{3e(3e+2b\log(f))}{4c\log(f)}} f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(-3e^{\frac{e(2e+b\log(f))}{c\log(f)}}\text{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e(e+b\log(f))}{c\log(f)}}\text{erfi}\left(\frac{-e+(b+2c}{2\sqrt{c}\sqrt{}}\right.\right.\right.}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x]^3,x]
```

```
[Out] (f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(-3*E^((e*(2*e + b*Log[f]))/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) - E^((3*b*e)/c)*Erfi[(-3*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d]))/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\text{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{-\frac{3\left(2\ln(f)be-4d\ln(f)c+3e^2\right)}{4\ln(f)c}}}{16\sqrt{-c\ln(f)}} + \frac{\text{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{\frac{3\ln(f}{2}}}{16\sqrt{-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(f)
)^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-3/4*(2*ln(f)*b*e-4*d*ln(f)*c+3*e^2
)/ln(f)/c)+1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(-c*ln(f))^(1/2))
/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*ln(f)*b*e-4*d*ln(f
)*c-3*e^2)/ln(f)/c)-3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))
^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)*b*e-4
*d*ln(f)*c-e^2)/ln(f)/c)+3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*l
n(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*ln(f)
*b*e-4*d*ln(f)*c+e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(247) = 494$.

Time = 0.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.67

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = -\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)^2+9e^2-6(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)^2+9e^2-6(2cd-be)\log}{4c\log(f)}\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
- 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*l
og(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)
*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) - 3*sqrt(-c*log(f))*(sqrt(pi)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f)))
 + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f
))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*log(f))
) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2
*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(
f) - e)*sqrt(-c*log(f))/(c*log(f))) - sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(
(b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt
(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c
*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/
(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx=\int f^{a+bx+cx^2}\sinh^3{(d+ex)}\,dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(e*x+d)**3,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-3e}{2\sqrt{-c\log(f)}}\right)e^{\left(-3d-\frac{(b\log(f)-3e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log(
f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^(d
- 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(
sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b*log(
f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f
))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f) - 3*e)
^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.08

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)-3\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+12\,cd\log(f)-6\,be\log(f)+9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}} + \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)+e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b\log(f)+3\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-12\,cd\log(f)+6\,be\log(f)+9\,e^2}{4\,c\log(f)}\right)}}{16\,\sqrt{-c\log\left(f\right)}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(e*x+d)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f))))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) + 9*
e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(
2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 +
4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(
pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^
2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f))
)/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f)
 + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f)
 + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^3(d+ex)\,dx = \int f^{c\,x^2+b\,x+a}\,{\sinh\left(d+e\,x\right)}^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x)^3, x)
```

3.360 $\int f^{a+bx+cx^2} \sinh (d+fx^2) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int f^{a+bx+cx^2} \sinh (d+fx^2) \, dx = \frac{e^{-d+\frac{b^2 \log^2(f)}{4f-4c\log(f)}} f^a \sqrt{\pi} \text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d-\frac{b^2 \log^2(f)}{4(f+c\log(f))}} f^a \sqrt{\pi} \text{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] 1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(f)
))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*b^2*ln(f)^2/
(f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(
1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2} \sinh (d+fx^2) \, dx = \frac{\sqrt{\pi} f^a e^{\frac{b^2 \log^2(f)}{4f-4c\log(f)}-d} \text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{\sqrt{\pi} f^a e^{d-\frac{b^2 \log^2(f)}{4(c\log(f)+f)}} \text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

[Out] (E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]}, Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z, x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+bx+cx^2}\right)\,dx\\
&= -\left(\frac{1}{2}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)\,dx\right)\\
&\quad+\frac{1}{2}\int \exp\left(d+a\log(f)+bx\log(f)+x^2(f+c\log(f))\right)\,dx
\end{aligned}$$

$$= -\left(\frac{1}{2}\left(e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right) + \frac{1}{2}\left(e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(f+c\log(f)))^2}{4(f+c\log(f))}\right)dx$$

$$= \frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.16

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)dx$$

$$= \frac{e^{-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(-e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\operatorname{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{2fx+(b+2cx}{2\sqrt{f+c\log}}\right.\right.}{4\sqrt{f-c\log(f)}\sqrt{f+c\log(f)}}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2],x]
```

```
[Out] (f^a*Sqrt[Pi]*(-(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] -
Sinh[d])) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[
f - c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*(f + c*Log[f]))
)*Sqrt[f - c*Log[f]]*Sqrt[f + c*Log[f]])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{b^2\ln(f)^2-4d\ln(f)c-4df}{4(f+c\ln(f))}}}{4\sqrt{-c\ln(f)-f}} + \frac{\operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{b^2\ln(f)^2+4}{4(c\ln}}}{4\sqrt{f-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)
^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))+1
/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2
)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)/(c*ln(f)-f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(131) = 262.

Time = 0.30 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.11

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx = \frac{\left(\sqrt{\pi}(c\log\left(f\right)+f\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4\,df+4\,(cd+af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-4}{4\,(c\log(}\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="fricas")
[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4
*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4*((
b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt
(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(
c*log(f) - f)) - (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2
 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*
sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f)
 + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*lo
g(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx = \int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d),x)
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log\left(f\right)-f}} - \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-f)}-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) - 1/4*sqrt
(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e^(-
1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)-4af\log(f)-4df}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log(f)-f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)+4af\log(f)-4df}{4(c\log(f)-f)}\right)}}{4\sqrt{-c\log(f)+f}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))
*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d
*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log
(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log
(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
 + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,\mathrm{sinh}\left(f\,x^2+d\right)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2),x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2), x)
```

3.361 $\int f^{a+bx+cx^2}\sinh^2(d+fx^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 225

$$\int f^{a+bx+cx^2}\sinh^2(d+fx^2)\,dx=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

```
[Out] -1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(
f)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(1/2)+1/8
*exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(b*ln(f)+2*x*(2*f+c*ln(f
)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

$= \{5623, 2266, 2235, 2325, 2236\}$

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}f^a e^{\frac{b^2\log^2(f)}{8f-4c\log(f)}-2d}\operatorname{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}} + \frac{\sqrt{\pi}f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}} - \frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*
f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])
])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*
f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f]]
)])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
```

```
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}f^{a+bx+cx^2}+\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}\right)\,dx\\
&= \frac{1}{4}\int e^{-2d-2fx^2}f^{a+bx+cx^2}\,dx+\frac{1}{4}\int e^{2d+2fx^2}f^{a+bx+cx^2}\,dx-\frac{1}{2}\int f^{a+bx+cx^2}\,dx\\
&= \frac{1}{4}\int \exp\left(-2d+a\log(f)+bx\log(f)-x^2(2f-c\log(f))\right)\,dx\\
&\quad+\frac{1}{4}\int \exp\left(2d+a\log(f)+bx\log(f)+x^2(2f+c\log(f))\right)\,dx-\frac{1}{2}f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx\\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}\\
&\quad+\frac{1}{4}\left(e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-2f+c\log(f)))^2}{4(-2f+c\log(f))}\right)\,dx\\
&\quad+\frac{1}{4}\left(e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(2f+c\log(f)))^2}{4(2f+c\log(f))}\right)\,dx\\
&= -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}\\
&\quad+\frac{e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\frac{1}{8}f^a\sqrt{\pi}\left(-\frac{2f^{-\frac{b^2}{4c}}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}}\right.$$
$$-\frac{e^{-\frac{b^2\log^2(f)}{8f+4c\log(f)}}\left(e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\mathrm{erf}\left(\frac{4fx-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2f+c\log(f))(\cosh(2d)-\sinh(2d))+e\right.}{-4f^2+c^2\log^2(f)}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^2,x]
```

```
[Out] (f^a*Sqrt[Pi]*((-2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*E
rf[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Log[
f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)*Lo
g[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh
[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Lo
g[f]^2))))/8
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+8d\ln(f)c-16df}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2}\ln(}{}}}{8\sqrt{-c\ln(f)-2f}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(
f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+8*d*ln(f)*c-16*d*f)/(c*ln(f)-2
*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-2*f)^(1/2))/(-c*
ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-8*d*ln(f)*c-16*d*f)/(2*
f+c*ln(f)))+1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f)
)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(185) = 370$.

Time = 0.32 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.07

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\cosh\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-16\,df+8\,(cd+af)\log(f)}{4\,(c\log(f)-2\,f)}\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log
(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*d +
 a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2
*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*
log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*
```

```
d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))/(c*log(
f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f))*sqrt
(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*co
sh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*sinh(-1/4
*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f
))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-2f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-2f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)+2f)}+2d\right)}}{8\sqrt{-c\log(f)-2f}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+2f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+2f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)-2f)}-2d\right)}}{8\sqrt{-c\log(f)+2f}} - \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
- 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f)
+ 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) + 2*f
) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/
(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-2\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)-8\,af\log(f)-16\,df}{4\,(c\log(f)+2\,f)}\right)}}{8\sqrt{-c\log\left(f\right)-2\,f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+2\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)-2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+8\,cd\log(f)+8\,af\log(f)-16\,df}{4\,(c\log(f)-2\,f)}\right)}}{8\sqrt{-c\log\left(f\right)+2\,f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2*
f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f) -
 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sq
rt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) - 2*f)
)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c)
)*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^2\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,{\mathrm{sinh}\left(f\,x^2+d\right)}^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^2, x)
```

