Computer Algebra Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/172-6.3.2-Hyperbolic-tangent-functions

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [247]. This is test number [172].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (247)	0.00 (0)
Mathematica	100.00 (247)	0.00 (0)
Fricas	84.62 (209)	15.38 (38)
Maple	83.00 (205)	17.00 (42)
Giac	75.30 (186)	24.70 (61)
Mupad	70.85 (175)	29.15 (72)
Maxima	61.13 (151)	38.87 (96)
Sympy	29.15 (72)	70.85 (175)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

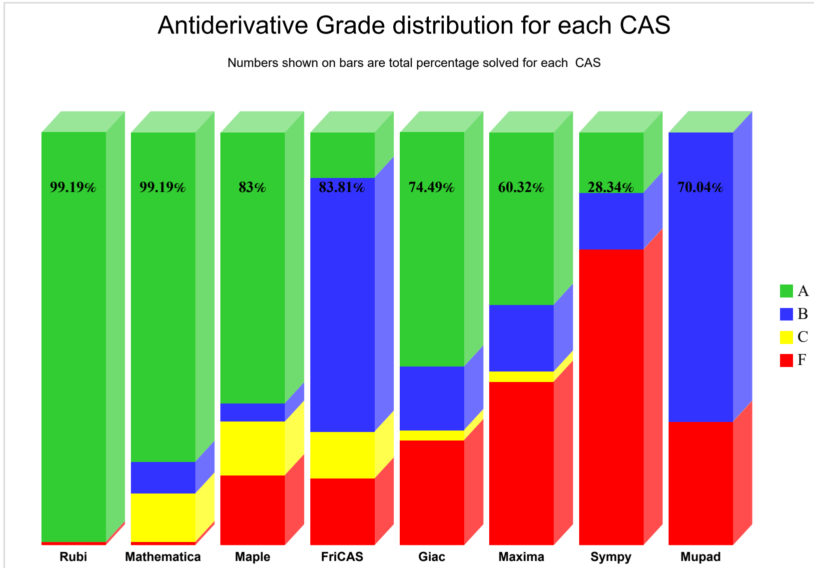
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

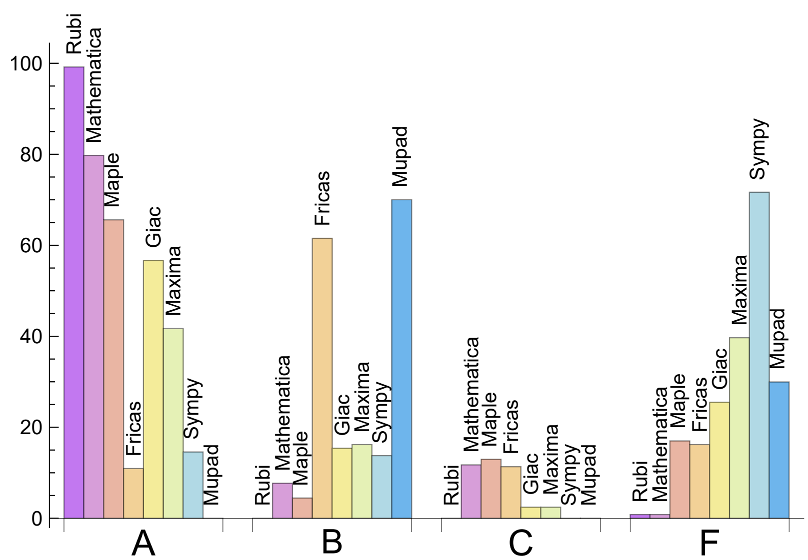
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.190	0.000	0.000	0.810
Mathematica	79.757	7.692	11.741	0.810
Maple	65.587	4.453	12.955	17.004
Giac	56.680	15.385	2.429	25.506
Maxima	41.700	16.194	2.429	39.676
Sympy	14.575	13.765	0.000	71.660
Fricas	10.931	61.538	11.336	16.194
Mupad	0.000	70.040	0.000	29.960

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	38	100.00	0.00	0.00
Maple	42	95.24	4.76	0.00
Giac	61	80.33	14.75	4.92
Mupad	72	0.00	100.00	0.00
Maxima	96	84.38	0.00	15.62
Sympy	175	96.57	3.43	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.08
Maxima	0.25
Giac	0.28
Fricas	0.29
Mathematica	1.00
Mupad	1.58
Maple	2.85
Sympy	3.45

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	75.76	1.06	48.00	0.95
Rubi	78.38	1.00	60.00	1.00
Mathematica	83.49	1.17	59.00	1.00
Mupad	86.13	1.37	48.00	0.95
Maxima	88.36	1.59	65.00	1.17
Giac	89.88	1.40	61.00	1.22
Sympy	454.44	4.61	69.00	1.52
Fricas	733.91	8.25	230.00	4.45

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

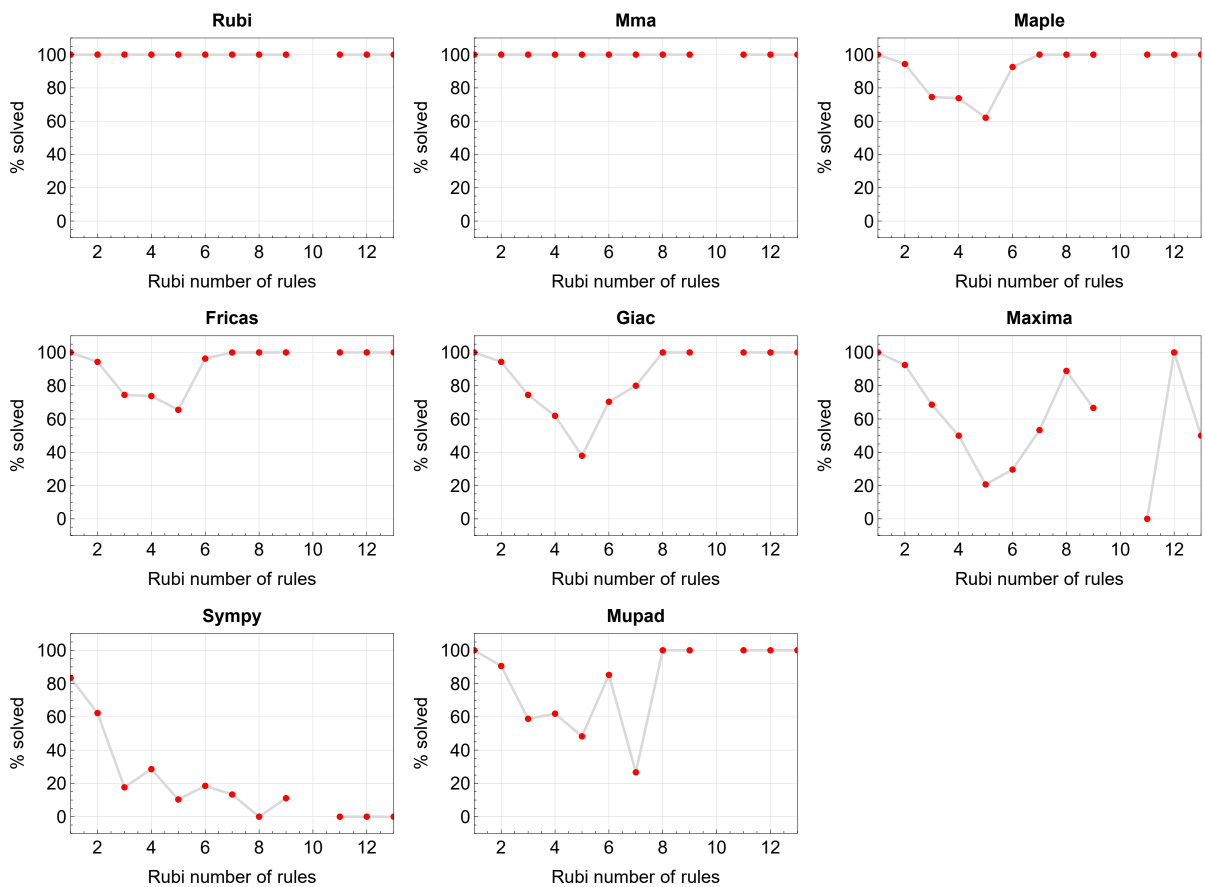


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

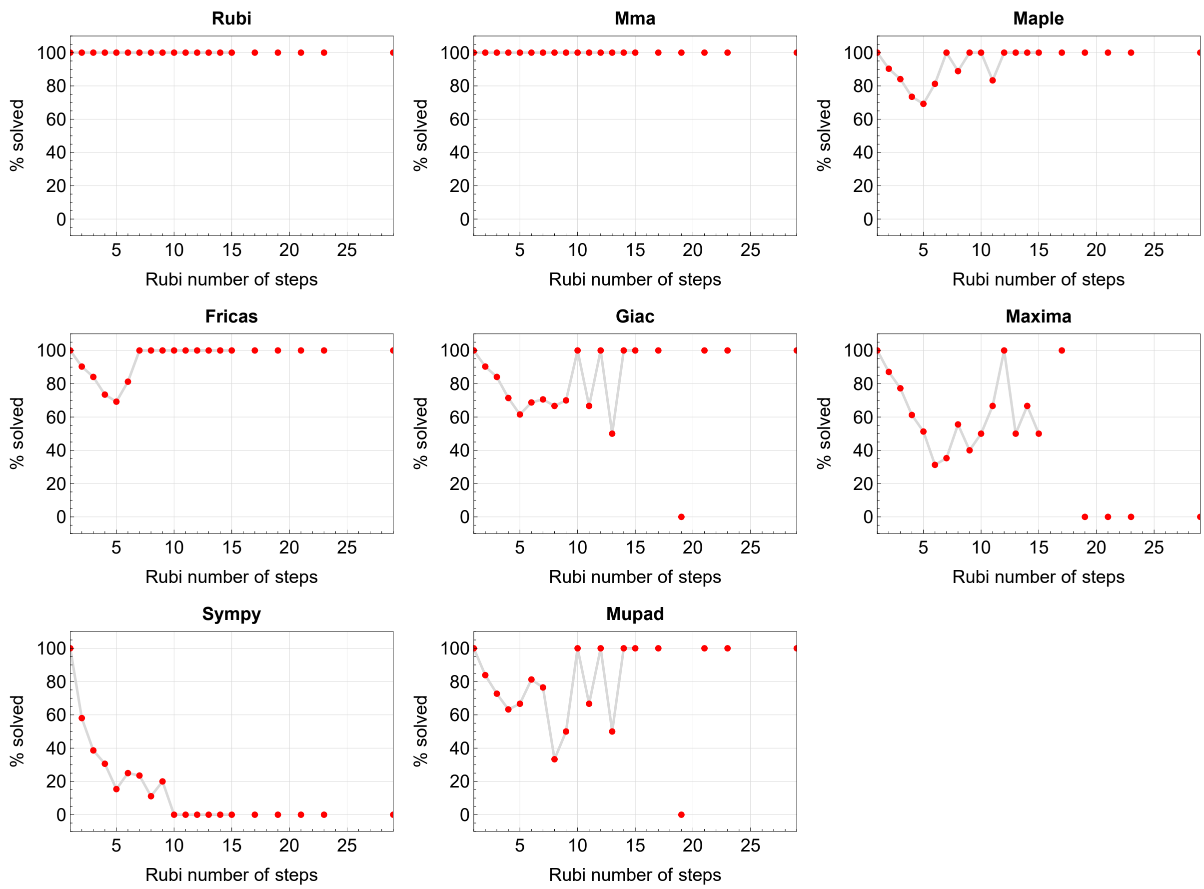


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

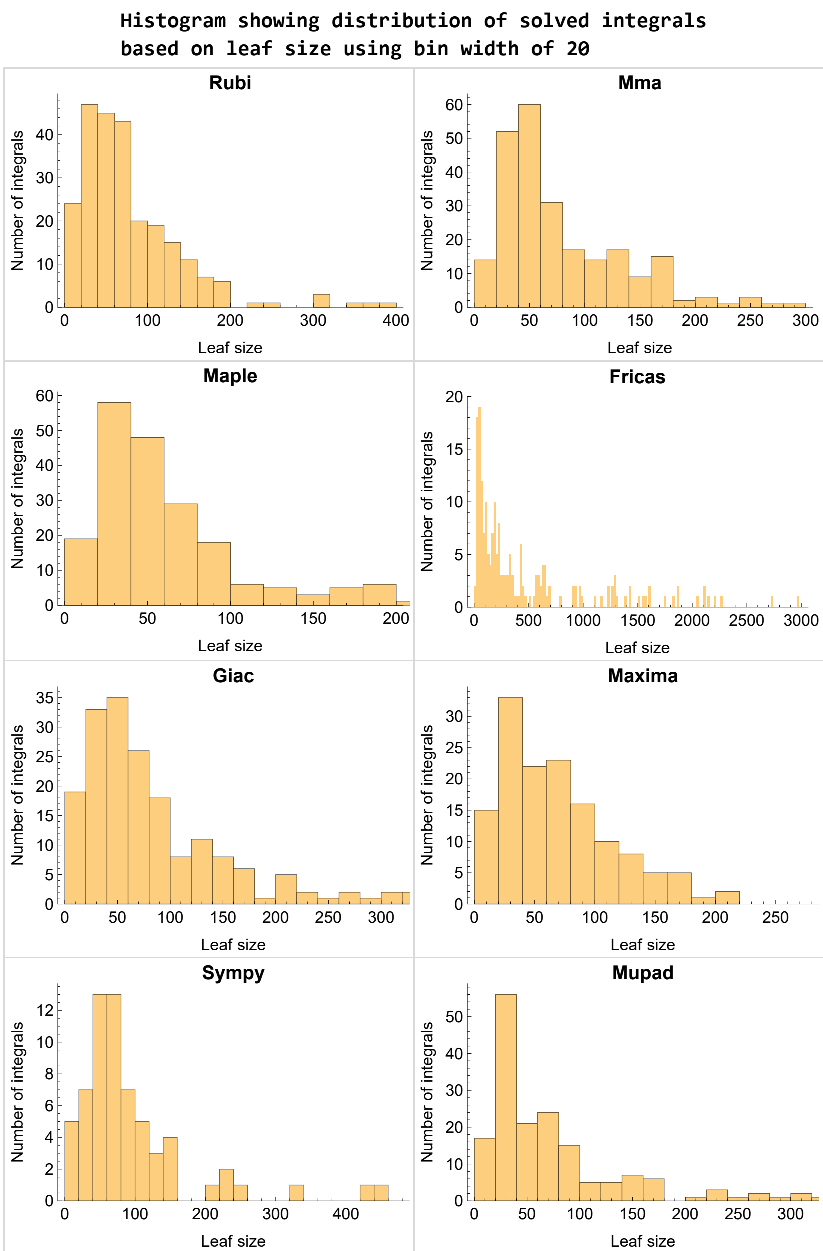


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

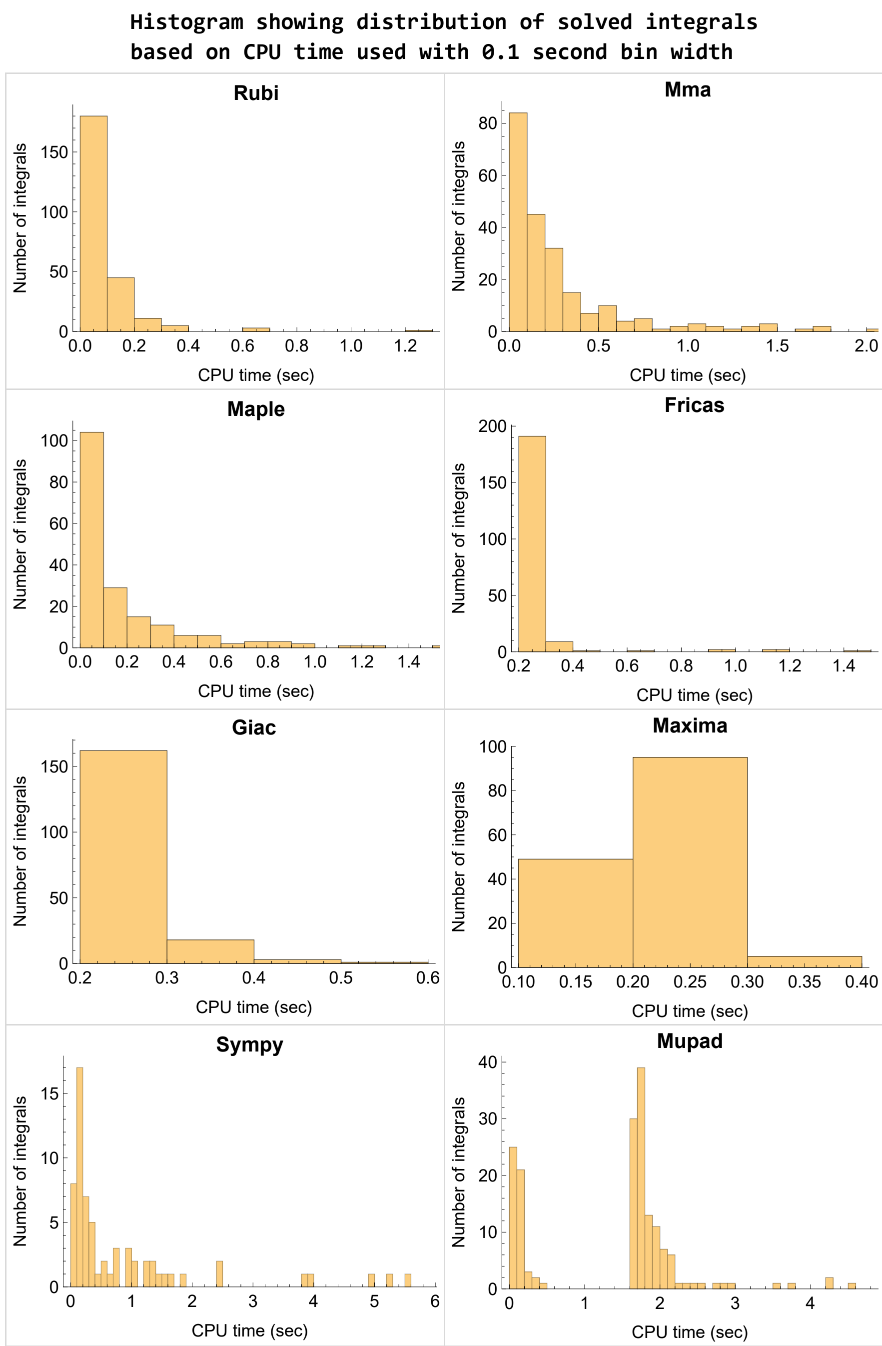


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

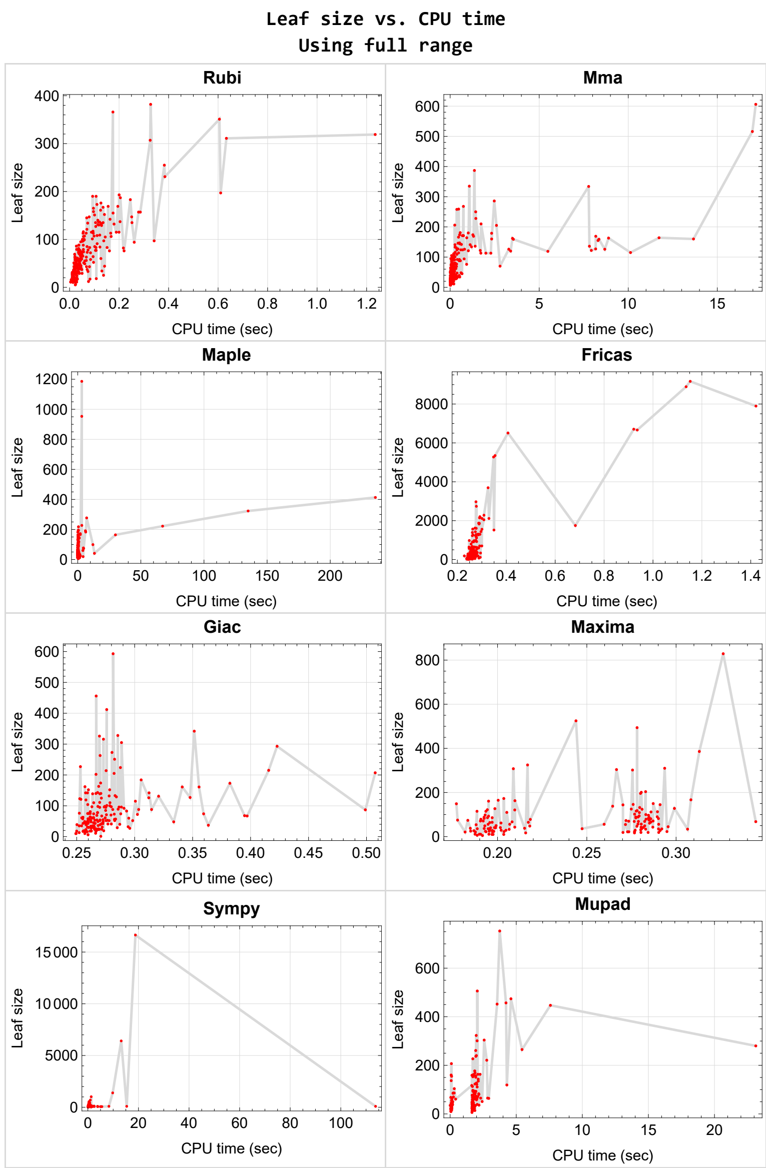


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{243, 247\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{163, 164, 168, 169, 170, 171, 192, 193, 238\}$

Maple $\{234, 235, 237, 238, 239\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

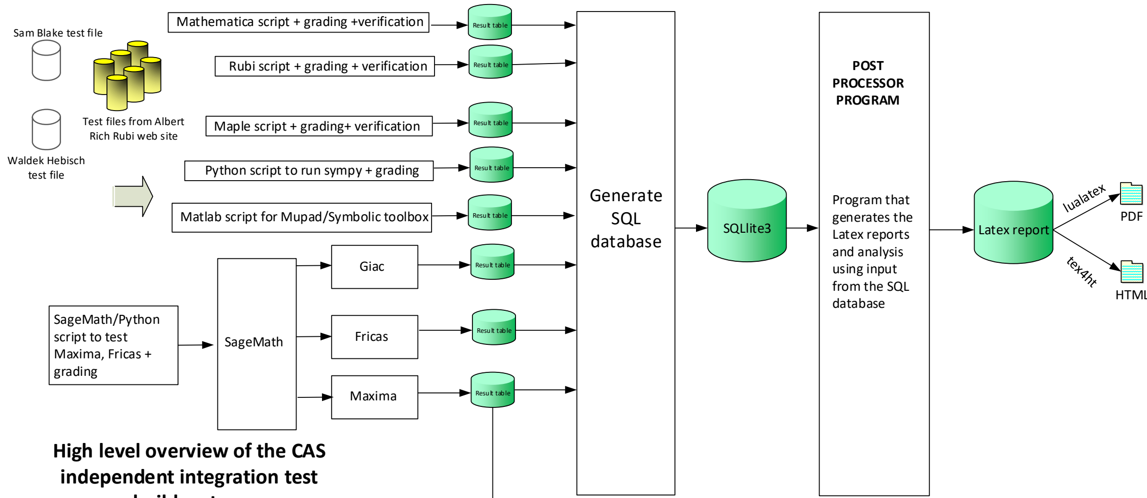
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 76, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 125, 126, 127, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 148, 150, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 176, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 215, 216, 226, 228, 229, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 244, 245, 246 }

B grade { 73, 75, 77, 79, 146, 163, 169, 170, 171, 172, 173, 174, 175, 177, 178, 190, 192, 230, 231 }

C grade { 8, 10, 12, 39, 54, 55, 56, 122, 123, 124, 128, 131, 147, 149, 151, 211, 212, 214, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 238 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 100, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 150, 153, 157, 176, 183, 186, 187, 188, 194, 195, 196, 197, 198, 199, 200, 201, 202, 205, 208, 209, 210, 211, 212, 213, 240, 241, 242, 244, 245, 246 }

B grade { 75, 89, 97, 99, 101, 102, 103, 104, 109, 144, 145 }

C grade { 147, 148, 149, 151, 152, 154, 155, 156, 158, 159, 206, 207, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239 }

F normal fail { 22, 23, 40, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 203, 204, 228, 229, 230, 231, 232, 233 }

F(-1) timedout fail { 243, 247 }

F(-2) exception fail { }

Fricas

A grade { 61, 65, 66, 71, 72, 84, 90, 93, 94, 95, 106, 112, 136, 137, 138, 139, 146, 148, 150, 152, 153, 155, 159, 209, 220, 236, 237 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 91, 92, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 157, 176, 183, 186, 187, 188, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 216, 217, 221, 234, 235, 238, 239 }

C grade { 27, 28, 29, 30, 31, 32, 147, 149, 151, 154, 156, 158, 214, 215, 218, 219, 222, 223, 224, 225, 226, 227, 240, 241, 242, 244, 245, 246 }

F normal fail { 22, 23, 40, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 228, 229, 230, 231, 232, 233 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 5, 6, 7, 8, 24, 25, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 56, 59, 60, 61, 62, 65, 66, 69, 70, 71, 72, 73, 74, 80, 82, 90, 91, 92, 93, 94, 95, 96, 105, 106, 107, 108, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 176, 183, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 234, 235, 236, 237, 238, 239 }

B grade { 1, 2, 3, 4, 9, 10, 11, 12, 26, 41, 42, 43, 51, 52, 53, 54, 55, 57, 58, 63, 64, 75, 76, 77, 78, 79, 85, 87, 89, 97, 98, 99, 100, 101, 102, 103, 104, 186, 187, 188 }

C grade { 27, 28, 29, 30, 31, 32 }

F normal fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 33, 34, 35, 36, 40, 125, 126, 127, 128, 129, 130, 131, 132, 144, 145, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 222, 223, 228, 229, 230, 231, 232, 233, 240, 241, 242, 244, 245, 246 }

F(-1) timedout fail { }

F(-2) exception fail { 67, 68, 81, 83, 84, 86, 88, 109, 110, 111, 112, 113, 114, 226, 227 }

Giac

A grade { 1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 16, 18, 19, 24, 26, 33, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 88, 90, 91, 92, 93, 94, 95, 98, 99, 100, 101, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 131, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 183, 187, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 234, 235, 236, 237, 238, 239 }

B grade { 6, 7, 13, 14, 15, 20, 21, 25, 34, 35, 50, 51, 55, 56, 75, 85, 87, 89, 96, 97, 102, 103, 104, 105, 109, 125, 126, 127, 128, 129, 130, 132, 143, 176, 186, 188, 226, 227 }

C grade { 27, 28, 29, 30, 31, 32 }

F normal fail { 22, 23, 40, 144, 145, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 200, 201, 205, 228, 229, 230, 231, 232, 233, 240, 241, 242, 244, 245, 246 }

F(-1) timedout fail { 194, 195, 196, 197, 198, 199, 202, 203, 204 }

F(-2) exception fail { 17, 67, 68 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 26, 30, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 176, 183, 186, 187, 188, 194, 195, 196, 197, 198, 199, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227 }

C grade { }

F normal fail { }

F(-1) timedout fail { 22, 23, 24, 25, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 144, 145, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 189, 190, 191, 192, 193, 200, 201, 202, 203, 204, 205, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 244, 245, 246 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 21, 41, 42, 43, 44, 57, 58, 59, 60, 65, 66, 95, 125, 126, 127, 128, 129, 130, 131, 132, 150, 157, 176, 186, 187, 188, 195, 196, 197, 198, 199 }

B grade { 6, 7, 8, 9, 10, 11, 12, 45, 46, 47, 48, 49, 61, 62, 63, 64, 70, 72, 92, 94, 106, 115, 116, 117, 118, 119, 120, 133, 134, 135, 136, 137, 138, 183 }

C grade { }

F normal fail { 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 50, 51, 52, 53, 54, 55, 56, 67, 68, 69, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 121, 122, 123, 124, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 180, 181, 182, 184, 185, 189, 190, 191, 192, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 240, 241, 242, 244, 245, 246 }

F(-1) timedout fail { 179, 193, 194, 234, 235, 239 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	53	39	115	254	39	74	34
N.S.	1	1.00	1.23	0.91	2.67	5.91	0.91	1.72	0.79
time (sec)	N/A	0.022	0.018	0.044	0.193	0.256	0.125	0.265	0.103

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	42	102	968	42	67	37
N.S.	1	1.00	0.88	1.00	2.43	23.05	1.00	1.60	0.88
time (sec)	N/A	0.026	0.083	0.034	0.277	0.268	0.107	0.272	0.111

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	38	27	71	119	27	52	24
N.S.	1	1.00	1.36	0.96	2.54	4.25	0.96	1.86	0.86
time (sec)	N/A	0.015	0.009	0.025	0.192	0.249	0.093	0.283	1.729

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	32	61	339	31	48	27
N.S.	1	1.00	1.00	1.19	2.26	12.56	1.15	1.78	1.00
time (sec)	N/A	0.014	0.013	0.024	0.285	0.252	0.077	0.262	1.735

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	23	17	25	33	15	24	13
N.S.	1	1.00	1.77	1.31	1.92	2.54	1.15	1.85	1.00
time (sec)	N/A	0.007	0.008	0.018	0.195	0.257	0.066	0.272	0.066

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	37	17	24	16
N.S.	1	1.00	1.00	1.09	1.00	3.36	1.55	2.18	1.45
time (sec)	N/A	0.005	0.007	0.033	0.195	0.261	0.067	0.266	1.694

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	12	11	37	27	25	21
N.S.	1	1.00	1.73	1.09	1.00	3.36	2.45	2.27	1.91
time (sec)	N/A	0.004	0.010	0.066	0.187	0.256	0.188	0.267	0.040

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	27	21	25	33	87	24	13
N.S.	1	1.00	2.08	1.62	1.92	2.54	6.69	1.85	1.00
time (sec)	N/A	0.007	0.012	0.028	0.185	0.239	0.529	0.265	1.662

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	38	79	346	112	49	68
N.S.	1	1.00	1.26	1.41	2.93	12.81	4.15	1.81	2.52
time (sec)	N/A	0.015	0.058	0.060	0.191	0.255	0.690	0.268	0.048

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	31	29	71	108	100	52	24
N.S.	1	1.00	1.11	1.04	2.54	3.86	3.57	1.86	0.86
time (sec)	N/A	0.014	0.012	0.053	0.190	0.249	1.017	0.278	0.058

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	44	48	122	978	126	70	159
N.S.	1	1.00	1.05	1.14	2.90	23.29	3.00	1.67	3.79
time (sec)	N/A	0.027	0.151	0.077	0.194	0.249	1.588	0.280	1.649

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	39	115	239	114	74	34
N.S.	1	1.00	0.72	0.91	2.67	5.56	2.65	1.72	0.79
time (sec)	N/A	0.022	0.009	0.071	0.188	0.246	2.420	0.288	0.066

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	80	80	0	1556	0	293	83
N.S.	1	1.00	0.82	0.82	0.00	16.04	0.00	3.02	0.86
time (sec)	N/A	0.055	0.291	0.097	0.000	0.301	0.000	0.423	2.158

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	63	0	980	0	173	62
N.S.	1	1.00	0.85	0.81	0.00	12.56	0.00	2.22	0.79
time (sec)	N/A	0.039	0.114	0.030	0.000	0.275	0.000	0.382	1.896

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	62	0	638	0	131	61
N.S.	1	1.00	0.88	0.83	0.00	8.51	0.00	1.75	0.81
time (sec)	N/A	0.037	0.055	0.025	0.000	0.272	0.000	0.321	1.798

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	47	0	593	0	88	41
N.S.	1	1.00	0.90	0.81	0.00	10.22	0.00	1.52	0.71
time (sec)	N/A	0.024	0.027	0.069	0.000	0.287	0.000	0.315	1.723

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	0	599	0	0	38
N.S.	1	1.00	0.86	0.81	0.00	10.51	0.00	0.00	0.67
time (sec)	N/A	0.022	0.026	0.049	0.000	0.262	0.000	0.000	1.793

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	74	65	0	924	0	48	64
N.S.	1	1.00	0.95	0.83	0.00	11.85	0.00	0.62	0.82
time (sec)	N/A	0.037	0.065	0.025	0.000	0.279	0.000	0.334	1.843

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	64	0	1436	0	87	63
N.S.	1	1.00	0.99	0.81	0.00	18.18	0.00	1.10	0.80
time (sec)	N/A	0.038	0.076	0.028	0.000	0.296	0.000	0.499	2.034

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	83	0	2144	0	207	80
N.S.	1	1.00	0.90	0.83	0.00	21.44	0.00	2.07	0.80
time (sec)	N/A	0.052	0.182	0.030	0.000	0.290	0.000	0.507	2.174

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	90	102	0	179	63	110	71
N.S.	1	1.00	1.30	1.48	0.00	2.59	0.91	1.59	1.03
time (sec)	N/A	0.043	0.083	0.053	0.000	0.255	1.281	0.279	2.091

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	49	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.096	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	30	42	467	0	52	0
N.S.	1	1.00	0.77	0.86	1.20	13.34	0.00	1.49	0.00
time (sec)	N/A	0.016	0.033	0.086	0.282	0.271	0.000	0.270	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	26	19	72	0	31	0
N.S.	1	1.00	1.00	1.62	1.19	4.50	0.00	1.94	0.00
time (sec)	N/A	0.011	0.009	0.043	0.284	0.266	0.000	0.265	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	20	29	31	76	0	1	14
N.S.	1	1.00	1.25	1.81	1.94	4.75	0.00	0.06	0.88
time (sec)	N/A	0.011	0.017	0.043	0.280	0.270	0.000	0.271	1.870

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	56	67	113	180	0	142	0
N.S.	1	1.00	0.64	0.76	1.28	2.05	0.00	1.61	0.00
time (sec)	N/A	0.038	0.231	0.090	0.290	0.265	0.000	0.312	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	46	53	66	100	0	92	0
N.S.	1	1.00	0.77	0.88	1.10	1.67	0.00	1.53	0.00
time (sec)	N/A	0.026	0.073	0.035	0.286	0.251	0.000	0.291	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	45	28	23	0	54	0
N.S.	1	1.00	1.00	1.45	0.90	0.74	0.00	1.74	0.00
time (sec)	N/A	0.014	0.027	0.041	0.291	0.262	0.000	0.267	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	52	45	23	0	61	24
N.S.	1	1.00	1.26	1.68	1.45	0.74	0.00	1.97	0.77
time (sec)	N/A	0.017	0.066	0.042	0.296	0.249	0.000	0.275	1.842

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	79	85	100	0	100	0
N.S.	1	1.00	0.85	1.32	1.42	1.67	0.00	1.67	0.00
time (sec)	N/A	0.027	0.100	0.039	0.277	0.255	0.000	0.279	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	C	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63	91	132	180	0	126	0
N.S.	1	1.00	0.72	1.03	1.50	2.05	0.00	1.43	0.00
time (sec)	N/A	0.037	0.180	0.040	0.280	0.284	0.000	0.312	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	38	43	0	106	0	55	0
N.S.	1	1.00	0.67	0.75	0.00	1.86	0.00	0.96	0.00
time (sec)	N/A	0.028	0.033	0.096	0.000	0.274	0.000	0.273	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	55	76	0	1269	0	342	0
N.S.	1	1.00	0.64	0.88	0.00	14.76	0.00	3.98	0.00
time (sec)	N/A	0.027	0.047	0.088	0.000	0.286	0.000	0.351	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	40	59	0	376	0	115	0
N.S.	1	1.00	0.63	0.94	0.00	5.97	0.00	1.83	0.00
time (sec)	N/A	0.021	0.027	0.044	0.000	0.268	0.000	0.301	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	53	65	0	516	0	41	0
N.S.	1	1.00	0.83	1.02	0.00	8.06	0.00	0.64	0.00
time (sec)	N/A	0.022	0.033	0.055	0.000	0.264	0.000	0.280	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	39	46	82	2114	0	45	0
N.S.	1	1.00	0.57	0.67	1.19	30.64	0.00	0.65	0.00
time (sec)	N/A	0.020	0.067	0.084	0.290	0.300	0.000	0.265	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	21	32	19	213	0	16	0
N.S.	1	1.00	0.68	1.03	0.61	6.87	0.00	0.52	0.00
time (sec)	N/A	0.011	0.014	0.044	0.290	0.280	0.000	0.259	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	26	32	23	238	0	19	0
N.S.	1	1.00	0.84	1.03	0.74	7.68	0.00	0.61	0.00
time (sec)	N/A	0.011	0.017	0.049	0.295	0.256	0.000	0.273	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.095	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	63	58	302	907	95	85	65
N.S.	1	1.00	0.63	0.58	3.02	9.07	0.95	0.85	0.65
time (sec)	N/A	0.059	0.249	0.103	0.276	0.262	0.131	0.275	0.148

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	196	562	76	71	53
N.S.	1	1.00	0.66	0.62	2.55	7.30	0.99	0.92	0.69
time (sec)	N/A	0.042	0.137	0.051	0.280	0.251	0.116	0.276	1.700

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	40	38	116	299	61	57	43
N.S.	1	1.00	0.71	0.68	2.07	5.34	1.09	1.02	0.77
time (sec)	N/A	0.028	0.166	0.041	0.278	0.249	0.100	0.284	1.685

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	29	28	50	117	44	39	33
N.S.	1	1.00	0.81	0.78	1.39	3.25	1.22	1.08	0.92
time (sec)	N/A	0.016	0.127	0.027	0.199	0.255	0.087	0.270	0.087

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	33	25	31	50	73	30	25
N.S.	1	1.00	1.18	0.89	1.11	1.79	2.61	1.07	0.89
time (sec)	N/A	0.010	0.070	0.031	0.196	0.261	0.361	0.264	1.693

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	42	43	101	223	42	41
N.S.	1	1.00	0.92	0.82	0.84	1.98	4.37	0.82	0.80
time (sec)	N/A	0.022	0.151	0.039	0.210	0.252	0.566	0.258	1.655

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	59	56	160	430	53	58
N.S.	1	1.00	0.81	0.81	0.77	2.19	5.89	0.73	0.79
time (sec)	N/A	0.037	0.191	0.041	0.207	0.263	0.748	0.266	1.739

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	96	76	67	220	694	64	75
N.S.	1	1.00	1.00	0.79	0.70	2.29	7.23	0.67	0.78
time (sec)	N/A	0.052	0.183	0.065	0.208	0.265	0.963	0.262	1.734

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	91	78	287	1018	75	92
N.S.	1	1.00	0.92	0.75	0.64	2.37	8.41	0.62	0.76
time (sec)	N/A	0.070	0.206	0.086	0.218	0.257	1.314	0.260	1.752

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	43	83	434	0	140	44
N.S.	1	1.00	0.82	0.75	1.46	7.61	0.00	2.46	0.77
time (sec)	N/A	0.033	0.389	0.054	0.284	0.274	0.000	0.267	0.168

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	70	252	0	96	54
N.S.	1	1.00	0.87	0.78	1.56	5.60	0.00	2.13	1.20
time (sec)	N/A	0.023	0.301	0.036	0.273	0.252	0.000	0.274	0.105

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	57	129	0	52	26
N.S.	1	1.00	1.00	0.82	1.73	3.91	0.00	1.58	0.79
time (sec)	N/A	0.017	0.229	0.028	0.281	0.270	0.000	0.275	1.707

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	43	50	0	27	16
N.S.	1	1.00	1.00	0.81	2.05	2.38	0.00	1.29	0.76
time (sec)	N/A	0.011	0.162	0.060	0.287	0.253	0.000	0.296	0.117

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	27	57	85	0	50	26
N.S.	1	1.00	0.81	0.84	1.78	2.66	0.00	1.56	0.81
time (sec)	N/A	0.017	0.193	0.052	0.279	0.249	0.000	0.259	0.120

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	35	69	166	0	95	32
N.S.	1	1.00	0.57	0.71	1.41	3.39	0.00	1.94	0.65
time (sec)	N/A	0.026	0.230	0.043	0.279	0.249	0.000	0.275	0.106

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	28	43	79	266	0	139	40
N.S.	1	1.00	0.46	0.70	1.30	4.36	0.00	2.28	0.66
time (sec)	N/A	0.032	0.271	0.044	0.283	0.263	0.000	0.270	1.674

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	114	182	310	2739	211	224	153
N.S.	1	1.00	0.80	1.28	2.18	19.29	1.49	1.58	1.08
time (sec)	N/A	0.163	0.476	0.085	0.294	0.278	0.147	0.288	1.720

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	132	201	1389	144	152	113
N.S.	1	1.00	0.90	1.31	1.99	13.75	1.43	1.50	1.12
time (sec)	N/A	0.092	0.248	0.054	0.281	0.261	0.117	0.278	1.686

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	93	118	646	100	97	77
N.S.	1	1.00	0.97	1.35	1.71	9.36	1.45	1.41	1.12
time (sec)	N/A	0.050	0.244	0.036	0.278	0.261	0.102	0.280	1.664

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	54	52	49	201	54	56	44
N.S.	1	1.00	1.42	1.37	1.29	5.29	1.42	1.47	1.16
time (sec)	N/A	0.018	0.081	0.023	0.193	0.242	0.080	0.261	0.076

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	64	55	56	62	224	62	60
N.S.	1	1.00	1.28	1.10	1.12	1.24	4.48	1.24	1.20
time (sec)	N/A	0.041	0.069	0.034	0.195	0.260	1.012	0.265	1.741

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	94	93	127	422	1389	132	127
N.S.	1	1.00	1.11	1.09	1.49	4.96	16.34	1.55	1.49
time (sec)	N/A	0.071	0.786	0.046	0.199	0.253	9.807	0.278	1.986

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	122	130	325	1427	6412	205	304
N.S.	1	1.00	0.95	1.01	2.52	11.06	49.71	1.59	2.36
time (sec)	N/A	0.129	1.689	0.089	0.217	0.274	13.181	0.282	2.573

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	160	163	525	3693	16643	305	452
N.S.	1	1.00	0.95	0.96	3.11	21.85	98.48	1.80	2.67
time (sec)	N/A	0.195	2.330	0.122	0.244	0.326	18.800	0.289	3.553

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	28	28	49	42	30	34
N.S.	1	1.00	1.71	0.90	0.90	1.58	1.35	0.97	1.10
time (sec)	N/A	0.030	0.029	0.030	0.197	0.260	0.263	0.265	0.128

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	28	29	48	42	25	33
N.S.	1	1.00	1.71	0.90	0.94	1.55	1.35	0.81	1.06
time (sec)	N/A	0.029	0.024	0.030	0.205	0.272	0.232	0.266	0.110

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	63	0	2203	0	0	151
N.S.	1	1.00	1.00	0.85	0.00	29.77	0.00	0.00	2.04
time (sec)	N/A	0.057	0.064	0.158	0.000	0.292	0.000	0.000	1.940

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	62	0	2279	0	0	240
N.S.	1	1.00	1.00	0.84	0.00	30.80	0.00	0.00	3.24
time (sec)	N/A	0.051	0.049	0.057	0.000	0.309	0.000	0.000	1.985

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	35	36	92	0	42	34
N.S.	1	1.00	0.70	0.58	0.60	1.53	0.00	0.70	0.57
time (sec)	N/A	0.048	0.099	1.610	0.189	0.260	0.000	0.272	1.905

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	24	27	56	134	25	23
N.S.	1	1.00	1.36	0.96	1.08	2.24	5.36	1.00	0.92
time (sec)	N/A	0.137	0.087	0.552	0.187	0.253	0.340	0.269	1.769

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	23	22	51	0	30	22
N.S.	1	1.00	0.63	0.61	0.58	1.34	0.00	0.79	0.58
time (sec)	N/A	0.045	0.015	0.230	0.194	0.246	0.000	0.260	1.698

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	12	11	23	48	11	11
N.S.	1	1.00	1.12	0.71	0.65	1.35	2.82	0.65	0.65
time (sec)	N/A	0.082	0.014	0.106	0.190	0.239	0.194	0.266	1.661

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	49	17	21	38	0	18	21
N.S.	1	1.00	4.08	1.42	1.75	3.17	0.00	1.50	1.75
time (sec)	N/A	0.077	0.111	0.086	0.188	0.249	0.000	0.262	0.057

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	24	29	77	0	29	23
N.S.	1	1.00	0.73	1.60	1.93	5.13	0.00	1.93	1.53
time (sec)	N/A	0.028	0.205	0.112	0.202	0.259	0.000	0.285	1.648

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	59	33	48	209	0	34	48
N.S.	1	1.00	3.28	1.83	2.67	11.61	0.00	1.89	2.67
time (sec)	N/A	0.108	0.219	0.214	0.218	0.245	0.000	0.262	1.649

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	19	75	84	0	18	18
N.S.	1	1.00	1.18	1.12	4.41	4.94	0.00	1.06	1.06
time (sec)	N/A	0.032	0.192	0.474	0.178	0.241	0.000	0.263	0.076

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	69	48	74	640	0	49	117
N.S.	1	1.00	2.03	1.41	2.18	18.82	0.00	1.44	3.44
time (sec)	N/A	0.131	0.353	0.995	0.183	0.248	0.000	0.257	1.634

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	27	25	149	185	0	24	24
N.S.	1	1.00	0.82	0.76	4.52	5.61	0.00	0.73	0.73
time (sec)	N/A	0.036	0.246	2.121	0.177	0.229	0.000	0.270	1.698

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	124	60	98	1260	0	61	207
N.S.	1	1.00	2.82	1.36	2.23	28.64	0.00	1.39	4.70
time (sec)	N/A	0.141	0.382	4.120	0.193	0.264	0.000	0.263	0.094

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	183	163	1226	0	214	135
N.S.	1	1.00	0.98	1.24	1.11	8.34	0.00	1.46	0.92
time (sec)	N/A	0.251	0.545	6.355	0.210	0.275	0.000	0.275	2.129

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	180	166	0	1861	0	163	261
N.S.	1	1.00	1.31	1.21	0.00	13.58	0.00	1.19	1.91
time (sec)	N/A	0.204	1.017	2.071	0.000	0.290	0.000	0.269	1.934

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	73	98	83	334	0	101	81
N.S.	1	1.00	0.87	1.17	0.99	3.98	0.00	1.20	0.96
time (sec)	N/A	0.123	0.154	0.564	0.193	0.261	0.000	0.276	1.795

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	427	0	60	157
N.S.	1	1.00	1.10	1.28	0.00	5.93	0.00	0.83	2.18
time (sec)	N/A	0.085	0.173	0.217	0.000	0.270	0.000	0.265	1.730

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	68	53	0	237	0	60	177
N.S.	1	1.00	1.31	1.02	0.00	4.56	0.00	1.15	3.40
time (sec)	N/A	0.097	0.170	0.157	0.000	0.271	0.000	0.263	1.858

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	50	65	122	0	78	323
N.S.	1	1.00	0.97	1.72	2.24	4.21	0.00	2.69	11.14
time (sec)	N/A	0.040	0.327	0.192	0.217	0.261	0.000	0.268	1.973

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	147	110	0	1165	0	125	506
N.S.	1	1.00	1.79	1.34	0.00	14.21	0.00	1.52	6.17
time (sec)	N/A	0.217	0.528	0.544	0.000	0.277	0.000	0.271	2.054

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	70	143	161	912	0	202	123
N.S.	1	1.00	0.90	1.83	2.06	11.69	0.00	2.59	1.58
time (sec)	N/A	0.075	2.801	1.271	0.195	0.267	0.000	0.267	1.877

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	335	227	0	5347	0	273	753
N.S.	1	1.00	1.31	0.89	0.00	20.97	0.00	1.07	2.95
time (sec)	N/A	0.383	1.078	3.251	0.000	0.354	0.000	0.280	3.750

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	119	277	308	2972	0	412	237
N.S.	1	1.00	0.92	2.13	2.37	22.86	0.00	3.17	1.82
time (sec)	N/A	0.118	5.487	7.081	0.209	0.276	0.000	0.276	1.945

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	46	29	34	43	0	28	61
N.S.	1	1.00	1.39	0.88	1.03	1.30	0.00	0.85	1.85
time (sec)	N/A	0.074	0.217	0.172	0.283	0.255	0.000	0.262	0.414

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	35	36	95	0	42	34
N.S.	1	1.00	0.88	0.58	0.60	1.58	0.00	0.70	0.57
time (sec)	N/A	0.044	0.087	1.126	0.195	0.245	0.000	0.273	1.895

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	36	30	33	60	134	31	29
N.S.	1	1.00	1.24	1.03	1.14	2.07	4.62	1.07	1.00
time (sec)	N/A	0.028	0.099	0.358	0.206	0.258	0.332	0.272	1.780

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	23	22	52	0	30	22
N.S.	1	1.00	0.84	0.61	0.58	1.37	0.00	0.79	0.58
time (sec)	N/A	0.035	0.091	0.162	0.197	0.246	0.000	0.260	0.121

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	18	17	25	48	19	17
N.S.	1	1.00	1.21	0.95	0.89	1.32	2.53	1.00	0.89
time (sec)	N/A	0.021	0.066	0.089	0.216	0.248	0.186	0.263	1.692

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	6	9	8	6	6
N.S.	1	1.00	0.70	0.70	0.60	0.90	0.80	0.60	0.60
time (sec)	N/A	0.014	0.026	0.177	0.189	0.244	0.156	0.260	1.651

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	7	6	5	20	0	13	13
N.S.	1	1.00	1.40	1.20	1.00	4.00	0.00	2.60	2.60
time (sec)	N/A	0.023	0.006	0.597	0.192	0.244	0.000	0.252	1.677

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	12	21	22	48	0	18	18
N.S.	1	1.00	2.00	3.50	3.67	8.00	0.00	3.00	3.00
time (sec)	N/A	0.024	0.102	1.713	0.274	0.238	0.000	0.267	0.072

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	10	37	53	0	10	16
N.S.	1	1.00	0.91	0.91	3.36	4.82	0.00	0.91	1.45
time (sec)	N/A	0.024	0.036	0.929	0.193	0.244	0.000	0.257	1.672

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	41	49	288	0	31	61
N.S.	1	1.00	1.00	1.71	2.04	12.00	0.00	1.29	2.54
time (sec)	N/A	0.029	0.100	13.255	0.282	0.243	0.000	0.259	1.692

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	19	93	140	0	18	18
N.S.	1	1.00	0.96	0.76	3.72	5.60	0.00	0.72	0.72
time (sec)	N/A	0.028	0.040	4.268	0.195	0.250	0.000	0.256	0.076

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	67	73	670	0	45	137
N.S.	1	1.00	1.00	1.97	2.15	19.71	0.00	1.32	4.03
time (sec)	N/A	0.034	0.114	0.059	0.272	0.260	0.000	0.259	0.079

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	127	413	386	5275	0	593	301
N.S.	1	1.00	0.91	2.95	2.76	37.68	0.00	4.24	2.15
time (sec)	N/A	0.114	0.439	235.720	0.313	0.348	0.000	0.281	2.039

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	80	222	204	1827	0	316	169
N.S.	1	1.00	0.96	2.67	2.46	22.01	0.00	3.81	2.04
time (sec)	N/A	0.081	0.239	67.233	0.283	0.289	0.000	0.273	1.885

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	99	89	430	0	104	88
N.S.	1	1.00	1.02	2.48	2.22	10.75	0.00	2.60	2.20
time (sec)	N/A	0.046	0.100	12.036	0.277	0.259	0.000	0.260	1.891

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	42	0	45	50
N.S.	1	1.00	1.00	1.09	1.00	3.82	0.00	4.09	4.55
time (sec)	N/A	0.029	0.032	1.589	0.191	0.261	0.000	0.257	0.194

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	49	42	41	42	146	43	35
N.S.	1	1.00	1.26	1.08	1.05	1.08	3.74	1.10	0.90
time (sec)	N/A	0.035	0.073	0.040	0.198	0.260	0.228	0.266	0.141

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	75	104	86	331	0	111	84
N.S.	1	1.00	0.82	1.14	0.95	3.64	0.00	1.22	0.92
time (sec)	N/A	0.111	0.167	0.590	0.197	0.267	0.000	0.270	1.894

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	206	191	165	1281	0	227	143
N.S.	1	1.00	1.33	1.23	1.06	8.26	0.00	1.46	0.92
time (sec)	N/A	0.176	0.259	6.091	0.200	0.277	0.000	0.253	2.124

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	187	166	323	0	6509	0	326	447
N.S.	1	1.19	1.06	2.06	0.00	41.46	0.00	2.08	2.85
time (sec)	N/A	0.205	0.469	134.960	0.000	0.407	0.000	0.270	7.585

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	109	116	164	0	2043	0	152	265
N.S.	1	1.07	1.14	1.61	0.00	20.03	0.00	1.49	2.60
time (sec)	N/A	0.122	0.285	29.880	0.000	0.308	0.000	0.260	5.439

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	65	77	0	309	0	63	119
N.S.	1	1.00	1.16	1.38	0.00	5.52	0.00	1.12	2.12
time (sec)	N/A	0.067	0.154	4.488	0.000	0.277	0.000	0.255	4.291

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	46	39	0	148	0	35	35
N.S.	1	1.00	1.24	1.05	0.00	4.00	0.00	0.95	0.95
time (sec)	N/A	0.024	0.030	0.401	0.000	0.249	0.000	0.252	0.118

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	93	0	435	0	61	157
N.S.	1	1.00	1.10	1.27	0.00	5.96	0.00	0.84	2.15
time (sec)	N/A	0.071	0.231	0.226	0.000	0.268	0.000	0.254	1.932

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	258	172	0	1871	0	162	221
N.S.	1	1.00	1.95	1.30	0.00	14.17	0.00	1.23	1.67
time (sec)	N/A	0.128	0.375	2.048	0.000	0.287	0.000	0.257	2.768

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	52	40	55	571	104	47	35
N.S.	1	1.00	1.21	0.93	1.28	13.28	2.42	1.09	0.81
time (sec)	N/A	0.068	0.134	0.059	0.272	0.248	0.210	0.260	0.097

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	45	30	43	354	85	39	29
N.S.	1	1.00	1.22	0.81	1.16	9.57	2.30	1.05	0.78
time (sec)	N/A	0.050	0.089	0.052	0.280	0.282	0.194	0.258	0.077

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	41	28	29	186	75	35	21
N.S.	1	1.00	1.32	0.90	0.94	6.00	2.42	1.13	0.68
time (sec)	N/A	0.037	0.117	0.049	0.277	0.262	0.179	0.254	0.069

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	34	18	17	73	61	17	21
N.S.	1	1.00	1.79	0.95	0.89	3.84	3.21	0.89	1.11
time (sec)	N/A	0.026	0.075	0.045	0.277	0.247	0.168	0.255	0.068

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	11	10	26	27	10	10
N.S.	1	1.00	0.88	0.69	0.62	1.62	1.69	0.62	0.62
time (sec)	N/A	0.015	0.077	0.044	0.197	0.246	0.159	0.249	0.064

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	11	10	26	27	10	10
N.S.	1	1.00	1.12	0.69	0.62	1.62	1.69	0.62	0.62
time (sec)	N/A	0.007	0.026	0.038	0.197	0.247	0.156	0.249	0.055

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	34	18	24	73	0	18	17
N.S.	1	1.00	1.79	0.95	1.26	3.84	0.00	0.95	0.89
time (sec)	N/A	0.030	0.101	0.128	0.193	0.259	0.000	0.250	0.062

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	47	30	38	196	0	36	29
N.S.	1	1.00	1.62	1.03	1.31	6.76	0.00	1.24	1.00
time (sec)	N/A	0.051	0.219	0.198	0.185	0.257	0.000	0.269	1.659

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	54	30	54	357	0	40	35
N.S.	1	1.00	1.46	0.81	1.46	9.65	0.00	1.08	0.95
time (sec)	N/A	0.071	0.221	0.213	0.188	0.252	0.000	0.287	0.075

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	53	44	64	582	0	48	69
N.S.	1	1.00	1.23	1.02	1.49	13.53	0.00	1.12	1.60
time (sec)	N/A	0.080	0.251	0.227	0.196	0.252	0.000	0.273	1.684

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	0	252	58	96	34
N.S.	1	1.00	0.87	0.78	0.00	5.60	1.29	2.13	0.76
time (sec)	N/A	0.040	0.508	0.057	0.000	0.266	3.902	0.287	1.740

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	0	129	48	53	25
N.S.	1	1.00	1.00	0.81	0.00	4.03	1.50	1.66	0.78
time (sec)	N/A	0.026	0.323	0.063	0.000	0.256	0.730	0.277	1.699

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	85	46	52	24
N.S.	1	1.00	1.00	0.83	0.00	2.83	1.53	1.73	0.80
time (sec)	N/A	0.028	0.373	0.068	0.000	0.263	1.268	0.262	0.124

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	36	35	0	168	60	73	32
N.S.	1	1.00	0.73	0.71	0.00	3.43	1.22	1.49	0.65
time (sec)	N/A	0.038	0.423	0.063	0.000	0.259	4.981	0.282	0.113

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	35	0	429	58	140	34
N.S.	1	1.00	1.00	0.78	0.00	9.53	1.29	3.11	0.76
time (sec)	N/A	0.046	0.667	0.057	0.000	0.268	5.294	0.271	1.756

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	26	0	237	49	96	25
N.S.	1	1.00	1.00	0.76	0.00	6.97	1.44	2.82	0.74
time (sec)	N/A	0.034	0.416	0.066	0.000	0.264	0.997	0.289	0.103

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	34	35	0	182	58	54	36
N.S.	1	1.00	0.81	0.83	0.00	4.33	1.38	1.29	0.86
time (sec)	N/A	0.045	0.537	0.067	0.000	0.254	1.473	0.270	0.118

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	35	0	168	60	95	31
N.S.	1	1.00	0.98	0.71	0.00	3.43	1.22	1.94	0.63
time (sec)	N/A	0.061	0.610	0.075	0.000	0.243	5.546	0.270	1.693

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	92	96	150	1296	546	142	85
N.S.	1	1.00	0.98	1.02	1.60	13.79	5.81	1.51	0.90
time (sec)	N/A	0.261	0.515	0.087	0.287	0.292	0.445	0.259	0.215

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	76	100	644	442	98	68
N.S.	1	1.00	1.01	1.00	1.32	8.47	5.82	1.29	0.89
time (sec)	N/A	0.161	0.362	0.077	0.283	0.268	0.385	0.268	1.761

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	67	71	264	330	75	59
N.S.	1	1.00	1.02	1.05	1.11	4.12	5.16	1.17	0.92
time (sec)	N/A	0.095	0.266	0.069	0.287	0.264	0.306	0.262	0.125

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	52	56	76	243	58	46
N.S.	1	1.00	0.94	0.83	0.89	1.21	3.86	0.92	0.73
time (sec)	N/A	0.068	0.150	0.048	0.260	0.262	0.259	0.255	1.741

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	40	40	43	141	43	36
N.S.	1	1.00	1.28	1.03	1.03	1.10	3.62	1.10	0.92
time (sec)	N/A	0.044	0.138	0.040	0.199	0.254	0.214	0.260	1.729

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	49	42	41	42	146	43	35
N.S.	1	1.00	1.26	1.08	1.05	1.08	3.74	1.10	0.90
time (sec)	N/A	0.039	0.007	0.028	0.202	0.258	0.210	0.255	0.002

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	65	56	65	73	0	58	58
N.S.	1	1.00	1.27	1.10	1.27	1.43	0.00	1.14	1.14
time (sec)	N/A	0.057	0.124	0.142	0.197	0.288	0.000	0.260	1.993

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	66	78	86	271	0	75	73
N.S.	1	1.00	1.10	1.30	1.43	4.52	0.00	1.25	1.22
time (sec)	N/A	0.126	0.240	0.136	0.194	0.277	0.000	0.259	1.967

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	92	121	641	0	97	111
N.S.	1	1.00	1.00	1.21	1.59	8.43	0.00	1.28	1.46
time (sec)	N/A	0.220	0.255	0.195	0.210	0.267	0.000	0.263	2.043

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	92	114	173	1299	0	142	163
N.S.	1	1.00	0.95	1.18	1.78	13.39	0.00	1.46	1.68
time (sec)	N/A	0.342	0.400	0.220	0.203	0.283	0.000	0.258	2.100

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	73	68	182	0	174	69
N.S.	1	1.00	0.89	1.33	1.24	3.31	0.00	3.16	1.25
time (sec)	N/A	0.062	0.132	4.689	0.345	0.257	0.000	0.271	1.794

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	250	953	0	1516	0	0	0
N.S.	1	1.00	1.08	4.13	0.00	6.56	0.00	0.00	0.00
time (sec)	N/A	0.385	1.433	3.199	0.000	0.350	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	268	1186	0	2110	0	0	0
N.S.	1	1.00	0.76	3.38	0.00	6.01	0.00	0.00	0.00
time (sec)	N/A	0.605	0.744	3.212	0.000	0.329	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	64	24	23	28	0	23	21
N.S.	1	1.00	2.21	0.83	0.79	0.97	0.00	0.79	0.72
time (sec)	N/A	0.038	0.024	0.205	0.204	0.258	0.000	0.258	1.713

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	64	37	128	113	0	123	47
N.S.	1	1.00	0.42	0.25	0.85	0.75	0.00	0.81	0.31
time (sec)	N/A	0.101	0.231	0.069	0.299	0.262	0.000	0.253	1.720

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	35	41	19	22	0	19	25
N.S.	1	1.00	1.52	1.78	0.83	0.96	0.00	0.83	1.09
time (sec)	N/A	0.017	0.144	0.091	0.270	0.245	0.000	0.256	1.689

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	58	33	124	92	0	119	44
N.S.	1	1.00	0.40	0.23	0.86	0.63	0.00	0.82	0.30
time (sec)	N/A	0.069	0.146	0.043	0.281	0.261	0.000	0.258	1.713

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	18	15	20	15
N.S.	1	1.00	1.00	0.92	0.83	1.50	1.25	1.67	1.25
time (sec)	N/A	0.011	0.022	0.048	0.203	0.248	0.093	0.266	1.785

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	59	42	125	120	0	121	45
N.S.	1	1.00	0.40	0.29	0.85	0.82	0.00	0.82	0.31
time (sec)	N/A	0.079	0.139	0.059	0.275	0.260	0.000	0.254	1.732

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	40	44	19	21	0	16	24
N.S.	1	1.00	2.00	2.20	0.95	1.05	0.00	0.80	1.20
time (sec)	N/A	0.020	0.131	0.064	0.280	0.251	0.000	0.258	1.711

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	86	42	40	58	0	39	39
N.S.	1	1.00	1.83	0.89	0.85	1.23	0.00	0.83	0.83
time (sec)	N/A	0.043	0.082	0.094	0.196	0.256	0.000	0.250	1.759

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	174	53	144	178	0	139	67
N.S.	1	1.00	1.01	0.31	0.83	1.03	0.00	0.80	0.39
time (sec)	N/A	0.111	0.531	0.067	0.270	0.263	0.000	0.264	1.764

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	41	57	35	50	0	35	41
N.S.	1	1.00	1.02	1.42	0.88	1.25	0.00	0.88	1.02
time (sec)	N/A	0.033	0.308	0.068	0.292	0.261	0.000	0.265	1.718

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	146	47	138	156	0	133	61
N.S.	1	1.00	0.88	0.28	0.84	0.95	0.00	0.81	0.37
time (sec)	N/A	0.096	0.442	0.051	0.264	0.253	0.000	0.283	1.722

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	24	13	21	28	12	19	28
N.S.	1	1.00	1.71	0.93	1.50	2.00	0.86	1.36	2.00
time (sec)	N/A	0.018	0.040	0.037	0.182	0.240	0.115	0.257	1.690

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	181	64	146	180	0	143	68
N.S.	1	1.00	0.95	0.34	0.77	0.95	0.00	0.75	0.36
time (sec)	N/A	0.093	0.565	0.056	0.276	0.256	0.000	0.268	1.769

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	40	66	37	51	0	39	47
N.S.	1	1.00	0.68	1.12	0.63	0.86	0.00	0.66	0.80
time (sec)	N/A	0.039	0.310	0.066	0.281	0.249	0.000	0.258	1.749

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	47	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.099	0.000	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.163	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	111	0	0	0	0	0	0
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	0.201	0.000	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	259	0	0	0	0	0	0
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.028	0.476	0.000	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	126	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.059	0.808	0.000	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	51	51	76	0	0	0	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	0.925	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	121	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	1.022	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	158	158	141	0	0	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	1.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	228	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.107	1.442	0.000	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.684	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.722	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	0.750	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	127	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	8.170	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	136	0	0	0	0	0	0
N.S.	1	1.00	2.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.046	7.819	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	122	0	0	0	0	0	0
N.S.	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	7.926	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	53	53	126	0	0	0	0	0	0
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	8.683	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	25	24	76	36	74	34
N.S.	1	1.00	0.96	1.00	0.96	3.04	1.44	2.96	1.36
time (sec)	N/A	0.015	0.061	0.097	0.198	0.269	1.385	0.360	1.681

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	126	0	0	0	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.043	3.292	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	56	56	120	0	0	0	0	0	0
N.S.	1	1.00	2.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.044	3.396	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	159	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.129	8.355	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	169	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	8.176	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	155	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.114	8.303	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	163	0	0	0	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	8.894	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	51	33	36	72	70	37	34
N.S.	1	1.00	1.82	1.18	1.29	2.57	2.50	1.32	1.21
time (sec)	N/A	0.021	0.123	0.100	0.247	0.253	2.498	0.364	1.686

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	162	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	3.500	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	159	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.122	3.550	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	47	304	566	68	127	94
N.S.	1	1.00	1.00	1.09	7.07	13.16	1.58	2.95	2.19
time (sec)	N/A	0.031	0.163	0.172	0.267	0.257	0.752	0.348	1.829

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	62	42	494	194	65	67	162
N.S.	1	1.00	1.38	0.93	10.98	4.31	1.44	1.49	3.60
time (sec)	N/A	0.033	0.097	0.328	0.278	0.269	1.652	0.397	1.761

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	55	62	829	1568	87	161	227
N.S.	1	1.00	0.83	0.94	12.56	23.76	1.32	2.44	3.44
time (sec)	N/A	0.043	0.215	0.648	0.326	0.263	3.837	0.356	1.716

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	160	0	0	0	0	0	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.055	13.664	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	516	0	0	0	0	0	0
N.S.	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	16.970	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	307	307	606	0	0	0	0	0	0
N.S.	1	1.00	1.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.326	17.161	0.000	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	387	0	0	0	0	0	0
N.S.	1	1.00	3.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	1.362	0.000	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	174	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	1.276	0.000	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	76	0	625	0	0	65
N.S.	1	1.00	0.85	1.04	0.00	8.56	0.00	0.00	0.89
time (sec)	N/A	0.045	0.324	0.329	0.000	0.267	0.000	0.000	2.834

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	74	0	334	87	0	51
N.S.	1	1.00	0.81	1.06	0.00	4.77	1.24	0.00	0.73
time (sec)	N/A	0.038	0.159	0.233	0.000	0.295	15.352	0.000	2.421

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	61	0	303	66	0	39
N.S.	1	1.00	0.90	1.27	0.00	6.31	1.38	0.00	0.81
time (sec)	N/A	0.030	0.093	0.239	0.000	0.265	0.980	0.000	2.046

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	37	0	305	66	0	36
N.S.	1	1.00	1.00	0.79	0.00	6.49	1.40	0.00	0.77
time (sec)	N/A	0.030	0.140	0.264	0.000	0.278	1.817	0.000	2.183

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	97	76	0	625	87	0	65
N.S.	1	1.00	1.37	1.07	0.00	8.80	1.23	0.00	0.92
time (sec)	N/A	0.040	0.171	0.266	0.000	0.279	8.316	0.000	2.309

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	101	74	0	1110	88	0	64
N.S.	1	1.00	1.40	1.03	0.00	15.42	1.22	0.00	0.89
time (sec)	N/A	0.036	0.274	0.276	0.000	0.278	113.756	0.000	2.919

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	136	149	0	8891	0	0	0
N.S.	1	1.00	1.01	1.10	0.00	65.86	0.00	0.00	0.00
time (sec)	N/A	0.252	1.404	0.825	0.000	1.136	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	105	90	0	6663	0	0	0
N.S.	1	1.00	1.00	0.86	0.00	63.46	0.00	0.00	0.00
time (sec)	N/A	0.144	0.079	0.714	0.000	0.936	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	0	1748	0	0	0
N.S.	1	1.00	1.00	0.90	0.00	30.14	0.00	0.00	0.00
time (sec)	N/A	0.085	0.041	0.770	0.000	0.683	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	109	0	0	6705	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	63.25	0.00	0.00	0.00
time (sec)	N/A	0.168	0.376	0.000	0.000	0.921	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	142	0	0	9168	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	50.10	0.00	0.00	0.00
time (sec)	N/A	0.245	0.631	0.000	0.000	1.152	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	131	165	0	7896	0	0	0
N.S.	1	1.00	0.99	1.25	0.00	59.82	0.00	0.00	0.00
time (sec)	N/A	0.180	0.341	0.746	0.000	1.421	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	76	92	94	604	0	68	155
N.S.	1	1.00	0.71	0.86	0.88	5.64	0.00	0.64	1.45
time (sec)	N/A	0.060	0.152	0.374	0.279	0.255	0.000	0.395	0.108

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	60	80	69	339	0	52	93
N.S.	1	1.00	0.78	1.04	0.90	4.40	0.00	0.68	1.21
time (sec)	N/A	0.042	0.106	0.334	0.277	0.266	0.000	0.298	1.721

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	48	47	147	0	41	58
N.S.	1	1.00	0.78	0.94	0.92	2.88	0.00	0.80	1.14
time (sec)	N/A	0.027	0.106	0.099	0.278	0.257	0.000	0.277	1.683

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	27	23	38	0	23	34
N.S.	1	1.00	0.88	1.08	0.92	1.52	0.00	0.92	1.36
time (sec)	N/A	0.011	0.023	0.088	0.277	0.246	0.000	0.268	0.048

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	22	27	38	49	0	32	38
N.S.	1	1.00	0.88	1.08	1.52	1.96	0.00	1.28	1.52
time (sec)	N/A	0.012	0.023	0.080	0.215	0.247	0.000	0.294	0.080

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	179	48	62	198	0	56	62
N.S.	1	1.00	3.38	0.91	1.17	3.74	0.00	1.06	1.17
time (sec)	N/A	0.026	2.328	0.090	0.202	0.254	0.000	0.283	1.760

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	286	77	88	459	0	72	97
N.S.	1	1.00	3.53	0.95	1.09	5.67	0.00	0.89	1.20
time (sec)	N/A	0.037	2.477	0.333	0.192	0.255	0.000	0.302	1.718

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	115	88	110	796	0	83	160
N.S.	1	1.00	1.02	0.78	0.97	7.04	0.00	0.73	1.42
time (sec)	N/A	0.054	10.127	0.360	0.205	0.256	0.000	0.293	0.074

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	48	35	89	397	0	89	86
N.S.	1	1.00	0.42	0.31	0.79	3.51	0.00	0.79	0.76
time (sec)	N/A	0.067	0.065	0.163	0.279	0.277	0.000	0.286	0.237

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	24	78	85	0	78	81
N.S.	1	1.00	1.00	0.25	0.82	0.89	0.00	0.82	0.85
time (sec)	N/A	0.045	0.040	0.127	0.285	0.266	0.000	0.268	1.836

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	22	31	0	23	26
N.S.	1	1.00	1.00	2.25	1.38	1.94	0.00	1.44	1.62
time (sec)	N/A	0.011	0.017	0.144	0.273	0.256	0.000	0.269	1.776

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	113	48	34	230	0	35	38
N.S.	1	1.00	3.23	1.37	0.97	6.57	0.00	1.00	1.09
time (sec)	N/A	0.021	1.724	0.160	0.306	0.261	0.000	0.264	0.186

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	59	81	547	0	81	86
N.S.	1	1.00	0.86	0.52	0.72	4.84	0.00	0.72	0.76
time (sec)	N/A	0.166	0.103	0.180	0.286	0.280	0.000	0.262	0.304

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	24	47	69	137	0	69	70
N.S.	1	1.00	0.25	0.48	0.71	1.41	0.00	0.71	0.72
time (sec)	N/A	0.138	0.017	0.163	0.282	0.273	0.000	0.267	0.249

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	22	138	75	113	0	76	81
N.S.	1	1.00	0.26	1.62	0.88	1.33	0.00	0.89	0.95
time (sec)	N/A	0.101	0.021	0.157	0.285	0.259	0.000	0.252	1.840

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	113	150	87	628	0	88	93
N.S.	1	1.00	1.05	1.39	0.81	5.81	0.00	0.81	0.86
time (sec)	N/A	0.111	2.285	0.171	0.285	0.270	0.000	0.281	1.947

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	51	36	0	1303	0	263	474
N.S.	1	1.00	0.13	0.09	0.00	3.41	0.00	0.69	1.24
time (sec)	N/A	0.327	0.074	0.167	0.000	0.287	0.000	0.270	4.599

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	24	24	0	171	0	251	457
N.S.	1	1.00	0.07	0.07	0.00	0.47	0.00	0.69	1.25
time (sec)	N/A	0.175	0.018	0.165	0.000	0.277	0.000	0.283	4.225

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	22	56	97	111	0	98	104
N.S.	1	1.00	0.19	0.48	0.84	0.96	0.00	0.84	0.90
time (sec)	N/A	0.056	0.023	0.168	0.285	0.294	0.000	0.274	0.312

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	113	68	109	939	0	110	122
N.S.	1	1.00	0.84	0.51	0.81	7.01	0.00	0.82	0.91
time (sec)	N/A	0.073	2.004	0.180	0.279	0.277	0.000	0.265	2.016

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	81	70	0	406	0	328	163
N.S.	1	1.00	0.76	0.65	0.00	3.79	0.00	3.07	1.52
time (sec)	N/A	0.099	0.092	0.344	0.000	0.278	0.000	0.285	2.264

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	107	191	0	1604	0	456	280
N.S.	1	1.00	0.70	1.26	0.00	10.55	0.00	3.00	1.84
time (sec)	N/A	0.141	0.204	0.404	0.000	0.277	0.000	0.267	23.125

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	205	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.135	2.601	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	169	0	0	0	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.095	1.314	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0	0
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.051	0.272	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	134	0	0	0	0	0	0
N.S.	1	1.00	2.06	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.054	1.199	0.000	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	164	0	0	0	0	0	0
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.096	0.932	0.000	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	161	161	210	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	1.735	0.000	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	311	311	133	195	145	1226	0	184	0
N.S.	1	1.00	0.43	0.63	0.47	3.94	0.00	0.59	0.00
time (sec)	N/A	0.634	0.275	0.600	0.291	0.261	0.000	0.306	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	193	104	131	90	458	0	129	0
N.S.	1	1.00	0.54	0.68	0.47	2.37	0.00	0.67	0.00
time (sec)	N/A	0.200	0.195	0.437	0.289	0.259	0.000	0.284	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	218	35	53	0	60	0
N.S.	1	1.00	0.61	2.63	0.42	0.64	0.00	0.72	0.00
time (sec)	N/A	0.107	0.059	0.582	0.290	0.257	0.000	0.295	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	51	48	56	70	0	88	0
N.S.	1	1.00	0.61	0.58	0.67	0.84	0.00	1.06	0.00
time (sec)	N/A	0.152	0.136	0.356	0.290	0.263	0.000	0.303	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	197	197	334	131	112	613	0	161	0
N.S.	1	1.00	1.70	0.66	0.57	3.11	0.00	0.82	0.00
time (sec)	N/A	0.610	7.778	0.467	0.286	0.267	0.000	0.341	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	319	319	164	195	167	1617	0	215	0
N.S.	1	1.00	0.51	0.61	0.52	5.07	0.00	0.67	0.00
time (sec)	N/A	1.235	11.729	0.460	0.308	0.272	0.000	0.416	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	697	0	0	0
N.S.	1	1.00	0.79	0.75	0.00	4.44	0.00	0.00	0.00
time (sec)	N/A	0.286	0.600	0.868	0.000	0.300	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	88	0	230	0	0	0
N.S.	1	1.00	0.77	0.77	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.189	0.505	0.399	0.000	0.264	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	58	0	215	0	0	0
N.S.	1	1.00	0.77	0.75	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.107	0.213	0.206	0.000	0.268	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	0	9	9	8	9	11
N.S.	1	1.00	1.29	0.00	1.29	1.29	1.14	1.29	1.57
time (sec)	N/A	0.064	3.032	0.000	0.645	0.268	10.759	0.359	2.633

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	698	0	0	0
N.S.	1	1.00	0.79	0.75	0.00	4.45	0.00	0.00	0.00
time (sec)	N/A	0.279	0.706	0.810	0.000	0.291	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	88	0	230	0	0	0
N.S.	1	1.00	0.77	0.77	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.200	0.574	0.350	0.000	0.273	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	216	0	0	0
N.S.	1	1.00	0.81	0.75	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	0.105	0.207	0.213	0.000	0.271	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	0	9	9	8	9	11
N.S.	1	1.00	1.29	0.00	1.29	1.29	1.14	1.29	1.57
time (sec)	N/A	0.059	5.059	0.000	0.391	0.266	2.170	0.337	2.355

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [224] had the largest ratio of [1.50000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	2	1.00	8	0.250
2	A	3	2	1.00	8	0.250
3	A	3	2	1.00	8	0.250
4	A	2	2	1.00	8	0.250
5	A	2	2	1.00	8	0.250
6	A	1	1	1.00	6	0.167
7	A	1	1	1.00	6	0.167
8	A	2	2	1.00	8	0.250
9	A	2	2	1.00	8	0.250
10	A	3	2	1.00	8	0.250
11	A	3	2	1.00	8	0.250
12	A	4	2	1.00	8	0.250
13	A	7	6	1.00	12	0.500
14	A	6	6	1.00	12	0.500
15	A	6	6	1.00	12	0.500
16	A	5	5	1.00	12	0.417
17	A	5	5	1.00	12	0.417
18	A	6	6	1.00	12	0.500
19	A	6	6	1.00	12	0.500
20	A	7	6	1.00	12	0.500
21	A	9	9	1.00	8	1.125
22	A	2	2	1.00	8	0.250
23	A	2	2	1.00	10	0.200
24	A	3	3	1.00	10	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	2	2	1.00	10	0.200
26	A	2	2	1.00	10	0.200
27	A	4	3	1.00	14	0.214
28	A	3	3	1.00	14	0.214
29	A	2	2	1.00	14	0.143
30	A	2	2	1.00	14	0.143
31	A	3	3	1.00	14	0.214
32	A	4	3	1.00	14	0.214
33	A	7	7	1.00	8	0.875
34	A	8	7	1.00	10	0.700
35	A	7	7	1.00	10	0.700
36	A	7	7	1.00	10	0.700
37	A	5	3	1.00	10	0.300
38	A	3	3	1.00	10	0.300
39	A	3	3	1.00	10	0.300
40	A	3	3	1.00	12	0.250
41	A	5	3	1.00	12	0.250
42	A	4	3	1.00	12	0.250
43	A	3	3	1.00	12	0.250
44	A	2	2	1.00	12	0.167
45	A	2	2	1.00	12	0.167
46	A	3	2	1.00	12	0.167
47	A	4	2	1.00	12	0.167
48	A	5	2	1.00	12	0.167
49	A	6	2	1.00	12	0.167
50	A	5	3	1.00	8	0.375
51	A	4	3	1.00	8	0.375
52	A	3	3	1.00	8	0.375
53	A	2	2	1.00	8	0.250
54	A	3	3	1.00	8	0.375
55	A	4	3	1.00	8	0.375
56	A	5	3	1.00	8	0.375
57	A	5	4	1.00	12	0.333
58	A	4	4	1.00	12	0.333
59	A	3	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	2	2	1.00	12	0.167
61	A	2	2	1.00	12	0.167
62	A	3	3	1.00	12	0.250
63	A	4	4	1.00	12	0.333
64	A	5	4	1.00	12	0.333
65	A	2	2	1.00	12	0.167
66	A	2	2	1.00	12	0.167
67	A	5	4	1.00	14	0.286
68	A	5	4	1.00	14	0.286
69	A	5	4	1.00	11	0.364
70	A	9	7	1.00	11	0.636
71	A	5	4	1.00	11	0.364
72	A	8	6	1.00	9	0.667
73	A	8	7	1.00	9	0.778
74	A	3	2	1.00	11	0.182
75	A	8	7	1.00	11	0.636
76	A	4	3	1.00	11	0.273
77	A	9	8	1.00	11	0.727
78	A	4	3	1.00	11	0.273
79	A	10	8	1.00	11	0.727
80	A	5	3	1.00	13	0.231
81	A	10	9	1.00	13	0.692
82	A	4	3	1.00	13	0.231
83	A	6	6	1.00	11	0.546
84	A	6	5	1.00	11	0.454
85	A	3	2	1.00	13	0.154
86	A	15	11	1.00	13	0.846
87	A	3	2	1.00	13	0.154
88	A	29	13	1.00	13	1.000
89	A	3	2	1.00	13	0.154
90	A	6	5	1.00	11	0.454
91	A	4	3	1.00	11	0.273
92	A	3	2	1.00	11	0.182
93	A	4	3	1.00	11	0.273
94	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	1	1	1.00	9	0.111
96	A	2	2	1.00	11	0.182
97	A	2	2	1.00	11	0.182
98	A	2	1	1.00	11	0.091
99	A	3	3	1.00	11	0.273
100	A	3	2	1.00	11	0.182
101	A	4	3	1.00	11	0.273
102	A	3	2	1.00	13	0.154
103	A	3	2	1.00	13	0.154
104	A	3	2	1.00	13	0.154
105	A	2	2	1.00	13	0.154
106	A	2	2	1.00	8	0.250
107	A	4	3	1.00	13	0.231
108	A	5	4	1.00	13	0.308
109	A	14	6	1.19	13	0.462
110	A	9	6	1.07	13	0.462
111	A	5	5	1.00	13	0.385
112	A	2	2	1.00	11	0.182
113	A	5	5	1.00	11	0.454
114	A	9	6	1.00	13	0.462
115	A	5	4	1.00	11	0.364
116	A	4	4	1.00	11	0.364
117	A	3	3	1.00	11	0.273
118	A	3	2	1.00	11	0.182
119	A	2	2	1.00	9	0.222
120	A	2	2	1.00	6	0.333
121	A	4	4	1.00	9	0.444
122	A	4	4	1.00	11	0.364
123	A	5	4	1.00	11	0.364
124	A	6	4	1.00	11	0.364
125	A	4	4	1.00	11	0.364
126	A	3	3	1.00	11	0.273
127	A	3	3	1.00	11	0.273
128	A	4	4	1.00	11	0.364
129	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	3	1.00	13	0.231
131	A	4	4	1.00	13	0.308
132	A	4	4	1.00	13	0.308
133	A	7	7	1.00	13	0.538
134	A	6	6	1.00	13	0.462
135	A	5	5	1.00	13	0.385
136	A	4	4	1.00	13	0.308
137	A	2	2	1.00	11	0.182
138	A	2	2	1.00	8	0.250
139	A	3	3	1.00	11	0.273
140	A	4	4	1.00	13	0.308
141	A	5	5	1.00	13	0.385
142	A	6	6	1.00	13	0.462
143	A	3	3	1.00	14	0.214
144	A	9	6	1.00	24	0.250
145	A	11	7	1.00	26	0.269
146	A	4	3	1.00	11	0.273
147	A	11	8	1.00	11	0.727
148	A	4	4	1.00	9	0.444
149	A	11	8	1.00	7	1.143
150	A	2	1	1.00	11	0.091
151	A	11	8	1.00	11	0.727
152	A	4	4	1.00	11	0.364
153	A	4	3	1.00	13	0.231
154	A	12	9	1.00	13	0.692
155	A	5	5	1.00	11	0.454
156	A	13	9	1.00	9	1.000
157	A	3	2	1.00	13	0.154
158	A	12	9	1.00	13	0.692
159	A	5	5	1.00	13	0.385
160	A	3	3	1.00	13	0.231
161	A	4	4	1.00	15	0.267
162	A	5	5	1.00	15	0.333
163	A	3	3	1.00	9	0.333
164	A	3	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	2	2	1.00	11	0.182
166	A	4	4	1.00	11	0.364
167	A	5	5	1.00	11	0.454
168	A	5	5	1.00	11	0.454
169	A	3	3	1.00	7	0.429
170	A	3	3	1.00	9	0.333
171	A	3	3	1.00	9	0.333
172	A	4	4	1.00	17	0.235
173	A	4	4	1.00	17	0.235
174	A	4	4	1.00	15	0.267
175	A	4	4	1.00	13	0.308
176	A	2	1	1.00	17	0.059
177	A	4	4	1.00	17	0.235
178	A	4	4	1.00	17	0.235
179	A	5	5	1.00	19	0.263
180	A	5	5	1.00	19	0.263
181	A	5	5	1.00	17	0.294
182	A	5	5	1.00	15	0.333
183	A	3	2	1.00	19	0.105
184	A	5	5	1.00	19	0.263
185	A	5	5	1.00	19	0.263
186	A	3	2	1.00	17	0.118
187	A	4	2	1.00	17	0.118
188	A	4	2	1.00	17	0.118
189	A	4	4	1.00	19	0.210
190	A	5	5	1.00	21	0.238
191	A	6	6	1.00	21	0.286
192	A	4	4	1.00	15	0.267
193	A	4	4	1.00	21	0.190
194	A	7	6	1.00	19	0.316
195	A	7	6	1.00	19	0.316
196	A	6	5	1.00	19	0.263
197	A	6	5	1.00	19	0.263
198	A	7	6	1.00	19	0.316
199	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	8	7	1.00	23	0.304
201	A	7	6	1.00	23	0.261
202	A	4	4	1.00	21	0.190
203	A	8	5	1.00	21	0.238
204	A	11	6	1.00	23	0.261
205	A	8	7	1.00	21	0.333
206	A	7	6	1.00	16	0.375
207	A	7	6	1.00	16	0.375
208	A	5	4	1.00	16	0.250
209	A	3	3	1.00	14	0.214
210	A	3	3	1.00	14	0.214
211	A	5	4	1.00	16	0.250
212	A	7	6	1.00	16	0.375
213	A	7	6	1.00	16	0.375
214	A	13	9	1.00	10	0.900
215	A	11	8	1.00	8	1.000
216	A	5	5	1.00	8	0.625
217	A	7	6	1.00	10	0.600
218	A	14	9	1.00	10	0.900
219	A	12	8	1.00	8	1.000
220	A	12	8	1.00	8	1.000
221	A	14	9	1.00	10	0.900
222	A	23	9	1.00	10	0.900
223	A	21	8	1.00	8	1.000
224	A	15	12	1.00	8	1.500
225	A	17	13	1.00	10	1.300
226	A	5	5	1.00	14	0.357
227	A	7	6	1.00	14	0.429
228	A	6	3	1.00	18	0.167
229	A	5	3	1.00	18	0.167
230	A	4	3	1.00	16	0.188
231	A	4	3	1.00	16	0.188
232	A	5	3	1.00	18	0.167
233	A	6	3	1.00	18	0.167
234	A	9	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	8	7	1.00	25	0.280
236	A	4	4	1.00	25	0.160
237	A	4	4	1.00	25	0.160
238	A	8	7	1.00	25	0.280
239	A	9	7	1.00	25	0.280
240	A	19	5	1.00	9	0.556
241	A	13	5	1.00	9	0.556
242	A	9	4	1.00	7	0.571
243	N/A	0	0	1.00	7	0.000
244	A	19	5	1.00	9	0.556
245	A	13	5	1.00	9	0.556
246	A	9	4	1.00	7	0.571
247	N/A	0	0	1.00	7	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \tanh^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \tanh^6(a+bx)\,dx = x - \frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b}$$

```
[Out] x-tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b-1/5*tanh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$\int \tanh^6(a+bx)\,dx = -\frac{\tanh^5(a+bx)}{5b} - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh(a+bx)}{b} + x$$

```
[In] Int[Tanh[a + b*x]^6,x]
```

```
[Out] x - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b) - Tanh[a + b*x]^5/(5*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\tanh^5(a+bx)}{5b} + \int \tanh^4(a+bx)\,dx \\
&= -\frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b} + \int \tanh^2(a+bx)\,dx \\
&= -\frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b} + \int 1\,dx \\
&= x - \frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$
\int \tanh^6(a+bx)\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))}{b} - \frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh^5(a+bx)}{5b}
$$

```
[In] Integrate[Tanh[a + b*x]^6,x]
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b) - Tanh[a
 + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
parallelrisch	$-\frac{3\tanh(bx+a)^5+5\tanh(bx+a)^3-15bx+15\tanh(bx+a)}{15b}$	39
derivativedivides	$\frac{-\frac{\tanh(bx+a)^5}{5}-\frac{\tanh(bx+a)^3}{3}-\tanh(bx+a)-\frac{\ln(-1+\tanh(bx+a))}{2}+\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	56
default	$\frac{-\frac{\tanh(bx+a)^5}{5}-\frac{\tanh(bx+a)^3}{3}-\tanh(bx+a)-\frac{\ln(-1+\tanh(bx+a))}{2}+\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	56
risch	$x+\frac{6\,e^{8bx+8a}+12\,e^{6bx+6a}+\frac{56\,e^{4bx+4a}}{3}+\frac{28\,e^{2bx+2a}}{3}+\frac{46}{15}}{b\left(1+e^{2bx+2a}\right)^5}$	67

```
[In] int(tanh(b*x+a)^6,x,method=_RETURNVERBOSE)
[Out] -1/15*(3*tanh(b*x+a)^5+5*tanh(b*x+a)^3-15*b*x+15*tanh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 254, normalized size of antiderivative = 5.91

$$\int \tanh^6(a+bx)\,dx = \frac{(15\,bx+23)\cosh\left(bx+a\right)^5 + 5\,(15\,bx+23)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^4 - 23\,\sinh\left(bx+a\right)^5 + 5\,(15\,bx+}{15\,\left(b\cosh\left(\right.\right.}$$

```
[In] integrate(tanh(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/15*((15*b*x + 23)*cosh(b*x + a)^5 + 5*(15*b*x + 23)*cosh(b*x + a)*sinh(b*
x + a)^4 - 23*sinh(b*x + a)^5 + 5*(15*b*x + 23)*cosh(b*x + a)^3 - 5*(46*cos
h(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 5*(2*(15*b*x + 23)*cosh(b*x + a)^3 + 3*
(15*b*x + 23)*cosh(b*x + a))*sinh(b*x + a)^2 + 10*(15*b*x + 23)*cosh(b*x +
a) - 5*(23*cosh(b*x + a)^4 + 15*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/(b*cos
h(b*x + a)^5 + 5*b*cosh(b*x + a)*sinh(b*x + a)^4 + 5*b*cosh(b*x + a)^3 + 5*
(2*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^2 + 10*b*cosh(b*x +
 a))
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \tanh^6(a+bx)\,dx = \begin{cases} x - \frac{\tanh^5\left(a+bx\right)}{5b} - \frac{\tanh^3\left(a+bx\right)}{3b} - \frac{\tanh\left(a+bx\right)}{b} & \text{for}\ b \neq 0 \\ x\tanh^6\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(b*x+a)**6,x)
```

```
[Out] Piecewise((x - tanh(a + b*x)**5/(5*b) - tanh(a + b*x)**3/(3*b) - tanh(a + b
*x)/b, Ne(b, 0)), (x*tanh(a)**6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(39) = 78.

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.67

$$\int \tanh^6(a+bx)\,dx = x + \frac{a}{b} - \frac{2\,\left(70\,e^{(-2\,bx-2\,a)} + 140\,e^{(-4\,bx-4\,a)} + 90\,e^{(-6\,bx-6\,a)} + 45\,e^{(-8\,bx-8\,a)} + 23\right)}{15\,b(5\,e^{(-2\,bx-2\,a)} + 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} + 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} + 1)}$$

```
[In] integrate(tanh(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] x + a/b - 2/15*(70*e^(-2*b*x - 2*a) + 140*e^(-4*b*x - 4*a) + 90*e^(-6*b*x -
 6*a) + 45*e^(-8*b*x - 8*a) + 23)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4
*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \tanh^6(a+bx)\,dx = \frac{15\,bx + 15\,a + \frac{2\,\left(45\,e^{\left(8\,bx+8\,a\right)}+90\,e^{\left(6\,bx+6\,a\right)}+140\,e^{\left(4\,bx+4\,a\right)}+70\,e^{\left(2\,bx+2\,a\right)}+23\right)}{\left(e^{\left(2\,bx+2\,a\right)}+1\right)^5}}{15\,b}$$

```
[In] integrate(tanh(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/15*(15*b*x + 15*a + 2*(45*e^(8*b*x + 8*a) + 90*e^(6*b*x + 6*a) + 140*e^(4
*b*x + 4*a) + 70*e^(2*b*x + 2*a) + 23)/(e^(2*b*x + 2*a) + 1)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \tanh^6(a+bx)\,dx = x - \frac{\frac{\tanh(a+b\,x)^5}{5} + \frac{\tanh(a+b\,x)^3}{3} + \tanh(a+b\,x)}{b}$$

```
[In] int(tanh(a + b*x)^6,x)
```

```
[Out] x - (tanh(a + b*x) + tanh(a + b*x)^3/3 + tanh(a + b*x)^5/5)/b
```

3.2 $\int \tanh^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \tanh^5(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b} - \frac{\tanh^4(a+bx)}{4b}$$

```
[Out] ln(cosh(b*x+a))/b-1/2*tanh(b*x+a)^2/b-1/4*tanh(b*x+a)^4/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \tanh^5(a+bx)\,dx = -\frac{\tanh^4(a+bx)}{4b} - \frac{\tanh^2(a+bx)}{2b} + \frac{\log(\cosh(a+bx))}{b}$$

```
[In] Int[Tanh[a + b*x]^5,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b) - Tanh[a + b*x]^4/(4*b)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\tanh^4(a+bx)}{4b} + \int \tanh^3(a+bx)\,dx \\
&= -\frac{\tanh^2(a+bx)}{2b} - \frac{\tanh^4(a+bx)}{4b} + \int \tanh(a+bx)\,dx \\
&= \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b} - \frac{\tanh^4(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \tanh^5(a+bx)\,dx = \frac{4\log(\cosh(a+bx)) - 2\tanh^2(a+bx) - \tanh^4(a+bx)}{4b}$$

```
[In] Integrate[Tanh[a + b*x]^5,x]
[Out] (4*Log[Cosh[a + b*x]] - 2*Tanh[a + b*x]^2 - Tanh[a + b*x]^4)/(4*b)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$-\frac{\tanh(bx+a)^4+4bx+2\tanh(bx+a)^2+4\ln(1-\tanh(bx+a))}{4b}$	42
derivativedivides	$\frac{-\frac{\tanh(bx+a)^4}{4}-\frac{\tanh(bx+a)^2}{2}-\frac{\ln(-1+\tanh(bx+a))}{2}-\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	48
default	$\frac{-\frac{\tanh(bx+a)^4}{4}-\frac{\tanh(bx+a)^2}{2}-\frac{\ln(-1+\tanh(bx+a))}{2}-\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	48
risch	$-x-\frac{2a}{b}+\frac{4\,e^{2bx+2a}\left(e^{4bx+4a}+e^{2bx+2a}+1\right)}{b\left(1+e^{2bx+2a}\right)^4}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	74

```
[In] int(tanh(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] -1/4*(tanh(b*x+a)^4+4*b*x+2*tanh(b*x+a)^2+4*ln(1-tanh(b*x+a)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 968 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 968, normalized size of antiderivative = 23.05

$$\int \tanh^5(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^8 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x
+ a)^8 + 4*(b*x - 1)*cosh(b*x + a)^6 + 4*(7*b*x*cosh(b*x + a)^2 + b*x - 1)*
sinh(b*x + a)^6 + 8*(7*b*x*cosh(b*x + a)^3 + 3*(b*x - 1)*cosh(b*x + a))*sin
h(b*x + a)^5 + 2*(3*b*x - 2)*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 +
30*(b*x - 1)*cosh(b*x + a)^2 + 3*b*x - 2)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b
*x + a)^5 + 10*(b*x - 1)*cosh(b*x + a)^3 + (3*b*x - 2)*cosh(b*x + a))*sinh(
b*x + a)^3 + 4*(b*x - 1)*cosh(b*x + a)^2 + 4*(7*b*x*cosh(b*x + a)^6 + 15*(b
*x - 1)*cosh(b*x + a)^4 + 3*(3*b*x - 2)*cosh(b*x + a)^2 + b*x - 1)*sinh(b*x
 + a)^2 + b*x - (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b
*x + a)^8 + 4*(7*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 +
 8*(7*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x +
 a)^4 + 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 + 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4
*(7*cosh(b*x + a)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x
+ a)^2 + 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cos
h(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(b*x + a)/(cosh(
b*x + a) - sinh(b*x + a))) + 8*(b*x*cosh(b*x + a)^7 + 3*(b*x - 1)*cosh(b*x
+ a)^5 + (3*b*x - 2)*cosh(b*x + a)^3 + (b*x - 1)*cosh(b*x + a))*sinh(b*x +
a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a
)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^6 + 8
*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x +
 a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)
^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sin
h(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 + 15*b*cosh(b*x
 + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 +
 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \tanh^5(a+bx)\,dx = \begin{cases} x - \frac{\log(\tanh(a+bx)+1)}{b} - \frac{\tanh^4(a+bx)}{4b} - \frac{\tanh^2(a+bx)}{2b} & \text{for } b \neq 0 \\ x\tanh^5(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(b*x+a)**5,x)
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b - tanh(a + b*x)**4/(4*b) - tanh(a +
 b*x)**2/(2*b), Ne(b, 0)), (x*tanh(a)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(38) = 76.

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.43

$$\int \tanh^5(a+bx)\,dx = x + \frac{a}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{4\left(e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(4\,e^{(-2\,bx-2\,a)}+6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}+e^{(-8\,bx-8\,a)}+1)}$$

```
[In] integrate(tanh(b*x+a)^5,x, algorithm="maxima")
[Out] x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 4*(e^(-2*b*x - 2*a) + e^(-4*b*x - 4
*a) + e^(-6*b*x - 6*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b*x - 4*a) + 4*e^(
-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.60

$$\int \tanh^5(a+bx)\,dx = -\frac{bx + a - \frac{4\left(e^{(6\,bx+6\,a)}+e^{(4\,bx+4\,a)}+e^{(2\,bx+2\,a)}\right)}{\left(e^{(2\,bx+2\,a)}+1\right)^4} - \log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

```
[In] integrate(tanh(b*x+a)^5,x, algorithm="giac")
[Out] -(b*x + a - 4*(e^(6*b*x + 6*a) + e^(4*b*x + 4*a) + e^(2*b*x + 2*a))/(e^(2*b
*x + 2*a) + 1)^4 - log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \tanh^5(a+bx)\,dx = x - \frac{\ln\left(\tanh(a+b\,x)+1\right) + \frac{\tanh(a+b\,x)^2}{2} + \frac{\tanh(a+b\,x)^4}{4}}{b}$$

```
[In] int(tanh(a + b*x)^5,x)
[Out] x - (log(tanh(a + b*x) + 1) + tanh(a + b*x)^2/2 + tanh(a + b*x)^4/4)/b
```

3.3 $\int \tanh^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \tanh^4(a+bx)\,dx = x-\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] x-tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \tanh^4(a+bx)\,dx = -\frac{\tanh^3(a+bx)}{3b}-\frac{\tanh(a+bx)}{b}+x$$

```
[In] Int[Tanh[a + b*x]^4,x]
```

```
[Out] x - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = -\frac{\tanh^3(a+bx)}{3b} + \int \tanh^2(a+bx)\,dx$$
$$= -\frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b} + \int 1\,dx$$
$$= x - \frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \tanh^4(a+bx)\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))}{b} - \frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}$$

```
[In] Integrate[Tanh[a + b*x]^4,x]
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$-\frac{\tanh(bx+a)^3-3bx+3\tanh(bx+a)}{3b}$	27
risch	$x+\frac{4\,e^{4bx+4a}+4\,e^{2bx+2a}+\frac{8}{3}}{b\left(1+e^{2bx+2a}\right)^3}$	45
derivativedivides	$\frac{-\frac{\tanh(bx+a)^3}{3}-\tanh(bx+a)-\frac{\ln(-1+\tanh(bx+a))}{2}+\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	46
default	$\frac{-\frac{\tanh(bx+a)^3}{3}-\tanh(bx+a)-\frac{\ln(-1+\tanh(bx+a))}{2}+\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	46

```
[In] int(tanh(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*(tanh(b*x+a)^3-3*b*x+3*tanh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 4.25

$$\int \tanh^4(a+bx)\,dx = \frac{(3\,bx+4)\cosh(bx+a)^3 + 3\,(3\,bx+4)\cosh(bx+a)\sinh(bx+a)^2 - 12\,\cosh(bx+a)^2\sinh(bx+a) - 4\,\text{si}}{3\,\left(b\cosh(bx+a)^3 + 3\,b\cosh(bx+a)\sinh(bx+a)^2 + 3\,b\cosh(bx+\right.}$$

```
[In] integrate(tanh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*x + 4)*cosh(b*x + a)^3 + 3*(3*b*x + 4)*cosh(b*x + a)*sinh(b*x + a
)^2 - 12*cosh(b*x + a)^2*sinh(b*x + a) - 4*sinh(b*x + a)^3 + 3*(3*b*x + 4)*
cosh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + 3*b
*cosh(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \tanh^4(a+bx)\,dx = \begin{cases} x - \frac{\tanh^3(a+bx)}{3b} - \frac{\tanh(a+bx)}{b} & \text{for } b \neq 0 \\ x\tanh^4(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(b*x+a)**4,x)
```

```
[Out] Piecewise((x - tanh(a + b*x)**3/(3*b) - tanh(a + b*x)/b, Ne(b, 0)), (x*tanh
(a)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \tanh^4(a+bx)\,dx = x + \frac{a}{b} - \frac{4\left(3\,e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} + 2\right)}{3\,b(3\,e^{(-2\,bx-2\,a)} + 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} + 1)}$$

```
[In] integrate(tanh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] x + a/b - 4/3*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + 2)/(b*(3*e^(-2*b*x
 - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \tanh^4(a+bx)\,dx = \frac{3\,bx + 3\,a + \frac{4\,\left(3\,e^{\left(4\,bx+4\,a\right)}+3\,e^{\left(2\,bx+2\,a\right)}+2\right)}{\left(e^{\left(2\,bx+2\,a\right)}+1\right)^{3}}}{3\,b}$$

```
[In] integrate(tanh(b*x+a)^4,x, algorithm="giac")
[Out] 1/3*(3*b*x + 3*a + 4*(3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) + 2)/(e^(2*b*x
+ 2*a) + 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \tanh^4(a+bx)\,dx = x - \frac{\frac{\tanh(a+b\,x)^3}{3} + \tanh(a+b\,x)}{b}$$

```
[In] int(tanh(a + b*x)^4,x)
[Out] x - (tanh(a + b*x) + tanh(a + b*x)^3/3)/b
```

3.4 $\int \tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

```
[Out] ln(cosh(b*x+a))/b-1/2*tanh(b*x+a)^2/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 3556}

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

```
[In] Int[Tanh[a + b*x]^3,x]
```

```
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = -\frac{\tanh^2(a+bx)}{2b} + \int \tanh(a+bx)\,dx$$
$$= \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tanh^3(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b} - \frac{\tanh^2(a+bx)}{2b}$$

```
[In] Integrate[Tanh[a + b*x]^3,x]
[Out] Log[Cosh[a + b*x]]/b - Tanh[a + b*x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result	size
parallelrisch	$-\frac{2bx+\tanh(bx+a)^2+2\ln(1-\tanh(bx+a))}{2b}$	32
derivativedivides	$\frac{-\frac{\tanh(bx+a)^2}{2}-\frac{\ln(-1+\tanh(bx+a))}{2}-\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	38
default	$\frac{-\frac{\tanh(bx+a)^2}{2}-\frac{\ln(-1+\tanh(bx+a))}{2}-\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	38
risch	$-x-\frac{2a}{b}+\frac{2\,e^{2bx+2a}}{b(1+e^{2bx+2a})^2}+\frac{\ln(1+e^{2bx+2a})}{b}$	54

```
[In] int(tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(2*b*x+tanh(b*x+a)^2+2*ln(1-tanh(b*x+a)))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 339, normalized size of antiderivative = 12.56

$$\int \tanh^3(a + bx)\, dx =$$

$$-\frac{bx \cosh\left(bx + a\right)^4 + 4\, bx \cosh\left(bx + a\right) \sinh\left(bx + a\right)^3 + bx \sinh\left(bx + a\right)^4 + 2\left(bx - 1\right) \cosh\left(bx + a\right)^2 + 2\,(}{}$$

```
[In] integrate(tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 + 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 + b*x - 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*cosh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 + (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \tanh^3(a + bx)\, dx = \begin{cases} x - \frac{\log\left(\tanh\left(a + bx\right) + 1\right)}{b} - \frac{\tanh^2\left(a + bx\right)}{2b} & \text{for}\ b \neq 0 \\ x \tanh^3\left(a\right) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(b*x+a)**3,x)
```

```
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b - tanh(a + b*x)**2/(2*b), Ne(b, 0))
, (x*tanh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(25) = 50.

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \tanh^3(a+bx)\,dx = x + \frac{a}{b} + \frac{\log\left(e^{(-2\,bx-2\,a)}+1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)}+e^{(-4\,bx-4\,a)}+1)}$$

```
[In] integrate(tanh(b*x+a)^3,x, algorithm="maxima")
[Out] x + a/b + log(e^(-2*b*x - 2*a) + 1)/b + 2*e^(-2*b*x - 2*a)/(b*(2*e^(-2*b*x
- 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \tanh^3(a+bx)\,dx = -\frac{bx+a-\frac{2\,e^{(2\,bx+2\,a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^2}-\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

```
[In] integrate(tanh(b*x+a)^3,x, algorithm="giac")
[Out] -(b*x + a - 2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) + 1)^2 - log(e^(2*b*x + 2*a)
 + 1))/b
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \tanh^3(a+bx)\,dx = x - \frac{\ln\left(\tanh(a+b\,x)+1\right)+\frac{\tanh(a+b\,x)^2}{2}}{b}$$

```
[In] int(tanh(a + b*x)^3,x)
[Out] x - (log(tanh(a + b*x) + 1) + tanh(a + b*x)^2/2)/b
```

3.5 $\int \tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \tanh^2(a+bx)\,dx = x - \frac{\tanh(a+bx)}{b}$$

```
[Out] x-tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$\int \tanh^2(a+bx)\,dx = x - \frac{\tanh(a+bx)}{b}$$

```
[In] Int[Tanh[a + b*x]^2,x]
```

```
[Out] x - Tanh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = -\frac{\tanh(a+bx)}{b} + \int 1\,dx$$
$$= x - \frac{\tanh(a+bx)}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \tanh^2(a+bx)\,dx = \frac{\operatorname{arctanh}(\tanh(a+bx))}{b} - \frac{\tanh(a+bx)}{b}$$

```
[In] Integrate[Tanh[a + b*x]^2,x]
[Out] ArcTanh[Tanh[a + b*x]]/b - Tanh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

method	result	size
parallelrisch	$-\frac{-bx+\tanh(bx+a)}{b}$	17
risch	$x+\frac{2}{b\left(1+e^{2bx+2a}\right)}$	21
derivativedivides	$\frac{-\tanh(bx+a)-\frac{\ln(-1+\tanh(bx+a))}{2}+\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	36
default	$\frac{-\tanh(bx+a)-\frac{\ln(-1+\tanh(bx+a))}{2}+\frac{\ln(1+\tanh(bx+a))}{2}}{b}$	36

```
[In] int(tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] -(-b*x+tanh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \tanh^2(a+bx)\,dx = \frac{(bx+1)\cosh(bx+a) - \sinh(bx+a)}{b\cosh(bx+a)}$$

```
[In] integrate(tanh(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + 1)*cosh(b*x + a) - sinh(b*x + a))/(b*cosh(b*x + a))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \tanh^2(a+bx)\,dx = \begin{cases} x - \frac{\tanh(a+bx)}{b} & \text{for } b \neq 0 \\ x\tanh^2(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(b*x+a)**2,x)
[Out] Piecewise((x - tanh(a + b*x)/b, Ne(b, 0)), (x*tanh(a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \tanh^2(a+bx)\,dx = x + \frac{a}{b} - \frac{2}{b(e^{(-2\,bx-2\,a)}+1)}$$

```
[In] integrate(tanh(b*x+a)^2,x, algorithm="maxima")
[Out] x + a/b - 2/(b*(e^(-2*b*x - 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \tanh^2(a+bx)\,dx = \frac{bx + a + \frac{2}{e^{(2\,bx+2\,a)}+1}}{b}$$

```
[In] integrate(tanh(b*x+a)^2,x, algorithm="giac")
[Out] (b*x + a + 2/(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \tanh^2(a+bx)\,dx = x - \frac{\tanh(a+b\,x)}{b}$$

```
[In] int(tanh(a + b*x)^2,x)
```

```
[Out] x - tanh(a + b*x)/b
```

3.6 $\int \tanh(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

```
[Out] ln(cosh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3556}

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(a+bx))}{b}$$

```
[In] Int[Tanh[a + b*x],x]
```

```
[Out] Log[Cosh[a + b*x]]/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\log(\cosh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \tanh(a+bx)\,dx=\frac{\log(\cosh(a+bx))}{b}$$

```
[In] Integrate[Tanh[a + b*x],x]
[Out] Log[Cosh[a + b*x]]/b
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\cosh(bx+a))}{b}$	12
default	$\frac{\ln(\cosh(bx+a))}{b}$	12
parallelrisch	$-\frac{bx+\ln(1-\tanh(bx+a))}{b}$	21
risch	$-x-\frac{2a}{b}+\frac{\ln(1+e^{2bx+2a})}{b}$	27

```
[In] int(tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(cosh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.36

$$\int \tanh(a+bx)\,dx=-\frac{bx-\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate(tanh(b*x+a),x, algorithm="fricas")
[Out] -(b*x - log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \tanh(a+bx)\,dx = \begin{cases} x - \frac{\log(\tanh(a+bx)+1)}{b} & \text{for } b \neq 0 \\ x\tanh(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(b*x+a),x)
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b, Ne(b, 0)), (x*tanh(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \tanh(a+bx)\,dx = \frac{\log(\cosh(bx+a))}{b}$$

```
[In] integrate(tanh(b*x+a),x, algorithm="maxima")
[Out] log(cosh(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \tanh(a+bx)\,dx = -\frac{bx+a-\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

```
[In] integrate(tanh(b*x+a),x, algorithm="giac")
[Out] -(b*x + a - log(e^(2*b*x + 2*a) + 1))/b
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \tanh(a+bx)\,dx = x - \frac{\ln(\tanh(a+b\,x)+1)}{b}$$

```
[In] int(tanh(a + b*x),x)
[Out] x - log(tanh(a + b*x) + 1)/b
```

3.7 $\int \coth(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] ln(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3556}

$$\int \coth(a+bx)\,dx=\frac{\log(\sinh(a+bx))}{b}$$

```
[In] Int[Coth[a + b*x],x]
```

```
[Out] Log[Sinh[a + b*x]]/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral}=\frac{\log(\sinh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \coth(a+bx)\,dx = \frac{\log(\cosh(a+bx)) + \log(\tanh(a+bx))}{b}$$

```
[In] Integrate[Coth[a + b*x],x]
[Out] (Log[Cosh[a + b*x]] + Log[Tanh[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(\sinh(bx+a))}{b}$	12
default	$\frac{\ln(\sinh(bx+a))}{b}$	12
risch	$-x-\frac{2a}{b}+\frac{\ln\left(e^{2bx+2a}-1\right)}{b}$	27
parallelrisch	$\frac{-bx+\ln(\tanh(bx+a))-\ln(1-\tanh(bx+a))}{b}$	30

```
[In] int(coth(b*x+a),x,method=_RETURNVERBOSE)
[Out] ln(sinh(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(11) = 22.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.36

$$\int \coth(a+bx)\,dx = -\frac{bx - \log\left(\frac{2\sinh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

```
[In] integrate(coth(b*x+a),x, algorithm="fricas")
[Out] -(b*x - log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(8) = 16$.

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \coth(a+bx)\,dx = \begin{cases} x - \frac{\log(\tanh(a+bx)+1)}{b} + \frac{\log(\tanh(a+bx))}{b} & \text{for } b \neq 0 \\ x\coth(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a),x)
[Out] Piecewise((x - log(tanh(a + b*x) + 1)/b + log(tanh(a + b*x))/b, Ne(b, 0)),
(x*coth(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \coth(a+bx)\,dx = \frac{\log(\sinh(bx+a))}{b}$$

```
[In] integrate(coth(b*x+a),x, algorithm="maxima")
[Out] log(sinh(b*x + a))/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \coth(a+bx)\,dx = -\frac{bx + a - \log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{b}$$

```
[In] integrate(coth(b*x+a),x, algorithm="giac")
[Out] -(b*x + a - log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.91

$$\int \coth(a+bx)\,dx = \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x}-1\right)}{b}-x$$

```
[In] int(coth(a + b*x),x)
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x
```

3.8 $\int \coth^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 13

$$\int \coth^2(a+bx)\,dx = x - \frac{\coth(a+bx)}{b}$$

```
[Out] x-coth(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \coth^2(a+bx)\,dx = x - \frac{\coth(a+bx)}{b}$$

```
[In] Int[Coth[a + b*x]^2,x]
[Out] x - Coth[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\text{integral} = -\frac{\coth(a+bx)}{b} + \int 1\,dx$$
$$= x - \frac{\coth(a+bx)}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \coth^2(a+bx)\,dx = -\frac{\coth(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(a+bx)\right)}{b}$$

```
[In] Integrate[Coth[a + b*x]^2,x]
[Out] -((Coth[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + b*x]^2])/b)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

method	result	size
risch	$x-\frac{2}{b\left(\mathrm{e}^{2bx+2a}-1\right)}$	21
parallelrisch	$\frac{-1+\tanh(bx+a)xb}{b\tanh(bx+a)}$	24
derivativedivides	$\frac{-\coth(bx+a)-\frac{\ln(\coth(bx+a)-1)}{2}+\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	36
default	$\frac{-\coth(bx+a)-\frac{\ln(\coth(bx+a)-1)}{2}+\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	36

```
[In] int(coth(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] x-2/b/(exp(2*b*x+2*a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \coth^2(a+bx)\,dx = \frac{(bx+1)\sinh(bx+a) - \cosh(bx+a)}{b\sinh(bx+a)}$$

```
[In] integrate(coth(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + 1)*sinh(b*x + a) - cosh(b*x + a))/(b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(8) = 16$.

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 6.69

$$\int \coth^2(a+bx)\,dx = \begin{cases} x\coth^2(a) & \text{for } b=0 \\ -\frac{\log(-e^{-bx})\coth^2(bx+\log(-e^{-bx}))}{b} & \text{for } a=\log(-e^{-bx}) \\ -\frac{\log(e^{-bx})\coth^2(bx+\log(e^{-bx}))}{b} & \text{for } a=\log(e^{-bx}) \\ x - \frac{1}{b\tanh(a+bx)} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)**2,x)
[Out] Piecewise((x*coth(a)**2, Eq(b, 0)), (-log(-exp(-b*x))*coth(b*x + log(-exp(-
b*x)))**2/b, Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))*coth(b*x + log(exp(-
b*x)))**2/b, Eq(a, log(exp(-b*x)))), (x - 1/(b*tanh(a + b*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \coth^2(a+bx)\,dx = x + \frac{a}{b} + \frac{2}{b(e^{(-2\,bx-2\,a)} - 1)}$$

```
[In] integrate(coth(b*x+a)^2,x, algorithm="maxima")
[Out] x + a/b + 2/(b*(e^(-2*b*x - 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \coth^2(a+bx)\,dx = \frac{bx + a - \frac{2}{e^{(2\,bx+2\,a)}-1}}{b}$$

```
[In] integrate(coth(b*x+a)^2,x, algorithm="giac")
[Out] (b*x + a - 2/(e^(2*b*x + 2*a) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \coth^2(a+bx)\,dx = x - \frac{\coth(a+b\,x)}{b}$$

```
[In] int(coth(a + b*x)^2,x)
[Out] x - coth(a + b*x)/b
```

3.9 $\int \coth^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \coth^3(a+bx)\,dx = -\frac{\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b+ln(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \coth^3(a+bx)\,dx = \frac{\log(\sinh(a+bx))}{b} - \frac{\coth^2(a+bx)}{2b}$$

```
[In] Int[Coth[a + b*x]^3,x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b + Log[Sinh[a + b*x]]/b
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth^2(a+bx)}{2b} + \int \coth(a+bx)\,dx \\
&= -\frac{\coth^2(a+bx)}{2b} + \frac{\log(\sinh(a+bx))}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$
\int \coth^3(a+bx)\,dx = -\frac{\coth^2(a+bx) - 2\log(\cosh(a+bx)) - 2\log(\tanh(a+bx))}{2b}
$$

```
[In] Integrate[Coth[a + b*x]^3,x]
[Out] -1/2*(Coth[a + b*x]^2 - 2*Log[Cosh[a + b*x]] - 2*Log[Tanh[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{-\frac{\coth(bx+a)^2}{2}-\frac{\ln(\coth(bx+a)-1)}{2}-\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	38
default	$\frac{-\frac{\coth(bx+a)^2}{2}-\frac{\ln(\coth(bx+a)-1)}{2}-\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	38
parallelrisch	$\frac{-2bx+2\ln(\tanh(bx+a))-2\ln(1-\tanh(bx+a))-\coth(bx+a)^2}{2b}$	43
risch	$-x-\frac{2a}{b}-\frac{2\,\mathrm{e}^{2bx+2a}}{b\left(\mathrm{e}^{2bx+2a}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	54

```
[In] int(coth(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/2*coth(b*x+a)^2-1/2*ln(coth(b*x+a)-1)-1/2*ln(coth(b*x+a)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 346, normalized size of antiderivative = 12.81

$$\int \coth^3(a+bx)\,dx =$$

$$-\frac{bx\cosh(bx+a)^4 + 4\,bx\cosh(bx+a)\sinh(bx+a)^3 + bx\sinh(bx+a)^4 - 2\,(bx-1)\cosh(bx+a)^2 + 2\,(}{}$$

```
[In] integrate(coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^4 + 4*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + b*x*sinh(b*x
+ a)^4 - 2*(b*x - 1)*cosh(b*x + a)^2 + 2*(3*b*x*cosh(b*x + a)^2 - b*x + 1)*
sinh(b*x + a)^2 + b*x - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3
+ sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x
+ a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(
b*x + a)/(cosh(b*x + a) - sinh(b*x + a))) + 4*(b*x*cosh(b*x + a)^3 - (b*x -
 1)*cosh(b*x + a))*sinh(b*x + a))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*si
nh(b*x + a)^3 + b*sinh(b*x + a)^4 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x +
 a)^2 - b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b
*x + a) + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(20) = 40$.

Time = 0.69 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.15

$$\int \coth^3(a+bx)\,dx = \begin{cases} x\coth^3(a) & \text{for } b=0 \\ -\frac{\log(-e^{-bx})\coth^3(bx+\log(-e^{-bx}))}{b} & \text{for } a=\log(-e^{-bx}) \\ -\frac{\log(e^{-bx})\coth^3(bx+\log(e^{-bx}))}{b} & \text{for } a=\log(e^{-bx}) \\ x-\frac{\log(\tanh(a+bx)+1)}{b}+\frac{\log(\tanh(a+bx))}{b}-\frac{1}{2b\tanh^2(a+bx)} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)**3,x)
```

```
[Out] Piecewise((x*coth(a)**3, Eq(b, 0)), (-log(-exp(-b*x))*coth(b*x + log(-exp(-
b*x)))**3/b, Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))*coth(b*x + log(exp(-
b*x)))**3/b, Eq(a, log(exp(-b*x)))), (x - log(tanh(a + b*x) + 1)/b + log(ta
nh(a + b*x))/b - 1/(2*b*tanh(a + b*x)**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.93

$$\int \coth^3(a+bx)\,dx = x + \frac{a}{b} + \frac{\log\left(e^{(-bx-a)}+1\right)}{b} + \frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{2\,e^{(-2\,bx-2\,a)}}{b(2\,e^{(-2\,bx-2\,a)} - e^{(-4\,bx-4\,a)} - 1)}$$

```
[In] integrate(coth(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 2*e^(-2*b*x -
 2*a)/(b*(2*e^(-2*b*x - 2*a) - e^(-4*b*x - 4*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \coth^3(a+bx)\,dx = -\frac{bx + a + \frac{2\,e^{(2\,bx+2\,a)}}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - \log\left(\left|e^{(2\,bx+2\,a)}-1\right|\right)}{b}$$

```
[In] integrate(coth(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -(b*x + a + 2*e^(2*b*x + 2*a)/(e^(2*b*x + 2*a) - 1)^2 - log(abs(e^(2*b*x +
2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.52

$$\int \coth^3(a+bx)\,dx = \frac{\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}-1\right)}{b} - x - \frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)} - \frac{2}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(coth(a + b*x)^3,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x - 2/(b*(exp(2*a + 2*b*x) - 1)) - 2/(b*(e
xp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1))
```

3.10 $\int \coth^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 28

$$\int \coth^4(a+bx)\,dx = x - \frac{\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b}$$

```
[Out] x-coth(b*x+a)/b-1/3*coth(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 8}

$$\int \coth^4(a+bx)\,dx = -\frac{\coth^3(a+bx)}{3b} - \frac{\coth(a+bx)}{b} + x$$

```
[In] Int[Coth[a + b*x]^4,x]
```

```
[Out] x - Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth^3(a+bx)}{3b} + \int \coth^2(a+bx)\,dx \\
&= -\frac{\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b} + \int 1\,dx \\
&= x - \frac{\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$
\int \coth^4(a+bx)\,dx = -\frac{\coth^3(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(a+bx)\right)}{3b}
$$

```
[In] Integrate[Coth[a + b*x]^4,x]
[Out] -1/3*(Coth[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$\frac{-\coth(bx+a)^3+3bx-3\coth(bx+a)}{3b}$	29
risch	$x-\frac{4\left(3\,\mathrm{e}^{4bx+4a}-3\,\mathrm{e}^{2bx+2a}+2\right)}{3b\left(\mathrm{e}^{2bx+2a}-1\right)^3}$	45
derivativedivides	$\frac{-\frac{\coth(bx+a)^3}{3}-\coth(bx+a)-\frac{\ln(\coth(bx+a)-1)}{2}+\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	46
default	$\frac{-\frac{\coth(bx+a)^3}{3}-\coth(bx+a)-\frac{\ln(\coth(bx+a)-1)}{2}+\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	46

```
[In] int(coth(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/3*(-coth(b*x+a)^3+3*b*x-3*coth(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.86

$$\int \coth^4(a+bx)\,dx$$
$$= \frac{(3\,bx+4)\sinh{(bx+a)}^3 - 4\,\cosh{(bx+a)}^3 - 12\,\cosh{(bx+a)}\sinh{(bx+a)}^2 + 3\,((3\,bx+4)\cosh{(bx+a)}^2}{3\,\left(b\sinh{(bx+a)}^3 + 3\,\left(b\cosh{(bx+a)}^2 - b\right)\sinh{(bx+a)}\right)}$$

```
[In] integrate(coth(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*x + 4)*sinh(b*x + a)^3 - 4*cosh(b*x + a)^3 - 12*cosh(b*x + a)*sin
h(b*x + a)^2 + 3*((3*b*x + 4)*cosh(b*x + a)^2 - 3*b*x - 4)*sinh(b*x + a))/(
b*sinh(b*x + a)^3 + 3*(b*cosh(b*x + a)^2 - b)*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(20) = 40$.

Time = 1.02 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.57

$$\int \coth^4(a+bx)\,dx = \begin{cases} x\coth^4{(a)} & \text{for } b=0 \\ -\frac{\log{\left(-e^{-bx}\right)}\coth^4{\left(bx+\log{\left(-e^{-bx}\right)}\right)}}{b} & \text{for } a=\log{\left(-e^{-bx}\right)} \\ -\frac{\log{\left(e^{-bx}\right)}\coth^4{\left(bx+\log{\left(e^{-bx}\right)}\right)}}{b} & \text{for } a=\log{\left(e^{-bx}\right)} \\ x-\frac{1}{b\tanh{(a+bx)}}-\frac{1}{3b\tanh^3{(a+bx)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)**4,x)
```

```
[Out] Piecewise((x*coth(a)**4, Eq(b, 0)), (-log(-exp(-b*x))*coth(b*x + log(-exp(-
b*x)))**4/b, Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))*coth(b*x + log(exp(-
b*x)))**4/b, Eq(a, log(exp(-b*x)))), (x - 1/(b*tanh(a + b*x)) - 1/(3*b*tanh
(a + b*x)**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(26) = 52$.

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \coth^4(a+bx)\,dx = x + \frac{a}{b} - \frac{4\left(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} - 2\right)}{3\,b(3\,e^{(-2\,bx-2\,a)} - 3\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)} - 1)}$$

```
[In] integrate(coth(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] x + a/b - 4/3*(3*e^(-2*b*x - 2*a) - 3*e^(-4*b*x - 4*a) - 2)/(b*(3*e^(-2*b*x
 - 2*a) - 3*e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \coth^4(a+bx)\,dx = \frac{3\,bx + 3\,a - \frac{4\,\left(3\,e^{(4\,bx+4\,a)} - 3\,e^{(2\,bx+2\,a)} + 2\right)}{\left(e^{(2\,bx+2\,a)} - 1\right)^3}}{3\,b}$$

```
[In] integrate(coth(b*x+a)^4,x, algorithm="giac")
[Out] 1/3*(3*b*x + 3*a - 4*(3*e^(4*b*x + 4*a) - 3*e^(2*b*x + 2*a) + 2)/(e^(2*b*x
+ 2*a) - 1)^3)/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \coth^4(a+bx)\,dx = x - \frac{\frac{\coth(a+b\,x)^3}{3} + \coth(a+b\,x)}{b}$$

```
[In] int(coth(a + b*x)^4,x)
[Out] x - (coth(a + b*x) + coth(a + b*x)^3/3)/b
```

3.11 $\int \coth^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int \coth^5(a+bx)\,dx=-\frac{\coth^2(a+bx)}{2b}-\frac{\coth^4(a+bx)}{4b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[Out] -1/2*coth(b*x+a)^2/b-1/4*coth(b*x+a)^4/b+ln(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3554, 3556}

$$\int \coth^5(a+bx)\,dx=-\frac{\coth^4(a+bx)}{4b}-\frac{\coth^2(a+bx)}{2b}+\frac{\log(\sinh(a+bx))}{b}$$

```
[In] Int[Coth[a + b*x]^5,x]
```

```
[Out] -1/2*Coth[a + b*x]^2/b - Coth[a + b*x]^4/(4*b) + Log[Sinh[a + b*x]]/b
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth^4(a+bx)}{4b} + \int \coth^3(a+bx)\,dx \\
&= -\frac{\coth^2(a+bx)}{2b} - \frac{\coth^4(a+bx)}{4b} + \int \coth(a+bx)\,dx \\
&= -\frac{\coth^2(a+bx)}{2b} - \frac{\coth^4(a+bx)}{4b} + \frac{\log(\sinh(a+bx))}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$
\int \coth^5(a+bx)\,dx = -\frac{2\coth^2(a+bx) + \coth^4(a+bx) - 4\log(\cosh(a+bx)) - 4\log(\tanh(a+bx))}{4b}
$$

```
[In] Integrate[Coth[a + b*x]^5,x]
[Out] -1/4*(2*Coth[a + b*x]^2 + Coth[a + b*x]^4 - 4*Log[Cosh[a + b*x]] - 4*Log[Ta
nh[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-\frac{\coth(bx+a)^4}{4}-\frac{\coth(bx+a)^2}{2}-\frac{\ln(\coth(bx+a)-1)}{2}-\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	48
default	$\frac{-\frac{\coth(bx+a)^4}{4}-\frac{\coth(bx+a)^2}{2}-\frac{\ln(\coth(bx+a)-1)}{2}-\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	48
parallelrisch	$\frac{-\coth(bx+a)^4-2\coth(bx+a)^2-4bx+4\ln(\tanh(bx+a))-4\ln(1-\tanh(bx+a))}{4b}$	53
risch	$-x-\frac{2a}{b}-\frac{4\,\mathrm{e}^{2bx+2a}\left(\mathrm{e}^{4bx+4a}-\mathrm{e}^{2bx+2a}+1\right)}{b\left(\mathrm{e}^{2bx+2a}-1\right)^4}+\frac{\ln\left(\mathrm{e}^{2bx+2a}-1\right)}{b}$	76

```
[In] int(coth(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/b*(-1/4*coth(b*x+a)^4-1/2*coth(b*x+a)^2-1/2*ln(coth(b*x+a)-1)-1/2*ln(coth
(b*x+a)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 978 vs. $2(38) = 76$.

Time = 0.25 (sec) , antiderivative size = 978, normalized size of antiderivative = 23.29

$$\int \coth^5(a + bx)\, dx = \text{Too large to display}$$

```
[In] integrate(coth(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -(b*x*cosh(b*x + a)^8 + 8*b*x*cosh(b*x + a)*sinh(b*x + a)^7 + b*x*sinh(b*x
+ a)^8 - 4*(b*x - 1)*cosh(b*x + a)^6 + 4*(7*b*x*cosh(b*x + a)^2 - b*x + 1)*
sinh(b*x + a)^6 + 8*(7*b*x*cosh(b*x + a)^3 - 3*(b*x - 1)*cosh(b*x + a))*sin
h(b*x + a)^5 + 2*(3*b*x - 2)*cosh(b*x + a)^4 + 2*(35*b*x*cosh(b*x + a)^4 -
30*(b*x - 1)*cosh(b*x + a)^2 + 3*b*x - 2)*sinh(b*x + a)^4 + 8*(7*b*x*cosh(b
*x + a)^5 - 10*(b*x - 1)*cosh(b*x + a)^3 + (3*b*x - 2)*cosh(b*x + a))*sinh(
b*x + a)^3 - 4*(b*x - 1)*cosh(b*x + a)^2 + 4*(7*b*x*cosh(b*x + a)^6 - 15*(b
*x - 1)*cosh(b*x + a)^4 + 3*(3*b*x - 2)*cosh(b*x + a)^2 - b*x + 1)*sinh(b*x
 + a)^2 + b*x - (cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b
*x + a)^8 + 4*(7*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^6 - 4*cosh(b*x + a)^6 +
 8*(7*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x +
 a)^4 - 30*cosh(b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*
cosh(b*x + a)^5 - 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4
*(7*cosh(b*x + a)^6 - 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 - 1)*sinh(b*x
+ a)^2 - 4*cosh(b*x + a)^2 + 8*(cosh(b*x + a)^7 - 3*cosh(b*x + a)^5 + 3*cos
h(b*x + a)^3 - cosh(b*x + a))*sinh(b*x + a) + 1)*log(2*sinh(b*x + a)/(cosh(
b*x + a) - sinh(b*x + a))) + 8*(b*x*cosh(b*x + a)^7 - 3*(b*x - 1)*cosh(b*x
+ a)^5 + (3*b*x - 2)*cosh(b*x + a)^3 - (b*x - 1)*cosh(b*x + a))*sinh(b*x +
a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(b*x + a
)^8 - 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^6 + 8
*(7*b*cosh(b*x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^5 + 6*b*cosh(b*x +
 a)^4 + 2*(35*b*cosh(b*x + a)^4 - 30*b*cosh(b*x + a)^2 + 3*b)*sinh(b*x + a)
^4 + 8*(7*b*cosh(b*x + a)^5 - 10*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sin
h(b*x + a)^3 - 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a)^6 - 15*b*cosh(b*x
 + a)^4 + 9*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 8*(b*cosh(b*x + a)^7 -
 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a)
 + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(32) = 64.

Time = 1.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.00

$$\int \coth^5(a+bx)\,dx$$
$$= \begin{cases} x\coth^5(a) & \text{for } b=0 \\ -\frac{\log(-e^{-bx})\coth^5(bx+\log(-e^{-bx}))}{b} & \text{for } a=\log(-e^{-bx}) \\ -\frac{\log(e^{-bx})\coth^5(bx+\log(e^{-bx}))}{b} & \text{for } a=\log(e^{-bx}) \\ x-\frac{\log(\tanh(a+bx)+1)}{b}+\frac{\log(\tanh(a+bx))}{b}-\frac{1}{2b\tanh^2(a+bx)}-\frac{1}{4b\tanh^4(a+bx)} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)**5,x)
```

```
[Out] Piecewise((x*coth(a)**5, Eq(b, 0)), (-log(-exp(-b*x))*coth(b*x + log(-exp(-
b*x)))**5/b, Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))*coth(b*x + log(exp(-
b*x)))**5/b, Eq(a, log(exp(-b*x)))), (x - log(tanh(a + b*x) + 1)/b + log(ta
nh(a + b*x))/b - 1/(2*b*tanh(a + b*x)**2) - 1/(4*b*tanh(a + b*x)**4), True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(38) = 76.

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.90

$$\int \coth^5(a+bx)\,dx = x+\frac{a}{b}+\frac{\log\left(e^{(-bx-a)}+1\right)}{b}+\frac{\log\left(e^{(-bx-a)}-1\right)}{b} + \frac{4\left(e^{(-2\,bx-2\,a)}-e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}\right)}{b(4\,e^{(-2\,bx-2\,a)}-6\,e^{(-4\,bx-4\,a)}+4\,e^{(-6\,bx-6\,a)}-e^{(-8\,bx-8\,a)}-1)}$$

```
[In] integrate(coth(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] x + a/b + log(e^(-b*x - a) + 1)/b + log(e^(-b*x - a) - 1)/b + 4*(e^(-2*b*x
- 2*a) - e^(-4*b*x - 4*a) + e^(-6*b*x - 6*a))/(b*(4*e^(-2*b*x - 2*a) - 6*e^
(-4*b*x - 4*a) + 4*e^(-6*b*x - 6*a) - e^(-8*b*x - 8*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \coth^5(a+bx)\,dx = -\frac{bx + a + \frac{4\left(e^{(6\,bx+6\,a)} - e^{(4\,bx+4\,a)} + e^{(2\,bx+2\,a)}\right)}{\left(e^{(2\,bx+2\,a)} - 1\right)^4} - \log\left(\left|e^{(2\,bx+2\,a)} - 1\right|\right)}{b}$$

```
[In] integrate(coth(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -(b*x + a + 4*(e^(6*b*x + 6*a) - e^(4*b*x + 4*a) + e^(2*b*x + 2*a))/(e^(2*b
*x + 2*a) - 1)^4 - log(abs(e^(2*b*x + 2*a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.79

$$\int \coth^5(a+bx)\,dx = \frac{\ln\left(e^{2\,a}\,e^{2\,b\,x} - 1\right)}{b} - x - \frac{4}{b\,\left(e^{2\,a+2\,b\,x} - 1\right)} - \frac{8}{b\,\left(e^{4\,a+4\,b\,x} - 2\,e^{2\,a+2\,b\,x} + 1\right)} - \frac{8}{b\,\left(3\,e^{2\,a+2\,b\,x} - 3\,e^{4\,a+4\,b\,x} + e^{6\,a+6\,b\,x} - 1\right)} - \frac{4}{b\,\left(6\,e^{4\,a+4\,b\,x} - 4\,e^{2\,a+2\,b\,x} - 4\,e^{6\,a+6\,b\,x} + e^{8\,a+8\,b\,x} + 1\right)}$$

```
[In] int(coth(a + b*x)^5,x)
```

```
[Out] log(exp(2*a)*exp(2*b*x) - 1)/b - x - 4/(b*(exp(2*a + 2*b*x) - 1)) - 8/(b*(e
xp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - 8/(b*(3*exp(2*a + 2*b*x) - 3*e
xp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1)) - 4/(b*(6*exp(4*a + 4*b*x) - 4*exp
(2*a + 2*b*x) - 4*exp(6*a + 6*b*x) + exp(8*a + 8*b*x) + 1))
```

3.12 $\int \coth^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \coth^6(a+bx)\,dx = x - \frac{\coth(a+bx)}{b} - \frac{\coth^3(a+bx)}{3b} - \frac{\coth^5(a+bx)}{5b}$$

```
[Out] x-coth(b*x+a)/b-1/3*coth(b*x+a)^3/b-1/5*coth(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3554, 8}

$$\int \coth^6(a+bx)\,dx = -\frac{\coth^5(a+bx)}{5b} - \frac{\coth^3(a+bx)}{3b} - \frac{\coth(a+bx)}{b} + x$$

```
[In] Int[Coth[a + b*x]^6,x]
```

```
[Out] x - Coth[a + b*x]/b - Coth[a + b*x]^3/(3*b) - Coth[a + b*x]^5/(5*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth^5(a+bx)}{5b}+\int \coth^4(a+bx)\,dx\\
&= -\frac{\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}+\int \coth^2(a+bx)\,dx\\
&= -\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}+\int 1\,dx\\
&= x-\frac{\coth(a+bx)}{b}-\frac{\coth^3(a+bx)}{3b}-\frac{\coth^5(a+bx)}{5b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$
\int \coth^6(a+bx)\,dx = -\frac{\coth^5(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\tanh^2(a+bx)\right)}{5b}
$$

```
[In] Integrate[Coth[a + b*x]^6,x]
[Out] -1/5*(Coth[a + b*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[a + b*x]^2])/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
parallelrisch	$\frac{-3\coth(bx+a)^5-5\coth(bx+a)^3+15bx-15\coth(bx+a)}{15b}$	39
derivativedivides	$\frac{-\frac{\coth(bx+a)^5}{5}-\frac{\coth(bx+a)^3}{3}-\coth(bx+a)-\frac{\ln(\coth(bx+a)-1)}{2}+\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	56
default	$\frac{-\frac{\coth(bx+a)^5}{5}-\frac{\coth(bx+a)^3}{3}-\coth(bx+a)-\frac{\ln(\coth(bx+a)-1)}{2}+\frac{\ln(\coth(bx+a)+1)}{2}}{b}$	56
risch	$x-\frac{2\left(45\,e^{8bx+8a}-90\,e^{6bx+6a}+140\,e^{4bx+4a}-70\,e^{2bx+2a}+23\right)}{15b\left(e^{2bx+2a}-1\right)^5}$	67

```
[In] int(coth(b*x+a)^6,x,method=_RETURNVERBOSE)
[Out] 1/15*(-3*coth(b*x+a)^5-5*coth(b*x+a)^3+15*b*x-15*coth(b*x+a))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(39) = 78.

Time = 0.25 (sec) , antiderivative size = 239, normalized size of antiderivative = 5.56

$$\int \coth^6(a+bx)\,dx$$
$$= \frac{(15\,bx+23)\sinh(bx+a)^5 - 23\,\cosh(bx+a)^5 - 115\,\cosh(bx+a)\sinh(bx+a)^4 + 5\,(2\,(15\,bx+23)\cosh$$

```
[In] integrate(coth(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/15*((15*b*x + 23)*sinh(b*x + a)^5 - 23*cosh(b*x + a)^5 - 115*cosh(b*x + a
)*sinh(b*x + a)^4 + 5*(2*(15*b*x + 23)*cosh(b*x + a)^2 - 15*b*x - 23)*sinh(
b*x + a)^3 + 25*cosh(b*x + a)^3 - 5*(46*cosh(b*x + a)^3 - 15*cosh(b*x + a))
*sinh(b*x + a)^2 + 5*((15*b*x + 23)*cosh(b*x + a)^4 - 3*(15*b*x + 23)*cosh(
b*x + a)^2 + 30*b*x + 46)*sinh(b*x + a) - 50*cosh(b*x + a))/(b*sinh(b*x + a
)^5 + 5*(2*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^3 + 5*(b*cosh(b*x + a)^4 -
3*b*cosh(b*x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(32) = 64.

Time = 2.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.65

$$\int \coth^6(a+bx)\,dx = \begin{cases} x\coth^6(a) & \text{for } b=0 \\ -\frac{\log(-e^{-bx})\coth^6(bx+\log(-e^{-bx}))}{b} & \text{for } a=\log(-e^{-bx}) \\ -\frac{\log(e^{-bx})\coth^6(bx+\log(e^{-bx}))}{b} & \text{for } a=\log(e^{-bx}) \\ x-\frac{1}{b\tanh(a+bx)}-\frac{1}{3b\tanh^3(a+bx)}-\frac{1}{5b\tanh^5(a+bx)} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(b*x+a)**6,x)
```

```
[Out] Piecewise((x*coth(a)**6, Eq(b, 0)), (-log(-exp(-b*x))*coth(b*x + log(-exp(-
b*x)))**6/b, Eq(a, log(-exp(-b*x)))), (-log(exp(-b*x))*coth(b*x + log(exp(-
b*x)))**6/b, Eq(a, log(exp(-b*x)))), (x - 1/(b*tanh(a + b*x)) - 1/(3*b*tanh
(a + b*x)**3) - 1/(5*b*tanh(a + b*x)**5), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(39) = 78.

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.67

$$\int \coth^6(a+bx)\,dx$$
$$= x + \frac{a}{b} - \frac{2\left(70\,e^{(-2\,bx-2\,a)} - 140\,e^{(-4\,bx-4\,a)} + 90\,e^{(-6\,bx-6\,a)} - 45\,e^{(-8\,bx-8\,a)} - 23\right)}{15\,b(5\,e^{(-2\,bx-2\,a)} - 10\,e^{(-4\,bx-4\,a)} + 10\,e^{(-6\,bx-6\,a)} - 5\,e^{(-8\,bx-8\,a)} + e^{(-10\,bx-10\,a)} - 1)}$$

```
[In] integrate(coth(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] x + a/b - 2/15*(70*e^(-2*b*x - 2*a) - 140*e^(-4*b*x - 4*a) + 90*e^(-6*b*x -
 6*a) - 45*e^(-8*b*x - 8*a) - 23)/(b*(5*e^(-2*b*x - 2*a) - 10*e^(-4*b*x - 4
*a) + 10*e^(-6*b*x - 6*a) - 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

$$\int \coth^6(a+bx)\,dx = \frac{15\,bx + 15\,a - \frac{2\left(45\,e^{(8\,bx+8\,a)} - 90\,e^{(6\,bx+6\,a)} + 140\,e^{(4\,bx+4\,a)} - 70\,e^{(2\,bx+2\,a)} + 23\right)}{\left(e^{(2\,bx+2\,a)} - 1\right)^5}}{15\,b}$$

```
[In] integrate(coth(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 1/15*(15*b*x + 15*a - 2*(45*e^(8*b*x + 8*a) - 90*e^(6*b*x + 6*a) + 140*e^(4
*b*x + 4*a) - 70*e^(2*b*x + 2*a) + 23)/(e^(2*b*x + 2*a) - 1)^5)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \coth^6(a+bx)\,dx = x - \frac{\frac{\coth(a+b\,x)^5}{5} + \frac{\coth(a+b\,x)^3}{3} + \coth(a+b\,x)}{b}$$

```
[In] int(coth(a + b*x)^6,x)
```

```
[Out] x - (coth(a + b*x) + coth(a + b*x)^3/3 + coth(a + b*x)^5/5)/b
```

3.13 $\int (b\tanh(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int (b\tanh(c+dx))^{7/2}\,dx = \frac{b^{7/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b^3\sqrt{b\tanh(c+dx)}}{d} - \frac{2b(b\tanh(c+dx))^{5/2}}{5d}$$

```
[Out] b^(7/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*tanh(d*x
+c))^(1/2)/b^(1/2))/d-2*b^3*(b*tanh(d*x+c))^(1/2)/d-2/5*b*(b*tanh(d*x+c))^(
5/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\int (b\tanh(c+dx))^{7/2}\,dx = \frac{b^{7/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b^3\sqrt{b\tanh(c+dx)}}{d} - \frac{2b(b\tanh(c+dx))^{5/2}}{5d}$$

```
[In] Int[(b*Tanh[c + d*x])^(7/2),x]
```

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Tanh[c + d*x]])/d - (2*b*(b*Tan
h[c + d*x])^(5/2))/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2b(b\tanh(c+dx))^{5/2}}{5d} + b^2\int (b\tanh(c+dx))^{3/2}\,dx$$

$$= -\frac{2b^3\sqrt{b\tanh(c+dx)}}{d} - \frac{2b(b\tanh(c+dx))^{5/2}}{5d} + b^4\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx$$

$$=-\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}-\frac{b^5\text{Subst}\left(\int\frac{1}{\sqrt{x}(-b^2+x^2)}\,dx,x,b\tanh(c+dx)\right)}{d}$$

$$=-\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}-\frac{(2b^5)\,\text{Subst}\left(\int\frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d}$$

$$=-\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}+\frac{b^4\text{Subst}\left(\int\frac{1}{b-x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d}+\frac{b^4\text{Subst}\left(\int\frac{1}{b+x^2}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{d}$$

$$=\frac{b^{7/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\tanh(c+dx)}}{d}-\frac{2b(b\tanh(c+dx))^{5/2}}{5d}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int(b\tanh(c+dx))^{7/2}\,dx=-\frac{(b\tanh(c+dx))^{7/2}\left(-\arctan\left(\sqrt{\tanh(c+dx)}\right)-\text{arctanh}\left(\sqrt{\tanh(c+dx)}\right)+2\sqrt{\tanh(c+dx)}+\frac{2}{5}\tanh^{5/2}(c+dx)\right)}{d\tanh^{\frac{7}{2}}(c+dx)}$$

```
[In] Integrate[(b*Tanh[c + d*x])^(7/2),x]
```

```
[Out] -(((b*Tanh[c + d*x])^(7/2)*(-ArcTan[Sqrt[Tanh[c + d*x]]] - ArcTanh[Sqrt[Tan
h[c + d*x]]] + 2*Sqrt[Tanh[c + d*x]] + (2*Tanh[c + d*x]^(5/2))/5))/(d*Tanh[
c + d*x]^(7/2)))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{b^{\frac{7}{2}} \arctan\left(\frac{\sqrt{b \tanh(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\tanh(dx+c)}}{d}-\frac{2b(b\tanh(dx+c))^{\frac{5}{2}}}{5d}$	80
default	$\frac{b^{\frac{7}{2}} \arctan\left(\frac{\sqrt{b \tanh(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \tanh(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\tanh(dx+c)}}{d}-\frac{2b(b\tanh(dx+c))^{\frac{5}{2}}}{5d}$	80

```
[In] int((b*tanh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^(7/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*tanh(d*x
+c))^(1/2)/b^(1/2))/d-2*b^3*(b*tanh(d*x+c))^(1/2)/d-2/5*b*(b*tanh(d*x+c))^(
5/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 1556, normalized size of antiderivative = 16.04

$$\int (b\tanh(c+dx))^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*tanh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3
*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 +
 b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - 5*(b^3*
cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4
 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x +
 c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b)
*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^4
+ 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*c
osh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*b^3*cosh(d*x + c)^
4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 + 4*b^3*co
sh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 + 2*b^3)*sinh(d*x + c)^2 +
 4*(3*b^3*cosh(d*x + c)^3 + 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh
(d*x + c)/cosh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
```

```
 c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)
+ d), -1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3
+ b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c
)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*si
nh(d*x + c))*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*
cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b))
 - 5*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(
d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 + b^3)*
sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)
)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b
*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + s
inh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2
+ 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(
d*x + c)/cosh(d*x + c)) - b) + 16*(3*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x
+ c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 + 4*b^3*cosh(d*x + c)^2 + 3*b^
3 + 2*(9*b^3*cosh(d*x + c)^2 + 2*b^3)*sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x +
 c)^3 + 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*sinh(d*x + c)/cosh(d*x +
 c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
 c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 +
 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)]
```

Sympy [F]

$$\int (b\tanh(c+dx))^{7/2}\,dx = \int (b\tanh(c+dx))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))**(7/2),x)
[Out] Integral((b*tanh(c + d*x))**(7/2), x)
```

Maxima [F]

$$\int (b\tanh(c+dx))^{7/2}\,dx = \int (b\tanh(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c))^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(79) = 158.

Time = 0.42 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.02

$$\int (b\tanh(c+dx))^{7/2}\,dx = \frac{10\,b^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)-5\,b^{\frac{7}{2}}\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)-\frac{16\,(\cdots}{\cdots}}{\cdots}$$

```
[In] integrate((b*tanh(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] 1/10*(10*b^(7/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
- b))/sqrt(b)) - 5*b^(7/2)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d
*x + 4*c) - b))) - 16*(5*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
- b))^4*b^4 + 10*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3*
b^(9/2) + 20*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*b^5
+ 10*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))*b^(11/2) + 3*b
^6)/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + sqrt(b))^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (b\tanh(c+dx))^{7/2}\,dx = \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{d}-\frac{2\,b^3\,\sqrt{b\tanh\left(c+d\,x\right)}}{d} - \frac{2\,b\,(b\tanh(c+d\,x))^{5/2}}{5\,d}-\frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}\,1\mathrm{i}}{\sqrt{b}}\right)\,1\mathrm{i}}{d}$$

```
[In] int((b*tanh(c + d*x))^(7/2),x)
```

```
[Out] (b^(7/2)*atan((b*tanh(c + d*x))^(1/2)/b^(1/2)))/d - (2*b^3*(b*tanh(c + d*x)
)^(1/2))/d - (2*b*(b*tanh(c + d*x))^(5/2))/(5*d) - (b^(7/2)*atan(((b*tanh(c
 + d*x))^(1/2)*1i)/b^(1/2))*1i)/d
```

3.14 $\int (b\tanh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int (b\tanh(c+dx))^{5/2}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\tanh(c+dx))^{3/2}}{3d}$$

```
[Out] -b^(5/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*tanh(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*tanh(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3557, 335, 304, 209, 212}

$$\int (b\tanh(c+dx))^{5/2}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\tanh(c+dx))^{3/2}}{3d}$$

```
[In] Int[(b*Tanh[c + d*x])^(5/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Tanh[c + d*x])^(3/2))/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2b(b\tanh(c+dx))^{3/2}}{3d} + b^2\int\sqrt{b\tanh(c+dx)}\,dx$$

$$= -\frac{2b(b\tanh(c+dx))^{3/2}}{3d} - \frac{b^3\text{Subst}\left(\int\frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{d}$$

$$= -\frac{2b(b\tanh(c+dx))^{3/2}}{3d} - \frac{(2b^3)\,\text{Subst}\left(\int\frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}$$

$$= -\frac{2b(b\tanh(c+dx))^{3/2}}{3d} + \frac{b^3\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d} - \frac{b^3\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}$$

$$= -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\tanh(c+dx))^{3/2}}{3d}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int (b\tanh(c+dx))^{5/2}\,dx = -\frac{(b\tanh(c+dx))^{5/2}\left(\arctan\left(\sqrt{\tanh(c+dx)}\right) - \text{arctanh}\left(\sqrt{\tanh(c+dx)}\right) + \frac{2}{3}\tanh^{\frac{3}{2}}(c+dx)\right)}{d\tanh^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(b*Tanh[c + d*x])^(5/2),x]
[Out] -(((b*Tanh[c + d*x])^(5/2)*(ArcTan[Sqrt[Tanh[c + d*x]]] - ArcTanh[Sqrt[Tanh
[c + d*x]]] + (2*Tanh[c + d*x]^(3/2))/3))/(d*Tanh[c + d*x]^(5/2)))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\tanh(dx+c))^{\frac{3}{2}}}{3d}$	63
default	$-\frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{5}{2}}\,\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b(b\tanh(dx+c))^{\frac{3}{2}}}{3d}$	63

```
[In] int((b*tanh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -b^(5/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*tanh(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*tanh(d*x+c))^(3/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 980, normalized size of antiderivative = 12.56

$$\int (b\tanh(c+dx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*tanh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*si
nh(d*x + c)^2 + b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/
(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 -
b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 + b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*s
inh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c
)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4))
 + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 - b^2)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)))/(d*cosh(d*x + c)^2 + 2
*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d), 1/12*(6*(b^2*cosh(
d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)
*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + 3*(b^2*co
sh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b
^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 1
2*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2
*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^
2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sin
h(d*x + c)/cosh(d*x + c)) - b) - 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x +
c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt(b*sinh(d*x + c)/cosh(d*x
 + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x +
 c)^2 + d)]
```

Sympy [F]

$$\int (b\tanh(c+dx))^{5/2}\,dx = \int \left(b\tanh\left(c+dx\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))**(5/2),x)
[Out] Integral((b*tanh(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\tanh(c+dx))^{5/2}\,dx = \int \left(b\tanh\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c))^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(62) = 124$.

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.22

$$\int (b\tanh(c+dx))^{5/2}\,dx = -\frac{6\,b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)+3\,b^{\frac{5}{2}}\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)+\frac{8\left(3\left(\sqrt{b}e^{(2\,dx+2\,c)}-\cdots\right.\right.}{\left(\sqrt{b}e^{(2\,dx+2\,c)}-\cdots\right.}}{6\,d}$$

```
[In] integrate((b*tanh(d*x+c))^(5/2),x, algorithm="giac")
[Out] -1/6*(6*b^(5/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b)) + 3*b^(5/2)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*
x + 4*c) - b))) + 8*(3*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
b))^2*b^3 + b^4)/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) + s
qrt(b))^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int (b\tanh(c+dx))^{5/2}\,dx = \frac{b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b\,(b\tanh(c+d\,x))^{3/2}}{3\,d}$$

```
[In] int((b*tanh(c + d*x))^(5/2),x)
```

```
[Out] (b^(5/2)*atanh((b*tanh(c + d*x))^(1/2)/b^(1/2)))/d - (b^(5/2)*atan((b*tanh(
c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*tanh(c + d*x))^(3/2))/(3*d)
```

3.15 $\int (b\tanh(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (b\tanh(c+dx))^{3/2}\,dx=\frac{b^{3/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\tanh(c+dx)}}{d}$$

```
[Out] b^(3/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*tanh(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*tanh(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\int (b\tanh(c+dx))^{3/2}\,dx=\frac{b^{3/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b\sqrt{b\tanh(c+dx)}}{d}$$

```
[In] Int[(b*Tanh[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Tanh[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Tanh[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2b\sqrt{b\tanh(c+dx)}}{d} + b^2\int\frac{1}{\sqrt{b\tanh(c+dx)}}\,dx$$

$$= -\frac{2b\sqrt{b\tanh(c+dx)}}{d} - \frac{b^3\text{Subst}\left(\int\frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\tanh(c+dx)\right)}{d}$$

$$= -\frac{2b\sqrt{b\tanh(c+dx)}}{d} - \frac{(2b^3)\,\text{Subst}\left(\int\frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}$$

$$= -\frac{2b\sqrt{b\tanh(c+dx)}}{d} + \frac{b^2\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d} + \frac{b^2\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}$$

$$= \frac{b^{3/2}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\tanh(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int (b\tanh(c+dx))^{3/2}\,dx = -\frac{\left(-\arctan\left(\sqrt{\tanh(c+dx)}\right) - \text{arctanh}\left(\sqrt{\tanh(c+dx)}\right) + 2\sqrt{\tanh(c+dx)}\right)(b\tanh(c+dx))^{3/2}}{d\tanh^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(b*Tanh[c + d*x])^(3/2),x]
[Out] -(((-ArcTan[Sqrt[Tanh[c + d*x]]] - ArcTanh[Sqrt[Tanh[c + d*x]]] + 2*Sqrt[Ta
nh[c + d*x]])*(b*Tanh[c + d*x])^(3/2))/(d*Tanh[c + d*x]^(3/2)))
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d} + \frac{b^{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\tanh(dx+c)}}{d}$	62
default	$\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d} + \frac{b^{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\tanh(dx+c)}}{d}$	62

```
[In] int((b*tanh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] b^(3/2)*arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*tanh(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*tanh(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(61) = 122$.

Time = 0.27 (sec) , antiderivative size = 638, normalized size of antiderivative = 8.51

$$\int (b\tanh(c+dx))^{3/2}\,dx = \left[-\frac{2\sqrt{-b}b\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\sinh(dx+c)}{\cosh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b}\right)-\sqrt{-b}b\log\left(-\cdots\right.}{\cdots}\right.$$

$$-\frac{2b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b}\sqrt{\frac{b\sinh(dx+c)}{\cosh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b}\right)-b^{\frac{3}{2}}\log\left(2b\cosh(dx+c)^4+8b\cosh(dx+c)^3\cdots\right.}{\cdots}$$

```
[In] integrate((b*tanh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - sqrt
(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(b*sinh(d*
x + c)/cosh(d*x + c)))/d, -1/4*(2*b^(3/2)*arctan(sqrt(b)*sqrt(b*sinh(d*x +
c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)) - b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)
^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)
*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c
)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
)*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - b) + 8*b*sqrt(b*sinh(d*x +
c)/cosh(d*x + c)))/d]
```

Sympy [F]

$$\int (b\tanh(c+dx))^{3/2}\,dx = \int (b\tanh(c+dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))**(3/2),x)
[Out] Integral((b*tanh(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\tanh(c+dx))^{3/2}\,dx = \int (b\tanh(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(61) = 122.

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.75

$$\int (b\tanh(c+dx))^{3/2}\,dx = \frac{\left(2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2dx+2c)}-\sqrt{be^{(4dx+4c)}-b}}{\sqrt{b}}\right) - \sqrt{b}\log\left(\left|-\sqrt{b}e^{(2dx+2c)}+\sqrt{be^{(4dx+4c)}-b}\right|\right) - \frac{\ldots}{\sqrt{b}\ldots}}{2d}$$

```
[In] integrate((b*tanh(d*x+c))^(3/2),x, algorithm="giac")
[Out] 1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
b))/sqrt(b)) - sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x +
 4*c) - b))) - 8*b/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) +
 sqrt(b)))*b/d
```

Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int (b\tanh(c+dx))^{3/2}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b\,\sqrt{b\tanh\left(c+d\,x\right)}}{d} + \frac{b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

```
[In] int((b*tanh(c + d*x))^(3/2),x)
```

```
[Out] (b^(3/2)*atan((b*tanh(c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*tanh(c + d*x))^
(1/2))/d + (b^(3/2)*atanh((b*tanh(c + d*x))^(1/2)/b^(1/2)))/d
```

3.16 $\int \sqrt{b \tanh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \sqrt{b \tanh(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3557, 335, 304, 209, 212}

$$\int \sqrt{b \tanh(c+dx)}\,dx = \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[In] Int[Sqrt[b*Tanh[c + d*x]],x]
```

```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Tanh[c + d*x]]/Sqrt[b]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{d}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}\\
&= \frac{b\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}\\
&= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \sqrt{b\tanh(c+dx)}\,dx = -\frac{\left(\arctan\left(\sqrt{\tanh(c+dx)}\right)-\operatorname{arctanh}\left(\sqrt{\tanh(c+dx)}\right)\right)\sqrt{b\tanh(c+dx)}}{d\sqrt{\tanh(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Tanh[c + d*x]],x]
[Out] -(((ArcTan[Sqrt[Tanh[c + d*x]]] - ArcTanh[Sqrt[Tanh[c + d*x]]])*Sqrt[b*Tanh
[c + d*x]])/(d*Sqrt[Tanh[c + d*x]]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}$	47
default	$-\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}$	47

```
[In] int((b*tanh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(46) = 92.

Time = 0.29 (sec) , antiderivative size = 593, normalized size of antiderivative = 10.22

$$\int \sqrt{b\tanh(c+dx)}\,dx = \left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\sinh(dx+c)}{\cosh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b}\right)-\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate((b*tanh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, 1/4*(2*sqrt(b)*arcta
n(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh
(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + sqrt(b)*log(2*b*cosh(d*
x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cos
h(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(
d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 +
cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) -
 b))/d]
```

Sympy [F]

$$\int \sqrt{b\tanh(c+dx)}\,dx = \int \sqrt{b\tanh{(c+dx)}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tanh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\tanh(c+dx)}\,dx = \int \sqrt{b\tanh{(dx+c)}}\,dx$$

```
[In] integrate((b*tanh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*tanh(d*x + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.52

$$\int \sqrt{b\tanh(c+dx)}\,dx$$
$$= -\frac{2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)+\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{2\,d}$$

```
[In] integrate((b*tanh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b)) + sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x
+ 4*c) - b))))/d
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \sqrt{b\tanh(c+dx)}\,dx = -\frac{\sqrt{b}\left(\operatorname{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)-\operatorname{atanh}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)\right)}{d}$$

```
[In] int((b*tanh(c + d*x))^(1/2),x)
```

```
[Out] -(b^(1/2)*(atan((b*tanh(c + d*x))^(1/2)/b^(1/2)) - atanh((b*tanh(c + d*x))^
(1/2)/b^(1/2))))/d
```

3.17 $\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[Out] arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*tanh(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3557, 335, 218, 212, 209}

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[In] Int[1/Sqrt[b*Tanh[c + d*x]],x]
```

```
[Out] ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Tanh[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\tanh(c+dx)\right)}{d}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}+\frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{d}\\
&= \frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = \frac{\left(\arctan\left(\sqrt{\tanh(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{\tanh(c+dx)}\right)\right)\sqrt{\tanh(c+dx)}}{d\sqrt{b\tanh(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Tanh[c + d*x]],x]
```

```
[Out] ((ArcTan[Sqrt[Tanh[c + d*x]]] + ArcTanh[Sqrt[Tanh[c + d*x]]])*Sqrt[Tanh[c +
 d*x]])/(d*Sqrt[b*Tanh[c + d*x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}$	46
default	$\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}$	46

```
[In] int(1/(b*tanh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*tanh(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(45) = 90$.

Time = 0.26 (sec) , antiderivative size = 599, normalized size of antiderivative = 10.51

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx$$
$$= \left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\sinh(dx+c)}{\cosh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b}\right)+\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4}{\ldots}\right.}{\ldots} \right.$$
$$-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{\frac{b\sinh(dx+c)}{\cosh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2-b}\right)-\sqrt{b}\log\left(2b\cosh(dx+c)^4+8b\cosh(dx\ldots\right.}{\ldots}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), -1/4*(2*sqrt(b)*
arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - sqrt(b)*log(2*b*co
sh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*si
nh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2
*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*
cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)
^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x +
c)) - b))/(b*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tanh{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*tanh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\tanh{(dx+c)}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*tanh(d*x + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)+\operatorname{atanh}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

```
[In] int(1/(b*tanh(c + d*x))^(1/2),x)
```

```
[Out] (atan((b*tanh(c + d*x))^(1/2)/b^(1/2)) + atanh((b*tanh(c + d*x))^(1/2)/b^(1
/2)))/(b^(1/2)*d)
```

3.18 $\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\tanh(c+dx)}}$$

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*tanh(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\tanh(c+dx)}}$$

```
[In] Int[(b*Tanh[c + d*x])^(-3/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Tanh[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Tanh[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2}{bd\sqrt{b\tanh(c+dx)}}+\frac{\int\sqrt{b\tanh(c+dx)}\,dx}{b^2}\\
&= -\frac{2}{bd\sqrt{b\tanh(c+dx)}}-\frac{\text{Subst}\left(\int\frac{\sqrt{x}}{-b^2+x^2}\,dx,x,b\tanh(c+dx)\right)}{bd}\\
&= -\frac{2}{bd\sqrt{b\tanh(c+dx)}}-\frac{2\text{Subst}\left(\int\frac{x^2}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{bd}
\end{aligned}
$$

$$= -\frac{2}{bd\sqrt{b\tanh(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{bd}$$

$$= -\frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\tanh(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = \frac{-2 - \arctan\left(\sqrt[4]{\tanh^2(c+dx)}\right)\sqrt[4]{\tanh^2(c+dx)} + \text{arctanh}\left(\sqrt[4]{\tanh^2(c+dx)}\right)}{bd\sqrt{b\tanh(c+dx)}}$$

```
[In] Integrate[(b*Tanh[c + d*x])^(-3/2),x]
```

```
[Out] (-2 - ArcTan[(Tanh[c + d*x]^2)^(1/4)]*(Tanh[c + d*x]^2)^(1/4) + ArcTanh[(Ta
nh[c + d*x]^2)^(1/4)]*(Tanh[c + d*x]^2)^(1/4))/(b*d*Sqrt[b*Tanh[c + d*x]])
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d} + \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d} - \frac{2}{bd\sqrt{b\tanh(dx+c)}}$	65
default	$-\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d} + \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d} - \frac{2}{bd\sqrt{b\tanh(dx+c)}}$	65

```
[In] int(1/(b*tanh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*tanh(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 924, normalized size of antiderivative = 11.85

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 - 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1
)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b*sinh(d*x + c)/cosh(d*x
 + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 - b^2*d), 1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)
/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 - b)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*si
nh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh
(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*si
nh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 +
 cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqr
t(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - b) - 8*(cosh(d*x + c)^2 + 2*cosh
(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b*sinh(d*x + c)/cosh(d*
x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2
*d*sinh(d*x + c)^2 - b^2*d)]
```

Sympy [F]

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tanh{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))**(3/2),x)
[Out] Integral((b*tanh(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\tanh{(dx+c)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c))^(-3/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = \frac{4}{\left(\sqrt{b}e^{(2\,dx+2\,c)} - \sqrt{be^{(4\,dx+4\,c)} - b} - \sqrt{b}\right)bd}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(3/2),x, algorithm="giac")
[Out] 4/((sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b))*b*d)
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx = \frac{\mathrm{atanh}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{\mathrm{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{2}{b\,d\,\sqrt{b\tanh{(c+d\,x)}}}$$

```
[In] int(1/(b*tanh(c + d*x))^(3/2),x)
[Out] atanh((b*tanh(c + d*x))^(1/2)/b^(1/2))/(b^(3/2)*d) - atan((b*tanh(c + d*x))
^(1/2)/b^(1/2))/(b^(3/2)*d) - 2/(b*d*(b*tanh(c + d*x))^(1/2))
```

3.19 $\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\tanh(c+dx))^{3/2}}$$

```
[Out] arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*tanh(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*tanh(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3555, 3557, 335, 218, 212, 209}

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\tanh(c+dx))^{3/2}}$$

```
[In] Int[(b*Tanh[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Tanh[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Tanh[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\tanh(c+dx)}}\,dx}{b^2}\\
&= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx,x,b\tanh(c+dx)\right)}{bd}\\
&= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}}-\frac{2\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,x,\sqrt{b\tanh(c+dx)}\right)}{bd}
\end{aligned}$$

$$= -\frac{2}{3bd(b\tanh(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{b^2 d} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{b^2 d}$$

$$= \frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\tanh(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \frac{-2 + 3\arctan\left(\sqrt[4]{\tanh^2(c+dx)}\right)\tanh^2(c+dx)^{3/4} + 3\text{arctanh}\left(\sqrt[4]{\tanh^2(c+dx}\right.}{3bd(b\tanh(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Tanh[c + d*x])^(-5/2),x]
```

```
[Out] (-2 + 3*ArcTan[(Tanh[c + d*x]^2)^(1/4)]*(Tanh[c + d*x]^2)^(3/4) + 3*ArcTanh
[(Tanh[c + d*x]^2)^(1/4)]*(Tanh[c + d*x]^2)^(3/4))/(3*b*d*(b*Tanh[c + d*x])
^(3/2))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d} - \frac{2}{3bd(b\tanh(dx+c))^{\frac{3}{2}}}$	64
default	$\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d} - \frac{2}{3bd(b\tanh(dx+c))^{\frac{3}{2}}}$	64

```
[In] int(1/(b*tanh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*tanh(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*tanh(d*x+c))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 691 vs. $2(63) = 126$.

Time = 0.30 (sec) , antiderivative size = 1436, normalized size of antiderivative = 18.18

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*
sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 - b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2
- 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1
)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) - 2*b)/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d
*x + c))*sinh(d*x + c) + 1)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)))/(b^3*d*cos
h(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4 - 2*b^3*d*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 - b^3*d)*s
inh(d*x + c)^2 + 4*(b^3*d*cosh(d*x + c)^3 - b^3*d*cosh(d*x + c))*sinh(d*x +
 c)), -1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(
b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) - 3*(cosh(d*x + c)^4 + 4*cosh(d*
x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d
*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*
x + c) + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x +
c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sq
rt(b*sinh(d*x + c)/cosh(d*x + c)) - b) + 8*(cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
```

```
c) + 1)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 - 2*b^3*d*cosh(d*x
 + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 - b^3*d)*sinh(d*x + c)^2 + 4*(
b^3*d*cosh(d*x + c)^3 - b^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tanh\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))**(5/2),x)
[Out] Integral((b*tanh(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\tanh\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c))^(-5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \frac{4\left(3\left(\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}\right)^{2}+b\right)}{3\left(\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}-\sqrt{b}\right)^{3}b^{2}d}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(5/2),x, algorithm="giac")
[Out] 4/3*(3*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2 + b)/((sqr
t(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b))^3*b^2*d)
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{1}{(b\tanh(c+dx))^{5/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d} - \frac{2}{3\,b\,d\,\left(b\tanh\left(c+d\,x\right)\right)^{3/2}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d}$$

```
[In] int(1/(b*tanh(c + d*x))^(5/2),x)
```

```
[Out] atan((b*tanh(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d) - 2/(3*b*d*(b*tanh(c + d*
x))^(3/2)) + atanh((b*tanh(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d)
```

3.20 $\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 100

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{5bd(b\tanh(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\tanh(c+dx)}}$$

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/b^3/d/(b*tanh(d*x+c))^(1/2)-2/5/b/d/(b*tanh(d*x+c)
)^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d}-\frac{2}{b^3d\sqrt{b\tanh(c+dx)}}-\frac{2}{5bd(b\tanh(c+dx))^{5/2}}$$

```
[In] Int[(b*Tanh[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Tanh[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Tanh[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Tanh[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Tanh[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}}+\frac{\int \frac{1}{(b\tanh(c+dx))^{3/2}}\,dx}{b^2}\\ &= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}}-\frac{2}{b^3d\sqrt{b\tanh(c+dx)}}+\frac{\int \sqrt{b\tanh(c+dx)}\,dx}{b^4}\end{aligned}$$

$$= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{b^3d} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\tanh(c+dx)}\right)}{b^3d}$$

$$= -\frac{\arctan\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{5bd(b\tanh(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\tanh(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx = \frac{-2\coth^2(c+dx) + 5\text{arctanh}\left(\sqrt[4]{\tanh^2(c+dx)}\right)\sqrt[4]{\tanh^2(c+dx)} - 5\left(2 + \text{arcta}\right.}{5b^3d\sqrt{b\tanh(c+dx)}}$$

```
[In] Integrate[(b*Tanh[c + d*x])^(-7/2),x]
```

```
[Out] (-2*Coth[c + d*x]^2 + 5*ArcTanh[(Tanh[c + d*x]^2)^(1/4)]*(Tanh[c + d*x]^2)^
(1/4) - 5*(2 + ArcTan[(Tanh[c + d*x]^2)^(1/4)]*(Tanh[c + d*x]^2)^(1/4)))/(5
*b^3*d*Sqrt[b*Tanh[c + d*x]])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}+\frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}-\frac{2}{b^3d\sqrt{b\tanh(dx+c)}}-\frac{2}{5bd(b\tanh(dx+c))^{\frac{5}{2}}}$	83
default	$-\frac{\arctan\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}+\frac{\text{arctanh}\left(\frac{\sqrt{b\tanh(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}-\frac{2}{b^3d\sqrt{b\tanh(dx+c)}}-\frac{2}{5bd(b\tanh(dx+c))^{\frac{5}{2}}}$	83

```
[In] int(1/(b*tanh(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((b*tanh(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*tanh(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/b^3/d/(b*tanh(d*x+c))^(1/2)-2/5/b/d/(b*tanh(d*x+c)
)^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 2144, normalized size of antiderivative = 21.44

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x +
c)^6 + 3*(5*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - 3*cosh(d*x + c)^4 + 4*(5
*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4
- 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x
+ c)^5 - 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) - 1)*sqrt(-b)*arc
tan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqr
t(-b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + 5*(cosh(d*x + c)^6 + 6*cosh
(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 - 1)*sin
h(d*x + c)^4 - 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 - 3*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 - 6*cosh(d*x + c)^2 + 1)*sinh(d*x +
c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 - 2*cosh(d*x + c)^3 + cosh(d*
x + c))*sinh(d*x + c) - 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*sqrt(b*sinh(d*x + c)/cosh
(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sin
h(d*x + c)^6 + (45*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 +
 4*(15*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)
^4 - 6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d
*x + c)^5 - 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 3)*sqrt(b*si
nh(d*x + c)/cosh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*
sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 - 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d
*cosh(d*x + c)^2 - b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 - b^4*d)*sinh(d*x + c
)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 - 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 3*(5*b^4*d*cosh(d*x + c)^4 - 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^2 + 6*(b^4*d*cosh(d*x + c)^5 - 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), 1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x +
 c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - 3*cos
```

```
h(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*
(5*cosh(d*x + c)^4 - 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x +
c)^2 + 6*(cosh(d*x + c)^5 - 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c
) - 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c))/(b*cosh(d
*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)) + 5*(
cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*
cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x +
c)^3 - 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 - 6*cosh(d*x
 + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 - 2*c
osh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) - 1)*sqrt(b)*log(2*b*cosh(d*x
 + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh
(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d
*x + c)^2 + 1)*sinh(d*x + c)^2 + cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 + c
osh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*sinh(d*x + c)/cosh(d*x + c)) -
b) - 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x
+ c)^6 + (45*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 + 4*(15
*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 - 6
*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d*x + c
)^5 - 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 3)*sqrt(b*sinh(d*x
 + c)/cosh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(d
*x + c)^5 + b^4*d*sinh(d*x + c)^6 - 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cosh(
d*x + c)^2 - b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 - b^4*d)*sinh(d*x + c)^4 +
4*(5*b^4*d*cosh(d*x + c)^3 - 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*
b^4*d*cosh(d*x + c)^4 - 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 +
6*(b^4*d*cosh(d*x + c)^5 - 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x + c))*s
inh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\tanh{(c+dx)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))**(7/2),x)
[Out] Integral((b*tanh(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\tanh(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*tanh(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c))^(-7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(82) = 164.

