Computer Algebra
Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.4-Hyperbolic-cotangent/175-6.4.2-Hyperbolic-cotangent-functions

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [224]. This is test number [175].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (224)	0.00 (0)
Mathematica	100.00 (224)	0.00 (0)
Fricas	79.91 (179)	20.09 (45)
Maple	73.66 (165)	26.34 (59)
Mupad	58.48 (131)	41.52 (93)
Giac	58.48 (131)	41.52 (93)
Maxima	46.88 (105)	53.12 (119)
Sympy	15.18 (34)	84.82 (190)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

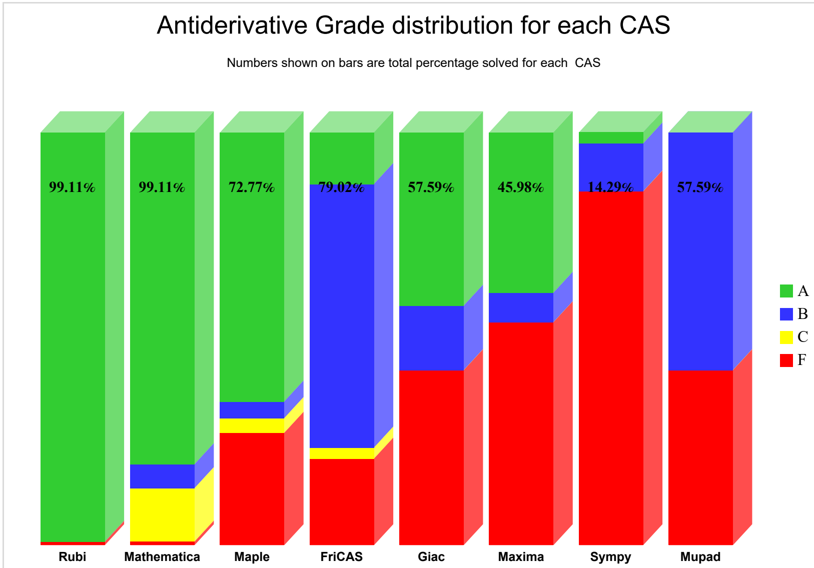
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

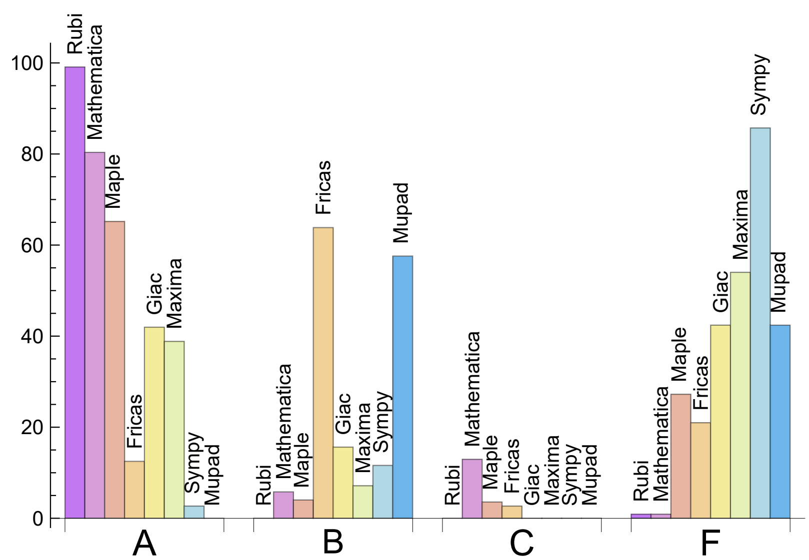
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.107	0.000	0.000	0.893
Mathematica	80.357	5.804	12.946	0.893
Maple	65.179	4.018	3.571	27.232
Giac	41.964	15.625	0.000	42.411
Maxima	38.839	7.143	0.000	54.018
Fricas	12.500	63.839	2.679	20.982
Sympy	2.679	11.607	0.000	85.714
Mupad	0.000	57.589	0.000	42.411