3.362 $\int f^{a+bx+cx^2} \sinh^3\left(d+fx^2\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)dx = -\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}} + \frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}} - \frac{3e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}} + \frac{e^{3d-\frac{b^2\log^2(f)}{4(3f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

```
[Out] -3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(f-c*ln(
f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+b^2*ln(f)^
2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(
1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)-3/16*exp(d-1/4*b^2*ln(f)^2/(f+c*ln(f)))*
f^a*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi^(1/2)/(f+c*ln(
f))^(1/2)+1/16*exp(3*d-1/4*b^2*ln(f)^2/(3*f+c*ln(f)))*f^a*erfi(1/2*(b*ln(f)
+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=-\frac{3\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^ae^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\operatorname{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3\sqrt{\pi}f^ae^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^ae^{3d-\frac{b^2\log^2(f)}{4(c\log(f)+3f)}}\operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^
(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*
x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) -
(3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] +
2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]]) + (E^(
3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*
x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3d-3fx^2}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+bx+cx^2}-\frac{3}{8}e^{d+fx^2}f^{a+bx+cx^2}\right.\\
&\qquad\left.+\frac{1}{8}e^{3d+3fx^2}f^{a+bx+cx^2}\right)dx\\
&= -\left(\frac{1}{8}\int e^{-3d-3fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{8}\int e^{3d+3fx^2}f^{a+bx+cx^2}\,dx\\
&\quad+\frac{3}{8}\int e^{-d-fx^2}f^{a+bx+cx^2}\,dx-\frac{3}{8}\int e^{d+fx^2}f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{8}\int \exp\left(-3d+a\log(f)+bx\log(f)-x^2(3f-c\log(f))\right)dx\right)\\
&\quad+\frac{1}{8}\int \exp\left(3d+a\log(f)+bx\log(f)+x^2(3f+c\log(f))\right)dx\\
&\quad+\frac{3}{8}\int \exp\left(-d+a\log(f)+bx\log(f)-x^2(f-c\log(f))\right)dx\\
&\quad-\frac{3}{8}\int \exp\left(d+a\log(f)+bx\log(f)+x^2(f+c\log(f))\right)dx\\
&= \frac{1}{8}\left(3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\\
&\quad-\frac{1}{8}\left(e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(-3f+c\log(f)))^2}{4(-3f+c\log(f))}\right)dx\\
&\quad-\frac{1}{8}\left(3e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(f+c\log(f)))^2}{4(f+c\log(f))}\right)dx\\
&\quad+\frac{1}{8}\left(e^{3d-\frac{b^2\log^2(f)}{4(3f+c\log(f))}}f^a\right)\int \exp\left(\frac{(b\log(f)+2x(3f+c\log(f)))^2}{4(3f+c\log(f))}\right)dx
\end{aligned}$$

$$= -\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}} + \frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}} - \frac{3e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}} + \frac{e^{3d-\frac{b^2\log^2(f)}{4(3f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

Mathematica [A] (verified)

Time = 4.73 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx = \frac{e^{-\frac{b^2\log^2(f)(2f+c\log(f))}{2(f+c\log(f))(3f+c\log(f))}}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(9f^3}{}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*(3*E^((b^2*Log[f]^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1
) + (3*f + c*Log[f])^(-1)))/4)*Erf[(2*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[f -
 c*Log[f]])]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 -
c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) - (f - c*Log[f])*(E^((b^2*Log[f]^2*((3*f
- c*Log[f])^(-1) + (f + c*Log[f])^(-1) + (3*f + c*Log[f])^(-1)))/4)*Erf[(6*
f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3
*f^2 + 4*c*f*Log[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[
f])*(3*E^((b^2*Log[f]^2)/(12*f + 4*c*Log[f]))*Erfi[(2*f*x + (b + 2*c*x)*Log
[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] +
 Sinh[d]) - E^((b^2*Log[f]^2)/(4*(f + c*Log[f])))*Erfi[(6*f*x + (b + 2*c*x)
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cos
h[3*d] + Sinh[3*d])))))/(16*E^((b^2*Log[f]^2*(2*f + c*Log[f]))/(2*(f + c*Lo
g[f])*(3*f + c*Log[f])))*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-12d\ln(f)c-36df}{4(3f+c\ln(f))}}}{16\sqrt{-c\ln(f)-3f}}+\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\cdots}{\cdots}}}{16\sqrt{3f-c\ln(f)}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*f)^(1/2))/(-c*ln(
f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-12*d*ln(f)*c-36*d*f)/(3*f+
c*ln(f)))+1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f-c*ln(f))^(1/2))/
(3*f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+12*d*ln(f)*c-36*d*f)
/(c*ln(f)-3*f))-3/16*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2)
)/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)/(
c*ln(f)-f))+3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))/
(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f
+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(275) = 550.

Time = 0.30 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.64

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log(f)
 - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3
)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*lo
g(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log(f))*s
qrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*
log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*
f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*
f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(-1/4*((b^2 -
 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*l
```

```
og(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*log(f) - f)/(c*log(
f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^
3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/(c*l
og(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) +
3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d + a*f)*log(f))/
(c*log(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f
))*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*log(
f)^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+d)**3,x)
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\begin{aligned}
&\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx\\
&=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-3f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log(f)-3f}}\\
&\quad-\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log(f)-f}}\\
&\quad+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log(f)+f}}\\
&\quad-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+3f}\,x-\frac{b\log(f)}{2\sqrt{-c\log(f)+3f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)-3f)}-3d\right)}}{16\sqrt{-c\log(f)+3f}}
\end{aligned}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="maxima")
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*f)
```

```
 - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 3/16
*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))
*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(p
i)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3*f))*e^
(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)+3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}} + \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x+\frac{b\log(f)}{c\log(f)-3\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+12\,cd\log(f)+12\,af\log(f)-36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) + 3
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log(f
) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi)*erf(-1/
2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^
2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) + f))/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + b*lo
g(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f)
 + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/16*sqrt(pi
)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f)))*e^(-1/4
*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 36*d*f)/(
c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+d\right)^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3,x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + f*x^2)^3, x)
```