Time = 0.51 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.07

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx = \frac{8\left(5\left(\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}\right)^4-10\left(\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}\right)^3 \cdots}{5\left(\sqrt{b}e^{(2\,dx+2\,c)} \cdots\right.}$$

```
[In] integrate(1/(b*tanh(d*x+c))^(7/2),x, algorithm="giac")
[Out] 8/5*(5*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4 - 10*(sqrt
(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3*sqrt(b) + 20*(sqrt(b)*
e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*b - 10*(sqrt(b)*e^(2*d*x +
 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))*b^(3/2) + 3*b^2)/((sqrt(b)*e^(2*d*x +
2*c) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b))^5*b^3*d)
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{1}{(b\tanh(c+dx))^{7/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\operatorname{atan}\left(\frac{\sqrt{b\tanh(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\frac{2}{5\,b}+\frac{2\tanh(c+d\,x)^2}{b}}{d\,(b\tanh(c+d\,x))^{5/2}}$$

```
[In] int(1/(b*tanh(c + d*x))^(7/2),x)
[Out] atanh((b*tanh(c + d*x))^(1/2)/b^(1/2))/(b^(7/2)*d) - atan((b*tanh(c + d*x))
^(1/2)/b^(1/2))/(b^(7/2)*d) - (2/(5*b) + (2*tanh(c + d*x)^2)/b)/(d*(b*tanh(
c + d*x))^(5/2))
```

3.21 $\int \sqrt[3]{\tanh(8x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int \sqrt[3]{\tanh(8x)}\,dx = -\frac{1}{16}\sqrt{3}\arctan\left(\frac{1+2\tanh^{\frac{2}{3}}(8x)}{\sqrt{3}}\right) - \frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right) + \frac{1}{32}\log\left(1+\tanh^{\frac{2}{3}}(8x)+\tanh^{\frac{4}{3}}(8x)\right)$$

```
[Out] -1/16*ln(1-tanh(8*x)^(2/3))+1/32*ln(1+tanh(8*x)^(2/3)+tanh(8*x)^(4/3))-1/16
*arctan(1/3*(1+2*tanh(8*x)^(2/3))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {3557, 335, 281, 298, 31, 648, 632, 210, 642}

$$\int \sqrt[3]{\tanh(8x)}\,dx = -\frac{1}{16}\sqrt{3}\arctan\left(\frac{2\tanh^{\frac{2}{3}}(8x)+1}{\sqrt{3}}\right) - \frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right) + \frac{1}{32}\log\left(\tanh^{\frac{4}{3}}(8x)+\tanh^{\frac{2}{3}}(8x)+1\right)$$

```
[In] Int[Tanh[8*x]^(1/3),x]
```

```
[Out] -1/16*(Sqrt[3]*ArcTan[(1 + 2*Tanh[8*x]^(2/3))/Sqrt[3]]) - Log[1 - Tanh[8*x]
^(2/3)]/16 + Log[1 + Tanh[8*x]^(2/3) + Tanh[8*x]^(4/3)]/32
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{\sqrt[3]{x}}{-1+x^2}\,dx, x, \tanh(8x)\right)\right)\\
&= -\left(\frac{3}{8}\text{Subst}\left(\int \frac{x^3}{-1+x^6}\,dx, x, \sqrt[3]{\tanh(8x)}\right)\right)\\
&= -\left(\frac{3}{16}\text{Subst}\left(\int \frac{x}{-1+x^3}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\right)\\
&= -\left(\frac{1}{16}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\right) + \frac{1}{16}\text{Subst}\left(\int \frac{-1+x}{1+x+x^2}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\\
&= -\frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right) + \frac{1}{32}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\\
&\quad - \frac{3}{32}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \tanh^{\frac{2}{3}}(8x)\right)\\
&= -\frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right) + \frac{1}{32}\log\left(1+\tanh^{\frac{2}{3}}(8x)+\tanh^{\frac{4}{3}}(8x)\right)\\
&\quad + \frac{3}{16}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\tanh^{\frac{2}{3}}(8x)\right)\\
&= -\frac{1}{16}\sqrt{3}\arctan\left(\frac{1+2\tanh^{\frac{2}{3}}(8x)}{\sqrt{3}}\right) - \frac{1}{16}\log\left(1-\tanh^{\frac{2}{3}}(8x)\right)\\
&\quad + \frac{1}{32}\log\left(1+\tanh^{\frac{2}{3}}(8x)+\tanh^{\frac{4}{3}}(8x)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \sqrt[3]{\tanh(8x)}\,dx = -\frac{\left(\log\left(1-\sqrt[3]{\tanh^2(8x)}\right) - \sqrt[3]{-1}\log\left(1+\sqrt[3]{-1}\sqrt[3]{\tanh^2(8x)}\right) + (-1)^{2/3}\log\left(1-(-1)^{2/3}\sqrt[3]{\tanh^2(8x)}\right.\right.}{16\tanh^2(8x)^{2/3}}$$

```
[In] Integrate[Tanh[8*x]^(1/3),x]
```

```
[Out] -1/16*((Log[1 - (Tanh[8*x]^2)^(1/3)] - (-1)^(1/3)*Log[1 + (-1)^(1/3)*(Tanh[
8*x]^2)^(1/3)] + (-1)^(2/3)*Log[1 - (-1)^(2/3)*(Tanh[8*x]^2)^(1/3)])*Tanh[8
*x]^(4/3))/(Tanh[8*x]^2)^(2/3)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48

method	result
derivativedivides	$-\frac{\ln\left(\tanh(8x)^{\frac{1}{3}}-1\right)}{16}+\frac{\ln\left(\tanh(8x)^{\frac{2}{3}}+\tanh(8x)^{\frac{1}{3}}+1\right)}{32}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2\tanh(8x)^{\frac{1}{3}}+1\right)\sqrt{3}}{3}\right)}{16}-\frac{\ln\left(\tanh(8x)^{\frac{1}{3}}+1\right.}{16}$
default	$-\frac{\ln\left(\tanh(8x)^{\frac{1}{3}}-1\right)}{16}+\frac{\ln\left(\tanh(8x)^{\frac{2}{3}}+\tanh(8x)^{\frac{1}{3}}+1\right)}{32}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2\tanh(8x)^{\frac{1}{3}}+1\right)\sqrt{3}}{3}\right)}{16}-\frac{\ln\left(\tanh(8x)^{\frac{1}{3}}+1\right.}{16}$

```
[In] int(tanh(8*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*ln(tanh(8*x)^(1/3)-1)+1/32*ln(tanh(8*x)^(2/3)+tanh(8*x)^(1/3)+1)+1/16
*3^(1/2)*arctan(1/3*(2*tanh(8*x)^(1/3)+1)*3^(1/2))-1/16*ln(tanh(8*x)^(1/3)+
1)+1/32*ln(tanh(8*x)^(2/3)-tanh(8*x)^(1/3)+1)-1/16*3^(1/2)*arctan(1/3*(2*ta
nh(8*x)^(1/3)-1)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(52) = 104$.

Time = 0.25 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.59

$$\int \sqrt[3]{\tanh(8x)}\,dx$$
$$= -\frac{1}{16}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\frac{\sinh(8x)}{\cosh(8x)}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)-\frac{1}{16}\log\left(\left(\frac{\sinh(8x)}{\cosh(8x)}\right)^{\frac{2}{3}}-1\right)$$
$$+\frac{1}{32}\log\left(\frac{\cosh(8x)^2+2\cosh(8x)\sinh(8x)+\sinh(8x)^2+\left(\cosh(8x)^2+2\cosh(8x)\sinh(8x)+\sinh\right.}{\cosh(8x)^2+2\cosh(8x)}\right.$$

```
[In] integrate(tanh(8*x)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(3)*arctan(2/3*sqrt(3)*(sinh(8*x)/cosh(8*x))^(2/3) + 1/3*sqrt(3))
 - 1/16*log((sinh(8*x)/cosh(8*x))^(2/3) - 1) + 1/32*log((cosh(8*x)^2 + 2*co
sh(8*x)*sinh(8*x) + sinh(8*x)^2 + (cosh(8*x)^2 + 2*cosh(8*x)*sinh(8*x) + si
nh(8*x)^2 + 1)*(sinh(8*x)/cosh(8*x))^(2/3) + (cosh(8*x)^2 + 2*cosh(8*x)*sin
h(8*x) + sinh(8*x)^2 - 1)*(sinh(8*x)/cosh(8*x))^(1/3) + 1)/(cosh(8*x)^2 + 2
*cosh(8*x)*sinh(8*x) + sinh(8*x)^2 + 1))
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \sqrt[3]{\tanh(8x)}\,dx = -\frac{\log\left(\tanh^{\frac{2}{3}}(8x)-1\right)}{16} + \frac{\log\left(\tanh^{\frac{4}{3}}(8x)+\tanh^{\frac{2}{3}}(8x)+1\right)}{32} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\left(\tanh^{\frac{2}{3}}(8x)+\frac{1}{2}\right)}{3}\right)}{16}$$

```
[In] integrate(tanh(8*x)**(1/3),x)
[Out] -log(tanh(8*x)**(2/3) - 1)/16 + log(tanh(8*x)**(4/3) + tanh(8*x)**(2/3) + 1
)/32 - sqrt(3)*atan(2*sqrt(3)*(tanh(8*x)**(2/3) + 1/2)/3)/16
```

Maxima [F]

$$\int \sqrt[3]{\tanh(8x)}\,dx = \int \tanh(8x)^{\frac{1}{3}}\,dx$$

```
[In] integrate(tanh(8*x)^(1/3),x, algorithm="maxima")
[Out] integrate(tanh(8*x)^(1/3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int \sqrt[3]{\tanh(8x)}\,dx = -\frac{1}{16}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\frac{e^{(16x)}-1}{e^{(16x)}+1}\right)^{\frac{2}{3}}+1\right)\right) + \frac{1}{32}\log\left(\left(\frac{e^{(16x)}-1}{e^{(16x)}+1}\right)^{\frac{2}{3}} + \frac{\left(\frac{e^{(16x)}-1}{e^{(16x)}+1}\right)^{\frac{1}{3}}\left(e^{(16x)}-1\right)}{e^{(16x)}+1}+1\right) - \frac{1}{16}\log\left(\left|\left(\frac{e^{(16x)}-1}{e^{(16x)}+1}\right)^{\frac{2}{3}}-1\right|\right)$$

```
[In] integrate(tanh(8*x)^(1/3),x, algorithm="giac")
[Out] -1/16*sqrt(3)*arctan(1/3*sqrt(3)*(2*((e^(16*x) - 1)/(e^(16*x) + 1))^(2/3) +
 1)) + 1/32*log(((e^(16*x) - 1)/(e^(16*x) + 1))^(2/3) + ((e^(16*x) - 1)/(e^
(16*x) + 1))^(1/3)*(e^(16*x) - 1)/(e^(16*x) + 1) + 1) - 1/16*log(abs(((e^(1
6*x) - 1)/(e^(16*x) + 1))^(2/3) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \sqrt[3]{\tanh(8x)}\,dx = -\frac{\ln\left(81\tanh(8\,x)^{2/3}-81\right)}{16} - \ln\left(162\tanh(8\,x)^{2/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-81\right)\left(-\frac{1}{32}+\frac{\sqrt{3}\,1\mathrm{i}}{32}\right) + \ln\left(-162\tanh(8\,x)^{2/3}\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-81\right)\left(\frac{1}{32}+\frac{\sqrt{3}\,1\mathrm{i}}{32}\right)$$

```
[In] int(tanh(8*x)^(1/3),x)
```

```
[Out] log(- 162*tanh(8*x)^(2/3)*((3^(1/2)*1i)/4 + 1/4) - 81)*((3^(1/2)*1i)/32 + 1
/32) - log(162*tanh(8*x)^(2/3)*((3^(1/2)*1i)/4 - 1/4) - 81)*((3^(1/2)*1i)/3
2 - 1/32) - log(81*tanh(8*x)^(2/3) - 81)/16
```

3.22 $\int \tanh^n(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \tanh^n(a+bx)\,dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\tanh^2(a+bx)\right)\tanh^{1+n}(a+bx)}{b(1+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],tanh(b*x+a)^2)*tanh(b*x+a)^(1+n)/b/(1+
n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3557, 371}

$$\int \tanh^n(a+bx)\,dx = \frac{\tanh^{n+1}(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\tanh^2(a+bx)\right)}{b(n+1)}$$

```
[In] Int[Tanh[a + b*x]^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[a + b*x]^2]*Tanh[a + b*x]^
(1 + n))/(b*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^n}{-1+x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \tanh^2(a+bx)\right)\tanh^{1+n}(a+bx)}{b(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \tanh^n(a+bx)\,dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \tanh^2(a+bx)\right)\tanh^{1+n}(a+bx)}{b(1+n)}$$

```
[In] Integrate[Tanh[a + b*x]^n,x]
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[a + b*x]^2]*Tanh[a + b*x]^
(1 + n))/(b*(1 + n))
```

Maple [F]

$$\int \tanh\left(bx+a\right)^{n}dx$$

```
[In] int(tanh(b*x+a)^n,x)
[Out] int(tanh(b*x+a)^n,x)
```

Fricas [F]

$$\int \tanh^n(a+bx)\,dx = \int \tanh\left(bx+a\right)^{n}\,dx$$

```
[In] integrate(tanh(b*x+a)^n,x, algorithm="fricas")
[Out] integral(tanh(b*x + a)^n, x)
```

Sympy [F]

$$\int \tanh^n(a + bx) \, dx = \int \tanh^n (a + bx) \, dx$$

```
[In] integrate(tanh(b*x+a)**n,x)
[Out] Integral(tanh(a + b*x)**n, x)
```

Maxima [F]

$$\int \tanh^n(a + bx) \, dx = \int \tanh (bx + a)^n \, dx$$

```
[In] integrate(tanh(b*x+a)^n,x, algorithm="maxima")
[Out] integrate(tanh(b*x + a)^n, x)
```

Giac [F]

$$\int \tanh^n(a + bx) \, dx = \int \tanh (bx + a)^n \, dx$$

```
[In] integrate(tanh(b*x+a)^n,x, algorithm="giac")
[Out] integrate(tanh(b*x + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^n(a + bx) \, dx = \int \tanh(a + b\,x)^n \, dx$$

```
[In] int(tanh(a + b*x)^n,x)
[Out] int(tanh(a + b*x)^n, x)
```

3.23 $\int (b\tanh(c+dx))^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int (b\tanh(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \tanh^2(c+dx)\right)(b\tanh(c+dx))^{1+n}}{bd(1+n)}$$

```
[Out] hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],tanh(d*x+c)^2)*(b*tanh(d*x+c))^(1+n)/b
/d/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3557, 371}

$$\int (b\tanh(c+dx))^n \, dx = \frac{(b\tanh(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1, \frac{n+1}{2}, \frac{n+3}{2}, \tanh^2(c+dx)\right)}{bd(n+1)}$$

```
[In] Int[(b*Tanh[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[c + d*x]^2]*(b*Tanh[c + d*
x])^(1 + n))/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{x^n}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{d}\\ &= \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \tanh^2(c+dx)\right)(b\tanh(c+dx))^{1+n}}{bd(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (b\tanh(c+dx))^n\,dx = \frac{\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \tanh^2(c+dx)\right)\tanh(c+dx)(b\tanh(c+dx))^n}{d(1+n)}$$

```
[In] Integrate[(b*Tanh[c + d*x])^n,x]
[Out] (Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Tanh[c + d*x]^2]*Tanh[c + d*x]*
(b*Tanh[c + d*x])^n)/(d*(1 + n))
```

Maple [F]

$$\int (b\tanh(dx+c))^n\,dx$$

```
[In] int((b*tanh(d*x+c))^n,x)
[Out] int((b*tanh(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\tanh(c+dx))^n\,dx = \int (b\tanh(dx+c))^n\,dx$$

```
[In] integrate((b*tanh(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*tanh(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\tanh(c+dx))^n\,dx = \int (b\tanh{(c+dx)})^n\,dx$$

```
[In] integrate((b*tanh(d*x+c))**n,x)
[Out] Integral((b*tanh(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\tanh(c+dx))^n\,dx = \int (b\tanh{(dx+c)})^n\,dx$$

```
[In] integrate((b*tanh(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c))^n, x)
```

Giac [F]

$$\int (b\tanh(c+dx))^n\,dx = \int (b\tanh{(dx+c)})^n\,dx$$

```
[In] integrate((b*tanh(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*tanh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tanh(c+dx))^n\,dx = \int (b\tanh(c+d\,x))^n\,dx$$

```
[In] int((b*tanh(c + d*x))^n,x)
[Out] int((b*tanh(c + d*x))^n, x)
```

3.24 $\int \left(a \tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \left(a \tanh^2(x)\right)^{3/2} dx = a \coth(x) \log(\cosh(x)) \sqrt{a \tanh^2(x)} - \frac{1}{2} a \tanh(x) \sqrt{a \tanh^2(x)}$$

```
[Out] a*coth(x)*ln(cosh(x))*(a*tanh(x)^2)^(1/2)-1/2*a*(a*tanh(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3739, 3554, 3556}

$$\int \left(a \tanh^2(x)\right)^{3/2} dx = a \coth(x) \sqrt{a \tanh^2(x)} \log(\cosh(x)) - \frac{1}{2} a \tanh(x) \sqrt{a \tanh^2(x)}$$

```
[In] Int[(a*Tanh[x]^2)^(3/2),x]
```

```
[Out] a*Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2] - (a*Tanh[x]*Sqrt[a*Tanh[x]^2])/2
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a\coth(x)\sqrt{a\tanh^2(x)}\right)\int \tanh^3(x)\,dx\\
&= -\frac{1}{2}a\tanh(x)\sqrt{a\tanh^2(x)} + \left(a\coth(x)\sqrt{a\tanh^2(x)}\right)\int \tanh(x)\,dx\\
&= a\coth(x)\log(\cosh(x))\sqrt{a\tanh^2(x)} - \frac{1}{2}a\tanh(x)\sqrt{a\tanh^2(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$
\int \left(a\tanh^2(x)\right)^{3/2}\,dx = \frac{1}{2}\coth(x)\left(-1+2\coth^2(x)\log(\cosh(x))\right)\left(a\tanh^2(x)\right)^{3/2}
$$

```
[In] Integrate[(a*Tanh[x]^2)^(3/2),x]
[Out] (Coth[x]*(-1 + 2*Coth[x]^2*Log[Cosh[x]])*(a*Tanh[x]^2)^(3/2))/2
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{\left(a\tanh(x)^2\right)^{\frac{3}{2}}\left(\tanh(x)^2+\ln(\tanh(x)-1)+\ln(1+\tanh(x))\right)}{2\tanh(x)^3}$	30
default	$-\frac{\left(a\tanh(x)^2\right)^{\frac{3}{2}}\left(\tanh(x)^2+\ln(\tanh(x)-1)+\ln(1+\tanh(x))\right)}{2\tanh(x)^3}$	30
risch	$\frac{a\sqrt{\frac{a\left(e^{2x}-1\right)^2}{\left(1+e^{2x}\right)^2}}\left(e^{4x}\ln(1+e^{2x})-e^{4x}x+2\,e^{2x}\ln(1+e^{2x})-2\,e^{2x}x+2\,e^{2x}+\ln(1+e^{2x})-x\right)}{(e^{2x}-1)(1+e^{2x})}$	95

[In] int((a*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(a*tanh(x)^2)^(3/2)*(tanh(x)^2+ln(tanh(x)-1)+ln(1+tanh(x)))/tanh(x)^3

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 467 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 467, normalized size of antiderivative = 13.34

$$\int \left(a\tanh^2(x)\right)^{3/2} dx = -\frac{\left(ax\cosh\left(x\right)^4 + \left(axe^{(2\,x)} + ax\right)\sinh\left(x\right)^4 + 4\left(ax\cosh\left(x\right)e^{(2\,x)} + ax\cosh\left(x\right)\right)\sinh\left(x\right)^3 + 2\left(ax - a\right)\cosh\left(x\right.}{}$$

[In] integrate((a*tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out] -(a*x*cosh(x)^4 + (a*x*e^(2*x) + a*x)*sinh(x)^4 + 4*(a*x*cosh(x)*e^(2*x) + a*x*cosh(x))*sinh(x)^3 + 2*(a*x - a)*cosh(x)^2 + 2*(3*a*x*cosh(x)^2 + a*x + (3*a*x*cosh(x)^2 + a*x - a)*e^(2*x) - a)*sinh(x)^2 + a*x + (a*x*cosh(x)^4 + 2*(a*x - a)*cosh(x)^2 + a*x)*e^(2*x) - (a*cosh(x)^4 + (a*e^(2*x) + a)*sinh(x)^4 + 4*(a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + (3*a*cosh(x)^2 + a)*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^4 + 2*a*cosh(x)^2 + a)*e^(2*x) + 4*(a*cosh(x)^3 + a*cosh(x) + (a*cosh(x)^3 + a*cosh(x))*e^(2*x))*sinh(x) + a)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 4*(a*x*cosh(x)^3 + (a*x - a)*cosh(x) + (a*x*cosh(x)^3 + (a*x - a)*cosh(x))*e^(2*x))*sinh(x))*sqrt((a*e^(4*x) - 2*a*e^(2*x) + a)/(e^(4*x) + 2*e^(2*x) + 1))/((e^(2*x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(cosh(x)*e^(2*x) - cosh(x))*sinh(x)^3 - 2*(3*cosh(x)^2 - (3*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^2 - 2*cosh(x)^2 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^(2*x) - 4*(cosh(x)^3 - (cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) - 1)

Sympy [F]

$$\int \left(a\tanh^2(x)\right)^{3/2} dx = \int \left(a\tanh^2\left(x\right)\right)^{\frac{3}{2}}\, dx$$

[In] integrate((a*tanh(x)**2)**(3/2),x)

[Out] Integral((a*tanh(x)**2)**(3/2), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \left(a\tanh^2(x)\right)^{3/2}\,dx = -a^{\frac{3}{2}}x - a^{\frac{3}{2}}\log\left(e^{(-2x)}+1\right) - \frac{2a^{\frac{3}{2}}e^{(-2x)}}{2e^{(-2x)}+e^{(-4x)}+1}$$

```
[In] integrate((a*tanh(x)^2)^(3/2),x, algorithm="maxima")
[Out] -a^(3/2)*x - a^(3/2)*log(e^(-2*x) + 1) - 2*a^(3/2)*e^(-2*x)/(2*e^(-2*x) + e
^(-4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int \left(a\tanh^2(x)\right)^{3/2}\,dx = -\left(x\operatorname{sgn}\left(e^{(4x)}-1\right) - \log\left(e^{(2x)}+1\right)\operatorname{sgn}\left(e^{(4x)}-1\right) - \frac{2e^{(2x)}\operatorname{sgn}\left(e^{(4x)}-1\right)}{\left(e^{(2x)}+1\right)^2}\right)a^{\frac{3}{2}}$$

```
[In] integrate((a*tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] -(x*sgn(e^(4*x) - 1) - log(e^(2*x) + 1)*sgn(e^(4*x) - 1) - 2*e^(2*x)*sgn(e^
(4*x) - 1)/(e^(2*x) + 1)^2)*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\tanh^2(x)\right)^{3/2}\,dx = \int \left(a\tanh(x)^2\right)^{3/2}\,dx$$

```
[In] int((a*tanh(x)^2)^(3/2),x)
[Out] int((a*tanh(x)^2)^(3/2), x)
```

3.25 $\int \sqrt{a \tanh^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{a \tanh^2(x)}\,dx = \coth(x)\log(\cosh(x))\sqrt{a \tanh^2(x)}$$

```
[Out] coth(x)*ln(cosh(x))*(a*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3739, 3556}

$$\int \sqrt{a \tanh^2(x)}\,dx = \coth(x)\sqrt{a \tanh^2(x)}\log(\cosh(x))$$

```
[In] Int[Sqrt[a*Tanh[x]^2],x]
```

```
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\coth(x)\sqrt{a\tanh^2(x)}\right)\int \tanh(x)\,dx\\ &= \coth(x)\log(\cosh(x))\sqrt{a\tanh^2(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{a\tanh^2(x)}\,dx = \coth(x)\log(\cosh(x))\sqrt{a\tanh^2(x)}$$

```
[In] Integrate[Sqrt[a*Tanh[x]^2],x]
[Out] Coth[x]*Log[Cosh[x]]*Sqrt[a*Tanh[x]^2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

method	result	size
derivativedivides	$-\frac{\sqrt{a\tanh(x)^{2}}\,(\ln(\tanh(x)-1)+\ln(1+\tanh(x)))}{2\tanh(x)}$	26
default	$-\frac{\sqrt{a\tanh(x)^{2}}\,(\ln(\tanh(x)-1)+\ln(1+\tanh(x)))}{2\tanh(x)}$	26
risch	$-\frac{\sqrt{\frac{a\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)x}{\mathrm{e}^{2x}-1}+\frac{\sqrt{\frac{a\left(\mathrm{e}^{2x}-1\right)^{2}}{\left(1+\mathrm{e}^{2x}\right)^{2}}}\,\left(1+\mathrm{e}^{2x}\right)\ln\left(1+\mathrm{e}^{2x}\right)}{\mathrm{e}^{2x}-1}$	81

```
[In] int((a*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*tanh(x)^2)^(1/2)*(ln(tanh(x)-1)+ln(1+tanh(x)))/tanh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 4.50

$$\int \sqrt{a \tanh^2(x)}\,dx = -\frac{\left(xe^{(2\,x)} - \left(e^{(2\,x)}+1\right)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right) + x\right)\sqrt{\frac{ae^{(4\,x)}-2\,ae^{(2\,x)}+a}{e^{(4\,x)}+2\,e^{(2\,x)}+1}}}{e^{(2\,x)}-1}$$

```
[In] integrate((a*tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] -(x*e^(2*x) - (e^(2*x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + x)*sqrt((a
*e^(4*x) - 2*a*e^(2*x) + a)/(e^(4*x) + 2*e^(2*x) + 1))/(e^(2*x) - 1)
```

Sympy [F]

$$\int \sqrt{a \tanh^2(x)}\,dx = \int \sqrt{a \tanh^2{(x)}}\,dx$$

```
[In] integrate((a*tanh(x)**2)**(1/2),x)
[Out] Integral(sqrt(a*tanh(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \sqrt{a \tanh^2(x)}\,dx = -\sqrt{a}x - \sqrt{a}\log\left(e^{(-2\,x)}+1\right)$$

```
[In] integrate((a*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(a)*x - sqrt(a)*log(e^(-2*x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \sqrt{a \tanh^2(x)}\,dx = -\left(x\mathrm{sgn}\left(e^{(4\,x)}-1\right) - \log\left(e^{(2\,x)}+1\right)\mathrm{sgn}\left(e^{(4\,x)}-1\right)\right)\sqrt{a}$$

```
[In] integrate((a*tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] -(x*sgn(e^(4*x) - 1) - log(e^(2*x) + 1)*sgn(e^(4*x) - 1))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a \tanh^2(x)}\, dx = \int \sqrt{a \tanh{(x)}^2}\, dx$$

```
[In] int((a*tanh(x)^2)^(1/2),x)
[Out] int((a*tanh(x)^2)^(1/2), x)
```

3.26 $\int \frac{1}{\sqrt{a \tanh^2(x)}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a \tanh^2(x)}} \, dx = \frac{\log(\sinh(x)) \tanh(x)}{\sqrt{a \tanh^2(x)}}$$

```
[Out] ln(sinh(x))*tanh(x)/(a*tanh(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3739, 3556}

$$\int \frac{1}{\sqrt{a \tanh^2(x)}} \, dx = \frac{\tanh(x) \log(\sinh(x))}{\sqrt{a \tanh^2(x)}}$$

```
[In] Int[1/Sqrt[a*Tanh[x]^2],x]
```

```
[Out] (Log[Sinh[x]]*Tanh[x])/Sqrt[a*Tanh[x]^2]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tanh(x)\int\coth(x)\,dx}{\sqrt{a\tanh^2(x)}}\\ &= \frac{\log(\sinh(x))\tanh(x)}{\sqrt{a\tanh^2(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int\frac{1}{\sqrt{a\tanh^2(x)}}\,dx = \frac{(\log(\cosh(x))+\log(\tanh(x)))\tanh(x)}{\sqrt{a\tanh^2(x)}}$$

```
[In] Integrate[1/Sqrt[a*Tanh[x]^2],x]
[Out] ((Log[Cosh[x]] + Log[Tanh[x]])*Tanh[x])/Sqrt[a*Tanh[x]^2]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

method	result	size
derivativedivides	$-\frac{\tanh(x)(\ln(1+\tanh(x))-2\ln(\tanh(x))+\ln(\tanh(x)-1))}{2\sqrt{a\tanh(x)^2}}$	29
default	$-\frac{\tanh(x)(\ln(1+\tanh(x))-2\ln(\tanh(x))+\ln(\tanh(x)-1))}{2\sqrt{a\tanh(x)^2}}$	29
risch	$-\frac{(e^{2x}-1)x}{\sqrt{\frac{a(e^{2x}-1)^2}{(1+e^{2x})^2}}(1+e^{2x})}+\frac{(e^{2x}-1)\ln(e^{2x}-1)}{\sqrt{\frac{a(e^{2x}-1)^2}{(1+e^{2x})^2}}(1+e^{2x})}$	81

```
[In] int(1/(a*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*tanh(x)*(ln(1+tanh(x))-2*ln(tanh(x))+ln(tanh(x)-1))/(a*tanh(x)^2)^(1/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.75

$$\int \frac{1}{\sqrt{a\tanh^2(x)}}\,dx = -\frac{\left(xe^{(2x)} - \left(e^{(2x)} + 1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right) + x\right)\sqrt{\frac{ae^{(4x)}-2ae^{(2x)}+a}{e^{(4x)}+2e^{(2x)}+1}}}{ae^{(2x)} - a}$$

```
[In] integrate(1/(a*tanh(x)^2)^(1/2),x, algorithm="fricas")
[Out] -(x*e^(2*x) - (e^(2*x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + x)*sqrt((a
*e^(4*x) - 2*a*e^(2*x) + a)/(e^(4*x) + 2*e^(2*x) + 1))/(a*e^(2*x) - a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\tanh^2(x)}}\,dx$$

```
[In] integrate(1/(a*tanh(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(a*tanh(x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{a\tanh^2(x)}}\,dx = -\frac{x}{\sqrt{a}} - \frac{\log\left(e^{(-x)} + 1\right)}{\sqrt{a}} - \frac{\log\left(e^{(-x)} - 1\right)}{\sqrt{a}}$$

```
[In] integrate(1/(a*tanh(x)^2)^(1/2),x, algorithm="maxima")
[Out] -x/sqrt(a) - log(e^(-x) + 1)/sqrt(a) - log(e^(-x) - 1)/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.06

$$\int \frac{1}{\sqrt{a \tanh^2(x)}}\, dx = 0$$

```
[In] integrate(1/(a*tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] 0
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{a \tanh^2(x)}}\, dx = \frac{\operatorname{atanh}\left(\frac{\tanh(x)}{\sqrt{\tanh(x)^2}}\right)}{\sqrt{a}}$$

```
[In] int(1/(a*tanh(x)^2)^(1/2),x)
[Out] atanh(tanh(x)/(tanh(x)^2)^(1/2))/a^(1/2)
```

3.27 $\int \left(-\tanh^2(c+dx)\right)^{5/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \left(-\tanh^2(c+dx)\right)^{5/2} dx = \frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d} - \frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d} - \frac{\tanh^3(c+dx)\sqrt{-\tanh^2(c+dx)}}{4d}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))*(-tanh(d*x+c)^2)^(1/2)/d-1/2*(-tanh(d*x+c)^2)^(
1/2)*tanh(d*x+c)/d-1/4*(-tanh(d*x+c)^2)^(1/2)*tanh(d*x+c)^3/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \left(-\tanh^2(c+dx)\right)^{5/2} dx = -\frac{\sqrt{-\tanh^2(c+dx)}\tanh(c+dx)}{2d} - \frac{\sqrt{-\tanh^2(c+dx)}\tanh^3(c+dx)}{4d} + \frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

```
[In] Int[(-Tanh[c + d*x]^2)^(5/2),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d - (Tanh[c + d*x
]*Sqrt[-Tanh[c + d*x]^2])/(2*d) - (Tanh[c + d*x]^3*Sqrt[-Tanh[c + d*x]^2])/
(4*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh^5(c+dx)\,dx \\
&= -\frac{\tanh^3(c+dx)\sqrt{-\tanh^2(c+dx)}}{4d}+\left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh^3(c+dx)\,dx \\
&= -\frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}-\frac{\tanh^3(c+dx)\sqrt{-\tanh^2(c+dx)}}{4d} \\
&\quad +\left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh(c+dx)\,dx \\
&= \frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d} \\
&\quad -\frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}-\frac{\tanh^3(c+dx)\sqrt{-\tanh^2(c+dx)}}{4d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \left(-\tanh^2(c+dx)\right)^{5/2}\,dx = \frac{\coth(c+dx)\left(-1-2\coth^2(c+dx)+4\coth^4(c+dx)\log(\cosh(c+dx))\right)\left(-\tanh^2(c+dx)\right)^{5/2}}{4d}$$

```
[In] Integrate[(-Tanh[c + d*x]^2)^(5/2),x]
```

```
[Out] (Coth[c + d*x]*(-1 - 2*Coth[c + d*x]^2 + 4*Coth[c + d*x]^4*Log[Cosh[c + d*x
]])*(-Tanh[c + d*x]^2)^(5/2))/(4*d)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\left(-\tanh(dx+c)^2\right)^{\frac{5}{2}}\left(\tanh(dx+c)^4+2\tanh(dx+c)^2+2\ln(\tanh(dx+c)-1)+2\ln(\tanh(dx+c)+1)\right)}{4d\tanh(dx+c)^5}$
default	$-\frac{\left(-\tanh(dx+c)^2\right)^{\frac{5}{2}}\left(\tanh(dx+c)^4+2\tanh(dx+c)^2+2\ln(\tanh(dx+c)-1)+2\ln(\tanh(dx+c)+1)\right)}{4d\tanh(dx+c)^5}$
risch	$\frac{(e^{2dx+2c}+1)\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,x}{e^{2dx+2c}-1}-\frac{2(e^{2dx+2c}+1)\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,(dx+c)}{(e^{2dx+2c}-1)d}+\frac{4\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,e^{2dx+2c}(e^{4dx+4c}}{(e^{2dx+2c}-1)(e^{2dx+2c}+1)}$

```
[In] int((-tanh(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(-tanh(d*x+c)^2)^(5/2)*(tanh(d*x+c)^4+2*tanh(d*x+c)^2+2*ln(tanh(d*x+
c)-1)+2*ln(tanh(d*x+c)+1))/tanh(d*x+c)^5
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.05

$$\int \left(-\tanh^2(c+dx)\right)^{5/2}\,dx = \frac{-i\,dxe^{(8\,dx+8\,c)}-i\,dx-4\,(i\,dx-i)e^{(6\,dx+6\,c)}-2\,(3i\,dx-2i)e^{(4\,dx+4\,c)}-4\,(i\,dx-i)e^{(2\,dx+2\,c)}+}{de^{(8\,dx+8\,c)}+4\,de^{(6\,dx+6\,c)}+6\,de^{(4\,dx+4}}$$

```
[In] integrate((-tanh(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*d*x*e^(8*d*x + 8*c) - I*d*x - 4*(I*d*x - I)*e^(6*d*x + 6*c) - 2*(3*I*d*
x - 2*I)*e^(4*d*x + 4*c) - 4*(I*d*x - I)*e^(2*d*x + 2*c) + (I*e^(8*d*x + 8*
c) + 4*I*e^(6*d*x + 6*c) + 6*I*e^(4*d*x + 4*c) + 4*I*e^(2*d*x + 2*c) + I)*l
og(e^(2*d*x + 2*c) + 1))/(d*e^(8*d*x + 8*c) + 4*d*e^(6*d*x + 6*c) + 6*d*e^(
4*d*x + 4*c) + 4*d*e^(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \left(-\tanh^2(c+dx)\right)^{5/2}\,dx=\int \left(-\tanh^2{(c+dx)}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((-tanh(d*x+c)**2)**(5/2),x)
[Out] Integral((-tanh(c + d*x)**2)**(5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \left(-\tanh^2(c+dx)\right)^{5/2}\,dx=-\frac{i\,(dx+c)}{d}-\frac{i\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(-i\,e^{(-2\,dx-2\,c)}-i\,e^{(-4\,dx-4\,c)}-i\,e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}$$

```
[In] integrate((-tanh(d*x+c)^2)^(5/2),x, algorithm="maxima")
[Out] -I*(d*x + c)/d - I*log(e^(-2*d*x - 2*c) + 1)/d + 4*(-I*e^(-2*d*x - 2*c) - I
*e^(-4*d*x - 4*c) - I*e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*
x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.61

$$\int \left(-\tanh^2(c+dx)\right)^{5/2}\,dx=\frac{i\,(dx+c)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)-i\,\log\left(e^{(2\,dx+2\,c)}+1\right)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)-\frac{4i\,\left(e^{(6\,dx+6\,c)}\mathrm{sgn}(-e^{(}\right.}{}}{d}$$

```
[In] integrate((-tanh(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] (I*(d*x + c)*sgn(-e^(4*d*x + 4*c) + 1) - I*log(e^(2*d*x + 2*c) + 1)*sgn(-e^
(4*d*x + 4*c) + 1) - 4*I*(e^(6*d*x + 6*c)*sgn(-e^(4*d*x + 4*c) + 1) + e^(4*
d*x + 4*c)*sgn(-e^(4*d*x + 4*c) + 1) + e^(2*d*x + 2*c)*sgn(-e^(4*d*x + 4*c)
 + 1))/(e^(2*d*x + 2*c) + 1)^4)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\tanh^2(c+dx)\right)^{5/2}\,dx = \int \left(-\tanh(c+d\,x)^2\right)^{5/2}\,dx$$

```
[In] int((-tanh(c + d*x)^2)^(5/2),x)
```

```
[Out] int((-tanh(c + d*x)^2)^(5/2), x)
```

3.28 $\int \left(-\tanh^2(c+dx)\right)^{3/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \left(-\tanh^2(c+dx)\right)^{3/2} dx = -\frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d} + \frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}$$

```
[Out] -coth(d*x+c)*ln(cosh(d*x+c))*(-tanh(d*x+c)^2)^(1/2)/d+1/2*(-tanh(d*x+c)^2)^
(1/2)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \left(-\tanh^2(c+dx)\right)^{3/2} dx = \frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d} - \frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

```
[In] Int[(-Tanh[c + d*x]^2)^(3/2),x]
```

```
[Out] -((Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d) + (Tanh[c +
d*x]*Sqrt[-Tanh[c + d*x]^2])/(2*d)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh^3(c+dx)\,dx\right)\\ &= \frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}-\left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh(c+dx)\,dx\\ &= -\frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d}+\frac{\tanh(c+dx)\sqrt{-\tanh^2(c+dx)}}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx = \frac{\coth(c+dx)\left(-1+2\coth^2(c+dx)\log(\cosh(c+dx))\right)\left(-\tanh^2(c+dx)\right)^{3/2}}{2d}$$

```
[In] Integrate[(-Tanh[c + d*x]^2)^(3/2),x]
[Out] (Coth[c + d*x]*(-1 + 2*Coth[c + d*x]^2*Log[Cosh[c + d*x]])*(-Tanh[c + d*x]^
2)^(3/2))/(2*d)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{\left(-\tanh(dx+c)^{2}\right)^{\frac{3}{2}}\left(\tanh(dx+c)^{2}+\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1)\right)}{2d\tanh(dx+c)^{3}}$
default	$-\frac{\left(-\tanh(dx+c)^{2}\right)^{\frac{3}{2}}\left(\tanh(dx+c)^{2}+\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1)\right)}{2d\tanh(dx+c)^{3}}$
risch	$-\frac{\sqrt{-\frac{\left(e^{2dx+2c}-1\right)^{2}}{\left(e^{2dx+2c}+1\right)^{2}}}\left(-e^{4dx+4c}dx+e^{4dx+4c}\ln\left(e^{2dx+2c}+1\right)-2e^{4dx+4c}c-2e^{2dx+2c}dx+2e^{2dx+2c}\ln\left(e^{2dx+2c}+1\right)-4e^{2}\right.}{\left(e^{2dx+2c}-1\right)\left(e^{2dx+2c}+1\right)d}$

[In] int((-tanh(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(-tanh(d*x+c)^2)^(3/2)*(tanh(d*x+c)^2+ln(tanh(d*x+c)-1)+ln(tanh(d*x+c)+1))/tanh(d*x+c)^3

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx=\frac{i\,dxe^{(4\,dx+4\,c)}+i\,dx-2\,(-i\,dx+i)e^{(2\,dx+2\,c)}+\left(-i\,e^{(4\,dx+4\,c)}-2i\,e^{(2\,dx+2\,c)}-i\right)\log\left(e^{(2\,dx+2\,c)}\right.}{de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d}$$

[In] integrate((-tanh(d*x+c)^2)^(3/2),x, algorithm="fricas")

[Out] (I*d*x*e^(4*d*x + 4*c) + I*d*x - 2*(-I*d*x + I)*e^(2*d*x + 2*c) + (-I*e^(4*d*x + 4*c) - 2*I*e^(2*d*x + 2*c) - I)*log(e^(2*d*x + 2*c) + 1))/(d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)

Sympy [F]

$$\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx=\int \left(-\tanh^{2}\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

[In] integrate((-tanh(d*x+c)**2)**(3/2),x)

[Out] Integral((-tanh(c + d*x)**2)**(3/2), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx = \frac{i\,(dx+c)}{d} + \frac{i\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2i\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}$$

```
[In] integrate((-tanh(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] I*(d*x + c)/d + I*log(e^(-2*d*x - 2*c) + 1)/d + 2*I*e^(-2*d*x - 2*c)/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx = \frac{-i\,(dx+c)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)+i\,\log\left(e^{(2\,dx+2\,c)}+1\right)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)+\frac{2i\,e^{(2\,dx+2\,c)}\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

```
[In] integrate((-tanh(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (-I*(d*x + c)*sgn(-e^(4*d*x + 4*c) + 1) + I*log(e^(2*d*x + 2*c) + 1)*sgn(-e
^(4*d*x + 4*c) + 1) + 2*I*e^(2*d*x + 2*c)*sgn(-e^(4*d*x + 4*c) + 1)/(e^(2*d
*x + 2*c) + 1)^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\tanh^2(c+dx)\right)^{3/2}\,dx = \int \left(-\tanh(c+d\,x)^2\right)^{3/2}\,dx$$

```
[In] int((-tanh(c + d*x)^2)^(3/2),x)
```

```
[Out] int((-tanh(c + d*x)^2)^(3/2), x)
```

3.29 $\int \sqrt{-\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \sqrt{-\tanh^2(c+dx)}\,dx=\frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))*(-tanh(d*x+c)^2)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3739, 3556}

$$\int \sqrt{-\tanh^2(c+dx)}\,dx=\frac{\sqrt{-\tanh^2(c+dx)}\coth(c+dx)\log(\cosh(c+dx))}{d}$$

```
[In] Int[Sqrt[-Tanh[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\coth(c+dx)\sqrt{-\tanh^2(c+dx)}\right)\int \tanh(c+dx)\,dx\\ &= \frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sqrt{-\tanh^2(c+dx)}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))\sqrt{-\tanh^2(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[-Tanh[c + d*x]^2],x]
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]]*Sqrt[-Tanh[c + d*x]^2])/d
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{\sqrt{-\tanh(dx+c)^2}\,(\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1))}{2d\tanh(dx+c)}$
default	$-\frac{\sqrt{-\tanh(dx+c)^2}\,(\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1))}{2d\tanh(dx+c)}$
risch	$\frac{(e^{2dx+2c}+1)\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,x}{e^{2dx+2c}-1}-\frac{2(e^{2dx+2c}+1)\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,(dx+c)}{(e^{2dx+2c}-1)d}+\frac{(e^{2dx+2c}+1)\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,\ln(e}{(e^{2dx+2c}-1)d}$

```
[In] int((-tanh(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*(-tanh(d*x+c)^2)^(1/2)*(ln(tanh(d*x+c)-1)+ln(tanh(d*x+c)+1))/tanh(d*
x+c)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \sqrt{-\tanh^2(c+dx)}\,dx = \frac{-i\,dx + i\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{d}$$

```
[In] integrate((-tanh(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] (-I*d*x + I*log(e^(2*d*x + 2*c) + 1))/d
```

Sympy [F]

$$\int \sqrt{-\tanh^2(c+dx)}\,dx = \int \sqrt{-\tanh^2\,(c+dx)}\,dx$$

```
[In] integrate((-tanh(d*x+c)**2)**(1/2),x)
[Out] Integral(sqrt(-tanh(c + d*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \sqrt{-\tanh^2(c+dx)}\,dx = -\frac{i\,(dx+c)}{d} - \frac{i\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}$$

```
[In] integrate((-tanh(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] -I*(d*x + c)/d - I*log(e^(-2*d*x - 2*c) + 1)/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{-\tanh^2(c+dx)}\,dx = \frac{i\,(dx+c)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right) - i\,\log\left(e^{(2\,dx+2\,c)}+1\right)\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}{d}$$

```
[In] integrate((-tanh(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] (I*(d*x + c)*sgn(-e^(4*d*x + 4*c) + 1) - I*log(e^(2*d*x + 2*c) + 1)*sgn(-e^
(4*d*x + 4*c) + 1))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\tanh^2(c+dx)}\,dx = \int \sqrt{-\tanh\left(c+d\,x\right)^2}\,dx$$

```
[In] int((-tanh(c + d*x)^2)^(1/2),x)
[Out] int((-tanh(c + d*x)^2)^(1/2), x)
```

3.30 $\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx = \frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[Out] ln(sinh(d*x+c))*tanh(d*x+c)/d/(-tanh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3739, 3556}

$$\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx = \frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[In] Int[1/Sqrt[-Tanh[c + d*x]^2],x]
```

```
[Out] (Log[Sinh[c + d*x]]*Tanh[c + d*x])/(d*Sqrt[-Tanh[c + d*x]^2])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tanh(c+dx)\int\coth(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}\\ &= \frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int\frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx=\frac{(\log(\cosh(c+dx))+\log(\tanh(c+dx)))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[In] Integrate[1/Sqrt[-Tanh[c + d*x]^2],x]
[Out] ((Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c + d*x])/(d*Sqrt[-Tanh[c +
 d*x]^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

method	result	size
derivativedivides	$-\frac{\tanh(dx+c)(\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1)-2\ln(\tanh(dx+c)))}{2d\sqrt{-\tanh(dx+c)^2}}$	52
default	$-\frac{\tanh(dx+c)(\ln(\tanh(dx+c)-1)+\ln(\tanh(dx+c)+1)-2\ln(\tanh(dx+c)))}{2d\sqrt{-\tanh(dx+c)^2}}$	52
risch	$\frac{(e^{2dx+2c}-1)x}{\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,(e^{2dx+2c}+1)}-\frac{2(e^{2dx+2c}-1)(dx+c)}{\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,(e^{2dx+2c}+1)d}+\frac{(e^{2dx+2c}-1)\ln(e^{2dx+2c}-1)}{\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,(e^{2dx+2c}+1)d}$	192

```
[In] int(1/(-tanh(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*tanh(d*x+c)*(ln(tanh(d*x+c)-1)+ln(tanh(d*x+c)+1)-2*ln(tanh(d*x+c)))/
(-tanh(d*x+c)^2)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx = \frac{i\,dx - i\,\log\left(e^{(2\,dx+2\,c)} - 1\right)}{d}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] (I*d*x - I*log(e^(2*d*x + 2*c) - 1))/d
```

Sympy [F]

$$\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{-\tanh^2{(c+dx)}}}\,dx$$

```
[In] integrate(1/(-tanh(d*x+c)**2)**(1/2),x)
[Out] Integral(1/sqrt(-tanh(c + d*x)**2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

$$\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx = \frac{i\,(dx+c)}{d} + \frac{i\,\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{i\,\log\left(e^{(-dx-c)} - 1\right)}{d}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] I*(d*x + c)/d + I*log(e^(-d*x - c) + 1)/d + I*log(e^(-d*x - c) - 1)/d
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx = -\frac{\frac{i\,dx+i\,c}{\operatorname{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)} - \frac{i\,\log\left(e^{(2\,dx+2\,c)}-1\right)}{\operatorname{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}}{d}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -((I*d*x + I*c)/sgn(-e^(4*d*x + 4*c) + 1) - I*log(e^(2*d*x + 2*c) - 1)/sgn(
-e^(4*d*x + 4*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{-\tanh^2(c+dx)}}\,dx = \frac{\operatorname{atan}\left(\frac{\tanh(c+d\,x)}{\sqrt{-\tanh(c+d\,x)^2}}\right)}{d}$$

```
[In] int(1/(-tanh(c + d*x)^2)^(1/2),x)
```

```
[Out] atan(tanh(c + d*x)/(-tanh(c + d*x)^2)^(1/2))/d
```

3.31 $\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx=\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[Out] 1/2*coth(d*x+c)/d/(-tanh(d*x+c)^2)^(1/2)-ln(sinh(d*x+c))*tanh(d*x+c)/d/(-ta
nh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx=\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[In] Int[(-Tanh[c + d*x]^2)^(-3/2),x]
```

```
[Out] Coth[c + d*x]/(2*d*Sqrt[-Tanh[c + d*x]^2]) - (Log[Sinh[c + d*x]]*Tanh[c + d
*x])/(d*Sqrt[-Tanh[c + d*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\tanh(c+dx)\int\coth^3(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}\\ &= \frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\tanh(c+dx)\int\coth(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}\\ &= \frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int\frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx=\frac{\coth(c+dx)-2(\log(\cosh(c+dx))+\log(\tanh(c+dx)))\tanh(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}$$

```
[In] Integrate[(-Tanh[c + d*x]^2)^(-3/2),x]
[Out] (Coth[c + d*x] - 2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c + d*x])
/(2*d*Sqrt[-Tanh[c + d*x]^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.32

method	result
derivativedivides	$-\frac{\tanh(dx+c)\left(\ln(\tanh(dx+c)-1)\tanh(dx+c)^2+\ln(\tanh(dx+c)+1)\tanh(dx+c)^2-2\ln(\tanh(dx+c))\tanh(dx+c)^2+1\right)}{2d\left(-\tanh(dx+c)^2\right)^{\frac{3}{2}}}$
default	$-\frac{\tanh(dx+c)\left(\ln(\tanh(dx+c)-1)\tanh(dx+c)^2+\ln(\tanh(dx+c)+1)\tanh(dx+c)^2-2\ln(\tanh(dx+c))\tanh(dx+c)^2+1\right)}{2d\left(-\tanh(dx+c)^2\right)^{\frac{3}{2}}}$
risch	$-\frac{-e^{4dx+4c}dx+e^{4dx+4c}\ln\left(e^{2dx+2c}-1\right)-2\,e^{4dx+4c}c+2\,e^{2dx+2c}dx-2\,e^{2dx+2c}\ln\left(e^{2dx+2c}-1\right)+4\,e^{2dx+2c}c-dx-2\,e^{2dx+}}{\left(e^{2dx+2c}-1\right)\left(e^{2dx+2c}+1\right)\sqrt{-\frac{\left(e^{2dx+2c}-1\right)^2}{\left(e^{2dx+2c}+1\right)^2}}\,d}$

[In] int(1/(-tanh(d*x+c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*tanh(d*x+c)*(ln(tanh(d*x+c)-1)*tanh(d*x+c)^2+ln(tanh(d*x+c)+1)*tanh(d*x+c)^2-2*ln(tanh(d*x+c))*tanh(d*x+c)^2+1)/(-tanh(d*x+c)^2)^(3/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx = \frac{-i\,dxe^{(4\,dx+4\,c)}-i\,dx-2\,(-i\,dx+i)e^{(2\,dx+2\,c)}+\left(i\,e^{(4\,dx+4\,c)}-2i\,e^{(2\,dx+2\,c)}+i\right.}{de^{(4\,dx+4\,c)}-2\,de^{(2\,dx+2\,c)}+d}$$

[In] integrate(1/(-tanh(d*x+c)^2)^(3/2),x, algorithm="fricas")

[Out] (-I*d*x*e^(4*d*x + 4*c) - I*d*x - 2*(-I*d*x + I)*e^(2*d*x + 2*c) + (I*e^(4*d*x + 4*c) - 2*I*e^(2*d*x + 2*c) + I)*log(e^(2*d*x + 2*c) - 1))/(d*e^(4*d*x + 4*c) - 2*d*e^(2*d*x + 2*c) + d)

Sympy [F]

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(-\tanh^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(-tanh(d*x+c)**2)**(3/2),x)

[Out] Integral((-tanh(c + d*x)**2)**(-3/2), x)

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx = -\frac{i\,(dx+c)}{d} - \frac{i\,\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{i\,\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2i\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -I*(d*x + c)/d - I*log(e^(-d*x - c) + 1)/d - I*log(e^(-d*x - c) - 1)/d - 2*
I*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx = \frac{\frac{i\,dx+i\,c}{\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)} - \frac{i\,\log\left(e^{(2\,dx+2\,c)}-1\right)}{\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)} + \frac{2i\,e^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}-1\right)^2\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}}{d}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((I*d*x + I*c)/sgn(-e^(4*d*x + 4*c) + 1) - I*log(e^(2*d*x + 2*c) - 1)/sgn(-
e^(4*d*x + 4*c) + 1) + 2*I*e^(2*d*x + 2*c)/((e^(2*d*x + 2*c) - 1)^2*sgn(-e^
(4*d*x + 4*c) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(-\mathrm{tanh}\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

```
[In] int(1/(-tanh(c + d*x)^2)^(3/2),x)
```

```
[Out] int(1/(-tanh(c + d*x)^2)^(3/2), x)
```

3.32 $\int \frac{1}{(-\tanh^2(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{1}{(-\tanh^2(c+dx))^{5/2}}\,dx = -\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}} - \frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}} + \frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[Out] -1/2*coth(d*x+c)/d/(-tanh(d*x+c)^2)^(1/2)-1/4*coth(d*x+c)^3/d/(-tanh(d*x+c)
^2)^(1/2)+ln(sinh(d*x+c))*tanh(d*x+c)/d/(-tanh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3739, 3554, 3556}

$$\int \frac{1}{(-\tanh^2(c+dx))^{5/2}}\,dx = -\frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}} - \frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}} + \frac{\tanh(c+dx)\log(\sinh(c+dx))}{d\sqrt{-\tanh^2(c+dx)}}$$

```
[In] Int[(-Tanh[c + d*x]^2)^(-5/2),x]
```

```
[Out] -1/2*Coth[c + d*x]/(d*Sqrt[-Tanh[c + d*x]^2]) - Coth[c + d*x]^3/(4*d*Sqrt[-
Tanh[c + d*x]^2]) + (Log[Sinh[c + d*x]]*Tanh[c + d*x])/(d*Sqrt[-Tanh[c + d*
x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tanh(c+dx)\int \coth^5(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}\\
&= -\frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}}+\frac{\tanh(c+dx)\int \coth^3(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}\\
&= -\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}}+\frac{\tanh(c+dx)\int \coth(c+dx)\,dx}{\sqrt{-\tanh^2(c+dx)}}\\
&= -\frac{\coth(c+dx)}{2d\sqrt{-\tanh^2(c+dx)}}-\frac{\coth^3(c+dx)}{4d\sqrt{-\tanh^2(c+dx)}}+\frac{\log(\sinh(c+dx))\tanh(c+dx)}{d\sqrt{-\tanh^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx=\frac{-2\coth(c+dx)-\coth^3(c+dx)+4(\log(\cosh(c+dx))+\log(\tanh(c+dx)))\,t}{4d\sqrt{-\tanh^2(c+dx)}}$$

```
[In] Integrate[(-Tanh[c + d*x]^2)^(-5/2),x]
[Out] (-2*Coth[c + d*x] - Coth[c + d*x]^3 + 4*(Log[Cosh[c + d*x]] + Log[Tanh[c +
d*x]])*Tanh[c + d*x])/(4*d*Sqrt[-Tanh[c + d*x]^2])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.03

method	result
derivativedivides	$-\frac{\tanh(dx+c)\left(2\ln(\tanh(dx+c)-1)\tanh(dx+c)^4+2\ln(\tanh(dx+c)+1)\tanh(dx+c)^4-4\ln(\tanh(dx+c))\tanh(dx+c)^4+\right.}{4d\left(-\tanh(dx+c)^2\right)^{\frac{5}{2}}}$
default	$-\frac{\tanh(dx+c)\left(2\ln(\tanh(dx+c)-1)\tanh(dx+c)^4+2\ln(\tanh(dx+c)+1)\tanh(dx+c)^4-4\ln(\tanh(dx+c))\tanh(dx+c)^4+\right.}{4d\left(-\tanh(dx+c)^2\right)^{\frac{5}{2}}}$
risch	$\frac{(e^{2dx+2c}-1)x}{\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,(e^{2dx+2c}+1)}-\frac{2(e^{2dx+2c}-1)(dx+c)}{\sqrt{-\frac{(e^{2dx+2c}-1)^2}{(e^{2dx+2c}+1)^2}}\,(e^{2dx+2c}+1)d}-\frac{4\,e^{2dx+2c}(e^{4dx+4c}-e^{2dx+2c}+1)}{(e^{2dx+2c}-1)^3(e^{2dx+2c}+1)\sqrt{-\frac{(e^{2dx+2c}-1}{(e^{2dx+2c}+1}}}$

```
[In] int(1/(-tanh(d*x+c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*tanh(d*x+c)*(2*ln(tanh(d*x+c)-1)*tanh(d*x+c)^4+2*ln(tanh(d*x+c)+1)*t
anh(d*x+c)^4-4*ln(tanh(d*x+c))*tanh(d*x+c)^4+2*tanh(d*x+c)^2+1)/(-tanh(d*x+
c)^2)^(5/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.05

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx = \frac{i\,dxe^{(8\,dx+8\,c)}+i\,dx-4\,(i\,dx-i)e^{(6\,dx+6\,c)}-2\,(-3i\,dx+2i)e^{(4\,dx+4\,c)}-4\,(i\,dx}{de^{(8\,dx+8\,c)}-4\,de^{(6\,d}}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (I*d*x*e^(8*d*x + 8*c) + I*d*x - 4*(I*d*x - I)*e^(6*d*x + 6*c) - 2*(-3*I*d*
x + 2*I)*e^(4*d*x + 4*c) - 4*(I*d*x - I)*e^(2*d*x + 2*c) + (-I*e^(8*d*x + 8
*c) + 4*I*e^(6*d*x + 6*c) - 6*I*e^(4*d*x + 4*c) + 4*I*e^(2*d*x + 2*c) - I)*
log(e^(2*d*x + 2*c) - 1))/(d*e^(8*d*x + 8*c) - 4*d*e^(6*d*x + 6*c) + 6*d*e^
(4*d*x + 4*c) - 4*d*e^(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(-\tanh^2\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(-tanh(d*x+c)**2)**(5/2),x)
[Out] Integral((-tanh(c + d*x)**2)**(-5/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.50

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx = \frac{i\,(dx+c)}{d} + \frac{i\,\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{i\,\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{4\left(-i\,e^{(-2\,dx-2\,c)}+i\,e^{(-4\,dx-4\,c)}-i\,e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(5/2),x, algorithm="maxima")
[Out] I*(d*x + c)/d + I*log(e^(-d*x - c) + 1)/d + I*log(e^(-d*x - c) - 1)/d - 4*(
-I*e^(-2*d*x - 2*c) + I*e^(-4*d*x - 4*c) - I*e^(-6*d*x - 6*c))/(d*(4*e^(-2*
d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1
))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.43

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx = -\frac{\frac{i\,dx+i\,c}{\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)} - \frac{i\,\log\left(e^{(2\,dx+2\,c)}-1\right)}{\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)} + \frac{4\left(i\,e^{(6\,dx+6\,c)}-i\,e^{(4\,dx+4\,c)}+i\,e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^4\mathrm{sgn}\left(-e^{(4\,dx+4\,c)}+1\right)}}{d}$$

```
[In] integrate(1/(-tanh(d*x+c)^2)^(5/2),x, algorithm="giac")
[Out] -((I*d*x + I*c)/sgn(-e^(4*d*x + 4*c) + 1) - I*log(e^(2*d*x + 2*c) - 1)/sgn(
-e^(4*d*x + 4*c) + 1) + 4*(I*e^(6*d*x + 6*c) - I*e^(4*d*x + 4*c) + I*e^(2*d
*x + 2*c))/((e^(2*d*x + 2*c) - 1)^4*sgn(-e^(4*d*x + 4*c) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\tanh^2(c+dx)\right)^{5/2}}\,dx = \int \frac{1}{\left(-\tanh\left(c+d\,x\right)^2\right)^{5/2}}\,dx$$

```
[In] int(1/(-tanh(c + d*x)^2)^(5/2),x)
[Out] int(1/(-tanh(c + d*x)^2)^(5/2), x)
```

3.33 $\int \sqrt{\tanh^3(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int \sqrt{\tanh^3(x)}\,dx = -2\coth(x)\sqrt{\tanh^3(x)} + \frac{\arctan\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}$$

```
[Out] -2*coth(x)*(tanh(x)^3)^(1/2)+arctan(tanh(x)^(1/2))*(tanh(x)^3)^(1/2)/tanh(x
)^(3/2)+arctanh(tanh(x)^(1/2))*(tanh(x)^3)^(1/2)/tanh(x)^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {3739, 3554, 3557, 335, 218, 212, 209}

$$\int \sqrt{\tanh^3(x)}\,dx = \frac{\sqrt{\tanh^3(x)}\arctan\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} + \frac{\sqrt{\tanh^3(x)}\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} - 2\sqrt{\tanh^3(x)}\coth(x)$$

```
[In] Int[Sqrt[Tanh[x]^3],x]
```

```
[Out] -2*Coth[x]*Sqrt[Tanh[x]^3] + (ArcTan[Sqrt[Tanh[x]]]*Sqrt[Tanh[x]^3])/Tanh[x
]^(3/2) + (ArcTanh[Sqrt[Tanh[x]]]*Sqrt[Tanh[x]^3])/Tanh[x]^(3/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{\tanh^3(x)} \int \tanh^{\frac{3}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} + \frac{\sqrt{\tanh^3(x)} \int \frac{1}{\sqrt{\tanh(x)}}\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} - \frac{\sqrt{\tanh^3(x)}\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(x)\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} - \frac{\left(2\sqrt{\tanh^3(x)}\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} + \frac{\sqrt{\tanh^3(x)}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&\quad + \frac{\sqrt{\tanh^3(x)}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{\tanh^3(x)} + \frac{\arctan\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{\text{arctanh}\left(\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$
\int \sqrt{\tanh^3(x)}\,dx = \frac{\left(\arctan\left(\sqrt{\tanh(x)}\right) + \text{arctanh}\left(\sqrt{\tanh(x)}\right) - 2\sqrt{\tanh(x)}\right)\sqrt{\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}
$$

```
[In] Integrate[Sqrt[Tanh[x]^3],x]
[Out] ((ArcTan[Sqrt[Tanh[x]]] + ArcTanh[Sqrt[Tanh[x]]] - 2*Sqrt[Tanh[x]])*Sqrt[Ta
nh[x]^3])/Tanh[x]^(3/2)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{\sqrt{\tanh(x)^{3}}\,\left(4\sqrt{\tanh(x)}+\ln\left(\sqrt{\tanh(x)}-1\right)-\ln\left(\sqrt{\tanh(x)}+1\right)-2\arctan\left(\sqrt{\tanh(x)}\right)\right)}{2\tanh(x)^{\frac{3}{2}}}$	43
default	$-\frac{\sqrt{\tanh(x)^{3}}\,\left(4\sqrt{\tanh(x)}+\ln\left(\sqrt{\tanh(x)}-1\right)-\ln\left(\sqrt{\tanh(x)}+1\right)-2\arctan\left(\sqrt{\tanh(x)}\right)\right)}{2\tanh(x)^{\frac{3}{2}}}$	43

```
[In] int((tanh(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(tanh(x)^3)^(1/2)*(4*tanh(x)^(1/2)+ln(tanh(x)^(1/2)-1)-ln(tanh(x)^(1/2
)+1)-2*arctan(tanh(x)^(1/2)))/tanh(x)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(43) = 86.

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.86

$$\int \sqrt{\tanh^3(x)}\,dx = -2\sqrt{\frac{\sinh(x)}{\cosh(x)}} + \arctan\left(-\cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\sqrt{\frac{\sinh(x)}{\cosh(x)}}\right) - \frac{1}{2}\log\left(-\cosh(x)^2 - 2\cosh(x)\sinh(x) - \sinh(x)^2 + \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\sqrt{\frac{\sinh(x)}{\cosh(x)}}\right)$$

```
[In] integrate((tanh(x)^3)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(sinh(x)/cosh(x)) + arctan(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^
2 + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(sinh(x)/cosh(x)))
- 1/2*log(-cosh(x)^2 - 2*cosh(x)*sinh(x) - sinh(x)^2 + (cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 + 1)*sqrt(sinh(x)/cosh(x)))
```

Sympy [F]

$$\int \sqrt{\tanh^3(x)}\,dx = \int \sqrt{\tanh^3{(x)}}\,dx$$

```
[In] integrate((tanh(x)**3)**(1/2),x)
[Out] Integral(sqrt(tanh(x)**3), x)
```

Maxima [F]

$$\int \sqrt{\tanh^3(x)}\,dx = \int \sqrt{\tanh{(x)}^3}\,dx$$

```
[In] integrate((tanh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(tanh(x)^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \sqrt{\tanh^3(x)}\,dx = \frac{4}{\sqrt{e^{(4x)}-1}-e^{(2x)}-1} + \arctan\left(\sqrt{e^{(4x)}-1}-e^{(2x)}\right) - \frac{1}{2}\log\left(-\sqrt{e^{(4x)}-1}+e^{(2x)}\right)$$

```
[In] integrate((tanh(x)^3)^(1/2),x, algorithm="giac")
[Out] 4/(sqrt(e^(4*x) - 1) - e^(2*x) - 1) + arctan(sqrt(e^(4*x) - 1) - e^(2*x)) -
 1/2*log(-sqrt(e^(4*x) - 1) + e^(2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\tanh^3(x)}\,dx = \int \sqrt{\tanh{(x)}^3}\,dx$$

```
[In] int((tanh(x)^3)^(1/2),x)
[Out] int((tanh(x)^3)^(1/2), x)
```

3.34 $\int \left(a\tanh^3(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \left(a\tanh^3(x)\right)^{3/2}\,dx = -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{a\arctan\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{a\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)}$$

```
[Out] -2/3*a*(a*tanh(x)^3)^(1/2)-a*arctan(tanh(x)^(1/2))*(a*tanh(x)^3)^(1/2)/tanh
(x)^(3/2)+a*arctanh(tanh(x)^(1/2))*(a*tanh(x)^3)^(1/2)/tanh(x)^(3/2)-2/7*a*
(a*tanh(x)^3)^(1/2)*tanh(x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3739, 3554, 3557, 335, 304, 209, 212}

$$\int \left(a\tanh^3(x)\right)^{3/2}\,dx = -\frac{a\sqrt{a\tanh^3(x)}\arctan\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} + \frac{a\sqrt{a\tanh^3(x)}\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} - \frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)}$$

```
[In] Int[(a*Tanh[x]^3)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a*Tanh[x]^3])/3 - (a*ArcTan[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Ta
nh[x]^(3/2) + (a*ArcTanh[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Tanh[x]^(3/2) -
(2*a*Tanh[x]^2*Sqrt[a*Tanh[x]^3])/7
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a\tanh^3(x)}\right)\int \tanh^{\frac{9}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} + \frac{\left(a\sqrt{a\tanh^3(x)}\right)\int \tanh^{\frac{5}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} + \frac{\left(a\sqrt{a\tanh^3(x)}\right)\int \sqrt{\tanh(x)}\,dx}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} - \frac{\left(a\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh(x)\right)}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} - \frac{\left(2a\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)} + \frac{\left(a\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} - \frac{\left(a\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}$$

$$= -\frac{2}{3}a\sqrt{a\tanh^3(x)} - \frac{a\arctan\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{a\text{arctanh}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} - \frac{2}{7}a\tanh^2(x)\sqrt{a\tanh^3(x)}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.64

$$\int \left(a\tanh^3(x)\right)^{3/2}\,dx = -\frac{\left(a\tanh^3(x)\right)^{3/2}\left(21\arctan\left(\sqrt{\tanh(x)}\right)-21\text{arctanh}\left(\sqrt{\tanh(x)}\right)+14\tanh^{\frac{3}{2}}(x)+6\tanh^{\frac{7}{2}}(x)\right)}{21\tanh^{\frac{9}{2}}(x)}$$

```
[In] Integrate[(a*Tanh[x]^3)^(3/2),x]
```

```
[Out] -1/21*((a*Tanh[x]^3)^(3/2)*(21*ArcTan[Sqrt[Tanh[x]]] - 21*ArcTanh[Sqrt[Tanh
[x]]] + 14*Tanh[x]^(3/2) + 6*Tanh[x]^(7/2)))/Tanh[x]^(9/2)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(a\tanh(x)^3\right)^{\frac{3}{2}}\left(21a^{\frac{7}{2}}\,\text{arctanh}\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)-21a^{\frac{7}{2}}\arctan\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)-6(a\tanh(x))^{\frac{7}{2}}-14a^2(a\tanh(x))^{\frac{3}{2}}\right)}{21\tanh(x)^3(a\tanh(x))^{\frac{3}{2}}a^2}$	76
default	$\frac{\left(a\tanh(x)^3\right)^{\frac{3}{2}}\left(21a^{\frac{7}{2}}\,\text{arctanh}\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)-21a^{\frac{7}{2}}\arctan\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)-6(a\tanh(x))^{\frac{7}{2}}-14a^2(a\tanh(x))^{\frac{3}{2}}\right)}{21\tanh(x)^3(a\tanh(x))^{\frac{3}{2}}a^2}$	76

```
[In] int((a*tanh(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*(a*tanh(x)^3)^(3/2)*(21*a^(7/2)*arctanh((a*tanh(x))^(1/2)/a^(1/2))-21*
a^(7/2)*arctan((a*tanh(x))^(1/2)/a^(1/2))-6*(a*tanh(x))^(7/2)-14*a^2*(a*tan
h(x))^(3/2))/tanh(x)^3/(a*tanh(x))^(3/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(66) = 132.

Time = 0.29 (sec) , antiderivative size = 1269, normalized size of antiderivative = 14.76

$$\int \left(a\tanh^3(x)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*tanh(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)
^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh
(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6
```

```
*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*arctan((co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt(a*sinh(x)/cosh(x))/(
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)) - 21*(a*cosh(x)^6 + 6
*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*
sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(
5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)
^3 + a*cosh(x))*sinh(x) + a)*sqrt(-a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)^3*sin
h(x) + 6*a*cosh(x)^2*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*(c
osh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(a*sinh(x)/cosh(
x)) - 2*a)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4)) + 16*(5*a*cosh(x)^6 + 30*a*cosh(x)*sinh(x)^5 +
 5*a*sinh(x)^6 - a*cosh(x)^4 + (75*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(25*a*cos
h(x)^3 - a*cosh(x))*sinh(x)^3 + a*cosh(x)^2 + (75*a*cosh(x)^4 - 6*a*cosh(x)
^2 + a)*sinh(x)^2 + 2*(15*a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x)
- 5*a)*sqrt(a*sinh(x)/cosh(x)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x
))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 +
6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/84*(42*(a*cosh(x)^6
+ 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 +
a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 +
3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh
(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*arctan((cosh(x)^2 + 2*cosh(x)*sinh(
x) + sinh(x)^2 + 1)*sqrt(a*sinh(x)/cosh(x))/sqrt(a)) - 21*(a*cosh(x)^6 + 6*
a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*s
inh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5
*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^
3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*log(2*a*cosh(x)^4 + 8*a*cosh(x)^3*sinh(
x) + 12*a*cosh(x)^2*sinh(x)^2 + 8*a*cosh(x)*sinh(x)^3 + 2*a*sinh(x)^4 + 2*(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 +
 cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(a)*sqrt(a*sinh(x)/cosh
(x)) - a) + 16*(5*a*cosh(x)^6 + 30*a*cosh(x)*sinh(x)^5 + 5*a*sinh(x)^6 - a*
cosh(x)^4 + (75*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(25*a*cosh(x)^3 - a*cosh(x))
*sinh(x)^3 + a*cosh(x)^2 + (75*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 +
 2*(15*a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - 5*a)*sqrt(a*sinh(
x)/cosh(x)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2
+ 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5
*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*co
sh(x)^3 + cosh(x))*sinh(x) + 1)]
```

Sympy [F]

$$\int \left(a\tanh^3(x)\right)^{3/2} dx = \int \left(a\tanh^3\left(x\right)\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a*tanh(x)**3)**(3/2),x)
[Out] Integral((a*tanh(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\tanh^3(x)\right)^{3/2} dx = \int \left(a\tanh\left(x\right)^3\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a*tanh(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*tanh(x)^3)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(66) = 132$.

Time = 0.35 (sec) , antiderivative size = 342, normalized size of antiderivative = 3.98

$$\int \left(a\tanh^3(x)\right)^{3/2} dx =$$

$$-\frac{1}{42}\left(42\sqrt{a}\arctan\left(-\frac{\sqrt{a}e^{(2x)} - \sqrt{ae^{(4x)} - a}}{\sqrt{a}}\right)\operatorname{sgn}\left(e^{(4x)} - 1\right) + 21\sqrt{a}\log\left(\left|-\sqrt{a}e^{(2x)} + \sqrt{ae^{(4x)} - a}\right|\right)\operatorname{sg}\right.$$

```
[In] integrate((a*tanh(x)^3)^(3/2),x, algorithm="giac")
[Out] -1/42*(42*sqrt(a)*arctan(-(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))/sqrt(a))*
sgn(e^(4*x) - 1) + 21*sqrt(a)*log(abs(-sqrt(a)*e^(2*x) + sqrt(a*e^(4*x) - a
)))*sgn(e^(4*x) - 1) + 16*(21*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^6*a*s
gn(e^(4*x) - 1) + 42*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^5*a^(3/2)*sgn(
e^(4*x) - 1) + 119*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^4*a^2*sgn(e^(4*x
) - 1) + 56*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^3*a^(5/2)*sgn(e^(4*x) -
 1) + 63*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))^2*a^3*sgn(e^(4*x) - 1) + 1
4*(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))*a^(7/2)*sgn(e^(4*x) - 1) + 5*a^4*
sgn(e^(4*x) - 1))/(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a) + sqrt(a))^7)*a
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\tanh^3(x)\right)^{3/2}\,dx=\int \left(a\tanh(x)^3\right)^{3/2}\,dx$$

```
[In] int((a*tanh(x)^3)^(3/2),x)
[Out] int((a*tanh(x)^3)^(3/2), x)
```

3.35 $\int \sqrt{a \tanh^3(x)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \sqrt{a \tanh^3(x)} \, dx = -2 \coth(x) \sqrt{a \tanh^3(x)} + \frac{\arctan\left(\sqrt{\tanh(x)}\right) \sqrt{a \tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right) \sqrt{a \tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}$$

```
[Out] -2*coth(x)*(a*tanh(x)^3)^(1/2)+arctan(tanh(x)^(1/2))*(a*tanh(x)^3)^(1/2)/ta
nh(x)^(3/2)+arctanh(tanh(x)^(1/2))*(a*tanh(x)^3)^(1/2)/tanh(x)^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3739, 3554, 3557, 335, 218, 212, 209}

$$\int \sqrt{a \tanh^3(x)} \, dx = \frac{\sqrt{a \tanh^3(x)} \arctan\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} + \frac{\sqrt{a \tanh^3(x)} \operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)} - 2 \coth(x) \sqrt{a \tanh^3(x)}$$

```
[In] Int[Sqrt[a*Tanh[x]^3],x]
```

```
[Out] -2*Coth[x]*Sqrt[a*Tanh[x]^3] + (ArcTan[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Ta
nh[x]^(3/2) + (ArcTanh[Sqrt[Tanh[x]]]*Sqrt[a*Tanh[x]^3])/Tanh[x]^(3/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a\tanh^3(x)}\int \tanh^{\frac{3}{2}}(x)\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} + \frac{\sqrt{a\tanh^3(x)}\int \frac{1}{\sqrt{\tanh(x)}}\,dx}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} - \frac{\sqrt{a\tanh^3(x)}\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(x)\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} - \frac{\left(2\sqrt{a\tanh^3(x)}\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} + \frac{\sqrt{a\tanh^3(x)}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&\quad + \frac{\sqrt{a\tanh^3(x)}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\tanh^{\frac{3}{2}}(x)}\\
&= -2\coth(x)\sqrt{a\tanh^3(x)} + \frac{\arctan\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}\\
&\quad + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$
\int \sqrt{a\tanh^3(x)}\,dx = \frac{\left(\arctan\left(\sqrt{\tanh(x)}\right) + \operatorname{arctanh}\left(\sqrt{\tanh(x)}\right) - 2\sqrt{\tanh(x)}\right)\sqrt{a\tanh^3(x)}}{\tanh^{\frac{3}{2}}(x)}
$$

```
[In] Integrate[Sqrt[a*Tanh[x]^3],x]
[Out] ((ArcTan[Sqrt[Tanh[x]]] + ArcTanh[Sqrt[Tanh[x]]] - 2*Sqrt[Tanh[x]])*Sqrt[a*
Tanh[x]^3])/Tanh[x]^(3/2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\sqrt{a\tanh(x)^{3}}\left(-2\sqrt{a\tanh(x)}+\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)+\sqrt{a}\,\arctan\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)\right)}{\tanh(x)\sqrt{a\tanh(x)}}$	59
default	$\frac{\sqrt{a\tanh(x)^{3}}\left(-2\sqrt{a\tanh(x)}+\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)+\sqrt{a}\,\arctan\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)\right)}{\tanh(x)\sqrt{a\tanh(x)}}$	59

```
[In] int((a*tanh(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] (a*tanh(x)^3)^(1/2)/tanh(x)/(a*tanh(x))^(1/2)*(-2*(a*tanh(x))^(1/2)+a^(1/2)
*arctanh((a*tanh(x))^(1/2)/a^(1/2))+a^(1/2)*arctan((a*tanh(x))^(1/2)/a^(1/2
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 376, normalized size of antiderivative = 5.97

$$\int\sqrt{a\tanh^{3}(x)}\,dx$$

$$= \left[-\frac{1}{2}\sqrt{-a}\arctan\left(\frac{\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}\right)\sqrt{-a}\sqrt{\frac{a\sinh(x)}{\cosh(x)}}}{a\cosh(x)^{2}+2a\cosh(x)\sinh(x)+a\sinh(x)^{2}-a}\right)\right.$$

$$+\frac{1}{4}\sqrt{-a}\log\left(-\frac{a\cosh(x)^{4}+4a\cosh(x)^{3}\sinh(x)+6a\cosh(x)^{2}\sinh(x)^{2}+4a\cosh(x)\sinh(x)^{3}+a\,\mathrm{si}}{\cosh(x)^{4}+4\cosh(x)^{3}\sinh(x)+6\cosh(x)^{2}\,\mathrm{si}}\right.$$

$$-2\sqrt{\frac{a\sinh(x)}{\cosh(x)}},-\frac{1}{2}\sqrt{a}\arctan\left(\frac{\sqrt{a}\sqrt{\frac{a\sinh(x)}{\cosh(x)}}}{a\cosh(x)^{2}+2a\cosh(x)\sinh(x)+a\sinh(x)^{2}-a}\right)$$

$$+\frac{1}{4}\sqrt{a}\log\left(2a\cosh(x)^{4}+8a\cosh(x)^{3}\sinh(x)+12a\cosh(x)^{2}\sinh(x)^{2}\right.$$

$$+8a\cosh(x)\sinh(x)^{3}+2a\sinh(x)^{4}$$

$$+2\left(\cosh(x)^{4}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+\left(6\cosh(x)^{2}+1\right)\sinh(x)^{2}+\cosh(x)^{2}+2\left(2\cosh(x)\right.\right.$$

$$\left.\left.-a\right)-2\sqrt{\frac{a\sinh(x)}{\cosh(x)}}\right]$$

```
[In] integrate((a*tanh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a)*arctan((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*
sqrt(a*sinh(x)/cosh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 -
a)) + 1/4*sqrt(-a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)^3*sinh(x) + 6*a*cosh(x)^
2*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(cosh(x)^2 + 2*cosh(x
)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(a*sinh(x)/cosh(x)) - 2*a)/(cosh(x)
^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + si
nh(x)^4)) - 2*sqrt(a*sinh(x)/cosh(x)), -1/2*sqrt(a)*arctan(sqrt(a)*sqrt(a*s
inh(x)/cosh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)) + 1/
4*sqrt(a)*log(2*a*cosh(x)^4 + 8*a*cosh(x)^3*sinh(x) + 12*a*cosh(x)^2*sinh(x
)^2 + 8*a*cosh(x)*sinh(x)^3 + 2*a*sinh(x)^4 + 2*(cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^
3 + cosh(x))*sinh(x))*sqrt(a)*sqrt(a*sinh(x)/cosh(x)) - a) - 2*sqrt(a*sinh(
x)/cosh(x))]
```

Sympy [F]

$$\int \sqrt{a\tanh^3(x)}\,dx = \int \sqrt{a\tanh^3{(x)}}\,dx$$

```
[In] integrate((a*tanh(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tanh(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\tanh^3(x)}\,dx = \int \sqrt{a\tanh{(x)}^3}\,dx$$

```
[In] integrate((a*tanh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*tanh(x)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(49) = 98.

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.83

$$\begin{aligned}\int \sqrt{a\tanh^3(x)}\,dx &= \sqrt{a}\arctan\left(-\frac{\sqrt{a}e^{(2x)} - \sqrt{ae^{(4x)} - a}}{\sqrt{a}}\right)\operatorname{sgn}\left(e^{(4x)} - 1\right) \\ &\quad - \frac{1}{2}\sqrt{a}\log\left(\left|-\sqrt{a}e^{(2x)} + \sqrt{ae^{(4x)} - a}\right|\right)\operatorname{sgn}\left(e^{(4x)} - 1\right) \\ &\quad - \frac{4\,a\operatorname{sgn}\left(e^{(4x)} - 1\right)}{\sqrt{a}e^{(2x)} - \sqrt{ae^{(4x)} - a} + \sqrt{a}}\end{aligned}$$

```
[In] integrate((a*tanh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*arctan(-(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a))/sqrt(a))*sgn(e^(4*x
) - 1) - 1/2*sqrt(a)*log(abs(-sqrt(a)*e^(2*x) + sqrt(a*e^(4*x) - a)))*sgn(e
^(4*x) - 1) - 4*a*sgn(e^(4*x) - 1)/(sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a) +
 sqrt(a))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\tanh^3(x)}\,dx = \int \sqrt{a\tanh(x)^3}\,dx$$

```
[In] int((a*tanh(x)^3)^(1/2),x)
```

```
[Out] int((a*tanh(x)^3)^(1/2), x)
```

3.36 $\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx = -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}} - \frac{\arctan\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}}$$

```
[Out] -2*tanh(x)/(a*tanh(x)^3)^(1/2)-arctan(tanh(x)^(1/2))*tanh(x)^(3/2)/(a*tanh(
x)^3)^(1/2)+arctanh(tanh(x)^(1/2))*tanh(x)^(3/2)/(a*tanh(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3739, 3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx = -\frac{\tanh^{\frac{3}{2}}(x)\arctan\left(\sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}} + \frac{\tanh^{\frac{3}{2}}(x)\operatorname{arctanh}\left(\sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}} - \frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}}$$

```
[In] Int[1/Sqrt[a*Tanh[x]^3],x]
```

```
[Out] (-2*Tanh[x])/Sqrt[a*Tanh[x]^3] - (ArcTan[Sqrt[Tanh[x]]]*Tanh[x]^(3/2))/Sqrt
[a*Tanh[x]^3] + (ArcTanh[Sqrt[Tanh[x]]]*Tanh[x]^(3/2))/Sqrt[a*Tanh[x]^3]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 304</u>

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

<u>Rule 335</u>

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 3555</u>

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

<u>Rule 3557</u>

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

<u>Rule 3739</u>

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\tanh^{\frac{3}{2}}(x)\int \frac{1}{\tanh^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\tanh^3(x)}}$$

$$= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}} + \frac{\tanh^{\frac{3}{2}}(x)\int \sqrt{\tanh(x)}\,dx}{\sqrt{a\tanh^3(x)}}$$

$$= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}} - \frac{\tanh^{\frac{3}{2}}(x)\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh(x)\right)}{\sqrt{a\tanh^3(x)}}$$

$$= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}} - \frac{\left(2\tanh^{\frac{3}{2}}(x)\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}}$$

$$= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}} + \frac{\tanh^{\frac{3}{2}}(x)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}} - \frac{\tanh^{\frac{3}{2}}(x)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{a\tanh^3(x)}}$$

$$= -\frac{2\tanh(x)}{\sqrt{a\tanh^3(x)}} - \frac{\arctan\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}} + \frac{\text{arctanh}\left(\sqrt{\tanh(x)}\right)\tanh^{\frac{3}{2}}(x)}{\sqrt{a\tanh^3(x)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx = -\frac{\tanh(x)\left(2+\arctan\left(\sqrt[4]{\tanh^2(x)}\right)\sqrt[4]{\tanh^2(x)} - \text{arctanh}\left(\sqrt[4]{\tanh^2(x)}\right)\sqrt[4]{\tanh^2(x)}\right)}{\sqrt{a\tanh^3(x)}}$$

```
[In] Integrate[1/Sqrt[a*Tanh[x]^3],x]
```

```
[Out] -((Tanh[x]*(2 + ArcTan[(Tanh[x]^2)^(1/4)]*(Tanh[x]^2)^(1/4) - ArcTanh[(Tanh
[x]^2)^(1/4)]*(Tanh[x]^2)^(1/4)))/Sqrt[a*Tanh[x]^3])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$-\frac{\tanh(x)\left(2a^{\frac{5}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)a^{2}\sqrt{a\tanh(x)}+\arctan\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)a^{2}\sqrt{a\tanh(x)}\right)}{\sqrt{a\tanh(x)^{3}}\,a^{\frac{5}{2}}}$	65
default	$-\frac{\tanh(x)\left(2a^{\frac{5}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)a^{2}\sqrt{a\tanh(x)}+\arctan\left(\frac{\sqrt{a\tanh(x)}}{\sqrt{a}}\right)a^{2}\sqrt{a\tanh(x)}\right)}{\sqrt{a\tanh(x)^{3}}\,a^{\frac{5}{2}}}$	65

```
[In] int(1/(a*tanh(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -tanh(x)*(2*a^(5/2)-arctanh((a*tanh(x))^(1/2)/a^(1/2))*a^2*(a*tanh(x))^(1/2
)+arctan((a*tanh(x))^(1/2)/a^(1/2))*a^2*(a*tanh(x))^(1/2))/(a*tanh(x)^3)^(1
/2)/a^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(50) = 100$.

Time = 0.26 (sec) , antiderivative size = 516, normalized size of antiderivative = 8.06

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx$$
$$= \left[-\frac{2\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\sqrt{-a}\arctan\left(\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2\right)\sqrt{-a}\sqrt{\frac{a\sinh(x)}{\cosh(x)}}}{a\cosh(x)^2+2a\cosh(x)\sinh(x)+a\sinh(x)^2-a}\right.}{}\right.$$
$$-\frac{2\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\sqrt{a}\arctan\left(\frac{\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\sqrt{\frac{a\sinh(x)}{\cosh(x)}}}{\sqrt{a}}\right.}{}$$

```
[In] integrate(1/(a*tanh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*arctan((c
osh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt(a*sinh(x)/cosh(x))/
(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)) + (cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a)*log(-(a*cosh(x)^4 + 4*a*cosh(x)^3*si
```

```
nh(x) + 6*a*cosh(x)^2*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(a*sinh(x)/cosh
(x)) - 2*a)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*co
sh(x)*sinh(x)^3 + sinh(x)^4)) + 8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 1)*sqrt(a*sinh(x)/cosh(x)))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh
(x)^2 - a), -1/4*(2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)
*arctan((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a*sinh(x)/cosh
(x))/sqrt(a)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a)*log
(2*a*cosh(x)^4 + 8*a*cosh(x)^3*sinh(x) + 12*a*cosh(x)^2*sinh(x)^2 + 8*a*cos
h(x)*sinh(x)^3 + 2*a*sinh(x)^4 + 2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(
x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*
sinh(x))*sqrt(a)*sqrt(a*sinh(x)/cosh(x)) - a) + 8*(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2 + 1)*sqrt(a*sinh(x)/cosh(x)))/(a*cosh(x)^2 + 2*a*cosh(x)*
sinh(x) + a*sinh(x)^2 - a)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\tanh^3{(x)}}}\,dx$$

```
[In] integrate(1/(a*tanh(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*tanh(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\tanh{(x)}^3}}\,dx$$

```
[In] integrate(1/(a*tanh(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*tanh(x)^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx = \frac{4}{\left(\sqrt{a}e^{(2\,x)} - \sqrt{ae^{(4\,x)} - a} - \sqrt{a}\right)\operatorname{sgn}\left(e^{(4\,x)} - 1\right)}$$

```
[In] integrate(1/(a*tanh(x)^3)^(1/2),x, algorithm="giac")
[Out] 4/((sqrt(a)*e^(2*x) - sqrt(a*e^(4*x) - a) - sqrt(a))*sgn(e^(4*x) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\tanh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\tanh(x)^3}}\,dx$$

```
[In] int(1/(a*tanh(x)^3)^(1/2),x)
[Out] int(1/(a*tanh(x)^3)^(1/2), x)
```

3.37 $\int \left(a\tanh^4(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \left(a\tanh^4(x)\right)^{3/2}\,dx = -a\coth(x)\sqrt{a\tanh^4(x)} + ax\coth^2(x)\sqrt{a\tanh^4(x)} - \frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)} - \frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}$$

```
[Out] -a*coth(x)*(a*tanh(x)^4)^(1/2)+a*x*coth(x)^2*(a*tanh(x)^4)^(1/2)-1/3*a*(a*t
anh(x)^4)^(1/2)*tanh(x)-1/5*a*(a*tanh(x)^4)^(1/2)*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3739, 3554, 8}

$$\int \left(a\tanh^4(x)\right)^{3/2}\,dx = -\frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)} - \frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)} + ax\coth^2(x)\sqrt{a\tanh^4(x)} - a\coth(x)\sqrt{a\tanh^4(x)}$$

```
[In] Int[(a*Tanh[x]^4)^(3/2),x]
```

```
[Out] -(a*Coth[x]*Sqrt[a*Tanh[x]^4]) + a*x*Coth[x]^2*Sqrt[a*Tanh[x]^4] - (a*Tanh[
x]*Sqrt[a*Tanh[x]^4])/3 - (a*Tanh[x]^3*Sqrt[a*Tanh[x]^4])/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^6(x)\,dx\\
&= -\frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)} + \left(a\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^4(x)\,dx\\
&= -\frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)} - \frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}\\
&\quad + \left(a\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^2(x)\,dx\\
&= -a\coth(x)\sqrt{a\tanh^4(x)} - \frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)}\\
&\quad - \frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)} + \left(a\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int 1\,dx\\
&= -a\coth(x)\sqrt{a\tanh^4(x)} + ax\coth^2(x)\sqrt{a\tanh^4(x)}\\
&\quad - \frac{1}{3}a\tanh(x)\sqrt{a\tanh^4(x)} - \frac{1}{5}a\tanh^3(x)\sqrt{a\tanh^4(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

$$\begin{aligned}
\int \left(a\tanh^4(x)\right)^{3/2}\,dx &= \frac{1}{15}\coth(x)\left(-3-5\coth^2(x)\right.\\
&\quad \left.-15\coth^4(x)+15\operatorname{arctanh}(\tanh(x))\coth^5(x)\right)\left(a\tanh^4(x)\right)^{3/2}
\end{aligned}$$

```
[In] Integrate[(a*Tanh[x]^4)^(3/2),x]
[Out] (Coth[x]*(-3 - 5*Coth[x]^2 - 15*Coth[x]^4 + 15*ArcTanh[Tanh[x]]*Coth[x]^5)*
(a*Tanh[x]^4)^(3/2))/15
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$-\frac{\left(a \tanh \left(x \right)^{4}\right)^{\frac{3}{2}} \left(6 \tanh \left(x \right)^{5}+10 \tanh \left(x \right)^{3}+15 \ln \left(\tanh \left(x \right)-1\right)-15 \ln \left(1+\tanh \left(x \right)\right)+30 \tanh \left(x \right)\right)}{30 \tanh \left(x \right)^{6}}$	46
default	$-\frac{\left(a \tanh \left(x \right)^{4}\right)^{\frac{3}{2}} \left(6 \tanh \left(x \right)^{5}+10 \tanh \left(x \right)^{3}+15 \ln \left(\tanh \left(x \right)-1\right)-15 \ln \left(1+\tanh \left(x \right)\right)+30 \tanh \left(x \right)\right)}{30 \tanh \left(x \right)^{6}}$	46
risch	$\frac{a \left(1+{\mathrm e}^{2 x}\right)^{2} \sqrt{\frac{a \left({\mathrm e}^{2 x}-1\right)^{4}}{\left(1+{\mathrm e}^{2 x}\right)^{4}}}\, x}{\left({\mathrm e}^{2 x}-1\right)^{2}}+\frac{2 a \sqrt{\frac{a \left({\mathrm e}^{2 x}-1\right)^{4}}{\left(1+{\mathrm e}^{2 x}\right)^{4}}}\, \left(45 \,{\mathrm e}^{8 x}+90 \,{\mathrm e}^{6 x}+140 \,{\mathrm e}^{4 x}+70 \,{\mathrm e}^{2 x}+23\right)}{15 \left({\mathrm e}^{2 x}-1\right)^{2} \left(1+{\mathrm e}^{2 x}\right)^{3}}$	106

```
[In] int((a*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(a*tanh(x)^4)^(3/2)*(6*tanh(x)^5+10*tanh(x)^3+15*ln(tanh(x)-1)-15*ln(
1+tanh(x))+30*tanh(x))/tanh(x)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2114 vs. 2(57) = 114.

Time = 0.30 (sec) , antiderivative size = 2114, normalized size of antiderivative = 30.64

$$\int \left(a \tanh^4(x)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate((a*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*a*x*cosh(x)^10 + 15*(a*x*e^(4*x) + 2*a*x*e^(2*x) + a*x)*sinh(x)^10
 + 150*(a*x*cosh(x)*e^(4*x) + 2*a*x*cosh(x)*e^(2*x) + a*x*cosh(x))*sinh(x)^
9 + 15*(5*a*x + 6*a)*cosh(x)^8 + 15*(45*a*x*cosh(x)^2 + 5*a*x + (45*a*x*cos
h(x)^2 + 5*a*x + 6*a)*e^(4*x) + 2*(45*a*x*cosh(x)^2 + 5*a*x + 6*a)*e^(2*x)
+ 6*a)*sinh(x)^8 + 120*(15*a*x*cosh(x)^3 + (5*a*x + 6*a)*cosh(x) + (15*a*x*
cosh(x)^3 + (5*a*x + 6*a)*cosh(x))*e^(4*x) + 2*(15*a*x*cosh(x)^3 + (5*a*x +
 6*a)*cosh(x))*e^(2*x))*sinh(x)^7 + 30*(5*a*x + 6*a)*cosh(x)^6 + 30*(105*a*
x*cosh(x)^4 + 14*(5*a*x + 6*a)*cosh(x)^2 + 5*a*x + (105*a*x*cosh(x)^4 + 14*
(5*a*x + 6*a)*cosh(x)^2 + 5*a*x + 6*a)*e^(4*x) + 2*(105*a*x*cosh(x)^4 + 14*
(5*a*x + 6*a)*cosh(x)^2 + 5*a*x + 6*a)*e^(2*x) + 6*a)*sinh(x)^6 + 60*(63*a*
x*cosh(x)^5 + 14*(5*a*x + 6*a)*cosh(x)^3 + 3*(5*a*x + 6*a)*cosh(x) + (63*a*
x*cosh(x)^5 + 14*(5*a*x + 6*a)*cosh(x)^3 + 3*(5*a*x + 6*a)*cosh(x))*e^(4*x)
 + 2*(63*a*x*cosh(x)^5 + 14*(5*a*x + 6*a)*cosh(x)^3 + 3*(5*a*x + 6*a)*cosh(
x))*e^(2*x))*sinh(x)^5 + 10*(15*a*x + 28*a)*cosh(x)^4 + 10*(315*a*x*cosh(x)
^6 + 105*(5*a*x + 6*a)*cosh(x)^4 + 45*(5*a*x + 6*a)*cosh(x)^2 + 15*a*x + (3
15*a*x*cosh(x)^6 + 105*(5*a*x + 6*a)*cosh(x)^4 + 45*(5*a*x + 6*a)*cosh(x)^2
```

+ 15*a*x + 28*a)*e^(4*x) + 2*(315*a*x*cosh(x)^6 + 105*(5*a*x + 6*a)*cosh(x
)^4 + 45*(5*a*x + 6*a)*cosh(x)^2 + 15*a*x + 28*a)*e^(2*x) + 28*a)*sinh(x)^4
+ 40*(45*a*x*cosh(x)^7 + 21*(5*a*x + 6*a)*cosh(x)^5 + 15*(5*a*x + 6*a)*cos
h(x)^3 + (15*a*x + 28*a)*cosh(x) + (45*a*x*cosh(x)^7 + 21*(5*a*x + 6*a)*cos
h(x)^5 + 15*(5*a*x + 6*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x))*e^(4*x) + 2*
(45*a*x*cosh(x)^7 + 21*(5*a*x + 6*a)*cosh(x)^5 + 15*(5*a*x + 6*a)*cosh(x)^3
+ (15*a*x + 28*a)*cosh(x))*e^(2*x))*sinh(x)^3 + 5*(15*a*x + 28*a)*cosh(x)^
2 + 5*(135*a*x*cosh(x)^8 + 84*(5*a*x + 6*a)*cosh(x)^6 + 90*(5*a*x + 6*a)*co
sh(x)^4 + 12*(15*a*x + 28*a)*cosh(x)^2 + 15*a*x + (135*a*x*cosh(x)^8 + 84*(
5*a*x + 6*a)*cosh(x)^6 + 90*(5*a*x + 6*a)*cosh(x)^4 + 12*(15*a*x + 28*a)*co
sh(x)^2 + 15*a*x + 28*a)*e^(4*x) + 2*(135*a*x*cosh(x)^8 + 84*(5*a*x + 6*a)*
cosh(x)^6 + 90*(5*a*x + 6*a)*cosh(x)^4 + 12*(15*a*x + 28*a)*cosh(x)^2 + 15*
a*x + 28*a)*e^(2*x) + 28*a)*sinh(x)^2 + 15*a*x + (15*a*x*cosh(x)^10 + 15*(5
*a*x + 6*a)*cosh(x)^8 + 30*(5*a*x + 6*a)*cosh(x)^6 + 10*(15*a*x + 28*a)*cos
h(x)^4 + 5*(15*a*x + 28*a)*cosh(x)^2 + 15*a*x + 46*a)*e^(4*x) + 2*(15*a*x*c
osh(x)^10 + 15*(5*a*x + 6*a)*cosh(x)^8 + 30*(5*a*x + 6*a)*cosh(x)^6 + 10*(1
5*a*x + 28*a)*cosh(x)^4 + 5*(15*a*x + 28*a)*cosh(x)^2 + 15*a*x + 46*a)*e^(2
x) + 10(15*a*x*cosh(x)^9 + 12*(5*a*x + 6*a)*cosh(x)^7 + 18*(5*a*x + 6*a)*
cosh(x)^5 + 4*(15*a*x + 28*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x) + (15*a*x
cosh(x)^9 + 12(5*a*x + 6*a)*cosh(x)^7 + 18*(5*a*x + 6*a)*cosh(x)^5 + 4*(1
5*a*x + 28*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x))*e^(4*x) + 2*(15*a*x*cosh
(x)^9 + 12*(5*a*x + 6*a)*cosh(x)^7 + 18*(5*a*x + 6*a)*cosh(x)^5 + 4*(15*a*x
+ 28*a)*cosh(x)^3 + (15*a*x + 28*a)*cosh(x))*e^(2*x))*sinh(x) + 46*a)*sqrt
((a*e^(8*x) - 4*a*e^(6*x) + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(e^(8*x) + 4*e^(
6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))/((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^10 +
cosh(x)^10 + 10*(cosh(x)*e^(4*x) - 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^9 +
5*(9*cosh(x)^2 + (9*cosh(x)^2 + 1)*e^(4*x) - 2*(9*cosh(x)^2 + 1)*e^(2*x) +
1)*sinh(x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + (3*cosh(x)^3 + cosh(x))*e^(
4*x) - 2*(3*cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x)^7 + 10*(21*cosh
(x)^4 + 14*cosh(x)^2 + (21*cosh(x)^4 + 14*cosh(x)^2 + 1)*e^(4*x) - 2*(21*co
sh(x)^4 + 14*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*c
osh(x)^5 + 70*cosh(x)^3 + (63*cosh(x)^5 + 70*cosh(x)^3 + 15*cosh(x))*e^(4*x
) - 2*(63*cosh(x)^5 + 70*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sinh
(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh(x)^2 + (21*cosh(x)^6 + 35
*cosh(x)^4 + 15*cosh(x)^2 + 1)*e^(4*x) - 2*(21*cosh(x)^6 + 35*cosh(x)^4 + 1
5*cosh(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 +
7*cosh(x)^5 + 5*cosh(x)^3 + (3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x)^3 + cosh
(x))*e^(4*x) - 2*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x)^3 + cosh(x))*e^(2*x
) + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*
cosh(x)^2 + (9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*
e^(4*x) - 2*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*
e^(2*x) + 1)*sinh(x)^2 + 5*cosh(x)^2 + (cosh(x)^10 + 5*cosh(x)^8 + 10*cosh(
x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(4*x) - 2*(cosh(x)^10 + 5*cosh(x)^
8 + 10*cosh(x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x) + 10*(cosh(x)^9
+ 4*cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + (cosh(x)^9 + 4*cosh(x)^7 + 6*co

```
sh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(4*x) - 2*(cosh(x)^9 + 4*cosh(x)^7 + 6*c
osh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(2*x) + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(a\tanh^4(x)\right)^{3/2} dx = \int \left(a\tanh^4{(x)}\right)^{\frac{3}{2}} dx$$

```
[In] integrate((a*tanh(x)**4)**(3/2),x)
[Out] Integral((a*tanh(x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.19

$$\int \left(a\tanh^4(x)\right)^{3/2} dx = a^{\frac{3}{2}}x - \frac{2\left(70\,a^{\frac{3}{2}}e^{(-2\,x)} + 140\,a^{\frac{3}{2}}e^{(-4\,x)} + 90\,a^{\frac{3}{2}}e^{(-6\,x)} + 45\,a^{\frac{3}{2}}e^{(-8\,x)} + 23\,a^{\frac{3}{2}}\right)}{15\left(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1\right)}$$

```
[In] integrate((a*tanh(x)^4)^(3/2),x, algorithm="maxima")
[Out] a^(3/2)*x - 2/15*(70*a^(3/2)*e^(-2*x) + 140*a^(3/2)*e^(-4*x) + 90*a^(3/2)*e
^(-6*x) + 45*a^(3/2)*e^(-8*x) + 23*a^(3/2))/(5*e^(-2*x) + 10*e^(-4*x) + 10*
e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.65

$$\int \left(a\tanh^4(x)\right)^{3/2} dx = \frac{1}{15}\,a^{\frac{3}{2}}\left(15\,x + \frac{2\left(45\,e^{(8\,x)} + 90\,e^{(6\,x)} + 140\,e^{(4\,x)} + 70\,e^{(2\,x)} + 23\right)}{\left(e^{(2\,x)} + 1\right)^{5}}\right)$$

```
[In] integrate((a*tanh(x)^4)^(3/2),x, algorithm="giac")
[Out] 1/15*a^(3/2)*(15*x + 2*(45*e^(8*x) + 90*e^(6*x) + 140*e^(4*x) + 70*e^(2*x)
+ 23)/(e^(2*x) + 1)^5)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a \tanh^4(x)\right)^{3/2} \, dx = \int \left(a \tanh(x)^4\right)^{3/2} dx$$

```
[In] int((a*tanh(x)^4)^(3/2),x)
[Out] int((a*tanh(x)^4)^(3/2), x)
```

3.38 $\int \sqrt{a \tanh^4(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \sqrt{a \tanh^4(x)}\, dx = -\coth(x)\sqrt{a \tanh^4(x)} + x \coth^2(x)\sqrt{a \tanh^4(x)}$$

```
[Out] -coth(x)*(a*tanh(x)^4)^(1/2)+x*coth(x)^2*(a*tanh(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3739, 3554, 8}

$$\int \sqrt{a \tanh^4(x)}\, dx = x \coth^2(x)\sqrt{a \tanh^4(x)} - \coth(x)\sqrt{a \tanh^4(x)}$$

```
[In] Int[Sqrt[a*Tanh[x]^4],x]
```

```
[Out] -(Coth[x]*Sqrt[a*Tanh[x]^4]) + x*Coth[x]^2*Sqrt[a*Tanh[x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int \tanh^2(x)\,dx\\ &= -\coth(x)\sqrt{a\tanh^4(x)}+\left(\coth^2(x)\sqrt{a\tanh^4(x)}\right)\int 1\,dx\\ &= -\coth(x)\sqrt{a\tanh^4(x)}+x\coth^2(x)\sqrt{a\tanh^4(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \sqrt{a\tanh^4(x)}\,dx = \coth(x)(-1+\operatorname{arctanh}(\tanh(x))\coth(x))\sqrt{a\tanh^4(x)}$$

```
[In] Integrate[Sqrt[a*Tanh[x]^4],x]
[Out] Coth[x]*(-1 + ArcTanh[Tanh[x]]*Coth[x])*Sqrt[a*Tanh[x]^4]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$-\frac{\sqrt{a\tanh(x)^4}\,(2\tanh(x)+\ln(\tanh(x)-1)-\ln(1+\tanh(x)))}{2\tanh(x)^2}$	32
default	$-\frac{\sqrt{a\tanh(x)^4}\,(2\tanh(x)+\ln(\tanh(x)-1)-\ln(1+\tanh(x)))}{2\tanh(x)^2}$	32
risch	$\frac{\sqrt{\frac{a\left(e^{2x}-1\right)^4}{\left(1+e^{2x}\right)^4}}\,\left(1+e^{2x}\right)^2 x}{\left(e^{2x}-1\right)^2}+\frac{2\sqrt{\frac{a\left(e^{2x}-1\right)^4}{\left(1+e^{2x}\right)^4}}\,\left(1+e^{2x}\right)}{\left(e^{2x}-1\right)^2}$	76

```
[In] int((a*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(a*tanh(x)^4)^(1/2)*(2*tanh(x)+ln(tanh(x)-1)-ln(1+tanh(x)))/tanh(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(27) = 54$.

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 6.87

$$\int \sqrt{a\tanh^4(x)}\,dx$$
$$= \frac{\left(x\cosh\left(x\right)^2 + \left(xe^{(4x)} + 2\,xe^{(2x)} + x\right)\sinh\left(x\right)^2 + \left(x\cosh\left(x\right)^2 + x + 2\right)e^{(4x)} + 2\left(x\cosh\left(x\right)^2 + x + 2\right)e^{(2x)}}{\left(e^{(4x)} - 2\,e^{(2x)} + 1\right)\sinh\left(x\right)^2 + \cosh\left(x\right)^2 + \left(\cosh\left(x\right)^2 + 1\right)e^{(4x)} - 2\left(\cosh\left(x\right.\right.}$$

```
[In] integrate((a*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^2 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 + x +
 2)*e^(4*x) + 2*(x*cosh(x)^2 + x + 2)*e^(2*x) + 2*(x*cosh(x)*e^(4*x) + 2*x*
cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) + x + 2)*sqrt((a*e^(8*x) - 4*a*e^(6*x)
 + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2
*x) + 1))/((e^(4*x) - 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1
)*e^(4*x) - 2*(cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(4*x) - 2*cosh(x)*e^(2
*x) + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \sqrt{a\tanh^4(x)}\,dx = \int \sqrt{a\tanh^4\left(x\right)}\,dx$$

```
[In] integrate((a*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*tanh(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sqrt{a\tanh^4(x)}\,dx = \sqrt{a}x - \frac{2\sqrt{a}}{e^{(-2x)} + 1}$$

```
[In] integrate((a*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*x - 2*sqrt(a)/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.52

$$\int \sqrt{a \tanh^4(x)}\, dx = \sqrt{a}\left(x + \frac{2}{e^{(2x)} + 1}\right)$$

```
[In] integrate((a*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] sqrt(a)*(x + 2/(e^(2*x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a \tanh^4(x)}\, dx = \int \sqrt{a \tanh(x)^4}\, dx$$

```
[In] int((a*tanh(x)^4)^(1/2),x)
[Out] int((a*tanh(x)^4)^(1/2), x)
```

3.39 $\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx=-\frac{\tanh(x)}{\sqrt{a\tanh^4(x)}}+\frac{x\tanh^2(x)}{\sqrt{a\tanh^4(x)}}$$

```
[Out] -tanh(x)/(a*tanh(x)^4)^(1/2)+x*tanh(x)^2/(a*tanh(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx=\frac{x\tanh^2(x)}{\sqrt{a\tanh^4(x)}}-\frac{\tanh(x)}{\sqrt{a\tanh^4(x)}}$$

```
[In] Int[1/Sqrt[a*Tanh[x]^4],x]
[Out] -(Tanh[x]/Sqrt[a*Tanh[x]^4]) + (x*Tanh[x]^2)/Sqrt[a*Tanh[x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tanh^2(x)\int \coth^2(x)\,dx}{\sqrt{a\tanh^4(x)}}\\
&= -\frac{\tanh(x)}{\sqrt{a\tanh^4(x)}}+\frac{\tanh^2(x)\int 1\,dx}{\sqrt{a\tanh^4(x)}}\\
&= -\frac{\tanh(x)}{\sqrt{a\tanh^4(x)}}+\frac{x\tanh^2(x)}{\sqrt{a\tanh^4(x)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$
\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right)\tanh(x)}{\sqrt{a\tanh^4(x)}}
$$

```
[In] Integrate[1/Sqrt[a*Tanh[x]^4],x]
[Out] -((Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Tanh[x])/Sqrt[a*Tanh[x]^4])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\tanh(x)(\ln(1+\tanh(x))\tanh(x)-\ln(\tanh(x)-1)\tanh(x)-2)}{2\sqrt{a\tanh(x)^4}}$	32
default	$\frac{\tanh(x)(\ln(1+\tanh(x))\tanh(x)-\ln(\tanh(x)-1)\tanh(x)-2)}{2\sqrt{a\tanh(x)^4}}$	32
risch	$\frac{e^{4x}x-2\,e^{2x}x-2\,e^{2x}+x+2}{\sqrt{\frac{a\left(e^{2x}-1\right)^4}{\left(1+e^{2x}\right)^4}}\,(1+e^{2x})^2}$	52

[In] int(1/(a*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*tanh(x)*(ln(1+tanh(x))*tanh(x)-ln(tanh(x)-1)*tanh(x)-2)/(a*tanh(x)^4)^(
1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 7.68

$$\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx = \frac{\left(x\cosh\left(x\right)^2+\left(xe^{(4\,x)}+2\,xe^{(2\,x)}+x\right)\sinh\left(x\right)^2+\left(x\cosh\left(x\right)^2-x-2\right)e^{(4\,x)}+2\,\left(x\cosh\left(x\right)^2-x-2\right)e^{(2\,x)}}{a\cosh\left(x\right)^2+\left(ae^{(4\,x)}-2\,ae^{(2\,x)}+a\right)\sinh\left(x\right)^2+\left(a\cosh\left(x\right)^2-a\right)e^{(4\,x)}-2\,\left(a\cosh\right.}$$

[In] integrate(1/(a*tanh(x)^4)^(1/2),x, algorithm="fricas")

[Out] (x*cosh(x)^2 + (x*e^(4*x) + 2*x*e^(2*x) + x)*sinh(x)^2 + (x*cosh(x)^2 - x -
 2)*e^(4*x) + 2*(x*cosh(x)^2 - x - 2)*e^(2*x) + 2*(x*cosh(x)*e^(4*x) + 2*x*
cosh(x)*e^(2*x) + x*cosh(x))*sinh(x) - x - 2)*sqrt((a*e^(8*x) - 4*a*e^(6*x)
 + 6*a*e^(4*x) - 4*a*e^(2*x) + a)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2
*x) + 1))/(a*cosh(x)^2 + (a*e^(4*x) - 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(
x)^2 - a)*e^(4*x) - 2*(a*cosh(x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) - 2*
a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x) - a)

Sympy [F]

$$\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx = \int \frac{1}{\sqrt{a\tanh^4{(x)}}}\,dx$$

```
[In] integrate(1/(a*tanh(x)**4)**(1/2),x)
[Out] Integral(1/sqrt(a*tanh(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx = \frac{x}{\sqrt{a}} + \frac{2\sqrt{a}}{ae^{(-2x)} - a}$$

```
[In] integrate(1/(a*tanh(x)^4)^(1/2),x, algorithm="maxima")
[Out] x/sqrt(a) + 2*sqrt(a)/(a*e^(-2*x) - a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx = \frac{x}{\sqrt{a}} - \frac{2}{\sqrt{a}(e^{(2x)} - 1)}$$

```
[In] integrate(1/(a*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] x/sqrt(a) - 2/(sqrt(a)*(e^(2*x) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\tanh^4(x)}}\,dx = \int \frac{1}{\sqrt{a\,{\tanh\left(x\right)}^4}}\,dx$$

```
[In] int(1/(a*tanh(x)^4)^(1/2),x)
[Out] int(1/(a*tanh(x)^4)^(1/2), x)
```

3.40 $\int (b\tanh^m(c+dx))^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (b\tanh^m(c+dx))^n \, dx = \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\tanh^2(c+dx)\right)\tanh(c+dx)\,(b\tanh^m(c+dx))^n}{d(1+mn)}$$

```
[Out] hypergeom([1, 1/2*m*n+1/2],[1/2*m*n+3/2],tanh(d*x+c)^2)*tanh(d*x+c)*(b*tanh
(d*x+c)^m)^n/d/(m*n+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3740, 3557, 371}

$$\int (b\tanh^m(c+dx))^n \, dx = \frac{\tanh(c+dx)\,(b\tanh^m(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(mn+1),\frac{1}{2}(mn+3),\tanh^2(c+dx)\right)}{d(mn+1)}$$

```
[In] Int[(b*Tanh[c + d*x]^m)^n,x]
```

```
[Out] (Hypergeometric2F1[1, (1 + m*n)/2, (3 + m*n)/2, Tanh[c + d*x]^2]*Tanh[c + d
*x]*(b*Tanh[c + d*x]^m)^n)/(d*(1 + m*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\tanh^{-mn}(c+dx)\,(b\tanh^m(c+dx))^n\right)\int\tanh^{mn}(c+dx)\,dx\\
&= -\frac{\left(\tanh^{-mn}(c+dx)\,(b\tanh^m(c+dx))^n\right)\text{Subst}\left(\int\frac{x^{mn}}{-1+x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\tanh^2(c+dx)\right)\tanh(c+dx)\,(b\tanh^m(c+dx))^n}{d(1+mn)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int(b\tanh^m(c+dx))^n\,dx$$
$$=\frac{\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\tanh^2(c+dx)\right)\tanh(c+dx)\,(b\tanh^m(c+dx))^n}{d(1+mn)}$$

```
[In] Integrate[(b*Tanh[c + d*x]^m)^n,x]
[Out] (Hypergeometric2F1[1, (1 + m*n)/2, (3 + m*n)/2, Tanh[c + d*x]^2]*Tanh[c + d
*x]*(b*Tanh[c + d*x]^m)^n)/(d*(1 + m*n))
```

Maple [F]

$$\int (b\tanh (dx+c)^m)^n\,dx$$

```
[In] int((b*tanh(d*x+c)^m)^n,x)
[Out] int((b*tanh(d*x+c)^m)^n,x)
```

Fricas [F]

$$\int (b\tanh^m(c+dx))^n\ dx = \int (b\tanh (dx+c)^m)^n\ dx$$

```
[In] integrate((b*tanh(d*x+c)^m)^n,x, algorithm="fricas")
[Out] integral((b*tanh(d*x + c)^m)^n, x)
```

Sympy [F]

$$\int (b\tanh^m(c+dx))^n\ dx = \int (b\tanh^m (c+dx))^n\ dx$$

```
[In] integrate((b*tanh(d*x+c)**m)**n,x)
[Out] Integral((b*tanh(c + d*x)**m)**n, x)
```

Maxima [F]

$$\int (b\tanh^m(c+dx))^n\ dx = \int (b\tanh (dx+c)^m)^n\ dx$$

```
[In] integrate((b*tanh(d*x+c)^m)^n,x, algorithm="maxima")
[Out] integrate((b*tanh(d*x + c)^m)^n, x)
```

Giac [F]

$$\int (b\tanh^m(c+dx))^n \, dx = \int (b\tanh{(dx+c)}^m)^n \, dx$$

```
[In] integrate((b*tanh(d*x+c)^m)^n,x, algorithm="giac")
[Out] integrate((b*tanh(d*x + c)^m)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\tanh^m(c+dx))^n \, dx = \int (b\tanh(c+d\,x)^m)^n \, dx$$

```
[In] int((b*tanh(c + d*x)^m)^n,x)
[Out] int((b*tanh(c + d*x)^m)^n, x)
```

3.41 $\int (a + a\tanh(c + dx))^5 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 100

$$\int (a + a\tanh(c + dx))^5 \, dx = 16a^5x + \frac{16a^5\log(\cosh(c + dx))}{d} - \frac{8a^5\tanh(c + dx)}{d} - \frac{2a^2(a + a\tanh(c + dx))^3}{3d} - \frac{a(a + a\tanh(c + dx))^4}{4d} - \frac{2a(a^2 + a^2\tanh(c + dx))^2}{d}$$

```
[Out] 16*a^5*x+16*a^5*ln(cosh(d*x+c))/d-8*a^5*tanh(d*x+c)/d-2/3*a^2*(a+a*tanh(d*x
+c))^3/d-1/4*a*(a+a*tanh(d*x+c))^4/d-2*a*(a^2+a^2*tanh(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3559, 3558, 3556}

$$\int (a + a\tanh(c + dx))^5 \, dx = -\frac{8a^5\tanh(c + dx)}{d} + \frac{16a^5\log(\cosh(c + dx))}{d} + 16a^5x - \frac{2a^2(a\tanh(c + dx) + a)^3}{3d} - \frac{2a(a^2\tanh(c + dx) + a^2)^2}{d} - \frac{a(a\tanh(c + dx) + a)^4}{4d}$$

```
[In] Int[(a + a*Tanh[c + d*x])^5,x]
```

```
[Out] 16*a^5*x + (16*a^5*Log[Cosh[c + d*x]])/d - (8*a^5*Tanh[c + d*x])/d - (2*a^2
*(a + a*Tanh[c + d*x])^3)/(3*d) - (a*(a + a*Tanh[c + d*x])^4)/(4*d) - (2*a*
(a^2 + a^2*Tanh[c + d*x])^2)/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a(a+a\tanh(c+dx))^4}{4d} + (2a)\int (a+a\tanh(c+dx))^4\,dx \\
&= -\frac{2a^2(a+a\tanh(c+dx))^3}{3d} - \frac{a(a+a\tanh(c+dx))^4}{4d} + (4a^2)\int (a+a\tanh(c+dx))^3\,dx \\
&= -\frac{2a^3(a+a\tanh(c+dx))^2}{d} - \frac{2a^2(a+a\tanh(c+dx))^3}{3d} \\
&\quad - \frac{a(a+a\tanh(c+dx))^4}{4d} + (8a^3)\int (a+a\tanh(c+dx))^2\,dx \\
&= 16a^5x - \frac{8a^5\tanh(c+dx)}{d} - \frac{2a^3(a+a\tanh(c+dx))^2}{d} - \frac{2a^2(a+a\tanh(c+dx))^3}{3d} \\
&\quad - \frac{a(a+a\tanh(c+dx))^4}{4d} + (16a^5)\int \tanh(c+dx)\,dx \\
&= 16a^5x + \frac{16a^5\log(\cosh(c+dx))}{d} - \frac{8a^5\tanh(c+dx)}{d} - \frac{2a^3(a+a\tanh(c+dx))^2}{d} \\
&\quad - \frac{2a^2(a+a\tanh(c+dx))^3}{3d} - \frac{a(a+a\tanh(c+dx))^4}{4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.63

$$\int (a + a\tanh(c+dx))^5\,dx =$$
$$-\frac{a^5(35 + 192\log(1-\tanh(c+dx)) + 180\tanh(c+dx) + 66\tanh^2(c+dx) + 20\tanh^3(c+dx) + 3\tanh^4}{12d}$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^5,x]
[Out] -1/12*(a^5*(35 + 192*Log[1 - Tanh[c + d*x]] + 180*Tanh[c + d*x] + 66*Tanh[c
 + d*x]^2 + 20*Tanh[c + d*x]^3 + 3*Tanh[c + d*x]^4))/d
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

method	result
derivativedivides	$\frac{a^5\left(-\frac{\tanh(dx+c)^4}{4}-\frac{5\tanh(dx+c)^3}{3}-\frac{11\tanh(dx+c)^2}{2}-15\tanh(dx+c)-16\ln(\tanh(dx+c)-1)\right)}{d}$
default	$\frac{a^5\left(-\frac{\tanh(dx+c)^4}{4}-\frac{5\tanh(dx+c)^3}{3}-\frac{11\tanh(dx+c)^2}{2}-15\tanh(dx+c)-16\ln(\tanh(dx+c)-1)\right)}{d}$
parallelrisch	$-\frac{3\tanh(dx+c)^4a^5+20\tanh(dx+c)^3a^5+66\tanh(dx+c)^2a^5+192\ln(1-\tanh(dx+c))a^5+180a^5\tanh(dx+c)}{12d}$
risch	$-\frac{32a^5c}{d}+\frac{4a^5\left(48\,e^{6dx+6c}+108\,e^{4dx+4c}+88\,e^{2dx+2c}+25\right)}{3d\left(e^{2dx+2c}+1\right)^4}+\frac{16a^5\ln\left(e^{2dx+2c}+1\right)}{d}$
parts	$a^5x+\frac{a^5\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{5a^5\ln(\cosh(dx+c))}{d}+\frac{10a^5\left(-\tan\right.}{}$

```
[In] int((a+a*tanh(d*x+c))^5,x,method=_RETURNVERBOSE)
[Out] 1/d*a^5*(-1/4*tanh(d*x+c)^4-5/3*tanh(d*x+c)^3-11/2*tanh(d*x+c)^2-15*tanh(d*
x+c)-16*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 907 vs. 2(96) = 192.

Time = 0.26 (sec) , antiderivative size = 907, normalized size of antiderivative = 9.07

$$\int (a+a\tanh(c+dx))^5\,dx = \text{Too large to display}$$

```
[In] integrate((a+a*tanh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 4/3*(48*a^5*cosh(d*x + c)^6 + 288*a^5*cosh(d*x + c)*sinh(d*x + c)^5 + 48*a^
5*sinh(d*x + c)^6 + 108*a^5*cosh(d*x + c)^4 + 88*a^5*cosh(d*x + c)^2 + 25*a
^5 + 36*(20*a^5*cosh(d*x + c)^2 + 3*a^5)*sinh(d*x + c)^4 + 48*(20*a^5*cosh(
d*x + c)^3 + 9*a^5*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(90*a^5*cosh(d*x + c)
^4 + 81*a^5*cosh(d*x + c)^2 + 11*a^5)*sinh(d*x + c)^2 + 12*(a^5*cosh(d*x +
c)^8 + 8*a^5*cosh(d*x + c)*sinh(d*x + c)^7 + a^5*sinh(d*x + c)^8 + 4*a^5*co
sh(d*x + c)^6 + 6*a^5*cosh(d*x + c)^4 + 4*a^5*cosh(d*x + c)^2 + 4*(7*a^5*co
sh(d*x + c)^2 + a^5)*sinh(d*x + c)^6 + 8*(7*a^5*cosh(d*x + c)^3 + 3*a^5*cos
h(d*x + c))*sinh(d*x + c)^5 + a^5 + 2*(35*a^5*cosh(d*x + c)^4 + 30*a^5*cosh
(d*x + c)^2 + 3*a^5)*sinh(d*x + c)^4 + 8*(7*a^5*cosh(d*x + c)^5 + 10*a^5*co
sh(d*x + c)^3 + 3*a^5*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^5*cosh(d*x +
c)^6 + 15*a^5*cosh(d*x + c)^4 + 9*a^5*cosh(d*x + c)^2 + a^5)*sinh(d*x + c)^
2 + 8*(a^5*cosh(d*x + c)^7 + 3*a^5*cosh(d*x + c)^5 + 3*a^5*cosh(d*x + c)^3
+ a^5*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - si
nh(d*x + c))) + 16*(18*a^5*cosh(d*x + c)^5 + 27*a^5*cosh(d*x + c)^3 + 11*a^
5*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh
(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x
 + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cos
h(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c
)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*
cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int (a+a\tanh(c+dx))^5\,dx$$

$$= \begin{cases} 32a^5x - \frac{16a^5\log(\tanh(c+dx)+1)}{d} - \frac{a^5\tanh^4(c+dx)}{4d} - \frac{5a^5\tanh^3(c+dx)}{3d} - \frac{11a^5\tanh^2(c+dx)}{2d} - \frac{15a^5\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\tanh(c)+a)^5 & \text{otherwis} \end{cases}$$

```
[In] integrate((a+a*tanh(d*x+c))**5,x)
```

```
[Out] Piecewise((32*a**5*x - 16*a**5*log(tanh(c + d*x) + 1)/d - a**5*tanh(c + d*x
)**4/(4*d) - 5*a**5*tanh(c + d*x)**3/(3*d) - 11*a**5*tanh(c + d*x)**2/(2*d)
 - 15*a**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a*tanh(c) + a)**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(96) = 192$.

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.02

$$\int (a + a\tanh(c + dx))^5 \, dx$$
$$= \frac{5}{3}a^5\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right)$$
$$+ a^5\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)}\right)$$
$$+ 10a^5\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right)$$
$$+ 10a^5\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^5x + \frac{5a^5\log\left(\cosh\left(dx + c\right)\right)}{d}$$

```
[In] integrate((a+a*tanh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a^5*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^5*(x
+ c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6
*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 10*a^5*(x + c/d + log(e^(-2*d*x - 2
*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
 1))) + 10*a^5*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x + 5*a^5*log
(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int (a + a\tanh(c + dx))^5 \, dx$$
$$= \frac{4\left(12a^5\log\left(e^{(2dx+2c)} + 1\right) + \frac{48a^5e^{(6dx+6c)} + 108a^5e^{(4dx+4c)} + 88a^5e^{(2dx+2c)} + 25a^5}{\left(e^{(2dx+2c)} + 1\right)^4}\right)}{3d}$$

```
[In] integrate((a+a*tanh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 4/3*(12*a^5*log(e^(2*d*x + 2*c) + 1) + (48*a^5*e^(6*d*x + 6*c) + 108*a^5*e^
(4*d*x + 4*c) + 88*a^5*e^(2*d*x + 2*c) + 25*a^5)/(e^(2*d*x + 2*c) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int (a + a\tanh(c + dx))^5 \, dx = 32\,a^5\,x - \frac{a^5\,\left(192\,\ln\left(\tanh(c+d\,x)+1\right) + 180\tanh(c+d\,x) + 66\tanh(c+d\,x)^2 + 20\tanh(c+d\,x)^3 + 3\tanh(c-\right.}{12\,d}$$

```
[In] int((a + a*tanh(c + d*x))^5,x)
```

```
[Out] 32*a^5*x - (a^5*(192*log(tanh(c + d*x) + 1) + 180*tanh(c + d*x) + 66*tanh(c
 + d*x)^2 + 20*tanh(c + d*x)^3 + 3*tanh(c + d*x)^4))/(12*d)
```

3.42 $\int (a+a\tanh(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int (a+a\tanh(c+dx))^4\,dx = 8a^4x+\frac{8a^4\log(\cosh(c+dx))}{d}-\frac{4a^4\tanh(c+dx)}{d}-\frac{a(a+a\tanh(c+dx))^3}{3d}-\frac{\left(a^2+a^2\tanh(c+dx)\right)^2}{d}$$

```
[Out] 8*a^4*x+8*a^4*ln(cosh(d*x+c))/d-4*a^4*tanh(d*x+c)/d-1/3*a*(a+a*tanh(d*x+c))
^3/d-(a^2+a^2*tanh(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3559, 3558, 3556}

$$\int (a+a\tanh(c+dx))^4\,dx = -\frac{4a^4\tanh(c+dx)}{d}+\frac{8a^4\log(\cosh(c+dx))}{d}+8a^4x-\frac{\left(a^2\tanh(c+dx)+a^2\right)^2}{d}-\frac{a(a\tanh(c+dx)+a)^3}{3d}$$

```
[In] Int[(a + a*Tanh[c + d*x])^4,x]
```

```
[Out] 8*a^4*x + (8*a^4*Log[Cosh[c + d*x]])/d - (4*a^4*Tanh[c + d*x])/d - (a*(a +
a*Tanh[c + d*x])^3)/(3*d) - (a^2 + a^2*Tanh[c + d*x])^2/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a(a+a\tanh(c+dx))^3}{3d} + (2a)\int (a+a\tanh(c+dx))^3\,dx \\
&= -\frac{a(a+a\tanh(c+dx))^3}{3d} - \frac{\left(a^2+a^2\tanh(c+dx)\right)^2}{d} + \left(4a^2\right)\int (a+a\tanh(c+dx))^2\,dx \\
&= 8a^4x - \frac{4a^4\tanh(c+dx)}{d} - \frac{a(a+a\tanh(c+dx))^3}{3d} \\
&\quad - \frac{\left(a^2+a^2\tanh(c+dx)\right)^2}{d} + \left(8a^4\right)\int \tanh(c+dx)\,dx \\
&= 8a^4x + \frac{8a^4\log(\cosh(c+dx))}{d} - \frac{4a^4\tanh(c+dx)}{d} \\
&\quad - \frac{a(a+a\tanh(c+dx))^3}{3d} - \frac{\left(a^2+a^2\tanh(c+dx)\right)^2}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$
\int (a+a\tanh(c+dx))^4\,dx = -\frac{a^4\left(4+24\log(1-\tanh(c+dx))+21\tanh(c+dx)+6\tanh^2(c+dx)+\tanh^3(c+dx)\right)}{3d}
$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^4,x]
[Out] -1/3*(a^4*(4 + 24*Log[1 - Tanh[c + d*x]] + 21*Tanh[c + d*x] + 6*Tanh[c + d*
x]^2 + Tanh[c + d*x]^3))/d
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{a^{4}\left(-\frac{\tanh(dx+c)^{3}}{3}-2\tanh(dx+c)^{2}-7\tanh(dx+c)-8\ln(\tanh(dx+c)-1)\right)}{d}$
default	$\frac{a^{4}\left(-\frac{\tanh(dx+c)^{3}}{3}-2\tanh(dx+c)^{2}-7\tanh(dx+c)-8\ln(\tanh(dx+c)-1)\right)}{d}$
parallelrisch	$-\frac{\tanh(dx+c)^{3}a^{4}+6\tanh(dx+c)^{2}a^{4}+24\ln(1-\tanh(dx+c))a^{4}+21a^{4}\tanh(dx+c)}{3d}$
risch	$-\frac{16a^{4}c}{d}+\frac{4a^{4}\left(18\,\mathrm{e}^{4dx+4c}+27\,\mathrm{e}^{2dx+2c}+11\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^{3}}+\frac{8a^{4}\ln\left(\mathrm{e}^{2dx+2c}+1\right)}{d}$
parts	$x\,a^{4}+\frac{a^{4}\left(-\frac{\tanh(dx+c)^{3}}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{4a^{4}\ln(\cosh(dx+c))}{d}+\frac{6a^{4}\left(-\tanh\right.}{}$

```
[In] int((a+a*tanh(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d*a^4*(-1/3*tanh(d*x+c)^3-2*tanh(d*x+c)^2-7*tanh(d*x+c)-8*ln(tanh(d*x+c)-
1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(75) = 150.

Time = 0.25 (sec) , antiderivative size = 562, normalized size of antiderivative = 7.30

$$\int (a+a\tanh(c+dx))^4\,dx$$
$$=\frac{4\left(18\,a^{4}\cosh\left(dx+c\right)^{4}+72\,a^{4}\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+18\,a^{4}\sinh\left(dx+c\right)^{4}+27\,a^{4}\cosh\left(dx+c\right)^{2}+1\right.}{}$$

```
[In] integrate((a+a*tanh(d*x+c))^4,x, algorithm="fricas")
[Out] 4/3*(18*a^4*cosh(d*x + c)^4 + 72*a^4*cosh(d*x + c)*sinh(d*x + c)^3 + 18*a^4
*sinh(d*x + c)^4 + 27*a^4*cosh(d*x + c)^2 + 11*a^4 + 27*(4*a^4*cosh(d*x + c
)^2 + a^4)*sinh(d*x + c)^2 + 6*(a^4*cosh(d*x + c)^6 + 6*a^4*cosh(d*x + c)*s
inh(d*x + c)^5 + a^4*sinh(d*x + c)^6 + 3*a^4*cosh(d*x + c)^4 + 3*a^4*cosh(d
*x + c)^2 + 3*(5*a^4*cosh(d*x + c)^2 + a^4)*sinh(d*x + c)^4 + a^4 + 4*(5*a^
4*cosh(d*x + c)^3 + 3*a^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^4*cosh(d*
x + c)^4 + 6*a^4*cosh(d*x + c)^2 + a^4)*sinh(d*x + c)^2 + 6*(a^4*cosh(d*x +
 c)^5 + 2*a^4*cosh(d*x + c)^3 + a^4*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 18*(4*a^4*cosh(d*x + c)^3 + 3
*a^4*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*s
inh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x
```

```
+ c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int (a+a\tanh(c+dx))^4\,dx = \begin{cases} 16a^4x - \frac{8a^4\log(\tanh(c+dx)+1)}{d} - \frac{a^4\tanh^3(c+dx)}{3d} - \frac{2a^4\tanh^2(c+dx)}{d} - \frac{7a^4\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\tanh(c)+a)^4 & \text{otherwise} \end{cases}$$

```
[In] integrate((a+a*tanh(d*x+c))**4,x)
```

```
[Out] Piecewise((16*a**4*x - 8*a**4*log(tanh(c + d*x) + 1)/d - a**4*tanh(c + d*x)
**3/(3*d) - 2*a**4*tanh(c + d*x)**2/d - 7*a**4*tanh(c + d*x)/d, Ne(d, 0)),
(x*(a*tanh(c) + a)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(75) = 150.

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.55

$$\begin{aligned}\int (a+a\tanh(c+dx))^4\,dx &= \frac{1}{3}a^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) \\ &\quad + 4a^4\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right) \\ &\quad + 6a^4\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^4x + \frac{4a^4\log(\cosh(dx+c))}{d}\end{aligned}$$

```
[In] integrate((a+a*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^4*(
x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x
- 2*c) + e^(-4*d*x - 4*c) + 1))) + 6*a^4*(x + c/d - 2/(d*(e^(-2*d*x - 2*c)
+ 1))) + a^4*x + 4*a^4*log(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int (a+a\tanh(c+dx))^4\,dx = \frac{4\left(6\,a^4\log\left(e^{(2\,dx+2\,c)}+1\right)+\frac{18\,a^4e^{(4\,dx+4\,c)}+27\,a^4e^{(2\,dx+2\,c)}+11\,a^4}{\left(e^{(2\,dx+2\,c)}+1\right)^3}\right)}{3\,d}$$

```
[In] integrate((a+a*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 4/3*(6*a^4*log(e^(2*d*x + 2*c) + 1) + (18*a^4*e^(4*d*x + 4*c) + 27*a^4*e^(2
*d*x + 2*c) + 11*a^4)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.69

$$\int (a+a\tanh(c+dx))^4\,dx = 16\,a^4\,x - \frac{a^4\left(24\,\ln\left(\tanh(c+d\,x)+1\right)+21\tanh(c+d\,x)+6\tanh(c+d\,x)^2+\tanh(c+d\,x)^3\right)}{3\,d}$$

```
[In] int((a + a*tanh(c + d*x))^4,x)
```

```
[Out] 16*a^4*x - (a^4*(24*log(tanh(c + d*x) + 1) + 21*tanh(c + d*x) + 6*tanh(c +
d*x)^2 + tanh(c + d*x)^3))/(3*d)
```

3.43 $\int (a+a\tanh(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int (a+a\tanh(c+dx))^3\,dx = 4a^3x+\frac{4a^3\log(\cosh(c+dx))}{d} - \frac{2a^3\tanh(c+dx)}{d} - \frac{a(a+a\tanh(c+dx))^2}{2d}$$

```
[Out] 4*a^3*x+4*a^3*ln(cosh(d*x+c))/d-2*a^3*tanh(d*x+c)/d-1/2*a*(a+a*tanh(d*x+c))
^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3559, 3558, 3556}

$$\int (a+a\tanh(c+dx))^3\,dx = -\frac{2a^3\tanh(c+dx)}{d}+\frac{4a^3\log(\cosh(c+dx))}{d} + 4a^3x-\frac{a(a\tanh(c+dx)+a)^2}{2d}$$

```
[In] Int[(a + a*Tanh[c + d*x])^3,x]
```

```
[Out] 4*a^3*x + (4*a^3*Log[Cosh[c + d*x]])/d - (2*a^3*Tanh[c + d*x])/d - (a*(a +
a*Tanh[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a(a+a\tanh(c+dx))^2}{2d}+(2a)\int(a+a\tanh(c+dx))^2\,dx\\ &= 4a^3x-\frac{2a^3\tanh(c+dx)}{d}-\frac{a(a+a\tanh(c+dx))^2}{2d}+\left(4a^3\right)\int\tanh(c+dx)\,dx\\ &= 4a^3x+\frac{4a^3\log(\cosh(c+dx))}{d}-\frac{2a^3\tanh(c+dx)}{d}-\frac{a(a+a\tanh(c+dx))^2}{2d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int(a+a\tanh(c+dx))^3\,dx=-\frac{a^3\left(8\log(1-\tanh(c+dx))+6\tanh(c+dx)+\tanh^2(c+dx)\right)}{2d}$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^3,x]
[Out] -1/2*(a^3*(8*Log[1 - Tanh[c + d*x]] + 6*Tanh[c + d*x] + Tanh[c + d*x]^2))/d
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a^3\left(-\frac{\tanh(dx+c)^2}{2}-3\tanh(dx+c)-4\ln(\tanh(dx+c)-1)\right)}{d}$
default	$\frac{a^3\left(-\frac{\tanh(dx+c)^2}{2}-3\tanh(dx+c)-4\ln(\tanh(dx+c)-1)\right)}{d}$
parallelrisch	$-\frac{\tanh(dx+c)^2a^3+8\ln(1-\tanh(dx+c))a^3+6a^3\tanh(dx+c)}{2d}$
risch	$-\frac{8a^3c}{d}+\frac{2a^3\left(4\,\mathrm{e}^{2dx+2c}+3\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}+\frac{4a^3\ln\left(\mathrm{e}^{2dx+2c}+1\right)}{d}$
parts	$a^3x+\frac{a^3\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{3a^3\ln(\cosh(dx+c))}{d}+\frac{3a^3\left(-\tanh(dx+c)-\frac{\ln(\right.}{}}{}$

[In] int((a+a*tanh(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*a^3*(-1/2*tanh(d*x+c)^2-3*tanh(d*x+c)-4*ln(tanh(d*x+c)-1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 299, normalized size of antiderivative = 5.34

$$\int (a+a\tanh(c+dx))^3\,dx$$
$$=\frac{2\left(4\,a^3\cosh\left(dx+c\right)^2+8\,a^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)+4\,a^3\sinh\left(dx+c\right)^2+3\,a^3+2\,\left(a^3\cosh\left(dx+c\right)^4\right.\right.}{d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(d\right.}$$

```
[In] integrate((a+a*tanh(d*x+c))^3,x, algorithm="fricas")

[Out] 2*(4*a^3*cosh(d*x + c)^2 + 8*a^3*cosh(d*x + c)*sinh(d*x + c) + 4*a^3*sinh(d
*x + c)^2 + 3*a^3 + 2*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)*sinh(d*x +
 c)^3 + a^3*sinh(d*x + c)^4 + 2*a^3*cosh(d*x + c)^2 + a^3 + 2*(3*a^3*cosh(d
*x + c)^2 + a^3)*sinh(d*x + c)^2 + 4*(a^3*cosh(d*x + c)^3 + a^3*cosh(d*x +
c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(d
*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 +
2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*co
sh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int (a+a\tanh(c+dx))^3\,dx = \begin{cases} 8a^3x - \frac{4a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} - \frac{3a^3\tanh(c+dx)}{d} & \text{for } d\neq 0 \\ x(a\tanh(c)+a)^3 & \text{otherwise} \end{cases}$$

```
[In] integrate((a+a*tanh(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*x - 4*a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)*
*2/(2*d) - 3*a**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a*tanh(c) + a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(54) = 108.

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.07

$$\int (a+a\tanh(c+dx))^3\,dx = a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) + 3\,a^3\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^3x+\frac{3\,a^3\log\left(\cosh\left(dx+c\right)\right)}{d}$$

```
[In] integrate((a+a*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^3*(x + c/d - 2/(d*(e^(-2*d*x -
2*c) + 1))) + a^3*x + 3*a^3*log(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int (a+a\tanh(c+dx))^3\,dx = \frac{2\left(2\,a^3\log\left(e^{(2\,dx+2\,c)}+1\right)+\frac{4\,a^3e^{(2\,dx+2\,c)}+3\,a^3}{\left(e^{(2\,dx+2\,c)}+1\right)^2}\right)}{d}$$

```
[In] integrate((a+a*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(e^(2*d*x + 2*c) + 1) + (4*a^3*e^(2*d*x + 2*c) + 3*a^3)/(e^(2*d
*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int (a+a\tanh(c+dx))^3\,dx$$
$$=8\,a^3\,x-\frac{a^3\,\left(8\,\ln\left(\tanh(c+d\,x)+1\right)+6\tanh(c+d\,x)+\tanh(c+d\,x)^2\right)}{2\,d}$$

```
[In] int((a + a*tanh(c + d*x))^3,x)
[Out] 8*a^3*x - (a^3*(8*log(tanh(c + d*x) + 1) + 6*tanh(c + d*x) + tanh(c + d*x)^
2))/(2*d)
```

3.44 $\int (a+a\tanh(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int (a+a\tanh(c+dx))^2\,dx = 2a^2x+\frac{2a^2\log(\cosh(c+dx))}{d}-\frac{a^2\tanh(c+dx)}{d}$$

```
[Out] 2*a^2*x+2*a^2*ln(cosh(d*x+c))/d-a^2*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3558, 3556}

$$\int (a+a\tanh(c+dx))^2\,dx = -\frac{a^2\tanh(c+dx)}{d}+\frac{2a^2\log(\cosh(c+dx))}{d}+2a^2x$$

```
[In] Int[(a + a*Tanh[c + d*x])^2,x]
```

```
[Out] 2*a^2*x + (2*a^2*Log[Cosh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2a^2x - \frac{a^2\tanh(c+dx)}{d} + (2a^2)\int \tanh(c+dx)\,dx\\ &= 2a^2x + \frac{2a^2\log(\cosh(c+dx))}{d} - \frac{a^2\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int (a+a\tanh(c+dx))^2\,dx = \frac{a(-2a\log(1-\tanh(c+dx))-a\tanh(c+dx))}{d}$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^2,x]
[Out] (a*(-2*a*Log[1 - Tanh[c + d*x]] - a*Tanh[c + d*x]))/d
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{a^2(-\tanh(dx+c)-2\ln(\tanh(dx+c)-1))}{d}$	28
default	$\frac{a^2(-\tanh(dx+c)-2\ln(\tanh(dx+c)-1))}{d}$	28
parallelrisch	$-\frac{2\ln(1-\tanh(dx+c))a^2+a^2\tanh(dx+c)}{d}$	33
risch	$-\frac{4a^2c}{d}+\frac{2a^2}{d(\mathrm{e}^{2dx+2c}+1)}+\frac{2a^2\ln(\mathrm{e}^{2dx+2c}+1)}{d}$	52
parts	$a^2x+\frac{a^2\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{2a^2\ln(\cosh(dx+c))}{d}$	60

```
[In] int((a+a*tanh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d*a^2*(-tanh(d*x+c)-2*ln(tanh(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.25

$$\int (a + a\tanh(c + dx))^2\,dx$$
$$= \frac{2\left(a^2 + \left(a^2\cosh(dx+c)^2 + 2a^2\cosh(dx+c)\sinh(dx+c) + a^2\sinh(dx+c)^2 + a^2\right)\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right.}{d\cosh(dx+c)^2 + 2d\cosh(dx+c)\sinh(dx+c) + d\sinh(dx+c)^2 + d}$$

```
[In] integrate((a+a*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 2*(a^2 + (a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sin
h(d*x + c)^2 + a^2)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(
d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d
)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int (a + a\tanh(c + dx))^2\,dx = \begin{cases} 4a^2x - \frac{2a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a\tanh(c) + a)^2 & \text{otherwise} \end{cases}$$

```
[In] integrate((a+a*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise((4*a**2*x - 2*a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)/
d, Ne(d, 0)), (x*(a*tanh(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int (a+a\tanh(c+dx))^2\,dx = a^2\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right) + a^2x + \frac{2\,a^2\log(\cosh(dx+c))}{d}$$

```
[In] integrate((a+a*tanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x + 2*a^2*log(cosh(d*x +
 c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int (a + a\tanh(c + dx))^2\, dx = \frac{2\left(a^2 \log\left(e^{(2\,dx+2\,c)} + 1\right) + \frac{a^2}{e^{(2\,dx+2\,c)}+1}\right)}{d}$$

```
[In] integrate((a+a*tanh(d*x+c))^2,x, algorithm="giac")
[Out] 2*(a^2*log(e^(2*d*x + 2*c) + 1) + a^2/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int (a + a\tanh(c + dx))^2\, dx = 4\,a^2\,x - \frac{a^2\,(2\,\ln(\tanh(c + d\,x) + 1) + \tanh(c + d\,x))}{d}$$

```
[In] int((a + a*tanh(c + d*x))^2,x)
[Out] 4*a^2*x - (a^2*(2*log(tanh(c + d*x) + 1) + tanh(c + d*x)))/d
```

3.45 $\int \frac{1}{a+a\tanh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx=\frac{x}{2a}-\frac{1}{2d(a+a\tanh(c+dx))}$$

```
[Out] 1/2*x/a-1/2/d/(a+a*tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3560, 8}

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx=\frac{x}{2a}-\frac{1}{2d(a\tanh(c+dx)+a)}$$

```
[In] Int[(a + a*Tanh[c + d*x])^(-1),x]
```

```
[Out] x/(2*a) - 1/(2*d*(a + a*Tanh[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a + 
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{1}{2d(a+a\tanh(c+dx))}+\frac{\int 1\,dx}{2a} \\ &= \frac{x}{2a}-\frac{1}{2d(a+a\tanh(c+dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx=\frac{\frac{\operatorname{arctanh}(\tanh(c+dx))}{a}-\frac{1}{a+a\tanh(c+dx)}}{2d}$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-1),x]
[Out] (ArcTanh[Tanh[c + d*x]]/a - (a + a*Tanh[c + d*x])^(-1))/(2*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x}{2a}-\frac{e^{-2dx-2c}}{4da}$	25
parallelrisch	$\frac{-1+\tanh(dx+c)xd+dx}{2da(\tanh(dx+c)+1)}$	33
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{4}-\frac{1}{2(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{4}}{da}$	43
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{4}-\frac{1}{2(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{4}}{da}$	43

```
[In] int(1/(a+a*tanh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*x/a-1/4/d/a*exp(-2*d*x-2*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx = \frac{(2\,dx-1)\cosh(dx+c)+(2\,dx+1)\sinh(dx+c)}{4\,(ad\cosh(dx+c)+ad\sinh(dx+c))}$$

```
[In] integrate(1/(a+a*tanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*((2*d*x - 1)*cosh(d*x + c) + (2*d*x + 1)*sinh(d*x + c))/(a*d*cosh(d*x +
 c) + a*d*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(19) = 38$.

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx = \begin{cases} \frac{dx\tanh(c+dx)}{2ad\tanh(c+dx)+2ad} + \frac{dx}{2ad\tanh(c+dx)+2ad} - \frac{1}{2ad\tanh(c+dx)+2ad} & \text{for } d \neq 0 \\ \frac{x}{a\tanh(c)+a} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+a*tanh(d*x+c)),x)
```

```
[Out] Piecewise((d*x*tanh(c + d*x)/(2*a*d*tanh(c + d*x) + 2*a*d) + d*x/(2*a*d*tan
h(c + d*x) + 2*a*d) - 1/(2*a*d*tanh(c + d*x) + 2*a*d), Ne(d, 0)), (x/(a*tan
h(c) + a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx = \frac{dx+c}{2\,ad} - \frac{e^{(-2\,dx-2\,c)}}{4\,ad}$$

```
[In] integrate(1/(a+a*tanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(d*x + c)/(a*d) - 1/4*e^(-2*d*x - 2*c)/(a*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx = \frac{\frac{2\,(dx+c)}{a} - \frac{e^{(-2\,dx-2\,c)}}{a}}{4\,d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c)),x, algorithm="giac")
[Out] 1/4*(2*(d*x + c)/a - e^(-2*d*x - 2*c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+a\tanh(c+dx)}\,dx = \frac{x}{2\,a} - \frac{1}{2\,a\,d\,\left(\tanh\left(c+d\,x\right)+1\right)}$$

```
[In] int(1/(a + a*tanh(c + d*x)),x)
[Out] x/(2*a) - 1/(2*a*d*(tanh(c + d*x) + 1))
```

3.46 $\int \frac{1}{(a+a\tanh(c+dx))^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{(a+a\tanh(c+dx))^2} dx = \frac{x}{4a^2} - \frac{1}{4d(a+a\tanh(c+dx))^2} - \frac{1}{4d(a^2+a^2\tanh(c+dx))}$$

```
[Out] 1/4*x/a^2-1/4/d/(a+a*tanh(d*x+c))^2-1/4/d/(a^2+a^2*tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3560, 8}

$$\int \frac{1}{(a+a\tanh(c+dx))^2} dx = -\frac{1}{4d(a^2\tanh(c+dx)+a^2)} + \frac{x}{4a^2} - \frac{1}{4d(a\tanh(c+dx)+a)^2}$$

```
[In] Int[(a + a*Tanh[c + d*x])^(-2),x]
```

```
[Out] x/(4*a^2) - 1/(4*d*(a + a*Tanh[c + d*x])^2) - 1/(4*d*(a^2 + a^2*Tanh[c + d*
x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4d(a+a\tanh(c+dx))^2}+\frac{\int \frac{1}{a+a\tanh(c+dx)}\,dx}{2a}\\
&= -\frac{1}{4d(a+a\tanh(c+dx))^2}-\frac{1}{4d\,(a^2+a^2\tanh(c+dx))}+\frac{\int 1\,dx}{4a^2}\\
&= \frac{x}{4a^2}-\frac{1}{4d(a+a\tanh(c+dx))^2}-\frac{1}{4d\,(a^2+a^2\tanh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx = -\frac{2+\tanh(c+dx)-\operatorname{arctanh}(\tanh(c+dx))(1+\tanh(c+dx))^2}{4a^2d(1+\tanh(c+dx))^2}$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-2),x]
[Out] -1/4*(2 + Tanh[c + d*x] - ArcTanh[Tanh[c + d*x]]*(1 + Tanh[c + d*x])^2)/(a^
2*d*(1 + Tanh[c + d*x])^2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{x}{4a^2}-\frac{e^{-2dx-2c}}{4a^2d}-\frac{e^{-4dx-4c}}{16a^2d}$	42
parallelrisch	$\frac{-2+\tanh(dx+c)^2xd+2\tanh(dx+c)xd+dx-\tanh(dx+c)}{4d\,a^2(\tanh(dx+c)+1)^2}$	53
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{8}-\frac{1}{4(\tanh(dx+c)+1)^2}-\frac{1}{4(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{8}}{d\,a^2}$	55
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{8}-\frac{1}{4(\tanh(dx+c)+1)^2}-\frac{1}{4(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{8}}{d\,a^2}$	55

```
[In] int(1/(a+a*tanh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x/a^2-1/4/a^2/d*exp(-2*d*x-2*c)-1/16/a^2/d*exp(-4*d*x-4*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(45) = 90$.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.98

$$\int \frac{1}{(a + a\tanh(c + dx))^2}\,dx = \frac{(4\,dx - 1)\cosh(dx + c)^2 + 2\,(4\,dx + 1)\cosh(dx + c)\sinh(dx + c) + (4\,dx - 1)\sinh(dx + c)^2 - 4}{16\,\left(a^2d\cosh(dx + c)^2 + 2\,a^2d\cosh(dx + c)\sinh(dx + c) + a^2d\sinh(dx + c)^2\right)}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*((4*d*x - 1)*cosh(d*x + c)^2 + 2*(4*d*x + 1)*cosh(d*x + c)*sinh(d*x +
c) + (4*d*x - 1)*sinh(d*x + c)^2 - 4)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh
(d*x + c)*sinh(d*x + c) + a^2*d*sinh(d*x + c)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(41) = 82$.

Time = 0.57 (sec) , antiderivative size = 223, normalized size of antiderivative = 4.37

$$\int \frac{1}{(a + a\tanh(c + dx))^2}\,dx = \begin{cases} \frac{dx\tanh^2(c+dx)}{4a^2d\tanh^2(c+dx)+8a^2d\tanh(c+dx)+4a^2d} + \frac{2dx\tanh(c+dx)}{4a^2d\tanh^2(c+dx)+8a^2d\tanh(c+dx)+4a^2d} + \frac{dx}{4a^2d\tanh^2(c+dx)+8a^2d\tanh(c+dx)+4a^2d} \\ \frac{x}{(a\tanh(c)+a)^2} \end{cases}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise(((d*x*tanh(c + d*x)**2/(4*a**2*d*tanh(c + d*x)**2 + 8*a**2*d*tanh(
c + d*x) + 4*a**2*d) + 2*d*x*tanh(c + d*x)/(4*a**2*d*tanh(c + d*x)**2 + 8*a
**2*d*tanh(c + d*x) + 4*a**2*d) + d*x/(4*a**2*d*tanh(c + d*x)**2 + 8*a**2*d
*tanh(c + d*x) + 4*a**2*d) - tanh(c + d*x)/(4*a**2*d*tanh(c + d*x)**2 + 8*a
**2*d*tanh(c + d*x) + 4*a**2*d) - 2/(4*a**2*d*tanh(c + d*x)**2 + 8*a**2*d*t
anh(c + d*x) + 4*a**2*d), Ne(d, 0)), (x/(a*tanh(c) + a)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx = \frac{dx+c}{4\,a^2d} - \frac{4\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)}}{16\,a^2d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^2,x, algorithm="maxima")
[Out] 1/4*(d*x + c)/(a^2*d) - 1/16*(4*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c))/(a^2*d
)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx = -\frac{\frac{(4\,e^{(2\,dx+2\,c)}+1)e^{(-4\,dx-4\,c)}}{a^2} - \frac{4\,(dx+c)}{a^2}}{16\,d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^2,x, algorithm="giac")
[Out] -1/16*((4*e^(2*d*x + 2*c) + 1)*e^(-4*d*x - 4*c)/a^2 - 4*(d*x + c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx = \frac{x}{4\,a^2} - \frac{\mathrm{e}^{-2\,c-2\,d\,x}}{4\,a^2\,d} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}}{16\,a^2\,d}$$

```
[In] int(1/(a + a*tanh(c + d*x))^2,x)
[Out] x/(4*a^2) - exp(- 2*c - 2*d*x)/(4*a^2*d) - exp(- 4*c - 4*d*x)/(16*a^2*d)
```

3.47 $\int \frac{1}{(a+a\tanh(c+dx))^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{(a+a\tanh(c+dx))^3} dx = \frac{x}{8a^3} - \frac{1}{6d(a+a\tanh(c+dx))^3} - \frac{1}{8ad(a+a\tanh(c+dx))^2} - \frac{1}{8d(a^3+a^3\tanh(c+dx))}$$

```
[Out] 1/8*x/a^3-1/6/d/(a+a*tanh(d*x+c))^3-1/8/a/d/(a+a*tanh(d*x+c))^2-1/8/d/(a^3+
a^3*tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3560, 8}

$$\int \frac{1}{(a+a\tanh(c+dx))^3} dx = -\frac{1}{8d(a^3\tanh(c+dx)+a^3)} + \frac{x}{8a^3} - \frac{1}{8ad(a\tanh(c+dx)+a)^2} - \frac{1}{6d(a\tanh(c+dx)+a)^3}$$

```
[In] Int[(a + a*Tanh[c + d*x])^(-3),x]
```

```
[Out] x/(8*a^3) - 1/(6*d*(a + a*Tanh[c + d*x])^3) - 1/(8*a*d*(a + a*Tanh[c + d*x]
)^2) - 1/(8*d*(a^3 + a^3*Tanh[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{6d(a+a\tanh(c+dx))^3}+\frac{\int\frac{1}{(a+a\tanh(c+dx))^2}\,dx}{2a}\\
&= -\frac{1}{6d(a+a\tanh(c+dx))^3}-\frac{1}{8ad(a+a\tanh(c+dx))^2}+\frac{\int\frac{1}{a+a\tanh(c+dx)}\,dx}{4a^2}\\
&= -\frac{1}{6d(a+a\tanh(c+dx))^3}-\frac{1}{8ad(a+a\tanh(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\tanh(c+dx)\right)}\\
&\quad+\frac{\int 1\,dx}{8a^3}\\
&= \frac{x}{8a^3}-\frac{1}{6d(a+a\tanh(c+dx))^3}-\frac{1}{8ad(a+a\tanh(c+dx))^2}-\frac{1}{8d\left(a^3+a^3\tanh(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int\frac{1}{(a+a\tanh(c+dx))^3}\,dx = -\frac{10+9\tanh(c+dx)+3\tanh^2(c+dx)-3\operatorname{arctanh}(\tanh(c+dx))(1+\tanh(c+dx))^3}{24a^3d(1+\tanh(c+dx))^3}$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-3),x]
[Out] -1/24*(10 + 9*Tanh[c + d*x] + 3*Tanh[c + d*x]^2 - 3*ArcTanh[Tanh[c + d*x]]*
(1 + Tanh[c + d*x])^3)/(a^3*d*(1 + Tanh[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{x}{8a^3}-\frac{3\,\mathrm{e}^{-2dx-2c}}{16a^3d}-\frac{3\,\mathrm{e}^{-4dx-4c}}{32a^3d}-\frac{\mathrm{e}^{-6dx-6c}}{48a^3d}$	59
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{16}-\frac{1}{6(\tanh(dx+c)+1)^3}-\frac{1}{8(\tanh(dx+c)+1)^2}-\frac{1}{8(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{16}}{d\,a^3}$	67
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{16}-\frac{1}{6(\tanh(dx+c)+1)^3}-\frac{1}{8(\tanh(dx+c)+1)^2}-\frac{1}{8(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{16}}{d\,a^3}$	67
parallelrisch	$\frac{-10-9\tanh(dx+c)+3\tanh(dx+c)^3xd+3dx+9\tanh(dx+c)^2xd+9\tanh(dx+c)xd-3\tanh(dx+c)^2}{24d\,a^3(\tanh(dx+c)+1)^3}$	77

```
[In] int(1/(a+a*tanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x/a^3-3/16/a^3/d*exp(-2*d*x-2*c)-3/32/a^3/d*exp(-4*d*x-4*c)-1/48/a^3/d*
exp(-6*d*x-6*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(65) = 130$.

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.19

$$\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx$$
$$=\frac{2\,(6\,dx-1)\cosh\left(dx+c\right)^3+6\,(6\,dx-1)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\,(6\,dx+1)\sinh\left(dx+c\right)^3+3\,(2\,(6}{96\,\left(a^3d\cosh\left(dx+c\right)^3+3\,a^3d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)+3\,a^3d\cosh\left(dx+c\right)\mathrm{s}\right.}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(2*(6*d*x - 1)*cosh(d*x + c)^3 + 6*(6*d*x - 1)*cosh(d*x + c)*sinh(d*x
+ c)^2 + 2*(6*d*x + 1)*sinh(d*x + c)^3 + 3*(2*(6*d*x + 1)*cosh(d*x + c)^2 -
 3)*sinh(d*x + c) - 27*cosh(d*x + c))/(a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*d*s
inh(d*x + c)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(60) = 120$.

Time = 0.75 (sec) , antiderivative size = 430, normalized size of antiderivative = 5.89

$$\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx$$
$$=\begin{cases}\frac{3dx\tanh^3(c+dx)}{24a^3d\tanh^3(c+dx)+72a^3d\tanh^2(c+dx)+72a^3d\tanh(c+dx)+24a^3d}+\frac{9dx\tanh^2(c+dx)}{24a^3d\tanh^3(c+dx)+72a^3d\tanh^2(c+dx)+72a^3d\tanh(c+dx)+24a^3d} \\ \frac{x}{(a\tanh(c)+a)^3}\end{cases}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))**3,x)
```

```
[Out] Piecewise((3*d*x*tanh(c + d*x)**3/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*t
anh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d) + 9*d*x*tanh(c + d*x
)**2/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*t
anh(c + d*x) + 24*a**3*d) + 9*d*x*tanh(c + d*x)/(24*a**3*d*tanh(c + d*x)**3
 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d) + 3*d*
x/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*tanh
(c + d*x) + 24*a**3*d) - 3*tanh(c + d*x)**2/(24*a**3*d*tanh(c + d*x)**3 + 7
2*a**3*d*tanh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d) - 9*tanh(c
 + d*x)/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*tanh(c + d*x)**2 + 72*a**3*
d*tanh(c + d*x) + 24*a**3*d) - 10/(24*a**3*d*tanh(c + d*x)**3 + 72*a**3*d*t
anh(c + d*x)**2 + 72*a**3*d*tanh(c + d*x) + 24*a**3*d), Ne(d, 0)), (x/(a*ta
nh(c) + a)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx = \frac{dx+c}{8\,a^3d} - \frac{18\,e^{(-2\,dx-2\,c)}+9\,e^{(-4\,dx-4\,c)}+2\,e^{(-6\,dx-6\,c)}}{96\,a^3d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)/(a^3*d) - 1/96*(18*e^(-2*d*x - 2*c) + 9*e^(-4*d*x - 4*c) + 2*
e^(-6*d*x - 6*c))/(a^3*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx = -\frac{\frac{\left(18\,e^{(4\,dx+4\,c)}+9\,e^{(2\,dx+2\,c)}+2\right)e^{(-6\,dx-6\,c)}}{a^3} - \frac{12\,(dx+c)}{a^3}}{96\,d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*((18*e^(4*d*x + 4*c) + 9*e^(2*d*x + 2*c) + 2)*e^(-6*d*x - 6*c)/a^3 -
12*(d*x + c)/a^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx=\frac{x}{8\,a^3}-\frac{3\,\mathrm{e}^{-2\,c-2\,d\,x}}{16\,a^3\,d}-\frac{3\,\mathrm{e}^{-4\,c-4\,d\,x}}{32\,a^3\,d}-\frac{\mathrm{e}^{-6\,c-6\,d\,x}}{48\,a^3\,d}$$

```
[In] int(1/(a + a*tanh(c + d*x))^3,x)
```

```
[Out] x/(8*a^3) - (3*exp(- 2*c - 2*d*x))/(16*a^3*d) - (3*exp(- 4*c - 4*d*x))/(32*
a^3*d) - exp(- 6*c - 6*d*x)/(48*a^3*d)
```

3.48 $\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx = \frac{x}{16a^4} - \frac{1}{8d(a+a\tanh(c+dx))^4} - \frac{1}{12ad(a+a\tanh(c+dx))^3} - \frac{1}{16d\left(a^2+a^2\tanh(c+dx)\right)^2} - \frac{1}{16d\left(a^4+a^4\tanh(c+dx)\right)}$$

```
[Out] 1/16*x/a^4-1/8/d/(a+a*tanh(d*x+c))^4-1/12/a/d/(a+a*tanh(d*x+c))^3-1/16/d/(a
^2+a^2*tanh(d*x+c))^2-1/16/d/(a^4+a^4*tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3560, 8}

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx = -\frac{1}{16d\left(a^4\tanh(c+dx)+a^4\right)} + \frac{x}{16a^4} - \frac{1}{16d\left(a^2\tanh(c+dx)+a^2\right)^2} - \frac{1}{12ad(a\tanh(c+dx)+a)^3} - \frac{1}{8d(a\tanh(c+dx)+a)^4}$$

```
[In] Int[(a + a*Tanh[c + d*x])^(-4),x]
```

```
[Out] x/(16*a^4) - 1/(8*d*(a + a*Tanh[c + d*x])^4) - 1/(12*a*d*(a + a*Tanh[c + d*
x])^3) - 1/(16*d*(a^2 + a^2*Tanh[c + d*x])^2) - 1/(16*d*(a^4 + a^4*Tanh[c +
 d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{8d(a+a\tanh(c+dx))^4}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx}{2a}\\
&= -\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx}{4a^2}\\
&= -\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}\\
&\quad -\frac{1}{16d\left(a^2+a^2\tanh(c+dx)\right)^2}+\frac{\int \frac{1}{a+a\tanh(c+dx)}\,dx}{8a^3}\\
&= -\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}\\
&\quad -\frac{1}{16d\left(a^2+a^2\tanh(c+dx)\right)^2}-\frac{1}{16d\left(a^4+a^4\tanh(c+dx)\right)}+\frac{\int 1\,dx}{16a^4}\\
&= \frac{x}{16a^4}-\frac{1}{8d(a+a\tanh(c+dx))^4}-\frac{1}{12ad(a+a\tanh(c+dx))^3}\\
&\quad -\frac{1}{16d\left(a^2+a^2\tanh(c+dx)\right)^2}-\frac{1}{16d\left(a^4+a^4\tanh(c+dx)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx = \frac{a\left(\frac{\operatorname{arctanh}(\tanh(c+dx))}{16a^5}-\frac{1}{8a(a+a\tanh(c+dx))^4}-\frac{1}{12a^2(a+a\tanh(c+dx))^3}-\frac{1}{16a^3(a+a\tanh(c+dx))^2}-\frac{1}{16a^4(a+a\tanh(c+dx))}\right)}{d}$$

```
[In] Integrate[(a + a*Tanh[c + d*x])^(-4),x]
```

```
[Out] (a*(ArcTanh[Tanh[c + d*x]]/(16*a^5) - 1/(8*a*(a + a*Tanh[c + d*x])^4) - 1/(
12*a^2*(a + a*Tanh[c + d*x])^3) - 1/(16*a^3*(a + a*Tanh[c + d*x])^2) - 1/(1
6*a^4*(a + a*Tanh[c + d*x]))))/d
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.79

method	result
risch	$\frac{x}{16a^4}-\frac{\mathrm{e}^{-2dx-2c}}{8a^4d}-\frac{3\,\mathrm{e}^{-4dx-4c}}{32a^4d}-\frac{\mathrm{e}^{-6dx-6c}}{24a^4d}-\frac{\mathrm{e}^{-8dx-8c}}{128a^4d}$
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{32}-\frac{1}{8(\tanh(dx+c)+1)^4}-\frac{1}{12(\tanh(dx+c)+1)^3}-\frac{1}{16(\tanh(dx+c)+1)^2}-\frac{1}{16(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{32}}{d\,a^4}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{32}-\frac{1}{8(\tanh(dx+c)+1)^4}-\frac{1}{12(\tanh(dx+c)+1)^3}-\frac{1}{16(\tanh(dx+c)+1)^2}-\frac{1}{16(\tanh(dx+c)+1)}+\frac{\ln(\tanh(dx+c)+1)}{32}}{d\,a^4}$
parallelrisch	$\frac{-16-19\tanh(dx+c)+3\tanh(dx+c)^4xd+12\tanh(dx+c)^3xd+3dx+18\tanh(dx+c)^2xd+12\tanh(dx+c)xd-3\tanh(dx+c}{48d\,a^4(\tanh(dx+c)+1)^4}$

```
[In] int(1/(a+a*tanh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x/a^4-1/8/a^4/d*exp(-2*d*x-2*c)-3/32/a^4/d*exp(-4*d*x-4*c)-1/24/a^4/d*
exp(-6*d*x-6*c)-1/128/a^4/d*exp(-8*d*x-8*c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(86) = 172.

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx$$
$$=\frac{3\,(8\,dx-1)\cosh\left(dx+c\right)^4+12\,(8\,dx+1)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+3\,(8\,dx-1)\sinh\left(dx+c\right)^4+2\,(9}{384\,\left(a^4d\cosh\left(dx+c\right)^4+4\,a^4d\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)+\right.}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(8*d*x - 1)*cosh(d*x + c)^4 + 12*(8*d*x + 1)*cosh(d*x + c)*sinh(d*
x + c)^3 + 3*(8*d*x - 1)*sinh(d*x + c)^4 + 2*(9*(8*d*x - 1)*cosh(d*x + c)^2
 - 32)*sinh(d*x + c)^2 - 64*cosh(d*x + c)^2 + 4*(3*(8*d*x + 1)*cosh(d*x + c
)^3 - 16*cosh(d*x + c))*sinh(d*x + c) - 36)/(a^4*d*cosh(d*x + c)^4 + 4*a^4*
d*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 +
 4*a^4*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^4*d*sinh(d*x + c)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(80) = 160$.

Time = 0.96 (sec) , antiderivative size = 694, normalized size of antiderivative = 7.23

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx = \begin{cases} \frac{3dx\tanh^4(c+dx)}{48a^4d\tanh^4(c+dx)+192a^4d\tanh^3(c+dx)+288a^4d\tanh^2(c+dx)+192a^4d\tanh(c+dx)+48a^4d} + \frac{12dx\,\mathrm{t}}{48a^4d\tanh^4(c+dx)+192a^4d\tanh^3(c+dx)+} \\ \frac{x}{(a\tanh(c)+a)^4} \end{cases}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))**4,x)
```

```
[Out] Piecewise((3*d*x*tanh(c + d*x)**4/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*
tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) +
 48*a**4*d) + 12*d*x*tanh(c + d*x)**3/(48*a**4*d*tanh(c + d*x)**4 + 192*a**
4*d*tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*
x) + 48*a**4*d) + 18*d*x*tanh(c + d*x)**2/(48*a**4*d*tanh(c + d*x)**4 + 192
*a**4*d*tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c
+ d*x) + 48*a**4*d) + 12*d*x*tanh(c + d*x)/(48*a**4*d*tanh(c + d*x)**4 + 19
2*a**4*d*tanh(c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c
 + d*x) + 48*a**4*d) + 3*d*x/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(
c + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a
**4*d) - 3*tanh(c + d*x)**3/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c
 + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a*
*4*d) - 12*tanh(c + d*x)**2/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c
 + d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a*
*4*d) - 19*tanh(c + d*x)/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c +
d*x)**3 + 288*a**4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a**4*
d) - 16/(48*a**4*d*tanh(c + d*x)**4 + 192*a**4*d*tanh(c + d*x)**3 + 288*a**
4*d*tanh(c + d*x)**2 + 192*a**4*d*tanh(c + d*x) + 48*a**4*d), Ne(d, 0)), (x
/(a*tanh(c) + a)**4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx = \frac{dx+c}{16\,a^4d} - \frac{48\,e^{(-2\,dx-2\,c)} + 36\,e^{(-4\,dx-4\,c)} + 16\,e^{(-6\,dx-6\,c)} + 3\,e^{(-8\,dx-8\,c)}}{384\,a^4d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/16*(d*x + c)/(a^4*d) - 1/384*(48*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 16*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c))/(a^4*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx=-\frac{\frac{\left(48\,e^{(6\,dx+6\,c)}+36\,e^{(4\,dx+4\,c)}+16\,e^{(2\,dx+2\,c)}+3\right)e^{(-8\,dx-8\,c)}}{a^4}-\frac{24\,(dx+c)}{a^4}}{384\,d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/384*((48*e^(6*d*x + 6*c) + 36*e^(4*d*x + 4*c) + 16*e^(2*d*x + 2*c) + 3)*
e^(-8*d*x - 8*c)/a^4 - 24*(d*x + c)/a^4)/d
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx=\frac{x}{16\,a^4}-\frac{e^{-2\,c-2\,d\,x}}{8\,a^4\,d}-\frac{3\,e^{-4\,c-4\,d\,x}}{32\,a^4\,d}-\frac{e^{-6\,c-6\,d\,x}}{24\,a^4\,d}-\frac{e^{-8\,c-8\,d\,x}}{128\,a^4\,d}$$

```
[In] int(1/(a + a*tanh(c + d*x))^4,x)
```

```
[Out] x/(16*a^4) - exp(- 2*c - 2*d*x)/(8*a^4*d) - (3*exp(- 4*c - 4*d*x))/(32*a^4*
d) - exp(- 6*c - 6*d*x)/(24*a^4*d) - exp(- 8*c - 8*d*x)/(128*a^4*d)
```

3.49 $\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx = \frac{x}{32a^5} - \frac{1}{10d(a+a\tanh(c+dx))^5} - \frac{1}{16ad(a+a\tanh(c+dx))^4} - \frac{1}{24a^2d(a+a\tanh(c+dx))^3} - \frac{1}{32ad\left(a^2+a^2\tanh(c+dx)\right)^2} - \frac{1}{32d\left(a^5+a^5\tanh(c+dx)\right)}$$

```
[Out] 1/32*x/a^5-1/10/d/(a+a*tanh(d*x+c))^5-1/16/a/d/(a+a*tanh(d*x+c))^4-1/24/a^2
/d/(a+a*tanh(d*x+c))^3-1/32/a/d/(a^2+a^2*tanh(d*x+c))^2-1/32/d/(a^5+a^5*tan
h(d*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3560, 8}

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx = -\frac{1}{32d\left(a^5\tanh(c+dx)+a^5\right)} + \frac{x}{32a^5} - \frac{1}{32ad\left(a^2\tanh(c+dx)+a^2\right)^2} - \frac{1}{24a^2d(a\tanh(c+dx)+a)^3} - \frac{1}{16ad(a\tanh(c+dx)+a)^4} - \frac{1}{10d(a\tanh(c+dx)+a)^5}$$

```
[In] Int[(a + a*Tanh[c + d*x])^(-5),x]
```

```
[Out] x/(32*a^5) - 1/(10*d*(a + a*Tanh[c + d*x])^5) - 1/(16*a*d*(a + a*Tanh[c + d
*x])^4) - 1/(24*a^2*d*(a + a*Tanh[c + d*x])^3) - 1/(32*a*d*(a^2 + a^2*Tanh[
c + d*x])^2) - 1/(32*d*(a^5 + a^5*Tanh[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{10d(a+a\tanh(c+dx))^5}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^4}\,dx}{2a}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^3}\,dx}{4a^2}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}\\
&\quad -\frac{1}{24a^2d(a+a\tanh(c+dx))^3}+\frac{\int \frac{1}{(a+a\tanh(c+dx))^2}\,dx}{8a^3}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}\\
&\quad -\frac{1}{24a^2d(a+a\tanh(c+dx))^3}-\frac{1}{32a^3d(a+a\tanh(c+dx))^2}+\frac{\int \frac{1}{a+a\tanh(c+dx)}\,dx}{16a^4}\\
&= -\frac{1}{10d(a+a\tanh(c+dx))^5}-\frac{1}{16ad(a+a\tanh(c+dx))^4}\\
&\quad -\frac{1}{24a^2d(a+a\tanh(c+dx))^3}-\frac{1}{32a^3d(a+a\tanh(c+dx))^2}\\
&\quad -\frac{1}{32d\left(a^5+a^5\tanh(c+dx)\right)}+\frac{\int 1\,dx}{32a^5}\\
&= \frac{x}{32a^5}-\frac{1}{10d(a+a\tanh(c+dx))^5}\\
&\quad -\frac{1}{16ad(a+a\tanh(c+dx))^4}-\frac{1}{24a^2d(a+a\tanh(c+dx))^3}\\
&\quad -\frac{1}{32a^3d(a+a\tanh(c+dx))^2}-\frac{1}{32d\left(a^5+a^5\tanh(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx$$
$$= \frac{\operatorname{sech}^5(c+dx)(-500\cosh(c+dx)-375\cosh(3(c+dx))-149\cosh(5(c+dx))-100\sinh(c+dx)-225\,\mathrm{si}}{3840a^5d(1+\tan}$$

[In] Integrate[(a + a*Tanh[c + d*x])^(-5),x]

[Out] (Sech[c + d*x]^5*(-500*Cosh[c + d*x] - 375*Cosh[3*(c + d*x)] - 149*Cosh[5*(c + d*x)] - 100*Sinh[c + d*x] - 225*Sinh[3*(c + d*x)] - 125*Sinh[5*(c + d*x)] + 120*ArcTanh[Tanh[c + d*x]]*(Cosh[5*(c + d*x)] + Sinh[5*(c + d*x)])))/(3840*a^5*d*(1 + Tanh[c + d*x])^5)

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{64}-\frac{1}{10(\tanh(dx+c)+1)^5}-\frac{1}{16(\tanh(dx+c)+1)^4}-\frac{1}{24(\tanh(dx+c)+1)^3}-\frac{1}{32(\tanh(dx+c)+1)^2}-\frac{1}{32(\tanh(dx+c)+1)}+}{d\,a^5}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{64}-\frac{1}{10(\tanh(dx+c)+1)^5}-\frac{1}{16(\tanh(dx+c)+1)^4}-\frac{1}{24(\tanh(dx+c)+1)^3}-\frac{1}{32(\tanh(dx+c)+1)^2}-\frac{1}{32(\tanh(dx+c)+1)}+}{d\,a^5}$
risch	$\frac{x}{32a^5}-\frac{5\,\mathrm{e}^{-2dx-2c}}{64a^5d}-\frac{5\,\mathrm{e}^{-4dx-4c}}{64a^5d}-\frac{5\,\mathrm{e}^{-6dx-6c}}{96a^5d}-\frac{5\,\mathrm{e}^{-8dx-8c}}{256a^5d}-\frac{\mathrm{e}^{-10dx-10c}}{320a^5d}$
parallelrisch	$\frac{-128-175\tanh(dx+c)+75\tanh(dx+c)^4xd+150\tanh(dx+c)^3xd+15dx-15\tanh(dx+c)^4+150\tanh(dx+c)^2xd+75\tanh(}{480d\,a^5(\tanh(dx+c)+1)^5}$

[In] int(1/(a+a*tanh(d*x+c))^5,x,method=_RETURNVERBOSE)

[Out] 1/d/a^5*(-1/64*ln(tanh(d*x+c)-1)-1/10/(tanh(d*x+c)+1)^5-1/16/(tanh(d*x+c)+1)^4-1/24/(tanh(d*x+c)+1)^3-1/32/(tanh(d*x+c)+1)^2-1/32/(tanh(d*x+c)+1)+1/64*ln(tanh(d*x+c)+1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(109) = 218.

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.37

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx = \frac{12\,(10\,dx-1)\cosh\left(dx+c\right)^5+60\,(10\,dx-1)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+12\,(10\,dx+1)\sinh\left(dx+c\right)^5}{3840\,\left(a^5d\cosh\left(dx+c\right)^5+5\right.}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(12*(10*d*x - 1)*cosh(d*x + c)^5 + 60*(10*d*x - 1)*cosh(d*x + c)*sin
h(d*x + c)^4 + 12*(10*d*x + 1)*sinh(d*x + c)^5 + 15*(8*(10*d*x + 1)*cosh(d*
x + c)^2 - 15)*sinh(d*x + c)^3 - 375*cosh(d*x + c)^3 + 15*(8*(10*d*x - 1)*c
osh(d*x + c)^3 - 75*cosh(d*x + c))*sinh(d*x + c)^2 + 5*(12*(10*d*x + 1)*cos
h(d*x + c)^4 - 135*cosh(d*x + c)^2 - 20)*sinh(d*x + c) - 500*cosh(d*x + c))
/(a^5*d*cosh(d*x + c)^5 + 5*a^5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^5*d*
cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^5*d*cosh(d*x + c)^2*sinh(d*x + c)^3
+ 5*a^5*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^5*d*sinh(d*x + c)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. 2(102) = 204.

Time = 1.31 (sec) , antiderivative size = 1018, normalized size of antiderivative = 8.41

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))**5,x)
```

```
[Out] Piecewise((15*d*x*tanh(c + d*x)**5/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5
*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d
*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) + 75*d*x*tanh(c + d*x)**4/
(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*t
anh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x)
+ 480*a**5*d) + 150*d*x*tanh(c + d*x)**3/(480*a**5*d*tanh(c + d*x)**5 + 240
0*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh
(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) + 150*d*x*tanh(c + d
*x)**2/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a
**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c
+ d*x) + 480*a**5*d) + 75*d*x*tanh(c + d*x)/(480*a**5*d*tanh(c + d*x)**5 +
2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*t
anh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) + 15*d*x/(480*a**
```

```
5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c +
d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a*
*5*d) - 15*tanh(c + d*x)**4/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh
(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2
+ 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) - 75*tanh(c + d*x)**3/(480*a**5*d
*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x
)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*
d) - 155*tanh(c + d*x)**2/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c
 + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 +
2400*a**5*d*tanh(c + d*x) + 480*a**5*d) - 175*tanh(c + d*x)/(480*a**5*d*tan
h(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**5*d*tanh(c + d*x)**3
 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c + d*x) + 480*a**5*d) -
 128/(480*a**5*d*tanh(c + d*x)**5 + 2400*a**5*d*tanh(c + d*x)**4 + 4800*a**
5*d*tanh(c + d*x)**3 + 4800*a**5*d*tanh(c + d*x)**2 + 2400*a**5*d*tanh(c +
d*x) + 480*a**5*d), Ne(d, 0)), (x/(a*tanh(c) + a)**5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx = \frac{dx+c}{32\,a^5d} - \frac{300\,e^{(-2\,dx-2\,c)}+300\,e^{(-4\,dx-4\,c)}+200\,e^{(-6\,dx-6\,c)}+75\,e^{(-8\,dx-8\,c)}+12\,e^{(-10\,dx-10\,c)}}{3840\,a^5d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/32*(d*x + c)/(a^5*d) - 1/3840*(300*e^(-2*d*x - 2*c) + 300*e^(-4*d*x - 4*c
) + 200*e^(-6*d*x - 6*c) + 75*e^(-8*d*x - 8*c) + 12*e^(-10*d*x - 10*c))/(a^
5*d)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx = -\frac{\frac{\left(300\,e^{(8\,dx+8\,c)}+300\,e^{(6\,dx+6\,c)}+200\,e^{(4\,dx+4\,c)}+75\,e^{(2\,dx+2\,c)}+12\right)e^{(-10\,dx-10\,c)}}{a^5} - \frac{120\,(dx+c)}{a^5}}{3840\,d}$$

```
[In] integrate(1/(a+a*tanh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/3840*((300*e^(8*d*x + 8*c) + 300*e^(6*d*x + 6*c) + 200*e^(4*d*x + 4*c) +
 75*e^(2*d*x + 2*c) + 12)*e^(-10*d*x - 10*c)/a^5 - 120*(d*x + c)/a^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+a\tanh(c+dx))^5}\,dx = \frac{x}{32\,a^5}-\frac{5\,\mathrm{e}^{-2\,c-2\,d\,x}}{64\,a^5\,d}-\frac{5\,\mathrm{e}^{-4\,c-4\,d\,x}}{64\,a^5\,d}-\frac{5\,\mathrm{e}^{-6\,c-6\,d\,x}}{96\,a^5\,d}-\frac{5\,\mathrm{e}^{-8\,c-8\,d\,x}}{256\,a^5\,d}-\frac{\mathrm{e}^{-10\,c-10\,d\,x}}{320\,a^5\,d}$$

```
[In] int(1/(a + a*tanh(c + d*x))^5,x)
```

```
[Out] x/(32*a^5) - (5*exp(- 2*c - 2*d*x))/(64*a^5*d) - (5*exp(- 4*c - 4*d*x))/(64
*a^5*d) - (5*exp(- 6*c - 6*d*x))/(96*a^5*d) - (5*exp(- 8*c - 8*d*x))/(256*a
^5*d) - exp(- 10*c - 10*d*x)/(320*a^5*d)
```

3.50 $\int (1+\tanh(x))^{7/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int (1+\tanh(x))^{7/2}\,dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 8\sqrt{1+\tanh(x)} - \frac{4}{3}(1+\tanh(x))^{3/2} - \frac{2}{5}(1+\tanh(x))^{5/2}$$

```
[Out] 8*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-8*(1+tanh(x))^(1/2)-4/3*(1
+tanh(x))^(3/2)-2/5*(1+tanh(x))^(5/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3559, 3561, 212}

$$\int (1+\tanh(x))^{7/2}\,dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right) - \frac{2}{5}(\tanh(x)+1)^{5/2} - \frac{4}{3}(\tanh(x)+1)^{3/2} - 8\sqrt{\tanh(x)+1}$$

```
[In] Int[(1 + Tanh[x])^(7/2),x]
```

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 8*Sqrt[1 + Tanh[x]] - (4*(1
+ Tanh[x])^(3/2))/3 - (2*(1 + Tanh[x])^(5/2))/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{5}(1+\tanh(x))^{5/2}+2\int(1+\tanh(x))^{5/2}\,dx\\
&= -\frac{4}{3}(1+\tanh(x))^{3/2}-\frac{2}{5}(1+\tanh(x))^{5/2}+4\int(1+\tanh(x))^{3/2}\,dx\\
&= -8\sqrt{1+\tanh(x)}-\frac{4}{3}(1+\tanh(x))^{3/2}-\frac{2}{5}(1+\tanh(x))^{5/2}+8\int\sqrt{1+\tanh(x)}\,dx\\
&= -8\sqrt{1+\tanh(x)}-\frac{4}{3}(1+\tanh(x))^{3/2}\\
&\quad-\frac{2}{5}(1+\tanh(x))^{5/2}+16\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-8\sqrt{1+\tanh(x)}-\frac{4}{3}(1+\tanh(x))^{3/2}-\frac{2}{5}(1+\tanh(x))^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int(1+\tanh(x))^{7/2}\,dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-\frac{2}{15}\sqrt{1+\tanh(x)}\left(73+16\tanh(x)+3\tanh^2(x)\right)$$

```
[In] Integrate[(1 + Tanh[x])^(7/2),x]
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*Sqrt[1 + Tanh[x]]*(73 + 1
6*Tanh[x] + 3*Tanh[x]^2))/15
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$8 \operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-8\sqrt{1+\tanh\left(x\right)}-\frac{4(1+\tanh(x))^{\frac{3}{2}}}{3}-\frac{2(1+\tanh(x))^{\frac{5}{2}}}{5}$	43
default	$8 \operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-8\sqrt{1+\tanh\left(x\right)}-\frac{4(1+\tanh(x))^{\frac{3}{2}}}{3}-\frac{2(1+\tanh(x))^{\frac{5}{2}}}{5}$	43

```
[In] int((1+tanh(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-8*(1+tanh(x))^(1/2)-4/3*(1
+tanh(x))^(3/2)-2/5*(1+tanh(x))^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(42) = 84.

Time = 0.27 (sec) , antiderivative size = 434, normalized size of antiderivative = 7.61

$$\int (1+\tanh(x))^{7/2}\,dx =$$

$$-\frac{4\left(2\sqrt{2}(23\sqrt{2}\cosh\left(x\right)^{5}+115\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{4}+23\sqrt{2}\sinh\left(x\right)^{5}+5\left(46\sqrt{2}\cosh\left(x\right)^{2}+7\sqrt{2}\right)\sinh\left(x\right.\right.}{}$$

```
[In] integrate((1+tanh(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^5 + 115*sqrt(2)*cosh(x)*sinh(x)^4 + 23
*sqrt(2)*sinh(x)^5 + 5*(46*sqrt(2)*cosh(x)^2 + 7*sqrt(2))*sinh(x)^3 + 35*sq
rt(2)*cosh(x)^3 + 5*(46*sqrt(2)*cosh(x)^3 + 21*sqrt(2)*cosh(x))*sinh(x)^2 +
 5*(23*sqrt(2)*cosh(x)^4 + 21*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x) + 15*s
qrt(2)*cosh(x))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^6 +
 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 +
 sqrt(2))*sinh(x)^4 + 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 + 3*sqrt
(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 + 6*sqrt(2)*cosh(x)^2 + sqr
t(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 + 2*sqrt(2)*co
sh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(
cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) -
 2*sinh(x)^2 - 1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh
(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
+ 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5
+ 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (1+\tanh(x))^{7/2}\,dx = \int (\tanh(x)+1)^{\frac{7}{2}}\,dx$$

```
[In] integrate((1+tanh(x))**(7/2),x)
[Out] Integral((tanh(x) + 1)**(7/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.46

$$\int (1+\tanh(x))^{7/2}\,dx = -4\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right) - \frac{8\sqrt{2}}{\sqrt{e^{(-2x)}+1}} - \frac{8\sqrt{2}}{3\left(e^{(-2x)}+1\right)^{\frac{3}{2}}} - \frac{8\sqrt{2}}{5\left(e^{(-2x)}+1\right)^{\frac{5}{2}}}$$

```
[In] integrate((1+tanh(x))^(7/2),x, algorithm="maxima")
[Out] -4*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/s
qrt(e^(-2*x) + 1))) - 8*sqrt(2)/sqrt(e^(-2*x) + 1) - 8/3*sqrt(2)/(e^(-2*x)
+ 1)^(3/2) - 8/5*sqrt(2)/(e^(-2*x) + 1)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(42) = 84.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.46

$$\int (1+\tanh(x))^{7/2}\,dx = \frac{4}{15}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^4 - 135\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3 + 170\left(\sqrt{e^{(4x)}-\right.\right.}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right.}\right.$$

```
[In] integrate((1+tanh(x))^(7/2),x, algorithm="giac")
[Out] 4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^4 - 135*(sqrt(e^(4*
x) + e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 10
0*sqrt(e^(4*x) + e^(2*x)) + 100*e^(2*x) + 23)/(sqrt(e^(4*x) + e^(2*x)) - e^
(2*x) - 1)^5 - 15*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int (1+\tanh(x))^{7/2}\,dx = -8\,\sqrt{\tanh\left(x\right)+1}-\frac{4\,{\left(\tanh(x)+1\right)}^{3/2}}{3}-\frac{2\,{\left(\tanh(x)+1\right)}^{5/2}}{5}-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{\tanh\left(x\right)+1}\,1{}\mathrm{i}}{2}\right)\,8{}\mathrm{i}$$

```
[In] int((tanh(x) + 1)^(7/2),x)
```

```
[Out] - 2^(1/2)*atan((2^(1/2)*(tanh(x) + 1)^(1/2)*1i)/2)*8i - 8*(tanh(x) + 1)^(1/
2) - (4*(tanh(x) + 1)^(3/2))/3 - (2*(tanh(x) + 1)^(5/2))/5
```

3.51 $\int (1+\tanh(x))^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int (1+\tanh(x))^{5/2}\,dx=4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-4\sqrt{1+\tanh(x)}-\frac{2}{3}(1+\tanh(x))^{3/2}$$

```
[Out] 4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-4*(1+tanh(x))^(1/2)-2/3*(1
+tanh(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3559, 3561, 212}

$$\int (1+\tanh(x))^{5/2}\,dx=4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}-4\sqrt{\tanh(x)+1}$$

```
[In] Int[(1 + Tanh[x])^(5/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 4*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{3}(1+\tanh(x))^{3/2} + 2\int (1+\tanh(x))^{3/2}\,dx \\
&= -4\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2} + 4\int \sqrt{1+\tanh(x)}\,dx \\
&= -4\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2} + 8\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right) \\
&= 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 4\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int (1+\tanh(x))^{5/2}\,dx = 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - \frac{2}{3}\sqrt{1+\tanh(x)}(7+\tanh(x))$$

```
[In] Integrate[(1 + Tanh[x])^(5/2),x]
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*Sqrt[1 + Tanh[x]]*(7 + Ta
nh[x]))/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$4 \operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-4\sqrt{1+\tanh\left(x\right)}-\frac{2(1+\tanh(x))^{\frac{3}{2}}}{3}$	35
default	$4 \operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-4\sqrt{1+\tanh\left(x\right)}-\frac{2(1+\tanh(x))^{\frac{3}{2}}}{3}$	35

```
[In] int((1+tanh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-4*(1+tanh(x))^(1/2)-2/3*(1
+tanh(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(34) = 68.

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 5.60

$$\int (1+\tanh(x))^{5/2}\,dx =$$
$$-\frac{2\left(2\sqrt{2}(4\sqrt{2}\cosh\left(x\right)^{3}+12\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+4\sqrt{2}\sinh\left(x\right)^{3}+3\left(4\sqrt{2}\cosh\left(x\right)^{2}+\sqrt{2}\right)\sinh\left(x\right)+3\right.}{}$$

```
[In] integrate((1+tanh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^3 + 12*sqrt(2)*cosh(x)*sinh(x)^2 + 4*sqr
t(2)*sinh(x)^3 + 3*(4*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 3*sqrt(2)*cosh
(x))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*si
nh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
 sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (1+\tanh(x))^{5/2}\,dx = \int (\tanh(x)+1)^{\frac{5}{2}}\,dx$$

```
[In] integrate((1+tanh(x))**(5/2),x)
[Out] Integral((tanh(x) + 1)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.56

$$\int (1+\tanh(x))^{5/2}\,dx = -2\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right) - \frac{4\sqrt{2}}{\sqrt{e^{(-2x)}+1}} - \frac{4\sqrt{2}}{3\left(e^{(-2x)}+1\right)^{\frac{3}{2}}}$$

```
[In] integrate((1+tanh(x))^(5/2),x, algorithm="maxima")
[Out] -2*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/s
qrt(e^(-2*x) + 1))) - 4*sqrt(2)/sqrt(e^(-2*x) + 1) - 4/3*sqrt(2)/(e^(-2*x)
+ 1)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.13

$$\int (1+\tanh(x))^{5/2}\,dx = \frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2-9\sqrt{e^{(4x)}+e^{(2x)}}+9e^{(2x)}+4\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^3} - 3\log\left(-2\sqrt{e^{(4}\right.\right.$$

```
[In] integrate((1+tanh(x))^(5/2),x, algorithm="giac")
[Out] 2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 9*sqrt(e^(4*x) +
e^(2*x)) + 9*e^(2*x) + 4)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1)^3 - 3*log
(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int (1+\tanh(x))^{5/2}\,dx = \sqrt{8}\,\ln\left(-2\sqrt{8}\,\sqrt{\tanh\left(x\right)+1}-8\right) - \frac{2\left(\tanh(x)+1\right)^{3/2}}{3} - 2\sqrt{2}\,\ln\left(4\sqrt{2}\,\sqrt{\tanh\left(x\right)+1}-8\right) - 4\sqrt{\tanh\left(x\right)+1}$$

```
[In] int((tanh(x) + 1)^(5/2),x)
```

```
[Out] 8^(1/2)*log(- 2*8^(1/2)*(tanh(x) + 1)^(1/2) - 8) - (2*(tanh(x) + 1)^(3/2))/
3 - 2*2^(1/2)*log(4*2^(1/2)*(tanh(x) + 1)^(1/2) - 8) - 4*(tanh(x) + 1)^(1/2
)
```

3.52 $\int (1+\tanh(x))^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int (1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)}$$

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3559, 3561, 212}

$$\int (1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\tanh(x)+1}$$

```
[In] Int[(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
```

```
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -2\sqrt{1+\tanh(x)}+2\int\sqrt{1+\tanh(x)}\,dx\\ &= -2\sqrt{1+\tanh(x)}+4\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\ &= 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-2\sqrt{1+\tanh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int(1+\tanh(x))^{3/2}\,dx=2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-2\sqrt{1+\tanh(x)}$$

```
[In] Integrate[(1 + Tanh[x])^(3/2),x]
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\,\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}$	27
default	$2\,\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}$	27

```
[In] int((1+tanh(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.91

$$\int (1+\tanh(x))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}+\cdots\right.}{\cosh\left(x\right)^{2}+2\cosh\left(x\right)\mathrm{s}\cdots}$$

```
[In] integrate((1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(cosh(x)/(cosh(x) - sin
h(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int (1+\tanh(x))^{3/2}\,dx = \int \left(\tanh\left(x\right)+1\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((1+tanh(x))**(3/2),x)
```

```
[Out] Integral((tanh(x) + 1)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int (1+\tanh(x))^{3/2}\,dx = -\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}}\right)-\frac{2\sqrt{2}}{\sqrt{e^{(-2\,x)}+1}}$$

```
[In] integrate((1+tanh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/sqr
t(e^(-2*x) + 1))) - 2*sqrt(2)/sqrt(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int (1 + \tanh(x))^{3/2}\, dx = \sqrt{2}\left(\frac{2}{\sqrt{e^{(4\,x)} + e^{(2\,x)}} - e^{(2\,x)} - 1} - \log\left(-2\,\sqrt{e^{(4\,x)} + e^{(2\,x)}} + 2\,e^{(2\,x)} + 1\right)\right)$$

```
[In] integrate((1+tanh(x))^(3/2),x, algorithm="giac")
[Out] sqrt(2)*(2/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1) - log(-2*sqrt(e^(4*x) +
e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (1 + \tanh(x))^{3/2}\, dx = 2\,\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh\left(x\right) + 1}}{2}\right) - 2\,\sqrt{\tanh\left(x\right) + 1}$$

```
[In] int((tanh(x) + 1)^(3/2),x)
[Out] 2*2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2) - 2*(tanh(x) + 1)^(1/2)
```

3.53 $\int \sqrt{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt{1+\tanh(x)}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)$$

```
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3561, 212}

$$\int \sqrt{1+\tanh(x)}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)$$

```
[In] Int[Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
```

```
b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right)$$
$$= \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sqrt{1+\tanh(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)$$

```
[In] Integrate[Sqrt[1 + Tanh[x]],x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	17
default	$\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	17

```
[In] int((1+tanh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \sqrt{1+\tanh(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x)) - 2\cosh(x)^2 - 4\cosh(x)\sinh(x) - 2\sinh(x)^2 - 1\right)$$

```
[In] integrate((1+tanh(x))^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1)
```

Sympy [F]

$$\int \sqrt{1+\tanh(x)}\,dx = \int \sqrt{\tanh(x)+1}\,dx$$

```
[In] integrate((1+tanh(x))**(1/2),x)
[Out] Integral(sqrt(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \sqrt{1+\tanh(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)$$

```
[In] integrate((1+tanh(x))^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)
/sqrt(e^(-2*x) + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \sqrt{1+\tanh(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)$$

```
[In] integrate((1+tanh(x))^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\tanh(x)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)$$

```
[In] int((tanh(x) + 1)^(1/2),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2)
```

3.54 $\int \frac{1}{\sqrt{1+\tanh(x)}} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int \frac{1}{\sqrt{1+\tanh(x)}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1+\tanh(x)}}$$

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3560, 3561, 212}

$$\int \frac{1}{\sqrt{1+\tanh(x)}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{\tanh(x)+1}}$$

```
[In] Int[1/Sqrt[1 + Tanh[x]],x]
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Tanh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
```

```
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{\sqrt{1+\tanh(x)}}+\frac{1}{2}\int\sqrt{1+\tanh(x)}\,dx\\
&= -\frac{1}{\sqrt{1+\tanh(x)}}+\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int\frac{1}{\sqrt{1+\tanh(x)}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\tanh(x))\right)}{\sqrt{1+\tanh(x)}}$$

```
[In] Integrate[1/Sqrt[1 + Tanh[x]],x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Tanh[x])/2]/Sqrt[1 + Tanh[x]])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\tanh(x)}}$	27
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\tanh(x)}}$	27

```
[In] int(1/(1+tanh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+tanh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.66

$$\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx = \frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(-2\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-2\cosh\left(x\right)^{2}-4\cosh\left(x\right)\sinh\right.}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(1/(1+tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(
x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*si
nh(x)^2 - 1) - 4*sqrt(cosh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx = \int \frac{1}{\sqrt{\tanh\left(x\right)+1}}\,dx$$

```
[In] integrate(1/(1+tanh(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx = -\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)-\frac{1}{2}\sqrt{2}\sqrt{e^{(-2x)}+1}$$

```
[In] integrate(1/(1+tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)
/sqrt(e^(-2*x) + 1))) - 1/2*sqrt(2)*sqrt(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx = -\frac{1}{4}\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}}+\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

```
[In] integrate(1/(1+tanh(x))^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*(2/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x)) + log(-2*sqrt(e^(4*x) +
 e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh(x)+1}}{2}\right)}{2} - \frac{1}{\sqrt{\tanh(x)+1}}$$

```
[In] int(1/(tanh(x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2))/2 - 1/(tanh(x) + 1)^(1/2)
```

3.55 $\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}$$

```
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/2/(1+tanh(x))^(1/2)-1/
3/(1+tanh(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3560, 3561, 212}

$$\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{2\sqrt{\tanh(x)+1}}-\frac{1}{3(\tanh(x)+1)^{3/2}}$$

```
[In] Int[(1 + Tanh[x])^(-3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Tanh[x])^(3/2))
- 1/(2*Sqrt[1 + Tanh[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3(1+\tanh(x))^{3/2}}+\frac{1}{2}\int\frac{1}{\sqrt{1+\tanh(x)}}\,dx\\
&= -\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{4}\int\sqrt{1+\tanh(x)}\,dx\\
&= -\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{2}\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int\frac{1}{(1+\tanh(x))^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\tanh(x))\right)}{3(1+\tanh(x))^{3/2}}$$

```
[In] Integrate[(1 + Tanh[x])^(-3/2),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Tanh[x])/2]/(1 + Tanh[x])^(3/2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\tanh(x)}}-\frac{1}{3(1+\tanh(x))^{\frac{3}{2}}}$	35
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\tanh(x)}}-\frac{1}{3(1+\tanh(x))^{\frac{3}{2}}}$	35

```
[In] int(1/(1+tanh(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/2/(1+tanh(x))^(1/2)-1/
3/(1+tanh(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(34) = 68.

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.39

$$\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx = -\frac{2\sqrt{2}\left(4\sqrt{2}\cosh(x)^2+8\sqrt{2}\cosh(x)\sinh(x)+4\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\cdots\right.}{24\left(\cos\cdots\right.}$$

```
[In] integrate(1/(1+tanh(x))^(3/2),x, algorithm="fricas")
[Out] -1/24*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^2 + 8*sqrt(2)*cosh(x)*sinh(x) + 4*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*
cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]

$$\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx = \int \frac{1}{(\tanh(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+tanh(x))**(3/2),x)
[Out] Integral((tanh(x) + 1)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx = -\frac{1}{12}\sqrt{2}\left(\frac{3}{e^{(-2x)}+1}+1\right)\left(e^{(-2x)}+1\right)^{\frac{3}{2}} - \frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)$$

```
[In] integrate(1/(1+tanh(x))^(3/2),x, algorithm="maxima")
[Out] -1/12*sqrt(2)*(3/(e^(-2*x) + 1) + 1)*(e^(-2*x) + 1)^(3/2) - 1/8*sqrt(2)*log
(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) + sqrt(2)/sqrt(e^(-2*x) +
 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.94

$$\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx = -\frac{1}{24}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2-3\sqrt{e^{(4x)}+e^{(2x)}}+3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3}+3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2\right.\right.$$

```
[In] integrate(1/(1+tanh(x))^(3/2),x, algorithm="giac")
[Out] -1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x)
+ e^(2*x)) + 3*e^(2*x) + 1)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^3 + 3*log(-
2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+\tanh(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{4} - \frac{\frac{\tanh(x)}{2}+\frac{5}{6}}{(\tanh(x)+1)^{3/2}}$$

```
[In] int(1/(tanh(x) + 1)^(3/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2))/4 - (tanh(x)/2 + 5/6)/(tan
h(x) + 1)^(3/2)
```

3.56 $\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 61

$$\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{5(1+\tanh(x))^{5/2}} - \frac{1}{6(1+\tanh(x))^{3/2}} - \frac{1}{4\sqrt{1+\tanh(x)}}$$

```
[Out] 1/8*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/4/(1+tanh(x))^(1/2)-1/
5/(1+tanh(x))^(5/2)-1/6/(1+tanh(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3560, 3561, 212}

$$\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{4\sqrt{\tanh(x)+1}} - \frac{1}{6(\tanh(x)+1)^{3/2}} - \frac{1}{5(\tanh(x)+1)^{5/2}}$$

```
[In] Int[(1 + Tanh[x])^(-5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Tanh[x])^(5/2))
- 1/(6*(1 + Tanh[x])^(3/2)) - 1/(4*Sqrt[1 + Tanh[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5(1+\tanh(x))^{5/2}}+\frac{1}{2}\int\frac{1}{(1+\tanh(x))^{3/2}}\,dx\\
&= -\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}+\frac{1}{4}\int\frac{1}{\sqrt{1+\tanh(x)}}\,dx\\
&= -\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}-\frac{1}{4\sqrt{1+\tanh(x)}}+\frac{1}{8}\int\sqrt{1+\tanh(x)}\,dx\\
&= -\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}-\frac{1}{4\sqrt{1+\tanh(x)}}\\
&\quad+\frac{1}{4}\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= \frac{\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{5(1+\tanh(x))^{5/2}}-\frac{1}{6(1+\tanh(x))^{3/2}}-\frac{1}{4\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

$$\int\frac{1}{(1+\tanh(x))^{5/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\frac{1}{2}(1+\tanh(x))\right)}{5(1+\tanh(x))^{5/2}}$$

```
[In] Integrate[(1 + Tanh[x])^(-5/2),x]
[Out] -1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Tanh[x])/2]/(1 + Tanh[x])^(5/2)
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{4\sqrt{1+\tanh(x)}}-\frac{1}{5(1+\tanh(x))^{\frac{5}{2}}}-\frac{1}{6(1+\tanh(x))^{\frac{3}{2}}}$	43
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{4\sqrt{1+\tanh(x)}}-\frac{1}{5(1+\tanh(x))^{\frac{5}{2}}}-\frac{1}{6(1+\tanh(x))^{\frac{3}{2}}}$	43

```
[In] int(1/(1+tanh(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/4/(1+tanh(x))^(1/2)-1/
5/(1+tanh(x))^(5/2)-1/6/(1+tanh(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(42) = 84.

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.36

$$\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx =$$
$$-\frac{2\sqrt{2}\left(23\sqrt{2}\cosh(x)^4+92\sqrt{2}\cosh(x)\sinh(x)^3+23\sqrt{2}\sinh(x)^4+\left(138\sqrt{2}\cosh(x)^2+11\sqrt{2}\right)\sinh(x)^2\right.}{}$$

```
[In] integrate(1/(1+tanh(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^4 + 92*sqrt(2)*cosh(x)*sinh(x)^3 + 23
*sqrt(2)*sinh(x)^4 + (138*sqrt(2)*cosh(x)^2 + 11*sqrt(2))*sinh(x)^2 + 11*sq
rt(2)*cosh(x)^2 + 2*(46*sqrt(2)*cosh(x)^3 + 11*sqrt(2)*cosh(x))*sinh(x) + 3
*sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^5 + 5*sqr
t(2)*cosh(x)^4*sinh(x) + 10*sqrt(2)*cosh(x)^3*sinh(x)^2 + 10*sqrt(2)*cosh(x
)^2*sinh(x)^3 + 5*sqrt(2)*cosh(x)*sinh(x)^4 + sqrt(2)*sinh(x)^5)*log(-2*sqr
t(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 -
4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10
*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(
x)^5)
```

Sympy [F]

$$\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx = \int \frac{1}{(\tanh{(x)}+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(1+tanh(x))**(5/2),x)
[Out] Integral((tanh(x) + 1)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.30

$$\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx = -\frac{1}{120}\sqrt{2}\left(\frac{5}{e^{(-2x)}+1} + \frac{15}{\left(e^{(-2x)}+1\right)^2} + 3\right)\left(e^{(-2x)}+1\right)^{\frac{5}{2}} - \frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-2x)}+1}}}\right)$$

```
[In] integrate(1/(1+tanh(x))^(5/2),x, algorithm="maxima")
[Out] -1/120*sqrt(2)*(5/(e^(-2*x) + 1) + 15/(e^(-2*x) + 1)^2 + 3)*(e^(-2*x) + 1)^
(5/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-2*x) + 1))/(sqrt(2) +
 sqrt(2)/sqrt(e^(-2*x) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(42) = 84.

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx = -\frac{1}{240}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^4 - 45\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3 + 35\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2 -\right.}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^5}\right.$$

```
[In] integrate(1/(1+tanh(x))^(5/2),x, algorithm="giac")
[Out] -1/240*sqrt(2)*(2*(45*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^4 - 45*(sqrt(e^(4
*x) + e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 15
*sqrt(e^(4*x) + e^(2*x)) + 15*e^(2*x) + 3)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*
x))^5 + 15*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{1}{(1+\tanh(x))^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh(x)+1}}{2}\right)}{8} - \frac{\frac{\tanh(x)}{6}+\frac{(\tanh(x)+1)^2}{4}+\frac{11}{30}}{(\tanh\left(x\right)+1)^{5/2}}$$

```
[In] int(1/(tanh(x) + 1)^(5/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2))/8 - (tanh(x)/6 + (tanh(x)
+ 1)^2/4 + 11/30)/(tanh(x) + 1)^(5/2)
```

3.57 $\int (a+b\tanh(c+dx))^5\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int (a+b\tanh(c+dx))^5\,dx = a(a^4+10a^2b^2+5b^4)\,x+\frac{b(5a^4+10a^2b^2+b^4)\log(\cosh(c+dx))}{d}-\frac{4ab^2(a^2+b^2)\tanh(c+dx)}{d}-\frac{b(3a^2+b^2)\,(a+b\tanh(c+dx))^2}{2d}-\frac{2ab(a+b\tanh(c+dx))^3}{3d}-\frac{b(a+b\tanh(c+dx))^4}{4d}$$

```
[Out] a*(a^4+10*a^2*b^2+5*b^4)*x+b*(5*a^4+10*a^2*b^2+b^4)*ln(cosh(d*x+c))/d-4*a*b
^2*(a^2+b^2)*tanh(d*x+c)/d-1/2*b*(3*a^2+b^2)*(a+b*tanh(d*x+c))^2/d-2/3*a*b*
(a+b*tanh(d*x+c))^3/d-1/4*b*(a+b*tanh(d*x+c))^4/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3563, 3609, 3606, 3556}

$$\int (a+b\tanh(c+dx))^5\,dx = -\frac{b(3a^2+b^2)\,(a+b\tanh(c+dx))^2}{2d}-\frac{4ab^2(a^2+b^2)\tanh(c+dx)}{d}+\frac{b(5a^4+10a^2b^2+b^4)\log(\cosh(c+dx))}{d}+ax(a^4+10a^2b^2+5b^4)-\frac{b(a+b\tanh(c+dx))^4}{4d}-\frac{2ab(a+b\tanh(c+dx))^3}{3d}$$

```
[In] Int[(a + b*Tanh[c + d*x])^5,x]
```

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x + (b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Cosh[c +
 d*x]])/d - (4*a*b^2*(a^2 + b^2)*Tanh[c + d*x])/d - (b*(3*a^2 + b^2)*(a + b
*Tanh[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Tanh[c + d*x])^3)/(3*d) - (b*(a +
b*Tanh[c + d*x])^4)/(4*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b(a+b\tanh(c+dx))^4}{4d} + \int (a+b\tanh(c+dx))^3\left(a^2+b^2+2ab\tanh(c+dx)\right)\,dx \\
&= -\frac{2ab(a+b\tanh(c+dx))^3}{3d} - \frac{b(a+b\tanh(c+dx))^4}{4d} \\
&\quad + \int (a+b\tanh(c+dx))^2\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\tanh(c+dx)\right)\,dx \\
&= -\frac{b(3a^2+b^2)(a+b\tanh(c+dx))^2}{2d} - \frac{2ab(a+b\tanh(c+dx))^3}{3d} \\
&\quad - \frac{b(a+b\tanh(c+dx))^4}{4d} + \int (a+b\tanh(c+dx))\left(a^4+6a^2b^2+b^4\right. \\
&\qquad \left. +4ab\left(a^2+b^2\right)\tanh(c+dx)\right)\,dx
\end{aligned}
$$

$$= a(a^4 + 10a^2b^2 + 5b^4)\,x - \frac{4ab^2(a^2+b^2)\tanh(c+dx)}{d} - \frac{b(3a^2+b^2)\,(a+b\tanh(c+dx))^2}{2d} - \frac{2ab(a+b\tanh(c+dx))^3}{3d} - \frac{b(a+b\tanh(c+dx))^4}{4d} + \left(b(5a^4+10a^2b^2+b^4)\right)\int \tanh(c+dx)\,dx$$

$$= a(a^4 + 10a^2b^2 + 5b^4)\,x + \frac{b(5a^4+10a^2b^2+b^4)\log(\cosh(c+dx))}{d} - \frac{4ab^2(a^2+b^2)\tanh(c+dx)}{d} - \frac{b(3a^2+b^2)\,(a+b\tanh(c+dx))^2}{2d} - \frac{2ab(a+b\tanh(c+dx))^3}{3d} - \frac{b(a+b\tanh(c+dx))^4}{4d}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int (a+b\tanh(c+dx))^5\,dx = -\frac{6(a+b)^5\log(1-\tanh(c+dx)) - 6(a-b)^5\log(1+\tanh(c+dx)) + 60ab^2(2a^2+b^2)\tanh(c+dx) + 6\cdots}{12d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^5,x]
[Out] -1/12*(6*(a + b)^5*Log[1 - Tanh[c + d*x]] - 6*(a - b)^5*Log[1 + Tanh[c + d*
x]] + 60*a*b^2*(2*a^2 + b^2)*Tanh[c + d*x] + 6*b^3*(10*a^2 + b^2)*Tanh[c +
d*x]^2 + 20*a*b^4*Tanh[c + d*x]^3 + 3*b^5*Tanh[c + d*x]^4)/d
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{-10a^3b^2\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5a\,b^4+b^5\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^5\tanh(dx+c)^2}{2}-\frac{b^5\tan\cdots}{}}{d}$
default	$\frac{-10a^3b^2\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5a\,b^4+b^5\right)\ln(\tanh(dx+c)-1)}{2}-\frac{b^5\tanh(dx+c)^2}{2}-\frac{b^5\tan\cdots}{}}{d}$
parallelrisch	$-\frac{3b^5\tanh(dx+c)^4+20a\,b^4\tanh(dx+c)^3-12a^5dx+60a^4bdx-120a^3b^2dx+120a^2b^3dx-60a\,b^4dx+12b^5dx+60a^2b^3\tanh(\cdots}{}$
parts	$a^5x+\frac{b^5\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{5a\,b^4\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)\cdots\right.}{}$
risch	$a^5x-5b\,a^4x+10a^3b^2x-10b^3a^2x+5a\,b^4x-b^5x-\frac{10b\,a^4c}{d}-\frac{20b^3a^2c}{d}-\frac{2b^5c}{d}+\frac{4b^2\left(15a^3\mathrm{e}^{6dx}\cdots\right.}{}$

```
[In] int((a+b*tanh(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-10*a^3*b^2*tanh(d*x+c)-5*a*b^4*tanh(d*x+c)-1/2*(a^5+5*a^4*b+10*a^3*b^
2+10*a^2*b^3+5*a*b^4+b^5)*ln(tanh(d*x+c)-1)-1/2*b^5*tanh(d*x+c)^2-1/4*b^5*t
anh(d*x+c)^4-5/3*a*b^4*tanh(d*x+c)^3-5*a^2*b^3*tanh(d*x+c)^2+1/2*(a^5-5*a^4
*b+10*a^3*b^2-10*a^2*b^3+5*a*b^4-b^5)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2739 vs. $2(136) = 272$.

Time = 0.28 (sec) , antiderivative size = 2739, normalized size of antiderivative = 19.29

$$\int (a + b\tanh(c + dx))^5 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d
*x + c)^8 + 24*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d*x*sinh(d*x + c)^8 + 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^
4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*co
sh(d*x + c)^6 + 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + 7*(a^5 - 5*a^4*
b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^2 + (a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*sinh(d*x + c)^6 + 24
*(7*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x
+ c)^3 + 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 60
*a^3*b^2 + 40*a*b^4 + 6*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^4
 + 6*(35*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh
(d*x + c)^4 + 30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b
 + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x + 30*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - 5*a^4*b + 10*a^3*b^2
 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^5 + 10*(5*a^3*b^2 + 5*a^2*
b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*d*x)*cosh(d*x + c)^3 + (30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
- b^5)*d*x + 4*(45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 3*(a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2 + 4*(21*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)
^6 + 45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 45*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
```

```
)*d*x)*cosh(d*x + c)^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*d*x + 9*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*
a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 3*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^8 + 8*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a^4*b + 10*a^2*b^3 +
 b^5)*sinh(d*x + c)^8 + 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 4*
(5*a^4*b + 10*a^2*b^3 + b^5 + 7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^3 + 3*
(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^4*b + 10*
a^2*b^3 + b^5 + 6*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 2*(15*a^4*
b + 30*a^2*b^3 + 3*b^5 + 35*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 +
30*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*
a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 10*(5*a^4*b + 10*a^2*b^3 + b^5)
*cosh(d*x + c)^3 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 4*(7*(5*a^4*b + 10
*a^2*b^3 + b^5)*cosh(d*x + c)^6 + 5*a^4*b + 10*a^2*b^3 + b^5 + 15*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 9*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
7 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 3*(5*a^4*b + 10*a^2*b^
3 + b^5)*cosh(d*x + c)^3 + (5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh
(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^5
 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^7 +
 9*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 1
0*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^5 + 3*(30*a^3*b^2 + 20*a^2*b^
3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d*x)*cosh(d*x + c)^3 + (45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5
+ 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^
7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)
^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d
*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.49

$$\int (a+b\tanh(c+dx))^5\,dx$$

$$= \begin{cases} a^5x + 5a^4bx - \frac{5a^4b\log(\tanh(c+dx)+1)}{d} + 10a^3b^2x - \frac{10a^3b^2\tanh(c+dx)}{d} + 10a^2b^3x - \frac{10a^2b^3\log(\tanh(c+dx)+1)}{d} - \frac{5a^2b^3}{} \\ x(a+b\tanh(c))^5 \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c))**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*log(tanh(c + d*x) + 1)/d + 10*a**
3*b**2*x - 10*a**3*b**2*tanh(c + d*x)/d + 10*a**2*b**3*x - 10*a**2*b**3*log
(tanh(c + d*x) + 1)/d - 5*a**2*b**3*tanh(c + d*x)**2/d + 5*a*b**4*x - 5*a*b
**4*tanh(c + d*x)**3/(3*d) - 5*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*log(t
anh(c + d*x) + 1)/d - b**5*tanh(c + d*x)**4/(4*d) - b**5*tanh(c + d*x)**2/(
2*d), Ne(d, 0)), (x*(a + b*tanh(c))**5, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(136) = 272.

Time = 0.29 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.18

$$\int (a+b\tanh(c+dx))^5\,dx$$

$$= \frac{5}{3}ab^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right)$$
$$+ b^5\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)}\right)$$
$$+ 10a^2b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right)$$
$$+ 10a^3b^2\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^5x + \frac{5a^4b\log(\cosh(dx+c))}{d}$$

```
[In] integrate((a+b*tanh(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d
*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + b^5*(
x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4
*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(
-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 10*a^2*b^3*(x + c/d + log(e^(-2*d
*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c) + 1))) + 10*a^3*b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x +
 5*a^4*b*log(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.58

$$\int (a+b\tanh(c+dx))^5\,dx$$
$$=\frac{3\left(a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5\right)(dx+c)+3\left(5\,a^4b+10\,a^2b^3+b^5\right)\log\left(e^{(2\,dx+2\,c)}+1\right)+\frac{4\,(15}{}}{3}$$

```
[In] integrate((a+b*tanh(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)
+ 3*(5*a^4*b + 10*a^2*b^3 + b^5)*log(e^(2*d*x + 2*c) + 1) + 4*(15*a^3*b^2 +
 10*a*b^4 + 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*c) + 3*(
15*a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) + (45*a^3*b^2 + 1
5*a^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.08

$$\int (a+b\tanh(c+dx))^5\,dx = x\left(a^5+5\,a^4\,b+10\,a^3\,b^2+10\,a^2\,b^3+5\,a\,b^4+b^5\right) - \frac{5\tanh(c+d\,x)\,\left(2\,a^3\,b^2+a\,b^4\right)}{d} - \frac{b^5\tanh(c+d\,x)^4}{4\,d} - \frac{\ln\left(\tanh(c+d\,x)+1\right)\,\left(5\,a^4\,b+10\,a^2\,b^3+b^5\right)}{d} - \frac{\tanh(c+d\,x)^2\left(10\,a^2\,b^3+b^5\right)}{2\,d} - \frac{5\,a\,b^4\tanh(c+d\,x)^3}{3\,d}$$

```
[In] int((a + b*tanh(c + d*x))^5,x)
```

```
[Out] x*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2) - (5*tanh(c + d
*x)*(a*b^4 + 2*a^3*b^2))/d - (b^5*tanh(c + d*x)^4)/(4*d) - (log(tanh(c + d*
x) + 1)*(5*a^4*b + b^5 + 10*a^2*b^3))/d - (tanh(c + d*x)^2*(b^5 + 10*a^2*b^
3))/(2*d) - (5*a*b^4*tanh(c + d*x)^3)/(3*d)
```

3.58 $\int (a+b\tanh(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int (a+b\tanh(c+dx))^4\,dx = (a^4+6a^2b^2+b^4)\,x+\frac{4ab(a^2+b^2)\log(\cosh(c+dx))}{d} - \frac{b^2(3a^2+b^2)\tanh(c+dx)}{d} - \frac{ab(a+b\tanh(c+dx))^2}{d} - \frac{b(a+b\tanh(c+dx))^3}{3d}$$

```
[Out] (a^4+6*a^2*b^2+b^4)*x+4*a*b*(a^2+b^2)*ln(cosh(d*x+c))/d-b^2*(3*a^2+b^2)*tan
h(d*x+c)/d-a*b*(a+b*tanh(d*x+c))^2/d-1/3*b*(a+b*tanh(d*x+c))^3/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3563, 3609, 3606, 3556}

$$\int (a+b\tanh(c+dx))^4\,dx = -\frac{b^2(3a^2+b^2)\tanh(c+dx)}{d} + \frac{4ab(a^2+b^2)\log(\cosh(c+dx))}{d} + x\left(a^4+6a^2b^2+b^4\right) - \frac{b(a+b\tanh(c+dx))^3}{3d} - \frac{ab(a+b\tanh(c+dx))^2}{d}$$

```
[In] Int[(a + b*Tanh[c + d*x])^4,x]
```

```
[Out] (a^4 + 6*a^2*b^2 + b^4)*x + (4*a*b*(a^2 + b^2)*Log[Cosh[c + d*x]])/d - (b^2
*(3*a^2 + b^2)*Tanh[c + d*x])/d - (a*b*(a + b*Tanh[c + d*x])^2)/d - (b*(a +
 b*Tanh[c + d*x])^3)/(3*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b(a+b\tanh(c+dx))^3}{3d}+\int (a+b\tanh(c+dx))^2\left(a^2+b^2+2ab\tanh(c+dx)\right)\,dx\\
&= -\frac{ab(a+b\tanh(c+dx))^2}{d}-\frac{b(a+b\tanh(c+dx))^3}{3d}\\
&\quad+\int (a+b\tanh(c+dx))\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\tanh(c+dx)\right)\,dx\\
&= \left(a^4+6a^2b^2+b^4\right)x-\frac{b^2(3a^2+b^2)\tanh(c+dx)}{d}-\frac{ab(a+b\tanh(c+dx))^2}{d}\\
&\quad-\frac{b(a+b\tanh(c+dx))^3}{3d}+\left(4ab\left(a^2+b^2\right)\right)\int \tanh(c+dx)\,dx\\
&= \left(a^4+6a^2b^2+b^4\right)x+\frac{4ab(a^2+b^2)\log(\cosh(c+dx))}{d}\\
&\quad-\frac{b^2(3a^2+b^2)\tanh(c+dx)}{d}-\frac{ab(a+b\tanh(c+dx))^2}{d}-\frac{b(a+b\tanh(c+dx))^3}{3d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int (a+b\tanh(c+dx))^4\,dx =$$

$$-\frac{3(a+b)^4\log(1-\tanh(c+dx))-3(a-b)^4\log(1+\tanh(c+dx))+6b^2(6a^2+b^2)\tanh(c+dx)+12ab^3}{6d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^4,x]
```

```
[Out] -1/6*(3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x
]] + 6*b^2*(6*a^2 + b^2)*Tanh[c + d*x] + 12*a*b^3*Tanh[c + d*x]^2 + 2*b^4*T
anh[c + d*x]^3)/d
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.31

method	result
parallelrisch	$-\left(b^4\tanh(dx+c)^3-3a^4dx+12a^3bdx-18a^2b^2dx+12a\,b^3dx-3b^4dx+6a\,b^3\tanh(dx+c)^2+12\ln(1-\tanh(dx+c))a^3b+12\ln(1\right.$ … $/(3d)$
derivativedivides	$\frac{-\frac{b^4\tanh(dx+c)^3}{3}-2a\,b^3\tanh(dx+c)^2-6a^2b^2\tanh(dx+c)-b^4\tanh(dx+c)-\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\ln(\tanh(dx+c)-1)}{2}+\ldots}{d}$
default	$\frac{-\frac{b^4\tanh(dx+c)^3}{3}-2a\,b^3\tanh(dx+c)^2-6a^2b^2\tanh(dx+c)-b^4\tanh(dx+c)-\frac{\left(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4\right)\ln(\tanh(dx+c)-1)}{2}+\ldots}{d}$
parts	$x\,a^4+\frac{b^4\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{4a\,b^3\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)}{2}\ldots\right.}{d}$
risch	$x\,a^4-4b\,a^3x+6a^2b^2x-4b^3ax+b^4x-\frac{8b\,a^3c}{d}-\frac{8b^3ac}{d}+\frac{4b^2\left(9a^2e^{4dx+4c}+6ab\,e^{4dx+4c}+3\,e^{4dx+4c}b^2+\ldots\right.}{\ldots}$

```
[In] int((a+b*tanh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b^4*tanh(d*x+c)^3-3*a^4*d*x+12*a^3*b*d*x-18*a^2*b^2*d*x+12*a*b^3*d*x-
3*b^4*d*x+6*a*b^3*tanh(d*x+c)^2+12*ln(1-tanh(d*x+c))*a^3*b+12*ln(1-tanh(d*x
+c))*a*b^3+18*a^2*b^2*tanh(d*x+c)+3*b^4*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. 2(99) = 198.

Time = 0.26 (sec) , antiderivative size = 1389, normalized size of antiderivative = 13.75

$$\int (a + b\tanh(c + dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 + 18
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c
)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*sinh(d*x + c)^6 + 3
*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4
)*d*x*cosh(d*x + c)^2 + 12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 + 36*a^2*b^2 + 8*b^4 + 12*(5
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 + (12*a^2*
b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x + 6*(12*a^2*b^2 + 8*a*b^3 + 4*b^4
+ 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b + a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6 + 3*(a^3*b + a*b^
3)*cosh(d*x + c)^4 + 3*(a^3*b + a*b^3 + 5*(a^3*b + a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + a^3*b + a*b^3 + 4*(5*(a^3*b + a*b^3)*cosh(d*x + c)^3 + 3*
(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3*b + a*b^3)*cosh(d*x
 + c)^2 + 3*(5*(a^3*b + a*b^3)*cosh(d*x + c)^4 + a^3*b + a*b^3 + 6*(a^3*b +
 a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((a^3*b + a*b^3)*cosh(d*x + c)
^5 + 2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*cosh(d*x + c))*sin
h(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 + 2*(12*a^2*b^
2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*co
sh(d*x + c)^3 + (24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4
 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 3
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x +
 c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 + 2
*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.43

$$\int (a+b\tanh(c+dx))^4\,dx$$
$$= \begin{cases} a^4x + 4a^3bx - \frac{4a^3b\log(\tanh(c+dx)+1)}{d} + 6a^2b^2x - \frac{6a^2b^2\tanh(c+dx)}{d} + 4ab^3x - \frac{4ab^3\log(\tanh(c+dx)+1)}{d} - \frac{2ab^3\tanh^2(c+\cdots}{d} \\ x(a+b\tanh(c))^4 \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c))**4,x)
```

```
[Out] Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 6*a**2
*b**2*x - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*x - 4*a*b**3*log(tanh(c +
d*x) + 1)/d - 2*a*b**3*tanh(c + d*x)**2/d + b**4*x - b**4*tanh(c + d*x)**3/
(3*d) - b**4*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c))**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(99) = 198.

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.99

$$\int (a+b\tanh(c+dx))^4\,dx$$
$$= \frac{1}{3}b^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2\right)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) + 4ab^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right) + 6a^2b^2\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^4x + \frac{4a^3b\log(\cosh(dx+c))}{d}$$

```
[In] integrate((a+b*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a*b^3
*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*
x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 6*a^2*b^2*(x + c/d - 2/(d*(e^(-2*d*x -
 2*c) + 1))) + a^4*x + 4*a^3*b*log(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.50

$$\int (a+b\tanh(c+dx))^4\,dx$$
$$=\frac{3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)(dx+c)+12\left(a^3b+ab^3\right)\log\left(e^{(2dx+2c)}+1\right)+\frac{4\left(9a^2b^2+2b^4+3\left(3a^2b^2+2ab^3\cdots\right.\right.}{\cdots}}{3d}$$

```
[In] integrate((a+b*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c) + 12*(a^3*b +
a*b^3)*log(e^(2*d*x + 2*c) + 1) + 4*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^2 + 2*a
*b^3 + b^4)*e^(4*d*x + 4*c) + 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*x + 2*c)
)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

$$\int (a+b\tanh(c+dx))^4\,dx = x\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)-\frac{b^4\tanh(c+dx)^3}{3d}-\frac{\ln\left(\tanh(c+dx)+1\right)\left(4a^3b+4ab^3\right)}{d}-\frac{2ab^3\tanh(c+dx)^2}{d}-\frac{b^2\tanh(c+dx)\left(6a^2+b^2\right)}{d}$$

```
[In] int((a + b*tanh(c + d*x))^4,x)
```

```
[Out] x*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) - (b^4*tanh(c + d*x)^3)/(3*d)
 - (log(tanh(c + d*x) + 1)*(4*a*b^3 + 4*a^3*b))/d - (2*a*b^3*tanh(c + d*x)^
2)/d - (b^2*tanh(c + d*x)*(6*a^2 + b^2))/d
```

3.59 $\int (a+b\tanh(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int (a+b\tanh(c+dx))^3\,dx = a\left(a^2+3b^2\right)x+\frac{b(3a^2+b^2)\log(\cosh(c+dx))}{d}-\frac{2ab^2\tanh(c+dx)}{d}-\frac{b(a+b\tanh(c+dx))^2}{2d}$$

```
[Out] a*(a^2+3*b^2)*x+b*(3*a^2+b^2)*ln(cosh(d*x+c))/d-2*a*b^2*tanh(d*x+c)/d-1/2*b
*(a+b*tanh(d*x+c))^2/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3563, 3606, 3556}

$$\int (a+b\tanh(c+dx))^3\,dx = \frac{b(3a^2+b^2)\log(\cosh(c+dx))}{d}+ax\left(a^2+3b^2\right)-\frac{2ab^2\tanh(c+dx)}{d}-\frac{b(a+b\tanh(c+dx))^2}{2d}$$

```
[In] Int[(a + b*Tanh[c + d*x])^3,x]
```

```
[Out] a*(a^2 + 3*b^2)*x + (b*(3*a^2 + b^2)*Log[Cosh[c + d*x]])/d - (2*a*b^2*Tanh[
c + d*x])/d - (b*(a + b*Tanh[c + d*x])^2)/(2*d)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{b(a+b\tanh(c+dx))^2}{2d} + \int (a+b\tanh(c+dx))\left(a^2+b^2+2ab\tanh(c+dx)\right)\,dx$$

$$= a\left(a^2+3b^2\right)x - \frac{2ab^2\tanh(c+dx)}{d} - \frac{b(a+b\tanh(c+dx))^2}{2d} + \left(b\left(3a^2+b^2\right)\right)\int \tanh(c+dx)\,dx$$

$$= a\left(a^2+3b^2\right)x + \frac{b(3a^2+b^2)\log(\cosh(c+dx))}{d} - \frac{2ab^2\tanh(c+dx)}{d} - \frac{b(a+b\tanh(c+dx))^2}{2d}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (a+b\tanh(c+dx))^3\,dx = -\frac{(a+b)^3\log(1-\tanh(c+dx)) - (a-b)^3\log(1+\tanh(c+dx)) + 6ab^2\tanh(c+dx) + b^3\tanh^2(c+dx)}{2d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^3,x]
[Out] -1/2*((a + b)^3*Log[1 - Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh[c + d*x]] +
 6*a*b^2*Tanh[c + d*x] + b^3*Tanh[c + d*x]^2)/d
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.35

method	result
derivativedivides	$\frac{-\frac{b^{3} \tanh(dx+c)^{2}}{2}-3a\,b^{2}\tanh(dx+c)-\frac{\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b^{3} \tanh(dx+c)^{2}}{2}-3a\,b^{2}\tanh(dx+c)-\frac{\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^{3}-3a^{2}b+3a\,b^{2}-b^{3}\right)\ln(\tanh(dx+c)+1)}{2}}{d}$
parallelrisch	$-\frac{-2a^{3}dx+6a^{2}bdx-6a\,b^{2}dx+2b^{3}dx+b^{3}\tanh(dx+c)^{2}+6\ln(1-\tanh(dx+c))a^{2}b+2\ln(1-\tanh(dx+c))b^{3}+6a\,b^{2}\tanh(dx+}{2d}$
parts	$a^{3}x+\frac{b^{3}\left(-\frac{\tanh(dx+c)^{2}}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{3a\,b^{2}\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(d}{2}\right.}{d}$
risch	$a^{3}x-3b\,a^{2}x+3a\,b^{2}x-b^{3}x-\frac{6bc\,a^{2}}{d}-\frac{2b^{3}c}{d}+\frac{2b^{2}\left(3\,e^{2dx+2c}a+b\,e^{2dx+2c}+3a\right)}{d\left(e^{2dx+2c}+1\right)^{2}}+\frac{3b\ln\left(e^{2dx+2c}+1\right)a^{2}}{d}$

```
[In] int((a+b*tanh(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2*b^3*tanh(d*x+c)^2-3*a*b^2*tanh(d*x+c)-1/2*(a^3+3*a^2*b+3*a*b^2+b^
3)*ln(tanh(d*x+c)-1)+1/2*(a^3-3*a^2*b+3*a*b^2-b^3)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 646, normalized size of antiderivative = 9.36

$$\int (a+b\tanh(c+dx))^3\,dx$$

$$=\frac{(a^3-3\,a^2b+3\,ab^2-b^3)dx\cosh{(dx+c)}^4+4\,(a^3-3\,a^2b+3\,ab^2-b^3)dx\cosh{(dx+c)}\sinh{(dx+c)}^3+(a^3}{}$$

```
[In] integrate((a+b*tanh(d*x+c))^3,x, algorithm="fricas")

[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x
+ 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 +
 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 + 3*a*b^2 + b^3 +
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b^3)*co
sh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b
+ b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 + 2*(3*a^2*b + b^3)*cosh(d*x + c)^2
+ 2*(3*a^2*b + b^3 + 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4
*((3*a^2*b + b^3)*cosh(d*x + c)^3 + (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x
 + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^3 - 3*a
```

```
^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 + (3*a*b^2 + b^3 + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4
*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2
+ 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*co
sh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.45

$$\int (a+b\tanh(c+dx))^3\,dx = \begin{cases} a^3x+3a^2bx-\frac{3a^2b\log(\tanh(c+dx)+1)}{d}+3ab^2x-\frac{3ab^2\tanh(c+dx)}{d}+b^3x-\frac{b^3\log(\tanh(c+dx)+1)}{d}-\frac{b^3\tanh^2(c+dx)}{2d} & \text{f} \\ x(a+b\tanh(c))^3 & \text{o} \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d + 3*a*b*
*2*x - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*log(tanh(c + d*x) + 1)/d -
b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c))**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.71

$$\int (a+b\tanh(c+dx))^3\,dx = b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) + 3\,ab^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^3x+\frac{3\,a^2b\log\left(\cosh\left(dx+c\right)\right)}{d}$$

```
[In] integrate((a+b*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a*b^2*(x + c/d - 2/(d*(e^(-2*d*x
- 2*c) + 1))) + a^3*x + 3*a^2*b*log(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int (a + b\tanh(c + dx))^3\,dx$$
$$= \frac{(a^3 - 3\,a^2b + 3\,ab^2 - b^3)(dx + c) + (3\,a^2b + b^3)\log\left(e^{(2\,dx + 2\,c)} + 1\right) + \frac{2\left(3\,ab^2 + \left(3\,ab^2 + b^3\right)e^{(2\,dx + 2\,c)}\right)}{\left(e^{(2\,dx + 2\,c)} + 1\right)^2}}{d}$$

```
[In] integrate((a+b*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c) + (3*a^2*b + b^3)*log(e^(2*d*x +
 2*c) + 1) + 2*(3*a*b^2 + (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c)
 + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int (a + b\tanh(c + dx))^3\,dx = x\left(a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3\right) - \frac{\ln\left(\tanh(c + d\,x) + 1\right)\,\left(3\,a^2\,b + b^3\right)}{d} - \frac{b^3\tanh(c + d\,x)^2}{2\,d} - \frac{3\,a\,b^2\tanh(c + d\,x)}{d}$$

```
[In] int((a + b*tanh(c + d*x))^3,x)
```

```
[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (log(tanh(c + d*x) + 1)*(3*a^2*b + b^3)
)/d - (b^3*tanh(c + d*x)^2)/(2*d) - (3*a*b^2*tanh(c + d*x))/d
```

3.60 $\int (a+b\tanh(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int (a+b\tanh(c+dx))^2\,dx = (a^2+b^2)\,x + \frac{2ab\log(\cosh(c+dx))}{d} - \frac{b^2\tanh(c+dx)}{d}$$

```
[Out] (a^2+b^2)*x+2*a*b*ln(cosh(d*x+c))/d-b^2*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3558, 3556}

$$\int (a+b\tanh(c+dx))^2\,dx = x(a^2+b^2) + \frac{2ab\log(\cosh(c+dx))}{d} - \frac{b^2\tanh(c+dx)}{d}$$

```
[In] Int[(a + b*Tanh[c + d*x])^2,x]
```

```
[Out] (a^2 + b^2)*x + (2*a*b*Log[Cosh[c + d*x]])/d - (b^2*Tanh[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a^2+b^2\right)x-\frac{b^2\tanh(c+dx)}{d}+(2ab)\int\tanh(c+dx)\,dx\\ &= \left(a^2+b^2\right)x+\frac{2ab\log(\cosh(c+dx))}{d}-\frac{b^2\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int(a+b\tanh(c+dx))^2\,dx = \frac{-(a+b)^2\log(1-\tanh(c+dx))+(a-b)^2\log(1+\tanh(c+dx))-2b^2\tanh(c+dx)}{2d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^2,x]
[Out] (-((a + b)^2*Log[1 - Tanh[c + d*x]]) + (a - b)^2*Log[1 + Tanh[c + d*x]] - 2
*b^2*Tanh[c + d*x])/(2*d)
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

method	result	size
parallelrisch	$-\frac{-a^2dx+2abdx-b^2dx+2\ln(1-\tanh(dx+c))ab+b^2\tanh(dx+c)}{d}$	52
parts	$a^2x+\frac{b^2\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{2ab\ln(\cosh(dx+c))}{d}$	59
derivativedivides	$\frac{-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2-2ab+b^2\right)\ln(\tanh(dx+c)+1)}{2}}{d}$	61
default	$\frac{-b^2\tanh(dx+c)-\frac{\left(a^2+2ab+b^2\right)\ln(\tanh(dx+c)-1)}{2}+\frac{\left(a^2-2ab+b^2\right)\ln(\tanh(dx+c)+1)}{2}}{d}$	61
risch	$a^2x-2abx+b^2x-\frac{4abc}{d}+\frac{2b^2}{d\left(e^{2dx+2c}+1\right)}+\frac{2ab\ln\left(e^{2dx+2c}+1\right)}{d}$	65

```
[In] int((a+b*tanh(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] -(-a^2*d*x+2*a*b*d*x-b^2*d*x+2*ln(1-tanh(d*x+c))*a*b+b^2*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(38) = 76$.

Time = 0.24 (sec) , antiderivative size = 201, normalized size of antiderivative = 5.29

$$\int (a+b\tanh(c+dx))^2\,dx = \frac{(a^2-2\,ab+b^2)dx\cosh{(dx+c)}^2+2\,(a^2-2\,ab+b^2)dx\cosh{(dx+c)}\sinh{(dx+c)}+(a^2-2\,ab+b^2)dx\,\mathrm{s}}{d\cosh{(dx+c}}$$

```
[In] integrate((a+b*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 + (a^2 - 2*
a*b + b^2)*d*x + 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(
d*x + c) + a*b*sinh(d*x + c)^2 + a*b)*log(2*cosh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*s
inh(d*x + c)^2 + d)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int (a+b\tanh(c+dx))^2\,dx = \begin{cases} a^2x+2abx-\frac{2ab\log{(\tanh{(c+dx)}+1)}}{d}+b^2x-\frac{b^2\tanh{(c+dx)}}{d} & \text{for } d\neq 0 \\ x(a+b\tanh{(c)})^2 & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d + b**2*x - b**
2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c))**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+b\tanh(c+dx))^2\,dx = b^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^2x+\frac{2\,ab\log{(\cosh{(dx+c)})}}{d}$$

```
[In] integrate((a+b*tanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x + 2*a*b*log(cosh(d*x +
 c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int (a+b\tanh(c+dx))^2\,dx = \frac{2\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)+(a^2-2\,ab+b^2)(dx+c)+\frac{2\,b^2}{e^{(2\,dx+2\,c)}+1}}{d}$$

```
[In] integrate((a+b*tanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b*log(e^(2*d*x + 2*c) + 1) + (a^2 - 2*a*b + b^2)*(d*x + c) + 2*b^2/(e^
(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int (a+b\tanh(c+dx))^2\,dx = x\,\left(a^2+2\,a\,b+b^2\right)-\frac{b^2\tanh(c+d\,x)}{d}-\frac{2\,a\,b\,\ln\left(\tanh(c+d\,x)+1\right)}{d}$$

```
[In] int((a + b*tanh(c + d*x))^2,x)
```

```
[Out] x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x))/d - (2*a*b*log(tanh(c + d*x) +
1))/d
```

3.61 $\int \frac{1}{a+b\tanh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 50

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] a*x/(a^2-b^2)-b*ln(a*cosh(d*x+c)+b*sinh(d*x+c))/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3565, 3611}

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\,(a^2-b^2)}$$

```
[In] Int[(a + b*Tanh[c + d*x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[c + d*x] + b*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
```

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib-ia\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{a^2-b^2} \\ &= \frac{ax}{a^2-b^2} - \frac{b\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)\,d} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx = \frac{(-a+b)\log(1-\tanh(c+dx))+(a+b)\log(1+\tanh(c+dx))-2b\log(a+b\tanh(c+dx))}{2(a-b)(a+b)d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-1),x]
[Out] ((-a + b)*Log[1 - Tanh[c + d*x]] + (a + b)*Log[1 + Tanh[c + d*x]] - 2*b*Log
[a + b*Tanh[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$-\frac{-adx-dxb-\ln(1-\tanh(dx+c))b+b\ln(a+b\tanh(dx+c))}{d(a^2-b^2)}$	55
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{b\ln(a+b\tanh(dx+c))}{(a-b)(a+b)}+\frac{\ln(\tanh(dx+c)+1)}{2a-2b}}{d}$	71
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2a+2b}-\frac{b\ln(a+b\tanh(dx+c))}{(a-b)(a+b)}+\frac{\ln(\tanh(dx+c)+1)}{2a-2b}}{d}$	71
risch	$\frac{x}{a+b}+\frac{2xb}{a^2-b^2}+\frac{2bc}{d(a^2-b^2)}-\frac{b\ln\left(e^{2dx+2c}+\frac{a-b}{a+b}\right)}{d(a^2-b^2)}$	81

```
[In] int(1/(a+b*tanh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -(-a*d*x-d*x*b-ln(1-tanh(d*x+c))*b+b*ln(a+b*tanh(d*x+c)))/d/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx = \frac{(a+b)dx - b\log\left(\frac{2\,(a\cosh(dx+c)+b\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2-b^2)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)),x, algorithm="fricas")
[Out] ((a + b)*d*x - b*log(2*(a*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 - b^2)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(37) = 74.

Time = 1.01 (sec) , antiderivative size = 224, normalized size of antiderivative = 4.48

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\tanh(c)} & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{x}{a} & \text{for } b=0 \\ -\frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)-2bd}+\frac{dx}{2bd\tanh(c+dx)-2bd}+\frac{1}{2bd\tanh(c+dx)-2bd} & \text{for } a=-b \\ \frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)+2bd}+\frac{dx}{2bd\tanh(c+dx)+2bd}-\frac{1}{2bd\tanh(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\tanh(c)} & \text{for } d=0 \\ \frac{adx}{a^2d-b^2d}-\frac{bdx}{a^2d-b^2d}-\frac{b\log\left(\frac{a}{b}+\tanh(c+dx)\right)}{a^2d-b^2d}+\frac{b\log(\tanh(c+dx)+1)}{a^2d-b^2d} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*tanh(d*x+c)),x)
[Out] Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0)),
 (-d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) - 2*b*d) + d*x/(2*b*d*tanh(c + d*
x) - 2*b*d) + 1/(2*b*d*tanh(c + d*x) - 2*b*d), Eq(a, -b)), (d*x*tanh(c + d*
x)/(2*b*d*tanh(c + d*x) + 2*b*d) + d*x/(2*b*d*tanh(c + d*x) + 2*b*d) - 1/(2
*b*d*tanh(c + d*x) + 2*b*d), Eq(a, b)), (x/(a + b*tanh(c)), Eq(d, 0)), (a*d
*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b**2*d) - b*log(a/b + tanh(c + d*x))
/(a**2*d - b**2*d) + b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,dx-2\,c)}-a-b\right)}{(a^2-b^2)d}+\frac{dx+c}{(a+b)d}$$

[In] integrate(1/(a+b*tanh(d*x+c)),x, algorithm="maxima")

[Out] -b*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^2 - b^2)*d) + (d*x + c)/((a + b)*d)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx = -\frac{\frac{b\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^2-b^2}-\frac{dx+c}{a-b}}{d}$$

[In] integrate(1/(a+b*tanh(d*x+c)),x, algorithm="giac")

[Out] -(b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b))/(a^2 - b^2) - (d*x + c)/(a - b))/d

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{1}{a+b\tanh(c+dx)}\,dx = \frac{a\,x-b\,x}{a^2-b^2}+\frac{b\left(\ln\left(\tanh(c+d\,x)+1\right)-\ln\left(a+b\tanh(c+d\,x)\right)\right)}{d\,\left(a^2-b^2\right)}$$

[In] int(1/(a + b*tanh(c + d*x)),x)

[Out] (a*x - b*x)/(a^2 - b^2) + (b*(log(tanh(c + d*x) + 1) - log(a + b*tanh(c + d*x))))/(d*(a^2 - b^2))

3.62 $\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{2ab\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)^2\,d} + \frac{b}{(a^2-b^2)\,d(a+b\tanh(c+dx))}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2-2*a*b*ln(a*cosh(d*x+c)+b*sinh(d*x+c))/(a^2-b^2)^2/d
+b/(a^2-b^2)/d/(a+b*tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3564, 3612, 3611}

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = \frac{b}{d\,(a^2-b^2)\,(a+b\tanh(c+dx))} - \frac{2ab\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\,(a^2-b^2)^2} + \frac{x(a^2+b^2)}{(a^2-b^2)^2}$$

```
[In] Int[(a + b*Tanh[c + d*x])^(-2),x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 - (2*a*b*Log[a*Cosh[c + d*x] + b*Sinh[c + d*x
]])/((a^2 - b^2)^2*d) + b/((a^2 - b^2)*d*(a + b*Tanh[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
```

```
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = \frac{b}{(a^2-b^2)\,d(a+b\tanh(c+dx))} + \frac{\int \frac{a-b\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{a^2-b^2}$$
$$= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,d(a+b\tanh(c+dx))} - \frac{(2iab)\int \frac{-ib-ia\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{(a^2-b^2)^2}$$
$$= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} - \frac{2ab\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)^2\,d} + \frac{b}{(a^2-b^2)\,d(a+b\tanh(c+dx))}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = \frac{-\frac{\log(1-\tanh(c+dx))}{(a+b)^2} + \frac{\log(1+\tanh(c+dx))}{(a-b)^2} + \frac{2b\left(-2a\log(a+b\tanh(c+dx))+\frac{a^2-b^2}{a+b\tanh(c+dx)}\right)}{(a^2-b^2)^2}}{2d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-2),x]
```

```
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 + (
2*b*(-2*a*Log[a + b*Tanh[c + d*x]] + (a^2 - b^2)/(a + b*Tanh[c + d*x])))/(a
^2 - b^2)^2)/(2*d)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{b}{(a-b)(a+b)(a+b\tanh(dx+c))}-\frac{2ab\ln(a+b\tanh(dx+c))}{(a+b)^2(a-b)^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a-b)^2}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^2}+\frac{b}{(a-b)(a+b)(a+b\tanh(dx+c))}-\frac{2ab\ln(a+b\tanh(dx+c))}{(a+b)^2(a-b)^2}+\frac{\ln(\tanh(dx+c)+1)}{2(a-b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{4abx}{a^4-2a^2b^2+b^4}+\frac{4abc}{d(a^4-2a^2b^2+b^4)}+\frac{2b^2}{(a-b)d(a^2+2ab+b^2)(e^{2dx+2c}a+b\,e^{2dx+2c}+a-b)}-\frac{2ab\ln\left(e\right.}{d(a^4-}$
parallelrisch	$-\frac{a^2b^2\tanh(dx+c)-b^4\tanh(dx+c)-a^2b^2dx-a^4dx-2a^3bdx-2\ln(1-\tanh(dx+c))a^3b+2\ln(a+b\tanh(dx+c))a^3b-2\ln(}{(a^4-2a^2}$

```
[In] int(1/(a+b*tanh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^2*ln(tanh(d*x+c)-1)+b/(a-b)/(a+b)/(a+b*tanh(d*x+c))-2*a*b/(
a+b)^2/(a-b)^2*ln(a+b*tanh(d*x+c))+1/2/(a-b)^2*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(85) = 170.

Time = 0.25 (sec) , antiderivative size = 422, normalized size of antiderivative = 4.96

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx$$
$$=\frac{(a^3+3\,a^2b+3\,ab^2+b^3)dx\cosh{(dx+c)}^2+2\,(a^3+3\,a^2b+3\,ab^2+b^3)dx\cosh{(dx+c)}\sinh{(dx+c)}+(a}{(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5)d\cosh{(dx+c}}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x*sinh(d*x + c)^2 + 2*a*b^2 - 2*b^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*
x - 2*(a^2*b - a*b^2 + (a^2*b + a*b^2)*cosh(d*x + c)^2 + 2*(a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c) + (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(a*cos
h(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a
^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^2 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1389 vs. 2(70) = 140.

Time = 9.81 (sec) , antiderivative size = 1389, normalized size of antiderivative = 16.34

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**2, Eq(b
, 0)), (d*x*tanh(c + d*x)**2/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*tanh(c +
 d*x) + 4*b**2*d) - 2*d*x*tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2
*d*tanh(c + d*x) + 4*b**2*d) + d*x/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*ta
nh(c + d*x) + 4*b**2*d) - tanh(c + d*x)/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2
*d*tanh(c + d*x) + 4*b**2*d) + 2/(4*b**2*d*tanh(c + d*x)**2 - 8*b**2*d*tanh
(c + d*x) + 4*b**2*d), Eq(a, -b)), (d*x*tanh(c + d*x)**2/(4*b**2*d*tanh(c +
 d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + 2*d*x*tanh(c + d*x)/(4*b**2
*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) + d*x/(4*b**2*d*ta
nh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - tanh(c + d*x)/(4*b**2
*d*tanh(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d) - 2/(4*b**2*d*tanh
(c + d*x)**2 + 8*b**2*d*tanh(c + d*x) + 4*b**2*d), Eq(a, b)), (x/(a + b*tan
h(c))**2, Eq(d, 0)), (a**3*d*x/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b*
*2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) + a**
2*b*d*x*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*
a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) - 2*a**2*b*d*x
/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c +
d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) - 2*a**2*b*log(a/b + tanh(c + d*x))
/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c +
d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) + 2*a**2*b*log(tanh(c + d*x) + 1)/(
a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*
x) + a*b**4*d + b**5*d*tanh(c + d*x)) + a**2*b/(a**5*d + a**4*b*d*tanh(c +
d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh
(c + d*x)) - 2*a*b**2*d*x*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) -
2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*
x)) + a*b**2*d*x/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*
b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) - 2*a*b**2*log(a/b
+ tanh(c + d*x))*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b*
*2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) + 2*a
*b**2*log(tanh(c + d*x) + 1)*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x)
 - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c +
 d*x)) + b**3*d*x*tanh(c + d*x)/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b
**2*d - 2*a**2*b**3*d*tanh(c + d*x) + a*b**4*d + b**5*d*tanh(c + d*x)) - b*
*3/(a**5*d + a**4*b*d*tanh(c + d*x) - 2*a**3*b**2*d - 2*a**2*b**3*d*tanh(c
+ d*x) + a*b**4*d + b**5*d*tanh(c + d*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.49

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = -\frac{2\,ab\log\left(-(a-b)e^{(-2\,dx-2\,c)}-a-b\right)}{(a^4-2\,a^2b^2+b^4)d} - \frac{2\,b^2}{(a^4-2\,a^2b^2+b^4+(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,dx-2\,c)})d} + \frac{dx+c}{(a^2+2\,ab+b^2)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^4 - 2*a^2*b^2 + b^4)*d) -
 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*x
- 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = -\frac{\frac{2\,ab\log\left(\left|-ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^4-2\,a^2b^2+b^4} - \frac{dx+c}{a^2-2\,ab+b^2} - \frac{2\,(ab^2-b^3)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right)(a+b)^2(a-b)^2}}{d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*b*log(abs(-a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - a + b))/(a^4 - 2*a
^2*b^2 + b^4) - (d*x + c)/(a^2 - 2*a*b + b^2) - 2*(a*b^2 - b^3)/((a*e^(2*d*
x + 2*c) + b*e^(2*d*x + 2*c) + a - b)*(a + b)^2*(a - b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.49

$$\int \frac{1}{(a+b\tanh(c+dx))^2}\,dx = \frac{\frac{a\,x}{{(a+b)}^2}+\frac{b\,x\,\mathrm{tanh}(c+d\,x)}{{(a+b)}^2}-\frac{b^2\,\mathrm{tanh}(c+d\,x)}{a\,d\,(a^2-b^2)}}{a+b\,\mathrm{tanh}\left(c+d\,x\right)} - \frac{2\,a\,b\,\ln\left(a+b\,\mathrm{tanh}(c+d\,x)\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)} + \frac{2\,a\,b\,\ln\left(\mathrm{tanh}(c+d\,x)+1\right)}{d\,{\left(a^2-b^2\right)}^2}$$

```
[In] int(1/(a + b*tanh(c + d*x))^2,x)
```

```
[Out] ((a*x)/(a + b)^2 + (b*x*tanh(c + d*x))/(a + b)^2 - (b^2*tanh(c + d*x))/(a*d
*(a^2 - b^2)))/(a + b*tanh(c + d*x)) - (2*a*b*log(a + b*tanh(c + d*x)))/(d*
(a^4 + b^4 - 2*a^2*b^2)) + (2*a*b*log(tanh(c + d*x) + 1))/(d*(a^2 - b^2)^2)
```

3.63 $\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 129

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx = \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} - \frac{b(3a^2+b^2)\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)^3\,d} + \frac{b}{2\,(a^2-b^2)\,d(a+b\tanh(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\tanh(c+dx))}$$

```
[Out] a*(a^2+3*b^2)*x/(a^2-b^2)^3-b*(3*a^2+b^2)*ln(a*cosh(d*x+c)+b*sinh(d*x+c))/(
a^2-b^2)^3/d+1/2*b/(a^2-b^2)/d/(a+b*tanh(d*x+c))^2+2*a*b/(a^2-b^2)^2/d/(a+b
*tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3564, 3610, 3612, 3611}

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx = \frac{2ab}{d\,(a^2-b^2)^2\,(a+b\tanh(c+dx))} + \frac{b}{2d\,(a^2-b^2)\,(a+b\tanh(c+dx))^2} - \frac{b(3a^2+b^2)\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\,(a^2-b^2)^3} + \frac{ax(a^2+3b^2)}{(a^2-b^2)^3}$$

```
[In] Int[(a + b*Tanh[c + d*x])^(-3),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 - (b*(3*a^2 + b^2)*Log[a*Cosh[c + d*x] +
b*Sinh[c + d*x]])/((a^2 - b^2)^3*d) + b/(2*(a^2 - b^2)*d*(a + b*Tanh[c + d*
x])^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Tanh[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b}{2\left(a^2-b^2\right)d(a+b\tanh(c+dx))^2}+\frac{\int \frac{a-b\tanh(c+dx)}{(a+b\tanh(c+dx))^2}\,dx}{a^2-b^2} \\
&= \frac{b}{2\left(a^2-b^2\right)d(a+b\tanh(c+dx))^2} \\
&\quad+\frac{2ab}{\left(a^2-b^2\right)^2d(a+b\tanh(c+dx))}+\frac{\int \frac{a^2+b^2-2ab\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{\left(a^2-b^2\right)^2}
\end{aligned}
$$

$$= \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} + \frac{b}{2\,(a^2-b^2)\,d(a+b\tanh(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\tanh(c+dx))} - \frac{(ib(3a^2+b^2))\int \frac{-ib-ia\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{(a^2-b^2)^3}$$

$$= \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} - \frac{b(3a^2+b^2)\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)^3\,d} + \frac{b}{2\,(a^2-b^2)\,d(a+b\tanh(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\tanh(c+dx))}$$

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx = \frac{-\frac{\log(1-\tanh(c+dx))}{(a+b)^3} + \frac{\log(1+\tanh(c+dx))}{(a-b)^3} + \frac{b\left(-2(3a^2+b^2)\log(a+b\tanh(c+dx)) + \frac{(a^2-b^2)(5a^2-b^2+4ab\tanh(c+dx))}{(a+b\tanh(c+dx))^2}\right)}{(a^2-b^2)^3}}{2d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-3),x]
```

```
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^3) + Log[1 + Tanh[c + d*x]]/(a - b)^3 + (
b*(-2*(3*a^2 + b^2)*Log[a + b*Tanh[c + d*x]] + ((a^2 - b^2)*(5*a^2 - b^2 +
4*a*b*Tanh[c + d*x]))/(a + b*Tanh[c + d*x])^2))/(a^2 - b^2)^3)/(2*d)
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b}{2(a-b)(a+b)(a+b\tanh(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\tanh(dx+c))}-\frac{b(3a^2+b^2)\ln(a+b\tanh(dx+c))}{(a+b)^3(a-b)^3}+\frac{\ln(\tan}{2}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^3}+\frac{b}{2(a-b)(a+b)(a+b\tanh(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\tanh(dx+c))}-\frac{b(3a^2+b^2)\ln(a+b\tanh(dx+c))}{(a+b)^3(a-b)^3}+\frac{\ln(\tan}{2}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6b\,a^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{6bc\,a^2}{d(a^6-3a^4b^2+3a^2b^4-b^6)}+\frac{2b^3}{d(a^6-3a^4b^2-}$
parallelrisch	$-\frac{6a^2b^5+2\ln(a+b\tanh(dx+c))\tanh(dx+c)^2b^7-b^7-4x\tanh(dx+c)a\,b^6d-4x\tanh(dx+c)a^4b^3d-12x\tanh(dx+c)a^3b^4d}{}$

```
[In] int(1/(a+b*tanh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^3*ln(tanh(d*x+c)-1)+1/2*b/(a-b)/(a+b)/(a+b*tanh(d*x+c))^2+2
*a*b/(a+b)^2/(a-b)^2/(a+b*tanh(d*x+c))-b*(3*a^2+b^2)/(a+b)^3/(a-b)^3*ln(a+b
*tanh(d*x+c))+1/2/(a-b)^3*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1427 vs. $2(127) = 254$.

Time = 0.27 (sec) , antiderivative size = 1427, normalized size of antiderivative = 11.06

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c
)^4 + 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*x*sinh(d*x + c)^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + (a^5 + a
^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*x + 2*(3*a^3*b^2 - a^2*b^3 -
3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x
)*cosh(d*x + c)^2 + 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 + 3*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^2 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*sinh(d*x + c)^2 - (3*
a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + 2*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^4*b + 6*a^3*b^2 + 4*
a^2*b^3 + 2*a*b^4 + b^5)*sinh(d*x + c)^4 + 2*(3*a^4*b - 2*a^2*b^3 - b^5)*co
sh(d*x + c)^2 + 2*(3*a^4*b - 2*a^2*b^3 - b^5 + 3*(3*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^3 + (3*a^4*b - 2*a^2*b^3
 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(a*cosh(d*x + c) + b*sinh(d*x +
 c))/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^3 + (3*a^3*b^2 - a^2*b^3 - 3*a*
b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3
*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^4 + 4*(a^8 + 2*a^7*b - 2*
a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*
a^2*b^6 - 2*a*b^7 - b^8)*d*sinh(d*x + c)^4 + 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*d*cosh(d*x + c)^2 + 2*(3*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6
*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^2 + (a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sinh(d*x + c)^2 + (a^8 - 2*a^
7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)*d + 4*
((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 -
 b^8)*d*cosh(d*x + c)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d
*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6412 vs. 2(107) = 214.

Time = 13.18 (sec) , antiderivative size = 6412, normalized size of antiderivative = 49.71

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))**3,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**3, Eq(b
, 0)), (-3*d*x*tanh(c + d*x)**3/(24*b**3*d*tanh(c + d*x)**3 - 72*b**3*d*tan
h(c + d*x)**2 + 72*b**3*d*tanh(c + d*x) - 24*b**3*d) + 9*d*x*tanh(c + d*x)*
*2/(24*b**3*d*tanh(c + d*x)**3 - 72*b**3*d*tanh(c + d*x)**2 + 72*b**3*d*tan
h(c + d*x) - 24*b**3*d) - 9*d*x*tanh(c + d*x)/(24*b**3*d*tanh(c + d*x)**3 -
 72*b**3*d*tanh(c + d*x)**2 + 72*b**3*d*tanh(c + d*x) - 24*b**3*d) + 3*d*x/
(24*b**3*d*tanh(c + d*x)**3 - 72*b**3*d*tanh(c + d*x)**2 + 72*b**3*d*tanh(c
 + d*x) - 24*b**3*d) + 3*tanh(c + d*x)**2/(24*b**3*d*tanh(c + d*x)**3 - 72*
b**3*d*tanh(c + d*x)**2 + 72*b**3*d*tanh(c + d*x) - 24*b**3*d) - 9*tanh(c +
 d*x)/(24*b**3*d*tanh(c + d*x)**3 - 72*b**3*d*tanh(c + d*x)**2 + 72*b**3*d*
tanh(c + d*x) - 24*b**3*d) + 10/(24*b**3*d*tanh(c + d*x)**3 - 72*b**3*d*tan
h(c + d*x)**2 + 72*b**3*d*tanh(c + d*x) - 24*b**3*d), Eq(a, -b)), (3*d*x*ta
nh(c + d*x)**3/(24*b**3*d*tanh(c + d*x)**3 + 72*b**3*d*tanh(c + d*x)**2 + 7
2*b**3*d*tanh(c + d*x) + 24*b**3*d) + 9*d*x*tanh(c + d*x)**2/(24*b**3*d*tan
h(c + d*x)**3 + 72*b**3*d*tanh(c + d*x)**2 + 72*b**3*d*tanh(c + d*x) + 24*b
**3*d) + 9*d*x*tanh(c + d*x)/(24*b**3*d*tanh(c + d*x)**3 + 72*b**3*d*tanh(c
 + d*x)**2 + 72*b**3*d*tanh(c + d*x) + 24*b**3*d) + 3*d*x/(24*b**3*d*tanh(c
 + d*x)**3 + 72*b**3*d*tanh(c + d*x)**2 + 72*b**3*d*tanh(c + d*x) + 24*b**3
*d) - 3*tanh(c + d*x)**2/(24*b**3*d*tanh(c + d*x)**3 + 72*b**3*d*tanh(c + d
*x)**2 + 72*b**3*d*tanh(c + d*x) + 24*b**3*d) - 9*tanh(c + d*x)/(24*b**3*d*
tanh(c + d*x)**3 + 72*b**3*d*tanh(c + d*x)**2 + 72*b**3*d*tanh(c + d*x) + 2
4*b**3*d) - 10/(24*b**3*d*tanh(c + d*x)**3 + 72*b**3*d*tanh(c + d*x)**2 + 7
2*b**3*d*tanh(c + d*x) + 24*b**3*d), Eq(a, b)), (x/(a + b*tanh(c))**3, Eq(d
, 0)), (2*a**5*d*x/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tan
h(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*
d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*
b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8
*d*tanh(c + d*x)**2) + 4*a**4*b*d*x*tanh(c + d*x)/(2*a**8*d + 4*a**7*b*d*ta
nh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3
*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3
*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*
a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) - 6*a**4*b*d*x/(2*a**8*
d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2
*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4
*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2
```

```
*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) - 6*a*
*4*b*log(a/b + tanh(c + d*x))/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6
*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6
*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x
) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*
x) - 2*b**8*d*tanh(c + d*x)**2) + 6*a**4*b*log(tanh(c + d*x) + 1)/(2*a**8*d
 + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*
d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*
b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*
a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) + 5*a**
4*b/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 -
 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)
**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c +
 d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x
)**2) + 2*a**3*b**2*d*x*tanh(c + d*x)**2/(2*a**8*d + 4*a**7*b*d*tanh(c + d*
x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c
 + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*t
anh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*
tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) - 12*a**3*b**2*d*x*tanh(c + d*x)
/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*
a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2
 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*
x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**
2) + 6*a**3*b**2*d*x/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*t
anh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**
4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**
2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b*
*8*d*tanh(c + d*x)**2) - 12*a**3*b**2*log(a/b + tanh(c + d*x))*tanh(c + d*x
)/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6
*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**
2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d
*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)*
*2) + 12*a**3*b**2*log(tanh(c + d*x) + 1)*tanh(c + d*x)/(2*a**8*d + 4*a**7*
b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**
5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 1
2*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*
d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) + 4*a**3*b**2*tan
h(c + d*x)/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*
x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c
 + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*t
anh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(
c + d*x)**2) - 6*a**2*b**3*d*x*tanh(c + d*x)**2/(2*a**8*d + 4*a**7*b*d*tanh
(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d
*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b
**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*
```

```
b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) + 12*a**2*b**3*d*x*tanh(c
 + d*x)/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)*
*2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c +
d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh
(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c +
 d*x)**2) - 2*a**2*b**3*d*x/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b
**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a
**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x)
+ 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x)
 - 2*b**8*d*tanh(c + d*x)**2) - 6*a**2*b**3*log(a/b + tanh(c + d*x))*tanh(c
 + d*x)**2/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*
x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c
 + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*t
anh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(
c + d*x)**2) - 2*a**2*b**3*log(a/b + tanh(c + d*x))/(2*a**8*d + 4*a**7*b*d*
tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b*
*3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a*
*3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d -
4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) + 6*a**2*b**3*log(tan
h(c + d*x) + 1)*tanh(c + d*x)**2/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a
**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x)
- 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c +
d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c +
 d*x) - 2*b**8*d*tanh(c + d*x)**2) + 2*a**2*b**3*log(tanh(c + d*x) + 1)/(2*
a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6
*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6
*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**
2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) -
 6*a**2*b**3/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c +
d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh
(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d
*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tan
h(c + d*x)**2) + 6*a*b**4*d*x*tanh(c + d*x)**2/(2*a**8*d + 4*a**7*b*d*tanh(
c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*
tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b*
*5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b
**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) - 4*a*b**4*d*x*tanh(c + d*
x)/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 -
6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)*
*2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c +
d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)
**2) - 4*a*b**4*log(a/b + tanh(c + d*x))*tanh(c + d*x)/(2*a**8*d + 4*a**7*b
*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5
*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12
*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d
```

```
 - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) + 4*a*b**4*log(tan
h(c + d*x) + 1)*tanh(c + d*x)/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6
*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6
*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x
) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*
x) - 2*b**8*d*tanh(c + d*x)**2) - 4*a*b**4*tanh(c + d*x)/(2*a**8*d + 4*a**7
*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a*
*5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d +
12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6
*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) - 2*b**5*d*x*tan
h(c + d*x)**2/(2*a**8*d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c +
 d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tan
h(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*
d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*ta
nh(c + d*x)**2) - 2*b**5*log(a/b + tanh(c + d*x))*tanh(c + d*x)**2/(2*a**8*
d + 4*a**7*b*d*tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2
*d - 12*a**5*b**3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4
*b**4*d + 12*a**3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2
*a**2*b**6*d - 4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) + 2*b*
*5*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(2*a**8*d + 4*a**7*b*d*tanh(c +
d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b**3*d*tanh
(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a**3*b**5*d
*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d - 4*a*b**7*
d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2) + b**5/(2*a**8*d + 4*a**7*b*d*
tanh(c + d*x) + 2*a**6*b**2*d*tanh(c + d*x)**2 - 6*a**6*b**2*d - 12*a**5*b*
*3*d*tanh(c + d*x) - 6*a**4*b**4*d*tanh(c + d*x)**2 + 6*a**4*b**4*d + 12*a*
*3*b**5*d*tanh(c + d*x) + 6*a**2*b**6*d*tanh(c + d*x)**2 - 2*a**2*b**6*d -
4*a*b**7*d*tanh(c + d*x) - 2*b**8*d*tanh(c + d*x)**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(127) = 254$.

Time = 0.22 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.52

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx = -\frac{(3\,a^2b+b^3)\log\left(-(a-b)e^{(-2\,dx-2\,c)}-a-b\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)d} - \frac{2\left(3\,a^2b^2+3\,ab^3+(3\,a^2b^2-2\,a\right.}{(a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7+2\,(a^7-a^6b-3\,a^5b^2+3\,a^4b^3+3\,a^3b^4-3\,a^2b^5} + \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 + (3*a^2*b^2 - 2*a*b^3 - b^4)
```

```
*e^(-2*d*x - 2*c))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7 + 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*e^(-2*d*x - 2*c) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^
4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^(-4*d*x - 4*c))*d) + (d*x +
c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx =$$

$$-\frac{\frac{(3a^2b+b^3)\log\left(\left|-ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{dx+c}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{2\left(\left(3\,a^2b^2-4\,ab^3+b^4\right)e^{(2\,dx+2\,c)}+\frac{3\left(a^3b^2-2\,a^2b^3+ab^4\right)}{a+b}\right)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right)^2(a+b)^2(a-b)^3}}{d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((3*a^2*b + b^3)*log(abs(-a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - a + b))/
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 -
b^3) - 2*((3*a^2*b^2 - 4*a*b^3 + b^4)*e^(2*d*x + 2*c) + 3*(a^3*b^2 - 2*a^2*
b^3 + a*b^4)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)^2*(a
 + b)^2*(a - b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.36

$$\int \frac{1}{(a+b\tanh(c+dx))^3}\,dx$$

$$=\frac{\tanh(c+d\,x)\,\left(\frac{1}{a\,d}-\frac{a^4+a^2\,b^2}{a\,d\,(a^4-2\,a^2\,b^2+b^4)}\right)+\frac{a^2\,x}{(a+b)\,(a^2+2\,a\,b+b^2)}+\frac{b^2\,x\tanh(c+d\,x)^2}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}+\frac{\tanh(c+d\,x)^2\,\left(\frac{b^5}{2}-\frac{5\,a^2\,b^3}{2}\right)}{a^2\,d\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\,a\,b}{a^3+3\ldots}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)+b^2\tanh\left(c+d\,x\right)^2}$$

$$-\frac{\ln\left(a+b\tanh(c+d\,x)\right)\,\left(3\,a^2\,b+b^3\right)}{d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\ln\left(\tanh(c+d\,x)+1\right)\,\left(3\,a^2\,b+b^3\right)}{d\,\left(a^2-b^2\right)^3}$$

```
[In] int(1/(a + b*tanh(c + d*x))^3,x)
```

```
[Out] (tanh(c + d*x)*(1/(a*d) - (a^4 + a^2*b^2)/(a*d*(a^4 + b^4 - 2*a^2*b^2))) +
(a^2*x)/((a + b)*(2*a*b + a^2 + b^2)) + (b^2*x*tanh(c + d*x)^2)/(3*a*b^2 +
3*a^2*b + a^3 + b^3) + (tanh(c + d*x)^2*(b^5/2 - (5*a^2*b^3)/2))/(a^2*d*(a^
4 + b^4 - 2*a^2*b^2)) + (2*a*b*x*tanh(c + d*x))/(3*a*b^2 + 3*a^2*b + a^3 +
b^3))/(a^2 + b^2*tanh(c + d*x)^2 + 2*a*b*tanh(c + d*x)) - (log(a + b*tanh(c
 + d*x))*(3*a^2*b + b^3))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (log(ta
nh(c + d*x) + 1)*(3*a^2*b + b^3))/(d*(a^2 - b^2)^3)
```

3.64 $\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 169

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx = \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4} - \frac{4ab(a^2+b^2)\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)^4\,d} + \frac{b}{3\,(a^2-b^2)\,d(a+b\tanh(c+dx))^3} + \frac{ab}{(a^2-b^2)^2\,d(a+b\tanh(c+dx))^2} + \frac{b(3a^2+b^2)}{(a^2-b^2)^3\,d(a+b\tanh(c+dx))}$$

```
[Out] (a^4+6*a^2*b^2+b^4)*x/(a^2-b^2)^4-4*a*b*(a^2+b^2)*ln(a*cosh(d*x+c)+b*sinh(d
*x+c))/(a^2-b^2)^4/d+1/3*b/(a^2-b^2)/d/(a+b*tanh(d*x+c))^3+a*b/(a^2-b^2)^2/
d/(a+b*tanh(d*x+c))^2+b*(3*a^2+b^2)/(a^2-b^2)^3/d/(a+b*tanh(d*x+c))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {3564, 3610, 3612, 3611}

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx = \frac{b(3a^2+b^2)}{d\left(a^2-b^2\right)^3(a+b\tanh(c+dx))} + \frac{ab}{d\left(a^2-b^2\right)^2(a+b\tanh(c+dx))^2} + \frac{b}{3d\left(a^2-b^2\right)(a+b\tanh(c+dx))^3} - \frac{4ab(a^2+b^2)\log(a\cosh(c+dx)+b\sinh(c+dx))}{d\left(a^2-b^2\right)^4} + \frac{x(a^4+6a^2b^2+b^4)}{\left(a^2-b^2\right)^4}$$

```
[In] Int[(a + b*Tanh[c + d*x])^(-4),x]
```

```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 - (4*a*b*(a^2 + b^2)*Log[a*Cosh[c
 + d*x] + b*Sinh[c + d*x]])/((a^2 - b^2)^4*d) + b/(3*(a^2 - b^2)*d*(a + b*T
anh[c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Tanh[c + d*x])^2) + (b*(3*
a^2 + b^2))/((a^2 - b^2)^3*d*(a + b*Tanh[c + d*x]))
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b}{3(a^2-b^2)d(a+b\tanh(c+dx))^3}+\frac{\int \frac{a-b\tanh(c+dx)}{(a+b\tanh(c+dx))^3}\,dx}{a^2-b^2}\\
&= \frac{b}{3(a^2-b^2)d(a+b\tanh(c+dx))^3}\\
&\quad+\frac{ab}{(a^2-b^2)^2 d(a+b\tanh(c+dx))^2}+\frac{\int \frac{a^2+b^2-2ab\tanh(c+dx)}{(a+b\tanh(c+dx))^2}\,dx}{(a^2-b^2)^2}\\
&= \frac{b}{3(a^2-b^2)d(a+b\tanh(c+dx))^3}+\frac{ab}{(a^2-b^2)^2 d(a+b\tanh(c+dx))^2}\\
&\quad+\frac{b(3a^2+b^2)}{(a^2-b^2)^3 d(a+b\tanh(c+dx))}+\frac{\int \frac{a(a^2+3b^2)-b(3a^2+b^2)\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{(a^2-b^2)^3}\\
&= \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4}+\frac{b}{3(a^2-b^2)d(a+b\tanh(c+dx))^3}\\
&\quad+\frac{ab}{(a^2-b^2)^2 d(a+b\tanh(c+dx))^2}+\frac{b(3a^2+b^2)}{(a^2-b^2)^3 d(a+b\tanh(c+dx))}\\
&\quad-\frac{(4iab(a^2+b^2))\int \frac{-ib-ia\tanh(c+dx)}{a+b\tanh(c+dx)}\,dx}{(a^2-b^2)^4}\\
&= \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4}-\frac{4ab(a^2+b^2)\log(a\cosh(c+dx)+b\sinh(c+dx))}{(a^2-b^2)^4 d}\\
&\quad+\frac{b}{3(a^2-b^2)d(a+b\tanh(c+dx))^3}\\
&\quad+\frac{ab}{(a^2-b^2)^2 d(a+b\tanh(c+dx))^2}+\frac{b(3a^2+b^2)}{(a^2-b^2)^3 d(a+b\tanh(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx$$
$$= \frac{-\frac{3\log(1-\tanh(c+dx))}{(a+b)^4} + \frac{3\log(1+\tanh(c+dx))}{(a-b)^4} + \frac{2b\left(-12a(a^2+b^2)\log(a+b\tanh(c+dx)) + \frac{(a^2-b^2)\left(13a^4-2a^2b^2+b^4+3ab\left(7a^2+b^2\right)\tanh(c+dx\cdots\right)}{(a+b\tanh(c+dx))^3}\right)}{(a^2-b^2)^4}}{6d}$$

```
[In] Integrate[(a + b*Tanh[c + d*x])^(-4),x]
```

```
[Out] ((-3*Log[1 - Tanh[c + d*x]])/(a + b)^4 + (3*Log[1 + Tanh[c + d*x]])/(a - b)
^4 + (2*b*(-12*a*(a^2 + b^2)*Log[a + b*Tanh[c + d*x]] + ((a^2 - b^2)*(13*a^
4 - 2*a^2*b^2 + b^4 + 3*a*b*(7*a^2 + b^2)*Tanh[c + d*x] + 3*b^2*(3*a^2 + b^
2)*Tanh[c + d*x]^2))/(a + b*Tanh[c + d*x])^3))/(a^2 - b^2)^4)/(6*d)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{b}{3(a-b)(a+b)(a+b\tanh(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\tanh(dx+c))^2}+\frac{b\left(3a^2+b^2\right)}{(a+b)^3(a-b)^3(a+b\tanh(dx+c))}-\frac{4ba(\cdots}{\cdots}}{d}$
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{2(a+b)^4}+\frac{b}{3(a-b)(a+b)(a+b\tanh(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\tanh(dx+c))^2}+\frac{b\left(3a^2+b^2\right)}{(a+b)^3(a-b)^3(a+b\tanh(dx+c))}-\frac{4ba(\cdots}{\cdots}}{d}$
risch	$\frac{x}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}+\frac{8b\,a^3x}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{8b^3ax}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{8b\,a^3c}{d(a^8-4a^6b^2+6a^4b^4\cdots}$
parallelrisch	$-\frac{12\ln(a+b\tanh(dx+c))a^8b^2+12\ln(a+b\tanh(dx+c))a^6b^4+7\tanh(dx+c)^3a^5b^5-6\tanh(dx+c)^3a^3b^7-\tanh(dx+c)^3a\,b\cdots}{\cdots}$

```
[In] int(1/(a+b*tanh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a+b)^4*ln(tanh(d*x+c)-1)+1/3*b/(a-b)/(a+b)/(a+b*tanh(d*x+c))^3+a
*b/(a+b)^2/(a-b)^2/(a+b*tanh(d*x+c))^2+b*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(a+b*t
anh(d*x+c))-4*b*a*(a^2+b^2)/(a+b)^4/(a-b)^4*ln(a+b*tanh(d*x+c))+1/2/(a-b)^4
*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3693 vs. $2(167) = 334$.

Time = 0.33 (sec) , antiderivative size = 3693, normalized size of antiderivative = 21.85

$$\int \frac{1}{(a + b\tanh(c + dx))^4}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 +
 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^6 + 18*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a
^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)*sinh(d*
x + c)^5 + 3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
*b^5 + 7*a*b^6 + b^7)*d*x*sinh(d*x + c)^6 + 36*a^5*b^2 - 108*a^4*b^3 + 116*
a^3*b^4 - 60*a^2*b^5 + 24*a*b^6 - 8*b^7 + 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^
3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*
b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^4 + 3*(12*a^5*b^2 + 4*a
^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^2 + 3
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 -
 b^7)*d*x)*sinh(d*x + c)^4 + 12*(5*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3
 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^3 + (12*a^5*b
^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^
2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7)*d*x + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2
*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^
4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c)^2 + 3*(24*a^5*b^2 - 32*a^4*
b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a
^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x
 + c)^4 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*
a*b^6 + b^7)*d*x + 6*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7
 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b
^6 - b^7)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*(a^6*b - 3*a^5*b^2 + 4
*a^4*b^3 - 4*a^3*b^4 + 3*a^2*b^5 - a*b^6 + (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 +
 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^6 + 6*(a^6*b + 3*a^5*b^2 + 4*
a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)*sinh(d*x + c)^5 + (a
^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*sinh(d*x + c)
^6 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^4 + 3*(a^6*b + a^5
*b^2 - a^2*b^5 - a*b^6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a
^2*b^5 + a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^6*b + 3*a^5*b^2
+ 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^3 + 3*(a^6*b + a
^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^6*b - a^5*b
^2 - a^2*b^5 + a*b^6)*cosh(d*x + c)^2 + 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^
```

```
6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(
d*x + c)^4 + 6*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 + 6*((a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^
6)*cosh(d*x + c)^5 + 2*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^3
+ (a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(
a*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*
(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^
6 + b^7)*d*x*cosh(d*x + c)^5 + 2*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a
*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2
*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^3 + (24*a^5*b^2 - 32*a^4*b^3 - 12*
a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/((a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14
*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x +
 c)^6 + 6*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5
+ 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b
^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10
+ b^11)*d*sinh(d*x + c)^6 + 3*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a
^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b
^10 - b^11)*d*cosh(d*x + c)^4 + 3*(5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^
3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9
+ 3*a*b^10 + b^11)*d*cosh(d*x + c)^2 + (a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b
^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*
b^9 - a*b^10 - b^11)*d)*sinh(d*x + c)^4 + 3*(a^11 - a^10*b - 5*a^9*b^2 + 5*
a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5
*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^11 + 3*a^10*b - a^9*b
^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*
b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^3 + 3*(a^11 + a^10*b - 5*a
^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*
a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5
*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^
4 + 6*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10
*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x +
 c)^2 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 -
10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d)*sinh(d*
x + c)^2 + (a^11 - 3*a^10*b - a^9*b^2 + 11*a^8*b^3 - 6*a^7*b^4 - 14*a^6*b^5
 + 14*a^5*b^6 + 6*a^4*b^7 - 11*a^3*b^8 + a^2*b^9 + 3*a*b^10 - b^11)*d + 6*(
(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b
^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^5
+ 2*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a
^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c
)^3 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10
*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x +
```

```
c))*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16643 vs. $2(144) = 288$.

Time = 18.80 (sec) , antiderivative size = 16643, normalized size of antiderivative = 98.48

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))**4,x)
```

```
[Out] Piecewise((zoo*x/tanh(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a**4, Eq(b
, 0)), (3*d*x*tanh(c + d*x)**4/(48*b**4*d*tanh(c + d*x)**4 - 192*b**4*d*tan
h(c + d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d*x) + 48
*b**4*d) - 12*d*x*tanh(c + d*x)**3/(48*b**4*d*tanh(c + d*x)**4 - 192*b**4*d
*tanh(c + d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d*x)
+ 48*b**4*d) + 18*d*x*tanh(c + d*x)**2/(48*b**4*d*tanh(c + d*x)**4 - 192*b*
*4*d*tanh(c + d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d
*x) + 48*b**4*d) - 12*d*x*tanh(c + d*x)/(48*b**4*d*tanh(c + d*x)**4 - 192*b
**4*d*tanh(c + d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c +
d*x) + 48*b**4*d) + 3*d*x/(48*b**4*d*tanh(c + d*x)**4 - 192*b**4*d*tanh(c +
 d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d*x) + 48*b**4
*d) - 3*tanh(c + d*x)**3/(48*b**4*d*tanh(c + d*x)**4 - 192*b**4*d*tanh(c +
d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d*x) + 48*b**4*
d) + 12*tanh(c + d*x)**2/(48*b**4*d*tanh(c + d*x)**4 - 192*b**4*d*tanh(c +
d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d*x) + 48*b**4*
d) - 19*tanh(c + d*x)/(48*b**4*d*tanh(c + d*x)**4 - 192*b**4*d*tanh(c + d*x
)**3 + 288*b**4*d*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d*x) + 48*b**4*d)
+ 16/(48*b**4*d*tanh(c + d*x)**4 - 192*b**4*d*tanh(c + d*x)**3 + 288*b**4*d
*tanh(c + d*x)**2 - 192*b**4*d*tanh(c + d*x) + 48*b**4*d), Eq(a, -b)), (3*d
*x*tanh(c + d*x)**4/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c + d*x)*
*3 + 288*b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b**4*d) +
12*d*x*tanh(c + d*x)**3/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c + d
*x)**3 + 288*b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b**4*d
) + 18*d*x*tanh(c + d*x)**2/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c
 + d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b*
*4*d) + 12*d*x*tanh(c + d*x)/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(
c + d*x)**3 + 288*b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b
**4*d) + 3*d*x/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c + d*x)**3 +
288*b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b**4*d) - 3*tan
h(c + d*x)**3/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c + d*x)**3 + 2
88*b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b**4*d) - 12*tan
h(c + d*x)**2/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c + d*x)**3 + 2
88*b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b**4*d) - 19*tan
```

h(c + d*x)/(48*b**4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c + d*x)**3 + 288*
b**4*d*tanh(c + d*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b**4*d) - 16/(48*b*
*4*d*tanh(c + d*x)**4 + 192*b**4*d*tanh(c + d*x)**3 + 288*b**4*d*tanh(c + d
*x)**2 + 192*b**4*d*tanh(c + d*x) + 48*b**4*d), Eq(a, b)), (x/(a + b*tanh(c
))**4, Eq(d, 0)), (3*a**7*d*x/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a*
*9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**
3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**
7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) +
 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c +
 d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 +
 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d
*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 9*a**6*b
*d*x*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*t
anh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8
*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d -
12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b*
*6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 -
36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**
8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b
10*d*tanh(c + d*x)2 + 3*b**11*d*tanh(c + d*x)**3) - 12*a**6*b*d*x/(3*a*
*11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**
9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) -
36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c +
d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 -
12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c +
d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tan
h(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2
+ 3*b**11*d*tanh(c + d*x)**3) - 12*a**6*b*log(a/b + tanh(c + d*x))/(3*a**11
*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b
2*d + 3*a8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*
a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x
)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*
a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x
) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c
 + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3
*b**11*d*tanh(c + d*x)**3) + 12*a**6*b*log(tanh(c + d*x) + 1)/(3*a**11*d +
9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d
 + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*
b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3
+ 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*
b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 3
6*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*
x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**1
1*d*tanh(c + d*x)**3) + 13*a**6*b/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) +
9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*

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x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18
*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*
x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh
(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)*
*2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c
 + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 9*a*
*5*b**2*d*x*tanh(c + d*x)**2/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**
9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3
 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7
*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) +
54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c +
d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 +
3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*
x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 36*a**5*b
**2*d*x*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*
d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a
**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d
 - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5
*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3
 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*
b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*
a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 18*a**5*b**2*d*x
/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 -
12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d
*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tan
h(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)
**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tan
h(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9
*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*
x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 36*a**5*b**2*log(a/b + tanh(c + d*x))
*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(
c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**
3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a
**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d
*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a
**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d
- 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10
*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 36*a**5*b**2*log(tanh(c
 + d*x) + 1)*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*
b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 -
 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b
**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54
*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*
x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*
a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x)
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 + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 21*a**5*b**
2*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh
(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b*
*3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*
a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*
d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*
a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d
 - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**1
0*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 3*a**4*b**3*d*x*tanh(c
 + d*x)**3/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c +
d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*
tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*
b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tan
h(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*
b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12
*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*t
anh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 36*a**4*b**3*d*x*tanh(c + d
*x)**2/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)
**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh
(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5
*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c
+ d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7
*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**
2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(
c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 54*a**4*b**3*d*x*tanh(c + d*x)/
(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 1
2*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*
x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh
(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)*
*2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh
(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*
d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x
)**2 + 3*b**11*d*tanh(c + d*x)**3) - 12*a**4*b**3*d*x/(3*a**11*d + 9*a**10*
b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**
8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*t
anh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**
6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d +
 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b
**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 +
9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh
(c + d*x)**3) - 36*a**4*b**3*log(a/b + tanh(c + d*x))*tanh(c + d*x)**2/(3*a
**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a*
*9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) -
 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c +
 d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 -
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12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c +
 d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*ta
nh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2
 + 3*b**11*d*tanh(c + d*x)**3) - 12*a**4*b**3*log(a/b + tanh(c + d*x))/(3*a
**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a*
*9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) -
 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c +
 d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 -
 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c +
 d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*ta
nh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2
 + 3*b**11*d*tanh(c + d*x)**3) + 36*a**4*b**3*log(tanh(c + d*x) + 1)*tanh(c
 + d*x)**2/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c +
d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*
tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*
b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tan
h(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*
b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12
*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*t
anh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 12*a**4*b**3*log(tanh(c + d
*x) + 1)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*
x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*ta
nh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b*
*5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(
c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b*
*7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a
**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tan
h(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 9*a**4*b**3*tanh(c + d*x)**2/
(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 1
2*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*
x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh
(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)*
*2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh
(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*
d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x
)**2 + 3*b**11*d*tanh(c + d*x)**3) - 15*a**4*b**3/(3*a**11*d + 9*a**10*b*d*
tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b*
*3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(
c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b*
*5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*
a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*
d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a*
*2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c +
 d*x)**3) - 12*a**3*b**4*d*x*tanh(c + d*x)**3/(3*a**11*d + 9*a**10*b*d*tanh
(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d
```

```
*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c +
d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d
*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4
*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*ta
nh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b
**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x
)**3) + 54*a**3*b**4*d*x*tanh(c + d*x)**2/(3*a**11*d + 9*a**10*b*d*tanh(c +
 d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tan
h(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)
**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tan
h(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**
7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c
 + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*
d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3
) - 36*a**3*b**4*d*x*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) +
 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d
*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 1
8*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d
*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tan
h(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)
**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(
c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 3*a
**3*b**4*d*x/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c
+ d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*
d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**
6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*t
anh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**
4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d -
12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d
*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 12*a**3*b**4*log(a/b + ta
nh(c + d*x))*tanh(c + d*x)**3/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a*
*9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**
3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**
7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) +
 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c +
 d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 +
 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d
*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 36*a**3*
b**4*log(a/b + tanh(c + d*x))*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c
 + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*t
anh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*
x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*t
anh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b
**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh
(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**
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9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)*
*3) + 12*a**3*b**4*log(tanh(c + d*x) + 1)*tanh(c + d*x)**3/(3*a**11*d + 9*a
**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d +
3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**
4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 5
4*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**
6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a
**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)*
*3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d
*tanh(c + d*x)**3) + 36*a**3*b**4*log(tanh(c + d*x) + 1)*tanh(c + d*x)/(3*a
**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a*
*9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) -
 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c +
 d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 -
 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c +
 d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*ta
nh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2
 + 3*b**11*d*tanh(c + d*x)**3) - 18*a**3*b**4*tanh(c + d*x)/(3*a**11*d + 9*
a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d +
 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b*
*4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 +
54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b*
*6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*
a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)
**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*
d*tanh(c + d*x)**3) + 18*a**2*b**5*d*x*tanh(c + d*x)**3/(3*a**11*d + 9*a**1
0*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a
**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d
*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a
**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d
 + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3
*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3
+ 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*ta
nh(c + d*x)**3) - 36*a**2*b**5*d*x*tanh(c + d*x)**2/(3*a**11*d + 9*a**10*b*
d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*
b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tan
h(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*
b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 1
8*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**
8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*
a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c
 + d*x)**3) + 9*a**2*b**5*d*x*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c
 + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*t
anh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*
x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*t
```

anh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b
7*d*tanh(c + d*x)3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh
(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**
9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)*
*3) - 36*a**2*b**5*log(a/b + tanh(c + d*x))*tanh(c + d*x)**2/(3*a**11*d + 9
*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d
+ 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b
4*d*tanh(c + d*x)2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 +
54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b
6*d + 18*a4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36
*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x
)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11
*d*tanh(c + d*x)**3) + 36*a**2*b**5*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2
/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 -
12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d
*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tan
h(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)
2 - 12*a5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tan
h(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9
*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*
x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 6*a**2*b**5*tanh(c + d*x)**2/(3*a**11
*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b
2*d + 3*a8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*
a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x
)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*
a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x
) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c
+ d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3
*b**11*d*tanh(c + d*x)**3) + 3*a**2*b**5/(3*a**11*d + 9*a**10*b*d*tanh(c +
d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh
(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)*
*2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh
(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7
*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c
+ d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d
*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3)
- 12*a*b**6*d*x*tanh(c + d*x)**3/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) +
9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*
x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18
*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*
x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh
(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)*
*2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c
+ d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 9*a*
b**6*d*x*tanh(c + d*x)**2/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b

```
**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 -
36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b*
*4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*
a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x
)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a
**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x)
+ 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 12*a*b**6*lo
g(a/b + tanh(c + d*x))*tanh(c + d*x)**3/(3*a**11*d + 9*a**10*b*d*tanh(c + d
*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(
c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**
2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(
c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*
d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c +
 d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*
tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3)
+ 12*a*b**6*log(tanh(c + d*x) + 1)*tanh(c + d*x)**3/(3*a**11*d + 9*a**10*b*
d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*
b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tan
h(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*
b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 1
8*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**
8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*
a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c
 + d*x)**3) - 3*a*b**6*tanh(c + d*x)/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x)
 + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c +
 d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 +
 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c +
 d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*t
anh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*
x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tan
h(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) + 3
*b**7*d*x*tanh(c + d*x)**3/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*
b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 -
 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b
**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54
*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*
x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*
a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x)
 + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) - 3*b**7*tanh
(c + d*x)**2/(3*a**11*d + 9*a**10*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c
+ d*x)**2 - 12*a**9*b**2*d + 3*a**8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*
d*tanh(c + d*x) - 36*a**7*b**4*d*tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**
6*b**5*d*tanh(c + d*x)**3 + 54*a**6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*t
anh(c + d*x)**2 - 12*a**5*b**6*d + 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**
4*b**7*d*tanh(c + d*x) - 36*a**3*b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d -
```

```
12*a**2*b**9*d*tanh(c + d*x)**3 + 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d
*tanh(c + d*x)**2 + 3*b**11*d*tanh(c + d*x)**3) - b**7/(3*a**11*d + 9*a**10
*b*d*tanh(c + d*x) + 9*a**9*b**2*d*tanh(c + d*x)**2 - 12*a**9*b**2*d + 3*a*
*8*b**3*d*tanh(c + d*x)**3 - 36*a**8*b**3*d*tanh(c + d*x) - 36*a**7*b**4*d*
tanh(c + d*x)**2 + 18*a**7*b**4*d - 12*a**6*b**5*d*tanh(c + d*x)**3 + 54*a*
*6*b**5*d*tanh(c + d*x) + 54*a**5*b**6*d*tanh(c + d*x)**2 - 12*a**5*b**6*d
+ 18*a**4*b**7*d*tanh(c + d*x)**3 - 36*a**4*b**7*d*tanh(c + d*x) - 36*a**3*
b**8*d*tanh(c + d*x)**2 + 3*a**3*b**8*d - 12*a**2*b**9*d*tanh(c + d*x)**3 +
 9*a**2*b**9*d*tanh(c + d*x) + 9*a*b**10*d*tanh(c + d*x)**2 + 3*b**11*d*tan
h(c + d*x)**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(167) = 334$.

Time = 0.24 (sec) , antiderivative size = 525, normalized size of antiderivative = 3.11

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx = -\frac{4\,(a^3b+ab^3)\log\left(-(a-b)e^{(-2\,dx-2\,c)}-a-b\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)d} - \frac{}{3\,(a^{10}+2\,a^9b-3\,a^8b^2-8\,a^7b^3+2\,a^6b^4+12\,a^5b^5+2\,a^4b^6-8\,a^3b^7-3\,a^2b^8+2\,ab^9+b^{10}+3\,(a^{10}-5\,a} + \frac{dx+c}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -4*(a^3*b + a*b^3)*log(-(a - b)*e^(-2*d*x - 2*c) - a - b)/((a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d) - 4/3*(9*a^4*b^2 + 18*a^3*b^3 + 11*a^2*b
^4 + 4*a*b^5 + 2*b^6 + 3*(6*a^4*b^2 + 2*a^3*b^3 - 5*a^2*b^4 - 2*a*b^5 - b^6
)*e^(-2*d*x - 2*c) + 3*(3*a^4*b^2 - 4*a^3*b^3 + b^6)*e^(-4*d*x - 4*c))/((a^
10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 -
 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 + 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4
- 10*a^4*b^6 + 5*a^2*b^8 - b^10)*e^(-2*d*x - 2*c) + 3*(a^10 - 2*a^9*b - 3*a
^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2
*b^8 - 2*a*b^9 + b^10)*e^(-4*d*x - 4*c) + (a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10)*
e^(-6*d*x - 6*c))*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.80

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx = -\frac{\frac{12\,(a^3b+ab^3)\log\left(\left|-ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{3\,(dx+c)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}-\frac{4\left(3\,(3\,a^4b^2-2\,a^3b^3-2\,a^2b^4+2\,ab^5-b^6)e^{(4\,dx+4\,c)}+3\cdots\right.}{(ae\cdots}}{3\,d}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3*b + a*b^3)*log(abs(-a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - a
 + b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 3*(d*x + c)/(a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - 4*(3*(3*a^4*b^2 - 2*a^3*b^3 - 2*a^2
*b^4 + 2*a*b^5 - b^6)*e^(4*d*x + 4*c) + 3*(6*a^4*b^2 - 14*a^3*b^3 + 11*a^2*
b^4 - 4*a*b^5 + b^6)*e^(2*d*x + 2*c) + (9*a^5*b^2 - 27*a^4*b^3 + 29*a^3*b^4
 - 15*a^2*b^5 + 6*a*b^6 - 2*b^7)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x
+ 2*c) + a - b)^3*(a + b)^3*(a - b)^4))/d
```

Mupad [B] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+b\tanh(c+dx))^4}\,dx = \frac{\ln\left(\tanh(c+d\,x)+1\right)}{2\,d\,a^4-8\,d\,a^3\,b+12\,d\,a^2\,b^2-8\,d\,a\,b^3+2\,d\,b^4}$$

$$-\frac{\frac{\tanh(c+d\,x)\,\left(6\,a^4\,b^2-3\,a^2\,b^4+b^6\right)}{a\,d\,\left(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6\right)}+\frac{\tanh(c+d\,x)^2\,\left(10\,a^4\,b^3-3\,a^2\,b^5+b^7\right)}{a^2\,d\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}+\frac{\tanh(c+d\,x)^3\,\left(\frac{13\,a^4\,b^4}{3}-\frac{2\,a^2\,b^6}{3}+\frac{b^8}{3}\right)}{a^3\,d\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}\cdots}{a^3+3\,a^2\,b\tanh\left(c+d\,x\right)+3\,a\,b^2\tanh\left(c+d\,x\right)^2+b^3\tanh\left(c+d\,x\right)^3}$$

$$-\frac{\ln\left(1-\tanh(c+d\,x)\right)}{2\,d\,a^4+8\,d\,a^3\,b+12\,d\,a^2\,b^2+8\,d\,a\,b^3+2\,d\,b^4}$$

$$-\frac{4\,\ln\left(a+b\tanh(c+d\,x)\right)\,\left(a^3\,b+a\,b^3\right)}{d\,\left(a^4-4\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3+b^4\right)\,\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)}$$

```
[In] int(1/(a + b*tanh(c + d*x))^4,x)
```

```
[Out] log(tanh(c + d*x) + 1)/(2*a^4*d + 2*b^4*d + 12*a^2*b^2*d - 8*a*b^3*d - 8*a^
3*b*d) - ((tanh(c + d*x)*(b^6 - 3*a^2*b^4 + 6*a^4*b^2))/(a*d*(a^6 - b^6 + 3
*a^2*b^4 - 3*a^4*b^2)) + (tanh(c + d*x)^2*(b^7 - 3*a^2*b^5 + 10*a^4*b^3))/(
a^2*d*(3*a*b^2 - 3*a^2*b + a^3 - b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (t
anh(c + d*x)^3*(b^8/3 - (2*a^2*b^6)/3 + (13*a^4*b^4)/3))/(a^3*d*(3*a*b^2 -
3*a^2*b + a^3 - b^3)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))/(a^3 + b^3*tanh(c +
d*x)^3 + 3*a*b^2*tanh(c + d*x)^2 + 3*a^2*b*tanh(c + d*x)) - log(1 - tanh(c
+ d*x))/(2*a^4*d + 2*b^4*d + 12*a^2*b^2*d + 8*a*b^3*d + 8*a^3*b*d) - (4*log
(a + b*tanh(c + d*x))*(a*b^3 + a^3*b))/(d*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 +
6*a^2*b^2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2))
```

3.65 $\int \frac{1}{4+6\tanh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = -\frac{x}{5}+\frac{3\log(2\cosh(c+dx)+3\sinh(c+dx))}{10d}$$

```
[Out] -1/5*x+3/10*ln(2*cosh(d*x+c)+3*sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3565, 3611}

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = \frac{3\log(3\sinh(c+dx)+2\cosh(c+dx))}{10d}-\frac{x}{5}$$

```
[In] Int[(4 + 6*Tanh[c + d*x])^(-1),x]
```

```
[Out] -1/5*x + (3*Log[2*Cosh[c + d*x] + 3*Sinh[c + d*x]])/(10*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{x}{5} + \frac{3}{10}i\int \frac{-6i - 4i\tanh(c+dx)}{4+6\tanh(c+dx)}\,dx$$
$$= -\frac{x}{5} + \frac{3\log(2\cosh(c+dx)+3\sinh(c+dx))}{10d}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = -\frac{\log(1-\tanh(c+dx))}{20d} - \frac{\log(1+\tanh(c+dx))}{4d} + \frac{3\log(2+3\tanh(c+dx))}{10d}$$

```
[In] Integrate[(4 + 6*Tanh[c + d*x])^(-1),x]
[Out] -1/20*Log[1 - Tanh[c + d*x]]/d - Log[1 + Tanh[c + d*x]]/(4*d) + (3*Log[2 + 3*Tanh[c + d*x]])/(10*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{x}{2}-\frac{3c}{5d}+\frac{3\ln\left(e^{2dx+2c}-\frac{1}{5}\right)}{10d}$	28
parallelrisch	$-\frac{5dx+3\ln(1-\tanh(dx+c))-3\ln\left(\frac{2}{3}+\tanh(dx+c)\right)}{10d}$	35
derivativedivides	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{10}+\frac{3\ln(2+3\tanh(dx+c))}{5}-\frac{\ln(\tanh(dx+c)+1)}{2}}{2d}$	42
default	$\frac{-\frac{\ln(\tanh(dx+c)-1)}{10}+\frac{3\ln(2+3\tanh(dx+c))}{5}-\frac{\ln(\tanh(dx+c)+1)}{2}}{2d}$	42

```
[In] int(1/(4+6*tanh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/2*x-3/5*c/d+3/10/d*ln(exp(2*d*x+2*c)-1/5)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = -\frac{5\,dx - 3\,\log\left(\frac{2\,(2\,\cosh(dx+c)+3\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

```
[In] integrate(1/(4+6*tanh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/10*(5*d*x - 3*log(2*(2*cosh(d*x + c) + 3*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/d
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = \begin{cases} \frac{x}{10} - \frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d} + \frac{3\log\left(3\tanh\left(c+dx\right)+2\right)}{10d} & \text{for}\ d \neq 0 \\ \frac{x}{6\tanh\left(c\right)+4} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(4+6*tanh(d*x+c)),x)
```

```
[Out] Piecewise((x/10 - 3*log(tanh(c + d*x) + 1)/(10*d) + 3*log(3*tanh(c + d*x) +
 2)/(10*d), Ne(d, 0)), (x/(6*tanh(c) + 4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = \frac{dx+c}{10\,d} + \frac{3\,\log\left(e^{(-2\,dx-2\,c)} - 5\right)}{10\,d}$$

```
[In] integrate(1/(4+6*tanh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) - 5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = -\frac{5\,dx+5\,c-3\,\log\left(\left|5\,e^{(2\,dx+2\,c)}-1\right|\right)}{10\,d}$$

```
[In] integrate(1/(4+6*tanh(d*x+c)),x, algorithm="giac")
[Out] -1/10*(5*d*x + 5*c - 3*log(abs(5*e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{4+6\tanh(c+dx)}\,dx = \frac{x}{10}-\frac{\frac{3\,\ln(\tanh(c+d\,x)+1)}{10}-\frac{3\,\ln(3\,\tanh(c+d\,x)+2)}{10}}{d}$$

```
[In] int(1/(6*tanh(c + d*x) + 4),x)
[Out] x/10 - ((3*log(tanh(c + d*x) + 1))/10 - (3*log(3*tanh(c + d*x) + 2))/10)/d
```

3.66 $\int \frac{1}{4-6\tanh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx=-\frac{x}{5}-\frac{3\log(2\cosh(c+dx)-3\sinh(c+dx))}{10d}$$

```
[Out] -1/5*x-3/10*ln(2*cosh(d*x+c)-3*sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3565, 3611}

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx=-\frac{3\log(2\cosh(c+dx)-3\sinh(c+dx))}{10d}-\frac{x}{5}$$

```
[In] Int[(4 - 6*Tanh[c + d*x])^(-1),x]
```

```
[Out] -1/5*x - (3*Log[2*Cosh[c + d*x] - 3*Sinh[c + d*x]])/(10*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{x}{5} - \frac{3}{10}i\int \frac{6i - 4i\tanh(c+dx)}{4-6\tanh(c+dx)}\,dx$$
$$= -\frac{x}{5} - \frac{3\log(2\cosh(c+dx) - 3\sinh(c+dx))}{10d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx = -\frac{3\log(2-3\tanh(c+dx))}{10d} + \frac{\log(1-\tanh(c+dx))}{4d} + \frac{\log(1+\tanh(c+dx))}{20d}$$

```
[In] Integrate[(4 - 6*Tanh[c + d*x])^(-1),x]
[Out] (-3*Log[2 - 3*Tanh[c + d*x]])/(10*d) + Log[1 - Tanh[c + d*x]]/(4*d) + Log[1
 + Tanh[c + d*x]]/(20*d)
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{x}{10}+\frac{3c}{5d}-\frac{3\ln\left(e^{2dx+2c}-5\right)}{10d}$	28
parallelrisch	$-\frac{-dx-3\ln(1-\tanh(dx+c))+3\ln\left(-\frac{2}{3}+\tanh(dx+c)\right)}{10d}$	35
derivativedivides	$\frac{\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{10}-\frac{3\ln(-2+3\tanh(dx+c))}{5}}{2d}$	42
default	$\frac{\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{10}-\frac{3\ln(-2+3\tanh(dx+c))}{5}}{2d}$	42

```
[In] int(1/(4-6*tanh(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/10*x+3/5*c/d-3/10/d*ln(exp(2*d*x+2*c)-5)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx = \frac{dx - 3\,\log\left(-\frac{2\,(2\,\cosh(dx+c)-3\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

```
[In] integrate(1/(4-6*tanh(d*x+c)),x, algorithm="fricas")
[Out] 1/10*(d*x - 3*log(-2*(2*cosh(d*x + c) - 3*sinh(d*x + c))/(cosh(d*x + c) - s
inh(d*x + c))))/d
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx = \begin{cases} -\frac{x}{2} + \frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d} - \frac{3\log\left(3\tanh\left(c+dx\right)-2\right)}{10d} & \text{for } d \neq 0 \\ \frac{x}{4-6\tanh\left(c\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(4-6*tanh(d*x+c)),x)
[Out] Piecewise((-x/2 + 3*log(tanh(c + d*x) + 1)/(10*d) - 3*log(3*tanh(c + d*x) -
 2)/(10*d), Ne(d, 0)), (x/(4 - 6*tanh(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx = -\frac{1}{2}\,x - \frac{c}{2\,d} - \frac{3\,\log\left(5\,e^{(-2\,dx-2\,c)}-1\right)}{10\,d}$$

```
[In] integrate(1/(4-6*tanh(d*x+c)),x, algorithm="maxima")
[Out] -1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) - 1)/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx = \frac{dx+c-3\,\log\left(\left|e^{(2\,dx+2\,c)}-5\right|\right)}{10\,d}$$

```
[In] integrate(1/(4-6*tanh(d*x+c)),x, algorithm="giac")
[Out] 1/10*(d*x + c - 3*log(abs(e^(2*d*x + 2*c) - 5)))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{1}{4-6\tanh(c+dx)}\,dx = \frac{\frac{3\,\ln(\tanh(c+d\,x)+1)}{10}-\frac{3\,\ln(3\tanh(c+d\,x)-2)}{10}}{d}-\frac{x}{2}$$

```
[In] int(-1/(6*tanh(c + d*x) - 4),x)
[Out] ((3*log(tanh(c + d*x) + 1))/10 - (3*log(3*tanh(c + d*x) - 2))/10)/d - x/2
```

3.67 $\int \sqrt{a+b\tanh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \sqrt{a+b\tanh(c+dx)}\,dx = -\frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] -arctanh((a+b*tanh(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+arctanh((a+b*ta
nh(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3566, 714, 1144, 213}

$$\int \sqrt{a+b\tanh(c+dx)}\,dx = \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d} - \frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

```
[In] Int[Sqrt[a + b*Tanh[c + d*x]],x]
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{\sqrt{a+x}}{-b^2+x^2}\,dx, x, b\tanh(c+dx)\right)}{d}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}\\
&= \frac{(a-b)\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}\\
&\quad - \frac{(a+b)\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}\\
&= -\frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\tanh(c+dx)}\,dx = -\frac{\sqrt{a-b}\mathrm{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\mathrm{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

[In] Integrate[Sqrt[a + b*Tanh[c + d*x]],x]

[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]])/d

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{a+b}}\right)\sqrt{a+b}}{d} - \frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{-a+b}}\right)}{d}$	63
default	$\frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{a+b}}\right)\sqrt{a+b}}{d} - \frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{-a+b}}\right)}{d}$	63

[In] int((a+b*tanh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] arctanh((a+b*tanh(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d-1/d*(-a+b)^(1/2)*arctan((a+b*tanh(d*x+c))^(1/2)/(-a+b)^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(62) = 124$.

Time = 0.29 (sec) , antiderivative size = 2203, normalized size of antiderivative = 29.77

$$\int \sqrt{a+b\tanh(c+dx)}\,dx = \text{Too large to display}$$

[In] integrate((a+b*tanh(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + (2*a

```
 + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)
^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b)*cosh(d*x + c))*sinh(d*x + c)
+ a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c)) +
8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 + 4*(a^2 - a*
b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*si
nh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + (2*a - b)*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 + (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((a*cosh(d*x +
 c) + b*sinh(d*x + c))/cosh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 +
2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x +
 c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 + a)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2)) -
sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 + 4*(a^2 - a*b)*cosh(d*
x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c
)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh
(d*x + c)^3 + a*sinh(d*x + c)^4 + (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x
 + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b*si
nh(d*x + c))/cosh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 + 2*(a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a - b)*sqrt(-a +
b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 - b^2)*cos
h(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh
(d*x + c)^2 + a^2 - 2*a*b + b^2)) - sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a
^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b^
2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 + (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x +
 c)^2 + 2*a + b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*si
nh(d*x + c))/cosh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/d, -1/2*(sqrt(-a + b)*arctan(-(a*cos
h(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a - b)
*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2
```

```
 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2
- b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + b^2)) + sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 + a)*sqrt(-a - b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2)))/d
]
```

Sympy [F]

$$\int \sqrt{a + b \tanh(c + dx)}\, dx = \int \sqrt{a + b \tanh{(c + dx)}}\, dx$$

```
[In] integrate((a+b*tanh(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*tanh(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a + b \tanh(c + dx)}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate((a+b*tanh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a + b \tanh(c + dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*tanh(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int \sqrt{a+b\tanh(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{b^2\sqrt{a+b}\sqrt{a+b\tanh(c+dx)}\,1\mathrm{i}-a\,b\sqrt{a+b}\sqrt{a+b\tanh(c+dx)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a+b}\,1\mathrm{i}}{d} + \frac{\operatorname{atan}\left(\frac{b^2\sqrt{a-b}\sqrt{a+b\tanh(c+dx)}\,1\mathrm{i}+a\,b\sqrt{a-b}\sqrt{a+b\tanh(c+dx)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a-b}\,1\mathrm{i}}{d}$$

```
[In] int((a + b*tanh(c + d*x))^(1/2),x)
```

```
[Out] (atan((b^2*(a + b)^(1/2)*(a + b*tanh(c + d*x))^(1/2)*1i - a*b*(a + b)^(1/2)
*(a + b*tanh(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a + b)^(1/2)*1i)/d + (atan
((b^2*(a - b)^(1/2)*(a + b*tanh(c + d*x))^(1/2)*1i + a*b*(a - b)^(1/2)*(a +
 b*tanh(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a - b)^(1/2)*1i)/d
```

3.68 $\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

```
[Out] -arctanh((a+b*tanh(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+arctanh((a+b*ta
nh(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3566, 722, 1107, 213}

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[In] Int[1/Sqrt[a + b*Tanh[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+x}\left(-b^2+x^2\right)}\,dx, x, b\tanh(c+dx)\right)}{d}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{1}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\tanh(c+dx)}\right)}{d}\\
&= -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

```
[In] Integrate[1/Sqrt[a + b*Tanh[c + d*x]],x]
[Out] -(ArcTanh[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Tanh[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$	62
default	$\frac{\arctan\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh(dx+c)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$	62

```
[In] int(1/(a+b*tanh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(-a+b)^(1/2)*arctan((a+b*tanh(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b*
tanh(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 2279, normalized size of antiderivative = 30.80

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*tanh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh
(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 + (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 + 2*a + b)*sinh(
d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b)*cosh(d*x + c))*sinh(d
*x + c) + a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x
+ c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c)) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 +
4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)
^4 + 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 + 2
*a^2 - 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + (2*a - b)*cosh(
d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 2*(2*a*cosh(
d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sq
rt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c)) + 4*((2*a^2 - b^2)*co
sh(d*x + c)^3 + 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
```

```
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a^2 - b^2)*d), -1/4*(
2*(a - b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
 + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a)*sqrt(-a - b)*sqrt((a*cos
h(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b +
 b^2)*sinh(d*x + c)^2 + a^2 - b^2)) - (a + b)*sqrt(a - b)*log(((2*a^2 - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2
- b^2)*sinh(d*x + c)^4 + 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)
*cosh(d*x + c)^2 + 2*a^2 - 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 -
 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 + (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
 c)^2 + 2*(2*a*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a
 - b)*sqrt(a - b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c)) +
 4*((2*a^2 - b^2)*cosh(d*x + c)^3 + 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a
^2 - b^2)*d), -1/4*(2*(a + b)*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a - b)*sqrt(-a + b)*sqrt
((a*cosh(d*x + c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 - b^2)*cosh(d*x +
 c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x +
c)^2 + a^2 - 2*a*b + b^2)) - sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(
a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 + 2*a^2 - b
^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x
+ c)^2 + 2*a + b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 + (2*a + b
)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((a*cosh(d*x + c) + b*s
inh(d*x + c))/cosh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/((a^2 - b^2)*d), -1/2*((a + b)*sqrt
(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 + a - b)*sqrt(-a + b)*sqrt((a*cosh(d*x + c) + b*sinh(d*x + c
))/cosh(d*x + c))/((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c
)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 2*a*b + b^2)) + (a -
b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a)*sqrt(-a - b)*sqrt((a*cosh(d*x +
 c) + b*sinh(d*x + c))/cosh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 + a^2 - b^2)))/((a^2 - b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\tanh{(c+dx)}}}\,dx$$

[In] integrate(1/(a+b*tanh(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt(a + b*tanh(c + d*x)), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(1/(a+b*tanh(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/(a+b*tanh(d*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 240, normalized size of antiderivative = 3.24

$$\int \frac{1}{\sqrt{a+b\tanh(c+dx)}}\,dx = \frac{\operatorname{atanh}\left(\frac{(a\,d^3+b\,d^3)\,\sqrt{a+b\tanh(c+d\,x)}}{b\,d^3\,\sqrt{a+b}}-\frac{16\,a\,b^2\,\sqrt{a+b\tanh(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3+b\,d^3}+\frac{16\,a\,b^3\,d^3}{a\,d^3+b\,d^3}\right)\sqrt{a+b}}\right)}{d\,\sqrt{a+b}} + \frac{\operatorname{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\tanh(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3-b\,d^3}-\frac{16\,a\,b^3\,d^3}{a\,d^3-b\,d^3}\right)\sqrt{a-b}}+\frac{(a\,d^3-b\,d^3)\,\sqrt{a+b\tanh(c+d\,x)}}{b\,d^3\,\sqrt{a-b}}\right)}{d\,\sqrt{a-b}}$$

```
[In] int(1/(a + b*tanh(c + d*x))^(1/2),x)
```

```
[Out] atanh(((a*d^3 + b*d^3)*(a + b*tanh(c + d*x))^(1/2))/(b*d^3*(a + b)^(1/2)) -
 (16*a*b^2*(a + b*tanh(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 + b*d^3) + (1
6*a*b^3*d^3)/(a*d^3 + b*d^3))*(a + b)^(1/2)))/(d*(a + b)^(1/2)) + atanh((16
*a*b^2*(a + b*tanh(c + d*x))^(1/2))/(((16*b^4*d^3)/(a*d^3 - b*d^3) - (16*a*
b^3*d^3)/(a*d^3 - b*d^3))*(a - b)^(1/2)) + ((a*d^3 - b*d^3)*(a + b*tanh(c +
 d*x))^(1/2))/(b*d^3*(a - b)^(1/2)))/(d*(a - b)^(1/2))
```

3.69 $\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = \frac{x}{16}+\frac{1}{32(1-\tanh(x))^2}-\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}+\frac{5}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tanh(x))}$$

```
[Out] 1/16*x+1/32/(1-tanh(x))^2-1/8/(1-tanh(x))-1/24/(1+tanh(x))^3+5/32/(1+tanh(x
))^2-3/16/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3597, 862, 90, 213}

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = \frac{x}{16}-\frac{1}{8(1-\tanh(x))}-\frac{3}{16(\tanh(x)+1)}+\frac{1}{32(1-\tanh(x))^2}+\frac{5}{32(\tanh(x)+1)^2}-\frac{1}{24(\tanh(x)+1)^3}$$

```
[In] Int[Sinh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] x/16 + 1/(32*(1 - Tanh[x])^2) - 1/(8*(1 - Tanh[x])) - 1/(24*(1 + Tanh[x])^3
) + 5/(32*(1 + Tanh[x])^2) - 3/(16*(1 + Tanh[x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^4}{(1+x)\left(-1+x^2\right)^3}\,dx, x, \tanh(x)\right)\\
&= -\text{Subst}\left(\int \frac{x^4}{(-1+x)^3(1+x)^4}\,dx, x, \tanh(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}-\frac{1}{8(1+x)^4}+\frac{5}{16(1+x)^3}-\frac{3}{16(1+x)^2}\right.\right.\\
&\qquad \left.\left.+\frac{1}{16\left(-1+x^2\right)}\right)dx, x, \tanh(x)\right)\\
&= \frac{1}{32(1-\tanh(x))^2}-\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}+\frac{5}{32(1+\tanh(x))^2}\\
&\quad -\frac{3}{16(1+\tanh(x))}-\frac{1}{16}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{x}{16}+\frac{1}{32(1-\tanh(x))^2}-\frac{1}{8(1-\tanh(x))}\\
&\quad -\frac{1}{24(1+\tanh(x))^3}+\frac{5}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tanh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = \frac{1}{192}(12x - 15\cosh(2x) + 6\cosh(4x) - \cosh(6x) - 3\sinh(2x) - 3\sinh(4x) + \sinh(6x))$$

```
[In] Integrate[Sinh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (12*x - 15*Cosh[2*x] + 6*Cosh[4*x] - Cosh[6*x] - 3*Sinh[2*x] - 3*Sinh[4*x]
+ Sinh[6*x])/192
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x}{16}+\frac{e^{4x}}{128}-\frac{3\,e^{2x}}{64}-\frac{e^{-2x}}{32}+\frac{3\,e^{-4x}}{128}-\frac{e^{-6x}}{192}$
parallelrisch	$-\frac{19}{96}+\frac{\cosh(4x)}{32}-\frac{\cosh(6x)}{192}-\frac{5\cosh(2x)}{64}-\frac{\sinh(4x)}{64}-\frac{\sinh(2x)}{64}+\frac{\sinh(6x)}{192}+\frac{\ln(1+\tanh(x))}{32}-\frac{\ln(1-\tanh(x))}{32}$
default	$-\frac{1}{3(\tanh(\frac{x}{2})+1)^6}+\frac{1}{(\tanh(\frac{x}{2})+1)^5}-\frac{7}{8(\tanh(\frac{x}{2})+1)^4}+\frac{1}{12(\tanh(\frac{x}{2})+1)^3}+\frac{1}{8(\tanh(\frac{x}{2})+1)^2}+\frac{\ln(\tanh(\frac{x}{2})+1)}{16}+$

```
[In] int(sinh(x)^4/(1+tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x+1/128*exp(4*x)-3/64*exp(2*x)-1/32*exp(-2*x)+3/128*exp(-4*x)-1/192*ex
p(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = \frac{\cosh(x)^5 + 5\cosh(x)\sinh(x)^4 + 5\sinh(x)^5 + \left(50\cosh(x)^2 - 27\right)\sinh(x)^3 - 9\cosh(x)^3 + \left(10\cosh(x\right.}{384\,(\cosh(x)}$$

```
[In] integrate(sinh(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + 5*sinh(x)^5 + (50*cosh(x)^2 - 27)*
sinh(x)^3 - 9*cosh(x)^3 + (10*cosh(x)^3 - 27*cosh(x))*sinh(x)^2 + 12*(2*x -
 1)*cosh(x) + (25*cosh(x)^4 - 81*cosh(x)^2 + 24*x + 12)*sinh(x))/(cosh(x) +
 sinh(x))
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = \int \frac{\sinh^4{(x)}}{\tanh{(x)}+1}\,dx$$

```
[In] integrate(sinh(x)**4/(1+tanh(x)),x)
[Out] Integral(sinh(x)**4/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = -\frac{1}{128}\left(6\,e^{(-2\,x)}-1\right)e^{(4\,x)}+\frac{1}{16}\,x-\frac{1}{32}\,e^{(-2\,x)}+\frac{3}{128}\,e^{(-4\,x)}-\frac{1}{192}\,e^{(-6\,x)}$$

```
[In] integrate(sinh(x)^4/(1+tanh(x)),x, algorithm="maxima")
[Out] -1/128*(6*e^(-2*x) - 1)*e^(4*x) + 1/16*x - 1/32*e^(-2*x) + 3/128*e^(-4*x) -
 1/192*e^(-6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = -\frac{1}{384}\left(22\,e^{(6\,x)}+12\,e^{(4\,x)}-9\,e^{(2\,x)}+2\right)e^{(-6\,x)}+\frac{1}{16}\,x+\frac{1}{128}\,e^{(4\,x)}-\frac{3}{64}\,e^{(2\,x)}$$

```
[In] integrate(sinh(x)^4/(1+tanh(x)),x, algorithm="giac")
[Out] -1/384*(22*e^(6*x) + 12*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/16*x + 1/128*
e^(4*x) - 3/64*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\sinh^4(x)}{1+\tanh(x)}\,dx = \frac{x}{16} - \frac{e^{-2x}}{32} - \frac{3e^{2x}}{64} + \frac{3e^{-4x}}{128} + \frac{e^{4x}}{128} - \frac{e^{-6x}}{192}$$

```
[In] int(sinh(x)^4/(tanh(x) + 1),x)
```

```
[Out] x/16 - exp(-2*x)/32 - (3*exp(2*x))/64 + (3*exp(-4*x))/128 + exp(4*x)/128 -
exp(-6*x)/192
```

3.70 $\int \frac{\sinh^3(x)}{1+\tanh(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{\sinh^3(x)}{1+\tanh(x)} dx = -\frac{1}{3}\cosh^3(x) + \frac{\cosh^5(x)}{5} - \frac{\sinh^5(x)}{5}$$

```
[Out] -1/3*cosh(x)^3+1/5*cosh(x)^5-1/5*sinh(x)^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {3599, 3187, 3186, 2645, 14, 2644, 30}

$$\int \frac{\sinh^3(x)}{1+\tanh(x)} dx = -\frac{\sinh^5(x)}{5} + \frac{\cosh^5(x)}{5} - \frac{\cosh^3(x)}{3}$$

```
[In] Int[Sinh[x]^3/(1 + Tanh[x]),x]
[Out] -1/3*Cosh[x]^3 + Cosh[x]^5/5 - Sinh[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(x)\sinh^3(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \cosh(x)(-i\cosh(x)+i\sinh(x))\sinh^3(x)\,dx \\
&= -\int \left(-\cosh^2(x)\sinh^3(x)+\cosh(x)\sinh^4(x)\right)\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= \int \cosh^2(x)\sinh^3(x)\,dx - \int \cosh(x)\sinh^4(x)\,dx\\
&= i\text{Subst}\left(\int x^4\,dx, x, i\sinh(x)\right) - \text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{5}\sinh^5(x) - \text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \cosh(x)\right)\\
&= -\frac{1}{3}\cosh^3(x) + \frac{\cosh^5(x)}{5} - \frac{\sinh^5(x)}{5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$
\int \frac{\sinh^3(x)}{1+\tanh(x)}\,dx = \frac{1}{120}(\cosh(x)-\sinh(x))(-20\cosh(2x)+4\cosh(4x)-10\sinh(2x) + \sinh(4x))
$$

```
[In] Integrate[Sinh[x]^3/(1 + Tanh[x]),x]
[Out] ((Cosh[x] - Sinh[x])*(-20*Cosh[2*x] + 4*Cosh[4*x] - 10*Sinh[2*x] + Sinh[4*x
]))/120
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result
risch	$\frac{e^{3x}}{48}-\frac{e^{x}}{8}-\frac{e^{-3x}}{24}+\frac{e^{-5x}}{80}$
parallelrisch	$-\frac{2}{15}-\frac{\cosh(x)}{8}-\frac{\sinh(x)}{8}-\frac{\sinh(5x)}{80}-\frac{\cosh(3x)}{48}+\frac{\cosh(5x)}{80}+\frac{\sinh(3x)}{16}$
default	$\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}+\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+$ 8 tan

```
[In] int(sinh(x)^3/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/48*exp(3*x)-1/8*exp(x)-1/24*exp(-3*x)+1/80*exp(-5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{\sinh^3(x)}{1+\tanh(x)}\,dx = \frac{\cosh(x)^4 + \cosh(x)\sinh(x)^3 + \sinh(x)^4 + \left(6\cosh(x)^2 - 5\right)\sinh(x)^2 - 5\cosh(x)^2 + \left(\cosh(x)^3 - 5\cos\right.}{30\left(\cosh(x) + \sinh(x)\right)}$$

```
[In] integrate(sinh(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 5)*sinh(x)
^2 - 5*cosh(x)^2 + (cosh(x)^3 - 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(19) = 38$.

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.36

$$\begin{aligned}\int \frac{\sinh^3(x)}{1+\tanh(x)}\,dx &= \frac{3\sinh^3(x)\tanh(x)}{15\tanh(x)+15} - \frac{3\sinh^3(x)}{15\tanh(x)+15} + \frac{6\sinh^2(x)\cosh(x)\tanh(x)}{15\tanh(x)+15} \\ &+ \frac{9\sinh^2(x)\cosh(x)}{15\tanh(x)+15} - \frac{6\sinh(x)\cosh^2(x)\tanh(x)}{15\tanh(x)+15} \\ &+ \frac{6\sinh(x)\cosh^2(x)}{15\tanh(x)+15} - \frac{8\cosh^3(x)\tanh(x)}{15\tanh(x)+15} - \frac{2\cosh^3(x)}{15\tanh(x)+15}\end{aligned}$$

```
[In] integrate(sinh(x)**3/(1+tanh(x)),x)
```

```
[Out] 3*sinh(x)**3*tanh(x)/(15*tanh(x) + 15) - 3*sinh(x)**3/(15*tanh(x) + 15) + 6
*sinh(x)**2*cosh(x)*tanh(x)/(15*tanh(x) + 15) + 9*sinh(x)**2*cosh(x)/(15*ta
nh(x) + 15) - 6*sinh(x)*cosh(x)**2*tanh(x)/(15*tanh(x) + 15) + 6*sinh(x)*co
sh(x)**2/(15*tanh(x) + 15) - 8*cosh(x)**3*tanh(x)/(15*tanh(x) + 15) - 2*cos
h(x)**3/(15*tanh(x) + 15)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(x)}{1+\tanh(x)}\,dx = -\frac{1}{48}\left(6\,e^{(-2\,x)}-1\right)e^{(3\,x)} - \frac{1}{24}\,e^{(-3\,x)} + \frac{1}{80}\,e^{(-5\,x)}$$

```
[In] integrate(sinh(x)^3/(1+tanh(x)),x, algorithm="maxima")
[Out] -1/48*(6*e^(-2*x) - 1)*e^(3*x) - 1/24*e^(-3*x) + 1/80*e^(-5*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{1+\tanh(x)}\,dx = -\frac{1}{240}\left(10\,e^{(2\,x)}-3\right)e^{(-5\,x)} + \frac{1}{48}\,e^{(3\,x)} - \frac{1}{8}\,e^{x}$$

```
[In] integrate(sinh(x)^3/(1+tanh(x)),x, algorithm="giac")
[Out] -1/240*(10*e^(2*x) - 3)*e^(-5*x) + 1/48*e^(3*x) - 1/8*e^x
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^3(x)}{1+\tanh(x)}\,dx = \frac{e^{3\,x}}{48} - \frac{e^{-3\,x}}{24} + \frac{e^{-5\,x}}{80} - \frac{e^{x}}{8}$$

```
[In] int(sinh(x)^3/(tanh(x) + 1),x)
[Out] exp(3*x)/48 - exp(-3*x)/24 + exp(-5*x)/80 - exp(x)/8
```

3.71 $\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx=-\frac{x}{8}+\frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}+\frac{1}{4(1+\tanh(x))}$$

```
[Out] -1/8*x+1/8/(1-tanh(x))-1/8/(1+tanh(x))^2+1/4/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3597, 862, 90, 213}

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx=-\frac{x}{8}+\frac{1}{8(1-\tanh(x))}+\frac{1}{4(\tanh(x)+1)}-\frac{1}{8(\tanh(x)+1)^2}$$

```
[In] Int[Sinh[x]^2/(1 + Tanh[x]),x]
[Out] -1/8*x + 1/(8*(1 - Tanh[x])) - 1/(8*(1 + Tanh[x])^2) + 1/(4*(1 + Tanh[x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^2}{(1+x)\left(-1+x^2\right)^2}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \frac{x^2}{(-1+x)^2(1+x)^3}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{8\left(-1+x^2\right)}\right)dx, x, \tanh(x)\right)\\
&= \frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}+\frac{1}{4(1+\tanh(x))}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\
&= -\frac{x}{8}+\frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}+\frac{1}{4(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx = \frac{1}{32}(-4x+4\cosh(2x)-\cosh(4x)+\sinh(4x))$$

```
[In] Integrate[Sinh[x]^2/(1 + Tanh[x]),x]
[Out] (-4*x + 4*Cosh[2*x] - Cosh[4*x] + Sinh[4*x])/32
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{x}{8}+\frac{{\mathrm e}^{2 x}}{16}+\frac{{\mathrm e}^{-2 x}}{16}-\frac{{\mathrm e}^{-4 x}}{32}$
parallelrisch	$-\frac{x}{8}+\frac{\sinh \left(4 x \right)}{32}-\frac{\cosh \left(4 x \right)}{32}+\frac{\cosh \left(2 x \right)}{8}-\frac{3}{32}$
default	$-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)+1\right)^{4}}+\frac{1}{\left(\tanh \left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{2 \left(\tanh \left(\frac{x}{2}\right)+1\right)^{2}}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4 \left(\tanh \left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{4 \tanh \left(\frac{x}{2}\right)-4}+\frac{\ln}{}$

[In] int(sinh(x)^2/(1+tanh(x)),x,method=_RETURNVERBOSE)

[Out] -1/8*x+1/16*exp(2*x)+1/16*exp(-2*x)-1/32*exp(-4*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx = \frac{\cosh(x)^3+3\cosh(x)\sinh(x)^2+3\sinh(x)^3-2(2x-1)\cosh(x)+\left(9\cosh(x)^2-4x-2\right)\sinh(x)}{32\left(\cosh(x)+\sinh(x)\right)}$$

[In] integrate(sinh(x)^2/(1+tanh(x)),x, algorithm="fricas")

[Out] 1/32*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*sinh(x)^3 - 2*(2*x - 1)*cosh(x) + (9*cosh(x)^2 - 4*x - 2)*sinh(x))/(cosh(x) + sinh(x))

Sympy [F]

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx = \int \frac{\sinh^2(x)}{\tanh(x)+1}\,dx$$

[In] integrate(sinh(x)**2/(1+tanh(x)),x)

[Out] Integral(sinh(x)**2/(tanh(x) + 1), x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx = -\frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)} + \frac{1}{16}\,e^{(-2\,x)} - \frac{1}{32}\,e^{(-4\,x)}$$

```
[In] integrate(sinh(x)^2/(1+tanh(x)),x, algorithm="maxima")
[Out] -1/8*x + 1/16*e^(2*x) + 1/16*e^(-2*x) - 1/32*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx = \frac{1}{32}\left(3\,e^{(4\,x)} + 2\,e^{(2\,x)} - 1\right)e^{(-4\,x)} - \frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

```
[In] integrate(sinh(x)^2/(1+tanh(x)),x, algorithm="giac")
[Out] 1/32*(3*e^(4*x) + 2*e^(2*x) - 1)*e^(-4*x) - 1/8*x + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{1+\tanh(x)}\,dx = \frac{\mathrm{e}^{-2\,x}}{16} - \frac{x}{8} + \frac{\mathrm{e}^{2\,x}}{16} - \frac{\mathrm{e}^{-4\,x}}{32}$$

```
[In] int(sinh(x)^2/(tanh(x) + 1),x)
[Out] exp(-2*x)/16 - x/8 + exp(2*x)/16 - exp(-4*x)/32
```

3.72 $\int \frac{\sinh(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx=\frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}$$

```
[Out] 1/3*cosh(x)^3-1/3*sinh(x)^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3599, 3187, 3186, 2645, 30, 2644}

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx=\frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}$$

```
[In] Int[Sinh[x]/(1 + Tanh[x]),x]
```

```
[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(x)\sinh(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \cosh(x)(-i\cosh(x)+i\sinh(x))\sinh(x)\,dx \\
&= \int \left(\cosh^2(x)\sinh(x)-\cosh(x)\sinh^2(x)\right)\,dx \\
&= \int \cosh^2(x)\sinh(x)\,dx-\int \cosh(x)\sinh^2(x)\,dx \\
&= -\left(i\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\right)+\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right) \\
&= \frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx = \frac{1}{12}(3\cosh(x)+\cosh(3x)-4\sinh^3(x))$$

```
[In] Integrate[Sinh[x]/(1 + Tanh[x]),x]
[Out] (3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{e^{x}}{4}+\frac{e^{-3x}}{12}$	12
parallelrisch	$\frac{\cosh(3x)}{12}+\frac{\cosh(x)}{4}-\frac{\sinh(3x)}{12}+\frac{\sinh(x)}{4}+\frac{2}{3}$	23
default	$\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	42

```
[In] int(sinh(x)/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*exp(x)+1/12*exp(-3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx = \frac{\cosh(x)^2+\cosh(x)\sinh(x)+\sinh(x)^2}{3(\cosh(x)+\sinh(x))}$$

```
[In] integrate(sinh(x)/(1+tanh(x)),x, algorithm="fricas")
[Out] 1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(12) = 24$.

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx = \frac{\sinh(x)\tanh(x)}{3\tanh(x)+3} - \frac{\sinh(x)}{3\tanh(x)+3} + \frac{2\cosh(x)\tanh(x)}{3\tanh(x)+3} + \frac{\cosh(x)}{3\tanh(x)+3}$$

```
[In] integrate(sinh(x)/(1+tanh(x)),x)
```

```
[Out] sinh(x)*tanh(x)/(3*tanh(x) + 3) - sinh(x)/(3*tanh(x) + 3) + 2*cosh(x)*tanh(
x)/(3*tanh(x) + 3) + cosh(x)/(3*tanh(x) + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx = \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

```
[In] integrate(sinh(x)/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx = \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

```
[In] integrate(sinh(x)/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\sinh(x)}{1+\tanh(x)}\,dx = \frac{e^{-3\,x}}{12} + \frac{e^{x}}{4}$$

```
[In] int(sinh(x)/(tanh(x) + 1),x)
```

```
[Out] exp(-3*x)/12 + exp(x)/4
```

3.73 $\int \frac{\text{csch}(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{\text{csch}(x)}{1+\tanh(x)}\,dx=-\text{arctanh}(\cosh(x))+\cosh(x)-\sinh(x)$$

```
[Out] -arctanh(cosh(x))+cosh(x)-sinh(x)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3599, 3187, 3186, 2717, 2672, 327, 212}

$$\int \frac{\text{csch}(x)}{1+\tanh(x)}\,dx=-\text{arctanh}(\cosh(x))-\sinh(x)+\cosh(x)$$

```
[In] Int[Csch[x]/(1 + Tanh[x]),x]
```

```
[Out] -ArcTanh[Cosh[x]] + Cosh[x] - Sinh[x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\coth(x)}{\cosh(x)+\sinh(x)}\,dx\\ &= i\int \coth(x)(-i\cosh(x)+i\sinh(x))\,dx\end{aligned}$$

$$= -\int (\cosh(x) - \cosh(x)\coth(x))\, dx$$

$$= -\int \cosh(x)\, dx + \int \cosh(x)\coth(x)\, dx$$

$$= -\sinh(x) - \text{Subst}\left(\int \frac{x^2}{1-x^2}\, dx, x, \cosh(x)\right)$$

$$= \cosh(x) - \sinh(x) - \text{Subst}\left(\int \frac{1}{1-x^2}\, dx, x, \cosh(x)\right)$$

$$= -\text{arctanh}(\cosh(x)) + \cosh(x) - \sinh(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int \frac{\text{csch}(x)}{1+\tanh(x)}\, dx = \frac{\cosh(x) - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right) - \left(\log\left(\cosh\left(\frac{x}{2}\right)\right) - \log\left(\sinh\left(\frac{x}{2}\right)\right) + \sinh(x)\right)\tanh(x)}{1+\tanh(x)}$$

```
[In] Integrate[Csch[x]/(1 + Tanh[x]),x]
[Out] (Cosh[x] - Log[Cosh[x/2]] + Log[Sinh[x/2]] - (Log[Cosh[x/2]] - Log[Sinh[x/2
]] + Sinh[x])*Tanh[x])/(1 + Tanh[x])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{2}{\tanh\left(\frac{x}{2}\right)+1}$	17
risch	$e^{-x} - \ln\left(e^{x}+1\right) + \ln\left(e^{x}-1\right)$	18

```
[In] int(csch(x)/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] ln(tanh(1/2*x))+2/(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

$$\int \frac{\operatorname{csch}(x)}{1+\tanh(x)}\,dx = -\frac{(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)+1)-(\cosh(x)+\sinh(x))\log(\cosh(x)+\sinh(x)-1)-}{\cosh(x)+\sinh(x)}$$

```
[In] integrate(csch(x)/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -((cosh(x) + sinh(x))*log(cosh(x) + sinh(x) + 1) - (cosh(x) + sinh(x))*log(
cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{csch}(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(csch(x)/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{csch}(x)}{1+\tanh(x)}\,dx = e^{(-x)} - \log\left(e^{(-x)}+1\right) + \log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] e^(-x) - log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{csch}(x)}{1+\tanh(x)}\,dx = e^{(-x)} - \log\left(e^x+1\right) + \log\left(|e^x-1|\right)$$

```
[In] integrate(csch(x)/(1+tanh(x)),x, algorithm="giac")
[Out] e^(-x) - log(e^x + 1) + log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{csch}(x)}{1+\tanh(x)}\,dx = \ln\left(2-2\,e^x\right) - \ln\left(-2\,e^x-2\right) + e^{-x}$$

```
[In] int(1/(sinh(x)*(tanh(x) + 1)),x)
[Out] log(2 - 2*exp(x)) - log(- 2*exp(x) - 2) + exp(-x)
```

3.74 $\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = -\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))$$

```
[Out] -coth(x)-ln(tanh(x))+ln(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3597, 46}

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = -\coth(x)-\log(\tanh(x))+\log(\tanh(x)+1)$$

```
[In] Int[Csch[x]^2/(1 + Tanh[x]),x]
```

```
[Out] -Coth[x] - Log[Tanh[x]] + Log[1 + Tanh[x]]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx, x, \tanh(x)\right)\\ &= -\coth(x)-\log(\tanh(x))+\log(1+\tanh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = x-\coth(x)-\log(\sinh(x))$$

```
[In] Integrate[Csch[x]^2/(1 + Tanh[x]),x]
[Out] x - Coth[x] - Log[Sinh[x]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

method	result	size
risch	$2x-\frac{2}{e^{2x}-1}-\ln\left(e^{2x}-1\right)$	24
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)+2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)$	32

```
[In] int(csch(x)^2/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 2*x-2/(exp(2*x)-1)-ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 5.13

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = \frac{2\,x\cosh\left(x\right)^{2} + 4\,x\cosh\left(x\right)\sinh\left(x\right) + 2\,x\sinh\left(x\right)^{2} - \left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 1\right)\log}{\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} - 1}$$

```
[In] integrate(csch(x)^2/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] (2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(
x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*x - 2)/(
cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{csch}^{2}\left(x\right)}{\tanh\left(x\right) + 1}\,dx$$

```
[In] integrate(csch(x)**2/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**2/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = \frac{2}{e^{\left(-2\,x\right)} - 1} - \log\left(e^{\left(-x\right)} + 1\right) - \log\left(e^{\left(-x\right)} - 1\right)$$

```
[In] integrate(csch(x)^2/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) - 1) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = 2\,x + \frac{e^{(2\,x)} - 3}{e^{(2\,x)} - 1} - \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(csch(x)^2/(1+tanh(x)),x, algorithm="giac")
[Out] 2*x + (e^(2*x) - 3)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\operatorname{csch}^2(x)}{1+\tanh(x)}\,dx = 2\,x - \ln\left(\mathrm{e}^{2\,x} - 1\right) - \frac{2}{\mathrm{e}^{2\,x} - 1}$$

```
[In] int(1/(sinh(x)^2*(tanh(x) + 1)),x)
[Out] 2*x - log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

3.75 $\int \frac{\text{csch}^3(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{\text{csch}^3(x)}{1+\tanh(x)}\,dx = -\frac{1}{2}\text{arctanh}(\cosh(x))+\text{csch}(x)-\frac{1}{2}\coth(x)\text{csch}(x)$$

```
[Out] -1/2*arctanh(cosh(x))+csch(x)-1/2*coth(x)*csch(x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3599, 3187, 3186, 2686, 8, 2691, 3855}

$$\int \frac{\text{csch}^3(x)}{1+\tanh(x)}\,dx = -\frac{1}{2}\text{arctanh}(\cosh(x))+\text{csch}(x)-\frac{1}{2}\coth(x)\text{csch}(x)$$

```
[In] Int[Csch[x]^3/(1 + Tanh[x]),x]
[Out] -1/2*ArcTanh[Cosh[x]] + Csch[x] - (Coth[x]*Csch[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{\coth(x)\text{csch}^2(x)}{\cosh(x)+\sinh(x)}\,dx\\ &= i\int \coth(x)\text{csch}^2(x)(-i\cosh(x)+i\sinh(x))\,dx\\ &= \int \left(-\coth(x)\text{csch}(x)+\coth^2(x)\text{csch}(x)\right)\,dx\end{aligned}$$

$$= -\int \coth(x)\text{csch}(x)\,dx + \int \coth^2(x)\text{csch}(x)\,dx$$

$$= -\frac{1}{2}\coth(x)\text{csch}(x) + i\text{Subst}\left(\int 1\,dx, x, -i\text{csch}(x)\right) + \frac{1}{2}\int \text{csch}(x)\,dx$$

$$= -\frac{1}{2}\text{arctanh}(\cosh(x)) + \text{csch}(x) - \frac{1}{2}\coth(x)\text{csch}(x)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 59 vs. $2(18) = 36$.

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{\text{csch}^3(x)}{1+\tanh(x)}\,dx = \frac{1}{8}\left(4\coth\left(\frac{x}{2}\right) - \text{csch}^2\left(\frac{x}{2}\right) - 4\log\left(\cosh\left(\frac{x}{2}\right)\right) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right) - \text{sech}^2\left(\frac{x}{2}\right) - 4\tanh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (4*Coth[x/2] - Csch[x/2]^2 - 4*Log[Cosh[x/2]] + 4*Log[Sinh[x/2]] - Sech[x/2
]^2 - 4*Tanh[x/2])/8
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

method	result	size
risch	$\frac{e^x(e^{2x}-3)}{(e^{2x}-1)^2} + \frac{\ln(e^x-1)}{2} - \frac{\ln(e^x+1)}{2}$	33
default	$\frac{\tanh(\frac{x}{2})^2}{8} - \frac{\tanh(\frac{x}{2})}{2} + \frac{\ln(\tanh(\frac{x}{2}))}{2} - \frac{1}{8\tanh(\frac{x}{2})^2} + \frac{1}{2\tanh(\frac{x}{2})}$	39

```
[In] int(csch(x)^3/(1+tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)*(exp(2*x)-3)/(exp(2*x)-1)^2+1/2*ln(exp(x)-1)-1/2*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(14) = 28.

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 11.61

$$\int \frac{\operatorname{csch}^3(x)}{1+\tanh(x)}\,dx = \frac{2\cosh(x)^3+6\cosh(x)\sinh(x)^2+2\sinh(x)^3-\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh\right.\right.}{}$$

```
[In] integrate(csch(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^4 + 4*cosh(
x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*
(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(
x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*
(cosh(x)^2 - 1)*sinh(x) - 6*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin
h(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(
x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(csch(x)**3/(1+tanh(x)),x)
```

```
[Out] Integral(csch(x)**3/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(14) = 28.

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \frac{\operatorname{csch}^3(x)}{1+\tanh(x)}\,dx = -\frac{e^{(-x)}-3\,e^{(-3\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) - 3*e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) - 1/2*log(e^(-x) + 1) +
1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{csch}^3(x)}{1+\tanh(x)}\,dx = \frac{e^{(3x)} - 3e^x}{\left(e^{(2x)} - 1\right)^2} - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(\left|e^x - 1\right|\right)$$

```
[In] integrate(csch(x)^3/(1+tanh(x)),x, algorithm="giac")
[Out] (e^(3*x) - 3*e^x)/(e^(2*x) - 1)^2 - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1)
)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int \frac{\operatorname{csch}^3(x)}{1+\tanh(x)}\,dx = \frac{\ln\left(1 - e^x\right)}{2} - \frac{\ln\left(-e^x - 1\right)}{2} + \frac{e^x}{e^{2x} - 1} - \frac{2e^x}{e^{4x} - 2e^{2x} + 1}$$

```
[In] int(1/(sinh(x)^3*(tanh(x) + 1)),x)
[Out] log(1 - exp(x))/2 - log(- exp(x) - 1)/2 + exp(x)/(exp(2*x) - 1) - (2*exp(x)
)/(exp(4*x) - 2*exp(2*x) + 1)
```

3.76 $\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx=\frac{\coth^2(x)}{2}-\frac{\coth^3(x)}{3}$$

```
[Out] 1/2*coth(x)^2-1/3*coth(x)^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3597, 862, 45}

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx=\frac{\coth^2(x)}{2}-\frac{\coth^3(x)}{3}$$

```
[In] Int[Csch[x]^4/(1 + Tanh[x]),x]
```

```
[Out] Coth[x]^2/2 - Coth[x]^3/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{-1+x^2}{x^4(1+x)}\,dx, x, \tanh(x)\right)\\
&= -\text{Subst}\left(\int \frac{-1+x}{x^4}\,dx, x, \tanh(x)\right)\\
&= -\text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x^3}\right)\,dx, x, \tanh(x)\right)\\
&= \frac{\coth^2(x)}{2}-\frac{\coth^3(x)}{3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx = -\frac{1}{6}\operatorname{csch}(x)(2\cosh(x)+(-3+2\coth(x))\operatorname{csch}(x))$$

```
[In] Integrate[Csch[x]^4/(1 + Tanh[x]),x]
[Out] -1/6*(Csch[x]*(2*Cosh[x] + (-3 + 2*Coth[x])*Csch[x]))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

method	result	size
risch	$-\frac{2(3\,e^{2x}+1)}{3(e^{2x}-1)^3}$	19
parallelrisch	$\frac{7\coth(x)^2}{12}-\frac{\coth(x)^3}{3}-\frac{\operatorname{csch}(x)^2}{12}$	20
default	$-\frac{\tanh(\frac{x}{2})^3}{24}+\frac{\tanh(\frac{x}{2})^2}{8}-\frac{\tanh(\frac{x}{2})}{8}+\frac{1}{8\tanh(\frac{x}{2})^2}-\frac{1}{24\tanh(\frac{x}{2})^3}-\frac{1}{8\tanh(\frac{x}{2})}$	48

```
[In] int(csch(x)^4/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -2/3*(3*exp(2*x)+1)/(exp(2*x)-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.94

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx = -\frac{4\,(2\,\cosh(x)+\sinh(x)}{3\,\left(\cosh(x)^5+5\,\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\,\cosh(x)^2-3\right)\sinh(x)^3-3\,\cosh(x)^3+\left(10\,\cosh(x\right.\right.}$$

```
[In] integrate(csch(x)^4/(1+tanh(x)),x, algorithm="fricas")
[Out] -4/3*(2*cosh(x) + sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 - 3)*sinh(x)^3 - 3*cosh(x)^3 + (10*cosh(x)^3 - 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 - 9*cosh(x)^2 + 4)*sinh(x) + 2*cosh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(csch(x)**4/(1+tanh(x)),x)
[Out] Integral(csch(x)**4/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(13) = 26$.

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx = -\frac{2\,e^{(-2\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1}+\frac{4\,e^{(-4\,x)}}{3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1} + \frac{2}{3\,\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}$$

```
[In] integrate(csch(x)^4/(1+tanh(x)),x, algorithm="maxima")
[Out] -2*e^(-2*x)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1) + 4*e^(-4*x)/(3*e^(-2*
x) - 3*e^(-4*x) + e^(-6*x) - 1) + 2/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) -
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx = -\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}-1\right)^3}$$

```
[In] integrate(csch(x)^4/(1+tanh(x)),x, algorithm="giac")
[Out] -2/3*(3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^4(x)}{1+\tanh(x)}\,dx = -\frac{2\left(3\,\mathrm{e}^{2\,x}+1\right)}{3\left(\mathrm{e}^{2\,x}-1\right)^3}$$

```
[In] int(1/(sinh(x)^4*(tanh(x) + 1)),x)
[Out] -(2*(3*exp(2*x) + 1))/(3*(exp(2*x) - 1)^3)
```

3.77 $\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)} \, dx = \frac{1}{8}\operatorname{arctanh}(\cosh(x)) - \frac{1}{8}\coth(x)\operatorname{csch}(x) + \frac{\operatorname{csch}^3(x)}{3} - \frac{1}{4}\coth(x)\operatorname{csch}^3(x)$$

```
[Out] 1/8*arctanh(cosh(x))-1/8*coth(x)*csch(x)+1/3*csch(x)^3-1/4*coth(x)*csch(x)^
3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3599, 3187, 3186, 2686, 30, 2691, 3853, 3855}

$$\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)} \, dx = \frac{1}{8}\operatorname{arctanh}(\cosh(x)) + \frac{\operatorname{csch}^3(x)}{3} - \frac{1}{4}\coth(x)\operatorname{csch}^3(x) - \frac{1}{8}\coth(x)\operatorname{csch}(x)$$

```
[In] Int[Csch[x]^5/(1 + Tanh[x]),x]
```

```
[Out] ArcTanh[Cosh[x]]/8 - (Coth[x]*Csch[x])/8 + Csch[x]^3/3 - (Coth[x]*Csch[x]^3
)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\coth(x)\text{csch}^4(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \coth(x)\text{csch}^4(x)(-i\cosh(x)+i\sinh(x))\,dx \\
&= -\int \left(\coth(x)\text{csch}^3(x)-\coth^2(x)\text{csch}^3(x)\right)\,dx \\
&= -\int \coth(x)\text{csch}^3(x)\,dx+\int \coth^2(x)\text{csch}^3(x)\,dx \\
&= -\frac{1}{4}\coth(x)\text{csch}^3(x)-i\text{Subst}\left(\int x^2\,dx,x,-i\text{csch}(x)\right)+\frac{1}{4}\int \text{csch}^3(x)\,dx \\
&= -\frac{1}{8}\coth(x)\text{csch}(x)+\frac{\text{csch}^3(x)}{3}-\frac{1}{4}\coth(x)\text{csch}^3(x)-\frac{1}{8}\int \text{csch}(x)\,dx \\
&= \frac{1}{8}\text{arctanh}(\cosh(x))-\frac{1}{8}\coth(x)\text{csch}(x)+\frac{\text{csch}^3(x)}{3}-\frac{1}{4}\coth(x)\text{csch}^3(x)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$
\begin{aligned}
\int \frac{\text{csch}^5(x)}{1+\tanh(x)}\,dx = \frac{1}{192}\text{csch}^4(x)\Big(&-42\cosh(x)-6\cosh(3x) \\
&+2\sinh(x)\left(32-9\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(x)\right. \\
&\left.+3\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\sinh(3x)\right)\Big)
\end{aligned}
$$

```
[In] Integrate[Csch[x]^5/(1 + Tanh[x]),x]
```

```
[Out] (Csch[x]^4*(-42*Cosh[x] - 6*Cosh[3*x] + 2*Sinh[x]*(32 - 9*(Log[Cosh[x/2]] -
 Log[Sinh[x/2]])*Sinh[x] + 3*(Log[Cosh[x/2]] - Log[Sinh[x/2]])*Sinh[3*x])))
/192
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

method	result	size
risch	$-\frac{e^{x}\left(3\,e^{6x}-11\,e^{4x}+53\,e^{2x}+3\right)}{12\left(e^{2x}-1\right)^{4}}-\frac{\ln\left(e^{x}-1\right)}{8}+\frac{\ln\left(e^{x}+1\right)}{8}$	48
default	$\frac{\tanh\left(\frac{x}{2}\right)^{4}}{64}-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{24}+\frac{\tanh\left(\frac{x}{2}\right)}{8}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8}+\frac{1}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{1}{64\tanh\left(\frac{x}{2}\right)^{4}}-\frac{1}{8\tanh\left(\frac{x}{2}\right)}$	55

```
[In] int(csch(x)^5/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -1/12*exp(x)*(3*exp(6*x)-11*exp(4*x)+53*exp(2*x)+3)/(exp(2*x)-1)^4-1/8*ln(e
xp(x)-1)+1/8*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 640, normalized size of antiderivative = 18.82

$$\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^5/(1+tanh(x)),x, algorithm="fricas")
[Out] -1/24*(6*cosh(x)^7 + 42*cosh(x)*sinh(x)^6 + 6*sinh(x)^7 + 2*(63*cosh(x)^2 -
 11)*sinh(x)^5 - 22*cosh(x)^5 + 10*(21*cosh(x)^3 - 11*cosh(x))*sinh(x)^4 +
2*(105*cosh(x)^4 - 110*cosh(x)^2 + 53)*sinh(x)^3 + 106*cosh(x)^3 + 2*(63*co
sh(x)^5 - 110*cosh(x)^3 + 159*cosh(x))*sinh(x)^2 - 3*(cosh(x)^8 + 8*cosh(x)
*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7
*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sin
h(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2
 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cos
h(x) + sinh(x) + 1) + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7
*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(
x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*c
osh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x
)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5
 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(21*c
osh(x)^6 - 55*cosh(x)^4 + 159*cosh(x)^2 + 3)*sinh(x) + 6*cosh(x))/(cosh(x)^
8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cos
h(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(
x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(
x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2
- 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x)
 + 1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{csch}^5(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(csch(x)**5/(1+tanh(x)),x)
[Out] Integral(csch(x)**5/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.18

$$\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)}\,dx = \frac{3\,e^{(-x)} - 11\,e^{(-3\,x)} + 53\,e^{(-5\,x)} + 3\,e^{(-7\,x)}}{12\,(4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1)} + \frac{1}{8}\,\log\left(e^{(-x)} + 1\right) - \frac{1}{8}\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(csch(x)^5/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/12*(3*e^(-x) - 11*e^(-3*x) + 53*e^(-5*x) + 3*e^(-7*x))/(4*e^(-2*x) - 6*e^
(-4*x) + 4*e^(-6*x) - e^(-8*x) - 1) + 1/8*log(e^(-x) + 1) - 1/8*log(e^(-x)
- 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)}\,dx = -\frac{3\,e^{(7\,x)} - 11\,e^{(5\,x)} + 53\,e^{(3\,x)} + 3\,e^{x}}{12\,(e^{(2\,x)} - 1)^4} + \frac{1}{8}\,\log\left(e^{x} + 1\right) - \frac{1}{8}\,\log\left(|e^{x} - 1|\right)$$

```
[In] integrate(csch(x)^5/(1+tanh(x)),x, algorithm="giac")
[Out] -1/12*(3*e^(7*x) - 11*e^(5*x) + 53*e^(3*x) + 3*e^x)/(e^(2*x) - 1)^4 + 1/8*l
og(e^x + 1) - 1/8*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.44

$$\int \frac{\operatorname{csch}^5(x)}{1+\tanh(x)}\,dx = \frac{\ln\left(\frac{e^x}{4}+\frac{1}{4}\right)}{8} - \frac{\ln\left(\frac{e^x}{4}-\frac{1}{4}\right)}{8} - \frac{e^x}{4\,\left(e^{2\,x}-1\right)} - \frac{2\,e^{3\,x}+2\,e^x}{6\,e^{4\,x}-4\,e^{2\,x}-4\,e^{6\,x}+e^{8\,x}+1} - \frac{4\,e^x}{3\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} + \frac{e^x}{6\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}$$

```
[In] int(1/(sinh(x)^5*(tanh(x) + 1)),x)
```

```
[Out] log(exp(x)/4 + 1/4)/8 - log(exp(x)/4 - 1/4)/8 - exp(x)/(4*(exp(2*x) - 1)) -
 (2*exp(3*x) + 2*exp(x))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) +
 1) - (4*exp(x))/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) + exp(x)/(6*(
exp(4*x) - 2*exp(2*x) + 1))
```

3.78 $\int \frac{\text{csch}^6(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{\text{csch}^6(x)}{1+\tanh(x)}\,dx = -\frac{1}{2}\coth^2(x)+\frac{\coth^3(x)}{3}+\frac{\coth^4(x)}{4}-\frac{\coth^5(x)}{5}$$

```
[Out] -1/2*coth(x)^2+1/3*coth(x)^3+1/4*coth(x)^4-1/5*coth(x)^5
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3597, 862, 76}

$$\int \frac{\text{csch}^6(x)}{1+\tanh(x)}\,dx = -\frac{1}{5}\coth^5(x)+\frac{\coth^4(x)}{4}+\frac{\coth^3(x)}{3}-\frac{\coth^2(x)}{2}$$

```
[In] Int[Csch[x]^6/(1 + Tanh[x]),x]
```

```
[Out] -1/2*Coth[x]^2 + Coth[x]^3/3 + Coth[x]^4/4 - Coth[x]^5/5
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{(-1+x^2)^2}{x^6(1+x)}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \frac{(-1+x)^2(1+x)}{x^6}\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{x^6}-\frac{1}{x^5}-\frac{1}{x^4}+\frac{1}{x^3}\right)\,dx, x, \tanh(x)\right)\\ &= -\frac{1}{2}\coth^2(x)+\frac{\coth^3(x)}{3}+\frac{\coth^4(x)}{4}-\frac{\coth^5(x)}{5}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{csch}^6(x)}{1+\tanh(x)}\,dx = \frac{1}{120}\operatorname{csch}^5(x)(-20\cosh(x)-5\cosh(3x)+\cosh(5x)+30\sinh(x))$$

```
[In] Integrate[Csch[x]^6/(1 + Tanh[x]),x]
[Out] (Csch[x]^5*(-20*Cosh[x] - 5*Cosh[3*x] + Cosh[5*x] + 30*Sinh[x]))/120
```

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{4(20\,e^{4x}+5\,e^{2x}-1)}{15(e^{2x}-1)^5}$
parallelrisch	$\frac{(384\coth(x)-636)\cosh(2x)+(-64\coth(x)+159)\cosh(4x)+448\coth(x)-483}{-1440-480\cosh(4x)+1920\cosh(2x)}$
default	$-\frac{\tanh(\frac{x}{2})^5}{160}+\frac{\tanh(\frac{x}{2})^4}{64}+\frac{\tanh(\frac{x}{2})^3}{96}-\frac{\tanh(\frac{x}{2})^2}{16}+\frac{\tanh(\frac{x}{2})}{16}+\frac{1}{64\tanh(\frac{x}{2})^4}-\frac{1}{16\tanh(\frac{x}{2})^2}+\frac{1}{96\tanh(\frac{x}{2})^3}-$

```
[In] int(csch(x)^6/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -4/15*(20*exp(4*x)+5*exp(2*x)-1)/(exp(2*x)-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(25) = 50.

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.61

$$\int \frac{\operatorname{csch}^6(x)}{1+\tanh(x)}\,dx =$$

$$-\frac{}{15\left(\cosh\left(x\right)^{8}+8\,\cosh\left(x\right)\sinh\left(x\right)^{7}+\sinh\left(x\right)^{8}+\left(28\,\cosh\left(x\right)^{2}-5\right)\sinh\left(x\right)^{6}-5\,\cosh\left(x\right)^{6}+2\,\left(28\,\cosh\right.}$$

```
[In] integrate(csch(x)^6/(1+tanh(x)),x, algorithm="fricas")
[Out] -4/15*(19*cosh(x)^2 + 42*cosh(x)*sinh(x) + 19*sinh(x)^2 + 5)/(cosh(x)^8 + 8
*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 5)*sinh(x)^6 - 5*cosh(x)^6
 + 2*(28*cosh(x)^3 - 15*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 15*cosh(x)^2
 + 2)*sinh(x)^4 + 10*cosh(x)^4 + 4*(14*cosh(x)^5 - 25*cosh(x)^3 + 10*cosh(x
))*sinh(x)^3 + (28*cosh(x)^6 - 75*cosh(x)^4 + 60*cosh(x)^2 - 11)*sinh(x)^2
- 11*cosh(x)^2 + 2*(4*cosh(x)^7 - 15*cosh(x)^5 + 20*cosh(x)^3 - 9*cosh(x))*
sinh(x) + 5)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^6(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{csch}^{6}\left(x\right)}{\tanh\left(x\right)+1}\,dx$$

```
[In] integrate(csch(x)**6/(1+tanh(x)),x)
[Out] Integral(csch(x)**6/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(25) = 50.

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.52

$$\begin{aligned}\int \frac{\operatorname{csch}^6(x)}{1+\tanh(x)}\,dx &= \frac{4\,e^{(-2\,x)}}{3\,\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}\\ &\quad-\frac{8\,e^{(-4\,x)}}{3\,\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}\\ &\quad+\frac{8\,e^{(-6\,x)}}{5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1}\\ &\quad-\frac{4}{15\,\left(5\,e^{(-2\,x)}-10\,e^{(-4\,x)}+10\,e^{(-6\,x)}-5\,e^{(-8\,x)}+e^{(-10\,x)}-1\right)}\end{aligned}$$

```
[In] integrate(csch(x)^6/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 4/3*e^(-2*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x) + e^(-10*
x) - 1) - 8/3*e^(-4*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*e^(-8*x)
 + e^(-10*x) - 1) + 8*e^(-6*x)/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) - 5*
e^(-8*x) + e^(-10*x) - 1) - 4/15/(5*e^(-2*x) - 10*e^(-4*x) + 10*e^(-6*x) -
5*e^(-8*x) + e^(-10*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{csch}^6(x)}{1+\tanh(x)}\,dx = -\frac{4\left(20\,e^{(4\,x)}+5\,e^{(2\,x)}-1\right)}{15\left(e^{(2\,x)}-1\right)^5}$$

```
[In] integrate(csch(x)^6/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -4/15*(20*e^(4*x) + 5*e^(2*x) - 1)/(e^(2*x) - 1)^5
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int \frac{\operatorname{csch}^6(x)}{1+\tanh(x)}\,dx = -\frac{4\left(5\,\mathrm{e}^{2\,x}+20\,\mathrm{e}^{4\,x}-1\right)}{15\left(\mathrm{e}^{2\,x}-1\right)^5}$$

```
[In] int(1/(sinh(x)^6*(tanh(x) + 1)),x)
```

```
[Out] -(4*(5*exp(2*x) + 20*exp(4*x) - 1))/(15*(exp(2*x) - 1)^5)
```

3.79 $\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)} \, dx = -\frac{1}{16}\operatorname{arctanh}(\cosh(x)) + \frac{1}{16}\coth(x)\operatorname{csch}(x) - \frac{1}{24}\coth(x)\operatorname{csch}^3(x) + \frac{\operatorname{csch}^5(x)}{5} - \frac{1}{6}\coth(x)\operatorname{csch}^5(x)$$

```
[Out] -1/16*arctanh(cosh(x))+1/16*coth(x)*csch(x)-1/24*coth(x)*csch(x)^3+1/5*csch
(x)^5-1/6*coth(x)*csch(x)^5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3599, 3187, 3186, 2686, 30, 2691, 3853, 3855}

$$\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)} \, dx = -\frac{1}{16}\operatorname{arctanh}(\cosh(x)) + \frac{\operatorname{csch}^5(x)}{5} - \frac{1}{6}\coth(x)\operatorname{csch}^5(x) - \frac{1}{24}\coth(x)\operatorname{csch}^3(x) + \frac{1}{16}\coth(x)\operatorname{csch}(x)$$

```
[In] Int[Csch[x]^7/(1 + Tanh[x]),x]
```

```
[Out] -1/16*ArcTanh[Cosh[x]] + (Coth[x]*Csch[x])/16 - (Coth[x]*Csch[x]^3)/24 + Cs
ch[x]^5/5 - (Coth[x]*Csch[x]^5)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\coth(x)\text{csch}^6(x)}{\cosh(x)+\sinh(x)}\,dx \\
&= i\int \coth(x)\text{csch}^6(x)(-i\cosh(x)+i\sinh(x))\,dx \\
&= \int \left(-\coth(x)\text{csch}^5(x)+\coth^2(x)\text{csch}^5(x)\right)\,dx \\
&= -\int \coth(x)\text{csch}^5(x)\,dx+\int \coth^2(x)\text{csch}^5(x)\,dx \\
&= -\frac{1}{6}\coth(x)\text{csch}^5(x)+i\text{Subst}\left(\int x^4\,dx,x,-i\text{csch}(x)\right)+\frac{1}{6}\int \text{csch}^5(x)\,dx \\
&= -\frac{1}{24}\coth(x)\text{csch}^3(x)+\frac{\text{csch}^5(x)}{5}-\frac{1}{6}\coth(x)\text{csch}^5(x)-\frac{1}{8}\int \text{csch}^3(x)\,dx \\
&= \frac{1}{16}\coth(x)\text{csch}(x)-\frac{1}{24}\coth(x)\text{csch}^3(x)+\frac{\text{csch}^5(x)}{5}-\frac{1}{6}\coth(x)\text{csch}^5(x)+\frac{1}{16}\int \text{csch}(x)\,dx \\
&= -\frac{1}{16}\text{arctanh}(\cosh(x))+\frac{1}{16}\coth(x)\text{csch}(x) \\
&\quad -\frac{1}{24}\coth(x)\text{csch}^3(x)+\frac{\text{csch}^5(x)}{5}-\frac{1}{6}\coth(x)\text{csch}^5(x)
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

$$
\int \frac{\text{csch}^7(x)}{1+\tanh(x)}\,dx = \frac{72\coth\left(\frac{x}{2}\right)+30\text{csch}^2\left(\frac{x}{2}\right)-120\log\left(\cosh\left(\frac{x}{2}\right)\right)+120\log\left(\sinh\left(\frac{x}{2}\right)\right)+30\text{sech}^2\left(\frac{x}{2}\right)-5\text{sech}^6\left(\frac{x}{2}\right)-288\text{csch}^3}{1920}
$$

```
[In] Integrate[Csch[x]^7/(1 + Tanh[x]),x]
```

```
[Out] (72*Coth[x/2] + 30*Csch[x/2]^2 - 120*Log[Cosh[x/2]] + 120*Log[Sinh[x/2]] +
30*Sech[x/2]^2 - 5*Sech[x/2]^6 - 288*Csch[x]^3*Sinh[x/2]^4 - 384*Csch[x]^5*
Sinh[x/2]^6 - 18*Csch[x/2]^4*Sinh[x] + Csch[x/2]^6*(-5 + 6*Sinh[x]) - 72*Ta
nh[x/2])/1920
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

method	result
risch	$\frac{e^{x}(15\,e^{10x}-85\,e^{8x}+198\,e^{6x}-1338\,e^{4x}-85\,e^{2x}+15)}{120(e^{2x}-1)^{6}}+\frac{\ln(e^{x}-1)}{16}-\frac{\ln(e^{x}+1)}{16}$
default	$\frac{\tanh(\frac{x}{2})^{6}}{384}-\frac{\tanh(\frac{x}{2})^{5}}{160}-\frac{\tanh(\frac{x}{2})^{4}}{128}+\frac{\tanh(\frac{x}{2})^{3}}{32}-\frac{\tanh(\frac{x}{2})^{2}}{128}-\frac{\tanh(\frac{x}{2})}{16}+\frac{\ln(\tanh(\frac{x}{2}))}{16}+\frac{1}{128\tanh(\frac{x}{2})^{4}}+\frac{1}{160\tanh(}$

```
[In] int(csch(x)^7/(1+tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*exp(x)*(15*exp(10*x)-85*exp(8*x)+198*exp(6*x)-1338*exp(4*x)-85*exp(2*
x)+15)/(exp(2*x)-1)^6+1/16*ln(exp(x)-1)-1/16*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 1260, normalized size of antiderivative = 28.64

$$\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^7/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/240*(30*cosh(x)^11 + 330*cosh(x)*sinh(x)^10 + 30*sinh(x)^11 + 10*(165*cos
h(x)^2 - 17)*sinh(x)^9 - 170*cosh(x)^9 + 90*(55*cosh(x)^3 - 17*cosh(x))*sin
h(x)^8 + 36*(275*cosh(x)^4 - 170*cosh(x)^2 + 11)*sinh(x)^7 + 396*cosh(x)^7
+ 84*(165*cosh(x)^5 - 170*cosh(x)^3 + 33*cosh(x))*sinh(x)^6 + 12*(1155*cosh
(x)^6 - 1785*cosh(x)^4 + 693*cosh(x)^2 - 223)*sinh(x)^5 - 2676*cosh(x)^5 +
60*(165*cosh(x)^7 - 357*cosh(x)^5 + 231*cosh(x)^3 - 223*cosh(x))*sinh(x)^4
+ 10*(495*cosh(x)^8 - 1428*cosh(x)^6 + 1386*cosh(x)^4 - 2676*cosh(x)^2 - 17
)*sinh(x)^3 - 170*cosh(x)^3 + 6*(275*cosh(x)^9 - 1020*cosh(x)^7 + 1386*cosh
(x)^5 - 4460*cosh(x)^3 - 85*cosh(x))*sinh(x)^2 - 15*(cosh(x)^12 + 12*cosh(x
)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10
+ 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2
 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x
))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x
)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cos
h(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(
x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*c
osh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh
(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(
x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(
x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 15*(cosh(x)^12 +
```

```
12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*co
sh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*
cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 +
 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 -
5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^
3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 -
 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)
^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10
- 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2
- 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5
+ 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(55*co
sh(x)^10 - 255*cosh(x)^8 + 462*cosh(x)^6 - 2230*cosh(x)^4 - 85*cosh(x)^2 +
5)*sinh(x) + 30*cosh(x))/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 +
 6*(11*cosh(x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh
(x))*sinh(x)^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x
)^8 + 24*(33*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(
x)^6 - 315*cosh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33
*cosh(x)^7 - 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*co
sh(x)^8 - 84*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*co
sh(x)^4 + 20*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3
*cosh(x))*sinh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*c
osh(x)^4 + 15*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*c
osh(x)^9 + 10*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1
)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{csch}^7(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(csch(x)**7/(1+tanh(x)),x)
[Out] Integral(csch(x)**7/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.23

$$\begin{aligned}&\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)}\,dx\\&= -\frac{15\,e^{(-x)} - 85\,e^{(-3\,x)} + 198\,e^{(-5\,x)} - 1338\,e^{(-7\,x)} - 85\,e^{(-9\,x)} + 15\,e^{(-11\,x)}}{120\,\left(6\,e^{(-2\,x)} - 15\,e^{(-4\,x)} + 20\,e^{(-6\,x)} - 15\,e^{(-8\,x)} + 6\,e^{(-10\,x)} - e^{(-12\,x)} - 1\right)}\\&\quad - \frac{1}{16}\,\log\left(e^{(-x)} + 1\right) + \frac{1}{16}\,\log\left(e^{(-x)} - 1\right)\end{aligned}$$

```
[In] integrate(csch(x)^7/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] -1/120*(15*e^(-x) - 85*e^(-3*x) + 198*e^(-5*x) - 1338*e^(-7*x) - 85*e^(-9*x
) + 15*e^(-11*x))/(6*e^(-2*x) - 15*e^(-4*x) + 20*e^(-6*x) - 15*e^(-8*x) + 6
*e^(-10*x) - e^(-12*x) - 1) - 1/16*log(e^(-x) + 1) + 1/16*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)}\,dx = \frac{15\,e^{(11\,x)} - 85\,e^{(9\,x)} + 198\,e^{(7\,x)} - 1338\,e^{(5\,x)} - 85\,e^{(3\,x)} + 15\,e^{x}}{120\,\left(e^{(2\,x)} - 1\right)^{6}} - \frac{1}{16}\,\log\left(e^{x} + 1\right) + \frac{1}{16}\,\log\left(|e^{x} - 1|\right)$$

```
[In] integrate(csch(x)^7/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/120*(15*e^(11*x) - 85*e^(9*x) + 198*e^(7*x) - 1338*e^(5*x) - 85*e^(3*x) +
 15*e^x)/(e^(2*x) - 1)^6 - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.70

$$\int \frac{\operatorname{csch}^7(x)}{1+\tanh(x)}\,dx = \frac{\ln\left(\frac{1}{8} - \frac{e^{x}}{8}\right)}{16} - \frac{\ln\left(-\frac{e^{x}}{8} - \frac{1}{8}\right)}{16} - \frac{\frac{16\,e^{3\,x}}{3} + \frac{16\,e^{5\,x}}{3}}{15\,e^{4\,x} - 6\,e^{2\,x} - 20\,e^{6\,x} + 15\,e^{8\,x} - 6\,e^{10\,x} + e^{12\,x} + 1} - \frac{\frac{8\,e^{3\,x}}{3} + \frac{8\,e^{x}}{5}}{5\,e^{2\,x} - 10\,e^{4\,x} + 10\,e^{6\,x} - 5\,e^{8\,x} + e^{10\,x} - 1} - \frac{6\,e^{x}}{5\,\left(6\,e^{4\,x} - 4\,e^{2\,x} - 4\,e^{6\,x} + e^{8\,x} + 1\right)} + \frac{e^{x}}{8\,\left(e^{2\,x} - 1\right)} + \frac{e^{x}}{15\,\left(3\,e^{2\,x} - 3\,e^{4\,x} + e^{6\,x} - 1\right)} - \frac{e^{x}}{12\,\left(e^{4\,x} - 2\,e^{2\,x} + 1\right)}$$

```
[In] int(1/(sinh(x)^7*(tanh(x) + 1)),x)
```

```
[Out] log(1/8 - exp(x)/8)/16 - log(- exp(x)/8 - 1/8)/16 - ((16*exp(3*x))/3 + (16*
exp(5*x))/3)/(15*exp(4*x) - 6*exp(2*x) - 20*exp(6*x) + 15*exp(8*x) - 6*exp(
10*x) + exp(12*x) + 1) - ((8*exp(3*x))/3 + (8*exp(x))/5)/(5*exp(2*x) - 10*e
xp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) - (6*exp(x))/(5*(6*exp(
4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) + exp(x)/(8*(exp(2*x) - 1))
 + exp(x)/(15*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - exp(x)/(12*(exp(4
*x) - 2*exp(2*x) + 1))
```

3.80 $\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 147

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = -\frac{a(3a+b)\log(1-\tanh(x))}{16(a+b)^3}+\frac{a(3a-b)\log(1+\tanh(x))}{16(a-b)^3} -\frac{a^4b\log(a+b\tanh(x))}{(a^2-b^2)^3}-\frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} +\frac{\cosh^2(x)\left(4b(2a^2-b^2)-a(5a^2-b^2)\tanh(x)\right)}{8(a^2-b^2)^2}$$

```
[Out] -1/16*a*(3*a+b)*ln(1-tanh(x))/(a+b)^3+1/16*a*(3*a-b)*ln(1+tanh(x))/(a-b)^3-
a^4*b*ln(a+b*tanh(x))/(a^2-b^2)^3-1/4*cosh(x)^4*(b-a*tanh(x))/(a^2-b^2)+1/8
*cosh(x)^2*(4*b*(2*a^2-b^2)-a*(5*a^2-b^2)*tanh(x))/(a^2-b^2)^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3597, 1661, 815}

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = -\frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} +\frac{\cosh^2(x)\left(4b(2a^2-b^2)-a(5a^2-b^2)\tanh(x)\right)}{8(a^2-b^2)^2} -\frac{a^4b\log(a+b\tanh(x))}{(a^2-b^2)^3}-\frac{a(3a+b)\log(1-\tanh(x))}{16(a+b)^3} +\frac{a(3a-b)\log(\tanh(x)+1)}{16(a-b)^3}$$

```
[In] Int[Sinh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] -1/16*(a*(3*a + b)*Log[1 - Tanh[x]])/(a + b)^3 + (a*(3*a - b)*Log[1 + Tanh[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Tanh[x]])/(a^2 - b^2)^3 - (Cosh[x]^4
*(b - a*Tanh[x]))/(4*(a^2 - b^2)) + (Cosh[x]^2*(4*b*(2*a^2 - b^2) - a*(5*a^
2 - b^2)*Tanh[x]))/(8*(a^2 - b^2)^2)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{x^4}{(a+x)\left(-b^2+x^2\right)^3}\,dx, x, b\tanh(x)\right)\right)\\
&= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2-b^2}-\frac{3ab^4x}{a^2-b^2}+4b^2x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\tanh(x)\right)}{4b}\\
&= -\frac{\cosh^4(x)(b-a\tanh(x))}{4\left(a^2-b^2\right)} + \frac{\cosh^2(x)\left(4b(2a^2-b^2)-a(5a^2-b^2)\tanh(x)\right)}{8\left(a^2-b^2\right)^2}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^2}-\frac{ab^4\left(5a^2-b^2\right)x}{\left(a^2-b^2\right)^2}}{(a+x)\left(-b^2+x^2\right)}\,dx, x, b\tanh(x)\right)}{8b^3}
\end{aligned}
$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} + \frac{\cosh^2(x)\left(4b(2a^2-b^2)-a(5a^2-b^2)\tanh(x)\right)}{8(a^2-b^2)^2} - \frac{\text{Subst}\left(\int\left(-\frac{ab^3(3a+b)}{2(a+b)^3(b-x)}+\frac{8a^4b^4}{(a-b)^3(a+b)^3(a+x)}-\frac{a(3a-b)b^3}{2(a-b)^3(b+x)}\right)dx,x,b\tanh(x)\right)}{8b^3}$$

$$= -\frac{a(3a+b)\log(1-\tanh(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\tanh(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\tanh(x))}{(a^2-b^2)^3} - \frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} + \frac{\cosh^2(x)\left(4b(2a^2-b^2)-a(5a^2-b^2)\tanh(x)\right)}{8(a^2-b^2)^2}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = \frac{12a^5x+24a^3b^2x-4ab^4x+4b(3a^4-4a^2b^2+b^4)\cosh(2x)-b(a^2-b^2)^2\cosh(4x)-32a^4b\log(a\cosh(x)+\ldots}{32(a-b)^3(a+b)^3}$$

```
[In] Integrate[Sinh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (12*a^5*x + 24*a^3*b^2*x - 4*a*b^4*x + 4*b*(3*a^4 - 4*a^2*b^2 + b^4)*Cosh[2
*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*a^4*b*Log[a*Cosh[x] + b*Sinh[x]] - 8*a
^3*(a^2 - b^2)*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh
[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [A] (verified)

Time = 6.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.24

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{axb}{8(a+b)^3}+\frac{e^{4x}}{64a+64b}-\frac{e^{2x}a}{8(a+b)^2}-\frac{e^{2x}b}{16(a+b)^2}+\frac{e^{-2x}a}{8(a-b)^2}-\frac{e^{-2x}b}{16(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2a^4bx}{a^6-3a^4b^2+3a^2b^4-b^6}-\ldots$
default	$-\frac{a^4b\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{(a-b)^3(a+b)^3}-\frac{8}{(32a-32b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{32}{(64a-64b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-a-b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\ldots$

```
[In] int(sinh(x)^4/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^2*x/(a+b)^3+1/8*a*x/(a+b)^3*b+1/64/(a+b)*exp(4*x)-1/8/(a+b)^2*exp(2*x
)*a-1/16/(a+b)^2*exp(2*x)*b+1/8/(a-b)^2*exp(-2*x)*a-1/16/(a-b)^2*exp(-2*x)*
b-1/64/(a-b)*exp(-4*x)+2*a^4*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*x-a^4*b/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. $2(139) = 278$.

Time = 0.27 (sec) , antiderivative size = 1226, normalized size of antiderivative = 8.34

$$\int \frac{\sinh^4(x)}{a + b\tanh(x)}\, dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(2*a^5 - 3*a^4*
b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^6 - 4*(2*a^5 - 3*a^4*b - 2*a^3*b^2
 + 4*a^2*b^3 - b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^4
+ 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(2
*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2
+ 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 30*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*
a^2*b^3 - b^5)*cosh(x)^2 + 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x)*sinh(
x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 -
 10*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^3 + 4*(3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x))*sinh(x)^3 + 4*(2*a^5 + 3*a^4*b - 2*
a^3*b^2 - 4*a^2*b^3 + b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 2*a^5 + 3*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 +
b^5 - 15*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^4 + 12*(3*
a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^4*b*cosh(
x)^4 + 4*a^4*b*cosh(x)^3*sinh(x) + 6*a^4*b*cosh(x)^2*sinh(x)^2 + 4*a^4*b*co
sh(x)*sinh(x)^3 + a^4*b*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) -
 sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^7 - 3*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^5 + 4*(3*a^5
 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^3 + (2*a^5 + 3*a^4*b - 2*a^3*b^2
- 4*a^2*b^3 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*si
nh(x)^4)
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = \int \frac{\sinh^4{(x)}}{a+b\tanh{(x)}}\,dx$$

```
[In] integrate(sinh(x)**4/(a+b*tanh(x)),x)
[Out] Integral(sinh(x)**4/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = -\frac{a^4 b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{(3\,a^2+ab)x}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)} - \frac{\left(4\,(2\,a+b)e^{(-2\,x)}-a-b\right)e^{(4\,x)}}{64\,(a^2+2\,ab+b^2)}+\frac{4\,(2\,a-b)e^{(-2\,x)}-(a-b)e^{(-4\,x)}}{64\,(a^2-2\,ab+b^2)}$$

```
[In] integrate(sinh(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
[Out] -a^4*b*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 1/8*(3*a^2 + a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2*a + b)*e^
(-2*x) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - b)*e^(-2*x) -
(a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = -\frac{a^4 b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{(3\,a^2-ab)x}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)} - \frac{\left(18\,a^2e^{(4\,x)}-6\,abe^{(4\,x)}-8\,a^2e^{(2\,x)}+12\,abe^{(2\,x)}-4\,b^2e^{(2\,x)}+a^2-2\,ab+b^2\right)e^{(-4\,x)}}{64\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{ae^{(4\,x)}+be^{(4\,x)}-8\,ae^{(2\,x)}-4\,be^{(2\,x)}}{64\,(a^2+2\,ab+b^2)}$$

```
[In] integrate(sinh(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -a^4*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + 1/8*(3*a^2 - a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/64*(18*a^
2*e^(4*x) - 6*a*b*e^(4*x) - 8*a^2*e^(2*x) + 12*a*b*e^(2*x) - 4*b^2*e^(2*x)
+ a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^(
4*x) + b*e^(4*x) - 8*a*e^(2*x) - 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^4(x)}{a+b\tanh(x)}\,dx = \frac{e^{4x}}{64\,a+64\,b}-\frac{e^{-4x}}{64\,a-64\,b}+\frac{e^{-2x}\,(2\,a-b)}{16\,(a-b)^2}-\frac{e^{2x}\,(2\,a+b)}{16\,(a+b)^2}-\frac{a^4\,b\,\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{a\,x\,(3\,a-b)}{8\,(a-b)^3}$$

```
[In] int(sinh(x)^4/(a + b*tanh(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (exp(-2*x)*(2*a - b))/(1
6*(a - b)^2) - (exp(2*x)*(2*a + b))/(16*(a + b)^2) - (a^4*b*log(a - b + a*e
xp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*x*(3*a - b)
)/(8*(a - b)^3)
```

3.81 $\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 137

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx=-\frac{a^3b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a\cosh(x)}{a^2-b^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}$$

```
[Out] -a^3*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a*b^2*
cosh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+1/3*a*cosh(x)^3/(a^2-b^2)+a^2*b*sin
h(x)/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3599, 3188, 2644, 30, 2713, 3178, 3153, 212, 2718}

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx=-\frac{b\sinh^3(x)}{3\left(a^2-b^2\right)}+\frac{a^2b\sinh(x)}{\left(a^2-b^2\right)^2}+\frac{a\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{a\cosh(x)}{a^2-b^2}-\frac{ab^2\cosh(x)}{\left(a^2-b^2\right)^2}-\frac{a^3b\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}$$

```
[In] Int[Sinh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -((a^3*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
) - (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3)
```

```
/(3*(a^2 - b^2)) + (a^2*b*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 -
b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(x)\sinh^3(x)}{a\cosh(x)+b\sinh(x)}\,dx \\
&= \frac{a\int \sinh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh(x)\sinh^2(x)\,dx}{a^2-b^2} + \frac{(ab)\int \frac{\sinh^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2} \\
&= \frac{a^2 b\sinh(x)}{(a^2-b^2)^2} - \frac{(a^3 b)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{(a^2-b^2)^2} - \frac{(ab^2)\int \sinh(x)\,dx}{(a^2-b^2)^2} \\
&\quad - \frac{a\mathrm{Subst}\left(\int (1-x^2)\,dx, x, \cosh(x)\right)}{a^2-b^2} - \frac{(ib)\mathrm{Subst}\left(\int x^2\,dx, x, i\sinh(x)\right)}{a^2-b^2} \\
&= -\frac{ab^2\cosh(x)}{(a^2-b^2)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2 b\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)} \\
&\quad - \frac{(ia^3 b)\,\mathrm{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{(a^2-b^2)^2} \\
&= -\frac{a^3 b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{ab^2\cosh(x)}{(a^2-b^2)^2} \\
&\quad - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{a^2 b\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx = \frac{-3a\sqrt{a-b}\sqrt{a+b}(3a^2+b^2)\cosh(x)+a\sqrt{a-b}\sqrt{a+b}(a^2-b^2)\cosh(3x)+b\Big(-24a^3\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{12(a-b)^{5/2}(a+b)^{5/2}}$$

[In] Integrate[Sinh[x]^3/(a + b*Tanh[x]),x]

[Out] (-3*a*Sqrt[a - b]*Sqrt[a + b]*(3*a^2 + b^2)*Cosh[x] + a*Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*Cosh[3*x] + b*(-24*a^3*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] + 3*Sqrt[a - b]*Sqrt[a + b]*(5*a^2 - b^2)*Sinh[x] - Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*Sinh[3*x]))/(12*(a - b)^(5/2)*(a + b)^(5/2))

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21

method	result
default	$-\frac{2a^3 b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{a^2-b^2}}-\frac{8}{(16a-16b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{16}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3(16a-16b)}-\frac{a}{2(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{}{3(\tan}$
risch	$\frac{e^{3x}}{24a+24b}-\frac{3\,e^{x}a}{8(a+b)^2}-\frac{e^{x}b}{8(a+b)^2}-\frac{3\,e^{-x}a}{8(a-b)^2}+\frac{e^{-x}b}{8(a-b)^2}+\frac{e^{-3x}}{24a-24b}-\frac{b\,a^3\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}+\frac{b\,a^3\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)}$

[In] int(sinh(x)^3/(a+b*tanh(x)),x,method=_RETURNVERBOSE)

[Out] -2*a^3*b/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-8/(16*a-16*b)/(tanh(1/2*x)+1)^2+16/3/(tanh(1/2*x)+1)^3/(16*a-16*b)-1/2*a/(a-b)^2/(tanh(1/2*x)+1)-16/3/(tanh(1/2*x)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*x)-1)^2+1/2*a/(a+b)^2/(tanh(1/2*x)-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(129) = 258$.

Time = 0.29 (sec) , antiderivative size = 1861, normalized size of antiderivative = 13.58

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh
(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/(
(a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))
 + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*
a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5
*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^
2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 +
 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*
a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*
b^4 + b^5 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(
x)^4 - 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 - 5*(a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 - 3*(3*a^5 - 5*
a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5
+ 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cos
h(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(a^2 - b^
```

```
2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx = \int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(sinh(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx = -\frac{2\,a^3b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} - \frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-12\,abe^x-3\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

```
[In] integrate(sinh(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) - a + b)*e^(-3*x)/(a^2 -
2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^x
- 12*a*b*e^x - 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.91

$$\int \frac{\sinh^3(x)}{a+b\tanh(x)}\,dx = \frac{e^{-3x}}{24a-24b}+\frac{e^{3x}}{24a+24b}-\frac{e^{x}(3a+b)}{8(a+b)^2}-\frac{e^{-x}(3a-b)}{8(a-b)^2} - \frac{2\,\mathrm{atan}\left(\frac{a^3\,b\,e^x\,\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}{a^5\sqrt{a^6b^2}-b^5\sqrt{a^6b^2}+2a^2b^3\sqrt{a^6b^2}-2a^3b^2\sqrt{a^6b^2}+ab^4\sqrt{a^6b^2}-a^4b\sqrt{a^6b^2}}\right)\sqrt{a^6b^2}}{\sqrt{a^{10}-5a^8b^2+10a^6b^4-10a^4b^6+5a^2b^8-b^{10}}}$$

```
[In] int(sinh(x)^3/(a + b*tanh(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(x)*(3*a + b))/(8*(a
 + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) - (2*atan((a^3*b*exp(x)*(a^10
- b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2))/(a^5*(a^6*
b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^2*(a
^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b^2)^(
1/2))/(a^10 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2)^(1/2)
```

3.82 $\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx=\frac{a\log(1-\tanh(x))}{4(a+b)^2}-\frac{a\log(1+\tanh(x))}{4(a-b)^2}+\frac{a^2b\log(a+b\tanh(x))}{(a^2-b^2)^2}-\frac{\cosh^2(x)(b-a\tanh(x))}{2(a^2-b^2)}$$

```
[Out] 1/4*a*ln(1-tanh(x))/(a+b)^2-1/4*a*ln(1+tanh(x))/(a-b)^2+a^2*b*ln(a+b*tanh(x
))/(a^2-b^2)^2-1/2*cosh(x)^2*(b-a*tanh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3597, 1661, 815}

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx=\frac{a^2b\log(a+b\tanh(x))}{(a^2-b^2)^2}-\frac{\cosh^2(x)(b-a\tanh(x))}{2(a^2-b^2)}+\frac{a\log(1-\tanh(x))}{4(a+b)^2}-\frac{a\log(\tanh(x)+1)}{4(a-b)^2}$$

```
[In] Int[Sinh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (a*Log[1 - Tanh[x]])/(4*(a + b)^2) - (a*Log[1 + Tanh[x]])/(4*(a - b)^2) + (
a^2*b*Log[a + b*Tanh[x]])/(a^2 - b^2)^2 - (Cosh[x]^2*(b - a*Tanh[x]))/(2*(a
^2 - b^2))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = b\text{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\tanh(x)\right)$$

$$= -\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}}{(a+x)\left(-b^2+x^2\right)}\,dx, x, b\tanh(x)\right)}{2b}$$

$$= -\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)} + \frac{\text{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)} + \frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)} - \frac{ab}{2(a-b)^2(b+x)}\right)dx, x, b\tanh(x)\right)}{2b}$$

$$= \frac{a\log(1-\tanh(x))}{4(a+b)^2} - \frac{a\log(1+\tanh(x))}{4(a-b)^2} + \frac{a^2b\log(a+b\tanh(x))}{\left(a^2-b^2\right)^2} - \frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx = \frac{(-a^2b+b^3)\cosh(2x)+a(-2(a^2+b^2)\,x+4ab\log(a\cosh(x)+b\sinh(x))+(a^2-b^2)\sinh(2x))}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[Sinh[x]^2/(a + b*Tanh[x]),x]
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(-2*(a^2 + b^2)*x + 4*a*b*Log[a*Cosh[x] + b
*Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{ax}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}-\frac{2a^2bx}{a^4-2a^2b^2+b^4}+\frac{a^2b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{a^2b\ln\left(\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a\right)}{(a-b)^2(a+b)^2}-\frac{4}{(8a-8b)(\tanh(\frac{x}{2})+1)^2}+\frac{8}{(16a-16b)(\tanh(\frac{x}{2})+1)}-\frac{a\ln(\tanh(\frac{x}{2})+1)}{2(a-b)^2}+\frac{4}{(8a+8b)(\tan$

```
[In] int(sinh(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*a*x/(a+b)^2+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)-2*a^2*b/(a^4-2*a^2*
b^2+b^4)*x+a^2*b/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(79) = 158.

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.98

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x}{}$$

```
[In] integrate(sinh(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 + 2*a^2*
```

```
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 + 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(a*cosh(x) + b*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx = \int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(sinh(x)**2/(a+b*tanh(x)),x)
[Out] Integral(sinh(x)**2/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx = \frac{a^2b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4-2\,a^2b^2+b^4} - \frac{ax}{2\,(a^2+2\,ab+b^2)} + \frac{e^{(2\,x)}}{8\,(a+b)} - \frac{e^{(-2\,x)}}{8\,(a-b)}$$

```
[In] integrate(sinh(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
[Out] a^2*b*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*a*x/(a^2
 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx = \frac{a^2b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^4-2\,a^2b^2+b^4} - \frac{ax}{2\,(a^2-2\,ab+b^2)} + \frac{\left(2\,ae^{(2\,x)}-a+b\right)e^{(-2\,x)}}{8\,(a^2-2\,ab+b^2)} + \frac{e^{(2\,x)}}{8\,(a+b)}$$

```
[In] integrate(sinh(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] a^2*b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*a*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - a + b)*e^(-2*x)/(a^2 - 2*a*b
+ b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.96

$$\int \frac{\sinh^2(x)}{a+b\tanh(x)}\,dx=\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}-\frac{ax}{2(a-b)^2}+\frac{a^2b\ln\left(a-b+ae^{2x}+be^{2x}\right)}{a^4-2a^2b^2+b^4}$$

```
[In] int(sinh(x)^2/(a + b*tanh(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (a*x)/(2*(a - b)^2) + (a^2*b
*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.83 $\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx=\frac{ab\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}$$

```
[Out] a*b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh(x)
/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3599, 3188, 2717, 2718, 3153, 212}

$$\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx=\frac{ab\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}$$

```
[In] Int[Sinh[x]/(a + b*Tanh[x]),x]
```

```
[Out] (a*b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) + (
a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\cosh(x)\sinh(x)}{a\cosh(x)+b\sinh(x)}\,dx\\
&= \frac{a\int \sinh(x)\,dx}{a^2-b^2}-\frac{b\int \cosh(x)\,dx}{a^2-b^2}+\frac{(ab)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^2-b^2}\\
&= \frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}+\frac{(iab)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a^2-b^2}\\
&= \frac{ab\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx = \frac{2ab\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} + \frac{a\cosh(x)}{a^2-b^2} + \frac{b\sinh(x)}{-a^2+b^2}$$

```
[In] Integrate[Sinh[x]/(a + b*Tanh[x]),x]
```

```
[Out] (2*a*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)*
(a + b)^(3/2)) + (a*Cosh[x])/(a^2 - b^2) + (b*Sinh[x])/(-a^2 + b^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{2ab\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a+b)(a-b)\sqrt{a^2-b^2}} + \frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	92
risch	$\frac{e^{x}}{2a+2b} + \frac{e^{-x}}{2a-2b} - \frac{ba\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)} + \frac{ba\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	120

```
[In] int(sinh(x)/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*b/(a+b)/(a-b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2
)^(1/2))+4/(4*a-4*b)/(tanh(1/2*x)+1)-4/(4*a+4*b)/(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 427, normalized size of antiderivative = 5.93

$$\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx = \left[\frac{a^3+a^2b-ab^2-b^3+(a^3-a^2b-ab^2+b^3)\cosh(x)^2+2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)+(a^3-a}{2((a^4-2a}\right.$$

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cos
```

```
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)
*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x
) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
2*a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^
2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F]

$$\int \frac{\sinh(x)}{a + b\tanh(x)}\,dx = \int \frac{\sinh(x)}{a + b\tanh(x)}\,dx$$

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(sinh(x)/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(x)}{a + b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\sinh(x)}{a + b\tanh(x)}\,dx = \frac{2\,ab\arctan\left(\frac{ae^x + be^x}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{\frac{3}{2}}} + \frac{e^{(-x)}}{2\,(a - b)} + \frac{e^x}{2\,(a + b)}$$

```
[In] integrate(sinh(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*a*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 1/2*e^(-x
)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.18

$$\int \frac{\sinh(x)}{a+b\tanh(x)}\,dx = \frac{e^x}{2\,a+2\,b}+\frac{e^{-x}}{2\,a-2\,b} + \frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}{a^3\,\sqrt{a^2\,b^2}+b^3\,\sqrt{a^2\,b^2}-a\,b^2\,\sqrt{a^2\,b^2}-a^2\,b\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}}$$

```
[In] int(sinh(x)/(a + b*tanh(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2) -
 a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(a^6 - b^
6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2)
```

3.84 $\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx=-\frac{b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\operatorname{arctanh}(\cosh(x))}{a}$$

```
[Out] -arctanh(cosh(x))/a-b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-
b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3599, 3189, 3855, 3153, 212}

$$\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx=-\frac{b\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\operatorname{arctanh}(\cosh(x))}{a}$$

```
[In] Int[Csch[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]))
- ArcTanh[Cosh[x]]/a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\coth(x)}{a\cosh(x)+b\sinh(x)}\,dx\\
&= i\int\left(-\frac{i\operatorname{csch}(x)}{a}+\frac{ib}{a(a\cosh(x)+b\sinh(x))}\right)dx\\
&= \frac{\int \operatorname{csch}(x)\,dx}{a}-\frac{b\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a}\\
&= -\frac{\operatorname{arctanh}(\cosh(x))}{a}-\frac{(ib)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-ib\cosh(x)-ia\sinh(x)\right)}{a}\\
&= -\frac{b\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}-\frac{\operatorname{arctanh}(\cosh(x))}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx = \frac{-\frac{2b\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} - \log\left(\cosh\left(\frac{x}{2}\right)\right) + \log\left(\sinh\left(\frac{x}{2}\right)\right)}{a}$$

```
[In] Integrate[Csch[x]/(a + b*Tanh[x]),x]
```

```
[Out] ((-2*b*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sq
rt[a + b]) - Log[Cosh[x/2]] + Log[Sinh[x/2]])/a
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{2b\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	53
risch	$-\frac{\ln(e^x+1)}{a}-\frac{b\ln\left(e^x+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a}+\frac{b\ln\left(e^x-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,a}+\frac{\ln(e^x-1)}{a}$	97

```
[In] int(csch(x)/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b/a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))+1/
a*ln(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 237, normalized size of antiderivative = 4.56

$$\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}b\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+2\,\sqrt{-a^2+b^2}(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b}\right)+(a^2-b^2)\,\mathrm{lo}}{a^3-ab^2}\right.$$

```
[In] integrate(csch(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) +
(a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b
```

```
)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + (a^
2 - b^2)*log(cosh(x) + sinh(x) + 1) - (a^2 - b^2)*log(cosh(x) + sinh(x) - 1
))/(a^3 - a*b^2), (2*sqrt(a^2 - b^2)*b*arctan(sqrt(a^2 - b^2)/((a + b)*cosh
(x) + (a + b)*sinh(x))) - (a^2 - b^2)*log(cosh(x) + sinh(x) + 1) + (a^2 - b
^2)*log(cosh(x) + sinh(x) - 1))/(a^3 - a*b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a + b\tanh(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a + b\tanh(x)}\,dx$$

```
[In] integrate(csch(x)/(a+b*tanh(x)),x)
[Out] Integral(csch(x)/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(x)}{a + b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)/(a+b*tanh(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}(x)}{a + b\tanh(x)}\,dx = -\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a} - \frac{\log\left(e^x+1\right)}{a} + \frac{\log\left(|e^x-1|\right)}{a}$$

```
[In] integrate(csch(x)/(a+b*tanh(x)),x, algorithm="giac")
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a) - log(e^x
+ 1)/a + log(abs(e^x - 1))/a
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.40

$$\int \frac{\operatorname{csch}(x)}{a+b\tanh(x)}\,dx = \frac{\ln\left(32\,a\,b-32\,a^2+32\,a^2\,\mathrm{e}^x-32\,a\,b\,\mathrm{e}^x\right)}{a} - \frac{\ln\left(32\,a\,b-32\,a^2-32\,a^2\,\mathrm{e}^x+32\,a\,b\,\mathrm{e}^x\right)}{a} - \frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x-32\,a\,b\,\sqrt{b^2-a^2}\right)\,\sqrt{b^2-a^2}}{a\,b^2-a^3} + \frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x+32\,a\,b\,\sqrt{b^2-a^2}\right)\,\sqrt{b^2-a^2}}{a\,b^2-a^3}$$

```
[In] int(1/(sinh(x)*(a + b*tanh(x))),x)
```

```
[Out] log(32*a*b - 32*a^2 + 32*a^2*exp(x) - 32*a*b*exp(x))/a - log(32*a*b - 32*a^
2 - 32*a^2*exp(x) + 32*a*b*exp(x))/a - (b*log(32*a*b^2*exp(x) + 32*a^2*b*ex
p(x) - 32*a*b*(b^2 - a^2)^(1/2))*(b^2 - a^2)^(1/2))/(a*b^2 - a^3) + (b*log(
32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(b^2 - a^2)^(1/2))*(b^2 - a^2)^(
1/2))/(a*b^2 - a^3)
```

3.85 $\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx=-\frac{\coth(x)}{a}-\frac{b\log(\tanh(x))}{a^2}+\frac{b\log(a+b\tanh(x))}{a^2}$$

```
[Out] -coth(x)/a-b*ln(tanh(x))/a^2+b*ln(a+b*tanh(x))/a^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3597, 46}

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx=-\frac{b\log(\tanh(x))}{a^2}+\frac{b\log(a+b\tanh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[In] Int[Csch[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -(Coth[x]/a) - (b*Log[Tanh[x]])/a^2 + (b*Log[a + b*Tanh[x]])/a^2
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= b\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\tanh(x)\right)\\ &= b\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx, x, b\tanh(x)\right)\\ &= -\frac{\coth(x)}{a}-\frac{b\log(\tanh(x))}{a^2}+\frac{b\log(a+b\tanh(x))}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx = -\frac{a\coth(x)+b\log(\sinh(x))-b\log(a\cosh(x)+b\sinh(x))}{a^2}$$

```
[In] Integrate[Csch[x]^2/(a + b*Tanh[x]),x]
[Out] -((a*Coth[x] + b*Log[Sinh[x]] - b*Log[a*Cosh[x] + b*Sinh[x]])/a^2)
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

method	result	size
risch	$-\frac{2}{a(e^{2x}-1)}+\frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2}-\frac{b\ln(e^{2x}-1)}{a^2}$	50
default	$-\frac{\tanh(\frac{x}{2})}{2a}+\frac{b\ln\left(\tanh(\frac{x}{2})^2a+2b\tanh(\frac{x}{2})+a\right)}{a^2}-\frac{1}{2a\tanh(\frac{x}{2})}-\frac{b\ln(\tanh(\frac{x}{2}))}{a^2}$	56

```
[In] int(csch(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -2/a/(exp(2*x)-1)+1/a^2*b*ln(exp(2*x)+(a-b)/(a+b))-1/a^2*b*ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.21

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx = \frac{\left(b\cosh\left(x\right)^2+2\,b\cosh\left(x\right)\sinh\left(x\right)+b\sinh\left(x\right)^2-b\right)\log\left(\frac{2\,\left(a\cosh(x)+b\sinh(x)\right)}{\cosh(x)-\sinh(x)}\right)-\left(b\cosh\left(x\right)^2+2\,b\cosh\left(x\right)\mathrm{si}}{a^2\cosh\left(x\right)^2+2\,a^2\cosh\left(x\right)\sinh\left(x\right)+a^2\sinh\left(x\right)^2-a^2}$$

```
[In] integrate(csch(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*log(2*(a*cosh(x) + b
*sinh(x))/(cosh(x) - sinh(x))) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sin
h(x)^2 - b)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 2*a)/(a^2*cosh(x)^2 + 2*a^
2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{csch}^2\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)**2/(a + b*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.24

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx = \frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2} - \frac{b\log\left(e^{(-x)}+1\right)}{a^2} - \frac{b\log\left(e^{(-x)}-1\right)}{a^2} + \frac{2}{ae^{(-2\,x)}-a}$$

```
[In] integrate(csch(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] b*log(-(a - b)*e^(-2*x) - a - b)/a^2 - b*log(e^(-x) + 1)/a^2 - b*log(e^(-x)
 - 1)/a^2 + 2/(a*e^(-2*x) - a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(29) = 58$.