3.363 $\int f^{a+bx+cx^2} \sinh(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=-\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

```
[Out] -1/4*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)+1/4*exp(d-1/4*(e+b*ln
(f))^2/(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1
/2))*Pi^(1/2)/(f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\,\mathrm{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] -1/4*(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*
Log[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] +
(E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f
] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx \\
&= -\left(\frac{1}{2}\int e^{-d-ex-fx^2}f^{a+bx+cx^2}\,dx\right)+\frac{1}{2}\int e^{d+ex+fx^2}f^{a+bx+cx^2}\,dx \\
&= -\left(\frac{1}{2}\int \exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx\right) \\
&\quad+\frac{1}{2}\int \exp\left(d+a\log(f)+x(e+b\log(f))+x^2(f+c\log(f))\right)dx
\end{aligned}
$$

$$= -\left(\frac{1}{2}\left(e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int \exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx\right) + \frac{1}{2}\left(e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\right)\int \exp\left(\frac{(e+b\log(f)+2x(f+c\log(f)))^2}{4(f+c\log(f))}\right)dx$$

$$= -\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

Mathematica [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.57

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

$$= \frac{e^{-\frac{e^2+b^2\log^2(f)}{4(f+c\log(f))}}f^{a+\frac{bef}{-f^2+c^2\log^2(f)}}\sqrt{\pi}\left(-e^{\frac{f\left(e^2+b^2\log^2(f)\right)}{2\left(f^2-c^2\log^2(f)\right)}}f^{\frac{be}{2(f+c\log(f))}}\mathrm{erf}\left(\frac{e+2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(f+c\log$$

$$4\left(f^2\right.$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2],x]
```

```
[Out] (f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(-(E^((f*(e^2 + b^2*Log[f]^
2))/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[
f])*(Cosh[d] - Sinh[d])) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x + (
b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]
]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2
 - c^2*Log[f]^2))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\mathrm{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\,f^a\mathrm{e}^{-\frac{b^2\ln(f)^2+2\ln(f)be-4d\ln(f)c-4df+e^2}{4(f+c\ln(f))}}}{4\sqrt{-c\ln(f)-f}}+\frac{\mathrm{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{b\ln(f)-e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\,f}{4\sqrt{f-}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

[Out] -1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-f)^(1/2))/(-c*ln(f
)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c-4*d*f
+e^2)/(f+c*ln(f)))+1/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))
^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*
d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(139) = 278$.

Time = 0.30 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.25

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = \frac{\left(\sqrt{\pi}(c\log(f)+f)\cosh\left(-\frac{(b^2-4\,ac)\log(f)^2+e^2-4\,df+2\,(2\,cd-be+2\,af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log(f)+f)\sinh\left(-\frac{(b^2-4\,ac)\log(f)^2+e^2-4\,df+2\,(2\,cd-be+2\,af)\log(f)}{4\,(c\log(f)-f)}\right)\right)\cdots}{\cdots}$$

[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="fricas")

[Out] 1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d
f + 2(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) +
 f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*
f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x +
b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) - (sqrt(pi)*(c*log(f) -
f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f
)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2 - 4*a*c
)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + f)))
*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqrt(-c*log(f
) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)

Sympy [F]

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx = \int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)dx$$

[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d),x)

[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2), x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int f^{a+bx+cx^2} \sinh \left(d+ex+fx^2\right) \, dx$$
$$= \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4\,(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log\left(f\right)-f}} - \frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4\,(c\log(f)-f)}-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*log
(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f)
- 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-c*l
og(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f
)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

$$\int f^{a+bx+cx^2} \sinh \left(d+ex+fx^2\right) \, dx =$$
$$-\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}$$
$$+\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
+ f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f)
 - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sq
rt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f)))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + 4*a*
f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\sinh\left(f\,x^2+e\,x+d\right)\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2),x)
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2), x)
```

3.364 $\int f^{a+bx+cx^2} \sinh^2(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 239

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{2e-b\log(f)+2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{8f+4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2e+b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

```
[Out] -1/4*f^(a-1/4*b^2/c)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))*Pi^(1/2)/c^(1/
2)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*erf(1/2*(2
*e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))*Pi^(1/2)/(2*f-c*ln(f))^(
1/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*erfi(1/2*(2*e+b*ln(f)
+2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))*Pi^(1/2)/(2*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used

$= \{5623, 2266, 2235, 2325, 2236\}$

$$\int f^{a+bx+cx^2} \sinh^2 \left(d+ex+fx^2\right) \, dx$$
$$= -\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right) \operatorname{erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}} + \frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)+8f}\right) \operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)+2e}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f
]))*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f -
c*Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f +
 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*
Sqrt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{2} f^{a+bx+cx^2} + \frac{1}{4} e^{-2d-2ex-2fx^2} f^{a+bx+cx^2} + \frac{1}{4} e^{2d+2ex+2fx^2} f^{a+bx+cx^2}\right) dx \\
&= \frac{1}{4} \int e^{-2d-2ex-2fx^2} f^{a+bx+cx^2} \, dx + \frac{1}{4} \int e^{2d+2ex+2fx^2} f^{a+bx+cx^2} \, dx - \frac{1}{2} \int f^{a+bx+cx^2} \, dx \\
&= \frac{1}{4} \int \exp\left(-2d + a\log(f) - x(2e - b\log(f)) - x^2(2f - c\log(f))\right) dx + \frac{1}{4} \int \exp\left(2d \right. \\
&\qquad \left. + a\log(f) + x(2e + b\log(f)) + x^2(2f + c\log(f))\right) dx - \frac{1}{2} f^{a-\frac{b^2}{4c}} \int f^{\frac{(b+2cx)^2}{4c}} \, dx \\
&= -\frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{1}{4}\left(\exp\left(-2d \right.\right. \\
&\qquad \left.\left. + \frac{(2e - b\log(f))^2}{8f - 4c\log(f)}\right) f^a\right) \int \exp\left(\frac{(-2e + b\log(f) + 2x(-2f + c\log(f)))^2}{4(-2f + c\log(f))}\right) dx \\
&\quad + \frac{1}{4}\left(\exp\left(2d \right.\right. \\
&\qquad \left.\left. - \frac{(2e + b\log(f))^2}{8f + 4c\log(f)}\right) f^a\right) \int \exp\left(\frac{(2e + b\log(f) + 2x(2f + c\log(f)))^2}{4(2f + c\log(f))}\right) dx \\
&= -\frac{f^{a-\frac{b^2}{4c}} \sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} \\
&\quad + \frac{\exp\left(-2d + \frac{(2e-b\log(f))^2}{8f-4c\log(f)}\right) f^a \sqrt{\pi}\,\text{erf}\left(\frac{2e-b\log(f)+2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f - c\log(f)}} \\
&\quad + \frac{\exp\left(2d - \frac{(2e+b\log(f))^2}{8f+4c\log(f)}\right) f^a \sqrt{\pi}\,\text{erfi}\left(\frac{2e+b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f + c\log(f)}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 4.46 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.42

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=-\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

$$-\frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f+4c\log(f)}}f^{a+\frac{4bef}{-4f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(4e^2+b^2\log^2(f)\right)}{4f^2-c^2\log^2(f)}}f^{\frac{be}{2f+c\log(f)}}\mathrm{erf}\left(\frac{2(e+2fx)-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2f+\cdots}{8\cdots}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^2,x]
```

```
[Out] -1/4*(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c]
)])/(Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqr
t[Pi]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f
+ c*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f
]])]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((b*
e)/(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2*f
+ c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d])
))/(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[f]^2)
)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\mathrm{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)-2e}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\,f^a\,\mathrm{e}^{-\frac{b^2\ln(f)^2-4\ln(f)be+8d\ln(f)c-16df+4e^2}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\mathrm{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)-2f}}\right)\cdots}{8\sqrt{-\cdots}}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-2*e)/(2*f-c*ln(f))^(1/2))/(2*f
-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*c-
16*d*f+4*e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*ln(
f))/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*l
n(f)^2+4*ln(f)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))+1/4*f^a*Pi^(1/2
)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*l
n(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(197) = 394$.

Time = 0.29 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.16

$$\int f^{a+bx+cx^2} \sinh^2\left(d+ex+fx^2\right) \, dx =$$

$$-\frac{\left(\sqrt{\pi}\left(c^2 \log\left(f\right)^2+2\,cf \log\left(f\right)\right) \cosh\left(-\frac{\left(b^2-4\,ac\right) \log(f)^2+4\,e^2-16\,df+4\,(2\,cd-be+2\,af) \log(f)}{4\,(c \log(f)-2\,f)}\right)+\sqrt{\pi}\left(c^2 \log\left(f\right)^2+\right.\right.}{}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(
f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)) +
 sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-
c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*log(f)
+ 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/
4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f)
)/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log(f) +
2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) - 2*(sqrt(pi)*(c^2*log(f)^2 -
4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt
(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2} \sinh^2\left(d+ex+fx^2\right) \, dx = \int f^{a+bx+cx^2} \sinh^2\left(d+ex+fx^2\right) dx$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(f**(a + b*x + c*x**2)*sinh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)-2\,f}x-\frac{b\log(f)+2\,e}{2\,\sqrt{-c\log(f)-2\,f}}\right)e^{\left(-\frac{(b\log(f)+2\,e)^2}{4\,(c\log(f)+2\,f)}+2\,d\right)}}{8\,\sqrt{-c\log(f)-2\,f}} + \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)+2\,f}x-\frac{b\log(f)-2\,e}{2\,\sqrt{-c\log(f)+2\,f}}\right)e^{\left(-\frac{(b\log(f)-2\,e)^2}{4\,(c\log(f)-2\,f)}-2\,d\right)}}{8\,\sqrt{-c\log(f)+2\,f}} - \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\,\sqrt{-c\log(f)}}\right)}{4\,\sqrt{-c\log(f)}f^{\frac{b^2}{4\,c}}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-c
*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*
log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*log(f
) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) - 2*f)
 - 2*d)/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/
2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.13

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)-2\,f}\left(2\,x+\frac{b\log(f)+2\,e}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)+4\,be\log(f)-8\,af\log(f)+4\,e^2-16\,df}{4\,(c\log(f)+2\,f)}\right)}}{8\,\sqrt{-c\log(f)-2\,f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)+2\,f}\left(2\,x+\frac{b\log(f)-2\,e}{c\log(f)-2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+8\,cd\log(f)-4\,be\log(f)+8\,af\log(f)+4\,e^2-16\,df}{4\,(c\log(f)-2\,f)}\right)}}{8\,\sqrt{-c\log(f)+2\,f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{4\,\sqrt{-c\log(f)}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*log
(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*
log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2*e)/
(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) -
4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) - 2*f))/sqrt(-c*log
(f) + 2*f) + 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^
2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\sinh\left(f\,x^2+e\,x+d\right)^2\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^2, x)
```