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx = \frac{(ab+b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^3+a^2b} - \frac{b\log\left(\left|e^{(2x)}-1\right|\right)}{a^2} + \frac{be^{(2x)}-2a-b}{a^2(e^{(2x)}-1)}$$

```
[In] integrate(csch(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^3 + a^2*b) - b*log(a
bs(e^(2*x) - 1))/a^2 + (b*e^(2*x) - 2*a - b)/(a^2*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 323, normalized size of antiderivative = 11.14

$$\int \frac{\operatorname{csch}^2(x)}{a+b\tanh(x)}\,dx = \frac{2\operatorname{atan}\left(\frac{b\left(a^4\left(b^2\right)^{3/2}-a^6\sqrt{b^2}\right)\left(a\,b^5\sqrt{-a^4}-b^6\sqrt{-a^4}+a^2\,b^4\sqrt{-a^4}-a^3\,b^3\sqrt{-a^4}+b^6\,\mathrm{e}^{2\,x}\sqrt{-a^4}-2\,a^2\,b^4\,\mathrm{e}^{2\,x}\sqrt{-a^4}+a^4\,b^2\,\mathrm{e}^{2\,x}\sqrt{-a^4}\right)+b^2\left(a^3\cdots\right.}{-a^{12}\,b^4+3\,a^{10}\,b^6-3\,a^8\cdots}\right.}{\sqrt{-a^4}} - \frac{2}{a\left(\mathrm{e}^{2\,x}-1\right)}$$

```
[In] int(1/(sinh(x)^2*(a + b*tanh(x))),x)
```

```
[Out] (2*atan((b*(a^4*(b^2)^(3/2) - a^6*(b^2)^(1/2))*(a*b^5*(-a^4)^(1/2) - b^6*(-
a^4)^(1/2) + a^2*b^4*(-a^4)^(1/2) - a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a
^4)^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)
) + b^2*(a^3*(b^2)^(3/2) - a^5*(b^2)^(1/2))*(a*b^5*(-a^4)^(1/2) - b^6*(-a^4
)^(1/2) + a^2*b^4*(-a^4)^(1/2) - a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a^4)
^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)))/
(a^6*b^10 - 3*a^8*b^8 + 3*a^10*b^6 - a^12*b^4))*(b^2)^(1/2))/(-a^4)^(1/2) -
 2/(a*(exp(2*x) - 1))
```

3.86 $\int \frac{\text{csch}^3(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{\text{csch}^3(x)}{a+b\tanh(x)}\,dx = \frac{b\sqrt{a^2-b^2}\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}+\frac{\text{arctanh}(\cosh(x))}{2a}-\frac{b^2\text{arctanh}(\cosh(x))}{a^3}+\frac{b\text{csch}(x)}{a^2}-\frac{\coth(x)\text{csch}(x)}{2a}$$

```
[Out] 1/2*arctanh(cosh(x))/a-b^2*arctanh(cosh(x))/a^3+b*csch(x)/a^2-1/2*coth(x)*c
sch(x)/a+b*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^
3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3599, 3189, 3853, 3855, 2701, 327, 213, 2702, 3183, 3153, 212}

$$\int \frac{\text{csch}^3(x)}{a+b\tanh(x)}\,dx = -\frac{b^2\text{arctanh}(\cosh(x))}{a^3}+\frac{b\text{csch}(x)}{a^2}+\frac{b\sqrt{a^2-b^2}\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}+\frac{\text{arctanh}(\cosh(x))}{2a}-\frac{\coth(x)\text{csch}(x)}{2a}$$

```
[In] Int[Csch[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (b*Sqrt[a^2 - b^2]*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 + A
rcTanh[Cosh[x]]/(2*a) - (b^2*ArcTanh[Cosh[x]])/a^3 + (b*Csch[x])/a^2 - (Cot
h[x]*Csch[x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\coth(x)\text{csch}^2(x)}{a\cosh(x)+b\sinh(x)}\,dx\\
&= -\left(i\int\left(\frac{i\text{csch}^3(x)}{a}-\frac{ib\text{csch}^2(x)\text{sech}(x)}{a^2}+\frac{ib^2\text{csch}(x)\text{sech}^2(x)}{a^3}-\frac{ib^3\text{sech}^2(x)}{a^3(a\cosh(x)+b\sinh(x))}\right)dx\right)\\
&= \frac{\int\text{csch}^3(x)\,dx}{a}-\frac{b\int\text{csch}^2(x)\text{sech}(x)\,dx}{a^2}+\frac{b^2\int\text{csch}(x)\text{sech}^2(x)\,dx}{a^3}-\frac{b^3\int\frac{\text{sech}^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^3}\\
&= -\frac{\coth(x)\text{csch}(x)}{2a}-\frac{b^2\text{sech}(x)}{a^3}-\frac{\int\text{csch}(x)\,dx}{2a}+\frac{(ib)\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,-i\text{csch}(x)\right)}{a^2}\\
&\quad-\frac{b\int\text{sech}(x)\,dx}{a^2}+\frac{b^2\text{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\text{sech}(x)\right)}{a^3}+\frac{(b(a^2-b^2))\int\frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^3}
\end{aligned}$$

$$= -\frac{b\arctan(\sinh(x))}{a^2} + \frac{\operatorname{arctanh}(\cosh(x))}{2a} + \frac{b\operatorname{csch}(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{(ib)\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(x)\right)}{a^2} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \operatorname{sech}(x)\right)}{a^3} + \frac{(ib(a^2-b^2))\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{a^3}$$

$$= \frac{b\sqrt{a^2-b^2}\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3} + \frac{\operatorname{arctanh}(\cosh(x))}{2a} - \frac{b^2\operatorname{arctanh}(\cosh(x))}{a^3} + \frac{b\operatorname{csch}(x)}{a^2} - \frac{\coth(x)\operatorname{csch}(x)}{2a}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.79

$$\int \frac{\operatorname{csch}^3(x)}{a+b\tanh(x)}\,dx = -\frac{-16\sqrt{a-b}b\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right) - 4ab\coth\left(\frac{x}{2}\right) + a^2\operatorname{csch}^2\left(\frac{x}{2}\right) - 4a^2\log\left(\cosh\left(\frac{x}{2}\right)\right) + 8b^2\log\left(\text{co}\right.}{8a^3}$$

```
[In] Integrate[Csch[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -1/8*(-16*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*S
qrt[a + b])] - 4*a*b*Coth[x/2] + a^2*Csch[x/2]^2 - 4*a^2*Log[Cosh[x/2]] + 8
*b^2*Log[Cosh[x/2]] + 4*a^2*Log[Sinh[x/2]] - 8*b^2*Log[Sinh[x/2]] + a^2*Sec
h[x/2]^2 + 4*a*b*Tanh[x/2])/a^3
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.34

method	result
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^2 a}{2} - 2b\tanh\left(\frac{x}{2}\right)}{4a^2} + \frac{2b\sqrt{a^2-b^2}\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{a^3} - \frac{1}{8a\tanh\left(\frac{x}{2}\right)^2} + \frac{(-2a^2+4b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4a^3} + \frac{b}{2a^2\tanh\left(\frac{x}{2}\right)}$
risch	$-\frac{e^x(a\,e^{2x}-2b\,e^{2x}+a+2b)}{(e^{2x}-1)^2 a^2} + \frac{\sqrt{-a^2+b^2}\,b\ln\left(e^x+\frac{\sqrt{-a^2+b^2}}{a+b}\right)}{a^3} - \frac{\sqrt{-a^2+b^2}\,b\ln\left(e^x-\frac{\sqrt{-a^2+b^2}}{a+b}\right)}{a^3} + \frac{\ln(e^x+1)}{2a} - \frac{\ln(e^x+1)b^2}{a^3} -$

```
[In] int(csch(x)^3/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*(1/2*tanh(1/2*x)^2*a-2*b*tanh(1/2*x))+2*b*(a^2-b^2)^(1/2)/a^3*arcta
n(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-1/8/a/tanh(1/2*x)^2+1/4/a^3*(-
2*a^2+4*b^2)*ln(tanh(1/2*x))+1/2/a^2*b/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 1165, normalized size of antiderivative = 14.21

$$\int \frac{\operatorname{csch}^3(x)}{a + b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2 - 2*a*b)*cosh(x)^3 + 6*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + 2*(a
^2 - 2*a*b)*sinh(x)^3 - 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^
4 - 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*co
sh(x))*sinh(x) + b)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cos
h(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) -
 a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
+ a - b)) + 2*(a^2 + 2*a*b)*cosh(x) - ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2
*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)
^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 +
4*((a^2 - 2*b^2)*cosh(x)^3 - (a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) +
sinh(x) + 1) + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3
 + (a^2 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)
*cosh(x)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x
)^3 - (a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(3*(a^
2 - 2*a*b)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)
*sinh(x)^3 + a^3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a
^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x)), -1/2*(2*(a^2 - 2*
a*b)*cosh(x)^3 + 6*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + 2*(a^2 - 2*a*b)*sinh(x
)^3 + 4*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*b*cosh(x)^2
+ 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x)^3 - b*cosh(x))*sinh(x) + b
)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)
)) + 2*(a^2 + 2*a*b)*cosh(x) - ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*c
osh(x)*sinh(x)^3 + (a^2 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*
(3*(a^2 - 2*b^2)*cosh(x)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2
 - 2*b^2)*cosh(x)^3 - (a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
 + 1) + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2
 - 2*b^2)*sinh(x)^4 - 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x
)^2 - a^2 + 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 - (
a^2 - 2*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(3*(a^2 - 2*a
*b)*cosh(x)^2 + a^2 + 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x
)^3 + a^3*sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sin
h(x)^2 + 4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{csch}^3(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(csch(x)**3/(a+b*tanh(x)),x)
[Out] Integral(csch(x)**3/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \frac{\operatorname{csch}^3(x)}{a+b\tanh(x)}\,dx = \frac{(a^2-2\,b^2)\log(e^x+1)}{2\,a^3} - \frac{(a^2-2\,b^2)\log(|e^x-1|)}{2\,a^3} + \frac{2\,(a^2b-b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^3} - \frac{ae^{(3\,x)}-2\,be^{(3\,x)}+ae^x+2\,be^x}{a^2\left(e^{(2\,x)}-1\right)^2}$$

```
[In] integrate(csch(x)^3/(a+b*tanh(x)),x, algorithm="giac")
[Out] 1/2*(a^2 - 2*b^2)*log(e^x + 1)/a^3 - 1/2*(a^2 - 2*b^2)*log(abs(e^x - 1))/a^
3 + 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2
)*a^3) - (a*e^(3*x) - 2*b*e^(3*x) + a*e^x + 2*b*e^x)/(a^2*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 506, normalized size of antiderivative = 6.17

$$\int \frac{\operatorname{csch}^3(x)}{a+b\tanh(x)}\,dx
= \frac{\ln\left(8\,a^3\,b-16\,a\,b^3-4\,a^4+8\,b^4+4\,a^2\,b^2-4\,a^4\,\mathrm{e}^x+8\,b^4\,\mathrm{e}^x-16\,a\,b^3\,\mathrm{e}^x+8\,a^3\,b\,\mathrm{e}^x+4\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a}
-\frac{2\,\mathrm{e}^x}{a-2\,a\,\mathrm{e}^{2\,x}+a\,\mathrm{e}^{4\,x}}
-\frac{\ln\left(16\,a\,b^3-8\,a^3\,b+4\,a^4-8\,b^4-4\,a^2\,b^2-4\,a^4\,\mathrm{e}^x+8\,b^4\,\mathrm{e}^x-16\,a\,b^3\,\mathrm{e}^x+8\,a^3\,b\,\mathrm{e}^x+4\,a^2\,b^2\,\mathrm{e}^x\right)}{2\,a}
-\frac{b^2\,\ln\left(8\,a^3\,b-16\,a\,b^3-4\,a^4+8\,b^4+4\,a^2\,b^2-4\,a^4\,\mathrm{e}^x+8\,b^4\,\mathrm{e}^x-16\,a\,b^3\,\mathrm{e}^x+8\,a^3\,b\,\mathrm{e}^x+4\,a^2\,b^2\,\mathrm{e}^x\right)}{a^3}
+\frac{b^2\,\ln\left(16\,a\,b^3-8\,a^3\,b+4\,a^4-8\,b^4-4\,a^2\,b^2-4\,a^4\,\mathrm{e}^x+8\,b^4\,\mathrm{e}^x-16\,a\,b^3\,\mathrm{e}^x+8\,a^3\,b\,\mathrm{e}^x+4\,a^2\,b^2\,\mathrm{e}^x\right)}{a^3}
-\frac{a\,\mathrm{e}^x}{a^2\,\mathrm{e}^{2\,x}-a^2}+\frac{2\,b\,\mathrm{e}^x}{a^2\,\mathrm{e}^{2\,x}-a^2}
-\frac{b\,\ln\left(8\,b^2\,\sqrt{b^2-a^2}-8\,b^3\,\mathrm{e}^x+8\,a^2\,b\,\mathrm{e}^x-8\,a\,b\,\sqrt{b^2-a^2}\right)\,\sqrt{b^2-a^2}}{a^3}
+\frac{b\,\ln\left(8\,b^2\,\sqrt{b^2-a^2}+8\,b^3\,\mathrm{e}^x-8\,a^2\,b\,\mathrm{e}^x-8\,a\,b\,\sqrt{b^2-a^2}\right)\,\sqrt{b^2-a^2}}{a^3}$$

```
[In] int(1/(sinh(x)^3*(a + b*tanh(x))),x)
```

```
[Out] log(8*a^3*b - 16*a*b^3 - 4*a^4 + 8*b^4 + 4*a^2*b^2 - 4*a^4*exp(x) + 8*b^4*e
xp(x) - 16*a*b^3*exp(x) + 8*a^3*b*exp(x) + 4*a^2*b^2*exp(x))/(2*a) - (2*exp
(x))/(a - 2*a*exp(2*x) + a*exp(4*x)) - log(16*a*b^3 - 8*a^3*b + 4*a^4 - 8*b
^4 - 4*a^2*b^2 - 4*a^4*exp(x) + 8*b^4*exp(x) - 16*a*b^3*exp(x) + 8*a^3*b*ex
p(x) + 4*a^2*b^2*exp(x))/(2*a) - (b^2*log(8*a^3*b - 16*a*b^3 - 4*a^4 + 8*b^
4 + 4*a^2*b^2 - 4*a^4*exp(x) + 8*b^4*exp(x) - 16*a*b^3*exp(x) + 8*a^3*b*exp
(x) + 4*a^2*b^2*exp(x)))/a^3 + (b^2*log(16*a*b^3 - 8*a^3*b + 4*a^4 - 8*b^4
- 4*a^2*b^2 - 4*a^4*exp(x) + 8*b^4*exp(x) - 16*a*b^3*exp(x) + 8*a^3*b*exp(x
) + 4*a^2*b^2*exp(x)))/a^3 - (a*exp(x))/(a^2*exp(2*x) - a^2) + (2*b*exp(x))
/(a^2*exp(2*x) - a^2) - (b*log(8*b^2*(b^2 - a^2)^(1/2) - 8*b^3*exp(x) + 8*a
^2*b*exp(x) - 8*a*b*(b^2 - a^2)^(1/2))*(b^2 - a^2)^(1/2))/a^3 + (b*log(8*b^
2*(b^2 - a^2)^(1/2) + 8*b^3*exp(x) - 8*a^2*b*exp(x) - 8*a*b*(b^2 - a^2)^(1/
2))*(b^2 - a^2)^(1/2))/a^3
```

3.87 $\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx = \frac{(a^2-b^2)\coth(x)}{a^3}+\frac{b\coth^2(x)}{2a^2}-\frac{\coth^3(x)}{3a}+\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\tanh(x))}{a^4}$$

```
[Out] (a^2-b^2)*coth(x)/a^3+1/2*b*coth(x)^2/a^2-1/3*coth(x)^3/a+b*(a^2-b^2)*ln(ta
nh(x))/a^4-b*(a^2-b^2)*ln(a+b*tanh(x))/a^4
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3597, 908}

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx = \frac{b\coth^2(x)}{2a^2}+\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\tanh(x))}{a^4}+\frac{(a^2-b^2)\coth(x)}{a^3}-\frac{\coth^3(x)}{3a}$$

```
[In] Int[Csch[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] ((a^2 - b^2)*Coth[x])/a^3 + (b*Coth[x]^2)/(2*a^2) - Coth[x]^3/(3*a) + (b*(a
^2 - b^2)*Log[Tanh[x]])/a^4 - (b*(a^2 - b^2)*Log[a + b*Tanh[x]])/a^4
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(b\text{Subst}\left(\int \frac{-b^2+x^2}{x^4(a+x)}\,dx, x, b\tanh(x)\right)\right)\\ &= -\left(b\text{Subst}\left(\int \left(-\frac{b^2}{ax^4}+\frac{b^2}{a^2x^3}+\frac{a^2-b^2}{a^3x^2}+\frac{-a^2+b^2}{a^4x}+\frac{a^2-b^2}{a^4(a+x)}\right)dx, x, b\tanh(x)\right)\right)\\ &= \frac{(a^2-b^2)\coth(x)}{a^3}+\frac{b\coth^2(x)}{2a^2}-\frac{\coth^3(x)}{3a}\\ &\quad+\frac{b(a^2-b^2)\log(\tanh(x))}{a^4}-\frac{b(a^2-b^2)\log(a+b\tanh(x))}{a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx = \frac{3a^2b\operatorname{csch}^2(x)-2\coth(x)\left(-2a^3+3ab^2+a^3\operatorname{csch}^2(x)\right)+6b(a^2-b^2)\left(\log(\sinh(x))-\log(a\cosh(x)+b\sinh(\right.}{6a^4}$$

```
[In] Integrate[Csch[x]^4/(a + b*Tanh[x]),x]
[Out] (3*a^2*b*Csch[x]^2 - 2*Coth[x]*(-2*a^3 + 3*a*b^2 + a^3*Csch[x]^2) + 6*b*(a^
2 - b^2)*(Log[Sinh[x]] - Log[a*Cosh[x] + b*Sinh[x]]))/(6*a^4)
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{2\left(-3ab\,e^{4x}+3b^{2}e^{4x}+6a^{2}e^{2x}+3b\,e^{2x}a-6b^{2}e^{2x}-2a^{2}+3b^{2}\right)}{3a^{3}\left(e^{2x}-1\right)^{3}}-\frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^{2}}+\frac{b^{3}\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^{4}}+\frac{b\ln\left(e^{2x}-1\right)}{a^{2}}-\frac{b^{3}\ln(e^{2x}}{a^{4}}$
default	$-\frac{\frac{a^{2}\tanh\left(\frac{x}{2}\right)^{3}}{3}-\tanh\left(\frac{x}{2}\right)^{2}ab-3a^{2}\tanh\left(\frac{x}{2}\right)+4b^{2}\tanh\left(\frac{x}{2}\right)}{8a^{3}}-\frac{2b\left(\frac{a^{2}}{2}-\frac{b^{2}}{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{a^{4}}-\frac{1}{24a\tanh\left(\frac{x}{2}\right)^{3}}-\frac{}{8}$

```
[In] int(csch(x)^4/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-3*a*b*exp(4*x)+3*b^2*exp(4*x)+6*a^2*exp(2*x)+3*b*exp(2*x)*a-6*b^2*ex
p(2*x)-2*a^2+3*b^2)/a^3/(exp(2*x)-1)^3-1/a^2*b*ln(exp(2*x)+(a-b)/(a+b))+1/a
^4*b^3*ln(exp(2*x)+(a-b)/(a+b))+1/a^2*b*ln(exp(2*x)-1)-1/a^4*b^3*ln(exp(2*x
)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 912, normalized size of antiderivative = 11.69

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 + 6
*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 - 6*(2*a^3 + a^2*b - 2*a*b^2)*
cosh(x)^2 - 6*(2*a^3 + a^2*b - 2*a*b^2 - 6*(a^2*b - a*b^2)*cosh(x)^2)*sinh(
x)^2 - 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^
2*b - b^3)*sinh(x)^6 - 3*(a^2*b - b^3)*cosh(x)^4 - 3*(a^2*b - b^3 - 5*(a^2*
b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 - 3*(a^2*b - b
^3)*cosh(x))*sinh(x)^3 - a^2*b + b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^
2*b - b^3)*cosh(x)^4 + a^2*b - b^3 - 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 +
 6*((a^2*b - b^3)*cosh(x)^5 - 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cos
h(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^2
*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*sin
h(x)^6 - 3*(a^2*b - b^3)*cosh(x)^4 - 3*(a^2*b - b^3 - 5*(a^2*b - b^3)*cosh(
x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 - 3*(a^2*b - b^3)*cosh(x))*s
inh(x)^3 - a^2*b + b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*cos
h(x)^4 + a^2*b - b^3 - 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b - b
^3)*cosh(x)^5 - 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x))
*log(2*sinh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^2)*cosh(x)^3 - (2*
a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sin
```

```
h(x)^5 + a^4*sinh(x)^6 - 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(
x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3
 + 3*(5*a^4*cosh(x)^4 - 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5
 - 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx=\int \frac{\operatorname{csch}^4\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

```
[In] integrate(csch(x)**4/(a+b*tanh(x)),x)
[Out] Integral(csch(x)**4/(a + b*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(74) = 148$.

Time = 0.19 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.06

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx=-\frac{2\left(2\,a^2-3\,b^2-3\,(2\,a^2-ab-2\,b^2)e^{(-2\,x)}-3\,(ab+b^2)e^{(-4\,x)}\right)}{3\,(3\,a^3e^{(-2\,x)}-3\,a^3e^{(-4\,x)}+a^3e^{(-6\,x)}-a^3)}-\frac{(a^2b-b^3)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^4}+\frac{(a^2b-b^3)\log\left(e^{(-x)}+1\right)}{a^4}+\frac{(a^2b-b^3)\log\left(e^{(-x)}-1\right)}{a^4}$$

```
[In] integrate(csch(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
[Out] -2/3*(2*a^2 - 3*b^2 - 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-
4*x))/(3*a^3*e^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(-6*x) - a^3) - (a^2*b - b^3
)*log(-(a - b)*e^(-2*x) - a - b)/a^4 + (a^2*b - b^3)*log(e^(-x) + 1)/a^4 +
(a^2*b - b^3)*log(e^(-x) - 1)/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(74) = 148$.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.59

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx=-\frac{(a^3b+a^2b^2-ab^3-b^4)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^5+a^4b}+\frac{(a^2b-b^3)\log\left(\left|e^{(2\,x)}-1\right|\right)}{a^4}-\frac{11\,a^2be^{(6\,x)}-11\,b^3e^{(6\,x)}-45\,a^2be^{(4\,x)}+12\,ab^2e^{(4\,x)}+33\,b^3e^{(4\,x)}+24\,a^3e^{(2\,x)}+45\,a^2be^{(2\,x)}-24\,ab^2e^{(2\,x)}-}{6\,a^4(e^{(2\,x)}-1)^3}$$

```
[In] integrate(csch(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -(a^3*b + a^2*b^2 - a*b^3 - b^4)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a
^5 + a^4*b) + (a^2*b - b^3)*log(abs(e^(2*x) - 1))/a^4 - 1/6*(11*a^2*b*e^(6*
x) - 11*b^3*e^(6*x) - 45*a^2*b*e^(4*x) + 12*a*b^2*e^(4*x) + 33*b^3*e^(4*x)
+ 24*a^3*e^(2*x) + 45*a^2*b*e^(2*x) - 24*a*b^2*e^(2*x) - 33*b^3*e^(2*x) - 8
*a^3 - 11*a^2*b + 12*a*b^2 + 11*b^3)/(a^4*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \frac{\operatorname{csch}^4(x)}{a+b\tanh(x)}\,dx = \frac{2\,b\,(a-b)}{a^3\,\left(e^{2\,x}-1\right)}-\frac{2\,(2\,a-b)}{a^2\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}-\frac{8}{3\,a\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} - \frac{b\,\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)\,(a+b)\,(a-b)}{a^4} + \frac{b\,\ln\left(e^{2\,x}-1\right)\,(a+b)\,(a-b)}{a^4}$$

```
[In] int(1/(sinh(x)^4*(a + b*tanh(x))),x)
```

```
[Out] (2*b*(a - b))/(a^3*(exp(2*x) - 1)) - (2*(2*a - b))/(a^2*(exp(4*x) - 2*exp(2
*x) + 1)) - 8/(3*a*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (b*log(a - b
 + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/a^4 + (b*log(exp(2*x) - 1)*(a
+ b)*(a - b))/a^4
```

3.88 $\int \frac{\text{csch}^5(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 255

$$\begin{aligned}\int \frac{\text{csch}^5(x)}{a+b\tanh(x)}\,dx = &-\frac{b\arctan(\sinh(x))}{a^2}+\frac{b^3\arctan(\sinh(x))}{a^4}+\frac{b(a^2-b^2)\arctan(\sinh(x))}{a^4}\\&-\frac{b(a^2-b^2)^{3/2}\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^5}-\frac{3\text{arctanh}(\cosh(x))}{8a}\\&+\frac{3b^2\text{arctanh}(\cosh(x))}{2a^3}-\frac{b^4\text{arctanh}(\cosh(x))}{a^5}-\frac{b\text{csch}(x)}{a^2}\\&+\frac{3b^3\text{csch}(x)}{2a^4}+\frac{3\coth(x)\text{csch}(x)}{8a}+\frac{b\text{csch}^3(x)}{3a^2}-\frac{\coth(x)\text{csch}^3(x)}{4a}\\&-\frac{3b^2\text{sech}(x)}{2a^3}+\frac{b^4\text{sech}(x)}{a^5}+\frac{b^2(a^2-b^2)\,\text{sech}(x)}{a^5}\\&-\frac{b^2\text{csch}^2(x)\text{sech}(x)}{2a^3}-\frac{b^3\text{csch}(x)\text{sech}^2(x)}{2a^4}-\frac{b^3\text{sech}(x)\tanh(x)}{2a^4}\end{aligned}$$

```
[Out] -b*arctan(sinh(x))/a^2+b^3*arctan(sinh(x))/a^4+b*(a^2-b^2)*arctan(sinh(x))/
a^4-b*(a^2-b^2)^(3/2)*arctan((b*cosh(x)+a*sinh(x))/(a^2-b^2)^(1/2))/a^5-3/8
*arctanh(cosh(x))/a+3/2*b^2*arctanh(cosh(x))/a^3-b^4*arctanh(cosh(x))/a^5-b
*csch(x)/a^2+3/2*b^3*csch(x)/a^4+3/8*coth(x)*csch(x)/a+1/3*b*csch(x)^3/a^2-
1/4*coth(x)*csch(x)^3/a-3/2*b^2*sech(x)/a^3+b^4*sech(x)/a^5+b^2*(a^2-b^2)*s
ech(x)/a^5-1/2*b^2*csch(x)^2*sech(x)/a^3-1/2*b^3*csch(x)*sech(x)^2/a^4-1/2*
b^3*sech(x)*tanh(x)/a^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3599, 3189, 3853, 3855, 2701, 308, 213, 2702, 294, 327, 3183, 3153, 212}

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx = -\frac{b^4\operatorname{arctanh}(\cosh(x))}{a^5}+\frac{b^4\operatorname{sech}(x)}{a^5}+\frac{b^3\arctan(\sinh(x))}{a^4}+\frac{3b^3\operatorname{csch}(x)}{2a^4} -\frac{b^3\operatorname{csch}(x)\operatorname{sech}^2(x)}{2a^4}-\frac{b^3\tanh(x)\operatorname{sech}(x)}{2a^4}+\frac{3b^2\operatorname{arctanh}(\cosh(x))}{2a^3} -\frac{3b^2\operatorname{sech}(x)}{2a^3}-\frac{b^2\operatorname{csch}^2(x)\operatorname{sech}(x)}{2a^3}-\frac{b\arctan(\sinh(x))}{a^2} +\frac{b\operatorname{csch}^3(x)}{3a^2}-\frac{b\operatorname{csch}(x)}{a^2}-\frac{b(a^2-b^2)^{3/2}\arctan\left(\frac{a\sinh(x)+b\cosh(x)}{\sqrt{a^2-b^2}}\right)}{a^5} +\frac{b^2(a^2-b^2)\operatorname{sech}(x)}{a^5}+\frac{b(a^2-b^2)\arctan(\sinh(x))}{a^4} -\frac{3\operatorname{arctanh}(\cosh(x))}{8a}-\frac{\coth(x)\operatorname{csch}^3(x)}{4a}+\frac{3\coth(x)\operatorname{csch}(x)}{8a}$$

```
[In] Int[Csch[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] -((b*ArcTan[Sinh[x]])/a^2) + (b^3*ArcTan[Sinh[x]])/a^4 + (b*(a^2 - b^2)*Arc
Tan[Sinh[x]])/a^4 - (b*(a^2 - b^2)^(3/2)*ArcTan[(b*Cosh[x] + a*Sinh[x])/Sqr
t[a^2 - b^2]])/a^5 - (3*ArcTanh[Cosh[x]])/(8*a) + (3*b^2*ArcTanh[Cosh[x]])/
(2*a^3) - (b^4*ArcTanh[Cosh[x]])/a^5 - (b*Csch[x])/a^2 + (3*b^3*Csch[x])/(2
*a^4) + (3*Coth[x]*Csch[x])/(8*a) + (b*Csch[x]^3)/(3*a^2) - (Coth[x]*Csch[x
]^3)/(4*a) - (3*b^2*Sech[x])/(2*a^3) + (b^4*Sech[x])/a^5 + (b^2*(a^2 - b^2)
*Sech[x])/a^5 - (b^2*Csch[x]^2*Sech[x])/(2*a^3) - (b^3*Csch[x]*Sech[x]^2)/(
2*a^4) - (b^3*Sech[x]*Tanh[x])/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\coth(x)\text{csch}^4(x)}{a\cosh(x)+b\sinh(x)}\,dx \\
&= i\int \left(-\frac{i\text{csch}^5(x)}{a}+\frac{ib\text{csch}^4(x)\text{sech}(x)}{a^2}-\frac{ib^2\text{csch}^3(x)\text{sech}^2(x)}{a^3}+\frac{ib^3\text{csch}^2(x)\text{sech}^3(x)}{a^4}\right. \\
&\qquad \left.-\frac{ib^4\text{csch}(x)\text{sech}^4(x)}{a^5}+\frac{ib^5\text{sech}^4(x)}{a^5(a\cosh(x)+b\sinh(x))}\right)dx \\
&= \frac{\int \text{csch}^5(x)\,dx}{a}-\frac{b\int \text{csch}^4(x)\text{sech}(x)\,dx}{a^2}+\frac{b^2\int \text{csch}^3(x)\text{sech}^2(x)\,dx}{a^3} \\
&\quad -\frac{b^3\int \text{csch}^2(x)\text{sech}^3(x)\,dx}{a^4}+\frac{b^4\int \text{csch}(x)\text{sech}^4(x)\,dx}{a^5}-\frac{b^5\int \frac{\text{sech}^4(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^5}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\coth(x)\text{csch}^3(x)}{4a} - \frac{b^4\text{sech}^3(x)}{3a^5} - \frac{3\int \text{csch}^3(x)\,dx}{4a} - \frac{(ib)\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, -i\text{csch}(x)\right)}{a^2} \\
&\quad - \frac{b^2\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \text{sech}(x)\right)}{a^3} + \frac{(ib^3)\,\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, -i\text{csch}(x)\right)}{a^4} \\
&\quad - \frac{b^3\int \text{sech}^3(x)\,dx}{a^4} + \frac{b^4\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \text{sech}(x)\right)}{a^5} + \frac{(b^3(a^2-b^2))\int \frac{\text{sech}^2(x)}{a\cosh(x)+b\sinh(x)}\,dx}{a^5} \\
&= \frac{3\coth(x)\text{csch}(x)}{8a} - \frac{\coth(x)\text{csch}^3(x)}{4a} + \frac{b^2(a^2-b^2)\,\text{sech}(x)}{a^5} \\
&\quad - \frac{b^2\text{csch}^2(x)\text{sech}(x)}{2a^3} - \frac{b^3\text{csch}(x)\text{sech}^2(x)}{2a^4} - \frac{b^4\text{sech}^3(x)}{3a^5} - \frac{b^3\text{sech}(x)\tanh(x)}{2a^4} \\
&\quad + \frac{3\int \text{csch}(x)\,dx}{8a} - \frac{(ib)\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, -i\text{csch}(x)\right)}{a^2} \\
&\quad - \frac{(3b^2)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{sech}(x)\right)}{2a^3} + \frac{(3ib^3)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\text{csch}(x)\right)}{2a^4} \\
&\quad - \frac{b^3\int \text{sech}(x)\,dx}{2a^4} + \frac{b^4\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \text{sech}(x)\right)}{a^5} \\
&\quad + \frac{(b(a^2-b^2))\int \text{sech}(x)\,dx}{a^4} - \frac{\left(b(a^2-b^2)^2\right)\int \frac{1}{a\cosh(x)+b\sinh(x)}\,dx}{a^5} \\
&= -\frac{b^3\arctan(\sinh(x))}{2a^4} + \frac{b(a^2-b^2)\arctan(\sinh(x))}{a^4} - \frac{3\text{arctanh}(\cosh(x))}{8a} \\
&\quad - \frac{b\text{csch}(x)}{a^2} + \frac{3b^3\text{csch}(x)}{2a^4} + \frac{3\coth(x)\text{csch}(x)}{8a} + \frac{b\text{csch}^3(x)}{3a^2} \\
&\quad - \frac{\coth(x)\text{csch}^3(x)}{4a} - \frac{3b^2\text{sech}(x)}{2a^3} + \frac{b^4\text{sech}(x)}{a^5} + \frac{b^2(a^2-b^2)\,\text{sech}(x)}{a^5} \\
&\quad - \frac{b^2\text{csch}^2(x)\text{sech}(x)}{2a^3} - \frac{b^3\text{csch}(x)\text{sech}^2(x)}{2a^4} - \frac{b^3\text{sech}(x)\tanh(x)}{2a^4} \\
&\quad - \frac{(ib)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(x)\right)}{a^2} - \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(x)\right)}{2a^3} \\
&\quad + \frac{(3ib^3)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\text{csch}(x)\right)}{2a^4} + \frac{b^4\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \text{sech}(x)\right)}{a^5} \\
&\quad - \frac{\left(ib(a^2-b^2)^2\right)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -ib\cosh(x)-ia\sinh(x)\right)}{a^5}
\end{aligned}
$$

$$= -\frac{b\arctan(\sinh(x))}{a^2} + \frac{b^3\arctan(\sinh(x))}{a^4} + \frac{b(a^2-b^2)\arctan(\sinh(x))}{a^4} - \frac{b(a^2-b^2)^{3/2}\arctan\left(\frac{b\cosh(x)+a\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^5} - \frac{3\text{arctanh}(\cosh(x))}{8a} + \frac{3b^2\text{arctanh}(\cosh(x))}{2a^3} - \frac{b^4\text{arctanh}(\cosh(x))}{a^5} - \frac{b\text{csch}(x)}{a^2} + \frac{3b^3\text{csch}(x)}{2a^4} + \frac{3\coth(x)\text{csch}(x)}{8a} + \frac{b\text{csch}^3(x)}{3a^2} - \frac{\coth(x)\text{csch}^3(x)}{4a} - \frac{3b^2\text{sech}(x)}{2a^3} + \frac{b^4\text{sech}(x)}{a^5} + \frac{b^2(a^2-b^2)\text{sech}(x)}{a^5} - \frac{b^2\text{csch}^2(x)\text{sech}(x)}{2a^3} - \frac{b^3\text{csch}(x)\text{sech}^2(x)}{2a^4} - \frac{b^3\text{sech}(x)\tanh(x)}{2a^4}$$

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.31

$$\int \frac{\text{csch}^5(x)}{a+b\tanh(x)}\,dx = \frac{-384a^2\sqrt{a-b}b\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right) + 384\sqrt{a-b}b^3\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right) - 16ab(7a^2-6b^2)}{}$$

```
[In] Integrate[Csch[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] (-384*a^2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*S
qrt[a + b])] + 384*Sqrt[a - b]*b^3*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sq
rt[a - b]*Sqrt[a + b])] - 16*a*b*(7*a^2 - 6*b^2)*Coth[x/2] + 6*a^2*(3*a^2 -
 4*b^2)*Csch[x/2]^2 - 72*a^4*Log[Cosh[x/2]] + 288*a^2*b^2*Log[Cosh[x/2]] -
192*b^4*Log[Cosh[x/2]] + 72*a^4*Log[Sinh[x/2]] - 288*a^2*b^2*Log[Sinh[x/2]]
 + 192*b^4*Log[Sinh[x/2]] + 18*a^4*Sech[x/2]^2 - 24*a^2*b^2*Sech[x/2]^2 + 3
*a^4*Sech[x/2]^4 + 64*a^3*b*Csch[x]^3*Sinh[x/2]^4 + a^3*Csch[x/2]^4*(-3*a +
 4*b*Sinh[x]) + 112*a^3*b*Tanh[x/2] - 96*a*b^3*Tanh[x/2])/(192*a^5)
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{a^3\tanh\left(\frac{x}{2}\right)^4}{4} - \frac{2b\tanh\left(\frac{x}{2}\right)^3a^2}{3} - 2a^3\tanh\left(\frac{x}{2}\right)^2 + 2a\,b^2\tanh\left(\frac{x}{2}\right)^2 + 10a^2b\tanh\left(\frac{x}{2}\right) - 8b^3\tanh\left(\frac{x}{2}\right)}{16a^4} - \frac{2b(a^4-2a^2b^2+b^4)\arctan\left(\frac{2a\tanh(}{2\sqrt{a}}\right.}{a^5\sqrt{a^2-b^2}}$
risch	$\frac{e^x\left(9a^3e^{6x} - 24e^{6x}a^2b - 12e^{6x}a\,b^2 + 24b^3e^{6x} - 33a^3e^{4x} + 104a^2b\,e^{4x} + 12a\,b^2e^{4x} - 72b^3e^{4x} - 33a^3e^{2x} - 104e^{2x}a^2b + 12e^{2x}a\,b^2 + 72b^3e^{2x} +\right.}{12a^4(e^{2x}-1)^4}$

```
[In] int(csch(x)^5/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a^4*(1/4*a^3*tanh(1/2*x)^4-2/3*b*tanh(1/2*x)^3*a^2-2*a^3*tanh(1/2*x)^2
+2*a*b^2*tanh(1/2*x)^2+10*a^2*b*tanh(1/2*x)-8*b^3*tanh(1/2*x))-2*b*(a^4-2*a
^2*b^2+b^4)/a^5/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^
(1/2))-1/64/a/tanh(1/2*x)^4-1/32*(-4*a^2+4*b^2)/a^3/tanh(1/2*x)^2+1/16/a^5*
(6*a^4-24*a^2*b^2+16*b^4)*ln(tanh(1/2*x))+1/24/a^2*b/tanh(1/2*x)^3-1/8*b*(5
*a^2-4*b^2)/a^4/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2646 vs. $2(231) = 462$.

Time = 0.35 (sec) , antiderivative size = 5347, normalized size of antiderivative = 20.97

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^5/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(csch(x)**5/(a+b*tanh(x)),x)
```

```
[Out] Integral(csch(x)**5/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)^5/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx = -\frac{(3\,a^4-12\,a^2b^2+8\,b^4)\log\left(e^x+1\right)}{8\,a^5} + \frac{(3\,a^4-12\,a^2b^2+8\,b^4)\log\left(|e^x-1|\right)}{8\,a^5} - \frac{2\,(a^4b-2\,a^2b^3+b^5)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^5} + \frac{9\,a^3e^{(7\,x)}-24\,a^2be^{(7\,x)}-12\,ab^2e^{(7\,x)}+24\,b^3e^{(7\,x)}-33\,a^3e^{(5\,x)}+104\,a^2be^{(5\,x)}+12\,ab^2e^{(5\,x)}-72\,b^3e^{(5\,x)}-}{12\,a^4(e^{(2\,x)}-}$$

```
[In] integrate(csch(x)^5/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -1/8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*log(e^x + 1)/a^5 + 1/8*(3*a^4 - 12*a^2*b^
2 + 8*b^4)*log(abs(e^x - 1))/a^5 - 2*(a^4*b - 2*a^2*b^3 + b^5)*arctan((a*e^
x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5) + 1/12*(9*a^3*e^(7*x) - 2
4*a^2*b*e^(7*x) - 12*a*b^2*e^(7*x) + 24*b^3*e^(7*x) - 33*a^3*e^(5*x) + 104*
a^2*b*e^(5*x) + 12*a*b^2*e^(5*x) - 72*b^3*e^(5*x) - 33*a^3*e^(3*x) - 104*a^
2*b*e^(3*x) + 12*a*b^2*e^(3*x) + 72*b^3*e^(3*x) + 9*a^3*e^x + 24*a^2*b*e^x
- 12*a*b^2*e^x - 24*b^3*e^x)/(a^4*(e^(2*x) - 1)^4)
```

Mupad [B] (verification not implemented)

Time = 3.75 (sec) , antiderivative size = 753, normalized size of antiderivative = 2.95

$$\int \frac{\operatorname{csch}^5(x)}{a+b\tanh(x)}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)\,\left(3\,a^4-12\,a^2\,b^2+8\,b^4\right)}{8\,a^5} - \frac{\ln\left(\mathrm{e}^x+1\right)\,\left(3\,a^4-12\,a^2\,b^2+8\,b^4\right)}{8\,a^5} - \frac{4\,\mathrm{e}^x}{a\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)} - \frac{2\,\mathrm{e}^x\,\left(9\,a-4\,b\right)}{3\,a^2\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)} - \frac{\mathrm{e}^x\,\left(3\,a^2-16\,a\,b+12\,b^2\right)}{6\,a^3\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} - \frac{\mathrm{e}^x\,\left(-3\,a^3+8\,a^2\,b+4\,a\,b^2-8\,b^3\right)}{4\,a^4\,\left(\mathrm{e}^{2\,x}-1\right)} + \frac{b\,\ln\left(\frac{b\,\mathrm{e}^x\,{\left(a-b\right)}^2\,\left(-9\,a^7-24\,a^6\,b+144\,a^5\,b^2+24\,a^4\,b^3-456\,a^3\,b^4+224\,a^2\,b^5+288\,a\,b^6-192\,b^7\right)}{2\,a^{12}\,\left(a+b\right)} - \frac{b\,\left(a-b\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,\left(8\,a^5\,b^3-9\,a^8\right.}{}\right.}{} - \frac{b\,\ln\left(\frac{b\,\mathrm{e}^x\,{\left(a-b\right)}^2\,\left(-9\,a^7-24\,a^6\,b+144\,a^5\,b^2+24\,a^4\,b^3-456\,a^3\,b^4+224\,a^2\,b^5+288\,a\,b^6-192\,b^7\right)}{2\,a^{12}\,\left(a+b\right)} - \frac{b\,\left(a-b\right)\,\sqrt{-{\left(a+b\right)}^3\,{\left(a-b\right)}^3}\,\left(9\,a^7\,b+9\,a^8-\right.}{}\right.}{}$$

```
[In] int(1/(sinh(x)^5*(a + b*tanh(x))),x)
```

```
[Out] (log(exp(x) - 1)*(3*a^4 + 8*b^4 - 12*a^2*b^2))/(8*a^5) - (log(exp(x) + 1)*(
3*a^4 + 8*b^4 - 12*a^2*b^2))/(8*a^5) - (4*exp(x))/(a*(6*exp(4*x) - 4*exp(2*
x) - 4*exp(6*x) + exp(8*x) + 1)) - (2*exp(x)*(9*a - 4*b))/(3*a^2*(3*exp(2*x
) - 3*exp(4*x) + exp(6*x) - 1)) - (exp(x)*(3*a^2 - 16*a*b + 12*b^2))/(6*a^3
*(exp(4*x) - 2*exp(2*x) + 1)) - (exp(x)*(4*a*b^2 + 8*a^2*b - 3*a^3 - 8*b^3)
)/(4*a^4*(exp(2*x) - 1)) + (b*log((b*exp(x)*(a - b)^2*(288*a*b^6 - 24*a^6*b
 - 9*a^7 - 192*b^7 + 224*a^2*b^5 - 456*a^3*b^4 + 24*a^4*b^3 + 144*a^5*b^2))
/(2*a^12*(a + b)) - (b*(a - b)*(-(a + b)^3*(a - b)^3)^(1/2)*(8*a^5*b^3 - 9*
a^8 - 9*a^7*b + 8*a^6*b^2 + 192*b^5*exp(x)*(-(a^2 - b^2)^3)^(1/2) - 224*a^2
*b^3*exp(x)*(-(a^2 - b^2)^3)^(1/2) - 88*a^3*b^2*exp(x)*(-(a^2 - b^2)^3)^(1/
2) + 96*a*b^4*exp(x)*(-(a^2 - b^2)^3)^(1/2) + 24*a^4*b*exp(x)*(-(a^2 - b^2)
^3)^(1/2)))/(2*a^12*(a + b)^4))*(-(a + b)^3*(a - b)^3)^(1/2))/a^5 - (b*log(
(b*exp(x)*(a - b)^2*(288*a*b^6 - 24*a^6*b - 9*a^7 - 192*b^7 + 224*a^2*b^5 -
 456*a^3*b^4 + 24*a^4*b^3 + 144*a^5*b^2))/(2*a^12*(a + b)) - (b*(a - b)*(-(
a + b)^3*(a - b)^3)^(1/2)*(9*a^7*b + 9*a^8 - 8*a^5*b^3 - 8*a^6*b^2 + 192*b^
5*exp(x)*(-(a^2 - b^2)^3)^(1/2) - 224*a^2*b^3*exp(x)*(-(a^2 - b^2)^3)^(1/2)
 - 88*a^3*b^2*exp(x)*(-(a^2 - b^2)^3)^(1/2) + 96*a*b^4*exp(x)*(-(a^2 - b^2)
^3)^(1/2) + 24*a^4*b*exp(x)*(-(a^2 - b^2)^3)^(1/2)))/(2*a^12*(a + b)^4))*(-
(a + b)^3*(a - b)^3)^(1/2))/a^5
```

3.89 $\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 130

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx = -\frac{(a^2-b^2)^2\coth(x)}{a^5}-\frac{b(2a^2-b^2)\coth^2(x)}{2a^4}+\frac{(2a^2-b^2)\coth^3(x)}{3a^3}+\frac{b\coth^4(x)}{4a^2}-\frac{\coth^5(x)}{5a}-\frac{b(a^2-b^2)^2\log(\tanh(x))}{a^6}+\frac{b(a^2-b^2)^2\log(a+b\tanh(x))}{a^6}$$

```
[Out] -(a^2-b^2)^2*coth(x)/a^5-1/2*b*(2*a^2-b^2)*coth(x)^2/a^4+1/3*(2*a^2-b^2)*co
th(x)^3/a^3+1/4*b*coth(x)^4/a^2-1/5*coth(x)^5/a-b*(a^2-b^2)^2*ln(tanh(x))/a
^6+b*(a^2-b^2)^2*ln(a+b*tanh(x))/a^6
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3597, 908}

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx = \frac{b\coth^4(x)}{4a^2}-\frac{b(a^2-b^2)^2\log(\tanh(x))}{a^6}+\frac{b(a^2-b^2)^2\log(a+b\tanh(x))}{a^6}-\frac{(a^2-b^2)^2\coth(x)}{a^5}-\frac{b(2a^2-b^2)\coth^2(x)}{2a^4}+\frac{(2a^2-b^2)\coth^3(x)}{3a^3}-\frac{\coth^5(x)}{5a}$$

```
[In] Int[Csch[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Coth[x])/a^5) - (b*(2*a^2 - b^2)*Coth[x]^2)/(2*a^4) + ((2*
a^2 - b^2)*Coth[x]^3)/(3*a^3) + (b*Coth[x]^4)/(4*a^2) - Coth[x]^5/(5*a) - (
b*(a^2 - b^2)^2*Log[Tanh[x]])/a^6 + (b*(a^2 - b^2)^2*Log[a + b*Tanh[x]])/a^
6
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= b\text{Subst}\left(\int \frac{(-b^2+x^2)^2}{x^6(a+x)}\,dx, x, b\tanh(x)\right)\\
&= b\text{Subst}\left(\int \left(\frac{b^4}{ax^6}-\frac{b^4}{a^2x^5}+\frac{-2a^2b^2+b^4}{a^3x^4}+\frac{2a^2b^2-b^4}{a^4x^3}+\frac{(a^2-b^2)^2}{a^5x^2}-\frac{(a^2-b^2)^2}{a^6x}\right.\right.\\
&\qquad\left.\left.+\frac{(a^2-b^2)^2}{a^6(a+x)}\right)\,dx, x, b\tanh(x)\right)\\
&= -\frac{(a^2-b^2)^2\coth(x)}{a^5}-\frac{b(2a^2-b^2)\coth^2(x)}{2a^4}+\frac{(2a^2-b^2)\coth^3(x)}{3a^3}+\frac{b\coth^4(x)}{4a^2}\\
&\quad-\frac{\coth^5(x)}{5a}-\frac{b(a^2-b^2)^2\log(\tanh(x))}{a^6}+\frac{b(a^2-b^2)^2\log(a+b\tanh(x))}{a^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx = \frac{-4\coth(x)\left(8a^5-25a^3b^2+15ab^4+(-4a^5+5a^3b^2)\operatorname{csch}^2(x)+3a^5\operatorname{csch}^4(x)\right)+15b\Big(-2a^2(a^2-b^2)\operatorname{csch}^2(x}{60a^6}$$

```
[In] Integrate[Csch[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] (-4*Coth[x]*(8*a^5 - 25*a^3*b^2 + 15*a*b^4 + (-4*a^5 + 5*a^3*b^2)*Csch[x]^2
 + 3*a^5*Csch[x]^4) + 15*b*(-2*a^2*(a^2 - b^2)*Csch[x]^2 + a^4*Csch[x]^4 -
4*(a^2 - b^2)^2*(Log[Sinh[x]] - Log[a*Cosh[x] + b*Sinh[x]])))/(60*a^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(122) = 244$.

Time = 7.08 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.13

method	result
default	$-\frac{\frac{a^4 \tanh\left(\frac{x}{2}\right)^5}{5}-\frac{b \tanh\left(\frac{x}{2}\right)^4 a^3}{2}-\frac{5 \tanh\left(\frac{x}{2}\right)^3 a^4}{3}+\frac{4a^2 b^2 \tanh\left(\frac{x}{2}\right)^3}{3}+6a^3 b \tanh\left(\frac{x}{2}\right)^2-4b^3 \tanh\left(\frac{x}{2}\right)^2 a+10a^4 \tanh\left(\frac{x}{2}\right)-28a^2 b^2 \tanh\left(\frac{x}{2}\right)+}{32a^5}$
risch	$-\frac{2\left(15a^3 b\, e^{8x}-15a^2 b^2 e^{8x}-15a\, b^3 e^{8x}+15b^4 e^{8x}-75a^3 b\, e^{6x}+90a^2 b^2 e^{6x}+45a\, b^3 e^{6x}-60b^4 e^{6x}+80\, e^{4x} a^4+75\, e^{4x} a^3 b-160\, e^{4x} a^2 b^2-45\right.}{15a^5 \left(e^{2x}-1\right)^5}$

```
[In] int(csch(x)^6/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/a^5*(1/5*a^4*tanh(1/2*x)^5-1/2*b*tanh(1/2*x)^4*a^3-5/3*tanh(1/2*x)^3*
a^4+4/3*a^2*b^2*tanh(1/2*x)^3+6*a^3*b*tanh(1/2*x)^2-4*b^3*tanh(1/2*x)^2*a+1
0*a^4*tanh(1/2*x)-28*a^2*b^2*tanh(1/2*x)+16*tanh(1/2*x)*b^4)+2/a^6*b*(1/2*a
^4-a^2*b^2+1/2*b^4)*ln(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)-1/160/a/tanh(1/2*
x)^5-1/96*(-5*a^2+4*b^2)/a^3/tanh(1/2*x)^3-1/32/a^5*(10*a^4-28*a^2*b^2+16*b
^4)/tanh(1/2*x)+1/64/a^2*b/tanh(1/2*x)^4-1/16/a^4*b*(3*a^2-2*b^2)/tanh(1/2*
x)^2-1/a^6*b*(a^4-2*a^2*b^2+b^4)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(122) = 244$.

Time = 0.28 (sec) , antiderivative size = 2972, normalized size of antiderivative = 22.86

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^6/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(30*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*cosh(x)^8 + 240*(a^4*b - a^3*
b^2 - a^2*b^3 + a*b^4)*cosh(x)*sinh(x)^7 + 30*(a^4*b - a^3*b^2 - a^2*b^3 +
a*b^4)*sinh(x)^8 - 30*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x)^6
 - 30*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4 - 28*(a^4*b - a^3*b^2 - a^
2*b^3 + a*b^4)*cosh(x)^2)*sinh(x)^6 + 60*(28*(a^4*b - a^3*b^2 - a^2*b^3 + a
*b^4)*cosh(x)^3 - 3*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x))*si
nh(x)^5 + 16*a^5 - 50*a^3*b^2 + 30*a*b^4 + 10*(16*a^5 + 15*a^4*b - 32*a^3*b
^2 - 9*a^2*b^3 + 18*a*b^4)*cosh(x)^4 + 10*(16*a^5 + 15*a^4*b - 32*a^3*b^2 -
 9*a^2*b^3 + 18*a*b^4 + 210*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*cosh(x)^4 -
```

```
 45*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(
42*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*cosh(x)^5 - 15*(5*a^4*b - 6*a^3*b^2
- 3*a^2*b^3 + 4*a*b^4)*cosh(x)^3 + (16*a^5 + 15*a^4*b - 32*a^3*b^2 - 9*a^2*
b^3 + 18*a*b^4)*cosh(x))*sinh(x)^3 - 10*(8*a^5 + 3*a^4*b - 22*a^3*b^2 - 3*a
^2*b^3 + 12*a*b^4)*cosh(x)^2 + 10*(84*(a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*c
osh(x)^6 - 8*a^5 - 3*a^4*b + 22*a^3*b^2 + 3*a^2*b^3 - 12*a*b^4 - 45*(5*a^4*
b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*b^4)*cosh(x)^4 + 6*(16*a^5 + 15*a^4*b - 32*
a^3*b^2 - 9*a^2*b^3 + 18*a*b^4)*cosh(x)^2)*sinh(x)^2 - 15*((a^4*b - 2*a^2*b
^3 + b^5)*cosh(x)^10 + 10*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)*sinh(x)^9 + (a^
4*b - 2*a^2*b^3 + b^5)*sinh(x)^10 - 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^8 -
 5*(a^4*b - 2*a^2*b^3 + b^5 - 9*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x
)^8 + 40*(3*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 - (a^4*b - 2*a^2*b^3 + b^5)
*cosh(x))*sinh(x)^7 + 10*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^6 + 10*(a^4*b -
2*a^2*b^3 + b^5 + 21*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 - 14*(a^4*b - 2*a^
2*b^3 + b^5)*cosh(x)^2)*sinh(x)^6 + 4*(63*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)
^5 - 70*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 + 15*(a^4*b - 2*a^2*b^3 + b^5)*
cosh(x))*sinh(x)^5 - a^4*b + 2*a^2*b^3 - b^5 - 10*(a^4*b - 2*a^2*b^3 + b^5)
*cosh(x)^4 + 10*(21*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^6 - a^4*b + 2*a^2*b^3
 - b^5 - 35*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 + 15*(a^4*b - 2*a^2*b^3 + b
^5)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 - 7*(a
^4*b - 2*a^2*b^3 + b^5)*cosh(x)^5 + 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 -
 (a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^3 + 5*(a^4*b - 2*a^2*b^3 + b^5)
*cosh(x)^2 + 5*(9*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^8 - 28*(a^4*b - 2*a^2*b
^3 + b^5)*cosh(x)^6 + a^4*b - 2*a^2*b^3 + b^5 + 30*(a^4*b - 2*a^2*b^3 + b^5
)*cosh(x)^4 - 12*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 10*((a^4*
b - 2*a^2*b^3 + b^5)*cosh(x)^9 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 + 6*
(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^5 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3
 + (a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 15*((a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^10 + 10*(a^4
*b - 2*a^2*b^3 + b^5)*cosh(x)*sinh(x)^9 + (a^4*b - 2*a^2*b^3 + b^5)*sinh(x)
^10 - 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^8 - 5*(a^4*b - 2*a^2*b^3 + b^5 -
9*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^4*b - 2*a^2*b^3
 + b^5)*cosh(x)^3 - (a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^7 + 10*(a^4*
b - 2*a^2*b^3 + b^5)*cosh(x)^6 + 10*(a^4*b - 2*a^2*b^3 + b^5 + 21*(a^4*b -
2*a^2*b^3 + b^5)*cosh(x)^4 - 14*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x
)^6 + 4*(63*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^5 - 70*(a^4*b - 2*a^2*b^3 + b
^5)*cosh(x)^3 + 15*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x))*sinh(x)^5 - a^4*b + 2
*a^2*b^3 - b^5 - 10*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 + 10*(21*(a^4*b - 2
*a^2*b^3 + b^5)*cosh(x)^6 - a^4*b + 2*a^2*b^3 - b^5 - 35*(a^4*b - 2*a^2*b^3
 + b^5)*cosh(x)^4 + 15*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2)*sinh(x)^4 + 40*
(3*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 - 7*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x
)^5 + 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 - (a^4*b - 2*a^2*b^3 + b^5)*cos
h(x))*sinh(x)^3 + 5*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^2 + 5*(9*(a^4*b - 2*a
^2*b^3 + b^5)*cosh(x)^8 - 28*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^6 + a^4*b -
2*a^2*b^3 + b^5 + 30*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^4 - 12*(a^4*b - 2*a^
```

```
2*b^3 + b^5)*cosh(x)^2)*sinh(x)^2 + 10*((a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^9
 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^7 + 6*(a^4*b - 2*a^2*b^3 + b^5)*cosh
(x)^5 - 4*(a^4*b - 2*a^2*b^3 + b^5)*cosh(x)^3 + (a^4*b - 2*a^2*b^3 + b^5)*c
osh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 20*(12*(a^4*b - a^3*b
^2 - a^2*b^3 + a*b^4)*cosh(x)^7 - 9*(5*a^4*b - 6*a^3*b^2 - 3*a^2*b^3 + 4*a*
b^4)*cosh(x)^5 + 2*(16*a^5 + 15*a^4*b - 32*a^3*b^2 - 9*a^2*b^3 + 18*a*b^4)*
cosh(x)^3 - (8*a^5 + 3*a^4*b - 22*a^3*b^2 - 3*a^2*b^3 + 12*a*b^4)*cosh(x))*
sinh(x))/(a^6*cosh(x)^10 + 10*a^6*cosh(x)*sinh(x)^9 + a^6*sinh(x)^10 - 5*a^
6*cosh(x)^8 + 10*a^6*cosh(x)^6 - 10*a^6*cosh(x)^4 + 5*(9*a^6*cosh(x)^2 - a^
6)*sinh(x)^8 + 5*a^6*cosh(x)^2 + 40*(3*a^6*cosh(x)^3 - a^6*cosh(x))*sinh(x)
^7 + 10*(21*a^6*cosh(x)^4 - 14*a^6*cosh(x)^2 + a^6)*sinh(x)^6 - a^6 + 4*(63
*a^6*cosh(x)^5 - 70*a^6*cosh(x)^3 + 15*a^6*cosh(x))*sinh(x)^5 + 10*(21*a^6*
cosh(x)^6 - 35*a^6*cosh(x)^4 + 15*a^6*cosh(x)^2 - a^6)*sinh(x)^4 + 40*(3*a^
6*cosh(x)^7 - 7*a^6*cosh(x)^5 + 5*a^6*cosh(x)^3 - a^6*cosh(x))*sinh(x)^3 +
5*(9*a^6*cosh(x)^8 - 28*a^6*cosh(x)^6 + 30*a^6*cosh(x)^4 - 12*a^6*cosh(x)^2
 + a^6)*sinh(x)^2 + 10*(a^6*cosh(x)^9 - 4*a^6*cosh(x)^7 + 6*a^6*cosh(x)^5 -
 4*a^6*cosh(x)^3 + a^6*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx=\int \frac{\operatorname{csch}^6\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

```
[In] integrate(csch(x)**6/(a+b*tanh(x)),x)
[Out] Integral(csch(x)**6/(a + b*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(122) = 244$.

Time = 0.21 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.37

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx = \frac{2\left(8\,a^4-25\,a^2b^2+15\,b^4-5\left(8\,a^4-3\,a^3b-22\,a^2b^2+3\,ab^3+12\,b^4\right)e^{(-2\,x)}+5\left(16\,a^4-15\,a^3b-32\,a^2b^2+\cdots\right.\right.}{15\left(5\,a^5e^{(-2\,x)}-10\,a^5e^{(-4\,x)}+10\,a^5e^{(-6\,x)}\cdots\right.} + \frac{\left(a^4b-2\,a^2b^3+b^5\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^6} - \frac{\left(a^4b-2\,a^2b^3+b^5\right)\log\left(e^{(-x)}+1\right)}{a^6} - \frac{\left(a^4b-2\,a^2b^3+b^5\right)\log\left(e^{(-x)}-1\right)}{a^6}$$

```
[In] integrate(csch(x)^6/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(8*a^4 - 25*a^2*b^2 + 15*b^4 - 5*(8*a^4 - 3*a^3*b - 22*a^2*b^2 + 3*a*b
^3 + 12*b^4)*e^(-2*x) + 5*(16*a^4 - 15*a^3*b - 32*a^2*b^2 + 9*a*b^3 + 18*b^
4)*e^(-4*x) + 15*(5*a^3*b + 6*a^2*b^2 - 3*a*b^3 - 4*b^4)*e^(-6*x) - 15*(a^3
*b + a^2*b^2 - a*b^3 - b^4)*e^(-8*x))/(5*a^5*e^(-2*x) - 10*a^5*e^(-4*x) + 1
0*a^5*e^(-6*x) - 5*a^5*e^(-8*x) + a^5*e^(-10*x) - a^5) + (a^4*b - 2*a^2*b^3
 + b^5)*log(-(a - b)*e^(-2*x) - a - b)/a^6 - (a^4*b - 2*a^2*b^3 + b^5)*log(
e^(-x) + 1)/a^6 - (a^4*b - 2*a^2*b^3 + b^5)*log(e^(-x) - 1)/a^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(122) = 244.

Time = 0.28 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.17

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx = \frac{(a^5b+a^4b^2-2\,a^3b^3-2\,a^2b^4+ab^5+b^6)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^7+a^6b} - \frac{(a^4b-2\,a^2b^3+b^5)\log\left(\left|e^{(2\,x)}-1\right|\right)}{a^6} + \frac{137\,a^4be^{(10\,x)}-274\,a^2b^3e^{(10\,x)}+137\,b^5e^{(10\,x)}-805\,a^4be^{(8\,x)}+120\,a^3b^2e^{(8\,x)}+1490\,a^2b^3e^{(8\,x)}-120\,ab^4e^{(8\,x)}}{}$$

```
[In] integrate(csch(x)^6/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a^5*b + a^4*b^2 - 2*a^3*b^3 - 2*a^2*b^4 + a*b^5 + b^6)*log(abs(a*e^(2*x) +
 b*e^(2*x) + a - b))/(a^7 + a^6*b) - (a^4*b - 2*a^2*b^3 + b^5)*log(abs(e^(2
*x) - 1))/a^6 + 1/60*(137*a^4*b*e^(10*x) - 274*a^2*b^3*e^(10*x) + 137*b^5*e
^(10*x) - 805*a^4*b*e^(8*x) + 120*a^3*b^2*e^(8*x) + 1490*a^2*b^3*e^(8*x) -
120*a*b^4*e^(8*x) - 685*b^5*e^(8*x) + 1970*a^4*b*e^(6*x) - 720*a^3*b^2*e^(6
*x) - 3100*a^2*b^3*e^(6*x) + 480*a*b^4*e^(6*x) + 1370*b^5*e^(6*x) - 640*a^5
*e^(4*x) - 1970*a^4*b*e^(4*x) + 1280*a^3*b^2*e^(4*x) + 3100*a^2*b^3*e^(4*x)
 - 720*a*b^4*e^(4*x) - 1370*b^5*e^(4*x) + 320*a^5*e^(2*x) + 805*a^4*b*e^(2*
x) - 880*a^3*b^2*e^(2*x) - 1490*a^2*b^3*e^(2*x) + 480*a*b^4*e^(2*x) + 685*b
^5*e^(2*x) - 64*a^5 - 137*a^4*b + 200*a^3*b^2 + 274*a^2*b^3 - 120*a*b^4 - 1
37*b^5)/(a^6*(e^(2*x) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.82

$$\int \frac{\operatorname{csch}^6(x)}{a+b\tanh(x)}\,dx = \frac{2\,(a-b)\,\left(a\,b-b^2\right)}{a^4\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)}-\frac{8\,\left(4\,a^2-3\,a\,b+b^2\right)}{3\,a^3\,\left(3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1\right)}-\frac{4\,(4\,a-b)}{a^2\,\left(6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}-\frac{32}{5\,a\,\left(5\,\mathrm{e}^{2\,x}-10\,\mathrm{e}^{4\,x}+10\,\mathrm{e}^{6\,x}-5\,\mathrm{e}^{8\,x}+\mathrm{e}^{10\,x}-1\right)}-\frac{2\,(a+b)\,(a-b)\,\left(a\,b-b^2\right)}{a^5\,\left(\mathrm{e}^{2\,x}-1\right)}+\frac{b\,\ln\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)\,{(a+b)}^2\,{(a-b)}^2}{a^6}-\frac{b\,\ln\left(\mathrm{e}^{2\,x}-1\right)\,{(a+b)}^2\,{(a-b)}^2}{a^6}$$

```
[In] int(1/(sinh(x)^6*(a + b*tanh(x))),x)
```

```
[Out] (2*(a - b)*(a*b - b^2))/(a^4*(exp(4*x) - 2*exp(2*x) + 1)) - (8*(4*a^2 - 3*a
*b + b^2))/(3*a^3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (4*(4*a - b))
/(a^2*(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1)) - 32/(5*a*(5*e
xp(2*x) - 10*exp(4*x) + 10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1)) - (2*(a
+ b)*(a - b)*(a*b - b^2))/(a^5*(exp(2*x) - 1)) + (b*log(a - b + a*exp(2*x)
+ b*exp(2*x))*(a + b)^2*(a - b)^2)/a^6 - (b*log(exp(2*x) - 1)*(a + b)^2*(a
- b)^2)/a^6
```

3.90 $\int \frac{\operatorname{csch}(x)}{i+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{\operatorname{csch}(x)}{i+\tanh(x)}\,dx = i\operatorname{arctanh}(\cosh(x)) - \frac{i\operatorname{arctanh}\left(\frac{\cosh(x)+i\sinh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] I*arctanh(cosh(x))-1/2*I*arctanh(1/2*(cosh(x)+I*sinh(x))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3599, 3189, 3855, 3153, 212}

$$\int \frac{\operatorname{csch}(x)}{i+\tanh(x)}\,dx = i\operatorname{arctanh}(\cosh(x)) - \frac{i\operatorname{arctanh}\left(\frac{\cosh(x)+i\sinh(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Int[Csch[x]/(I + Tanh[x]),x]
```

```
[Out] I*ArcTanh[Cosh[x]] - (I*ArcTanh[(Cosh[x] + I*Sinh[x])/Sqrt[2]])/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{\coth(x)}{i\cosh(x)+\sinh(x)}\,dx \\
&= i\int \left(-\text{csch}(x)-\frac{i}{\cosh(x)-i\sinh(x)}\right)\,dx \\
&= -(i\int \text{csch}(x)\,dx)+\int \frac{1}{\cosh(x)-i\sinh(x)}\,dx \\
&= i\text{arctanh}(\cosh(x))+i\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,-\cosh(x)-i\sinh(x)\right) \\
&= i\text{arctanh}(\cosh(x))-\frac{i\text{arctanh}\left(\frac{\cosh(x)+i\sinh(x)}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{\operatorname{csch}(x)}{i+\tanh(x)}\,dx = -i\left(\sqrt{2}\operatorname{arctanh}\left(\frac{1+i\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)$$

```
[In] Integrate[Csch[x]/(I + Tanh[x]),x]
[Out] (-I)*(Sqrt[2]*ArcTanh[(1 + I*Tanh[x/2])/Sqrt[2]] - Log[Cosh[x/2]] + Log[Sin
h[x/2]])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$-i\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\sqrt{2}\,\arctan\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)-2i\right)\sqrt{2}}{4}\right)$	29
risch	$i\ln\left(\mathrm{e}^{x}+1\right)-i\ln\left(\mathrm{e}^{x}-1\right)+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{x}-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{x}+\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{2}$	60

```
[In] int(csch(x)/(I+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -I*ln(tanh(1/2*x))+2^(1/2)*arctan(1/4*(2*tanh(1/2*x)-2*I)*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{\operatorname{csch}(x)}{i+\tanh(x)}\,dx = -\frac{1}{2}i\sqrt{2}\log\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}+e^{x}\right) + \frac{1}{2}i\sqrt{2}\log\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}+e^{x}\right)+i\log\left(e^{x}+1\right)-i\log\left(e^{x}-1\right)$$

```
[In] integrate(csch(x)/(I+tanh(x)),x, algorithm="fricas")
[Out] -1/2*I*sqrt(2)*log(-(1/2*I - 1/2)*sqrt(2) + e^x) + 1/2*I*sqrt(2)*log((1/2*I
 - 1/2)*sqrt(2) + e^x) + I*log(e^x + 1) - I*log(e^x - 1)
```

Sympy [F]

$$\int \frac{\mathrm{csch}(x)}{i+\tanh(x)}\,dx = \int \frac{\mathrm{csch}\,(x)}{\tanh\,(x)+i}\,dx$$

```
[In] integrate(csch(x)/(I+tanh(x)),x)
[Out] Integral(csch(x)/(tanh(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{\mathrm{csch}(x)}{i+\tanh(x)}\,dx = -\sqrt{2}\arctan\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}e^{(-x)}\right)+i\,\log\left(e^{(-x)}+1\right)-i\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)/(I+tanh(x)),x, algorithm="maxima")
[Out] -sqrt(2)*arctan((1/2*I + 1/2)*sqrt(2)*e^(-x)) + I*log(e^(-x) + 1) - I*log(e
^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{\mathrm{csch}(x)}{i+\tanh(x)}\,dx = \sqrt{2}\arctan\left(-\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}e^{x}\right)+i\,\log\left(e^{x}+1\right)-i\,\log\left(|e^{x}-1|\right)$$

```
[In] integrate(csch(x)/(I+tanh(x)),x, algorithm="giac")
[Out] sqrt(2)*arctan(-(1/2*I - 1/2)*sqrt(2)*e^x) + I*log(e^x + 1) - I*log(abs(e^x
 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{\mathrm{csch}(x)}{i+\tanh(x)}\,dx = \ln\left(-8\,\mathrm{e}^{x}-8\right)\,1\mathrm{i}-\ln\left(8-8\,\mathrm{e}^{x}\right)\,1\mathrm{i} - \frac{\sqrt{2}\,\ln\left(\mathrm{e}^{x}\,(4-4\mathrm{i})-\sqrt{2}\,4\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{x}\,(4-4\mathrm{i})+\sqrt{2}\,4\mathrm{i}\right)\,1\mathrm{i}}{2}$$

```
[In] int(1/(sinh(x)*(tanh(x) + 1i)),x)
[Out] log(- 8*exp(x) - 8)*1i - log(8 - 8*exp(x))*1i - (2^(1/2)*log(exp(x)*(4 - 4i
) - 2^(1/2)*4i)*1i)/2 + (2^(1/2)*log(exp(x)*(4 - 4i) + 2^(1/2)*4i)*1i)/2
```

3.91 $\int \frac{\cosh^4(x)}{1+\tanh(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\cosh^4(x)}{1+\tanh(x)} dx = \frac{5x}{16} + \frac{1}{32(1-\tanh(x))^2} + \frac{1}{8(1-\tanh(x))} - \frac{1}{24(1+\tanh(x))^3} - \frac{3}{32(1+\tanh(x))^2} - \frac{3}{16(1+\tanh(x))}$$

```
[Out] 5/16*x+1/32/(1-tanh(x))^2+1/8/(1-tanh(x))-1/24/(1+tanh(x))^3-3/32/(1+tanh(x
))^2-3/16/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3568, 46, 213}

$$\int \frac{\cosh^4(x)}{1+\tanh(x)} dx = \frac{5x}{16} + \frac{1}{8(1-\tanh(x))} - \frac{3}{16(\tanh(x)+1)} + \frac{1}{32(1-\tanh(x))^2} - \frac{3}{32(\tanh(x)+1)^2} - \frac{1}{24(\tanh(x)+1)^3}$$

```
[In] Int[Cosh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (5*x)/16 + 1/(32*(1 - Tanh[x])^2) + 1/(8*(1 - Tanh[x])) - 1/(24*(1 + Tanh[x
])^3) - 3/(32*(1 + Tanh[x])^2) - 3/(16*(1 + Tanh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x)^3(1+x)^4}\,dx, x, \tanh(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}+\frac{1}{8(1+x)^4}+\frac{3}{16(1+x)^3}+\frac{3}{16(1+x)^2}-\frac{5}{16\left(-1+x^2\right)}\right)dx, x, \tanh(x)\right)\\
&= \frac{1}{32(1-\tanh(x))^2}+\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}-\frac{3}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tanh(x))}-\frac{5}{16}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right)\\
&= \frac{5x}{16}+\frac{1}{32(1-\tanh(x))^2}+\frac{1}{8(1-\tanh(x))}-\frac{1}{24(1+\tanh(x))^3}-\frac{3}{32(1+\tanh(x))^2}-\frac{3}{16(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^4(x)}{1+\tanh(x)}\,dx = \frac{\text{sech}(x)(-80\cosh(x)+15\cosh(3x)+\cosh(5x)+40\sinh(x)+120\text{arctanh}(\tanh(x))(\cosh(x)+\sinh(x))}{384(1+\tanh(x))}$$

```
[In] Integrate[Cosh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (Sech[x]*(-80*Cosh[x] + 15*Cosh[3*x] + Cosh[5*x] + 40*Sinh[x] + 120*ArcTanh
[Tanh[x]]*(Cosh[x] + Sinh[x]) + 45*Sinh[3*x] + 5*Sinh[5*x]))/(384*(1 + Tanh
[x]))
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result
risch	$\frac{5x}{16}+\frac{e^{4x}}{128}+\frac{5\,e^{2x}}{64}-\frac{5\,e^{-2x}}{32}-\frac{5\,e^{-4x}}{128}-\frac{e^{-6x}}{192}$
parallelrisch	$-\frac{13}{96}-\frac{\cosh(4x)}{32}-\frac{\cosh(6x)}{192}-\frac{5\cosh(2x)}{64}+\frac{5\ln(1+\tanh(x))}{32}-\frac{5\ln(1-\tanh(x))}{32}+\frac{3\sinh(4x)}{64}+\frac{15\sinh(2x)}{64}+$ sinh
default	$-\frac{1}{3(\tanh(\frac{x}{2})+1)^6}+\frac{1}{(\tanh(\frac{x}{2})+1)^5}-\frac{15}{8(\tanh(\frac{x}{2})+1)^4}+\frac{25}{12(\tanh(\frac{x}{2})+1)^3}-\frac{15}{8(\tanh(\frac{x}{2})+1)^2}+\frac{1}{\tanh(\frac{x}{2})+1}+$ 5 ln(

```
[In] int(cosh(x)^4/(1+tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*x+1/128*exp(4*x)+5/64*exp(2*x)-5/32*exp(-2*x)-5/128*exp(-4*x)-1/192*ex
p(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(44) = 88.

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^4(x)}{1+\tanh(x)}\,dx$$
$$=\frac{\cosh(x)^5+5\,\cosh(x)\sinh(x)^4+5\,\sinh(x)^5+5\left(10\,\cosh(x)^2+9\right)\sinh(x)^3+15\,\cosh(x)^3+5\left(2\,\cosh(x\right.}{384\,(\cosh(x)+}$$

```
[In] integrate(cosh(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + 5*sinh(x)^5 + 5*(10*cosh(x)^2 + 9)
*sinh(x)^3 + 15*cosh(x)^3 + 5*(2*cosh(x)^3 + 9*cosh(x))*sinh(x)^2 + 60*(2*x
 - 1)*cosh(x) + 5*(5*cosh(x)^4 + 27*cosh(x)^2 + 24*x + 12)*sinh(x))/(cosh(x
) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{1+\tanh(x)}\,dx = \int \frac{\cosh^4{(x)}}{\tanh{(x)}+1}\,dx$$

```
[In] integrate(cosh(x)**4/(1+tanh(x)),x)
[Out] Integral(cosh(x)**4/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^4(x)}{1+\tanh(x)}\,dx = \frac{1}{128}\left(10\,e^{(-2\,x)}+1\right)e^{(4\,x)} + \frac{5}{16}\,x - \frac{5}{32}\,e^{(-2\,x)} - \frac{5}{128}\,e^{(-4\,x)} - \frac{1}{192}\,e^{(-6\,x)}$$

```
[In] integrate(cosh(x)^4/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/128*(10*e^(-2*x) + 1)*e^(4*x) + 5/16*x - 5/32*e^(-2*x) - 5/128*e^(-4*x) -
 1/192*e^(-6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^4(x)}{1+\tanh(x)}\,dx = -\frac{1}{384}\left(110\,e^{(6\,x)}+60\,e^{(4\,x)}+15\,e^{(2\,x)}+2\right)e^{(-6\,x)} + \frac{5}{16}\,x + \frac{1}{128}\,e^{(4\,x)} + \frac{5}{64}\,e^{(2\,x)}$$

```
[In] integrate(cosh(x)^4/(1+tanh(x)),x, algorithm="giac")
[Out] -1/384*(110*e^(6*x) + 60*e^(4*x) + 15*e^(2*x) + 2)*e^(-6*x) + 5/16*x + 1/12
8*e^(4*x) + 5/64*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\cosh^4(x)}{1+\tanh(x)}\,dx = \frac{5\,x}{16} - \frac{5\,\mathrm{e}^{-2\,x}}{32} + \frac{5\,\mathrm{e}^{2\,x}}{64} - \frac{5\,\mathrm{e}^{-4\,x}}{128} + \frac{\mathrm{e}^{4\,x}}{128} - \frac{\mathrm{e}^{-6\,x}}{192}$$

```
[In] int(cosh(x)^4/(tanh(x) + 1),x)
[Out] (5*x)/16 - (5*exp(-2*x))/32 + (5*exp(2*x))/64 - (5*exp(-4*x))/128 + exp(4*x
)/128 - exp(-6*x)/192
```

3.92 $\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = \frac{4\sinh(x)}{5}+\frac{4\sinh^3(x)}{15}-\frac{\cosh^3(x)}{5(1+\tanh(x))}$$

```
[Out] 4/5*sinh(x)+4/15*sinh(x)^3-1/5*cosh(x)^3/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3583, 2713}

$$\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = \frac{4\sinh^3(x)}{15}+\frac{4\sinh(x)}{5}-\frac{\cosh^3(x)}{5(\tanh(x)+1)}$$

```
[In] Int[Cosh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (4*Sinh[x])/5 + (4*Sinh[x]^3)/15 - Cosh[x]^3/(5*(1 + Tanh[x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
```

```
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh^3(x)}{5(1+\tanh(x))}+\frac{4}{5}\int \cosh^3(x)\,dx \\
&= -\frac{\cosh^3(x)}{5(1+\tanh(x))}+\frac{4}{5}i\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right) \\
&= \frac{4\sinh(x)}{5}+\frac{4\sinh^3(x)}{15}-\frac{\cosh^3(x)}{5(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = \frac{\text{sech}(x)(-45+20\cosh(2x)+\cosh(4x)+40\sinh(2x)+4\sinh(4x))}{120(1+\tanh(x))}$$

```
[In] Integrate[Cosh[x]^3/(1 + Tanh[x]),x]
[Out] (Sech[x]*(-45 + 20*Cosh[2*x] + Cosh[4*x] + 40*Sinh[2*x] + 4*Sinh[4*x]))/(12
0*(1 + Tanh[x]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result
risch	$\frac{e^{3x}}{48}+\frac{e^{x}}{4}-\frac{3\,e^{-x}}{8}-\frac{e^{-3x}}{12}-\frac{e^{-5x}}{80}$
parallelrisch	$-\frac{\cosh(3x)}{16}-\frac{\cosh(x)}{8}+\frac{5\sinh(3x)}{48}+\frac{5\sinh(x)}{8}-\frac{\cosh(5x)}{80}+\frac{\sinh(5x)}{80}+\frac{7}{15}$
default	$-\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^5}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}-\frac{5}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{11}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-$ 4(

```
[In] int(cosh(x)^3/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/48*exp(3*x)+1/4*exp(x)-3/8*exp(-x)-1/12*exp(-3*x)-1/80*exp(-5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(23) = 46$.

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = \frac{\cosh(x)^4 + 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 + 10\right)\sinh(x)^2 + 20\cosh(x)^2 + 16\left(\cosh(x\right.}{120\left(\cosh(x) + \sinh(x)\right)}$$

```
[In] integrate(cosh(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 10)*
sinh(x)^2 + 20*cosh(x)^2 + 16*(cosh(x)^3 + 5*cosh(x))*sinh(x) - 45)/(cosh(x
) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(26) = 52$.

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 4.62

$$\begin{aligned}\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = &-\frac{8\sinh^3(x)\tanh(x)}{15\tanh(x)+15} - \frac{2\sinh^3(x)}{15\tanh(x)+15} - \frac{6\sinh^2(x)\cosh(x)\tanh(x)}{15\tanh(x)+15} \\ &+ \frac{6\sinh^2(x)\cosh(x)}{15\tanh(x)+15} + \frac{6\sinh(x)\cosh^2(x)\tanh(x)}{15\tanh(x)+15} \\ &+ \frac{9\sinh(x)\cosh^2(x)}{15\tanh(x)+15} + \frac{3\cosh^3(x)\tanh(x)}{15\tanh(x)+15} - \frac{3\cosh^3(x)}{15\tanh(x)+15}\end{aligned}$$

```
[In] integrate(cosh(x)**3/(1+tanh(x)),x)
```

```
[Out] -8*sinh(x)**3*tanh(x)/(15*tanh(x) + 15) - 2*sinh(x)**3/(15*tanh(x) + 15) -
6*sinh(x)**2*cosh(x)*tanh(x)/(15*tanh(x) + 15) + 6*sinh(x)**2*cosh(x)/(15*t
anh(x) + 15) + 6*sinh(x)*cosh(x)**2*tanh(x)/(15*tanh(x) + 15) + 9*sinh(x)*c
osh(x)**2/(15*tanh(x) + 15) + 3*cosh(x)**3*tanh(x)/(15*tanh(x) + 15) - 3*co
sh(x)**3/(15*tanh(x) + 15)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = \frac{1}{48}\left(12\,e^{(-2\,x)}+1\right)e^{(3\,x)} - \frac{3}{8}\,e^{(-x)} - \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{80}\,e^{(-5\,x)}$$

```
[In] integrate(cosh(x)^3/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/48*(12*e^(-2*x) + 1)*e^(3*x) - 3/8*e^(-x) - 1/12*e^(-3*x) - 1/80*e^(-5*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = -\frac{1}{240}\left(90\,e^{(4\,x)}+20\,e^{(2\,x)}+3\right)e^{(-5\,x)} + \frac{1}{48}\,e^{(3\,x)} + \frac{1}{4}\,e^{x}$$

```
[In] integrate(cosh(x)^3/(1+tanh(x)),x, algorithm="giac")
[Out] -1/240*(90*e^(4*x) + 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{1+\tanh(x)}\,dx = \frac{e^{3\,x}}{48} - \frac{e^{-3\,x}}{12} - \frac{3\,e^{-x}}{8} - \frac{e^{-5\,x}}{80} + \frac{e^{x}}{4}$$

```
[In] int(cosh(x)^3/(tanh(x) + 1),x)
[Out] exp(3*x)/48 - exp(-3*x)/12 - (3*exp(-x))/8 - exp(-5*x)/80 + exp(x)/4
```

3.93 $\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = \frac{3x}{8} + \frac{1}{8(1-\tanh(x))} - \frac{1}{8(1+\tanh(x))^2} - \frac{1}{4(1+\tanh(x))}$$

```
[Out] 3/8*x+1/8/(1-tanh(x))-1/8/(1+tanh(x))^2-1/4/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3568, 46, 213}

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = \frac{3x}{8} + \frac{1}{8(1-\tanh(x))} - \frac{1}{4(\tanh(x)+1)} - \frac{1}{8(\tanh(x)+1)^2}$$

```
[In] Int[Cosh[x]^2/(1 + Tanh[x]),x]
[Out] (3*x)/8 + 1/(8*(1 - Tanh[x])) - 1/(8*(1 + Tanh[x])^2) - 1/(4*(1 + Tanh[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x)^2(1+x)^3}\,dx, x, \tanh(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\,(-1+x^2)}\right)dx, x, \tanh(x)\right) \\
&= \frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}-\frac{1}{4(1+\tanh(x))}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(x)\right) \\
&= \frac{3x}{8}+\frac{1}{8(1-\tanh(x))}-\frac{1}{8(1+\tanh(x))^2}-\frac{1}{4(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = \frac{1}{16}(6\text{arctanh}(\tanh(x)) + \cosh(x)(\cosh(x)-\sinh(x))(-5+\cosh(2x)+3\sinh(2x)))$$

```
[In] Integrate[Cosh[x]^2/(1 + Tanh[x]),x]
[Out] (6*ArcTanh[Tanh[x]] + Cosh[x]*(Cosh[x] - Sinh[x])*(-5 + Cosh[2*x] + 3*Sinh[
2*x]))/16
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result
risch	$\frac{3x}{8}+\frac{{e}^{2x}}{16}-\frac{3\,{e}^{-2x}}{16}-\frac{{e}^{-4x}}{32}$
parallelrisch	$\frac{3x}{8}-\frac{\cosh(4x)}{32}-\frac{\cosh(2x)}{8}+\frac{\sinh(4x)}{32}+\frac{\sinh(2x)}{4}+\frac{5}{32}$
default	$-\frac{1}{2(\tanh(\frac{x}{2})+1)^{4}}+\frac{1}{(\tanh(\frac{x}{2})+1)^{3}}-\frac{3}{2(\tanh(\frac{x}{2})+1)^{2}}+\frac{1}{\tanh(\frac{x}{2})+1}+\frac{3\ln(\tanh(\frac{x}{2})+1)}{8}+\frac{1}{4(\tanh(\frac{x}{2})-1)^{2}}+\frac{1}{4\mathrm{t}}$

[In] int(cosh(x)^2/(1+tanh(x)),x,method=_RETURNVERBOSE)

[Out] 3/8*x+1/16*exp(2*x)-3/16*exp(-2*x)-1/32*exp(-4*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.37

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = \frac{\cosh{(x)}^{3}+3\,\cosh{(x)}\sinh{(x)}^{2}+3\,\sinh{(x)}^{3}+6\,(2\,x-1)\cosh{(x)}+3\,\left(3\,\cosh{(x)}^{2}+4\,x+2\right)\sinh{(x)}}{32\,(\cosh{(x)}+\sinh{(x)})}$$

[In] integrate(cosh(x)^2/(1+tanh(x)),x, algorithm="fricas")

[Out] 1/32*(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + 3*sinh(x)^3 + 6*(2*x - 1)*cosh(x) + 3*(3*cosh(x)^2 + 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))

Sympy [F]

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = \int \frac{\cosh^{2}{(x)}}{\tanh{(x)}+1}\,dx$$

[In] integrate(cosh(x)**2/(1+tanh(x)),x)

[Out] Integral(cosh(x)**2/(tanh(x) + 1), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = \frac{3}{8}\,x + \frac{1}{16}\,e^{(2\,x)} - \frac{3}{16}\,e^{(-2\,x)} - \frac{1}{32}\,e^{(-4\,x)}$$

```
[In] integrate(cosh(x)^2/(1+tanh(x)),x, algorithm="maxima")
[Out] 3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) - 1/32*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = -\frac{1}{32}\left(9\,e^{(4\,x)} + 6\,e^{(2\,x)} + 1\right)e^{(-4\,x)} + \frac{3}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

```
[In] integrate(cosh(x)^2/(1+tanh(x)),x, algorithm="giac")
[Out] -1/32*(9*e^(4*x) + 6*e^(2*x) + 1)*e^(-4*x) + 3/8*x + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\cosh^2(x)}{1+\tanh(x)}\,dx = \frac{3\,x}{8} - \frac{3\,\mathrm{e}^{-2\,x}}{16} + \frac{\mathrm{e}^{2\,x}}{16} - \frac{\mathrm{e}^{-4\,x}}{32}$$

```
[In] int(cosh(x)^2/(tanh(x) + 1),x)
[Out] (3*x)/8 - (3*exp(-2*x))/16 + exp(2*x)/16 - exp(-4*x)/32
```

3.94 $\int \frac{\cosh(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx=\frac{2\sinh(x)}{3}-\frac{\cosh(x)}{3(1+\tanh(x))}$$

```
[Out] 2/3*sinh(x)-1/3*cosh(x)/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3583, 2717}

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx=\frac{2\sinh(x)}{3}-\frac{\cosh(x)}{3(\tanh(x)+1)}$$

```
[In] Int[Cosh[x]/(1 + Tanh[x]),x]
```

```
[Out] (2*Sinh[x])/3 - Cosh[x]/(3*(1 + Tanh[x]))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
```

```
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\text{integral} = -\frac{\cosh(x)}{3(1+\tanh(x))} + \frac{2}{3}\int \cosh(x)\,dx$$
$$= \frac{2\sinh(x)}{3} - \frac{\cosh(x)}{3(1+\tanh(x))}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx = \frac{1}{12}(-3\cosh(x) - \cosh(3x) + 9\sinh(x) + \sinh(3x))$$

```
[In] Integrate[Cosh[x]/(1 + Tanh[x]),x]
[Out] (-3*Cosh[x] - Cosh[3*x] + 9*Sinh[x] + Sinh[3*x])/12
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{e^{x}}{4}-\frac{e^{-x}}{2}-\frac{e^{-3x}}{12}$	18
parallelrisch	$-\frac{\cosh(3x)}{12}-\frac{\cosh(x)}{4}+\frac{\sinh(3x)}{12}+\frac{3\sinh(x)}{4}+\frac{1}{3}$	23
default	$-\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	40

```
[In] int(cosh(x)/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*exp(x)-1/2*exp(-x)-1/12*exp(-3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx = \frac{\cosh{(x)}^2 + 4\,\cosh{(x)}\sinh{(x)} + \sinh{(x)}^2 - 3}{6\left(\cosh{(x)} + \sinh{(x)}\right)}$$

```
[In] integrate(cosh(x)/(1+tanh(x)),x, algorithm="fricas")
[Out] 1/6*(cosh(x)^2 + 4*cosh(x)*sinh(x) + sinh(x)^2 - 3)/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(15) = 30$.

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx = \frac{2\sinh{(x)}\tanh{(x)}}{3\tanh{(x)} + 3} + \frac{\sinh{(x)}}{3\tanh{(x)} + 3} + \frac{\cosh{(x)}\tanh{(x)}}{3\tanh{(x)} + 3} - \frac{\cosh{(x)}}{3\tanh{(x)} + 3}$$

```
[In] integrate(cosh(x)/(1+tanh(x)),x)
[Out] 2*sinh(x)*tanh(x)/(3*tanh(x) + 3) + sinh(x)/(3*tanh(x) + 3) + cosh(x)*tanh(
x)/(3*tanh(x) + 3) - cosh(x)/(3*tanh(x) + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx = -\frac{1}{2}\,e^{(-x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

```
[In] integrate(cosh(x)/(1+tanh(x)),x, algorithm="maxima")
[Out] -1/2*e^(-x) - 1/12*e^(-3*x) + 1/4*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx = -\frac{1}{12}\left(6\,e^{(2\,x)}+1\right)e^{(-3\,x)}+\frac{1}{4}\,e^{x}$$

```
[In] integrate(cosh(x)/(1+tanh(x)),x, algorithm="giac")
[Out] -1/12*(6*e^(2*x) + 1)*e^(-3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(x)}{1+\tanh(x)}\,dx = \frac{e^{x}}{4}-\frac{e^{-3\,x}}{12}-\frac{e^{-x}}{2}$$

```
[In] int(cosh(x)/(tanh(x) + 1),x)
[Out] exp(x)/4 - exp(-3*x)/12 - exp(-x)/2
```

3.95 $\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx=-\frac{\operatorname{sech}(x)}{1+\tanh(x)}$$

```
[Out] -sech(x)/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3569}

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx=-\frac{\operatorname{sech}(x)}{\tanh(x)+1}$$

```
[In] Int[Sech[x]/(1 + Tanh[x]),x]
```

```
[Out] -(Sech[x]/(1 + Tanh[x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral}=-\frac{\operatorname{sech}(x)}{1+\tanh(x)}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx = -\cosh(x)+\sinh(x)$$

```
[In] Integrate[Sech[x]/(1 + Tanh[x]),x]
[Out] -Cosh[x] + Sinh[x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
risch	$-{\mathrm e}^{-x}$	7
gosper	$-\frac{\operatorname{sech}(x)}{1+\tanh(x)}$	11
default	$-\frac{2}{\tanh\left(\frac{x}{2}\right)+1}$	11

```
[In] int(sech(x)/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -exp(-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx = -\frac{1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

```
[In] integrate(sech(x)/(1+tanh(x)),x, algorithm="fricas")
[Out] -1/(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx = -\frac{\operatorname{sech}(x)}{\tanh(x)+1}$$

```
[In] integrate(sech(x)/(1+tanh(x)),x)
[Out] -sech(x)/(tanh(x) + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx = -e^{(-x)}$$

```
[In] integrate(sech(x)/(1+tanh(x)),x, algorithm="maxima")
[Out] -e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx = -e^{(-x)}$$

```
[In] integrate(sech(x)/(1+tanh(x)),x, algorithm="giac")
[Out] -e^(-x)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{sech}(x)}{1+\tanh(x)}\,dx = -\mathrm{e}^{-x}$$

```
[In] int(1/(cosh(x)*(tanh(x) + 1)),x)
[Out] -exp(-x)
```

3.96 $\int \frac{\text{sech}^2(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 5

$$\int \frac{\text{sech}^2(x)}{1+\tanh(x)}\,dx = \log(1+\tanh(x))$$

```
[Out] ln(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3568, 31}

$$\int \frac{\text{sech}^2(x)}{1+\tanh(x)}\,dx = \log(\tanh(x)+1)$$

```
[In] Int[Sech[x]^2/(1 + Tanh[x]),x]
```

```
[Out] Log[1 + Tanh[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tanh(x)\right)\\ &= \log(1+\tanh(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx = x - \log(\cosh(x))$$

```
[In] Integrate[Sech[x]^2/(1 + Tanh[x]),x]
[Out] x - Log[Cosh[x]]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(1+\tanh(x))$	6
default	$\ln(1+\tanh(x))$	6
risch	$2x - \ln(1+e^{2x})$	14

```
[In] int(sech(x)^2/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] ln(1+tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(5) = 10.

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx = 2\,x - \log\left(\frac{2\,\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

```
[In] integrate(sech(x)^2/(1+tanh(x)),x, algorithm="fricas")
[Out] 2*x - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(sech(x)**2/(1+tanh(x)),x)
[Out] Integral(sech(x)**2/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx = \log(\tanh(x)+1)$$

```
[In] integrate(sech(x)^2/(1+tanh(x)),x, algorithm="maxima")
[Out] log(tanh(x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.60

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx = 2x - \log\left(e^{(2x)}+1\right)$$

```
[In] integrate(sech(x)^2/(1+tanh(x)),x, algorithm="giac")
[Out] 2*x - log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 13, normalized size of antiderivative = 2.60

$$\int \frac{\operatorname{sech}^2(x)}{1+\tanh(x)}\,dx = 2x - \ln\left(e^{2x}+1\right)$$

```
[In] int(1/(cosh(x)^2*(tanh(x) + 1)),x)
[Out] 2*x - log(exp(2*x) + 1)
```

3.97 $\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx = \arctan(\sinh(x)) + \operatorname{sech}(x)$$

```
[Out] arctan(sinh(x))+sech(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3582, 3855}

$$\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx = \arctan(\sinh(x)) + \operatorname{sech}(x)$$

```
[In] Int[Sech[x]^3/(1 + Tanh[x]),x]
```

```
[Out] ArcTan[Sinh[x]] + Sech[x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{sech}(x) + \int \text{sech}(x)\,dx \\ &= \arctan(\sinh(x)) + \text{sech}(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\text{sech}^3(x)}{1+\tanh(x)}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \text{sech}(x)$$

```
[In] Integrate[Sech[x]^3/(1 + Tanh[x]),x]
[Out] 2*ArcTan[Tanh[x/2]] + Sech[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(6) = 12.

Time = 1.71 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.50

method	result	size
default	$\frac{2}{1+\tanh\left(\frac{x}{2}\right)^2} + 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	21
risch	$\frac{2\,\mathrm{e}^{x}}{1+\mathrm{e}^{2x}} + i\ln\left(\mathrm{e}^{x}+i\right) - i\ln\left(\mathrm{e}^{x}-i\right)$	32

```
[In] int(sech(x)^3/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 2/(1+tanh(1/2*x)^2)+2*arctan(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(6) = 12$.

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 8.00

$$\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx = \frac{2\left(\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\arctan\left(\cosh(x)+\sinh(x)\right)+\cosh(x)+\sinh(x)\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}$$

```
[In] integrate(sech(x)^3/(1+tanh(x)),x, algorithm="fricas")
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(cosh(x) + sinh(x)
) + cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(sech(x)**3/(1+tanh(x)),x)
[Out] Integral(sech(x)**3/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 3.67

$$\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx = \frac{2\,e^{(-x)}}{e^{(-2\,x)}+1} - 2\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(sech(x)^3/(1+tanh(x)),x, algorithm="maxima")
[Out] 2*e^(-x)/(e^(-2*x) + 1) - 2*arctan(e^(-x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx = \frac{2\,e^x}{e^{(2\,x)}+1} + 2\,\arctan\left(e^x\right)$$

```
[In] integrate(sech(x)^3/(1+tanh(x)),x, algorithm="giac")
[Out] 2*e^x/(e^(2*x) + 1) + 2*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{sech}^3(x)}{1+\tanh(x)}\,dx = 2\,\mathrm{atan}(\mathrm{e}^x) + \frac{2\,\mathrm{e}^x}{\mathrm{e}^{2\,x}+1}$$

```
[In] int(1/(cosh(x)^3*(tanh(x) + 1)),x)
[Out] 2*atan(exp(x)) + (2*exp(x))/(exp(2*x) + 1)
```

3.98 $\int \frac{\mathrm{sech}^4(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\mathrm{sech}^4(x)}{1+\tanh(x)}\,dx = \tanh(x)-\frac{\tanh^2(x)}{2}$$

```
[Out] tanh(x)-1/2*tanh(x)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3568}

$$\int \frac{\mathrm{sech}^4(x)}{1+\tanh(x)}\,dx = \tanh(x)-\frac{\tanh^2(x)}{2}$$

```
[In] Int[Sech[x]^4/(1 + Tanh[x]),x]
```

```
[Out] Tanh[x] - Tanh[x]^2/2
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int (1-x)\,dx, x, \tanh(x)\right)\\ &= \tanh(x)-\frac{\tanh^2(x)}{2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^4(x)}{1+\tanh(x)}\,dx = -\frac{1}{2}(-2+\tanh(x))\tanh(x)$$

```
[In] Integrate[Sech[x]^4/(1 + Tanh[x]),x]
[Out] -1/2*((-2 + Tanh[x])*Tanh[x])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\tanh(x)-\frac{\tanh(x)^2}{2}$	10
default	$\tanh(x)-\frac{\tanh(x)^2}{2}$	10
risch	$-\frac{2}{(1+e^{2x})^2}$	11
parallelrisch	$-\frac{11}{18}+\tanh(x)+\frac{\operatorname{sech}(x)^2}{2}$	11

```
[In] int(sech(x)^4/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] tanh(x)-1/2*tanh(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.82

$$\int \frac{\operatorname{sech}^4(x)}{1+\tanh(x)}\,dx = -\frac{2}{\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2+1\right)\sinh(x)^2+2\cosh(x)^2+4\left(\cosh(x)^3+\right.}$$

```
[In] integrate(sech(x)^4/(1+tanh(x)),x, algorithm="fricas")
[Out] -2/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(
x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{sech}^4{(x)}}{\tanh{(x)}+1}\,dx$$

```
[In] integrate(sech(x)**4/(1+tanh(x)),x)
[Out] Integral(sech(x)**4/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(9) = 18.

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.36

$$\int \frac{\operatorname{sech}^4(x)}{1+\tanh(x)}\,dx = \frac{4\,e^{(-2\,x)}}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1} + \frac{2}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1}$$

```
[In] integrate(sech(x)^4/(1+tanh(x)),x, algorithm="maxima")
[Out] 4*e^(-2*x)/(2*e^(-2*x) + e^(-4*x) + 1) + 2/(2*e^(-2*x) + e^(-4*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^4(x)}{1+\tanh(x)}\,dx = -\frac{2}{\left(e^{(2\,x)}+1\right)^2}$$

```
[In] integrate(sech(x)^4/(1+tanh(x)),x, algorithm="giac")
[Out] -2/(e^(2*x) + 1)^2
```

Mupad [B] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}^4(x)}{1+\tanh(x)}\,dx = -\frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

```
[In] int(1/(cosh(x)^4*(tanh(x) + 1)),x)
[Out] -2/(2*exp(2*x) + exp(4*x) + 1)
```

3.99 $\int \frac{\text{sech}^5(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{\text{sech}^5(x)}{1+\tanh(x)}\,dx=\frac{1}{2}\arctan(\sinh(x))+\frac{\text{sech}^3(x)}{3}+\frac{1}{2}\text{sech}(x)\tanh(x)$$

```
[Out] 1/2*arctan(sinh(x))+1/3*sech(x)^3+1/2*sech(x)*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3582, 3853, 3855}

$$\int \frac{\text{sech}^5(x)}{1+\tanh(x)}\,dx=\frac{1}{2}\arctan(\sinh(x))+\frac{\text{sech}^3(x)}{3}+\frac{1}{2}\tanh(x)\text{sech}(x)$$

```
[In] Int[Sech[x]^5/(1 + Tanh[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/2 + Sech[x]^3/3 + (Sech[x]*Tanh[x])/2
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\operatorname{sech}^3(x)}{3} + \int \operatorname{sech}^3(x)\,dx \\ &= \frac{\operatorname{sech}^3(x)}{3} + \frac{1}{2}\operatorname{sech}(x)\tanh(x) + \frac{1}{2}\int \operatorname{sech}(x)\,dx \\ &= \frac{1}{2}\arctan(\sinh(x)) + \frac{\operatorname{sech}^3(x)}{3} + \frac{1}{2}\operatorname{sech}(x)\tanh(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx = \arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{\operatorname{sech}^3(x)}{3} + \frac{1}{2}\operatorname{sech}(x)\tanh(x)$$

```
[In] Integrate[Sech[x]^5/(1 + Tanh[x]),x]
[Out] ArcTan[Tanh[x/2]] + Sech[x]^3/3 + (Sech[x]*Tanh[x])/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(18) = 36.

Time = 13.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)^5+2\tanh\left(\frac{x}{2}\right)^4+\tanh\left(\frac{x}{2}\right)+\frac{2}{3}}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}+\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	41
risch	$\frac{e^x\left(3\,e^{4x}+8\,e^{2x}-3\right)}{3\left(1+e^{2x}\right)^3}+\frac{i\ln\left(e^x+i\right)}{2}-\frac{i\ln\left(e^x-i\right)}{2}$	46

```
[In] int(sech(x)^5/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 2*(-1/2*tanh(1/2*x)^5+tanh(1/2*x)^4+1/2*tanh(1/2*x)+1/3)/(1+tanh(1/2*x)^2)^
3+arctan(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(18) = 36$.

Time = 0.24 (sec) , antiderivative size = 288, normalized size of antiderivative = 12.00

$$\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx = \frac{3 \cosh(x)^5 + 15 \cosh(x)\sinh(x)^4 + 3 \sinh(x)^5 + 2\left(15 \cosh(x)^2 + 4\right)\sinh(x)^3 + 8 \cosh(x)^3 + 6\left(5 \cosh\right.}{}$$

```
[In] integrate(sech(x)^5/(1+tanh(x)),x, algorithm="fricas")
[Out] 1/3*(3*cosh(x)^5 + 15*cosh(x)*sinh(x)^4 + 3*sinh(x)^5 + 2*(15*cosh(x)^2 + 4
)*sinh(x)^3 + 8*cosh(x)^3 + 6*(5*cosh(x)^3 + 4*cosh(x))*sinh(x)^2 + 3*(cosh
(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3
*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cos
h(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))
*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(5*cosh(x)^4 + 8*cosh(x)^2 - 1)
*sinh(x) - 3*cosh(x))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*c
osh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)
^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)
^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{sech}^5(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(sech(x)**5/(1+tanh(x)),x)
[Out] Integral(sech(x)**5/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx = \frac{3\,e^{(-x)}+8\,e^{(-3\,x)}-3\,e^{(-5\,x)}}{3\,(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1)} - \arctan\left(e^{(-x)}\right)$$

```
[In] integrate(sech(x)^5/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/3*(3*e^(-x) + 8*e^(-3*x) - 3*e^(-5*x))/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x
) + 1) - arctan(e^(-x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx = \frac{3\,e^{(5\,x)}+8\,e^{(3\,x)}-3\,e^{x}}{3\,(e^{(2\,x)}+1)^3} + \arctan\left(e^{x}\right)$$

```
[In] integrate(sech(x)^5/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 1/3*(3*e^(5*x) + 8*e^(3*x) - 3*e^x)/(e^(2*x) + 1)^3 + arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{sech}^5(x)}{1+\tanh(x)}\,dx = \operatorname{atan}(\mathrm{e}^{x}) + \frac{\mathrm{e}^{x}}{\mathrm{e}^{2\,x}+1} - \frac{8\,\mathrm{e}^{x}}{3\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)} + \frac{2\,\mathrm{e}^{x}}{3\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

```
[In] int(1/(cosh(x)^5*(tanh(x) + 1)),x)
```

```
[Out] atan(exp(x)) + exp(x)/(exp(2*x) + 1) - (8*exp(x))/(3*(3*exp(2*x) + 3*exp(4*
x) + exp(6*x) + 1)) + (2*exp(x))/(3*(2*exp(2*x) + exp(4*x) + 1))
```

3.100 $\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx=-\frac{2}{3}(1-\tanh(x))^3+\frac{1}{4}(1-\tanh(x))^4$$

```
[Out] -2/3*(1-tanh(x))^3+1/4*(1-tanh(x))^4
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3568, 45}

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx=\frac{1}{4}(1-\tanh(x))^4-\frac{2}{3}(1-\tanh(x))^3$$

```
[In] Int[Sech[x]^6/(1 + Tanh[x]),x]
```

```
[Out] (-2*(1 - Tanh[x])^3)/3 + (1 - Tanh[x])^4/4
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
```

```
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int (1-x)^2(1+x)\,dx, x, \tanh(x)\right)\\ &= \text{Subst}\left(\int \left(2(1-x)^2-(1-x)^3\right)\,dx, x, \tanh(x)\right)\\ &= -\frac{2}{3}(1-\tanh(x))^3+\frac{1}{4}(1-\tanh(x))^4\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx = \frac{1}{12}\tanh(x)\left(12-6\tanh(x)-4\tanh^2(x)+3\tanh^3(x)\right)$$

```
[In] Integrate[Sech[x]^6/(1 + Tanh[x]),x]
[Out] (Tanh[x]*(12 - 6*Tanh[x] - 4*Tanh[x]^2 + 3*Tanh[x]^3))/12
```

Maple [A] (verified)

Time = 4.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{4(4\,\mathrm{e}^{2x}+1)}{3(1+\mathrm{e}^{2x})^4}$	19
parallelrisch	$-\frac{39}{100}+\frac{2\tanh(x)}{3}+\frac{\tanh(x)\operatorname{sech}(x)^2}{3}+\frac{\operatorname{sech}(x)^4}{4}$	21
derivativedivides	$\frac{\tanh(x)^4}{4}-\frac{\tanh(x)^3}{3}-\frac{\tanh(x)^2}{2}+\tanh(x)$	22
default	$\frac{\tanh(x)^4}{4}-\frac{\tanh(x)^3}{3}-\frac{\tanh(x)^2}{2}+\tanh(x)$	22

```
[In] int(sech(x)^6/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -4/3*(4*exp(2*x)+1)/(1+exp(2*x))^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 5.60

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx =$$

$$-\frac{}{3\left(\cosh(x)^7 + 7\cosh(x)\sinh(x)^6 + \sinh(x)^7 + \left(21\cosh(x)^2 + 4\right)\sinh(x)^5 + 4\cosh(x)^5 + 5\left(7\cosh(x\right.\right.}$$

```
[In] integrate(sech(x)^6/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -4/3*(5*cosh(x) + 3*sinh(x))/(cosh(x)^7 + 7*cosh(x)*sinh(x)^6 + sinh(x)^7 +
 (21*cosh(x)^2 + 4)*sinh(x)^5 + 4*cosh(x)^5 + 5*(7*cosh(x)^3 + 4*cosh(x))*s
inh(x)^4 + (35*cosh(x)^4 + 40*cosh(x)^2 + 6)*sinh(x)^3 + 6*cosh(x)^3 + (21*
cosh(x)^5 + 40*cosh(x)^3 + 18*cosh(x))*sinh(x)^2 + (7*cosh(x)^6 + 20*cosh(x
)^4 + 18*cosh(x)^2 + 3)*sinh(x) + 5*cosh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx = \int \frac{\operatorname{sech}^6(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(sech(x)**6/(1+tanh(x)),x)
```

```
[Out] Integral(sech(x)**6/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.72

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx = \frac{16\,e^{(-2\,x)}}{3\left(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1\right)} + \frac{8\,e^{(-4\,x)}}{4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1} + \frac{4}{3\left(4\,e^{(-2\,x)} + 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} + e^{(-8\,x)} + 1\right)}$$

```
[In] integrate(sech(x)^6/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 8*e^(
-4*x)/(4*e^(-2*x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) + 4/3/(4*e^(-2*
x) + 6*e^(-4*x) + 4*e^(-6*x) + e^(-8*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx = -\frac{4\left(4\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}+1\right)^4}$$

```
[In] integrate(sech(x)^6/(1+tanh(x)),x, algorithm="giac")
[Out] -4/3*(4*e^(2*x) + 1)/(e^(2*x) + 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{\operatorname{sech}^6(x)}{1+\tanh(x)}\,dx = -\frac{4\left(4\,\mathrm{e}^{2\,x}+1\right)}{3\left(\mathrm{e}^{2\,x}+1\right)^4}$$

```
[In] int(1/(cosh(x)^6*(tanh(x) + 1)),x)
[Out] -(4*(4*exp(2*x) + 1))/(3*(exp(2*x) + 1)^4)
```

3.101 $\int \frac{\operatorname{sech}^7(x)}{1+\tanh(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int \frac{\operatorname{sech}^7(x)}{1+\tanh(x)} \, dx = \frac{3}{8}\arctan(\sinh(x)) + \frac{\operatorname{sech}^5(x)}{5} + \frac{3}{8}\operatorname{sech}(x)\tanh(x) + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x)$$

```
[Out] 3/8*arctan(sinh(x))+1/5*sech(x)^5+3/8*sech(x)*tanh(x)+1/4*sech(x)^3*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3582, 3853, 3855}

$$\int \frac{\operatorname{sech}^7(x)}{1+\tanh(x)} \, dx = \frac{3}{8}\arctan(\sinh(x)) + \frac{\operatorname{sech}^5(x)}{5} + \frac{1}{4}\tanh(x)\operatorname{sech}^3(x) + \frac{3}{8}\tanh(x)\operatorname{sech}(x)$$

```
[In] Int[Sech[x]^7/(1 + Tanh[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 + Sech[x]^5/5 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Ta
nh[x])/4
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^5(x)}{5} + \int \operatorname{sech}^5(x)\,dx \\
&= \frac{\operatorname{sech}^5(x)}{5} + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x) + \frac{3}{4}\int \operatorname{sech}^3(x)\,dx \\
&= \frac{\operatorname{sech}^5(x)}{5} + \frac{3}{8}\operatorname{sech}(x)\tanh(x) + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x) + \frac{3}{8}\int \operatorname{sech}(x)\,dx \\
&= \frac{3}{8}\arctan(\sinh(x)) + \frac{\operatorname{sech}^5(x)}{5} + \frac{3}{8}\operatorname{sech}(x)\tanh(x) + \frac{1}{4}\operatorname{sech}^3(x)\tanh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^7(x)}{1+\tanh(x)}\,dx = \frac{1}{40}\left(30\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + 8\operatorname{sech}^5(x) + 15\operatorname{sech}(x)\tanh(x) + 10\operatorname{sech}^3(x)\tanh(x)\right)$$

```
[In] Integrate[Sech[x]^7/(1 + Tanh[x]),x]
[Out] (30*ArcTan[Tanh[x/2]] + 8*Sech[x]^5 + 15*Sech[x]*Tanh[x] + 10*Sech[x]^3*Tan
h[x])/40
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.97

$$\frac{-\frac{5\tanh\left(\frac{x}{2}\right)^{9}}{4}+2\tanh\left(\frac{x}{2}\right)^{8}-\frac{\tanh\left(\frac{x}{2}\right)^{7}}{2}+4\tanh\left(\frac{x}{2}\right)^{4}+\frac{\tanh\left(\frac{x}{2}\right)^{3}}{2}+\frac{5\tanh\left(\frac{x}{2}\right)}{4}+\frac{2}{5}}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{5}}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}$$

```
[In] int(sech(x)^7/(1+tanh(x)),x)
```

```
[Out] 2*(-5/8*tanh(1/2*x)^9+tanh(1/2*x)^8-1/4*tanh(1/2*x)^7+2*tanh(1/2*x)^4+1/4*t
anh(1/2*x)^3+5/8*tanh(1/2*x)+1/5)/(1+tanh(1/2*x)^2)^5+3/4*arctan(tanh(1/2*x
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 670, normalized size of antiderivative = 19.71

$$\int \frac{\operatorname{sech}^7(x)}{1+\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^7/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/20*(15*cosh(x)^9 + 135*cosh(x)*sinh(x)^8 + 15*sinh(x)^9 + 10*(54*cosh(x)^
2 + 7)*sinh(x)^7 + 70*cosh(x)^7 + 70*(18*cosh(x)^3 + 7*cosh(x))*sinh(x)^6 +
 2*(945*cosh(x)^4 + 735*cosh(x)^2 + 64)*sinh(x)^5 + 128*cosh(x)^5 + 10*(189
*cosh(x)^5 + 245*cosh(x)^3 + 64*cosh(x))*sinh(x)^4 + 10*(126*cosh(x)^6 + 24
5*cosh(x)^4 + 128*cosh(x)^2 - 7)*sinh(x)^3 - 70*cosh(x)^3 + 10*(54*cosh(x)^
7 + 147*cosh(x)^5 + 128*cosh(x)^3 - 21*cosh(x))*sinh(x)^2 + 15*(cosh(x)^10
+ 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 1)*sinh(x)^8 + 5*cos
h(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21*cosh(x)^4 + 14*cosh(
x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5 + 70*cosh(x)^3 + 15*co
sh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh(x)^2 + 1)*sinh
(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cosh(x)^3 + cosh(x
))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4 + 12*cosh(x)^2
+ 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*cosh(x)^7 + 6*cosh(x)^5 +
4*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*(27*cosh(
x)^8 + 98*cosh(x)^6 + 128*cosh(x)^4 - 42*cosh(x)^2 - 3)*sinh(x) - 15*cosh(x
))/(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 + 1)*si
nh(x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21*cosh(
x)^4 + 14*cosh(x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5 + 70*co
sh(x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 + 15*cosh
(x)^2 + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^5 + 5*cos
```

```
h(x)^3 + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*cosh(x)^4
+ 12*cosh(x)^2 + 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*cosh(x)^7 +
 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{\mathrm{sech}^7(x)}{1+\tanh(x)}\,dx = \int \frac{\mathrm{sech}^7\left(x\right)}{\tanh\left(x\right)+1}\,dx$$

```
[In] integrate(sech(x)**7/(1+tanh(x)),x)
[Out] Integral(sech(x)**7/(tanh(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int \frac{\mathrm{sech}^7(x)}{1+\tanh(x)}\,dx = \frac{15\,e^{(-x)} + 70\,e^{(-3\,x)} + 128\,e^{(-5\,x)} - 70\,e^{(-7\,x)} - 15\,e^{(-9\,x)}}{20\,(5\,e^{(-2\,x)} + 10\,e^{(-4\,x)} + 10\,e^{(-6\,x)} + 5\,e^{(-8\,x)} + e^{(-10\,x)} + 1)} - \frac{3}{4}\,\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(sech(x)^7/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/20*(15*e^(-x) + 70*e^(-3*x) + 128*e^(-5*x) - 70*e^(-7*x) - 15*e^(-9*x))/(
5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) - 3/4*
arctan(e^(-x))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.32

$$\int \frac{\mathrm{sech}^7(x)}{1+\tanh(x)}\,dx = \frac{15\,e^{(9\,x)} + 70\,e^{(7\,x)} + 128\,e^{(5\,x)} - 70\,e^{(3\,x)} - 15\,e^{x}}{20\,{\left(e^{(2\,x)} + 1\right)}^{5}} + \frac{3}{4}\,\arctan\left(e^{x}\right)$$

```
[In] integrate(sech(x)^7/(1+tanh(x)),x, algorithm="giac")
[Out] 1/20*(15*e^(9*x) + 70*e^(7*x) + 128*e^(5*x) - 70*e^(3*x) - 15*e^x)/(e^(2*x)
 + 1)^5 + 3/4*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.03

$$\int \frac{\operatorname{sech}^7(x)}{1+\tanh(x)}\,dx = \frac{3\,\mathrm{atan}(\mathrm{e}^x)}{4} - \frac{32\,\mathrm{e}^{3\,x}}{5\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+10\,\mathrm{e}^{6\,x}+5\,\mathrm{e}^{8\,x}+\mathrm{e}^{10\,x}+1\right)} - \frac{12\,\mathrm{e}^x}{5\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)} + \frac{3\,\mathrm{e}^x}{4\,\left(\mathrm{e}^{2\,x}+1\right)} + \frac{2\,\mathrm{e}^x}{5\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)} + \frac{\mathrm{e}^x}{2\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

```
[In] int(1/(cosh(x)^7*(tanh(x) + 1)),x)
```

```
[Out] (3*atan(exp(x)))/4 - (32*exp(3*x))/(5*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*
x) + 5*exp(8*x) + exp(10*x) + 1)) - (12*exp(x))/(5*(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1)) + (3*exp(x))/(4*(exp(2*x) + 1)) + (2*exp(x))
/(5*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) + exp(x)/(2*(2*exp(2*x) + exp
(4*x) + 1))
```

3.102 $\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)}\,dx=-\frac{(a^2-b^2)^3\log(a+b\tanh(x))}{b^7}+\frac{a(a^4-3a^2b^2+3b^4)\tanh(x)}{b^6}-\frac{(a^4-3a^2b^2+3b^4)\tanh^2(x)}{2b^5}+\frac{a(a^2-3b^2)\tanh^3(x)}{3b^4}-\frac{(a^2-3b^2)\tanh^4(x)}{4b^3}+\frac{a\tanh^5(x)}{5b^2}-\frac{\tanh^6(x)}{6b}$$

```
[Out] -(a^2-b^2)^3*ln(a+b*tanh(x))/b^7+a*(a^4-3*a^2*b^2+3*b^4)*tanh(x)/b^6-1/2*(a
^4-3*a^2*b^2+3*b^4)*tanh(x)^2/b^5+1/3*a*(a^2-3*b^2)*tanh(x)^3/b^4-1/4*(a^2-
3*b^2)*tanh(x)^4/b^3+1/5*a*tanh(x)^5/b^2-1/6*tanh(x)^6/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 711}

$$\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)}\,dx=-\frac{(a^2-b^2)^3\log(a+b\tanh(x))}{b^7}+\frac{a(a^2-3b^2)\tanh^3(x)}{3b^4}-\frac{(a^2-3b^2)\tanh^4(x)}{4b^3}+\frac{a(a^4-3a^2b^2+3b^4)\tanh(x)}{b^6}-\frac{(a^4-3a^2b^2+3b^4)\tanh^2(x)}{2b^5}+\frac{a\tanh^5(x)}{5b^2}-\frac{\tanh^6(x)}{6b}$$

```
[In] Int[Sech[x]^8/(a + b*Tanh[x]),x]
```

```
[Out] -(((a^2 - b^2)^3*Log[a + b*Tanh[x]])/b^7) + (a*(a^4 - 3*a^2*b^2 + 3*b^4)*Ta
nh[x])/b^6 - ((a^4 - 3*a^2*b^2 + 3*b^4)*Tanh[x]^2)/(2*b^5) + (a*(a^2 - 3*b^
2)*Tanh[x]^3)/(3*b^4) - ((a^2 - 3*b^2)*Tanh[x]^4)/(4*b^3) + (a*Tanh[x]^5)/(
5*b^2) - Tanh[x]^6/(6*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{b^2}\right)^3}{a+x}\,dx, x, b\tanh(x)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^5-3a^3b^2+3ab^4}{b^6} - \frac{(a^4-3a^2b^2+3b^4)x}{b^6} + \frac{a(a^2-3b^2)x^2}{b^6} + \frac{(-a^2+3b^2)x^3}{b^6} + \frac{ax^4}{b^6} - \frac{x^5}{b^6} + \frac{(-a^2+b^2)^3}{b^6(a+x)}\right)\,dx, x, b\right.}{b}$$

$$= -\frac{(a^2-b^2)^3\log(a+b\tanh(x))}{b^7} + \frac{a(a^4-3a^2b^2+3b^4)\tanh(x)}{b^6} - \frac{(a^4-3a^2b^2+3b^4)\tanh^2(x)}{2b^5} + \frac{a(a^2-3b^2)\tanh^3(x)}{3b^4} - \frac{(a^2-3b^2)\tanh^4(x)}{4b^3} + \frac{a\tanh^5(x)}{5b^2} - \frac{\tanh^6(x)}{6b}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{\text{sech}^8(x)}{a+b\tanh(x)}\,dx$$

$$= \frac{-60(a^2-b^2)^3\log(a+b\tanh(x)) + 15b^4(-a^2+b^2)\,\text{sech}^4(x) + 10b^6\text{sech}^6(x) + 60ab(a^4-3a^2b^2+3b^4)\tanh}{60b^7}$$

```
[In] Integrate[Sech[x]^8/(a + b*Tanh[x]),x]
```

```
[Out] (-60*(a^2 - b^2)^3*Log[a + b*Tanh[x]] + 15*b^4*(-a^2 + b^2)*Sech[x]^4 + 10*
b^6*Sech[x]^6 + 60*a*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Tanh[x] - 30*b^2*(a^2 - b^
2)^2*Tanh[x]^2 + 20*a*b^3*(a^2 - 3*b^2)*Tanh[x]^3 + 12*a*b^5*Tanh[x]^5)/(60
*b^7)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(130) = 260$.

Time = 235.72 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.95

method	result
default	$\frac{2\left(\left(a^5b-3a^3b^3+3a\,b^5\right)\tanh\left(\frac{x}{2}\right)^{11}+\left(-a^4b^2+3a^2b^4-3b^6\right)\tanh\left(\frac{x}{2}\right)^{10}+\left(5a^5b-\frac{41}{3}a^3b^3+11a\,b^5\right)\tanh\left(\frac{x}{2}\right)^9+\left(-4a^4b^2+10a^2b^4-6b^6\right)\tanh\left(\frac{x}{2}\right)^8+}{}$
risch	$-\frac{2(330a\,b^4e^{6x}+150a^5e^{4x}+75a^5e^{2x}-400a^3b^2e^{6x}+105a\,b^4e^{8x}+30a^2b^3e^{2x}-15a^4b\,e^{10x}-30a^3b^2e^{10x}+30a^2b^3e^{10x}+15a\,b^4e^{10x}-60a^4}{}$

```
[In] int(sech(x)^8/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^7*(((a^5*b-3*a^3*b^3+3*a*b^5)*tanh(1/2*x)^11+(-a^4*b^2+3*a^2*b^4-3*b^6)
*tanh(1/2*x)^10+(5*a^5*b-41/3*a^3*b^3+11*a*b^5)*tanh(1/2*x)^9+(-4*a^4*b^2+1
0*a^2*b^4-6*b^6)*tanh(1/2*x)^8+(10*a^5*b-26*a^3*b^3+106/5*a*b^5)*tanh(1/2*x
)^7+(-6*a^4*b^2+14*a^2*b^4-34/3*b^6)*tanh(1/2*x)^6+(10*a^5*b-26*a^3*b^3+106
/5*a*b^5)*tanh(1/2*x)^5+(-4*a^4*b^2+10*a^2*b^4-6*b^6)*tanh(1/2*x)^4+(5*a^5*
b-41/3*a^3*b^3+11*a*b^5)*tanh(1/2*x)^3+(-a^4*b^2+3*a^2*b^4-3*b^6)*tanh(1/2*
x)^2+(a^5*b-3*a^3*b^3+3*a*b^5)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^6+1/2*(a^6-3*
a^4*b^2+3*a^2*b^4-b^6)*ln(1+tanh(1/2*x)^2))-(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b
^7*ln(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5275 vs. $2(130) = 260$.

Time = 0.35 (sec) , antiderivative size = 5275, normalized size of antiderivative = 37.68

$$\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^8/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^8{(x)}}{a+b\tanh{(x)}}\,dx$$

```
[In] integrate(sech(x)**8/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**8/(a + b*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(130) = 260.

Time = 0.31 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.76

$$\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)}\,dx$$

$$= \frac{2\left(15\,a^5-40\,a^3b^2+33\,ab^4+3\,(25\,a^5+5\,a^4b-70\,a^3b^2-10\,a^2b^3+61\,ab^4+5\,b^5)e^{(-2\,x)}+30\,(5\,a^5+2\,a^4b-\right.}{}$$

$$-\frac{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{b^7}$$

$$+\frac{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\log\left(e^{(-2\,x)}+1\right)}{b^7}$$

```
[In] integrate(sech(x)^8/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*a^5 - 40*a^3*b^2 + 33*a*b^4 + 3*(25*a^5 + 5*a^4*b - 70*a^3*b^2 - 1
0*a^2*b^3 + 61*a*b^4 + 5*b^5)*e^(-2*x) + 30*(5*a^5 + 2*a^4*b - 14*a^3*b^2 -
 5*a^2*b^3 + 13*a*b^4 + 3*b^5)*e^(-4*x) + 10*(15*a^5 + 9*a^4*b - 40*a^3*b^2
 - 24*a^2*b^3 + 33*a*b^4 + 23*b^5)*e^(-6*x) + 15*(5*a^5 + 4*a^4*b - 12*a^3*
b^2 - 10*a^2*b^3 + 7*a*b^4 + 6*b^5)*e^(-8*x) + 15*(a^5 + a^4*b - 2*a^3*b^2
- 2*a^2*b^3 + a*b^4 + b^5)*e^(-10*x))/(6*b^6*e^(-2*x) + 15*b^6*e^(-4*x) + 2
0*b^6*e^(-6*x) + 15*b^6*e^(-8*x) + 6*b^6*e^(-10*x) + b^6*e^(-12*x) + b^6) -
 (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(-(a - b)*e^(-2*x) - a - b)/b^7 + (
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(e^(-2*x) + 1)/b^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(130) = 260$.

Time = 0.28 (sec) , antiderivative size = 593, normalized size of antiderivative = 4.24

$$\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)}\,dx = -\frac{(a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{ab^7+b^8} + \frac{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\log\left(e^{(2\,x)}+1\right)}{b^7} - \frac{147\,a^6e^{(12\,x)}-441\,a^4b^2e^{(12\,x)}+441\,a^2b^4e^{(12\,x)}-147\,b^6e^{(12\,x)}+882\,a^6e^{(10\,x)}+120\,a^5be^{(10\,x)}-2766\,a^4b^2e^{(}}{}$$

```
[In] integrate(sech(x)^8/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^2*b^5 - a*b^6 - b^7
)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b^7 + b^8) + (a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*log(e^(2*x) + 1)/b^7 - 1/60*(147*a^6*e^(12*x) - 441*a^4*
b^2*e^(12*x) + 441*a^2*b^4*e^(12*x) - 147*b^6*e^(12*x) + 882*a^6*e^(10*x) +
 120*a^5*b*e^(10*x) - 2766*a^4*b^2*e^(10*x) - 240*a^3*b^3*e^(10*x) + 2886*a
^2*b^4*e^(10*x) + 120*a*b^5*e^(10*x) - 1002*b^6*e^(10*x) + 2205*a^6*e^(8*x)
 + 600*a^5*b*e^(8*x) - 7095*a^4*b^2*e^(8*x) - 1440*a^3*b^3*e^(8*x) + 7815*a
^2*b^4*e^(8*x) + 840*a*b^5*e^(8*x) - 2925*b^6*e^(8*x) + 2940*a^6*e^(6*x) +
1200*a^5*b*e^(6*x) - 9540*a^4*b^2*e^(6*x) - 3200*a^3*b^3*e^(6*x) + 10740*a^
2*b^4*e^(6*x) + 2640*a*b^5*e^(6*x) - 4780*b^6*e^(6*x) + 2205*a^6*e^(4*x) +
1200*a^5*b*e^(4*x) - 7095*a^4*b^2*e^(4*x) - 3360*a^3*b^3*e^(4*x) + 7815*a^2
*b^4*e^(4*x) + 3120*a*b^5*e^(4*x) - 2925*b^6*e^(4*x) + 882*a^6*e^(2*x) + 60
0*a^5*b*e^(2*x) - 2766*a^4*b^2*e^(2*x) - 1680*a^3*b^3*e^(2*x) + 2886*a^2*b^
4*e^(2*x) + 1464*a*b^5*e^(2*x) - 1002*b^6*e^(2*x) + 147*a^6 + 120*a^5*b - 4
41*a^4*b^2 - 320*a^3*b^3 + 441*a^2*b^4 + 264*a*b^5 - 147*b^6)/(b^7*(e^(2*x)
 + 1)^6)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{sech}^8(x)}{a+b\tanh(x)}\,dx = \frac{\ln\left(e^{2x}+1\right)\,(a+b)^3\,(a-b)^3}{b^7} - \frac{32\,(a-5\,b)}{5\,b^2\,\left(5\,e^{2x}+10\,e^{4x}+10\,e^{6x}+5\,e^{8x}+e^{10x}+1\right)} - \frac{4\,(a^2-4\,a\,b+7\,b^2)}{b^3\,\left(4\,e^{2x}+6\,e^{4x}+4\,e^{6x}+e^{8x}+1\right)} - \frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\,(a+b)^3\,(a-b)^3}{b^7} - \frac{32}{3\,b\,\left(6\,e^{2x}+15\,e^{4x}+20\,e^{6x}+15\,e^{8x}+6\,e^{10x}+e^{12x}+1\right)} - \frac{8\,(a-b)\,(a^2-2\,a\,b+b^2)}{3\,b^4\,\left(3\,e^{2x}+3\,e^{4x}+e^{6x}+1\right)} - \frac{2\,(a+b)^2\,(a-b)\,(a^2-2\,a\,b+b^2)}{b^6\,\left(e^{2x}+1\right)} - \frac{2\,(a+b)\,(a-b)\,(a^2-2\,a\,b+b^2)}{b^5\,\left(2\,e^{2x}+e^{4x}+1\right)}$$

```
[In] int(1/(cosh(x)^8*(a + b*tanh(x))),x)
```

```
[Out] (log(exp(2*x) + 1)*(a + b)^3*(a - b)^3)/b^7 - (32*(a - 5*b))/(5*b^2*(5*exp(
2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (4*(a^2 -
 4*a*b + 7*b^2))/(b^3*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)
) - (log(a - b + a*exp(2*x) + b*exp(2*x))*(a + b)^3*(a - b)^3)/b^7 - 32/(3*
b*(6*exp(2*x) + 15*exp(4*x) + 20*exp(6*x) + 15*exp(8*x) + 6*exp(10*x) + exp
(12*x) + 1)) - (8*(a - b)*(a^2 - 2*a*b + b^2))/(3*b^4*(3*exp(2*x) + 3*exp(4
*x) + exp(6*x) + 1)) - (2*(a + b)^2*(a - b)*(a^2 - 2*a*b + b^2))/(b^6*(exp(
2*x) + 1)) - (2*(a + b)*(a - b)*(a^2 - 2*a*b + b^2))/(b^5*(2*exp(2*x) + exp
(4*x) + 1))
```

3.103 $\int \frac{\text{sech}^6(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\text{sech}^6(x)}{a+b\tanh(x)}\,dx=\frac{(a^2-b^2)^2\log(a+b\tanh(x))}{b^5}-\frac{a(a^2-2b^2)\tanh(x)}{b^4}+\frac{(a^2-2b^2)\tanh^2(x)}{2b^3}-\frac{a\tanh^3(x)}{3b^2}+\frac{\tanh^4(x)}{4b}$$

```
[Out] (a^2-b^2)^2*ln(a+b*tanh(x))/b^5-a*(a^2-2*b^2)*tanh(x)/b^4+1/2*(a^2-2*b^2)*t
anh(x)^2/b^3-1/3*a*tanh(x)^3/b^2+1/4*tanh(x)^4/b
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 711}

$$\int \frac{\text{sech}^6(x)}{a+b\tanh(x)}\,dx=\frac{(a^2-b^2)^2\log(a+b\tanh(x))}{b^5}-\frac{a(a^2-2b^2)\tanh(x)}{b^4}+\frac{(a^2-2b^2)\tanh^2(x)}{2b^3}-\frac{a\tanh^3(x)}{3b^2}+\frac{\tanh^4(x)}{4b}$$

```
[In] Int[Sech[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] ((a^2 - b^2)^2*Log[a + b*Tanh[x]])/b^5 - (a*(a^2 - 2*b^2)*Tanh[x])/b^4 + ((
a^2 - 2*b^2)*Tanh[x]^2)/(2*b^3) - (a*Tanh[x]^3)/(3*b^2) + Tanh[x]^4/(4*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
```

```
x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1-\frac{x^2}{b^2}\right)^2}{a+x}\,dx, x, b\tanh(x)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{-a^3+2ab^2}{b^4} - \frac{(-a^2+2b^2)x}{b^4} - \frac{ax^2}{b^4} + \frac{x^3}{b^4} + \frac{(-a^2+b^2)^2}{b^4(a+x)}\right)dx, x, b\tanh(x)\right)}{b} \\
&= \frac{(a^2-b^2)^2\log(a+b\tanh(x))}{b^5} - \frac{a(a^2-2b^2)\tanh(x)}{b^4} \\
&\quad + \frac{(a^2-2b^2)\tanh^2(x)}{2b^3} - \frac{a\tanh^3(x)}{3b^2} + \frac{\tanh^4(x)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{\text{sech}^6(x)}{a+b\tanh(x)}\,dx = \frac{12(a^2-b^2)^2\log(a+b\tanh(x)) + 3b^4\text{sech}^4(x) - 12ab(a^2-2b^2)\tanh(x) + 6b^2(a^2-b^2)\tanh^2(x) - 4ab^3\tan}{12b^5}$$

```
[In] Integrate[Sech[x]^6/(a + b*Tanh[x]),x]
```

```
[Out] (12*(a^2 - b^2)^2*Log[a + b*Tanh[x]] + 3*b^4*Sech[x]^4 - 12*a*b*(a^2 - 2*b^
2)*Tanh[x] + 6*b^2*(a^2 - b^2)*Tanh[x]^2 - 4*a*b^3*Tanh[x]^3)/(12*b^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(77) = 154$.

Time = 67.23 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.67

method	result
default	$\frac{(a^4-2a^2b^2+b^4)\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{b^5}-\frac{2\left(\frac{\left(a^3b-2a\,b^3\right)\tanh\left(\frac{x}{2}\right)^7+\left(-a^2b^2+2b^4\right)\tanh\left(\frac{x}{2}\right)^6+\left(3a^3b-\frac{14}{3}a\,b^3\right)\tanh\left(\frac{x}{2}\right)^5+\left(-\right.}{}\right.}{}$
risch	$\frac{2a^3\mathrm{e}^{6x}-2\,\mathrm{e}^{6x}a^2b-2\,\mathrm{e}^{6x}a\,b^2+2b^3\mathrm{e}^{6x}+6a^3\mathrm{e}^{4x}-4a^2b\,\mathrm{e}^{4x}-10a\,b^2\mathrm{e}^{4x}+8b^3\mathrm{e}^{4x}+6a^3\mathrm{e}^{2x}-2\,\mathrm{e}^{2x}a^2b-\frac{34\,\mathrm{e}^{2x}a\,b^2}{3}+2b^3\mathrm{e}^{2x}+2a^3-\frac{10a\,b^2}{3}}{b^4(1+\mathrm{e}^{2x})^4}-$

```
[In] int(sech(x)^6/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (a^4-2*a^2*b^2+b^4)/b^5*ln(tanh(1/2*x)^2*a+2*b*tanh(1/2*x)+a)-2/b^5*(((a^3*
b-2*a*b^3)*tanh(1/2*x)^7+(-a^2*b^2+2*b^4)*tanh(1/2*x)^6+(3*a^3*b-14/3*a*b^3
)*tanh(1/2*x)^5+(-2*a^2*b^2+2*b^4)*tanh(1/2*x)^4+(3*a^3*b-14/3*a*b^3)*tanh(
1/2*x)^3+(-a^2*b^2+2*b^4)*tanh(1/2*x)^2+(a^3*b-2*a*b^3)*tanh(1/2*x))/(1+tan
h(1/2*x)^2)^4+1/2*(a^4-2*a^2*b^2+b^4)*ln(1+tanh(1/2*x)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 1827, normalized size of antiderivative = 22.01

$$\int \frac{\operatorname{sech}^6(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^6/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(6*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^6 + 36*(a^3*b - a^2*b^2 - a*
b^3 + b^4)*cosh(x)*sinh(x)^5 + 6*(a^3*b - a^2*b^2 - a*b^3 + b^4)*sinh(x)^6
+ 6*(3*a^3*b - 2*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x)^4 + 6*(3*a^3*b - 2*a^2*
b^2 - 5*a*b^3 + 4*b^4 + 15*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^2)*sinh(
x)^4 + 6*a^3*b - 10*a*b^3 + 24*(5*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^3
 + (3*a^3*b - 2*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x))*sinh(x)^3 + 2*(9*a^3*b
- 3*a^2*b^2 - 17*a*b^3 + 3*b^4)*cosh(x)^2 + 2*(45*(a^3*b - a^2*b^2 - a*b^3
+ b^4)*cosh(x)^4 + 9*a^3*b - 3*a^2*b^2 - 17*a*b^3 + 3*b^4 + 18*(3*a^3*b - 2
*a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(x)^2)*sinh(x)^2 + 3*((a^4 - 2*a^2*b^2 + b^
4)*cosh(x)^8 + 8*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b
^2 + b^4)*sinh(x)^8 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*
b^2 + b^4 + 7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6
*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^
```

```
4 + 3*a^4 - 6*a^2*b^2 + 3*b^4 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(
x)^4 + a^4 - 2*a^2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*
(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh
(x)^3 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*
cosh(x)^6 + 15*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
9*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)
*cosh(x)^7 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4
)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x) +
b*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*
(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)
^8 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a^
4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*co
sh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2
*b^2 + 3*b^4 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^
2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2
+ b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 -
2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^
4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) +
 4*(9*(a^3*b - a^2*b^2 - a*b^3 + b^4)*cosh(x)^5 + 6*(3*a^3*b - 2*a^2*b^2 -
5*a*b^3 + 4*b^4)*cosh(x)^3 + (9*a^3*b - 3*a^2*b^2 - 17*a*b^3 + 3*b^4)*cosh(
x))*sinh(x))/(b^5*cosh(x)^8 + 8*b^5*cosh(x)*sinh(x)^7 + b^5*sinh(x)^8 + 4*b
^5*cosh(x)^6 + 6*b^5*cosh(x)^4 + 4*b^5*cosh(x)^2 + 4*(7*b^5*cosh(x)^2 + b^5
)*sinh(x)^6 + 8*(7*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^5 + b^5 + 2*(35*b
^5*cosh(x)^4 + 30*b^5*cosh(x)^2 + 3*b^5)*sinh(x)^4 + 8*(7*b^5*cosh(x)^5 + 1
0*b^5*cosh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 4*(7*b^5*cosh(x)^6 + 15*b^5*co
sh(x)^4 + 9*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 8*(b^5*cosh(x)^7 + 3*b^5*cosh(
x)^5 + 3*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^6(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(sech(x)**6/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**6/(a + b*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(77) = 154$.

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.46

$$\int \frac{\operatorname{sech}^6(x)}{a+b\tanh(x)}\,dx = -\frac{2\,\left(3\,a^3-5\,ab^2+(9\,a^3+3\,a^2b-17\,ab^2-3\,b^3)e^{(-2\,x)}+3\,(3\,a^3+2\,a^2b-5\,ab^2-4\,b^3)e^{(-4\,x)}+3\,(a^3+a^2\ldots\right.}{3\,(4\,b^4e^{(-2\,x)}+6\,b^4e^{(-4\,x)}+4\,b^4e^{(-6\,x)}+b^4e^{(-8\,x)}+b^4)} + \frac{(a^4-2\,a^2b^2+b^4)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{b^5} - \frac{(a^4-2\,a^2b^2+b^4)\log\left(e^{(-2\,x)}+1\right)}{b^5}$$

```
[In] integrate(sech(x)^6/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a^3 - 5*a*b^2 + (9*a^3 + 3*a^2*b - 17*a*b^2 - 3*b^3)*e^(-2*x) + 3*(
3*a^3 + 2*a^2*b - 5*a*b^2 - 4*b^3)*e^(-4*x) + 3*(a^3 + a^2*b - a*b^2 - b^3)
*e^(-6*x))/(4*b^4*e^(-2*x) + 6*b^4*e^(-4*x) + 4*b^4*e^(-6*x) + b^4*e^(-8*x)
 + b^4) + (a^4 - 2*a^2*b^2 + b^4)*log(-(a - b)*e^(-2*x) - a - b)/b^5 - (a^4
 - 2*a^2*b^2 + b^4)*log(e^(-2*x) + 1)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(77) = 154$.

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 3.81

$$\int \frac{\operatorname{sech}^6(x)}{a+b\tanh(x)}\,dx = \frac{(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{ab^5+b^6} - \frac{(a^4-2\,a^2b^2+b^4)\log\left(e^{(2\,x)}+1\right)}{b^5} + \frac{25\,a^4e^{(8\,x)}-50\,a^2b^2e^{(8\,x)}+25\,b^4e^{(8\,x)}+100\,a^4e^{(6\,x)}+24\,a^3be^{(6\,x)}-224\,a^2b^2e^{(6\,x)}-24\,ab^3e^{(6\,x)}+124\,b^4e\ldots}{\ldots}$$

```
[In] integrate(sech(x)^6/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*log(abs(a*e^(2*x) + b*e
^(2*x) + a - b))/(a*b^5 + b^6) - (a^4 - 2*a^2*b^2 + b^4)*log(e^(2*x) + 1)/b
^5 + 1/12*(25*a^4*e^(8*x) - 50*a^2*b^2*e^(8*x) + 25*b^4*e^(8*x) + 100*a^4*e
^(6*x) + 24*a^3*b*e^(6*x) - 224*a^2*b^2*e^(6*x) - 24*a*b^3*e^(6*x) + 124*b^
4*e^(6*x) + 150*a^4*e^(4*x) + 72*a^3*b*e^(4*x) - 348*a^2*b^2*e^(4*x) - 120*
a*b^3*e^(4*x) + 246*b^4*e^(4*x) + 100*a^4*e^(2*x) + 72*a^3*b*e^(2*x) - 224*
a^2*b^2*e^(2*x) - 136*a*b^3*e^(2*x) + 124*b^4*e^(2*x) + 25*a^4 + 24*a^3*b -
 50*a^2*b^2 - 40*a*b^3 + 25*b^4)/(b^5*(e^(2*x) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.04

$$\int \frac{\operatorname{sech}^6(x)}{a+b\tanh(x)}\,dx = \frac{4}{b\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}+\frac{2\,{\left(a-b\right)}^2}{b^3\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)} +\frac{8\,\left(a-3\,b\right)}{3\,b^2\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}+\frac{2\,\left(a+b\right)\,{\left(a-b\right)}^2}{b^4\,\left(\mathrm{e}^{2\,x}+1\right)} +\frac{\ln\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)\,{\left(a+b\right)}^2\,{\left(a-b\right)}^2}{b^5} -\frac{\ln\left(\mathrm{e}^{2\,x}+1\right)\,{\left(a+b\right)}^2\,{\left(a-b\right)}^2}{b^5}$$

```
[In] int(1/(cosh(x)^6*(a + b*tanh(x))),x)
```

```
[Out] 4/(b*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) + (2*(a - b)^2)
/(b^3*(2*exp(2*x) + exp(4*x) + 1)) + (8*(a - 3*b))/(3*b^2*(3*exp(2*x) + 3*e
xp(4*x) + exp(6*x) + 1)) + (2*(a + b)*(a - b)^2)/(b^4*(exp(2*x) + 1)) + (lo
g(a - b + a*exp(2*x) + b*exp(2*x))*(a + b)^2*(a - b)^2)/b^5 - (log(exp(2*x)
 + 1)*(a + b)^2*(a - b)^2)/b^5
```

3.104 $\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx=-\frac{(a^2-b^2)\log(a+b\tanh(x))}{b^3}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

```
[Out] -(a^2-b^2)*ln(a+b*tanh(x))/b^3+a*tanh(x)/b^2-1/2*tanh(x)^2/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 711}

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx=-\frac{(a^2-b^2)\log(a+b\tanh(x))}{b^3}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

```
[In] Int[Sech[x]^4/(a + b*Tanh[x]),x]
[Out] -(((a^2 - b^2)*Log[a + b*Tanh[x]])/b^3) + (a*Tanh[x])/b^2 - Tanh[x]^2/(2*b)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{b^2}}{a+x}\,dx, x, b\tanh(x)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{b^2}-\frac{x}{b^2}+\frac{-a^2+b^2}{b^2(a+x)}\right)\,dx, x, b\tanh(x)\right)}{b} \\
&= -\frac{(a^2-b^2)\log(a+b\tanh(x))}{b^3}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{\text{sech}^4(x)}{a+b\tanh(x)}\,dx = \frac{2(-a^2+b^2)\log(a+b\tanh(x))+2ab\tanh(x)-b^2\tanh^2(x)}{2b^3}$$

```
[In] Integrate[Sech[x]^4/(a + b*Tanh[x]),x]
[Out] (2*(-a^2 + b^2)*Log[a + b*Tanh[x]] + 2*a*b*Tanh[x] - b^2*Tanh[x]^2)/(2*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(38) = 76.

Time = 12.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

method	result	size
default	$\frac{\frac{2\left(ab\tanh\left(\frac{x}{2}\right)^3-\tanh\left(\frac{x}{2}\right)^2b^2+ab\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+(a^2-b^2)\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{b^3}-\frac{(a^2-b^2)\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{b^3}$	99
risch	$-\frac{2(a\,e^{2x}-b\,e^{2x}+a)}{(1+e^{2x})^2b^2}+\frac{\ln(1+e^{2x})a^2}{b^3}-\frac{\ln(1+e^{2x})}{b}-\frac{\ln\left(e^{2x}+\frac{a-b}{a+b}\right)a^2}{b^3}+\frac{\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{b}$	102

```
[In] int(sech(x)^4/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] 2/b^3*((a*b*tanh(1/2*x)^3-tanh(1/2*x)^2*b^2+a*b*tanh(1/2*x))/(1+tanh(1/2*x)
^2)^2+1/2*(a^2-b^2)*ln(1+tanh(1/2*x)^2))-(a^2-b^2)/b^3*ln(tanh(1/2*x)^2*a+2
*b*tanh(1/2*x)+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 430, normalized size of antiderivative = 10.75

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx = -\frac{2\,(ab-b^2)\cosh(x)^2+4\,(ab-b^2)\cosh(x)\sinh(x)+2\,(ab-b^2)\sinh(x)^2+2\,ab+\left((a^2-b^2)\cosh(x)^4\right.}{}$$

```
[In] integrate(sech(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -(2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*s
inh(x)^2 + 2*a*b + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3
 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(
x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 -
 b^2)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x)))
- ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*si
nh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)
*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*si
nh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)*s
inh(x)^3 + b^3*sinh(x)^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3
)*sinh(x)^2 + 4*(b^3*cosh(x)^3 + b^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(sech(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**4/(a + b*tanh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx = \frac{2\left((a+b)e^{(-2\,x)}+a\right)}{2\,b^2e^{(-2\,x)}+b^2e^{(-4\,x)}+b^2} - \frac{(a^2-b^2)\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{b^3} + \frac{(a^2-b^2)\log\left(e^{(-2\,x)}+1\right)}{b^3}$$

```
[In] integrate(sech(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] 2*((a + b)*e^(-2*x) + a)/(2*b^2*e^(-2*x) + b^2*e^(-4*x) + b^2) - (a^2 - b^2
)*log(-(a - b)*e^(-2*x) - a - b)/b^3 + (a^2 - b^2)*log(e^(-2*x) + 1)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.60

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx = -\frac{(a^3+a^2b-ab^2-b^3)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{ab^3+b^4} + \frac{(a^2-b^2)\log\left(e^{(2x)}+1\right)}{b^3} - \frac{2\left(ab+(ab-b^2)e^{(2x)}\right)}{b^3\left(e^{(2x)}+1\right)^2}$$

```
[In] integrate(sech(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + a^2*b - a*b^2 - b^3)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b^3
 + b^4) + (a^2 - b^2)*log(e^(2*x) + 1)/b^3 - 2*(a*b + (a*b - b^2)*e^(2*x))/
(b^3*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int \frac{\operatorname{sech}^4(x)}{a+b\tanh(x)}\,dx = \frac{\ln\left(e^{2x}+1\right)\,(a+b)\,(a-b)}{b^3} - \frac{2\,(a-b)}{b^2\,\left(e^{2x}+1\right)} - \frac{\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)\,(a+b)\,(a-b)}{b^3} - \frac{2}{b\,\left(2\,e^{2x}+e^{4x}+1\right)}$$

```
[In] int(1/(cosh(x)^4*(a + b*tanh(x))),x)
```

```
[Out] (log(exp(2*x) + 1)*(a + b)*(a - b))/b^3 - (2*(a - b))/(b^2*(exp(2*x) + 1))
- (log(a - b + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/b^3 - 2/(b*(2*exp(
2*x) + exp(4*x) + 1))
```

3.105 $\int \frac{\text{sech}^2(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\text{sech}^2(x)}{a+b\tanh(x)}\,dx=\frac{\log(a+b\tanh(x))}{b}$$

```
[Out] ln(a+b*tanh(x))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 31}

$$\int \frac{\text{sech}^2(x)}{a+b\tanh(x)}\,dx=\frac{\log(a+b\tanh(x))}{b}$$

```
[In] Int[Sech[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] Log[a + b*Tanh[x]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
```

```
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tanh(x)\right)}{b}\\ &= \frac{\log(a+b\tanh(x))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log(a+b\tanh(x))}{b}$$

```
[In] Integrate[Sech[x]^2/(a + b*Tanh[x]),x]
[Out] Log[a + b*Tanh[x]]/b
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\tanh(x))}{b}$	12
default	$\frac{\ln(a+b\tanh(x))}{b}$	12
risch	$-\frac{\ln(1+e^{2x})}{b}+\frac{\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{b}$	35

```
[In] int(sech(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] ln(a+b*tanh(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.82

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right) - \log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
[Out] (log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x
) - sinh(x))))/b
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(sech(x)**2/(a+b*tanh(x)),x)
[Out] Integral(sech(x)**2/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{\log\left(b\tanh(x)+a\right)}{b}$$

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
[Out] log(b*tanh(x) + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.09

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx = \frac{(a+b)\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{ab+b^2} - \frac{\log\left(e^{(2x)}+1\right)}{b}$$

```
[In] integrate(sech(x)^2/(a+b*tanh(x)),x, algorithm="giac")
[Out] (a + b)*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a*b + b^2) - log(e^(2*x) +
 1)/b
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \frac{\operatorname{sech}^2(x)}{a+b\tanh(x)}\,dx=-\frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-b^2}+a\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}+b\,\mathrm{e}^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/(cosh(x)^2*(a + b*tanh(x))),x)
```

```
[Out] -(2*atan((a*(-b^2)^(1/2) + a*exp(2*x)*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

3.106 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3565, 3611}

$$\int \frac{1}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[In] Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2} \\ &= \frac{ax}{a^2-b^2} - \frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(a+b\tanh(x))}{2(a-b)(a+b)}$$

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
[Out] ((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[a + b*Tanh[
x]])/(2*(a - b)*(a + b))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$-\frac{-\ln(1-\tanh(x))b+b\ln(a+b\tanh(x))-ax-bx}{a^2-b^2}$	42
derivativedivides	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2a+2b}-\frac{b\ln(a+b\tanh(x))}{(a-b)(a+b)}$	55
default	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2a+2b}-\frac{b\ln(a+b\tanh(x))}{(a-b)(a+b)}$	55
risch	$\frac{x}{a+b}+\frac{2xb}{a^2-b^2}-\frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	55

```
[In] int(1/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -(-ln(1-tanh(x))*b+b*ln(a+b*tanh(x))-a*x-b*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(a+b)x - b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(29) = 58.

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.74

$$\int \frac{1}{a+b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}(x - \log(\tanh(x)+1) + \log(\tanh(x))) & \text{for } a=0 \wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b} + \frac{x}{2b\tanh(x)-2b} + \frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} - \frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2} - \frac{bx}{a^2-b^2} - \frac{b\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2-b^2} + \frac{b\log(\tanh(x)+1)}{a^2-b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*tanh(x)),x)
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) +
 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b
*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - b*
x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(
a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="giac")
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{a\,x-b\,(x-\ln(\tanh(x)+1)+\ln(a+b\tanh(x)))}{a^2-b^2}$$

```
[In] int(1/(a + b*tanh(x)),x)
[Out] (a*x - b*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)
```

3.107 $\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx=-\frac{(a+2b)\log(1-\tanh(x))}{4(a+b)^2}+\frac{(a-2b)\log(1+\tanh(x))}{4(a-b)^2}+\frac{b^3\log(a+b\tanh(x))}{(a^2-b^2)^2}-\frac{\cosh^2(x)(b-a\tanh(x))}{2(a^2-b^2)}$$

```
[Out] -1/4*(a+2*b)*ln(1-tanh(x))/(a+b)^2+1/4*(a-2*b)*ln(1+tanh(x))/(a-b)^2+b^3*ln
(a+b*tanh(x))/(a^2-b^2)^2-1/2*cosh(x)^2*(b-a*tanh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3587, 755, 815}

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx=-\frac{\cosh^2(x)(b-a\tanh(x))}{2(a^2-b^2)}+\frac{b^3\log(a+b\tanh(x))}{(a^2-b^2)^2}-\frac{(a+2b)\log(1-\tanh(x))}{4(a+b)^2}+\frac{(a-2b)\log(\tanh(x)+1)}{4(a-b)^2}$$

```
[In] Int[Cosh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -1/4*((a + 2*b)*Log[1 - Tanh[x]])/(a + b)^2 + ((a - 2*b)*Log[1 + Tanh[x]])/
(4*(a - b)^2) + (b^3*Log[a + b*Tanh[x]])/(a^2 - b^2)^2 - (Cosh[x]^2*(b - a*
Tanh[x]))/(2*(a^2 - b^2))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\tanh(x)\right)}{b}\\
&= -\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)} + \frac{b\text{Subst}\left(\int \frac{-2+\frac{a^2}{b^2}+\frac{ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\tanh(x)\right)}{2\left(a^2-b^2\right)}\\
&= -\frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}\\
&\quad + \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right)dx, x, b\tanh(x)\right)}{2\left(a^2-b^2\right)}\\
&= -\frac{(a+2b)\log(1-\tanh(x))}{4(a+b)^2} + \frac{(a-2b)\log(1+\tanh(x))}{4(a-b)^2}\\
&\quad + \frac{b^3\log(a+b\tanh(x))}{\left(a^2-b^2\right)^2} - \frac{\cosh^2(x)(b-a\tanh(x))}{2\left(a^2-b^2\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx = \frac{2a^3x-6ab^2x+(-a^2b+b^3)\cosh(2x)+4b^3\log(a\cosh(x)+b\sinh(x))+a(a^2-b^2)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[Cosh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (2*a^3*x - 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] + 4*b^3*Log[a*Cosh[x] + b
*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{xb}{(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}-\frac{2b^3x}{a^4-2a^2b^2+b^4}+\frac{b^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$\frac{b^3\ln\left(\tanh\left(\frac{x}{2}\right)^2a+2b\tanh\left(\frac{x}{2}\right)+a\right)}{(a-b)^2(a+b)^2}+\frac{1}{(2a+2b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{(-a-2b)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2}-\frac{}{(2a-2b)}$

```
[In] int(cosh(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/(a+b)^2+x/(a+b)^2*b+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)-2*b^3/(a
^4-2*a^2*b^2+b^4)*x+b^3/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(86) = 172.

Time = 0.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.64

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x}{}$$

```
[In] integrate(cosh(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 - 3*a*b^
```

```
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 + 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(a*cosh(x) + b*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx = \int \frac{\cosh^2{(x)}}{a+b\tanh{(x)}}\,dx$$

```
[In] integrate(cosh(x)**2/(a+b*tanh(x)),x)
[Out] Integral(cosh(x)**2/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx = \frac{b^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-2a^2b^2+b^4} + \frac{(a+2b)x}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

```
[In] integrate(cosh(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
[Out] b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a + 2*b)*
x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx = \frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4} + \frac{(a-2b)x}{2(a^2-2ab+b^2)} - \frac{\left(2ae^{(2x)}-4be^{(2x)}+a-b\right)e^{(-2x)}}{8(a^2-2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)}$$

```
[In] integrate(cosh(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(
a - 2*b)*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) + a - b)*e^
(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(x)}{a+b\tanh(x)}\,dx=\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8a-8b}+\frac{b^3\,\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2\,a^2\,b^2+b^4}+\frac{x\,(a-2\,b)}{2\,(a-b)^2}$$

```
[In] int(cosh(x)^2/(a + b*tanh(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (b^3*log(a - b + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) + (x*(a - 2*b))/(2*(a - b)^2)
```

3.108 $\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 155

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx = -\frac{(3a^2+9ab+8b^2)\log(1-\tanh(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(1+\tanh(x))}{16(a-b)^3} - \frac{b^5\log(a+b\tanh(x))}{(a^2-b^2)^3} - \frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} + \frac{\cosh^2(x)\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\tanh(x)\right)}{8(a^2-b^2)^2}$$

```
[Out] -1/16*(3*a^2+9*a*b+8*b^2)*ln(1-tanh(x))/(a+b)^3+1/16*(3*a^2-9*a*b+8*b^2)*ln
(1+tanh(x))/(a-b)^3-b^5*ln(a+b*tanh(x))/(a^2-b^2)^3-1/4*cosh(x)^4*(b-a*tanh
(x))/(a^2-b^2)+1/8*cosh(x)^2*(4*b^3-a*(7-3*a^2/b^2)*b^2*tanh(x))/(a^2-b^2)^
2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3587, 755, 837, 815}

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx = -\frac{(3a^2+9ab+8b^2)\log(1-\tanh(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(\tanh(x)+1)}{16(a-b)^3} - \frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} - \frac{b^5\log(a+b\tanh(x))}{(a^2-b^2)^3} + \frac{\cosh^2(x)\left(4b^3-ab^2\left(7-\frac{3a^2}{b^2}\right)\tanh(x)\right)}{8(a^2-b^2)^2}$$

```
[In] Int[Cosh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] -1/16*((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Tanh[x]])/(a + b)^3 + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Tanh[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Tanh[x]])/(a^2
 - b^2)^3 - (Cosh[x]^4*(b - a*Tanh[x]))/(4*(a^2 - b^2)) + (Cosh[x]^2*(4*b^3
 - a*(7 - (3*a^2)/b^2)*b^2*Tanh[x]))/(8*(a^2 - b^2)^2)
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^3}\,dx, x, b\tanh(x)\right)}{b}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} + \frac{b\text{Subst}\left(\int \frac{-4+\frac{3a^2}{b^2}+\frac{3ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}dx, x, b\tanh(x)\right)}{4(a^2-b^2)}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} + \frac{\cosh^2(x)\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\tanh(x)\right)}{8(a^2-b^2)^2} - \frac{b^5\text{Subst}\left(\int \frac{-\frac{3a^4-7a^2b^2+8b^4}{b^6}+\frac{a\left(7-\frac{3a^2}{b^2}\right)x}{b^4}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}dx, x, b\tanh(x)\right)}{8(a^2-b^2)^2}$$

$$= -\frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} + \frac{\cosh^2(x)\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\tanh(x)\right)}{8(a^2-b^2)^2} - \frac{b^5\text{Subst}\left(\int\left(-\frac{(a-b)^2(3a^2+9ab+8b^2)}{2b^5(a+b)(b-x)}+\frac{8}{(a-b)(a+b)(a+x)}-\frac{(a+b)^2(3a^2-9ab+8b^2)}{2(a-b)b^5(b+x)}\right)dx, x, b\tanh(x)\right)}{8(a^2-b^2)^2}$$

$$= -\frac{(3a^2+9ab+8b^2)\log(1-\tanh(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(1+\tanh(x))}{16(a-b)^3} - \frac{b^5\log(a+b\tanh(x))}{(a^2-b^2)^3} - \frac{\cosh^4(x)(b-a\tanh(x))}{4(a^2-b^2)} + \frac{\cosh^2(x)\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\tanh(x)\right)}{8(a^2-b^2)^2}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.33

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}dx = \frac{8b^3(a^2-b^2)\cosh^2(x)-4b(a^2-b^2)^2\cosh^4(x)-3a^5\log(1-\tanh(x))+10a^3b^2\log(1-\tanh(x))-15ab^4\log}{}$$

```
[In] Integrate[Cosh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (8*b^3*(a^2 - b^2)*Cosh[x]^2 - 4*b*(a^2 - b^2)^2*Cosh[x]^4 - 3*a^5*Log[1 -
Tanh[x]] + 10*a^3*b^2*Log[1 - Tanh[x]] - 15*a*b^4*Log[1 - Tanh[x]] + 8*b^5*
Log[1 - Tanh[x]] + 3*a^5*Log[1 + Tanh[x]] - 10*a^3*b^2*Log[1 + Tanh[x]] + 1
5*a*b^4*Log[1 + Tanh[x]] + 8*b^5*Log[1 + Tanh[x]] - 16*b^5*Log[a + b*Tanh[x
]] + 4*a*(a^2 - b^2)^2*Cosh[x]^3*Sinh[x] + a*(3*a^4 - 10*a^2*b^2 + 7*b^4)*S
inh[2*x])/(16*(a - b)^3*(a + b)^3)
```

Maple [A] (verified)

Time = 6.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.23

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{9axb}{8(a+b)^3}+\frac{x\,b^2}{(a+b)^3}+\frac{e^{4x}}{64a+64b}+\frac{e^{2x}a}{8(a+b)^2}+\frac{3\,e^{2x}b}{16(a+b)^2}-\frac{e^{-2x}a}{8(a-b)^2}+\frac{3\,e^{-2x}b}{16(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2b^5x}{a^6-3a^4b^2+3a}$
default	$-\frac{1}{2(2a-2b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{2}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-5a+7b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{7a-9b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{(3a^2-9ab+8b^2)\,l}{8(a-}$

```
[In] int(cosh(x)^4/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^2*x/(a+b)^3+9/8*a*x/(a+b)^3*b+x/(a+b)^3*b^2+1/64/(a+b)*exp(4*x)+1/8/(
a+b)^2*exp(2*x)*a+3/16/(a+b)^2*exp(2*x)*b-1/8/(a-b)^2*exp(-2*x)*a+3/16/(a-b
)^2*exp(-2*x)*b-1/64/(a-b)*exp(-4*x)+2*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*x-
b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. 2(147) = 294.

Time = 0.28 (sec) , antiderivative size = 1281, normalized size of antiderivative = 8.26

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(2*a^5 - a^4*b
- 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^6 + 4*(2*a^5 - a^4*b - 6
*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5 + 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 + 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^
5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 +
 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 30*(
2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^2 + 4*(3*a
^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 10*(2*a^5 - a^4*b - 6*a^3*b^2
+ 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^3 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x))*sinh(x)^3 - 4*(2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 +
 4*a*b^4 + 3*b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^6 - 2*a^5 - a^4*b + 6*a^3*b^2 + 4*a^2*b^3 - 4*a*b^4 - 3
*b^5 + 15*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
```

```
^4 + 12*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^2)*sinh(x)^2 - 64
*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b
^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x
) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^7 + 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^5 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^3 - (2*a^5 + a^4*b
 - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^
2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx=\int \frac{\cosh^4\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

```
[In] integrate(cosh(x)**4/(a+b*tanh(x)),x)
[Out] Integral(cosh(x)**4/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx=&-\frac{b^5\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2+9ab+8b^2)x}{8(a^3+3a^2b+3ab^2+b^3)}\\&+\frac{\left(4(2a+3b)e^{(-2x)}+a+b\right)e^{(4x)}}{64(a^2+2ab+b^2)}\\&-\frac{4(2a-3b)e^{(-2x)}+(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}\end{aligned}$$

```
[In] integrate(cosh(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
[Out] -b^5*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1
/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*(2*a
 + 3*b)*e^(-2*x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - 3*b)
*e^(-2*x) + (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.46

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx = -\frac{b^5\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2-9ab+8b^2)x}{8(a^3-3a^2b+3ab^2-b^3)} - \frac{\left(18a^2e^{(4x)}-54abe^{(4x)}+48b^2e^{(4x)}+8a^2e^{(2x)}-20abe^{(2x)}+12b^2e^{(2x)}+a^2-2ab+b^2\right)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)} + \frac{ae^{(4x)}+be^{(4x)}+8ae^{(2x)}+12be^{(2x)}}{64(a^2+2ab+b^2)}$$

```
[In] integrate(cosh(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6) + 1/8*(3*a^2 - 9*a*b + 8*b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/6
4*(18*a^2*e^(4*x) - 54*a*b*e^(4*x) + 48*b^2*e^(4*x) + 8*a^2*e^(2*x) - 20*a*
b*e^(2*x) + 12*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3
*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x) + 8*a*e^(2*x) + 12*b*e^(2*x))/(
a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^4(x)}{a+b\tanh(x)}\,dx = \frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}-\frac{e^{-2x}(2a-3b)}{16(a-b)^2} - \frac{b^5\ln\left(a-b+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{x(3a^2-9ab+8b^2)}{8(a-b)^3}+\frac{e^{2x}(2a+3b)}{16(a+b)^2}$$

```
[In] int(cosh(x)^4/(a + b*tanh(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) - (exp(-2*x)*(2*a - 3*b))/
(16*(a - b)^2) - (b^5*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) + (x*(3*a^2 - 9*a*b + 8*b^2))/(8*(a - b)^3) + (exp(2*x
)*(2*a + 3*b))/(16*(a + b)^2)
```

3.109 $\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 157

$$\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx = \frac{a(8a^4-20a^2b^2+15b^4)\arctan(\sinh(x))}{8b^6} - \frac{(a^2-b^2)^{5/2}\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^6} + \frac{(a^2-b^2)^2\operatorname{sech}(x)}{b^5} - \frac{(a^2-b^2)\operatorname{sech}^3(x)}{3b^3} + \frac{\operatorname{sech}^5(x)}{5b} - \frac{a(4a^2-7b^2)\operatorname{sech}(x)\tanh(x)}{8b^4} + \frac{a\operatorname{sech}^3(x)\tanh(x)}{4b^2}$$

```
[Out] 1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*arctan(sinh(x))/b^6-(a^2-b^2)^(5/2)*arctan(
cosh(x)*(b+a*tanh(x))/(a^2-b^2)^(1/2))/b^6+(a^2-b^2)^2*sech(x)/b^5-1/3*(a^2
-b^2)*sech(x)^3/b^3+1/5*sech(x)^5/b-1/8*a*(4*a^2-7*b^2)*sech(x)*tanh(x)/b^4
+1/4*a*sech(x)^3*tanh(x)/b^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used

= {3591, 3567, 3853, 3855, 3590, 212}

$$\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx = \frac{a(a^2-b^2)^2\arctan(\sinh(x))}{b^6} - \frac{(a^2-b^2)^{5/2}\arctan\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^6} - \frac{a(a^2-b^2)\arctan(\sinh(x))}{2b^4} + \frac{(a^2-b^2)^2\operatorname{sech}(x)}{b^5} - \frac{a(a^2-b^2)\tanh(x)\operatorname{sech}(x)}{2b^4} - \frac{(a^2-b^2)\operatorname{sech}^3(x)}{3b^3} + \frac{3a\arctan(\sinh(x))}{8b^2} + \frac{a\tanh(x)\operatorname{sech}^3(x)}{4b^2} + \frac{3a\tanh(x)\operatorname{sech}(x)}{8b^2} + \frac{\operatorname{sech}^5(x)}{5b}$$

```
[In] Int[Sech[x]^7/(a + b*Tanh[x]),x]
```

```
[Out] (3*a*ArcTan[Sinh[x]])/(8*b^2) - (a*(a^2 - b^2)*ArcTan[Sinh[x]])/(2*b^4) + (
a*(a^2 - b^2)^2*ArcTan[Sinh[x]])/b^6 - ((a^2 - b^2)^(5/2)*ArcTan[(Cosh[x]*(
b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/b^6 + ((a^2 - b^2)^2*Sech[x])/b^5 - ((a^2
 - b^2)*Sech[x]^3)/(3*b^3) + Sech[x]^5/(5*b) + (3*a*Sech[x]*Tanh[x])/(8*b^2
) - (a*(a^2 - b^2)*Sech[x]*Tanh[x])/(2*b^4) + (a*Sech[x]^3*Tanh[x])/(4*b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
```

```
, 0] && IGtQ[m, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \operatorname{sech}^5(x)(a-b\tanh(x))\,dx}{b^2}-\frac{(a^2-b^2)\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx}{b^2}\\
&= \frac{\operatorname{sech}^5(x)}{5b}+\frac{a\int \operatorname{sech}^5(x)\,dx}{b^2}-\frac{(a^2-b^2)\int \operatorname{sech}^3(x)(a-b\tanh(x))\,dx}{b^4}\\
&\quad+\frac{(a^2-b^2)^2\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx}{b^4}\\
&= -\frac{(a^2-b^2)\operatorname{sech}^3(x)}{3b^3}+\frac{\operatorname{sech}^5(x)}{5b}+\frac{a\operatorname{sech}^3(x)\tanh(x)}{4b^2}\\
&\quad+\frac{(3a)\int \operatorname{sech}^3(x)\,dx}{4b^2}-\frac{(a(a^2-b^2))\int \operatorname{sech}^3(x)\,dx}{b^4}\\
&\quad+\frac{(a^2-b^2)^2\int \operatorname{sech}(x)(a-b\tanh(x))\,dx}{b^6}-\frac{(a^2-b^2)^3\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{b^6}\\
&= \frac{(a^2-b^2)^2\operatorname{sech}(x)}{b^5}-\frac{(a^2-b^2)\operatorname{sech}^3(x)}{3b^3}+\frac{\operatorname{sech}^5(x)}{5b}+\frac{3a\operatorname{sech}(x)\tanh(x)}{8b^2}\\
&\quad-\frac{a(a^2-b^2)\operatorname{sech}(x)\tanh(x)}{2b^4}+\frac{a\operatorname{sech}^3(x)\tanh(x)}{4b^2}+\frac{(3a)\int \operatorname{sech}(x)\,dx}{8b^2}\\
&\quad-\frac{(a(a^2-b^2))\int \operatorname{sech}(x)\,dx}{2b^4}+\frac{\left(a(a^2-b^2)^2\right)\int \operatorname{sech}(x)\,dx}{b^6}\\
&\quad-\frac{\left(i(a^2-b^2)^3\right)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,\cosh(x)(-ib-ia\tanh(x))\right)}{b^6}
\end{aligned}$$

$$= \frac{3a\arctan(\sinh(x))}{8b^2} - \frac{a(a^2-b^2)\arctan(\sinh(x))}{2b^4} + \frac{a(a^2-b^2)^2\arctan(\sinh(x))}{b^6} - \frac{(a^2-b^2)^{5/2}\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^6} + \frac{(a^2-b^2)^2\operatorname{sech}(x)}{b^5} - \frac{(a^2-b^2)\operatorname{sech}^3(x)}{3b^3} + \frac{\operatorname{sech}^5(x)}{5b} + \frac{3a\operatorname{sech}(x)\tanh(x)}{8b^2} - \frac{a(a^2-b^2)\operatorname{sech}(x)\tanh(x)}{2b^4} + \frac{a\operatorname{sech}^3(x)\tanh(x)}{4b^2}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx = \frac{30\left(a(8a^4-20a^2b^2+15b^4)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - 8\sqrt{a-b}\sqrt{a+b}(a^2-b^2)^2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\right) + 24b^5\operatorname{sec}}{120}$$

```
[In] Integrate[Sech[x]^7/(a + b*Tanh[x]),x]
```

```
[Out] (30*(a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Tanh[x/2]] - 8*Sqrt[a - b]*Sqrt
[a + b]*(a^2 - b^2)^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])
+ 24*b^5*Sech[x]^5 + 10*b^3*Sech[x]^3*(-4*a^2 + 4*b^2 + 3*a*b*Tanh[x]) + 15
*b*Sech[x]*(8*(a^2 - b^2)^2 + (-4*a^3*b + 7*a*b^3)*Tanh[x]))/(120*b^6)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(143) = 286.

Time = 134.96 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.06

method	result
default	$\frac{2(-a^6+3a^4b^2-3a^2b^4+b^6)\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^6\sqrt{a^2-b^2}} + \frac{2\left(\left(\frac{1}{2}a^3b^2-\frac{9}{8}a\,b^4\right)\tanh\left(\frac{x}{2}\right)^9+\left(a^4b-3a^2b^3+3b^5\right)\tanh\left(\frac{x}{2}\right)^8+\left(a^3b^2-\frac{5}{4}a\,b^4\right)\tanh\right.}{}$
risch	$\frac{e^x\left(120a^4e^{8x}-60a^3b\,e^{8x}-240a^2b^2e^{8x}+105a\,b^3e^{8x}+120b^4e^{8x}+480a^4e^{6x}-120a^3b\,e^{6x}-1120a^2b^2e^{6x}+330a\,b^3e^{6x}+640b^4e^{6x}+720\,e^{4x}\right.}{60b^5(}$

```
[In] int(sech(x)^7/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-a^6+3*a^4*b^2-3*a^2*b^4+b^6)/b^6/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1
/2*x)+2*b)/(a^2-b^2)^(1/2))+2/b^6*(((1/2*a^3*b^2-9/8*a*b^4)*tanh(1/2*x)^9+(
a^4*b-3*a^2*b^3+3*b^5)*tanh(1/2*x)^8+(a^3*b^2-5/4*a*b^4)*tanh(1/2*x)^7+(4*a
^4*b-10*a^2*b^3+6*b^5)*tanh(1/2*x)^6+(6*a^4*b-40/3*a^2*b^3+28/3*b^5)*tanh(1
/2*x)^4+(-a^3*b^2+5/4*a*b^4)*tanh(1/2*x)^3+(4*a^4*b-26/3*a^2*b^3+14/3*b^5)*
tanh(1/2*x)^2+(-1/2*a^3*b^2+9/8*a*b^4)*tanh(1/2*x)+a^4*b-7/3*a^2*b^3+23/15*
```

```
b^5)/(1+tanh(1/2*x)^2)^5+1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*arctan(tanh(1/2*x)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3227 vs. $2(143) = 286$.

Time = 0.41 (sec) , antiderivative size = 6509, normalized size of antiderivative = 41.46

$$\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^7/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^7{(x)}}{a+b\tanh{(x)}}\,dx$$

```
[In] integrate(sech(x)**7/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**7/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^7(x)}{a+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^7/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(143) = 286$.