3.365 $\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 344

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx=\frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{e^{-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3e-b\log(f)+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}-\frac{3e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d-\frac{(3e+b\log(f))^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

```
[Out] 3/16*exp(-d+1/4*(e-b*ln(f))^2/(f-c*ln(f)))*f^a*erf(1/2*(e-b*ln(f)+2*x*(f-c*
ln(f)))/(f-c*ln(f))^(1/2))*Pi^(1/2)/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+(3*e-b*
ln(f))^2/(12*f-4*c*ln(f)))*f^a*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*ln(f)))/(3*f
-c*ln(f))^(1/2))*Pi^(1/2)/(3*f-c*ln(f))^(1/2)-3/16*exp(d-1/4*(e+b*ln(f))^2/
(f+c*ln(f)))*f^a*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))*Pi
^(1/2)/(f+c*ln(f))^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/(3*f+c*ln(f)))*f^
a*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))*Pi^(1/2)/(3
*f+c*ln(f))^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5623, 2325, 2266, 2236, 2235}

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}f^a\exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\text{erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}} + \frac{3\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}} + \frac{\sqrt{\pi}f^a\exp\left(3d-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}\right)\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}} - \frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}$$

```
[In] Int[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log
[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]])
- (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e
- b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f
- c*Log[f]]) - (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*
Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[
f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])
])/(16*Sqrt[3*f + c*Log[f]])
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 2266

```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[F^(a - b^2/
(4*c)), Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Rule 2325

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 5623

```
Int[(F_)^(u_)*Sinh[v_]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^u, Sinh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{8}e^{-3(d+ex+fx^2)}f^{a+bx+cx^2}+\frac{3}{8}\exp\left(2d+2ex+2fx^2-3(d+ex+fx^2)\right)f^{a+bx+cx^2} \right.\\
&\qquad -\frac{3}{8}\exp\left(4d+4ex+4fx^2-3(d+ex+fx^2)\right)f^{a+bx+cx^2}\\
&\qquad \left. +\frac{1}{8}\exp\left(6d+6ex+6fx^2-3(d+ex+fx^2)\right)f^{a+bx+cx^2}\right)dx\\
&= -\left(\frac{1}{8}\int e^{-3(d+ex+fx^2)}f^{a+bx+cx^2}\,dx\right)\\
&\quad +\frac{1}{8}\int \exp\left(6d+6ex+6fx^2-3(d+ex+fx^2)\right)f^{a+bx+cx^2}\,dx\\
&\quad +\frac{3}{8}\int \exp\left(2d+2ex+2fx^2-3(d+ex+fx^2)\right)f^{a+bx+cx^2}\,dx\\
&\quad -\frac{3}{8}\int \exp\left(4d+4ex+4fx^2-3(d+ex+fx^2)\right)f^{a+bx+cx^2}\,dx\\
&= -\left(\frac{1}{8}\int \exp\left(-3d+a\log(f)-x(3e-b\log(f))-x^2(3f-c\log(f))\right)dx\right)\\
&\quad +\frac{1}{8}\int \exp\left(3d+a\log(f)+x(3e+b\log(f))+x^2(3f+c\log(f))\right)dx\\
&\quad +\frac{3}{8}\int \exp\left(-d+a\log(f)-x(e-b\log(f))-x^2(f-c\log(f))\right)dx\\
&\quad -\frac{3}{8}\int \exp\left(d+a\log(f)+x(e+b\log(f))+x^2(f+c\log(f))\right)dx
\end{aligned}$$

$$= -\left(\frac{1}{8}\left(\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\right)\int\exp\left(\frac{(-3e+b\log(f)+2x(-3f+c\log(f)))^2}{4(-3f+c\log(f))}\right)dx\right)$$
$$+\frac{1}{8}\left(3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\right)\int\exp\left(\frac{(-e+b\log(f)+2x(-f+c\log(f)))^2}{4(-f+c\log(f))}\right)dx$$
$$-\frac{1}{8}\left(3e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\right)\int\exp\left(\frac{(e+b\log(f)+2x(f+c\log(f)))^2}{4(f+c\log(f))}\right)dx$$
$$+\frac{1}{8}\left(\exp\left(3d-\frac{(3e+b\log(f))^2}{4(3f+c\log(f))}\right)f^a\right)\int\exp\left(\frac{(3e+b\log(f)+2x(3f+c\log(f)))^2}{4(3f+c\log(f))}\right)dx$$
$$=\frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}$$
$$-\frac{\exp\left(-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}\right)f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3e-b\log(f)+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}$$
$$-\frac{3e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}$$
$$+\frac{\exp\left(3d-\frac{(3e+b\log(f))^2}{4(3f+c\log(f))}\right)f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2991 vs. $2(344) = 688$.

Time = 6.46 (sec) , antiderivative size = 2991, normalized size of antiderivative = 8.69

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx = \text{Result too large to show}$$

```
[In] Integrate[f^(a + b*x + c*x^2)*Sinh[d + e*x + f*x^2]^3,x]
```

```
[Out] (f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log
[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] -
2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e^2 +
 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*Erf[(e
 + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*Sqrt[f
 - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) -
 (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log
[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)
```

/(4*(f - c*Log[f]))) - (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x
*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e
*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c*f^2*Cosh[3*d]*Erf[(3*e +
 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f
 - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f])
)) + (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt
[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f]
 - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c^3*Cosh[3*d]*Erf[(3*e + 6*f*x -
b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Lo
g[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (2
7*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[
f]])]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*
Log[f]))) + (27*c*f^2*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2
*Sqrt[f + c*Log[f]])]*Log[f]*Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b
^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Cosh[d]*Erfi[(e + 2*f*x + b*Log
[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]])/E^
((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Cosh[d]*E
rfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*
Sqrt[f + c*Log[f]])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]
))) + (3*f^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt
[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log
[f]^2)/(4*(3*f + c*Log[f]))) - (c*f^2*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f
] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]*Sqrt[3*f + c*Log[f]])/E^
((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Cos
h[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]]
)]*Log[f]^2*Sqrt[3*f + c*Log[f]])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/
(4*(3*f + c*Log[f]))) + (c^3*Cosh[3*d]*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x
*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*Sqrt[3*f + c*Log[f]])/E^((9*e^2
 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (27*f^3*Erf[(e + 2*
f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*S
inh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*c
*f^2*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[
f]*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(
f - c*Log[f]))) + (3*c^2*f*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqr
t[f - c*Log[f]])]*Log[f]^2*Sqrt[f - c*Log[f]]*Sinh[d])/E^((-e^2 + 2*b*e*Log
[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) + (3*c^3*Erf[(e + 2*f*x - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]]*Sinh[d]
)/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (27*f^3*Erf
i[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*
Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f])))
 + (27*c*f^2*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f
]])]*Log[f]*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]
^2)/(4*(f + c*Log[f]))) + (3*c^2*f*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log[f
])/(2*Sqrt[f + c*Log[f]])]*Log[f]^2*Sqrt[f + c*Log[f]]*Sinh[d])/E^((e^2 + 2
*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) - (3*c^3*Erfi[(e + 2*f*x +

```
b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Log[f]^3*Sqrt[f + c*Log[f]
]*Sinh[d])/E^((e^2 + 2*b*e*Log[f] + b^2*Log[f]^2)/(4*(f + c*Log[f]))) + (3*
f^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*S
qrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4
*(3*f - c*Log[f]))) + (c*f^2*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2
*Sqrt[3*f - c*Log[f]])]*Log[f]*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 +
 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (3*c^2*f*Erf[(3*e + 6
*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f
 - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f -
 c*Log[f]))) - (c^3*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f
 - c*Log[f]])]*Log[f]^3*Sqrt[3*f - c*Log[f]]*Sinh[3*d])/E^((-9*e^2 + 6*b*e*
Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (3*f^3*Erfi[(3*e + 6*f*x + b
*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Sqrt[3*f + c*Log[f]]*Sinh
[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) - (c*
f^2*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*
Log[f]*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f
]^2)/(4*(3*f + c*Log[f]))) - (3*c^2*f*Erfi[(3*e + 6*f*x + b*Log[f] + 2*c*x*
Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^2*Sqrt[3*f + c*Log[f]]*Sinh[3*d])/
E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4*(3*f + c*Log[f]))) + (c^3*Erfi[
(3*e + 6*f*x + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*Log[f]^3*
Sqrt[3*f + c*Log[f]]*Sinh[3*d])/E^((9*e^2 + 6*b*e*Log[f] + b^2*Log[f]^2)/(4
*(3*f + c*Log[f])))))/(16*(f - c*Log[f])*(3*f - c*Log[f])*(f + c*Log[f])*(3
*f + c*Log[f]))
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e+b\ln(f)}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+6\ln(f)be-12d\ln(f)c-36df+9e^{2}}{4(3f+c\ln(f))}}}{16\sqrt{-c\ln(f)-3f}}+\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{b\ln(f)-3e}{2\sqrt{3f-c\ln(f}}\right.}{}$

```
[In] int(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f)-3*f)^(1/2))/(
-c*ln(f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+6*ln(f)*b*e-12*d*ln(
f)*c-36*d*f+9*e^2)/(3*f+c*ln(f)))+1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*ln
(f)-3*e)/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^
2*ln(f)^2-6*ln(f)*b*e+12*d*ln(f)*c-36*d*f+9*e^2)/(c*ln(f)-3*f))-3/16*erf(-x
*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^
(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)
-f))+3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-f)^(1/2))/(-c
*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*ln(f)*c-
4*d*f+e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 940 vs. 2(295) = 590.

Time = 0.30 (sec) , antiderivative size = 940, normalized size of antiderivative = 2.73

$$\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right) dx = \text{Too large to display}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] 1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*f)*
log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f
^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(
2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-
1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*
f)) - 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e + 2*a*f)*l
og(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*
log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d
- b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x
 - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi
)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(-1/4*((b^2
- 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f)
 + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*
sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*l
og(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*lo
g(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) - (sqrt(pi)*(c^3*log(f)^3 - 3
*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(
pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b^
2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*l
og(f) + 3*f)))*sqrt(-c*log(f) - 3*f)*erf(1/2*(6*f*x + (2*c*x + b)*log(f) +
3*e)*sqrt(-c*log(f) - 3*f)/(c*log(f) + 3*f)))/(c^4*log(f)^4 - 10*c^2*f^2*lo
g(f)^2 + 9*f^4)
```

Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \sinh^3\left(d+ex+fx^2\right) dx = \text{Timed out}$$

```
[In] integrate(f**(c*x**2+b*x+a)*sinh(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.92

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x-\frac{b\log(f)+3\,e}{2\,\sqrt{-c\log(f)-3\,f}}\right)e^{\left(-\frac{(b\log(f)+3\,e)^2}{4\,(c\log(f)+3\,f)}+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}$$
$$-\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)+e}{2\,\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4\,(c\log(f)+f)}+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}$$
$$+\frac{3\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)-e}{2\,\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4\,(c\log(f)-f)}-d\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}$$
$$-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x-\frac{b\log(f)-3\,e}{2\,\sqrt{-c\log(f)+3\,f}}\right)e^{\left(-\frac{(b\log(f)-3\,e)^2}{4\,(c\log(f)-3\,f)}-3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(-
c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c
*log(f) - 3*f) - 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f
) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sq
rt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*lo
g(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)
/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*
(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/(c*log(f
) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)+3e}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)-12af\log(f)+9e^2-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}} + \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)-4af\log(f)+e^2-4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+4af\log(f)+e^2-4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+3f}\left(2x+\frac{b\log(f)-3e}{c\log(f)-3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+12af\log(f)+9e^2-36df}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

```
[In] integrate(f^(c*x^2+b*x+a)*sinh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*lo
g(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) + 6*b*
e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f)
 - 3*f) + 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/
(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*
b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f
) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log
(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*lo
g(f) + 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) + 1/
16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e)/(c*log(f
) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*l
og(f) + 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) +
3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\sinh^3\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,{\mathrm{sinh}\left(f\,x^2+e\,x+d\right)}^3\,dx$$