Time = 0.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.08

$$\int \frac{\operatorname{sech}^7(x)}{a + b\tanh(x)}\,dx$$
$$= \frac{(8\,a^5 - 20\,a^3b^2 + 15\,ab^4)\arctan\left(e^x\right)}{4\,b^6} - \frac{2\,(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\arctan\left(\frac{ae^x + be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^6}$$
$$+ \frac{120\,a^4e^{(9\,x)} - 60\,a^3be^{(9\,x)} - 240\,a^2b^2e^{(9\,x)} + 105\,ab^3e^{(9\,x)} + 120\,b^4e^{(9\,x)} + 480\,a^4e^{(7\,x)} - 120\,a^3be^{(7\,x)} - 112}{}$$

```
[In] integrate(sech(x)^7/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 1/4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*arctan(e^x)/b^6 - 2*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b
^6) + 1/60*(120*a^4*e^(9*x) - 60*a^3*b*e^(9*x) - 240*a^2*b^2*e^(9*x) + 105*
a*b^3*e^(9*x) + 120*b^4*e^(9*x) + 480*a^4*e^(7*x) - 120*a^3*b*e^(7*x) - 112
0*a^2*b^2*e^(7*x) + 330*a*b^3*e^(7*x) + 640*b^4*e^(7*x) + 720*a^4*e^(5*x) -
 1760*a^2*b^2*e^(5*x) + 1424*b^4*e^(5*x) + 480*a^4*e^(3*x) + 120*a^3*b*e^(3
*x) - 1120*a^2*b^2*e^(3*x) - 330*a*b^3*e^(3*x) + 640*b^4*e^(3*x) + 120*a^4*
e^x + 60*a^3*b*e^x - 240*a^2*b^2*e^x - 105*a*b^3*e^x + 120*b^4*e^x)/(b^5*(e
^(2*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 7.59 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.85

$$\int \frac{\operatorname{sech}^7(x)}{a + b\tanh(x)}\,dx = \frac{32\,e^x}{5\,b\,\left(5\,e^{2\,x} + 10\,e^{4\,x} + 10\,e^{6\,x} + 5\,e^{8\,x} + e^{10\,x} + 1\right)}$$
$$- \frac{\ln\left(\sqrt{-(a+b)^5\,(a-b)^5} + a^5\,e^x + b^5\,e^x + a\,b^4\,e^x + a^4\,b\,e^x - 2\,a^2\,b^3\,e^x - 2\,a^3\,b^2\,e^x\right)\,\sqrt{-(a+b)^5\,(a-b}}{b^6}$$
$$+ \frac{\ln\left(a^5\,e^x - \sqrt{-(a+b)^5\,(a-b)^5} + b^5\,e^x + a\,b^4\,e^x + a^4\,b\,e^x - 2\,a^2\,b^3\,e^x - 2\,a^3\,b^2\,e^x\right)\,\sqrt{-(a+b)^5\,(a-b}}{b^6}$$
$$- \frac{e^x\,\left(-12\,a^3 + 16\,a^2\,b + 9\,a\,b^2 - 16\,b^3\right)}{6\,b^4\,\left(2\,e^{2\,x} + e^{4\,x} + 1\right)} + \frac{e^x\,\left(8\,a^4 - 4\,a^3\,b - 16\,a^2\,b^2 + 7\,a\,b^3 + 8\,b^4\right)}{4\,b^5\,\left(e^{2\,x} + 1\right)}$$
$$+ \frac{4\,e^x\,(5\,a - 16\,b)}{5\,b^2\,\left(4\,e^{2\,x} + 6\,e^{4\,x} + 4\,e^{6\,x} + e^{8\,x} + 1\right)} + \frac{2\,e^x\,\left(20\,a^2 - 45\,a\,b + 28\,b^2\right)}{15\,b^3\,\left(3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1\right)}$$
$$- \frac{a\,\ln\left(e^x - \mathrm{i}\right)\,\left(8\,a^4 - 20\,a^2\,b^2 + 15\,b^4\right)\,1\mathrm{i}}{8\,b^6} + \frac{a\,\ln\left(e^x + 1\mathrm{i}\right)\,\left(8\,a^4 - 20\,a^2\,b^2 + 15\,b^4\right)\,1\mathrm{i}}{8\,b^6}$$

```
[In] int(1/(cosh(x)^7*(a + b*tanh(x))),x)
```

```
[Out] (32*exp(x))/(5*b*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp
(10*x) + 1)) - (log((-(a + b)^5*(a - b)^5)^(1/2) + a^5*exp(x) + b^5*exp(x)
+ a*b^4*exp(x) + a^4*b*exp(x) - 2*a^2*b^3*exp(x) - 2*a^3*b^2*exp(x))*(-(a +
 b)^5*(a - b)^5)^(1/2))/b^6 + (log(a^5*exp(x) - (-(a + b)^5*(a - b)^5)^(1/2
) + b^5*exp(x) + a*b^4*exp(x) + a^4*b*exp(x) - 2*a^2*b^3*exp(x) - 2*a^3*b^2
*exp(x))*(-(a + b)^5*(a - b)^5)^(1/2))/b^6 - (exp(x)*(9*a*b^2 + 16*a^2*b -
12*a^3 - 16*b^3))/(6*b^4*(2*exp(2*x) + exp(4*x) + 1)) + (exp(x)*(7*a*b^3 -
4*a^3*b + 8*a^4 + 8*b^4 - 16*a^2*b^2))/(4*b^5*(exp(2*x) + 1)) - (a*log(exp(
x) - 1i)*(8*a^4 + 15*b^4 - 20*a^2*b^2)*1i)/(8*b^6) + (a*log(exp(x) + 1i)*(8
*a^4 + 15*b^4 - 20*a^2*b^2)*1i)/(8*b^6) + (4*exp(x)*(5*a - 16*b))/(5*b^2*(4
*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) + (2*exp(x)*(20*a^2 -
45*a*b + 28*b^2))/(15*b^3*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1))
```

3.110 $\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 102

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = -\frac{a(2a^2-3b^2)\arctan(\sinh(x))}{2b^4} + \frac{(a^2-b^2)^{3/2}\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^4} - \frac{(a^2-b^2)\operatorname{sech}(x)}{b^3} + \frac{\operatorname{sech}^3(x)}{3b} + \frac{a\operatorname{sech}(x)\tanh(x)}{2b^2}$$

```
[Out] -1/2*a*(2*a^2-3*b^2)*arctan(sinh(x))/b^4+(a^2-b^2)^(3/2)*arctan(cosh(x)*(b+
a*tanh(x))/(a^2-b^2)^(1/2))/b^4-(a^2-b^2)*sech(x)/b^3+1/3*sech(x)^3/b+1/2*a
*sech(x)*tanh(x)/b^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3591, 3567, 3853, 3855, 3590, 212}

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = -\frac{a(a^2-b^2)\arctan(\sinh(x))}{b^4} + \frac{(a^2-b^2)^{3/2}\arctan\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^4} - \frac{(a^2-b^2)\operatorname{sech}(x)}{b^3} + \frac{a\arctan(\sinh(x))}{2b^2} + \frac{a\tanh(x)\operatorname{sech}(x)}{2b^2} + \frac{\operatorname{sech}^3(x)}{3b}$$

```
[In] Int[Sech[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] (a*ArcTan[Sinh[x]])/(2*b^2) - (a*(a^2 - b^2)*ArcTan[Sinh[x]])/b^4 + ((a^2 -
 b^2)^(3/2)*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/b^4 - ((a^2
- b^2)*Sech[x])/b^3 + Sech[x]^3/(3*b) + (a*Sech[x]*Tanh[x])/(2*b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral} = \frac{\int \operatorname{sech}^3(x)(a-b\tanh(x))\,dx}{b^2} - \frac{(a^2-b^2)\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx}{b^2}$$

$$= \frac{\operatorname{sech}^3(x)}{3b} + \frac{a\int \operatorname{sech}^3(x)\,dx}{b^2} - \frac{(a^2-b^2)\int \operatorname{sech}(x)(a-b\tanh(x))\,dx}{b^4} + \frac{(a^2-b^2)^2\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{b^4}$$

$$= -\frac{(a^2-b^2)\operatorname{sech}(x)}{b^3} + \frac{\operatorname{sech}^3(x)}{3b} + \frac{a\operatorname{sech}(x)\tanh(x)}{2b^2} + \frac{a\int \operatorname{sech}(x)\,dx}{2b^2} - \frac{(a(a^2-b^2))\int \operatorname{sech}(x)\,dx}{b^4} + \frac{\left(i(a^2-b^2)^2\right)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, \cosh(x)(-ib-ia\tanh(x))\right)}{b^4}$$

$$= \frac{a\arctan(\sinh(x))}{2b^2} - \frac{a(a^2-b^2)\arctan(\sinh(x))}{b^4} + \frac{(a^2-b^2)^{3/2}\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^4} - \frac{(a^2-b^2)\operatorname{sech}(x)}{b^3} + \frac{\operatorname{sech}^3(x)}{3b} + \frac{a\operatorname{sech}(x)\tanh(x)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = \frac{-6\left(a(2a^2-3b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + 2\sqrt{a-b}\sqrt{a+b}(-a^2+b^2)\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)\right) + 2b^3\operatorname{sech}^3(x) + 3}{6b^4}$$

```
[In] Integrate[Sech[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] (-6*(a*(2*a^2 - 3*b^2)*ArcTan[Tanh[x/2]] + 2*Sqrt[a - b]*Sqrt[a + b]*(-a^2
+ b^2)*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])]) + 2*b^3*Sech[x]
^3 + 3*b*Sech[x]*(-2*a^2 + 2*b^2 + a*b*Tanh[x]))/(6*b^4)
```

Maple [A] (verified)

Time = 29.88 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.61

method	result
default	$\frac{2\left(a^4-2a^2b^2+b^4\right)\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}}-\frac{2\left(\frac{\frac{a\,b^2\tanh\left(\frac{x}{2}\right)^5}{2}+\left(a^2b-2b^3\right)\tanh\left(\frac{x}{2}\right)^4+\left(2a^2b-2b^3\right)\tanh\left(\frac{x}{2}\right)^2-\frac{a\,b^2\tanh\left(\frac{x}{2}\right)}{2}+a^2b-\frac{4b^3}{3}}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}\right.}{b^4}$
risch	$-\frac{e^x\left(6a^2e^{4x}-3ab\,e^{4x}-6b^2e^{4x}+12a^2e^{2x}-20b^2e^{2x}+6a^2+3ab-6b^2\right)}{3b^3\left(1+e^{2x}\right)^3}+\frac{ia^3\ln\left(e^x-i\right)}{b^4}-\frac{3ia\ln\left(e^x-i\right)}{2b^2}-\frac{ia^3\ln\left(e^x+i\right)}{b^4}+\frac{3ia\ln\left(e^x+\right.}{2b^2}$

[In] int(sech(x)^5/(a+b*tanh(x)),x,method=_RETURNVERBOSE)

[Out] 2*(a^4-2*a^2*b^2+b^4)/b^4/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))-2/b^4*((1/2*a*b^2*tanh(1/2*x)^5+(a^2*b-2*b^3)*tanh(1/2*x)^4+(2*a^2*b-2*b^3)*tanh(1/2*x)^2-1/2*a*b^2*tanh(1/2*x)+a^2*b-4/3*b^3)/(1+tanh(1/2*x)^2)^3+1/2*a*(2*a^2-3*b^2)*arctan(tanh(1/2*x)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(92) = 184$.

Time = 0.31 (sec) , antiderivative size = 2043, normalized size of antiderivative = 20.03

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

[In] integrate(sech(x)^5/(a+b*tanh(x)),x, algorithm="fricas")

[Out] [-1/3*(3*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^5 + 15*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)*sinh(x)^4 + 3*(2*a^2*b - a*b^2 - 2*b^3)*sinh(x)^5 + 4*(3*a^2*b - 5*b^3)*cosh(x)^3 + 2*(6*a^2*b - 10*b^3 + 15*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^3 + 6*(5*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^3 + 2*(3*a^2*b - 5*b^3)*cosh(x))*sinh(x)^2 + 3*((a^2 - b^2)*cosh(x)^6 + 6*(a^2 - b^2)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4 + 3*(5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(5*(a^2 - b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 3*((2*a^3 - 3*a*b^2)*cosh(x)^6 + 6*(2*a^3 - 3*a*b^2)*cosh(x)*sinh(x)^5 + (2*a^3 - 3*a*b^2)*sinh(x)^6 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^4 + 3*(2*a^3 - 3*a*b^2 + 5*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(2*a^3 - 3*a*b^2)*cosh(x)^3 + 3*(2*a^3 - 3*a*b^2)*cosh(x))*sinh(x)^3 + 2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2 + 3*(5*(2*a^3 - 3*a*b^2)*cosh(x)^4 + 2*a^3 - 3*a*b^2 + 6*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((2*a^3 - 3*a*b^2)*cosh(x)^5 + 2*

```
(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*sinh(x))*arctan(co
sh(x) + sinh(x)) + 3*(2*a^2*b + a*b^2 - 2*b^3)*cosh(x) + 3*(5*(2*a^2*b - a*
b^2 - 2*b^3)*cosh(x)^4 + 2*a^2*b + a*b^2 - 2*b^3 + 4*(3*a^2*b - 5*b^3)*cosh
(x)^2)*sinh(x))/(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 +
3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 + b^4)*sinh(x)^4 + b
^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 + 6
*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*b^4*cosh(x)^3 + b^4*
cosh(x))*sinh(x)), -1/3*(3*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^5 + 15*(2*a^2*
b - a*b^2 - 2*b^3)*cosh(x)*sinh(x)^4 + 3*(2*a^2*b - a*b^2 - 2*b^3)*sinh(x)^
5 + 4*(3*a^2*b - 5*b^3)*cosh(x)^3 + 2*(6*a^2*b - 10*b^3 + 15*(2*a^2*b - a*b
^2 - 2*b^3)*cosh(x)^2)*sinh(x)^3 + 6*(5*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^3
 + 2*(3*a^2*b - 5*b^3)*cosh(x))*sinh(x)^2 + 6*((a^2 - b^2)*cosh(x)^6 + 6*(a
^2 - b^2)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)
^4 + 3*(5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*c
osh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(
5*(a^2 - b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 +
a^2 - b^2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b^2
)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x)
 + (a + b)*sinh(x))) + 3*((2*a^3 - 3*a*b^2)*cosh(x)^6 + 6*(2*a^3 - 3*a*b^2)
*cosh(x)*sinh(x)^5 + (2*a^3 - 3*a*b^2)*sinh(x)^6 + 3*(2*a^3 - 3*a*b^2)*cosh
(x)^4 + 3*(2*a^3 - 3*a*b^2 + 5*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(
5*(2*a^3 - 3*a*b^2)*cosh(x)^3 + 3*(2*a^3 - 3*a*b^2)*cosh(x))*sinh(x)^3 + 2*
a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2 + 3*(5*(2*a^3 - 3*a*b^2)*cosh
(x)^4 + 2*a^3 - 3*a*b^2 + 6*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 6*((2*
a^3 - 3*a*b^2)*cosh(x)^5 + 2*(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2
)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(2*a^2*b + a*b^2 - 2*b^3)
*cosh(x) + 3*(5*(2*a^2*b - a*b^2 - 2*b^3)*cosh(x)^4 + 2*a^2*b + a*b^2 - 2*b
^3 + 4*(3*a^2*b - 5*b^3)*cosh(x)^2)*sinh(x))/(b^4*cosh(x)^6 + 6*b^4*cosh(x)
*sinh(x)^5 + b^4*sinh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*c
osh(x)^2 + b^4)*sinh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(
x)^3 + 3*(5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(
x)^5 + 2*b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(sech(x)**5/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)**5/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^5/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx = -\frac{(2\,a^3-3\,ab^2)\arctan(e^x)}{b^4} + \frac{2\,(a^4-2\,a^2b^2+b^4)\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^4} - \frac{6\,a^2e^{(5\,x)}-3\,abe^{(5\,x)}-6\,b^2e^{(5\,x)}+12\,a^2e^{(3\,x)}-20\,b^2e^{(3\,x)}+6\,a^2e^x+3\,abe^x-6\,b^2e^x}{3\,b^3\left(e^{(2\,x)}+1\right)^3}$$

```
[In] integrate(sech(x)^5/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -(2*a^3 - 3*a*b^2)*arctan(e^x)/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((a*e^
x + b*e^x)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^4) - 1/3*(6*a^2*e^(5*x) - 3*
a*b*e^(5*x) - 6*b^2*e^(5*x) + 12*a^2*e^(3*x) - 20*b^2*e^(3*x) + 6*a^2*e^x +
 3*a*b*e^x - 6*b^2*e^x)/(b^3*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.60

$$\int \frac{\operatorname{sech}^5(x)}{a+b\tanh(x)}\,dx$$
$$= \frac{\ln\left(\sqrt{-(a+b)^3\,(a-b)^3}+a^3\,\mathrm{e}^x-b^3\,\mathrm{e}^x-a\,b^2\,\mathrm{e}^x+a^2\,b\,\mathrm{e}^x\right)\,\sqrt{-(a+b)^3\,(a-b)^3}}{b^4}$$
$$-\frac{8\,\mathrm{e}^x}{3\,b\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}$$
$$-\frac{\ln\left(\sqrt{-(a+b)^3\,(a-b)^3}-a^3\,\mathrm{e}^x+b^3\,\mathrm{e}^x+a\,b^2\,\mathrm{e}^x-a^2\,b\,\mathrm{e}^x\right)\,\sqrt{-(a+b)^3\,(a-b)^3}}{b^4}$$
$$-\frac{2\,\mathrm{e}^x\,(3\,a-4\,b)}{3\,b^2\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}+\frac{\mathrm{e}^x\,\left(-2\,a^2+a\,b+2\,b^2\right)}{b^3\,\left(\mathrm{e}^{2\,x}+1\right)}$$
$$+\frac{a\,\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,\left(2\,a^2-3\,b^2\right)\,1\mathrm{i}}{2\,b^4}-\frac{a\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,\left(2\,a^2-3\,b^2\right)\,1\mathrm{i}}{2\,b^4}$$

```
[In] int(1/(cosh(x)^5*(a + b*tanh(x))),x)
```

```
[Out] (log((-(a + b)^3*(a - b)^3)^(1/2) + a^3*exp(x) - b^3*exp(x) - a*b^2*exp(x)
+ a^2*b*exp(x))*(-(a + b)^3*(a - b)^3)^(1/2))/b^4 - (8*exp(x))/(3*b*(3*exp(
2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (log((-(a + b)^3*(a - b)^3)^(1/2) - a^
3*exp(x) + b^3*exp(x) + a*b^2*exp(x) - a^2*b*exp(x))*(-(a + b)^3*(a - b)^3)
^(1/2))/b^4 - (2*exp(x)*(3*a - 4*b))/(3*b^2*(2*exp(2*x) + exp(4*x) + 1)) +
(exp(x)*(a*b - 2*a^2 + 2*b^2))/(b^3*(exp(2*x) + 1)) + (a*log(exp(x) - 1i)*(
2*a^2 - 3*b^2)*1i)/(2*b^4) - (a*log(exp(x) + 1i)*(2*a^2 - 3*b^2)*1i)/(2*b^4
)
```

3.111 $\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx = \frac{a\arctan(\sinh(x))}{b^2} - \frac{\sqrt{a^2-b^2}\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^2} + \frac{\operatorname{sech}(x)}{b}$$

```
[Out] a*arctan(sinh(x))/b^2+sech(x)/b-arctan(cosh(x)*(b+a*tanh(x))/(a^2-b^2)^(1/2
))*(a^2-b^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3591, 3567, 3855, 3590, 212}

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx = -\frac{\sqrt{a^2-b^2}\arctan\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2} + \frac{a\arctan(\sinh(x))}{b^2} + \frac{\operatorname{sech}(x)}{b}$$

```
[In] Int[Sech[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (a*ArcTan[Sinh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTan[(Cosh[x]*(b + a*Tanh[x]))
/Sqrt[a^2 - b^2]])/b^2 + Sech[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \operatorname{sech}(x)(a-b\tanh(x))\,dx}{b^2}-\frac{(a^2-b^2)\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{b^2}\\
&= \frac{\operatorname{sech}(x)}{b}+\frac{a\int \operatorname{sech}(x)\,dx}{b^2}-\frac{(i(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,\cosh(x)(-ib-ia\tanh(x))\right)}{b^2}\\
&= \frac{a\arctan(\sinh(x))}{b^2}-\frac{\sqrt{a^2-b^2}\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{\operatorname{sech}(x)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx=\frac{2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)+b\operatorname{sech}(x)}{b^2}$$

```
[In] Integrate[Sech[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (2*a*ArcTan[Tanh[x/2]] - 2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])
/(Sqrt[a - b]*Sqrt[a + b])] + b*Sech[x])/b^2
```

Maple [A] (verified)

Time = 4.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\frac{2b}{1+\tanh\left(\frac{x}{2}\right)^2}+2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}+\frac{2\left(-a^2+b^2\right)\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}$	77
risch	$\frac{2\,\mathrm{e}^x}{b(1+\mathrm{e}^{2x})}+\frac{ia\ln(\mathrm{e}^x+i)}{b^2}-\frac{ia\ln(\mathrm{e}^x-i)}{b^2}+\frac{\sqrt{-a^2+b^2}\,\ln\left(\mathrm{e}^x-\frac{\sqrt{-a^2+b^2}}{a+b}\right)}{b^2}-\frac{\sqrt{-a^2+b^2}\,\ln\left(\mathrm{e}^x+\frac{\sqrt{-a^2+b^2}}{a+b}\right)}{b^2}$	117

```
[In] int(sech(x)^3/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^2*(b/(1+tanh(1/2*x)^2)+a*arctan(tanh(1/2*x)))+2*(-a^2+b^2)/b^2/(a^2-b^2
)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(52) = 104.

Time = 0.28 (sec) , antiderivative size = 309, normalized size of antiderivative = 5.52

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx$$
$$=\left[\frac{\sqrt{-a^2+b^2}\left(\cosh(x)^2+2\,\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x}\right.}{b^2\cosh(}\right.$$

```
[In] integrate(sech(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(((a
+ b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^
2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x
```

```
)*sinh(x) + (a + b)*sinh(x)^2 + a - b)) + 2*(a*cosh(x)^2 + 2*a*cosh(x)*sinh
(x) + a*sinh(x)^2 + a)*arctan(cosh(x) + sinh(x)) + 2*b*cosh(x) + 2*b*sinh(x
))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2), 2*(sqrt(a
^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(sqrt(a^2 -
 b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + (a*cosh(x)^2 + 2*a*cosh(x)*sin
h(x) + a*sinh(x)^2 + a)*arctan(cosh(x) + sinh(x)) + b*cosh(x) + b*sinh(x))/
(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}^3\left(x\right)}{a+b\tanh\left(x\right)}\,dx$$

```
[In] integrate(sech(x)**3/(a+b*tanh(x)),x)
[Out] Integral(sech(x)**3/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx = \frac{2\,a\arctan\left(e^{x}\right)}{b^2} - \frac{2\,\sqrt{a^2-b^2}\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{b^2} + \frac{2\,e^x}{b(e^{(2\,x)}+1)}$$

```
[In] integrate(sech(x)^3/(a+b*tanh(x)),x, algorithm="giac")
[Out] 2*a*arctan(e^x)/b^2 - 2*sqrt(a^2 - b^2)*arctan((a*e^x + b*e^x)/sqrt(a^2 - b
^2))/b^2 + 2*e^x/(b*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.12

$$\int \frac{\operatorname{sech}^3(x)}{a+b\tanh(x)}\,dx = \frac{\ln\left(a\,\mathrm{e}^x+b\,\mathrm{e}^x-\sqrt{b^2-a^2}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}}{b^2} - \frac{\ln\left(a\,\mathrm{e}^x+b\,\mathrm{e}^x+\sqrt{b^2-a^2}\right)\,\sqrt{-\left(a+b\right)\,\left(a-b\right)}}{b^2} + \frac{2\,\mathrm{e}^x}{b\,\left(\mathrm{e}^{2\,x}+1\right)} - \frac{a\,\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}}{b^2} + \frac{a\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{b^2}$$

```
[In] int(1/(cosh(x)^3*(a + b*tanh(x))),x)
```

```
[Out] (a*log(exp(x) + 1i)*1i)/b^2 - (a*log(exp(x) - 1i)*1i)/b^2 - (log(a*exp(x) +
 b*exp(x) + (b^2 - a^2)^(1/2))*(-(a + b)*(a - b))^(1/2))/b^2 + (log(a*exp(x
) + b*exp(x) - (b^2 - a^2)^(1/2))*(-(a + b)*(a - b))^(1/2))/b^2 + (2*exp(x)
)/(b*(exp(2*x) + 1))
```

3.112 $\int \frac{\text{sech}(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\text{sech}(x)}{a+b\tanh(x)}\,dx=\frac{\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] arctan(cosh(x)*(b+a*tanh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3590, 212}

$$\int \frac{\text{sech}(x)}{a+b\tanh(x)}\,dx=\frac{\arctan\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] Int[Sech[x]/(a + b*Tanh[x]),x]
[Out] ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
```

```
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= i\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, \cosh(x)(-ib-ia\tanh(x))\right)\\ &= \frac{\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx = \frac{2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Integrate[Sech[x]/(a + b*Tanh[x]),x]
[Out] (2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/(Sqrt[a - b]*Sqrt[a
 + b])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
default	$\frac{2\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	39
risch	$-\frac{\ln\left(\mathrm{e}^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	70

```
[In] int(sech(x)/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{-a^2+b^2}(\cosh(x)+\sinh(x))-a+b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2+a-b}\right)}{a^2-b^2}, -\frac{2\,\arctan\left(\frac{\sqrt{a^2-b^2}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

```
[In] integrate(sech(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
+ b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x)) - a + b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a - b))/(a^2 - b
^2), -2*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/sqrt(a^
2 - b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(sech(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx = \frac{2\,\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] integrate(sech(x)/(a+b*tanh(x)),x, algorithm="giac")
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2-b^2}}{a-b}\right)}{\sqrt{a^2-b^2}}$$

```
[In] int(1/(cosh(x)*(a + b*tanh(x))),x)
[Out] (2*atan((exp(x)*(a^2 - b^2)^(1/2))/(a - b)))/(a^2 - b^2)^(1/2)
```

3.113 $\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 73

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx=-\frac{b^2\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

```
[Out] -b^2*arctan(cosh(x)*(b+a*tanh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3592, 3567, 2717, 3590, 212}

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx=-\frac{b^2\arctan\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

```
[In] Int[Cosh[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b^2*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2)
) - (b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cosh(x)(a-b\tanh(x))\,dx}{a^2-b^2}-\frac{b^2\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\
&=-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{(ib^2)\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,\cosh(x)(-ib-ia\tanh(x))\right)}{a^2-b^2}\\
&=-\frac{b^2\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx = -\frac{2b^2\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} + \frac{b\cosh(x)}{-a^2+b^2} + \frac{a\sinh(x)}{a^2-b^2}$$

```
[In] Integrate[Cosh[x]/(a + b*Tanh[x]),x]
```

```
[Out] (-2*b^2*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])])/((a - b)^(3/2)
*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

method	result	size
default	$-\frac{2b^2\arctan\left(\frac{2a\tanh\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{2}{(2a-2b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2}{(2a+2b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	93
risch	$\frac{e^{x}}{2a+2b}-\frac{e^{-x}}{2(a-b)}-\frac{b^2\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{b^2\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	122

```
[In] int(cosh(x)/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tanh(1/2*x)+2*b)/(a^2-b^
2)^(1/2))-2/(2*a-2*b)/(tanh(1/2*x)+1)-2/(2*a+2*b)/(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.96

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx$$

$$= \left[-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2((a^4-}\right.$$

$$-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2((a^4-2a^2b^2+b^4)\cosh(x)+}$$

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*log(((a + b)*co
sh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2
)*(cosh(x) + sinh(x)) - a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + a - b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 -
 2*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b
- a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) -
(a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 - 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(
a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F]

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx=\int \frac{\cosh{(x)}}{a+b\tanh{(x)}}\,dx$$

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx=-\frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

```
[In] integrate(cosh(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -2*b^2*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) - 1/2*e^(-
x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

$$\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx = \frac{e^x}{2a+2b} - \frac{e^{-x}}{2a-2b} - \frac{b^2 \ln\left(-\frac{2b^2}{(a+b)^{5/2}\sqrt{b-a}} - \frac{2b^2 e^x}{-a^3-a^2b+ab^2+b^3}\right)}{(a+b)^{3/2}(b-a)^{3/2}} + \frac{b^2 \ln\left(\frac{2b^2}{(a+b)^{5/2}\sqrt{b-a}} - \frac{2b^2 e^x}{-a^3-a^2b+ab^2+b^3}\right)}{(a+b)^{3/2}(b-a)^{3/2}}$$

```
[In] int(cosh(x)/(a + b*tanh(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) - (b^2*log(- (2*b^2)/((a + b)^(5/2
)*(b - a)^(1/2)) - (2*b^2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3
/2)*(b - a)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(b - a)^(1/2)) - (2*b^
2*exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(b - a)^(3/2))
```

3.114 $\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 132

$$\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx = \frac{b^4\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{b^3\cosh(x)}{(a^2-b^2)^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}-\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh(x)}{a^2-b^2}+\frac{a\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] b^4*arctan(cosh(x)*(b+a*tanh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+b^3*cosh(
x)/(a^2-b^2)^2-1/3*b*cosh(x)^3/(a^2-b^2)-a*b^2*sinh(x)/(a^2-b^2)^2+a*sinh(x
)/(a^2-b^2)+1/3*a*sinh(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3592, 3567, 2713, 2717, 3590, 212}

$$\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx = \frac{b^4\arctan\left(\frac{\cosh(x)(a\tanh(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{a\sinh^3(x)}{3(a^2-b^2)}-\frac{ab^2\sinh(x)}{(a^2-b^2)^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh^3(x)}{3(a^2-b^2)}+\frac{b^3\cosh(x)}{(a^2-b^2)^2}$$

```
[In] Int[Cosh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (b^4*ArcTan[(Cosh[x]*(b + a*Tanh[x]))/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) +
 (b^3*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) - (a*b^2*Sinh[
x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \cosh^3(x)(a-b\tanh(x))\,dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\
&= -\frac{b\cosh^3(x)}{3\left(a^2-b^2\right)}-\frac{b^2\int \cosh(x)(a-b\tanh(x))\,dx}{\left(a^2-b^2\right)^2}+\frac{b^4\int \frac{\operatorname{sech}(x)}{a+b\tanh(x)}\,dx}{\left(a^2-b^2\right)^2}+\frac{a\int \cosh^3(x)\,dx}{a^2-b^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{b^3\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} - \frac{(ab^2)\int \cosh(x)\,dx}{(a^2-b^2)^2} \\
&\quad + \frac{(ib^4)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, \cosh(x)(-ib-ia\tanh(x))\right)}{(a^2-b^2)^2} \\
&\quad + \frac{(ia)\text{Subst}\left(\int (1-x^2)\,dx, x, -i\sinh(x)\right)}{a^2-b^2} \\
&= \frac{b^4\arctan\left(\frac{\cosh(x)(b+a\tanh(x))}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{b^3\cosh(x)}{(a^2-b^2)^2} \\
&\quad - \frac{b\cosh^3(x)}{3(a^2-b^2)} - \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.95

$$
\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx
$$

$$
= \frac{24b^4\sqrt{a+b}\arctan\left(\frac{b+a\tanh\left(\frac{x}{2}\right)}{\sqrt{a-b}\sqrt{a+b}}\right) - 3\sqrt{a-b}b(a^3+a^2b-5ab^2-5b^3)\cosh(x) - (a-b)^{3/2}b(a+b)^2\cosh(3x)}{}
$$

```
[In] Integrate[Cosh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (24*b^4*Sqrt[a + b]*ArcTan[(b + a*Tanh[x/2])/(Sqrt[a - b]*Sqrt[a + b])] - 3
*Sqrt[a - b]*b*(a^3 + a^2*b - 5*a*b^2 - 5*b^3)*Cosh[x] - (a - b)^(3/2)*b*(a
 + b)^2*Cosh[3*x] + 9*a^4*Sqrt[a - b]*Sinh[x] + 9*a^3*Sqrt[a - b]*b*Sinh[x]
 - 21*a^2*Sqrt[a - b]*b^2*Sinh[x] - 21*a*Sqrt[a - b]*b^3*Sinh[x] + a^4*Sqrt
[a - b]*Sinh[3*x] + a^3*Sqrt[a - b]*b*Sinh[3*x] - a^2*Sqrt[a - b]*b^2*Sinh[
3*x] - a*Sqrt[a - b]*b^3*Sinh[3*x])/(12*(a - b)^(5/2)*(a + b)^3)
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30

method	result
risch	$\frac{e^{3x}}{24a+24b}+\frac{3\,e^{x}a}{8(a+b)^2}+\frac{5\,e^{x}b}{8(a+b)^2}-\frac{3\,e^{-x}a}{8(a-b)^2}+\frac{5\,e^{-x}b}{8(a-b)^2}-\frac{e^{-3x}}{24(a-b)}-\frac{b^4\ln\left(e^{x}-\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}+\frac{b^4\ln\left(e^{x}+\frac{a-b}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2}$
default	$-\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3(2a-2b)}+\frac{1}{(2a-2b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{2a-3b}{2(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^3(2a+2b)}-\frac{1}{(2a+2b)(\tanh(\frac{x}{2}}$

```
[In] int(cosh(x)^3/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/(a+b)*exp(x)^3+3/8/(a+b)^2*exp(x)*a+5/8/(a+b)^2*exp(x)*b-3/8/(a-b)^2/e
xp(x)*a+5/8/(a-b)^2/exp(x)*b-1/24/(a-b)/exp(x)^3-1/(-a^2+b^2)^(1/2)*b^4/(a+
b)^2/(a-b)^2*ln(exp(x)-(a-b)/(-a^2+b^2)^(1/2))+1/(-a^2+b^2)^(1/2)*b^4/(a+b)
^2/(a-b)^2*ln(exp(x)+(a-b)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(124) = 248$.

Time = 0.29 (sec) , antiderivative size = 1871, normalized size of antiderivative = 14.17

$$\int \frac{\cosh^3(x)}{a + b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 - 5*b^5)*cosh(x)^4 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 - 5*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^
5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4
 + 5*b^5)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 +
 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 -
6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sin
h(x)^2 - 24*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x
)^2 + b^4*sinh(x)^3)*sqrt(-a^2 + b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*co
sh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(-a^2 + b^2)*(cosh(x) + sinh(x))
- a + b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2
 + a - b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^5 + 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^3
 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*sinh
(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*
cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((
a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2
+ 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a
*b^4 - 5*b^5)*cosh(x)^4 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 - 5*b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^
2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cos
```

```
h(x)^3 + 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(
x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5
)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 -
 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5
 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sinh(x)^2 -
 48*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^
4*sinh(x)^3)*sqrt(a^2 - b^2)*arctan(sqrt(a^2 - b^2)/((a + b)*cosh(x) + (a +
 b)*sinh(x))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 + 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x
)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a + b\tanh(x)}\,dx = \int \frac{\cosh^3{(x)}}{a + b\tanh{(x)}}\,dx$$

```
[In] integrate(cosh(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(cosh(x)**3/(a + b*tanh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{a + b\tanh(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.23

$$\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx = \frac{2\,b^4\arctan\left(\frac{ae^x+be^x}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} - \frac{\left(9\,ae^{(2\,x)}-15\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+9\,a^2e^x+24\,abe^x+15\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

```
[In] integrate(cosh(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((a*e^x + b*e^x)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt
(a^2 - b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) + a - b)*e^(-3*x)/(a^2 - 2*
a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 9*a^2*e^x +
24*a*b*e^x + 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.67

$$\int \frac{\cosh^3(x)}{a+b\tanh(x)}\,dx = \frac{e^{3\,x}}{24\,a+24\,b} - \frac{e^{-3\,x}}{24\,a-24\,b} - \frac{e^{-x}\,(3\,a-5\,b)}{8\,(a-b)^2} + \frac{e^{x}\,(3\,a+5\,b)}{8\,(a+b)^2} - \frac{b^4\,\ln\left(-\frac{2\,b^4\,e^x}{a^5+a^4\,b-2\,a^3\,b^2-2\,a^2\,b^3+a\,b^4+b^5}-\frac{2\,b^4}{(a+b)^{7/2}\,(b-a)^{3/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}} + \frac{b^4\,\ln\left(\frac{2\,b^4}{(a+b)^{7/2}\,(b-a)^{3/2}}-\frac{2\,b^4\,e^x}{a^5+a^4\,b-2\,a^3\,b^2-2\,a^2\,b^3+a\,b^4+b^5}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}$$

```
[In] int(cosh(x)^3/(a + b*tanh(x)),x)
```

```
[Out] exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) - (exp(-x)*(3*a - 5*b))/(8
*(a - b)^2) + (exp(x)*(3*a + 5*b))/(8*(a + b)^2) - (b^4*log(- (2*b^4*exp(x)
)/(a*b^4 + a^4*b + a^5 + b^5 - 2*a^2*b^3 - 2*a^3*b^2) - (2*b^4)/((a + b)^(7
/2)*(b - a)^(3/2))))/((a + b)^(5/2)*(b - a)^(5/2)) + (b^4*log((2*b^4)/((a +
 b)^(7/2)*(b - a)^(3/2)) - (2*b^4*exp(x))/(a*b^4 + a^4*b + a^5 + b^5 - 2*a^
2*b^3 - 2*a^3*b^2)))/((a + b)^(5/2)*(b - a)^(5/2))
```

3.115 $\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx=\frac{5x}{2}-2\log(\cosh(x))-\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^4(x)}{2(1+\tanh(x))}$$

```
[Out] 5/2*x-2*ln(cosh(x))-5/2*tanh(x)+tanh(x)^2-5/6*tanh(x)^3+1/2*tanh(x)^4/(1+ta
nh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3631, 3609, 3606, 3556}

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx=\frac{5x}{2}+\frac{\tanh^4(x)}{2(\tanh(x)+1)}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))$$

```
[In] Int[Tanh[x]^5/(1 + Tanh[x]),x]
```

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^4/(2*(1 + Tanh[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tanh^4(x)}{2(1+\tanh(x))} - \frac{1}{2}\int (4-5\tanh(x))\tanh^3(x)\,dx \\
&= -\frac{5}{6}\tanh^3(x) + \frac{\tanh^4(x)}{2(1+\tanh(x))} + \frac{1}{2}i\int (-5i+4i\tanh(x))\tanh^2(x)\,dx \\
&= \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^4(x)}{2(1+\tanh(x))} + \frac{1}{2}\int \tanh(x)(-4+5\tanh(x))\,dx \\
&= \frac{5x}{2} - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^4(x)}{2(1+\tanh(x))} - 2\int \tanh(x)\,dx \\
&= \frac{5x}{2} - 2\log(\cosh(x)) - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^4(x)}{2(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx = \frac{-12\log(\cosh(x)) - 3(5+4\log(\cosh(x)))\tanh(x) - 9\tanh^2(x) + \tanh^3(x) - 2\tanh^4(x) + 15\text{arctanh}(\tanh(x))(1+\tanh(x))}{6(1+\tanh(x))}$$

[In] Integrate[Tanh[x]^5/(1 + Tanh[x]),x]

[Out] (-12*Log[Cosh[x]] - 3*(5 + 4*Log[Cosh[x]])*Tanh[x] - 9*Tanh[x]^2 + Tanh[x]^3 - 2*Tanh[x]^4 + 15*ArcTanh[Tanh[x]]*(1 + Tanh[x]))/(6*(1 + Tanh[x]))

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\tanh(x)^3}{3}+\frac{\tanh(x)^2}{2}-2\tanh(x)+\frac{1}{2+2\tanh(x)}+\frac{9\ln(1+\tanh(x))}{4}-\frac{\ln(\tanh(x)-1)}{4}$	40
default	$-\frac{\tanh(x)^3}{3}+\frac{\tanh(x)^2}{2}-2\tanh(x)+\frac{1}{2+2\tanh(x)}+\frac{9\ln(1+\tanh(x))}{4}-\frac{\ln(\tanh(x)-1)}{4}$	40
risch	$\frac{9x}{2}+\frac{e^{-2x}}{4}+\frac{4e^{4x}+6e^{2x}+\frac{14}{3}}{(1+e^{2x})^3}-2\ln(1+e^{2x})$	44
parallelrisch	$-\frac{2\tanh(x)^4-15-\tanh(x)^3-12\ln(1-\tanh(x))\tanh(x)-27\tanh(x)x+9\tanh(x)^2-12\ln(1-\tanh(x))-27x}{6(1+\tanh(x))}$	57

[In] int(tanh(x)^5/(1+tanh(x)),x,method=_RETURNVERBOSE)

[Out] -1/3*tanh(x)^3+1/2*tanh(x)^2-2*tanh(x)+1/2/(1+tanh(x))+9/4*ln(1+tanh(x))-1/4*ln(tanh(x)-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 571, normalized size of antiderivative = 13.28

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx = \text{Too large to display}$$

[In] integrate(tanh(x)^5/(1+tanh(x)),x, algorithm="fricas")

[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x + 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*cosh(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*

```
cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^3 +
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh(x)^
7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^
3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh
(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx = \frac{3x\tanh{(x)}}{6\tanh{(x)}+6} + \frac{3x}{6\tanh{(x)}+6} + \frac{12\log{(\tanh{(x)}+1)}\tanh{(x)}}{6\tanh{(x)}+6} + \frac{12\log{(\tanh{(x)}+1)}}{6\tanh{(x)}+6} - \frac{2\tanh^4{(x)}}{6\tanh{(x)}+6} + \frac{\tanh^3{(x)}}{6\tanh{(x)}+6} - \frac{9\tanh^2{(x)}}{6\tanh{(x)}+6} + \frac{15}{6\tanh{(x)}+6}$$

```
[In] integrate(tanh(x)**5/(1+tanh(x)),x)
```

```
[Out] 3*x*tanh(x)/(6*tanh(x) + 6) + 3*x/(6*tanh(x) + 6) + 12*log(tanh(x) + 1)*tan
h(x)/(6*tanh(x) + 6) + 12*log(tanh(x) + 1)/(6*tanh(x) + 6) - 2*tanh(x)**4/(
6*tanh(x) + 6) + tanh(x)**3/(6*tanh(x) + 6) - 9*tanh(x)**2/(6*tanh(x) + 6)
+ 15/(6*tanh(x) + 6)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x - \frac{2\left(15\,e^{(-2\,x)} + 12\,e^{(-4\,x)} + 7\right)}{3\left(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1\right)} + \frac{1}{4}\,e^{(-2\,x)} - 2\,\log\left(e^{(-2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)^5/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-
6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx = \frac{9}{2}\,x + \frac{\left(51\,e^{(6\,x)} + 81\,e^{(4\,x)} + 65\,e^{(2\,x)} + 3\right)e^{(-2\,x)}}{12\left(e^{(2\,x)} + 1\right)^3} - 2\,\log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)^5/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x) +
 1)^3 - 2*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\tanh^5(x)}{1+\tanh(x)}\,dx = \frac{x}{2} + 2\,\ln\left(\tanh(x)+1\right) - 2\tanh(x) + \frac{\tanh(x)^2}{2} - \frac{\tanh(x)^3}{3} + \frac{1}{2\,\left(\tanh\left(x\right)+1\right)}$$

```
[In] int(tanh(x)^5/(tanh(x) + 1),x)
```

```
[Out] x/2 + 2*log(tanh(x) + 1) - 2*tanh(x) + tanh(x)^2/2 - tanh(x)^3/3 + 1/(2*(ta
nh(x) + 1))
```

3.116 $\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx=-\frac{3x}{2}+2\log(\cosh(x))+\frac{3\tanh(x)}{2}-\tanh^2(x)+\frac{\tanh^3(x)}{2(1+\tanh(x))}$$

```
[Out] -3/2*x+2*ln(cosh(x))+3/2*tanh(x)-tanh(x)^2+1/2*tanh(x)^3/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3631, 3609, 3606, 3556}

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx=-\frac{3x}{2}+\frac{\tanh^3(x)}{2(\tanh(x)+1)}-\tanh^2(x)+\frac{3\tanh(x)}{2}+2\log(\cosh(x))$$

```
[In] Int[Tanh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^3/(2*(1 + T
anh[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
```

```
f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tanh^3(x)}{2(1+\tanh(x))} - \frac{1}{2}\int (3-4\tanh(x))\tanh^2(x)\,dx \\
&= -\tanh^2(x) + \frac{\tanh^3(x)}{2(1+\tanh(x))} + \frac{1}{2}i\int (-4i+3i\tanh(x))\tanh(x)\,dx \\
&= -\frac{3x}{2} + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^3(x)}{2(1+\tanh(x))} + 2\int \tanh(x)\,dx \\
&= -\frac{3x}{2} + 2\log(\cosh(x)) + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^3(x)}{2(1+\tanh(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$
\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx = \frac{4\log(\cosh(x)) + (3+4\log(\cosh(x)))\tanh(x) + \tanh^2(x) - \tanh^3(x) - 3\operatorname{arctanh}(\tanh(x))(1+\tanh(x))}{2(1+\tanh(x))}
$$

```
[In] Integrate[Tanh[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (4*Log[Cosh[x]] + (3 + 4*Log[Cosh[x]])*Tanh[x] + Tanh[x]^2 - Tanh[x]^3 - 3*
ArcTanh[Tanh[x]]*(1 + Tanh[x]))/(2*(1 + Tanh[x]))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{7x}{2}-\frac{e^{-2x}}{4}-\frac{2}{(1+e^{2x})^2}+2\ln\left(1+e^{2x}\right)$	30
derivativedivides	$-\frac{\tanh(x)^2}{2}+\tanh\left(x\right)-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{2(1+\tanh(x))}-\frac{7\ln(1+\tanh(x))}{4}$	32
default	$-\frac{\tanh(x)^2}{2}+\tanh\left(x\right)-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{2(1+\tanh(x))}-\frac{7\ln(1+\tanh(x))}{4}$	32
parallelrisch	$-\frac{3+\tanh(x)^3+4\ln(1-\tanh(x))\tanh(x)+7\tanh(x)x-\tanh(x)^2+4\ln(1-\tanh(x))+7x}{2(1+\tanh(x))}$	49

```
[In] int(tanh(x)^4/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -7/2*x-1/4*exp(-2*x)-2/(1+exp(2*x))^2+2*ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(31) = 62.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 9.57

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx =$$
$$-\frac{14\,x\cosh\left(x\right)^6+84\,x\cosh\left(x\right)\sinh\left(x\right)^5+14\,x\sinh\left(x\right)^6+\left(28\,x+1\right)\cosh\left(x\right)^4+\left(210\,x\cosh\left(x\right)^2+28\,x\right.}{}$$

```
[In] integrate(tanh(x)^4/(1+tanh(x)),x, algorithm="fricas")
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 + (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 +
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh
(x)^3 + 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 + (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh(x)^3 + 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(34) = 68$.

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.30

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx = \frac{x\tanh(x)}{2\tanh(x)+2} + \frac{x}{2\tanh(x)+2} - \frac{4\log(\tanh(x)+1)\tanh(x)}{2\tanh(x)+2} - \frac{4\log(\tanh(x)+1)}{2\tanh(x)+2} - \frac{\tanh^3(x)}{2\tanh(x)+2} + \frac{\tanh^2(x)}{2\tanh(x)+2} - \frac{3}{2\tanh(x)+2}$$

```
[In] integrate(tanh(x)**4/(1+tanh(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 4*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) - 4*log(tanh(x) + 1)/(2*tanh(x) + 2) - tanh(x)**3/(2*tanh(x
) + 2) + tanh(x)**2/(2*tanh(x) + 2) - 3/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x + \frac{2\left(2\,e^{(-2\,x)}+1\right)}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1} - \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*l
og(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx = -\frac{7}{2}\,x - \frac{\left(e^{(4\,x)}+10\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}+1\right)^2} + 2\,\log\left(e^{(2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] -7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e^
(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\tanh^4(x)}{1+\tanh(x)}\,dx = \frac{x}{2} - 2\,\ln\left(\tanh(x)+1\right) + \tanh(x) - \frac{\tanh(x)^2}{2} - \frac{1}{2\,\left(\tanh\left(x\right)+1\right)}$$

```
[In] int(tanh(x)^4/(tanh(x) + 1),x)
[Out] x/2 - 2*log(tanh(x) + 1) + tanh(x) - tanh(x)^2/2 - 1/(2*(tanh(x) + 1))
```

3.117 $\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx=\frac{3x}{2}-\log(\cosh(x))-\frac{3\tanh(x)}{2}+\frac{\tanh^2(x)}{2(1+\tanh(x))}$$

```
[Out] 3/2*x-ln(cosh(x))-3/2*tanh(x)+1/2*tanh(x)^2/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3631, 3606, 3556}

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx=\frac{3x}{2}+\frac{\tanh^2(x)}{2(\tanh(x)+1)}-\frac{3\tanh(x)}{2}-\log(\cosh(x))$$

```
[In] Int[Tanh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]^2/(2*(1 + Tanh[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tanh^2(x)}{2(1+\tanh(x))} - \frac{1}{2}\int (2-3\tanh(x))\tanh(x)\,dx \\ &= \frac{3x}{2} - \frac{3\tanh(x)}{2} + \frac{\tanh^2(x)}{2(1+\tanh(x))} - \int \tanh(x)\,dx \\ &= \frac{3x}{2} - \log(\cosh(x)) - \frac{3\tanh(x)}{2} + \frac{\tanh^2(x)}{2(1+\tanh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx = -\frac{2\log(\cosh(x)) + (3+2\log(\cosh(x)))\tanh(x) + 2\tanh^2(x) - 3\text{arctanh}(\tanh(x))(1+\tanh(x))}{2(1+\tanh(x))}$$

```
[In] Integrate[Tanh[x]^3/(1 + Tanh[x]),x]
```

```
[Out] -1/2*(2*Log[Cosh[x]] + (3 + 2*Log[Cosh[x]])*Tanh[x] + 2*Tanh[x]^2 - 3*ArcTa
nh[Tanh[x]]*(1 + Tanh[x]))/(1 + Tanh[x])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\tanh(x)-\frac{\ln(\tanh(x)-1)}{4}+\frac{1}{2+2\tanh(x)}+\frac{5\ln(1+\tanh(x))}{4}$	28
default	$-\tanh(x)-\frac{\ln(\tanh(x)-1)}{4}+\frac{1}{2+2\tanh(x)}+\frac{5\ln(1+\tanh(x))}{4}$	28
risch	$\frac{5x}{2}+\frac{e^{-2x}}{4}+\frac{2}{1+e^{2x}}-\ln(1+e^{2x})$	30
parallelrisch	$-\frac{-3-2\ln(1-\tanh(x))\tanh(x)-5\tanh(x)x+2\tanh(x)^2-2\ln(1-\tanh(x))-5x}{2(1+\tanh(x))}$	45

```
[In] int(tanh(x)^3/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -tanh(x)-1/4*ln(tanh(x)-1)+1/2/(1+tanh(x))+5/4*ln(1+tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 6.00

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx = \frac{10\,x\cosh(x)^4+40\,x\cosh(x)\sinh(x)^3+10\,x\sinh(x)^4+(10\,x+9)\cosh(x)^2+\left(60\,x\cosh(x)^2+10\,x+9\right)}{4\left(\cosh(x)\right.}$$

```
[In] integrate(tanh(x)^3/(1+tanh(x)),x, algorithm="fricas")
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*co
sh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + c
osh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.42

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx = \frac{x\tanh(x)}{2\tanh(x)+2}+\frac{x}{2\tanh(x)+2}+\frac{2\log(\tanh(x)+1)\tanh(x)}{2\tanh(x)+2} + \frac{2\log(\tanh(x)+1)}{2\tanh(x)+2}-\frac{2\tanh^2(x)}{2\tanh(x)+2}+\frac{3}{2\tanh(x)+2}$$

```
[In] integrate(tanh(x)**3/(1+tanh(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 2*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) + 2*log(tanh(x) + 1)/(2*tanh(x) + 2) - 2*tanh(x)**2/(2*tanh
(x) + 2) + 3/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x - \frac{2}{e^{(-2\,x)}+1} + \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/(e^(-2*x) + 1) + 1/4*e^(-2*x) - log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx = \frac{5}{2}\,x + \frac{\left(9\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}+1\right)} - \log\left(e^{(2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^3/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\tanh^3(x)}{1+\tanh(x)}\,dx = \frac{x}{2} + \ln\left(\tanh(x)+1\right) - \tanh(x) + \frac{1}{2\,\left(\tanh\left(x\right)+1\right)}$$

```
[In] int(tanh(x)^3/(tanh(x) + 1),x)
```

```
[Out] x/2 + log(tanh(x) + 1) - tanh(x) + 1/(2*(tanh(x) + 1))
```

3.118 $\int \frac{\tanh^2(x)}{1+\tanh(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\tanh^2(x)}{1+\tanh(x)} dx = -\frac{x}{2} + \log(\cosh(x)) - \frac{1}{2(1+\tanh(x))}$$

```
[Out] -1/2*x+ln(cosh(x))-1/2/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3621, 3556}

$$\int \frac{\tanh^2(x)}{1+\tanh(x)} dx = -\frac{x}{2} - \frac{1}{2(\tanh(x)+1)} + \log(\cosh(x))$$

```
[In] Int[Tanh[x]^2/(1 + Tanh[x]),x]
[Out] -1/2*x + Log[Cosh[x]] - 1/(2*(1 + Tanh[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2(1+\tanh(x))} - \frac{1}{2}\int (1-2\tanh(x))\,dx \\ &= -\frac{x}{2} - \frac{1}{2(1+\tanh(x))} + \int \tanh(x)\,dx \\ &= -\frac{x}{2} + \log(\cosh(x)) - \frac{1}{2(1+\tanh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx = \frac{2\log(\cosh(x)) + \tanh(x) + 2\log(\cosh(x))\tanh(x) - \operatorname{arctanh}(\tanh(x))(1+\tanh(x))}{2(1+\tanh(x))}$$

```
[In] Integrate[Tanh[x]^2/(1 + Tanh[x]),x]
[Out] (2*Log[Cosh[x]] + Tanh[x] + 2*Log[Cosh[x]]*Tanh[x] - ArcTanh[Tanh[x]]*(1 +
Tanh[x]))/(2*(1 + Tanh[x]))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{3x}{2} - \frac{e^{-2x}}{4} + \ln(1+e^{2x})$	18
derivativedivides	$-\frac{1}{2(1+\tanh(x))} - \frac{3\ln(1+\tanh(x))}{4} - \frac{\ln(\tanh(x)-1)}{4}$	24
default	$-\frac{1}{2(1+\tanh(x))} - \frac{3\ln(1+\tanh(x))}{4} - \frac{\ln(\tanh(x)-1)}{4}$	24
parallelrisch	$-\frac{1+2\ln(1-\tanh(x))\tanh(x)+3\tanh(x)x+2\ln(1-\tanh(x))+3x}{2(1+\tanh(x))}$	39

```
[In] int(tanh(x)^2/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -3/2*x-1/4*exp(-2*x)+ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx = -\frac{6\,x\cosh\left(x\right)^2+12\,x\cosh\left(x\right)\sinh\left(x\right)+6\,x\sinh\left(x\right)^2-4\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)\log\left(\right.}{4\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2\right)}$$

```
[In] integrate(tanh(x)^2/(1+tanh(x)),x, algorithm="fricas")
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(15) = 30.

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx = \frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2\tanh\left(x\right)+2} - \frac{2\log\left(\tanh\left(x\right)+1\right)}{2\tanh\left(x\right)+2}-\frac{1}{2\tanh\left(x\right)+2}$$

```
[In] integrate(tanh(x)**2/(1+tanh(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) - 2*log(tanh(x) + 1)/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x-\frac{1}{4}\,e^{\left(-2\,x\right)}+\log\left(e^{\left(-2\,x\right)}+1\right)$$

```
[In] integrate(tanh(x)^2/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx = -\frac{3}{2}\,x - \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)^2/(1+tanh(x)),x, algorithm="giac")
[Out] -3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^2(x)}{1+\tanh(x)}\,dx = \frac{x}{2} - \ln\left(\tanh(x)+1\right) - \frac{1}{2\,\left(\tanh\left(x\right)+1\right)}$$

```
[In] int(tanh(x)^2/(tanh(x) + 1),x)
[Out] x/2 - log(tanh(x) + 1) - 1/(2*(tanh(x) + 1))
```

3.119 $\int \frac{\tanh(x)}{1+\tanh(x)} dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{\tanh(x)}{1+\tanh(x)} dx = \frac{x}{2} + \frac{1}{2(1+\tanh(x))}$$

```
[Out] 1/2*x+1/2/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3607, 8}

$$\int \frac{\tanh(x)}{1+\tanh(x)} dx = \frac{x}{2} + \frac{1}{2(\tanh(x)+1)}$$

```
[In] Int[Tanh[x]/(1 + Tanh[x]),x]
```

```
[Out] x/2 + 1/(2*(1 + Tanh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2(1+\tanh(x))} + \frac{\int 1\,dx}{2}\\ &= \frac{x}{2} + \frac{1}{2(1+\tanh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tanh(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\left(\operatorname{arctanh}(\tanh(x)) + \frac{1}{1+\tanh(x)}\right)$$

```
[In] Integrate[Tanh[x]/(1 + Tanh[x]),x]
[Out] (ArcTanh[Tanh[x]] + (1 + Tanh[x])^(-1))/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}+\frac{e^{-2x}}{4}$	11
parallelrisch	$-\frac{-1-\tanh(x)x-x}{2(1+\tanh(x))}$	19
derivativedivides	$\frac{1}{2+2\tanh(x)}+\frac{\ln(1+\tanh(x))}{4}-\frac{\ln(\tanh(x)-1)}{4}$	24
default	$\frac{1}{2+2\tanh(x)}+\frac{\ln(1+\tanh(x))}{4}-\frac{\ln(\tanh(x)-1)}{4}$	24

```
[In] int(tanh(x)/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\tanh(x)}{1+\tanh(x)}\,dx = \frac{(2\,x+1)\cosh\left(x\right)+(2\,x-1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(tanh(x)/(1+tanh(x)),x, algorithm="fricas")
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{\tanh(x)}{1+\tanh(x)}\,dx = \frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}+\frac{1}{2\tanh\left(x\right)+2}$$

```
[In] integrate(tanh(x)/(1+tanh(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\tanh(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x+\frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(tanh(x)/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\tanh(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(tanh(x)/(1+tanh(x)),x, algorithm="giac")
[Out] 1/2*x + 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\tanh(x)}{1+\tanh(x)}\,dx = \frac{x}{2} + \frac{\mathrm{e}^{-2\,x}}{4}$$

```
[In] int(tanh(x)/(tanh(x) + 1),x)
[Out] x/2 + exp(-2*x)/4
```

3.120 $\int \frac{1}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \frac{1}{1+\tanh(x)}\,dx=\frac{x}{2}-\frac{1}{2(1+\tanh(x))}$$

```
[Out] 1/2*x-1/2/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\int \frac{1}{1+\tanh(x)}\,dx=\frac{x}{2}-\frac{1}{2(\tanh(x)+1)}$$

```
[In] Int[(1 + Tanh[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Tanh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2(1+\tanh(x))}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2(1+\tanh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{1+\tanh(x)}\,dx = \frac{1}{2}\text{arctanh}(\tanh(x))-\frac{1}{2(1+\tanh(x))}$$

```
[In] Integrate[(1 + Tanh[x])^(-1),x]
[Out] ArcTanh[Tanh[x]]/2 - 1/(2*(1 + Tanh[x]))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}-\frac{e^{-2x}}{4}$	11
parallelrisch	$-\frac{1-\tanh(x)x-x}{2(1+\tanh(x))}$	19
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{2(1+\tanh(x))}+\frac{\ln(1+\tanh(x))}{4}$	24
default	$-\frac{\ln(\tanh(x)-1)}{4}-\frac{1}{2(1+\tanh(x))}+\frac{\ln(1+\tanh(x))}{4}$	24

```
[In] int(1/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{1+\tanh(x)}\,dx = \frac{(2\,x-1)\cosh\left(x\right)+(2\,x+1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(1/(1+tanh(x)),x, algorithm="fricas")
[Out] 1/4*((2*x - 1)*cosh(x) + (2*x + 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{1+\tanh(x)}\,dx = \frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{1}{2\tanh\left(x\right)+2}$$

```
[In] integrate(1/(1+tanh(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\tanh(x)}\,dx = \frac{1}{2}\,x-\frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(1/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/2*x - 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\tanh(x)}\,dx = \frac{1}{2}\,x - \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(1/(1+tanh(x)),x, algorithm="giac")
[Out] 1/2*x - 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\tanh(x)}\,dx = \frac{x}{2} - \frac{\mathrm{e}^{-2\,x}}{4}$$

```
[In] int(1/(tanh(x) + 1),x)
[Out] x/2 - exp(-2*x)/4
```

3.121 $\int \frac{\coth(x)}{1+\tanh(x)} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{\coth(x)}{1+\tanh(x)} \, dx = -\frac{x}{2} + \log(\sinh(x)) + \frac{1}{2(1+\tanh(x))}$$

```
[Out] -1/2*x+ln(sinh(x))+1/2/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3632, 3560, 8, 3556}

$$\int \frac{\coth(x)}{1+\tanh(x)} \, dx = -\frac{x}{2} + \frac{1}{2(\tanh(x)+1)} + \log(\sinh(x))$$

```
[In] Int[Coth[x]/(1 + Tanh[x]),x]
[Out] -1/2*x + Log[Sinh[x]] + 1/(2*(1 + Tanh[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3632

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \coth(x)\,dx - \int \frac{1}{1+\tanh(x)}\,dx\\ &= \log(\sinh(x)) + \frac{1}{2(1+\tanh(x))} - \frac{\int 1\,dx}{2}\\ &= -\frac{x}{2} + \log(\sinh(x)) + \frac{1}{2(1+\tanh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\coth(x)}{1+\tanh(x)}\,dx = -\frac{1}{4}\log(1-\tanh(x)) + \log(\tanh(x)) - \frac{3}{4}\log(1+\tanh(x)) + \frac{1}{2(1+\tanh(x))}$$

```
[In] Integrate[Coth[x]/(1 + Tanh[x]),x]
[Out] -1/4*Log[1 - Tanh[x]] + Log[Tanh[x]] - (3*Log[1 + Tanh[x]])/4 + 1/(2*(1 + T
anh[x]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{3x}{2}+\frac{e^{-2x}}{4}+\ln\left(e^{2x}-1\right)$	18
default	$\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}$	43
parallelrisch	$\frac{(-2-2\tanh(x))\ln(1-\tanh(x))+(2+2\tanh(x))\ln(\tanh(x))-3\tanh(x)x-3x+1}{2+2\tanh(x)}$	44

```
[In] int(coth(x)/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -3/2*x+1/4*exp(-2*x)+ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\coth(x)}{1+\tanh(x)}\,dx = -\frac{6\,x\cosh\left(x\right)^{2} + 12\,x\cosh\left(x\right)\sinh\left(x\right) + 6\,x\sinh\left(x\right)^{2} - 4\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)\log\left(\right.}{4\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)}$$

```
[In] integrate(coth(x)/(1+tanh(x)),x, algorithm="fricas")
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F]

$$\int \frac{\coth(x)}{1+\tanh(x)}\,dx = \int \frac{\coth\left(x\right)}{\tanh\left(x\right)+1}\,dx$$

```
[In] integrate(coth(x)/(1+tanh(x)),x)
[Out] Integral(coth(x)/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\coth(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{\left(-2\,x\right)} + \log\left(e^{\left(-x\right)} + 1\right) + \log\left(e^{\left(-x\right)} - 1\right)$$

```
[In] integrate(coth(x)/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/4*e^(-2*x) + log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\coth(x)}{1+\tanh(x)}\,dx = -\frac{3}{2}\,x + \frac{1}{4}\,e^{(-2\,x)} + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(coth(x)/(1+tanh(x)),x, algorithm="giac")
[Out] -3/2*x + 1/4*e^(-2*x) + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\coth(x)}{1+\tanh(x)}\,dx = \ln\left(e^{2\,x} - 1\right) - \frac{3\,x}{2} + \frac{e^{-2\,x}}{4}$$

```
[In] int(coth(x)/(tanh(x) + 1),x)
[Out] log(exp(2*x) - 1) - (3*x)/2 + exp(-2*x)/4
```

3.122 $\int \frac{\coth^2(x)}{1+\tanh(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\coth^2(x)}{1+\tanh(x)} dx = \frac{3x}{2} - \frac{3\coth(x)}{2} - \log(\sinh(x)) + \frac{\coth(x)}{2(1+\tanh(x))}$$

```
[Out] 3/2*x-3/2*coth(x)-ln(sinh(x))+1/2*coth(x)/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3633, 3610, 3612, 3556}

$$\int \frac{\coth^2(x)}{1+\tanh(x)} dx = \frac{3x}{2} - \frac{3\coth(x)}{2} - \log(\sinh(x)) + \frac{\coth(x)}{2(\tanh(x)+1)}$$

```
[In] Int[Coth[x]^2/(1 + Tanh[x]),x]
```

```
[Out] (3*x)/2 - (3*Coth[x])/2 - Log[Sinh[x]] + Coth[x]/(2*(1 + Tanh[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth(x)}{2(1+\tanh(x))} - \frac{1}{2}\int \coth^2(x)(-3+2\tanh(x))\,dx \\
&= -\frac{3\coth(x)}{2} + \frac{\coth(x)}{2(1+\tanh(x))} - \frac{1}{2}i\int \coth(x)(-2i+3i\tanh(x))\,dx \\
&= \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth(x)}{2(1+\tanh(x))} - \int \coth(x)\,dx \\
&= \frac{3x}{2} - \frac{3\coth(x)}{2} - \log(\sinh(x)) + \frac{\coth(x)}{2(1+\tanh(x))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\coth^2(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\Bigg(\coth^2(x) + \frac{\coth^4(x)}{1+\coth(x)} - \coth^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(x)\right) - 2(\log(\cosh(x)) + \log(\tanh(x)))\Bigg)$$

[In] Integrate[Coth[x]^2/(1 + Tanh[x]),x]

[Out] (Coth[x]^2 + Coth[x]^4/(1 + Coth[x]) - Coth[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[x]^2] - 2*(Log[Cosh[x]] + Log[Tanh[x]]))/2

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result
risch	$\frac{5x}{2}-\frac{{\mathrm e}^{-2x}}{4}-\frac{2}{{\mathrm e}^{2x}-1}-\ln\left({\mathrm e}^{2x}-1\right)$
parallelrisch	$\frac{(2+2\tanh(x))\ln(1-\tanh(x))+(-2-2\tanh(x))\ln(\tanh(x))+5\tanh(x)x+5x-2\coth(x)-3}{2+2\tanh(x)}$
default	$-\frac{\tanh(\frac{x}{2})}{2}-\frac{\ln(\tanh(\frac{x}{2})-1)}{2}-\frac{1}{2\tanh(\frac{x}{2})}-\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{(\tanh(\frac{x}{2})+1)^{2}}+\frac{1}{\tanh(\frac{x}{2})+1}+\frac{5\ln(\tanh(\frac{x}{2})+1)}{2}$

[In] int(coth(x)^2/(1+tanh(x)),x,method=_RETURNVERBOSE)

[Out] 5/2*x-1/4*exp(-2*x)-2/(exp(2*x)-1)-ln(exp(2*x)-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(23) = 46.

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 6.76

$$\int\frac{\coth^2(x)}{1+\tanh(x)}\,dx$$
$$=\frac{10\,x\cosh{(x)}^{4}+40\,x\cosh{(x)}\sinh{(x)}^{3}+10\,x\sinh{(x)}^{4}-(10\,x+9)\cosh{(x)}^{2}+\left(60\,x\cosh{(x)}^{2}-10\,x-9\right)}{4\left(\cosh{(x)}\right.}$$

[In] integrate(coth(x)^2/(1+tanh(x)),x, algorithm="fricas")

[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 - (10*x + 9)*cosh(x)^2 + (60*x*cosh(x)^2 - 10*x - 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(20*x*cosh(x)^3 - (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - cosh(x))*sinh(x))

Sympy [F]

$$\int \frac{\coth^2(x)}{1+\tanh(x)}\,dx = \int \frac{\coth^2(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(coth(x)**2/(1+tanh(x)),x)
[Out] Integral(coth(x)**2/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{\coth^2(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x + \frac{2}{e^{(-2\,x)}-1} - \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^2/(1+tanh(x)),x, algorithm="maxima")
[Out] 1/2*x + 2/(e^(-2*x) - 1) - 1/4*e^(-2*x) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\coth^2(x)}{1+\tanh(x)}\,dx = \frac{5}{2}\,x - \frac{\left(9\,e^{(2\,x)}-1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}-1\right)} - \log\left(\left|e^{(2\,x)}-1\right|\right)$$

```
[In] integrate(coth(x)^2/(1+tanh(x)),x, algorithm="giac")
[Out] 5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\coth^2(x)}{1+\tanh(x)}\,dx = \frac{5\,x}{2} - \ln\left(\mathrm{e}^{2\,x}-1\right) - \frac{\mathrm{e}^{-2\,x}}{4} - \frac{2}{\mathrm{e}^{2\,x}-1}$$

```
[In] int(coth(x)^2/(tanh(x) + 1),x)
[Out] (5*x)/2 - log(exp(2*x) - 1) - exp(-2*x)/4 - 2/(exp(2*x) - 1)
```

3.123 $\int \frac{\coth^3(x)}{1+\tanh(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\coth^3(x)}{1+\tanh(x)} dx = -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + 2\log(\sinh(x)) + \frac{\coth^2(x)}{2(1+\tanh(x))}$$

```
[Out] -3/2*x+3/2*coth(x)-coth(x)^2+2*ln(sinh(x))+1/2*coth(x)^2/(1+tanh(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3633, 3610, 3612, 3556}

$$\int \frac{\coth^3(x)}{1+\tanh(x)} dx = -\frac{3x}{2} - \coth^2(x) + \frac{3\coth(x)}{2} + 2\log(\sinh(x)) + \frac{\coth^2(x)}{2(\tanh(x)+1)}$$

```
[In] Int[Coth[x]^3/(1 + Tanh[x]),x]
```

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + 2*Log[Sinh[x]] + Coth[x]^2/(2*(1 + T
anh[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth^2(x)}{2(1+\tanh(x))} - \frac{1}{2}\int \coth^3(x)(-4+3\tanh(x))\,dx \\
&= -\coth^2(x) + \frac{\coth^2(x)}{2(1+\tanh(x))} - \frac{1}{2}i\int \coth^2(x)(-3i+4i\tanh(x))\,dx \\
&= \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^2(x)}{2(1+\tanh(x))} + \frac{1}{2}\int \coth(x)(4-3\tanh(x))\,dx \\
&= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^2(x)}{2(1+\tanh(x))} + 2\int \coth(x)\,dx \\
&= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + 2\log(\sinh(x)) + \frac{\coth^2(x)}{2(1+\tanh(x))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\left(-2\coth^2(x)-\coth^4(x)+\frac{\coth^5(x)}{1+\coth(x)} + \coth^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(x)\right) + 4(\log(\cosh(x))+\log(\tanh(x)))\right)$$

```
[In] Integrate[Coth[x]^3/(1 + Tanh[x]),x]
[Out] (-2*Coth[x]^2 - Coth[x]^4 + Coth[x]^5/(1 + Coth[x]) + Coth[x]^3*Hypergeomet
ric2F1[-3/2, 1, -1/2, Tanh[x]^2] + 4*(Log[Cosh[x]] + Log[Tanh[x]]))/2
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{7x}{2}+\frac{e^{-2x}}{4}-\frac{2}{(e^{2x}-1)^2}+2\ln\left(e^{2x}-1\right)$
parallelrisch	$\frac{(-4\tanh(x)-4)\ln(1-\tanh(x))+(4\tanh(x)+4)\ln(\tanh(x))-7\tanh(x)x-\coth(x)^2-7x+\coth(x)+3}{2+2\tanh(x)}$
default	$\frac{1}{(\tanh(\frac{x}{2})+1)^2}-\frac{1}{\tanh(\frac{x}{2})+1}-\frac{7\ln(\tanh(\frac{x}{2})+1)}{2}-\frac{\tanh(\frac{x}{2})^2}{8}+\frac{\tanh(\frac{x}{2})}{2}-\frac{1}{8\tanh(\frac{x}{2})^2}+\frac{1}{2\tanh(\frac{x}{2})}+2\ln\left(\tan\right.$

```
[In] int(coth(x)^3/(1+tanh(x)),x,method=_RETURNVERBOSE)
[Out] -7/2*x+1/4*exp(-2*x)-2/(exp(2*x)-1)^2+2*ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 357, normalized size of antiderivative = 9.65

$$\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx = -\frac{14\,x\cosh(x)^6+84\,x\cosh(x)\sinh(x)^5+14\,x\sinh(x)^6-(28\,x+1)\cosh(x)^4+\left(210\,x\cosh(x)^2-28\,x-\right.}{}$$

```
[In] integrate(coth(x)^3/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 - (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 - 28*x - 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 - (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 -
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh
(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*sinh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 - (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) - 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx = \int \frac{\coth^3(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(coth(x)**3/(1+tanh(x)),x)
```

```
[Out] Integral(coth(x)**3/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx = \frac{1}{2}x + \frac{2\left(2e^{(-2x)}-1\right)}{2e^{(-2x)}-e^{(-4x)}-1} + \frac{1}{4}e^{(-2x)} + 2\log\left(e^{(-x)}+1\right) + 2\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^3/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) - 1)/(2*e^(-2*x) - e^(-4*x) - 1) + 1/4*e^(-2*x) + 2*l
og(e^(-x) + 1) + 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx = -\frac{7}{2}\,x + \frac{\left(e^{(4\,x)} - 10\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)} - 1\right)^2} + 2\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(coth(x)^3/(1+tanh(x)),x, algorithm="giac")
[Out] -7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(ab
s(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\coth^3(x)}{1+\tanh(x)}\,dx = 2\,\ln\left(\mathrm{e}^{2\,x} - 1\right) - \frac{7\,x}{2} + \frac{\mathrm{e}^{-2\,x}}{4} - \frac{2}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1}$$

```
[In] int(coth(x)^3/(tanh(x) + 1),x)
[Out] 2*log(exp(2*x) - 1) - (7*x)/2 + exp(-2*x)/4 - 2/(exp(4*x) - 2*exp(2*x) + 1)
```

3.124 $\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx=\frac{5x}{2}-\frac{5\coth(x)}{2}+\coth^2(x)-\frac{5\coth^3(x)}{6}-2\log(\sinh(x))+\frac{\coth^3(x)}{2(1+\tanh(x))}$$

```
[Out] 5/2*x-5/2*coth(x)+coth(x)^2-5/6*coth(x)^3-2*ln(sinh(x))+1/2*coth(x)^3/(1+ta
nh(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3633, 3610, 3612, 3556}

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx=\frac{5x}{2}-\frac{5\coth^3(x)}{6}+\coth^2(x)-\frac{5\coth(x)}{2}-2\log(\sinh(x))+\frac{\coth^3(x)}{2(\tanh(x)+1)}$$

```
[In] Int[Coth[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (5*x)/2 - (5*Coth[x])/2 + Coth[x]^2 - (5*Coth[x]^3)/6 - 2*Log[Sinh[x]] + Co
th[x]^3/(2*(1 + Tanh[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth^3(x)}{2(1+\tanh(x))} - \frac{1}{2}\int \coth^4(x)(-5+4\tanh(x))\,dx \\
&= -\frac{5}{6}\coth^3(x) + \frac{\coth^3(x)}{2(1+\tanh(x))} - \frac{1}{2}i\int \coth^3(x)(-4i+5i\tanh(x))\,dx \\
&= \coth^2(x) - \frac{5\coth^3(x)}{6} + \frac{\coth^3(x)}{2(1+\tanh(x))} + \frac{1}{2}\int \coth^2(x)(5-4\tanh(x))\,dx \\
&= -\frac{5\coth(x)}{2} + \coth^2(x) - \frac{5\coth^3(x)}{6} + \frac{\coth^3(x)}{2(1+\tanh(x))} + \frac{1}{2}i\int \coth(x)(4i-5i\tanh(x))\,dx \\
&= \frac{5x}{2} - \frac{5\coth(x)}{2} + \coth^2(x) - \frac{5\coth^3(x)}{6} + \frac{\coth^3(x)}{2(1+\tanh(x))} - 2\int \coth(x)\,dx \\
&= \frac{5x}{2} - \frac{5\coth(x)}{2} + \coth^2(x) - \frac{5\coth^3(x)}{6} - 2\log(\sinh(x)) + \frac{\coth^3(x)}{2(1+\tanh(x))}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.23

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\left(2\coth^2(x)+\coth^4(x)+\frac{\coth^6(x)}{1+\coth(x)} - \coth^5(x)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\tanh^2(x)\right) - 4(\log(\cosh(x))+\log(\tanh(x)))\right)$$

```
[In] Integrate[Coth[x]^4/(1 + Tanh[x]),x]
```

```
[Out] (2*Coth[x]^2 + Coth[x]^4 + Coth[x]^6/(1 + Coth[x]) - Coth[x]^5*Hypergeometr
ic2F1[-5/2, 1, -3/2, Tanh[x]^2] - 4*(Log[Cosh[x]] + Log[Tanh[x]]))/2
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result
risch	$\frac{9x}{2}-\frac{e^{-2x}}{4}-\frac{2(6\,e^{4x}-9\,e^{2x}+7)}{3(e^{2x}-1)^3}-2\ln\left(e^{2x}-1\right)$
parallelrisch	$\frac{(12\tanh(x)+12)\ln(1-\tanh(x))+(-12\tanh(x)-12)\ln(\tanh(x))-2\coth(x)^3+27\tanh(x)x+\coth(x)^2+27x-9\coth(x)-15}{6+6\tanh(x)}$
default	$-\frac{\tanh(\frac{x}{2})^3}{24}+\frac{\tanh(\frac{x}{2})^2}{8}-\frac{9\tanh(\frac{x}{2})}{8}-\frac{1}{(\tanh(\frac{x}{2})+1)^2}+\frac{1}{\tanh(\frac{x}{2})+1}+\frac{9\ln(\tanh(\frac{x}{2})+1)}{2}-\frac{1}{24\tanh(\frac{x}{2})^3}+\frac{\cdots}{8\,\mathrm{ta}\cdots}$

```
[In] int(coth(x)^4/(1+tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 9/2*x-1/4*exp(-2*x)-2/3*(6*exp(4*x)-9*exp(2*x)+7)/(exp(2*x)-1)^3-2*ln(exp(2
*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. 2(35) = 70.

Time = 0.25 (sec) , antiderivative size = 582, normalized size of antiderivative = 13.53

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^4/(1+tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 - 3*(54*x +
 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 - 54*x - 17)*sinh(x)^6 + 18*(168*x*cosh
(x)^3 - (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*
cosh(x)^4 - 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 - 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 - (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 - 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 - 54*x - 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 -
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 - 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 - 45*cosh(x)^4 + 18*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(4*cosh(x)^
7 - 9*cosh(x)^5 + 6*cosh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 - 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 - (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^
3 - 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 - 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 - 45*cosh(x)^4 + 18*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(4*cosh
(x)^7 - 9*cosh(x)^5 + 6*cosh(x)^3 - cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx = \int \frac{\coth^4(x)}{\tanh(x)+1}\,dx$$

```
[In] integrate(coth(x)**4/(1+tanh(x)),x)
```

```
[Out] Integral(coth(x)**4/(tanh(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.49

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx = \frac{1}{2}\,x - \frac{2\left(15\,e^{(-2\,x)} - 12\,e^{(-4\,x)} - 7\right)}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} - \frac{1}{4}\,e^{(-2\,x)} - 2\,\log\left(e^{(-x)} + 1\right) - 2\,\log\left(e^{(-x)} - 1\right)$$

```
[In] integrate(coth(x)^4/(1+tanh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 2/3*(15*e^(-2*x) - 12*e^(-4*x) - 7)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-
6*x) - 1) - 1/4*e^(-2*x) - 2*log(e^(-x) + 1) - 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx = \frac{9}{2}\,x - \frac{\left(51\,e^{(6\,x)} - 81\,e^{(4\,x)} + 65\,e^{(2\,x)} - 3\right)e^{(-2\,x)}}{12\,\left(e^{(2\,x)} - 1\right)^{3}} - 2\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(coth(x)^4/(1+tanh(x)),x, algorithm="giac")
```

```
[Out] 9/2*x - 1/12*(51*e^(6*x) - 81*e^(4*x) + 65*e^(2*x) - 3)*e^(-2*x)/(e^(2*x) -
 1)^3 - 2*log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{\coth^4(x)}{1+\tanh(x)}\,dx = \frac{9\,x}{2} - 2\,\ln\left(\mathrm{e}^{2\,x} - 1\right) - \frac{\mathrm{e}^{-2\,x}}{4} - \frac{8}{3\,\left(3\,\mathrm{e}^{2\,x} - 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} - 1\right)} - \frac{2}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{4}{\mathrm{e}^{2\,x} - 1}$$

```
[In] int(coth(x)^4/(tanh(x) + 1),x)
```

```
[Out] (9*x)/2 - 2*log(exp(2*x) - 1) - exp(-2*x)/4 - 8/(3*(3*exp(2*x) - 3*exp(4*x)
 + exp(6*x) - 1)) - 2/(exp(4*x) - 2*exp(2*x) + 1) - 4/(exp(2*x) - 1)
```

3.125 $\int \tanh(x)(1+\tanh(x))^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2}$$

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)-2/3*(1
+tanh(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3608, 3559, 3561, 212}

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right) - \frac{2}{3}(\tanh(x)+1)^{3/2} - 2\sqrt{\tanh(x)+1}$$

```
[In] Int[Tanh[x]*(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{3}(1+\tanh(x))^{3/2} + \int (1+\tanh(x))^{3/2}\,dx \\
&= -2\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2} + 2\int \sqrt{1+\tanh(x)}\,dx \\
&= -2\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2} + 4\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right) \\
&= 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)} - \frac{2}{3}(1+\tanh(x))^{3/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - \frac{2}{3}\sqrt{1+\tanh(x)}(4+\tanh(x))$$

```
[In] Integrate[Tanh[x]*(1 + Tanh[x])^(3/2),x]
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*Sqrt[1 + Tanh[x]]*(4 + Ta
nh[x]))/3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\tanh\left(x\right)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}-\frac{2\left(1+\tanh\left(x\right)\right)^{\frac{3}{2}}}{3}$	35
default	$2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\tanh\left(x\right)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}-\frac{2\left(1+\tanh\left(x\right)\right)^{\frac{3}{2}}}{3}$	35

```
[In] int(tanh(x)*(1+tanh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)-2/3*(1
+tanh(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 5.60

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh\left(x\right)^{3}+15\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+5\sqrt{2}\sinh\left(x\right)^{3}+3\left(5\sqrt{2}\cosh\left(x\right)^{2}+\sqrt{2}\right)\sinh\left(x\right)+3\sqrt{2}\,c}{}$$

```
[In] integrate(tanh(x)*(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x)*sinh(x)^2 + 5*sqr
t(2)*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 3*sqrt(2)*cosh
(x))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*si
nh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
 sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^4
+ 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(
x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 3.90 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = -\sqrt{2}\left(\log\left(\sqrt{\tanh(x)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh(x)+1}+\sqrt{2}\right)\right) - \frac{2(\tanh(x)+1)^{\frac{3}{2}}}{3} - 2\sqrt{\tanh(x)+1}$$

```
[In] integrate(tanh(x)*(1+tanh(x))**(3/2),x)
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
))) - 2*(tanh(x) + 1)**(3/2)/3 - 2*sqrt(tanh(x) + 1)
```

Maxima [F]

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = \int (\tanh(x)+1)^{\frac{3}{2}}\tanh(x)\,dx$$

```
[In] integrate(tanh(x)*(1+tanh(x))^(3/2),x, algorithm="maxima")
[Out] -2/3*sqrt(2)/(e^(-2*x) + 1)^(3/2) + integrate(2*sqrt(2)*e^(-x)/((e^(-x) + e
^(-3*x))*(e^(-2*x) + 1)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.13

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = \frac{1}{3}\sqrt{2}\left(\frac{2\left(9\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2-12\sqrt{e^{(4x)}+e^{(2x)}}+12e^{(2x)}+5\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^3} - 3\log\left(-2\sqrt{\right.\right.$$

```
[In] integrate(tanh(x)*(1+tanh(x))^(3/2),x, algorithm="giac")
[Out] 1/3*sqrt(2)*(2*(9*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 12*sqrt(e^(4*x) +
 e^(2*x)) + 12*e^(2*x) + 5)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1)^3 - 3*l
og(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \tanh(x)(1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh(x)+1}}{2}\right) - 2\sqrt{\tanh(x)+1} - \frac{2\,(\tanh(x)+1)^{3/2}}{3}$$

```
[In] int(tanh(x)*(tanh(x) + 1)^(3/2),x)
[Out] 2*2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2) - 2*(tanh(x) + 1)^(1/2) -
(2*(tanh(x) + 1)^(3/2))/3
```

3.126 $\int \tanh(x)\sqrt{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)}$$

```
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3608, 3561, 212}

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\tanh(x)+1}$$

```
[In] Int[Tanh[x]*Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
```

```
b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -2\sqrt{1+\tanh(x)} + \int \sqrt{1+\tanh(x)}\,dx \\ &= -2\sqrt{1+\tanh(x)} + 2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right) \\ &= \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)}$$

```
[In] Integrate[Tanh[x]*Sqrt[1 + Tanh[x]],x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}$	26
default	$\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}$	26

```
[In] int((1+tanh(x))^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.03

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx = -\frac{4\sqrt{2}\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right.}{2\left(\cosh\left(x\right)^{2}+2\cosh\right.}$$

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(cosh(x)/(cosh(x) -
 sinh(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
 sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{\tanh\left(x\right)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh\left(x\right)+1}+\sqrt{2}\right)\right)}{2}-2\sqrt{\tanh\left(x\right)+1}$$

```
[In] integrate((1+tanh(x))**(1/2)*tanh(x),x)
```

```
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
)))/2 - 2*sqrt(tanh(x) + 1)
```

Maxima [F]

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx = \int \sqrt{\tanh\left(x\right)+1}\tanh\left(x\right)\,dx$$

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)/sqrt(e^(-2*x) + 1) + integrate(sqrt(2)*e^(-x)/((e^(-x) + e^(-3*x))
*sqrt(e^(-2*x) + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.66

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx$$
$$= \frac{1}{2}\sqrt{2}\left(\frac{4}{\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1} - \log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(4/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1) - log(-2*sqrt(e^(4*x
) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \tanh(x)\sqrt{1+\tanh(x)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right) - 2\sqrt{\tanh(x)+1}$$

```
[In] int(tanh(x)*(tanh(x) + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2) - 2*(tanh(x) + 1)^(1/2)
```

3.127 $\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\tanh(x)}}$$

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3607, 3561, 212}

$$\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{\tanh(x)+1}}$$

```
[In] Int[Tanh[x]/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Tanh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{\sqrt{1+\tanh(x)}}+\frac{1}{2}\int\sqrt{1+\tanh(x)}\,dx\\
&= \frac{1}{\sqrt{1+\tanh(x)}}+\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int\frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\tanh(x)}}$$

```
[In] Integrate[Tanh[x]/Sqrt[1 + Tanh[x]],x]
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Tanh[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1}{\sqrt{1+\tanh(x)}}$	25
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1}{\sqrt{1+\tanh(x)}}$	25

```
[In] int(tanh(x)/(1+tanh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+tanh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.83

$$\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx$$
$$= \frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(-2\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)-2\cosh\left(x\right)^{2}-4\cosh\left(x\right)\sin\right.}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(tanh(x)/(1+tanh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(
x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*si
nh(x)^2 - 1) + 4*sqrt(cosh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{\tanh\left(x\right)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh\left(x\right)+1}+\sqrt{2}\right)\right)}{4} + \frac{1}{\sqrt{\tanh\left(x\right)+1}}$$

```
[In] integrate(tanh(x)/(1+tanh(x))**(1/2),x)
```

```
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
)))/4 + 1/sqrt(tanh(x) + 1)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{\tanh(x)+1}}\,dx$$

```
[In] integrate(tanh(x)/(1+tanh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*sqrt(e^(-2*x) + 1) + integrate(e^(-x)/(sqrt(2)*e^(-x)/sqrt(e^(-
2*x) + 1) + sqrt(2)*e^(-3*x)/sqrt(e^(-2*x) + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx = \frac{1}{4}\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}} - \log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

```
[In] integrate(tanh(x)/(1+tanh(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(2/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x)) - log(-2*sqrt(e^(4*x) +
e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right)}{2} + \frac{1}{\sqrt{\tanh(x)+1}}$$

```
[In] int(tanh(x)/(tanh(x) + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2))/2 + 1/(tanh(x) + 1)^(1/2)
```

3.128 $\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2\sqrt{1+\tanh(x)}}$$

```
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/2/(1+tanh(x))^(1/2)+1/
3/(1+tanh(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3607, 3560, 3561, 212}

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{2\sqrt{\tanh(x)+1}}+\frac{1}{3(\tanh(x)+1)^{3/2}}$$

```
[In] Int[Tanh[x]/(1 + Tanh[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Tanh[x])^(3/2))
- 1/(2*Sqrt[1 + Tanh[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3(1+\tanh(x))^{3/2}} + \frac{1}{2}\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx \\
&= \frac{1}{3(1+\tanh(x))^{3/2}} - \frac{1}{2\sqrt{1+\tanh(x)}} + \frac{1}{4}\int \sqrt{1+\tanh(x)}\,dx \\
&= \frac{1}{3(1+\tanh(x))^{3/2}} - \frac{1}{2\sqrt{1+\tanh(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(x)}\right) \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{3(1+\tanh(x))^{3/2}} - \frac{1}{2\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx = \frac{2 - 3\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \frac{1}{2}(1+\tanh(x))\right)(1+\tanh(x))}{6(1+\tanh(x))^{3/2}}$$

```
[In] Integrate[Tanh[x]/(1 + Tanh[x])^(3/2),x]
[Out] (2 - 3*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Tanh[x])/2]*(1 + Tanh[x]))/(6*(
1 + Tanh[x])^(3/2))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{3(1+\tanh(x))^{\frac{3}{2}}}$	35
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\tanh(x)}}+\frac{1}{3(1+\tanh(x))^{\frac{3}{2}}}$	35

```
[In] int(tanh(x)/(1+tanh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/2/(1+tanh(x))^(1/2)+1/
3/(1+tanh(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.43

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}\left(2\sqrt{2}\cosh(x)^2+4\sqrt{2}\cosh(x)\sinh(x)+2\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\right.}{24(c}$$

```
[In] integrate(tanh(x)/(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(2*sqrt(2)*cosh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x) + 2*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*
cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [A] (verification not implemented)

Time = 4.98 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{\tanh(x)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh(x)+1}+\sqrt{2}\right)\right)}{8} - \frac{1}{2\sqrt{\tanh(x)+1}} + \frac{1}{3\left(\tanh(x)+1\right)^{\frac{3}{2}}}$$

```
[In] integrate(tanh(x)/(1+tanh(x))**(3/2),x)
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
)))/8 - 1/(2*sqrt(tanh(x) + 1)) + 1/(3*(tanh(x) + 1)**(3/2))
```

Maxima [F]

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(\tanh(x)+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)/(1+tanh(x))^(3/2),x, algorithm="maxima")
[Out] 1/12*sqrt(2)*(e^(-2*x) + 1)^(3/2) + integrate(1/2*e^(-x)/(sqrt(2)*e^(-x)/(e
^(-2*x) + 1)^(3/2) + sqrt(2)*e^(-3*x)/(e^(-2*x) + 1)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx = -\frac{1}{24}\sqrt{2}\left(\frac{2\left(3\sqrt{e^{(4x)}+e^{(2x)}}-3e^{(2x)}-1\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^{3}} + 3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}+1\right)\right)$$

```
[In] integrate(tanh(x)/(1+tanh(x))^(3/2),x, algorithm="giac")
[Out] -1/24*sqrt(2)*(2*(3*sqrt(e^(4*x) + e^(2*x)) - 3*e^(2*x) - 1)/(sqrt(e^(4*x)
+ e^(2*x)) - e^(2*x))^3 + 3*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1)
)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{\tanh(x)}{(1+\tanh(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh(x)+1}}{2}\right)}{4} - \frac{\frac{\tanh(x)}{2}+\frac{1}{6}}{\left(\tanh\left(x\right)+1\right)^{3/2}}$$

```
[In] int(tanh(x)/(tanh(x) + 1)^(3/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2))/4 - (tanh(x)/2 + 1/6)/(tan
h(x) + 1)^(3/2)
```

3.129 $\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\tanh(x)} - \frac{2}{5}(1+\tanh(x))^{5/2}$$

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)-2/5*(1
+tanh(x))^(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3624, 3559, 3561, 212}

$$\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right) - \frac{2}{5}(\tanh(x)+1)^{5/2} - 2\sqrt{\tanh(x)+1}$$

```
[In] Int[Tanh[x]^2*(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(5/2))/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && !LeQ
[m, -1] && !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{5}(1+\tanh(x))^{5/2}+\int(1+\tanh(x))^{3/2}\,dx\\
&= -2\sqrt{1+\tanh(x)}-\frac{2}{5}(1+\tanh(x))^{5/2}+2\int\sqrt{1+\tanh(x)}\,dx\\
&= -2\sqrt{1+\tanh(x)}-\frac{2}{5}(1+\tanh(x))^{5/2}+4\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-2\sqrt{1+\tanh(x)}-\frac{2}{5}(1+\tanh(x))^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int\tanh^2(x)(1+\tanh(x))^{3/2}\,dx=2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-2\sqrt{1+\tanh(x)}-\frac{2}{5}(1+\tanh(x))^{5/2}$$

```
[In] Integrate[Tanh[x]^2*(1 + Tanh[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - 2*Sqrt[1 + Tanh[x]] - (2*(1
+ Tanh[x])^(5/2))/5
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}-\frac{2(1+\tanh(x))^{\frac{5}{2}}}{5}$	35
default	$2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\tanh\left(x\right)}-\frac{2(1+\tanh(x))^{\frac{5}{2}}}{5}$	35

```
[In] int(tanh(x)^2*(1+tanh(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+tanh(x))^(1/2)-2/5*(1
+tanh(x))^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 429, normalized size of antiderivative = 9.53

$$\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}\big(9\sqrt{2}\cosh\left(x\right)^5+45\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^4+9\sqrt{2}\sinh\left(x\right)^5+10\left(9\sqrt{2}\cosh\left(x\right)^2+\sqrt{2}\right)\sinh\left(x\right)^3+10}{}$$

```
[In] integrate(tanh(x)^2*(1+tanh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(2)*(9*sqrt(2)*cosh(x)^5 + 45*sqrt(2)*cosh(x)*sinh(x)^4 + 9*sqr
t(2)*sinh(x)^5 + 10*(9*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^3 + 10*sqrt(2)*
cosh(x)^3 + 30*(3*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x)^2 + 5*(9*sqr
t(2)*cosh(x)^4 + 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 5*sqrt(2)*cosh(x)
)*sqrt(cosh(x)/(cosh(x) - sinh(x))) - 5*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh
(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(
x)^4 + 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x))*si
nh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 + 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2
 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 + 2*sqrt(2)*cosh(x)^3 + sqrt(
2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(
x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 -
1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh
(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^
```

```
4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 +
 cosh(x))*sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx = -\sqrt{2}\Big(\log\left(\sqrt{\tanh(x)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh(x)+1}+\sqrt{2}\right)\Big) - \frac{2(\tanh(x)+1)^{\frac{5}{2}}}{5} - 2\sqrt{\tanh(x)+1}$$

```
[In] integrate(tanh(x)**2*(1+tanh(x))**(3/2),x)
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
))) - 2*(tanh(x) + 1)**(5/2)/5 - 2*sqrt(tanh(x) + 1)
```

Maxima [F]

$$\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx = \int (\tanh(x)+1)^{\frac{3}{2}}\tanh(x)^2\,dx$$

```
[In] integrate(tanh(x)^2*(1+tanh(x))^(3/2),x, algorithm="maxima")
[Out] integrate((tanh(x) + 1)^(3/2)*tanh(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.11

$$\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx = \frac{1}{5}\sqrt{2}\left(\frac{2\left(25\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^4-60\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3+70\left(\sqrt{e^{(4x)}+e^{(}\right.\right.}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^5}\right.$$

```
[In] integrate(tanh(x)^2*(1+tanh(x))^(3/2),x, algorithm="giac")
[Out] 1/5*sqrt(2)*(2*(25*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^4 - 60*(sqrt(e^(4*x)
 + e^(2*x)) - e^(2*x))^3 + 70*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 40*sq
rt(e^(4*x) + e^(2*x)) + 40*e^(2*x) + 9)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x)
- 1)^5 - 5*log(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \tanh^2(x)(1+\tanh(x))^{3/2}\,dx = 2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh(x)+1}}{2}\right) - 2\sqrt{\tanh(x)+1} - \frac{2\,(\tanh(x)+1)^{5/2}}{5}$$

```
[In] int(tanh(x)^2*(tanh(x) + 1)^(3/2),x)
```

```
[Out] 2*2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2) - 2*(tanh(x) + 1)^(1/2) -
(2*(tanh(x) + 1)^(5/2))/5
```

3.130 $\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\tanh(x))^{3/2}$$

```
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2/3*(1+tanh(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3624, 3561, 212}

$$\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)-\frac{2}{3}(\tanh(x)+1)^{3/2}$$

```
[In] Int[Tanh[x]^2*Sqrt[1 + Tanh[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*(1 + Tanh[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
```

```
b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2}{3}(1+\tanh(x))^{3/2}+\int\sqrt{1+\tanh(x)}\,dx\\ &= -\frac{2}{3}(1+\tanh(x))^{3/2}+2\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\ &= \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\tanh(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int\tanh^2(x)\sqrt{1+\tanh(x)}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\tanh(x))^{3/2}$$

```
[In] Integrate[Tanh[x]^2*Sqrt[1 + Tanh[x]],x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]] - (2*(1 + Tanh[x])^(3/2))/3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-\frac{2(1+\tanh(x))^{\frac{3}{2}}}{3}$	26
default	$\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-\frac{2(1+\tanh(x))^{\frac{3}{2}}}{3}$	26

```
[In] int((1+tanh(x))^(1/2)*tanh(x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-2/3*(1+tanh(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 6.97

$$\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx =$$

$$-\frac{8\sqrt{2}\left(\sqrt{2}\cosh(x)^3+3\sqrt{2}\cosh(x)^2\sinh(x)+3\sqrt{2}\cosh(x)\sinh(x)^2+\sqrt{2}\sinh(x)^3\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-}{}$$

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(8*sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2
)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*sqrt(cosh(x)/(cosh(x) - sinh(x)))
- 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt
(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*sqrt(2)*sqrt(cos
h(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sin
h(x) - 2*sinh(x)^2 - 1))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(
3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x)
+ 1)
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{\tanh(x)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh(x)+1}+\sqrt{2}\right)\right)}{2}-\frac{2(\tanh(x)+1)^{\frac{3}{2}}}{3}$$

```
[In] integrate((1+tanh(x))**(1/2)*tanh(x)**2,x)
```

```
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
)))/2 - 2*(tanh(x) + 1)**(3/2)/3
```

Maxima [F]

$$\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx = \int \sqrt{\tanh(x)+1}\tanh(x)^2\,dx$$

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x)^2,x, algorithm="maxima")
[Out] integrate(sqrt(tanh(x) + 1)*tanh(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.82

$$\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx$$
$$= \frac{1}{6}\sqrt{2}\left(\frac{8\left(3\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2-3\sqrt{e^{(4x)}+e^{(2x)}}+3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}-1\right)^3}-3\log\left(-2\sqrt{e^{(4x)}+e^{(2x)}}+2e^{(2x)}\right.\right.$$

```
[In] integrate((1+tanh(x))^(1/2)*tanh(x)^2,x, algorithm="giac")
[Out] 1/6*sqrt(2)*(8*(3*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x) +
e^(2*x)) + 3*e^(2*x) + 1)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x) - 1)^3 - 3*log
(-2*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \tanh^2(x)\sqrt{1+\tanh(x)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\tanh(x)+1}}{2}\right) - \frac{2(\tanh(x)+1)^{3/2}}{3}$$

```
[In] int(tanh(x)^2*(tanh(x) + 1)^(1/2),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2) - (2*(tanh(x) + 1)^(3/2))/3
```

3.131 $\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1+\tanh(x)}} - 2\sqrt{1+\tanh(x)}$$

```
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+tanh(x))^(1/2)-2*(1
+tanh(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3624, 3560, 3561, 212}

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} - 2\sqrt{\tanh(x)+1} - \frac{1}{\sqrt{\tanh(x)+1}}$$

```
[In] Int[Tanh[x]^2/Sqrt[1 + Tanh[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Tanh[x]] - 2*Sqrt[1
 + Tanh[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -2\sqrt{1+\tanh(x)}+\int \frac{1}{\sqrt{1+\tanh(x)}}\,dx \\
&= -\frac{1}{\sqrt{1+\tanh(x)}}-2\sqrt{1+\tanh(x)}+\frac{1}{2}\int \sqrt{1+\tanh(x)}\,dx \\
&= -\frac{1}{\sqrt{1+\tanh(x)}}-2\sqrt{1+\tanh(x)}+\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right) \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\tanh(x)}}-2\sqrt{1+\tanh(x)}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$
\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = \frac{-\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\tanh(x))\right)-2(1+\tanh(x))}{\sqrt{1+\tanh(x)}}
$$

```
[In] Integrate[Tanh[x]^2/Sqrt[1 + Tanh[x]],x]
[Out] (-Hypergeometric2F1[-1/2, 1, 1/2, (1 + Tanh[x])/2] - 2*(1 + Tanh[x]))/Sqrt[
1 + Tanh[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\tanh(x)}}-2\sqrt{1+\tanh\left(x\right)}$	35
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\tanh(x)}}-2\sqrt{1+\tanh\left(x\right)}$	35

```
[In] int(tanh(x)^2/(1+tanh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+tanh(x))^(1/2)-2*(1
+tanh(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(34) = 68.

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.33

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx =$$
$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh\left(x\right)^2+10\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+5\sqrt{2}\sinh\left(x\right)^2+\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh\left(x\right)^3\right.}{}$$

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(1/2),x, algorithm="fricas")
[Out] -1/4*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^2 + 10*sqrt(2)*cosh(x)*sinh(x) + 5*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x
)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^
2 + sqrt(2))*sinh(x) + sqrt(2)*cosh(x))*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sin
h(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 +
1)*sinh(x) + cosh(x))
```

Sympy [A] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{\tanh(x)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh(x)+1}+\sqrt{2}\right)\right)}{4} - 2\sqrt{\tanh(x)+1} - \frac{1}{\sqrt{\tanh(x)+1}}$$

```
[In] integrate(tanh(x)**2/(1+tanh(x))**(1/2),x)
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
)))/4 - 2*sqrt(tanh(x) + 1) - 1/sqrt(tanh(x) + 1)
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{\tanh(x)+1}}\,dx$$

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^2/sqrt(tanh(x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = -\frac{1}{4}\sqrt{2}\log\left(-4\sqrt{e^{(4x)}+e^{(2x)}}+4e^{(2x)}+2\right) - \frac{5\sqrt{2}e^{(2x)}+\sqrt{2}}{2\sqrt{e^{(4x)}+e^{(2x)}}}$$

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*log(-4*sqrt(e^(4*x) + e^(2*x)) + 4*e^(2*x) + 2) - 1/2*(5*sqrt(
2)*e^(2*x) + sqrt(2))/sqrt(e^(4*x) + e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^2(x)}{\sqrt{1+\tanh(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh(x)+1}}{2}\right)}{2} - \frac{3}{\sqrt{\tanh\left(x\right)+1}} - \frac{2\tanh(x)}{\sqrt{\tanh\left(x\right)+1}}$$

```
[In] int(tanh(x)^2/(tanh(x) + 1)^(1/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2))/2 - 3/(tanh(x) + 1)^(1/2)
- (2*tanh(x))/(tanh(x) + 1)^(1/2)
```

3.132 $\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\tanh(x))^{3/2}} + \frac{3}{2\sqrt{1+\tanh(x)}}$$

```
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)+3/2/(1+tanh(x))^(1/2)-1/
3/(1+tanh(x))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3621, 3607, 3561, 212}

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\tanh(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{3}{2\sqrt{\tanh(x)+1}} - \frac{1}{3(\tanh(x)+1)^{3/2}}$$

```
[In] Int[Tanh[x]^2/(1 + Tanh[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Tanh[x])^(3/2))
+ 3/(2*Sqrt[1 + Tanh[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3(1+\tanh(x))^{3/2}}-\frac{1}{2}\int\frac{1-2\tanh(x)}{\sqrt{1+\tanh(x)}}\,dx\\
&= -\frac{1}{3(1+\tanh(x))^{3/2}}+\frac{3}{2\sqrt{1+\tanh(x)}}+\frac{1}{4}\int\sqrt{1+\tanh(x)}\,dx\\
&= -\frac{1}{3(1+\tanh(x))^{3/2}}+\frac{3}{2\sqrt{1+\tanh(x)}}+\frac{1}{2}\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\tanh(x)}\right)\\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\tanh(x))^{3/2}}+\frac{3}{2\sqrt{1+\tanh(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int\frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx=\frac{14+18\tanh(x)+3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}}{\sqrt{2}}\right)(1+\tanh(x))^{3/2}}{12(1+\tanh(x))^{3/2}}$$

```
[In] Integrate[Tanh[x]^2/(1 + Tanh[x])^(3/2),x]
[Out] (14 + 18*Tanh[x] + 3*Sqrt[2]*ArcTanh[Sqrt[1 + Tanh[x]]/Sqrt[2]]*(1 + Tanh[x
])^(3/2))/(12*(1 + Tanh[x])^(3/2))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{3}{2\sqrt{1+\tanh(x)}}-\frac{1}{3(1+\tanh(x))^{\frac{3}{2}}}$	35
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\tanh(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{3}{2\sqrt{1+\tanh(x)}}-\frac{1}{3(1+\tanh(x))^{\frac{3}{2}}}$	35

```
[In] int(tanh(x)^2/(1+tanh(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/4*arctanh(1/2*(1+tanh(x))^(1/2)*2^(1/2))*2^(1/2)+3/2/(1+tanh(x))^(1/2)-1/
3/(1+tanh(x))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(34) = 68.

Time = 0.24 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.43

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = \frac{2\sqrt{2}\left(8\sqrt{2}\cosh(x)^2+16\sqrt{2}\cosh(x)\sinh(x)+8\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}}{}$$

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(3/2),x, algorithm="fricas")
[Out] 1/24*(2*sqrt(2)*(8*sqrt(2)*cosh(x)^2 + 16*sqrt(2)*cosh(x)*sinh(x) + 8*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(-2*sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sin
h(x)) - 2*cosh(x)^2 - 4*cosh(x)*sinh(x) - 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*
cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [A] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.22

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\log\left(\sqrt{\tanh(x)+1}-\sqrt{2}\right)-\log\left(\sqrt{\tanh(x)+1}+\sqrt{2}\right)\right)}{8} + \frac{3}{2\sqrt{\tanh(x)+1}} - \frac{1}{3(\tanh(x)+1)^{\frac{3}{2}}}$$

```
[In] integrate(tanh(x)**2/(1+tanh(x))**(3/2),x)
[Out] -sqrt(2)*(log(sqrt(tanh(x) + 1) - sqrt(2)) - log(sqrt(tanh(x) + 1) + sqrt(2
)))/8 + 3/(2*sqrt(tanh(x) + 1)) - 1/(3*(tanh(x) + 1)**(3/2))
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = \int \frac{\tanh\left(x\right)^2}{\left(\tanh\left(x\right)+1\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(x)^2/(tanh(x) + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.94

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = \frac{1}{24}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^2+3\sqrt{e^{(4x)}+e^{(2x)}}-3e^{(2x)}-1\right)}{\left(\sqrt{e^{(4x)}+e^{(2x)}}-e^{(2x)}\right)^3}-3\log\left(\right.\right.$$

```
[In] integrate(tanh(x)^2/(1+tanh(x))^(3/2),x, algorithm="giac")
[Out] 1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x) +
 e^(2*x)) - 3*e^(2*x) - 1)/(sqrt(e^(4*x) + e^(2*x)) - e^(2*x))^3 - 3*log(-2
*sqrt(e^(4*x) + e^(2*x)) + 2*e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{\tanh^2(x)}{(1+\tanh(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\tanh(x)+1}}{2}\right)}{4}+\frac{\frac{3\tanh(x)}{2}+\frac{7}{6}}{\left(\tanh\left(x\right)+1\right)^{3/2}}$$

```
[In] int(tanh(x)^2/(tanh(x) + 1)^(3/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(tanh(x) + 1)^(1/2))/2))/4 + ((3*tanh(x))/2 + 7/6)/
(tanh(x) + 1)^(3/2)
```

3.133 $\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{a\log(\cosh(x))}{a^2-b^2}+\frac{a^5\log(a+b\tanh(x))}{b^4(a^2-b^2)} - \frac{(a^2+b^2)\tanh(x)}{b^3}+\frac{a\tanh^2(x)}{2b^2}-\frac{\tanh^3(x)}{3b}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(cosh(x))/(a^2-b^2)+a^5*ln(a+b*tanh(x))/b^4/(a^2-b^2)-(a
^2+b^2)*tanh(x)/b^3+1/2*a*tanh(x)^2/b^2-1/3*tanh(x)^3/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3647, 3728, 3729, 3707, 3698, 31, 3556}

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{a\log(\cosh(x))}{a^2-b^2}-\frac{(a^2+b^2)\tanh(x)}{b^3} +\frac{a^5\log(a+b\tanh(x))}{b^4(a^2-b^2)}+\frac{a\tanh^2(x)}{2b^2}-\frac{\tanh^3(x)}{3b}$$

```
[In] Int[Tanh[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[Cosh[x]])/(a^2 - b^2) + (a^5*Log[a + b*Tanh[x
]])/(b^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/b^3 + (a*Tanh[x]^2)/(2*b^2) -
 Tanh[x]^3/(3*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\tanh^3(x)}{3b} - \frac{\int \frac{\tanh^2(x)(-3a-3b\tanh(x)+3a\tanh^2(x))}{a+b\tanh(x)}\,dx}{3b} \\
&= \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} - \frac{\int \frac{\tanh(x)(6a^2-6(a^2+b^2)\tanh^2(x))}{a+b\tanh(x)}\,dx}{6b^2} \\
&= -\frac{(a^2+b^2)\tanh(x)}{b^3} + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} - \frac{\int \frac{-6a(a^2+b^2)-6b^3\tanh(x)+6a(a^2+b^2)\tanh^2(x)}{a+b\tanh(x)}\,dx}{6b^3} \\
&= -\frac{bx}{a^2-b^2} - \frac{(a^2+b^2)\tanh(x)}{b^3} + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} + \frac{a\int \tanh(x)\,dx}{a^2-b^2} + \frac{a^5\int \frac{1-\tanh^2(x)}{a+b\tanh(x)}\,dx}{b^3(a^2-b^2)} \\
&= -\frac{bx}{a^2-b^2} + \frac{a\log(\cosh(x))}{a^2-b^2} - \frac{(a^2+b^2)\tanh(x)}{b^3} \\
&\quad + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b} + \frac{a^5\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tanh(x)\right)}{b^4(a^2-b^2)} \\
&= -\frac{bx}{a^2-b^2} + \frac{a\log(\cosh(x))}{a^2-b^2} + \frac{a^5\log(a+b\tanh(x))}{b^4(a^2-b^2)} \\
&\quad - \frac{(a^2+b^2)\tanh(x)}{b^3} + \frac{a\tanh^2(x)}{2b^2} - \frac{\tanh^3(x)}{3b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = \frac{1}{6}\left(-\frac{3\log(1-\tanh(x))}{a+b}-\frac{3\log(1+\tanh(x))}{a-b}+\frac{6a^5\log(a+b\tanh(x))}{b^4(a^2-b^2)} -\frac{6(a^2+b^2)\tanh(x)}{b^3}+\frac{3a\tanh^2(x)}{b^2}-\frac{2\tanh^3(x)}{b}\right)$$

```
[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]),x]
```

```
[Out] ((-3*Log[1 - Tanh[x]])/(a + b) - (3*Log[1 + Tanh[x]])/(a - b) + (6*a^5*Log[
a + b*Tanh[x]])/(b^4*(a^2 - b^2)) - (6*(a^2 + b^2)*Tanh[x])/b^3 + (3*a*Tanh
[x]^2)/b^2 - (2*Tanh[x]^3)/b)/6
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{\tanh(x)^3}{3b}+\frac{a\tanh(x)^2}{2b^2}-\frac{a^2\tanh(x)}{b^3}-\frac{\tanh(x)}{b}-\frac{\ln(\tanh(x)-1)}{2a+2b}+\frac{a^5\ln(a+b\tanh(x))}{b^4(a+b)(a-b)}-\frac{\ln(1+\tanh(x))}{2a-2b}$
default	$-\frac{\tanh(x)^3}{3b}+\frac{a\tanh(x)^2}{2b^2}-\frac{a^2\tanh(x)}{b^3}-\frac{\tanh(x)}{b}-\frac{\ln(\tanh(x)-1)}{2a+2b}+\frac{a^5\ln(a+b\tanh(x))}{b^4(a+b)(a-b)}-\frac{\ln(1+\tanh(x))}{2a-2b}$
parallelrisch	$-\frac{2\tanh(x)^3a^2b^3-2\tanh(x)^3b^5-3\tanh(x)^2b^2a^3+3\tanh(x)^2ab^4+6\ln(1-\tanh(x))ab^4-6a^5\ln(a+b\tanh(x))+6ab^4x+}{6b^4(a^2-b^2)}$
risch	$\frac{x}{a+b}+\frac{2xa^3}{b^4}+\frac{2ax}{b^2}-\frac{2xa^5}{b^4(a^2-b^2)}+\frac{2a^2e^{4x}-2ab\,e^{4x}+4b^2e^{4x}+4a^2e^{2x}-2b\,e^{2x}a+4b^2e^{2x}+2a^2+\frac{8b^2}{3}}{b^3(1+e^{2x})^3}-\frac{a^3\ln(1+e}{b^4}$

```
[In] int(tanh(x)^5/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*tanh(x)^3/b+1/2*a*tanh(x)^2/b^2-1/b^3*a^2*tanh(x)-tanh(x)/b-1/(2*a+2*b
)*ln(tanh(x)-1)+1/b^4*a^5/(a+b)/(a-b)*ln(a+b*tanh(x))-1/(2*a-2*b)*ln(1+tanh
(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1296 vs. $2(90) = 180$.

Time = 0.29 (sec) , antiderivative size = 1296, normalized size of antiderivative = 13.79

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 +
3*(a*b^4 + b^5)*x*sinh(x)^6 - 6*a^4*b - 2*a^2*b^3 + 8*b^5 - 3*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 - 3*(2
*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 15*(a*b^4 + b^5)*x*cosh(
x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3 - (2*
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x
))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x
)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2 - 2*a*b
^4 + 4*b^5 - 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^
4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 + 3*(a*b^4 + b^5)*x -
3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 + 3*a^5*cosh(x)^
4 + 3*a^5*cosh(x)^2 + a^5 + 3*(5*a^5*cosh(x)^2 + a^5)*sinh(x)^4 + 4*(5*a^5*
cosh(x)^3 + 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 + 6*a^5*cosh(x)^2
 + a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 + 2*a^5*cosh(x)^3 + a^5*cosh(x))*sinh(
x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^5 - a*b^4)*c
osh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4)*sinh(x)^6 + a^
5 - a*b^4 + 3*(a^5 - a*b^4)*cosh(x)^4 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*
cosh(x)^4 + 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^5 - a*b^4)*cosh(x)
^5 + 2*(a^5 - a*b^4)*cosh(x)^3 + (a^5 - a*b^4)*cosh(x))*sinh(x))*log(2*cosh
(x)/(cosh(x) - sinh(x))) + 6*(3*(a*b^4 + b^5)*x*cosh(x)^5 - 2*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^3 - (4*a
^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x))/(
(a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^2*b^4
- b^6)*sinh(x)^6 + a^2*b^4 - b^6 + 3*(a^2*b^4 - b^6)*cosh(x)^4 + 3*(a^2*b^4
 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b^4 - b^6)*cosh
(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2
+ 3*(a^2*b^4 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^4 + 6*(a^2*b^4 - b^6)*cosh(x
)^2)*sinh(x)^2 + 6*((a^2*b^4 - b^6)*cosh(x)^5 + 2*(a^2*b^4 - b^6)*cosh(x)^3
 + (a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(78) = 156$.

Time = 0.45 (sec) , antiderivative size = 546, normalized size of antiderivative = 5.81

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx$$

$$= \begin{cases} \tilde{\infty}\left(x - \frac{\tanh^3(x)}{3} - \tanh(x)\right) \\ \frac{x-\log(\tanh(x)+1)-\frac{\tanh^4(x)}{4}-\frac{\tanh^2(x)}{2}}{a} \\ \frac{27x\tanh(x)}{6b\tanh(x)-6b} - \frac{27x}{6b\tanh(x)-6b} - \frac{12\log(\tanh(x)+1)\tanh(x)}{6b\tanh(x)-6b} + \frac{12\log(\tanh(x)+1)}{6b\tanh(x)-6b} - \frac{2\tanh^4(x)}{6b\tanh(x)-6b} - \frac{\tanh^3(x)}{6b\tanh(x)-6b} - \frac{9\tan}{6b\tanh} \\ \frac{3x\tanh(x)}{6b\tanh(x)+6b} + \frac{3x}{6b\tanh(x)+6b} + \frac{12\log(\tanh(x)+1)\tanh(x)}{6b\tanh(x)+6b} + \frac{12\log(\tanh(x)+1)}{6b\tanh(x)+6b} - \frac{2\tanh^4(x)}{6b\tanh(x)+6b} + \frac{\tanh^3(x)}{6b\tanh(x)+6b} - \frac{9\tan}{6b\tanh} \\ \frac{6a^5\log\left(\frac{a}{b}+\tanh(x)\right)}{6a^2b^4-6b^6} - \frac{6a^4b\tanh(x)}{6a^2b^4-6b^6} + \frac{3a^3b^2\tanh^2(x)}{6a^2b^4-6b^6} - \frac{2a^2b^3\tanh^3(x)}{6a^2b^4-6b^6} + \frac{6ab^4x}{6a^2b^4-6b^6} - \frac{6ab^4\log(\tanh(x)+1)}{6a^2b^4-6b^6} - \frac{3ab^4\tanh^2(x)}{6a^2b^4-6b^6} \end{cases}$$

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - tanh(x)**3/3 - tanh(x)), Eq(a, 0) & Eq(b, 0)), ((x - lo
g(tanh(x) + 1) - tanh(x)**4/4 - tanh(x)**2/2)/a, Eq(b, 0)), (27*x*tanh(x)/(
6*b*tanh(x) - 6*b) - 27*x/(6*b*tanh(x) - 6*b) - 12*log(tanh(x) + 1)*tanh(x)
/(6*b*tanh(x) - 6*b) + 12*log(tanh(x) + 1)/(6*b*tanh(x) - 6*b) - 2*tanh(x)*
*4/(6*b*tanh(x) - 6*b) - tanh(x)**3/(6*b*tanh(x) - 6*b) - 9*tanh(x)**2/(6*b
*tanh(x) - 6*b) + 15/(6*b*tanh(x) - 6*b), Eq(a, -b)), (3*x*tanh(x)/(6*b*tan
h(x) + 6*b) + 3*x/(6*b*tanh(x) + 6*b) + 12*log(tanh(x) + 1)*tanh(x)/(6*b*ta
nh(x) + 6*b) + 12*log(tanh(x) + 1)/(6*b*tanh(x) + 6*b) - 2*tanh(x)**4/(6*b*
tanh(x) + 6*b) + tanh(x)**3/(6*b*tanh(x) + 6*b) - 9*tanh(x)**2/(6*b*tanh(x)
 + 6*b) + 15/(6*b*tanh(x) + 6*b), Eq(a, b)), (6*a**5*log(a/b + tanh(x))/(6*
a**2*b**4 - 6*b**6) - 6*a**4*b*tanh(x)/(6*a**2*b**4 - 6*b**6) + 3*a**3*b**2
*tanh(x)**2/(6*a**2*b**4 - 6*b**6) - 2*a**2*b**3*tanh(x)**3/(6*a**2*b**4 -
6*b**6) + 6*a*b**4*x/(6*a**2*b**4 - 6*b**6) - 6*a*b**4*log(tanh(x) + 1)/(6*
a**2*b**4 - 6*b**6) - 3*a*b**4*tanh(x)**2/(6*a**2*b**4 - 6*b**6) - 6*b**5*x
/(6*a**2*b**4 - 6*b**6) + 2*b**5*tanh(x)**3/(6*a**2*b**4 - 6*b**6) + 6*b**5
*tanh(x)/(6*a**2*b**4 - 6*b**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.60

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = \frac{a^5\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2b^4-b^6} - \frac{2\left(3a^2+4b^2+3(2a^2+ab+2b^2)e^{(-2x)}+3(a^2+ab+2b^2)e^{(-4x)}\right)}{3\left(3b^3e^{(-2x)}+3b^3e^{(-4x)}+b^3e^{(-6x)}+b^3\right)} + \frac{x}{a+b} - \frac{(a^3+ab^2)\log\left(e^{(-2x)}+1\right)}{b^4}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] a^5*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b^4 - b^6) - 2/3*(3*a^2 + 4*b^2 + 3
*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*e^
(-2*x) + 3*b^3*e^(-4*x) + b^3*e^(-6*x) + b^3) + x/(a + b) - (a^3 + a*b^2)*l
og(e^(-2*x) + 1)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.51

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = \frac{a^5\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^2b^4-b^6} - \frac{x}{a-b} - \frac{(a^3+ab^2)\log\left(e^{(2x)}+1\right)}{b^4} + \frac{2\left(3a^2b+4b^3+3(a^2b-ab^2+2b^3)e^{(4x)}+3(2a^2b-ab^2+2b^3)e^{(2x)}\right)}{3b^4\left(e^{(2x)}+1\right)^3}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] a^5*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b^4 - b^6) - x/(a - b) - (
a^3 + a*b^2)*log(e^(2*x) + 1)/b^4 + 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b - a*b^2
 + 2*b^3)*e^(4*x) + 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2*x) + 1)
^3)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

$$\int \frac{\tanh^5(x)}{a+b\tanh(x)}\,dx = \frac{x}{a+b} - \frac{\tanh(x)^3}{3b} - \frac{a\,\ln(\tanh(x)+1)}{a^2-b^2} + \frac{a\tanh(x)^2}{2b^2} - \frac{\tanh(x)\,(a^2+b^2)}{b^3} + \frac{a^5\,\ln(a+b\tanh(x))}{b^4\,(a^2-b^2)}$$

```
[In] int(tanh(x)^5/(a + b*tanh(x)),x)
```

```
[Out] x/(a + b) - tanh(x)^3/(3*b) - (a*log(tanh(x) + 1))/(a^2 - b^2) + (a*tanh(x)
^2)/(2*b^2) - (tanh(x)*(a^2 + b^2))/b^3 + (a^5*log(a + b*tanh(x)))/(b^4*(a^
2 - b^2))
```

3.134 $\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2-b^2}-\frac{a^4\log(a+b\tanh(x))}{b^3(a^2-b^2)}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

```
[Out] a*x/(a^2-b^2)-b*ln(cosh(x))/(a^2-b^2)-a^4*ln(a+b*tanh(x))/b^3/(a^2-b^2)+a*t
anh(x)/b^2-1/2*tanh(x)^2/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3647, 3728, 3708, 3698, 31, 3556}

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2-b^2}-\frac{a^4\log(a+b\tanh(x))}{b^3(a^2-b^2)}+\frac{a\tanh(x)}{b^2}-\frac{\tanh^2(x)}{2b}$$

```
[In] Int[Tanh[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/(a^2 - b^2) - (a^4*Log[a + b*Tanh[x]])
/(b^3*(a^2 - b^2)) + (a*Tanh[x])/b^2 - Tanh[x]^2/(2*b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3708

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[b*((A - C)/(a^2 + b^2)), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
 f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\text{integral} = -\frac{\tanh^2(x)}{2b} - \frac{\int \frac{\tanh(x)\left(-2a-2b\tanh(x)+2a\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{2b}$$

$$= \frac{a\tanh(x)}{b^2} - \frac{\tanh^2(x)}{2b} - \frac{\int \frac{2a^2-2(a^2+b^2)\tanh^2(x)}{a+b\tanh(x)}\,dx}{2b^2}$$

$$= \frac{ax}{a^2-b^2} + \frac{a\tanh(x)}{b^2} - \frac{\tanh^2(x)}{2b} - \frac{a^4\int \frac{1-\tanh^2(x)}{a+b\tanh(x)}\,dx}{b^2(a^2-b^2)} - \frac{b\int \tanh(x)\,dx}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(\cosh(x))}{a^2-b^2} + \frac{a\tanh(x)}{b^2} - \frac{\tanh^2(x)}{2b} - \frac{a^4\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tanh(x)\right)}{b^3(a^2-b^2)}$$

$$= \frac{ax}{a^2-b^2} - \frac{b\log(\cosh(x))}{a^2-b^2} - \frac{a^4\log(a+b\tanh(x))}{b^3(a^2-b^2)} + \frac{a\tanh(x)}{b^2} - \frac{\tanh^2(x)}{2b}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx = -\frac{\log(1-\tanh(x))}{2(a+b)} + \frac{\log(1+\tanh(x))}{2(a-b)} - \frac{a^4\log(a+b\tanh(x))}{b^3(a^2-b^2)} + \frac{a\tanh(x)}{b^2} - \frac{\tanh^2(x)}{2b}$$

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]),x]
[Out] -1/2*Log[1 - Tanh[x]]/(a + b) + Log[1 + Tanh[x]]/(2*(a - b)) - (a^4*Log[a + b*Tanh[x]])/(b^3*(a^2 - b^2)) + (a*Tanh[x])/b^2 - Tanh[x]^2/(2*b)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{\tanh(x)^2}{2b}+\frac{a\tanh(x)}{b^2}-\frac{a^4\ln(a+b\tanh(x))}{b^3(a+b)(a-b)}+\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2a+2b}$
default	$-\frac{\tanh(x)^2}{2b}+\frac{a\tanh(x)}{b^2}-\frac{a^4\ln(a+b\tanh(x))}{b^3(a+b)(a-b)}+\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2a+2b}$
parallelrisch	$-\frac{\tanh(x)^2a^2b^2-\tanh(x)^2b^4-2\ln(1-\tanh(x))b^4+2a^4\ln(a+b\tanh(x))-2b^3ax-2b^4x-2b\tanh(x)a^3+2\tanh(x)ab^3}{2b^3(a^2-b^2)}$
risch	$\frac{x}{a+b}-\frac{2xa^2}{b^3}-\frac{2x}{b}+\frac{2xa^4}{b^3(a^2-b^2)}-\frac{2(a\,e^{2x}-b\,e^{2x}+a)}{(1+e^{2x})^2b^2}+\frac{\ln(1+e^{2x})a^2}{b^3}+\frac{\ln(1+e^{2x})}{b}-\frac{a^4\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{b^3(a^2-b^2)}$

```
[In] int(tanh(x)^4/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*tanh(x)^2/b+a*tanh(x)/b^2-1/b^3*a^4/(a+b)/(a-b)*ln(a+b*tanh(x))+1/(2*a-2*b)*ln(1+tanh(x))-1/(2*a+2*b)*ln(tanh(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 644, normalized size of antiderivative = 8.47

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx$$
$$= \frac{(ab^3+b^4)x\cosh(x)^4+4\,(ab^3+b^4)x\cosh(x)\sinh(x)^3+(ab^3+b^4)x\sinh(x)^4-2\,a^3b+2\,ab^3-2\,(a^3b-a^2}{}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
[Out] ((a*b^3 + b^4)*x*cosh(x)^4 + 4*(a*b^3 + b^4)*x*cosh(x)*sinh(x)^3 + (a*b^3 +
 b^4)*x*sinh(x)^4 - 2*a^3*b + 2*a*b^3 - 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 -
(a*b^3 + b^4)*x)*cosh(x)^2 - 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 - 3*(a*b^3 +
b^4)*x*cosh(x)^2 - (a*b^3 + b^4)*x)*sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*cosh
(x)^4 + 4*a^4*cosh(x)*sinh(x)^3 + a^4*sinh(x)^4 + 2*a^4*cosh(x)^2 + a^4 + 2
*(3*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 4*(a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x
))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) + ((a^4 - b^4)*cosh(x
)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4 +
 2*(a^4 - b^4)*cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)^
2 + 4*((a^4 - b^4)*cosh(x)^3 + (a^4 - b^4)*cosh(x))*sinh(x))*log(2*cosh(x)/
(cosh(x) - sinh(x))) + 4*((a*b^3 + b^4)*x*cosh(x)^3 - (a^3*b - a^2*b^2 - a*
b^3 + b^4 - (a*b^3 + b^4)*x)*cosh(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 -
b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh
(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5 + 3*(a^2*b^3 - b^5)*
cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(
x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(61) = 122.

Time = 0.39 (sec) , antiderivative size = 442, normalized size of antiderivative = 5.82

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(x-\log(\tanh(x)+1)-\frac{\tanh^2(x)}{2}\right) \\ \frac{x-\frac{\tanh^3(x)}{3}-\tanh(x)}{a} \\ \frac{7x\tanh(x)}{2b\tanh(x)-2b}-\frac{7x}{2b\tanh(x)-2b}-\frac{4\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b}+\frac{4\log(\tanh(x)+1)}{2b\tanh(x)-2b}-\frac{\tanh^3(x)}{2b\tanh(x)-2b}-\frac{\tanh^2(x)}{2b\tanh(x)-2b}+\frac{3}{2b\tanh(x)} \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{4\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b}-\frac{4\log(\tanh(x)+1)}{2b\tanh(x)+2b}-\frac{\tanh^3(x)}{2b\tanh(x)+2b}+\frac{\tanh^2(x)}{2b\tanh(x)+2b}-\frac{3}{2b\tanh(x)} \\ -\frac{2a^4\log\left(\frac{a}{b}+\tanh(x)\right)}{2a^2b^3-2b^5}+\frac{2a^3b\tanh(x)}{2a^2b^3-2b^5}-\frac{a^2b^2\tanh^2(x)}{2a^2b^3-2b^5}+\frac{2ab^3x}{2a^2b^3-2b^5}-\frac{2ab^3\tanh(x)}{2a^2b^3-2b^5}-\frac{2b^4x}{2a^2b^3-2b^5}+\frac{2b^4\log(\tanh(x)+1)}{2a^2b^3-2b^5}+\frac{b^4\tan}{2a^2} \end{cases}$$

```
[In] integrate(tanh(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) - tanh(x)**2/2), Eq(a, 0) & Eq(b, 0)),
 ((x - tanh(x)**3/3 - tanh(x))/a, Eq(b, 0)), (7*x*tanh(x)/(2*b*tanh(x) - 2*
b) - 7*x/(2*b*tanh(x) - 2*b) - 4*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) - 2*
b) + 4*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) - tanh(x)**3/(2*b*tanh(x) - 2*b
) - tanh(x)**2/(2*b*tanh(x) - 2*b) + 3/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*
tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 4*log(tanh(x) + 1)*ta
nh(x)/(2*b*tanh(x) + 2*b) - 4*log(tanh(x) + 1)/(2*b*tanh(x) + 2*b) - tanh(x
)**3/(2*b*tanh(x) + 2*b) + tanh(x)**2/(2*b*tanh(x) + 2*b) - 3/(2*b*tanh(x)
+ 2*b), Eq(a, b)), (-2*a**4*log(a/b + tanh(x))/(2*a**2*b**3 - 2*b**5) + 2*a
**3*b*tanh(x)/(2*a**2*b**3 - 2*b**5) - a**2*b**2*tanh(x)**2/(2*a**2*b**3 -
2*b**5) + 2*a*b**3*x/(2*a**2*b**3 - 2*b**5) - 2*a*b**3*tanh(x)/(2*a**2*b**3
 - 2*b**5) - 2*b**4*x/(2*a**2*b**3 - 2*b**5) + 2*b**4*log(tanh(x) + 1)/(2*a
**2*b**3 - 2*b**5) + b**4*tanh(x)**2/(2*a**2*b**3 - 2*b**5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx = -\frac{a^4\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2b^3-b^5} + \frac{2\left((a+b)e^{(-2\,x)}+a\right)}{2\,b^2e^{(-2\,x)}+b^2e^{(-4\,x)}+b^2} + \frac{x}{a+b} + \frac{\left(a^2+b^2\right)\log\left(e^{(-2\,x)}+1\right)}{b^3}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -a^4*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b^3 - b^5) + 2*((a + b)*e^(-2*x) +
 a)/(2*b^2*e^(-2*x) + b^2*e^(-4*x) + b^2) + x/(a + b) + (a^2 + b^2)*log(e^(
-2*x) + 1)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx = -\frac{a^4\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2b^3-b^5} + \frac{x}{a-b} + \frac{\left(a^2+b^2\right)\log\left(e^{(2\,x)}+1\right)}{b^3} - \frac{2\left(ab+\left(ab-b^2\right)e^{(2\,x)}\right)}{b^3\left(e^{(2\,x)}+1\right)^2}$$

```
[In] integrate(tanh(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -a^4*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b^3 - b^5) + x/(a - b) +
(a^2 + b^2)*log(e^(2*x) + 1)/b^3 - 2*(a*b + (a*b - b^2)*e^(2*x))/(b^3*(e^(2
*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{\tanh^4(x)}{a+b\tanh(x)}\,dx = \frac{x}{a+b} - \frac{\tanh(x)^2}{2\,b} + \frac{b\,\ln\left(\tanh(x)+1\right)}{a^2-b^2} + \frac{a\tanh(x)}{b^2} - \frac{a^4\,\ln\left(a+b\tanh(x)\right)}{b^3\,\left(a^2-b^2\right)}$$

```
[In] int(tanh(x)^4/(a + b*tanh(x)),x)
```

```
[Out] x/(a + b) - tanh(x)^2/(2*b) + (b*log(tanh(x) + 1))/(a^2 - b^2) + (a*tanh(x)
)/b^2 - (a^4*log(a + b*tanh(x)))/(b^3*(a^2 - b^2))
```

3.135 $\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{a\log(\cosh(x))}{a^2-b^2}+\frac{a^3\log(a+b\tanh(x))}{b^2(a^2-b^2)}-\frac{\tanh(x)}{b}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(cosh(x))/(a^2-b^2)+a^3*ln(a+b*tanh(x))/b^2/(a^2-b^2)-ta
nh(x)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3647, 3707, 3698, 31, 3556}

$$\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{a\log(\cosh(x))}{a^2-b^2}+\frac{a^3\log(a+b\tanh(x))}{b^2(a^2-b^2)}-\frac{\tanh(x)}{b}$$

```
[In] Int[Tanh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[Cosh[x]])/(a^2 - b^2) + (a^3*Log[a + b*Tanh[x
]])/(b^2*(a^2 - b^2)) - Tanh[x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\tanh(x)}{b} - \frac{\int \frac{-a-b\tanh(x)+a\tanh^2(x)}{a+b\tanh(x)}\,dx}{b}\\
&= -\frac{bx}{a^2-b^2} - \frac{\tanh(x)}{b} + \frac{a\int \tanh(x)\,dx}{a^2-b^2} + \frac{a^3\int \frac{1-\tanh^2(x)}{a+b\tanh(x)}\,dx}{b\,(a^2-b^2)}\\
&= -\frac{bx}{a^2-b^2} + \frac{a\log(\cosh(x))}{a^2-b^2} - \frac{\tanh(x)}{b} + \frac{a^3\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\tanh(x)\right)}{b^2\,(a^2-b^2)}\\
&= -\frac{bx}{a^2-b^2} + \frac{a\log(\cosh(x))}{a^2-b^2} + \frac{a^3\log(a+b\tanh(x))}{b^2\,(a^2-b^2)} - \frac{\tanh(x)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx = -\frac{\log(1-\tanh(x))}{2(a+b)}-\frac{\log(1+\tanh(x))}{2(a-b)}+\frac{a^3\log(a+b\tanh(x))}{b^2\,(a^2-b^2)}-\frac{\tanh(x)}{b}$$

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -1/2*Log[1 - Tanh[x]]/(a + b) - Log[1 + Tanh[x]]/(2*(a - b)) + (a^3*Log[a +
 b*Tanh[x]])/(b^2*(a^2 - b^2)) - Tanh[x]/b
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{\tanh(x)}{b}-\frac{\ln(1+\tanh(x))}{2a-2b}+\frac{a^3\ln(a+b\tanh(x))}{b^2(a+b)(a-b)}-\frac{\ln(\tanh(x)-1)}{2a+2b}$	67
default	$-\frac{\tanh(x)}{b}-\frac{\ln(1+\tanh(x))}{2a-2b}+\frac{a^3\ln(a+b\tanh(x))}{b^2(a+b)(a-b)}-\frac{\ln(\tanh(x)-1)}{2a+2b}$	67
parallelrisch	$-\frac{\ln(1-\tanh(x))a\,b^2-a^3\ln(a+b\tanh(x))+a\,b^2x+b^3x+\tanh(x)a^2b-\tanh(x)b^3}{b^2(a^2-b^2)}$	67
risch	$\frac{x}{a+b}-\frac{2a^3x}{b^2(a^2-b^2)}+\frac{2ax}{b^2}+\frac{2}{b(1+e^{2x})}+\frac{a^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{b^2(a^2-b^2)}-\frac{a\ln(1+e^{2x})}{b^2}$	97

```
[In] int(tanh(x)^3/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -tanh(x)/b-1/(2*a-2*b)*ln(1+tanh(x))+1/b^2*a^3/(a+b)/(a-b)*ln(a+b*tanh(x))-
1/(2*a+2*b)*ln(tanh(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.12

$$\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx = -\frac{(ab^2+b^3)x\cosh(x)^2+2\,(ab^2+b^3)x\cosh(x)\sinh(x)+(ab^2+b^3)x\sinh(x)^2-2\,a^2b+2\,b^3+(ab^2+b^3}{}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 +
b^3)*x*sinh(x)^2 - 2*a^2*b + 2*b^3 + (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*a
^3*cosh(x)*sinh(x) + a^3*sinh(x)^2 + a^3)*log(2*(a*cosh(x) + b*sinh(x))/(co
sh(x) - sinh(x))) + (a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 - a*b^2
)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(
x))))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^4)*cosh(x
)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(49) = 98$.

Time = 0.31 (sec) , antiderivative size = 330, normalized size of antiderivative = 5.16

$$\int \frac{\tanh^3(x)}{a + b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}(x - \tanh(x)) & \text{for } a = 0 \\ \frac{x - \log(\tanh(x)+1) - \frac{\tanh^2(x)}{2}}{a} & \text{for } b = 0 \\ \frac{5x\tanh(x)}{2b\tanh(x)-2b} - \frac{5x}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x)+1)}{2b\tanh(x)-2b} - \frac{2\tanh^2(x)}{2b\tanh(x)-2b} + \frac{3}{2b\tanh(x)-2b} & \text{for } a = - \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x)+1)}{2b\tanh(x)+2b} - \frac{2\tanh^2(x)}{2b\tanh(x)+2b} + \frac{3}{2b\tanh(x)+2b} & \text{for } a = b \\ \frac{a^3\log(\frac{a}{b}+\tanh(x))}{a^2b^2-b^4} - \frac{a^2b\tanh(x)}{a^2b^2-b^4} + \frac{ab^2x}{a^2b^2-b^4} - \frac{ab^2\log(\tanh(x)+1)}{a^2b^2-b^4} - \frac{b^3x}{a^2b^2-b^4} + \frac{b^3\tanh(x)}{a^2b^2-b^4} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - tanh(x)), Eq(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1)
- tanh(x)**2/2)/a, Eq(b, 0)), (5*x*tanh(x)/(2*b*tanh(x) - 2*b) - 5*x/(2*b*t
anh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) - 2*b) + 2*log(tanh
(x) + 1)/(2*b*tanh(x) - 2*b) - 2*tanh(x)**2/(2*b*tanh(x) - 2*b) + 3/(2*b*ta
nh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) +
 2*b) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) + 2*log(tanh(x) + 1)
/(2*b*tanh(x) + 2*b) - 2*tanh(x)**2/(2*b*tanh(x) + 2*b) + 3/(2*b*tanh(x) +
2*b), Eq(a, b)), (a**3*log(a/b + tanh(x))/(a**2*b**2 - b**4) - a**2*b*tanh(
x)/(a**2*b**2 - b**4) + a*b**2*x/(a**2*b**2 - b**4) - a*b**2*log(tanh(x) +
1)/(a**2*b**2 - b**4) - b**3*x/(a**2*b**2 - b**4) + b**3*tanh(x)/(a**2*b**2
 - b**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx = \frac{a^3\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2b^2-b^4}+\frac{x}{a+b} - \frac{a\log\left(e^{(-2\,x)}+1\right)}{b^2}-\frac{2}{be^{(-2\,x)}+b}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] a^3*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(-
2*x) + 1)/b^2 - 2/(b*e^(-2*x) + b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx = \frac{a^3\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2b^2-b^4}-\frac{x}{a-b} - \frac{a\log\left(e^{(2\,x)}+1\right)}{b^2}+\frac{2}{b(e^{(2\,x)}+1)}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] a^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b^2 - b^4) - x/(a - b) - a
*log(e^(2*x) + 1)/b^2 + 2/(b*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{\tanh^3(x)}{a+b\tanh(x)}\,dx = \frac{x}{a+b}-\frac{\tanh(x)}{b}-\frac{a\,\ln(\tanh(x)+1)}{a^2-b^2}+\frac{a^3\,\ln(a+b\tanh(x))}{b^2\,(a^2-b^2)}$$

```
[In] int(tanh(x)^3/(a + b*tanh(x)),x)
```

```
[Out] x/(a + b) - tanh(x)/b - (a*log(tanh(x) + 1))/(a^2 - b^2) + (a^3*log(a + b*t
anh(x)))/(b^2*(a^2 - b^2))
```

3.136 $\int \frac{\tanh^2(x)}{a+b\tanh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)} dx = -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\cosh(x))}{b} - \frac{a^2\log(a\cosh(x)+b\sinh(x))}{b(a^2-b^2)}$$

```
[Out] -a*x/b^2+a^3*x/b^2/(a^2-b^2)+ln(cosh(x))/b-a^2*ln(a*cosh(x)+b*sinh(x))/b/(a
^2-b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3622, 3556, 3565, 3611}

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)} dx = -\frac{a^2\log(a\cosh(x)+b\sinh(x))}{b(a^2-b^2)} + \frac{a^3x}{b^2(a^2-b^2)} - \frac{ax}{b^2} + \frac{\log(\cosh(x))}{b}$$

```
[In] Int[Tanh[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Cosh[x]]/b - (a^2*Log[a*Cosh
[x] + b*Sinh[x]])/(b*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
```

```
d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{ax}{b^2} + \frac{a^2 \int \frac{1}{a+b\tanh(x)}\,dx}{b^2} + \frac{\int \tanh(x)\,dx}{b} \\ &= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\cosh(x))}{b} - \frac{(ia^2)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{b(a^2-b^2)} \\ &= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\cosh(x))}{b} - \frac{a^2\log(a\cosh(x)+b\sinh(x))}{b(a^2-b^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx = -\frac{\log(1-\tanh(x))}{2(a+b)} + \frac{\log(1+\tanh(x))}{2(a-b)} - \frac{a^2\log(a+b\tanh(x))}{b(a^2-b^2)}$$

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]),x]
[Out] -1/2*Log[1 - Tanh[x]]/(a + b) + Log[1 + Tanh[x]]/(2*(a - b)) - (a^2*Log[a +
 b*Tanh[x]])/(b*(a^2 - b^2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$-\frac{-\ln(1-\tanh(x))b^2+a^2\ln(a+b\tanh(x))-abx-b^2x}{b(a^2-b^2)}$	52
derivativedivides	$-\frac{a^2\ln(a+b\tanh(x))}{(a+b)(a-b)b}+\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2a+2b}$	60
default	$-\frac{a^2\ln(a+b\tanh(x))}{(a+b)(a-b)b}+\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{\ln(\tanh(x)-1)}{2a+2b}$	60
risch	$\frac{x}{a+b}+\frac{2x\,a^2}{b(a^2-b^2)}-\frac{2x}{b}-\frac{a^2\ln\left(\mathrm{e}^{2x}+\frac{a-b}{a+b}\right)}{b(a^2-b^2)}+\frac{\ln(1+\mathrm{e}^{2x})}{b}$	82

```
[In] int(tanh(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -(-ln(1-tanh(x))*b^2+a^2*ln(a+b*tanh(x))-a*b*x-b^2*x)/b/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx = -\frac{a^2\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-(ab+b^2)x-(a^2-b^2)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^2b-b^3}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
[Out] -(a^2*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - (a*b + b^2)*x -
(a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(51) = 102.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.86

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)) & \text{for } a=0\wedge b=0 \\ \frac{x-\tanh(x)}{a} & \text{for } b=0 \\ \frac{3x\tanh(x)}{2b\tanh(x)-2b}-\frac{3x}{2b\tanh(x)-2b}-\frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b}+\frac{2\log(\tanh(x)+1)}{2b\tanh(x)-2b}+\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b}-\frac{2\log(\tanh(x)+1)}{2b\tanh(x)+2b}-\frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ -\frac{a^2\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2b-b^3}+\frac{abx}{a^2b-b^3}-\frac{b^2x}{a^2b-b^3}+\frac{b^2\log(\tanh(x)+1)}{a^2b-b^3} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), ((x - tanh(x))
/a, Eq(b, 0)), (3*x*tanh(x)/(2*b*tanh(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) -
 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*t
anh(x) - 2*b) + 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x)
+ 2*b) + x/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) +
2*b) - 2*log(tanh(x) + 1)/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a
, b)), (-a**2*log(a/b + tanh(x))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) -
b**2*x/(a**2*b - b**3) + b**2*log(tanh(x) + 1)/(a**2*b - b**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx = -\frac{a^2\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2b-b^3}+\frac{x}{a+b}+\frac{\log\left(e^{(-2x)}+1\right)}{b}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log(-(a - b)*e^(-2*x) - a - b)/(a^2*b - b^3) + x/(a + b) + log(e^(-2*x
) + 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx = -\frac{a^2\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2b-b^3}+\frac{x}{a-b}+\frac{\log\left(e^{(2\,x)}+1\right)}{b}$$

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2*b - b^3) + x/(a - b) + lo
g(e^(2*x) + 1)/b
```

Mupad [B] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{\tanh^2(x)}{a+b\tanh(x)}\,dx = -\frac{b^2\,\left(x-\ln\left(\tanh(x)+1\right)\right)+a^2\,\ln\left(a+b\tanh(x)\right)-a\,b\,x}{b\,\left(a^2-b^2\right)}$$

```
[In] int(tanh(x)^2/(a + b*tanh(x)),x)
```

```
[Out] -(b^2*(x - log(tanh(x) + 1)) + a^2*log(a + b*tanh(x)) - a*b*x)/(b*(a^2 - b^
2))
```

3.137 $\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3612, 3611}

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = \frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[In] Int[Tanh[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{bx}{a^2-b^2} + \frac{(ia)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}$$
$$= -\frac{bx}{a^2-b^2} + \frac{a\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))-(a+b)\log(1+\tanh(x))+2a\log(a+b\tanh(x))}{2(a-b)(a+b)}$$

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]),x]
[Out] ((-a + b)*Log[1 - Tanh[x]] - (a + b)*Log[1 + Tanh[x]] + 2*a*Log[a + b*Tanh[
x]])/(2*(a - b)*(a + b))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$-\frac{a\ln(1-\tanh(x))-a\ln(a+b\tanh(x))+ax+bx}{a^2-b^2}$	40
risch	$\frac{x}{a+b}-\frac{2ax}{a^2-b^2}+\frac{a\ln\left(\mathrm{e}^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	54
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{2a+2b}+\frac{a\ln(a+b\tanh(x))}{(a+b)(a-b)}-\frac{\ln(1+\tanh(x))}{2a-2b}$	55
default	$-\frac{\ln(\tanh(x)-1)}{2a+2b}+\frac{a\ln(a+b\tanh(x))}{(a+b)(a-b)}-\frac{\ln(1+\tanh(x))}{2a-2b}$	55

```
[In] int(tanh(x)/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -(a*ln(1-tanh(x))-a*ln(a+b*tanh(x))+a*x+b*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = -\frac{(a+b)x - a\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a + b)*x - a*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(29) = 58.

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.62

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0\wedge b=0 \\ \frac{x-\log(\tanh(x)+1)}{a} & \text{for } b=0 \\ \frac{x\tanh(x)}{2b\tanh(x)-2b} - \frac{x}{2b\tanh(x)-2b} + \frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} + \frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2} + \frac{a\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2-b^2} - \frac{a\log(\tanh(x)+1)}{a^2-b^2} - \frac{bx}{a^2-b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1))/a, Eq(b, 0)
), (x*tanh(x)/(2*b*tanh(x) - 2*b) - x/(2*b*tanh(x) - 2*b) + 1/(2*b*tanh(x)
- 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b)
+ 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) + a*log(a/b + tanh(x
))/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) - b*x/(a**2 - b**2), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = \frac{a\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x, algorithm="maxima")
[Out] a*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = \frac{a\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^2-b^2}-\frac{x}{a-b}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)),x, algorithm="giac")
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) - x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\tanh(x)}{a+b\tanh(x)}\,dx = -\frac{b\,x-a\left(x-\ln\left(\tanh(x)+1\right)+\ln\left(a+b\tanh(x)\right)\right)}{a^2-b^2}$$

```
[In] int(tanh(x)/(a + b*tanh(x)),x)
[Out] -(b*x - a*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)
```

3.138 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3565, 3611}

$$\int \frac{1}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[In] Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2} - \frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(a+b\tanh(x))}{2(a-b)(a+b)}$$

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
[Out] ((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[a + b*Tanh[
x]])/(2*(a - b)*(a + b))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$-\frac{-\ln(1-\tanh(x))b+b\ln(a+b\tanh(x))-ax-bx}{a^2-b^2}$	42
derivativedivides	$\frac{\ln(1+\tanh(x))}{2a-2b} - \frac{\ln(\tanh(x)-1)}{2a+2b} - \frac{b\ln(a+b\tanh(x))}{(a-b)(a+b)}$	55
default	$\frac{\ln(1+\tanh(x))}{2a-2b} - \frac{\ln(\tanh(x)-1)}{2a+2b} - \frac{b\ln(a+b\tanh(x))}{(a-b)(a+b)}$	55
risch	$\frac{x}{a+b} + \frac{2xb}{a^2-b^2} - \frac{b\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	55

```
[In] int(1/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] -(-ln(1-tanh(x))*b+b*ln(a+b*tanh(x))-a*x-b*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{(a+b)x-b\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(29) = 58$.

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.74

$$\int \frac{1}{a+b\tanh(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)+\log(\tanh(x))) & \text{for } a=0 \wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b}+\frac{x}{2b\tanh(x)-2b}+\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2}-\frac{bx}{a^2-b^2}-\frac{b\log\left(\frac{a}{b}+\tanh(x)\right)}{a^2-b^2}+\frac{b\log(\tanh(x)+1)}{a^2-b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) +
 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b
*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - b*
x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(
a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\tanh(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^2-b^2}+\frac{x}{a-b}$$

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="giac")
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{a+b\tanh(x)}\,dx = \frac{a\,x-b\,(x-\ln\left(\tanh(x)+1\right)+\ln\left(a+b\tanh(x)\right))}{a^2-b^2}$$

```
[In] int(1/(a + b*tanh(x)),x)
[Out] (a*x - b*(x - log(tanh(x) + 1) + log(a + b*tanh(x))))/(a^2 - b^2)
```

3.139 $\int \frac{\coth(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{\coth(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{\log(\sinh(x))}{a}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{a\,(a^2-b^2)}$$

```
[Out] -b*x/(a^2-b^2)+ln(sinh(x))/a+b^2*ln(a*cosh(x)+b*sinh(x))/a/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3652, 3611, 3556}

$$\int \frac{\coth(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{a\,(a^2-b^2)}+\frac{\log(\sinh(x))}{a}$$

```
[In] Int[Coth[x]/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Sinh[x]]/a + (b^2*Log[a*Cosh[x] + b*Sinh[x]])/(a
*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
```

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3652

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{bx}{a^2-b^2}+\frac{\int \coth(x)\,dx}{a}+\frac{(ib^2)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a\,(a^2-b^2)}\\ &= -\frac{bx}{a^2-b^2}+\frac{\log(\sinh(x))}{a}+\frac{b^2\log(a\cosh(x)+b\sinh(x))}{a\,(a^2-b^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{\coth(x)}{a+b\tanh(x)}\,dx = &-\frac{\log(1-\tanh(x))}{2(a+b)}+\frac{\log(\tanh(x))}{a}\\ &-\frac{\log(1+\tanh(x))}{2(a-b)}+\frac{b^2\log(a+b\tanh(x))}{a\,(a^2-b^2)}\end{aligned}$$

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]),x]
```

```
[Out] -1/2*Log[1 - Tanh[x]]/(a + b) + Log[Tanh[x]]/a - Log[1 + Tanh[x]]/(2*(a - b
)) + (b^2*Log[a + b*Tanh[x]])/(a*(a^2 - b^2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$\frac{b^{2} \ln(a+b\tanh(x))-a^{2} \ln(1-\tanh(x))-(a+b)((-a+b)\ln(\tanh(x))+ax)}{a^{3}-a\,b^{2}}$	56
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{2a+2b}+\frac{b^{2} \ln(a+b\tanh(x))}{a(a+b)(a-b)}+\frac{\ln(\tanh(x))}{a}-\frac{\ln(1+\tanh(x))}{2a-2b}$	67
default	$-\frac{\ln(\tanh(x)-1)}{2a+2b}+\frac{b^{2} \ln(a+b\tanh(x))}{a(a+b)(a-b)}+\frac{\ln(\tanh(x))}{a}-\frac{\ln(1+\tanh(x))}{2a-2b}$	67
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^{2}}{a(a^{2}-b^{2})}+\frac{\ln(e^{2x}-1)}{a}+\frac{b^{2} \ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a(a^{2}-b^{2})}$	81

```
[In] int(coth(x)/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
[Out] (b^2*ln(a+b*tanh(x))-a^2*ln(1-tanh(x))-(a+b)*((-a+b)*ln(tanh(x))+a*x))/(a^3
-a*b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\coth(x)}{a+b\tanh(x)}\,dx=\frac{b^{2}\log\left(\frac{2\,(a\cosh(x)+b\sinh(x))}{\cosh(x)-\sinh(x)}\right)-(a^{2}+ab)x+(a^{2}-b^{2})\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{a^{3}-ab^{2}}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)),x, algorithm="fricas")
[Out] (b^2*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x + (
a^2 - b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
```

Sympy [F]

$$\int \frac{\coth(x)}{a+b\tanh(x)}\,dx=\int \frac{\coth{(x)}}{a+b\tanh{(x)}}\,dx$$

```
[In] integrate(coth(x)/(a+b*tanh(x)),x)
[Out] Integral(coth(x)/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int \frac{\coth(x)}{a+b\tanh(x)}\,dx = \frac{b^2\log\left(-(a-b)e^{(-2\,x)}-a-b\right)}{a^3-ab^2}+\frac{x}{a+b}+\frac{\log\left(e^{(-x)}+1\right)}{a}+\frac{\log\left(e^{(-x)}-1\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-(a - b)*e^(-2*x) - a - b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-x) +
 1)/a + log(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{\coth(x)}{a+b\tanh(x)}\,dx = \frac{b^2\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}+a-b\right|\right)}{a^3-ab^2}-\frac{x}{a-b}+\frac{\log\left(\left|e^{(2\,x)}-1\right|\right)}{a}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^3 - a*b^2) - x/(a - b) + log
(abs(e^(2*x) - 1))/a
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{\coth(x)}{a+b\tanh(x)}\,dx = \frac{\ln\left(e^{2\,x}-1\right)}{a}-\frac{x}{a-b}-\frac{b^2\,\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a\,b^2-a^3}$$

```
[In] int(coth(x)/(a + b*tanh(x)),x)
```

```
[Out] log(exp(2*x) - 1)/a - x/(a - b) - (b^2*log(a - b + a*exp(2*x) + b*exp(2*x))
)/(a*b^2 - a^3)
```

3.140 $\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{\coth(x)}{a}-\frac{b\log(\sinh(x))}{a^2}-\frac{b^3\log(a\cosh(x)+b\sinh(x))}{a^2\,(a^2-b^2)}$$

```
[Out] a*x/(a^2-b^2)-coth(x)/a-b*ln(sinh(x))/a^2-b^3*ln(a*cosh(x)+b*sinh(x))/a^2/(
a^2-b^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3650, 3732, 3611, 3556}

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b^3\log(a\cosh(x)+b\sinh(x))}{a^2\,(a^2-b^2)}-\frac{b\log(\sinh(x))}{a^2}-\frac{\coth(x)}{a}$$

```
[In] Int[Coth[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - Coth[x]/a - (b*Log[Sinh[x]])/a^2 - (b^3*Log[a*Cosh[x] +
 b*Sinh[x]])/(a^2*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
```

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth(x)}{a} - \frac{i\int \frac{\coth(x)\left(-ib+ia\tanh(x)+ib\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{a} \\
&= \frac{ax}{a^2-b^2} - \frac{\coth(x)}{a} - \frac{b\int \coth(x)\,dx}{a^2} - \frac{(ib^3)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2\left(a^2-b^2\right)} \\
&= \frac{ax}{a^2-b^2} - \frac{\coth(x)}{a} - \frac{b\log(\sinh(x))}{a^2} - \frac{b^3\log(a\cosh(x)+b\sinh(x))}{a^2\left(a^2-b^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx = -\frac{\coth(x)}{a} - \frac{\log(1-\coth(x))}{2(a+b)} + \frac{\log(1+\coth(x))}{2(a-b)} - \frac{b^3\log(b+a\coth(x))}{a^2(a^2-b^2)}$$

```
[In] Integrate[Coth[x]^2/(a + b*Tanh[x]),x]
```

```
[Out] -(Coth[x]/a) - Log[1 - Coth[x]]/(2*(a + b)) + Log[1 + Coth[x]]/(2*(a - b))
- (b^3*Log[b + a*Coth[x]])/(a^2*(a^2 - b^2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{b\ln(\tanh(x))}{a^2} - \frac{1}{a\tanh(x)} + \frac{\ln(1+\tanh(x))}{2a-2b} - \frac{\ln(\tanh(x)-1)}{2a+2b} - \frac{b^3\ln(a+b\tanh(x))}{a^2(a-b)(a+b)}$
default	$-\frac{b\ln(\tanh(x))}{a^2} - \frac{1}{a\tanh(x)} + \frac{\ln(1+\tanh(x))}{2a-2b} - \frac{\ln(\tanh(x)-1)}{2a+2b} - \frac{b^3\ln(a+b\tanh(x))}{a^2(a-b)(a+b)}$
parallelrisch	$\frac{-b^3\ln(a+b\tanh(x))\tanh(x)+\ln(1-\tanh(x))\tanh(x)a^2b+(a+b)(-b\tanh(x)(a-b)\ln(\tanh(x))+a(ax\tanh(x)-a+b))}{(a^4-a^2b^2)\tanh(x)}$
risch	$\frac{x}{a+b} + \frac{2xb}{a^2} + \frac{2x\,b^3}{a^2(a^2-b^2)} - \frac{2}{a(e^{2x}-1)} - \frac{b\ln(e^{2x}-1)}{a^2} - \frac{b^3\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^2(a^2-b^2)}$

```
[In] int(coth(x)^2/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b*ln(tanh(x))/a^2-1/a/tanh(x)+1/(2*a-2*b)*ln(1+tanh(x))-1/(2*a+2*b)*ln(tan
h(x)-1)-b^3/a^2/(a-b)/(a+b)*ln(a+b*tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.52

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx =$$

$$-\frac{(a^3+a^2b)x\cosh(x)^2 + 2(a^3+a^2b)x\cosh(x)\sinh(x) + (a^3+a^2b)x\sinh(x)^2 - 2a^3 + 2ab^2 - (a^3+a^2}{}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^
2*b)*x*sinh(x)^2 - 2*a^3 + 2*a*b^2 - (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*b
```

```
^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 - b^3)*log(2*(a*cosh(x) + b*sinh(x))/(co
sh(x) - sinh(x))) + (a^2*b - b^3 - (a^2*b - b^3)*cosh(x)^2 - 2*(a^2*b - b^3
)*cosh(x)*sinh(x) - (a^2*b - b^3)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(
x))))/(a^4 - a^2*b^2 - (a^4 - a^2*b^2)*cosh(x)^2 - 2*(a^4 - a^2*b^2)*cosh(x
)*sinh(x) - (a^4 - a^2*b^2)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx = \int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx$$

```
[In] integrate(coth(x)**2/(a+b*tanh(x)),x)
[Out] Integral(coth(x)**2/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx = -\frac{b^3\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^4-a^2b^2}+\frac{x}{a+b} - \frac{b\log\left(e^{(-x)}+1\right)}{a^2}-\frac{b\log\left(e^{(-x)}-1\right)}{a^2}+\frac{2}{ae^{(-2x)}-a}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)),x, algorithm="maxima")
[Out] -b^3*log(-(a - b)*e^(-2*x) - a - b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^(
-x) + 1)/a^2 - b*log(e^(-x) - 1)/a^2 + 2/(a*e^(-2*x) - a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx = -\frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^4-a^2b^2}+\frac{x}{a-b} - \frac{b\log\left(\left|e^{(2x)}-1\right|\right)}{a^2}-\frac{2}{a(e^{(2x)}-1)}$$

```
[In] integrate(coth(x)^2/(a+b*tanh(x)),x, algorithm="giac")
[Out] -b^3*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^4 - a^2*b^2) + x/(a - b) -
b*log(abs(e^(2*x) - 1))/a^2 - 2/(a*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{\coth^2(x)}{a+b\tanh(x)}\,dx = \frac{x}{a-b} - \frac{2}{a\,\left(\mathrm{e}^{2\,x}-1\right)} - \frac{b^3\,\ln\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^4-a^2\,b^2} - \frac{b\,\ln\left(\mathrm{e}^{2\,x}-1\right)}{a^2}$$

```
[In] int(coth(x)^2/(a + b*tanh(x)),x)
```

```
[Out] x/(a - b) - 2/(a*(exp(2*x) - 1)) - (b^3*log(a - b + a*exp(2*x) + b*exp(2*x)
))/(a^4 - a^2*b^2) - (b*log(exp(2*x) - 1))/a^2
```

3.141 $\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{b\coth(x)}{a^2}-\frac{\coth^2(x)}{2a}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}+\frac{b^4\log(a\cosh(x)+b\sinh(x))}{a^3(a^2-b^2)}$$

```
[Out] -b*x/(a^2-b^2)+b*coth(x)/a^2-1/2*coth(x)^2/a+(a^2+b^2)*ln(sinh(x))/a^3+b^4*
ln(a*cosh(x)+b*sinh(x))/a^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3650, 3730, 3733, 3611, 3556}

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{b\coth(x)}{a^2}+\frac{(a^2+b^2)\log(\sinh(x))}{a^3}+\frac{b^4\log(a\cosh(x)+b\sinh(x))}{a^3(a^2-b^2)}-\frac{\coth^2(x)}{2a}$$

```
[In] Int[Coth[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (b*Coth[x])/a^2 - Coth[x]^2/(2*a) + ((a^2 + b^2)*Log
[Sinh[x]])/a^3 + (b^4*Log[a*Cosh[x] + b*Sinh[x]])/(a^3*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3733

```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(a*(
A*c - c*C) - b*(A*d - C*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
 + f*x]), x], x] - Dist[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\coth^2(x)}{2a} - \frac{i\int \frac{\coth^2(x)\left(-2ib+2ia\tanh(x)+2ib\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{2a}$$

$$= \frac{b\coth(x)}{a^2} - \frac{\coth^2(x)}{2a} - \frac{\int \frac{\coth(x)\left(-2(a^2+b^2)+2b^2\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{2a^2}$$

$$= -\frac{bx}{a^2-b^2} + \frac{b\coth(x)}{a^2} - \frac{\coth^2(x)}{2a} + \frac{(ib^4)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^3\,(a^2-b^2)} + \frac{(a^2+b^2)\int \coth(x)\,dx}{a^3}$$

$$= -\frac{bx}{a^2-b^2} + \frac{b\coth(x)}{a^2} - \frac{\coth^2(x)}{2a} + \frac{(a^2+b^2)\log(\sinh(x))}{a^3} + \frac{b^4\log(a\cosh(x)+b\sinh(x))}{a^3\,(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = \frac{b\coth(x)}{a^2} - \frac{\coth^2(x)}{2a} - \frac{\log(1-\coth(x))}{2(a+b)} - \frac{\log(1+\coth(x))}{2(a-b)} + \frac{b^4\log(b+a\coth(x))}{a^3\,(a^2-b^2)}$$

```
[In] Integrate[Coth[x]^3/(a + b*Tanh[x]),x]
```

```
[Out] (b*Coth[x])/a^2 - Coth[x]^2/(2*a) - Log[1 - Coth[x]]/(2*(a + b)) - Log[1 + Coth[x]]/(2*(a - b)) + (b^4*Log[b + a*Coth[x]])/(a^3*(a^2 - b^2))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{2\ln(a+b\tanh(x))b^4-2\ln(1-\tanh(x))a^4+(2a^4-2b^4)\ln(\tanh(x))-2\left(\frac{\coth(x)^2 a(a-b)}{2}-b\coth(x)(a-b)+a^2x\right)(a+b)a}{2a^5-2a^3b^2}$
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{2a+2b}-\frac{\ln(1+\tanh(x))}{2a-2b}+\frac{b^4\ln(a+b\tanh(x))}{a^3(a+b)(a-b)}+\frac{b}{a^2\tanh(x)}-\frac{(-a^2-b^2)\ln(\tanh(x))}{a^3}-\frac{1}{2a\tanh(x)^2}$
default	$-\frac{\ln(\tanh(x)-1)}{2a+2b}-\frac{\ln(1+\tanh(x))}{2a-2b}+\frac{b^4\ln(a+b\tanh(x))}{a^3(a+b)(a-b)}+\frac{b}{a^2\tanh(x)}-\frac{(-a^2-b^2)\ln(\tanh(x))}{a^3}-\frac{1}{2a\tanh(x)^2}$
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^2}{a^3}-\frac{2x\,b^4}{a^3(a^2-b^2)}-\frac{2(a\,e^{2x}-b\,e^{2x}+b)}{(e^{2x}-1)^2a^2}+\frac{\ln(e^{2x}-1)}{a}+\frac{\ln(e^{2x}-1)b^2}{a^3}+\frac{b^4\ln\left(e^{2x}+\frac{a-b}{a+b}\right)}{a^3(a^2-b^2)}$

```
[In] int(coth(x)^3/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*ln(a+b*tanh(x))*b^4-2*ln(1-tanh(x))*a^4+(2*a^4-2*b^4)*ln(tanh(x))-2*(1/2*coth(x)^2*a*(a-b)-b*coth(x)*(a-b)+a^2*x)*(a+b)*a)/(2*a^5-2*a^3*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 641, normalized size of antiderivative = 8.43

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = -\frac{(a^4+a^3b)x\cosh{(x)}^4+4\,(a^4+a^3b)x\cosh{(x)}\sinh{(x)}^3+(a^4+a^3b)x\sinh{(x)}^4+2\,a^3b-2\,ab^3+2\,(a^4-}{}$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + a^3*b)*x*cosh(x)^4 + 4*(a^4 + a^3*b)*x*cosh(x)*sinh(x)^3 + (a^4 +
a^3*b)*x*sinh(x)^4 + 2*a^3*b - 2*a*b^3 + 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 -
 (a^4 + a^3*b)*x)*cosh(x)^2 + 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 + 3*(a^4 + a
^3*b)*x*cosh(x)^2 - (a^4 + a^3*b)*x)*sinh(x)^2 + (a^4 + a^3*b)*x - (b^4*cos
h(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 - 2*b^4*cosh(x)^2 + b^4 +
2*(3*b^4*cosh(x)^2 - b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - b^4*cosh(x))*sinh(
x))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - ((a^4 - b^4)*cosh(
x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4
- 2*(a^4 - b^4)*cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 - b^4)*cosh(x)^3 - (a^4 - b^4)*cosh(x))*sinh(x))*log(2*sinh(x)
/(cosh(x) - sinh(x))) + 4*((a^4 + a^3*b)*x*cosh(x)^3 + (a^4 - a^3*b - a^2*b
^2 + a*b^3 - (a^4 + a^3*b)*x)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3
*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sin
h(x)^4 - 2*(a^5 - a^3*b^2)*cosh(x)^2 - 2*(a^5 - a^3*b^2 - 3*(a^5 - a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 - (a^5 - a^3*b^2)*cosh
(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = \int \frac{\coth^3{(x)}}{a+b\tanh{(x)}}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*tanh(x)),x)
```

```
[Out] Integral(coth(x)**3/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.59

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = \frac{b^4\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^5-a^3b^2} + \frac{2\left((a+b)e^{(-2x)}-b\right)}{2a^2e^{(-2x)}-a^2e^{(-4x)}-a^2} + \frac{x}{a+b} + \frac{(a^2+b^2)\log\left(e^{(-x)}+1\right)}{a^3} + \frac{(a^2+b^2)\log\left(e^{(-x)}-1\right)}{a^3}$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-(a - b)*e^(-2*x) - a - b)/(a^5 - a^3*b^2) + 2*((a + b)*e^(-2*x) -
b)/(2*a^2*e^(-2*x) - a^2*e^(-4*x) - a^2) + x/(a + b) + (a^2 + b^2)*log(e^(-
x) + 1)/a^3 + (a^2 + b^2)*log(e^(-x) - 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = \frac{b^4\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^5-a^3b^2} - \frac{x}{a-b} + \frac{(a^2+b^2)\log\left(\left|e^{(2x)}-1\right|\right)}{a^3} - \frac{2\left(ab+(a^2-ab)e^{(2x)}\right)}{a^3\left(e^{(2x)}-1\right)^2}$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^5 - a^3*b^2) - x/(a - b) + (
a^2 + b^2)*log(abs(e^(2*x) - 1))/a^3 - 2*(a*b + (a^2 - a*b)*e^(2*x))/(a^3*(
e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int \frac{\coth^3(x)}{a+b\tanh(x)}\,dx = \frac{\ln\left(\mathrm{e}^{2x}-1\right)\,\left(a^2+b^2\right)}{a^3} - \frac{x}{a-b} - \frac{2}{a\,\left(\mathrm{e}^{4x}-2\,\mathrm{e}^{2x}+1\right)} + \frac{b^4\,\ln\left(a-b+a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}\right)}{a^5-a^3\,b^2} - \frac{2\,\left(a^2-b^2\right)}{a^2\,\left(a+b\right)\,\left(\mathrm{e}^{2x}-1\right)}$$

```
[In] int(coth(x)^3/(a + b*tanh(x)),x)
```

```
[Out] (log(exp(2*x) - 1)*(a^2 + b^2))/a^3 - x/(a - b) - 2/(a*(exp(4*x) - 2*exp(2*
x) + 1)) + (b^4*log(a - b + a*exp(2*x) + b*exp(2*x)))/(a^5 - a^3*b^2) - (2*
(a^2 - b^2))/(a^2*(a + b)*(exp(2*x) - 1))
```

3.142 $\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{a^3} + \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} - \frac{b(a^2+b^2)\log(\sinh(x))}{a^4} - \frac{b^5\log(a\cosh(x)+b\sinh(x))}{a^4(a^2-b^2)}$$

```
[Out] a*x/(a^2-b^2)-(a^2+b^2)*coth(x)/a^3+1/2*b*coth(x)^2/a^2-1/3*coth(x)^3/a-b*(
a^2+b^2)*ln(sinh(x))/a^4-b^5*ln(a*cosh(x)+b*sinh(x))/a^4/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3650, 3730, 3731, 3732, 3611, 3556}

$$\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx = \frac{ax}{a^2-b^2} + \frac{b\coth^2(x)}{2a^2} - \frac{b(a^2+b^2)\log(\sinh(x))}{a^4} - \frac{b^5\log(a\cosh(x)+b\sinh(x))}{a^4(a^2-b^2)} - \frac{(a^2+b^2)\coth(x)}{a^3} - \frac{\coth^3(x)}{3a}$$

```
[In] Int[Coth[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - ((a^2 + b^2)*Coth[x])/a^3 + (b*Coth[x]^2)/(2*a^2) - Cot
h[x]^3/(3*a) - (b*(a^2 + b^2)*Log[Sinh[x]])/a^4 - (b^5*Log[a*Cosh[x] + b*Si
nh[x]])/(a^4*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth^3(x)}{3a} - \frac{i\int \frac{\coth^3(x)\left(-3ib+3ia\tanh(x)+3ib\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{3a} \\
&= \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} - \frac{\int \frac{\coth^2(x)\left(-6\left(a^2+b^2\right)+6b^2\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{6a^2} \\
&= -\frac{\left(a^2+b^2\right)\coth(x)}{a^3} + \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} \\
&\quad + \frac{i\int \frac{\coth(x)\left(6ib\left(a^2+b^2\right)-6ia^3\tanh(x)-6ib\left(a^2+b^2\right)\tanh^2(x)\right)}{a+b\tanh(x)}\,dx}{6a^3} \\
&= \frac{ax}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{a^3} + \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} \\
&\quad - \frac{\left(ib^5\right)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^4\left(a^2-b^2\right)} - \frac{\left(b\left(a^2+b^2\right)\right)\int \coth(x)\,dx}{a^4} \\
&= \frac{ax}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{a^3} + \frac{b\coth^2(x)}{2a^2} - \frac{\coth^3(x)}{3a} \\
&\quad - \frac{b\left(a^2+b^2\right)\log(\sinh(x))}{a^4} - \frac{b^5\log(a\cosh(x)+b\sinh(x))}{a^4\left(a^2-b^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx = \frac{1}{6}\left(-\frac{6(a^2+b^2)\coth(x)}{a^3}+\frac{3b\coth^2(x)}{a^2}-\frac{2\coth^3(x)}{a} -\frac{3\log(1-\coth(x))}{a+b}+\frac{3\log(1+\coth(x))}{a-b}+\frac{6b^5\log(b+a\coth(x))}{a^4\left(-a^2+b^2\right)}\right)$$

```
[In] Integrate[Coth[x]^4/(a + b*Tanh[x]),x]
```

```
[Out] ((-6*(a^2 + b^2)*Coth[x])/a^3 + (3*b*Coth[x]^2)/a^2 - (2*Coth[x]^3)/a - (3*
Log[1 - Coth[x]])/(a + b) + (3*Log[1 + Coth[x]])/(a - b) + (6*b^5*Log[b + a
*Coth[x]])/(a^4*(-a^2 + b^2)))/6
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{b^5\ln(a+b\tanh(x))}{a^4(a+b)(a-b)}+\frac{b}{2a^2\tanh(x)^2}+\frac{-a^2-b^2}{a^3\tanh(x)}-\frac{(a^2+b^2)b\ln(\tanh(x))}{a^4}-\frac{1}{3a\tanh(x)^3}-\ln$
default	$\frac{\ln(1+\tanh(x))}{2a-2b}-\frac{b^5\ln(a+b\tanh(x))}{a^4(a+b)(a-b)}+\frac{b}{2a^2\tanh(x)^2}+\frac{-a^2-b^2}{a^3\tanh(x)}-\frac{(a^2+b^2)b\ln(\tanh(x))}{a^4}-\frac{1}{3a\tanh(x)^3}-\ln$
parallelrisch	$\frac{-6\ln(a+b\tanh(x))b^5+6\ln(1-\tanh(x))a^4b+\left(-6a^4b+6b^5\right)\ln(\tanh(x))+\left(-2a^5+2a^3b^2\right)\coth(x)^3+\left(3a^4b-3a^2b^3\right)\coth(x)}{6a^6-6a^4b^2}$
risch	$\frac{x}{a+b}+\frac{2xb}{a^2}+\frac{2x\,b^3}{a^4}+\frac{2x\,b^5}{a^4(a^2-b^2)}-\frac{2\left(6a^2e^{4x}-3ab\,e^{4x}+3b^2e^{4x}-6a^2e^{2x}+3b\,e^{2x}a-6b^2e^{2x}+4a^2+3b^2\right)}{3a^3\left(e^{2x}-1\right)^3}-\frac{b\ln(e^{2x}-}{a^2}$

```
[In] int(coth(x)^4/(a+b*tanh(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2*a-2*b)*ln(1+tanh(x))-b^5/a^4/(a+b)/(a-b)*ln(a+b*tanh(x))+1/2/a^2*b/tan
h(x)^2+(-a^2-b^2)/a^3/tanh(x)-(a^2+b^2)/a^4*b*ln(tanh(x))-1/3/a/tanh(x)^3-1
/(2*a+2*b)*ln(tanh(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(93) = 186.

Time = 0.28 (sec) , antiderivative size = 1299, normalized size of antiderivative = 13.39

$$\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^4/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 + a^4*b)*x*cosh(x)^6 + 18*(a^5 + a^4*b)*x*cosh(x)*sinh(x)^5 + 3
*(a^5 + a^4*b)*x*sinh(x)^6 - 8*a^5 + 2*a^3*b^2 + 6*a*b^4 - 3*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^4 - 3*(4*
a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 - 15*(a^5 + a^4*b)*x*cosh(x
)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^4 + 12*(5*(a^5 + a^4*b)*x*cosh(x)^3 - (4*a
^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)
*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*a^2*b^3
- 4*a*b^4 - 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 +
 a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 - 3*(a^5 + a^4*b)*x - 3
*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 - 3*b^5*cosh(x)^4
 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*cosh(x)^2 - b^5)*sinh(x)^4 + 4*(5*b^5*c
osh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 - 6*b^5*cosh(x)^2
+ b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 - 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x
))*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4*b - b^5)*co
sh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)*sinh(x)^6 - a^4
*b + b^5 - 3*(a^4*b - b^5)*cosh(x)^4 - 3*(a^4*b - b^5 - 5*(a^4*b - b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 - 3*(a^4*b - b^5)*cosh(x))
*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*c
osh(x)^4 - 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b - b^5)*cosh(x)^
5 - 2*(a^4*b - b^5)*cosh(x)^3 + (a^4*b - b^5)*cosh(x))*sinh(x))*log(2*sinh(
x)/(cosh(x) - sinh(x))) + 6*(3*(a^5 + a^4*b)*x*cosh(x)^5 - 2*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^3 + (4*a^
5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x))*sinh(x))/((
a^6 - a^4*b^2)*cosh(x)^6 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4
*b^2)*sinh(x)^6 - a^6 + a^4*b^2 - 3*(a^6 - a^4*b^2)*cosh(x)^4 - 3*(a^6 - a^
4*b^2 - 5*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(
x)^3 - 3*(a^6 - a^4*b^2)*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 +
 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^4 - 6*(a^6 - a^4*b^2)*cosh(x)
^2)*sinh(x)^2 + 6*((a^6 - a^4*b^2)*cosh(x)^5 - 2*(a^6 - a^4*b^2)*cosh(x)^3
+ (a^6 - a^4*b^2)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a + b\tanh(x)}\,dx = \int \frac{\coth^4{(x)}}{a + b\tanh{(x)}}\,dx$$

```
[In] integrate(coth(x)**4/(a+b*tanh(x)),x)
```

```
[Out] Integral(coth(x)**4/(a + b*tanh(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.78

$$\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx = -\frac{b^5\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^6-a^4b^2} + \frac{2\left(4a^2+3b^2-3(2a^2+ab+2b^2)e^{(-2x)}+3(2a^2+ab+b^2)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}-3a^3e^{(-4x)}+a^3e^{(-6x)}-a^3\right)} + \frac{x}{a+b} - \frac{(a^2b+b^3)\log\left(e^{(-x)}+1\right)}{a^4} - \frac{(a^2b+b^3)\log\left(e^{(-x)}-1\right)}{a^4}$$

```
[In] integrate(coth(x)^4/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) - a - b)/(a^6 - a^4*b^2) + 2/3*(4*a^2 + 3*b^2 -
3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(2*a^2 + a*b + b^2)*e^(-4*x))/(3*a^3*e
^(-2*x) - 3*a^3*e^(-4*x) + a^3*e^(-6*x) - a^3) + x/(a + b) - (a^2*b + b^3)*
log(e^(-x) + 1)/a^4 - (a^2*b + b^3)*log(e^(-x) - 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.46

$$\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx = -\frac{b^5\log\left(\left|ae^{(2x)}+be^{(2x)}+a-b\right|\right)}{a^6-a^4b^2} + \frac{x}{a-b} - \frac{(a^2b+b^3)\log\left(\left|e^{(2x)}-1\right|\right)}{a^4} - \frac{2\left(4a^3+3ab^2+3(2a^3-a^2b+ab^2)e^{(4x)}-3(2a^3-a^2b+2ab^2)e^{(2x)}\right)}{3a^4\left(e^{(2x)}-1\right)^3}$$

```
[In] integrate(coth(x)^4/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^6 - a^4*b^2) + x/(a - b) -
(a^2*b + b^3)*log(abs(e^(2*x) - 1))/a^4 - 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 -
 a^2*b + a*b^2)*e^(4*x) - 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x
) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int \frac{\coth^4(x)}{a+b\tanh(x)}\,dx = \frac{x}{a-b} - \frac{8}{3\,a\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)} - \frac{b^5\,\ln\left(a-b+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^6-a^4\,b^2} - \frac{\ln\left(e^{2\,x}-1\right)\,\left(a^2\,b+b^3\right)}{a^4} - \frac{2\,\left(2\,a^3+a^2\,b+b^3\right)}{a^3\,\left(a+b\right)\,\left(e^{2\,x}-1\right)} - \frac{2\,\left(2\,a^2+a\,b-b^2\right)}{a^2\,\left(a+b\right)\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}$$

```
[In] int(coth(x)^4/(a + b*tanh(x)),x)
```

```
[Out] x/(a - b) - 8/(3*a*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - (b^5*log(a -
 b + a*exp(2*x) + b*exp(2*x)))/(a^6 - a^4*b^2) - (log(exp(2*x) - 1)*(a^2*b
+ b^3))/a^4 - (2*(a^2*b + 2*a^3 + b^3))/(a^3*(a + b)*(exp(2*x) - 1)) - (2*(
a*b + 2*a^2 - b^2))/(a^2*(a + b)*(exp(4*x) - 2*exp(2*x) + 1))
```

3.143 $\int \frac{x\text{sech}^2(x)}{(a+b\tanh(x))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{x\text{sech}^2(x)}{(a+b\tanh(x))^2}\,dx=\frac{ax}{b\,(a^2-b^2)}-\frac{\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{x}{b(a+b\tanh(x))}$$

```
[Out] a*x/b/(a^2-b^2)-ln(a*cosh(x)+b*sinh(x))/(a^2-b^2)-x/b/(a+b*tanh(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5574, 3565, 3611}

$$\int \frac{x\text{sech}^2(x)}{(a+b\tanh(x))^2}\,dx=\frac{ax}{b\,(a^2-b^2)}-\frac{\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{x}{b(a+b\tanh(x))}$$

```
[In] Int[(x*Sech[x]^2)/(a + b*Tanh[x])^2,x]
```

```
[Out] (a*x)/(b*(a^2 - b^2)) - Log[a*Cosh[x] + b*Sinh[x]]/(a^2 - b^2) - x/(b*(a +
b*Tanh[x]))
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 5574

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^2*((a_) + (b_.)*Tan
h[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Tanh[c
+ d*x])^(n + 1)/(b*d*(n + 1))), x] - Dist[f*(m/(b*d*(n + 1))), Int[(e + f*x
)^(m - 1)*(a + b*Tanh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x}{b(a+b\tanh(x))}+\frac{\int \frac{1}{a+b\tanh(x)}\,dx}{b}\\ &= \frac{ax}{b\,(a^2-b^2)}-\frac{x}{b(a+b\tanh(x))}-\frac{i\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{b\,(a^2-b^2)}-\frac{\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}-\frac{x}{b(a+b\tanh(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{x\text{sech}^2(x)}{(a+b\tanh(x))^2}\,dx=\frac{bx-a\log(a\cosh(x)+b\sinh(x))}{a^3-ab^2}+\frac{x\sinh(x)}{a^2\cosh(x)+ab\sinh(x)}$$

```
[In] Integrate[(x*Sech[x]^2)/(a + b*Tanh[x])^2,x]
[Out] (b*x - a*Log[a*Cosh[x] + b*Sinh[x]])/(a^3 - a*b^2) + (x*Sinh[x])/(a^2*Cosh[
x] + a*b*Sinh[x])
```

Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.33

method	result	size
risch	$\frac{2x}{a^2-b^2}-\frac{2x}{(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}+a-b)(a+b)}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{a-b}{a+b}\right)}{a^2-b^2}$	73

```
[In] int(x*sech(x)^2/(a+b*tanh(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2-b^2)*x-2*x/(a*exp(2*x)+b*exp(2*x)+a-b)/(a+b)-1/(a^2-b^2)*ln(exp(2*x)
+(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(55) = 110$.

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.31

$$\int \frac{x\text{sech}^2(x)}{(a+b\tanh(x))^2}\,dx$$
$$= \frac{2\,(a+b)x\cosh(x)^2 + 4\,(a+b)x\cosh(x)\sinh(x) + 2\,(a+b)x\sinh(x)^2 - ((a+b)\cosh(x)^2 + 2\,(a+b)\,\text{cos}}{a^3 - a^2b - ab^2 + b^3 + (a^3 + a^2b - ab^2 - b^3)\cosh(x)^2 + 2\,(a^3 + a^2b - ab^2 - b^3)\cosh(}$$

```
[In] integrate(x*sech(x)^2/(a+b*tanh(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(a + b)*x*cosh(x)^2 + 4*(a + b)*x*cosh(x)*sinh(x) + 2*(a + b)*x*sinh(x)^
2 - ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a
- b)*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^3 - a^2*b - a*b
^2 + b^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 -
 b^3)*cosh(x)*sinh(x) + (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{x\text{sech}^2(x)}{(a+b\tanh(x))^2}\,dx = \int \frac{x\,\text{sech}^2\,(x)}{(a+b\tanh\,(x))^2}\,dx$$

```
[In] integrate(x*sech(x)**2/(a+b*tanh(x))**2,x)
```

```
[Out] Integral(x*sech(x)**2/(a + b*tanh(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{x\text{sech}^2(x)}{(a+b\tanh(x))^2}\,dx = \frac{2\,xe^{(2\,x)}}{a^2 - 2\,ab + b^2 + (a^2 - b^2)e^{(2\,x)}} - \frac{\log\left(\frac{(a+b)e^{(2\,x)} + a - b}{a+b}\right)}{a^2 - b^2}$$

```
[In] integrate(x*sech(x)^2/(a+b*tanh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(2*x)/(a^2 - 2*a*b + b^2 + (a^2 - b^2)*e^(2*x)) - log(((a + b)*e^(2*x
) + a - b)/(a + b))/(a^2 - b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.16

$$\int \frac{x\mathrm{sech}^2(x)}{(a+b\tanh(x))^2}\,dx = \frac{2\,axe^{(2\,x)} + 2\,bxe^{(2\,x)} - ae^{(2\,x)}\log\left(-ae^{(2\,x)} - be^{(2\,x)} - a + b\right) - be^{(2\,x)}\log\left(-ae^{(2\,x)} - be^{(2\,x)} - a + b\right) - a\,\mathrm{lo}}{a^3e^{(2\,x)} + a^2be^{(2\,x)} - ab^2e^{(2\,x)} - b^3e^{(2\,x)} + a^3 - a^2b -}$$

```
[In] integrate(x*sech(x)^2/(a+b*tanh(x))^2,x, algorithm="giac")
```

```
[Out] (2*a*x*e^(2*x) + 2*b*x*e^(2*x) - a*e^(2*x)*log(-a*e^(2*x) - b*e^(2*x) - a +
 b) - b*e^(2*x)*log(-a*e^(2*x) - b*e^(2*x) - a + b) - a*log(-a*e^(2*x) - b*
e^(2*x) - a + b) + b*log(-a*e^(2*x) - b*e^(2*x) - a + b))/(a^3*e^(2*x) + a^
2*b*e^(2*x) - a*b^2*e^(2*x) - b^3*e^(2*x) + a^3 - a^2*b - a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{x\mathrm{sech}^2(x)}{(a+b\tanh(x))^2}\,dx = \frac{2\,x}{a^2-b^2} - \frac{\ln\left(a-b+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2} - \frac{2\,x}{(a+b)\,\left(a-b+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

```
[In] int(x/(cosh(x)^2*(a + b*tanh(x))^2),x)
```

```
[Out] (2*x)/(a^2 - b^2) - log(a - b + a*exp(2*x) + b*exp(2*x))/(a^2 - b^2) - (2*x
)/((a + b)*(a - b + exp(2*x)*(a + b)))
```

3.144 $\int \frac{x\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 231

$$\int \frac{x\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} + \frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^2} - \frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^2}$$

```
[Out] 1/2*x*ln(1+(a+b)*exp(2*d*x+2*c)/(a-b-2*(-a)^(1/2)*b^(1/2)))/d/(-a)^(1/2)/b^
(1/2)-1/2*x*ln(1+(a+b)*exp(2*d*x+2*c)/(a-b+2*(-a)^(1/2)*b^(1/2)))/d/(-a)^(1
/2)/b^(1/2)+1/4*polylog(2,-(a+b)*exp(2*d*x+2*c)/(a-b-2*(-a)^(1/2)*b^(1/2)))
/d^2/(-a)^(1/2)/b^(1/2)-1/4*polylog(2,-(a+b)*exp(2*d*x+2*c)/(a-b+2*(-a)^(1/
2)*b^(1/2)))/d^2/(-a)^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5751, 3401, 2296, 2221, 2317, 2438}

$$\int \frac{x\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-b-2\sqrt{-a}\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}d^2} - \frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-b+2\sqrt{-a}\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}d^2} + \frac{x\log\left(\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d}$$

```
[In] Int[(x*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (x*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b)])/(2*Sqrt
[-a]*Sqrt[b]*d) - (x*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt
[b] - b)])/(2*Sqrt[-a]*Sqrt[b]*d) + PolyLog[2, -(((a + b)*E^(2*c + 2*d*x))/
(a - 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^2) - PolyLog[2, -(((a
+ b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^
2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5751

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sech[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*T
anh[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b - c + (
b + c)*Cosh[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m
```

```
, 0]
```

Rubi steps

$$\text{integral} = 2\int \frac{x}{a-b+(a+b)\cosh(2c+2dx)}\,dx$$

$$= 4\int \frac{e^{2c+2dx}x}{a+b+2(a-b)e^{2c+2dx}+(a+b)e^{2(2c+2dx)}}\,dx$$

$$= \frac{(2(a+b))\int \frac{e^{2c+2dx}x}{2(a-b)-4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}} - \frac{(2(a+b))\int \frac{e^{2c+2dx}x}{2(a-b)+4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}}$$

$$= \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{\int \log\left(1+\frac{2(a+b)e^{2c+2dx}}{2(a-b)-4\sqrt{-a}\sqrt{b}}\right)\,dx}{2\sqrt{-a}\sqrt{b}d} + \frac{\int \log\left(1+\frac{2(a+b)e^{2c+2dx}}{2(a-b)+4\sqrt{-a}\sqrt{b}}\right)\,dx}{2\sqrt{-a}\sqrt{b}d}$$

$$= \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2(a+b)x}{2(a-b)-4\sqrt{-a}\sqrt{b}}\right)}{x}\,dx, x, e^{2c+2dx}\right)}{4\sqrt{-a}\sqrt{b}d^2} + \frac{\text{Subst}\left(\int \frac{\log\left(1+\frac{2(a+b)x}{2(a-b)+4\sqrt{-a}\sqrt{b}}\right)}{x}\,dx, x, e^{2c+2dx}\right)}{4\sqrt{-a}\sqrt{b}d^2}$$

$$= \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} + \frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^2} - \frac{\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^2}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.08

$$\int \frac{x\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-4\sqrt{-a}c\arctan\left(\frac{a-b+(a+b)e^{2(c+dx)}}{2\sqrt{a}\sqrt{b}}\right)+2\sqrt{a}(c+dx)\log\left(1+\frac{(a+b)e^{2(c+dx)}}{a-2\sqrt{-a}\sqrt{b}-b}\right)-2\sqrt{a}(c+dx)\log\left(1+\frac{(a+b)e^{2(c+dx)}}{a+2\sqrt{-a}\sqrt{b}-b}\right)+\ldots}{4\sqrt{-a^2}\sqrt{b}d^2}$$

```
[In] Integrate[(x*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (-4*Sqrt[-a]*c*ArcTan[(a - b + (a + b)*E^(2*(c + d*x)))/(2*Sqrt[a]*Sqrt[b])
] + 2*Sqrt[a]*(c + d*x)*Log[1 + ((a + b)*E^(2*(c + d*x)))/(a - 2*Sqrt[-a]*S
qrt[b] - b)] - 2*Sqrt[a]*(c + d*x)*Log[1 + ((a + b)*E^(2*(c + d*x)))/(a + 2
*Sqrt[-a]*Sqrt[b] - b)] + Sqrt[a]*PolyLog[2, -(((a + b)*E^(2*(c + d*x)))/(a
 - 2*Sqrt[-a]*Sqrt[b] - b))] - Sqrt[a]*PolyLog[2, -(((a + b)*E^(2*(c + d*x)
))/(a + 2*Sqrt[-a]*Sqrt[b] - b))])/(4*Sqrt[-a^2]*Sqrt[b]*d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 952 vs. 2(187) = 374.