```
[In] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3,x)
```

```
[Out] int(f^(a + b*x + c*x^2)*sinh(d + e*x + f*x^2)^3, x)
```

3.366 $\int (x+\sinh(x))^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 30

$$\int (x+\sinh(x))^2 \, dx = -\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)$$

```
[Out] -1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6874, 3377, 2717, 2715, 8}

$$\int (x+\sinh(x))^2 \, dx = \frac{x^3}{3}-\frac{x}{2}-2\sinh(x)+2x\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[In] Int[(x + Sinh[x])^2,x]
```

```
[Out] -1/2*x + x^3/3 + 2*x*Cosh[x] - 2*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(x^2+2x\sinh(x)+\sinh^2(x)\right)\,dx \\
&= \frac{x^3}{3}+2\int x\sinh(x)\,dx+\int \sinh^2(x)\,dx \\
&= \frac{x^3}{3}+2x\cosh(x)+\frac{1}{2}\cosh(x)\sinh(x)-\frac{\int 1\,dx}{2}-2\int \cosh(x)\,dx \\
&= -\frac{x}{2}+\frac{x^3}{3}+2x\cosh(x)-2\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (x+\sinh(x))^2\,dx=\frac{1}{6}x\left(-3+2x^2\right)+2x\cosh(x)-2\sinh(x)+\frac{1}{4}\sinh(2x)$$

```
[In] Integrate[(x + Sinh[x])^2,x]
[Out] (x*(-3 + 2*x^2))/6 + 2*x*Cosh[x] - 2*Sinh[x] + Sinh[2*x]/4
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{x}{2}+\frac{x^{3}}{3}+2x\cosh\left(x\right)-2\sinh\left(x\right)+\frac{\cosh(x)\sinh(x)}{2}$	25
parallelrisch	$-\frac{x}{2}+\frac{x^{3}}{3}+2x\cosh\left(x\right)-2\sinh\left(x\right)+\frac{\sinh(2x)}{4}$	25
parts	$-\frac{x}{2}+\frac{x^{3}}{3}+2x\cosh\left(x\right)-2\sinh\left(x\right)+\frac{\cosh(x)\sinh(x)}{2}$	25
risch	$\frac{x^{3}}{3}-\frac{x}{2}+\frac{e^{2x}}{8}+\left(x-1\right)e^{x}+\left(1+x\right)e^{-x}-\frac{e^{-2x}}{8}$	36

```
[In] int((x+sinh(x))^2,x,method=_RETURNVERBOSE)
[Out] -1/2*x+1/3*x^3+2*x*cosh(x)-2*sinh(x)+1/2*cosh(x)*sinh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int (x+\sinh(x))^2\,dx=\frac{1}{3}\,x^{3}+2\,x\cosh\left(x\right)+\frac{1}{2}\left(\cosh\left(x\right)-4\right)\sinh\left(x\right)-\frac{1}{2}\,x$$

```
[In] integrate((x+sinh(x))^2,x, algorithm="fricas")
[Out] 1/3*x^3 + 2*x*cosh(x) + 1/2*(cosh(x) - 4)*sinh(x) - 1/2*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (x+\sinh(x))^2\,dx=\frac{x^{3}}{3}+\frac{x\sinh^{2}\left(x\right)}{2}-\frac{x\cosh^{2}\left(x\right)}{2}+2x\cosh\left(x\right)+\frac{\sinh\left(x\right)\cosh\left(x\right)}{2}-2\sinh\left(x\right)$$

```
[In] integrate((x+sinh(x))**2,x)
[Out] x**3/3 + x*sinh(x)**2/2 - x*cosh(x)**2/2 + 2*x*cosh(x) + sinh(x)*cosh(x)/2
- 2*sinh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int (x+\sinh(x))^2\,dx = \frac{1}{3}\,x^3 + (x+1)e^{(-x)} + (x-1)e^x - \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate((x+sinh(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int (x+\sinh(x))^2\,dx = \frac{1}{3}\,x^3 + (x+1)e^{(-x)} + (x-1)e^x - \frac{1}{2}\,x + \frac{1}{8}\,e^{(2\,x)} - \frac{1}{8}\,e^{(-2\,x)}$$

```
[In] integrate((x+sinh(x))^2,x, algorithm="giac")
[Out] 1/3*x^3 + (x + 1)*e^(-x) + (x - 1)*e^x - 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sinh(x))^2\,dx = \frac{\cosh(x)\,\sinh(x)}{2} - 2\sinh(x) - \frac{x}{2} + 2\,x\cosh(x) + \frac{x^3}{3}$$

```
[In] int((x + sinh(x))^2,x)
[Out] (cosh(x)*sinh(x))/2 - 2*sinh(x) - x/2 + 2*x*cosh(x) + x^3/3
```

3.367 $\int (x+\sinh(x))^3 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 56

$$\int (x+\sinh(x))^3 \, dx = -\frac{3x^2}{4}+\frac{x^4}{4}+5\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3} - 6x\sinh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}$$

```
[Out] -3/4*x^2+1/4*x^4+5*cosh(x)+3*x^2*cosh(x)+1/3*cosh(x)^3-6*x*sinh(x)+3/2*x*co
sh(x)*sinh(x)-3/4*sinh(x)^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6874, 3377, 2718, 3391, 30, 2713}

$$\int (x+\sinh(x))^3 \, dx = \frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh(x)-\frac{3\sinh^2(x)}{4}-6x\sinh(x) +\frac{\cosh^3(x)}{3}+5\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

```
[In] Int[(x + Sinh[x])^3,x]
```

```
[Out] (-3*x^2)/4 + x^4/4 + 5*Cosh[x] + 3*x^2*Cosh[x] + Cosh[x]^3/3 - 6*x*Sinh[x]
+ (3*x*Cosh[x]*Sinh[x])/2 - (3*Sinh[x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(x^3+3x^2\sinh(x)+3x\sinh^2(x)+\sinh^3(x)\right)\,dx \\
&= \frac{x^4}{4}+3\int x^2\sinh(x)\,dx+3\int x\sinh^2(x)\,dx+\int \sinh^3(x)\,dx \\
&= \frac{x^4}{4}+3x^2\cosh(x)+\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}-\frac{3\int x\,dx}{2} \\
&\quad -6\int x\cosh(x)\,dx-\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh(x)\right) \\
&= -\frac{3x^2}{4}+\frac{x^4}{4}-\cosh(x)+3x^2\cosh(x)+\frac{\cosh^3(x)}{3}-6x\sinh(x) \\
&\quad +\frac{3}{2}x\cosh(x)\sinh(x)-\frac{3\sinh^2(x)}{4}+6\int \sinh(x)\,dx
\end{aligned}
$$

$$= -\frac{3x^2}{4} + \frac{x^4}{4} + 5\cosh(x) + 3x^2\cosh(x) + \frac{\cosh^3(x)}{3} - 6x\sinh(x) + \frac{3}{2}x\cosh(x)\sinh(x) - \frac{3\sinh^2(x)}{4}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (x+\sinh(x))^3\,dx = \frac{1}{24}\left(18\left(7+4x^2\right)\cosh(x) - 9\cosh(2x) + 2\cosh(3x) + 6x\left(-3x+x^3-24\sinh(x)+3\sinh(2x)\right)\right)$$

```
[In] Integrate[(x + Sinh[x])^3,x]
```

```
[Out] (18*(7 + 4*x^2)*Cosh[x] - 9*Cosh[2*x] + 2*Cosh[3*x] + 6*x*(-3*x + x^3 - 24*
Sinh[x] + 3*Sinh[2*x]))/24
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result
parallelrisch	$-\frac{47}{24}+\frac{x^4}{4}-\frac{3x^2}{4}+3x^2\cosh\left(x\right)-6x\sinh\left(x\right)+\frac{3x\sinh(2x)}{4}+\frac{\cosh(3x)}{12}+\frac{21\cosh(x)}{4}-\frac{3\cosh(2x)}{8}$
default	$\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh\left(x\right)+\frac{3x\cosh(x)\sinh(x)}{2}-\frac{3x^2}{4}-\frac{3\cosh(x)^2}{4}+3x^2\cosh\left(x\right)-6x\sinh\left(x\right)+6$ c
parts	$\left(-\frac{2}{3}+\frac{\sinh(x)^2}{3}\right)\cosh\left(x\right)+\frac{3x\cosh(x)\sinh(x)}{2}-\frac{3x^2}{4}-\frac{3\cosh(x)^2}{4}+3x^2\cosh\left(x\right)-6x\sinh\left(x\right)+6$ c
risch	$\frac{x^4}{4}-\frac{3x^2}{4}+\frac{9}{16}+\frac{e^{3x}}{24}+\left(-\frac{3}{16}+\frac{3x}{8}\right)e^{2x}+\left(\frac{21}{8}-3x+\frac{3}{2}x^2\right)e^{x}+\left(\frac{21}{8}+3x+\frac{3}{2}x^2\right)e^{-x}+\left(-\frac{3}{16}\right.$

```
[In] int((x+sinh(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -47/24+1/4*x^4-3/4*x^2+3*x^2*cosh(x)-6*x*sinh(x)+3/4*x*sinh(2*x)+1/12*cosh(
3*x)+21/4*cosh(x)-3/8*cosh(2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int (x+\sinh(x))^3\,dx = \frac{1}{4}\,x^4+\frac{1}{12}\,\cosh\left(x\right)^3+\frac{1}{8}\,(2\,\cosh\left(x\right)-3)\sinh\left(x\right)^2-\frac{3}{4}\,x^2 +\frac{3}{4}\,\left(4\,x^2+7\right)\cosh\left(x\right)-\frac{3}{8}\,\cosh\left(x\right)^2+\frac{3}{2}\,(x\cosh\left(x\right)-4\,x)\sinh\left(x\right)$$

[In] integrate((x+sinh(x))^3,x, algorithm="fricas")

[Out] 1/4*x^4 + 1/12*cosh(x)^3 + 1/8*(2*cosh(x) - 3)*sinh(x)^2 - 3/4*x^2 + 3/4*(4*x^2 + 7)*cosh(x) - 3/8*cosh(x)^2 + 3/2*(x*cosh(x) - 4*x)*sinh(x)

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (x+\sinh(x))^3\,dx = \frac{x^4}{4}+\frac{3x^2\sinh^2\left(x\right)}{4}-\frac{3x^2\cosh^2\left(x\right)}{4}+3x^2\cosh\left(x\right)+\frac{3x\sinh\left(x\right)\cosh\left(x\right)}{2} -6x\sinh\left(x\right)+\sinh^2\left(x\right)\cosh\left(x\right)-\frac{2\cosh^3\left(x\right)}{3}-\frac{3\cosh^2\left(x\right)}{4}+6\cosh\left(x\right)$$

[In] integrate((x+sinh(x))**3,x)

[Out] x**4/4 + 3*x**2*sinh(x)**2/4 - 3*x**2*cosh(x)**2/4 + 3*x**2*cosh(x) + 3*x*sinh(x)*cosh(x)/2 - 6*x*sinh(x) + sinh(x)**2*cosh(x) - 2*cosh(x)**3/3 - 3*cosh(x)**2/4 + 6*cosh(x)

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int (x+\sinh(x))^3\,dx = \frac{1}{4}\,x^4-\frac{3}{4}\,x^2+\frac{3}{16}\,(2\,x-1)e^{(2\,x)}+\frac{3}{2}\,\left(x^2+2\,x+2\right)e^{(-x)} -\frac{3}{16}\,(2\,x+1)e^{(-2\,x)}+\frac{3}{2}\,\left(x^2-2\,x+2\right)e^x +\frac{1}{24}\,e^{(3\,x)}-\frac{3}{8}\,e^{(-x)}+\frac{1}{24}\,e^{(-3\,x)}-\frac{3}{8}\,e^x$$

[In] integrate((x+sinh(x))^3,x, algorithm="maxima")

[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/2*(x^2 + 2*x + 2)*e^(-x) - 3/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(-x) + 1/24*e^(-3*x) - 3/8*e^x

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int (x+\sinh(x))^3\,dx = \frac{1}{4}\,x^4 - \frac{3}{4}\,x^2 + \frac{3}{16}\,(2\,x-1)e^{(2\,x)} + \frac{3}{8}\,(4\,x^2+8\,x+7)e^{(-x)} - \frac{3}{16}\,(2\,x+1)e^{(-2\,x)} + \frac{3}{8}\,(4\,x^2-8\,x+7)e^{x} + \frac{1}{24}\,e^{(3\,x)} + \frac{1}{24}\,e^{(-3\,x)}$$

```
[In] integrate((x+sinh(x))^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) + 3/8*(4*x^2 + 8*x + 7)*e^(-x) -
 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 7)*e^x + 1/24*e^(3*x) + 1/24*
e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (x+\sinh(x))^3\,dx = 5\cosh(x) + 3\,x^2\cosh(x) - \frac{3\cosh(x)^2}{4} + \frac{\cosh(x)^3}{3} - 6\,x\sinh(x) - \frac{3\,x^2}{4} + \frac{x^4}{4} + \frac{3\,x\cosh(x)\,\sinh(x)}{2}$$

```
[In] int((x + sinh(x))^3,x)
```

```
[Out] 5*cosh(x) + 3*x^2*cosh(x) - (3*cosh(x)^2)/4 + cosh(x)^3/3 - 6*x*sinh(x) - (
3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2
```

3.368 $\int \frac{\sinh(a+bx)}{c+dx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = -\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}$$

```
[Out] 1/2*cosh(a+b*(-c)^(1/2)/d^(1/2))*Shi(b*x-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d
^(1/2)-1/2*cosh(a-b*(-c)^(1/2)/d^(1/2))*Shi(b*x+b*(-c)^(1/2)/d^(1/2))/(-c)^
(1/2)/d^(1/2)-1/2*Chi(b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a-b*(-c)^(1/2)/d^(1/2)
)/(-c)^(1/2)/d^(1/2)+1/2*Chi(-b*x+b*(-c)^(1/2)/d^(1/2))*sinh(a+b*(-c)^(1/2)
/d^(1/2))/(-c)^(1/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5388, 3384, 3379, 3382}

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = -\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