Time = 3.20 (sec) , antiderivative size = 953, normalized size of antiderivative = 4.13

method	result
risch	$-\frac{c^2}{d^2(-2\sqrt{-ab}-a+b)}+\frac{\operatorname{polylog}\left(2,\frac{(a+b)e^{2dx+2c}}{-2\sqrt{-ab}-a+b}\right)}{2d^2(-2\sqrt{-ab}-a+b)}-\frac{c^2}{2d^2\sqrt{-ab}}+\frac{\operatorname{polylog}\left(2,\frac{(a+b)e^{2dx+2c}}{2\sqrt{-ab}-a+b}\right)}{4d^2\sqrt{-ab}}-\frac{a\,x^2}{2\sqrt{-ab}\,(-2\sqrt{-ab}-a+b)}+$

```
[In] int(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2/(-2*(-a*b)^(1/2)-a+b)*c^2+1/2/d^2/(-2*(-a*b)^(1/2)-a+b)*polylog(2,(a
+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))-1/2/d^2/(-a*b)^(1/2)*c^2+1/4/d^2/
(-a*b)^(1/2)*polylog(2,(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))-1/2/(-a*b
)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*x^2+1/2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*
b*x^2+1/d^2/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2
)-a+b))*c+1/2/d/(-a*b)^(1/2)*ln(1-(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b)
)*x-1/d/(-a*b)^(1/2)*c*x-1/d^2*c/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*exp(2*d*x+
2*c)+2*a-2*b)/(a*b)^(1/2))+1/d/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2
*c)/(-2*(-a*b)^(1/2)-a+b))*x-2/d/(-2*(-a*b)^(1/2)-a+b)*c*x+1/2/d^2/(-a*b)^(
1/2)*ln(1-(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))*c-1/(-2*(-a*b)^(1/2)-a
+b)*x^2-1/2/(-a*b)^(1/2)*x^2-1/2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*c
^2+1/2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*c^2*b+1/4/d^2/(-a*b)^(1/2)/(-
2*(-a*b)^(1/2)-a+b)*polylog(2,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*a
-1/4/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*polylog(2,(a+b)*exp(2*d*x+2*c)/
```

```
(-2*(-a*b)^(1/2)-a+b))*b+1/2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a
+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*a*c-1/2/d^2/(-a*b)^(1/2)/(-2*(-a*
b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*b*c+1/2/d/(-
a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)
-a+b))*a*x-1/2/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*
c)/(-2*(-a*b)^(1/2)-a+b))*b*x-1/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*c*x+
1/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*c*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1516 vs. $2(185) = 370$.

Time = 0.35 (sec) , antiderivative size = 1516, normalized size of antiderivative = 6.56

$$\int \frac{x \operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*((a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(2*sqrt(-(2*(a + b)*sqrt(
-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x
+ c)) + (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(-2*sqrt(-(2*(a + b)*sq
rt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d
*x + c)) - (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(2*sqrt((2*(a + b)*s
qrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(
d*x + c)) - (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*c*log(-2*sqrt((2*(a + b)
*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sin
h(d*x + c)) - (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*dilog(-(((a - b)*cosh(
d*x + c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(
d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 +
 2*a*b + b^2)) + a - b)/(a + b)) + a + b)/(a + b) + 1) - (a + b)*sqrt(-a*b/
(a^2 + 2*a*b + b^2))*dilog((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c)
- 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b
+ b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b))
- a - b)/(a + b) + 1) + (a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*dilog(-(((a
- b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a
+ b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a
*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + a + b)/(a + b) + 1) + (a + b)*s
qrt(-a*b/(a^2 + 2*a*b + b^2))*dilog((((a - b)*cosh(d*x + c) + (a - b)*sinh(
d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2
 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(
a + b)) - a - b)/(a + b) + 1) - ((a + b)*d*x + (a + b)*c)*sqrt(-a*b/(a^2 +
2*a*b + b^2))*log((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) - 2*((a +
 b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*
sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) + a + b)/
```

```
(a + b)) - ((a + b)*d*x + (a + b)*c)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log(-((
(a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) +
(a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqr
t(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b)) - a - b)/(a + b)) + ((a + b)*
d*x + (a + b)*c)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log((((a - b)*cosh(d*x + c)
 + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c)
)*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b +
b^2)) - a + b)/(a + b)) + a + b)/(a + b)) + ((a + b)*d*x + (a + b)*c)*sqrt(
-a*b/(a^2 + 2*a*b + b^2))*log(-(((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x +
 c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*
a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b
)) - a - b)/(a + b)))/(a*b*d^2)
```

Sympy [F]

$$\int \frac{x\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{x\,\text{sech}^2\,(c+dx)}{a+b\tanh^2\,(c+dx)}\,dx$$

```
[In] integrate(x*sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(x*sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{x\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{x\,\text{sech}\,(dx+c)^2}{b\tanh\,(dx+c)^2+a}\,dx$$

```
[In] integrate(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] integrate(x*sech(d*x + c)^2/(b*tanh(d*x + c)^2 + a), x)
```

Giac [F]

$$\int \frac{x\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{x\,\text{sech}\,(dx+c)^2}{b\tanh\,(dx+c)^2+a}\,dx$$

```
[In] integrate(x*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{x}{\cosh\left(c+d\,x\right)^2\,\left(b\tanh\left(c+d\,x\right)^2+a\right)}\,dx$$

```
[In] int(x/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
[Out] int(x/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)), x)
```

3.145 $\int \frac{x^2\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 351

$$\int \frac{x^2\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} - \frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d} + \frac{x\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2} - \frac{x\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2} - \frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^3} + \frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^3}$$

```
[Out] 1/2*x^2*ln(1+(a+b)*exp(2*d*x+2*c)/(a-b-2*(-a)^(1/2)*b^(1/2)))/d/(-a)^(1/2)/
b^(1/2)-1/2*x^2*ln(1+(a+b)*exp(2*d*x+2*c)/(a-b+2*(-a)^(1/2)*b^(1/2)))/d/(-a
)^(1/2)/b^(1/2)+1/2*x*polylog(2,-(a+b)*exp(2*d*x+2*c)/(a-b-2*(-a)^(1/2)*b^(
1/2)))/d^2/(-a)^(1/2)/b^(1/2)-1/2*x*polylog(2,-(a+b)*exp(2*d*x+2*c)/(a-b+2*
(-a)^(1/2)*b^(1/2)))/d^2/(-a)^(1/2)/b^(1/2)-1/4*polylog(3,-(a+b)*exp(2*d*x+
2*c)/(a-b-2*(-a)^(1/2)*b^(1/2)))/d^3/(-a)^(1/2)/b^(1/2)+1/4*polylog(3,-(a+b
)*exp(2*d*x+2*c)/(a-b+2*(-a)^(1/2)*b^(1/2)))/d^3/(-a)^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5751, 3401, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{x^2\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{a-b-2\sqrt{-a}\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}d^3}+\frac{\text{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{a-b+2\sqrt{-a}\sqrt{b}}\right)}{4\sqrt{-a}\sqrt{b}d^3}+\frac{x\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-b-2\sqrt{-a}\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-b+2\sqrt{-a}\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}d^2}+\frac{x^2\log\left(\frac{(a+b)e^{2c+2dx}}{-2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{x^2\log\left(\frac{(a+b)e^{2c+2dx}}{2\sqrt{-a}\sqrt{b}+a-b}+1\right)}{2\sqrt{-a}\sqrt{b}d}$$

```
[In] Int[(x^2*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (x^2*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b)])/(2*Sq
rt[-a]*Sqrt[b]*d) - (x^2*Log[1 + ((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*
Sqrt[b] - b)])/(2*Sqrt[-a]*Sqrt[b]*d) + (x*PolyLog[2, -(((a + b)*E^(2*c + 2
*d*x))/(a - 2*Sqrt[-a]*Sqrt[b] - b))])/(2*Sqrt[-a]*Sqrt[b]*d^2) - (x*PolyLo
g[2, -(((a + b)*E^(2*c + 2*d*x))/(a + 2*Sqrt[-a]*Sqrt[b] - b))])/(2*Sqrt[-a
]*Sqrt[b]*d^2) - PolyLog[3, -(((a + b)*E^(2*c + 2*d*x))/(a - 2*Sqrt[-a]*Sqr
t[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^3) + PolyLog[3, -(((a + b)*E^(2*c + 2*d*x
))/(a + 2*Sqrt[-a]*Sqrt[b] - b))]/(4*Sqrt[-a]*Sqrt[b]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 5751

```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sech[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*T
anh[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Dist[2, Int[(f + g*x)^m/(b - c + (
b + c)*Cosh[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m
, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{x^2}{a-b+(a+b)\cosh(2c+2dx)}\,dx \\
&= 4\int \frac{e^{2c+2dx}x^2}{a+b+2(a-b)e^{2c+2dx}+(a+b)e^{2(2c+2dx)}}\,dx \\
&= \frac{(2(a+b))\int \frac{e^{2c+2dx}x^2}{2(a-b)-4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}} - \frac{(2(a+b))\int \frac{e^{2c+2dx}x^2}{2(a-b)+4\sqrt{-a}\sqrt{b}+2(a+b)e^{2c+2dx}}\,dx}{\sqrt{-a}\sqrt{b}}
\end{aligned}$$

$$=\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{\int x\log\left(1+\frac{2(a+b)e^{2c+2dx}}{2(a-b)-4\sqrt{-a}\sqrt{b}}\right)dx}{\sqrt{-a}\sqrt{b}d}+\frac{\int x\log\left(1+\frac{2(a+b)e^{2c+2dx}}{2(a-b)+4\sqrt{-a}\sqrt{b}}\right)dx}{\sqrt{-a}\sqrt{b}d}$$

$$=\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}+\frac{x\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{\int\operatorname{PolyLog}\left(2,-\frac{2(a+b)e^{2c+2dx}}{2(a-b)-4\sqrt{-a}\sqrt{b}}\right)dx}{2\sqrt{-a}\sqrt{b}d^2}+\frac{\int\operatorname{PolyLog}\left(2,-\frac{2(a+b)e^{2c+2dx}}{2(a-b)+4\sqrt{-a}\sqrt{b}}\right)dx}{2\sqrt{-a}\sqrt{b}d^2}$$

$$=\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}+\frac{x\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{(a+b)x}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{x}dx,x,e^{2c+2dx}\right)}{4\sqrt{-a}\sqrt{b}d^3}+\frac{\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,-\frac{(a+b)x}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{x}dx,x,e^{2c+2dx}\right)}{4\sqrt{-a}\sqrt{b}d^3}$$

$$=\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}-\frac{x^2\log\left(1+\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d}+\frac{x\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{2\sqrt{-a}\sqrt{b}d^2}-\frac{\operatorname{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{a-2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^3}+\frac{\operatorname{PolyLog}\left(3,-\frac{(a+b)e^{2c+2dx}}{a+2\sqrt{-a}\sqrt{b}-b}\right)}{4\sqrt{-a}\sqrt{b}d^3}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.76

$$\int \frac{x^2\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2d^2x^2\log\left(1+\frac{(a+b)e^{2(c+dx)}}{a-2\sqrt{-a}\sqrt{b}-b}\right)-2d^2x^2\log\left(1+\frac{(a+b)e^{2(c+dx)}}{a+2\sqrt{-a}\sqrt{b}-b}\right)+2dx\,\text{PolyLog}\left(2,-\frac{(a+b)e^{2(c+dx)}}{a-2\sqrt{-a}\sqrt{b}-b}\right)-2dx\,\text{Poly}}{4\sqrt{-a}\sqrt{b}d^3}$$

```
[In] Integrate[(x^2*Sech[c + d*x]^2)/(a + b*Tanh[c + d*x]^2),x]
```

```
[Out] (2*d^2*x^2*Log[1 + ((a + b)*E^(2*(c + d*x)))/(a - 2*Sqrt[-a]*Sqrt[b] - b)]
- 2*d^2*x^2*Log[1 + ((a + b)*E^(2*(c + d*x)))/(a + 2*Sqrt[-a]*Sqrt[b] - b)]
 + 2*d*x*PolyLog[2, -(((a + b)*E^(2*(c + d*x)))/(a - 2*Sqrt[-a]*Sqrt[b] - b
))] - 2*d*x*PolyLog[2, -(((a + b)*E^(2*(c + d*x)))/(a + 2*Sqrt[-a]*Sqrt[b]
- b))] - PolyLog[3, -(((a + b)*E^(2*(c + d*x)))/(a - 2*Sqrt[-a]*Sqrt[b] - b
))] + PolyLog[3, -(((a + b)*E^(2*(c + d*x)))/(a + 2*Sqrt[-a]*Sqrt[b] - b))]
)/(4*Sqrt[-a]*Sqrt[b]*d^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. 2(285) = 570.

Time = 3.21 (sec) , antiderivative size = 1186, normalized size of antiderivative = 3.38

method	result	size
risch	Expression too large to display	1186

```
[In] int(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*c^2/(a*b)^(1/2)*arctan(1/4*(2*(a+b)*exp(2*d*x+2*c)+2*a-2*b)/(a*b)^(1/
2))-1/2/d^3/(-a*b)^(1/2)*ln(1-(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))*c^
2+1/d^2/(-a*b)^(1/2)*c^2*x+1/2/d/(-a*b)^(1/2)*ln(1-(a+b)*exp(2*d*x+2*c)/(2*
(-a*b)^(1/2)-a+b))*x^2+1/2/d^2/(-a*b)^(1/2)*polylog(2,(a+b)*exp(2*d*x+2*c)/
(2*(-a*b)^(1/2)-a+b))*x-1/d^3/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*
c)/(-2*(-a*b)^(1/2)-a+b))*c^2+1/d^2/(-2*(-a*b)^(1/2)-a+b)*polylog(2,(a+b)*e
xp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x+2/d^2/(-2*(-a*b)^(1/2)-a+b)*c^2*x+4/
3/d^3/(-2*(-a*b)^(1/2)-a+b)*c^3-1/2/d^3/(-2*(-a*b)^(1/2)-a+b)*polylog(3,(a+
b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))+2/3/d^3/(-a*b)^(1/2)*c^3-1/4/d^3/(
-a*b)^(1/2)*polylog(3,(a+b)*exp(2*d*x+2*c)/(2*(-a*b)^(1/2)-a+b))+1/2/d^2/(-
a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*polylog(2,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b
)^(1/2)-a+b))*x-1/2/d/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*ln(1-(a+b)*exp(2
*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x^2-1/2/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-
a+b)*b*polylog(2,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x+1/d^2/(-a*b)
^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*c^2*x-1/2/d^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-
```

```
a+b)*a*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*c^2+1/2/d^3/(-a*b)^
(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+
b))*c^2+1/d/(-2*(-a*b)^(1/2)-a+b)*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2
)-a+b))*x^2+1/3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*x^3-1/3/(-a*b)^(1/2)/(
-2*(-a*b)^(1/2)-a+b)*a*x^3+2/3/d^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*a*c^3
-2/3/d^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*c^3-1/4/d^3/(-a*b)^(1/2)/(-2*
(-a*b)^(1/2)-a+b)*a*polylog(3,(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))+1
/4/d^3/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*polylog(3,(a+b)*exp(2*d*x+2*c)/
(-2*(-a*b)^(1/2)-a+b))-1/3/(-a*b)^(1/2)*x^3-2/3/(-2*(-a*b)^(1/2)-a+b)*x^3-1
/d^2/(-a*b)^(1/2)/(-2*(-a*b)^(1/2)-a+b)*b*c^2*x+1/2/d/(-a*b)^(1/2)/(-2*(-a*
b)^(1/2)-a+b)*a*ln(1-(a+b)*exp(2*d*x+2*c)/(-2*(-a*b)^(1/2)-a+b))*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2110 vs. $2(283) = 566$.

Time = 0.33 (sec) , antiderivative size = 2110, normalized size of antiderivative = 6.01

$$\int \frac{x^2 \operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*d*x*dilog(-(((a - b)*cosh(d*x
 + c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x
 + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*
a*b + b^2)) + a - b)/(a + b)) + a + b)/(a + b) + 1) + 2*(a + b)*sqrt(-a*b/(
a^2 + 2*a*b + b^2))*d*x*dilog((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x +
c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a
*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b
)) - a - b)/(a + b) + 1) - 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*d*x*dil
og(-(((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x +
 c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b
)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) + a + b)/(a + b) + 1) -
2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*d*x*dilog((((a - b)*cosh(d*x + c)
+ (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))
*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b
^2)) - a + b)/(a + b)) - a - b)/(a + b) + 1) + (a + b)*sqrt(-a*b/(a^2 + 2*a
*b + b^2))*c^2*log(2*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a -
b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) + (a + b)*sqrt(-a*b/(a^2 +
 2*a*b + b^2))*c^2*log(-2*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) +
 a - b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) - (a + b)*sqrt(-a*b/(
a^2 + 2*a*b + b^2))*c^2*log(2*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)
) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) - (a + b)*sqrt(-a*
b/(a^2 + 2*a*b + b^2))*c^2*log(-2*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b +
```

```
b^2)) - a + b)/(a + b)) + 2*cosh(d*x + c) + 2*sinh(d*x + c)) + ((a + b)*d^2
*x^2 - (a + b)*c^2)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log((((a - b)*cosh(d*x +
 c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x +
 c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*
b + b^2)) + a - b)/(a + b)) + a + b)/(a + b)) + ((a + b)*d^2*x^2 - (a + b)*
c^2)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*log(-(((a - b)*cosh(d*x + c) + (a - b)*
sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b
/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a
- b)/(a + b)) - a - b)/(a + b)) - ((a + b)*d^2*x^2 - (a + b)*c^2)*sqrt(-a*b
/(a^2 + 2*a*b + b^2))*log((((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) +
 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b +
 b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) +
a + b)/(a + b)) - ((a + b)*d^2*x^2 - (a + b)*c^2)*sqrt(-a*b/(a^2 + 2*a*b +
b^2))*log(-(((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cos
h(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2
*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b)) - a - b)/(a + b))
 - 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*polylog(3, ((a - b)*cosh(d*x +
c) + (a - b)*sinh(d*x + c) - 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x +
c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b
 + b^2)) + a - b)/(a + b))/(a + b)) - 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^
2))*polylog(3, -((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) - 2*((a + b)
*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqr
t(-(2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) + a - b)/(a + b))/(a + b)) + 2
*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*polylog(3, ((a - b)*cosh(d*x + c) +
 (a - b)*sinh(d*x + c) + 2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*
sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^
2)) - a + b)/(a + b))/(a + b)) + 2*(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2))*p
olylog(3, -((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c) + 2*((a + b)*cosh
(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-a*b/(a^2 + 2*a*b + b^2)))*sqrt((2*
(a + b)*sqrt(-a*b/(a^2 + 2*a*b + b^2)) - a + b)/(a + b))/(a + b)))/(a*b*d^3
)
```

Sympy [F]

$$\int \frac{x^2 \operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{x^2 \operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

```
[In] integrate(x**2*sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
[Out] Integral(x**2*sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{x^2 \text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{x^2 \operatorname{sech}\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^2+a}\,dx$$

```
[In] integrate(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
[Out] integrate(x^2*sech(d*x + c)^2/(b*tanh(d*x + c)^2 + a), x)
```

Giac [F]

$$\int \frac{x^2 \text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{x^2 \operatorname{sech}\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^2+a}\,dx$$

```
[In] integrate(x^2*sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{x^2}{\cosh\left(c+d\,x\right)^2\,\left(b\tanh\left(c+d\,x\right)^2+a\right)}\,dx$$

```
[In] int(x^2/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
[Out] int(x^2/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)), x)
```

3.146 $\int x^3 \tanh(a + 2\log(x))\, dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int x^3 \tanh(a + 2\log(x))\, dx = \frac{x^4}{4} - \frac{1}{2} e^{-2a} \log\left(1 + e^{2a} x^4\right)$$

```
[Out] 1/4*x^4-1/2*ln(1+exp(2*a)*x^4)/exp(2*a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5656, 455, 45}

$$\int x^3 \tanh(a + 2\log(x))\, dx = \frac{x^4}{4} - \frac{1}{2} e^{-2a} \log\left(e^{2a} x^4 + 1\right)$$

```
[In] Int[x^3*Tanh[a + 2*Log[x]],x]
```

```
[Out] x^4/4 - Log[1 + E^(2*a)*x^4]/(2*E^(2*a))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^3(-1+e^{2a}x^4)}{1+e^{2a}x^4}\,dx\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{-1+e^{2a}x}{1+e^{2a}x}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(1-\frac{2}{1+e^{2a}x}\right)\,dx, x, x^4\right)\\ &= \frac{x^4}{4}-\frac{1}{2}e^{-2a}\log\left(1+e^{2a}x^4\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(29) = 58.

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.21

$$\int x^3\tanh(a+2\log(x))\,dx = \frac{x^4}{4}-\frac{1}{2}\cosh(2a)\log\left(\cosh(a)+x^4\cosh(a)-\sinh(a)+x^4\sinh(a)\right) + \frac{1}{2}\log\left(\cosh(a)+x^4\cosh(a)-\sinh(a)+x^4\sinh(a)\right)\sinh(2a)$$

```
[In] Integrate[x^3*Tanh[a + 2*Log[x]],x]
[Out] x^4/4 - (Cosh[2*a]*Log[Cosh[a] + x^4*Cosh[a] - Sinh[a] + x^4*Sinh[a]])/2 +
(Log[Cosh[a] + x^4*Cosh[a] - Sinh[a] + x^4*Sinh[a]]*Sinh[2*a])/2
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x^4}{4}-\frac{\mathrm{e}^{-2a}\ln\left(1+\mathrm{e}^{2a}x^4\right)}{2}$	24

```
[In] int(x^3*tanh(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

[Out] 1/4*x^4-1/2*exp(-2*a)*ln(1+exp(2*a)*x^4)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int x^3 \tanh(a + 2\log(x))\,dx = \frac{1}{4}\left(x^4 e^{(2a)} - 2\log\left(x^4 e^{(2a)} + 1\right)\right)e^{(-2a)}$$

[In] integrate(x^3*tanh(a+2*log(x)),x, algorithm="fricas")

[Out] 1/4*(x^4*e^(2*a) - 2*log(x^4*e^(2*a) + 1))*e^(-2*a)

Sympy [F]

$$\int x^3 \tanh(a + 2\log(x))\,dx = \int x^3 \tanh\left(a + 2\log\left(x\right)\right)dx$$

[In] integrate(x**3*tanh(a+2*ln(x)),x)

[Out] Integral(x**3*tanh(a + 2*log(x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^3 \tanh(a + 2\log(x))\,dx = \frac{1}{4}x^4 - \frac{1}{2}e^{(-2a)}\log\left(x^4 e^{(2a)} + 1\right)$$

[In] integrate(x^3*tanh(a+2*log(x)),x, algorithm="maxima")

[Out] 1/4*x^4 - 1/2*e^(-2*a)*log(x^4*e^(2*a) + 1)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int x^3 \tanh(a + 2\log(x))\, dx = \frac{1}{4}\, x^4 - \frac{1}{2}\, e^{(-2\, a)} \log\left(x^4 e^{(2\, a)} + 1\right)$$

```
[In] integrate(x^3*tanh(a+2*log(x)),x, algorithm="giac")
[Out] 1/4*x^4 - 1/2*e^(-2*a)*log(x^4*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int x^3 \tanh(a + 2\log(x))\, dx = \frac{x^4}{4} - \frac{\mathrm{e}^{-2\, a}\, \ln\left(x^4 + \mathrm{e}^{-2\, a}\right)}{2}$$

```
[In] int(x^3*tanh(a + 2*log(x)),x)
[Out] x^4/4 - (exp(-2*a)*log(exp(-2*a) + x^4))/2
```

3.147 $\int x^2 \tanh(a + 2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 151

$$\int x^2 \tanh(a + 2\log(x))\,dx = \frac{x^3}{3} + \frac{e^{-3a/2}\arctan\left(1 - \sqrt{2}e^{a/2}x\right)}{\sqrt{2}} - \frac{e^{-3a/2}\arctan\left(1 + \sqrt{2}e^{a/2}x\right)}{\sqrt{2}} - \frac{e^{-3a/2}\log\left(1 - \sqrt{2}e^{a/2}x + e^a x^2\right)}{2\sqrt{2}} + \frac{e^{-3a/2}\log\left(1 + \sqrt{2}e^{a/2}x + e^a x^2\right)}{2\sqrt{2}}$$

```
[Out] 1/3*x^3-1/2*arctan(-1+exp(1/2*a)*x*2^(1/2))/exp(3/2*a)*2^(1/2)-1/2*arctan(1
+exp(1/2*a)*x*2^(1/2))/exp(3/2*a)*2^(1/2)-1/4*ln(1+exp(a)*x^2-exp(1/2*a)*x*
2^(1/2))/exp(3/2*a)*2^(1/2)+1/4*ln(1+exp(a)*x^2+exp(1/2*a)*x*2^(1/2))/exp(3
/2*a)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {5656, 470, 303, 1176, 631, 210, 1179, 642}

$$\int x^2 \tanh(a + 2\log(x))\,dx = \frac{e^{-3a/2}\arctan\left(1 - \sqrt{2}e^{a/2}x\right)}{\sqrt{2}} - \frac{e^{-3a/2}\arctan\left(\sqrt{2}e^{a/2}x + 1\right)}{\sqrt{2}} - \frac{e^{-3a/2}\log\left(e^a x^2 - \sqrt{2}e^{a/2}x + 1\right)}{2\sqrt{2}} + \frac{e^{-3a/2}\log\left(e^a x^2 + \sqrt{2}e^{a/2}x + 1\right)}{2\sqrt{2}} + \frac{x^3}{3}$$

```
[In] Int[x^2*Tanh[a + 2*Log[x]],x]
```

```
[Out] x^3/3 + ArcTan[1 - Sqrt[2]*E^(a/2)*x]/(Sqrt[2]*E^((3*a)/2)) - ArcTan[1 + Sq
rt[2]*E^(a/2)*x]/(Sqrt[2]*E^((3*a)/2)) - Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^
2]/(2*Sqrt[2]*E^((3*a)/2)) + Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2]/(2*Sqrt[2
]*E^((3*a)/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(-1+e^{2a}x^4)}{1+e^{2a}x^4}\,dx\\
&= \frac{x^3}{3}-2\int \frac{x^2}{1+e^{2a}x^4}\,dx\\
&= \frac{x^3}{3}+e^{-a}\int \frac{1-e^ax^2}{1+e^{2a}x^4}\,dx-e^{-a}\int \frac{1+e^ax^2}{1+e^{2a}x^4}\,dx\\
&= \frac{x^3}{3}-\frac{1}{2}e^{-2a}\int \frac{1}{e^{-a}-\sqrt{2}e^{-a/2}x+x^2}\,dx-\frac{1}{2}e^{-2a}\int \frac{1}{e^{-a}+\sqrt{2}e^{-a/2}x+x^2}\,dx\\
&\quad -\frac{e^{-3a/2}\int \frac{\sqrt{2}e^{-a/2}+2x}{-e^{-a}-\sqrt{2}e^{-a/2}x-x^2}\,dx}{2\sqrt{2}}-\frac{e^{-3a/2}\int \frac{\sqrt{2}e^{-a/2}-2x}{-e^{-a}+\sqrt{2}e^{-a/2}x-x^2}\,dx}{2\sqrt{2}}\\
&= \frac{x^3}{3}-\frac{e^{-3a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}+\frac{e^{-3a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}\\
&\quad -\frac{e^{-3a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}\\
&\quad +\frac{e^{-3a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}\\
&= \frac{x^3}{3}+\frac{e^{-3a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}-\frac{e^{-3a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}\\
&\quad -\frac{e^{-3a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}+\frac{e^{-3a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.42

$$\int x^2 \tanh(a+2\log(x))\,dx = \frac{1}{6}\left(2x^3 + 3\text{RootSum}\left[\cosh(a)-\sinh(a)+\cosh(a)\#1^4 + \sinh(a)\#1^4\&, \frac{\log(x)-\log(x-\#1)}{\#1}\&\right](\cosh(2a) - \sinh(2a))\right)$$

```
[In] Integrate[x^2*Tanh[a + 2*Log[x]],x]
[Out] (2*x^3 + 3*RootSum[Cosh[a] - Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log
[x] - Log[x - #1])/#1 & ]*(Cosh[2*a] - Sinh[2*a]))/6
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.25

method	result	size
risch	$\frac{x^3}{3} - \frac{e^{-2a}\left(\sum_{\_R=\text{RootOf}\left(e^{2a}\_Z^4+1\right)} \frac{\ln\left(x-\_R\right)}{\_R}\right)}{2}$	37

```
[In] int(x^2*tanh(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3-1/2*exp(-2*a)*sum(1/_R*ln(x-_R),_R=RootOf(exp(2*a)*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int x^2 \tanh(a+2\log(x))\,dx = \frac{1}{3}x^3 - \frac{1}{2}\left(-e^{(-6a)}\right)^{\frac{1}{4}}\log\left(\left(-e^{(-6a)}\right)^{\frac{3}{4}}e^{(4a)}+x\right) + \frac{1}{2}i\left(-e^{(-6a)}\right)^{\frac{1}{4}}\log\left(i\left(-e^{(-6a)}\right)^{\frac{3}{4}}e^{(4a)}+x\right) - \frac{1}{2}i\left(-e^{(-6a)}\right)^{\frac{1}{4}}\log\left(-i\left(-e^{(-6a)}\right)^{\frac{3}{4}}e^{(4a)}+x\right) + \frac{1}{2}\left(-e^{(-6a)}\right)^{\frac{1}{4}}\log\left(-\left(-e^{(-6a)}\right)^{\frac{3}{4}}e^{(4a)}+x\right)$$

```
[In] integrate(x^2*tanh(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 1/2*(-e^(-6*a))^(1/4)*log((-e^(-6*a))^(3/4)*e^(4*a) + x) + 1/2*I*
(-e^(-6*a))^(1/4)*log(I*(-e^(-6*a))^(3/4)*e^(4*a) + x) - 1/2*I*(-e^(-6*a))^
(1/4)*log(-I*(-e^(-6*a))^(3/4)*e^(4*a) + x) + 1/2*(-e^(-6*a))^(1/4)*log(-(-
e^(-6*a))^(3/4)*e^(4*a) + x)
```

Sympy [F]

$$\int x^2 \tanh(a + 2\log(x))\,dx = \int x^2 \tanh\left(a + 2\log\left(x\right)\right)dx$$

```
[In] integrate(x**2*tanh(a+2*ln(x)),x)
```

```
[Out] Integral(x**2*tanh(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int x^2 \tanh(a + 2\log(x))\,dx = \frac{1}{3}\,x^3 - \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,xe^a + \sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\Big)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} - \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\Big(2\,xe^a - \sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\Big)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{1}{4}\,\sqrt{2}e^{\left(-\frac{3}{2}\,a\right)}\log\left(x^2e^a + \sqrt{2}xe^{\left(\frac{1}{2}\,a\right)} + 1\right) - \frac{1}{4}\,\sqrt{2}e^{\left(-\frac{3}{2}\,a\right)}\log\left(x^2e^a - \sqrt{2}xe^{\left(\frac{1}{2}\,a\right)} + 1\right)$$

```
[In] integrate(x^2*tanh(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-
1/2*a))*e^(-3/2*a) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1
/2*a))*e^(-1/2*a))*e^(-3/2*a) + 1/4*sqrt(2)*e^(-3/2*a)*log(x^2*e^a + sqrt(2
)*x*e^(1/2*a) + 1) - 1/4*sqrt(2)*e^(-3/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*
a) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int x^2 \tanh(a+2\log(x))\,dx = \frac{1}{3}\,x^3 - \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)} + 2\,x\right)e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} - \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)} - 2\,x\right)e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{1}{4}\,\sqrt{2}e^{\left(-\frac{3}{2}\,a\right)}\log\left(\sqrt{2}xe^{\left(-\frac{1}{2}\,a\right)} + x^2 + e^{(-a)}\right) - \frac{1}{4}\,\sqrt{2}e^{\left(-\frac{3}{2}\,a\right)}\log\left(-\sqrt{2}xe^{\left(-\frac{1}{2}\,a\right)} + x^2 + e^{(-a)}\right)$$

```
[In] integrate(x^2*tanh(a+2*log(x)),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*
a))*e^(-3/2*a) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)
*e^(1/2*a))*e^(-3/2*a) + 1/4*sqrt(2)*e^(-3/2*a)*log(sqrt(2)*x*e^(-1/2*a) +
x^2 + e^(-a)) - 1/4*sqrt(2)*e^(-3/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^
(-a))
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.31

$$\int x^2 \tanh(a+2\log(x))\,dx = \frac{\operatorname{atan}\left(x\left(-\mathrm{e}^{2\,a}\right)^{1/4}\right)}{\left(-\mathrm{e}^{2\,a}\right)^{3/4}} - \frac{\operatorname{atanh}\left(x\left(-\mathrm{e}^{2\,a}\right)^{1/4}\right)}{\left(-\mathrm{e}^{2\,a}\right)^{3/4}} + \frac{x^3}{3}$$

```
[In] int(x^2*tanh(a + 2*log(x)),x)
```

```
[Out] atan(x*(-exp(2*a))^(1/4))/(-exp(2*a))^(3/4) - atanh(x*(-exp(2*a))^(1/4))/(-
exp(2*a))^(3/4) + x^3/3
```

3.148 $\int x \tanh(a + 2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \tanh(a + 2\log(x))\,dx = \frac{x^2}{2} - e^{-a}\arctan\left(e^a x^2\right)$$

```
[Out] 1/2*x^2-arctan(exp(a)*x^2)/exp(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5656, 470, 281, 209}

$$\int x \tanh(a + 2\log(x))\,dx = \frac{x^2}{2} - e^{-a}\arctan\left(e^a x^2\right)$$

```
[In] Int[x*Tanh[a + 2*Log[x]],x]
[Out] x^2/2 - ArcTan[E^a*x^2]/E^a
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x(-1+e^{2a}x^4)}{1+e^{2a}x^4}\,dx \\
&= \frac{x^2}{2} - 2\int \frac{x}{1+e^{2a}x^4}\,dx \\
&= \frac{x^2}{2} - \text{Subst}\left(\int \frac{1}{1+e^{2a}x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} - e^{-a}\arctan\left(e^a x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int x\tanh(a+2\log(x))\,dx = \frac{x^2}{2} - \arctan\left(x^2(\cosh(a)+\sinh(a))\right)\cosh(a) + \arctan\left(x^2(\cosh(a)+\sinh(a))\right)\sinh(a)$$

```
[In] Integrate[x*Tanh[a + 2*Log[x]],x]
```

```
[Out] x^2/2 - ArcTan[x^2*(Cosh[a] + Sinh[a])]*Cosh[a] + ArcTan[x^2*(Cosh[a] + Sin
h[a])]*Sinh[a]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

method	result	size
risch	$\frac{x^2}{2}+\frac{i e^{-a} \ln(e^a x^2-i)}{2}-\frac{i e^{-a} \ln(e^a x^2+i)}{2}$	41

```
[In] int(x*tanh(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/2*I/exp(a)*ln(exp(a)*x^2-I)-1/2*I/exp(a)*ln(exp(a)*x^2+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x \tanh(a+2\log(x))\,dx = \frac{1}{2}\left(x^2e^a - 2\arctan\left(x^2e^a\right)\right)e^{(-a)}$$

```
[In] integrate(x*tanh(a+2*log(x)),x, algorithm="fricas")
[Out] 1/2*(x^2*e^a - 2*arctan(x^2*e^a))*e^(-a)
```

Sympy [F]

$$\int x \tanh(a+2\log(x))\,dx = \int x \tanh\left(a+2\log\left(x\right)\right)\,dx$$

```
[In] integrate(x*tanh(a+2*ln(x)),x)
[Out] Integral(x*tanh(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \tanh(a+2\log(x))\,dx = \frac{1}{2}x^2 - \arctan\left(x^2e^a\right)e^{(-a)}$$

```
[In] integrate(x*tanh(a+2*log(x)),x, algorithm="maxima")
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\tanh(a+2\log(x))\,dx = \frac{1}{2}\,x^2 - \arctan\left(x^2e^a\right)e^{(-a)}$$

```
[In] integrate(x*tanh(a+2*log(x)),x, algorithm="giac")
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a)
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\tanh(a+2\log(x))\,dx = \frac{x^2}{2} - \frac{\mathrm{atan}\left(x^2\sqrt{\mathrm{e}^{2\,a}}\right)}{\sqrt{\mathrm{e}^{2\,a}}}$$

```
[In] int(x*tanh(a + 2*log(x)),x)
[Out] x^2/2 - atan(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

3.149 $\int \tanh(a+2\log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 145

$$\int \tanh(a+2\log(x))\,dx = x+\frac{e^{-a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}-\frac{e^{-a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}+\frac{e^{-a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}-\frac{e^{-a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}$$

```
[Out] x-1/2*arctan(-1+exp(1/2*a)*x*2^(1/2))/exp(1/2*a)*2^(1/2)-1/2*arctan(1+exp(1
/2*a)*x*2^(1/2))/exp(1/2*a)*2^(1/2)+1/4*ln(1+exp(a)*x^2-exp(1/2*a)*x*2^(1/2
))/exp(1/2*a)*2^(1/2)-1/4*ln(1+exp(a)*x^2+exp(1/2*a)*x*2^(1/2))/exp(1/2*a)*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.143, Rules used = {5652, 396, 217, 1179, 642, 1176, 631, 210}

$$\int \tanh(a+2\log(x))\,dx = \frac{e^{-a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}-\frac{e^{-a/2}\arctan\left(\sqrt{2}e^{a/2}x+1\right)}{\sqrt{2}}+\frac{e^{-a/2}\log\left(e^ax^2-\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}-\frac{e^{-a/2}\log\left(e^ax^2+\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}+x$$

```
[In] Int[Tanh[a + 2*Log[x]],x]
```

```
[Out] x + ArcTan[1 - Sqrt[2]*E^(a/2)*x]/(Sqrt[2]*E^(a/2)) - ArcTan[1 + Sqrt[2]*E^
(a/2)*x]/(Sqrt[2]*E^(a/2)) + Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^2]/(2*Sqrt[2
]*E^(a/2)) - Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2]/(2*Sqrt[2]*E^(a/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{-1+e^{2a}x^4}{1+e^{2a}x^4}\,dx \\
&= x-2\int \frac{1}{1+e^{2a}x^4}\,dx \\
&= x-\int \frac{1-e^{a}x^2}{1+e^{2a}x^4}\,dx-\int \frac{1+e^{a}x^2}{1+e^{2a}x^4}\,dx \\
&= x-\frac{1}{2}e^{-a}\int \frac{1}{e^{-a}-\sqrt{2}e^{-a/2}x+x^2}\,dx-\frac{1}{2}e^{-a}\int \frac{1}{e^{-a}+\sqrt{2}e^{-a/2}x+x^2}\,dx \\
&\quad+\frac{e^{-a/2}\int \frac{\sqrt{2}e^{-a/2}+2x}{-e^{-a}-\sqrt{2}e^{-a/2}x-x^2}\,dx}{2\sqrt{2}}+\frac{e^{-a/2}\int \frac{\sqrt{2}e^{-a/2}-2x}{-e^{-a}+\sqrt{2}e^{-a/2}x-x^2}\,dx}{2\sqrt{2}} \\
&= x+\frac{e^{-a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^{a}x^2\right)}{2\sqrt{2}}-\frac{e^{-a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^{a}x^2\right)}{2\sqrt{2}} \\
&\quad-\frac{e^{-a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} \\
&\quad+\frac{e^{-a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} \\
&= x+\frac{e^{-a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}-\frac{e^{-a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} \\
&\quad+\frac{e^{-a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^{a}x^2\right)}{2\sqrt{2}}-\frac{e^{-a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^{a}x^2\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.40

$$\int \tanh(a+2\log(x))\,dx = x+\frac{1}{2}\text{RootSum}\Big[\cosh(a)-\sinh(a)+\cosh(a)\#1^4 + \sinh(a)\#1^4\&, \frac{\log(x)-\log(x-\#1)}{\#1^3}\&\Big](\cosh(2a)-\sinh(2a))$$

```
[In] Integrate[Tanh[a + 2*Log[x]],x]
[Out] x + (RootSum[Cosh[a] - Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log[x] -
Log[x - #1])/#1^3 & ]*(Cosh[2*a] - Sinh[2*a]))/2
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.23

method	result	size
risch	$x-\frac{e^{-2a}\left(\sum_{\_R=\text{RootOf}\left(e^{2a}\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{2}$	33

```
[In] int(tanh(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] x-1/2*exp(-2*a)*sum(1/_R^3*ln(x-_R),_R=RootOf(exp(2*a)*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \tanh(a+2\log(x))\,dx = -\frac{1}{2}\left(-e^{(-2a)}\right)^{\frac{1}{4}}\log\left(x+\left(-e^{(-2a)}\right)^{\frac{1}{4}}\right) - \frac{1}{2}i\left(-e^{(-2a)}\right)^{\frac{1}{4}}\log\left(x+i\left(-e^{(-2a)}\right)^{\frac{1}{4}}\right) + \frac{1}{2}i\left(-e^{(-2a)}\right)^{\frac{1}{4}}\log\left(x-i\left(-e^{(-2a)}\right)^{\frac{1}{4}}\right) + \frac{1}{2}\left(-e^{(-2a)}\right)^{\frac{1}{4}}\log\left(x-\left(-e^{(-2a)}\right)^{\frac{1}{4}}\right)+x$$

```
[In] integrate(tanh(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(-e^(-2*a))^(1/4)*log(x + (-e^(-2*a))^(1/4)) - 1/2*I*(-e^(-2*a))^(1/4)
*log(x + I*(-e^(-2*a))^(1/4)) + 1/2*I*(-e^(-2*a))^(1/4)*log(x - I*(-e^(-2*a
))^(1/4)) + 1/2*(-e^(-2*a))^(1/4)*log(x - (-e^(-2*a))^(1/4)) + x
```

Sympy [F]

$$\int \tanh(a+2\log(x))\,dx = \int \tanh\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(tanh(a+2*ln(x)),x)
```

```
[Out] Integral(tanh(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

$$\begin{aligned}\int \tanh(a+2\log(x))\,dx = &-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,xe^{a}+\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)} \\ &-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,xe^{a}-\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)} \\ &-\frac{1}{4}\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)}\log\left(x^{2}e^{a}+\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+1\right) \\ &+\frac{1}{4}\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)}\log\left(x^{2}e^{a}-\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+1\right)+x\end{aligned}$$

```
[In] integrate(tanh(a+2*log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-1/2*a))*e
^(-1/2*a) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1/2*a))*e^
(-1/2*a))*e^(-1/2*a) - 1/4*sqrt(2)*e^(-1/2*a)*log(x^2*e^a + sqrt(2)*x*e^(1/
2*a) + 1) + 1/4*sqrt(2)*e^(-1/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*a) + 1) +
 x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82

$$\int \tanh(a+2\log(x))\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} - \frac{1}{4}\sqrt{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right) + \frac{1}{4}\sqrt{2}e^{\left(-\frac{1}{2}a\right)}\log\left(-\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right)+x$$

```
[In] integrate(tanh(a+2*log(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(-1
/2*a) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a
))*e^(-1/2*a) - 1/4*sqrt(2)*e^(-1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(
-a)) + 1/4*sqrt(2)*e^(-1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + x
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int \tanh(a+2\log(x))\,dx = x - \frac{\mathrm{atan}\left(x\,{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}} - \frac{\mathrm{atanh}\left(x\,{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}}$$

```
[In] int(tanh(a + 2*log(x)),x)
```

```
[Out] x - atan(x*(-exp(2*a))^(1/4))/(-exp(2*a))^(1/4) - atanh(x*(-exp(2*a))^(1/4)
)/(-exp(2*a))^(1/4)
```

3.150 $\int \frac{\tanh(a+2\log(x))}{x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \frac{1}{2}\log(\cosh(a+2\log(x)))$$

```
[Out] 1/2*ln(cosh(a+2*ln(x)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3556}

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \frac{1}{2}\log(\cosh(a+2\log(x)))$$

```
[In] Int[Tanh[a + 2*Log[x]]/x,x]
```

```
[Out] Log[Cosh[a + 2*Log[x]]]/2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \tanh(a+2x)\,dx, x, \log(x)\right) \\ &= \frac{1}{2}\log(\cosh(a+2\log(x))) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \frac{1}{2}\log(\cosh(a+2\log(x)))$$

```
[In] Integrate[Tanh[a + 2*Log[x]]/x,x]
[Out] Log[Cosh[a + 2*Log[x]]]/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln(\cosh(a+2\ln(x)))}{2}$	11
default	$\frac{\ln(\cosh(a+2\ln(x)))}{2}$	11
risch	$-\ln(x)+\frac{\ln(-e^{2a}x^4-1)}{2}$	20
parallelrisch	$-\ln(x)-\frac{\ln(1-\tanh(a+2\ln(x)))}{2}$	20

```
[In] int(tanh(a+2*ln(x))/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(cosh(a+2*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \frac{1}{2}\log\left(x^4e^{(2a)}+1\right)-\log(x)$$

```
[In] integrate(tanh(a+2*log(x))/x,x, algorithm="fricas")
[Out] 1/2*log(x^4*e^(2*a) + 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \log(x) - \frac{\log(\tanh(a+2\log(x))+1)}{2}$$

```
[In] integrate(tanh(a+2*ln(x))/x,x)
[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \frac{1}{2}\log(\cosh(a+2\log(x)))$$

```
[In] integrate(tanh(a+2*log(x))/x,x, algorithm="maxima")
[Out] 1/2*log(cosh(a + 2*log(x)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \frac{1}{2}\log\left(x^4e^{(2a)}+1\right) - \frac{1}{4}\log\left(x^4\right)$$

```
[In] integrate(tanh(a+2*log(x))/x,x, algorithm="giac")
[Out] 1/2*log(x^4*e^(2*a) + 1) - 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\tanh(a+2\log(x))}{x}\,dx = \ln(x) - \frac{\ln(\tanh(a+2\ln(x))+1)}{2}$$

```
[In] int(tanh(a + 2*log(x))/x,x)
[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2
```

3.151 $\int \frac{\tanh(a+2\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 147

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = \frac{1}{x} - \frac{e^{a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} + \frac{e^{a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} + \frac{e^{a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}} - \frac{e^{a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}$$

```
[Out] 1/x+1/2*exp(1/2*a)*arctan(-1+exp(1/2*a)*x*2^(1/2))*2^(1/2)+1/2*exp(1/2*a)*a
rctan(1+exp(1/2*a)*x*2^(1/2))*2^(1/2)+1/4*exp(1/2*a)*ln(1+exp(a)*x^2-exp(1/
2*a)*x*2^(1/2))*2^(1/2)-1/4*exp(1/2*a)*ln(1+exp(a)*x^2+exp(1/2*a)*x*2^(1/2)
)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5656, 464, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = -\frac{e^{a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} + \frac{e^{a/2}\arctan\left(\sqrt{2}e^{a/2}x+1\right)}{\sqrt{2}} + \frac{e^{a/2}\log\left(e^ax^2-\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}} - \frac{e^{a/2}\log\left(e^ax^2+\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}} + \frac{1}{x}$$

```
[In] Int[Tanh[a + 2*Log[x]]/x^2,x]
```

```
[Out] x^(-1) - (E^(a/2)*ArcTan[1 - Sqrt[2]*E^(a/2)*x])/Sqrt[2] + (E^(a/2)*ArcTan[
1 + Sqrt[2]*E^(a/2)*x])/Sqrt[2] + (E^(a/2)*Log[1 - Sqrt[2]*E^(a/2)*x + E^a*
```

```
x^2])/(2*Sqrt[2]) - (E^(a/2)*Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2])/(2*Sqrt[
2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{-1+e^{2a}x^4}{x^2\left(1+e^{2a}x^4\right)}\,dx \\
&= \frac{1}{x}+\left(2e^{2a}\right)\int \frac{x^2}{1+e^{2a}x^4}\,dx \\
&= \frac{1}{x}-e^a\int \frac{1-e^ax^2}{1+e^{2a}x^4}\,dx+e^a\int \frac{1+e^ax^2}{1+e^{2a}x^4}\,dx \\
&= \frac{1}{x}+\frac{1}{2}\int \frac{1}{e^{-a}-\sqrt{2}e^{-a/2}x+x^2}\,dx+\frac{1}{2}\int \frac{1}{e^{-a}+\sqrt{2}e^{-a/2}x+x^2}\,dx \\
&\quad+\frac{e^{a/2}\int \frac{\sqrt{2}e^{-a/2}+2x}{-e^{-a}-\sqrt{2}e^{-a/2}x-x^2}\,dx}{2\sqrt{2}}+\frac{e^{a/2}\int \frac{\sqrt{2}e^{-a/2}-2x}{-e^{-a}+\sqrt{2}e^{-a/2}x-x^2}\,dx}{2\sqrt{2}} \\
&= \frac{1}{x}+\frac{e^{a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}-\frac{e^{a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}} \\
&\quad+\frac{e^{a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}-\frac{e^{a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} \\
&= \frac{1}{x}-\frac{e^{a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{\sqrt{2}}+\frac{e^{a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{\sqrt{2}} \\
&\quad+\frac{e^{a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}-\frac{e^{a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.40

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = \frac{2 - x\text{RootSum}\left[\cosh(a)+\sinh(a)+\cosh(a)\#1^4-\sinh(a)\#1^4\&,\frac{\log(x)+\log\left(\frac{1}{x}-\#1\right)}{\#1^3}\&\right](\cosh(a)+\sinh(a))}{2x}$$

```
[In] Integrate[Tanh[a + 2*Log[x]]/x^2,x]
[Out] (2 - x*RootSum[Cosh[a] + Sinh[a] + Cosh[a]*#1^4 - Sinh[a]*#1^4 & , (Log[x]
+ Log[x^(-1) - #1])/#1^3 & ]*(Cosh[a] + Sinh[a])^2)/(2*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.29

method	result	size
risch	$\frac{1}{x}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+e^{2a}\right)}\_R\ln\left(\left(5\_R^4+4\,e^{2a}\right)x-\_R^3\right)\right)}{2}$	42

```
[In] int(tanh(a+2*ln(x))/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x+1/2*sum(_R*ln((5*_R^4+4*exp(2*a))*x-_R^3),_R=RootOf(_Z^4+exp(2*a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.82

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = \frac{x\left(-e^{(2a)}\right)^{\frac{1}{4}}\log\left(xe^{(2a)}+\left(-e^{(2a)}\right)^{\frac{3}{4}}\right)-i\,x\left(-e^{(2a)}\right)^{\frac{1}{4}}\log\left(xe^{(2a)}+i\left(-e^{(2a)}\right)^{\frac{3}{4}}\right)+i\,x\left(-e^{(2a)}\right)^{\frac{1}{4}}\log\left(xe^{(2a}\right.}{2\,x}$$

```
[In] integrate(tanh(a+2*log(x))/x^2,x, algorithm="fricas")
[Out] 1/2*(x*(-e^(2*a))^(1/4)*log(x*e^(2*a) + (-e^(2*a))^(3/4)) - I*x*(-e^(2*a))^
(1/4)*log(x*e^(2*a) + I*(-e^(2*a))^(3/4)) + I*x*(-e^(2*a))^(1/4)*log(x*e^(2
```

```
*a) - I*(-e^(2*a))^(3/4)) - x*(-e^(2*a))^(1/4)*log(x*e^(2*a) - (-e^(2*a))^(
3/4)) + 2)/x
```

Sympy [F]

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = \int \frac{\tanh\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(tanh(a+2*ln(x))/x**2,x)
```

```
[Out] Integral(tanh(a + 2*log(x))/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\begin{aligned}\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = &-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}a\right)}+\frac{2}{x}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}\\ &-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}a\right)}-\frac{2}{x}\right)e^{\left(-\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}\\ &-\frac{1}{4}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\sqrt{2}e^{\left(\frac{1}{2}a\right)}}{x}+\frac{1}{x^2}+e^a\right)\\ &+\frac{1}{4}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\left(-\frac{\sqrt{2}e^{\left(\frac{1}{2}a\right)}}{x}+\frac{1}{x^2}+e^a\right)+\frac{1}{x}\end{aligned}$$

```
[In] integrate(tanh(a+2*log(x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) + 2/x)*e^(-1/2*a))*e^(1/
2*a) - 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) - 2/x)*e^(-1/2*a)
)*e^(1/2*a) - 1/4*sqrt(2)*e^(1/2*a)*log(sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a)
+ 1/4*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a) + 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} - \frac{1}{4}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\left(\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right) + \frac{1}{4}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\left(-\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right) + \frac{1}{x}$$

```
[In] integrate(tanh(a+2*log(x))/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(1/2
*a) + 1/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a))
*e^(1/2*a) - 1/4*sqrt(2)*e^(1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a))
 + 1/4*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + 1/x
```

Mupad [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.31

$$\int \frac{\tanh(a+2\log(x))}{x^2}\,dx = \mathrm{atan}\left(x\left(-\mathrm{e}^{2a}\right)^{1/4}\right)\left(-\mathrm{e}^{2a}\right)^{1/4} - \mathrm{atanh}\left(x\left(-\mathrm{e}^{2a}\right)^{1/4}\right)\left(-\mathrm{e}^{2a}\right)^{1/4} + \frac{1}{x}$$

```
[In] int(tanh(a + 2*log(x))/x^2,x)
```

```
[Out] atan(x*(-exp(2*a))^(1/4))*(-exp(2*a))^(1/4) - atanh(x*(-exp(2*a))^(1/4))*(-
exp(2*a))^(1/4) + 1/x
```

3.152 $\int \frac{\tanh(a+2\log(x))}{x^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx = \frac{1}{2x^2} + e^a \arctan\left(e^a x^2\right)$$

```
[Out] 1/2/x^2+exp(a)*arctan(exp(a)*x^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5656, 464, 281, 209}

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx = e^a \arctan\left(e^a x^2\right) + \frac{1}{2x^2}$$

```
[In] Int[Tanh[a + 2*Log[x]]/x^3,x]
```

```
[Out] 1/(2*x^2) + E^a*ArcTan[E^a*x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{-1+e^{2a}x^4}{x^3\left(1+e^{2a}x^4\right)}\,dx\\ &= \frac{1}{2x^2}+\left(2e^{2a}\right)\int \frac{x}{1+e^{2a}x^4}\,dx\\ &= \frac{1}{2x^2}+e^{2a}\text{Subst}\left(\int \frac{1}{1+e^{2a}x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2x^2}+e^{a}\arctan\left(e^{a}x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx = \frac{1}{2x^2}-\arctan\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)\cosh(a) - \arctan\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)\sinh(a)$$

```
[In] Integrate[Tanh[a + 2*Log[x]]/x^3,x]
```

```
[Out] 1/(2*x^2) - ArcTan[(Cosh[a] - Sinh[a])/x^2]*Cosh[a] - ArcTan[(Cosh[a] - Sin
h[a])/x^2]*Sinh[a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

method	result	size
risch	$\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\mathrm{e}^{2a}+\_Z^2\right)}\_R\ln\left(\left(4\,\mathrm{e}^{2a}+5\_R^2\right)x^2-\_R\right)\right)}{2}$	44

```
[In] int(tanh(a+2*ln(x))/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2/x^2+1/2*sum(_R*ln((4*exp(2*a)+5*_R^2)*x^2-_R),_R=RootOf(exp(2*a)+_Z^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx=\frac{2\,x^2\arctan\left(x^2e^a\right)e^a+1}{2\,x^2}$$

```
[In] integrate(tanh(a+2*log(x))/x^3,x, algorithm="fricas")
[Out] 1/2*(2*x^2*arctan(x^2*e^a)*e^a + 1)/x^2
```

Sympy [F]

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx=\int \frac{\tanh\left(a+2\log\left(x\right)\right)}{x^3}\,dx$$

```
[In] integrate(tanh(a+2*ln(x))/x**3,x)
[Out] Integral(tanh(a + 2*log(x))/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx = -\arctan\left(\frac{e^{(-a)}}{x^2}\right)e^a + \frac{1}{2\,x^2}$$

```
[In] integrate(tanh(a+2*log(x))/x^3,x, algorithm="maxima")
[Out] -arctan(e^(-a)/x^2)*e^a + 1/2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx = \arctan\left(x^2e^a\right)e^a + \frac{1}{2\,x^2}$$

```
[In] integrate(tanh(a+2*log(x))/x^3,x, algorithm="giac")
[Out] arctan(x^2*e^a)*e^a + 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\tanh(a+2\log(x))}{x^3}\,dx = \mathrm{atan}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)\,\sqrt{\mathrm{e}^{2\,a}} + \frac{1}{2\,x^2}$$

```
[In] int(tanh(a + 2*log(x))/x^3,x)
[Out] atan(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2) + 1/(2*x^2)
```

3.153 $\int x^3 \tanh^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int x^3 \tanh^2(a+2\log(x))\,dx = \frac{x^4}{4} - \frac{e^{-2a}}{1+e^{2a}x^4} - e^{-2a}\log\left(1+e^{2a}x^4\right)$$

```
[Out] 1/4*x^4-1/exp(2*a)/(1+exp(2*a)*x^4)-ln(1+exp(2*a)*x^4)/exp(2*a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5656, 455, 45}

$$\int x^3 \tanh^2(a+2\log(x))\,dx = -\frac{e^{-2a}}{e^{2a}x^4+1} - e^{-2a}\log\left(e^{2a}x^4+1\right) + \frac{x^4}{4}$$

```
[In] Int[x^3*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 - 1/(E^(2*a)*(1 + E^(2*a)*x^4)) - Log[1 + E^(2*a)*x^4]/E^(2*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(-1+e^{2a}x^4)^2}{(1+e^{2a}x^4)^2}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{(-1+e^{2a}x)^2}{(1+e^{2a}x)^2}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(1+\frac{4}{(1+e^{2a}x)^2}-\frac{4}{1+e^{2a}x}\right)\,dx, x, x^4\right) \\
&= \frac{x^4}{4}-\frac{e^{-2a}}{1+e^{2a}x^4}-e^{-2a}\log\left(1+e^{2a}x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.83

$$\begin{aligned}
\int x^3\tanh^2(a+2\log(x))\,dx &= \frac{x^4}{4}-\cosh(2a)\log\left(\left(1+x^4\right)\cosh(a)+\left(-1+x^4\right)\sinh(a)\right) \\
&\quad+\log\left(\left(1+x^4\right)\cosh(a)+\left(-1+x^4\right)\sinh(a)\right)\sinh(2a) \\
&\quad+\frac{-\cosh(3a)+\sinh(3a)}{(1+x^4)\cosh(a)+(-1+x^4)\sinh(a)}
\end{aligned}$$

```
[In] Integrate[x^3*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 - Cosh[2*a]*Log[(1 + x^4)*Cosh[a] + (-1 + x^4)*Sinh[a]] + Log[(1 + x^
4)*Cosh[a] + (-1 + x^4)*Sinh[a]]*Sinh[2*a] + (-Cosh[3*a] + Sinh[3*a])/((1 +
 x^4)*Cosh[a] + (-1 + x^4)*Sinh[a])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{x^4}{4}-\frac{e^{-2a}}{1+e^{2a}x^4}-e^{-2a}\ln\left(1+e^{2a}x^4\right)$	42

```
[In] int(x^3*tanh(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^4-exp(-2*a)/(1+exp(2*a)*x^4)-exp(-2*a)*ln(1+exp(2*a)*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int x^3\tanh^2(a+2\log(x))\,dx=\frac{x^8e^{(4a)}+x^4e^{(2a)}-4\left(x^4e^{(2a)}+1\right)\log\left(x^4e^{(2a)}+1\right)-4}{4\left(x^4e^{(4a)}+e^{(2a)}\right)}$$

```
[In] integrate(x^3*tanh(a+2*log(x))^2,x, algorithm="fricas")
[Out] 1/4*(x^8*e^(4*a) + x^4*e^(2*a) - 4*(x^4*e^(2*a) + 1)*log(x^4*e^(2*a) + 1) -
 4)/(x^4*e^(4*a) + e^(2*a))
```

Sympy [F]

$$\int x^3\tanh^2(a+2\log(x))\,dx=\int x^3\tanh^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(x**3*tanh(a+2*ln(x))**2,x)
[Out] Integral(x**3*tanh(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^3\tanh^2(a+2\log(x))\,dx=\frac{1}{4}x^4-e^{(-2a)}\log\left(x^4e^{(2a)}+1\right)-\frac{1}{x^4e^{(4a)}+e^{(2a)}}$$

```
[In] integrate(x^3*tanh(a+2*log(x))^2,x, algorithm="maxima")
[Out] 1/4*x^4 - e^(-2*a)*log(x^4*e^(2*a) + 1) - 1/(x^4*e^(4*a) + e^(2*a))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^3 \tanh^2(a+2\log(x))\,dx = \frac{1}{4}\,x^4 + \frac{x^4}{x^4e^{(2\,a)}+1} - e^{(-2\,a)}\log\left(x^4e^{(2\,a)}+1\right)$$

```
[In] integrate(x^3*tanh(a+2*log(x))^2,x, algorithm="giac")
[Out] 1/4*x^4 + x^4/(x^4*e^(2*a) + 1) - e^(-2*a)*log(x^4*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^3 \tanh^2(a+2\log(x))\,dx = \frac{x^4}{4} - \frac{\mathrm{e}^{-2\,a}}{\mathrm{e}^{2\,a}\,x^4+1} - \mathrm{e}^{-2\,a}\,\ln\left(x^4+\mathrm{e}^{-2\,a}\right)$$

```
[In] int(x^3*tanh(a + 2*log(x))^2,x)
[Out] x^4/4 - exp(-2*a)/(x^4*exp(2*a) + 1) - exp(-2*a)*log(exp(-2*a) + x^4)
```

3.154 $\int x^2 \tanh^2(a + 2\log(x))\, dx$

Optimal result

Integrand size = 13, antiderivative size = 173

$$\int x^2 \tanh^2(a+2\log(x))\,dx = \frac{x^3}{3}+\frac{x^3}{1+e^{2a}x^4}+\frac{3e^{-3a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{3e^{-3a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{3e^{-3a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^a x^2\right)}{4\sqrt{2}} + \frac{3e^{-3a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^a x^2\right)}{4\sqrt{2}}$$

```
[Out] 1/3*x^3+x^3/(1+exp(2*a)*x^4)-3/4*arctan(-1+exp(1/2*a)*x*2^(1/2))/exp(3/2*a)
*2^(1/2)-3/4*arctan(1+exp(1/2*a)*x*2^(1/2))/exp(3/2*a)*2^(1/2)-3/8*ln(1+exp
(a)*x^2-exp(1/2*a)*x*2^(1/2))/exp(3/2*a)*2^(1/2)+3/8*ln(1+exp(a)*x^2+exp(1/
2*a)*x*2^(1/2))/exp(3/2*a)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {5656, 474, 470, 303, 1176, 631, 210, 1179, 642}

$$\int x^2 \tanh^2(a+2\log(x))\,dx = \frac{3e^{-3a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{3e^{-3a/2}\arctan\left(\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}} - \frac{3e^{-3a/2}\log\left(e^a x^2-\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}} + \frac{3e^{-3a/2}\log\left(e^a x^2+\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}} + \frac{x^3}{e^{2a}x^4+1}+\frac{x^3}{3}$$

```
[In] Int[x^2*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] x^3/3 + x^3/(1 + E^(2*a)*x^4) + (3*ArcTan[1 - Sqrt[2]*E^(a/2)*x])/(2*Sqrt[2
]*E^((3*a)/2)) - (3*ArcTan[1 + Sqrt[2]*E^(a/2)*x])/(2*Sqrt[2]*E^((3*a)/2))
- (3*Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sqrt[2]*E^((3*a)/2)) + (3*Log
[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sqrt[2]*E^((3*a)/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(-1+e^{2a}x^4)^2}{(1+e^{2a}x^4)^2}\,dx \\
&= \frac{x^3}{1+e^{2a}x^4} - \frac{1}{4}e^{-4a}\int \frac{x^2(8e^{4a}-4e^{6a}x^4)}{1+e^{2a}x^4}\,dx \\
&= \frac{x^3}{3} + \frac{x^3}{1+e^{2a}x^4} - 3\int \frac{x^2}{1+e^{2a}x^4}\,dx \\
&= \frac{x^3}{3} + \frac{x^3}{1+e^{2a}x^4} + \frac{1}{2}\left(3e^{-a}\right)\int \frac{1-e^a x^2}{1+e^{2a}x^4}\,dx - \frac{1}{2}\left(3e^{-a}\right)\int \frac{1+e^a x^2}{1+e^{2a}x^4}\,dx \\
&= \frac{x^3}{3} + \frac{x^3}{1+e^{2a}x^4} - \frac{1}{4}\left(3e^{-2a}\right)\int \frac{1}{e^{-a}-\sqrt{2}e^{-a/2}x+x^2}\,dx \\
&\quad - \frac{1}{4}\left(3e^{-2a}\right)\int \frac{1}{e^{-a}+\sqrt{2}e^{-a/2}x+x^2}\,dx \\
&\quad - \frac{\left(3e^{-3a/2}\right)\int \frac{\sqrt{2}e^{-a/2}+2x}{-e^{-a}-\sqrt{2}e^{-a/2}x-x^2}\,dx}{4\sqrt{2}} - \frac{\left(3e^{-3a/2}\right)\int \frac{\sqrt{2}e^{-a/2}-2x}{-e^{-a}+\sqrt{2}e^{-a/2}x-x^2}\,dx}{4\sqrt{2}}
\end{aligned}$$

$$= \frac{x^3}{3} + \frac{x^3}{1+e^{2a}x^4} - \frac{3e^{-3a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}} + \frac{3e^{-3a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}} - \frac{\left(3e^{-3a/2}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} + \frac{\left(3e^{-3a/2}\right)\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}$$

$$= \frac{x^3}{3} + \frac{x^3}{1+e^{2a}x^4} + \frac{3e^{-3a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{3e^{-3a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{3e^{-3a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}} + \frac{3e^{-3a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.01

$$\int x^2\tanh^2(a+2\log(x))\,dx = \frac{1}{12}\left(4x^3 + \frac{12x^3}{1+e^{2a}x^4} + 9(-1)^{3/4}e^{-3a/2}\log\left(\sqrt[4]{-1}e^{-3a/2}-e^{-a}x\right) + 9\sqrt[4]{-1}e^{-3a/2}\log\left((-1)^{3/4}e^{-3a/2}-e^{-a}x\right) - 9(-1)^{3/4}e^{-3a/2}\log\left(\sqrt[4]{\right.\right.$$

```
[In] Integrate[x^2*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] (4*x^3 + (12*x^3)/(1 + E^(2*a)*x^4) + (9*(-1)^(3/4)*Log[(-1)^(1/4)/E^((3*a)
/2) - x/E^a])/E^((3*a)/2) + (9*(-1)^(1/4)*Log[(-1)^(3/4)/E^((3*a)/2) - x/E^
a])/E^((3*a)/2) - (9*(-1)^(3/4)*Log[(-1)^(1/4)/E^((3*a)/2) + x/E^a])/E^((3*
a)/2) - (9*(-1)^(1/4)*Log[(-1)^(3/4)/E^((3*a)/2) + x/E^a])/E^((3*a)/2))/12
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.31

method	result	size
risch	$\frac{x^3}{3}+\frac{x^3}{1+e^{2a}x^4}-\frac{3\,e^{-2a}\left(\sum_{\_R=\text{RootOf}\left(e^{2a}\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{4}$	53

```
[In] int(x^2*tanh(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+x^3/(1+exp(2*a)*x^4)-3/4*exp(-2*a)*sum(1/_R*ln(x-_R),_R=RootOf(exp(
2*a)*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03

$$\int x^2 \tanh^2(a+2\log(x))\,dx$$
$$= \frac{4\,x^7e^{(2\,a)} + 16\,x^3 - 9\left(x^4e^{(2\,a)}+1\right)\left(-e^{(-6\,a)}\right)^{\frac{1}{4}}\log\left(\left(-e^{(-6\,a)}\right)^{\frac{3}{4}}e^{(4\,a)}+x\right) - 9\left(-i\,x^4e^{(2\,a)}-i\right)\left(-e^{(-6\,a)}\right)^{\frac{1}{4}}\text{lo}}{}$$

```
[In] integrate(x^2*tanh(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^7*e^(2*a) + 16*x^3 - 9*(x^4*e^(2*a) + 1)*(-e^(-6*a))^(1/4)*log((-
e^(-6*a))^(3/4)*e^(4*a) + x) - 9*(-I*x^4*e^(2*a) - I)*(-e^(-6*a))^(1/4)*log
(I*(-e^(-6*a))^(3/4)*e^(4*a) + x) - 9*(I*x^4*e^(2*a) + I)*(-e^(-6*a))^(1/4)
*log(-I*(-e^(-6*a))^(3/4)*e^(4*a) + x) + 9*(x^4*e^(2*a) + 1)*(-e^(-6*a))^(1
/4)*log(-(-e^(-6*a))^(3/4)*e^(4*a) + x))/(x^4*e^(2*a) + 1)
```

Sympy [F]

$$\int x^2 \tanh^2(a+2\log(x))\,dx = \int x^2 \tanh^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(x**2*tanh(a+2*ln(x))**2,x)
```

```
[Out] Integral(x**2*tanh(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\int x^2 \tanh^2(a+2\log(x))\,dx = \frac{1}{3}\,x^3 - \frac{3}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,xe^a+\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} - \frac{3}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,xe^a-\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{3}{8}\,\sqrt{2}e^{\left(-\frac{3}{2}\,a\right)}\log\left(x^2e^a+\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+1\right) - \frac{3}{8}\,\sqrt{2}e^{\left(-\frac{3}{2}\,a\right)}\log\left(x^2e^a-\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+1\right) + \frac{x^3}{x^4e^{(2\,a)}+1}$$

```
[In] integrate(x^2*tanh(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 3/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-
1/2*a))*e^(-3/2*a) - 3/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1
/2*a))*e^(-1/2*a))*e^(-3/2*a) + 3/8*sqrt(2)*e^(-3/2*a)*log(x^2*e^a + sqrt(2
)*x*e^(1/2*a) + 1) - 3/8*sqrt(2)*e^(-3/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*
a) + 1) + x^3/(x^4*e^(2*a) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.80

$$\begin{aligned}\int x^2\tanh^2(a+2\log(x))\,dx &= \frac{1}{3}x^3 - \frac{3}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} \\ &\quad - \frac{3}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{3}{2}a\right)} \\ &\quad + \frac{3}{8}\sqrt{2}e^{\left(-\frac{3}{2}a\right)}\log\left(\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right) \\ &\quad - \frac{3}{8}\sqrt{2}e^{\left(-\frac{3}{2}a\right)}\log\left(-\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right) + \frac{x^3}{x^4e^{(2a)}+1}\end{aligned}$$

```
[In] integrate(x^2*tanh(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 3/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*
a))*e^(-3/2*a) - 3/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)
*e^(1/2*a))*e^(-3/2*a) + 3/8*sqrt(2)*e^(-3/2*a)*log(sqrt(2)*x*e^(-1/2*a) +
x^2 + e^(-a)) - 3/8*sqrt(2)*e^(-3/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^
(-a)) + x^3/(x^4*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.39

$$\begin{aligned}\int x^2\tanh^2(a+2\log(x))\,dx &= \frac{x^3}{\mathrm{e}^{2a}\,x^4+1} + \frac{3\,\mathrm{atan}\left(x\,{\left(-\mathrm{e}^{2a}\right)}^{1/4}\right)}{2\,{\left(-\mathrm{e}^{2a}\right)}^{3/4}} \\ &\quad + \frac{x^3}{3} + \frac{\mathrm{atan}\left(x\,{\left(-\mathrm{e}^{2a}\right)}^{1/4}\,1\mathrm{i}\right)\,3\mathrm{i}}{2\,{\left(-\mathrm{e}^{2a}\right)}^{3/4}}\end{aligned}$$

```
[In] int(x^2*tanh(a + 2*log(x))^2,x)
```

```
[Out] x^3/(x^4*exp(2*a) + 1) + (3*atan(x*(-exp(2*a))^(1/4)))/(2*(-exp(2*a))^(3/4)
) + (atan(x*(-exp(2*a))^(1/4)*1i)*3i)/(2*(-exp(2*a))^(3/4)) + x^3/3
```

3.155 $\int x \tanh^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int x \tanh^2(a+2\log(x))\,dx=\frac{x^2}{2}+\frac{x^2}{1+e^{2a}x^4}-e^{-a}\arctan\left(e^ax^2\right)$$

```
[Out] 1/2*x^2+x^2/(1+exp(2*a)*x^4)-arctan(exp(a)*x^2)/exp(a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5656, 474, 470, 281, 209}

$$\int x \tanh^2(a+2\log(x))\,dx=-e^{-a}\arctan\left(e^ax^2\right)+\frac{x^2}{e^{2a}x^4+1}+\frac{x^2}{2}$$

```
[In] Int[x*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] x^2/2 + x^2/(1 + E^(2*a)*x^4) - ArcTan[E^a*x^2]/E^a
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{x(-1+e^{2a}x^4)^2}{(1+e^{2a}x^4)^2}\,dx \\
&= \frac{x^2}{1+e^{2a}x^4} - \frac{1}{4}e^{-4a}\int \frac{x(4e^{4a}-4e^{6a}x^4)}{1+e^{2a}x^4}\,dx \\
&= \frac{x^2}{2} + \frac{x^2}{1+e^{2a}x^4} - 2\int \frac{x}{1+e^{2a}x^4}\,dx \\
&= \frac{x^2}{2} + \frac{x^2}{1+e^{2a}x^4} - \text{Subst}\left(\int \frac{1}{1+e^{2a}x^2}\,dx, x, x^2\right) \\
&= \frac{x^2}{2} + \frac{x^2}{1+e^{2a}x^4} - e^{-a}\arctan\left(e^a x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int x\tanh^2(a+2\log(x))\,dx=\frac{x^2}{2}+\frac{x^2}{1+e^{2(a+2\log(x))}}-e^{-a}\arctan\left(e^ax^2\right)$$

```
[In] Integrate[x*Tanh[a + 2*Log[x]]^2,x]
[Out] x^2/2 + x^2/(1 + E^(2*(a + 2*Log[x]))) - ArcTan[E^a*x^2]/E^a
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{x^2}{2}+\frac{x^2}{1+e^{2a}x^4}+\frac{ie^{-a}\ln\left(e^ax^2-i\right)}{2}-\frac{ie^{-a}\ln\left(e^ax^2+i\right)}{2}$	57

```
[In] int(x*tanh(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+x^2/(exp(a)^2*x^4+1)+1/2*I/exp(a)*ln(exp(a)*x^2-I)-1/2*I/exp(a)*ln(
exp(a)*x^2+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int x\tanh^2(a+2\log(x))\,dx=\frac{x^6e^{(3a)}+3x^2e^a-2\left(x^4e^{(2a)}+1\right)\arctan\left(x^2e^a\right)}{2\left(x^4e^{(3a)}+e^a\right)}$$

```
[In] integrate(x*tanh(a+2*log(x))^2,x, algorithm="fricas")
[Out] 1/2*(x^6*e^(3*a) + 3*x^2*e^a - 2*(x^4*e^(2*a) + 1)*arctan(x^2*e^a))/(x^4*e^
(3*a) + e^a)
```

Sympy [F]

$$\int x\tanh^2(a+2\log(x))\,dx = \int x\tanh^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(x*tanh(a+2*ln(x))**2,x)
[Out] Integral(x*tanh(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x\tanh^2(a+2\log(x))\,dx = \frac{1}{2}x^2 - \arctan\left(x^2e^a\right)e^{(-a)} + \frac{x^2}{x^4e^{(2a)}+1}$$

```
[In] integrate(x*tanh(a+2*log(x))^2,x, algorithm="maxima")
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a) + x^2/(x^4*e^(2*a) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int x\tanh^2(a+2\log(x))\,dx = \frac{1}{2}x^2 - \arctan\left(x^2e^a\right)e^{(-a)} + \frac{x^2}{x^4e^{(2a)}+1}$$

```
[In] integrate(x*tanh(a+2*log(x))^2,x, algorithm="giac")
[Out] 1/2*x^2 - arctan(x^2*e^a)*e^(-a) + x^2/(x^4*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int x\tanh^2(a+2\log(x))\,dx = \frac{x^2}{\mathrm{e}^{2a}\,x^4+1} - \frac{\mathrm{atan}\left(x^2\,\sqrt{\mathrm{e}^{2a}}\right)}{\sqrt{\mathrm{e}^{2a}}} + \frac{x^2}{2}$$

```
[In] int(x*tanh(a + 2*log(x))^2,x)
[Out] x^2/(x^4*exp(2*a) + 1) - atan(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2) + x^2/2
```

3.156 $\int \tanh^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 165

$$\int \tanh^2(a+2\log(x))\,dx = x+\frac{x}{1+e^{2a}x^4}+\frac{e^{-a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{e^{-a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}+\frac{e^{-a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}} - \frac{e^{-a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}}$$

```
[Out] x+x/(1+exp(2*a)*x^4)-1/4*arctan(-1+exp(1/2*a)*x*2^(1/2))/exp(1/2*a)*2^(1/2)
-1/4*arctan(1+exp(1/2*a)*x*2^(1/2))/exp(1/2*a)*2^(1/2)+1/8*ln(1+exp(a)*x^2-
exp(1/2*a)*x*2^(1/2))/exp(1/2*a)*2^(1/2)-1/8*ln(1+exp(a)*x^2+exp(1/2*a)*x*2
^(1/2))/exp(1/2*a)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5652, 398, 294, 217, 1179, 642, 1176, 631, 210}

$$\int \tanh^2(a+2\log(x))\,dx = \frac{e^{-a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{e^{-a/2}\arctan\left(\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}} + \frac{x}{e^{2a}x^4+1}+\frac{e^{-a/2}\log\left(e^ax^2-\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}} - \frac{e^{-a/2}\log\left(e^ax^2+\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}}+x$$

```
[In] Int[Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] x + x/(1 + E^(2*a)*x^4) + ArcTan[1 - Sqrt[2]*E^(a/2)*x]/(2*Sqrt[2]*E^(a/2))
 - ArcTan[1 + Sqrt[2]*E^(a/2)*x]/(2*Sqrt[2]*E^(a/2)) + Log[1 - Sqrt[2]*E^(a
/2)*x + E^a*x^2]/(4*Sqrt[2]*E^(a/2)) - Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2]
/(4*Sqrt[2]*E^(a/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(-1+e^{2a}x^4)^2}{(1+e^{2a}x^4)^2}\,dx \\
&= \int \left(1-\frac{4e^{2a}x^4}{(1+e^{2a}x^4)^2}\right)dx \\
&= x-\left(4e^{2a}\right)\int \frac{x^4}{(1+e^{2a}x^4)^2}\,dx \\
&= x+\frac{x}{1+e^{2a}x^4}-\int \frac{1}{1+e^{2a}x^4}\,dx \\
&= x+\frac{x}{1+e^{2a}x^4}-\frac{1}{2}\int \frac{1-e^a x^2}{1+e^{2a}x^4}\,dx-\frac{1}{2}\int \frac{1+e^a x^2}{1+e^{2a}x^4}\,dx \\
&= x+\frac{x}{1+e^{2a}x^4}-\frac{1}{4}e^{-a}\int \frac{1}{e^{-a}-\sqrt{2}e^{-a/2}x+x^2}\,dx-\frac{1}{4}e^{-a}\int \frac{1}{e^{-a}+\sqrt{2}e^{-a/2}x+x^2}\,dx \\
&\quad+\frac{e^{-a/2}\int \frac{\sqrt{2}e^{-a/2}+2x}{-e^{-a}-\sqrt{2}e^{-a/2}x-x^2}\,dx}{4\sqrt{2}}+\frac{e^{-a/2}\int \frac{\sqrt{2}e^{-a/2}-2x}{-e^{-a}+\sqrt{2}e^{-a/2}x-x^2}\,dx}{4\sqrt{2}} \\
&= x+\frac{x}{1+e^{2a}x^4}+\frac{e^{-a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^a x^2\right)}{4\sqrt{2}}-\frac{e^{-a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^a x^2\right)}{4\sqrt{2}} \\
&\quad-\frac{e^{-a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}+\frac{e^{-a/2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}
\end{aligned}$$

$$= x + \frac{x}{1+e^{2a}x^4} + \frac{e^{-a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{e^{-a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} + \frac{e^{-a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^a x^2\right)}{4\sqrt{2}} - \frac{e^{-a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^a x^2\right)}{4\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \tanh^2(a+2\log(x))\,dx = \frac{1}{4}\Bigg(4x + \frac{4x}{1+e^{2a}x^4} + \sqrt[4]{-1}e^{-a/2}\log\left(\sqrt[4]{-1}e^{-a/2}-x\right) + (-1)^{3/4}e^{-a/2}\log\left((-1)^{3/4}e^{-a/2}-x\right) - \sqrt[4]{-1}e^{-a/2}\log\left(\sqrt[4]{-1}e^{-a/2}+x\right) - (\cdot$$

```
[In] Integrate[Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] (4*x + (4*x)/(1 + E^(2*a)*x^4) + ((-1)^(1/4)*Log[(-1)^(1/4)/E^(a/2) - x])/E
^(a/2) + ((-1)^(3/4)*Log[(-1)^(3/4)/E^(a/2) - x])/E^(a/2) - ((-1)^(1/4)*Log
[(-1)^(1/4)/E^(a/2) + x])/E^(a/2) - ((-1)^(3/4)*Log[(-1)^(3/4)/E^(a/2) + x]
)/E^(a/2))/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.28

method	result	size
risch	$x+\frac{x}{1+\mathrm{e}^{2a}x^4}-\frac{\mathrm{e}^{-2a}\left(\sum_{\_R=\mathrm{RootOf}\left(\mathrm{e}^{2a}\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{4}$	47

```
[In] int(tanh(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x+x/(1+exp(2*a)*x^4)-1/4*exp(-2*a)*sum(1/_R^3*ln(x-_R),_R=RootOf(exp(2*a)*_
Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.95

$$\int \tanh^2(a+2\log(x))\,dx$$
$$= \frac{4\,x^5e^{(2\,a)} - \left(x^4e^{(2\,a)}+1\right)\left(-e^{(-2\,a)}\right)^{\frac{1}{4}}\log\left(x+\left(-e^{(-2\,a)}\right)^{\frac{1}{4}}\right)+\left(-i\,x^4e^{(2\,a)}-i\right)\left(-e^{(-2\,a)}\right)^{\frac{1}{4}}\log\left(x+i\left(-e^{(-2}\right.\right.}{4\,(x}$$

```
[In] integrate(tanh(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^5*e^(2*a) - (x^4*e^(2*a) + 1)*(-e^(-2*a))^(1/4)*log(x + (-e^(-2*a)
)^(1/4)) + (-I*x^4*e^(2*a) - I)*(-e^(-2*a))^(1/4)*log(x + I*(-e^(-2*a))^(1/
4)) + (I*x^4*e^(2*a) + I)*(-e^(-2*a))^(1/4)*log(x - I*(-e^(-2*a))^(1/4)) +
(x^4*e^(2*a) + 1)*(-e^(-2*a))^(1/4)*log(x - (-e^(-2*a))^(1/4)) + 8*x)/(x^4*
e^(2*a) + 1)
```

Sympy [F]

$$\int \tanh^2(a+2\log(x))\,dx = \int \tanh^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(tanh(a+2*ln(x))**2,x)
```

```
[Out] Integral(tanh(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \tanh^2(a+2\log(x))\,dx = &-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,xe^a+\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\\ &-\frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,xe^a-\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)}\\ &-\frac{1}{8}\,\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)}\log\left(x^2e^a+\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+1\right)\\ &+\frac{1}{8}\,\sqrt{2}e^{\left(-\frac{1}{2}\,a\right)}\log\left(x^2e^a-\sqrt{2}xe^{\left(\frac{1}{2}\,a\right)}+1\right)+x+\frac{x}{x^4e^{(2\,a)}+1}\end{aligned}$$

```
[In] integrate(tanh(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a + sqrt(2)*e^(1/2*a))*e^(-1/2*a))*e
^(-1/2*a) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e^a - sqrt(2)*e^(1/2*a))*e^
(-1/2*a))*e^(-1/2*a) - 1/8*sqrt(2)*e^(-1/2*a)*log(x^2*e^a + sqrt(2)*x*e^(1/
2*a) + 1) + 1/8*sqrt(2)*e^(-1/2*a)*log(x^2*e^a - sqrt(2)*x*e^(1/2*a) + 1) +
 x + x/(x^4*e^(2*a) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.81

$$\begin{aligned}\int \tanh^2(a+2\log(x))\,dx = &-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\\ &-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)}\\ &-\frac{1}{8}\sqrt{2}e^{\left(-\frac{1}{2}a\right)}\log\left(\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right)\\ &+\frac{1}{8}\sqrt{2}e^{\left(-\frac{1}{2}a\right)}\log\left(-\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right)+x+\frac{x}{x^4e^{(2a)}+1}\end{aligned}$$

```
[In] integrate(tanh(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(-1
/2*a) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a
))*e^(-1/2*a) - 1/8*sqrt(2)*e^(-1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(
-a)) + 1/8*sqrt(2)*e^(-1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) + x
 + x/(x^4*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \tanh^2(a+2\log(x))\,dx = x-\frac{\mathrm{atan}\left(x\,{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{2\,{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}}+\frac{x}{\mathrm{e}^{2\,a}\,x^4+1}+\frac{\mathrm{atan}\left(x\,{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,{\left(-\mathrm{e}^{2\,a}\right)}^{1/4}}$$

```
[In] int(tanh(a + 2*log(x))^2,x)
```

```
[Out] x - atan(x*(-exp(2*a))^(1/4))/(2*(-exp(2*a))^(1/4)) + (atan(x*(-exp(2*a))^(
1/4)*1i)*1i)/(2*(-exp(2*a))^(1/4)) + x/(x^4*exp(2*a) + 1)
```

3.157 $\int \frac{\tanh^2(a+2\log(x))}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\tanh^2(a+2\log(x))}{x} dx = \log(x) - \frac{1}{2}\tanh(a+2\log(x))$$

```
[Out] ln(x)-1/2*tanh(a+2*ln(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3554, 8}

$$\int \frac{\tanh^2(a+2\log(x))}{x} dx = \log(x) - \frac{1}{2}\tanh(a+2\log(x))$$

```
[In] Int[Tanh[a + 2*Log[x]]^2/x,x]
[Out] Log[x] - Tanh[a + 2*Log[x]]/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \tanh^2(a+2x)\,dx, x, \log(x)\right)\\ &= -\frac{1}{2}\tanh(a+2\log(x)) + \text{Subst}(\int 1\,dx, x, \log(x))\\ &= \log(x) - \frac{1}{2}\tanh(a+2\log(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\tanh^2(a+2\log(x))}{x}\,dx = \frac{1}{2}\text{arctanh}(\tanh(a+2\log(x))) - \frac{1}{2}\tanh(a+2\log(x))$$

```
[In] Integrate[Tanh[a + 2*Log[x]]^2/x,x]
[Out] ArcTanh[Tanh[a + 2*Log[x]]]/2 - Tanh[a + 2*Log[x]]/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\ln(x) - \frac{\tanh(a+2\ln(x))}{2}$	13
risch	$\frac{1}{1+e^{2a}x^4} + \ln(x)$	16
derivativedivides	$-\frac{\tanh(a+2\ln(x))}{2} - \frac{\ln(\tanh(a+2\ln(x))-1)}{4} + \frac{\ln(\tanh(a+2\ln(x))+1)}{4}$	35
default	$-\frac{\tanh(a+2\ln(x))}{2} - \frac{\ln(\tanh(a+2\ln(x))-1)}{4} + \frac{\ln(\tanh(a+2\ln(x))+1)}{4}$	35

```
[In] int(tanh(a+2*ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] ln(x)-1/2*tanh(a+2*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\tanh^2(a+2\log(x))}{x}\,dx = \frac{\left(x^4e^{(2\,a)}+1\right)\log\left(x\right)+1}{x^4e^{(2\,a)}+1}$$

```
[In] integrate(tanh(a+2*log(x))^2/x,x, algorithm="fricas")
[Out] ((x^4*e^(2*a) + 1)*log(x) + 1)/(x^4*e^(2*a) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^2(a+2\log(x))}{x}\,dx = \log\left(x\right) - \frac{\tanh\left(a+2\log\left(x\right)\right)}{2}$$

```
[In] integrate(tanh(a+2*ln(x))**2/x,x)
[Out] log(x) - tanh(a + 2*log(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\tanh^2(a+2\log(x))}{x}\,dx = \frac{1}{2}\,a - \frac{1}{e^{(-2\,a-4\,\log(x))}+1} + \log\left(x\right)$$

```
[In] integrate(tanh(a+2*log(x))^2/x,x, algorithm="maxima")
[Out] 1/2*a - 1/(e^(-2*a - 4*log(x)) + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\tanh^2(a+2\log(x))}{x}\,dx = \frac{1}{x^4e^{(2\,a)}+1} + \frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate(tanh(a+2*log(x))^2/x,x, algorithm="giac")
[Out] 1/(x^4*e^(2*a) + 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\tanh^2(a+2\log(x))}{x}\,dx = \ln{(x)} - \frac{x^4\,\mathrm{e}^{2\,a}-1}{2\,\left(\mathrm{e}^{2\,a}\,x^4+1\right)}$$

```
[In] int(tanh(a + 2*log(x))^2/x,x)
[Out] log(x) - (x^4*exp(2*a) - 1)/(2*(x^4*exp(2*a) + 1))
```

3.158 $\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 190

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx = -\frac{1}{x(1+e^{2a}x^4)}-\frac{2e^{2a}x^3}{1+e^{2a}x^4}+\frac{e^{a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}-\frac{e^{a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}-\frac{e^{a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}}+\frac{e^{a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}}$$

```
[Out] -1/x/(1+exp(2*a)*x^4)-2*exp(2*a)*x^3/(1+exp(2*a)*x^4)-1/4*exp(1/2*a)*arctan
(-1+exp(1/2*a)*x*2^(1/2))*2^(1/2)-1/4*exp(1/2*a)*arctan(1+exp(1/2*a)*x*2^(1
/2))*2^(1/2)-1/8*exp(1/2*a)*ln(1+exp(a)*x^2-exp(1/2*a)*x*2^(1/2))*2^(1/2)+1
/8*exp(1/2*a)*ln(1+exp(a)*x^2+exp(1/2*a)*x*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {5656, 473, 468, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx = \frac{e^{a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}-\frac{e^{a/2}\arctan\left(\sqrt{2}e^{a/2}x+1\right)}{2\sqrt{2}}-\frac{1}{x(e^{2a}x^4+1)}-\frac{e^{a/2}\log\left(e^ax^2-\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}}+\frac{e^{a/2}\log\left(e^ax^2+\sqrt{2}e^{a/2}x+1\right)}{4\sqrt{2}}-\frac{2e^{2a}x^3}{e^{2a}x^4+1}$$

```
[In] Int[Tanh[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] -(1/(x*(1 + E^(2*a)*x^4))) - (2*E^(2*a)*x^3)/(1 + E^(2*a)*x^4) + (E^(a/2)*A
rcTan[1 - Sqrt[2]*E^(a/2)*x])/(2*Sqrt[2]) - (E^(a/2)*ArcTan[1 + Sqrt[2]*E^(
a/2)*x])/(2*Sqrt[2]) - (E^(a/2)*Log[1 - Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sq
rt[2]) + (E^(a/2)*Log[1 + Sqrt[2]*E^(a/2)*x + E^a*x^2])/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(-1+e^{2a}x^4)^2}{x^2\left(1+e^{2a}x^4\right)^2}\,dx \\
&= -\frac{1}{x\left(1+e^{2a}x^4\right)} + \int \frac{x^2(-7e^{2a}+e^{4a}x^4)}{(1+e^{2a}x^4)^2}\,dx \\
&= -\frac{1}{x\left(1+e^{2a}x^4\right)} - \frac{2e^{2a}x^3}{1+e^{2a}x^4} - e^{2a}\int \frac{x^2}{1+e^{2a}x^4}\,dx \\
&= -\frac{1}{x\left(1+e^{2a}x^4\right)} - \frac{2e^{2a}x^3}{1+e^{2a}x^4} + \frac{1}{2}e^a\int \frac{1-e^ax^2}{1+e^{2a}x^4}\,dx - \frac{1}{2}e^a\int \frac{1+e^ax^2}{1+e^{2a}x^4}\,dx \\
&= -\frac{1}{x\left(1+e^{2a}x^4\right)} - \frac{2e^{2a}x^3}{1+e^{2a}x^4} - \frac{1}{4}\int \frac{1}{e^{-a}-\sqrt{2}e^{-a/2}x+x^2}\,dx \\
&\quad - \frac{1}{4}\int \frac{1}{e^{-a}+\sqrt{2}e^{-a/2}x+x^2}\,dx \\
&\quad - \frac{e^{a/2}\int \frac{\sqrt{2}e^{-a/2}+2x}{-e^{-a}-\sqrt{2}e^{-a/2}x-x^2}\,dx}{4\sqrt{2}} - \frac{e^{a/2}\int \frac{\sqrt{2}e^{-a/2}-2x}{-e^{-a}+\sqrt{2}e^{-a/2}x-x^2}\,dx}{4\sqrt{2}}
\end{aligned}
$$

$$= -\frac{1}{x(1+e^{2a}x^4)} - \frac{2e^{2a}x^3}{1+e^{2a}x^4} - \frac{e^{a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}} + \frac{e^{a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}} - \frac{e^{a/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} + \frac{e^{a/2}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}}$$

$$= -\frac{1}{x(1+e^{2a}x^4)} - \frac{2e^{2a}x^3}{1+e^{2a}x^4} + \frac{e^{a/2}\arctan\left(1-\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{e^{a/2}\arctan\left(1+\sqrt{2}e^{a/2}x\right)}{2\sqrt{2}} - \frac{e^{a/2}\log\left(1-\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}} + \frac{e^{a/2}\log\left(1+\sqrt{2}e^{a/2}x+e^ax^2\right)}{4\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95

$$\int\frac{\tanh^2(a+2\log(x))}{x^2}\,dx = \frac{1}{4}\left(-\frac{4}{x} - \frac{4}{\frac{e^{-2a}}{x^3}+x} + (-1)^{3/4}e^{a/2}\log\left(\frac{e^{-2a}\left(\sqrt[4]{-1}-e^{a/2}x\right)}{x^4}\right) + \sqrt[4]{-1}e^{a/2}\log\left(\frac{e^{-2a}\left((-1)^{3/4}-e^{a/2}x\right)}{x^4}\right) - (-1)^{3/4}e^{a/2}\log\left(\frac{e^{-2a}\ldots}{\ldots}\right)\right.$$

```
[In] Integrate[Tanh[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] (-4/x - 4/(1/(E^(2*a)*x^3) + x) + (-1)^(3/4)*E^(a/2)*Log[((-1)^(1/4) - E^(a
/2)*x)/(E^(2*a)*x^4)] + (-1)^(1/4)*E^(a/2)*Log[((-1)^(3/4) - E^(a/2)*x)/(E^
(2*a)*x^4)] - (-1)^(3/4)*E^(a/2)*Log[((-1)^(1/4) + E^(a/2)*x)/(E^(2*a)*x^4)
] - (-1)^(1/4)*E^(a/2)*Log[((-1)^(3/4) + E^(a/2)*x)/(E^(2*a)*x^4)])/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.34

method	result	size
risch	$\frac{-2\,e^{2a}x^4-1}{x(1+e^{2a}x^4)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+e^{2a}\right)}\_R\ln\left(\left(5\_R^4+4\,e^{2a}\right)x+\_R^3\right)\right)}{4}$	64

```
[In] int(tanh(a+2*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*exp(2*a)*x^4-1)/x/(1+exp(2*a)*x^4)+1/4*sum(_R*ln((5*_R^4+4*exp(2*a))*x+
_R^3),_R=RootOf(_Z^4+exp(2*a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.95

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx = -\frac{8\,x^4e^{(2\,a)} + \left(x^5e^{(2\,a)} + x\right)\left(-e^{(2\,a)}\right)^{\frac{1}{4}}\log\left(xe^{(2\,a)} + \left(-e^{(2\,a)}\right)^{\frac{3}{4}}\right) - \left(i\,x^5e^{(2\,a)} + i\,x\right)\left(-e^{(2\,a)}\right)^{\frac{1}{4}}\log\left(xe^{(2\,a)} + i\right.}{}$$

```
[In] integrate(tanh(a+2*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^4*e^(2*a) + (x^5*e^(2*a) + x)*(-e^(2*a))^(1/4)*log(x*e^(2*a) + (-
e^(2*a))^(3/4)) - (I*x^5*e^(2*a) + I*x)*(-e^(2*a))^(1/4)*log(x*e^(2*a) + I*
(-e^(2*a))^(3/4)) - (-I*x^5*e^(2*a) - I*x)*(-e^(2*a))^(1/4)*log(x*e^(2*a) -
 I*(-e^(2*a))^(3/4)) - (x^5*e^(2*a) + x)*(-e^(2*a))^(1/4)*log(x*e^(2*a) - (
-e^(2*a))^(3/4)) + 4)/(x^5*e^(2*a) + x)
```

Sympy [F]

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx = \int \frac{\tanh^2\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(tanh(a+2*ln(x))**2/x**2,x)
```

```
[Out] Integral(tanh(a + 2*log(x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.77

$$\begin{aligned}\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx ={}& \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}\,a\right)} + \frac{2}{x}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} \\ &+ \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(\frac{1}{2}\,a\right)} - \frac{2}{x}\right)e^{\left(-\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} \\ &+ \frac{1}{8}\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}}{x} + \frac{1}{x^2} + e^a\right) \\ &- \frac{1}{8}\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}\log\left(-\frac{\sqrt{2}e^{\left(\frac{1}{2}\,a\right)}}{x} + \frac{1}{x^2} + e^a\right) - \frac{1}{x} - \frac{e^{(2\,a)}}{x\left(\frac{1}{x^4} + e^{(2\,a)}\right)}\end{aligned}$$

```
[In] integrate(tanh(a+2*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) + 2/x)*e^(-1/2*a))*e^(1/2
*a) + 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(1/2*a) - 2/x)*e^(-1/2*a))
*e^(1/2*a) + 1/8*sqrt(2)*e^(1/2*a)*log(sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a) -
 1/8*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*e^(1/2*a)/x + 1/x^2 + e^a) - 1/x - e^(2
*a)/(x*(1/x^4 + e^(2*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.75

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}+2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}e^{\left(-\frac{1}{2}a\right)}-2x\right)e^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} + \frac{1}{8}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\left(\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right) - \frac{1}{8}\sqrt{2}e^{\left(\frac{1}{2}a\right)}\log\left(-\sqrt{2}xe^{\left(-\frac{1}{2}a\right)}+x^2+e^{(-a)}\right) - \frac{2x^4e^{(2a)}+1}{x^5e^{(2a)}+x}$$

```
[In] integrate(tanh(a+2*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) + 2*x)*e^(1/2*a))*e^(1/
2*a) - 1/4*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*e^(-1/2*a) - 2*x)*e^(1/2*a)
)*e^(1/2*a) + 1/8*sqrt(2)*e^(1/2*a)*log(sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)
) - 1/8*sqrt(2)*e^(1/2*a)*log(-sqrt(2)*x*e^(-1/2*a) + x^2 + e^(-a)) - (2*x^
4*e^(2*a) + 1)/(x^5*e^(2*a) + x)
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.36

$$\int \frac{\tanh^2(a+2\log(x))}{x^2}\,dx = \frac{\operatorname{atanh}\left(x\left(-e^{2a}\right)^{1/4}\right)\left(-e^{2a}\right)^{1/4}}{2} - \frac{\operatorname{atan}\left(x\left(-e^{2a}\right)^{1/4}\right)\left(-e^{2a}\right)^{1/4}}{2} - \frac{2e^{2a}x^4+1}{e^{2a}x^5+x}$$

```
[In] int(tanh(a + 2*log(x))^2/x^2,x)
```

```
[Out] (atanh(x*(-exp(2*a))^(1/4))*(-exp(2*a))^(1/4))/2 - (atan(x*(-exp(2*a))^(1/4
))*(-exp(2*a))^(1/4))/2 - (2*x^4*exp(2*a) + 1)/(x + x^5*exp(2*a))
```

3.159 $\int \frac{\tanh^2(a+2\log(x))}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\tanh^2(a+2\log(x))}{x^3} \, dx = -\frac{1}{2x^2(1+e^{2a}x^4)} - \frac{3e^{2a}x^2}{2(1+e^{2a}x^4)} - e^a \arctan\left(e^a x^2\right)$$

```
[Out] -1/2/x^2/(1+exp(2*a)*x^4)-3/2*exp(2*a)*x^2/(1+exp(2*a)*x^4)-exp(a)*arctan(e
xp(a)*x^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5656, 473, 468, 281, 209}

$$\int \frac{\tanh^2(a+2\log(x))}{x^3} \, dx = -e^a \arctan\left(e^a x^2\right) - \frac{3e^{2a}x^2}{2(e^{2a}x^4+1)} - \frac{1}{2x^2(e^{2a}x^4+1)}$$

```
[In] Int[Tanh[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] -1/2*1/(x^2*(1 + E^(2*a)*x^4)) - (3*E^(2*a)*x^2)/(2*(1 + E^(2*a)*x^4)) - E^
a*ArcTan[E^a*x^2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\left(-1+e^{2a}x^4\right)^2}{x^3\left(1+e^{2a}x^4\right)^2}\,dx\\
&= -\frac{1}{2x^2\left(1+e^{2a}x^4\right)}+\frac{1}{2}\int \frac{x\left(-10e^{2a}+2e^{4a}x^4\right)}{\left(1+e^{2a}x^4\right)^2}\,dx\\
&= -\frac{1}{2x^2\left(1+e^{2a}x^4\right)}-\frac{3e^{2a}x^2}{2\left(1+e^{2a}x^4\right)}-\left(2e^{2a}\right)\int \frac{x}{1+e^{2a}x^4}\,dx\\
&= -\frac{1}{2x^2\left(1+e^{2a}x^4\right)}-\frac{3e^{2a}x^2}{2\left(1+e^{2a}x^4\right)}-e^{2a}\text{Subst}\left(\int \frac{1}{1+e^{2a}x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2\left(1+e^{2a}x^4\right)}-\frac{3e^{2a}x^2}{2\left(1+e^{2a}x^4\right)}-e^{a}\arctan\left(e^{a}x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx = \frac{-1-\frac{2}{1+e^{-2(a+2\log(x))}}}{2x^2}+e^a\arctan\left(\frac{e^{-a}}{x^2}\right)$$

```
[In] Integrate[Tanh[a + 2*Log[x]]^2/x^3,x]
[Out] (-1 - 2/(1 + E^(-2*(a + 2*Log[x]))))/(2*x^2) + E^a*ArcTan[1/(E^a*x^2)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{-\frac{3\,\mathrm{e}^{2a}x^4}{2}-\frac{1}{2}}{x^2\left(1+\mathrm{e}^{2a}x^4\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\mathrm{e}^{2a}+\_Z^2\right)}\_R\ln\left(\left(-4\,\mathrm{e}^{2a}-5\_R^2\right)x^2-\_R\right)\right)}{2}$	66

```
[In] int(tanh(a+2*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
[Out] (-3/2*exp(2*a)*x^4-1/2)/x^2/(1+exp(2*a)*x^4)+1/2*sum(_R*ln((-4*exp(2*a)-5*_
R^2)*x^2-_R),_R=RootOf(exp(2*a)+_Z^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx = -\frac{3\,x^4e^{(2\,a)}+2\left(x^6e^{(3\,a)}+x^2e^a\right)\arctan\left(x^2e^a\right)+1}{2\left(x^6e^{(2\,a)}+x^2\right)}$$

```
[In] integrate(tanh(a+2*log(x))^2/x^3,x, algorithm="fricas")
[Out] -1/2*(3*x^4*e^(2*a) + 2*(x^6*e^(3*a) + x^2*e^a)*arctan(x^2*e^a) + 1)/(x^6*e
^(2*a) + x^2)
```

Sympy [F]

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx = \int \frac{\tanh^2\left(a+2\log\left(x\right)\right)}{x^3}\,dx$$

```
[In] integrate(tanh(a+2*ln(x))**2/x**3,x)
[Out] Integral(tanh(a + 2*log(x))**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.63

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx = \arctan\left(\frac{e^{(-a)}}{x^2}\right)e^a - \frac{1}{2\,x^2} - \frac{e^{(2\,a)}}{x^2\left(\frac{1}{x^4}+e^{(2\,a)}\right)}$$

```
[In] integrate(tanh(a+2*log(x))^2/x^3,x, algorithm="maxima")
[Out] arctan(e^(-a)/x^2)*e^a - 1/2/x^2 - e^(2*a)/(x^2*(1/x^4 + e^(2*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx = -\arctan\left(x^2e^a\right)e^a - \frac{3\,x^4e^{(2\,a)}+1}{2\left(x^6e^{(2\,a)}+x^2\right)}$$

```
[In] integrate(tanh(a+2*log(x))^2/x^3,x, algorithm="giac")
[Out] -arctan(x^2*e^a)*e^a - 1/2*(3*x^4*e^(2*a) + 1)/(x^6*e^(2*a) + x^2)
```

Mupad [B] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{\tanh^2(a+2\log(x))}{x^3}\,dx = -\mathrm{atan}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)\,\sqrt{\mathrm{e}^{2\,a}} - \frac{\frac{3\,\mathrm{e}^{2\,a}\,x^4}{2}+\frac{1}{2}}{\mathrm{e}^{2\,a}\,x^6+x^2}$$

```
[In] int(tanh(a + 2*log(x))^2/x^3,x)
[Out] - atan(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2) - ((3*x^4*exp(2*a))/2 + 1/2)/(x^6
*exp(2*a) + x^2)
```

3.160 $\int (ex)^m \tanh(a+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int (ex)^m \tanh(a+2\log(x))\,dx = \frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-e^{2a}x^4\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-exp
(2*a)*x^4)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5656, 470, 371}

$$\int (ex)^m \tanh(a+2\log(x))\,dx = \frac{(ex)^{m+1}}{e(m+1)} - \frac{2(ex)^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-e^{2a}x^4\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*Tanh[a + 2*Log[x]],x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4
, (5 + m)/4, -(E^(2*a)*x^4)])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{(ex)^m\left(-1+e^{2a}x^4\right)}{1+e^{2a}x^4}\,dx\\ &= \frac{(ex)^{1+m}}{e(1+m)}-2\int \frac{(ex)^m}{1+e^{2a}x^4}\,dx\\ &= \frac{(ex)^{1+m}}{e(1+m)}-\frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-e^{2a}x^4\right)}{e(1+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int (ex)^m \tanh(a+2\log(x))\,dx = -\frac{x(ex)^m\left(-1+2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-x^4(\cosh(2a)+\sinh(2a))\right)\right)}{1+m}$$

```
[In] Integrate[(e*x)^m*Tanh[a + 2*Log[x]],x]
[Out] -((x*(e*x)^m*(-1 + 2*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(x^4*(Cosh
[2*a] + Sinh[2*a]))]))/(1 + m))
```

Maple [F]

$$\int (ex)^m \tanh (a + 2\ln (x))\, dx$$

```
[In] int((e*x)^m*tanh(a+2*ln(x)),x)
[Out] int((e*x)^m*tanh(a+2*ln(x)),x)
```

Fricas [F]

$$\int (ex)^m \tanh(a + 2\log(x))\, dx = \int (ex)^m \tanh (a + 2\ \log (x))\ dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x)),x, algorithm="fricas")
[Out] integral((e*x)^m*tanh(a + 2*log(x)), x)
```

Sympy [F]

$$\int (ex)^m \tanh(a + 2\log(x))\, dx = \int (ex)^m \tanh (a + 2\log (x))\, dx$$

```
[In] integrate((e*x)**m*tanh(a+2*ln(x)),x)
[Out] Integral((e*x)**m*tanh(a + 2*log(x)), x)
```

Maxima [F]

$$\int (ex)^m \tanh(a + 2\log(x))\, dx = \int (ex)^m \tanh (a + 2\ \log (x))\ dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x)),x, algorithm="maxima")
[Out] integrate((e*x)^m*tanh(a + 2*log(x)), x)
```

Giac [F]

$$\int (ex)^m \tanh(a+2\log(x))\,dx = \int (ex)^m \tanh\left(a+2\,\log\left(x\right)\right)\,dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x)),x, algorithm="giac")
[Out] integrate((e*x)^m*tanh(a + 2*log(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh(a+2\log(x))\,dx = \int \tanh(a+2\,\ln\left(x\right))\,\left(e\,x\right)^m\,dx$$

```
[In] int(tanh(a + 2*log(x))*(e*x)^m,x)
[Out] int(tanh(a + 2*log(x))*(e*x)^m, x)
```

3.161 $\int (ex)^m \tanh^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = \frac{(ex)^{1+m}}{e(1+m)} + \frac{(ex)^{1+m}}{e\,(1+e^{2a}x^4)} - \frac{(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-e^{2a}x^4\right)}{e}$$

```
[Out] (e*x)^(1+m)/e/(1+m)+(e*x)^(1+m)/e/(1+exp(2*a)*x^4)-(e*x)^(1+m)*hypergeom([1
, 1/4+1/4*m],[5/4+1/4*m],-exp(2*a)*x^4)/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {5656, 474, 470, 371}

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = -\frac{(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-e^{2a}x^4\right)}{e} + \frac{(ex)^{m+1}}{e\,(e^{2a}x^4+1)} + \frac{(ex)^{m+1}}{e(m+1)}$$

```
[In] Int[(e*x)^m*Tanh[a + 2*Log[x]]^2,x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) + (e*x)^(1 + m)/(e*(1 + E^(2*a)*x^4)) - ((e*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(E^(2*a)*x^4)])/e
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(ex)^m \left(-1+e^{2a}x^4\right)^2}{\left(1+e^{2a}x^4\right)^2}\,dx \\
&= \frac{(ex)^{1+m}}{e\left(1+e^{2a}x^4\right)} - \frac{1}{4}e^{-4a}\int \frac{(ex)^m \left(4e^{4a}m-4e^{6a}x^4\right)}{1+e^{2a}x^4}\,dx \\
&= \frac{(ex)^{1+m}}{e(1+m)} + \frac{(ex)^{1+m}}{e\left(1+e^{2a}x^4\right)} - (1+m)\int \frac{(ex)^m}{1+e^{2a}x^4}\,dx \\
&= \frac{(ex)^{1+m}}{e(1+m)} + \frac{(ex)^{1+m}}{e\left(1+e^{2a}x^4\right)} - \frac{(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-e^{2a}x^4\right)}{e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = -\frac{x(ex)^m \left(-1+4\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-x^4(\cosh(2a)+\sinh(2a))\right)-4\,\text{Hypergeometric2F1}\left(\right.\right.}{1+m}$$

```
[In] Integrate[(e*x)^m*Tanh[a + 2*Log[x]]^2,x]
[Out] -((x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(x^4*(Cosh
[2*a] + Sinh[2*a]))] - 4*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -(x^4*(
Cosh[2*a] + Sinh[2*a]))]))/(1 + m))
```

Maple [F]

$$\int (ex)^m \tanh\left(a+2\ln(x)\right)^2 dx$$

```
[In] int((e*x)^m*tanh(a+2*ln(x))^2,x)
[Out] int((e*x)^m*tanh(a+2*ln(x))^2,x)
```

Fricas [F]

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = \int (ex)^m \tanh\left(a+2\,\log\left(x\right)\right)^2\,dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^2,x, algorithm="fricas")
[Out] integral((e*x)^m*tanh(a + 2*log(x))^2, x)
```

Sympy [F]

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = \int (ex)^m \tanh^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate((e*x)**m*tanh(a+2*ln(x))**2,x)
[Out] Integral((e*x)**m*tanh(a + 2*log(x))**2, x)
```

Maxima [F]

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = \int (ex)^m \tanh\left(a+2\,\log\left(x\right)\right)^2\,dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^2,x, algorithm="maxima")
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^2, x)
```

Giac [F]

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = \int (ex)^m \tanh\left(a+2\,\log\left(x\right)\right)^2\,dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh^2(a+2\log(x))\,dx = \int \tanh(a+2\,\ln\left(x\right))^2\,(e\,x)^m\,dx$$

```
[In] int(tanh(a + 2*log(x))^2*(e*x)^m,x)
[Out] int(tanh(a + 2*log(x))^2*(e*x)^m, x)
```

3.162 $\int (ex)^m \tanh^3(a+2\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 176

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx = \frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)} - \frac{(ex)^{1+m}\left(1-e^{2a}x^4\right)^2}{4e\left(1+e^{2a}x^4\right)^2} - \frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)+e^{4a}(5+m)x^4\right)}{8e\left(1+e^{2a}x^4\right)} - \frac{\left(9+2m+m^2\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-e^{2a}x^4\right)}{4e(1+m)}$$

```
[Out] 1/8*(3+m)*(5+m)*(e*x)^(1+m)/e/(1+m)-1/4*(e*x)^(1+m)*(1-exp(2*a)*x^4)^2/e/(1
+exp(2*a)*x^4)^2-1/8*(e*x)^(1+m)*(exp(2*a)*(3-m)+exp(4*a)*(5+m)*x^4)/e/exp(
2*a)/(1+exp(2*a)*x^4)-1/4*(m^2+2*m+9)*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],
[5/4+1/4*m],-exp(2*a)*x^4)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{5656, 479, 591, 470, 371\}$

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx$$
$$= -\frac{(m^2+2m+9)\,(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},-e^{2a}x^4\right)}{4e(m+1)} - \frac{e^{-2a}(e^{4a}(m+5)x^4+e^{2a}(3-m))\,(ex)^{m+1}}{8e\,(e^{2a}x^4+1)} - \frac{(1-e^{2a}x^4)^2\,(ex)^{m+1}}{4e\,(e^{2a}x^4+1)^2} + \frac{(m+3)(m+5)(ex)^{m+1}}{8e(m+1)}$$

```
[In] Int[(e*x)^m*Tanh[a + 2*Log[x]]^3,x]
```

```
[Out] ((3 + m)*(5 + m)*(e*x)^(1 + m))/(8*e*(1 + m)) - ((e*x)^(1 + m)*(1 - E^(2*a)
*x^4)^2)/(4*e*(1 + E^(2*a)*x^4)^2) - ((e*x)^(1 + m)*(E^(2*a)*(3 - m) + E^(4
*a)*(5 + m)*x^4))/(8*e*E^(2*a)*(1 + E^(2*a)*x^4)) - ((9 + 2*m + m^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(E^(2*a)*x^4)])/(4*e*(1
 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(ex)^m \left(-1+e^{2a}x^4\right)^3}{\left(1+e^{2a}x^4\right)^3}\,dx \\
&= -\frac{(ex)^{1+m}\left(1-e^{2a}x^4\right)^2}{4e\left(1+e^{2a}x^4\right)^2} - \frac{1}{8}e^{-2a}\int \frac{(ex)^m\left(-1+e^{2a}x^4\right)\left(-2e^{2a}(3-m)-2e^{4a}(5+m)x^4\right)}{\left(1+e^{2a}x^4\right)^2}\,dx \\
&= -\frac{(ex)^{1+m}\left(1-e^{2a}x^4\right)^2}{4e\left(1+e^{2a}x^4\right)^2} - \frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)+e^{4a}(5+m)x^4\right)}{8e\left(1+e^{2a}x^4\right)} \\
&\quad + \frac{1}{32}e^{-4a}\int \frac{(ex)^m\left(-4e^{4a}(1-m)(3-m)+4e^{6a}(3+m)(5+m)x^4\right)}{1+e^{2a}x^4}\,dx \\
&= \frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)} - \frac{(ex)^{1+m}\left(1-e^{2a}x^4\right)^2}{4e\left(1+e^{2a}x^4\right)^2} \\
&\quad - \frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)+e^{4a}(5+m)x^4\right)}{8e\left(1+e^{2a}x^4\right)} + \frac{1}{4}\left(-9-2m-m^2\right)\int \frac{(ex)^m}{1+e^{2a}x^4}\,dx \\
&= \frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)} - \frac{(ex)^{1+m}\left(1-e^{2a}x^4\right)^2}{4e\left(1+e^{2a}x^4\right)^2} \\
&\quad - \frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)+e^{4a}(5+m)x^4\right)}{8e\left(1+e^{2a}x^4\right)} \\
&\quad - \frac{\left(9+2m+m^2\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-e^{2a}x^4\right)}{4e(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.63

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx = -\frac{x(ex)^m\left(-1+6\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},-x^4(\cosh(2a)+\sinh(2a))\right)-12\,\text{Hypergeometric2F}$$

```
[In] Integrate[(e*x)^m*Tanh[a + 2*Log[x]]^3,x]
[Out] -((x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, -(x^4*(Cosh
[2*a] + Sinh[2*a]))] - 12*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -(x^4*
(Cosh[2*a] + Sinh[2*a]))] + 8*Hypergeometric2F1[3, (1 + m)/4, (5 + m)/4, -(
x^4*(Cosh[2*a] + Sinh[2*a]))]))/(1 + m))
```

Maple [F]

$$\int (ex)^m \tanh(a+2\ln(x))^3\,dx$$

```
[In] int((e*x)^m*tanh(a+2*ln(x))^3,x)
[Out] int((e*x)^m*tanh(a+2*ln(x))^3,x)
```

Fricas [F]

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx = \int (ex)^m \tanh(a+2\log(x))^3\,dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^3,x, algorithm="fricas")
[Out] integral((e*x)^m*tanh(a + 2*log(x))^3, x)
```

Sympy [F]

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx = \int (ex)^m \tanh^3(a+2\log(x))\,dx$$

```
[In] integrate((e*x)**m*tanh(a+2*ln(x))**3,x)
[Out] Integral((e*x)**m*tanh(a + 2*log(x))**3, x)
```

Maxima [F]

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx = \int (ex)^m \tanh\left(a+2\,\log\left(x\right)\right)^3\,dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^3,x, algorithm="maxima")
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^3, x)
```

Giac [F]

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx = \int (ex)^m \tanh\left(a+2\,\log\left(x\right)\right)^3\,dx$$

```
[In] integrate((e*x)^m*tanh(a+2*log(x))^3,x, algorithm="giac")
[Out] integrate((e*x)^m*tanh(a + 2*log(x))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh^3(a+2\log(x))\,dx = \int \tanh(a+2\,\ln\left(x\right))^3\,(e\,x)^m\,dx$$

```
[In] int(tanh(a + 2*log(x))^3*(e*x)^m,x)
[Out] int(tanh(a + 2*log(x))^3*(e*x)^m, x)
```

3.163 $\int \tanh^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 79

$$\int \tanh^p(a+b\log(x))\,dx = x\left(1-e^{2a}x^{2b}\right)^{-p}\left(-1+e^{2a}x^{2b}\right)^p \operatorname{AppellF1}\left(\frac{1}{2b},-p,p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[Out] x*(-1+exp(2*a)*x^(2*b))^p*AppellF1(1/2/b,-p,p,1+1/2/b,exp(2*a)*x^(2*b),-exp
(2*a)*x^(2*b))/((1-exp(2*a)*x^(2*b))^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5652, 441, 440}

$$\int \tanh^p(a+b\log(x))\,dx = x\left(1-e^{2a}x^{2b}\right)^{-p}\left(e^{2a}x^{2b}-1\right)^p \operatorname{AppellF1}\left(\frac{1}{2b},-p,p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[In] Int[Tanh[a + b*Log[x]]^p,x]
```

```
[Out] (x*(-1 + E^(2*a)*x^(2*b))^p*AppellF1[1/(2*b), -p, p, (2 + b^(-1))/2, E^(2*a
)*x^(2*b), -(E^(2*a)*x^(2*b))])/(1 - E^(2*a)*x^(2*b))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-1+e^{2a}x^{2b}\right)^p \left(1+e^{2a}x^{2b}\right)^{-p}\,dx\\ &= \left(\left(1-e^{2a}x^{2b}\right)^{-p}\left(-1+e^{2a}x^{2b}\right)^p\right)\int \left(1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\,dx\\ &= x\left(1-e^{2a}x^{2b}\right)^{-p}\left(-1+e^{2a}x^{2b}\right)^p \operatorname{AppellF1}\left(\frac{1}{2b},-p,p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. $2(79) = 158$.

Time = 0.48 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.28

$$\int \tanh^p(a+b\log(x))\,dx$$

$$= \frac{(1+2b)x\left(\frac{-1+e^{2a}x^{2b}}{1+e^{2a}x^{2b}}\right)^p \operatorname{AppellF1}\left(\frac{1}{2b},-p,p,1+\frac{1}{2b}\right.}{-2be^{2a}px^{2b}\operatorname{AppellF1}\left(1+\frac{1}{2b},1-p,p,2+\frac{1}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)-2be^{2a}px^{2b}\operatorname{AppellF1}\left(1+\frac{1}{2b},-p,1+p,2+\right.}$$

```
[In] Integrate[Tanh[a + b*Log[x]]^p,x]
```

```
[Out] ((1 + 2*b)*x*((-1 + E^(2*a)*x^(2*b))/(1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(2
*b), -p, p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(-2*b*E^(2*a
)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 - p, p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -
(E^(2*a)*x^(2*b))] - 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), -p, 1 + p,
 2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1[1/(
2*b), -p, p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

Maple [F]

$$\int \tanh(a + b\ln(x))^p \, dx$$

```
[In] int(tanh(a+b*ln(x))^p,x)
[Out] int(tanh(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^p(a + b\log(x)) \, dx = \int \tanh(b\log(x) + a)^p \, dx$$

```
[In] integrate(tanh(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int \tanh^p(a + b\log(x)) \, dx = \int \tanh^p(a + b\log(x)) \, dx$$

```
[In] integrate(tanh(a+b*ln(x))**p,x)
[Out] Integral(tanh(a + b*log(x))**p, x)
```

Maxima [F]

$$\int \tanh^p(a + b\log(x)) \, dx = \int \tanh(b\log(x) + a)^p \, dx$$

```
[In] integrate(tanh(a+b*log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(b*log(x) + a)^p, x)
```

Giac [F]

$$\int \tanh^p(a + b\log(x))\, dx = \int \tanh\left(b\log\left(x\right) + a\right)^p\, dx$$

```
[In] integrate(tanh(a+b*log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p(a + b\log(x))\, dx = \int \tanh(a + b\,\ln\left(x\right))^p\, dx$$

```
[In] int(tanh(a + b*log(x))^p,x)
[Out] int(tanh(a + b*log(x))^p, x)
```

3.164 $\int (ex)^m \tanh^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int (ex)^m \tanh^p(a+b\log(x))\,dx = \frac{(ex)^{1+m}\left(1-e^{2a}x^{2b}\right)^{-p}\left(-1+e^{2a}x^{2b}\right)^p \operatorname{AppellF1}\left(\frac{1+m}{2b},-p,p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(-1+exp(2*a)*x^(2*b))^p*AppellF1(1/2*(1+m)/b,-p,p,1+1/2*(1+m)/b
,exp(2*a)*x^(2*b),-exp(2*a)*x^(2*b))/e/(1+m)/((1-exp(2*a)*x^(2*b))^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5656, 525, 524}

$$\int (ex)^m \tanh^p(a+b\log(x))\,dx = \frac{(ex)^{m+1}\left(1-e^{2a}x^{2b}\right)^{-p}\left(e^{2a}x^{2b}-1\right)^p \operatorname{AppellF1}\left(\frac{m+1}{2b},-p,p,\frac{m+1}{2b}+1,e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*Tanh[a + b*Log[x]]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(-1 + E^(2*a)*x^(2*b))^p*AppellF1[(1 + m)/(2*b), -p, p, 1 +
(1 + m)/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(e*(1 + m)*(1 - E^(2*a
)*x^(2*b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (ex)^m \left(-1+e^{2a}x^{2b}\right)^p \left(1+e^{2a}x^{2b}\right)^{-p} \, dx\\ &= \left(\left(1-e^{2a}x^{2b}\right)^{-p}\left(-1+e^{2a}x^{2b}\right)^p\right)\int (ex)^m \left(1-e^{2a}x^{2b}\right)^p \left(1+e^{2a}x^{2b}\right)^{-p} \, dx\\ &= \frac{(ex)^{1+m}\left(1-e^{2a}x^{2b}\right)^{-p}\left(-1+e^{2a}x^{2b}\right)^p \text{AppellF1}\left(\frac{1+m}{2b},-p,p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(1+m)}\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\int (ex)^m \tanh^p(a+b\log(x))\, dx = \frac{x(ex)^m \left(1-e^{2a}x^{2b}\right)^{-p}\left(\frac{-1+e^{2a}x^{2b}}{1+e^{2a}x^{2b}}\right)^p \left(1+e^{2a}x^{2b}\right)^p \text{AppellF1}\left(\frac{1+m}{2b},-p,p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{1+m}$$

```
[In] Integrate[(e*x)^m*Tanh[a + b*Log[x]]^p,x]
```

```
[Out] (x*(e*x)^m*((-1 + E^(2*a)*x^(2*b))/(1 + E^(2*a)*x^(2*b)))^p*(1 + E^(2*a)*x^
(2*b))^p*AppellF1[(1 + m)/(2*b), -p, p, 1 + (1 + m)/(2*b), E^(2*a)*x^(2*b),
 -(E^(2*a)*x^(2*b))])/((1 + m)*(1 - E^(2*a)*x^(2*b))^p)
```

Maple [F]

$$\int (ex)^m \tanh\left(a + b\ln\left(x\right)\right)^p dx$$

```
[In] int((e*x)^m*tanh(a+b*ln(x))^p,x)
[Out] int((e*x)^m*tanh(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int (ex)^m \tanh^p(a + b\log(x))\, dx = \int (ex)^m \tanh\left(b\log\left(x\right) + a\right)^p\, dx$$

```
[In] integrate((e*x)^m*tanh(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*tanh(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int (ex)^m \tanh^p(a + b\log(x))\, dx = \int (ex)^m \tanh^p\left(a + b\log\left(x\right)\right) dx$$

```
[In] integrate((e*x)**m*tanh(a+b*ln(x))**p,x)
[Out] Integral((e*x)**m*tanh(a + b*log(x))**p, x)
```

Maxima [F]

$$\int (ex)^m \tanh^p(a + b\log(x))\, dx = \int (ex)^m \tanh\left(b\log\left(x\right) + a\right)^p\, dx$$

```
[In] integrate((e*x)^m*tanh(a+b*log(x))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*tanh(b*log(x) + a)^p, x)
```

Giac [F]

$$\int (ex)^m \tanh^p(a+b\log(x))\,dx = \int (ex)^m \tanh\left(b\log\left(x\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^m*tanh(a+b*log(x))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*tanh(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh^p(a+b\log(x))\,dx = \int \tanh(a+b\,\ln\left(x\right))^p\,(e\,x)^m\,dx$$

```
[In] int(tanh(a + b*log(x))^p*(e*x)^m,x)
[Out] int(tanh(a + b*log(x))^p*(e*x)^m, x)
```

3.165 $\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx = \frac{2^{-p}e^{-2a}(-1+e^{2a}x)^{1+p}\,\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}(1-e^{2a}x)\right)}{1+p}$$

```
[Out] (-1+exp(2*a)*x)^(p+1)*hypergeom([p, p+1],[2+p],1/2-1/2*exp(2*a)*x)/(2^p)/ex
p(2*a)/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5652, 71}

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx = \frac{e^{-2a}2^{-p}(e^{2a}x-1)^{p+1}\,\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}(1-e^{2a}x)\right)}{p+1}$$

```
[In] Int[Tanh[a + Log[x]/2]^p,x]
```

```
[Out] ((-1 + E^(2*a)*x)^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 - E^(2*a)*x
)/2])/(2^p*E^(2*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-1+e^{2a}x\right)^p \left(1+e^{2a}x\right)^{-p} \, dx \\ &= \frac{2^{-p}e^{-2a}(-1+e^{2a}x)^{1+p} \, \text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}(1-e^{2a}x)\right)}{1+p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right) \, dx = \frac{2^{-p}e^{-2a}\left(\frac{-1+e^{2a}x}{1+e^{2a}x}\right)^{1+p} \left(1+e^{2a}x\right)^{1+p} \text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}-\frac{1}{2}e^{2a}x\right)}{1+p}$$

```
[In] Integrate[Tanh[a + Log[x]/2]^p,x]
[Out] (((-1 + E^(2*a)*x)/(1 + E^(2*a)*x))^(1 + p)*(1 + E^(2*a)*x)^(1 + p)*Hyperge
ometric2F1[p, 1 + p, 2 + p, 1/2 - (E^(2*a)*x)/2])/(2^p*E^(2*a)*(1 + p))
```

Maple [F]

$$\int \tanh\left(a+\frac{\ln(x)}{2}\right)^p dx$$

```
[In] int(tanh(a+1/2*ln(x))^p,x)
[Out] int(tanh(a+1/2*ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \tanh\left(a+\frac{1}{2}\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+1/2*log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(a + 1/2*log(x))^p, x)
```

Sympy [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \tanh^p\left(a+\frac{\log\left(x\right)}{2}\right)\,dx$$

```
[In] integrate(tanh(a+1/2*ln(x))**p,x)
[Out] Integral(tanh(a + log(x)/2)**p, x)
```

Maxima [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \tanh\left(a+\frac{1}{2}\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+1/2*log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(a + 1/2*log(x))^p, x)
```

Giac [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \tanh\left(a+\frac{1}{2}\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+1/2*log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(a + 1/2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \tanh\left(a+\frac{\ln(x)}{2}\right)^p\,dx$$

```
[In] int(tanh(a + log(x)/2)^p,x)
[Out] int(tanh(a + log(x)/2)^p, x)
```

3.166 $\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 106

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx = e^{-4a}\left(-1+e^{2a}\sqrt{x}\right)^{1+p}\left(1+e^{2a}\sqrt{x}\right)^{1-p} - \frac{2^{1-p}e^{-4a}p\left(-1+e^{2a}\sqrt{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1-e^{2a}\sqrt{x}\right)\right)}{1+p}$$

```
[Out] -2^(1-p)*p*hypergeom([p, p+1],[2+p],1/2-1/2*exp(2*a)*x^(1/2))*(-1+exp(2*a)*
x^(1/2))^(p+1)/exp(4*a)/(p+1)+(-1+exp(2*a)*x^(1/2))^(p+1)*(1+exp(2*a)*x^(1/
2))^(1-p)/exp(4*a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5652, 383, 81, 71}

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx = e^{-4a}\left(e^{2a}\sqrt{x}-1\right)^{p+1}\left(e^{2a}\sqrt{x}+1\right)^{1-p} - \frac{e^{-4a}2^{1-p}p\left(e^{2a}\sqrt{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\left(1-e^{2a}\sqrt{x}\right)\right)}{p+1}$$

```
[In] Int[Tanh[a + Log[x]/4]^p,x]
```

```
[Out] ((-1 + E^(2*a)*Sqrt[x])^(1 + p)*(1 + E^(2*a)*Sqrt[x])^(1 - p))/E^(4*a) - (2
^(1 - p)*p*(-1 + E^(2*a)*Sqrt[x])^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p
, (1 - E^(2*a)*Sqrt[x])/2])/(E^(4*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-1+e^{2a}\sqrt{x}\right)^p \left(1+e^{2a}\sqrt{x}\right)^{-p}\, dx\\
&= 2\text{Subst}\left(\int x\left(-1+e^{2a}x\right)^p \left(1+e^{2a}x\right)^{-p}\, dx, x, \sqrt{x}\right)\\
&= e^{-4a}\left(-1+e^{2a}\sqrt{x}\right)^{1+p}\left(1+e^{2a}\sqrt{x}\right)^{1-p}\\
&\quad -\left(2e^{-2a}p\right)\text{Subst}\left(\int \left(-1+e^{2a}x\right)^p \left(1+e^{2a}x\right)^{-p}\, dx, x, \sqrt{x}\right)\\
&= e^{-4a}\left(-1+e^{2a}\sqrt{x}\right)^{1+p}\left(1+e^{2a}\sqrt{x}\right)^{1-p}\\
&\quad -\frac{2^{1-p}e^{-4a}p\left(-1+e^{2a}\sqrt{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1-e^{2a}\sqrt{x}\right)\right)}{1+p}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx = \frac{e^{-4a}\left(-1+e^{2a}\sqrt{x}\right)\left(\frac{-1+e^{2a}\sqrt{x}}{2+2e^{2a}\sqrt{x}}\right)^p\left(2^p(1+p)\left(1+e^{2a}\sqrt{x}\right)-2p\left(1+e^{2a}\sqrt{x}\right)^p \text{Hypergeometric2F1}\left(p,1+p,\right.\right.}{1+p}$$

```
[In] Integrate[Tanh[a + Log[x]/4]^p,x]
[Out] ((-1 + E^(2*a)*Sqrt[x])*((-1 + E^(2*a)*Sqrt[x])/(2 + 2*E^(2*a)*Sqrt[x]))^p*
(2^p*(1 + p)*(1 + E^(2*a)*Sqrt[x]) - 2*p*(1 + E^(2*a)*Sqrt[x])^p*Hypergeome
tric2F1[p, 1 + p, 2 + p, 1/2 - (E^(2*a)*Sqrt[x])/2]))/(E^(4*a)*(1 + p))
```

Maple [F]

$$\int \tanh\left(a+\frac{\ln(x)}{4}\right)^p\,dx$$

```
[In] int(tanh(a+1/4*ln(x))^p,x)
[Out] int(tanh(a+1/4*ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx = \int \tanh\left(a+\frac{1}{4}\log(x)\right)^p\,dx$$

```
[In] integrate(tanh(a+1/4*log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(a + 1/4*log(x))^p, x)
```

Sympy [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx = \int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx$$

```
[In] integrate(tanh(a+1/4*ln(x))**p,x)
[Out] Integral(tanh(a + log(x)/4)**p, x)
```

Maxima [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \tanh\left(a+\frac{1}{4}\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+1/4*log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(a + 1/4*log(x))^p, x)
```

Giac [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \tanh\left(a+\frac{1}{4}\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+1/4*log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(a + 1/4*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \tanh\left(a+\frac{\ln\left(x\right)}{4}\right)^p\,dx$$

```
[In] int(tanh(a + log(x)/4)^p,x)
[Out] int(tanh(a + log(x)/4)^p, x)
```

3.167 $\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 158

$$\begin{aligned}\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx &= -e^{-6a}p\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x}\\ &\quad+\frac{2^{-p}e^{-6a}(1+2p^2)\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1-e^{2a}\sqrt[3]{x}\right)\right)}{1+p}\end{aligned}$$

```
[Out] -p*(-1+exp(2*a)*x^(1/3))^(p+1)*(1+exp(2*a)*x^(1/3))^(1-p)/exp(6*a)+(-1+exp(
2*a)*x^(1/3))^(p+1)*(1+exp(2*a)*x^(1/3))^(1-p)*x^(1/3)/exp(4*a)+(2*p^2+1)*(
-1+exp(2*a)*x^(1/3))^(p+1)*hypergeom([p, p+1],[2+p],1/2-1/2*exp(2*a)*x^(1/3
))/(2^p)/exp(6*a)/(p+1)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5652, 383, 92, 81, 71}

$$\begin{aligned}\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx &= \frac{e^{-6a}2^{-p}(2p^2+1)\left(e^{2a}\sqrt[3]{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\left(1-e^{2a}\sqrt[3]{x}\right)\right)}{p+1}\\ &\quad-e^{-6a}p\left(e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(e^{2a}\sqrt[3]{x}+1\right)^{1-p}+e^{-4a}\sqrt[3]{x}\left(e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(e^{2a}\sqrt[3]{x}+1\right)^{1-p}\end{aligned}$$

```
[In] Int[Tanh[a + Log[x]/6]^p,x]
```

```
[Out] -((p*(-1 + E^(2*a)*x^(1/3))^(1 + p)*(1 + E^(2*a)*x^(1/3))^(1 - p))/E^(6*a))
 + ((-1 + E^(2*a)*x^(1/3))^(1 + p)*(1 + E^(2*a)*x^(1/3))^(1 - p)*x^(1/3))/E
^(4*a) + ((1 + 2*p^2)*(-1 + E^(2*a)*x^(1/3))^(1 + p)*Hypergeometric2F1[p, 1
 + p, 2 + p, (1 - E^(2*a)*x^(1/3))/2])/(2^p*E^(6*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\text{integral} = \int \left(-1+e^{2a}\sqrt[3]{x}\right)^p \left(1+e^{2a}\sqrt[3]{x}\right)^{-p}\,dx$$

$$
\begin{aligned}
&= 3\text{Subst}\left(\int x^2\left(-1+e^{2a}x\right)^p\left(1+e^{2a}x\right)^{-p}\,dx, x, \sqrt[3]{x}\right)\\
&= e^{-4a}\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x}\\
&\quad + e^{-4a}\text{Subst}\left(\int\left(-1+e^{2a}x\right)^p\left(1+e^{2a}x\right)^{-p}\left(1-2e^{2a}px\right)\,dx, x, \sqrt[3]{x}\right)\\
&= -e^{-6a}p\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x}\\
&\quad + \left(e^{-4a}\left(1+2p^2\right)\right)\text{Subst}\left(\int\left(-1+e^{2a}x\right)^p\left(1+e^{2a}x\right)^{-p}\,dx, x, \sqrt[3]{x}\right)\\
&= -e^{-6a}p\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x}\\
&\quad + \frac{2^{-p}e^{-6a}\left(1+2p^2\right)\left(-1+e^{2a}\sqrt[3]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1-e^{2a}\sqrt[3]{x}\right)\right)}{1+p}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.89

$$
\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx
$$

$$
= \frac{e^{-6a}\left(-1+e^{2a}\sqrt[3]{x}\right)\left(\frac{-1+e^{2a}\sqrt[3]{x}}{2+2e^{2a}\sqrt[3]{x}}\right)^p\left(2^p(1+p)\left(1+e^{2a}\sqrt[3]{x}\right)\left(-p+e^{2a}\sqrt[3]{x}\right)+\left(1+2p^2\right)\left(1+e^{2a}\sqrt[3]{x}\right)^p\text{Hyper}\right.}{1+p}
$$

```
[In] Integrate[Tanh[a + Log[x]/6]^p,x]
```

```
[Out] ((-1 + E^(2*a)*x^(1/3))*((-1 + E^(2*a)*x^(1/3))/(2 + 2*E^(2*a)*x^(1/3)))^p*
(2^p*(1 + p)*(1 + E^(2*a)*x^(1/3))*(-p + E^(2*a)*x^(1/3)) + (1 + 2*p^2)*(1
+ E^(2*a)*x^(1/3))^p*Hypergeometric2F1[p, 1 + p, 2 + p, 1/2 - (E^(2*a)*x^(1
/3))/2]))/(E^(6*a)*(1 + p))
```

Maple [F]

$$
\int \tanh\left(a+\frac{\ln(x)}{6}\right)^p\,dx
$$

```
[In] int(tanh(a+1/6*ln(x))^p,x)
```

```
[Out] int(tanh(a+1/6*ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{6}\right) dx = \int \tanh\left(a+\frac{1}{6}\log(x)\right)^p dx$$

```
[In] integrate(tanh(a+1/6*log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(a + 1/6*log(x))^p, x)
```

Sympy [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{6}\right) dx = \int \tanh^p\left(a+\frac{\log(x)}{6}\right) dx$$

```
[In] integrate(tanh(a+1/6*ln(x))**p,x)
[Out] Integral(tanh(a + log(x)/6)**p, x)
```

Maxima [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{6}\right) dx = \int \tanh\left(a+\frac{1}{6}\log(x)\right)^p dx$$

```
[In] integrate(tanh(a+1/6*log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(a + 1/6*log(x))^p, x)
```

Giac [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{6}\right) dx = \int \tanh\left(a+\frac{1}{6}\log(x)\right)^p dx$$

```
[In] integrate(tanh(a+1/6*log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(a + 1/6*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \tanh\left(a+\frac{\ln(x)}{6}\right)^p\,dx$$

```
[In] int(tanh(a + log(x)/6)^p,x)
[Out] int(tanh(a + log(x)/6)^p, x)
```

3.168 $\int \tanh^p\left(a+\frac{\log(x)}{8}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 190

$$\int \tanh^p\left(a+\frac{\log(x)}{8}\right)\,dx=\frac{1}{3}e^{-12a}\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}\left(3+2p^2\right)-2e^{6a}p\sqrt[4]{x}\right)+e^{-4a}\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x}-\frac{2^{2-p}e^{-8a}p(2+p^2)\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1-e^{2a}\sqrt[4]{x}\right)\right)}{3(1+p)}$$

```
[Out] 1/3*(-1+exp(2*a)*x^(1/4))^(p+1)*(1+exp(2*a)*x^(1/4))^(1-p)*(exp(4*a)*(2*p^2
+3)-2*exp(6*a)*p*x^(1/4))/exp(12*a)-1/3*2^(2-p)*p*(p^2+2)*(-1+exp(2*a)*x^(1
/4))^(p+1)*hypergeom([p, p+1],[2+p],1/2-1/2*exp(2*a)*x^(1/4))/exp(8*a)/(p+1
)+(-1+exp(2*a)*x^(1/4))^(p+1)*(1+exp(2*a)*x^(1/4))^(1-p)*x^(1/2)/exp(4*a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5652, 383, 102, 152, 71}

$$\int \tanh^p\left(a+\frac{\log(x)}{8}\right)\,dx$$
$$=-\frac{e^{-8a}2^{2-p}p(p^2+2)\left(e^{2a}\sqrt[4]{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\left(1-e^{2a}\sqrt[4]{x}\right)\right)}{3(p+1)}+\frac{1}{3}e^{-12a}\left(e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{4a}\left(2p^2+3\right)-2e^{6a}p\sqrt[4]{x}\right)\left(e^{2a}\sqrt[4]{x}+1\right)^{1-p}+e^{-4a}\sqrt{x}\left(e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{2a}\sqrt[4]{x}+1\right)^{1-p}$$

```
[In] Int[Tanh[a + Log[x]/8]^p,x]
```

```
[Out] ((-1 + E^(2*a)*x^(1/4))^(1 + p)*(1 + E^(2*a)*x^(1/4))^(1 - p)*(E^(4*a)*(3 +
 2*p^2) - 2*E^(6*a)*p*x^(1/4)))/(3*E^(12*a)) + ((-1 + E^(2*a)*x^(1/4))^(1 +
 p)*(1 + E^(2*a)*x^(1/4))^(1 - p)*Sqrt[x])/E^(4*a) - (2^(2 - p)*p*(2 + p^2)
*(-1 + E^(2*a)*x^(1/4))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 - E^(
2*a)*x^(1/4))/2])/(3*E^(8*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-1+e^{2a}\sqrt[4]{x}\right)^p \left(1+e^{2a}\sqrt[4]{x}\right)^{-p} dx \\
&= 4\text{Subst}\left(\int x^3\left(-1+e^{2a}x\right)^p \left(1+e^{2a}x\right)^{-p} dx, x, \sqrt[4]{x}\right) \\
&= e^{-4a}\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p} \left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x} \\
&\quad + e^{-4a}\text{Subst}\left(\int x\left(-1+e^{2a}x\right)^p \left(1+e^{2a}x\right)^{-p}\left(2-2e^{2a}px\right) dx, x, \sqrt[4]{x}\right) \\
&= \frac{1}{3}e^{-12a}\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}\left(3+2p^2\right)-2e^{6a}p\sqrt[4]{x}\right) \\
&\quad + e^{-4a}\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x} \\
&\quad - \frac{1}{3}\left(4e^{-6a}p\left(2+p^2\right)\right)\text{Subst}\left(\int \left(-1+e^{2a}x\right)^p\left(1+e^{2a}x\right)^{-p} dx, x, \sqrt[4]{x}\right) \\
&= \frac{1}{3}e^{-12a}\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}\left(3+2p^2\right)-2e^{6a}p\sqrt[4]{x}\right) \\
&\quad + e^{-4a}\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x} \\
&\quad - \frac{2^{2-p}e^{-8a}p\left(2+p^2\right)\left(-1+e^{2a}\sqrt[4]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1-e^{2a}\sqrt[4]{x}\right)\right)}{3(1+p)}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 1.44 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.20

$$\int \tanh^p\left(a+\frac{\log(x)}{8}\right) dx = \frac{e^{-8a}\left(-1+e^{2a}\sqrt[4]{x}\right)\left(\frac{-1+e^{2a}\sqrt[4]{x}}{2+2e^{2a}\sqrt[4]{x}}\right)^p\left(-8p\left(1+e^{2a}\sqrt[4]{x}\right)^p\text{Hypergeometric2F1}\left(-2+p,1+p,2+p,\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt[4]{}\right.\right.}{}$$

```
[In] Integrate[Tanh[a + Log[x]/8]^p,x]
```

```
[Out] ((-1 + E^(2*a)*x^(1/4))*((-1 + E^(2*a)*x^(1/4))/(2 + 2*E^(2*a)*x^(1/4)))^p*
(-8*p*(1 + E^(2*a)*x^(1/4))^p*Hypergeometric2F1[-2 + p, 1 + p, 2 + p, 1/2 -
 (E^(2*a)*x^(1/4))/2] + 4*(1 + 2*p)*(1 + E^(2*a)*x^(1/4))^p*Hypergeometric2
F1[-1 + p, 1 + p, 2 + p, 1/2 - (E^(2*a)*x^(1/4))/2] + (1 + p)*(2^p*E^(4*a)*
(1 + E^(2*a)*x^(1/4))*Sqrt[x] - 2*(1 + E^(2*a)*x^(1/4))^p*Hypergeometric2F1
[p, 1 + p, 2 + p, 1/2 - (E^(2*a)*x^(1/4))/2])))/(E^(8*a)*(1 + p))
```

Maple [F]

$$\int \tanh\left(a+\frac{\ln\left(x\right)}{8}\right)^{p}dx$$

```
[In] int(tanh(a+1/8*ln(x))^p,x)
[Out] int(tanh(a+1/8*ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^{p}\left(a+\frac{\log(x)}{8}\right)\,dx = \int \tanh\left(a+\frac{1}{8}\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+1/8*log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(a + 1/8*log(x))^p, x)
```

Sympy [F]

$$\int \tanh^{p}\left(a+\frac{\log(x)}{8}\right)\,dx = \int \tanh^{p}\left(a+\frac{\log\left(x\right)}{8}\right)\,dx$$

```
[In] integrate(tanh(a+1/8*ln(x))**p,x)
[Out] Integral(tanh(a + log(x)/8)**p, x)
```

Maxima [F]

$$\int \tanh^{p}\left(a+\frac{\log(x)}{8}\right)\,dx = \int \tanh\left(a+\frac{1}{8}\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+1/8*log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(a + 1/8*log(x))^p, x)
```

Giac [F]

$$\int \tanh^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \tanh\left(a+\frac{1}{8}\,\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+1/8*log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(a + 1/8*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \tanh\left(a+\frac{\ln\left(x\right)}{8}\right)^p\,dx$$

```
[In] int(tanh(a + log(x)/8)^p,x)
[Out] int(tanh(a + log(x)/8)^p, x)
```

3.169 $\int \tanh^p(a+\log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 61

$$\int \tanh^p(a+\log(x))\,dx = x\left(1-e^{2a}x^2\right)^{-p}\left(-1+e^{2a}x^2\right)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)$$

```
[Out] x*(-1+exp(2*a)*x^2)^p*AppellF1(1/2,-p,p,3/2,exp(2*a)*x^2,-exp(2*a)*x^2)/((1
-exp(2*a)*x^2)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5652, 441, 440}

$$\int \tanh^p(a+\log(x))\,dx = x\left(1-e^{2a}x^2\right)^{-p}\left(e^{2a}x^2-1\right)^p \operatorname{AppellF1}\left(\frac{1}{2},-p,p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)$$

```
[In] Int[Tanh[a + Log[x]]^p,x]
```

```
[Out] (x*(-1 + E^(2*a)*x^2)^p*AppellF1[1/2, -p, p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x^
2)])/(1 - E^(2*a)*x^2)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-1+e^{2a}x^2\right)^p \left(1+e^{2a}x^2\right)^{-p}\, dx\\ &= \left(\left(1-e^{2a}x^2\right)^{-p} \left(-1+e^{2a}x^2\right)^p\right) \int \left(1-e^{2a}x^2\right)^p \left(1+e^{2a}x^2\right)^{-p}\, dx\\ &= x\left(1-e^{2a}x^2\right)^{-p} \left(-1+e^{2a}x^2\right)^p \text{AppellF1}\left(\frac{1}{2},-p,p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.

Time = 0.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \tanh^p(a+\log(x))\, dx$$

$$= \frac{3x\left(\frac{-1+e^{2a}x^2}{1+e^{2a}x^2}\right)^p \text{AppellF1}\left(\frac{1}{2},-p,p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)}{3\,\text{AppellF1}\left(\frac{1}{2},-p,p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)-2e^{2a}px^2\left(\text{AppellF1}\left(\frac{3}{2},1-p,p,\frac{5}{2},e^{2a}x^2,-e^{2a}x^2\right)+\text{AppellF1}\left(\frac{3}{2},-\right.\right.}$$

```
[In] Integrate[Tanh[a + Log[x]]^p,x]
```

```
[Out] (3*x*((-1 + E^(2*a)*x^2)/(1 + E^(2*a)*x^2))^p*AppellF1[1/2, -p, p, 3/2, E^(
2*a)*x^2, -(E^(2*a)*x^2)])/(3*AppellF1[1/2, -p, p, 3/2, E^(2*a)*x^2, -(E^(2
*a)*x^2)] - 2*E^(2*a)*p*x^2*(AppellF1[3/2, 1 - p, p, 5/2, E^(2*a)*x^2, -(E^
(2*a)*x^2)] + AppellF1[3/2, -p, 1 + p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]))
```

Maple [F]

$$\int \tanh\left(a+\ln\left(x\right)\right)^{p}dx$$

```
[In] int(tanh(a+ln(x))^p,x)
[Out] int(tanh(a+ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^{p}(a+\log(x))\,dx = \int \tanh\left(a+\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(a + log(x))^p, x)
```

Sympy [F]

$$\int \tanh^{p}(a+\log(x))\,dx = \int \tanh^{p}\left(a+\log\left(x\right)\right)dx$$

```
[In] integrate(tanh(a+ln(x))**p,x)
[Out] Integral(tanh(a + log(x))**p, x)
```

Maxima [F]

$$\int \tanh^{p}(a+\log(x))\,dx = \int \tanh\left(a+\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(a + log(x))^p, x)
```

Giac [F]

$$\int \tanh^p(a+\log(x))\,dx = \int \tanh\left(a+\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(a + log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p(a+\log(x))\,dx = \int \tanh(a+\ln\left(x\right))^p\,dx$$

```
[In] int(tanh(a + log(x))^p,x)
[Out] int(tanh(a + log(x))^p, x)
```

3.170 $\int \tanh^p(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 61

$$\int \tanh^p(a+2\log(x))\,dx = x(1-e^{2a}x^4)^{-p}(-1+e^{2a}x^4)^p \operatorname{AppellF1}\left(\frac{1}{4},-p,p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)$$

```
[Out] x*(-1+exp(2*a)*x^4)^p*AppellF1(1/4,-p,p,5/4,exp(2*a)*x^4,-exp(2*a)*x^4)/((1
-exp(2*a)*x^4)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5652, 441, 440}

$$\int \tanh^p(a+2\log(x))\,dx = x(1-e^{2a}x^4)^{-p}(e^{2a}x^4-1)^p \operatorname{AppellF1}\left(\frac{1}{4},-p,p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)$$

```
[In] Int[Tanh[a + 2*Log[x]]^p,x]
```

```
[Out] (x*(-1 + E^(2*a)*x^4)^p*AppellF1[1/4, -p, p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x^
4)])/(1 - E^(2*a)*x^4)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-1+e^{2a}x^4\right)^p \left(1+e^{2a}x^4\right)^{-p}\,dx\\ &= \left(\left(1-e^{2a}x^4\right)^{-p}\left(-1+e^{2a}x^4\right)^p\right)\int \left(1-e^{2a}x^4\right)^p \left(1+e^{2a}x^4\right)^{-p}\,dx\\ &= x\left(1-e^{2a}x^4\right)^{-p}\left(-1+e^{2a}x^4\right)^p \operatorname{AppellF1}\left(\frac{1}{4},-p,p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.

Time = 0.72 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \tanh^p(a+2\log(x))\,dx = \frac{5x\left(\frac{-1+e^{2a}x^4}{1+e^{2a}x^4}\right)^p \operatorname{AppellF1}\left(\frac{1}{4},-p,p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)}{5\operatorname{AppellF1}\left(\frac{1}{4},-p,p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)-4e^{2a}px^4\left(\operatorname{AppellF1}\left(\frac{5}{4},1-p,p,\frac{9}{4},e^{2a}x^4,-e^{2a}x^4\right)+\operatorname{AppellF1}\left(\frac{5}{4},-p,1+p,\frac{9}{4},e^{2a}x^4,-e^{2a}x^4\right)\right)}$$

```
[In] Integrate[Tanh[a + 2*Log[x]]^p,x]
```

```
[Out] (5*x*((-1 + E^(2*a)*x^4)/(1 + E^(2*a)*x^4))^p*AppellF1[1/4, -p, p, 5/4, E^(
2*a)*x^4, -(E^(2*a)*x^4)])/(5*AppellF1[1/4, -p, p, 5/4, E^(2*a)*x^4, -(E^(2
*a)*x^4)] - 4*E^(2*a)*p*x^4*(AppellF1[5/4, 1 - p, p, 9/4, E^(2*a)*x^4, -(E^
(2*a)*x^4)] + AppellF1[5/4, -p, 1 + p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]))
```

Maple [F]

$$\int \tanh\left(a+2\ln\left(x\right)\right)^{p}dx$$

```
[In] int(tanh(a+2*ln(x))^p,x)
[Out] int(tanh(a+2*ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^{p}(a+2\log(x))\,dx = \int \tanh\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+2*log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(a + 2*log(x))^p, x)
```

Sympy [F]

$$\int \tanh^{p}(a+2\log(x))\,dx = \int \tanh^{p}\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(tanh(a+2*ln(x))**p,x)
[Out] Integral(tanh(a + 2*log(x))**p, x)
```

Maxima [F]

$$\int \tanh^{p}(a+2\log(x))\,dx = \int \tanh\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+2*log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(a + 2*log(x))^p, x)
```

Giac [F]

$$\int \tanh^p(a + 2\log(x))\,dx = \int \tanh\left(a + 2\,\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(tanh(a+2*log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(a + 2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p(a + 2\log(x))\,dx = \int \tanh(a + 2\,\ln\left(x\right))^p\,dx$$

```
[In] int(tanh(a + 2*log(x))^p,x)
[Out] int(tanh(a + 2*log(x))^p, x)
```

3.171 $\int \tanh^p(a+3\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 61

$$\int \tanh^p(a+3\log(x))\,dx = x\left(1-e^{2a}x^6\right)^{-p}\left(-1+e^{2a}x^6\right)^p \operatorname{AppellF1}\left(\frac{1}{6},-p,p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)$$

```
[Out] x*(-1+exp(2*a)*x^6)^p*AppellF1(1/6,-p,p,7/6,exp(2*a)*x^6,-exp(2*a)*x^6)/((1
-exp(2*a)*x^6)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5652, 441, 440}

$$\int \tanh^p(a+3\log(x))\,dx = x\left(1-e^{2a}x^6\right)^{-p}\left(e^{2a}x^6-1\right)^p \operatorname{AppellF1}\left(\frac{1}{6},-p,p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)$$

```
[In] Int[Tanh[a + 3*Log[x]]^p,x]
```

```
[Out] (x*(-1 + E^(2*a)*x^6)^p*AppellF1[1/6, -p, p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x^
6)])/(1 - E^(2*a)*x^6)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5652

```
Int[Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 + E^(2*
a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-1+e^{2a}x^6\right)^p \left(1+e^{2a}x^6\right)^{-p}\,dx\\
&= \left(\left(1-e^{2a}x^6\right)^{-p}\left(-1+e^{2a}x^6\right)^p\right)\int \left(1-e^{2a}x^6\right)^p \left(1+e^{2a}x^6\right)^{-p}\,dx\\
&= x\left(1-e^{2a}x^6\right)^{-p}\left(-1+e^{2a}x^6\right)^p \operatorname{AppellF1}\left(\frac{1}{6},-p,p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.

Time = 0.75 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \tanh^p(a+3\log(x))\,dx$$

$$= \frac{7x\left(\frac{-1+e^{2a}x^6}{1+e^{2a}x^6}\right)^p \operatorname{AppellF1}\left(\frac{1}{6},-p,p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)}{7\operatorname{AppellF1}\left(\frac{1}{6},-p,p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)-6e^{2a}px^6\left(\operatorname{AppellF1}\left(\frac{7}{6},1-p,p,\frac{13}{6},e^{2a}x^6,-e^{2a}x^6\right)+\operatorname{AppellF1}\left(\frac{7}{6},\right.\right.}$$

```
[In] Integrate[Tanh[a + 3*Log[x]]^p,x]
```

```
[Out] (7*x*((-1 + E^(2*a)*x^6)/(1 + E^(2*a)*x^6))^p*AppellF1[1/6, -p, p, 7/6, E^(
2*a)*x^6, -(E^(2*a)*x^6)])/(7*AppellF1[1/6, -p, p, 7/6, E^(2*a)*x^6, -(E^(2
*a)*x^6)] - 6*E^(2*a)*p*x^6*(AppellF1[7/6, 1 - p, p, 13/6, E^(2*a)*x^6, -(E
^(2*a)*x^6)] + AppellF1[7/6, -p, 1 + p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)])
)
```

Maple [F]

$$\int \tanh\left(a+3\ln\left(x\right)\right)^{p}dx$$

```
[In] int(tanh(a+3*ln(x))^p,x)
[Out] int(tanh(a+3*ln(x))^p,x)
```

Fricas [F]

$$\int \tanh^{p}(a+3\log(x))\,dx = \int \tanh\left(a+3\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+3*log(x))^p,x, algorithm="fricas")
[Out] integral(tanh(a + 3*log(x))^p, x)
```

Sympy [F]

$$\int \tanh^{p}(a+3\log(x))\,dx = \int \tanh^{p}\left(a+3\log\left(x\right)\right)dx$$

```
[In] integrate(tanh(a+3*ln(x))**p,x)
[Out] Integral(tanh(a + 3*log(x))**p, x)
```

Maxima [F]

$$\int \tanh^{p}(a+3\log(x))\,dx = \int \tanh\left(a+3\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(tanh(a+3*log(x))^p,x, algorithm="maxima")
[Out] integrate(tanh(a + 3*log(x))^p, x)
```

Giac [F]

$$\int \tanh^p(a + 3\log(x))\, dx = \int \tanh\left(a + 3\,\log\left(x\right)\right)^p\, dx$$

```
[In] integrate(tanh(a+3*log(x))^p,x, algorithm="giac")
[Out] integrate(tanh(a + 3*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p(a + 3\log(x))\, dx = \int \tanh(a + 3\,\ln\left(x\right))^p\, dx$$

```
[In] int(tanh(a + 3*log(x))^p,x)
[Out] int(tanh(a + 3*log(x))^p, x)
```

3.172 $\int x^3 \tanh(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int x^3 \tanh(d(a+b\log(cx^n)))\,dx = \frac{x^4}{4} - \frac{1}{2}x^4\,\text{Hypergeometric2F1}\left(1, \frac{2}{bdn}, 1+\frac{2}{bdn}, -e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/4*x^4-1/2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b
*d))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5658, 5656, 470, 371}

$$\int x^3 \tanh(d(a+b\log(cx^n)))\,dx = \frac{x^4}{4} - \frac{1}{2}x^4\,\text{Hypergeometric2F1}\left(1, \frac{2}{bdn}, 1+\frac{2}{bdn}, -e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[x^3*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^4/4 - (x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), -(E^(2*a*d)*(c*
x^n)^(2*b*d))])/2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int x^{-1+\frac{4}{n}}\tanh(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{4}{n}}\left(-1+e^{2ad}x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^4}{4}-\frac{\left(2x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{4}{n}}}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^4}{4}-\frac{1}{2}x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},-e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 127 vs. 2(59) = 118.

Time = 8.17 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.15

$$\int x^3 \tanh(d(a+b\log(cx^n)))\,dx$$
$$=\frac{x^4\left(2e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1+\frac{2}{bdn},2+\frac{2}{bdn},-e^{2d(a+b\log(cx^n))}\right)-(2+bdn)\text{Hypergeometric}\right.}{8+4bdn}$$

```
[In] Integrate[x^3*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x^4*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - (2 + b*d*n)*Hypergeometric2F1[1,
2/(b*d*n), 1 + 2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]))/(8 + 4*b*d*n)
```

Maple [F]

$$\int x^3 \tanh(d(a+b\ln(c\,x^n)))\,dx$$

```
[In] int(x^3*tanh(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*tanh(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^3 \tanh(d(a+b\log(cx^n)))\,dx=\int x^3 \tanh((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^3*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^3 \tanh(d(a+b\log(cx^n)))\,dx=\int x^3 \tanh(ad+bd\log(cx^n))\,dx$$

```
[In] integrate(x**3*tanh(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(x**3*tanh(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^3 \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \tanh\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] 1/4*x^4 - 2*integrate(x^3/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Giac [F]

$$\int x^3 \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \tanh\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^3*tanh((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \tanh(d\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

```
[In] int(x^3*tanh(d*(a + b*log(c*x^n))),x)
[Out] int(x^3*tanh(d*(a + b*log(c*x^n))), x)
```

3.173 $\int x^2 \tanh(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int x^2 \tanh(d(a+b\log(cx^n)))\,dx = \frac{x^3}{3} - \frac{2}{3}x^3 \text{Hypergeometric2F1}\left(1, \frac{3}{2bdn}, 1+\frac{3}{2bdn}, -e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/3*x^3-2/3*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],-exp(2*a*d)*(c*x^n)^
(2*b*d))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5658, 5656, 470, 371}

$$\int x^2 \tanh(d(a+b\log(cx^n)))\,dx = \frac{x^3}{3} - \frac{2}{3}x^3 \text{Hypergeometric2F1}\left(1, \frac{3}{2bdn}, 1+\frac{3}{2bdn}, -e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[x^2*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^3/3 - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), -(E^(2*a*
d)*(c*x^n)^(2*b*d))])/3
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^3 (cx^n)^{-3/n}\right) \text{Subst}\left(\int x^{-1+\frac{3}{n}} \tanh(d(a+b\log(x)))\,dx, x, cx^n\right)}{n} \\
&= \frac{\left(x^3 (cx^n)^{-3/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}\left(-1+e^{2ad}x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{n} \\
&= \frac{x^3}{3} - \frac{\left(2x^3 (cx^n)^{-3/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{n} \\
&= \frac{x^3}{3} - \frac{2}{3}x^3 \,\text{Hypergeometric2F1}\left(1, \frac{3}{2bdn}, 1+\frac{3}{2bdn}, -e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 136 vs. 2(63) = 126.

Time = 7.82 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.16

$$\int x^2 \tanh \left(d(a + b \log \left(cx^n\right))\right) \, dx$$
$$= \frac{x^3 \left(3 e^{2d(a+b\log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 + \frac{3}{2bdn}, 2 + \frac{3}{2bdn}, -e^{2d(a+b\log(cx^n))}\right) - (3 + 2bdn) \text{Hypergeomet}}{9 + 6bdn}$$

```
[In] Integrate[x^2*Tanh[d*(a + b*Log[c*x^n])],x]
[Out] (x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
+ 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - (3 + 2*b*d*n)*Hypergeometric2
F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]))/(9 + 6*b
*d*n)
```

Maple [F]

$$\int x^2 \tanh \left(d(a + b \ln \left(c x^n\right))\right) dx$$

```
[In] int(x^2*tanh(d*(a+b*ln(c*x^n))),x)
[Out] int(x^2*tanh(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2 \tanh \left(d(a + b \log \left(cx^n\right))\right) \, dx = \int x^2 \tanh \left((b \log \left(cx^n\right) + a)d\right) \, dx$$

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x^2*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^2 \tanh \left(d(a + b \log \left(cx^n\right))\right) \, dx = \int x^2 \tanh \left(ad + bd \log \left(cx^n\right)\right) dx$$

```
[In] integrate(x**2*tanh(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**2*tanh(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \tanh \left(d(a + b \log \left(cx^n\right))\right) \, dx = \int x^2 \tanh \left((b \log \left(cx^n\right) + a)d\right) \, dx$$

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] 1/3*x^3 - 2*integrate(x^2/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Giac [F]

$$\int x^2 \tanh \left(d(a + b \log \left(cx^n\right))\right) \, dx = \int x^2 \tanh \left((b \log \left(cx^n\right) + a)d\right) \, dx$$

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^2*tanh((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh \left(d(a + b \log \left(cx^n\right))\right) \, dx = \int x^2 \tanh(d\,(a + b\,\ln\left(c\,x^n\right))) \, dx$$

```
[In] int(x^2*tanh(d*(a + b*log(c*x^n))),x)
[Out] int(x^2*tanh(d*(a + b*log(c*x^n))), x)
```

3.174 $\int x \tanh (d(a+b\log (cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int x \tanh (d(a+b\log (cx^n)))\,dx=\frac{x^2}{2}-x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/2*x^2-x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5658, 5656, 470, 371}

$$\int x \tanh (d(a+b\log (cx^n)))\,dx=\frac{x^2}{2}-x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[x*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^2/2 - x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), -(E^(2*a*d)*(c*x
^n)^(2*b*d))]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\tanh(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}\left(-1+e^{2ad}x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^2}{2}-\frac{\left(2x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^2}{2}-x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 122 vs. 2(55) = 110.

Time = 7.93 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.22

$$\int x \tanh (d(a + b \log (cx^n))) \, dx = \frac{x^2 \left(e^{2d(a+b\log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 + \frac{1}{bdn}, 2 + \frac{1}{bdn}, -e^{2d(a+b\log(cx^n))}\right) - (1 + bdn) \text{Hypergeometric2}\right.}{2 + 2bdn}$$

```
[In] Integrate[x*Tanh[d*(a + b*Log[c*x^n])],x]
[Out] (x^2*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 + 1/
(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - (1 + b*d*n)*Hypergeometric2F1[1, 1/
(b*d*n), 1 + 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]))/(2 + 2*b*d*n)
```

Maple [F]

$$\int x \tanh (d(a + b \ln (c \, x^n))) \, dx$$

```
[In] int(x*tanh(d*(a+b*ln(c*x^n))),x)
[Out] int(x*tanh(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x \tanh (d(a + b \log (cx^n))) \, dx = \int x \tanh ((b \log (cx^n) + a)d) \, dx$$

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral(x*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x \tanh (d(a + b \log (cx^n))) \, dx = \int x \tanh (ad + bd \log (cx^n)) \, dx$$

```
[In] integrate(x*tanh(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x*tanh(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x\tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tanh\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] 1/2*x^2 - 2*integrate(x/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Giac [F]

$$\int x\tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tanh\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*tanh((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x\tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int x\tanh(d\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

```
[In] int(x*tanh(d*(a + b*log(c*x^n))),x)
[Out] int(x*tanh(d*(a + b*log(c*x^n))), x)
```

3.175 $\int \tanh(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \tanh(d(a+b\log(cx^n)))\,dx = x - 2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] x-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5654, 5656, 470, 371}

$$\int \tanh(d(a+b\log(cx^n)))\,dx = x - 2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x - 2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), -(E^(2*a*d)*(c*x
^n)^(2*b*d))]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5654

```
Int[Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Tanh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\tanh(d(a+b\log(x)))\,dx, x, cx^n\right)}{n} \\
&= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}\left(-1+e^{2ad}x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{n} \\
&= x-\frac{\left(2x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{n} \\
&= x-2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. $2(53) = 106$.

Time = 8.68 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.38

$$\int \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{e^{2d(a+b\log(cx^n))}x\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bdn},2+\frac{1}{2bdn},-e^{2d(a+b\log(cx^n))}\right)}{1+2bdn} - x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},-e^{2d(a+b\log(cx^n))}\right)$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1/(
2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n) - x*Hypergeometric2F1
[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]
```

Maple [F]

$$\int \tanh\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

```
[In] int(tanh(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh\left(d(a+b\log\left(cx^n\right))\right)dx$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n))),x)
[Out] Integral(tanh(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] x - 2*integrate(1/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*d) + 1), x)
```

Giac [F]

$$\int \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(tanh((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh(d\left(a+b\,\ln\left(c\,x^n\right)\right))\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n))),x)
[Out] int(tanh(d*(a + b*log(c*x^n))), x)
```

3.176 $\int \frac{\tanh(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x}\,dx = \frac{\log(\cosh(ad+bd\log(cx^n)))}{bdn}$$

```
[Out] ln(cosh(a*d+b*d*ln(c*x^n)))/b/d/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3556}

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x}\,dx = \frac{\log(\cosh(ad+bd\log(cx^n)))}{bdn}$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Log[Cosh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \tanh(d(a+bx))\,dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\log(\cosh(ad+bd\log(cx^n)))}{bdn} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\tanh (d(a+b\log (cx^n)))}{x}\,dx=\frac{\log (\cosh (d(a+b\log (cx^n))))}{bdn}$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]/x,x]
[Out] Log[Cosh[d*(a + b*Log[c*x^n])]]/(b*d*n)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\ln(\cosh(d(a+b\ln(c\,x^n))))}{nbd}$
default	$\frac{\ln(\cosh(d(a+b\ln(c\,x^n))))}{nbd}$
parallelrisch	$-\frac{\ln(x)dbn+\ln(1-\tanh(d(a+b\ln(c\,x^n))))}{dbn}$
risch	$\ln(x)-\frac{2a}{bn}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\,\mathrm{csgn}(ic}{n}$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b/d*ln(cosh(d*(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{\tanh (d(a+b\log (cx^n)))}{x}\,dx = -\frac{bdn\log (x)-\log \left(\frac{2\cosh(bdn\log(x)+bd\log(c)+ad)}{\cosh(bdn\log(x)+bd\log(c)+ad)-\sinh(bdn\log(x)+bd\log(c)+ad)}\right)}{bdn}$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
[Out] -(b*d*n*log(x) - log(2*cosh(b*d*n*log(x) + b*d*log(c) + a*d)/(cosh(b*d*n*lo
g(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(x) + b*d*log(c) + a*d))))/(b*d*n)
```

Sympy [A] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\tanh\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=-\frac{\log\left(bdn\tanh^2\left(ad+bd\log\left(cx^n\right)\right)-bdn\right)}{2bdn}$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))/x,x)
[Out] -log(b*d*n*tanh(a*d + b*d*log(c*x**n))**2 - b*d*n)/(2*b*d*n)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\tanh\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx=\frac{\log\left(\cosh\left((b\log\left(cx^n\right)+a)d\right)\right)}{bdn}$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
[Out] log(cosh((b*log(c*x^n) + a)*d))/(b*d*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(25) = 50.

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.96

$$\int \frac{\tanh\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx =\frac{\log\left(\sqrt{2\,x^{2\,bdn}|c|^{2\,bd}\cos\left(\pi bd\mathrm{sgn}\left(c\right)-\pi bd\right)e^{(2\,ad)}+x^{4\,bdn}|c|^{4\,bd}e^{(4\,ad)}+1}\right)}{bdn}-\log\left(x\right)$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
[Out] log(sqrt(2*x^(2*b*d*n)*abs(c)^(2*b*d)*cos(pi*b*d*sgn(c) - pi*b*d)*e^(2*a*d)
 + x^(4*b*d*n)*abs(c)^(4*b*d)*e^(4*a*d) + 1))/(b*d*n) - log(x)
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\tanh\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{\ln\left(\mathrm{e}^{2\,a\,d}\,\left(c\,x^n\right)^{2\,b\,d}+1\right)}{b\,d\,n} - \ln\left(x\right)$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))/x,x)
[Out] log(exp(2*a*d)*(c*x^n)^(2*b*d) + 1)/(b*d*n) - log(x)
```

3.177 $\int \frac{\tanh(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{1}{x} + \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{x}$$

```
[Out] -1/x+2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b*d))
/x
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5658, 5656, 470, 371}

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x^2}\,dx = \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{x} - \frac{1}{x}$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -x^(-1) + (2*Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), -(E^(2*a
*d)*(c*x^n)^(2*b*d))])/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int x^{-1-\frac{1}{n}} \tanh(d(a+b\log(x)))\,dx, x, cx^n\right)}{nx} \\
&= \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}\left(-1+e^{2ad}x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{nx} \\
&= -\frac{1}{x} - \frac{\left(2(cx^n)^{\frac{1}{n}}\right) \operatorname{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{nx} \\
&= -\frac{1}{x} + \frac{2\,\text{Hypergeometric2F1}\left(1, -\frac{1}{2bdn}, 1-\frac{1}{2bdn}, -e^{2ad}(cx^n)^{2bd}\right)}{x}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(59) = 118.

Time = 3.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.14

$$\int \frac{\tanh (d(a+b\log (cx^n)))}{x^2}\,dx = \frac{\frac{e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1-\frac{1}{2bdn},2-\frac{1}{2bdn},-e^{2d(a+b\log(cx^n))}\right)}{-1+2bdn}+\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},-e^{2d}\right.}{x}$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]/x^2,x]
[Out] ((E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2
*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/(-1 + 2*b*d*n) + Hypergeometric2F1[1
, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/x
```

Maple [F]

$$\int \frac{\tanh (d(a+b\ln (c\,x^n)))}{x^2}dx$$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))/x^2,x)
[Out] int(tanh(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\tanh (d(a+b\log (cx^n)))}{x^2}\,dx = \int \frac{\tanh ((b\log (cx^n)+a)d)}{x^2}\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
[Out] integral(tanh(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]

$$\int \frac{\tanh (d(a+b\log (cx^n)))}{x^2}\,dx = \int \frac{\tanh (ad+bd\log (cx^n))}{x^2}\,dx$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))/x**2,x)
[Out] Integral(tanh(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\tanh \left(d(a+b\log \left(cx^{n}\right))\right)}{x^{2}}\,dx=\int \frac{\tanh \left((b\log \left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
[Out] -1/x - 2*integrate(1/(c^(2*b*d)*x^2*e^(2*b*d*log(x^n) + 2*a*d) + x^2), x)
```

Giac [F]

$$\int \frac{\tanh \left(d(a+b\log \left(cx^{n}\right))\right)}{x^{2}}\,dx=\int \frac{\tanh \left((b\log \left(cx^{n}\right)+a)d\right)}{x^{2}}\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
[Out] integrate(tanh((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh \left(d(a+b\log \left(cx^{n}\right))\right)}{x^{2}}\,dx=\int \frac{\tanh\left(d\,\left(a+b\,\ln\left(c\,x^{n}\right)\right)\right)}{x^{2}}\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))/x^2,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))/x^2, x)
```

3.178 $\int \frac{\tanh(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x^3}\,dx = -\frac{1}{2x^2}+\frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{x^2}$$

```
[Out] -1/2/x^2+hypergeom([1, -1/b/d/n],[1-1/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b*d))/x
^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5658, 5656, 470, 371}

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{x^2}-\frac{1}{2x^2}$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/2*1/x^2 + Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), -(E^(2*a*d)*
(c*x^n)^(2*b*d))]/x^2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int x^{-1-\frac{2}{n}}\tanh(d(a+b\log(x)))\,dx, x, cx^n\right)}{nx^2} \\
&= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}\left(-1+e^{2ad}x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{nx^2} \\
&= -\frac{1}{2x^2} - \frac{\left(2(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{nx^2} \\
&= -\frac{1}{2x^2} + \frac{\text{Hypergeometric2F1}\left(1, -\frac{1}{bdn}, 1-\frac{1}{bdn}, -e^{2ad}(cx^n)^{2bd}\right)}{x^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 120 vs. 2(56) = 112.

Time = 3.40 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.14

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x^3}\,dx = \frac{\frac{e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{bdn},2-\frac{1}{bdn},-e^{2d(a+b\log(cx^n))}\right)}{-1+bdn}+\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},-e^{2d(a+b\log(cx^n))}\right)}{2x^2}$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]/x^3,x]
[Out] ((E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d
*n), -E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/
(b*d*n)), 1 - 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))])/(2*x^2)
```

Maple [F]

$$\int \frac{\tanh(d(a+b\ln(c\,x^n)))}{x^3}dx$$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))/x^3,x)
[Out] int(tanh(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tanh((b\log(cx^n)+a)d)}{x^3}\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
[Out] integral(tanh(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]

$$\int \frac{\tanh(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tanh(ad+bd\log(cx^n))}{x^3}\,dx$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))/x**3,x)
[Out] Integral(tanh(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\tanh \left(d(a + b \log \left(cx^n\right))\right)}{x^3} \, dx = \int \frac{\tanh \left((b \log \left(cx^n\right) + a)d\right)}{x^3} \, dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
[Out] -1/2/x^2 - 2*integrate(1/(c^(2*b*d)*x^3*e^(2*b*d*log(x^n) + 2*a*d) + x^3),
x)
```

Giac [F]

$$\int \frac{\tanh \left(d(a + b \log \left(cx^n\right))\right)}{x^3} \, dx = \int \frac{\tanh \left((b \log \left(cx^n\right) + a)d\right)}{x^3} \, dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
[Out] integrate(tanh((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh \left(d(a + b \log \left(cx^n\right))\right)}{x^3} \, dx = \int \frac{\tanh\left(d\left(a + b\,\ln\left(c\,x^n\right)\right)\right)}{x^3} \, dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))/x^3,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))/x^3, x)
```

3.179 $\int x^3 \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int x^3 \tanh^2(d(a+b\log(cx^n)))\,dx = \frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4 + \frac{x^4\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/4*(1+4/b/d/n)*x^4+x^4*(1-exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1+exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],-exp(2*a*d)*(c*x^
n)^(2*b*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5658, 5656, 516, 470, 371}

$$\int x^3 \tanh^2(d(a+b\log(cx^n)))\,dx = -\frac{2x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^4\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(e^{2ad}(cx^n)^{2bd}+1\right)} + \frac{1}{4}x^4\left(\frac{4}{bdn}+1\right)$$

```
[In] Int[x^3*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 4/(b*d*n))*x^4)/4 + (x^4*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2
/(b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int x^{-1+\frac{4}{n}}\tanh^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}$$

$$=\frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int\frac{x^{-1+\frac{4}{n}}\left(-1+e^{2ad}x^{2bd}\right)^2}{\left(1+e^{2ad}x^{2bd}\right)^2}\,dx,x,cx^n\right)}{n}$$

$$=\frac{x^4\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{\left(e^{-2ad}x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int\frac{x^{-1+\frac{4}{n}}\left(\frac{2e^{2ad}(4-bdn)}{n}-\frac{2e^{4ad}(4+bdn)x^{2bd}}{n}\right)}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{2bdn}$$

$$=\frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4+\frac{x^4\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{\left(8x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int\frac{x^{-1+\frac{4}{n}}}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{bdn^2}$$

$$=\frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4+\frac{x^4\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{2x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 8.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20

$$\int x^3\tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\frac{x^4\left(8e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{2}{bdn},2+\frac{2}{bdn},-e^{2d(a+b\log(cx^n))}\right)+(2+bdn)\left(bdn-4\,\text{Hyperg}\right.\right.}{4bdn(2+bdn)}$$

```
[In] Integrate[x^3*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^4*(8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Hypergeome
tric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - 4*Tanh[
d*(a + b*Log[c*x^n])])))/(4*b*d*n*(2 + b*d*n))
```

Maple [F]

$$\int x^{3}\tanh \left(d(a+b\ln \left(c\,x^{n}\right))\right)^{2}dx$$

```
[In] int(x^3*tanh(d*(a+b*ln(c*x^n)))^2,x)
[Out] int(x^3*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^3 \tanh^2 \left(d(a + b\log \left(cx^n\right))\right)\, dx = \int x^3 \tanh \left((b\log \left(cx^n\right) + a)d\right)^2\, dx$$

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(x^3*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \tanh^2 \left(d(a + b\log \left(cx^n\right))\right)\, dx = \text{Timed out}$$

```
[In] integrate(x**3*tanh(d*(a+b*ln(c*x**n)))**2,x)
[Out] Timed out
```

Maxima [F]

$$\int x^3 \tanh^2 \left(d(a + b\log \left(cx^n\right))\right)\, dx = \int x^3 \tanh \left((b\log \left(cx^n\right) + a)d\right)^2\, dx$$

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] 1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 8)*x^4)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 8*integrate(x^3/(b*c^(2*
b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Giac [F]

$$\int x^3 \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3 \tanh\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

```
[In] integrate(x^3*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate(x^3*tanh((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^3\,\tanh(d\left(a+b\,\ln\left(c\,x^n\right)\right))^2\,dx$$

```
[In] int(x^3*tanh(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x^3*tanh(d*(a + b*log(c*x^n)))^2, x)
```

3.180 $\int x^2 \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int x^2 \tanh^2(d(a+b\log(cx^n)))\,dx = \frac{1}{3}\left(1+\frac{3}{bdn}\right)x^3 + \frac{x^3\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/3*(1+3/b/d/n)*x^3+x^3*(1-exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1+exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],-exp(2*a*d)*(
c*x^n)^(2*b*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5658, 5656, 516, 470, 371}

$$\int x^2 \tanh^2(d(a+b\log(cx^n)))\,dx = -\frac{2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^3\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(e^{2ad}(cx^n)^{2bd}+1\right)} + \frac{1}{3}x^3\left(\frac{3}{bdn}+1\right)$$

```
[In] Int[x^2*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 3/(b*d*n))*x^3)/3 + (x^3*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 +
 3/(2*b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\tanh^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^3 (cx^n)^{-3/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}} \left(-1+e^{2ad} x^{2bd}\right)^2}{\left(1+e^{2ad} x^{2bd}\right)^2} \, dx, x, cx^n\right)}{n}$$

$$= \frac{x^3 \left(1 - e^{2ad} (cx^n)^{2bd}\right)}{bdn \left(1 + e^{2ad} (cx^n)^{2bd}\right)} - \frac{\left(e^{-2ad} x^3 (cx^n)^{-3/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}} \left(\frac{2e^{2ad}(3-bdn)}{n} - \frac{2e^{4ad}(3+bdn)x^{2bd}}{n}\right)}{1+e^{2ad} x^{2bd}} \, dx, x, cx^n\right)}{2bdn}$$

$$= \frac{1}{3} \left(1 + \frac{3}{bdn}\right) x^3 + \frac{x^3 \left(1 - e^{2ad} (cx^n)^{2bd}\right)}{bdn \left(1 + e^{2ad} (cx^n)^{2bd}\right)} - \frac{\left(6x^3 (cx^n)^{-3/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}}{1+e^{2ad} x^{2bd}} \, dx, x, cx^n\right)}{bdn^2}$$

$$= \frac{1}{3} \left(1 + \frac{3}{bdn}\right) x^3 + \frac{x^3 \left(1 - e^{2ad} (cx^n)^{2bd}\right)}{bdn \left(1 + e^{2ad} (cx^n)^{2bd}\right)} - \frac{2x^3 \text{Hypergeometric2F1}\left(1, \frac{3}{2bdn}, 1 + \frac{3}{2bdn}, -e^{2ad} (cx^n)^{2bd}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 8.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.23

$$\int x^2 \tanh^2 (d(a + b \log (cx^n))) \, dx = \frac{x^3 \left(9e^{2d(a+b\log(cx^n))} \text{Hypergeometric2F1}\left(1, 1 + \frac{3}{2bdn}, 2 + \frac{3}{2bdn}, -e^{2d(a+b\log(cx^n))}\right) + (3 + 2bdn) \left(bdn - 3 \text{Hype}\right.\right.}{3bdn(3 + 2bdn)}$$

```
[In] Integrate[x^2*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^3*(9*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
+ 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(b*d*n - 3*Hype
rgeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))]
 - 3*Tanh[d*(a + b*Log[c*x^n])])))/(3*b*d*n*(3 + 2*b*d*n))
```

Maple [F]

$$\int x^2 \tanh\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

```
[In] int(x^2*tanh(d*(a+b*ln(c*x^n)))^2,x)
[Out] int(x^2*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^2 \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \tanh\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(x^2*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^2 \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \tanh^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

```
[In] integrate(x**2*tanh(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral(x**2*tanh(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^2 \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int x^2 \tanh\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] 1/3*(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 6)*x^3)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 6*integrate(x^2/(b*c^(2*
b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Giac [F]

$$\int x^2 \tanh^2(d(a + b\log(cx^n)))\, dx = \int x^2 \tanh((b\log(cx^n) + a)d)^2\, dx$$

```
[In] integrate(x^2*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate(x^2*tanh((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \tanh^2(d(a + b\log(cx^n)))\, dx = \int x^2 \tanh(d(a + b\ln(c x^n)))^2\, dx$$

```
[In] int(x^2*tanh(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x^2*tanh(d*(a + b*log(c*x^n)))^2, x)
```

3.181 $\int x \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 131

$$\int x \tanh^2(d(a+b\log(cx^n)))\,dx = \frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2+\frac{x^2\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/2*(1+2/b/d/n)*x^2+x^2*(1-exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1+exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],-exp(2*a*d)*(c*x^
n)^(2*b*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5658, 5656, 516, 470, 371}

$$\int x \tanh^2(d(a+b\log(cx^n)))\,dx = -\frac{2x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^2\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(e^{2ad}(cx^n)^{2bd}+1\right)}+\frac{1}{2}x^2\left(\frac{2}{bdn}+1\right)$$

```
[In] Int[x*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 2/(b*d*n))*x^2)/2 + (x^2*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1
/(b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\tanh^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}$$

$$
= \frac{\left(x^2 (cx^n)^{-2/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}\left(-1+e^{2ad}x^{2bd}\right)^2}{\left(1+e^{2ad}x^{2bd}\right)^2}\,dx, x, cx^n\right)}{n}
$$

$$
= \frac{x^2\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(e^{-2ad}x^2(cx^n)^{-2/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}\left(\frac{2e^{2ad}(2-bdn)}{n}-\frac{2e^{4ad}(2+bdn)x^{2bd}}{n}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{2bdn}
$$

$$
= \frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2 + \frac{x^2\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(4x^2(cx^n)^{-2/n}\right) \text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{bdn^2}
$$

$$
= \frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2 + \frac{x^2\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}
$$

Mathematica [A] (verified)

Time = 8.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$
\int x \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx
$$

$$
= \frac{x^2\left(2e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{bdn},2+\frac{1}{bdn},-e^{2d(a+b\log(cx^n))}\right)+(1+bdn)\left(bdn-2\,\text{Hyperg}\right.\right.}{2bdn(1+bdn)}
$$

```
[In] Integrate[x*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^2*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(b*d*n - 2*Hypergeome
tric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - 2*Tanh[
d*(a + b*Log[c*x^n])])))/(2*b*d*n*(1 + b*d*n))
```

Maple [F]

$$\int x\tanh\left(d(a+b\ln\left(c\,x^{n}\right))\right)^{2}dx$$

```
[In] int(x*tanh(d*(a+b*ln(c*x^n)))^2,x)
[Out] int(x*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x\tanh^{2}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx=\int x\tanh\left((b\log\left(cx^{n}\right)+a)d\right)^{2}\,dx$$

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(x*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x\tanh^{2}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx=\int x\tanh^{2}\left(ad+bd\log\left(cx^{n}\right)\right)dx$$

```
[In] integrate(x*tanh(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral(x*tanh(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x\tanh^{2}\left(d(a+b\log\left(cx^{n}\right))\right)\,dx=\int x\tanh\left((b\log\left(cx^{n}\right)+a)d\right)^{2}\,dx$$

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] 1/2*(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 4)*x^2)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 4*integrate(x/(b*c^(2*b*
d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Giac [F]

$$\int x \tanh^2(d(a + b\log(cx^n))) \, dx = \int x \tanh((b\log(cx^n) + a)d)^2 \, dx$$

```
[In] integrate(x*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate(x*tanh((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \tanh^2(d(a + b\log(cx^n))) \, dx = \int x \tanh(d(a + b \ln(c x^n)))^2 \, dx$$

```
[In] int(x*tanh(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x*tanh(d*(a + b*log(c*x^n)))^2, x)
```

3.182 $\int \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx=\left(1+\frac{1}{bdn}\right)x+\frac{x\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}(cx^n)^{2bd}\right)}-\frac{2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] (1+1/b/d/n)*x+x*(1-exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1+exp(2*a*d)*(c*x^n)^
(2*b*d))-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],-exp(2*a*d)*(c*x^n)^(2*
b*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5654, 5656, 516, 470, 371}

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx=-\frac{2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(e^{2ad}(cx^n)^{2bd}+1\right)}+x\left(\frac{1}{bdn}+1\right)$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (1 + 1/(b*d*n))*x + (x*(1 - E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 + E^(2*a*
d)*(c*x^n)^(2*b*d))) - (2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*
n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5654

```
Int[Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Tanh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\tanh^2(d(a+b\log(x)))\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}\left(-1+e^{2ad}x^{2bd}\right)^2}{\left(1+e^{2ad}x^{2bd}\right)^2}\,dx, x, cx^n\right)}{n}$$

$$= \frac{x\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(e^{-2ad}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}\left(\frac{2e^{2ad}(1-bdn)}{n}-\frac{2e^{4ad}(1+bdn)x^{2bd}}{n}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{2bdn}$$

$$= \left(1+\frac{1}{bdn}\right)x+\frac{x\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{\left(2x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{bdn^2}$$

$$= \left(1+\frac{1}{bdn}\right)x+\frac{x\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 8.89 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.28

$$\int \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$= \frac{x\left(e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bdn},2+\frac{1}{2bdn},-e^{2d(a+b\log(cx^n))}\right)+(1+2bdn)\left(bdn-\text{Hypergeo}\right.\right.}{bdn(1+2bdn)}$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1/
(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (1 + 2*b*d*n)*(b*d*n - Hypergeome
tric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - Tan
h[d*(a + b*Log[c*x^n])])))/(b*d*n*(1 + 2*b*d*n))
```

Maple [F]

$$\int \tanh\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx = \int \tanh((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx = \int \tanh^2(d(a+b\log(cx^n)))\,dx$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral(tanh(d*(a + b*log(c*x**n)))**2, x)
```

Maxima [F]

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx = \int \tanh((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] (b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d) + (b*d*n + 2)*x)/(b*c^(2*b*d)
*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) - 2*integrate(1/(b*c^(2*b*d)*d*n*e
^(2*b*d*log(x^n) + 2*a*d) + b*d*n), x)
```

Giac [F]

$$\int \tanh^2(d(a+b\log(cx^n)))\,dx = \int \tanh((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^2 (d(a + b\log (cx^n)))\ dx = \int \tanh(d\,(a + b\,\ln (c\,x^n)))^2\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^2,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))^2, x)
```

3.183 $\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx = \log(x)-\frac{\tanh(ad+bd\log(cx^n))}{bdn}$$

```
[Out] ln(x)-tanh(a*d+b*d*ln(c*x^n))/b/d/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3554, 8}

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx = \log(x)-\frac{\tanh(ad+bd\log(cx^n))}{bdn}$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] Log[x] - Tanh[a*d + b*d*Log[c*x^n]]/(b*d*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \tanh^2(d(a+bx))\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\tanh(ad+bd\log(cx^n))}{bdn} + \frac{\text{Subst}\left(\int 1\,dx, x, \log(cx^n)\right)}{n} \\
&= \log(x) - \frac{\tanh(ad+bd\log(cx^n))}{bdn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.82

$$
\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{\text{arctanh}(\tanh(ad+bd\log(cx^n)))}{bdn} - \frac{\tanh(ad+bd\log(cx^n))}{bdn}
$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2/x,x]
[Out] ArcTanh[Tanh[a*d + b*d*Log[c*x^n]]]/(b*d*n) - Tanh[a*d + b*d*Log[c*x^n]]/(b
*d*n)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

method	result
parallelrisch	$-\frac{-\ln(x)dbn+\tanh(d(a+b\ln(cx^n)))}{dbn}$
derivativedivides	$\frac{-\tanh(d(a+b\ln(cx^n)))-\frac{\ln(\tanh(d(a+b\ln(cx^n)))-1)}{2}+\frac{\ln(\tanh(d(a+b\ln(cx^n)))+1)}{2}}{nbd}$
default	$\frac{-\tanh(d(a+b\ln(cx^n)))-\frac{\ln(\tanh(d(a+b\ln(cx^n)))-1)}{2}+\frac{\ln(\tanh(d(a+b\ln(cx^n)))+1)}{2}}{nbd}$
risch	$\ln(x)+\frac{2}{dbn\left(c^{2bd}(x^n)^{2bd}\mathrm{e}^{d\left(ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)^2-ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(icx^n)\,\mathrm{csgn}(ic)-ib\pi\,\mathrm{csgn}(icx^n)^3+ib\pi\,\mathrm{csgn}(icx^n)^2\right.}\right.}$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
[Out] -(-ln(x)*d*b*n+tanh(d*(a+b*ln(c*x^n))))/d/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(28) = 56.

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

$$\int \frac{\tanh^2\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{(bdn\log\left(x\right)+1)\cosh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)-\sinh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)}{bdn\cosh\left(bdn\log\left(x\right)+bd\log\left(c\right)+ad\right)}$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
```

```
[Out] ((b*d*n*log(x) + 1)*cosh(b*d*n*log(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(
x) + b*d*log(c) + a*d))/(b*d*n*cosh(b*d*n*log(x) + b*d*log(c) + a*d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(22) = 44.

Time = 2.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.50

$$\int \frac{\tanh^2\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = -\frac{\log\left(\tanh\left(ad+bd\log\left(cx^n\right)\right)-1\right)}{2bdn} + \frac{\log\left(\tanh\left(ad+bd\log\left(cx^n\right)\right)+1\right)}{2bdn} - \frac{\tanh\left(ad+bd\log\left(cx^n\right)\right)}{bdn}$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2/x,x)
```

```
[Out] -log(tanh(a*d + b*d*log(c*x**n)) - 1)/(2*b*d*n) + log(tanh(a*d + b*d*log(c*
x**n)) + 1)/(2*b*d*n) - tanh(a*d + b*d*log(c*x**n))/(b*d*n)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\tanh^2\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{2}{bc^{2\,bd}dne^{\left(2\,bd\log\left(x^n\right)+2\,ad\right)}+bdn} + \log\left(x\right)$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
```

```
[Out] 2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{2}{(c^{2\,bd}x^{2\,bdn}e^{(2\,ad)}+1)bdn} + \log(x)$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
[Out] 2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) + 1)*b*d*n) + log(x)
```

Mupad [B] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x}\,dx = \ln(x) + \frac{2}{b\,d\,n\,\left(e^{2\,a\,d}\,(c\,x^n)^{2\,b\,d}+1\right)}$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^2/x,x)
[Out] log(x) + 2/(b*d*n*(exp(2*a*d)*(c*x^n)^(2*b*d) + 1))
```

3.184 $\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 135

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{1-\frac{1}{bdn}}{x}+\frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdnx}$$

```
[Out] (-1+1/b/d/n)/x+(1-exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x/(1+exp(2*a*d)*(c*x^n)
^(2*b*d))-2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],-exp(2*a*d)*(c*x^n)^(2*
b*d))/b/d/n/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5658, 5656, 516, 470, 371}

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdnx} + \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx\left(e^{2ad}(cx^n)^{2bd}+1\right)} - \frac{1-\frac{1}{bdn}}{x}$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] -((1 - 1/(b*d*n))/x) + (1 - E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x*(1 + E^(2*a
*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*
d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\tanh^2(d(a+b\log(x)))\,dx, x, cx^n\right)}{nx}$$

$$= \frac{(cx^n)^{\frac{1}{n}} \operatorname{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}\left(-1+e^{2ad}x^{2bd}\right)^2}{\left(1+e^{2ad}x^{2bd}\right)^2}\,dx, x, cx^n\right)}{nx}$$

$$= \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(e^{-2ad}(cx^n)^{\frac{1}{n}}\right) \operatorname{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}\left(-\frac{2e^{2ad}(1+bdn)}{n}-2e^{4ad}\left(bd-\frac{1}{n}\right)x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{2bdnx}$$

$$= -\frac{1-\frac{1}{bdn}}{x} + \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(2(cx^n)^{\frac{1}{n}}\right) \operatorname{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{bdn^2x}$$

$$= -\frac{1-\frac{1}{bdn}}{x} + \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdnx}$$

Mathematica [A] (verified)

Time = 3.50 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^2\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = -\frac{e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2bdn},2-\frac{1}{2bdn},-e^{2d(a+b\log(cx^n))}\right)+(-1+2bdn)\left(bdn+\text{Hyper}\right.}{bdn(-1+2bdn)x}$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] -((E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(
2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (-1 + 2*b*d*n)*(b*d*n + Hypergeome
tric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] +
Tanh[d*(a + b*Log[c*x^n])]))/(b*d*n*(-1 + 2*b*d*n)*x))
```

Maple [F]

$$\int \frac{\tanh \left(d(a+b\ln \left(c\, x^{n}\right))\right)^{2}}{x^{2}}dx$$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2/x^2,x)
[Out] int(tanh(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\tanh^{2}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx = \int \frac{\tanh\left((b\log\left(cx^{n}\right)+a)d\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
[Out] integral(tanh(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\tanh^{2}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx = \int \frac{\tanh^{2}\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{2}}\,dx$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2/x**2,x)
[Out] Integral(tanh(a*d + b*d*log(c*x**n))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\tanh^{2}\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{2}}\,dx = \int \frac{\tanh\left((b\log\left(cx^{n}\right)+a)d\right)^{2}}{x^{2}}\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
[Out] -(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n - 2)/(b*c^(2*b*d)*d*n*
x*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x) + 2*integrate(1/(b*c^(2*b*d)*d*n*x^
2*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x^2), x)
```

Giac [F]

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\tanh\left((b\log(cx^n)+a)d\right)^2}{x^2}\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\tanh(d\,(a+b\,\ln(c\,x^n)))^2}{x^2}\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^2/x^2,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))^2/x^2, x)
```

3.185 $\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{\tanh^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \frac{2-bdn}{2bdnx^2}+\frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}$$

```
[Out] 1/2*(-b*d*n+2)/b/d/n/x^2+(1-exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x^2/(1+exp(2*
a*d)*(c*x^n)^(2*b*d))-2*hypergeom([1, -1/b/d/n],[1-1/b/d/n],-exp(2*a*d)*(c*
x^n)^(2*b*d))/b/d/n/x^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5658, 5656, 516, 470, 371}

$$\int \frac{\tanh^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2} + \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(e^{2ad}\left(cx^n\right)^{2bd}+1\right)}+\frac{2-bdn}{2bdnx^2}$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2 - b*d*n)/(2*b*d*n*x^2) + (1 - E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x^2*(1 +
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/
(b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*n*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int x^{-1-\frac{2}{n}}\tanh^2(d(a+b\log(x)))\,dx, x, cx^n\right)}{nx^2}$$

$$= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}\left(-1+e^{2ad}x^{2bd}\right)^2}{\left(1+e^{2ad}x^{2bd}\right)^2}\,dx, x, cx^n\right)}{nx^2}$$

$$= \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(e^{-2ad}(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}\left(-\frac{2e^{2ad}(2+bdn)}{n}+\frac{2e^{4ad}(2-bdn)x^{2bd}}{n}\right)}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{2bdnx^2}$$

$$= \frac{2-bdn}{2bdnx^2} + \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(4(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{bdn^2x^2}$$

$$= \frac{2-bdn}{2bdnx^2} + \frac{1-e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}$$

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = -\frac{2e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{bdn},2-\frac{1}{bdn},-e^{2d(a+b\log(cx^n))}\right)+(-1+bdn)\left(bdn+2\,\text{Hyperg}\right.}{2bdn(-1+bdn)x^2}$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] -1/2*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 -
1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + (-1 + b*d*n)*(b*d*n + 2*Hypergeom
etric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + 2*T
anh[d*(a + b*Log[c*x^n])]))/(b*d*n*(-1 + b*d*n)*x^2)
```

Maple [F]

$$\int \frac{\tanh\left(d(a+b\ln\left(c\,x^n\right))\right)^2}{x^3}dx$$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tanh\left((b\log(cx^n)+a)d\right)^2}{x^3}\,dx$$

[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")

[Out] integral(tanh(b*d*log(c*x^n) + a*d)^2/x^3, x)

Sympy [F]

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tanh^2(ad+bd\log(cx^n))}{x^3}\,dx$$

[In] integrate(tanh(d*(a+b*ln(c*x**n)))**2/x**3,x)

[Out] Integral(tanh(a*d + b*d*log(c*x**n))**2/x**3, x)

Maxima [F]

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tanh\left((b\log(cx^n)+a)d\right)^2}{x^3}\,dx$$

[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")

[Out] -1/2*(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n - 4)/(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x^2) + 4*integrate(1/(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) + b*d*n*x^3), x)

Giac [F]

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tanh\left((b\log(cx^n)+a)d\right)^2}{x^3}\,dx$$

[In] integrate(tanh(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")

[Out] integrate(tanh((b*log(c*x^n) + a)*d)^2/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\tanh(d\,(a+b\,\ln(c\,x^n)))^2}{x^3}\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^2/x^3,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))^2/x^3, x)
```

3.186 $\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx = \frac{\log(\cosh(a+b\log(cx^n)))}{bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn}$$

```
[Out] ln(cosh(a+b*ln(c*x^n)))/b/n-1/2*tanh(a+b*ln(c*x^n))^2/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3554, 3556}

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx = \frac{\log(\cosh(a+b\log(cx^n)))}{bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn}$$

```
[In] Int[Tanh[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \tanh^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\tanh^2(a+b\log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int \tanh(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= \frac{\log(\cosh(a+b\log(cx^n)))}{bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx = \frac{\log(\cosh(a+b\log(cx^n)))}{bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn}$$

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^3/x,x]
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n)
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result
parallelrisch	$-\frac{2\ln(x)bn+\tanh(a+b\ln(c\,x^n))^2+2\ln(1-\tanh(a+b\ln(c\,x^n)))}{2bn}$
derivativedivides	$\frac{-\frac{\tanh(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\tanh(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))+1)}{2}}{nb}$
risch	$\ln(x)-\frac{2a}{bn}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\ \text{csgn}(ix^n)\,\text{csgn}(icx^n)^2}{n}+\frac{i\pi\ \text{csgn}(ix^n)\,\text{csgn}(icx^n)\,\text{csgn}(ic)}{n}+\frac{i\pi\,\text{csgn}(ic}{n}$

```
[In] int(tanh(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
[Out] -1/2*(2*ln(x)*b*n+tanh(a+b*ln(c*x^n))^2+2*ln(1-tanh(a+b*ln(c*x^n))))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 566, normalized size of antiderivative = 13.16

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx =$$

$$-\frac{bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^4\log\left(x\right)+4\,bn\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\log\left(x\right)\sinh\left(bn\log\left(x\right)\right.}{}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 4*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^4 + 2*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^2 + b*n*log(x) + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) + b*n
*log(x) - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c
) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) +
 a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*cosh(b*n*log(x) + b*log(c
) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)))
 + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) + (b*n*log(x) - 1)*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cos
h(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) +
b*log(c) + a)^3 + b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*
log(c) + a))
```

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\tanh^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\tanh^3(a+b\log(c)) & \text{for } n=0 \\ \frac{\log(cx^n)}{n} - \frac{\log(\tanh(a+b\log(cx^n))+1)}{bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Piecewise((log(x)*tanh(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*ta
nh(a + b*log(c))**3, Eq(n, 0)), (log(c*x**n)/n - log(tanh(a + b*log(c*x**n)
) + 1)/(b*n) - tanh(a + b*log(c*x**n))**2/(2*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 304, normalized size of antiderivative = 7.07

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx = \frac{4\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+3}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} - \frac{2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+3}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \frac{3\,(2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+1)}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} - \frac{3}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \frac{\log\left(\frac{\left(c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+1\right)e^{(-2\,a)}}{c^{2\,b}}\right)}{bn} - \log(x)$$

```
[In] integrate(tanh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] 1/4*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4
*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 1/4*(2*c^(2*b)*e^(2*b*l
og(x^n) + 2*a) + 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(
2*b*log(x^n) + 2*a) + b*n) + 3/4*(2*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*
c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b
*n) - 3/4/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^
n) + 2*a) + b*n) + log((c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)*e^(-2*a)/c^(2*b
))/(b*n) - log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(41) = 82$.

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.95

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(\sqrt{2\,x^{2\,bn}|c|^{2\,b}\cos(\pi b\mathrm{sgn}(c)-\pi b)\,e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1}\right)}{bn} - \frac{3\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{2\,(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1)^2bn} - \log(x)$$

```
[In] integrate(tanh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] log(sqrt(2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b*
n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/2*(3*c^(4*b)*x^(4*b*n)*e^(4*a) + 2*
c^(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^2*b*n) - lo
g(x)
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.19

$$\int \frac{\tanh^3(a+b\log(cx^n))}{x}\,dx = \frac{2}{b\,n+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}-\ln(x) - \frac{2}{b\,n+2\,b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}} + \frac{\ln\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}{b\,n}$$

```
[In] int(tanh(a + b*log(c*x^n))^3/x,x)
```

```
[Out] 2/(b*n + b*n*exp(2*a)*(c*x^n)^(2*b)) - log(x) - 2/(b*n + 2*b*n*exp(2*a)*(c*
x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*b) + 1)/
(b*n)
```

3.187 $\int \frac{\tanh^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{\tanh^4(a+b\log(cx^n))}{x}\,dx = \log(x)-\frac{\tanh(a+b\log(cx^n))}{bn}-\frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

```
[Out] ln(x)-tanh(a+b*ln(c*x^n))/b/n-1/3*tanh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3554, 8}

$$\int \frac{\tanh^4(a+b\log(cx^n))}{x}\,dx = -\frac{\tanh^3(a+b\log(cx^n))}{3bn}-\frac{\tanh(a+b\log(cx^n))}{bn}+\log(x)$$

```
[In] Int[Tanh[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Log[x] - Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \tanh^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\text{Subst}\left(\int \tanh^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\text{Subst}\left(\int 1\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \log(x) - \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$
\int \frac{\tanh^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\text{arctanh}(\tanh\left(a+b\log\left(cx^n\right)\right))}{bn} - \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
$$

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^4/x,x]
[Out] ArcTanh[Tanh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh
[a + b*Log[c*x^n]]^3/(3*b*n)
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

method	result
parallelrisch	$-\frac{-3\ln(x)bn+\tanh(a+b\ln(c\,x^n))^3+3\tanh(a+b\ln(c\,x^n))}{3bn}$
derivativedivides	$\frac{-\frac{\tanh(a+b\ln(c\,x^n))^3}{3}-\tanh(a+b\ln(c\,x^n))-\frac{\ln(\tanh(a+b\ln(c\,x^n))-1)}{2}+\frac{\ln(\tanh(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\tanh(a+b\ln(c\,x^n))^3}{3}-\tanh(a+b\ln(c\,x^n))-\frac{\ln(\tanh(a+b\ln(c\,x^n))-1)}{2}+\frac{\ln(\tanh(a+b\ln(c\,x^n))+1)}{2}}{nb}$
risch	$\ln(x)+\frac{4(x^n)^{4b}c^{4b}e^{4a}e^{2ib\pi\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2}e^{-2ib\pi\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)\,\text{csgn}(ic)}e^{-2ib\pi\,\text{csgn}(ic\,x^n)^3}e^{2ib\pi\,\text{csgn}(ic\,x\cdots}}{bn\left((x^n)^{2b}c^{2b}e^{2a}e^{ib\pi\,\text{csgn}(ix^n)\,\text{csgn}(ic\,x^n)^2}e^{-ib\pi\,\text{c}\cdots}\right.}$

```
[In] int(tanh(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
[Out] -1/3*(-3*ln(x)*b*n+tanh(a+b*ln(c*x^n))^3+3*tanh(a+b*ln(c*x^n)))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(43) = 86$.

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 4.31

$$\int \frac{\tanh^4(a + b\log(cx^n))}{x}\,dx$$
$$= \frac{(3\,bn\log(x) + 4)\cosh(bn\log(x) + b\log(c) + a)^3 + 3\,(3\,bn\log(x) + 4)\cosh(bn\log(x) + b\log(c) + a)\sinh}{3\,\left(bn\cosh(bn\log(x) + b\log(c) +\right.}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(3*b*n*log(x)
 + 4)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - 1
2*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) - 4*sin
h(b*n*log(x) + b*log(c) + a)^3 + 3*(3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*l
og(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*cosh(b*n*log(x) +
 b*log(c) + a))
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{\tanh^4(a + b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\tanh^4(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\tanh^4(a + b\log(c)) & \text{for } n = 0 \\ \frac{\log(cx^n)}{n} - \frac{\tanh^3(a+b\log(cx^n))}{3bn} - \frac{\tanh(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Piecewise((log(x)*tanh(a)**4, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*ta
nh(a + b*log(c))**4, Eq(n, 0)), (log(c*x**n)/n - tanh(a + b*log(c*x**n))**3
/(3*b*n) - tanh(a + b*log(c*x**n))/(b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(43) = 86.

Time = 0.28 (sec) , antiderivative size = 494, normalized size of antiderivative = 10.98

$$\int \frac{\tanh^4(a+b\log(cx^n))}{x}\,dx = \frac{18\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}+27\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+11}{12\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \frac{6\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}+15\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+11}{12\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \frac{2\,(3\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)}+3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+1)}{3\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} - \frac{3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}+1}{2\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \frac{2}{3\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)}+3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}+3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \log(x)$$

```
[In] integrate(tanh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] 1/12*(18*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 27*c^(2*b)*e^(2*b*log(x^n) + 2*a)
 + 11)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n)
+ 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/12*(6*c^(4*b)*e^(4
*b*log(x^n) + 4*a) + 15*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 11)/(b*c^(6*b)*n*e
^(6*b*log(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*
n*e^(2*b*log(x^n) + 2*a) + b*n) + 2/3*(3*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 3
*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) +
3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a)
 + b*n) - 1/2*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*lo
g(x^n) + 6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b
*log(x^n) + 2*a) + b*n) + 2/3/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 3*b*c^(
4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
 + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \frac{\tanh^4(a+b\log(cx^n))}{x}\,dx = \frac{4\,(3\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+2)}{3\,(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1)^3bn} + \log(x)$$

```
[In] integrate(tanh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] 4/3*(3*c^(4*b)*x^(4*b*n)*e^(4*a) + 3*c^(2*b)*x^(2*b*n)*e^(2*a) + 2)/((c^(2*
b)*x^(2*b*n)*e^(2*a) + 1)^3*b*n) + log(x)
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.60

$$\int \frac{\tanh^4(a+b\log(cx^n))}{x}\,dx = \ln(x) + \frac{\frac{4}{3\,b\,n}+\frac{4\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{3\,b\,n}}{3\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+3\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}+\mathrm{e}^{6\,a}\,{\left(c\,x^n\right)}^{6\,b}+1} + \frac{4}{3\,b\,n\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)} + \frac{4\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{3\,b\,n\,\left(2\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}+1\right)}$$

```
[In] int(tanh(a + b*log(c*x^n))^4/x,x)
```

```
[Out] log(x) + (4/(3*b*n) + (4*exp(4*a)*(c*x^n)^(4*b))/(3*b*n))/(3*exp(2*a)*(c*x^
n)^(2*b) + 3*exp(4*a)*(c*x^n)^(4*b) + exp(6*a)*(c*x^n)^(6*b) + 1) + 4/(3*b*
n*(exp(2*a)*(c*x^n)^(2*b) + 1)) + (4*exp(2*a)*(c*x^n)^(2*b))/(3*b*n*(2*exp(
2*a)*(c*x^n)^(2*b) + exp(4*a)*(c*x^n)^(4*b) + 1))
```

3.188 $\int \frac{\tanh^5(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\tanh^5(a+b\log(cx^n))}{x} dx = \frac{\log(\cosh(a+b\log(cx^n)))}{bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn} - \frac{\tanh^4(a+b\log(cx^n))}{4bn}$$

```
[Out] ln(cosh(a+b*ln(c*x^n)))/b/n-1/2*tanh(a+b*ln(c*x^n))^2/b/n-1/4*tanh(a+b*ln(c
*x^n))^4/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3554, 3556}

$$\int \frac{\tanh^5(a+b\log(cx^n))}{x} dx = -\frac{\tanh^4(a+b\log(cx^n))}{4bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn} + \frac{\log(\cosh(a+b\log(cx^n)))}{bn}$$

```
[In] Int[Tanh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] Log[Cosh[a + b*Log[c*x^n]]]/(b*n) - Tanh[a + b*Log[c*x^n]]^2/(2*b*n) - Tanh
[a + b*Log[c*x^n]]^4/(4*b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \tanh^5(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\tanh^4(a+b\log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \tanh^3(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{\tanh^2(a+b\log(cx^n))}{2bn} - \frac{\tanh^4(a+b\log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \tanh(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{\log(\cosh(a+b\log(cx^n)))}{bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn} - \frac{\tanh^4(a+b\log(cx^n))}{4bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(a+b\log(cx^n))}{x}\,dx = \frac{4\log(\cosh(a+b\log(cx^n))) - 2\tanh^2(a+b\log(cx^n)) - \tanh^4(a+b\log(cx^n))}{4bn}$$

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (4*Log[Cosh[a + b*Log[c*x^n]]] - 2*Tanh[a + b*Log[c*x^n]]^2 - Tanh[a + b*Lo
g[c*x^n]]^4)/(4*b*n)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

method	result
parallelrisch	$-\frac{\tanh(a+b\ln(c\,x^n))^4+4\ln(x)bn+2\tanh(a+b\ln(c\,x^n))^2+4\ln(1-\tanh(a+b\ln(c\,x^n)))}{4bn}$
derivativedivides	$\frac{-\frac{\tanh(a+b\ln(c\,x^n))^4}{4}-\frac{\tanh(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\tanh(a+b\ln(c\,x^n))^4}{4}-\frac{\tanh(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\tanh(a+b\ln(c\,x^n))+1)}{2}}{nb}$
risch	$\ln(x)-\frac{2a}{bn}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\,\mathrm{csgn}(}{n}$

```
[In] int(tanh(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(tanh(a+b*ln(c*x^n))^4+4*ln(x)*b*n+2*tanh(a+b*ln(c*x^n))^2+4*ln(1-tanh
(a+b*ln(c*x^n))))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1568 vs. $2(62) = 124$.

Time = 0.26 (sec) , antiderivative size = 1568, normalized size of antiderivative = 23.76

$$\int \frac{\tanh^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^8*log(x) + 8*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^8 + 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) + b*n*log(x) - 1)*
sinh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a
)^3*log(x) + 3*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*l
og(x) + b*log(c) + a)^5 + 2*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) +
 a)^4 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 30*(b*n*log(x)
 - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*log(x) - 2)*sinh(b*n*log(x)
 + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5*log(x) + 10
*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^3 + (3*b*n*log(x) - 2)*co
sh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(b*n*l
og(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n*log(x) + 4*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^6*log(x) + 15*(b*n*log(x) - 1)*cosh(b*n*log(x) +
b*log(c) + a)^4 + 3*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a)^2 +
b*n*log(x) - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*lo
g(c) + a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c)
+ a)^7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c
) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^6 + 4*cosh(b*n*log(x) + b*log
```

```
(c) + a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b
*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b
*log(c) + a)^4 + 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x)
+ b*log(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log
(x) + b*log(c) + a)^5 + 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*l
og(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*lo
g(x) + b*log(c) + a)^6 + 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*
log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*
n*log(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 + 3*cosh(
b*n*log(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b
*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*cosh(
b*n*log(x) + b*log(c) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(
x) + b*log(c) + a))) + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7*log(x) + 3*
(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^5 + (3*b*n*log(x) - 2)*cos
h(b*n*log(x) + b*log(c) + a)^3 + (b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c
) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) +
a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^7 + b*n*sinh(b*n*log(x) + b*log(c) + a)^8 + 4*b*n*cosh(b*n*log(x) + b*log
(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log
(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*
cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) +
a)^4 + 30*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) +
b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^5 + 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*
(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 + 15*b*n*cosh(b*n*log(x) + b*log(c
) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 + 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3
+ b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [A] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^5(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\tanh^5(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\tanh^5(a+b\log(c)) & \text{for } n=0 \\ \frac{\log(cx^n)}{n} - \frac{\log(\tanh(a+b\log(cx^n))+1)}{bn} - \frac{\tanh^4(a+b\log(cx^n))}{4bn} - \frac{\tanh^2(a+b\log(cx^n))}{2bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Piecewise((log(x)*tanh(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*ta
nh(a + b*log(c))**5, Eq(n, 0)), (log(c*x**n)/n - log(tanh(a + b*log(c*x**n)
) + 1)/(b*n) - tanh(a + b*log(c*x**n))**4/(4*b*n) - tanh(a + b*log(c*x**n))
**2/(2*b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(62) = 124$.

Time = 0.33 (sec) , antiderivative size = 829, normalized size of antiderivative = 12.56

$$\int \frac{\tanh^5(a + b\log(cx^n))}{x} \, dx = \text{Too large to display}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 1/24*(48*c^(6*b)*e^(6*b*log(x^n) + 6*a) + 108*c^(4*b)*e^(4*b*log(x^n) + 4*a
) + 88*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) +
8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n)
 + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 1/24*(12*c^(6*b)*e^
(6*b*log(x^n) + 6*a) + 42*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 52*c^(2*b)*e^(2*
b*log(x^n) + 2*a) + 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n
*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b
)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 5/8*(4*c^(6*b)*e^(6*b*log(x^n) + 6*a) +
 6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(
b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) +
 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a
) + b*n) - 5/12*(6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^
n) + 2*a) + 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*l
og(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*
b*log(x^n) + 2*a) + b*n) + 5/12*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c
^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*
b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) +
 b*n) - 5/8/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(
x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*l
og(x^n) + 2*a) + b*n) + log((c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)*e^(-2*a)/c
^(2*b))/(b*n) - log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(62) = 124$.

Time = 0.36 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.44

$$\int \frac{\tanh^5(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(\sqrt{2\,x^{2\,bn}|c|^{2\,b}\cos(\pi b\mathrm{sgn}(c)-\pi b)\,e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1}\right)}{bn} - \frac{25\,c^{8\,b}x^{8\,bn}e^{(8\,a)}+52\,c^{6\,b}x^{6\,bn}e^{(6\,a)}+102\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+52\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+25}{12\,(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1)^4bn} - \log(x)$$

```
[In] integrate(tanh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] log(sqrt(2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b*
n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/12*(25*c^(8*b)*x^(8*b*n)*e^(8*a) +
52*c^(6*b)*x^(6*b*n)*e^(6*a) + 102*c^(4*b)*x^(4*b*n)*e^(4*a) + 52*c^(2*b)*x
^(2*b*n)*e^(2*a) + 25)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^4*b*n) - log(x)
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.44

$$\int \frac{\tanh^5(a+b\log(cx^n))}{x}\,dx = \frac{8}{b\,n+3\,b\,n\,e^{2\,a}\,(c\,x^n)^{2\,b}+3\,b\,n\,e^{4\,a}\,(c\,x^n)^{4\,b}+b\,n\,e^{6\,a}\,(c\,x^n)^{6\,b}} - \ln(x) + \frac{4}{b\,n+b\,n\,e^{2\,a}\,(c\,x^n)^{2\,b}} - \frac{4}{b\,n+4\,b\,n\,e^{2\,a}\,(c\,x^n)^{2\,b}+6\,b\,n\,e^{4\,a}\,(c\,x^n)^{4\,b}+4\,b\,n\,e^{6\,a}\,(c\,x^n)^{6\,b}+b\,n\,e^{8\,a}\,(c\,x^n)^{8\,b}} - \frac{8}{b\,n+2\,b\,n\,e^{2\,a}\,(c\,x^n)^{2\,b}+b\,n\,e^{4\,a}\,(c\,x^n)^{4\,b}} + \frac{\ln\left(e^{2\,a}\,(c\,x^n)^{2\,b}+1\right)}{b\,n}$$

```
[In] int(tanh(a + b*log(c*x^n))^5/x,x)
```

```
[Out] 8/(b*n + 3*b*n*exp(2*a)*(c*x^n)^(2*b) + 3*b*n*exp(4*a)*(c*x^n)^(4*b) + b*n*
exp(6*a)*(c*x^n)^(6*b)) - log(x) + 4/(b*n + b*n*exp(2*a)*(c*x^n)^(2*b)) - 4
/(b*n + 4*b*n*exp(2*a)*(c*x^n)^(2*b) + 6*b*n*exp(4*a)*(c*x^n)^(4*b) + 4*b*n
*exp(6*a)*(c*x^n)^(6*b) + b*n*exp(8*a)*(c*x^n)^(8*b)) - 8/(b*n + 2*b*n*exp(
2*a)*(c*x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*
b) + 1)/(b*n)
```

3.189 $\int (ex)^m \tanh(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 88

$$\int (ex)^m \tanh(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+
m)/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b*d))/e/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5658, 5656, 470, 371}

$$\int (ex)^m \tanh(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{m+1}}{e(m+1)} - \frac{2(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2bdn},\frac{m+1}{2bdn}+1,-e^{2ad}(cx^n)^{2bd}\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(
2*b*d*n), 1 + (1 + m)/(2*b*d*n), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1+m}{n}}\tanh(d(a+b\log(x)))\,dx,x,cx^n\right)}{en}\\
&= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int\frac{x^{-1+\frac{1+m}{n}}\left(-1+e^{2ad}x^{2bd}\right)}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{en}\\
&= \frac{(ex)^{1+m}}{e(1+m)}-\frac{\left(2(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int\frac{x^{-1+\frac{1+m}{n}}}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{en}\\
&= \frac{(ex)^{1+m}}{e(1+m)}-\frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},-e^{2ad}\left(cx^n\right)^{2bd}\right)}{e(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 13.66 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

$$\int (ex)^m \tanh(d(a+b\log(cx^n)))\,dx$$
$$= \frac{x(ex)^m\left(-\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},-e^{2d(a+b\log(cx^n))}\right)+\frac{e^{2ad}(1+m)(cx^n)^{2bd}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2}\right.}{1+m+2bdn}\right.}{1+m}$$

```
[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])],x]
[Out] (x*(e*x)^m*(-Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n),
 -E^(2*d*(a + b*Log[c*x^n]))] + (E^(2*a*d)*(1 + m)*(c*x^n)^(2*b*d)*Hypergeo
metric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), -(E^
(2*a*d)*(c*x^n)^(2*b*d))])/(1 + m + 2*b*d*n)))/(1 + m)
```

Maple [F]

$$\int (ex)^m \tanh(d(a+b\ln(c\,x^n)))\,dx$$

```
[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n))),x)
[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \tanh(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tanh((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n))),x, algorithm="fricas")
[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \tanh(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tanh(ad+bd\log(cx^n))\,dx$$

```
[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n))),x)
[Out] Integral((e*x)**m*tanh(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \tanh (d(a + b\log (cx^n)))\, dx = \int (ex)^m \tanh ((b\log (cx^n) + a)d)\, dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] e^m*x*x^m/(m + 1) - 2*e^m*integrate(x^m/(c^(2*b*d)*e^(2*b*d*log(x^n) + 2*a*
d) + 1), x)
```

Giac [F]

$$\int (ex)^m \tanh (d(a + b\log (cx^n)))\, dx = \int (ex)^m \tanh ((b\log (cx^n) + a)d)\, dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh (d(a + b\log (cx^n)))\, dx = \int \tanh(d\,(a + b\,\ln (c\,x^n)))\,(e\,x)^m\, dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(tanh(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.190 $\int (ex)^m \tanh^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int (ex)^m \tanh^2(d(a+b\log(cx^n)))\,dx = \frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n}+\frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bden}$$

```
[Out] (b*d*n+m+1)*(e*x)^(1+m)/b/d/e/(1+m)/n+(e*x)^(1+m)*(1-exp(2*a*d)*(c*x^n)^(2*
b*d))/b/d/e/n/(1+exp(2*a*d)*(c*x^n)^(2*b*d))-2*(e*x)^(1+m)*hypergeom([1, 1/
2*(1+m)/b/d/n],[1+1/2*(1+m)/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/e/n
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5658, 5656, 516, 470, 371}

$$\int (ex)^m \tanh^2(d(a+b\log(cx^n)))\,dx = -\frac{2(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2bdn},\frac{m+1}{2bdn}+1,-e^{2ad}(cx^n)^{2bd}\right)}{bden} + \frac{(ex)^{m+1}\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bden\left(e^{2ad}(cx^n)^{2bd}+1\right)}+\frac{(ex)^{m+1}(bdn+m+1)}{bde(m+1)n}$$

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + m + b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + ((e*x)^(1 + m)*(1 - E^(
2*a*d)*(c*x^n)^(2*b*d)))/(b*d*e*n*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), -
(E^(2*a*d)*(c*x^n)^(2*b*d))])/(b*d*e*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\tanh^2(d(a+b\log(x)))\,dx, x, cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-1+e^{2ad}x^{2bd}\right)^2}{\left(1+e^{2ad}x^{2bd}\right)^2}\,dx, x, cx^n\right)}{en}$$

$$= \frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{\left(e^{-2ad}(ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(\frac{2e^{2ad}(1+m-bdn)}{n}-\frac{2e^{4ad}(1+m+bdn)x^{2bd}}{n}\right)}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{2bden}$$

$$= \frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n} + \frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{\left(2(1+m)(ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{1+e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{bden^2}$$

$$= \frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n} + \frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{bden}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 516 vs. 2(169) = 338.

Time = 16.97 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.05

$$\int (ex)^m\tanh^2(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m}{1+m} - \frac{x(ex)^m\text{sech}(d(a+b(-n\log(x)+\log(cx^n))))\,\text{sech}(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))\sinh(b}{bdn} + \frac{(1+m)x^{-m}(ex)^m\text{sech}(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\text{sech}(d(a+b\log(cx^n)))\sinh(bdn\log(x))}{1+m} - \frac{e^{-\frac{(1+2m)}{}}}{}\right.}{}$$

```
[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(e*x)^m)/(1 + m) - (x*(e*x)^m*Sech[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]
*Sech[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x
]])/(b*d*n) + ((1 + m)*(e*x)^m*Sech[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(
(x^(1 + m)*Sech[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(1 + m) - (Cosh[d
*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(E^((a + 2*a*m + b*(1 + m)*n*Log[x] +
b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Hypergeome
tric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), -E^(2*d*(a + b*Log[c*x
^n]))] - E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1 + m + 2*b*d*n)*Log[x] + ((1
+ 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1
, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), -E^(2*d*(a + b*
Log[c*x^n]))] + E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x
]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Tanh[d*(a + b*Log[c*x^n])]))/(E^
(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + 2*b
*d*n))))/(b*d*n*x^m)
```

Maple [F]

$$\int (ex)^m \tanh\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

```
[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^2,x)
[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int (ex)^m \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \tanh\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int (ex)^m \tanh^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \tanh^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

```
[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral((e*x)**m*tanh(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^m \tanh^2 (d(a + b\log (cx^n)))\ dx = \int (ex)^m \tanh ((b\log (cx^n) + a)d)^2\ dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] -2*e^m*(m + 1)*integrate(x^m/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) +
b*d*n), x) + (b*c^(2*b*d)*d*e^m*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x)) +
 (b*d*e^m*n + 2*e^m*(m + 1))*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log(x
^n) + 2*a*d) + (m*n + n)*b*d)
```

Giac [F]

$$\int (ex)^m \tanh^2 (d(a + b\log (cx^n)))\ dx = \int (ex)^m \tanh ((b\log (cx^n) + a)d)^2\ dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh^2 (d(a + b\log (cx^n)))\ dx = \int \tanh(d\,(a + b\,\ln (c\,x^n)))^2\,(e\,x)^m\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

3.191 $\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 307

$$\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx = \frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} - \frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1+e^{2ad}(cx^n)^{2bd}\right)^2} + \frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}-\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1+e^{2ad}(cx^n)^{2bd}\right)} - \frac{(1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(1+m)n^2}$$

```
[Out] 1/2*(b*d*n+m+1)*(2*b*d*n+m+1)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2-1/2*(e*x)^(1+
m)*(1-exp(2*a*d)*(c*x^n)^(2*b*d))^2/b/d/e/n/(1+exp(2*a*d)*(c*x^n)^(2*b*d))^
2+1/2*(e*x)^(1+m)*(exp(2*a*d)*(-2*b*d*n+m+1)/n-exp(4*a*d)*(2*b*d*n+m+1)*(c*
x^n)^(2*b*d)/n)/b^2/d^2/e/exp(2*a*d)/n/(1+exp(2*a*d)*(c*x^n)^(2*b*d))-(2*b^
2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+m
)/b/d/n],-exp(2*a*d)*(c*x^n)^(2*b*d))/b^2/d^2/e/(1+m)/n^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used

$= \{5658, 5656, 516, 608, 470, 371\}$

$$\int (ex)^m \tanh^3 (d(a + b \log (cx^n)))\, dx =$$

$$-\frac{(ex)^{m+1} (2b^2d^2n^2 + m^2 + 2m + 1) \text{Hypergeometric2F1}\left(1, \frac{m+1}{2bdn}, \frac{m+1}{2bdn} + 1, -e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(m+1)n^2}$$
$$+ \frac{e^{-2ad}(ex)^{m+1}\left(\frac{e^{2ad}(-2bdn+m+1)}{n} - \frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(e^{2ad}(cx^n)^{2bd} + 1\right)}$$
$$- \frac{(ex)^{m+1}\left(1 - e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(e^{2ad}(cx^n)^{2bd} + 1\right)^2} + \frac{(ex)^{m+1}(bdn + m + 1)(2bdn + m + 1)}{2b^2d^2e(m+1)n^2}$$

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] ((1 + m + b*d*n)*(1 + m + 2*b*d*n)*(e*x)^(1 + m))/(2*b^2*d^2*e*(1 + m)*n^2)
 - ((e*x)^(1 + m)*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^2)/(2*b*d*e*n*(1 + E^(2*a
*d)*(c*x^n)^(2*b*d))^2) + ((e*x)^(1 + m)*((E^(2*a*d)*(1 + m - 2*b*d*n))/n -
 (E^(4*a*d)*(1 + m + 2*b*d*n)*(c*x^n)^(2*b*d))/n))/(2*b^2*d^2*e*E^(2*a*d)*n
*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), -(E^
(2*a*d)*(c*x^n)^(2*b*d))])/(b^2*d^2*e*(1 + m)*n^2)
```

<u>Rule 371</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

<u>Rule 470</u>

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

<u>Rule 516</u>

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
```

```
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 608

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ[
p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\tanh^3(d(a+b\log(x)))\,dx,x,cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-1+e^{2ad}x^{2bd}\right)^3}{\left(1+e^{2ad}x^{2bd}\right)^3}\,dx,x,cx^n\right)}{en}$$

$$= -\frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)^2}$$

$$-\frac{\left(e^{-2ad}(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-1+e^{2ad}x^{2bd}\right)\left(\frac{2e^{2ad}(1+m-2bdn)}{n}-\frac{2e^{4ad}(1+m+2bdn)x^{2bd}}{n}\right)}{\left(1+e^{2ad}x^{2bd}\right)^2}\,dx,x,\right.}{4bden}$$

$$=-\frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}+\frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}-\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1+e^{2ad}(cx^n)^{2bd}\right)}$$

$$+\frac{\left(e^{-4ad}(ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int\frac{x^{-1+\frac{1+m}{n}}\left(-\frac{4e^{4ad}(1+m-2bdn)(1+m-bdn)}{n^2}+\frac{4e^{6ad}(1+m+bdn)(1+m+2bdn)x^{2bd}}{n^2}\right)}{1+e^{2ad}x^{2bd}}\right.}{8b^2d^2en}$$

$$=\frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2}-\frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}$$
$$+\frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}-\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1+e^{2ad}(cx^n)^{2bd}\right)}$$
$$-\frac{\left((1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\operatorname{Subst}\left(\int\frac{x^{-1+\frac{1+m}{n}}}{1+e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{b^2d^2en^3}$$

$$=\frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2}-\frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}$$
$$+\frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}-\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1+e^{2ad}(cx^n)^{2bd}\right)}$$
$$-\frac{(1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}\operatorname{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},-e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(1+m)n^2}$$

Mathematica [A] (verified)

Time = 17.16 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.97

$$\int(ex)^m\tanh^3(d(a+b\log(cx^n)))\,dx$$
$$=\frac{x(ex)^m\operatorname{sech}^2(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2bdn}$$
$$-\frac{(1+m)x(ex)^m\operatorname{sech}(d(a+b(-n\log(x)+\log(cx^n))))\operatorname{sech}(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))}{2b^2d^2n^2}$$
$$+\frac{(1+2m+m^2+2b^2d^2n^2)x^{-m}(ex)^m\operatorname{sech}(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\operatorname{sech}(d(a+b\log(cx^n)))\sinh(}{1+m}\right.}{}$$
$$+\frac{x(ex)^m\tanh(d(a+b(-n\log(x)+\log(cx^n))))}{1+m}$$

[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^3,x]

[Out] (x*(e*x)^m*Sech[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) - ((1 + m)*x*(e*x)^m*Sech[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sech[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(2*b^2*d^2*n^2) + ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Sech[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*((x^(1 + m)*Sech[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(1 + m) - (Cosh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*(E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] - E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1 + m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n)*(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), -E^(2*d*(a + b*Log[c*x^n]))] + E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Tanh[d*(a + b*Log[c*x^n])]))/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 + m)*(1 + m + 2*b*d*n))))/(2*b^2*d^2*n^2*x^m) + (x*(e*x)^m*Tanh[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m)

Maple [F]

$$\int (ex)^m \tanh \left(d(a+b\ln \left(c\,x^n\right))\right)^3 dx$$

[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^3,x)

[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^3,x)

Fricas [F]

$$\int (ex)^m \tanh^3\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \tanh\left((b\log\left(cx^n\right)+a)d\right)^3\,dx$$

[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")

[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d)^3, x)

Sympy [F]

$$\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tanh^3(ad+bd\log(cx^n))\,dx$$

[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n)))**3,x)

[Out] Integral((e*x)**m*tanh(a*d + b*d*log(c*x**n))**3, x)

Maxima [F]

$$\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tanh\left((b\log(cx^n)+a)d\right)^3\,dx$$

[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")

[Out] -(2*b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*integrate(x^m/(b^2*c^(2*b*d)*d^2*n^2*e^(2*b*d*log(x^n) + 2*a*d) + b^2*d^2*n^2), x) + (b^2*c^(4*b*d)*d^2*e^m*n^2*x*e^(4*b*d*log(x^n) + 4*a*d + m*log(x)) + (b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*x*x^m + (2*b^2*c^(2*b*d)*d^2*e^m*n^2*e^(2*a*d) + 2*(m*n + n)*b*c^(2*b*d)*d*e^m*e^(2*a*d) + (m^2 + 2*m + 1)*c^(2*b*d)*e^m*e^(2*a*d))*x*e^(2*b*d*log(x^n) + m*log(x)))/((m*n^2 + n^2)*b^2*c^(4*b*d)*d^2*e^(4*b*d*log(x^n) + 4*a*d) + 2*(m*n^2 + n^2)*b^2*c^(2*b*d)*d^2*e^(2*b*d*log(x^n) + 2*a*d) + (m*n^2 + n^2)*b^2*d^2)

Giac [F]

$$\int (ex)^m \tanh^3(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tanh\left((b\log(cx^n)+a)d\right)^3\,dx$$

[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")

[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^3, x)

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh^3 (d(a+b\log (cx^n)))\ dx = \int \tanh(d\,(a+b\,\ln (c\,x^n)))^3\,(e\,x)^m\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

3.192 $\int \tanh^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \tanh^p(d(a+b\log(cx^n)))\,dx = x\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(-1+e^{2ad}(cx^n)^{2bd}\right)^p \text{AppellF1}\left(\frac{1}{2bdn},-p,p,1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] x*(-1+exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2/b/d/n,-p,p,1+1/2/b/d/n,exp
(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/((1-exp(2*a*d)*(c*x^n)
^(2*b*d))^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5654, 5656, 525, 524}

$$\int \tanh^p(d(a+b\log(cx^n)))\,dx = x\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(e^{2ad}(cx^n)^{2bd}-1\right)^p \text{AppellF1}\left(\frac{1}{2bdn},-p,p,1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(-1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[1/(2*b*d*n), -p, p, 1 + 1/(2
*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(1 - E^(
2*a*d)*(c*x^n)^(2*b*d))^p
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5654

```
Int[Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Tanh[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\tanh^p(d(a+b\log(x)))\,dx, x, cx^n\right)}{n} \\
&= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\left(-1+e^{2ad}x^{2bd}\right)^p\left(1+e^{2ad}x^{2bd}\right)^{-p}\,dx, x, cx^n\right)}{n} \\
&= \frac{\left(x(cx^n)^{-1/n}\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(-1+e^{2ad}(cx^n)^{2bd}\right)^p\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\left(1-e^{2ad}x^{2bd}\right)^p\left(1+e^{2ad}\right.\right.}{n} \\
&= x\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(-1+e^{2ad}(cx^n)^{2bd}\right)^p\text{AppellF1}\left(\frac{1}{2bdn}, -p, p, 1\right. \\
&\qquad \left. + \frac{1}{2bdn}, e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 387 vs. 2(115) = 230.

Time = 1.36 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.37

$$\int \tanh^p(d(a+b\log(cx^n)))\,dx$$

$$= \frac{(1+2bdn)x\left(\frac{-1+e^{2ad}(cx^n}{1+e^{2ad}(cx^n}\right.}{-2bde^{2ad}np\,(cx^n)^{2bd}\,\mathrm{AppellF1}\left(1+\frac{1}{2bdn},1-p,p,2+\frac{1}{2bdn},e^{2ad}\,(cx^n)^{2bd},-e^{2ad}\,(cx^n)^{2bd}\right)-2bde^{2ad}np\,(cx^n}$$

```
[In] Integrate[Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((1 + 2*b*d*n)*x*((-1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(1 + E^(2*a*d)*(c*x^n)^(
2*b*d)))^p*AppellF1[1/(2*b*d*n), -p, p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^
(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(-2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*
d)*AppellF1[1 + 1/(2*b*d*n), 1 - p, p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(
2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] - 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d)
*AppellF1[1 + 1/(2*b*d*n), -p, 1 + p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2
*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*d*n),
-p, p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b
*d))])
```

Maple [F]

$$\int \tanh\left(d\left(a+b\ln\left(c\,x^{n}\right)\right)\right)^{p}dx$$

```
[In] int(tanh(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(tanh(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int \tanh^p(d(a+b\log(cx^n)))\,dx = \int \tanh\left((b\log(cx^n)+a)d\right)^p\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(tanh(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int \tanh^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh^p\left(d(a+b\log\left(cx^n\right))\right)dx$$

```
[In] integrate(tanh(d*(a+b*ln(c*x**n)))**p,x)
[Out] Integral(tanh(d*(a + b*log(c*x**n)))**p, x)
```

Maxima [F]

$$\int \tanh^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh\left((b\log\left(cx^n\right)+a)d\right)^p\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int \tanh^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh\left((b\log\left(cx^n\right)+a)d\right)^p\,dx$$

```
[In] integrate(tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate(tanh((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \tanh(d\,(a+b\,\ln\left(c\,x^n\right)))^p\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^p,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))^p, x)
```

3.193 $\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(-1+e^{2ad}(cx^n)^{2bd}\right)^p \text{AppellF1}\left(\frac{1+m}{2bdn},-p,p,1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(c$$
$$\overline{\qquad e(1+m) \qquad}$$

```
[Out] (e*x)^(1+m)*(-1+exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2*(1+m)/b/d/n,-p,p
,1+1/2*(1+m)/b/d/n,exp(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/
e/(1+m)/((1-exp(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5658, 5656, 525, 524}

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{m+1}\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(e^{2ad}(cx^n)^{2bd}-1\right)^p \text{AppellF1}\left(\frac{m+1}{2bdn},-p,p,\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n$$
$$\overline{\qquad e(m+1) \qquad}$$

```
[In] Int[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(-1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n
), -p, p, 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*
x^n)^(2*b*d))])/(e*(1 + m)*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5656

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Int[(e*x)^m*((-1 + E^(2*a*d)*x^(2*b*d))^p/(1 + E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5658

```
Int[((e_.)*(x_))^(m_.)*Tanh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Tanh[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\tanh^p(d(a+b\log(x)))\,dx,x,cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\left(-1+e^{2ad}x^{2bd}\right)^p\left(1+e^{2ad}x^{2bd}\right)^{-p}\,dx,x,cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^{-p}\left(-1+e^{2ad}\left(cx^n\right)^{2bd}\right)^p\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\left(1-e^{2ad}x^{2bd}\right)^{p}\right.}{en}$$

$$= \frac{(ex)^{1+m}\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^{-p}\left(-1+e^{2ad}\left(cx^n\right)^{2bd}\right)^p\text{AppellF1}\left(\frac{1+m}{2bdn},-p,p,1+\frac{1+m}{2bdn},e^{2ad}\left(cx^n\right)^{2bd},-\right.}{e(1+m)}$$

Mathematica [A] (warning: unable to verify)

Time = 1.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx$$
$$=\frac{x(ex)^m\left(1-e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(\frac{-1+e^{2ad}(cx^n)^{2bd}}{1+e^{2ad}(cx^n)^{2bd}}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^p \text{AppellF1}\left(\frac{1+m}{2bdn},-p,p,1+\frac{1+m}{2bdn},e^{2ad}(c\right.}{1+m}$$

```
[In] Integrate[(e*x)^m*Tanh[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*((-1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(1 + E^(2*a*d)*(c*x^n)^(2*b*d)
))^p*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n), -p, p, 1
 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d
))])/((1 + m)*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Maple [F]

$$\int (ex)^m \tanh(d(a+b\ln(c\,x^n)))^p dx$$

```
[In] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*tanh(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx=\int (ex)^m \tanh((b\log(cx^n)+a)d)^p\,dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*tanh(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx=\text{Timed out}$$

```
[In] integrate((e*x)**m*tanh(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tanh((b\log(cx^n)+a)d)^p\,dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \tanh((b\log(cx^n)+a)d)^p\,dx$$

```
[In] integrate((e*x)^m*tanh(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*tanh((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \tanh^p(d(a+b\log(cx^n)))\,dx = \int \tanh(d\,(a+b\,\ln(c\,x^n)))^p\,(e\,x)^m\,dx$$

```
[In] int(tanh(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
[Out] int(tanh(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

3.194 $\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

```
[Out] -arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(tanh(a+b*ln(c*x^n))^(1/2))/b
/n-2/3*tanh(a+b*ln(c*x^n))^(3/2)/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3554, 3557, 335, 304, 209, 212}

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

```
[In] Int[Tanh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n) - (2*Tanh[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \tanh^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int\sqrt{\tanh(a+bx)}\,dx,x,\log(cx^n)\right)}{n}$$

$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int\frac{\sqrt{x}}{-1+x^2}\,dx,x,\tanh(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{2\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int\frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right) - \operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right) + \frac{2}{3}\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{bn}$$

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^(5/2)/x,x]
[Out] -((ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]] - ArcTanh[Sqrt[Tanh[a + b*Log[c*x^n
]]]] + (2*Tanh[a + b*Log[c*x^n]]^(3/2))/3)/(b*n))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{2\tanh(a+b\ln(c\,x^n))^{\frac{3}{2}}}{3}-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	76
default	$\frac{-\frac{2\tanh(a+b\ln(c\,x^n))^{\frac{3}{2}}}{3}-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	76

```
[In] int(tanh(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2/3*tanh(a+b*ln(c*x^n))^(3/2)-1/2*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+1
/2*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)-arctan(tanh(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 625, normalized size of antiderivative = 8.56

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1
)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 +
(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(s
inh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + 4*cosh(b
*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh
(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(
x) + b*log(c) + a)^2 + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b
*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x)
 + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x)
 + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(
c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log
(c) + a))) + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a) + 4*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) +
 a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
 sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a
)/cosh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a
)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(a+b*ln(c*x**n))**(5/2)/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\tanh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
[Out] integrate(tanh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\tanh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\tanh(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{\operatorname{atan}\left(\sqrt{\tanh(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{2\tanh(a+b\,\ln(c\,x^n))^{3/2}}{3\,b\,n}$$

```
[In] int(tanh(a + b*log(c*x^n))^(5/2)/x,x)
[Out] atanh(tanh(a + b*log(c*x^n))^(1/2))/(b*n) - atan(tanh(a + b*log(c*x^n))^(1/
2))/(b*n) - (2*tanh(a + b*log(c*x^n))^(3/2))/(3*b*n)
```

3.195 $\int \frac{\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn}$$

```
[Out] arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(tanh(a+b*ln(c*x^n))^(1/2))/b/
n-2*tanh(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\int \frac{\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn}$$

```
[In] Int[Tanh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \tanh^{\frac{3}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tanh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn} - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2\sqrt{\tanh(a+b\log(cx^n))}}{bn}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{\tanh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right) + \text{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right) - 2\sqrt{\tanh(a+b\log(cx^n))}}{bn}$$

```
[In] Integrate[Tanh[a + b*Log[c*x^n]]^(3/2)/x,x]
[Out] (ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Tanh[a + b*Log[c*x^n]]
]] - 2*Sqrt[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{-2\sqrt{\tanh(a+b\ln(c\,x^{n}))}-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}-1\right)}{2}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}+1\right)}{2}+\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}\right)}{nb}$	74
default	$\frac{-2\sqrt{\tanh(a+b\ln(c\,x^{n}))}-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}-1\right)}{2}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}+1\right)}{2}+\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}\right)}{nb}$	74

```
[In] int(tanh(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2*tanh(a+b*ln(c*x^n))^(1/2)-1/2*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+1/2
*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)+arctan(tanh(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(64) = 128$.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.77

$$\int \frac{\tanh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = -\frac{4\sqrt{\frac{\sinh(bn\log(x)+b\log(c)+a)}{\cosh(bn\log(x)+b\log(c)+a)}} - 2\arctan\left(-\cosh(bn\log(x) + b\log(c) + a)^2 - 2\cosh(bn\log(x) + b\log(c)\right.}{}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a
)) - 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*s
qrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*
log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 15.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

$$\int \frac{\tanh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = -\frac{\log\left(\sqrt{\tanh(a + b\log(cx^n))} - 1\right)}{2bn} + \frac{\log\left(\sqrt{\tanh(a + b\log(cx^n))} + 1\right)}{2bn} - \frac{2\sqrt{\tanh(a + b\log(cx^n))}}{bn} + \frac{\operatorname{atan}\left(\sqrt{\tanh(a + b\log(cx^n))}\right)}{bn}$$

```
[In] integrate(tanh(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] -log(sqrt(tanh(a + b*log(c*x**n))) - 1)/(2*b*n) + log(sqrt(tanh(a + b*log(c
*x**n))) + 1)/(2*b*n) - 2*sqrt(tanh(a + b*log(c*x**n)))/(b*n) + atan(sqrt(t
anh(a + b*log(c*x**n))))/(b*n)
```

Maxima [F]

$$\int \frac{\tanh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\tanh(b\log(cx^n) + a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(tanh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{\tanh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \frac{\operatorname{atan}\left(\sqrt{\tanh(a + b\,\ln(c\,x^n))}\right) + \operatorname{atanh}\left(\sqrt{\tanh(a + b\,\ln(c\,x^n))}\right) - 2\,\sqrt{\tanh(a + b\,\ln(c\,x^n))}}{b\,n}$$

```
[In] int(tanh(a + b*log(c*x^n))^(3/2)/x,x)
[Out] (atan(tanh(a + b*log(c*x^n))^(1/2)) + atanh(tanh(a + b*log(c*x^n))^(1/2)) -
 2*tanh(a + b*log(c*x^n))^(1/2))/(b*n)
```

3.196 $\int \frac{\sqrt{\tanh(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\sqrt{\tanh(a+b\log(cx^n))}}{x}\,dx = -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] -arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(tanh(a+b*ln(c*x^n))^(1/2))/b
/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3557, 335, 304, 209, 212}

$$\int \frac{\sqrt{\tanh(a+b\log(cx^n))}}{x}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

```
[In] Int[Sqrt[Tanh[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{\tanh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}\\
&= -\frac{\arctan\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{\arctan\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)-\operatorname{arctanh}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[In] Integrate[Sqrt[Tanh[a + b*Log[c*x^n]]]/x,x]
[Out] -((ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]] - ArcTanh[Sqrt[Tanh[a + b*Log[c*x^n
]]]])/(b*n))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	61
default	$\frac{-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	61

```
[In] int(tanh(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b*(-1/2*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+1/2*ln(tanh(a+b*ln(c*x^n))^(1/2
)+1)-arctan(tanh(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(44) = 88$.

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 6.31

$$\int \frac{\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\right.\right.}{}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
[Out] -1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a
```

```
)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*
sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + lo
g(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b
*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n
*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}-1\right)}{2bn} + \frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}+1\right)}{2bn} - \frac{\operatorname{atan}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[In] integrate(tanh(a+b*ln(c*x**n))**(1/2)/x,x)
[Out] -log(sqrt(tanh(a + b*log(c*x**n))) - 1)/(2*b*n) + log(sqrt(tanh(a + b*log(c
*x**n))) + 1)/(2*b*n) - atan(sqrt(tanh(a + b*log(c*x**n))))/(b*n)
```

Maxima [F]

$$\int \frac{\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = \int \frac{\sqrt{\tanh\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(tanh(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\tanh(a+b\log(cx^n))}}{x}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\tanh(a+b\log(cx^n))}}{x}\,dx = -\frac{\operatorname{atan}\left(\sqrt{\tanh(a+b\,\ln(c\,x^n))}\right)-\operatorname{atanh}\left(\sqrt{\tanh(a+b\,\ln(c\,x^n))}\right)}{b\,n}$$

```
[In] int(tanh(a + b*log(c*x^n))^(1/2)/x,x)
```

```
[Out] -(atan(tanh(a + b*log(c*x^n))^(1/2)) - atanh(tanh(a + b*log(c*x^n))^(1/2)))
/(b*n)
```

3.197 $\int \frac{1}{x\sqrt{\tanh(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{1}{x\sqrt{\tanh(a+b\log(cx^n))}}\,dx = \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(tanh(a+b*ln(c*x^n))^(1/2))/b/
n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3557, 335, 218, 212, 209}

$$\int \frac{1}{x\sqrt{\tanh(a+b\log(cx^n))}}\,dx = \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

```
[In] Int[1/(x*Sqrt[Tanh[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\tanh(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} \\
&= \frac{\arctan\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\arctan\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

```
[In] Integrate[1/(x*Sqrt[Tanh[a + b*Log[c*x^n]]]),x]
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)+\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	37
default	$\frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)+\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	37

```
[In] int(1/x/tanh(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/n/b*(arctanh(tanh(a+b*ln(c*x^n))^(1/2))+arctan(tanh(a+b*ln(c*x^n))^(1/2))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(43) = 86.

Time = 0.28 (sec) , antiderivative size = 305, normalized size of antiderivative = 6.49

$$\int \frac{1}{x\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{2\arctan\left(-\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2-2\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)\right.\right.}{}$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
```

```
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*s
qrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) - log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*
log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx = -\frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}-1\right)}{2bn} + \frac{\log\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}+1\right)}{2bn} + \frac{\operatorname{atan}\left(\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}\right)}{bn}$$

```
[In] integrate(1/x/tanh(a+b*ln(c*x**n))**(1/2),x)
[Out] -log(sqrt(tanh(a + b*log(c*x**n))) - 1)/(2*b*n) + log(sqrt(tanh(a + b*log(c
*x**n))) + 1)/(2*b*n) + atan(sqrt(tanh(a + b*log(c*x**n))))/(b*n)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\tanh\left(a+b\log\left(cx^{n}\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\tanh\left(b\log\left(cx^{n}\right)+a\right)}}\,dx$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(tanh(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{\tanh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \frac{\operatorname{atan}\left(\sqrt{\tanh\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)+\operatorname{atanh}\left(\sqrt{\tanh\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n}$$

```
[In] int(1/(x*tanh(a + b*log(c*x^n))^(1/2)),x)
[Out] (atan(tanh(a + b*log(c*x^n))^(1/2)) + atanh(tanh(a + b*log(c*x^n))^(1/2)))/
(b*n)
```

3.198 $\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}}$$

```
[Out] -arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(tanh(a+b*ln(c*x^n))^(1/2))/b
/n-2/b/n/tanh(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}}$$

```
[In] Int[1/(x*Tanh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -(ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Tanh[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Tanh[a + b*Log[c*x^n]]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\tanh^{\frac{3}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\tanh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{-2-\arctan\left(\sqrt[4]{\tanh^2(a+b\log(cx^n))}\right)\sqrt[4]{\tanh^2(a+b\log(cx^n))}+\text{arctanh}\left(\sqrt[4]{\tanh^2(a+b\log(cx^n))}\right)\sqrt[4]{\tanh^2(a+b\log(cx^n))}}{bn\sqrt{\tanh(a+b\log(cx^n))}}$$

```
[In] Integrate[1/(x*Tanh[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2 - ArcTan[(Tanh[a + b*Log[c*x^n]]^2)^(1/4)]*(Tanh[a + b*Log[c*x^n]]^2)^(
1/4) + ArcTanh[(Tanh[a + b*Log[c*x^n]]^2)^(1/4)]*(Tanh[a + b*Log[c*x^n]]^2)
^(1/4))/(b*n*Sqrt[Tanh[a + b*Log[c*x^n]]])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}-1\right)}{2}-\frac{2}{\sqrt{\tanh(a+b\ln(c\,x^n))}}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	76
default	$\frac{-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}-1\right)}{2}-\frac{2}{\sqrt{\tanh(a+b\ln(c\,x^n))}}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^n))}\right)}{nb}$	76

```
[In] int(1/x/tanh(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/n/b*(-1/2*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)-2/tanh(a+b*ln(c*x^n))^(1/2)+1/2
*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)-arctan(tanh(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(65) = 130.

Time = 0.28 (sec) , antiderivative size = 625, normalized size of antiderivative = 8.80

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
[Out] -1/2*(2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1
)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 +
(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(s
inh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) + 4*cosh(b
*n*log(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b
*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x)
 + b*log(c) + a)^2 - 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n
*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) +
 b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) +
 b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
 + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c
) + a))) + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
) + 4*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a
)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + s
inh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/
cosh(b*n*log(x) + b*log(c) + a)) - 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^
2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
 b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [A] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{\log\left(\sqrt{\tanh(a+b\log(cx^n))}-1\right)}{2bn} + \frac{\log\left(\sqrt{\tanh(a+b\log(cx^n))}+1\right)}{2bn} - \frac{\operatorname{atan}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{bn\sqrt{\tanh(a+b\log(cx^n))}}$$

```
[In] integrate(1/x/tanh(a+b*ln(c*x**n))**(3/2),x)
[Out] -log(sqrt(tanh(a + b*log(c*x**n))) - 1)/(2*b*n) + log(sqrt(tanh(a + b*log(c
*x**n))) + 1)/(2*b*n) - atan(sqrt(tanh(a + b*log(c*x**n))))/(b*n) - 2/(b*n*
sqrt(tanh(a + b*log(c*x**n))))
```

Maxima [F]

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\tanh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*tanh(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\tanh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\tanh\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n} - \frac{\operatorname{atan}\left(\sqrt{\tanh\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}{b\,n} - \frac{2}{b\,n\,\sqrt{\tanh\left(a+b\,\ln\left(c\,x^n\right)\right)}}$$

```
[In] int(1/(x*tanh(a + b*log(c*x^n))^(3/2)),x)
```

```
[Out] atanh(tanh(a + b*log(c*x^n))^(1/2))/(b*n) - atan(tanh(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(b*n*tanh(a + b*log(c*x^n))^(1/2))
```

3.199 $\int \frac{1}{x \tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{x \tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[Out] arctan(tanh(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(tanh(a+b*ln(c*x^n))^(1/2))/b/
n-2/3/b/n/tanh(a+b*ln(c*x^n))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3555, 3557, 335, 218, 212, 209}

$$\int \frac{1}{x \tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[In] Int[1/(x*Tanh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[Sqrt[Tanh[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Tanh[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Tanh[a + b*Log[c*x^n]]^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\tanh^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{\tanh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \tanh(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{2\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\tanh(a+b\log(cx^n))}\right)}{bn}$$

$$= \frac{\arctan\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} + \frac{\mathrm{arctanh}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$

$$= \frac{-2+3\arctan\left(\sqrt[4]{\tanh^2(a+b\log(cx^n))}\right)\tanh^2(a+b\log(cx^n))^{3/4}+3\mathrm{arctanh}\left(\sqrt[4]{\tanh^2(a+b\log(cx^n}\right.}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[In] Integrate[1/(x*Tanh[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2 + 3*ArcTan[(Tanh[a + b*Log[c*x^n]]^2)^(1/4)]*(Tanh[a + b*Log[c*x^n]]^2)
^(3/4) + 3*ArcTanh[(Tanh[a + b*Log[c*x^n]]^2)^(1/4)]*(Tanh[a + b*Log[c*x^n]
]^2)^(3/4))/(3*b*n*Tanh[a + b*Log[c*x^n]]^(3/2))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{-\frac{2}{3\tanh(a+b\ln(c\,x^{n}))^{\frac{3}{2}}}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}+1\right)}{2}-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}-1\right)}{2}+\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}\right)}{nb}$	74
default	$\frac{-\frac{2}{3\tanh(a+b\ln(c\,x^{n}))^{\frac{3}{2}}}+\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}+1\right)}{2}-\frac{\ln\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}-1\right)}{2}+\arctan\left(\sqrt{\tanh(a+b\ln(c\,x^{n}))}\right)}{nb}$	74

```
[In] int(1/x/tanh(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2/3/tanh(a+b*ln(c*x^n))^(3/2)+1/2*ln(tanh(a+b*ln(c*x^n))^(1/2)+1)-1
/2*ln(tanh(a+b*ln(c*x^n))^(1/2)-1)+arctan(tanh(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1110 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 1110, normalized size of antiderivative = 15.42

$$\int \frac{1}{x\tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*n*log(x) + b*log(c) + a)^4 + 16*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*sinh(b*n*log(x) + b*log(c) + a)^4
 + 8*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 - 6*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c)
 + a)^3 - cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)
+ 1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*s
inh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqr
t(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b*log(c) + a))) - 8*cos
h(b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sin
h(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(c
osh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*co
```

```
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*lo
g(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*
log(c) + a)^2 + 1)*sqrt(sinh(b*n*log(x) + b*log(c) + a)/cosh(b*n*log(x) + b
*log(c) + a))) + 16*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) +
b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 4*(cosh(b*n*log(x) + b*log
(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*sqrt(sinh(b*n*log(x) + b*log(
c) + a)/cosh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cosh(b*n*log(x) +
b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*
n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3
 - b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [A] (verification not implemented)

Time = 113.76 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{\log\left(\sqrt{\tanh(a+b\log(cx^n))}-1\right)}{2bn} + \frac{\log\left(\sqrt{\tanh(a+b\log(cx^n))}+1\right)}{2bn} + \frac{\operatorname{atan}\left(\sqrt{\tanh(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\tanh^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[In] integrate(1/x/tanh(a+b*ln(c*x**n))**(5/2),x)

[Out] -log(sqrt(tanh(a + b*log(c*x**n))) - 1)/(2*b*n) + log(sqrt(tanh(a + b*log(c
*x**n))) + 1)/(2*b*n) + atan(sqrt(tanh(a + b*log(c*x**n))))/(b*n) - 2/(3*b*
n*tanh(a + b*log(c*x**n))**(3/2))
```

Maxima [F]

$$\int \frac{1}{x \tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x \tanh(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*tanh(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x \tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/tanh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{x \tanh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\operatorname{atan}\left(\sqrt{\tanh(a+b\,\ln(c\,x^n))}\right)}{b\,n} + \frac{\operatorname{atanh}\left(\sqrt{\tanh(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{2}{3\,b\,n\tanh(a+b\,\ln(c\,x^n))^{3/2}}$$

```
[In] int(1/(x*tanh(a + b*log(c*x^n))^(5/2)),x)
[Out] atan(tanh(a + b*log(c*x^n))^(1/2))/(b*n) + atanh(tanh(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(3*b*n*tanh(a + b*log(c*x^n))^(3/2))
```

3.200 $\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{(b-2c)\operatorname{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4c^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c}$$

```
[Out] 1/4*(b-2*c)*arctanh(1/2*(b+2*c*tanh(x)^2)/c^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^
4)^(1/2))/c^(3/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*tanh(x)^2)/(a+b+c)^(1/2)/(
a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*tanh(x)^2+c*tanh(x
)^4)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used

$= \{3781, 1265, 1667, 857, 635, 212, 738\}$

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{(b-2c)\operatorname{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4c^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c}$$

```
[In] Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*
Tanh[x]^4])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Tanh[x]^2 + c*Tanh[x]^4]/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right)\\
&= -\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c} - \frac{\text{Subst}\left(\int \frac{\frac{b}{2}+\frac{1}{2}(b-2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)}{2c}\\
&= -\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c}\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&\quad -\frac{(b-2c)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)}{4c}
\end{aligned}
$$

$$= -\frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c} - \frac{(b-2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2c} + \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)$$

$$= \frac{(b-2c)\text{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4c^{3/2}} + \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2c}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{1}{4}\left(\frac{(-b+2c)\text{arctanh}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{c^{3/2}} + \frac{2\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{a+b+c}} - \frac{2\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{c}\right)$$

```
[In] Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] (((-b + 2*c)*ArcTanh[(-b - 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 +
 c*Tanh[x]^4])])/c^(3/2) + (2*ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sq
rt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])])/Sqrt[a + b + c] - (2*S
qrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])/c)/4
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{\ln\left(\frac{\frac{b}{2}+c\tanh(x)^2}{\sqrt{c}}+\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}\right)}{2\sqrt{c}}-\frac{\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\tanh(x)^2}{\sqrt{c}}+\sqrt{a+b\tan}\right.}{4c^{\frac{3}{2}}}$
default	$-\frac{\ln\left(\frac{\frac{b}{2}+c\tanh(x)^2}{\sqrt{c}}+\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}\right)}{2\sqrt{c}}-\frac{\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\tanh(x)^2}{\sqrt{c}}+\sqrt{a+b\tan}\right.}{4c^{\frac{3}{2}}}$

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((1/2*b+c*tanh(x)^2)/c^(1/2)+(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))/c^(1
/2)-1/2*(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)/c+1/4*b/c^(3/2)*ln((1/2*b+c*tanh(
x)^2)/c^(1/2)+(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))+1/2/(a+b+c)^(1/2)*arctanh(
1/2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*tanh(x)^2+c*tanh(x
)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2075 vs. 2(111) = 222.

Time = 1.14 (sec) , antiderivative size = 8891, normalized size of antiderivative = 65.86

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$$

```
[In] integrate(tanh(x)**5/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
```

```
[Out] Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh\left(x\right)^5}{\sqrt{c\tanh\left(x\right)^4+b\tanh\left(x\right)^2+a}}\,dx$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima"
)
[Out] integrate(tanh(x)^5/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh\left(x\right)^5}{\sqrt{c\tanh\left(x\right)^4+b\tanh\left(x\right)^2+a}}\,dx$$

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(tanh(x)^5/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh(x)^5}{\sqrt{c\tanh\left(x\right)^4+b\tanh\left(x\right)^2+a}}\,dx$$

```
[In] int(tanh(x)^5/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2),x)
[Out] int(tanh(x)^5/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2), x)
```

3.201 $\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] -1/2*arctanh(1/2*(b+2*c*tanh(x)^2)/c^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2
))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*tanh(x)^2)/(a+b+c)^(1/2)/(a+b*tan
h(x)^2+c*tanh(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3781, 1265, 857, 635, 212, 738}

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\operatorname{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{c}}$$

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x
]^4])]/Sqrt[c] + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)\\
&= \frac{\operatorname{arctanh}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$
\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{a+b+c}}\right)
$$

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
[Out] (ArcTanh[(-b - 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4
])]/Sqrt[c] + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sq
rt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/Sqrt[a + b + c])/2
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{\frac{b}{2}+c\tanh(x)^2}{\sqrt{c}}+\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b\tanh(x)^2+2c\tanh(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}\right)}{2\sqrt{a+b+c}}$	90
default	$-\frac{\ln\left(\frac{\frac{b}{2}+c\tanh(x)^2}{\sqrt{c}}+\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b\tanh(x)^2+2c\tanh(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}\right)}{2\sqrt{a+b+c}}$	90

```
[In] int(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln((1/2*b+c*tanh(x)^2)/c^(1/2)+(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))/c^(1
/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)
^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. 2(85) = 170.

Time = 0.94 (sec) , antiderivative size = 6663, normalized size of antiderivative = 63.46

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas"
)
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
[Out] Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx=\int \frac{\tanh\left(x\right)^3}{\sqrt{c\tanh\left(x\right)^4+b\tanh\left(x\right)^2+a}}\,dx$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(tanh(x)^3/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx=\int \frac{\tanh\left(x\right)^3}{\sqrt{c\tanh\left(x\right)^4+b\tanh\left(x\right)^2+a}}\,dx$$

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(x)^3/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx=\int \frac{\tanh(x)^3}{\sqrt{c\tanh\left(x\right)^4+b\tanh\left(x\right)^2+a}}\,dx$$

```
[In] int(tanh(x)^3/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2),x)
```

```
[Out] int(tanh(x)^3/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2), x)
```

3.202 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}} dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}} dx = \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] 1/2*arctanh(1/2*(2*a+b+(b+2*c)*tanh(x)^2)/(a+b+c)^(1/2)/(a+b*tanh(x)^2+c*ta
nh(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3781, 1261, 738, 212}

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}} dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right) \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right) \\
&= \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right) \\
&= \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{b\tanh(x)^2+2c\tanh(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}\right)}{2\sqrt{a+b+c}}$	52
default	$\frac{\operatorname{arctanh}\left(\frac{b\tanh(x)^2+2c\tanh(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}\right)}{2\sqrt{a+b+c}}$	52

```
[In] int(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*tanh(x)^2+2*c*tanh(x)^2+2*a+b)/(a+b+c)^(1/
2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. 2(48) = 96.

Time = 0.68 (sec) , antiderivative size = 1748, normalized size of antiderivative = 30.14

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x
```

)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
sinh(x)^3 + 4(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
 a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 + 2*a - 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 + (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sqrt(a + b + c), -
1/2*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 + 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 + a - c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 + (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 + 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 +
 2*a - 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 + 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)^3 + 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 + 30*(a^2 + a*b - b*c - c^
2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(
7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 + 10*(a^2 + a*b - b*c -
 c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*si
nh(x)^3 + 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^6 + 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 + a^2 + a*b - b*c - c^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2
 + 2*(a + b)*c + c^2)*cosh(x)^7 + 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 + (a^2 + a*b - b*c - c
^2)*cosh(x))*sinh(x)))/(a + b + c)]

Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{a+b\tanh^2{(x)}+c\tanh^4{(x)}}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh{(x)}}{\sqrt{c\tanh{(x)}^4+b\tanh{(x)}^2+a}}\,dx$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{c\tanh{(x)}^4+b\tanh{(x)}^2+a}}\,dx$$

```
[In] int(tanh(x)/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2),x)
[Out] int(tanh(x)/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2), x)
```

3.203 $\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] -1/2*arctanh(1/2*(2*a+b*tanh(x)^2)/a^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2
))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*tanh(x)^2)/(a+b+c)^(1/2)/(a+b*tan
h(x)^2+c*tanh(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3781, 1265, 974, 738, 212}

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\operatorname{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}}$$

```
[In] Int[Coth[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x
]^4])]/Sqrt[a] + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x\,(1+x^2)\,\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a-bx+cx^2}} + \frac{1}{x\sqrt{a-bx+cx^2}}\right)\,dx, x, -\tanh^2(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)$$

$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right) - \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{-2a-b+(-b-2c)\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)$$

$$= -\frac{\text{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} - \frac{\text{arctanh}\left(\frac{-2a-b+(-b-2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = -\frac{\text{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} - \frac{\text{arctanh}\left(\frac{-2a-b-(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[In] Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x
]^4])]/Sqrt[a] - ArcTanh[(-2*a - b - (b + 2*c)*Tanh[x]^2)/(2*Sqrt[a + b + c
]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c])
```

Maple [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}dx$$

```
[In] int(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
[Out] int(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1520 vs. 2(86) = 172.

Time = 0.92 (sec) , antiderivative size = 6705, normalized size of antiderivative = 63.25

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$$

```
[In] integrate(coth(x)/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{c\tanh(x)^4+b\tanh(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(coth(x)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{c\tanh(x)^4+b\tanh(x)^2+a}}\,dx$$

```
[In] int(coth(x)/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2),x)
[Out] int(coth(x)/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2), x)
```

3.204 $\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} + \frac{b\operatorname{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2a}$$

```
[Out] 1/4*b*arctanh(1/2*(2*a+b*tanh(x)^2)/a^(1/2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/
2))/a^(3/2)-1/2*arctanh(1/2*(2*a+b*tanh(x)^2)/a^(1/2)/(a+b*tanh(x)^2+c*tanh
(x)^4)^(1/2))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*tanh(x)^2)/(a+b+c)^(1/
2)/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*coth(x)^2*(a+b*tanh
(x)^2+c*tanh(x)^4)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3781, 1265, 974, 744, 738, 212}

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \frac{b\operatorname{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2a}$$

```
[In] Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x
]^4])]/Sqrt[a] + (b*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[
x]^2 + c*Tanh[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b +
c]) - (Coth[x]^2*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
```

```
m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^3(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, i\tanh(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{a-bx+cx^2}}-\frac{1}{x\sqrt{a-bx+cx^2}}+\frac{1}{(1+x)\sqrt{a-bx+cx^2}}\right) dx, x, -\tanh^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\right)\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&\quad-\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)
\end{aligned}$$

$$= -\frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2a} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)}{4a}$$
$$- \text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)$$
$$+ \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)$$
$$= -\frac{\text{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} + \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$
$$- \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2a} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2a}$$
$$= -\frac{\text{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a}} + \frac{b\text{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}}$$
$$+ \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2a}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = -\frac{(2a-b)\text{arctanh}\left(\frac{2a+b\tanh^2(x)}{2\sqrt{a}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4a^{3/2}}$$
$$+ \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{2\sqrt{a+b+c}}$$
$$- \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}{2a}$$

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -1/4*((2*a - b)*ArcTanh[(2*a + b*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Tanh[x]^2
 + c*Tanh[x]^4])])/a^(3/2) + ArcTanh[(2*a + b + (b + 2*c)*Tanh[x]^2)/(2*Sqr
t[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])]/(2*Sqrt[a + b + c]) - (C
oth[x]^2*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])/(2*a)
```

Maple [F]

$$\int \frac{\coth(x)^3}{\sqrt{a+b\tanh(x)^2+c\tanh(x)^4}}dx$$

```
[In] int(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
[Out] int(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2136 vs. $2(149) = 298$.

Time = 1.15 (sec) , antiderivative size = 9168, normalized size of antiderivative = 50.10

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="fricas"
)
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*tanh(x)**2+c*tanh(x)**4)**(1/2),x)
[Out] Integral(coth(x)**3/sqrt(a + b*tanh(x)**2 + c*tanh(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{c\tanh(x)^4+b\tanh(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="maxima"
)
[Out] integrate(coth(x)^3/sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(coth(x)^3/(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{c\tanh(x)^4+b\tanh(x)^2+a}}\,dx$$

```
[In] int(coth(x)^3/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2),x)
[Out] int(coth(x)^3/(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2), x)
```

3.205 $\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\begin{aligned}&\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx\\&=-\frac{(b+2c)\text{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4\sqrt{c}}\\&\quad+\frac{1}{2}\sqrt{a+b+c}\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)\\&\quad-\frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\end{aligned}$$

```
[Out] -1/4*(b+2*c)*arctanh(1/2*(b+2*c*tanh(x)^2)/c^(1/2)/(a+b*tanh(x)^2+c*tanh(x)
^4)^(1/2))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*tanh(x)^2)/(a+b+c)^(1/2)/
(a+b*tanh(x)^2+c*tanh(x)^4)^(1/2))*(a+b+c)^(1/2)-1/2*(a+b*tanh(x)^2+c*tanh(
x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

$= \{3781, 1261, 748, 857, 635, 212, 738\}$

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx$$
$$= -\frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4\sqrt{c}} + \frac{1}{2}\sqrt{a+b+c}\operatorname{arctanh}\left(\frac{2a+(b+2c)\tanh^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}$$

```
[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] -1/4*((b + 2*c)*ArcTanh[(b + 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2
 + c*Tanh[x]^4)])])/Sqrt[c] + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])])/2 - Sq
rt[a + b*Tanh[x]^2 + c*Tanh[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
```

```
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3781

```
Int[tan[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*tan[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*tan[(d_.) + (e_.)*(x_)])^(n2_.))^(p_), x_Symbol]
 :> Dist[f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x]
, x, f*Tan[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n2
, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x\sqrt{a-bx^2+cx^4}}{1+x^2}\,dx, x, i\tanh(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a-bx+cx^2}}{1+x}\,dx, x, -\tanh^2(x)\right)\right)\\
&= -\frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}+\frac{1}{4}\text{Subst}\left(\int \frac{-2a-b+(b+2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&= -\frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\\
&\quad+\frac{1}{2}(-a-b-c)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)\\
&\quad+\frac{1}{4}(b+2c)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\tanh^2(x)\right)
\end{aligned}$$

$$= -\frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)} + (a+b+c)\text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right) + \frac{1}{2}(b+2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\tanh^2(x)}{\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)$$

$$= -\frac{(b+2c)\text{arctanh}\left(\frac{b+2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{4\sqrt{c}} + \frac{1}{2}\sqrt{a+b+c}\,\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.99

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx = \frac{1}{4}\left(\frac{(b+2c)\text{arctanh}\left(\frac{-b-2c\tanh^2(x)}{2\sqrt{c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right)}{\sqrt{c}} + 2\sqrt{a+b+c}\,\text{arctanh}\left(\frac{2a+b+(b+2c)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}}\right) - 2\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\right)$$

```
[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4],x]
```

```
[Out] (((b + 2*c)*ArcTanh[(-b - 2*c*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Tanh[x]^2 +
c*Tanh[x]^4])])/Sqrt[c] + 2*Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*Ta
nh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Tanh[x]^2 + c*Tanh[x]^4])] - 2*Sqrt[
a + b*Tanh[x]^2 + c*Tanh[x]^4])/4
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{\sqrt{\left(\tanh(x)^2-1\right)^2 c+(b+2c)\left(\tanh(x)^2-1\right)+a+b+c}}{2}-\frac{(b+2c)\ln\left(\frac{\frac{b}{2}+c+c\left(\tanh(x)^2-1\right)}{\sqrt{c}}+\sqrt{\left(\tanh(x)^2-1\right)^2 c+(b+2c)\left(\cdots\right.}\right.}{4\sqrt{c}}$
default	$-\frac{\sqrt{\left(\tanh(x)^2-1\right)^2 c+(b+2c)\left(\tanh(x)^2-1\right)+a+b+c}}{2}-\frac{(b+2c)\ln\left(\frac{\frac{b}{2}+c+c\left(\tanh(x)^2-1\right)}{\sqrt{c}}+\sqrt{\left(\tanh(x)^2-1\right)^2 c+(b+2c)\left(\cdots\right.}\right.}{4\sqrt{c}}$

```
[In] int((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((tanh(x)^2-1)^2*c+(b+2*c)*(tanh(x)^2-1)+a+b+c)^(1/2)-1/4*(b+2*c)*ln((
1/2*b+c+c*(tanh(x)^2-1))/c^(1/2)+((tanh(x)^2-1)^2*c+(b+2*c)*(tanh(x)^2-1)+a
+b+c)^(1/2))/c^(1/2)+1/2*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(tanh(x)^2-1
)+2*(a+b+c)^(1/2)*((tanh(x)^2-1)^2*c+(b+2*c)*(tanh(x)^2-1)+a+b+c)^(1/2))/(t
anh(x)^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1827 vs. 2(108) = 216.

Time = 1.42 (sec) , antiderivative size = 7896, normalized size of antiderivative = 59.82

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}+c\tanh^4{(x)}}\tanh{(x)}\,dx$$

```
[In] integrate((a+b*tanh(x)**2+c*tanh(x)**4)**(1/2)*tanh(x),x)
```

```
[Out] Integral(sqrt(a + b*tanh(x)**2 + c*tanh(x)**4)*tanh(x), x)
```

Maxima [F]

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx = \int \sqrt{c\tanh(x)^4+b\tanh(x)^2+a}\tanh(x)\,dx$$

```
[In] integrate((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x, algorithm="maxima")
[Out] integrate(sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a)*tanh(x), x)
```

Giac [F]

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx = \int \sqrt{c\tanh(x)^4+b\tanh(x)^2+a}\tanh(x)\,dx$$

```
[In] integrate((a+b*tanh(x)^2+c*tanh(x)^4)^(1/2)*tanh(x),x, algorithm="giac")
[Out] integrate(sqrt(c*tanh(x)^4 + b*tanh(x)^2 + a)*tanh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)+c\tanh^4(x)}\,dx = \int \tanh(x)\,\sqrt{c\tanh(x)^4+b\tanh(x)^2+a}\,dx$$

```
[In] int(tanh(x)*(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2),x)
[Out] int(tanh(x)*(a + b*tanh(x)^2 + c*tanh(x)^4)^(1/2), x)
```

3.206 $\int e^{a+bx} \tanh^4(a+bx)\, dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int e^{a+bx} \tanh^4(a+bx)\, dx = \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\,(1+e^{2a+2bx})^3} - \frac{14e^{a+bx}}{3b\,(1+e^{2a+2bx})^2} + \frac{5e^{a+bx}}{b\,(1+e^{2a+2bx})} - \frac{3\arctan\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b+8/3*exp(b*x+a)/b/(1+exp(2*b*x+2*a))^3-14/3*exp(b*x+a)/b/(1+exp
(2*b*x+2*a))^2+5*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-3*arctan(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 398, 1272, 1171, 393, 209}

$$\int e^{a+bx} \tanh^4(a+bx)\, dx = -\frac{3\arctan\left(e^{a+bx}\right)}{b} + \frac{e^{a+bx}}{b} + \frac{5e^{a+bx}}{b\,(e^{2a+2bx}+1)} - \frac{14e^{a+bx}}{3b\,(e^{2a+2bx}+1)^2} + \frac{8e^{a+bx}}{3b\,(e^{2a+2bx}+1)^3}$$

```
[In] Int[E^(a + b*x)*Tanh[a + b*x]^4,x]
```

```
[Out] E^(a + b*x)/b + (8*E^(a + b*x))/(3*b*(1 + E^(2*a + 2*b*x))^3) - (14*E^(a +
b*x))/(3*b*(1 + E^(2*a + 2*b*x))^2) + (5*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*
x))) - (3*ArcTan[E^(a + b*x)])/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1272

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sy
mbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^
(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^
2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e^(2*p + m/2)*(q + 1)*x^
m*(a + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*(d + e*(2*q + 3)*x^2))],
 x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[
m/2, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(1+x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{8x^2(1+x^4)}{(1+x^2)^4}\right)\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} - \frac{8\text{Subst}\left(\int \frac{x^2(1+x^4)}{(1+x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\,(1+e^{2a+2bx})^3} + \frac{4\text{Subst}\left(\int \frac{-2+6x^2-6x^4}{(1+x^2)^3}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\,(1+e^{2a+2bx})^3} - \frac{14e^{a+bx}}{3b\,(1+e^{2a+2bx})^2} - \frac{\text{Subst}\left(\int \frac{-6+24x^2}{(1+x^2)^2}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\,(1+e^{2a+2bx})^3} - \frac{14e^{a+bx}}{3b\,(1+e^{2a+2bx})^2} + \frac{5e^{a+bx}}{b\,(1+e^{2a+2bx})} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\,(1+e^{2a+2bx})^3} - \frac{14e^{a+bx}}{3b\,(1+e^{2a+2bx})^2} + \frac{5e^{a+bx}}{b\,(1+e^{2a+2bx})} - \frac{3\arctan\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.71

$$\int e^{a+bx}\tanh^4(a+bx)\,dx = \frac{e^{a+bx}\left(12+25e^{2(a+bx)}+24e^{4(a+bx)}+3e^{6(a+bx)}\right)}{3b\,\left(1+e^{2(a+bx)}\right)^3} - \frac{3\arctan\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x]^4,x]
[Out] (E^(a + b*x)*(12 + 25*E^(2*(a + b*x)) + 24*E^(4*(a + b*x)) + 3*E^(6*(a + b*
x))))/(3*b*(1 + E^(2*(a + b*x)))^3) - (3*ArcTan[E^(a + b*x)])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

method	result	si
risch	$\frac{\mathrm{e}^{bx+a}}{b}+\frac{\mathrm{e}^{bx+a}\left(15\,\mathrm{e}^{4bx+4a}+16\,\mathrm{e}^{2bx+2a}+9\right)}{3b\left(1+\mathrm{e}^{2bx+2a}\right)^{3}}+\frac{3i\ln\left(\mathrm{e}^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(\mathrm{e}^{bx+a}+i\right)}{2b}$	92
derivativedivides	$\frac{\frac{\sinh(bx+a)^{4}}{\cosh(bx+a)^{3}}+\frac{4\sinh(bx+a)^{2}}{\cosh(bx+a)^{3}}+\frac{8}{3\cosh(bx+a)^{3}}+\frac{\sinh(bx+a)^{3}}{\cosh(bx+a)^{2}}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^{2}}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	10
default	$\frac{\frac{\sinh(bx+a)^{4}}{\cosh(bx+a)^{3}}+\frac{4\sinh(bx+a)^{2}}{\cosh(bx+a)^{3}}+\frac{8}{3\cosh(bx+a)^{3}}+\frac{\sinh(bx+a)^{3}}{\cosh(bx+a)^{2}}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^{2}}-\frac{3\,\mathrm{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(\mathrm{e}^{bx+a}\right)}{b}$	10

```
[In] int(exp(b*x+a)*tanh(b*x+a)^4,x,method=_RETURNVERBOSE)

[Out] exp(b*x+a)/b+1/3*exp(b*x+a)*(15*exp(4*b*x+4*a)+16*exp(2*b*x+2*a)+9)/b/(1+ex
p(2*b*x+2*a))^3+3/2*I/b*ln(exp(b*x+a)-I)-3/2*I/b*ln(exp(b*x+a)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(95) = 190.

Time = 0.25 (sec) , antiderivative size = 604, normalized size of antiderivative = 5.64

$$\int e^{a+bx}\tanh^4(a+bx)\,dx$$
$$=\frac{3\cosh(bx+a)^7+21\cosh(bx+a)\sinh(bx+a)^6+3\sinh(bx+a)^7+3\left(21\cosh(bx+a)^2+8\right)\sinh(bx}{}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^4,x, algorithm="fricas")

[Out] 1/3*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + 3*(21*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^5 + 24*cosh(b*x + a)^5 + 15*(
7*cosh(b*x + a)^3 + 8*cosh(b*x + a))*sinh(b*x + a)^4 + 5*(21*cosh(b*x + a)^
4 + 48*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 25*cosh(b*x + a)^3 + 3*(21*co
sh(b*x + a)^5 + 80*cosh(b*x + a)^3 + 25*cosh(b*x + a))*sinh(b*x + a)^2 - 9*
(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*
x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*
cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) +
sinh(b*x + a)) + 3*(7*cosh(b*x + a)^6 + 40*cosh(b*x + a)^4 + 25*cosh(b*x +
a)^2 + 4)*sinh(b*x + a) + 12*cosh(b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b
*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*
cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*
```

```
x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*
b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*
x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int e^{a+bx}\tanh^4(a+bx)\,dx = e^a\int e^{bx}\tanh^4{(a+bx)}\,dx$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)**4,x)
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\tanh^4(a+bx)\,dx = -\frac{3\arctan\left(e^{(bx+a)}\right)}{b} + \frac{e^{(bx+a)}}{b} + \frac{15\,e^{(5\,bx+5\,a)} + 16\,e^{(3\,bx+3\,a)} + 9\,e^{(bx+a)}}{3\,b\left(e^{(6\,bx+6\,a)} + 3\,e^{(4\,bx+4\,a)} + 3\,e^{(2\,bx+2\,a)} + 1\right)}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^4,x, algorithm="maxima")
[Out] -3*arctan(e^(b*x + a))/b + e^(b*x + a)/b + 1/3*(15*e^(5*b*x + 5*a) + 16*e^(
3*b*x + 3*a) + 9*e^(b*x + a))/(b*(e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) + 3*e
^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int e^{a+bx}\tanh^4(a+bx)\,dx = \frac{\frac{15\,e^{(5\,bx+5\,a)}+16\,e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^3} - 9\arctan\left(e^{(bx+a)}\right) + 3\,e^{(bx+a)}}{3\,b}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^4,x, algorithm="giac")
[Out] 1/3*((15*e^(5*b*x + 5*a) + 16*e^(3*b*x + 3*a) + 9*e^(b*x + a))/(e^(2*b*x +
2*a) + 1)^3 - 9*arctan(e^(b*x + a)) + 3*e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int e^{a+bx}\tanh^4(a+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}+\frac{\frac{4\,\mathrm{e}^{a+b\,x}}{3\,b}+\frac{4\,\mathrm{e}^{5\,a+5\,b\,x}}{3\,b}}{3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)}+\frac{11\,\mathrm{e}^{a+b\,x}}{3\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(exp(a + b*x)*tanh(a + b*x)^4,x)
```

```
[Out] exp(a + b*x)/b - (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) + ((
4*exp(a + b*x))/(3*b) + (4*exp(5*a + 5*b*x))/(3*b))/(3*exp(2*a + 2*b*x) + 3
*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1) - (2*exp(a + b*x))/(b*(2*exp(2*a
+ 2*b*x) + exp(4*a + 4*b*x) + 1)) + (11*exp(a + b*x))/(3*b*(exp(2*a + 2*b*x
) + 1))
```

3.207 $\int e^{a+bx} \tanh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int e^{a+bx}\tanh^3(a+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}-\frac{3\arctan\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1+exp(2*b*
x+2*a))-3*arctan(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 398, 1172, 12, 294, 209}

$$\int e^{a+bx}\tanh^3(a+bx)\,dx=-\frac{3\arctan\left(e^{a+bx}\right)}{b}+\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}-\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

```
[In] Int[E^(a + b*x)*Tanh[a + b*x]^3,x]
```

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 + E^(2*a + 2*b*x))) - (3*ArcTan[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{2\left(1+3x^4\right)}{\left(1+x^2\right)^3}\right)\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b}-\frac{2\text{Subst}\left(\int \frac{1+3x^4}{\left(1+x^2\right)^3}\,dx, x, e^{a+bx}\right)}{b}
\end{aligned}
$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{\text{Subst}\left(\int -\frac{12x^2}{(1+x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} - \frac{6\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} - \frac{3\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2} + \frac{3e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} - \frac{3\arctan\left(e^{a+bx}\right)}{b}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\tanh^3(a+bx)\,dx = \frac{e^{a+bx}\left(2+5e^{2(a+bx)}+e^{4(a+bx)}\right)}{b\left(1+e^{2(a+bx)}\right)^2} - \frac{3\arctan\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x]^3,x]
[Out] (E^(a + b*x)*(2 + 5*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(b*(1 + E^(2*(a + b
*x)))^2) - (3*ArcTan[E^(a + b*x)])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{e^{bx+a}}{b}+\frac{e^{bx+a}\left(3\,e^{2bx+2a}+1\right)}{b\left(1+e^{2bx+2a}\right)^2}+\frac{3i\ln\left(e^{bx+a}-i\right)}{2b}-\frac{3i\ln\left(e^{bx+a}+i\right)}{2b}$	80
derivativedivides	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)+\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	89
default	$\frac{\frac{\sinh(bx+a)^3}{\cosh(bx+a)^2}+\frac{3\sinh(bx+a)}{\cosh(bx+a)^2}-\frac{3\,\operatorname{sech}(bx+a)\tanh(bx+a)}{2}-3\arctan\left(e^{bx+a}\right)+\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}}{b}$	89

```
[In] int(exp(b*x+a)*tanh(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] exp(b*x+a)/b+exp(b*x+a)*(3*exp(2*b*x+2*a)+1)/b/(1+exp(2*b*x+2*a))^2+3/2*I/b
*ln(exp(b*x+a)-I)-3/2*I/b*ln(exp(b*x+a)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(71) = 142.

Time = 0.27 (sec) , antiderivative size = 339, normalized size of antiderivative = 4.40

$$\int e^{a+bx}\tanh^3(a+bx)\,dx = \frac{\cosh(bx+a)^5 + 5\cosh(bx+a)\sinh(bx+a)^4 + \sinh(bx+a)^5 + 5\left(2\cosh(bx+a)^2+1\right)\sinh(bx+a)^3}{}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^5 + 5*cosh(b*x + a)*sinh(b*x + a)^4 + sinh(b*x + a)^5 + 5*(2
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^3 + 5*cosh(b*x + a)^3 + 5*(2*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^4 + 4*cosh(b*x
+ a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x
 + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x
+ a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (5*cosh(b*x + a)^4 + 15*c
osh(b*x + a)^2 + 2)*sinh(b*x + a) + 2*cosh(b*x + a))/(b*cosh(b*x + a)^4 + 4
*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2
+ 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*co
sh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int e^{a+bx}\tanh^3(a+bx)\,dx = e^a\int e^{bx}\tanh^3(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)**3,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

$$\int e^{a+bx}\tanh^3(a+bx)\,dx = -\frac{3\arctan\left(e^{(bx+a)}\right)}{b} + \frac{e^{(bx+a)}}{b} + \frac{3e^{(3bx+3a)} + e^{(bx+a)}}{b(e^{(4bx+4a)} + 2e^{(2bx+2a)} + 1)}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*x + a))/b + e^(b*x + a)/b + (3*e^(3*b*x + 3*a) + e^(b*x + a)
)/(b*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int e^{a+bx}\tanh^3(a+bx)\,dx = \frac{\frac{3\,e^{(3\,bx+3\,a)}+e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^2} - 3\,\arctan\left(e^{(bx+a)}\right) + e^{(bx+a)}}{b}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] ((3*e^(3*b*x + 3*a) + e^(b*x + a))/(e^(2*b*x + 2*a) + 1)^2 - 3*arctan(e^(b*
x + a)) + e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int e^{a+bx}\tanh^3(a+bx)\,dx = \frac{e^{a+b\,x}}{b} - \frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)} + \frac{3\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(exp(a + b*x)*tanh(a + b*x)^3,x)
```

```
[Out] exp(a + b*x)/b - (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2
*exp(a + b*x))/(b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + (3*exp(a +
 b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.208 $\int e^{a+bx}\tanh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 51

$$\int e^{a+bx}\tanh^2(a+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}-\frac{2\arctan\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b+2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))-2*arctan(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 398, 294, 209}

$$\int e^{a+bx}\tanh^2(a+bx)\,dx=-\frac{2\arctan\left(e^{a+bx}\right)}{b}+\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(e^{2a+2bx}+1\right)}$$

```
[In] Int[E^(a + b*x)*Tanh[a + b*x]^2,x]
```

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 + E^(2*a + 2*b*x))) - (2*ArcTan[E^(a
+ b*x)])/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{4x^2}{\left(1+x^2\right)^2}\right)\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} - \frac{4\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} - \frac{2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)} - \frac{2\arctan\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\tanh^2(a+bx)\,dx=\frac{e^{a+bx}\left(1+\frac{2}{1+e^{2(a+bx)}}\right)-2\arctan\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x]^2,x]
[Out] (E^(a + b*x)*(1 + 2/(1 + E^(2*(a + b*x)))) - 2*ArcTan[E^(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}+\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	48
default	$\frac{\frac{\sinh(bx+a)^2}{\cosh(bx+a)}+\frac{2}{\cosh(bx+a)}+\sinh(bx+a)-2\arctan\left(e^{bx+a}\right)}{b}$	48
risch	$\frac{e^{bx+a}}{b}+\frac{2\,e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}+\frac{i\ln\left(e^{bx+a}-i\right)}{b}-\frac{i\ln\left(e^{bx+a}+i\right)}{b}$	68

```
[In] int(exp(b*x+a)*tanh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)^2/cosh(b*x+a)+2/cosh(b*x+a)+sinh(b*x+a)-2*arctan(exp(b*x+a
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(47) = 94$.

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.88

$$\int e^{a+bx}\tanh^2(a+bx)\,dx$$
$$=\frac{\cosh(bx+a)^3+3\cosh(bx+a)\sinh(bx+a)^2+\sinh(bx+a)^3-2\left(\cosh(bx+a)^2+2\cosh(bx+a)\sinh\right.}{b\cosh(bx+a)^2+2\,b\cos}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^2,x, algorithm="fricas")
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - 2*(c
osh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)*arcta
n(cosh(b*x + a) + sinh(b*x + a)) + 3*(cosh(b*x + a)^2 + 1)*sinh(b*x + a) +
3*cosh(b*x + a))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*s
inh(b*x + a)^2 + b)
```

Sympy [F]

$$\int e^{a+bx}\tanh^2(a+bx)\,dx=e^a\int e^{bx}\tanh^2{(a+bx)}\,dx$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)**2,x)
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\tanh^2(a+bx)\,dx=-\frac{2\arctan\left(e^{(bx+a)}\right)}{b}+\frac{e^{(bx+a)}}{b}+\frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}+1)}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^2,x, algorithm="maxima")
[Out] -2*arctan(e^(b*x + a))/b + e^(b*x + a)/b + 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a
) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\tanh^2(a+bx)\,dx=\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}-2\arctan\left(e^{(bx+a)}\right)+e^{(bx+a)}}{b}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a)^2,x, algorithm="giac")
[Out] (2*e^(b*x + a)/(e^(2*b*x + 2*a) + 1) - 2*arctan(e^(b*x + a)) + e^(b*x + a))
/b
```

Mupad [B] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int e^{a+bx}\tanh^2(a+bx)\,dx = \frac{e^{a+b\,x}}{b} - \frac{2\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} + \frac{2\,e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(exp(a + b*x)*tanh(a + b*x)^2,x)
```

```
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) + (2
*exp(a + b*x))/(b*(exp(2*a + 2*b*x) + 1))
```

3.209 $\int e^{a+bx} \tanh(a+bx)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int e^{a+bx} \tanh(a+bx)\, dx = \frac{e^{a+bx}}{b} - \frac{2\arctan\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*arctan(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2320, 396, 209}

$$\int e^{a+bx} \tanh(a+bx)\, dx = \frac{e^{a+bx}}{b} - \frac{2\arctan\left(e^{a+bx}\right)}{b}$$

```
[In] Int[E^(a + b*x)*Tanh[a + b*x],x]
```

```
[Out] E^(a + b*x)/b - (2*ArcTan[E^(a + b*x)])/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} - \frac{2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} - \frac{2\arctan\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\tanh(a+bx)\,dx = \frac{e^{a+bx} - 2\arctan\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Tanh[a + b*x],x]
[Out] (E^(a + b*x) - 2*ArcTan[E^(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\sinh(bx+a)-2\arctan\left(\mathrm{e}^{bx+a}\right)+\cosh(bx+a)}{b}$	27
default	$\frac{\sinh(bx+a)-2\arctan\left(\mathrm{e}^{bx+a}\right)+\cosh(bx+a)}{b}$	27
risch	$\frac{\mathrm{e}^{bx+a}}{b}+\frac{i\ln\left(\mathrm{e}^{bx+a}-i\right)}{b}-\frac{i\ln\left(\mathrm{e}^{bx+a}+i\right)}{b}$	44

```
[In] int(exp(b*x+a)*tanh(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)-2*arctan(exp(b*x+a))+cosh(b*x+a))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int e^{a+bx}\tanh(a+bx)\,dx = -\frac{2\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)-\cosh\left(bx+a\right)-\sinh\left(bx+a\right)}{b}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x, algorithm="fricas")
[Out] -(2*arctan(cosh(b*x + a) + sinh(b*x + a)) - cosh(b*x + a) - sinh(b*x + a))/
b
```

Sympy [F]

$$\int e^{a+bx}\tanh(a+bx)\,dx = e^{a}\int e^{bx}\tanh\left(a+bx\right)dx$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*tanh(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\tanh(a+bx)\,dx = -\frac{2\arctan\left(e^{(bx+a)}\right)}{b}+\frac{e^{(bx+a)}}{b}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x, algorithm="maxima")
[Out] -2*arctan(e^(b*x + a))/b + e^(b*x + a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\tanh(a+bx)\,dx = -\frac{2\,\arctan\left(e^{(bx+a)}\right)-e^{(bx+a)}}{b}$$

```
[In] integrate(exp(b*x+a)*tanh(b*x+a),x, algorithm="giac")
[Out] -(2*arctan(e^(b*x + a)) - e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\tanh(a+bx)\,dx = \frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

```
[In] int(exp(a + b*x)*tanh(a + b*x),x)
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
```

3.210 $\int e^{a+bx} \coth(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 25

$$\int e^{a+bx} \coth(a+bx)\,dx = \frac{e^{a+bx}}{b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2320, 396, 212}

$$\int e^{a+bx} \coth(a+bx)\,dx = \frac{e^{a+bx}}{b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x],x]
```

```
[Out] E^(a + b*x)/b - (2*ArcTanh[E^(a + b*x)])/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{e^{a+bx}}{b} - \frac{2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\ &= \frac{e^{a+bx}}{b} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\coth(a+bx)\,dx = \frac{e^{a+bx} - 2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x],x]
[Out] (E^(a + b*x) - 2*ArcTanh[E^(a + b*x)])/b
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\sinh(bx+a)+\cosh(bx+a)-2\,\text{arctanh}\left(e^{bx+a}\right)}{b}$	27
default	$\frac{\sinh(bx+a)+\cosh(bx+a)-2\,\text{arctanh}\left(e^{bx+a}\right)}{b}$	27
risch	$\frac{e^{bx+a}}{b}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	39

```
[In] int(exp(b*x+a)*coth(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(sinh(b*x+a)+cosh(b*x+a)-2*arctanh(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int e^{a+bx}\coth(a+bx)\,dx$$
$$= \frac{\cosh(bx+a) - \log(\cosh(bx+a) + \sinh(bx+a) + 1) + \log(\cosh(bx+a) + \sinh(bx+a) - 1) + \sinh(bx}{b}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x, algorithm="fricas")
[Out] (cosh(b*x + a) - log(cosh(b*x + a) + sinh(b*x + a) + 1) + log(cosh(b*x + a)
 + sinh(b*x + a) - 1) + sinh(b*x + a))/b
```

Sympy [F]

$$\int e^{a+bx}\coth(a+bx)\,dx = e^{a}\int e^{bx}\coth(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x)
[Out] exp(a)*Integral(exp(b*x)*coth(a + b*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int e^{a+bx}\coth(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)} + 1\right)}{b} + \frac{\log\left(e^{(bx+a)} - 1\right)}{b}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x, algorithm="maxima")
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{e^{(bx+a)}-\log\left(e^{(bx+a)}+1\right)+\log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a),x, algorithm="giac")
[Out] (e^(b*x + a) - log(e^(b*x + a) + 1) + log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int e^{a+bx}\coth(a+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}$$

```
[In] int(coth(a + b*x)*exp(a + b*x),x)
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2)
```

3.211 $\int e^{a+bx}\coth^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int e^{a+bx}\coth^2(a+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b+2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-2*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2320, 398, 294, 212}

$$\int e^{a+bx}\coth^2(a+bx)\,dx=-\frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{e^{a+bx}}{b}+\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] E^(a + b*x)/b + (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))) - (2*ArcTanh[E^(a
 + b*x)])/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{4x^2}{(1-x^2)^2}\right)\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{4\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{2\text{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.38

$$\int e^{a+bx}\coth^2(a+bx)\,dx$$
$$=\frac{e^{a+bx}\left(\frac{1}{48}e^{-4(a+bx)}\left(-375-713e^{2(a+bx)}-181e^{4(a+bx)}+61e^{6(a+bx)}+\frac{3\left(125+196e^{2(a+bx)}-14e^{4(a+bx)}-52e^{6(a+bx)}+e^{8(a+}\right.}{\sqrt{e^{2(a+bx)}}}\right.\right.}{b}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^2,x]
```

```
[Out] (E^(a + b*x)*((-375 - 713*E^(2*(a + b*x)) - 181*E^(4*(a + b*x)) + 61*E^(6*(
a + b*x)) + (3*(125 + 196*E^(2*(a + b*x)) - 14*E^(4*(a + b*x)) - 52*E^(6*(a
 + b*x)) + E^(8*(a + b*x)))*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a +
b*x))])/(48*E^(4*(a + b*x))) + (4*(E^(a + b*x) + E^(3*(a + b*x)))^2*Hyperge
ometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*(a + b*x))])/105))/b
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)+\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	48
default	$\frac{\cosh(bx+a)-2\,\operatorname{arctanh}\left(e^{bx+a}\right)+\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}}{b}$	48
risch	$\frac{e^{bx+a}}{b}-\frac{2\,e^{bx+a}}{b\left(e^{2bx+2a}-1\right)}+\frac{\ln\left(e^{bx+a}-1\right)}{b}-\frac{\ln\left(e^{bx+a}+1\right)}{b}$	63

```
[In] int(exp(b*x+a)*coth(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(cosh(b*x+a)-2*arctanh(exp(b*x+a))+cosh(b*x+a)^2/sinh(b*x+a)-2/sinh(b*x
+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(47) = 94$.

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.74

$$\int e^{a+bx}\coth^2(a+bx)\,dx$$
$$= \frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 - \left(\cosh(bx+a)^2 + 2\cosh(bx+a)\sinh\right.}{}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 - (cos
h(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*log(cos
h(b*x + a) + sinh(b*x + a) + 1) + (cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 3*(
cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^2 +
2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 - b)
```

Sympy [F]

$$\int e^{a+bx}\coth^2(a+bx)\,dx = e^a\int e^{bx}\coth^2(a+bx)\,dx$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)**2,x)
```

```
[Out] exp(a)*Integral(exp(b*x)*coth(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{\log\left(e^{(bx+a)}+1\right)}{b} + \frac{\log\left(e^{(bx+a)}-1\right)}{b} - \frac{2\,e^{(bx+a)}}{b\left(e^{(2\,bx+2\,a)}-1\right)}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] e^(b*x + a)/b - log(e^(b*x + a) + 1)/b + log(e^(b*x + a) - 1)/b - 2*e^(b*x
+ a)/(b*(e^(2*b*x + 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int e^{a+bx}\coth^2(a+bx)\,dx = -\frac{\frac{2\,e^{(bx+a)}}{e^{(2\,bx+2\,a)}-1} - e^{(bx+a)} + \log\left(e^{(bx+a)}+1\right) - \log\left(\left|e^{(bx+a)}-1\right|\right)}{b}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*e^(b*x + a)/(e^(2*b*x + 2*a) - 1) - e^(b*x + a) + log(e^(b*x + a) + 1)
- log(abs(e^(b*x + a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int e^{a+bx}\coth^2(a+bx)\,dx = \frac{\mathrm{e}^{a+b\,x}}{b} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(coth(a + b*x)^2*exp(a + b*x),x)
```

```
[Out] exp(a + b*x)/b - (2*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
(2*exp(a + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.212 $\int e^{a+bx}\coth^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int e^{a+bx}\coth^3(a+bx)\,dx=\frac{e^{a+bx}}{b}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b-2*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^2+3*exp(b*x+a)/b/(1-exp(2*b*
x+2*a))-3*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 398, 1172, 12, 294, 213}

$$\int e^{a+bx}\coth^3(a+bx)\,dx=-\frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}+\frac{e^{a+bx}}{b}+\frac{3e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{2e^{a+bx}}{b\left(1-e^{2a+2bx}\right)^2}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] E^(a + b*x)/b - (2*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*x))^2) + (3*E^(a + b*x
))/(b*(1 - E^(2*a + 2*b*x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)\,dx, x, e^{a+bx}\right)}{b} \\
&= \frac{e^{a+bx}}{b} + \frac{2\text{Subst}\left(\int \frac{1+3x^4}{(-1+x^2)^3}\,dx, x, e^{a+bx}\right)}{b}
\end{aligned}
$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b(1-e^{2a+2bx})^2} + \frac{\text{Subst}\left(\int \frac{12x^2}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{2b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b(1-e^{2a+2bx})^2} + \frac{6\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b(1-e^{2a+2bx})^2} + \frac{3e^{a+bx}}{b(1-e^{2a+2bx})} + \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{a+bx}\right)}{b}$$

$$= \frac{e^{a+bx}}{b} - \frac{2e^{a+bx}}{b(1-e^{2a+2bx})^2} + \frac{3e^{a+bx}}{b(1-e^{2a+2bx})} - \frac{3\text{arctanh}\left(e^{a+bx}\right)}{b}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.48 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.53

$$\int e^{a+bx}\coth^3(a+bx)\,dx =$$

$$-\frac{e^{-5(a+bx)}\Big(-21\big(252105 + 507305e^{2(a+bx)} + 173916e^{4(a+bx)} - 154296e^{6(a+bx)} - 73885e^{8(a+bx)} + 4887e^{10(a}}{}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^3,x]
```

```
[Out] -1/60480*(-21*(252105 + 507305*E^(2*(a + b*x)) + 173916*E^(4*(a + b*x)) - 1
54296*E^(6*(a + b*x)) - 73885*E^(8*(a + b*x)) + 4887*E^(10*(a + b*x))) - (3
15*(-16807 - 28218*E^(2*(a + b*x)) + 1173*E^(4*(a + b*x)) + 17748*E^(6*(a +
 b*x)) + 4299*E^(8*(a + b*x)) - 1434*E^(10*(a + b*x)) + 7*E^(12*(a + b*x)))
*ArcTanh[Sqrt[E^(2*(a + b*x))]])/Sqrt[E^(2*(a + b*x))] + 384*E^(8*(a + b*x)
)*(1 + E^(2*(a + b*x)))^2*(7 + 5*E^(2*(a + b*x)))*HypergeometricPFQ[{3/2, 2
, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*(a + b*x))] + 256*E^(8*(a + b*x))*(1 + E^
(2*(a + b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}
, E^(2*(a + b*x))])/(b*E^(5*(a + b*x)))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{e^{bx+a}}{b}-\frac{e^{bx+a}(3\,e^{2bx+2a}-1)}{b(e^{2bx+2a}-1)^2}-\frac{3\ln(e^{bx+a}+1)}{2b}+\frac{3\ln(e^{bx+a}-1)}{2b}$	77
derivativedivides	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}+\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\,\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\,\operatorname{arctanh}(e^{bx+a})}{b}$	89
default	$\frac{\frac{\cosh(bx+a)^2}{\sinh(bx+a)}-\frac{2}{\sinh(bx+a)}+\frac{\cosh(bx+a)^3}{\sinh(bx+a)^2}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^2}+\frac{3\,\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\,\operatorname{arctanh}(e^{bx+a})}{b}$	89

```
[In] int(exp(b*x+a)*coth(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] exp(b*x+a)/b-exp(b*x+a)*(3*exp(2*b*x+2*a)-1)/b/(exp(2*b*x+2*a)-1)^2-3/2/b*l
n(exp(b*x+a)+1)+3/2/b*ln(exp(b*x+a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.67

$$\int e^{a+bx}\coth^3(a+bx)\,dx$$
$$= \frac{2\cosh(bx+a)^5+10\cosh(bx+a)\sinh(bx+a)^4+2\sinh(bx+a)^5+10\left(2\cosh(bx+a)^2-1\right)\sinh(bx+\cdots}{\cdots}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*x + a)^5 + 10*cosh(b*x + a)*sinh(b*x + a)^4 + 2*sinh(b*x + a)
^5 + 10*(2*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^3 - 10*cosh(b*x + a)^3 + 10*(
2*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^2 - 3*(cosh(b*x + a)^4 +
 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x + a)^2 -
 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh(b*x + a
))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + 3*(cosh(b*x
+ a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cosh(b*x
+ a)^2 - 1)*sinh(b*x + a)^2 - 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - cosh
(b*x + a))*sinh(b*x + a) + 1)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + 2*(5
*cosh(b*x + a)^4 - 15*cosh(b*x + a)^2 + 2)*sinh(b*x + a) + 4*cosh(b*x + a))
/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4
 - 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^2 + 4*(b
*cosh(b*x + a)^3 - b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int e^{a+bx}\coth^3(a+bx)\,dx = e^a\int e^{bx}\coth^3(a+bx)\,dx$$

[In] integrate(exp(b*x+a)*coth(b*x+a)**3,x)

[Out] exp(a)*Integral(exp(b*x)*coth(a + b*x)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int e^{a+bx}\coth^3(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{3\,\log\left(e^{(bx+a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(bx+a)}-1\right)}{2\,b} - \frac{3\,e^{(3\,bx+3\,a)} - e^{(bx+a)}}{b(e^{(4\,bx+4\,a)} - 2\,e^{(2\,bx+2\,a)} + 1)}$$

[In] integrate(exp(b*x+a)*coth(b*x+a)^3,x, algorithm="maxima")

[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b - (3*e^(3*b*x + 3*a) - e^(b*x + a))/(b*(e^(4*b*x + 4*a) - 2*e^(2*b*x + 2*a) + 1))

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int e^{a+bx}\coth^3(a+bx)\,dx = -\frac{\frac{2\,\left(3\,e^{(3\,bx+3\,a)}-e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^2} - 2\,e^{(bx+a)} + 3\,\log\left(e^{(bx+a)}+1\right) - 3\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{2\,b}$$

[In] integrate(exp(b*x+a)*coth(b*x+a)^3,x, algorithm="giac")

[Out] -1/2*(2*(3*e^(3*b*x + 3*a) - e^(b*x + a))/(e^(2*b*x + 2*a) - 1)^2 - 2*e^(b*x + a) + 3*log(e^(b*x + a) + 1) - 3*log(abs(e^(b*x + a) - 1)))/b

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

$$\int e^{a+bx}\coth^3(a+bx)\,dx=\frac{\mathrm{e}^{a+b\,x}}{b}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{4\,a+4\,b\,x}-2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{3\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(coth(a + b*x)^3*exp(a + b*x),x)
```

```
[Out] exp(a + b*x)/b - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
(2*exp(a + b*x))/(b*(exp(4*a + 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (3*exp(a
 + b*x))/(b*(exp(2*a + 2*b*x) - 1))
```

3.213 $\int e^{a+bx}\coth^4(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 113

$$\int e^{a+bx}\coth^4(a+bx)\,dx=\frac{e^{a+bx}}{b}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}-\frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2}+\frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}$$

```
[Out] exp(b*x+a)/b+8/3*exp(b*x+a)/b/(1-exp(2*b*x+2*a))^3-14/3*exp(b*x+a)/b/(1-exp
(2*b*x+2*a))^2+5*exp(b*x+a)/b/(1-exp(2*b*x+2*a))-3*arctanh(exp(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2320, 398, 1272, 1171, 393, 212}

$$\int e^{a+bx}\coth^4(a+bx)\,dx=-\frac{3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}+\frac{e^{a+bx}}{b}+\frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)}-\frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2}+\frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3}$$

```
[In] Int[E^(a + b*x)*Coth[a + b*x]^4,x]
```

```
[Out] E^(a + b*x)/b + (8*E^(a + b*x))/(3*b*(1 - E^(2*a + 2*b*x))^3) - (14*E^(a +
b*x))/(3*b*(1 - E^(2*a + 2*b*x))^2) + (5*E^(a + b*x))/(b*(1 - E^(2*a + 2*b*
x))) - (3*ArcTanh[E^(a + b*x)])/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1272

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sy
mbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*((d + e*x^2)^(q + 1)/(2*e^
(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^
2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e^(2*p + m/2)*(q + 1)*x^
m*(a + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*(d + e*(2*q + 3)*x^2))],
 x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[
m/2, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{8x^2(1+x^4)}{(1-x^2)^4}\right)\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{8\text{Subst}\left(\int \frac{x^2(1+x^4)}{(1-x^2)^4}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3} + \frac{4\text{Subst}\left(\int \frac{-2-6x^2-6x^4}{(1-x^2)^3}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2} - \frac{\text{Subst}\left(\int \frac{-6-24x^2}{(1-x^2)^2}\,dx, x, e^{a+bx}\right)}{3b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2} + \frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{a+bx}\right)}{b}\\
&= \frac{e^{a+bx}}{b} + \frac{8e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^3} - \frac{14e^{a+bx}}{3b\left(1-e^{2a+2bx}\right)^2} + \frac{5e^{a+bx}}{b\left(1-e^{2a+2bx}\right)} - \frac{3\operatorname{arctanh}\left(e^{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 10.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int e^{a+bx}\coth^4(a+bx)\,dx$$
$$= \frac{-24e^{a+bx}+50e^{3(a+bx)}-48e^{5(a+bx)}+6e^{7(a+bx)}+9\left(-1+e^{2(a+bx)}\right)^3\log\left(1-e^{a+bx}\right)-9\left(-1+e^{2(a+bx)}\right)^3\log}{6b\left(-1+e^{2(a+bx)}\right)^3}$$

```
[In] Integrate[E^(a + b*x)*Coth[a + b*x]^4,x]
```

```
[Out] (-24*E^(a + b*x) + 50*E^(3*(a + b*x)) - 48*E^(5*(a + b*x)) + 6*E^(7*(a + b*
x)) + 9*(-1 + E^(2*(a + b*x)))^3*Log[1 - E^(a + b*x)] - 9*(-1 + E^(2*(a + b
*x)))^3*Log[1 + E^(a + b*x)])/(6*b*(-1 + E^(2*(a + b*x)))^3)
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

method	result	si
risch	$\frac{e^{bx+a}}{b}-\frac{e^{bx+a}\left(15\,e^{4bx+4a}-16\,e^{2bx+2a}+9\right)}{3b\left(e^{2bx+2a}-1\right)^{3}}-\frac{3\ln\left(e^{bx+a}+1\right)}{2b}+\frac{3\ln\left(e^{bx+a}-1\right)}{2b}$	88
derivativedivides	$\frac{\frac{\cosh(bx+a)^{3}}{\sinh(bx+a)^{2}}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^{2}}+\frac{3\,\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\,\operatorname{arctanh}\left(e^{bx+a}\right)+\frac{\cosh(bx+a)^{4}}{\sinh(bx+a)^{3}}-\frac{4\cosh(bx+a)^{2}}{\sinh(bx+a)^{3}}+\frac{8}{3\sinh(bx+a)^{3}}}{b}$	10
default	$\frac{\frac{\cosh(bx+a)^{3}}{\sinh(bx+a)^{2}}-\frac{3\cosh(bx+a)}{\sinh(bx+a)^{2}}+\frac{3\,\operatorname{csch}(bx+a)\coth(bx+a)}{2}-3\,\operatorname{arctanh}\left(e^{bx+a}\right)+\frac{\cosh(bx+a)^{4}}{\sinh(bx+a)^{3}}-\frac{4\cosh(bx+a)^{2}}{\sinh(bx+a)^{3}}+\frac{8}{3\sinh(bx+a)^{3}}}{b}$	10

```
[In] int(exp(b*x+a)*coth(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] exp(b*x+a)/b-1/3*exp(b*x+a)*(15*exp(4*b*x+4*a)-16*exp(2*b*x+2*a)+9)/b/(exp(
2*b*x+2*a)-1)^3-3/2/b*ln(exp(b*x+a)+1)+3/2/b*ln(exp(b*x+a)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 796, normalized size of antiderivative = 7.04

$$\int e^{a+bx}\coth^4(a+bx)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*cosh(b*x + a)^7 + 42*cosh(b*x + a)*sinh(b*x + a)^6 + 6*sinh(b*x + a)
^7 + 6*(21*cosh(b*x + a)^2 - 8)*sinh(b*x + a)^5 - 48*cosh(b*x + a)^5 + 30*(
7*cosh(b*x + a)^3 - 8*cosh(b*x + a))*sinh(b*x + a)^4 + 10*(21*cosh(b*x + a)
^4 - 48*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 50*cosh(b*x + a)^3 + 6*(21*c
osh(b*x + a)^5 - 80*cosh(b*x + a)^3 + 25*cosh(b*x + a))*sinh(b*x + a)^2 - 9
*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(
5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x
+ a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 - 6*cosh(b
*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 - 2
*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*log(cosh(b*x + a) + si
nh(b*x + a) + 1) + 9*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + s
inh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 - 1)*sinh(b*x + a)^4 - 3*cosh(b*x + a
)^4 + 4*(5*cosh(b*x + a)^3 - 3*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b
*x + a)^4 - 6*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*
(cosh(b*x + a)^5 - 2*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) - 1)*lo
g(cosh(b*x + a) + sinh(b*x + a) - 1) + 6*(7*cosh(b*x + a)^6 - 40*cosh(b*x +
 a)^4 + 25*cosh(b*x + a)^2 - 4)*sinh(b*x + a) - 24*cosh(b*x + a))/(b*cosh(b
*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 - 3*b*cos
```

```
h(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 - b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*
x + a)^3 - 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*
b*cosh(b*x + a)^4 - 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*
x + a)^5 - 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) - b)
```

Sympy [F]

$$\int e^{a+bx}\coth^4(a+bx)\,dx = e^a\int e^{bx}\coth^4\left(a+bx\right)dx$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)**4,x)
[Out] exp(a)*Integral(exp(b*x)*coth(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97

$$\int e^{a+bx}\coth^4(a+bx)\,dx = \frac{e^{(bx+a)}}{b} - \frac{3\,\log\left(e^{(bx+a)}+1\right)}{2\,b} + \frac{3\,\log\left(e^{(bx+a)}-1\right)}{2\,b} - \frac{15\,e^{(5\,bx+5\,a)} - 16\,e^{(3\,bx+3\,a)} + 9\,e^{(bx+a)}}{3\,b(e^{(6\,bx+6\,a)} - 3\,e^{(4\,bx+4\,a)} + 3\,e^{(2\,bx+2\,a)} - 1)}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^4,x, algorithm="maxima")
[Out] e^(b*x + a)/b - 3/2*log(e^(b*x + a) + 1)/b + 3/2*log(e^(b*x + a) - 1)/b - 1
/3*(15*e^(5*b*x + 5*a) - 16*e^(3*b*x + 3*a) + 9*e^(b*x + a))/(b*(e^(6*b*x +
 6*a) - 3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\coth^4(a+bx)\,dx = -\frac{\frac{2\,\left(15\,e^{(5\,bx+5\,a)}-16\,e^{(3\,bx+3\,a)}+9\,e^{(bx+a)}\right)}{\left(e^{(2\,bx+2\,a)}-1\right)^3} - 6\,e^{(bx+a)} + 9\,\log\left(e^{(bx+a)}+1\right) - 9\,\log\left(\left|e^{(bx+a)}-1\right|\right)}{6\,b}$$

```
[In] integrate(exp(b*x+a)*coth(b*x+a)^4,x, algorithm="giac")
[Out] -1/6*(2*(15*e^(5*b*x + 5*a) - 16*e^(3*b*x + 3*a) + 9*e^(b*x + a))/(e^(2*b*x
 + 2*a) - 1)^3 - 6*e^(b*x + a) + 9*log(e^(b*x + a) + 1) - 9*log(abs(e^(b*x
+ a) - 1)))/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

$$\int e^{a+bx}\coth^4(a+bx)\,dx = \frac{e^{a+b\,x}}{b}-\frac{3\,\mathrm{atan}\left(\frac{e^{b\,x}\,e^{a}\,\sqrt{-b^2}}{b}\right)}{\sqrt{-b^2}}-\frac{\frac{4\,e^{a+b\,x}}{3\,b}+\frac{4\,e^{5\,a+5\,b\,x}}{3\,b}}{3\,e^{2\,a+2\,b\,x}-3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}-1}-\frac{2\,e^{a+b\,x}}{b\,\left(e^{4\,a+4\,b\,x}-2\,e^{2\,a+2\,b\,x}+1\right)}-\frac{11\,e^{a+b\,x}}{3\,b\,\left(e^{2\,a+2\,b\,x}-1\right)}$$

```
[In] int(coth(a + b*x)^4*exp(a + b*x),x)
```

```
[Out] exp(a + b*x)/b - (3*atan((exp(b*x)*exp(a)*(-b^2)^(1/2))/b))/(-b^2)^(1/2) -
((4*exp(a + b*x))/(3*b) + (4*exp(5*a + 5*b*x))/(3*b))/(3*exp(2*a + 2*b*x) -
 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) - 1) - (2*exp(a + b*x))/(b*(exp(4*a
+ 4*b*x) - 2*exp(2*a + 2*b*x) + 1)) - (11*exp(a + b*x))/(3*b*(exp(2*a + 2*b
*x) - 1))
```

3.214 $\int e^x \tanh^2(2x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int e^x \tanh^2(2x)\,dx = e^x + \frac{e^x}{1+e^{4x}} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

```
[Out] exp(x)+exp(x)/(1+exp(4*x))-1/4*arctan(-1+exp(x)*2^(1/2))*2^(1/2)-1/4*arctan
(1+exp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/8*ln
(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2320, 398, 294, 217, 1179, 642, 1176, 631, 210}

$$\int e^x \tanh^2(2x)\,dx = \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\arctan\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}} + e^x + \frac{e^x}{e^{4x}+1} + \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}} - \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}$$

```
[In] Int[E^x*Tanh[2*x]^2,x]
```

```
[Out] E^x + E^x/(1 + E^(4*x)) + ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) - ArcTan[1 +
Sqrt[2]*E^x]/(2*Sqrt[2]) + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2]) - Log
[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^4)^2}{(1+x^4)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1-\frac{4x^4}{(1+x^4)^2}\right)\,dx, x, e^x\right)\\
&= e^x - 4\text{Subst}\left(\int \frac{x^4}{(1+x^4)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{1+e^{4x}} - \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{1+e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{1+e^{4x}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}}\\
&= e^x + \frac{e^x}{1+e^{4x}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{2\sqrt{2}}
\end{aligned}$$

$$= e^x + \frac{e^x}{1+e^{4x}} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.42

$$\int e^x \tanh^2(2x)\,dx = e^x + \frac{e^x}{1+e^{4x}} + \frac{1}{4}\text{RootSum}\left[1+\#1^4\&,\frac{x-\log\left(e^x-\#1\right)}{\#1^3}\&\right]$$

```
[In] Integrate[E^x*Tanh[2*x]^2,x]
[Out] E^x + E^x/(1 + E^(4*x)) + RootSum[1 + #1^4 & , (x - Log[E^x - #1])/#1^3 & ]
/4
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.31

method	result
risch	$\mathrm{e}^x + \frac{\mathrm{e}^x}{1+\mathrm{e}^{4x}} + \left(\sum_{\_R=\mathrm{RootOf}(256\_Z^4+1)} \_R\ln\left(\mathrm{e}^x-4\_R\right)\right)$
default	$-\frac{2}{\tanh(\frac{x}{2})-1} - \frac{2\left(-\frac{\tanh(\frac{x}{2})^3}{2}-\frac{3\tanh(\frac{x}{2})^2}{2}+\frac{\tanh(\frac{x}{2})}{2}-\frac{1}{2}\right)}{\tanh(\frac{x}{2})^4+6\tanh(\frac{x}{2})^2+1} + \frac{\sqrt{2}\ln\left(\tanh(\frac{x}{2})^2+3-2\sqrt{2}\right)}{8} + \frac{\left(\sqrt{2}-2\right)\arctan\left(\frac{2\tanh(\frac{x}{2})}{2\sqrt{2}-2}\right)}{4\sqrt{2}-4} -$

```
[In] int(exp(x)*tanh(2*x)^2,x,method=_RETURNVERBOSE)
[Out] exp(x)+exp(x)/(1+exp(4*x))+sum(_R*ln(exp(x)-4*_R),_R=RootOf(256*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.51

$$\int e^x \tanh^2(2x)\,dx$$
$$= \frac{8\cosh(x)^5 + 80\cosh(x)^3\sinh(x)^2 + 80\cosh(x)^2\sinh(x)^3 + 40\cosh(x)\sinh(x)^4 + 8\sinh(x)^5 + (-(i \cdots}{}$$

```
[In] integrate(exp(x)*tanh(2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*cosh(x)^5 + 80*cosh(x)^3*sinh(x)^2 + 80*cosh(x)^2*sinh(x)^3 + 40*cos
h(x)*sinh(x)^4 + 8*sinh(x)^5 + (-(I + 1)*sqrt(2)*cosh(x)^4 - (4*I + 4)*sqrt
(2)*cosh(x)^3*sinh(x) - (6*I + 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 - (4*I + 4)*s
qrt(2)*cosh(x)*sinh(x)^3 - (I + 1)*sqrt(2)*sinh(x)^4 - (I + 1)*sqrt(2))*log
((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + ((I - 1)*sqrt(2)*cosh(x)^4 + (4
*I - 4)*sqrt(2)*cosh(x)^3*sinh(x) + (6*I - 6)*sqrt(2)*cosh(x)^2*sinh(x)^2 +
 (4*I - 4)*sqrt(2)*cosh(x)*sinh(x)^3 + (I - 1)*sqrt(2)*sinh(x)^4 + (I - 1)*
sqrt(2))*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (-(I - 1)*sqrt(2)*
cosh(x)^4 - (4*I - 4)*sqrt(2)*cosh(x)^3*sinh(x) - (6*I - 6)*sqrt(2)*cosh(x)
^2*sinh(x)^2 - (4*I - 4)*sqrt(2)*cosh(x)*sinh(x)^3 - (I - 1)*sqrt(2)*sinh(x
)^4 - (I - 1)*sqrt(2))*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + ((I +
 1)*sqrt(2)*cosh(x)^4 + (4*I + 4)*sqrt(2)*cosh(x)^3*sinh(x) + (6*I + 6)*sqr
t(2)*cosh(x)^2*sinh(x)^2 + (4*I + 4)*sqrt(2)*cosh(x)*sinh(x)^3 + (I + 1)*sq
rt(2)*sinh(x)^4 + (I + 1)*sqrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sin
h(x)) + 8*(5*cosh(x)^4 + 2)*sinh(x) + 16*cosh(x))/(cosh(x)^4 + 4*cosh(x)^3*
sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 1)
```

Sympy [F]

$$\int e^x \tanh^2(2x)\,dx = \int e^x \tanh^2(2x)\,dx$$

```
[In] integrate(exp(x)*tanh(2*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(2*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int e^x \tanh^2(2x)\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{e^x}{e^{(4x)}+1} + e^x$$

```
[In] integrate(exp(x)*tanh(2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x/(e^(4*x) + 1) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.79

$$\int e^x \tanh^2(2x)\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) + \frac{e^x}{e^{(4x)}+1} + e^x$$

```
[In] integrate(exp(x)*tanh(2*x)^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x/(e^(4*x) + 1) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int e^x \tanh^2(2x)\,dx = \mathrm{e}^x + \frac{\mathrm{e}^x}{\mathrm{e}^{4x}+1} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\mathrm{e}^x - \frac{\sqrt{2}}{2}\right)\right)}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\mathrm{e}^x + \frac{\sqrt{2}}{2}\right)\right)}{4} + \frac{\sqrt{2}\,\ln\left(\left(\mathrm{e}^x - \frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2}\right)}{8} - \frac{\sqrt{2}\,\ln\left(\left(\mathrm{e}^x + \frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2}\right)}{8}$$

```
[In] int(tanh(2*x)^2*exp(x),x)
```

```
[Out] exp(x) + exp(x)/(exp(4*x) + 1) - (2^(1/2)*atan(2^(1/2)*(exp(x) - 2^(1/2)/2)
))/4 - (2^(1/2)*atan(2^(1/2)*(exp(x) + 2^(1/2)/2)))/4 + (2^(1/2)*log((exp(x
) - 2^(1/2)/2)^2 + 1/2))/8 - (2^(1/2)*log((exp(x) + 2^(1/2)/2)^2 + 1/2))/8
```

3.215 $\int e^x \tanh(2x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 95

$$\int e^x \tanh(2x)\, dx = e^x + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}$$

```
[Out] exp(x)-1/2*arctan(-1+exp(x)*2^(1/2))*2^(1/2)-1/2*arctan(1+exp(x)*2^(1/2))*2
^(1/2)+1/4*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/4*ln(1+exp(2*x)+exp(x)*2
^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2320, 396, 217, 1179, 642, 1176, 631, 210}

$$\int e^x \tanh(2x)\, dx = \frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} - \frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}} + e^x + \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}} - \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}$$

```
[In] Int[E^x*Tanh[2*x],x]
```

```
[Out] E^x + ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + L
og[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/
(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+x^4}{1+x^4}\,dx, x, e^x\right)\\
&= e^x - 2\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= e^x - \text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)\\
&= e^x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{2\sqrt{2}}\\
&= e^x + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{\sqrt{2}}\\
&= e^x + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\\
&\quad + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int e^x \tanh(2x)\,dx = e^x &+ \frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\\
&+ \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{2\sqrt{2}}
\end{aligned}$$

```
[In] Integrate[E^x*Tanh[2*x],x]
```

```
[Out] E^x + ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2] - ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2] + L
og[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/
(2*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.25

method	result
risch	$e^{x}+\left(\sum_{\_R=\operatorname{RootOf}(16\_Z^{4}+1)}\_R\ln\left(e^{x}-2\_R\right)\right)$
default	$-\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+3+2\sqrt{2}\right)}{4}+\frac{\left(-\sqrt{2}-2\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{2+2\sqrt{2}}+\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+3-2\sqrt{2}\right)}{4}-\frac{\left(2-\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2\sqrt{2}-2}\right)}{2\sqrt{2}-2}$

```
[In] int(exp(x)*tanh(2*x),x,method=_RETURNVERBOSE)
[Out] exp(x)+sum(_R*ln(exp(x)-2*_R),_R=RootOf(16*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\begin{aligned}\int e^{x}\tanh(2x)\,dx = &-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\\&+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\\&-\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\\&+\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right)\\&+\cosh(x)+\sinh(x)\end{aligned}$$

```
[In] integrate(exp(x)*tanh(2*x),x, algorithm="fricas")
[Out] -(1/4*I + 1/4)*sqrt(2)*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (1/4*
I - 1/4)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (1/4*I - 1
/4)*sqrt(2)*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (1/4*I + 1/4)*sq
rt(2)*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + cosh(x) + sinh(x)
```

Sympy [F]

$$\int e^x \tanh(2x)\, dx = \int e^x \tanh{(2x)}\, dx$$

```
[In] integrate(exp(x)*tanh(2*x),x)
```

```
[Out] Integral(exp(x)*tanh(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int e^x \tanh(2x)\, dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) + e^x$$

```
[In] integrate(exp(x)*tanh(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int e^x \tanh(2x)\, dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) + e^x$$

```
[In] integrate(exp(x)*tanh(2*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) + e^x
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int e^x \tanh(2x)\,dx = e^x - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(2\,e^x-\sqrt{2}\right)}{2}\right)}{2} + \frac{\sqrt{2}\,\ln\left(\left(2\,e^x-\sqrt{2}\right)^2+2\right)}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(2\,e^x+\sqrt{2}\right)}{2}\right)}{2} - \frac{\sqrt{2}\,\ln\left(\left(2\,e^x+\sqrt{2}\right)^2+2\right)}{4}$$

```
[In] int(tanh(2*x)*exp(x),x)
```

```
[Out] exp(x) - (2^(1/2)*atan((2^(1/2)*(2*exp(x) - 2^(1/2)))/2))/2 + (2^(1/2)*log(
(2*exp(x) - 2^(1/2))^2 + 2))/4 - (2^(1/2)*atan((2^(1/2)*(2*exp(x) + 2^(1/2)
))/2))/2 - (2^(1/2)*log((2*exp(x) + 2^(1/2))^2 + 2))/4
```

3.216 $\int e^x \coth(2x)\, dx$

Optimal result

Integrand size = 8, antiderivative size = 16

$$\int e^x \coth(2x)\, dx = e^x - \arctan(e^x) - \operatorname{arctanh}(e^x)$$

```
[Out] exp(x)-arctan(exp(x))-arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {2320, 396, 218, 212, 209}

$$\int e^x \coth(2x)\, dx = -\arctan(e^x) - \operatorname{arctanh}(e^x) + e^x$$

```
[In] Int[E^x*Coth[2*x],x]
```

```
[Out] E^x - ArcTan[E^x] - ArcTanh[E^x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1-x^4}{1-x^4}\,dx, x, e^x\right)\\
&= e^x - 2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)\\
&= e^x - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&= e^x - \arctan\left(e^x\right) - \operatorname{arctanh}(e^x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^x \coth(2x)\,dx = e^x - \arctan\left(e^x\right) - \operatorname{arctanh}(e^x)$$

```
[In] Integrate[E^x*Coth[2*x],x]
[Out] E^x - ArcTan[E^x] - ArcTanh[E^x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

method	result	size
risch	$\mathrm{e}^{x}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{2}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{2}+\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{2}-\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{2}$	36

```
[In] int(exp(x)*coth(2*x),x,method=_RETURNVERBOSE)
[Out] exp(x)-1/2*ln(exp(x)+1)+1/2*ln(exp(x)-1)+1/2*I*ln(exp(x)-I)-1/2*I*ln(exp(x)
+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int e^{x}\coth(2x)\,dx = -\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+\cosh\left(x\right)-\frac{1}{2}\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right) + \frac{1}{2}\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+\sinh\left(x\right)$$

```
[In] integrate(exp(x)*coth(2*x),x, algorithm="fricas")
[Out] -arctan(cosh(x) + sinh(x)) + cosh(x) - 1/2*log(cosh(x) + sinh(x) + 1) + 1/2
*log(cosh(x) + sinh(x) - 1) + sinh(x)
```

Sympy [F]

$$\int e^{x}\coth(2x)\,dx = \int e^{x}\coth\left(2x\right)\,dx$$

```
[In] integrate(exp(x)*coth(2*x),x)
[Out] Integral(exp(x)*coth(2*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int e^x \coth(2x)\, dx = -\arctan\left(e^x\right) + e^x - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(e^x - 1\right)$$

```
[In] integrate(exp(x)*coth(2*x),x, algorithm="maxima")
[Out] -arctan(e^x) + e^x - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int e^x \coth(2x)\, dx = -\arctan\left(e^x\right) + e^x - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(|e^x - 1|\right)$$

```
[In] integrate(exp(x)*coth(2*x),x, algorithm="giac")
[Out] -arctan(e^x) + e^x - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int e^x \coth(2x)\, dx = \frac{\ln\left(2 - 2\,\mathrm{e}^x\right)}{2} - \frac{\ln\left(-2\,\mathrm{e}^x - 2\right)}{2} - \mathrm{atan}(\mathrm{e}^x) + \mathrm{e}^x$$

```
[In] int(coth(2*x)*exp(x),x)
[Out] log(2 - 2*exp(x))/2 - log(- 2*exp(x) - 2)/2 - atan(exp(x)) + exp(x)
```

3.217 $\int e^x \coth^2(2x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int e^x \coth^2(2x)\,dx = e^x + \frac{e^x}{1-e^{4x}} - \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2}$$

```
[Out] exp(x)+exp(x)/(1-exp(4*x))-1/2*arctan(exp(x))-1/2*arctanh(exp(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2320, 398, 294, 218, 212, 209}

$$\int e^x \coth^2(2x)\,dx = -\frac{1}{2}\arctan(e^x) - \frac{\operatorname{arctanh}(e^x)}{2} + e^x + \frac{e^x}{1-e^{4x}}$$

```
[In] Int[E^x*Coth[2*x]^2,x]
```

```
[Out] E^x + E^x/(1 - E^(4*x)) - ArcTan[E^x]/2 - ArcTanh[E^x]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x^4)^2}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{4x^4}{(1-x^4)^2}\right)\,dx, x, e^x\right)\\
&= e^x + 4\text{Subst}\left(\int \frac{x^4}{(1-x^4)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{1-e^{4x}} - \text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right)
\end{aligned}
$$

$$= e^x + \frac{e^x}{1-e^{4x}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)$$

$$= e^x + \frac{e^x}{1-e^{4x}} - \frac{\arctan(e^x)}{2} - \frac{\text{arctanh}(e^x)}{2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.72 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.23

$$\int e^x \coth^2(2x)\,dx = \frac{1}{640}e^{-7x}\Big(-3645 - 6769e^{4x} - 1483e^{8x} + 681e^{12x} + 5\big(729 + 1208e^{4x} + 102e^{8x} - 248e^{12x} + e^{16x}\big)\,\text{Hypergeometric2F1}\left(\frac{1}{4}, 1, \frac{5}{4}, e^{4x}\right)\Big) + \frac{16}{585}e^{5x}\left(1+e^{4x}\right)^2 {}_4F_3\left(\frac{5}{4}, 2, 2, 2; 1, 1, \frac{17}{4}; e^{4x}\right)$$

```
[In] Integrate[E^x*Coth[2*x]^2,x]
```

```
[Out] (-3645 - 6769*E^(4*x) - 1483*E^(8*x) + 681*E^(12*x) + 5*(729 + 1208*E^(4*x)
 + 102*E^(8*x) - 248*E^(12*x) + E^(16*x))*Hypergeometric2F1[1/4, 1, 5/4, E^
(4*x)])/(640*E^(7*x)) + (16*E^(5*x)*(1 + E^(4*x))^2*HypergeometricPFQ[{5/4,
 2, 2, 2}, {1, 1, 17/4}, E^(4*x)])/585
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

method	result	size
risch	$\mathrm{e}^{x}-\frac{\mathrm{e}^{x}}{\mathrm{e}^{4 x}-1}-\frac{\ln\left(\mathrm{e}^{x}+1\right)}{4}+\frac{i \ln\left(\mathrm{e}^{x}-i\right)}{4}-\frac{i \ln\left(\mathrm{e}^{x}+i\right)}{4}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{4}$	48

```
[In] int(exp(x)*coth(2*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)-exp(x)/(exp(4*x)-1)-1/4*ln(exp(x)+1)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp
(x)+I)+1/4*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 6.57

$$\int e^x \coth^2(2x)\,dx = \frac{4\cosh(x)^5 + 40\cosh(x)^3\sinh(x)^2 + 40\cosh(x)^2\sinh(x)^3 + 20\cosh(x)\sinh(x)^4 + 4\sinh(x)^5 - 2\,(\cos$$

```
[In] integrate(exp(x)*coth(2*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*cosh(x)^5 + 40*cosh(x)^3*sinh(x)^2 + 40*cosh(x)^2*sinh(x)^3 + 20*cos
h(x)*sinh(x)^4 + 4*sinh(x)^5 - 2*(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(
x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*arctan(cosh(x) + sinh
(x)) - (cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)
*sinh(x)^3 + sinh(x)^4 - 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x)^4 + 4*cos
h(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 -
1)*log(cosh(x) + sinh(x) - 1) + 4*(5*cosh(x)^4 - 2)*sinh(x) - 8*cosh(x))/(c
osh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^
3 + sinh(x)^4 - 1)
```

Sympy [F]

$$\int e^x \coth^2(2x)\,dx = \int e^x \coth^2(2x)\,dx$$

```
[In] integrate(exp(x)*coth(2*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(2*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int e^x \coth^2(2x)\,dx = -\frac{e^x}{e^{(4x)} - 1} - \frac{1}{2}\arctan\left(e^x\right) + e^x - \frac{1}{4}\log\left(e^x + 1\right) + \frac{1}{4}\log\left(e^x - 1\right)$$

```
[In] integrate(exp(x)*coth(2*x)^2,x, algorithm="maxima")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) + e^x - 1/4*log(e^x + 1) + 1/4*log(e^x
 - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int e^x \coth^2(2x)\,dx = -\frac{e^x}{e^{(4x)}-1} - \frac{1}{2}\arctan\left(e^x\right) + e^x - \frac{1}{4}\log\left(e^x+1\right) + \frac{1}{4}\log\left(\left|e^x-1\right|\right)$$

```
[In] integrate(exp(x)*coth(2*x)^2,x, algorithm="giac")
```

```
[Out] -e^x/(e^(4*x) - 1) - 1/2*arctan(e^x) + e^x - 1/4*log(e^x + 1) + 1/4*log(abs
(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int e^x \coth^2(2x)\,dx = \frac{\ln\left(1-e^x\right)}{4} - \frac{\ln\left(-e^x-1\right)}{4} - \frac{\mathrm{atan}(e^x)}{2} + e^x - \frac{e^x}{e^{4x}-1}$$

```
[In] int(coth(2*x)^2*exp(x),x)
```

```
[Out] log(1 - exp(x))/4 - log(- exp(x) - 1)/4 - atan(exp(x))/2 + exp(x) - exp(x)/
(exp(4*x) - 1)
```

3.218 $\int e^x \tanh^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 113

$$\int e^x \tanh^2(3x)\,dx = e^x + \frac{2e^x}{3(1+e^{6x})} - \frac{2\arctan(e^x)}{9} + \frac{1}{9}\arctan\left(\sqrt{3}-2e^x\right) - \frac{1}{9}\arctan\left(\sqrt{3}+2e^x\right) + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}}$$

```
[Out] exp(x)+2/3*exp(x)/(1+exp(6*x))-2/9*arctan(exp(x))-1/9*arctan(2*exp(x)-3^(1/
2))-1/9*arctan(2*exp(x)+3^(1/2))+1/18*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)
-1/18*ln(1+exp(2*x)+exp(x)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2320, 398, 294, 215, 648, 632, 210, 642, 209}

$$\int e^x \tanh^2(3x)\,dx = -\frac{2}{9}\arctan(e^x) + \frac{1}{9}\arctan\left(\sqrt{3}-2e^x\right) - \frac{1}{9}\arctan\left(2e^x+\sqrt{3}\right) + e^x + \frac{2e^x}{3(e^{6x}+1)} + \frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}} - \frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{6\sqrt{3}}$$

```
[In] Int[E^x*Tanh[3*x]^2,x]
```

```
[Out] E^x + (2*E^x)/(3*(1 + E^(6*x))) - (2*ArcTan[E^x])/9 + ArcTan[Sqrt[3] - 2*E^
x]/9 - ArcTan[Sqrt[3] + 2*E^x]/9 + Log[1 - Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3
]) - Log[1 + Sqrt[3]*E^x + E^(2*x)]/(6*Sqrt[3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^6)^2}{(1+x^6)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1-\frac{4x^6}{(1+x^6)^2}\right)\,dx, x, e^x\right)\\
&= e^x - 4\text{Subst}\left(\int \frac{x^6}{(1+x^6)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{2e^x}{3(1+e^{6x})} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^6}\,dx, x, e^x\right)\\
&= e^x + \frac{2e^x}{3(1+e^{6x})} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subst}\left(\int \frac{1+\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, e^x\right)\\
&= e^x + \frac{2e^x}{3(1+e^{6x})} - \frac{2\arctan(e^x)}{9} - \frac{1}{18}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{18}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right)}{6\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, e^x\right)}{6\sqrt{3}}\\
&= e^x + \frac{2e^x}{3(1+e^{6x})} - \frac{2\arctan(e^x)}{9} + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}}\\
&\quad + \frac{1}{9}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2e^x\right) + \frac{1}{9}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2e^x\right)
\end{aligned}$$

$$= e^x + \frac{2e^x}{3(1+e^{6x})} - \frac{2\arctan(e^x)}{9} + \frac{1}{9}\arctan\left(\sqrt{3}-2e^x\right) - \frac{1}{9}\arctan\left(\sqrt{3}+2e^x\right) + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{6\sqrt{3}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\int e^x \tanh^2(3x)\,dx = e^x + \frac{2e^x}{3(1+e^{6x})} - \frac{2\arctan(e^x)}{9} - \frac{1}{9}\text{RootSum}\left[1-\#1^2 + \#1^4\&, \frac{-2x+2\log(e^x-\#1)+x\#1^2-\log(e^x-\#1)\#1^2}{-\#1+2\#1^3}\&\right]$$

```
[In] Integrate[E^x*Tanh[3*x]^2,x]
```

```
[Out] E^x + (2*E^x)/(3*(1 + E^(6*x))) - (2*ArcTan[E^x])/9 - RootSum[1 - #1^2 + #1
^4 & , (-2*x + 2*Log[E^x - #1] + x*#1^2 - Log[E^x - #1]*#1^2)/(-#1 + 2*#1^3
) & ]/9
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.52

method	result	size
risch	$\mathrm{e}^{x}+\frac{2\,\mathrm{e}^{x}}{3(1+\mathrm{e}^{6x})}+\frac{i\ln(\mathrm{e}^{x}-i)}{9}-\frac{i\ln(\mathrm{e}^{x}+i)}{9}+\left(\sum_{\_R=\mathrm{RootOf}(6561\_Z^4-81\_Z^2+1)}\_R\ln\left(\mathrm{e}^{x}-9\_R\right)\right)$	59

```
[In] int(exp(x)*tanh(3*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)+2/3*exp(x)/(1+exp(6*x))+1/9*I*ln(exp(x)-I)-1/9*I*ln(exp(x)+I)+sum(_R
*ln(exp(x)-9*_R),_R=RootOf(6561*_Z^4-81*_Z^2+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.84

$$\int e^x \tanh^2(3x)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*tanh(3*x)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*cosh(x)^7 + 378*cosh(x)^5*sinh(x)^2 + 630*cosh(x)^4*sinh(x)^3 + 63
0*cosh(x)^3*sinh(x)^4 + 378*cosh(x)^2*sinh(x)^5 + 126*cosh(x)*sinh(x)^6 + 1
8*sinh(x)^7 - (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 2
0*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6 + 1)*sqrt(2*I*sqrt(3) + 2)*log(sqrt(2*I*sqrt(3) + 2) + 2*cosh(x) + 2*
sinh(x)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*c
osh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)
^6 + 1)*sqrt(2*I*sqrt(3) + 2)*log(-sqrt(2*I*sqrt(3) + 2) + 2*cosh(x) + 2*si
nh(x)) - (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cos
h(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
 + 1)*sqrt(-2*I*sqrt(3) + 2)*log(sqrt(-2*I*sqrt(3) + 2) + 2*cosh(x) + 2*sin
h(x)) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
+ 1)*sqrt(-2*I*sqrt(3) + 2)*log(-sqrt(-2*I*sqrt(3) + 2) + 2*cosh(x) + 2*sin
h(x)) - 4*(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*co
sh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6 + 1)*arctan(cosh(x) + sinh(x)) + 6*(21*cosh(x)^6 + 5)*sinh(x) + 30*cosh(x
))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 1)
```

Sympy [F]

$$\int e^x \tanh^2(3x)\,dx = \int e^x \tanh^2(3x)\,dx$$

```
[In] integrate(exp(x)*tanh(3*x)**2,x)
```

```
[Out] Integral(exp(x)*tanh(3*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int e^x \tanh^2(3x)\,dx = -\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) + \frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) + \frac{2e^x}{3\left(e^{(6x)} + 1\right)} - \frac{1}{9}\arctan\left(\sqrt{3} + 2e^x\right) - \frac{1}{9}\arctan\left(-\sqrt{3} + 2e^x\right) - \frac{2}{9}\arctan\left(e^x\right) + e^x$$

```
[In] integrate(exp(x)*tanh(3*x)^2,x, algorithm="maxima")
```

```
[Out] -1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/18*sqrt(3)*log(-sqrt(3)*e^
x + e^(2*x) + 1) + 2/3*e^x/(e^(6*x) + 1) - 1/9*arctan(sqrt(3) + 2*e^x) - 1/
9*arctan(-sqrt(3) + 2*e^x) - 2/9*arctan(e^x) + e^x
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.72

$$\int e^x \tanh^2(3x)\,dx = -\frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) + \frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) + \frac{2e^x}{3\left(e^{(6x)} + 1\right)} - \frac{1}{9}\arctan\left(\sqrt{3} + 2e^x\right) - \frac{1}{9}\arctan\left(-\sqrt{3} + 2e^x\right) - \frac{2}{9}\arctan\left(e^x\right) + e^x$$

```
[In] integrate(exp(x)*tanh(3*x)^2,x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/18*sqrt(3)*log(-sqrt(3)*e^
x + e^(2*x) + 1) + 2/3*e^x/(e^(6*x) + 1) - 1/9*arctan(sqrt(3) + 2*e^x) - 1/
9*arctan(-sqrt(3) + 2*e^x) - 2/9*arctan(e^x) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int e^x \tanh^2(3x)\,dx = e^x - \frac{\operatorname{atan}\left(2\,e^x + \sqrt{3}\right)}{9} - \frac{\operatorname{atan}\left(2\,e^x - \sqrt{3}\right)}{9} - \frac{2\operatorname{atan}(e^x)}{9} + \frac{2\,e^x}{3\,\left(e^{6\,x}+1\right)} + \frac{\sqrt{3}\,\ln\left(\left(\frac{2\,e^x}{3} - \frac{\sqrt{3}}{3}\right)^2 + \frac{1}{9}\right)}{18} - \frac{\sqrt{3}\,\ln\left(\left(\frac{2\,e^x}{3} + \frac{\sqrt{3}}{3}\right)^2 + \frac{1}{9}\right)}{18}$$

```
[In] int(tanh(3*x)^2*exp(x),x)
```

```
[Out] exp(x) - atan(2*exp(x) + 3^(1/2))/9 - atan(2*exp(x) - 3^(1/2))/9 - (2*atan(
exp(x)))/9 + (2*exp(x))/(3*(exp(6*x) + 1)) + (3^(1/2)*log(((2*exp(x))/3 - 3
^(1/2)/3)^2 + 1/9))/18 - (3^(1/2)*log(((2*exp(x))/3 + 3^(1/2)/3)^2 + 1/9))/
18
```

3.219 $\int e^x \tanh(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 97

$$\int e^x \tanh(3x)\,dx = e^x - \frac{2\arctan(e^x)}{3} + \frac{1}{3}\arctan\left(\sqrt{3}-2e^x\right) - \frac{1}{3}\arctan\left(\sqrt{3}+2e^x\right) + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}}$$

```
[Out] exp(x)-2/3*arctan(exp(x))-1/3*arctan(2*exp(x)-3^(1/2))-1/3*arctan(2*exp(x)+
3^(1/2))+1/6*ln(1+exp(2*x)-exp(x)*3^(1/2))*3^(1/2)-1/6*ln(1+exp(2*x)+exp(x)
*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2320, 396, 215, 648, 632, 210, 642, 209}

$$\int e^x \tanh(3x)\,dx = -\frac{2}{3}\arctan(e^x) + \frac{1}{3}\arctan\left(\sqrt{3}-2e^x\right) - \frac{1}{3}\arctan\left(2e^x+\sqrt{3}\right) + e^x + \frac{\log\left(-\sqrt{3}e^x+e^{2x}+1\right)}{2\sqrt{3}} - \frac{\log\left(\sqrt{3}e^x+e^{2x}+1\right)}{2\sqrt{3}}$$

```
[In] Int[E^x*Tanh[3*x],x]
```

```
[Out] E^x - (2*ArcTan[E^x])/3 + ArcTan[Sqrt[3] - 2*E^x]/3 - ArcTan[Sqrt[3] + 2*E^
x]/3 + Log[1 - Sqrt[3]*E^x + E^(2*x)]/(2*Sqrt[3]) - Log[1 + Sqrt[3]*E^x + E
^(2*x)]/(2*Sqrt[3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1+x^6}{1+x^6}\,dx, x, e^x\right)\\
&= e^x - 2\text{Subst}\left(\int \frac{1}{1+x^6}\,dx, x, e^x\right)\\
&= e^x - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{2}{3}\text{Subst}\left(\int \frac{1+\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx, x, e^x\right)\\
&= e^x - \frac{2\arctan(e^x)}{3} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{-\sqrt{3}+2x}{1-\sqrt{3}x+x^2}\,dx, x, e^x\right)}{2\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{1+\sqrt{3}x+x^2}\,dx, x, e^x\right)}{2\sqrt{3}}\\
&= e^x - \frac{2\arctan(e^x)}{3} + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}}\\
&\quad + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2e^x\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2e^x\right)\\
&= e^x - \frac{2\arctan(e^x)}{3} + \frac{1}{3}\arctan\left(\sqrt{3}-2e^x\right) - \frac{1}{3}\arctan\left(\sqrt{3}+2e^x\right)\\
&\quad + \frac{\log\left(1-\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}} - \frac{\log\left(1+\sqrt{3}e^x+e^{2x}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.25

$$\int e^x \tanh(3x)\,dx = e^x - 2e^x \operatorname{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{6x}\right)$$

```
[In] Integrate[E^x*Tanh[3*x],x]
[Out] E^x - 2*E^x*Hypergeometric2F1[1/6, 1, 7/6, -E^(6*x)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.48

method	result	size
risch	$\mathrm{e}^{x}+\left(\sum_{\_R=\operatorname{RootOf}(81\_Z^4-9\_Z^2+1)}\_R\ln\left(\mathrm{e}^{x}-3\_R\right)\right)+\frac{i\ln(\mathrm{e}^{x}-i)}{3}-\frac{i\ln(\mathrm{e}^{x}+i)}{3}$	47

```
[In] int(exp(x)*tanh(3*x),x,method=_RETURNVERBOSE)
[Out] exp(x)+sum(_R*ln(exp(x)-3*_R),_R=RootOf(81*_Z^4-9*_Z^2+1))+1/3*I*ln(exp(x)-
I)-1/3*I*ln(exp(x)+I)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\begin{aligned}\int e^x \tanh(3x)\,dx = &-\frac{1}{6}\sqrt{2i\sqrt{3}+2}\log\left(\sqrt{2i\sqrt{3}+2}+2\cosh(x)+2\sinh(x)\right)\\&+\frac{1}{6}\sqrt{2i\sqrt{3}+2}\log\left(-\sqrt{2i\sqrt{3}+2}+2\cosh(x)+2\sinh(x)\right)\\&-\frac{1}{6}\sqrt{-2i\sqrt{3}+2}\log\left(\sqrt{-2i\sqrt{3}+2}+2\cosh(x)+2\sinh(x)\right)\\&+\frac{1}{6}\sqrt{-2i\sqrt{3}+2}\log\left(-\sqrt{-2i\sqrt{3}+2}+2\cosh(x)+2\sinh(x)\right)\\&-\frac{2}{3}\arctan\left(\cosh(x)+\sinh(x)\right)+\cosh(x)+\sinh(x)\end{aligned}$$

```
[In] integrate(exp(x)*tanh(3*x),x, algorithm="fricas")
```

[Out] -1/6*sqrt(2*I*sqrt(3) + 2)*log(sqrt(2*I*sqrt(3) + 2) + 2*cosh(x) + 2*sinh(x
)) + 1/6*sqrt(2*I*sqrt(3) + 2)*log(-sqrt(2*I*sqrt(3) + 2) + 2*cosh(x) + 2*s
inh(x)) - 1/6*sqrt(-2*I*sqrt(3) + 2)*log(sqrt(-2*I*sqrt(3) + 2) + 2*cosh(x)
 + 2*sinh(x)) + 1/6*sqrt(-2*I*sqrt(3) + 2)*log(-sqrt(-2*I*sqrt(3) + 2) + 2*
cosh(x) + 2*sinh(x)) - 2/3*arctan(cosh(x) + sinh(x)) + cosh(x) + sinh(x)

Sympy [F]

$$\int e^x \tanh(3x)\,dx = \int e^x \tanh(3x)\,dx$$

[In] integrate(exp(x)*tanh(3*x),x)

[Out] Integral(exp(x)*tanh(3*x), x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\int e^x \tanh(3x)\,dx = -\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) + \frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{1}{3}\arctan\left(\sqrt{3} + 2e^x\right) - \frac{1}{3}\arctan\left(-\sqrt{3} + 2e^x\right) - \frac{2}{3}\arctan\left(e^x\right) + e^x$$

[In] integrate(exp(x)*tanh(3*x),x, algorithm="maxima")

[Out] -1/6*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*sqrt(3)*log(-sqrt(3)*e^x
+ e^(2*x) + 1) - 1/3*arctan(sqrt(3) + 2*e^x) - 1/3*arctan(-sqrt(3) + 2*e^x)
 - 2/3*arctan(e^x) + e^x

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.71

$$\int e^x \tanh(3x)\,dx = -\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) + \frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{1}{3}\arctan\left(\sqrt{3} + 2e^x\right) - \frac{1}{3}\arctan\left(-\sqrt{3} + 2e^x\right) - \frac{2}{3}\arctan\left(e^x\right) + e^x$$

[In] integrate(exp(x)*tanh(3*x),x, algorithm="giac")

[Out] -1/6*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*sqrt(3)*log(-sqrt(3)*e^x
+ e^(2*x) + 1) - 1/3*arctan(sqrt(3) + 2*e^x) - 1/3*arctan(-sqrt(3) + 2*e^x)
 - 2/3*arctan(e^x) + e^x

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.72

$$\int e^x \tanh(3x)\,dx = e^x - \frac{\operatorname{atan}\left(2\,e^x + \sqrt{3}\right)}{3} - \frac{\operatorname{atan}\left(2\,e^x - \sqrt{3}\right)}{3} - \frac{2\operatorname{atan}(e^x)}{3} + \frac{\sqrt{3}\,\ln\left(\left(2\,e^x - \sqrt{3}\right)^2 + 1\right)}{6} - \frac{\sqrt{3}\,\ln\left(\left(2\,e^x + \sqrt{3}\right)^2 + 1\right)}{6}$$

```
[In] int(tanh(3*x)*exp(x),x)
```

```
[Out] exp(x) - atan(2*exp(x) + 3^(1/2))/3 - atan(2*exp(x) - 3^(1/2))/3 - (2*atan(
exp(x)))/3 + (3^(1/2)*log((2*exp(x) - 3^(1/2))^2 + 1))/6 - (3^(1/2)*log((2*
exp(x) + 3^(1/2))^2 + 1))/6
```

3.220 $\int e^x \coth(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 85

$$\int e^x \coth(3x)\,dx = e^x + \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+2e^x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2\text{arctanh}(e^x)}{3} + \frac{1}{6}\log\left(1-e^x+e^{2x}\right) - \frac{1}{6}\log\left(1+e^x+e^{2x}\right)$$

```
[Out] exp(x)-2/3*arctanh(exp(x))+1/6*ln(1-exp(x)+exp(2*x))-1/6*ln(1+exp(x)+exp(2*
x))+1/3*arctan(1/3*(1-2*exp(x))*3^(1/2))*3^(1/2)-1/3*arctan(1/3*(1+2*exp(x)
)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2320, 396, 216, 648, 632, 210, 642, 212}

$$\int e^x \coth(3x)\,dx = \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{2e^x+1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2\text{arctanh}(e^x)}{3} + e^x + \frac{1}{6}\log\left(-e^x+e^{2x}+1\right) - \frac{1}{6}\log\left(e^x+e^{2x}+1\right)$$

```
[In] Int[E^x*Coth[3*x],x]
```

```
[Out] E^x + ArcTan[(1 - 2*E^x)/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2*E^x)/Sqrt[3]]/Sqr
t[3] - (2*ArcTanh[E^x])/3 + Log[1 - E^x + E^(2*x)]/6 - Log[1 + E^x + E^(2*x
)]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1-x^6}{1-x^6}\,dx, x, e^x\right)\\
&= e^x - 2\text{Subst}\left(\int \frac{1}{1-x^6}\,dx, x, e^x\right)\\
&= e^x - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{2}{3}\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, e^x\right)\\
&= e^x - \frac{2\text{arctanh}(e^x)}{3} + \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, e^x\right)\\
&= e^x - \frac{2\text{arctanh}(e^x)}{3} + \frac{1}{6}\log\left(1-e^x+e^{2x}\right) - \frac{1}{6}\log\left(1+e^x+e^{2x}\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2e^x\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2e^x\right)\\
&= e^x - \frac{\arctan\left(\frac{-1+2e^x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\arctan\left(\frac{1+2e^x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2\text{arctanh}(e^x)}{3}\\
&\quad + \frac{1}{6}\log\left(1-e^x+e^{2x}\right) - \frac{1}{6}\log\left(1+e^x+e^{2x}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int e^x \coth(3x)\,dx = e^x - 2e^x \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},e^{6x}\right)$$

```
[In] Integrate[E^x*Coth[3*x],x]
[Out] E^x - 2*E^x*Hypergeometric2F1[1/6, 1, 7/6, E^(6*x)]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.62

method	result
risch	$\mathrm{e}^{x}+\frac{\ln(\mathrm{e}^{x}-1)}{3}+\frac{\ln\left(\mathrm{e}^{x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(\mathrm{e}^{x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\mathrm{e}^{x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(\mathrm{e}^{x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln\left(\mathrm{e}^{x}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$

```
[In] int(exp(x)*coth(3*x),x,method=_RETURNVERBOSE)
[Out] exp(x)+1/3*ln(exp(x)-1)+1/6*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/6*I*ln(exp(x)-1/
2-1/2*I*3^(1/2))*3^(1/2)+1/6*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/6*I*ln(exp(x)-1
/2+1/2*I*3^(1/2))*3^(1/2)-1/6*ln(exp(x)+1/2-1/2*I*3^(1/2))+1/6*I*ln(exp(x)+
1/2-1/2*I*3^(1/2))*3^(1/2)-1/6*ln(exp(x)+1/2+1/2*I*3^(1/2))-1/6*I*ln(exp(x)
+1/2+1/2*I*3^(1/2))*3^(1/2)-1/3*ln(exp(x)+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.33

$$\begin{aligned}\int e^x \coth(3x)\,dx = &-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\cosh(x)+\frac{2}{3}\sqrt{3}\sinh(x)+\frac{1}{3}\sqrt{3}\right)\\&-\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\cosh(x)+\frac{2}{3}\sqrt{3}\sinh(x)-\frac{1}{3}\sqrt{3}\right)+\cosh(x)\\&-\frac{1}{6}\log\left(\frac{2\cosh(x)+1}{\cosh(x)-\sinh(x)}\right)+\frac{1}{6}\log\left(\frac{2\cosh(x)-1}{\cosh(x)-\sinh(x)}\right)\\&-\frac{1}{3}\log(\cosh(x)+\sinh(x)+1)+\frac{1}{3}\log(\cosh(x)+\sinh(x)-1)+\sinh(x)\end{aligned}$$

```
[In] integrate(exp(x)*coth(3*x),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)
) - 1/3*sqrt(3)*arctan(2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) - 1/3*sqrt
(3)) + cosh(x) - 1/6*log((2*cosh(x) + 1)/(cosh(x) - sinh(x))) + 1/6*log((2*
cosh(x) - 1)/(cosh(x) - sinh(x))) - 1/3*log(cosh(x) + sinh(x) + 1) + 1/3*lo
g(cosh(x) + sinh(x) - 1) + sinh(x)
```

Sympy [F]

$$\int e^x \coth(3x)\,dx = \int e^x \coth(3x)\,dx$$

```
[In] integrate(exp(x)*coth(3*x),x)
```

```
[Out] Integral(exp(x)*coth(3*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\begin{aligned}\int e^x \coth(3x)\,dx = &-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right)\\ &+ e^x - \frac{1}{6}\log\left(e^{(2x)}+e^x+1\right) + \frac{1}{6}\log\left(e^{(2x)}-e^x+1\right)\\ &- \frac{1}{3}\log\left(e^x+1\right) + \frac{1}{3}\log\left(e^x-1\right)\end{aligned}$$

```
[In] integrate(exp(x)*coth(3*x),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) + e^x - 1/6*log(e^(2*x) + e^x + 1) + 1/6*log(e^(2*x) - e^x
+ 1) - 1/3*log(e^x + 1) + 1/3*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\begin{aligned}\int e^x \coth(3x)\,dx = &-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right)\\ &+ e^x - \frac{1}{6}\log\left(e^{(2x)}+e^x+1\right) + \frac{1}{6}\log\left(e^{(2x)}-e^x+1\right)\\ &- \frac{1}{3}\log\left(e^x+1\right) + \frac{1}{3}\log\left(|e^x-1|\right)\end{aligned}$$

```
[In] integrate(exp(x)*coth(3*x),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) + e^x - 1/6*log(e^(2*x) + e^x + 1) + 1/6*log(e^(2*x) - e^x
+ 1) - 1/3*log(e^x + 1) + 1/3*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int e^x \coth(3x)\,dx = \frac{\ln(2-2\,e^x)}{3} - \frac{\ln(-2\,e^x-2)}{3} + \frac{\ln\left((2\,e^x-1)^2+3\right)}{6} - \frac{\ln\left((2\,e^x+1)^2+3\right)}{6} + e^x - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,e^x-1)}{3}\right)}{3} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\,e^x+1)}{3}\right)}{3}$$

```
[In] int(coth(3*x)*exp(x),x)
```

```
[Out] log(2 - 2*exp(x))/3 - log(- 2*exp(x) - 2)/3 + log((2*exp(x) - 1)^2 + 3)/6 -
 log((2*exp(x) + 1)^2 + 3)/6 + exp(x) - (3^(1/2)*atan((3^(1/2)*(2*exp(x) -
1))/3))/3 - (3^(1/2)*atan((3^(1/2)*(2*exp(x) + 1))/3))/3
```

3.221 $\int e^x \coth^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 108

$$\int e^x \coth^2(3x)\,dx = e^x + \frac{2e^x}{3(1-e^{6x})} + \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2\text{arctanh}(e^x)}{9} + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right)$$

```
[Out] exp(x)+2/3*exp(x)/(1-exp(6*x))-2/9*arctanh(exp(x))+1/18*ln(1-exp(x)+exp(2*x
))-1/18*ln(1+exp(x)+exp(2*x))+1/9*arctan(1/3*(1-2*exp(x))*3^(1/2))*3^(1/2)-
1/9*arctan(1/3*(1+2*exp(x))*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2320, 398, 294, 216, 648, 632, 210, 642, 212}

$$\int e^x \coth^2(3x)\,dx = \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{2e^x+1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2\text{arctanh}(e^x)}{9} + e^x + \frac{2e^x}{3(1-e^{6x})} + \frac{1}{18}\log\left(-e^x+e^{2x}+1\right) - \frac{1}{18}\log\left(e^x+e^{2x}+1\right)$$

```
[In] Int[E^x*Coth[3*x]^2,x]
```

```
[Out] E^x + (2*E^x)/(3*(1 - E^(6*x))) + ArcTan[(1 - 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) -
 ArcTan[(1 + 2*E^x)/Sqrt[3]]/(3*Sqrt[3]) - (2*ArcTanh[E^x])/9 + Log[1 - E^x
 + E^(2*x)]/18 - Log[1 + E^x + E^(2*x)]/18
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x^6)^2}{(1-x^6)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{4x^6}{(1-x^6)^2}\right)\,dx, x, e^x\right)\\
&= e^x + 4\text{Subst}\left(\int \frac{x^6}{(1-x^6)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{2e^x}{3\,(1-e^{6x})} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x^6}\,dx, x, e^x\right)\\
&= e^x + \frac{2e^x}{3\,(1-e^{6x})} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right)\\
&\quad - \frac{2}{9}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, e^x\right) - \frac{2}{9}\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, e^x\right)\\
&= e^x + \frac{2e^x}{3\,(1-e^{6x})} - \frac{2\text{arctanh}(e^x)}{9} + \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{18}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, e^x\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, e^x\right)\\
&= e^x + \frac{2e^x}{3\,(1-e^{6x})} - \frac{2\text{arctanh}(e^x)}{9} + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right)\\
&\quad + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2e^x\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2e^x\right)
\end{aligned}
$$

$$= e^x + \frac{2e^x}{3(1-e^{6x})} + \frac{\arctan\left(\frac{1-2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\arctan\left(\frac{1+2e^x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{2\text{arctanh}(e^x)}{9} + \frac{1}{18}\log\left(1-e^x+e^{2x}\right) - \frac{1}{18}\log\left(1+e^x+e^{2x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

$$\int e^x \coth^2(3x)\,dx$$
$$= \frac{e^{-11x}\left(-15379-28153e^{6x}-5633e^{12x}+3109e^{18x}+7(2197+3708e^{6x}+538e^{12x}-684e^{18x}+e^{24x})\,\text{Hyperg}\right.}{3024} + \frac{36e^{7x}(1+e^{6x})^2\,{}_4F_3\left(\frac{7}{6},2,2,2;1,1,\frac{25}{6};e^{6x}\right)}{1729}$$

```
[In] Integrate[E^x*Coth[3*x]^2,x]
```

```
[Out] (-15379 - 28153*E^(6*x) - 5633*E^(12*x) + 3109*E^(18*x) + 7*(2197 + 3708*E^
(6*x) + 538*E^(12*x) - 684*E^(18*x) + E^(24*x))*Hypergeometric2F1[1/6, 1, 7
/6, E^(6*x)])/(3024*E^(11*x)) + (36*E^(7*x)*(1 + E^(6*x))^2*HypergeometricP
FQ[{7/6, 2, 2, 2}, {1, 1, 25/6}, E^(6*x)])/1729
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.39

method	result
risch	$\mathrm{e}^x - \frac{2\,\mathrm{e}^x}{3(\mathrm{e}^{6x}-1)} - \frac{\ln(\mathrm{e}^x+1)}{9} + \frac{\ln(\mathrm{e}^x-1)}{9} + \frac{\ln\left(\mathrm{e}^x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{18} + \frac{i\ln\left(\mathrm{e}^x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{18} + \frac{\ln\left(\mathrm{e}^x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{18} - \frac{i\ln\left(\mathrm{e}^x-\frac{1}{2}+\frac{i\sqrt{}}{2}\right.}{18}$

```
[In] int(exp(x)*coth(3*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)-2/3*exp(x)/(exp(6*x)-1)-1/9*ln(exp(x)+1)+1/9*ln(exp(x)-1)+1/18*ln(ex
p(x)-1/2-1/2*I*3^(1/2))+1/18*I*3^(1/2)*ln(exp(x)-1/2-1/2*I*3^(1/2))+1/18*ln
(exp(x)-1/2+1/2*I*3^(1/2))-1/18*I*3^(1/2)*ln(exp(x)-1/2+1/2*I*3^(1/2))-1/18
*ln(exp(x)+1/2-1/2*I*3^(1/2))+1/18*I*3^(1/2)*ln(exp(x)+1/2-1/2*I*3^(1/2))-1
/18*ln(exp(x)+1/2+1/2*I*3^(1/2))-1/18*I*3^(1/2)*ln(exp(x)+1/2+1/2*I*3^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 628 vs. $2(78) = 156$.