```
[In] Int[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] -1/2*(CoshIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sinh[a - (b*Sqrt[-c])/Sqrt[d
]])/(Sqrt[-c]*Sqrt[d]) + (CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sinh[a +
 (b*Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a + (b*Sqrt[-c])/Sqrt[
d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cosh[
a - (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt
[-c]*Sqrt[d])
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{-c}\sinh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\sinh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)\,dx\\
&= -\frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}}-\frac{\int \frac{\sinh(a+bx)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}\\
&= -\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}+\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\sinh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}}\\
&\quad -\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{\sqrt{-c}+\sqrt{d}x}\,dx}{2\sqrt{-c}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\int \frac{\cosh\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{\sqrt{-c}-\sqrt{d}x}\,dx}{2\sqrt{-c}}
\end{aligned}
$$

$$= -\frac{\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right)\sinh\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\mathrm{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)\sinh\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a + \frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}} - bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a - \frac{b\sqrt{-c}}{\sqrt{d}}\right)\mathrm{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}} + bx\right)}{2\sqrt{-c}\sqrt{d}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = -\frac{ie^{-a-\frac{ib\sqrt{c}}{\sqrt{d}}}\left(e^{2a+\frac{2ib\sqrt{c}}{\sqrt{d}}}\,\mathrm{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right) - e^{2a}\,\mathrm{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right) + e^{\frac{2ib\sqrt{c}}{\sqrt{d}}}\,\mathrm{ExpInteg}\right.}{4\sqrt{c}\sqrt{d}}$$

```
[In] Integrate[Sinh[a + b*x]/(c + d*x^2),x]
```

```
[Out] ((-1/4*I)*E^(-a - (I*b*Sqrt[c])/Sqrt[d])*(E^(2*a + ((2*I)*b*Sqrt[c])/Sqrt[d
])*ExpIntegralEi[b*(((-I)*Sqrt[c])/Sqrt[d] + x)] - E^(2*a)*ExpIntegralEi[b*
((I*Sqrt[c])/Sqrt[d] + x)] + E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[((
-I)*b*Sqrt[c])/Sqrt[d] - b*x] - ExpIntegralEi[(I*b*Sqrt[c])/Sqrt[d] - b*x])
)/(Sqrt[c]*Sqrt[d])
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\mathrm{e}^{-\frac{-b\sqrt{-cd}+ad}{d}}\,\mathrm{Ei}_1\left(\frac{b\sqrt{-cd}+d(bx+a)-ad}{d}\right)}{4\sqrt{-cd}} - \frac{\mathrm{e}^{\frac{b\sqrt{-cd}+ad}{d}}\,\mathrm{Ei}_1\left(\frac{b\sqrt{-cd}-d(bx+a)+ad}{d}\right)}{4\sqrt{-cd}} + \frac{\mathrm{e}^{\frac{-b\sqrt{-cd}+ad}{d}}\,\mathrm{Ei}_1\left(-\frac{b\sqrt{-cd}+d(bx+a)-ad}{d}\right)}{4\sqrt{-cd}}$

```
[In] int(sinh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+a*d)/d)*Ei(1,(b*(-c*d)^(1/2)+d*(b*x
+a)-a*d)/d)-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+a*d)/d)*Ei(1,(b*(-c*d)^(1/
2)-d*(b*x+a)+a*d)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+a*d)/d)*Ei(1,-(b
*(-c*d)^(1/2)+d*(b*x+a)-a*d)/d)+1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+a*d)/
d)*Ei(1,-(b*(-c*d)^(1/2)-d*(b*x+a)+a*d)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.

Time = 0.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = -\frac{\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)-\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{\right.\right.}{}$$

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] -1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2*c/
d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x + s
qrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b
^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-b^2*c/
d)))/(b*c)
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \frac{\sinh(a+bx)}{c+dx^2}\,dx$$

```
[In] integrate(sinh(b*x+a)/(d*x**2+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \frac{\sinh(bx+a)}{dx^2+c}\,dx$$

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \frac{\sinh(bx+a)}{dx^2+c}\,dx$$

```
[In] integrate(sinh(b*x+a)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(sinh(b*x + a)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{c+dx^2}\,dx = \int \frac{\sinh(a+b\,x)}{d\,x^2+c}\,dx$$

```
[In] int(sinh(a + b*x)/(c + d*x^2),x)
[Out] int(sinh(a + b*x)/(c + d*x^2), x)
```

3.369 $\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 271

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \frac{\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} - \frac{\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}} + \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

```
[Out] cosh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*Shi(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))
/e)/(-4*c*e+d^2)^(1/2)-cosh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Shi(b*x+1/2*b
*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Chi(b*x+1/2*b*(d-(-4*c*e+d^2)
^(1/2))/e)*sinh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Chi(b*
x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sinh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-
4*c*e+d^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {6860, 3384, 3379, 3382}

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} + \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

```
[In] Int[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (CoshIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d - Sqr
t[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CoshIntegral[(b*(d + Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x]*Sinh[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[
d^2 - 4*c*e] + (Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b
*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d
 + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e
) + b*x])/Sqrt[d^2 - 4*c*e]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)} - \frac{2e\sinh(a+bx)}{\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}+2ex\right)}\right)dx$$

$$= \frac{(2e)\int \frac{\sinh(a+bx)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{\sinh(a+bx)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\left(2e\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sinh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$- \frac{\left(2e\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\sinh\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$+ \frac{\left(2e\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cosh\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d-\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$- \frac{\left(2e\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\right)\int \frac{\cosh\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{d+\sqrt{d^2-4ce}+2ex}\,dx}{\sqrt{d^2-4ce}}$$

$$= \frac{\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}}$$

$$- \frac{\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}}$$

$$+ \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

$$- \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.81

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \frac{e^{-a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}}\left(-e^{\frac{bd}{e}}\,\text{ExpIntegralEi}\left(-\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)+e^{2a+\frac{b\sqrt{d^2-4ce}}{e}}\,\text{ExpIntegralEi}\left(\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right.\right.}{2\sqrt{d^2-4c}}$$

```
[In] Integrate[Sinh[a + b*x]/(c + d*x + e*x^2),x]
```

```
[Out] (E^(-a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e))*(-(E^((b*d)/e)*ExpIntegralEi[-1
/2*(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e]) + E^(2*a + (b*Sqrt[d^2 - 4*c*e])
/e)*ExpIntegralEi[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] + E^((b*(d + S
qrt[d^2 - 4*c*e]))/e)*ExpIntegralEi[-1/2*(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x)
)/e] - E^(2*a)*ExpIntegralEi[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]))/(
2*Sqrt[d^2 - 4*c*e])
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{b\,\mathrm{e}^{\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{Ei}_1\left(\frac{-2e(bx+a)+2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}+\frac{b\,\mathrm{e}^{\frac{2ae-bd-\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{Ei}_1\left(-\frac{2e(bx+a)-2ae+bd+\sqrt{-4b^2c}}{2e}\right.}{2\sqrt{-4b^2ce+b^2d^2}}$

```
[In] int(sinh(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,1/2*(-2*e*(b*x+a)+2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/
2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*a*e-b*d-(-4*b^2*c*e+b^2*d^2)^(1
/2)))*Ei(1,-1/2*(2*e*(b*x+a)-2*a*e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+1/2*b
/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2
)))*Ei(1,-1/2*(-2*e*(b*x+a)+2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-1/2*b/
(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*a*e-b*d-(-4*b^2*c*e+b^2*d^2)^(1/2)
))*Ei(1,1/2*(2*e*(b*x+a)-2*a*e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(231) = 462.

Time = 0.29 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.48

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx =$$
$$-\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\mathrm{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)-e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\mathrm{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{b}}{2\,e}\right.}{}$$

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^
2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*
b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a*e
+ e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2
)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b
^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c
*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e
) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b^2*
d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*
e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d - 2*a*e +
e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*
Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2
*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e
)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))
/(b*d^2 - 4*b*c*e)
```

Sympy [F]

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx$$

```
[In] integrate(sinh(b*x+a)/(e*x**2+d*x+c),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*c*e-d^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\sinh(bx+a)}{ex^2+dx+c}\,dx$$

```
[In] integrate(sinh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(e*x^2 + d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\sinh(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

```
[In] int(sinh(a + b*x)/(c + d*x + e*x^2),x)
```

```
[Out] int(sinh(a + b*x)/(c + d*x + e*x^2), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```