Time = 0.27 (sec) , antiderivative size = 628, normalized size of antiderivative = 5.81

$$\int e^x \coth^2(3x)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*coth(3*x)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(18*cosh(x)^7 + 378*cosh(x)^5*sinh(x)^2 + 630*cosh(x)^4*sinh(x)^3 + 63
0*cosh(x)^3*sinh(x)^4 + 378*cosh(x)^2*sinh(x)^5 + 126*cosh(x)*sinh(x)^6 + 1
8*sinh(x)^7 - 2*(sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(
3)*cosh(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x
)^2*sinh(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*
arctan(2/3*sqrt(3)*cosh(x) + 2/3*sqrt(3)*sinh(x) + 1/3*sqrt(3)) - 2*(sqrt(3
)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2
+ 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(
3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 - sqrt(3))*arctan(2/3*sqrt(3)*cosh
(x) + 2/3*sqrt(3)*sinh(x) - 1/3*sqrt(3)) - (cosh(x)^6 + 6*cosh(x)^5*sinh(x)
 + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4
 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log((2*cosh(x) + 1)/(cosh(x) - sinh
(x))) + (cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh
(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6
- 1)*log((2*cosh(x) - 1)/(cosh(x) - sinh(x))) - 2*(cosh(x)^6 + 6*cosh(x)^5*
sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*si
nh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*log(cosh(x) + sinh(x) + 1) +
 2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)*
log(cosh(x) + sinh(x) - 1) + 6*(21*cosh(x)^6 - 5)*sinh(x) - 30*cosh(x))/(co
sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(
x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 - 1)
```

Sympy [F]

$$\int e^x \coth^2(3x)\,dx = \int e^x \coth^2{(3x)}\,dx$$

```
[In] integrate(exp(x)*coth(3*x)**2,x)
```

```
[Out] Integral(exp(x)*coth(3*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int e^x \coth^2(3x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) - \frac{2e^x}{3\left(e^{(6x)}-1\right)} + e^x - \frac{1}{18}\log\left(e^{(2x)}+e^x+1\right) + \frac{1}{18}\log\left(e^{(2x)}-e^x+1\right) - \frac{1}{9}\log\left(e^x+1\right) + \frac{1}{9}\log\left(e^x-1\right)$$

```
[In] integrate(exp(x)*coth(3*x)^2,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) + e^x - 1/18*log(e^(2*x) + e^x + 1)
 + 1/18*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int e^x \coth^2(3x)\,dx = -\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+1)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x-1)\right) - \frac{2e^x}{3\left(e^{(6x)}-1\right)} + e^x - \frac{1}{18}\log\left(e^{(2x)}+e^x+1\right) + \frac{1}{18}\log\left(e^{(2x)}-e^x+1\right) - \frac{1}{9}\log\left(e^x+1\right) + \frac{1}{9}\log\left(|e^x-1|\right)$$

```
[In] integrate(exp(x)*coth(3*x)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 1)) - 1/9*sqrt(3)*arctan(1/3*sqrt(
3)*(2*e^x - 1)) - 2/3*e^x/(e^(6*x) - 1) + e^x - 1/18*log(e^(2*x) + e^x + 1)
 + 1/18*log(e^(2*x) - e^x + 1) - 1/9*log(e^x + 1) + 1/9*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int e^x \coth^2(3x)\,dx = \frac{\ln\left(\frac{2}{3}-\frac{2\,e^x}{3}\right)}{9}-\frac{\ln\left(-\frac{2\,e^x}{3}-\frac{2}{3}\right)}{9}+\frac{\ln\left(\left(\frac{2\,e^x}{3}-\frac{1}{3}\right)^2+\frac{1}{3}\right)}{18}-\frac{\ln\left(\left(\frac{2\,e^x}{3}+\frac{1}{3}\right)^2+\frac{1}{3}\right)}{18}+e^x-\frac{2\,e^x}{3\,\left(e^{6\,x}-1\right)}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,e^x}{3}-\frac{1}{3}\right)\right)}{9}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\,\left(\frac{2\,e^x}{3}+\frac{1}{3}\right)\right)}{9}$$

```
[In] int(coth(3*x)^2*exp(x),x)
```

```
[Out] log(2/3 - (2*exp(x))/3)/9 - log(- (2*exp(x))/3 - 2/3)/9 + log(((2*exp(x))/3
 - 1/3)^2 + 1/3)/18 - log(((2*exp(x))/3 + 1/3)^2 + 1/3)/18 + exp(x) - (2*ex
p(x))/(3*(exp(6*x) - 1)) - (3^(1/2)*atan(3^(1/2)*((2*exp(x))/3 - 1/3)))/9 -
 (3^(1/2)*atan(3^(1/2)*((2*exp(x))/3 + 1/3)))/9
```

3.222 $\int e^x \tanh^2(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 382

$$\begin{aligned}\int e^x \tanh^2(4x)\,dx = {} & e^x+\frac{e^x}{2\left(1+e^{8x}\right)}+\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}} \\ & -\frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}} \\ & +\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) \\ & -\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) \\ & +\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right) \\ & -\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)\end{aligned}$$

```
[Out] exp(x)+1/2*exp(x)/(1+exp(8*x))+1/32*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))
*(2-2^(1/2))^(1/2)-1/32*ln(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))*(2-2^(1/2))
^(1/2)+1/8*arctan((-2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(
1/2))^(1/2)-1/8*arctan((2*exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2
*2^(1/2))^(1/2)+1/32*ln(1+exp(2*x)-exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1
/2)-1/32*ln(1+exp(2*x)+exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/8*arct
an((-2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/8
*arctan((2*exp(x)+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2320, 398, 294, 219, 1183, 648, 632, 210, 642}

$$\int e^x \tanh^2(4x)\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{8\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{8\sqrt{2\left(2+\sqrt{2}\right)}}+e^x + \frac{e^x}{2\left(e^{8x}+1\right)}+\frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right) - \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right) + \frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(-\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right) - \frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)$$

```
[In] Int[E^x*Tanh[4*x]^2,x]
```

```
[Out] E^x + E^x/(2*(1 + E^(8*x))) + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + S
qrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqr
t[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E
^x)/Sqrt[2 + Sqrt[2]]]/(8*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]
] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/(8*Sqrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]
]*Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 - Sqrt[2]]*Log[1 +
 Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/32 + (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 +
 Sqrt[2]]*E^x + E^(2*x)])/32 - (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]
*E^x + E^(2*x)])/32
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
```

```
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1-x^8)^2}{(1+x^8)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1-\frac{4x^8}{(1+x^8)^2}\right)\,dx, x, e^x\right)\\
&= e^x - 4\text{Subst}\left(\int \frac{x^8}{(1+x^8)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1+e^{8x})} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^8}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1+e^{8x})} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{4\sqrt{2}}\\
&= e^x + \frac{e^x}{2(1+e^{8x})} - \frac{\text{Subst}\left(\int \frac{\sqrt{2(2-\sqrt{2})}-(-1+\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2(2-\sqrt{2})}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2(2-\sqrt{2})}+(-1+\sqrt{2})x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2(2-\sqrt{2})}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2(2+\sqrt{2})}-(1+\sqrt{2})x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2(2+\sqrt{2})}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2(2+\sqrt{2})}+(1+\sqrt{2})x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{8\sqrt{2(2+\sqrt{2})}}
\end{aligned}$$

$$
\begin{aligned}
&= e^x + \frac{e^x}{2(1+e^{8x})} - \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{16}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{1}{32}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{32}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{1}{32}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{32}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{16}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{16}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)
\end{aligned}
$$

$$
\begin{aligned}
&= e^x + \frac{e^x}{2(1+e^{8x})} + \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)\\
&\quad - \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)\\
&\quad + \frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)\\
&\quad - \frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)\\
&\quad + \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, -\sqrt{2+\sqrt{2}}+2e^x\right)\\
&\quad + \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, \sqrt{2+\sqrt{2}}+2e^x\right)\\
&\quad + \frac{1}{8}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, -\sqrt{2-\sqrt{2}}+2e^x\right)\\
&\quad + \frac{1}{8}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, \sqrt{2-\sqrt{2}}+2e^x\right)
\end{aligned}
$$

$$= e^x + \frac{e^x}{2(1+e^{8x})} + \frac{1}{16}\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)$$
$$+ \frac{1}{16}\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)$$
$$- \frac{1}{16}\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)$$
$$- \frac{1}{16}\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)$$
$$+ \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)$$
$$- \frac{1}{32}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right)$$
$$+ \frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)$$
$$- \frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.13

$$\int e^x \tanh^2(4x)\,dx = e^x + \frac{e^x}{2(1+e^{8x})} + \frac{1}{16}\text{RootSum}\left[1+\#1^8\&, \frac{x-\log(e^x-\#1)}{\#1^7}\&\right]$$

```
[In] Integrate[E^x*Tanh[4*x]^2,x]
[Out] E^x + E^x/(2*(1 + E^(8*x))) + RootSum[1 + #1^8 & , (x - Log[E^x - #1])/#1^7
 & ]/16
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.09

method	result	size
risch	$\mathrm{e}^{x}+\frac{\mathrm{e}^{x}}{2+2\,\mathrm{e}^{8x}}+\left(\sum_{\_R=\mathrm{RootOf}(4294967296\_Z^{8}+1)}\_R\ln\left(\mathrm{e}^{x}-16\_R\right)\right)$	36

```
[In] int(exp(x)*tanh(4*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)+1/2*exp(x)/(1+exp(8*x))+sum(_R*ln(exp(x)-16*_R),_R=RootOf(4294967296
*_Z^8+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 1303, normalized size of antiderivative = 3.41

$$\int e^x \tanh^2(4x)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*tanh(4*x)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*cosh(x)^9 + 1152*cosh(x)^7*sinh(x)^2 + 2688*cosh(x)^6*sinh(x)^3 +
4032*cosh(x)^5*sinh(x)^4 + 4032*cosh(x)^4*sinh(x)^5 + 2688*cosh(x)^3*sinh(x
)^6 + 1152*cosh(x)^2*sinh(x)^7 + 288*cosh(x)*sinh(x)^8 + 32*sinh(x)^9 + (-(
I + 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 - (8*I + 8)*sqrt(2)*(-1)^(1/8)*cosh(x)^
7*sinh(x) - (28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*sinh(x)^2 - (56*I + 56
)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 - (70*I + 70)*sqrt(2)*(-1)^(1/8)*c
osh(x)^4*sinh(x)^4 - (56*I + 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^3*sinh(x)^5 - (
28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 - (8*I + 8)*sqrt(2)*(-1)^
(1/8)*cosh(x)*sinh(x)^7 - (I + 1)*sqrt(2)*(-1)^(1/8)*sinh(x)^8 - (I + 1)*sq
rt(2)*(-1)^(1/8))*log((I + 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x) + 2*sinh(x)) +
 ((I - 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 + (8*I - 8)*sqrt(2)*(-1)^(1/8)*cosh(
x)^7*sinh(x) + (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*sinh(x)^2 + (56*I -
 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 + (70*I - 70)*sqrt(2)*(-1)^(1/8
)*cosh(x)^4*sinh(x)^4 + (56*I - 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^3*sinh(x)^5
+ (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 + (8*I - 8)*sqrt(2)*(-
1)^(1/8)*cosh(x)*sinh(x)^7 + (I - 1)*sqrt(2)*(-1)^(1/8)*sinh(x)^8 + (I - 1)
*sqrt(2)*(-1)^(1/8))*log(-(I - 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x) + 2*sinh(x
)) + (-(I - 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 - (8*I - 8)*sqrt(2)*(-1)^(1/8)*
cosh(x)^7*sinh(x) - (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*sinh(x)^2 - (5
6*I - 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 - (70*I - 70)*sqrt(2)*(-1)
^(1/8)*cosh(x)^4*sinh(x)^4 - (56*I - 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^3*sinh(
x)^5 - (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 - (8*I - 8)*sqrt(
2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 - (I - 1)*sqrt(2)*(-1)^(1/8)*sinh(x)^8 - (I
 - 1)*sqrt(2)*(-1)^(1/8))*log((I - 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x) + 2*si
nh(x)) + ((I + 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 + (8*I + 8)*sqrt(2)*(-1)^(1/
8)*cosh(x)^7*sinh(x) + (28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*sinh(x)^2 +
 (56*I + 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 + (70*I + 70)*sqrt(2)*(
-1)^(1/8)*cosh(x)^4*sinh(x)^4 + (56*I + 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^3*si
nh(x)^5 + (28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 + (8*I + 8)*sq
rt(2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*(-1)^(1/8)*sinh(x)^8 +
```

```
(I + 1)*sqrt(2)*(-1)^(1/8))*log(-(I + 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x) +
2*sinh(x)) - 2*((-1)^(1/8)*cosh(x)^8 + 8*(-1)^(1/8)*cosh(x)^7*sinh(x) + 28*
(-1)^(1/8)*cosh(x)^6*sinh(x)^2 + 56*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 + 70*(-1
)^(1/8)*cosh(x)^4*sinh(x)^4 + 56*(-1)^(1/8)*cosh(x)^3*sinh(x)^5 + 28*(-1)^(
1/8)*cosh(x)^2*sinh(x)^6 + 8*(-1)^(1/8)*cosh(x)*sinh(x)^7 + (-1)^(1/8)*sinh
(x)^8 + (-1)^(1/8))*log((-1)^(1/8) + cosh(x) + sinh(x)) - 2*(I*(-1)^(1/8)*c
osh(x)^8 + 8*I*(-1)^(1/8)*cosh(x)^7*sinh(x) + 28*I*(-1)^(1/8)*cosh(x)^6*sin
h(x)^2 + 56*I*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 + 70*I*(-1)^(1/8)*cosh(x)^4*si
nh(x)^4 + 56*I*(-1)^(1/8)*cosh(x)^3*sinh(x)^5 + 28*I*(-1)^(1/8)*cosh(x)^2*s
inh(x)^6 + 8*I*(-1)^(1/8)*cosh(x)*sinh(x)^7 + I*(-1)^(1/8)*sinh(x)^8 + I*(-
1)^(1/8))*log(I*(-1)^(1/8) + cosh(x) + sinh(x)) - 2*(-I*(-1)^(1/8)*cosh(x)^
8 - 8*I*(-1)^(1/8)*cosh(x)^7*sinh(x) - 28*I*(-1)^(1/8)*cosh(x)^6*sinh(x)^2
- 56*I*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 - 70*I*(-1)^(1/8)*cosh(x)^4*sinh(x)^4
 - 56*I*(-1)^(1/8)*cosh(x)^3*sinh(x)^5 - 28*I*(-1)^(1/8)*cosh(x)^2*sinh(x)^
6 - 8*I*(-1)^(1/8)*cosh(x)*sinh(x)^7 - I*(-1)^(1/8)*sinh(x)^8 - I*(-1)^(1/8
))*log(-I*(-1)^(1/8) + cosh(x) + sinh(x)) + 2*((-1)^(1/8)*cosh(x)^8 + 8*(-1
)^(1/8)*cosh(x)^7*sinh(x) + 28*(-1)^(1/8)*cosh(x)^6*sinh(x)^2 + 56*(-1)^(1/
8)*cosh(x)^5*sinh(x)^3 + 70*(-1)^(1/8)*cosh(x)^4*sinh(x)^4 + 56*(-1)^(1/8)*
cosh(x)^3*sinh(x)^5 + 28*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 + 8*(-1)^(1/8)*cosh
(x)*sinh(x)^7 + (-1)^(1/8)*sinh(x)^8 + (-1)^(1/8))*log(-(-1)^(1/8) + cosh(x
) + sinh(x)) + 48*(6*cosh(x)^8 + 1)*sinh(x) + 48*cosh(x))/(cosh(x)^8 + 8*co
sh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh
(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(
x)*sinh(x)^7 + sinh(x)^8 + 1)
```

Sympy [F]

$$\int e^x \tanh^2(4x)\,dx = \int e^x \tanh^2{(4x)}\,dx$$

```
[In] integrate(exp(x)*tanh(4*x)**2,x)
[Out] Integral(exp(x)*tanh(4*x)**2, x)
```

Maxima [F]

$$\int e^x \tanh^2(4x)\,dx = \int e^x \tanh\left(4\,x\right)^2\,dx$$

```
[In] integrate(exp(x)*tanh(4*x)^2,x, algorithm="maxima")
[Out] 1/2*(2*e^(9*x) + 3*e^x)/(e^(8*x) + 1) - integrate(1/2*e^x/(e^(8*x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.69

$$\int e^x \tanh^2(4x)\,dx = -\frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{16}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) - \frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^x}{\sqrt{\sqrt{2}+2}}\right) - \frac{1}{16}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^x}{\sqrt{\sqrt{2}+2}}\right) - \frac{1}{32}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) + \frac{1}{32}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) - \frac{1}{32}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) + \frac{1}{32}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) + \frac{e^x}{2\,(e^{(8\,x)}+1)} + e^x$$

```
[In] integrate(exp(x)*tanh(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
 2)) - 1/16*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sq
rt(2) + 2)) - 1/16*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sq
rt(sqrt(2) + 2)) - 1/16*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e
^x)/sqrt(sqrt(2) + 2)) - 1/32*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x +
 e^(2*x) + 1) + 1/32*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x)
 + 1) - 1/32*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) +
 1/32*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/2*e
^x/(e^(8*x) + 1) + e^x
```

Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.24

$$\int e^x \tanh^2(4x)\,dx = e^x + \frac{e^x}{2\,(e^{8\,x}+1) } + \ln\left(\frac{e^x}{2} - \frac{\sqrt{\sqrt{2}+2}}{4} - \frac{\sqrt{2-\sqrt{2}}\,1i}{4}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right) - \ln\left(\frac{e^x}{2} + \frac{\sqrt{\sqrt{2}+2}}{4} + \frac{\sqrt{2-\sqrt{2}}\,1i}{4}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right) + \ln\left(\frac{e^x}{2} + \frac{\sqrt{2-\sqrt{2}}}{4} - \frac{\sqrt{\sqrt{2}+2}\,1i}{4}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{32} + \frac{\sqrt{\sqrt{2}+2}\,1i}{32}\right) - \ln\left(\frac{e^x}{2} - \frac{\sqrt{2-\sqrt{2}}}{4} + \frac{\sqrt{\sqrt{2}+2}\,1i}{4}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{32} + \frac{\sqrt{\sqrt{2}+2}\,1i}{32}\right) + \sqrt{2}\,\ln\left(\frac{e^x}{2} + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(-4-4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(\frac{1}{2}+\frac{1}{2}i\right) + \sqrt{2}\,\ln\left(\frac{e^x}{2} + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(-4+4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(\frac{1}{2}-\frac{1}{2}i\right) + \sqrt{2}\,\ln\left(\frac{e^x}{2} + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(4-4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(-\frac{1}{2}+\frac{1}{2}i\right) + \sqrt{2}\,\ln\left(\frac{e^x}{2} + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)(4+4i)\right)\left(\frac{\sqrt{\sqrt{2}+2}}{32} + \frac{\sqrt{2-\sqrt{2}}\,1i}{32}\right)\left(-\frac{1}{2}-\frac{1}{2}i\right)$$

```
[In] int(tanh(4*x)^2*exp(x),x)
[Out] exp(x) + exp(x)/(2*(exp(8*x) + 1)) + log(exp(x)/2 - (2^(1/2) + 2)^(1/2)/4 -
 ((2 - 2^(1/2))^(1/2)*1i)/4)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)
```

```
*1i)/32) - log(exp(x)/2 + (2^(1/2) + 2)^(1/2)/4 + ((2 - 2^(1/2))^(1/2)*1i)/
4)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) + log(exp(x)/2 -
((2^(1/2) + 2)^(1/2)*1i)/4 + (2 - 2^(1/2))^(1/2)/4)*(((2^(1/2) + 2)^(1/2)*1
i)/32 - (2 - 2^(1/2))^(1/2)/32) - log(exp(x)/2 + ((2^(1/2) + 2)^(1/2)*1i)/4
 - (2 - 2^(1/2))^(1/2)/4)*(((2^(1/2) + 2)^(1/2)*1i)/32 - (2 - 2^(1/2))^(1/2
)/32) + 2^(1/2)*log(exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1
/2))^(1/2)*1i)/32)*(4 + 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)
*1i)/32)*(1/2 + 1i/2) + 2^(1/2)*log(exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)
/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2
- 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(exp(x)/2 + 2^(1/2)*((2^
(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i))*((2^(1/2) + 2)
^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(exp(x)/
2 + 2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 + 4i)
)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 + 1i/2)
```

3.223 $\int e^x \tanh(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 366

$$\int e^x \tanh(4x)\,dx = e^x + \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)$$

```
[Out] exp(x)+1/8*ln(1+exp(2*x)-exp(x)*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)-1/8*ln
(1+exp(2*x)+exp(x)*(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/2*arctan((-2*exp(
x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctan((2*
exp(x)+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/8*ln(1+e
xp(2*x)-exp(x)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)-1/8*ln(1+exp(2*x)+exp(x
)*(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)+1/2*arctan((-2*exp(x)+(2+2^(1/2))^(1
/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)-1/2*arctan((2*exp(x)+(2+2^(1/2)
)^(1/2))/(2-2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2320, 396, 219, 1183, 648, 632, 210, 642}

$$\int e^x \tanh(4x)\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}+e^x+\frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)-\frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)+\frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(-\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)-\frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)$$

```
[In] Int[E^x*Tanh[4*x],x]
```

```
[Out] E^x + ArcTan[(Sqrt[2 - Sqrt[2]] - 2*E^x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 -
Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] - 2*E^x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[
2*(2 + Sqrt[2])]) - ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/(
2*Sqrt[2*(2 - Sqrt[2])]) - ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt
[2]]]/(2*Sqrt[2*(2 + Sqrt[2])]) + (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[
2]]*E^x + E^(2*x)])/8 - (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*E^x +
E^(2*x)])/8 + (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/
8 - (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 219

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{-1+x^8}{1+x^8}\,dx, x, e^x\right)\\ &= e^x - 2\text{Subst}\left(\int \frac{1}{1+x^8}\,dx, x, e^x\right)\end{aligned}$$

$$= e^x - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx, x, e^x\right)}{\sqrt{2}}$$

$$= e^x - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$
$$- \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= e^x - \frac{1}{4}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$
$$+ \frac{1}{8}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$
$$- \frac{1}{8}\sqrt{2-\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$
$$+ \frac{1}{8}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{-\sqrt{2+\sqrt{2}}+2x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$
$$- \frac{1}{8}\sqrt{2+\sqrt{2}}\text{Subst}\left(\int \frac{\sqrt{2+\sqrt{2}}+2x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$
$$- \frac{1}{4}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx, x, e^x\right)$$

$$= e^x + \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, -\sqrt{2+\sqrt{2}}+2e^x\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2+\sqrt{2}-x^2}\,dx, x, \sqrt{2+\sqrt{2}}+2e^x\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, -\sqrt{2-\sqrt{2}}+2e^x\right) + \frac{1}{2}\sqrt{\frac{1}{2}\left(3+2\sqrt{2}\right)}\text{Subst}\left(\int \frac{1}{-2-\sqrt{2}-x^2}\,dx, x, \sqrt{2-\sqrt{2}}+2e^x\right)$$

$$= e^x + \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{4}\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}e^x+e^{2x}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(1-\sqrt{2+\sqrt{2}}e^x+e^{2x}\right) - \frac{1}{8}\sqrt{2+\sqrt{2}}\log\left(1+\sqrt{2+\sqrt{2}}e^x+e^{2x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.07

$$\int e^x \tanh(4x)\,dx = e^x - 2e^x \,\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{8x}\right)$$

```
[In] Integrate[E^x*Tanh[4*x],x]
[Out] E^x - 2*E^x*Hypergeometric2F1[1/8, 1, 9/8, -E^(8*x)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.07

method	result	size
risch	$e^{x}+\left(\sum_{\_R=\operatorname{RootOf}\left(65536\_Z^{8}+1\right)}\_R\ln\left(e^{x}-4\_R\right)\right)$	24

```
[In] int(exp(x)*tanh(4*x),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)+sum(_R*ln(exp(x)-4*_R),_R=RootOf(65536*_Z^8+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.47

$$\begin{aligned}\int e^{x}\tanh(4x)\,dx= & -\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i+1)\sqrt{2}(-1)^{\frac{1}{8}}+2\cosh(x)+2\sinh(x)\right)\\ & +\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i-1)\sqrt{2}(-1)^{\frac{1}{8}}+2\cosh(x)+2\sinh(x)\right)\\ & -\left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i-1)\sqrt{2}(-1)^{\frac{1}{8}}+2\cosh(x)+2\sinh(x)\right)\\ & +\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i+1)\sqrt{2}(-1)^{\frac{1}{8}}+2\cosh(x)+2\sinh(x)\right)\\ & -\frac{1}{4}(-1)^{\frac{1}{8}}\log\left((-1)^{\frac{1}{8}}+\cosh(x)+\sinh(x)\right)\\ & -\frac{1}{4}i(-1)^{\frac{1}{8}}\log\left(i(-1)^{\frac{1}{8}}+\cosh(x)+\sinh(x)\right)\\ & +\frac{1}{4}i(-1)^{\frac{1}{8}}\log\left(-i(-1)^{\frac{1}{8}}+\cosh(x)+\sinh(x)\right)\\ & +\frac{1}{4}(-1)^{\frac{1}{8}}\log\left(-(-1)^{\frac{1}{8}}+\cosh(x)+\sinh(x)\right)+\cosh(x)+\sinh(x)\end{aligned}$$

```
[In] integrate(exp(x)*tanh(4*x),x, algorithm="fricas")
```

```
[Out] -(1/8*I + 1/8)*sqrt(2)*(-1)^(1/8)*log((I + 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x
) + 2*sinh(x)) + (1/8*I - 1/8)*sqrt(2)*(-1)^(1/8)*log(-(I - 1)*sqrt(2)*(-1)
^(1/8) + 2*cosh(x) + 2*sinh(x)) - (1/8*I - 1/8)*sqrt(2)*(-1)^(1/8)*log((I -
 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x) + 2*sinh(x)) + (1/8*I + 1/8)*sqrt(2)*(-1
)^(1/8)*log(-(I + 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x) + 2*sinh(x)) - 1/4*(-1)
^(1/8)*log((-1)^(1/8) + cosh(x) + sinh(x)) - 1/4*I*(-1)^(1/8)*log(I*(-1)^(1
/8) + cosh(x) + sinh(x)) + 1/4*I*(-1)^(1/8)*log(-I*(-1)^(1/8) + cosh(x) + s
```

```
inh(x)) + 1/4*(-1)^(1/8)*log(-(-1)^(1/8) + cosh(x) + sinh(x)) + cosh(x) + s
inh(x)
```

Sympy [F]

$$\int e^x \tanh(4x)\, dx = \int e^x \tanh{(4x)}\, dx$$

```
[In] integrate(exp(x)*tanh(4*x),x)
[Out] Integral(exp(x)*tanh(4*x), x)
```

Maxima [F]

$$\int e^x \tanh(4x)\, dx = \int e^x \tanh\left(4\,x\right)\, dx$$

```
[In] integrate(exp(x)*tanh(4*x),x, algorithm="maxima")
[Out] e^x - 2*integrate(e^x/(e^(8*x) + 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.69

$$\begin{aligned}\int e^x \tanh(4x)\, dx = &-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^x}{\sqrt{-\sqrt{2}+2}}\right)\\ &-\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^x}{\sqrt{-\sqrt{2}+2}}\right)\\ &-\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^x}{\sqrt{\sqrt{2}+2}}\right)\\ &-\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^x}{\sqrt{\sqrt{2}+2}}\right)\\ &-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)\\ &+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)\\ &-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)\\ &+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)+e^x\end{aligned}$$

```
[In] integrate(exp(x)*tanh(4*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt
(2) + 2)) - 1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt(
sqrt(2) + 2)) - 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^x)/
sqrt(sqrt(2) + 2)) - 1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x + e^(2
*x) + 1) + 1/8*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + 1/8*sq
rt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) + e^x
```

Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.25

$$\int e^x \tanh(4x)\,dx = \mathrm{e}^x - \ln\left(2\,\mathrm{e}^x + \sqrt{\sqrt{2}+2} + \sqrt{2-\sqrt{2}}\,1\mathrm{i}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right) + \ln\left(2\,\mathrm{e}^x + \sqrt{2-\sqrt{2}} - \sqrt{\sqrt{2}+2}\,1\mathrm{i}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{8} + \frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right) + \ln\left(2\,\mathrm{e}^x - \sqrt{\sqrt{2}+2} - \sqrt{2-\sqrt{2}}\,1\mathrm{i}\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right) - \ln\left(2\,\mathrm{e}^x - \sqrt{2-\sqrt{2}} + \sqrt{\sqrt{2}+2}\,1\mathrm{i}\right)\left(-\frac{\sqrt{2-\sqrt{2}}}{8} + \frac{\sqrt{\sqrt{2}+2}\,1\mathrm{i}}{8}\right) + \sqrt{2}\,\ln\left(2\,\mathrm{e}^x + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)(-4-4\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\ln\left(2\,\mathrm{e}^x + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)(-4+4\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\ln\left(2\,\mathrm{e}^x + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)(4-4\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(-\frac{1}{2} + \frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\ln\left(2\,\mathrm{e}^x + \sqrt{2}\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)(4+4\mathrm{i})\right)\left(\frac{\sqrt{\sqrt{2}+2}}{8} + \frac{\sqrt{2-\sqrt{2}}\,1\mathrm{i}}{8}\right)\left(-\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)$$

```
[In] int(tanh(4*x)*exp(x),x)
```

```
[Out] exp(x) - log(2*exp(x) + (2^(1/2) + 2)^(1/2) + (2 - 2^(1/2))^(1/2)*1i)*((2^(
1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) + log(2*exp(x) - (2^(1/2) +
 2)^(1/2)*1i + (2 - 2^(1/2))^(1/2))*(((2^(1/2) + 2)^(1/2)*1i)/8 - (2 - 2^(1
```

```
/2))^(1/2)/8) + log(2*exp(x) - (2^(1/2) + 2)^(1/2) - (2 - 2^(1/2))^(1/2)*1i
)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(2*exp(x) + (2^
(1/2) + 2)^(1/2)*1i - (2 - 2^(1/2))^(1/2))*(((2^(1/2) + 2)^(1/2)*1i)/8 - (2
 - 2^(1/2))^(1/2)/8) + 2^(1/2)*log(2*exp(x) - 2^(1/2)*((2^(1/2) + 2)^(1/2)/
8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(4 + 4i))*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^
(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) + 2^(1/2)*log(2*exp(x) - 2^(1/2)*((2^(1/2)
 + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(4 - 4i))*((2^(1/2) + 2)^(1/2)/
8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 - 1i/2) - 2^(1/2)*log(2*exp(x) + 2^(1/
2)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(4 - 4i))*((2^(1/2)
 + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 - 1i/2) - 2^(1/2)*log(2*ex
p(x) + 2^(1/2)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(4 + 4i
))*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2)
```

3.224 $\int e^x \coth(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 116

$$\int e^x \coth(4x)\,dx = e^x - \frac{\arctan(e^x)}{2} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{2} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}$$

```
[Out] exp(x)-1/2*arctan(exp(x))-1/2*arctanh(exp(x))-1/4*arctan(-1+exp(x)*2^(1/2))
*2^(1/2)-1/4*arctan(1+exp(x)*2^(1/2))*2^(1/2)+1/8*ln(1+exp(2*x)-exp(x)*2^(1
/2))*2^(1/2)-1/8*ln(1+exp(2*x)+exp(x)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {2320, 396, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int e^x \coth(4x)\,dx = -\frac{1}{2}\arctan(e^x) + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\arctan\left(\sqrt{2}e^x+1\right)}{2\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{2} + e^x + \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}} - \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{4\sqrt{2}}$$

```
[In] Int[E^x*Coth[4*x],x]
```

```
[Out] E^x - ArcTan[E^x]/2 + ArcTan[1 - Sqrt[2]*E^x]/(2*Sqrt[2]) - ArcTan[1 + Sqrt
[2]*E^x]/(2*Sqrt[2]) - ArcTanh[E^x]/2 + Log[1 - Sqrt[2]*E^x + E^(2*x)]/(4*S
qrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(4*Sqrt[2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{-1-x^8}{1-x^8}\,dx, x, e^x\right)\\
&= e^x - 2\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)\\
&= e^x - \text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= e^x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)
\end{aligned}$$

$$
\begin{aligned}
&= e^x - \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) \\
&\quad - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right) \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{4\sqrt{2}} \\
&= e^x - \frac{\arctan(e^x)}{2} - \frac{\operatorname{arctanh}(e^x)}{2} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{2\sqrt{2}} \\
&= e^x - \frac{\arctan(e^x)}{2} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} \\
&\quad - \frac{\operatorname{arctanh}(e^x)}{2} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.19

$$
\int e^x \coth(4x)\,dx = e^x - 2e^x \text{Hypergeometric2F1}\left(\frac{1}{8}, 1, \frac{9}{8}, e^{8x}\right)
$$

```
[In] Integrate[E^x*Coth[4*x],x]
[Out] E^x - 2*E^x*Hypergeometric2F1[1/8, 1, 9/8, E^(8*x)]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.48

method	result	size
risch	$\mathrm{e}^{x}+\frac{i\ln(\mathrm{e}^{x}-i)}{4}-\frac{i\ln(\mathrm{e}^{x}+i)}{4}-\frac{\ln(\mathrm{e}^{x}+1)}{4}+\frac{\ln(\mathrm{e}^{x}-1)}{4}+\left(\sum_{\_R=\mathrm{RootOf}(256\_Z^{4}+1)}\_R\ln\left(\mathrm{e}^{x}-4\_R\right)\right)$	56

```
[In] int(exp(x)*coth(4*x),x,method=_RETURNVERBOSE)
[Out] exp(x)+1/4*I*ln(exp(x)-I)-1/4*I*ln(exp(x)+I)-1/4*ln(exp(x)+1)+1/4*ln(exp(x)
-1)+sum(_R*ln(exp(x)-4*_R),_R=RootOf(256*_Z^4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int e^x \coth(4x)\,dx = -\left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left((i+1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left(-(i-1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right) - \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}\log\left((i-1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right) + \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}\log\left(-(i+1)\sqrt{2}+2\cosh(x)+2\sinh(x)\right) - \frac{1}{2}\arctan\left(\cosh(x)+\sinh(x)\right)+\cosh(x) - \frac{1}{4}\log\left(\cosh(x)+\sinh(x)+1\right)+\frac{1}{4}\log\left(\cosh(x)+\sinh(x)-1\right)+\sinh(x)$$

```
[In] integrate(exp(x)*coth(4*x),x, algorithm="fricas")
```

```
[Out] -(1/8*I + 1/8)*sqrt(2)*log((I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (1/8*
I - 1/8)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - (1/8*I - 1
/8)*sqrt(2)*log((I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (1/8*I + 1/8)*sq
rt(2)*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - 1/2*arctan(cosh(x) +
sinh(x)) + cosh(x) - 1/4*log(cosh(x) + sinh(x) + 1) + 1/4*log(cosh(x) + sin
h(x) - 1) + sinh(x)
```

Sympy [F]

$$\int e^x \coth(4x)\,dx = \int e^x \coth(4x)\,dx$$

```
[In] integrate(exp(x)*coth(4*x),x)
```

```
[Out] Integral(exp(x)*coth(4*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int e^x \coth(4x)\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{1}{2}\arctan\left(e^x\right)+e^x-\frac{1}{4}\log\left(e^x+1\right)+\frac{1}{4}\log\left(e^x-1\right)$$

```
[In] integrate(exp(x)*coth(4*x),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) + e^x - 1/4*
log(e^x + 1) + 1/4*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int e^x \coth(4x)\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right) - \frac{1}{2}\arctan\left(e^x\right)+e^x-\frac{1}{4}\log\left(e^x+1\right)+\frac{1}{4}\log\left(|e^x-1|\right)$$

```
[In] integrate(exp(x)*coth(4*x),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1) +
 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*arctan(e^x) + e^x - 1/4*
log(e^x + 1) + 1/4*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int e^x \coth(4x)\,dx = \frac{\ln(2-2\,e^x)}{4} - \frac{\ln(-2\,e^x-2)}{4} - \frac{\mathrm{atan}(e^x)}{2} + e^x - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(2\,e^x-\sqrt{2}\right)}{2}\right)}{4} + \frac{\sqrt{2}\,\ln\left(\left(2\,e^x-\sqrt{2}\right)^2+2\right)}{8} - \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(2\,e^x+\sqrt{2}\right)}{2}\right)}{4} - \frac{\sqrt{2}\,\ln\left(\left(2\,e^x+\sqrt{2}\right)^2+2\right)}{8}$$

```
[In] int(coth(4*x)*exp(x),x)
```

```
[Out] log(2 - 2*exp(x))/4 - log(- 2*exp(x) - 2)/4 - atan(exp(x))/2 + exp(x) - (2^
(1/2)*atan((2^(1/2)*(2*exp(x) - 2^(1/2)))/2))/4 + (2^(1/2)*log((2*exp(x) -
2^(1/2))^2 + 2))/8 - (2^(1/2)*atan((2^(1/2)*(2*exp(x) + 2^(1/2)))/2))/4 - (
2^(1/2)*log((2*exp(x) + 2^(1/2))^2 + 2))/8
```

3.225 $\int e^x \coth^2(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 134

$$\int e^x \coth^2(4x)\,dx = e^x + \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{8} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}$$

```
[Out] exp(x)+1/2*exp(x)/(1-exp(8*x))-1/8*arctan(exp(x))-1/8*arctanh(exp(x))-1/16*
arctan(-1+exp(x)*2^(1/2))*2^(1/2)-1/16*arctan(1+exp(x)*2^(1/2))*2^(1/2)+1/3
2*ln(1+exp(2*x)-exp(x)*2^(1/2))*2^(1/2)-1/32*ln(1+exp(2*x)+exp(x)*2^(1/2))*
2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.300, Rules used = {2320, 398, 294, 220, 218, 212, 209, 217, 1179, 642, 1176, 631, 210}

$$\int e^x \coth^2(4x)\,dx = -\frac{1}{8}\arctan(e^x) + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\arctan\left(\sqrt{2}e^x+1\right)}{8\sqrt{2}} - \frac{\operatorname{arctanh}(e^x)}{8} + e^x + \frac{e^x}{2(1-e^{8x})} + \frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}} - \frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{16\sqrt{2}}$$

```
[In] Int[E^x*Coth[4*x]^2,x]
```

```
[Out] E^x + E^x/(2*(1 - E^(8*x))) - ArcTan[E^x]/8 + ArcTan[1 - Sqrt[2]*E^x]/(8*Sq
rt[2]) - ArcTan[1 + Sqrt[2]*E^x]/(8*Sqrt[2]) - ArcTanh[E^x]/8 + Log[1 - Sqr
t[2]*E^x + E^(2*x)]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^(2*x)]/(16*Sqrt[
2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 220

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2)),
x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x] &
& IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{(1+x^8)^2}{(1-x^8)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{4x^8}{(1-x^8)^2}\right)\,dx, x, e^x\right)\\
&= e^x + 4\text{Subst}\left(\int \frac{x^8}{(1-x^8)^2}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^8}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, e^x\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\
&\quad - \frac{1}{8}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, e^x\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, e^x\right)\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} - \frac{\operatorname{arctanh}(e^x)}{8}\\
&\quad - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, e^x\right) - \frac{1}{16}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, e^x\right)\\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, e^x\right)}{16\sqrt{2}}\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} - \frac{\operatorname{arctanh}(e^x)}{8}\\
&\quad + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}\\
&\quad - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}e^x\right)}{8\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}e^x\right)}{8\sqrt{2}}\\
&= e^x + \frac{e^x}{2(1-e^{8x})} - \frac{\arctan(e^x)}{8} + \frac{\arctan\left(1-\sqrt{2}e^x\right)}{8\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}e^x\right)}{8\sqrt{2}}\\
&\quad - \frac{\operatorname{arctanh}(e^x)}{8} + \frac{\log\left(1-\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}e^x+e^{2x}\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.84

$$\int e^x \coth^2(4x)\,dx$$
$$= \frac{e^{-15x}(-44217 - 80225e^{8x} - 15127e^{16x} + 9361e^{24x} + 9(4913 + 8368e^{8x} + 1486e^{16x} - 1456e^{24x} + e^{32x})\,\mathrm{Hy}}{9216}$$
$$+ \frac{64e^{9x}(1+e^{8x})^2\ {}_4F_3\left(\frac{9}{8},2,2,2;1,1,\frac{33}{8};e^{8x}\right)}{3825}$$

```
[In] Integrate[E^x*Coth[4*x]^2,x]
```

```
[Out] (-44217 - 80225*E^(8*x) - 15127*E^(16*x) + 9361*E^(24*x) + 9*(4913 + 8368*E
^(8*x) + 1486*E^(16*x) - 1456*E^(24*x) + E^(32*x))*Hypergeometric2F1[1/8, 1
, 9/8, E^(8*x)])/(9216*E^(15*x)) + (64*E^(9*x)*(1 + E^(8*x))^2*Hypergeometr
icPFQ[{9/8, 2, 2, 2}, {1, 1, 33/8}, E^(8*x)])/3825
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

method	result
risch	$e^x - \frac{e^x}{2(e^{8x}-1)} + \frac{i\ln(e^x-i)}{16} - \frac{i\ln(e^x+i)}{16} - \frac{\ln(e^x+1)}{16} + \frac{\ln(e^x-1)}{16} + \left(\sum_{\_R=\mathrm{RootOf}(65536\_Z^4+1)} \_R\ln(e^x - 16\_\right.$

```
[In] int(exp(x)*coth(4*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)-1/2*exp(x)/(exp(8*x)-1)+1/16*I*ln(exp(x)-I)-1/16*I*ln(exp(x)+I)-1/16
*ln(exp(x)+1)+1/16*ln(exp(x)-1)+sum(_R*ln(exp(x)-16*_R),_R=RootOf(65536*_Z^
4+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 939, normalized size of antiderivative = 7.01

$$\int e^x \coth^2(4x)\,dx = \text{Too large to display}$$

```
[In] integrate(exp(x)*coth(4*x)^2,x, algorithm="fricas")
[Out] 1/32*(32*cosh(x)^9 + 1152*cosh(x)^7*sinh(x)^2 + 2688*cosh(x)^6*sinh(x)^3 +
4032*cosh(x)^5*sinh(x)^4 + 4032*cosh(x)^4*sinh(x)^5 + 2688*cosh(x)^3*sinh(x
)^6 + 1152*cosh(x)^2*sinh(x)^7 + 288*cosh(x)*sinh(x)^8 + 32*sinh(x)^9 - 4*(
cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cosh(x)^5*sin
h(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sin
h(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 - 1)*arctan(cosh(x) + sinh(x)) + (
-(I + 1)*sqrt(2)*cosh(x)^8 - (8*I + 8)*sqrt(2)*cosh(x)^7*sinh(x) - (28*I +
28)*sqrt(2)*cosh(x)^6*sinh(x)^2 - (56*I + 56)*sqrt(2)*cosh(x)^5*sinh(x)^3 -
 (70*I + 70)*sqrt(2)*cosh(x)^4*sinh(x)^4 - (56*I + 56)*sqrt(2)*cosh(x)^3*si
nh(x)^5 - (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I + 8)*sqrt(2)*cosh(
x)*sinh(x)^7 - (I + 1)*sqrt(2)*sinh(x)^8 + (I + 1)*sqrt(2))*log((I + 1)*sqr
t(2) + 2*cosh(x) + 2*sinh(x)) + ((I - 1)*sqrt(2)*cosh(x)^8 + (8*I - 8)*sqrt
(2)*cosh(x)^7*sinh(x) + (28*I - 28)*sqrt(2)*cosh(x)^6*sinh(x)^2 + (56*I - 5
6)*sqrt(2)*cosh(x)^5*sinh(x)^3 + (70*I - 70)*sqrt(2)*cosh(x)^4*sinh(x)^4 +
(56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I - 28)*sqrt(2)*cosh(x)^2*sin
h(x)^6 + (8*I - 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I - 1)*sqrt(2)*sinh(x)^8 -
(I - 1)*sqrt(2))*log(-(I - 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) + (-(I - 1)*
sqrt(2)*cosh(x)^8 - (8*I - 8)*sqrt(2)*cosh(x)^7*sinh(x) - (28*I - 28)*sqrt(
2)*cosh(x)^6*sinh(x)^2 - (56*I - 56)*sqrt(2)*cosh(x)^5*sinh(x)^3 - (70*I -
70)*sqrt(2)*cosh(x)^4*sinh(x)^4 - (56*I - 56)*sqrt(2)*cosh(x)^3*sinh(x)^5 -
 (28*I - 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 - (8*I - 8)*sqrt(2)*cosh(x)*sinh(x
)^7 - (I - 1)*sqrt(2)*sinh(x)^8 + (I - 1)*sqrt(2))*log((I - 1)*sqrt(2) + 2*
cosh(x) + 2*sinh(x)) + ((I + 1)*sqrt(2)*cosh(x)^8 + (8*I + 8)*sqrt(2)*cosh(
x)^7*sinh(x) + (28*I + 28)*sqrt(2)*cosh(x)^6*sinh(x)^2 + (56*I + 56)*sqrt(2
)*cosh(x)^5*sinh(x)^3 + (70*I + 70)*sqrt(2)*cosh(x)^4*sinh(x)^4 + (56*I + 5
6)*sqrt(2)*cosh(x)^3*sinh(x)^5 + (28*I + 28)*sqrt(2)*cosh(x)^2*sinh(x)^6 +
(8*I + 8)*sqrt(2)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*sinh(x)^8 - (I + 1)*s
qrt(2))*log(-(I + 1)*sqrt(2) + 2*cosh(x) + 2*sinh(x)) - 2*(cosh(x)^8 + 8*co
sh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh
(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(
x)*sinh(x)^7 + sinh(x)^8 - 1)*log(cosh(x) + sinh(x) + 1) + 2*(cosh(x)^8 + 8
*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*c
osh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*co
sh(x)*sinh(x)^7 + sinh(x)^8 - 1)*log(cosh(x) + sinh(x) - 1) + 48*(6*cosh(x)
^8 - 1)*sinh(x) - 48*cosh(x))/(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)
```

```
^6*sinh(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)
^3*sinh(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 - 1
)
```

Sympy [F]

$$\int e^x \coth^2(4x)\,dx = \int e^x \coth^2(4x)\,dx$$

```
[In] integrate(exp(x)*coth(4*x)**2,x)
[Out] Integral(exp(x)*coth(4*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81

$$\begin{aligned}\int e^x \coth^2(4x)\,dx = &-\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\&-\frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)-\frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)\\&+\frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)-\frac{e^x}{2\left(e^{(8x)}-1\right)}\\&-\frac{1}{8}\arctan\left(e^x\right)+e^x-\frac{1}{16}\log\left(e^x+1\right)+\frac{1}{16}\log\left(e^x-1\right)\end{aligned}$$

```
[In] integrate(exp(x)*coth(4*x)^2,x, algorithm="maxima")
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) + e^x - 1/16*log(e^x + 1) + 1/16*log(e^x - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.82

$$\int e^x \coth^2(4x)\,dx = -\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) - \frac{1}{16}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) - \frac{1}{32}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) + \frac{1}{32}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^x}{2\left(e^{(8x)}-1\right)} - \frac{1}{8}\arctan\left(e^x\right) + e^x - \frac{1}{16}\log\left(e^x+1\right) + \frac{1}{16}\log\left(|e^x-1|\right)$$

```
[In] integrate(exp(x)*coth(4*x)^2,x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) - 1/16*sqrt(2)*arctan(-
1/2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/32*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1
) + 1/32*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - 1/2*e^x/(e^(8*x) - 1) -
1/8*arctan(e^x) + e^x - 1/16*log(e^x + 1) + 1/16*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int e^x \coth^2(4x)\,dx = \frac{\ln\left(\frac{1}{2}-\frac{e^x}{2}\right)}{16} - \frac{\ln\left(-\frac{e^x}{2}-\frac{1}{2}\right)}{16} - \frac{\mathrm{atan}(e^x)}{8} + e^x - \frac{e^x}{2\,\left(e^{8x}-1\right)} - \frac{\sqrt{2}\,\mathrm{atan}\left(2\sqrt{2}\left(\frac{e^x}{2}-\frac{\sqrt{2}}{4}\right)\right)}{16} - \frac{\sqrt{2}\,\mathrm{atan}\left(2\sqrt{2}\left(\frac{e^x}{2}+\frac{\sqrt{2}}{4}\right)\right)}{16} + \frac{\sqrt{2}\,\ln\left(\left(\frac{e^x}{2}-\frac{\sqrt{2}}{4}\right)^2+\frac{1}{8}\right)}{32} - \frac{\sqrt{2}\,\ln\left(\left(\frac{e^x}{2}+\frac{\sqrt{2}}{4}\right)^2+\frac{1}{8}\right)}{32}$$

```
[In] int(coth(4*x)^2*exp(x),x)
```

```
[Out] log(1/2 - exp(x)/2)/16 - log(- exp(x)/2 - 1/2)/16 - atan(exp(x))/8 + exp(x)
 - exp(x)/(2*(exp(8*x) - 1)) - (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 - 2^(1/2)/
4)))/16 - (2^(1/2)*atan(2*2^(1/2)*(exp(x)/2 + 2^(1/2)/4)))/16 + (2^(1/2)*lo
g((exp(x)/2 - 2^(1/2)/4)^2 + 1/8))/32 - (2^(1/2)*log((exp(x)/2 + 2^(1/2)/4)
^2 + 1/8))/32
```

3.226 $\int \frac{e^x}{a-\tanh(2x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{e^x}{a-\tanh(2x)}\,dx=-\frac{e^x}{1-a}+\frac{\arctan\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{(1-a)\sqrt{1+a}\sqrt[4]{1-a^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{(1-a)\sqrt{1+a}\sqrt[4]{1-a^2}}$$

```
[Out] -exp(x)/(1-a)+arctan((1-a)^(1/4)*exp(x)/(1+a)^(1/4))/(1-a)/(-a^2+1)^(1/4)/(
1+a)^(1/2)+arctanh((1-a)^(1/4)*exp(x)/(1+a)^(1/4))/(1-a)/(-a^2+1)^(1/4)/(1+
a)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2320, 396, 218, 214, 211}

$$\int \frac{e^x}{a-\tanh(2x)}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{(1-a)\sqrt{a+1}\sqrt[4]{1-a^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{(1-a)\sqrt{a+1}\sqrt[4]{1-a^2}}-\frac{e^x}{1-a}$$

```
[In] Int[E^x/(a - Tanh[2*x]),x]
```

```
[Out] -(E^x/(1 - a)) + ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)]/((1 - a)*Sqrt[1
+ a]*(1 - a^2)^(1/4)) + ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)]/((1 - a)
*Sqrt[1 + a]*(1 - a^2)^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1+x^4}{1+a-(1-a)x^4}\,dx,x,e^x\right)\\
&= -\frac{e^x}{1-a}+\frac{2\text{Subst}\left(\int \frac{1}{1+a+(-1+a)x^4}\,dx,x,e^x\right)}{1-a}\\
&= -\frac{e^x}{1-a}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a}-\sqrt{1-a}x^2}\,dx,x,e^x\right)}{(1-a)\sqrt{1+a}}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+a}+\sqrt{1-a}x^2}\,dx,x,e^x\right)}{(1-a)\sqrt{1+a}}\\
&= -\frac{e^x}{1-a}+\frac{\arctan\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{(1-a)\sqrt{1+a}\sqrt[4]{1-a^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{(1-a)\sqrt{1+a}\sqrt[4]{1-a^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int \frac{e^x}{a-\tanh(2x)}\,dx=\frac{-\sqrt[4]{1-a}(1+a)^{3/4}e^x+\arctan\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)+\operatorname{arctanh}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{(1-a)^{5/4}(1+a)^{3/4}}$$

```
[In] Integrate[E^x/(a - Tanh[2*x]),x]
```

```
[Out] (-((1 - a)^(1/4)*(1 + a)^(3/4)*E^x) + ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1
/4)] + ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/((1 - a)^(5/4)*(1 + a)^(
3/4))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.65

method	result	si
risch	$\frac{e^x}{-1+a}+\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(16a^8-32a^7-32a^6+96a^5-96a^3+32a^2+32a-16\right)\_Z^4\right)}\_R\ln\left(e^x+\left(-2a^2+2\right)\_R\right)\right)$	7
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-4\_Z^3+6a\_Z^2-4\_Z+a\right)}\frac{\left(-\_R^2+2\_R-1\right)\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)}{\_R^3a-3\_R^2+3\_Ra-1}}{-2+2a}-\frac{2}{(-1+a)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	8

```
[In] int(exp(x)/(a-tanh(2*x)),x,method=_RETURNVERBOSE)
```

```
[Out] exp(x)/(-1+a)+sum(_R*ln(exp(x)+(-2*a^2+2)*_R),_R=RootOf(1+(16*a^8-32*a^7-32
*a^6+96*a^5-96*a^3+32*a^2+32*a-16)*_Z^4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.79

$$\int \frac{e^x}{a-\tanh(2x)}\,dx=$$

$$-\frac{(a-1)\left(-\frac{1}{a^8-2a^7-2a^6+6a^5-6a^3+2a^2+2a-1}\right)^{\frac{1}{4}}\log\left(\left(a^2-1\right)\left(-\frac{1}{a^8-2a^7-2a^6+6a^5-6a^3+2a^2+2a-1}\right)^{\frac{1}{4}}+\cosh(x)+\right.}{}$$

```
[In] integrate(exp(x)/(a-tanh(2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*((a - 1)*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^
(1/4)*log((a^2 - 1)*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a
- 1))^(1/4) + cosh(x) + sinh(x)) - (a - 1)*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5
 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4)*log(-(a^2 - 1)*(-1/(a^8 - 2*a^7 - 2*a^6
+ 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4) + cosh(x) + sinh(x)) - (I*a - I)*
(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4)*log(-(I*
a^2 - I)*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4)
 + cosh(x) + sinh(x)) - (-I*a + I)*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a^5 - 6*a^3
 + 2*a^2 + 2*a - 1))^(1/4)*log(-(-I*a^2 + I)*(-1/(a^8 - 2*a^7 - 2*a^6 + 6*a
^5 - 6*a^3 + 2*a^2 + 2*a - 1))^(1/4) + cosh(x) + sinh(x)) - 2*cosh(x) - 2*s
inh(x))/(a - 1)
```

Sympy [F]

$$\int \frac{e^x}{a - \tanh(2x)}\,dx = \int \frac{e^x}{a - \tanh{(2x)}}\,dx$$

```
[In] integrate(exp(x)/(a-tanh(2*x)),x)
```

```
[Out] Integral(exp(x)/(a - tanh(2*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^x}{a - \tanh(2x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(x)/(a-tanh(2*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(83) = 166$.

Time = 0.29 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.07

$$\int \frac{e^x}{a-\tanh(2x)}\,dx = -\frac{(a^4-2\,a^3+2\,a-1)^{\frac14}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac14}+2\,e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac14}}\right)}{\sqrt{2}a^3-\sqrt{2}a^2-\sqrt{2}a+\sqrt{2}} - \frac{(a^4-2\,a^3+2\,a-1)^{\frac14}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac14}-2\,e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac14}}\right)}{\sqrt{2}a^3-\sqrt{2}a^2-\sqrt{2}a+\sqrt{2}} - \frac{(a^4-2\,a^3+2\,a-1)^{\frac14}\log\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac14}e^x+\sqrt{\frac{a+1}{a-1}}+e^{(2\,x)}\right)}{2\left(\sqrt{2}a^3-\sqrt{2}a^2-\sqrt{2}a+\sqrt{2}\right)} + \frac{(a^4-2\,a^3+2\,a-1)^{\frac14}\log\left(-\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac14}e^x+\sqrt{\frac{a+1}{a-1}}+e^{(2\,x)}\right)}{2\left(\sqrt{2}a^3-\sqrt{2}a^2-\sqrt{2}a+\sqrt{2}\right)} + \frac{e^x}{a-1}$$

```
[In] integrate(exp(x)/(a-tanh(2*x)),x, algorithm="giac")
```

```
[Out] -(a^4 - 2*a^3 + 2*a - 1)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*((a + 1)/(a - 1)
)^(1/4) + 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^3 - sqrt(2)*a^2 - sqrt
(2)*a + sqrt(2)) - (a^4 - 2*a^3 + 2*a - 1)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(
2)*((a + 1)/(a - 1))^(1/4) - 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^3 -
 sqrt(2)*a^2 - sqrt(2)*a + sqrt(2)) - 1/2*(a^4 - 2*a^3 + 2*a - 1)^(1/4)*log
(sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a + 1)/(a - 1)) + e^(2*x))/(sq
rt(2)*a^3 - sqrt(2)*a^2 - sqrt(2)*a + sqrt(2)) + 1/2*(a^4 - 2*a^3 + 2*a - 1
)^(1/4)*log(-sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a + 1)/(a - 1)) +
e^(2*x))/(sqrt(2)*a^3 - sqrt(2)*a^2 - sqrt(2)*a + sqrt(2)) + e^x/(a - 1)
```

Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.52

$$\int \frac{e^x}{a-\tanh(2x)}\,dx = \frac{\ln\left(8\,a\,(-a-1)^{1/4}+8\,\mathrm{e}^x\,(a-1)^{5/4}-8\,(-a-1)^{1/4}\right)-\ln\left(8\,\mathrm{e}^x\,(a-1)^{5/4}-8\,a\,(-a-1)^{1/4}+8\,(-a-1\right.}{}$$

```
[In] int(exp(x)/(a - tanh(2*x)),x)
```

```
[Out] (log(8*a*(- a - 1)^(1/4) + 8*exp(x)*(a - 1)^(5/4) - 8*(- a - 1)^(1/4)) - lo
g(8*exp(x)*(a - 1)^(5/4) - 8*a*(- a - 1)^(1/4) + 8*(- a - 1)^(1/4)) - log(8
*exp(x)*(a - 1)^(5/4) - a*(- a - 1)^(1/4)*8i + (- a - 1)^(1/4)*8i)*1i + log
(a*(- a - 1)^(1/4)*8i + 8*exp(x)*(a - 1)^(5/4) - (- a - 1)^(1/4)*8i)*1i + 2
*exp(x)*(a - 1)^(1/4)*(- a - 1)^(3/4))/(2*(a - 1)^(5/4)*(- a - 1)^(3/4))
```

3.227 $\int \frac{e^x}{(a-\tanh(2x))^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 152

$$\int \frac{e^x}{(a-\tanh(2x))^2} dx = \frac{e^x}{(1-a)^2} + \frac{e^x}{(1-a)^2(1+a)\left(1+a+(-1+a)e^{4x}\right)} - \frac{(1+4a)\arctan\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{2(1-a)^2(1+a)^{3/2}\sqrt[4]{1-a^2}} - \frac{(1+4a)\text{arctanh}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{2(1-a)^2(1+a)^{3/2}\sqrt[4]{1-a^2}}$$

```
[Out] exp(x)/(1-a)^2+exp(x)/(1-a)^2/(1+a)/(1+a+(-1+a)*exp(4*x))-1/2*(1+4*a)*arcta
n((1-a)^(1/4)*exp(x)/(1+a)^(1/4))/(1-a)^2/(1+a)^(3/2)/(-a^2+1)^(1/4)-1/2*(1
+4*a)*arctanh((1-a)^(1/4)*exp(x)/(1+a)^(1/4))/(1-a)^2/(1+a)^(3/2)/(-a^2+1)^
(1/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2320, 398, 393, 218, 214, 211}

$$\int \frac{e^x}{(a-\tanh(2x))^2} dx = -\frac{(4a+1)\arctan\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{2(1-a)^2(a+1)^{3/2}\sqrt[4]{1-a^2}} - \frac{(4a+1)\text{arctanh}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{a+1}}\right)}{2(1-a)^2(a+1)^{3/2}\sqrt[4]{1-a^2}} + \frac{e^x}{(1-a)^2} + \frac{e^x}{(1-a)^2(a+1)\left((a-1)e^{4x}+a+1\right)}$$

```
[In] Int[E^x/(a - Tanh[2*x])^2,x]
```

```
[Out] E^x/(1 - a)^2 + E^x/((1 - a)^2*(1 + a)*(1 + a + (-1 + a)*E^(4*x))) - ((1 +
4*a)*ArcTan[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/(2*(1 - a)^2*(1 + a)^(3/2)*
```

```
(1 - a^2)^(1/4)) - ((1 + 4*a)*ArcTanh[((1 - a)^(1/4)*E^x)/(1 + a)^(1/4)])/(
2*(1 - a)^2*(1 + a)^(3/2)*(1 - a^2)^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{(1+x^4)^2}{(1+a-(1-a)x^4)^2}\,dx, x, e^x\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{(-1+a)^2} - \frac{4(a-(1-a)x^4)}{(-1+a)^2\left(1+a+(-1+a)x^4\right)^2}\right) dx, x, e^x\right)$$

$$= \frac{e^x}{(1-a)^2} - \frac{4\text{Subst}\left(\int \frac{a-(1-a)x^4}{(1+a+(-1+a)x^4)^2} dx, x, e^x\right)}{(1-a)^2}$$

$$= \frac{e^x}{(1-a)^2} + \frac{e^x}{(1-a)^2(1+a)\left(1+a-(1-a)e^{4x}\right)} - \frac{(1+4a)\text{Subst}\left(\int \frac{1}{1+a+(-1+a)x^4} dx, x, e^x\right)}{(1-a)^2(1+a)}$$

$$= \frac{e^x}{(1-a)^2} + \frac{e^x}{(1-a)^2(1+a)\left(1+a-(1-a)e^{4x}\right)} - \frac{(1+4a)\text{Subst}\left(\int \frac{1}{\sqrt{1+a}-\sqrt{1-a}x^2} dx, x, e^x\right)}{2(1-a)^2(1+a)^{3/2}} - \frac{(1+4a)\text{Subst}\left(\int \frac{1}{\sqrt{1+a}+\sqrt{1-a}x^2} dx, x, e^x\right)}{2(1-a)^2(1+a)^{3/2}}$$

$$= \frac{e^x}{(1-a)^2} + \frac{e^x}{(1-a)^2(1+a)\left(1+a-(1-a)e^{4x}\right)} - \frac{(1+4a)\arctan\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{2(1-a)^2(1+a)^{3/2}\sqrt[4]{1-a^2}} - \frac{(1+4a)\text{arctanh}\left(\frac{\sqrt[4]{1-a}e^x}{\sqrt[4]{1+a}}\right)}{2(1-a)^2(1+a)^{3/2}\sqrt[4]{1-a^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.70

$$\int \frac{e^x}{(a-\tanh(2x))^2} dx = \frac{\frac{4(-1+a)e^x\left(2+2a-e^{4x}+a^2(1+e^{4x})\right)}{1+a-e^{4x}+ae^{4x}} + (1+4a)\text{RootSum}\left[1+a-\#1^4+a\#1^4\&, \frac{x-\log\left(e^x-\#1\right)}{\#1^3}\&\right]}{4(-1+a)^3(1+a)}$$

```
[In] Integrate[E^x/(a - Tanh[2*x])^2,x]
```

```
[Out] ((4*(-1 + a)*E^x*(2 + 2*a - E^(4*x) + a^2*(1 + E^(4*x))))/(1 + a - E^(4*x)
+ a*E^(4*x)) + (1 + 4*a)*RootSum[1 + a - #1^4 + a*#1^4 & , (x - Log[E^x - #
1])/#1^3 & ])/(4*(-1 + a)^3*(1 + a))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.26

method	result
default	$-\frac{2}{(-1+a)^2(\tanh(\frac{x}{2})-1)}-\frac{2\left(\frac{-\frac{(a-2)\tanh(\frac{x}{2})^3}{2a(a+1)}-\frac{3\tanh(\frac{x}{2})^2}{2(a+1)}+\frac{(a+2)\tanh(\frac{x}{2})}{2a(a+1)}-\frac{1}{2(a+1)}}{\tanh(\frac{x}{2})^4a+6\tanh(\frac{x}{2})^2a-4\tanh(\frac{x}{2})^3+a-4\tanh(\frac{x}{2})}+\frac{(1+4a)\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^4-4\_Z^3+6a\_Z^2\right.}\right.}{}\right.}{(-1+a)^2}$
risch	$\frac{e^x}{a^2-2a+1}+\frac{e^x}{(a+1)(a^2-2a+1)(a\,e^{4x}-e^{4x}+a+1)}+\left(\sum_{\_R=\mathrm{RootOf}((256a^{16}-512a^{15}-1536a^{14}+3584a^{13}+3584a^{12}-10752a^{11}-358\right.$

[In] int(exp(x)/(a-tanh(2*x))^2,x,method=_RETURNVERBOSE)

[Out] -2/(-1+a)^2/(tanh(1/2*x)-1)-2/(-1+a)^2*((-1/2*(a-2)/a/(a+1)*tanh(1/2*x)^3-3/2/(a+1)*tanh(1/2*x)^2+1/2*(a+2)/a/(a+1)*tanh(1/2*x)-1/2/(a+1))/(tanh(1/2*x)^4*a+6*tanh(1/2*x)^2*a-4*tanh(1/2*x)^3+a-4*tanh(1/2*x))+1/8*(1+4*a)/(a+1)*sum((_R^2-2*_R+1)/(_R^3*a-3*_R^2+3*_R*a-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^4*a-4*_Z^3+6*_Z^2*a-4*_Z+a)))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 1604, normalized size of antiderivative = 10.55

$$\int \frac{e^x}{(a-\tanh(2x))^2}\,dx = \text{Too large to display}$$

[In] integrate(exp(x)/(a-tanh(2*x))^2,x, algorithm="fricas")

[Out] 1/4*(4*(a^2 - 1)*cosh(x)^5 + 40*(a^2 - 1)*cosh(x)^3*sinh(x)^2 + 40*(a^2 - 1)*cosh(x)^2*sinh(x)^3 + 20*(a^2 - 1)*cosh(x)*sinh(x)^4 + 4*(a^2 - 1)*sinh(x)^5 - ((a^4 - 2*a^3 + 2*a - 1)*cosh(x)^4 + 4*(a^4 - 2*a^3 + 2*a - 1)*cosh(x)^3*sinh(x) + 6*(a^4 - 2*a^3 + 2*a - 1)*cosh(x)^2*sinh(x)^2 + 4*(a^4 - 2*a^3 + 2*a - 1)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^3 + 2*a - 1)*sinh(x)^4 + a^4 - 2*a^2 + 1)*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)*log((4*a + 1)*cosh(x) + (4*a + 1)*sinh(x) + (a^4 - 2*a^2 + 1)*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)) + ((

```
a^4 - 2*a^3 + 2*a - 1)*cosh(x)^4 + 4*(a^4 - 2*a^3 + 2*a - 1)*cosh(x)^3*sinh
(x) + 6*(a^4 - 2*a^3 + 2*a - 1)*cosh(x)^2*sinh(x)^2 + 4*(a^4 - 2*a^3 + 2*a
- 1)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^3 + 2*a - 1)*sinh(x)^4 + a^4 - 2*a^2 +
1)*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a
^13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*
a^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)*log((4*a + 1)*cosh(x) + (4*a + 1)*si
nh(x) - (a^4 - 2*a^2 + 1)*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 -
 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7
+ 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)) + ((I*a^4 -
2*I*a^3 + 2*I*a - I)*cosh(x)^4 - 4*(-I*a^4 + 2*I*a^3 - 2*I*a + I)*cosh(x)^3
*sinh(x) - 6*(-I*a^4 + 2*I*a^3 - 2*I*a + I)*cosh(x)^2*sinh(x)^2 - 4*(-I*a^4
 + 2*I*a^3 - 2*I*a + I)*cosh(x)*sinh(x)^3 + (I*a^4 - 2*I*a^3 + 2*I*a - I)*s
inh(x)^4 + I*a^4 - 2*I*a^2 + I)*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(
a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 7
0*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)*log((4*
a + 1)*cosh(x) + (4*a + 1)*sinh(x) - (I*a^4 - 2*I*a^2 + I)*(-(256*a^4 + 256
*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12 - 42*
a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 + 6*a^
2 + 2*a - 1))^(1/4)) + ((-I*a^4 + 2*I*a^3 - 2*I*a + I)*cosh(x)^4 - 4*(I*a^4
 - 2*I*a^3 + 2*I*a - I)*cosh(x)^3*sinh(x) - 6*(I*a^4 - 2*I*a^3 + 2*I*a - I)
*cosh(x)^2*sinh(x)^2 - 4*(I*a^4 - 2*I*a^3 + 2*I*a - I)*cosh(x)*sinh(x)^3 +
(-I*a^4 + 2*I*a^3 - 2*I*a + I)*sinh(x)^4 - I*a^4 + 2*I*a^2 - I)*(-(256*a^4
+ 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 - 6*a^14 + 14*a^13 + 14*a^12
- 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6 + 42*a^5 - 14*a^4 - 14*a^3 +
 6*a^2 + 2*a - 1))^(1/4)*log((4*a + 1)*cosh(x) + (4*a + 1)*sinh(x) - (-I*a^
4 + 2*I*a^2 - I)*(-(256*a^4 + 256*a^3 + 96*a^2 + 16*a + 1)/(a^16 - 2*a^15 -
 6*a^14 + 14*a^13 + 14*a^12 - 42*a^11 - 14*a^10 + 70*a^9 - 70*a^7 + 14*a^6
+ 42*a^5 - 14*a^4 - 14*a^3 + 6*a^2 + 2*a - 1))^(1/4)) + 4*(a^2 + 2*a + 2)*c
osh(x) + 4*(5*(a^2 - 1)*cosh(x)^4 + a^2 + 2*a + 2)*sinh(x))/((a^4 - 2*a^3 +
 2*a - 1)*cosh(x)^4 + 4*(a^4 - 2*a^3 + 2*a - 1)*cosh(x)^3*sinh(x) + 6*(a^4
- 2*a^3 + 2*a - 1)*cosh(x)^2*sinh(x)^2 + 4*(a^4 - 2*a^3 + 2*a - 1)*cosh(x)*
sinh(x)^3 + (a^4 - 2*a^3 + 2*a - 1)*sinh(x)^4 + a^4 - 2*a^2 + 1)
```

Sympy [F]

$$\int \frac{e^x}{(a-\tanh(2x))^2}\,dx = \int \frac{e^x}{\left(a-\tanh\left(2x\right)\right)^2}\,dx$$

```
[In] integrate(exp(x)/(a-tanh(2*x))**2,x)
[Out] Integral(exp(x)/(a - tanh(2*x))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{e^x}{(a-\tanh(2x))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(exp(x)/(a-tanh(2*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a-1>0)', see `assume?` for more det
ails)Is
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(119) = 238.

Time = 0.27 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.00

$$\int \frac{e^x}{(a-\tanh(2x))^2}\,dx = -\frac{(a^4-2\,a^3+2\,a-1)^{\frac{1}{4}}(4\,a+1)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}+2\,e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^5-\sqrt{2}a^4-2\sqrt{2}a^3+2\sqrt{2}a^2+\sqrt{2}a-\sqrt{2}\right)} - \frac{(a^4-2\,a^3+2\,a-1)^{\frac{1}{4}}(4\,a+1)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}-2\,e^x\right)}{2\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^5-\sqrt{2}a^4-2\sqrt{2}a^3+2\sqrt{2}a^2+\sqrt{2}a-\sqrt{2}\right)} - \frac{(a^4-2\,a^3+2\,a-1)^{\frac{1}{4}}(4\,a+1)\log\left(\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}e^x+\sqrt{\frac{a+1}{a-1}}+e^{(2\,x)}\right)}{4\left(\sqrt{2}a^5-\sqrt{2}a^4-2\sqrt{2}a^3+2\sqrt{2}a^2+\sqrt{2}a-\sqrt{2}\right)} + \frac{(a^4-2\,a^3+2\,a-1)^{\frac{1}{4}}(4\,a+1)\log\left(-\sqrt{2}\left(\frac{a+1}{a-1}\right)^{\frac{1}{4}}e^x+\sqrt{\frac{a+1}{a-1}}+e^{(2\,x)}\right)}{4\left(\sqrt{2}a^5-\sqrt{2}a^4-2\sqrt{2}a^3+2\sqrt{2}a^2+\sqrt{2}a-\sqrt{2}\right)} + \frac{e^x}{a^2-2\,a+1} + \frac{e^x}{(a^3-a^2-a+1)(ae^{(4\,x)}+a-e^{(4\,x)}+1)}$$

```
[In] integrate(exp(x)/(a-tanh(2*x))^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^4 - 2*a^3 + 2*a - 1)^(1/4)*(4*a + 1)*arctan(1/2*sqrt(2)*(sqrt(2)*((
a + 1)/(a - 1))^(1/4) + 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^5 - sqrt
(2)*a^4 - 2*sqrt(2)*a^3 + 2*sqrt(2)*a^2 + sqrt(2)*a - sqrt(2)) - 1/2*(a^4 -
```

```
2*a^3 + 2*a - 1)^(1/4)*(4*a + 1)*arctan(-1/2*sqrt(2)*(sqrt(2)*((a + 1)/(a
- 1))^(1/4) - 2*e^x)/((a + 1)/(a - 1))^(1/4))/(sqrt(2)*a^5 - sqrt(2)*a^4 -
2*sqrt(2)*a^3 + 2*sqrt(2)*a^2 + sqrt(2)*a - sqrt(2)) - 1/4*(a^4 - 2*a^3 + 2
*a - 1)^(1/4)*(4*a + 1)*log(sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a +
 1)/(a - 1)) + e^(2*x))/(sqrt(2)*a^5 - sqrt(2)*a^4 - 2*sqrt(2)*a^3 + 2*sqrt
(2)*a^2 + sqrt(2)*a - sqrt(2)) + 1/4*(a^4 - 2*a^3 + 2*a - 1)^(1/4)*(4*a + 1
)*log(-sqrt(2)*((a + 1)/(a - 1))^(1/4)*e^x + sqrt((a + 1)/(a - 1)) + e^(2*x
))/(sqrt(2)*a^5 - sqrt(2)*a^4 - 2*sqrt(2)*a^3 + 2*sqrt(2)*a^2 + sqrt(2)*a -
 sqrt(2)) + e^x/(a^2 - 2*a + 1) + e^x/((a^3 - a^2 - a + 1)*(a*e^(4*x) + a -
 e^(4*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.84

$$\int \frac{e^x}{(a-\tanh(2x))^2}\,dx = \frac{e^x}{(a-1)^2} + \frac{\ln\left(\frac{4a+1}{(a-1)^{13/4}(-a-1)^{3/4}} + \frac{e^x\,(4a+1)}{a^4-2a^3+2a-1}\right)(4a+1)}{4\,(a-1)^{9/4}\,(-a-1)^{7/4}} - \frac{\ln\left(\frac{e^x\,(4a+1)}{a^4-2a^3+2a-1} - \frac{4a+1}{(a-1)^{13/4}(-a-1)^{3/4}}\right)(4a+1)}{4\,(a-1)^{9/4}\,(-a-1)^{7/4}} + \frac{e^x}{(a-1)^2\,(a+1)\,(a+e^{4x}\,(a-1)+1)} - \frac{\ln\left(\frac{e^x\,(4a+1)}{(a-1)^3\,(a+1)} - \frac{(4a+1)\,1i}{(a-1)^{13/4}(-a-1)^{3/4}}\right)(4a+1)\,1i}{4\,(a-1)^{9/4}\,(-a-1)^{7/4}} + \frac{\ln\left(\frac{e^x\,(4a+1)}{(a-1)^3\,(a+1)} + \frac{(4a+1)\,1i}{(a-1)^{13/4}(-a-1)^{3/4}}\right)(4a+1)\,1i}{4\,(a-1)^{9/4}\,(-a-1)^{7/4}}$$

```
[In] int(exp(x)/(a - tanh(2*x))^2,x)
```

```
[Out] exp(x)/(a - 1)^2 - (log((exp(x)*(4*a + 1))/((a - 1)^3*(a + 1)) - ((4*a + 1)
*1i)/((a - 1)^(13/4)*(- a - 1)^(3/4)))*(4*a + 1)*1i)/(4*(a - 1)^(9/4)*(- a
- 1)^(7/4)) + (log(((4*a + 1)*1i)/((a - 1)^(13/4)*(- a - 1)^(3/4)) + (exp(x
)*(4*a + 1))/((a - 1)^3*(a + 1)))*(4*a + 1)*1i)/(4*(a - 1)^(9/4)*(- a - 1)^
(7/4)) + (log((4*a + 1)/((a - 1)^(13/4)*(- a - 1)^(3/4)) + (exp(x)*(4*a + 1
))/(2*a - 2*a^3 + a^4 - 1))*(4*a + 1))/(4*(a - 1)^(9/4)*(- a - 1)^(7/4)) -
(log((exp(x)*(4*a + 1))/(2*a - 2*a^3 + a^4 - 1) - (4*a + 1)/((a - 1)^(13/4)
*(- a - 1)^(3/4)))*(4*a + 1))/(4*(a - 1)^(9/4)*(- a - 1)^(7/4)) + exp(x)/((
a - 1)^2*(a + 1)*(a + exp(4*x)*(a - 1) + 1))
```

3.228 $\int e^{c(a+bx)} \tanh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 167

$$\int e^{c(a+bx)} \tanh^3(d+ex)\,dx = \frac{e^{c(a+bx)}}{bc} - \frac{6e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc} + \frac{12e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc} - \frac{8e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc}$$

```
[Out] exp(c*(b*x+a))/b/c-6*exp(c*(b*x+a))*hypergeom([1, 1/2*b*c/e],[1+1/2*b*c/e],
-exp(2*e*x+2*d))/b/c+12*exp(c*(b*x+a))*hypergeom([2, 1/2*b*c/e],[1+1/2*b*c/
e],-exp(2*e*x+2*d))/b/c-8*exp(c*(b*x+a))*hypergeom([3, 1/2*b*c/e],[1+1/2*b*
c/e],-exp(2*e*x+2*d))/b/c
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5592, 2225, 2283}

$$\int e^{c(a+bx)} \tanh^3(d+ex)\,dx = -\frac{6e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},\frac{bc}{2e}+1,-e^{2(d+ex)}\right)}{bc} + \frac{12e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},\frac{bc}{2e}+1,-e^{2(d+ex)}\right)}{bc} - \frac{8e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc}{2e},\frac{bc}{2e}+1,-e^{2(d+ex)}\right)}{bc} + \frac{e^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (6*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) + (12*E^(c*(a + b*x))*Hypergeom
etric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) - (8*E^(
c*(a + b*x))*Hypergeometric2F1[3, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d +
e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5592

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{c(a+bx)} - \frac{8e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^3} + \frac{12e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2} - \frac{6e^{c(a+bx)}}{1+e^{2(d+ex)}}\right) dx \\
&= -\left(6\int \frac{e^{c(a+bx)}}{1+e^{2(d+ex)}}\,dx\right) - 8\int \frac{e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^3}\,dx + 12\int \frac{e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2}\,dx + \int e^{c(a+bx)}\,dx \\
&= \frac{e^{c(a+bx)}}{bc} - \frac{6e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc} \\
&\quad + \frac{12e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc} \\
&\quad - \frac{8e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.23

$$\int e^{c(a+bx)}\tanh^3(d+ex)\,dx$$
$$=\frac{1}{2}e^{ac}\left(\frac{2(b^2c^2+2e^2)\,e^{2d}\left(\frac{e^{(bc+2e)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{bc}{2e},2+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc+2e}-\frac{e^{bcx}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc}\right)}{e^2\,(1+e^{2d})}+\frac{e^{bcx}\text{sech}^2(d+ex)}{e}-\frac{bce^{bcx}\text{sech}(d)\text{sech}(d+ex)\sinh(ex)}{e^2}+\frac{2e^{bcx}\tanh(d)}{bc}\right)$$

```
[In] Integrate[E^(c*(a + b*x))*Tanh[d + e*x]^3,x]
```

```
[Out] (E^(a*c)*((2*(b^2*c^2 + 2*e^2)*E^(2*d)*((E^((b*c + 2*e)*x)*Hypergeometric2F
1[1, 1 + (b*c)/(2*e), 2 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c + 2*e) - (E^
(b*c*x)*Hypergeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))
])/(b*c)))/(e^2*(1 + E^(2*d))) + (E^(b*c*x)*Sech[d + e*x]^2)/e - (b*c*E^(b*
c*x)*Sech[d]*Sech[d + e*x]*Sinh[e*x])/e^2 + (2*E^(b*c*x)*Tanh[d])/(b*c)))/2
```

Maple [F]

$$\int \mathrm{e}^{c(bx+a)}\tanh\left(ex+d\right)^3dx$$

```
[In] int(exp(c*(b*x+a))*tanh(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*tanh(e*x+d)^3,x)
```

Fricas [F]

$$\int e^{c(a+bx)}\tanh^3(d+ex)\,dx=\int e^{((bx+a)c)}\tanh\left(ex+d\right)^3\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(b*c*x + a*c)*tanh(e*x + d)^3, x)
```

Sympy [F]

$$\int e^{c(a+bx)} \tanh^3(d+ex)\,dx = e^{ac} \int e^{bcx} \tanh^3{(d+ex)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)**3,x)
[Out] exp(a*c)*Integral(exp(b*c*x)*tanh(d + e*x)**3, x)
```

Maxima [F]

$$\int e^{c(a+bx)} \tanh^3(d+ex)\,dx = \int e^{((bx+a)c)} \tanh{(ex+d)}^3\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="maxima")
[Out] 48*(b^2*c^2*e*e^(a*c) + 2*e^3*e^(a*c))*integrate(e^(b*c*x)/(b^3*c^3 - 12*b^
2*c^2*e + 44*b*c*e^2 - 48*e^3 + (b^3*c^3*e^(8*d) - 12*b^2*c^2*e*e^(8*d) + 4
4*b*c*e^2*e^(8*d) - 48*e^3*e^(8*d))*e^(8*e*x) + 4*(b^3*c^3*e^(6*d) - 12*b^2
*c^2*e*e^(6*d) + 44*b*c*e^2*e^(6*d) - 48*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^
3*e^(4*d) - 12*b^2*c^2*e*e^(4*d) + 44*b*c*e^2*e^(4*d) - 48*e^3*e^(4*d))*e^(
4*e*x) + 4*(b^3*c^3*e^(2*d) - 12*b^2*c^2*e*e^(2*d) + 44*b*c*e^2*e^(2*d) - 4
8*e^3*e^(2*d))*e^(2*e*x)), x) - (b^3*c^3*e^(a*c) + 36*b^2*c^2*e*e^(a*c) + 4
4*b*c*e^2*e^(a*c) + 48*e^3*e^(a*c) - (b^3*c^3*e^(a*c + 6*d) - 12*b^2*c^2*e*
e^(a*c + 6*d) + 44*b*c*e^2*e^(a*c + 6*d) - 48*e^3*e^(a*c + 6*d))*e^(6*e*x)
+ 3*(b^3*c^3*e^(a*c + 4*d) - 8*b^2*c^2*e*e^(a*c + 4*d) + 4*b*c*e^2*e^(a*c +
 4*d) + 48*e^3*e^(a*c + 4*d))*e^(4*e*x) - 3*(b^3*c^3*e^(a*c + 2*d) - 28*b*c
*e^2*e^(a*c + 2*d) - 48*e^3*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^4*c^4 -
12*b^3*c^3*e + 44*b^2*c^2*e^2 - 48*b*c*e^3 + (b^4*c^4*e^(6*d) - 12*b^3*c^3*
e*e^(6*d) + 44*b^2*c^2*e^2*e^(6*d) - 48*b*c*e^3*e^(6*d))*e^(6*e*x) + 3*(b^4
*c^4*e^(4*d) - 12*b^3*c^3*e*e^(4*d) + 44*b^2*c^2*e^2*e^(4*d) - 48*b*c*e^3*e
^(4*d))*e^(4*e*x) + 3*(b^4*c^4*e^(2*d) - 12*b^3*c^3*e*e^(2*d) + 44*b^2*c^2*
e^2*e^(2*d) - 48*b*c*e^3*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int e^{c(a+bx)} \tanh^3(d+ex)\,dx = \int e^{((bx+a)c)} \tanh{(ex+d)}^3\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^3,x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tanh(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \tanh^3(d+ex)\,dx = \int e^{c\,(a+b\,x)} \tanh(d+e\,x)^3\,dx$$

```
[In] int(exp(c*(a + b*x))*tanh(d + e*x)^3,x)
[Out] int(exp(c*(a + b*x))*tanh(d + e*x)^3, x)
```

3.229 $\int e^{c(a+bx)} \tanh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 117

$$\int e^{c(a+bx)} \tanh^2(d+ex)\,dx = \frac{e^{c(a+bx)}}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc}$$

```
[Out] exp(c*(b*x+a))/b/c-4*exp(c*(b*x+a))*hypergeom([1, 1/2*b*c/e],[1+1/2*b*c/e],
-exp(2*e*x+2*d))/b/c+4*exp(c*(b*x+a))*hypergeom([2, 1/2*b*c/e],[1+1/2*b*c/e
],-exp(2*e*x+2*d))/b/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5592, 2225, 2283}

$$\int e^{c(a+bx)} \tanh^2(d+ex)\,dx = -\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},\frac{bc}{2e}+1,-e^{2(d+ex)}\right)}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},\frac{bc}{2e}+1,-e^{2(d+ex)}\right)}{bc} + \frac{e^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (4*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c) + (4*E^(c*(a + b*x))*Hypergeome
tric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5592

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(e^{c(a+bx)} + \frac{4e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2} - \frac{4e^{c(a+bx)}}{1+e^{2(d+ex)}}\right) dx \\
&= 4\int \frac{e^{c(a+bx)}}{\left(1+e^{2(d+ex)}\right)^2}\, dx - 4\int \frac{e^{c(a+bx)}}{1+e^{2(d+ex)}}\, dx + \int e^{c(a+bx)}\, dx \\
&= \frac{e^{c(a+bx)}}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc}{2e}, 1+\frac{bc}{2e}, -e^{2(d+ex)}\right)}{bc} \\
&\quad + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2, \frac{bc}{2e}, 1+\frac{bc}{2e}, -e^{2(d+ex)}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.44

$$\int e^{c(a+bx)} \tanh^2(d+ex)\, dx$$

$$= \frac{e^{c(a+bx)}\left(2b^2c^2e^{2(d+ex)}\,\text{Hypergeometric2F1}\left(1, 1+\frac{bc}{2e}, 2+\frac{bc}{2e}, -e^{2(d+ex)}\right) - (bc+2e)\left(2bce^{2d}\,\text{Hypergeometric}\right.\right.}{bce(bc+2e)\left(1+e^{2d}\right)}$$

```
[In] Integrate[E^(c*(a + b*x))*Tanh[d + e*x]^2,x]
```

```
[Out] (E^(c*(a + b*x))*(2*b^2*c^2*E^(2*(d + e*x))*Hypergeometric2F1[1, 1 + (b*c)/
(2*e), 2 + (b*c)/(2*e), -E^(2*(d + e*x))] - (b*c + 2*e)*(2*b*c*E^(2*d)*Hype
rgeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))] - (1 + E^(
2*d))*(e - b*c*Sech[d]*Sech[d + e*x]*Sinh[e*x]))))/(b*c*e*(b*c + 2*e)*(1 +
E^(2*d)))
```

Maple [F]

$$\int \mathrm{e}^{c(bx+a)} \tanh (ex+d)^2 \, dx$$

```
[In] int(exp(c*(b*x+a))*tanh(e*x+d)^2,x)
[Out] int(exp(c*(b*x+a))*tanh(e*x+d)^2,x)
```

Fricas [F]

$$\int e^{c(a+bx)} \tanh^2(d+ex) \, dx = \int e^{((bx+a)c)} \tanh (ex+d)^2 \, dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="fricas")
[Out] integral(e^(b*c*x + a*c)*tanh(e*x + d)^2, x)
```

Sympy [F]

$$\int e^{c(a+bx)} \tanh^2(d+ex) \, dx = e^{ac} \int e^{bcx} \tanh^2 (d+ex) \, dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)**2,x)
[Out] exp(a*c)*Integral(exp(b*c*x)*tanh(d + e*x)**2, x)
```

Maxima [F]

$$\int e^{c(a+bx)} \tanh^2(d+ex) \, dx = \int e^{((bx+a)c)} \tanh (ex+d)^2 \, dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="maxima")
[Out] -16*b*c*e*integrate(e^(b*c*x + a*c)/(b^2*c^2 - 6*b*c*e + 8*e^2 + (b^2*c^2*e
^(6*d) - 6*b*c*e*e^(6*d) + 8*e^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d) -
6*b*c*e*e^(4*d) + 8*e^2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d) - 6*b*c*e*e
```

```
^(2*d) + 8*e^2*e^(2*d))*e^(2*e*x)), x) + (b^2*c^2*e^(a*c) + 10*b*c*e*e^(a*c
) + 8*e^2*e^(a*c) + (b^2*c^2*e^(a*c + 4*d) - 6*b*c*e*e^(a*c + 4*d) + 8*e^2*
e^(a*c + 4*d))*e^(4*e*x) - 2*(b^2*c^2*e^(a*c + 2*d) - 2*b*c*e*e^(a*c + 2*d)
 - 8*e^2*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^3*c^3 - 6*b^2*c^2*e + 8*b*c
*e^2 + (b^3*c^3*e^(4*d) - 6*b^2*c^2*e*e^(4*d) + 8*b*c*e^2*e^(4*d))*e^(4*e*x
) + 2*(b^3*c^3*e^(2*d) - 6*b^2*c^2*e*e^(2*d) + 8*b*c*e^2*e^(2*d))*e^(2*e*x)
)
```

Giac [F]

$$\int e^{c(a+bx)}\tanh^2(d+ex)\,dx=\int e^{((bx+a)c)}\tanh\left(ex+d\right)^2\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d)^2,x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tanh(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tanh^2(d+ex)\,dx=\int \mathrm{e}^{c\,(a+b\,x)}\,\mathrm{tanh}(d+e\,x)^2\,dx$$

```
[In] int(exp(c*(a + b*x))*tanh(d + e*x)^2,x)
[Out] int(exp(c*(a + b*x))*tanh(d + e*x)^2, x)
```

3.230 $\int e^{c(a+bx)}\tanh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int e^{c(a+bx)}\tanh(d+ex)\,dx=\frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},1+\frac{bc}{2e},-e^{2(d+ex)}\right)}{bc}$$

```
[Out] exp(c*(b*x+a))/b/c-2*exp(c*(b*x+a))*hypergeom([1, 1/2*b*c/e],[1+1/2*b*c/e],
-exp(2*e*x+2*d))/b/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5592, 2225, 2283}

$$\int e^{c(a+bx)}\tanh(d+ex)\,dx=\frac{e^{c(a+bx)}}{bc}-\frac{2e^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},\frac{bc}{2e}+1,-e^{2(d+ex)}\right)}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Tanh[d + e*x],x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (2*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), -E^(2*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5592

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tanh[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((-1 + E^(2*(d + e*x)))^n/(1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(e^{c(a+bx)} - \frac{2e^{c(a+bx)}}{1+e^{2(d+ex)}}\right) dx \\ &= -\left(2\int \frac{e^{c(a+bx)}}{1+e^{2(d+ex)}}\, dx\right) + \int e^{c(a+bx)}\, dx \\ &= \frac{e^{c(a+bx)}}{bc} - \frac{2e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc}{2e}, 1+\frac{bc}{2e}, -e^{2(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int e^{c(a+bx)}\tanh(d+ex)\, dx$$
$$= \frac{e^{c(a+bx)}\left(2bce^{2(d+ex)}\,\text{Hypergeometric2F1}\left(1, 1+\frac{bc}{2e}, 2+\frac{bc}{2e}, -e^{2(d+ex)}\right) - (bc+2e)\left(1-e^{2d}+2e^{2d}\,\text{Hypergeor}\right.\right.}{bc(bc+2e)\left(1+e^{2d}\right)}$$

```
[In] Integrate[E^(c*(a + b*x))*Tanh[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*(2*b*c*E^(2*(d + e*x))*Hypergeometric2F1[1, 1 + (b*c)/(2*e
), 2 + (b*c)/(2*e), -E^(2*(d + e*x))] - (b*c + 2*e)*(1 - E^(2*d) + 2*E^(2*d
)*Hypergeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), -E^(2*(d + e*x))])))/(
b*c*(b*c + 2*e)*(1 + E^(2*d)))
```

Maple [F]

$$\int e^{c(bx+a)} \tanh(ex+d)\,dx$$

[In] int(exp(c*(b*x+a))*tanh(e*x+d),x)

[Out] int(exp(c*(b*x+a))*tanh(e*x+d),x)

Fricas [F]

$$\int e^{c(a+bx)} \tanh(d+ex)\,dx = \int e^{((bx+a)c)} \tanh(ex+d)\,dx$$

[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x, algorithm="fricas")

[Out] integral(e^(b*c*x + a*c)*tanh(e*x + d), x)

Sympy [F]

$$\int e^{c(a+bx)} \tanh(d+ex)\,dx = e^{ac} \int e^{bcx} \tanh(d+ex)\,dx$$

[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x)

[Out] exp(a*c)*Integral(exp(b*c*x)*tanh(d + e*x), x)

Maxima [F]

$$\int e^{c(a+bx)} \tanh(d+ex)\,dx = \int e^{((bx+a)c)} \tanh(ex+d)\,dx$$

[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x, algorithm="maxima")

[Out] 4*e*integrate(e^(b*c*x + a*c)/(b*c + (b*c*e^(4*d) - 2*e*e^(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d) - 2*e*e^(2*d))*e^(2*e*x) - 2*e), x) - (b*c*e^(a*c) + 2*e*e^(a*c) - (b*c*e^(a*c + 2*d) - 2*e*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^2*c^2 - 2*b*c*e + (b^2*c^2*e^(2*d) - 2*b*c*e*e^(2*d))*e^(2*e*x))

Giac [F]

$$\int e^{c(a+bx)}\tanh(d+ex)\,dx = \int e^{((bx+a)c)}\tanh(ex+d)\,dx$$

```
[In] integrate(exp(c*(b*x+a))*tanh(e*x+d),x, algorithm="giac")
[Out] integrate(e^((b*x + a)*c)*tanh(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tanh(d+ex)\,dx = \int \mathrm{e}^{c\,(a+b\,x)}\tanh(d+e\,x)\,dx$$

```
[In] int(exp(c*(a + b*x))*tanh(d + e*x),x)
[Out] int(exp(c*(a + b*x))*tanh(d + e*x), x)
```

3.231 $\int e^{c(a+bx)} \coth(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int e^{c(a+bx)} \coth(d+ex)\,dx = \frac{e^{c(a+bx)}}{bc} - \frac{2e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc}{2e}, 1+\frac{bc}{2e}, e^{2(d+ex)}\right)}{bc}$$

```
[Out] exp(c*(b*x+a))/b/c-2*exp(c*(b*x+a))*hypergeom([1, 1/2*b*c/e],[1+1/2*b*c/e],
exp(2*e*x+2*d))/b/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5593, 2225, 2283}

$$\int e^{c(a+bx)} \coth(d+ex)\,dx = \frac{e^{c(a+bx)}}{bc} - \frac{2e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc}{2e}, \frac{bc}{2e}+1, e^{2(d+ex)}\right)}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Coth[d + e*x],x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (2*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5593

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(e^{c(a+bx)} + \frac{2e^{c(a+bx)}}{-1+e^{2(d+ex)}}\right) dx \\
&= 2\int \frac{e^{c(a+bx)}}{-1+e^{2(d+ex)}}\,dx + \int e^{c(a+bx)}\,dx \\
&= \frac{e^{c(a+bx)}}{bc} - \frac{2e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc}{2e}, 1+\frac{bc}{2e}, e^{2(d+ex)}\right)}{bc}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 134 vs. $2(65) = 130$.

Time = 1.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.06

$$
\int e^{c(a+bx)} \coth(d+ex)\,dx
= \frac{e^{c(a+bx)}\left(2bce^{2(d+ex)}\,\text{Hypergeometric2F1}\left(1, 1+\frac{bc}{2e}, 2+\frac{bc}{2e}, e^{2(d+ex)}\right) + (bc+2e)\left(1+e^{2d} - 2e^{2d}\,\text{Hypergeome}\right.\right.}{bc(bc+2e)\left(-1+e^{2d}\right)}
$$

```
[In] Integrate[E^(c*(a + b*x))*Coth[d + e*x],x]
```

```
[Out] (E^(c*(a + b*x))*(2*b*c*E^(2*(d + e*x))*Hypergeometric2F1[1, 1 + (b*c)/(2*e
), 2 + (b*c)/(2*e), E^(2*(d + e*x))] + (b*c + 2*e)*(1 + E^(2*d) - 2*E^(2*d)
*Hypergeometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])))/(b*
c*(b*c + 2*e)*(-1 + E^(2*d)))
```

Maple [F]

$$\int e^{c(bx+a)} \coth(ex+d) \, dx$$

[In] int(exp(c*(b*x+a))*coth(e*x+d),x)

[Out] int(exp(c*(b*x+a))*coth(e*x+d),x)

Fricas [F]

$$\int e^{c(a+bx)} \coth(d+ex) \, dx = \int \coth(ex+d) \, e^{((bx+a)c)} \, dx$$

[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x, algorithm="fricas")

[Out] integral(coth(e*x + d)*e^(b*c*x + a*c), x)

Sympy [F]

$$\int e^{c(a+bx)} \coth(d+ex) \, dx = e^{ac} \int e^{bcx} \coth(d+ex) \, dx$$

[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x)

[Out] exp(a*c)*Integral(exp(b*c*x)*coth(d + e*x), x)

Maxima [F]

$$\int e^{c(a+bx)} \coth(d+ex) \, dx = \int \coth(ex+d) \, e^{((bx+a)c)} \, dx$$

[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x, algorithm="maxima")

[Out] 4*e*integrate(e^(b*c*x + a*c)/(b*c + (b*c*e^(4*d) - 2*e*e^(4*d))*e^(4*e*x) - 2*(b*c*e^(2*d) - 2*e*e^(2*d))*e^(2*e*x) - 2*e), x) - (b*c*e^(a*c) + 2*e*e^(a*c) + (b*c*e^(a*c + 2*d) - 2*e*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^2*c^2 - 2*b*c*e - (b^2*c^2*e^(2*d) - 2*b*c*e*e^(2*d))*e^(2*e*x))

Giac [F]

$$\int e^{c(a+bx)}\coth(d+ex)\,dx = \int \coth\left(ex+d\right)e^{\left((bx+a)c\right)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(coth(e*x + d)*e^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth(d+ex)\,dx = \int \coth(d+e\,x)\;\mathrm{e}^{c\,(a+b\,x)}\,dx$$

```
[In] int(coth(d + e*x)*exp(c*(a + b*x)),x)
```

```
[Out] int(coth(d + e*x)*exp(c*(a + b*x)), x)
```

3.232 $\int e^{c(a+bx)} \coth^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int e^{c(a+bx)} \coth^2(d+ex)\,dx = \frac{e^{c(a+bx)}}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},1+\frac{bc}{2e},e^{2(d+ex)}\right)}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},1+\frac{bc}{2e},e^{2(d+ex)}\right)}{bc}$$

```
[Out] exp(c*(b*x+a))/b/c-4*exp(c*(b*x+a))*hypergeom([1, 1/2*b*c/e],[1+1/2*b*c/e],
exp(2*e*x+2*d))/b/c+4*exp(c*(b*x+a))*hypergeom([2, 1/2*b*c/e],[1+1/2*b*c/e]
,exp(2*e*x+2*d))/b/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5593, 2225, 2283}

$$\int e^{c(a+bx)} \coth^2(d+ex)\,dx = -\frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},\frac{bc}{2e}+1,e^{2(d+ex)}\right)}{bc} + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},\frac{bc}{2e}+1,e^{2(d+ex)}\right)}{bc} + \frac{e^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (4*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) + (4*E^(c*(a + b*x))*Hypergeomet
ric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5593

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(e^{c(a+bx)} + \frac{4e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2} + \frac{4e^{c(a+bx)}}{-1+e^{2(d+ex)}}\right) dx \\ &= 4\int \frac{e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2}\, dx + 4\int \frac{e^{c(a+bx)}}{-1+e^{2(d+ex)}}\, dx + \int e^{c(a+bx)}\, dx \\ &= \frac{e^{c(a+bx)}}{bc} - \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc}{2e}, 1+\frac{bc}{2e}, e^{2(d+ex)}\right)}{bc} \\ &\quad + \frac{4e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2, \frac{bc}{2e}, 1+\frac{bc}{2e}, e^{2(d+ex)}\right)}{bc}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45

$$\int e^{c(a+bx)}\coth^2(d+ex)\,dx$$

$$= \frac{e^{c(a+bx)}\left(2b^2c^2e^{2(d+ex)}\,\text{Hypergeometric2F1}\left(1, 1+\frac{bc}{2e}, 2+\frac{bc}{2e}, e^{2(d+ex)}\right) - (bc+2e)\left(2bce^{2d}\,\text{Hypergeometric2F}\right.\right.}{bce(bc+2e)\left(-1+e^{2d}\right)}$$

```
[In] Integrate[E^(c*(a + b*x))*Coth[d + e*x]^2,x]
```

```
[Out] (E^(c*(a + b*x))*(2*b^2*c^2*E^(2*(d + e*x))*Hypergeometric2F1[1, 1 + (b*c)/
(2*e), 2 + (b*c)/(2*e), E^(2*(d + e*x))] - (b*c + 2*e)*(2*b*c*E^(2*d)*Hyper
geometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))] - (-1 + E^(2
*d))*(e + b*c*Csch[d]*Csch[d + e*x]*Sinh[e*x]))))/(b*c*e*(b*c + 2*e)*(-1 +
E^(2*d)))
```

Maple [F]

$$\int \mathrm{e}^{c(bx+a)} \coth (ex+d)^2 \, dx$$

```
[In] int(exp(c*(b*x+a))*coth(e*x+d)^2,x)
[Out] int(exp(c*(b*x+a))*coth(e*x+d)^2,x)
```

Fricas [F]

$$\int e^{c(a+bx)} \coth^2(d+ex) \, dx = \int \coth (ex+d)^2 \, e^{((bx+a)c)} \, dx$$

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="fricas")
[Out] integral(coth(e*x + d)^2*e^(b*c*x + a*c), x)
```

Sympy [F]

$$\int e^{c(a+bx)} \coth^2(d+ex) \, dx = e^{ac} \int e^{bcx} \coth^2 (d+ex) \, dx$$

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)**2,x)
[Out] exp(a*c)*Integral(exp(b*c*x)*coth(d + e*x)**2, x)
```

Maxima [F]

$$\int e^{c(a+bx)} \coth^2(d+ex) \, dx = \int \coth (ex+d)^2 \, e^{((bx+a)c)} \, dx$$

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="maxima")
[Out] 16*b*c*e*integrate(-e^(b*c*x + a*c)/(b^2*c^2 - 6*b*c*e + 8*e^2 - (b^2*c^2*e
^(6*d) - 6*b*c*e*e^(6*d) + 8*e^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d) -
6*b*c*e*e^(4*d) + 8*e^2*e^(4*d))*e^(4*e*x) - 3*(b^2*c^2*e^(2*d) - 6*b*c*e*e
^(2*d) + 8*e^2*e^(2*d))*e^(2*e*x)), x) + (b^2*c^2*e^(a*c) + 10*b*c*e*e^(a*c
) + 8*e^2*e^(a*c) + (b^2*c^2*e^(a*c + 4*d) - 6*b*c*e*e^(a*c + 4*d) + 8*e^2*
e^(a*c + 4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(a*c + 2*d) - 2*b*c*e*e^(a*c + 2*d)
 - 8*e^2*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^3*c^3 - 6*b^2*c^2*e + 8*b*c
*e^2 + (b^3*c^3*e^(4*d) - 6*b^2*c^2*e*e^(4*d) + 8*b*c*e^2*e^(4*d))*e^(4*e*x
) - 2*(b^3*c^3*e^(2*d) - 6*b^2*c^2*e*e^(2*d) + 8*b*c*e^2*e^(2*d))*e^(2*e*x)
)
```

Giac [F]

$$\int e^{c(a+bx)}\coth^2(d+ex)\,dx = \int \coth(ex+d)^2 e^{((bx+a)c)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^2,x, algorithm="giac")
[Out] integrate(coth(e*x + d)^2*e^((b*x + a)*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth^2(d+ex)\,dx = \int \coth(d+e\,x)^2\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

```
[In] int(coth(d + e*x)^2*exp(c*(a + b*x)),x)
[Out] int(coth(d + e*x)^2*exp(c*(a + b*x)), x)
```

3.233 $\int e^{c(a+bx)}\coth^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 161

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx=\frac{e^{c(a+bx)}}{bc}-\frac{6e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},1+\frac{bc}{2e},e^{2(d+ex)}\right)}{bc}+\frac{12e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},1+\frac{bc}{2e},e^{2(d+ex)}\right)}{bc}-\frac{8e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc}{2e},1+\frac{bc}{2e},e^{2(d+ex)}\right)}{bc}$$

```
[Out] exp(c*(b*x+a))/b/c-6*exp(c*(b*x+a))*hypergeom([1, 1/2*b*c/e],[1+1/2*b*c/e],
exp(2*e*x+2*d))/b/c+12*exp(c*(b*x+a))*hypergeom([2, 1/2*b*c/e],[1+1/2*b*c/e
],exp(2*e*x+2*d))/b/c-8*exp(c*(b*x+a))*hypergeom([3, 1/2*b*c/e],[1+1/2*b*c/
e],exp(2*e*x+2*d))/b/c
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5593, 2225, 2283}

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx=-\frac{6e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{bc}{2e},\frac{bc}{2e}+1,e^{2(d+ex)}\right)}{bc}+\frac{12e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,\frac{bc}{2e},\frac{bc}{2e}+1,e^{2(d+ex)}\right)}{bc}-\frac{8e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3,\frac{bc}{2e},\frac{bc}{2e}+1,e^{2(d+ex)}\right)}{bc}+\frac{e^{c(a+bx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Coth[d + e*x]^3,x]
```

```
[Out] E^(c*(a + b*x))/(b*c) - (6*E^(c*(a + b*x))*Hypergeometric2F1[1, (b*c)/(2*e)
, 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) + (12*E^(c*(a + b*x))*Hypergeome
tric2F1[2, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))])/(b*c) - (8*E^(c*
(a + b*x))*Hypergeometric2F1[3, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x
))])/(b*c)
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 5593

```
Int[Coth[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Sym
bol] :> Int[ExpandIntegrand[F^(c*(a + b*x))*((1 + E^(2*(d + e*x)))^n/(-1 +
E^(2*(d + e*x)))^n), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \int \left(e^{c(a+bx)} + \frac{8e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^3} + \frac{12e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2} + \frac{6e^{c(a+bx)}}{-1+e^{2(d+ex)}}\right) dx$$

$$= 6\int \frac{e^{c(a+bx)}}{-1+e^{2(d+ex)}}\,dx + 8\int \frac{e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^3}\,dx + 12\int \frac{e^{c(a+bx)}}{\left(-1+e^{2(d+ex)}\right)^2}\,dx + \int e^{c(a+bx)}\,dx$$

$$= \frac{e^{c(a+bx)}}{bc} - \frac{6e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1, \frac{bc}{2e}, 1+\frac{bc}{2e}, e^{2(d+ex)}\right)}{bc} + \frac{12e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2, \frac{bc}{2e}, 1+\frac{bc}{2e}, e^{2(d+ex)}\right)}{bc} - \frac{8e^{c(a+bx)}\,\text{Hypergeometric2F1}\left(3, \frac{bc}{2e}, 1+\frac{bc}{2e}, e^{2(d+ex)}\right)}{bc}$$

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.30

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx=\frac{e^{c(a+bx)}\coth(d)}{bc}-\frac{e^{c(a+bx)}\text{csch}^2(d+ex)}{2e}$$
$$+\frac{(b^2c^2+2e^2)\,e^{ac+2d+bcx}\left(bce^{2ex}\text{ Hypergeometric2F1}\left(1,1+\frac{bc}{2e},2+\frac{bc}{2e},e^{2(d+ex)}\right)-(bc+2e)\text{ Hypergeometr}\right.}{bce^2(bc+2e)\left(-1+e^{2d}\right)}$$
$$+\frac{bce^{c(a+bx)}\text{csch}(d)\text{csch}(d+ex)\sinh(ex)}{2e^2}$$

```
[In] Integrate[E^(c*(a + b*x))*Coth[d + e*x]^3,x]
```

```
[Out] (E^(c*(a + b*x))*Coth[d])/(b*c) - (E^(c*(a + b*x))*Csch[d + e*x]^2)/(2*e) +
 ((b^2*c^2 + 2*e^2)*E^(a*c + 2*d + b*c*x)*(b*c*E^(2*e*x)*Hypergeometric2F1[
1, 1 + (b*c)/(2*e), 2 + (b*c)/(2*e), E^(2*(d + e*x))] - (b*c + 2*e)*Hyperge
ometric2F1[1, (b*c)/(2*e), 1 + (b*c)/(2*e), E^(2*(d + e*x))]))/(b*c*e^2*(b*
c + 2*e)*(-1 + E^(2*d))) + (b*c*E^(c*(a + b*x))*Csch[d]*Csch[d + e*x]*Sinh[
e*x])/(2*e^2)
```

Maple [F]

$$\int \mathrm{e}^{c(bx+a)}\coth\left(ex+d\right)^{3}dx$$

```
[In] int(exp(c*(b*x+a))*coth(e*x+d)^3,x)
```

```
[Out] int(exp(c*(b*x+a))*coth(e*x+d)^3,x)
```

Fricas [F]

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx=\int \coth\left(ex+d\right)^{3}e^{\left((bx+a)c\right)}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] integral(coth(e*x + d)^3*e^(b*c*x + a*c), x)
```

Sympy [F]

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx = e^{ac}\int e^{bcx}\coth^3{(d+ex)}\,dx$$

[In] integrate(exp(c*(b*x+a))*coth(e*x+d)**3,x)

[Out] exp(a*c)*Integral(exp(b*c*x)*coth(d + e*x)**3, x)

Maxima [F]

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx = \int \coth{(ex+d)}^3 e^{((bx+a)c)}\,dx$$

[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="maxima")

[Out] 48*(b^2*c^2*e*e^(a*c) + 2*e^3*e^(a*c))*integrate(e^(b*c*x)/(b^3*c^3 - 12*b^2*c^2*e + 44*b*c*e^2 - 48*e^3 + (b^3*c^3*e^(8*d) - 12*b^2*c^2*e*e^(8*d) + 44*b*c*e^2*e^(8*d) - 48*e^3*e^(8*d))*e^(8*e*x) - 4*(b^3*c^3*e^(6*d) - 12*b^2*c^2*e*e^(6*d) + 44*b*c*e^2*e^(6*d) - 48*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d) - 12*b^2*c^2*e*e^(4*d) + 44*b*c*e^2*e^(4*d) - 48*e^3*e^(4*d))*e^(4*e*x) - 4*(b^3*c^3*e^(2*d) - 12*b^2*c^2*e*e^(2*d) + 44*b*c*e^2*e^(2*d) - 48*e^3*e^(2*d))*e^(2*e*x)), x) - (b^3*c^3*e^(a*c) + 36*b^2*c^2*e*e^(a*c) + 44*b*c*e^2*e^(a*c) + 48*e^3*e^(a*c) + (b^3*c^3*e^(a*c + 6*d) - 12*b^2*c^2*e*e^(a*c + 6*d) + 44*b*c*e^2*e^(a*c + 6*d) - 48*e^3*e^(a*c + 6*d))*e^(6*e*x) + 3*(b^3*c^3*e^(a*c + 4*d) - 8*b^2*c^2*e*e^(a*c + 4*d) + 4*b*c*e^2*e^(a*c + 4*d) + 48*e^3*e^(a*c + 4*d))*e^(4*e*x) + 3*(b^3*c^3*e^(a*c + 2*d) - 28*b*c*e^2*e^(a*c + 2*d) - 48*e^3*e^(a*c + 2*d))*e^(2*e*x))*e^(b*c*x)/(b^4*c^4 - 12*b^3*c^3*e + 44*b^2*c^2*e^2 - 48*b*c*e^3 - (b^4*c^4*e^(6*d) - 12*b^3*c^3*e*e^(6*d) + 44*b^2*c^2*e^2*e^(6*d) - 48*b*c*e^3*e^(6*d))*e^(6*e*x) + 3*(b^4*c^4*e^(4*d) - 12*b^3*c^3*e*e^(4*d) + 44*b^2*c^2*e^2*e^(4*d) - 48*b*c*e^3*e^(4*d))*e^(4*e*x) - 3*(b^4*c^4*e^(2*d) - 12*b^3*c^3*e*e^(2*d) + 44*b^2*c^2*e^2*e^(2*d) - 48*b*c*e^3*e^(2*d))*e^(2*e*x))

Giac [F]

$$\int e^{c(a+bx)}\coth^3(d+ex)\,dx = \int \coth{(ex+d)}^3 e^{((bx+a)c)}\,dx$$

[In] integrate(exp(c*(b*x+a))*coth(e*x+d)^3,x, algorithm="giac")

[Out] integrate(coth(e*x + d)^3*e^((b*x + a)*c), x)

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \coth^3(d+ex)\,dx = \int \coth(d+e\,x)^3\,\mathrm{e}^{c\,(a+b\,x)}\,dx$$

```
[In] int(coth(d + e*x)^3*exp(c*(a + b*x)),x)
[Out] int(coth(d + e*x)^3*exp(c*(a + b*x)), x)
```

3.234 $\int e^{c(a+bx)} \tanh^2(ac+bcx)^{5/2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 311

$$\begin{aligned}\int e^{c(a+bx)} \tanh^2(ac+bcx)^{5/2} \, dx &= \frac{e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} \\ &- \frac{4e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} \\ &+ \frac{26e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3} \\ &- \frac{55e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{6bc\left(1+e^{2c(a+bx)}\right)^2} \\ &+ \frac{25e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{4bc\left(1+e^{2c(a+bx)}\right)} \\ &- \frac{15\arctan\left(e^{c(a+bx)}\right) \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{4bc}\end{aligned}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c-4*exp(c*(b*x+a
))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4+26/
3*exp(c*(b*x+a))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(
b*x+a)))^3-55/6*exp(c*(b*x+a))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/
c/(1+exp(2*c*(b*x+a)))^2+25/4*exp(c*(b*x+a))*coth(b*c*x+a*c)*(tanh(b*c*x+a*
c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))-15/4*arctan(exp(c*(b*x+a)))*coth(b*c*x
+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6852, 2320, 398, 1828, 1171, 393, 209}

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{5/2}\,dx = -\frac{15\arctan\left(e^{c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{4bc} + \frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc} + \frac{25e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)} - \frac{55e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{6bc\left(e^{2c(a+bx)}+1\right)^2} + \frac{26e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3} - \frac{4e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4}$$

```
[In] Int[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c
*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(4*b*c*(1 + E^(
2*c*(a + b*x)))) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x]*Sqrt[Tanh[
a*c + b*c*x]^2])/(4*b*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\int e^{c(a+bx)}\tanh^5(ac+bcx)\,dx$$

$$= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{(-1+x^2)^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \left(1-\frac{2(1+10x^4+5x^8)}{(1+x^2)^5}\right)\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{\left(2\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{1+10x^4+5x^8}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} + \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{8-120x^2+40x^4-40x^6}{(1+x^2)^4}\,dx, x, e^{c(a+bx)}\right)}{4bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3} - \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{160-480x^2+240x^4}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{24bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3} - \frac{55e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{6bc\left(1+e^{2c(a+bx)}\right)^2} + \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{240-960x^2}{(1+x^2)^2}\,dx, x, e^{c(a+bx)}\right)}{96bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3} - \frac{55e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{6bc\left(1+e^{2c(a+bx)}\right)^2} + \frac{25e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{4bc\left(1+e^{2c(a+bx)}\right)} - \frac{\left(15\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{4bc}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{4e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3} - \frac{55e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{6bc\left(1+e^{2c(a+bx)}\right)^2} + \frac{25e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{4bc\left(1+e^{2c(a+bx)}\right)} - \frac{15\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{4bc}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.43

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{5/2}\,dx = \frac{\left(e^{c(a+bx)}\left(33+157e^{2c(a+bx)}+187e^{4c(a+bx)}+123e^{6c(a+bx)}+12e^{8c(a+bx)}\right)-45\left(1+e^{2c(a+bx)}\right)^4 \text{ar}}{12bc\left(1+e^{2c(a+bx)}\right)^4}$$

```
[In] Integrate[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 123
*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4*A
rcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)]*Sqrt[Tanh[c*(a + b*x)]^2])/(12*b*
c*(1 + E^(2*c*(a + b*x)))^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.63

method	result
default	$\frac{\operatorname{csgn}(\tanh(c(bx+a)))\left(\frac{\sinh(bcx+ac)^4}{\cosh(bcx+ac)^3}+\frac{4\sinh(bcx+ac)^2}{\cosh(bcx+ac)^3}+\frac{8}{3\cosh(bcx+ac)^3}+\frac{\sinh(bcx+ac)^5}{\cosh(bcx+ac)^4}+\frac{5\sinh(bcx+ac)^3}{\cosh(bcx+ac)^4}+\frac{5\sinh(bcx+ac)}{\cosh(bcx+ac)^4}-5\left(\frac{\operatorname{sech}(bcx+c}{4}\right.\right.}{cb}$
risch	$\frac{\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}\, e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)bc}+\frac{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}\, e^{c(bx+a)}\left(75\, e^{6c(bx+a)}+115\, e^{4c(bx+a)}+109\, e^{2c(bx+a)}+21\right)}{12\left(e^{2c(bx+a)}-1\right)\left(1+e^{2c(bx+a)}\right)^3 cb}+\frac{15i(1-}{}$

```
[In] int(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(tanh(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)^4/cosh(b*c*x+a*c)^3+4*sinh(b*c*x
+a*c)^2/cosh(b*c*x+a*c)^3+8/3/cosh(b*c*x+a*c)^3+sinh(b*c*x+a*c)^5/cosh(b*c*
x+a*c)^4+5*sinh(b*c*x+a*c)^3/cosh(b*c*x+a*c)^4+5*sinh(b*c*x+a*c)/cosh(b*c*x
+a*c)^4-5*(1/4*sech(b*c*x+a*c)^3+3/8*sech(b*c*x+a*c))*tanh(b*c*x+a*c)-15/4*
arctan(exp(b*c*x+a*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. $2(281) = 562$.

Time = 0.26 (sec) , antiderivative size = 1226, normalized size of antiderivative = 3.94

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*c)
^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*x +
a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*cosh(b*
c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x
 + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1870*cos
h(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)^3 + (43
2*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3
 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8
*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*
c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b
*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x
 + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x
+ a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x
 + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 15*cosh(b*c*x + a
*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4*cosh(b*c*x + a*c
)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^
3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + si
```

```
nh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*cosh(b*c*x + a*c)^6 + 935
*cosh(b*c*x + a*c)^4 + 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 33
*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh
(b*c*x + a*c)^7 + b*c*sinh(b*c*x + a*c)^8 + 4*b*c*cosh(b*c*x + a*c)^6 + 4*(
7*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a
*c)^4 + 8*(7*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^5 + 2*(35*b*c*cosh(b*c*x + a*c)^4 + 30*b*c*cosh(b*c*x + a*c)^2 + 3*b
*c)*sinh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x +
 a*c)^5 + 10*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*b*c*cosh(b*c*x + a*c)^6 + 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*
c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x
+ a*c)^7 + 3*b*c*cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh
(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)**2)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.47

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{5/2}\,dx = -\frac{15\arctan\left(e^{(bcx+ac)}\right)}{4\,bc} + \frac{12\,e^{(9\,bcx+9\,ac)} + 123\,e^{(7\,bcx+7\,ac)} + 187\,e^{(5\,bcx+5\,ac)} + 157\,e^{(3\,bcx+3\,ac)} + 33\,e^{(bcx+ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)} + 4\,e^{(6\,bcx+6\,ac)} + 6\,e^{(4\,bcx+4\,ac)} + 4\,e^{(2\,bcx+2\,ac)} + 1\right)}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
[Out] -15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e^
(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 33*
e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4
*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.59

$$\int e^{c(a+bx)} \tanh^2(ac+bcx)^{5/2}\,dx =$$

$$-\frac{45 \arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-12\,e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-\frac{75\,e^{(7\,bcx+7\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+115}{}}{12\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(45*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - 12*e^(b*c*
x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (75*e^(7*b*c*x + 7*a*c)*sgn(e^(2*b*
c*x + 2*a*c) - 1) + 115*e^(5*b*c*x + 5*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) +
109*e^(3*b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 21*e^(b*c*x + a*c)*s
gn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) + 1)^4)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \tanh^2(ac+bcx)^{5/2}\,dx = \int \mathrm{e}^{c\,(a+b\,x)}\,\left(\tanh(a\,c+b\,c\,x)^2\right)^{5/2}\,dx$$

```
[In] int(exp(c*(a + b*x))*(tanh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(tanh(a*c + b*c*x)^2)^(5/2), x)
```

3.235 $\int e^{c(a+bx)} \tanh^2(ac+bcx)^{3/2} dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int e^{c(a+bx)} \tanh^2(ac+bcx)^{3/2} dx = \frac{e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2} + \frac{3e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)} - \frac{3\arctan\left(e^{c(a+bx)}\right) \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c-2*exp(c*(b*x+a
))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^2+3*e
xp(c*(b*x+a))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x
+a)))-3*arctan(exp(c*(b*x+a)))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/
c
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

$= \{6852, 2320, 398, 1172, 12, 294, 209\}$

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx = -\frac{3\arctan\left(e^{c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc} + \frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc} + \frac{3e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)} - \frac{2e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

```
[In] Int[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*
x]^2])/(b*c*(1 + E^(2*c*(a + b*x)))) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\int e^{c(a+bx)}\tanh^3(ac+bcx)\,dx \\
&= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int\frac{(-1+x^2)^3}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int\left(1-\frac{2(1+3x^4)}{(1+x^2)^3}\right)\,dx,x,e^{c(a+bx)}\right)}{bc} \\
&= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} \\
&\quad -\frac{\left(2\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int\frac{1+3x^4}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc} \\
&= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc}-\frac{2e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\,(1+e^{2c(a+bx)})^2} \\
&\quad +\frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int-\frac{12x^2}{(1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{2bc}
\end{aligned}
$$

$$= \frac{e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2} - \frac{\left(6\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2} + \frac{3e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)} - \frac{\left(3\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} - \frac{2e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2} + \frac{3e^{c(a+bx)} \coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)} - \frac{3\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.54

$$\int e^{c(a+bx)} \tanh^2(ac+bcx)^{3/2}\,dx = \frac{\left(e^{c(a+bx)}\left(2+5e^{2c(a+bx)}+e^{4c(a+bx)}\right)-3\left(1+e^{2c(a+bx)}\right)^2\arctan\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))\sqrt{\tan}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

```
[In] Integrate[E^(c*(a + b*x))*(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(2 + 5*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))) - 3*(1 + E^
(2*c*(a + b*x)))^2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)]*Sqrt[Tanh[c*(
a + b*x)]^2])/(b*c*(1 + E^(2*c*(a + b*x)))^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

method	result
default	$\frac{\operatorname{csgn}(\tanh(c(bx+a)))\left(\frac{\sinh(bcx+ac)^2}{\cosh(bcx+ac)}+\frac{2}{\cosh(bcx+ac)}+\frac{\sinh(bcx+ac)^3}{\cosh(bcx+ac)^2}+\frac{3\sinh(bcx+ac)}{\cosh(bcx+ac)^2}-\frac{3\operatorname{sech}(bcx+ac)\tanh(bcx+ac)}{2}-3\arctan\left(e^{bcx+ac}\right)\right)}{cb}$
risch	$\frac{\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}\left(3ie^{4c(bx+a)}\ln\left(e^{c(bx+a)}-i\right)-3ie^{4c(bx+a)}\ln\left(e^{c(bx+a)}+i\right)+2e^{5c(bx+a)}+6ie^{2c(bx+a)}\ln\left(e^{c(bx+a)}-i\right)-6ie^{2c(bx+a)}\ln\right.}{2\left(e^{2c(bx+a)}-1\right)\left(1+e^{2c(bx+a)}\right)cb}$

```
[In] int(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(tanh(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)^2/cosh(b*c*x+a*c)+2/cosh(b*c*x+a
*c)+sinh(b*c*x+a*c)^3/cosh(b*c*x+a*c)^2+3*sinh(b*c*x+a*c)/cosh(b*c*x+a*c)^2
-3/2*sech(b*c*x+a*c)*tanh(b*c*x+a*c)-3*arctan(exp(b*c*x+a*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(179) = 358$.

Time = 0.26 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.37

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx = \frac{\cosh(bcx+ac)^5+5\cosh(bcx+ac)\sinh(bcx+ac)^4+\sinh(bcx+ac)^5+5\left(2\cosh(bcx+ac\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*x
 + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*c*
x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a
*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 2
*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c
*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*cosh(b*c*
x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh(b*c*x +
 a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*
c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3
+ b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx = -\frac{3\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(5\,bcx+5\,ac)} + 5\,e^{(3\,bcx+3\,ac)} + 2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)} + 2\,e^{(2\,bcx+2\,ac)} + 1\right)}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*c*x + a*c))/(b*c) + (e^(5*b*c*x + 5*a*c) + 5*e^(3*b*c*x + 3*
a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c)
 + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.67

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx =$$

$$-\frac{3\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - \frac{3\,e^{(3\,bcx+3\,ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+e^{(bcx+ac)}}{\left(e^{(2\,bcx+2\,ac)}+1\right)^2}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1) - (3*e^(3*b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*
c) - 1) + e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c
) + 1)^2)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\tanh^2(ac+bcx)^{3/2}\,dx=\int \mathrm{e}^{c\,(a+b\,x)}\,\left(\tanh(a\,c+b\,c\,x)^2\right)^{3/2}\,dx$$

```
[In] int(exp(c*(a + b*x))*(tanh(a*c + b*c*x)^2)^(3/2),x)
[Out] int(exp(c*(a + b*x))*(tanh(a*c + b*c*x)^2)^(3/2), x)
```

3.236 $\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx=\frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc}-\frac{2\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c-2*arctan(exp(c
*(b*x+a)))*coth(b*c*x+a*c)*(tanh(b*c*x+a*c)^2)^(1/2)/b/c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 396, 209}

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx=\frac{e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}-\frac{2\arctan\left(e^{c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}\coth(ac+bcx)}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Sqrt[Tanh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x]*Sqrt[Tanh[a*c + b*c*x]^2])/(b*c)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\int e^{c(a+bx)}\tanh(ac+bcx)\,dx \\
&= \frac{\left(\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} \\
&\quad - \frac{\left(2\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{e^{c(a+bx)}\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc} \\
&\quad - \frac{2\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)\sqrt{\tanh^2(ac+bcx)}}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx = \frac{\left(e^{c(a+bx)}-2\arctan\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))\sqrt{\tanh^2(c(a+bx))}}{bc}$$

[In] Integrate[E^(c*(a + b*x))*Sqrt[Tanh[a*c + b*c*x]^2],x]

[Out] ((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)]*Sqrt[Tanh[c*(a + b*x)]^2])/(b*c)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.63

method	result
risch	$\frac{\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}\,e^{c(bx+a)}}{\left(e^{2c(bx+a)}-1\right)bc}+\frac{i\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}\,\ln\left(e^{c(bx+a)}-i\right)}{\left(e^{2c(bx+a)}-1\right)cb}-\frac{i\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)}}}{\left(e^{2c(bx+a)}-1\right)c}$

[In] int(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)*exp(c*(b*x+a))/b/c+I*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp(c*(b*x+a))-I)-I*((exp(2*c*(b*x+a))-1)^2/(1+exp(2*c*(b*x+a)))^2)^(1/2)/(exp(2*c*(b*x+a))-1)*(1+exp(2*c*(b*x+a)))/c/b*ln(exp(c*(b*x+a))+I)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx = -\frac{2\arctan\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}{bc}$$

[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")

[Out] -(2*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - cosh(b*c*x + a*c) - sinh(b*c*x + a*c))/(b*c)

Sympy [F]

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx = e^{ac}\int\sqrt{\tanh^2(ac+bcx)}e^{bcx}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(sqrt(tanh(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx = -\frac{2\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(bcx+ac)}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] -2*arctan(e^(b*c*x + a*c))/(b*c) + e^(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx = -\frac{2\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] -(2*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sqrt{\tanh^2(ac+bcx)}\,dx = \int e^{c\,(a+b\,x)}\,\sqrt{\tanh\left(a\,c+b\,c\,x\right)^2}\,dx$$

```
[In] int(exp(c*(a + b*x))*(tanh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))*(tanh(a*c + b*c*x)^2)^(1/2), x)
```

3.237 $\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx=\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{2\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

```
[Out] exp(c*(b*x+a))*tanh(b*c*x+a*c)/b/c/(tanh(b*c*x+a*c)^2)^(1/2)-2*arctanh(exp(
c*(b*x+a)))*tanh(b*c*x+a*c)/b/c/(tanh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 396, 212}

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx=\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{2\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/Sqrt[Tanh[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (2*Ar
cTanh[E^(c*(a + b*x))]*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tanh(ac+bcx)\int e^{c(a+bx)}\coth(ac+bcx)\,dx}{\sqrt{\tanh^2(ac+bcx)}}\\
&= \frac{\tanh(ac+bcx)\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}\\
&= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{(2\tanh(ac+bcx))\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}\\
&= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{2\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx = \frac{\left(e^{c(a+bx)} - 2\text{arctanh}\left(e^{c(a+bx)}\right)\right)\tanh(c(a+bx))}{bc\sqrt{\tanh^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Tanh[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(b*c*Sqr
t[Tanh[c*(a + b*x)]^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\operatorname{csgn}(\tanh(c(bx+a)))(\sinh(bcx+ac)+\cosh(bcx+ac)-2\,\operatorname{arctanh}(e^{bcx+ac}))}{cb}$	48
risch	$\frac{(e^{2c(bx+a)}-1)e^{c(bx+a)}}{\sqrt{\frac{(e^{2c(bx+a)}-1)^2}{(1+e^{2c(bx+a)})^2}}\,(1+e^{2c(bx+a)})bc}+\frac{(e^{2c(bx+a)}-1)\ln(e^{c(bx+a)}-1)}{\sqrt{\frac{(e^{2c(bx+a)}-1)^2}{(1+e^{2c(bx+a)})^2}}\,(1+e^{2c(bx+a)})cb}-\frac{(e^{2c(bx+a)}-1)\ln(e^{c(bx+a)}+1)}{\sqrt{\frac{(e^{2c(bx+a)}-1)^2}{(1+e^{2c(bx+a)})^2}}\,(1+e^{2c(bx+a)})cb}$	213

```
[In] int(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(tanh(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)+cosh(b*c*x+a*c)-2*arctanh(exp(b*
c*x+a*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx$$
$$= \frac{\cosh(bcx+ac) - \log(\cosh(bcx+ac) + \sinh(bcx+ac) + 1) + \log(\cosh(bcx+ac) + \sinh(bcx+ac) - 1}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c) - log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + log(c
osh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + sinh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\tanh^2\left(ac+bcx\right)}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(tanh(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx = \frac{e^{(bcx+ac)}}{bc} - \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] e^(b*c*x + a*c)/(b*c) - log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c
) - 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.06

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx = \frac{e^{(bcx+ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - \log\left(e^{(bcx+ac)}+1\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) + \log\left(\left|e^{(bcx+ac)}-1\right|\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}\right.}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] (e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - log(e^(b*c*x + a*c) + 1)*sg
n(e^(2*b*c*x + 2*a*c) - 1) + log(abs(e^(b*c*x + a*c) - 1))*sgn(e^(2*b*c*x +
 2*a*c) - 1))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\tanh^2(ac+bcx)}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\sqrt{\tanh\left(a\,c+b\,c\,x\right)^2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(tanh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(tanh(a*c + b*c*x)^2)^(1/2), x)
```

3.238 $\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}} - \frac{3\operatorname{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

```
[Out] exp(c*(b*x+a))*tanh(b*c*x+a*c)/b/c/(tanh(b*c*x+a*c)^2)^(1/2)-2*exp(c*(b*x+a
))*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(tanh(b*c*x+a*c)^2)^(1/2)+3*e
xp(c*(b*x+a))*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))/(tanh(b*c*x+a*c)^2)^
(1/2)-3*arctanh(exp(c*(b*x+a)))*tanh(b*c*x+a*c)/b/c/(tanh(b*c*x+a*c)^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

= {6852, 2320, 398, 1172, 12, 294, 213}

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = -\frac{3\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} + \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt[Tanh[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a
 + b*x)))*Sqrt[Tanh[a*c + b*c*x]^2]) - (3*ArcTanh[E^(c*(a + b*x))]*Tanh[a*c
 + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tanh(ac+bcx)\int e^{c(a+bx)}\coth^3(ac+bcx)\,dx}{\sqrt{\tanh^2(ac+bcx)}}\\
&= \frac{\tanh(ac+bcx)\text{Subst}\left(\int \frac{(1+x^2)^3}{(-1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}\\
&= \frac{\tanh(ac+bcx)\text{Subst}\left(\int \left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}\\
&= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}+\frac{(2\tanh(ac+bcx))\text{Subst}\left(\int \frac{1+3x^4}{(-1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}\\
&= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}}\\
&\quad+\frac{\tanh(ac+bcx)\text{Subst}\left(\int \frac{12x^2}{(-1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{2bc\sqrt{\tanh^2(ac+bcx)}}
\end{aligned}
$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{(6\tanh(ac+bcx))\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}} + \frac{(3\tanh(ac+bcx))\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}} - \frac{3\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 7.78 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = -\frac{e^{-5c(a+bx)}\left(-21\left(252105+507305e^{2c(a+bx)}+173916e^{4c(a+bx)}-154296e^{6c(a+bx)}-73885e^{8c(a+bx)}+4887e^{10c(a+bx)}\right.\right.}{}$$

```
[In] Integrate[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -1/60480*((-21*(252105 + 507305*E^(2*c*(a + b*x)) + 173916*E^(4*c*(a + b*x)
) - 154296*E^(6*c*(a + b*x)) - 73885*E^(8*c*(a + b*x)) + 4887*E^(10*c*(a +
b*x))) - (315*(-16807 - 28218*E^(2*c*(a + b*x)) + 1173*E^(4*c*(a + b*x)) +
17748*E^(6*c*(a + b*x)) + 4299*E^(8*c*(a + b*x)) - 1434*E^(10*c*(a + b*x))
+ 7*E^(12*c*(a + b*x)))*ArcTanh[Sqrt[E^(2*c*(a + b*x))]])/Sqrt[E^(2*c*(a +
b*x))] + 384*E^(8*c*(a + b*x))*(1 + E^(2*c*(a + b*x)))^2*(7 + 5*E^(2*c*(a +
 b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, E^(2*c*(a + b
*x))] + 256*E^(8*c*(a + b*x))*(1 + E^(2*c*(a + b*x)))^3*HypergeometricPFQ[{
3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*c*(a + b*x))])*Tanh[c*(a + b*
x)]^3)/(b*c*E^(5*c*(a + b*x))*(Tanh[c*(a + b*x)]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
default	$\frac{\operatorname{csgn}(\tanh(c(bx+a)))\left(\frac{\cosh(bcx+ac)^3}{\sinh(bcx+ac)^2}-\frac{3\cosh(bcx+ac)}{\sinh(bcx+ac)^2}+\frac{3\operatorname{csch}(bcx+ac)\coth(bcx+ac)}{2}-3\operatorname{arctanh}\left(e^{bcx+ac}\right)+\frac{\cosh(bcx+ac)^2}{\sinh(bcx+ac)}-\frac{2}{\sinh(bcx+ac)}\right)}{cb}$
risch	$\frac{2e^{5c(bx+a)}+3e^{4c(bx+a)}\ln\left(e^{c(bx+a)}-1\right)-3e^{4c(bx+a)}\ln\left(e^{c(bx+a)}+1\right)-10e^{3c(bx+a)}-6e^{2c(bx+a)}\ln\left(e^{c(bx+a)}-1\right)+6e^{2c(bx+a)}\ln\left(e^{c(bx+}\right.}{2\left(e^{2c(bx+a)}-1\right)\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{\left(e^{2c(bx+a)}-1\right)^2}{\left(1+e^{2c(bx+a)}\right)^2}}\,cb}$

[In] int(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)

[Out] csgn(tanh(c*(b*x+a)))/c/b*(cosh(b*c*x+a*c)^3/sinh(b*c*x+a*c)^2-3/sinh(b*c*x+a*c)^2*cosh(b*c*x+a*c)+3/2*csch(b*c*x+a*c)*coth(b*c*x+a*c)-3*arctanh(exp(b*c*x+a*c))+1/sinh(b*c*x+a*c)*cosh(b*c*x+a*c)^2-2/sinh(b*c*x+a*c))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(179) = 358.

Time = 0.27 (sec) , antiderivative size = 613, normalized size of antiderivative = 3.11

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = \frac{2\cosh(bcx+ac)^5+10\cosh(bcx+ac)\sinh(bcx+ac)^4+2\sinh(bcx+ac)^5+10}{}$$

[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")

[Out] 1/2*(2*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + 2*sinh(b*c*x + a*c)^5 + 10*(2*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^3 - 10*cosh(b*c*x + a*c)^3 + 10*(2*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + 2*(5*cosh(b*c*x + a*c)^4 - 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 4*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + b*c*sinh(b*c*x + a*c)^4 - 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*c*x + a*c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\tanh^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(tanh(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.57

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = -\frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{2\,bc} + \frac{3\,\log\left(e^{(bcx+ac)}-1\right)}{2\,bc} + \frac{e^{(5\,bcx+5\,ac)}-5\,e^{(3\,bcx+3\,ac)}+2\,e^{(bcx+ac)}}{bc(e^{(4\,bcx+4\,ac)}-2\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(b*c*x + a*c) + 1)/(b*c) + 3/2*log(e^(b*c*x + a*c) - 1)/(b*c) +
(e^(5*b*c*x + 5*a*c) - 5*e^(3*b*c*x + 3*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(
4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = \frac{2\,e^{(bcx+ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - 3\,\log\left(e^{(bcx+ac)}+1\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) + 3\,\mathrm{lo}}{}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 3*log(e^(b*c*x + a*c)
 + 1)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 3*log(abs(e^(b*c*x + a*c) - 1))*sgn(e^
(2*b*c*x + 2*a*c) - 1) - 2*(3*e^(3*b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*c) -
 1) - e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) -
1)^2)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{3/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\tanh\left(a\,c+b\,c\,x\right)^2\right)^{3/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(tanh(a*c + b*c*x)^2)^(3/2),x)
[Out] int(exp(c*(a + b*x))/(tanh(a*c + b*c*x)^2)^(3/2), x)
```

3.239 $\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 319

$$\begin{aligned}\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx &= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} \\ &- \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\tanh^2(ac+bcx)}} \\ &+ \frac{26e^{c(a+bx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\tanh^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\tanh(ac+bcx)}{6bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} \\ &+ \frac{25e^{c(a+bx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}} - \frac{15\operatorname{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{4bc\sqrt{\tanh^2(ac+bcx)}}\end{aligned}$$

```
[Out] exp(c*(b*x+a))*tanh(b*c*x+a*c)/b/c/(tanh(b*c*x+a*c)^2)^(1/2)-4*exp(c*(b*x+a
))*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4/(tanh(b*c*x+a*c)^2)^(1/2)+26/
3*exp(c*(b*x+a))*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^3/(tanh(b*c*x+a*c
)^2)^(1/2)-55/6*exp(c*(b*x+a))*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2/(
tanh(b*c*x+a*c)^2)^(1/2)+25/4*exp(c*(b*x+a))*tanh(b*c*x+a*c)/b/c/(1-exp(2*c
*(b*x+a)))/(tanh(b*c*x+a*c)^2)^(1/2)-15/4*arctanh(exp(c*(b*x+a)))*tanh(b*c*
x+a*c)/b/c/(tanh(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6852, 2320, 398, 1828, 1171, 393, 213}

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = -\frac{15\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{4bc\sqrt{\tanh^2(ac+bcx)}} + \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} + \frac{25e^{c(a+bx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)\sqrt{\tanh^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\tanh(ac+bcx)}{6bc\left(1-e^{2c(a+bx)}\right)^2\sqrt{\tanh^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3\sqrt{\tanh^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4\sqrt{\tanh^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*Sqrt[Tanh[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))^4*Sqrt[Tanh[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Tanh[a*c + b*c*x])/(3*b*c*(1 - E^(2*c
*(a + b*x)))^3*Sqrt[Tanh[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Tanh[a*c +
b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2*Sqrt[Tanh[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(2*c*(a + b*x)))*Sqrt[Tanh[
a*c + b*c*x]^2]) - (15*ArcTanh[E^(c*(a + b*x))]*Tanh[a*c + b*c*x])/(4*b*c*S
qrt[Tanh[a*c + b*c*x]^2])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \frac{\tanh(ac+bcx)\int e^{c(a+bx)}\coth^5(ac+bcx)\,dx}{\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{\tanh(ac+bcx)\text{Subst}\left(\int \frac{(1+x^2)^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$=\frac{\tanh(ac+bcx)\text{Subst}\left(\int\left(1+\frac{2(1+10x^4+5x^8)}{(-1+x^2)^5}\right)\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$=\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}+\frac{(2\tanh(ac+bcx))\text{Subst}\left(\int\frac{1+10x^4+5x^8}{(-1+x^2)^5}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\tanh^2(ac+bcx)}}$$

$$=\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\,(1-e^{2c(a+bx)})^4\sqrt{\tanh^2(ac+bcx)}}+\frac{\tanh(ac+bcx)\text{Subst}\left(\int\frac{8+120x^2+40x^4+40x^6}{(-1+x^2)^4}\,dx,x,e^{c(a+bx)}\right)}{4bc\sqrt{\tanh^2(ac+bcx)}}$$

$$=\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\,(1-e^{2c(a+bx)})^4\sqrt{\tanh^2(ac+bcx)}}+\frac{26e^{c(a+bx)}\tanh(ac+bcx)}{3bc\,(1-e^{2c(a+bx)})^3\sqrt{\tanh^2(ac+bcx)}}+\frac{\tanh(ac+bcx)\text{Subst}\left(\int\frac{160+480x^2+240x^4}{(-1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{24bc\sqrt{\tanh^2(ac+bcx)}}$$

$$=\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\,(1-e^{2c(a+bx)})^4\sqrt{\tanh^2(ac+bcx)}}+\frac{26e^{c(a+bx)}\tanh(ac+bcx)}{3bc\,(1-e^{2c(a+bx)})^3\sqrt{\tanh^2(ac+bcx)}}-\frac{55e^{c(a+bx)}\tanh(ac+bcx)}{6bc\,(1-e^{2c(a+bx)})^2\sqrt{\tanh^2(ac+bcx)}}+\frac{\tanh(ac+bcx)\text{Subst}\left(\int\frac{240+960x^2}{(-1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{96bc\sqrt{\tanh^2(ac+bcx)}}$$

$$=\frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}}-\frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\,(1-e^{2c(a+bx)})^4\sqrt{\tanh^2(ac+bcx)}}+\frac{26e^{c(a+bx)}\tanh(ac+bcx)}{3bc\,(1-e^{2c(a+bx)})^3\sqrt{\tanh^2(ac+bcx)}}-\frac{55e^{c(a+bx)}\tanh(ac+bcx)}{6bc\,(1-e^{2c(a+bx)})^2\sqrt{\tanh^2(ac+bcx)}}+\frac{25e^{c(a+bx)}\tanh(ac+bcx)}{4bc\,(1-e^{2c(a+bx)})\sqrt{\tanh^2(ac+bcx)}}+\frac{(15\tanh(ac+bcx))\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,e^{c(a+bx)}\right)}{4bc\sqrt{\tanh^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\tanh(ac+bcx)}{bc\sqrt{\tanh^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)}{bc\,(1-e^{2c(a+bx)})^4\sqrt{\tanh^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\tanh(ac+bcx)}{3bc\,(1-e^{2c(a+bx)})^3\sqrt{\tanh^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\tanh(ac+bcx)}{6bc\,(1-e^{2c(a+bx)})^2\sqrt{\tanh^2(ac+bcx)}} + \frac{25e^{c(a+bx)}\tanh(ac+bcx)}{4bc\,(1-e^{2c(a+bx)})\sqrt{\tanh^2(ac+bcx)}} - \frac{15\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)}{4bc\sqrt{\tanh^2(ac+bcx)}}$$

Mathematica [A] (verified)

Time = 11.73 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.51

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = \frac{\left(66e^{c(a+bx)} - 314e^{3c(a+bx)} + 374e^{5c(a+bx)} - 246e^{7c(a+bx)} + 24e^{9c(a+bx)} + 45(-1+\right.}{24bc\,(-1+e^{2c(a+b}}$$

[In] Integrate[E^(c*(a + b*x))/(Tanh[a*c + b*c*x]^2)^(5/2),x]

[Out] ((66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 374*E^(5*c*(a + b*x)) - 246*E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(-1 + E^(2*c*(a + b*x)))^4*Log[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a + b*x)))^4*Log[1 + E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(24*b*c*(-1 + E^(2*c*(a + b*x)))^4*Sqrt[Tanh[c*(a + b*x)]^2])

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.46 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.61

method	result
default	$\frac{\mathrm{csgn}(\tanh(c(bx+a)))\left(\frac{\cosh(bcx+ac)^5}{\sinh(bcx+ac)^4}-\frac{5\cosh(bcx+ac)^3}{\sinh(bcx+ac)^4}+\frac{5\cosh(bcx+ac)}{\sinh(bcx+ac)^4}+5\left(-\frac{\mathrm{csch}(bcx+ac)^3}{4}+\frac{3\,\mathrm{csch}(bcx+ac)}{8}\right)\coth(bcx+ac)-\frac{15\,\mathrm{arctan}}{}\right.}{cb}$
risch	$\frac{(e^{2c(bx+a)}-1)e^{c(bx+a)}}{\sqrt{\frac{(e^{2c(bx+a)}-1)^2}{(1+e^{2c(bx+a)})^2}}(1+e^{2c(bx+a)})bc} - \frac{e^{c(bx+a)}(75e^{6c(bx+a)}-115e^{4c(bx+a)}+109e^{2c(bx+a)}-21)}{12(e^{2c(bx+a)}-1)^3(1+e^{2c(bx+a)})\sqrt{\frac{(e^{2c(bx+a)}-1)^2}{(1+e^{2c(bx+a)})^2}}cb} - \frac{15(e^{2c(bx+a)}-1)\ln(e^{c(bx+}}{8\sqrt{\frac{(e^{2c(bx+a)}-1)^2}{(1+e^{2c(bx+a)})^2}}(1+e^{2c(b}}$

[In] int(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] csgn(tanh(c*(b*x+a)))/c/b*(cosh(b*c*x+a*c)^5/sinh(b*c*x+a*c)^4-5/sinh(b*c*x+a*c)^4*cosh(b*c*x+a*c)^3+5/sinh(b*c*x+a*c)^4*cosh(b*c*x+a*c)+5*(-1/4*csch(b*c*x+a*c)^3+3/8*csch(b*c*x+a*c))*coth(b*c*x+a*c)-15/4*arctanh(exp(b*c*x+a*

```
c))+1/sinh(b*c*x+a*c)^3*cosh(b*c*x+a*c)^4-4/sinh(b*c*x+a*c)^3*cosh(b*c*x+a*
c)^2+8/3/sinh(b*c*x+a*c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. 2(281) = 562.

Time = 0.27 (sec) , antiderivative size = 1617, normalized size of antiderivative = 5.07

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*c)
^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*c*x
+ a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(1512*cos
h(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)^4 + 18
70*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x + a*c)^3
 + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x
+ a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*
c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7
*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(
7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*co
sh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cos
h(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b
*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c
*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x
 + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c)
 + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*si
nh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(
b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh
(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x
 + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x
+ a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c
))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) +
2*(108*cosh(b*c*x + a*c)^8 - 861*cosh(b*c*x + a*c)^6 + 935*cosh(b*c*x + a*c
)^4 - 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 66*cosh(b*c*x + a*c
```

```
))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 +
 b*c*sinh(b*c*x + a*c)^8 - 4*b*c*cosh(b*c*x + a*c)^6 + 4*(7*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)^4 + 8*(7*b*c*
cosh(b*c*x + a*c)^3 - 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*
b*c*cosh(b*c*x + a*c)^4 - 30*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x +
a*c)^4 - 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x + a*c)^5 - 10*b*c*
cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*b
*c*cosh(b*c*x + a*c)^6 - 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*c*cosh(b*c*x + a*
c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x + a*c)^7 - 3*b*c*
cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.52

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = -\frac{15\log\left(e^{(bcx+ac)}+1\right)}{8\,bc} + \frac{15\log\left(e^{(bcx+ac)}-1\right)}{8\,bc} + \frac{12\,e^{(9\,bcx+9\,ac)} - 123\,e^{(7\,bcx+7\,ac)} + 187\,e^{(5\,bcx+5\,ac)} - 157\,e^{(3\,bcx+3\,ac)} + 33\,e^{(bcx+ac)}}{12\,bc(e^{(8\,bcx+8\,ac)} - 4\,e^{(6\,bcx+6\,ac)} + 6\,e^{(4\,bcx+4\,ac)} - 4\,e^{(2\,bcx+2\,ac)} + 1)}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
+ 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x +
 5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x +
8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a
*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.67

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = \frac{24\,e^{(bcx+ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)-45\,\log\left(e^{(bcx+ac)}+1\right)\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+45\,\mathrm{l}}{}$$

```
[In] integrate(exp(c*(b*x+a))/(tanh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*e^(b*c*x + a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 45*log(e^(b*c*x + a
*c) + 1)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 45*log(abs(e^(b*c*x + a*c) - 1))*sg
n(e^(2*b*c*x + 2*a*c) - 1) - 2*(75*e^(7*b*c*x + 7*a*c)*sgn(e^(2*b*c*x + 2*a
*c) - 1) - 115*e^(5*b*c*x + 5*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 109*e^(3*
b*c*x + 3*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 21*e^(b*c*x + a*c)*sgn(e^(2*b
*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) - 1)^4)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\tanh^2(ac+bcx)^{5/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{{\left({\tanh\left(a\,c+b\,c\,x\right)}^2\right)}^{5/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(tanh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(tanh(a*c + b*c*x)^2)^(5/2), x)
```

3.240 $\int \sin^3(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 157

$$\int \sin^3(\tanh(a+bx))\,dx = -\frac{3\,\text{CosIntegral}(1-\tanh(a+bx))\sin(1)}{8b} - \frac{3\,\text{CosIntegral}(1+\tanh(a+bx))\sin(1)}{8b} + \frac{\text{CosIntegral}(3-3\tanh(a+bx))\sin(3)}{8b} + \frac{\text{CosIntegral}(3+3\tanh(a+bx))\sin(3)}{8b} - \frac{\cos(3)\text{Si}(3-3\tanh(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1-\tanh(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1+\tanh(a+bx))}{8b} - \frac{\cos(3)\text{Si}(3+3\tanh(a+bx))}{8b}$$

```
[Out] 1/8*cos(3)*Si(-3+3*tanh(b*x+a))/b-3/8*cos(1)*Si(-1+tanh(b*x+a))/b+3/8*cos(1
)*Si(1+tanh(b*x+a))/b-1/8*cos(3)*Si(3+3*tanh(b*x+a))/b-3/8*Ci(1-tanh(b*x+a)
)*sin(1)/b-3/8*Ci(1+tanh(b*x+a))*sin(1)/b+1/8*Ci(3-3*tanh(b*x+a))*sin(3)/b+
1/8*Ci(3+3*tanh(b*x+a))*sin(3)/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used

$= \{6857, 3393, 3384, 3380, 3383\}$

$$\int \sin^3(\tanh(a+bx))\,dx = \frac{\sin(3)\,\text{CosIntegral}(3-3\tanh(a+bx))}{8b} + \frac{\sin(3)\,\text{CosIntegral}(3\tanh(a+bx)+3)}{8b} - \frac{3\sin(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{8b} - \frac{3\sin(1)\,\text{CosIntegral}(\tanh(a+bx)+1)}{8b} - \frac{\cos(3)\text{Si}(3-3\tanh(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1-\tanh(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(\tanh(a+bx)+1)}{8b} - \frac{\cos(3)\text{Si}(3\tanh(a+bx)+3)}{8b}$$

```
[In] Int[Sin[Tanh[a + b*x]]^3,x]
```

```
[Out] (-3*CosIntegral[1 - Tanh[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Tanh[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Tanh[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Tanh[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Tanh[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Tanh[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Tanh[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Tanh[a + b*x]])/(8*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sin^3(x)}{2(-1+x)}+\frac{\sin^3(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(-1+x)}-\frac{\sin(3x)}{4(-1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{2b} \\
&\quad +\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(1+x)}-\frac{\sin(3x)}{4(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{2b} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b}-\frac{\text{Subst}\left(\int \frac{\sin(3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} \\
&\quad -\frac{3\text{Subst}\left(\int \frac{\sin(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b}+\frac{3\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b}
\end{aligned}
$$

$$= \frac{(3\cos(1))\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{(3\cos(1))\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{\cos(3)\text{Subst}\left(\int \frac{\sin(3-3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{\cos(3)\text{Subst}\left(\int \frac{\sin(3+3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{(3\sin(1))\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{(3\sin(1))\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{\sin(3)\text{Subst}\left(\int \frac{\cos(3-3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{\sin(3)\text{Subst}\left(\int \frac{\cos(3+3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b}$$

$$= -\frac{3\,\text{CosIntegral}(1-\tanh(a+bx))\sin(1)}{8b} - \frac{3\,\text{CosIntegral}(1+\tanh(a+bx))\sin(1)}{8b} + \frac{\text{CosIntegral}(3-3\tanh(a+bx))\sin(3)}{8b} + \frac{\text{CosIntegral}(3+3\tanh(a+bx))\sin(3)}{8b} - \frac{\cos(3)\text{Si}(3-3\tanh(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1-\tanh(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1+\tanh(a+bx))}{8b} - \frac{\cos(3)\text{Si}(3+3\tanh(a+bx))}{8b}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \sin^3(\tanh(a+bx))\,dx$$
$$= \frac{-6\,\text{CosIntegral}(1-\tanh(a+bx))\sin(1) - 6\,\text{CosIntegral}(1+\tanh(a+bx))\sin(1) + 2\,\text{CosIntegral}(3-3\,\text{ta}}{}$$

```
[In] Integrate[Sin[Tanh[a + b*x]]^3,x]
```

```
[Out] (-6*CosIntegral[1 - Tanh[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Tanh[a + b*x]
]*Sin[1] + 2*CosIntegral[3 - 3*Tanh[a + b*x]]*Sin[3] + 2*CosIntegral[3 + 3*
Tanh[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Tanh[a + b*x]] + 6*Cos[1
]*SinIntegral[1 - Tanh[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Tanh[a + b*x]]
- 2*Cos[3]*SinIntegral[3 + 3*Tanh[a + b*x]])/(16*b)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{\mathrm{Si}(3+3\tanh(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(3+3\tanh(bx+a))\sin(3)}{8}+\frac{\mathrm{Si}(-3+3\tanh(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(-3+3\tanh(bx+a))\sin(3)}{8}+\frac{3\ \mathrm{Si}(1+\mathrm{tan}\cdots}{\cdots}}{b}$
default	$\frac{-\frac{\mathrm{Si}(3+3\tanh(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(3+3\tanh(bx+a))\sin(3)}{8}+\frac{\mathrm{Si}(-3+3\tanh(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(-3+3\tanh(bx+a))\sin(3)}{8}+\frac{3\ \mathrm{Si}(1+\mathrm{tan}\cdots}{\cdots}}{b}$
risch	$-\frac{i\mathrm{e}^{-3i}\,\mathrm{Ei}_1\left(\frac{6i}{1+\mathrm{e}^{2bx+2a}}-6i\right)}{16b}-\frac{i\mathrm{e}^{-3i}\,\mathrm{Ei}_1\left(-\frac{6i}{1+\mathrm{e}^{2bx+2a}}\right)}{16b}+\frac{i\mathrm{e}^{3i}\,\mathrm{Ei}_1\left(\frac{6i}{1+\mathrm{e}^{2bx+2a}}\right)}{16b}+\frac{i\mathrm{e}^{3i}\,\mathrm{Ei}_1\left(-\frac{6i}{1+\mathrm{e}^{2bx+2a}}+6i\right)}{16b}-$

```
[In] int(sin(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/8*Si(3+3*tanh(b*x+a))*cos(3)+1/8*Ci(3+3*tanh(b*x+a))*sin(3)+1/8*Si(
-3+3*tanh(b*x+a))*cos(3)+1/8*Ci(-3+3*tanh(b*x+a))*sin(3)+3/8*Si(1+tanh(b*x+
a))*cos(1)-3/8*Ci(1+tanh(b*x+a))*sin(1)-3/8*Si(-1+tanh(b*x+a))*cos(1)-3/8*C
i(-1+tanh(b*x+a))*sin(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 697, normalized size of antiderivative = 4.44

$$\int \sin^3(\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(sin(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*((-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*sin(3)^2 - 2*I*(I*cos(3)*c
os(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I)*sin(1) + I*cos(1))*cos_
integral(3*(cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) - 3*(2*cos(3)*cos
(1)*sin(1) + I*cos(3)*sin(1)^2 + (-I*cos(1)^2 + I)*cos(3) + I*(-I*cos(1)^2
+ 2*cos(1)*sin(1) + I*sin(1)^2 + I)*sin(3))*cos_integral((cosh(b*x + a) + s
inh(b*x + a))/cosh(b*x + a)) + (-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*s
in(3)^2 - 2*I*(I*cos(3)*cos(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I
)*sin(1) + I*cos(1))*cos_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh
(b*x + a) + sinh(b*x + a)^2 + 1)) - 3*(2*cos(3)*cos(1)*sin(1) + I*cos(3)*si
n(1)^2 + (-I*cos(1)^2 + I)*cos(3) + I*(-I*cos(1)^2 + 2*cos(1)*sin(1) + I*si
n(1)^2 + I)*sin(3))*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 + 1)) - (cos(3)^2*cos(1) - (cos(1) + I*sin(1))*s
in(3)^2 + 2*I*(cos(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*s
in(1) + cos(1))*sin_integral(3*(cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a
)) - 3*(-2*I*cos(3)*cos(1)*sin(1) + cos(3)*sin(1)^2 - (cos(1)^2 + 1)*cos(3)
 - I*(cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*sin(3))*sin_integral((co
sh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) - (cos(3)^2*cos(1) - (cos(1) +
```

```
I*sin(1))*sin(3)^2 + 2*I*(cos(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(cos(
3)^2 + 1)*sin(1) + cos(1))*sin_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) - 3*(-2*I*cos(3)*cos(1)*sin(1) + co
s(3)*sin(1)^2 - (cos(1)^2 + 1)*cos(3) - I*(cos(1)^2 + 2*I*cos(1)*sin(1) - s
in(1)^2 + 1)*sin(3))*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh
(b*x + a) + sinh(b*x + a)^2 + 1)))/(b*cos(3)*cos(1) + I*b*cos(3)*sin(1) + I
*(b*cos(1) + I*b*sin(1))*sin(3))
```

Sympy [F]

$$\int \sin^3(\tanh(a+bx))\,dx = \int \sin^3\left(\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(sin(tanh(b*x+a))**3,x)
[Out] Integral(sin(tanh(a + b*x))**3, x)
```

Maxima [F]

$$\int \sin^3(\tanh(a+bx))\,dx = \int \sin\left(\tanh\left(bx+a\right)\right)^3\,dx$$

```
[In] integrate(sin(tanh(b*x+a))^3,x, algorithm="maxima")
[Out] integrate(sin(tanh(b*x + a))^3, x)
```

Giac [F]

$$\int \sin^3(\tanh(a+bx))\,dx = \int \sin\left(\tanh\left(bx+a\right)\right)^3\,dx$$

```
[In] integrate(sin(tanh(b*x+a))^3,x, algorithm="giac")
[Out] integrate(sin(tanh(b*x + a))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^3(\tanh(a+bx))\,dx = \int \sin\left(\tanh(a+b\,x)\right)^3\,dx$$

```
[In] int(sin(tanh(a + b*x))^3,x)
[Out] int(sin(tanh(a + b*x))^3, x)
```

3.241 $\int \sin^2(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \sin^2(\tanh(a+bx))\,dx = \frac{\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))}{4b} - \frac{\cos(2)\,\text{CosIntegral}(2+2\tanh(a+bx))}{4b} - \frac{\log(1-\tanh(a+bx))}{4b} + \frac{\log(1+\tanh(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2+2\tanh(a+bx))}{4b}$$

```
[Out] 1/4*Ci(2-2*tanh(b*x+a))*cos(2)/b-1/4*Ci(2+2*tanh(b*x+a))*cos(2)/b-1/4*ln(1-
tanh(b*x+a))/b+1/4*ln(1+tanh(b*x+a))/b-1/4*Si(-2+2*tanh(b*x+a))*sin(2)/b-1/
4*Si(2+2*tanh(b*x+a))*sin(2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6857, 3393, 3384, 3380, 3383}

$$\int \sin^2(\tanh(a+bx))\,dx = \frac{\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))}{4b} - \frac{\cos(2)\,\text{CosIntegral}(2\tanh(a+bx)+2)}{4b} + \frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2\tanh(a+bx)+2)}{4b} - \frac{\log(1-\tanh(a+bx))}{4b} + \frac{\log(\tanh(a+bx)+1)}{4b}$$

```
[In] Int[Sin[Tanh[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2
*Tanh[a + b*x]])/(4*b) - Log[1 - Tanh[a + b*x]]/(4*b) + Log[1 + Tanh[a + b*
x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Tanh[a + b*x]])/(4*b) - (Sin[2]*SinIn
tegral[2 + 2*Tanh[a + b*x]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sin^2(x)}{2(-1+x)}+\frac{\sin^2(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}-\frac{\cos(2x)}{2(-1+x)}\right)\,dx,x,\tanh(a+bx)\right)}{2b} \\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}-\frac{\cos(2x)}{2(1+x)}\right)\,dx,x,\tanh(a+bx)\right)}{2b} \\
&= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b} \\
&\quad+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{4b}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b} \\
&\quad+\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&\quad-\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2+2x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&\quad+\frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2-2x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&\quad-\frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2+2x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&= \frac{\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))}{4b}-\frac{\cos(2)\,\text{CosIntegral}(2+2\tanh(a+bx))}{4b} \\
&\quad-\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b} \\
&\quad+\frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b}-\frac{\sin(2)\text{Si}(2+2\tanh(a+bx))}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$
\int \sin^2(\tanh(a+bx))\,dx
= \frac{\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))-\cos(2)\,\text{CosIntegral}(2(1+\tanh(a+bx)))-\log(1-\tanh(a+bx)}{4b}
$$

```
[In] Integrate[Sin[Tanh[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Tanh[a
 + b*x])] - Log[1 - Tanh[a + b*x]] + Log[1 + Tanh[a + b*x]] + Sin[2]*SinInt
egral[2 - 2*Tanh[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Tanh[a + b*x])])/(4*
b)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{\ln(-1+\tanh(bx+a))}{4}+\frac{\ln(1+\tanh(bx+a))}{4}-\frac{\mathrm{Si}(-2+2\tanh(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(-2+2\tanh(bx+a))\cos(2)}{4}-\frac{\mathrm{Si}(2+2\tanh(bx+a))\sin(2)}{4}$
default	$\frac{-\frac{\ln(-1+\tanh(bx+a))}{4}+\frac{\ln(1+\tanh(bx+a))}{4}-\frac{\mathrm{Si}(-2+2\tanh(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(-2+2\tanh(bx+a))\cos(2)}{4}-\frac{\mathrm{Si}(2+2\tanh(bx+a))\sin(2)}{4}$
risch	$\frac{e^{2i}\,\mathrm{Ei}_1\left(-\frac{4i}{1+e^{2bx+2a}}+4i\right)}{8b}-\frac{e^{2i}\,\mathrm{Ei}_1\left(\frac{4i}{1+e^{2bx+2a}}\right)}{8b}-\frac{e^{-2i}\,\mathrm{Ei}_1\left(-\frac{4i}{1+e^{2bx+2a}}\right)}{8b}+\frac{e^{-2i}\,\mathrm{Ei}_1\left(\frac{4i}{1+e^{2bx+2a}}-4i\right)}{8b}+\frac{x}{2}$

```
[In] int(sin(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4*ln(-1+tanh(b*x+a))+1/4*ln(1+tanh(b*x+a))-1/4*Si(-2+2*tanh(b*x+a))
*sin(2)+1/4*Ci(-2+2*tanh(b*x+a))*cos(2)-1/4*Si(2+2*tanh(b*x+a))*sin(2)-1/4*
Ci(2+2*tanh(b*x+a))*cos(2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.00

$$\int \sin^2(\tanh(a+bx))\,dx$$
$$=\frac{4\,bx\cos(2)+4i\,bx\sin(2)-\left(\cos(2)^2+2i\,\cos(2)\sin(2)-\sin(2)^2+1\right)\mathrm{Ci}\left(\frac{2(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)}\right)+(\cos$$

```
[In] integrate(sin(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cos(2) + 4*I*b*x*sin(2) - (cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2)
^2 + 1)*cos_integral(2*(cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) + (co
s(2)^2 + 2*I*cos(2)*sin(2) - sin(2)^2 + 1)*cos_integral(4/(cosh(b*x + a)^2
+ 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + (I*cos(2)^2 - 2*c
os(2)*sin(2) - I*sin(2)^2 - I)*sin_integral(2*(cosh(b*x + a) + sinh(b*x + a
))/cosh(b*x + a)) + (-I*cos(2)^2 + 2*cos(2)*sin(2) + I*sin(2)^2 + I)*sin_in
tegral(4/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 + 1)))/(b*cos(2) + I*b*sin(2))
```

Sympy [F]

$$\int \sin^2(\tanh(a+bx))\,dx = \int \sin^2\left(\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(sin(tanh(b*x+a))**2,x)
[Out] Integral(sin(tanh(a + b*x))**2, x)
```

Maxima [F]

$$\int \sin^2(\tanh(a+bx))\,dx = \int \sin\left(\tanh\left(bx+a\right)\right)^2\,dx$$

```
[In] integrate(sin(tanh(b*x+a))^2,x, algorithm="maxima")
[Out] 1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)), x
)
```

Giac [F]

$$\int \sin^2(\tanh(a+bx))\,dx = \int \sin\left(\tanh\left(bx+a\right)\right)^2\,dx$$

```
[In] integrate(sin(tanh(b*x+a))^2,x, algorithm="giac")
[Out] integrate(sin(tanh(b*x + a))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(\tanh(a+bx))\,dx = \int \sin\left(\tanh(a+b\,x)\right)^2dx$$

```
[In] int(sin(tanh(a + b*x))^2,x)
[Out] int(sin(tanh(a + b*x))^2, x)
```

3.242 $\int \sin(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \sin(\tanh(a+bx))\,dx = -\frac{\text{CosIntegral}(1-\tanh(a+bx))\sin(1)}{2b} - \frac{\text{CosIntegral}(1+\tanh(a+bx))\sin(1)}{2b} + \frac{\cos(1)\text{Si}(1-\tanh(a+bx))}{2b} + \frac{\cos(1)\text{Si}(1+\tanh(a+bx))}{2b}$$

```
[Out] -1/2*cos(1)*Si(-1+tanh(b*x+a))/b+1/2*cos(1)*Si(1+tanh(b*x+a))/b-1/2*Ci(1-ta
nh(b*x+a))*sin(1)/b-1/2*Ci(1+tanh(b*x+a))*sin(1)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3414, 3384, 3380, 3383}

$$\int \sin(\tanh(a+bx))\,dx = -\frac{\sin(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{2b} - \frac{\sin(1)\,\text{CosIntegral}(\tanh(a+bx)+1)}{2b} + \frac{\cos(1)\text{Si}(1-\tanh(a+bx))}{2b} + \frac{\cos(1)\text{Si}(\tanh(a+bx)+1)}{2b}$$

```
[In] Int[Sin[Tanh[a + b*x]],x]
```

```
[Out] -1/2*(CosIntegral[1 - Tanh[a + b*x]]*Sin[1])/b - (CosIntegral[1 + Tanh[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Tanh[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Tanh[a + b*x]])/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{2(1-x)}+\frac{\sin(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&= -\frac{\cos(1)\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&\quad+\frac{\cos(1)\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&\quad+\frac{\sin(1)\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&\quad-\frac{\sin(1)\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}
\end{aligned}
$$

$$= -\frac{\text{CosIntegral}(1-\tanh(a+bx))\sin(1)}{2b} - \frac{\text{CosIntegral}(1+\tanh(a+bx))\sin(1)}{2b} + \frac{\cos(1)\text{Si}(1-\tanh(a+bx))}{2b} + \frac{\cos(1)\text{Si}(1+\tanh(a+bx))}{2b}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \sin(\tanh(a+bx))\,dx = -\frac{\text{CosIntegral}(1-\tanh(a+bx))\sin(1)+\text{CosIntegral}(1+\tanh(a+bx))\sin(1)-\cos(1)(\text{Si}(1-\tanh(a-}{2b}$$

```
[In] Integrate[Sin[Tanh[a + b*x]],x]
```

```
[Out] -1/2*(CosIntegral[1 - Tanh[a + b*x]]*Sin[1] + CosIntegral[1 + Tanh[a + b*x]
]*Sin[1] - Cos[1]*(SinIntegral[1 - Tanh[a + b*x]] + SinIntegral[1 + Tanh[a
+ b*x]]))/b
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result	si
derivativedivides	$\frac{\frac{\text{Si}(1+\tanh(bx+a))\cos(1)}{2}-\frac{\text{Ci}(1+\tanh(bx+a))\sin(1)}{2}-\frac{\text{Si}(-1+\tanh(bx+a))\cos(1)}{2}-\frac{\text{Ci}(-1+\tanh(bx+a))\sin(1)}{2}}{b}$	58
default	$\frac{\frac{\text{Si}(1+\tanh(bx+a))\cos(1)}{2}-\frac{\text{Ci}(1+\tanh(bx+a))\sin(1)}{2}-\frac{\text{Si}(-1+\tanh(bx+a))\cos(1)}{2}-\frac{\text{Ci}(-1+\tanh(bx+a))\sin(1)}{2}}{b}$	58
risch	$-\frac{i\,\mathrm{e}^{i}\,\text{Ei}_1\left(-\frac{2i}{1+\mathrm{e}^{2bx+2a}}+2i\right)}{4b}-\frac{i\,\mathrm{e}^{i}\,\text{Ei}_1\left(\frac{2i}{1+\mathrm{e}^{2bx+2a}}\right)}{4b}+\frac{i\,\mathrm{e}^{-i}\,\text{Ei}_1\left(-\frac{2i}{1+\mathrm{e}^{2bx+2a}}\right)}{4b}+\frac{i\,\mathrm{e}^{-i}\,\text{Ei}_1\left(\frac{2i}{1+\mathrm{e}^{2bx+2a}}-2i\right)}{4b}$	11

```
[In] int(sin(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*Si(1+tanh(b*x+a))*cos(1)-1/2*Ci(1+tanh(b*x+a))*sin(1)-1/2*Si(-1+ta
nh(b*x+a))*cos(1)-1/2*Ci(-1+tanh(b*x+a))*sin(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.79

$$\int \sin(\tanh(a+bx))\,dx = \frac{\left(i\cos(1)^2 - 2\cos(1)\sin(1) - i\sin(1)^2 - i\right)\operatorname{Ci}\left(\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)}\right) + \left(i\cos(1)^2 - 2\cos(1)\sin(1) - i\sin\right.}{}$$

```
[In] integrate(sin(tanh(b*x+a)),x, algorithm="fricas")
```

```
[Out] 1/4*((I*cos(1)^2 - 2*cos(1)*sin(1) - I*sin(1)^2 - I)*cos_integral((cosh(b*x
 + a) + sinh(b*x + a))/cosh(b*x + a)) + (I*cos(1)^2 - 2*cos(1)*sin(1) - I*s
in(1)^2 - I)*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 + 1)) + (cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*s
in_integral((cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) + (cos(1)^2 + 2*
I*cos(1)*sin(1) - sin(1)^2 + 1)*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/(b*cos(1) + I*b*sin(1))
```

Sympy [F]

$$\int \sin(\tanh(a+bx))\,dx = \int \sin\left(\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(sin(tanh(b*x+a)),x)
```

```
[Out] Integral(sin(tanh(a + b*x)), x)
```

Maxima [F]

$$\int \sin(\tanh(a+bx))\,dx = \int \sin\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sin(tanh(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(sin(tanh(b*x + a)), x)
```

Giac [F]

$$\int \sin(\tanh(a+bx))\,dx = \int \sin\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sin(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(sin(tanh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(\tanh(a+bx))\,dx = \int \sin\left(\tanh(a+b\,x)\right)\,dx$$

```
[In] int(sin(tanh(a + b*x)),x)
[Out] int(sin(tanh(a + b*x)), x)
```

3.243 $\int \csc(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \csc(\tanh(a+bx))\,dx = -\frac{1}{2}\text{Int}\left(\frac{\csc(\tanh(a+bx))\text{sech}^2(a+bx)}{-1+\tanh(a+bx)},x\right) + \frac{1}{2}\text{Int}\left(\frac{\csc(\tanh(a+bx))\text{sech}^2(a+bx)}{1+\tanh(a+bx)},x\right)$$

```
[Out] -1/2*Unintegrable(csc(tanh(b*x+a))*sech(b*x+a)^2/(-1+tanh(b*x+a)),x)+1/2*Un
integrable(csc(tanh(b*x+a))*sech(b*x+a)^2/(1+tanh(b*x+a)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc(\tanh(a+bx))\,dx = \int \csc(\tanh(a+bx))\,dx$$

```
[In] Int[Csc[Tanh[a + b*x]],x]
```

```
[Out] -1/2*Defer[Subst][Defer[Int][Csc[x]/(-1 + x), x], x, Tanh[a + b*x]]/b + Def
er[Subst][Defer[Int][Csc[x]/(1 + x), x], x, Tanh[a + b*x]]/(2*b)
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\csc(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\csc(x)}{2(-1+x)}+\frac{\csc(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\csc(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\csc(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}$$

Mathematica [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\tanh(a+bx))\,dx=\int \csc(\tanh(a+bx))\,dx$$

```
[In] Integrate[Csc[Tanh[a + b*x]],x]
[Out] Integrate[Csc[Tanh[a + b*x]], x]
```

Maple [F(-1)]

Timed out.

$$\int \csc\left(\tanh\left(bx+a\right)\right)dx$$

```
[In] int(csc(tanh(b*x+a)),x)
[Out] int(csc(tanh(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\tanh(a+bx))\,dx=\int \csc\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(csc(tanh(b*x+a)),x, algorithm="fricas")
[Out] integral(csc(tanh(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 10.76 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \csc(\tanh(a+bx))\,dx = \int \csc\left(\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(csc(tanh(b*x+a)),x)
[Out] Integral(csc(tanh(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\tanh(a+bx))\,dx = \int \csc\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(csc(tanh(b*x+a)),x, algorithm="maxima")
[Out] integrate(csc(tanh(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\tanh(a+bx))\,dx = \int \csc\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(csc(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(csc(tanh(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \csc(\tanh(a+bx))\,dx = \int \frac{1}{\sin\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

```
[In] int(1/sin(tanh(a + b*x)),x)
[Out] int(1/sin(tanh(a + b*x)), x)
```

3.244 $\int \cos^3(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 157

$$\int \cos^3(\tanh(a+bx))\,dx = -\frac{\cos(3)\,\text{CosIntegral}(3-3\tanh(a+bx))}{8b} - \frac{3\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{8b} + \frac{3\cos(1)\,\text{CosIntegral}(1+\tanh(a+bx))}{8b} + \frac{\cos(3)\,\text{CosIntegral}(3+3\tanh(a+bx))}{8b} - \frac{\sin(3)\text{Si}(3-3\tanh(a+bx))}{8b} - \frac{3\sin(1)\text{Si}(1-\tanh(a+bx))}{8b} + \frac{3\sin(1)\text{Si}(1+\tanh(a+bx))}{8b} + \frac{\sin(3)\text{Si}(3+3\tanh(a+bx))}{8b}$$

```
[Out] -3/8*Ci(1-tanh(b*x+a))*cos(1)/b+3/8*Ci(1+tanh(b*x+a))*cos(1)/b-1/8*Ci(3-3*t
anh(b*x+a))*cos(3)/b+1/8*Ci(3+3*tanh(b*x+a))*cos(3)/b+3/8*Si(-1+tanh(b*x+a)
)*sin(1)/b+3/8*Si(1+tanh(b*x+a))*sin(1)/b+1/8*Si(-3+3*tanh(b*x+a))*sin(3)/b
+1/8*Si(3+3*tanh(b*x+a))*sin(3)/b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used

$= \{6857, 3393, 3384, 3380, 3383\}$

$$\int \cos^3(\tanh(a+bx))\,dx = -\frac{\cos(3)\,\text{CosIntegral}(3-3\tanh(a+bx))}{8b} - \frac{3\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{8b} + \frac{3\cos(1)\,\text{CosIntegral}(\tanh(a+bx)+1)}{8b} + \frac{\cos(3)\,\text{CosIntegral}(3\tanh(a+bx)+3)}{8b} - \frac{\sin(3)\text{Si}(3-3\tanh(a+bx))}{8b} - \frac{3\sin(1)\text{Si}(1-\tanh(a+bx))}{8b} + \frac{3\sin(1)\text{Si}(\tanh(a+bx)+1)}{8b} + \frac{\sin(3)\text{Si}(3\tanh(a+bx)+3)}{8b}$$

```
[In] Int[Cos[Tanh[a + b*x]]^3,x]
```

```
[Out] -1/8*(Cos[3]*CosIntegral[3 - 3*Tanh[a + b*x]])/b - (3*Cos[1]*CosIntegral[1
- Tanh[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Tanh[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Tanh[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Tanh[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Tanh[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Tanh[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Tanh[a + b*x]])/(8*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cos^3(x)}{2(-1+x)}+\frac{\cos^3(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(-1+x)}+\frac{\cos(3x)}{4(-1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{2b}\\
&\quad +\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(1+x)}+\frac{\cos(3x)}{4(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b}+\frac{\text{Subst}\left(\int \frac{\cos(3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b}\\
&\quad -\frac{3\text{Subst}\left(\int \frac{\cos(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b}+\frac{3\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b}
\end{aligned}
$$

$$= -\frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{\cos(3)\text{Subst}\left(\int \frac{\cos(3-3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{\cos(3)\text{Subst}\left(\int \frac{\cos(3+3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{(3\sin(1))\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{(3\sin(1))\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b} - \frac{\sin(3)\text{Subst}\left(\int \frac{\sin(3-3x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{8b} + \frac{\sin(3)\text{Subst}\left(\int \frac{\sin(3+3x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{8b}$$

$$= -\frac{\cos(3)\,\text{CosIntegral}(3-3\tanh(a+bx))}{8b} - \frac{3\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{8b} + \frac{3\cos(1)\,\text{CosIntegral}(1+\tanh(a+bx))}{8b} + \frac{\cos(3)\,\text{CosIntegral}(3+3\tanh(a+bx))}{8b} - \frac{\sin(3)\text{Si}(3-3\tanh(a+bx))}{8b} - \frac{3\sin(1)\text{Si}(1-\tanh(a+bx))}{8b} + \frac{3\sin(1)\text{Si}(1+\tanh(a+bx))}{8b} + \frac{\sin(3)\text{Si}(3+3\tanh(a+bx))}{8b}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \cos^3(\tanh(a+bx))\,dx = \frac{-2\cos(3)\,\text{CosIntegral}(3-3\tanh(a+bx)) - 6\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx)) + 6\cos(1)\,\text{CosIntegra}}{}$$

```
[In] Integrate[Cos[Tanh[a + b*x]]^3,x]
```

```
[Out] (-2*Cos[3]*CosIntegral[3 - 3*Tanh[a + b*x]] - 6*Cos[1]*CosIntegral[1 - Tanh
[a + b*x]] + 6*Cos[1]*CosIntegral[1 + Tanh[a + b*x]] + 2*Cos[3]*CosIntegral
```

```
[3 + 3*Tanh[a + b*x]] - 2*Sin[3]*SinIntegral[3 - 3*Tanh[a + b*x]] - 6*Sin[1
]*SinIntegral[1 - Tanh[a + b*x]] + 6*Sin[1]*SinIntegral[1 + Tanh[a + b*x]]
+ 2*Sin[3]*SinIntegral[3 + 3*Tanh[a + b*x]])/(16*b)
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(-3+3\tanh(bx+a))\sin(3)}{8}-\frac{\mathrm{Ci}(-3+3\tanh(bx+a))\cos(3)}{8}+\frac{\mathrm{Si}(3+3\tanh(bx+a))\sin(3)}{8}+\frac{\mathrm{Ci}(3+3\tanh(bx+a))\cos(3)}{8}+\frac{3\ \mathrm{Si}(-1+\tanh}{}}{b}$
default	$\frac{\frac{\mathrm{Si}(-3+3\tanh(bx+a))\sin(3)}{8}-\frac{\mathrm{Ci}(-3+3\tanh(bx+a))\cos(3)}{8}+\frac{\mathrm{Si}(3+3\tanh(bx+a))\sin(3)}{8}+\frac{\mathrm{Ci}(3+3\tanh(bx+a))\cos(3)}{8}+\frac{3\ \mathrm{Si}(-1+\tanh}{}}{b}$
risch	$-\frac{e^{3i}\,\mathrm{Ei}_1\left(-\frac{6i}{1+e^{2bx+2a}}+6i\right)}{16b}+\frac{e^{3i}\,\mathrm{Ei}_1\left(\frac{6i}{1+e^{2bx+2a}}\right)}{16b}+\frac{e^{-3i}\,\mathrm{Ei}_1\left(-\frac{6i}{1+e^{2bx+2a}}\right)}{16b}-\frac{e^{-3i}\,\mathrm{Ei}_1\left(\frac{6i}{1+e^{2bx+2a}}-6i\right)}{16b}-$

```
[In] int(cos(tanh(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*Si(-3+3*tanh(b*x+a))*sin(3)-1/8*Ci(-3+3*tanh(b*x+a))*cos(3)+1/8*Si
(3+3*tanh(b*x+a))*sin(3)+1/8*Ci(3+3*tanh(b*x+a))*cos(3)+3/8*Si(-1+tanh(b*x+
a))*sin(1)-3/8*Ci(-1+tanh(b*x+a))*cos(1)+3/8*Si(1+tanh(b*x+a))*sin(1)+3/8*C
i(1+tanh(b*x+a))*cos(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.45

$$\int \cos^3(\tanh(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(cos(tanh(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*((cos(3)^2*cos(1) - (cos(1) + I*sin(1))*sin(3)^2 + 2*I*(cos(3)*cos(1)
+ I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*sin(1) + cos(1))*cos_integral(
3*(cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) - 3*(-2*I*cos(3)*cos(1)*si
n(1) + cos(3)*sin(1)^2 - (cos(1)^2 + 1)*cos(3) - I*(cos(1)^2 + 2*I*cos(1)*s
in(1) - sin(1)^2 + 1)*sin(3))*cos_integral((cosh(b*x + a) + sinh(b*x + a))/
cosh(b*x + a)) - (cos(3)^2*cos(1) - (cos(1) + I*sin(1))*sin(3)^2 + 2*I*(cos
(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*sin(1) + cos(1))*co
s_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 + 1)) - 3*(2*I*cos(3)*cos(1)*sin(1) - cos(3)*sin(1)^2 + (cos(1)^2 + 1)
*cos(3) + I*(cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*sin(3))*cos_integ
ral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 +
1)) + (-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*sin(3)^2 - 2*I*(I*cos(3)*c
os(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I)*sin(1) + I*cos(1))*sin_
```

```
integral(3*(cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) + 3*(2*cos(3)*cos
(1)*sin(1) + I*cos(3)*sin(1)^2 - (I*cos(1)^2 - I)*cos(3) - I*(I*cos(1)^2 -
2*cos(1)*sin(1) - I*sin(1)^2 - I)*sin(3))*sin_integral((cosh(b*x + a) + sin
h(b*x + a))/cosh(b*x + a)) + (I*cos(3)^2*cos(1) - (I*cos(1) - sin(1))*sin(3
)^2 - 2*I*(-I*cos(3)*cos(1) + cos(3)*sin(1))*sin(3) + I*(I*cos(3)^2 - I)*si
n(1) - I*cos(1))*sin_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
 + a) + sinh(b*x + a)^2 + 1)) - 3*(2*cos(3)*cos(1)*sin(1) + I*cos(3)*sin(1)
^2 + (-I*cos(1)^2 + I)*cos(3) + I*(-I*cos(1)^2 + 2*cos(1)*sin(1) + I*sin(1)
^2 + I)*sin(3))*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 + 1)))/(b*cos(3)*cos(1) + I*b*cos(3)*sin(1) + I*(b*c
os(1) + I*b*sin(1))*sin(3))
```

Sympy [F]

$$\int \cos^3(\tanh(a+bx))\,dx = \int \cos^3\left(\tanh\left(a+bx\right)\right)dx$$

[In] integrate(cos(tanh(b*x+a))**3,x)

[Out] Integral(cos(tanh(a + b*x))**3, x)

Maxima [F]

$$\int \cos^3(\tanh(a+bx))\,dx = \int \cos\left(\tanh\left(bx+a\right)\right)^3\,dx$$

[In] integrate(cos(tanh(b*x+a))^3,x, algorithm="maxima")

[Out] integrate(cos(tanh(b*x + a))^3, x)

Giac [F]

$$\int \cos^3(\tanh(a+bx))\,dx = \int \cos\left(\tanh\left(bx+a\right)\right)^3\,dx$$

[In] integrate(cos(tanh(b*x+a))^3,x, algorithm="giac")

[Out] integrate(cos(tanh(b*x + a))^3, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^3(\tanh(a+bx))\,dx=\int \cos\left(\tanh(a+b\,x)\right)^3\,dx$$

```
[In] int(cos(tanh(a + b*x))^3,x)
[Out] int(cos(tanh(a + b*x))^3, x)
```

3.245 $\int \cos^2(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \cos^2(\tanh(a+bx))\,dx = -\frac{\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))}{4b} + \frac{\cos(2)\,\text{CosIntegral}(2+2\tanh(a+bx))}{4b} - \frac{\log(1-\tanh(a+bx))}{4b} + \frac{\log(1+\tanh(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2+2\tanh(a+bx))}{4b}$$

```
[Out] -1/4*Ci(2-2*tanh(b*x+a))*cos(2)/b+1/4*Ci(2+2*tanh(b*x+a))*cos(2)/b-1/4*ln(1
-tanh(b*x+a))/b+1/4*ln(1+tanh(b*x+a))/b+1/4*Si(-2+2*tanh(b*x+a))*sin(2)/b+1
/4*Si(2+2*tanh(b*x+a))*sin(2)/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6857, 3393, 3384, 3380, 3383}

$$\int \cos^2(\tanh(a+bx))\,dx = -\frac{\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))}{4b} + \frac{\cos(2)\,\text{CosIntegral}(2\tanh(a+bx)+2)}{4b} - \frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2\tanh(a+bx)+2)}{4b} - \frac{\log(1-\tanh(a+bx))}{4b} + \frac{\log(\tanh(a+bx)+1)}{4b}$$

```
[In] Int[Cos[Tanh[a + b*x]]^2,x]
```

```
[Out] -1/4*(Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]])/b + (Cos[2]*CosIntegral[2 +
2*Tanh[a + b*x]])/(4*b) - Log[1 - Tanh[a + b*x]]/(4*b) + Log[1 + Tanh[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Tanh[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Tanh[a + b*x]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cos^2(x)}{2(-1+x)}+\frac{\cos^2(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}+\frac{\cos(2x)}{2(-1+x)}\right)\,dx,x,\tanh(a+bx)\right)}{2b} \\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}+\frac{\cos(2x)}{2(1+x)}\right)\,dx,x,\tanh(a+bx)\right)}{2b} \\
&= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b} \\
&\quad-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{4b}+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&= -\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b} \\
&\quad-\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&\quad+\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2+2x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&\quad-\frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2-2x)}{-1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&\quad+\frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2+2x)}{1+x}\,dx,x,\tanh(a+bx)\right)}{4b} \\
&= -\frac{\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))}{4b}+\frac{\cos(2)\,\text{CosIntegral}(2+2\tanh(a+bx))}{4b} \\
&\quad-\frac{\log(1-\tanh(a+bx))}{4b}+\frac{\log(1+\tanh(a+bx))}{4b} \\
&\quad-\frac{\sin(2)\text{Si}(2-2\tanh(a+bx))}{4b}+\frac{\sin(2)\text{Si}(2+2\tanh(a+bx))}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$
\int \cos^2(\tanh(a+bx))\,dx = \frac{-\cos(2)\,\text{CosIntegral}(2-2\tanh(a+bx))+\cos(2)\,\text{CosIntegral}(2(1+\tanh(a+bx)))-\log(1-\tanh(a+b
$$

$$4b$$

```
[In] Integrate[Cos[Tanh[a + b*x]]^2,x]
[Out] (-(Cos[2]*CosIntegral[2 - 2*Tanh[a + b*x]]) + Cos[2]*CosIntegral[2*(1 + Tan
h[a + b*x])] - Log[1 - Tanh[a + b*x]] + Log[1 + Tanh[a + b*x]] - Sin[2]*Sin
Integral[2 - 2*Tanh[a + b*x]] + Sin[2]*SinIntegral[2*(1 + Tanh[a + b*x])])/
(4*b)
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(-2+2\tanh(bx+a))\sin(2)}{4}-\frac{\mathrm{Ci}(-2+2\tanh(bx+a))\cos(2)}{4}+\frac{\mathrm{Si}(2+2\tanh(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(2+2\tanh(bx+a))\cos(2)}{4}-\frac{\ln(-1+\tanh(}{4}}{b}$
default	$\frac{\frac{\mathrm{Si}(-2+2\tanh(bx+a))\sin(2)}{4}-\frac{\mathrm{Ci}(-2+2\tanh(bx+a))\cos(2)}{4}+\frac{\mathrm{Si}(2+2\tanh(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(2+2\tanh(bx+a))\cos(2)}{4}-\frac{\ln(-1+\tanh(}{4}}{b}$
risch	$-\frac{e^{-2i}\,\mathrm{Ei}_1\left(\frac{4i}{1+e^{2bx+2a}}-4i\right)}{8b}+\frac{e^{2i}\,\mathrm{Ei}_1\left(\frac{4i}{1+e^{2bx+2a}}\right)}{8b}-\frac{e^{2i}\,\mathrm{Ei}_1\left(-\frac{4i}{1+e^{2bx+2a}}+4i\right)}{8b}+\frac{e^{-2i}\,\mathrm{Ei}_1\left(-\frac{4i}{1+e^{2bx+2a}}\right)}{8b}+$

```
[In] int(cos(tanh(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*Si(-2+2*tanh(b*x+a))*sin(2)-1/4*Ci(-2+2*tanh(b*x+a))*cos(2)+1/4*Si
(2+2*tanh(b*x+a))*sin(2)+1/4*Ci(2+2*tanh(b*x+a))*cos(2)-1/4*ln(-1+tanh(b*x+
a))+1/4*ln(1+tanh(b*x+a)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.00

$$\int \cos^2(\tanh(a+bx))\,dx$$
$$=\frac{4\,bx\cos(2)+4i\,bx\sin(2)+\left(\cos(2)^2+2i\,\cos(2)\sin(2)-\sin(2)^2+1\right)\mathrm{Ci}\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\cosh(bx+a)}\right)-(\cos}{}$$

```
[In] integrate(cos(tanh(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cos(2) + 4*I*b*x*sin(2) + (cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2)
^2 + 1)*cos_integral(2*(cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) - (co
s(2)^2 + 2*I*cos(2)*sin(2) - sin(2)^2 + 1)*cos_integral(4/(cosh(b*x + a)^2
+ 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + (-I*cos(2)^2 + 2*
cos(2)*sin(2) + I*sin(2)^2 + I)*sin_integral(2*(cosh(b*x + a) + sinh(b*x +
a))/cosh(b*x + a)) + (I*cos(2)^2 - 2*cos(2)*sin(2) - I*sin(2)^2 - I)*sin_in
tegral(4/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
 + 1)))/(b*cos(2) + I*b*sin(2))
```

Sympy [F]

$$\int \cos^2(\tanh(a+bx))\,dx = \int \cos^2\left(\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(cos(tanh(b*x+a))**2,x)
```

```
[Out] Integral(cos(tanh(a + b*x))**2, x)
```

Maxima [F]

$$\int \cos^2(\tanh(a+bx))\,dx = \int \cos\left(\tanh\left(bx+a\right)\right)^2\,dx$$

```
[In] integrate(cos(tanh(b*x+a))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*integrate(cos(2*(e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)), x
)
```

Giac [F]

$$\int \cos^2(\tanh(a+bx))\,dx = \int \cos\left(\tanh\left(bx+a\right)\right)^2\,dx$$

```
[In] integrate(cos(tanh(b*x+a))^2,x, algorithm="giac")
```

```
[Out] integrate(cos(tanh(b*x + a))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(\tanh(a+bx))\,dx = \int \cos\left(\tanh(a+b\,x)\right)^2\,dx$$

```
[In] int(cos(tanh(a + b*x))^2,x)
```

```
[Out] int(cos(tanh(a + b*x))^2, x)
```

3.246 $\int \cos(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \cos(\tanh(a+bx))\,dx = -\frac{\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{2b} + \frac{\cos(1)\,\text{CosIntegral}(1+\tanh(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\tanh(a+bx))}{2b} + \frac{\sin(1)\text{Si}(1+\tanh(a+bx))}{2b}$$

```
[Out] -1/2*Ci(1-tanh(b*x+a))*cos(1)/b+1/2*Ci(1+tanh(b*x+a))*cos(1)/b+1/2*Si(-1+ta
nh(b*x+a))*sin(1)/b+1/2*Si(1+tanh(b*x+a))*sin(1)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3415, 3384, 3380, 3383}

$$\int \cos(\tanh(a+bx))\,dx = -\frac{\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{2b} + \frac{\cos(1)\,\text{CosIntegral}(\tanh(a+bx)+1)}{2b} - \frac{\sin(1)\text{Si}(1-\tanh(a+bx))}{2b} + \frac{\sin(1)\text{Si}(\tanh(a+bx)+1)}{2b}$$

```
[In] Int[Cos[Tanh[a + b*x]],x]
```

```
[Out] -1/2*(Cos[1]*CosIntegral[1 - Tanh[a + b*x]])/b + (Cos[1]*CosIntegral[1 + Ta
nh[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Tanh[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Tanh[a + b*x]])/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{2(1-x)}+\frac{\cos(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b} \\
&= \frac{\cos(1)\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b} \\
&\quad+\frac{\cos(1)\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b} \\
&\quad+\frac{\sin(1)\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \tanh(a+bx)\right)}{2b} \\
&\quad+\frac{\sin(1)\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}
\end{aligned}$$

$$= -\frac{\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx))}{2b} + \frac{\cos(1)\,\text{CosIntegral}(1+\tanh(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\tanh(a+bx))}{2b} + \frac{\sin(1)\text{Si}(1+\tanh(a+bx))}{2b}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \cos(\tanh(a+bx))\,dx$$
$$= \frac{-\cos(1)\,\text{CosIntegral}(1-\tanh(a+bx)) + \cos(1)\,\text{CosIntegral}(1+\tanh(a+bx)) - \sin(1)\text{Si}(1-\tanh(a+}{2b}$$

```
[In] Integrate[Cos[Tanh[a + b*x]],x]
```

```
[Out] (-(Cos[1]*CosIntegral[1 - Tanh[a + b*x]]) + Cos[1]*CosIntegral[1 + Tanh[a +
 b*x]] - Sin[1]*SinIntegral[1 - Tanh[a + b*x]] + Sin[1]*SinIntegral[1 + Tan
h[a + b*x]])/(2*b)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\frac{\text{Si}(1+\tanh(bx+a))\sin(1)}{2}+\frac{\text{Ci}(1+\tanh(bx+a))\cos(1)}{2}+\frac{\text{Si}(-1+\tanh(bx+a))\sin(1)}{2}-\frac{\text{Ci}(-1+\tanh(bx+a))\cos(1)}{2}}{b}$	58
default	$\frac{\frac{\text{Si}(1+\tanh(bx+a))\sin(1)}{2}+\frac{\text{Ci}(1+\tanh(bx+a))\cos(1)}{2}+\frac{\text{Si}(-1+\tanh(bx+a))\sin(1)}{2}-\frac{\text{Ci}(-1+\tanh(bx+a))\cos(1)}{2}}{b}$	58
risch	$-\frac{e^{i}\,\text{Ei}_1\left(-\frac{2i}{1+e^{2bx+2a}}+2i\right)}{4b}+\frac{e^{i}\,\text{Ei}_1\left(\frac{2i}{1+e^{2bx+2a}}\right)}{4b}+\frac{e^{-i}\,\text{Ei}_1\left(-\frac{2i}{1+e^{2bx+2a}}\right)}{4b}-\frac{e^{-i}\,\text{Ei}_1\left(\frac{2i}{1+e^{2bx+2a}}-2i\right)}{4b}$	112

```
[In] int(cos(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*Si(1+tanh(b*x+a))*sin(1)+1/2*Ci(1+tanh(b*x+a))*cos(1)+1/2*Si(-1+ta
nh(b*x+a))*sin(1)-1/2*Ci(-1+tanh(b*x+a))*cos(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.81

$$\int \cos(\tanh(a+bx))\,dx$$
$$= \frac{\left(\cos(1)^2 + 2i\cos(1)\sin(1) - \sin(1)^2 + 1\right)\operatorname{Ci}\left(\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)}\right) - \left(\cos(1)^2 + 2i\cos(1)\sin(1) - \sin(1\right.}{}$$

```
[In] integrate(cos(tanh(b*x+a)),x, algorithm="fricas")
[Out] 1/4*((cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*cos_integral((cosh(b*x +
 a) + sinh(b*x + a))/cosh(b*x + a)) - (cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1
)^2 + 1)*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 + 1)) + (-I*cos(1)^2 + 2*cos(1)*sin(1) + I*sin(1)^2 + I)*si
n_integral((cosh(b*x + a) + sinh(b*x + a))/cosh(b*x + a)) + (I*cos(1)^2 - 2
*cos(1)*sin(1) - I*sin(1)^2 - I)*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)))/(b*cos(1) + I*b*sin(1))
```

Sympy [F]

$$\int \cos(\tanh(a+bx))\,dx = \int \cos(\tanh(a+bx))\,dx$$

```
[In] integrate(cos(tanh(b*x+a)),x)
[Out] Integral(cos(tanh(a + b*x)), x)
```

Maxima [F]

$$\int \cos(\tanh(a+bx))\,dx = \int \cos(\tanh(bx+a))\,dx$$

```
[In] integrate(cos(tanh(b*x+a)),x, algorithm="maxima")
[Out] integrate(cos(tanh(b*x + a)), x)
```

Giac [F]

$$\int \cos(\tanh(a+bx))\,dx = \int \cos\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(cos(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(cos(tanh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(\tanh(a+bx))\,dx = \int \cos\left(\tanh(a+b\,x)\right)\,dx$$

```
[In] int(cos(tanh(a + b*x)),x)
[Out] int(cos(tanh(a + b*x)), x)
```

3.247 $\int \sec(\tanh(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \sec(\tanh(a+bx))\,dx = -\frac{1}{2}\text{Int}\left(\frac{\sec(\tanh(a+bx))\text{sech}^2(a+bx)}{-1+\tanh(a+bx)},x\right) + \frac{1}{2}\text{Int}\left(\frac{\sec(\tanh(a+bx))\text{sech}^2(a+bx)}{1+\tanh(a+bx)},x\right)$$

```
[Out] -1/2*Unintegrable(sec(tanh(b*x+a))*sech(b*x+a)^2/(-1+tanh(b*x+a)),x)+1/2*Un
integrable(sec(tanh(b*x+a))*sech(b*x+a)^2/(1+tanh(b*x+a)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec(\tanh(a+bx))\,dx = \int \sec(\tanh(a+bx))\,dx$$

```
[In] Int[Sec[Tanh[a + b*x]],x]
```

```
[Out] -1/2*Defer[Subst][Defer[Int][Sec[x]/(-1 + x), x], x, Tanh[a + b*x]]/b + Def
er[Subst][Defer[Int][Sec[x]/(1 + x), x], x, Tanh[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sec(x)}{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{\sec(x)}{2(-1+x)} + \frac{\sec(x)}{2(1+x)}\right)\,dx, x, \tanh(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \frac{\sec(x)}{-1+x}\,dx, x, \tanh(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sec(x)}{1+x}\,dx, x, \tanh(a+bx)\right)}{2b}\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 5.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\tanh(a+bx))\,dx = \int \sec(\tanh(a+bx))\,dx$$

```
[In] Integrate[Sec[Tanh[a + b*x]],x]
[Out] Integrate[Sec[Tanh[a + b*x]], x]
```

Maple [F(-1)]

Timed out.

$$\int \sec(\tanh(bx+a))\,dx$$

```
[In] int(sec(tanh(b*x+a)),x)
[Out] int(sec(tanh(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\tanh(a+bx))\,dx = \int \sec(\tanh(bx+a))\,dx$$

```
[In] integrate(sec(tanh(b*x+a)),x, algorithm="fricas")
[Out] integral(sec(tanh(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \sec(\tanh(a+bx))\,dx = \int \sec\left(\tanh\left(a+bx\right)\right)dx$$

```
[In] integrate(sec(tanh(b*x+a)),x)
[Out] Integral(sec(tanh(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\tanh(a+bx))\,dx = \int \sec\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sec(tanh(b*x+a)),x, algorithm="maxima")
[Out] integrate(sec(tanh(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\tanh(a+bx))\,dx = \int \sec\left(\tanh\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sec(tanh(b*x+a)),x, algorithm="giac")
[Out] integrate(sec(tanh(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \sec(\tanh(a+bx))\,dx = \int \frac{1}{\cos\left(\tanh\left(a+b\,x\right)\right)}\,dx$$

```
[In] int(1/cos(tanh(a + b*x)),x)
[Out] int(1/cos(tanh(a + b*x)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                 end if
             else #result contains complex but optimal is not
                 if debug then
                         print("result contains complex but optimal is not");
                 fi;
                 return "C","Result contains complex when optimal does not.";
             fi;
         else # result do not contain complex
              # this assumes optimal do not as well. No check is needed here.
             if debug then
                    print("result do not contain complex, this assumes optimal do not as well")
             fi;
             if leaf_count_result<=2*leaf_count_optimal then
                 if debug then
                     print("leaf_count_result<=2*leaf_count_optimal");
                 fi;
                 return "A"," ";
             else
                 if debug then
                     print("leaf_count_result>2*leaf_count_optimal");
                 fi;
                 return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
             fi;
         fi;
     else #ExpnType(result) > ExpnType(optimal)
         if debug then
             print("ExpnType(result) > ExpnType(optimal)");
         fi;
         return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
     fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
   #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

   return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.a
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```