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	45	77.78	0.00	22.22
Maple	59	100.00	0.00	0.00
Mupad	93	0.00	100.00	0.00
Giac	93	77.42	9.68	12.90
Maxima	119	93.28	0.00	6.72
Sympy	190	90.53	6.32	3.16

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.09
Maxima	0.23
Giac	0.30
Fricas	0.30
Maple	0.53
Mathematica	1.20
Mupad	1.80
Sympy	3.21

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	69.42	1.12	55.00	0.95
Mupad	72.60	1.31	42.00	0.96
Maxima	82.47	1.54	47.00	1.13
Rubi	85.48	1.00	60.00	1.00
Giac	88.30	1.54	62.00	1.26
Mathematica	92.87	1.25	62.50	1.00
Sympy	211.44	4.00	130.00	3.62
Fricas	1016.08	8.88	287.00	4.94

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

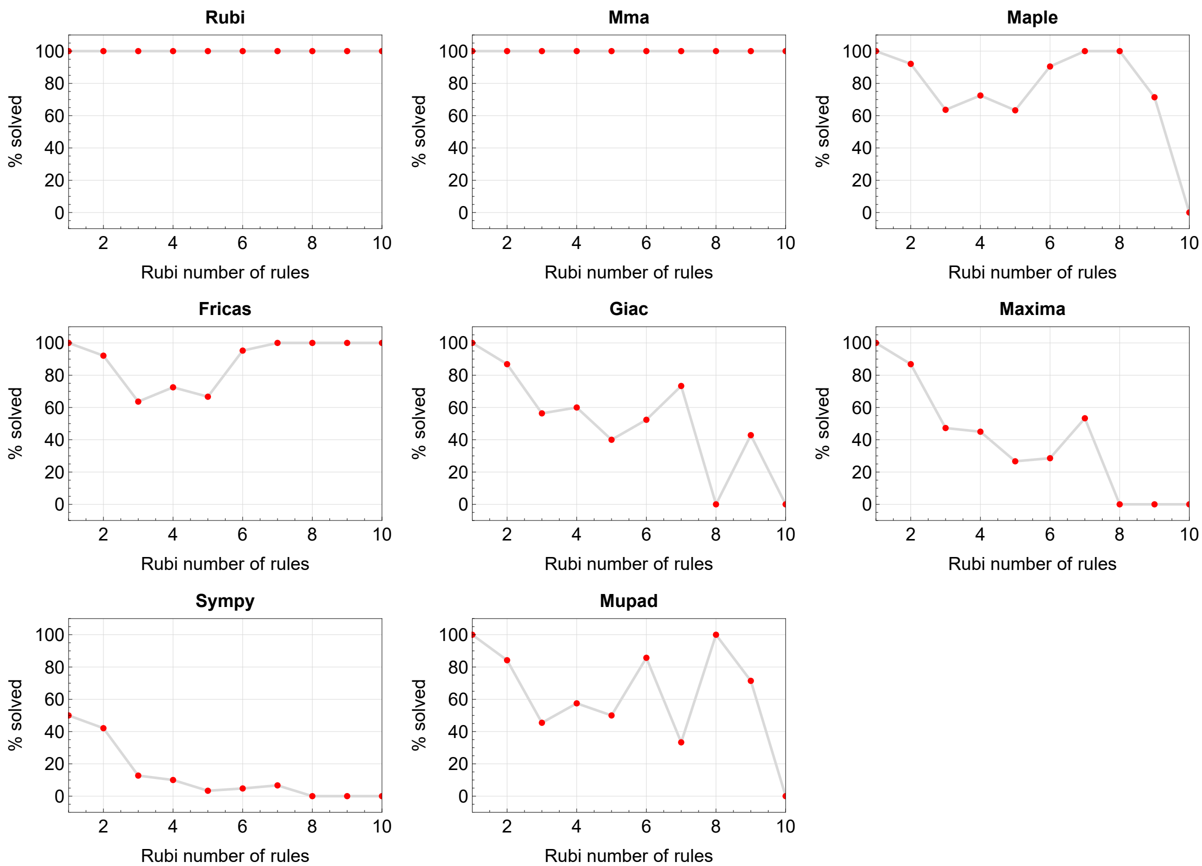


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

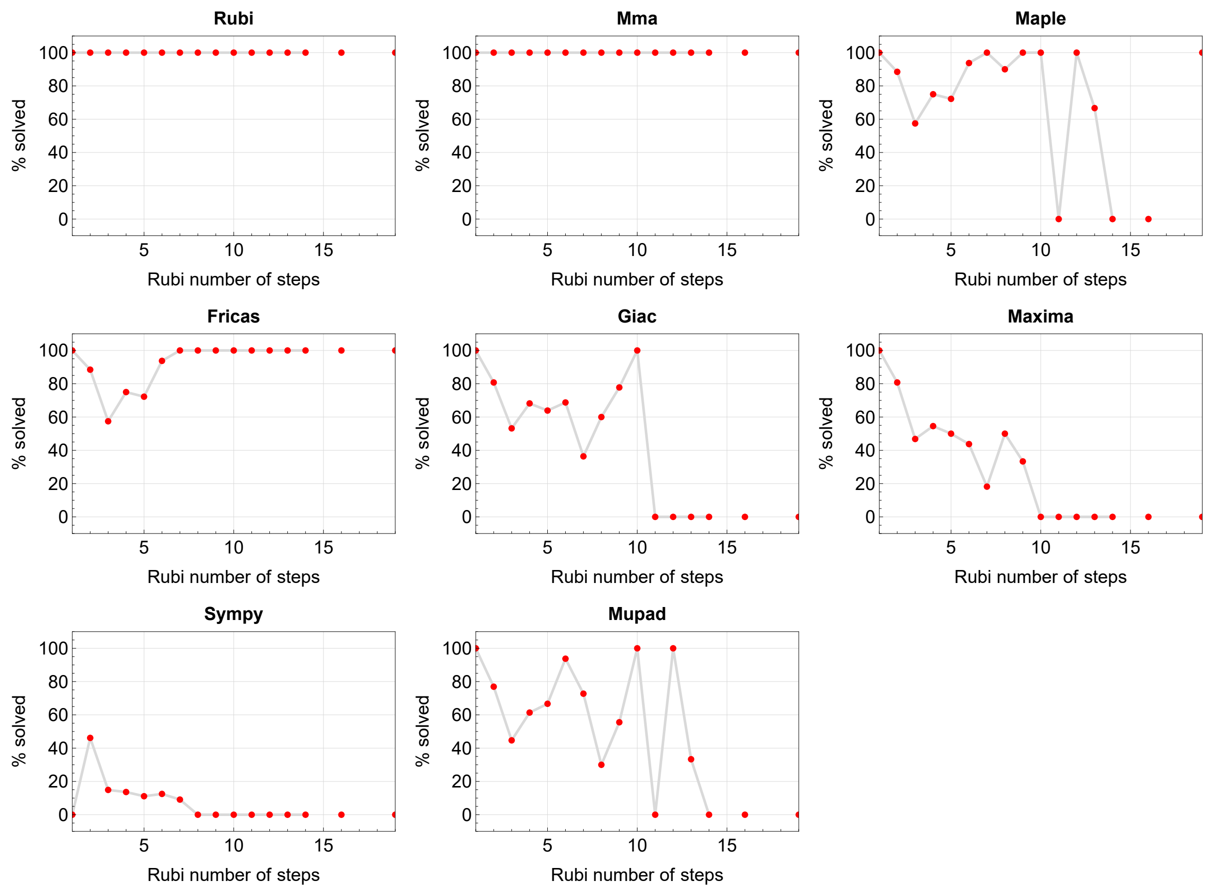


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

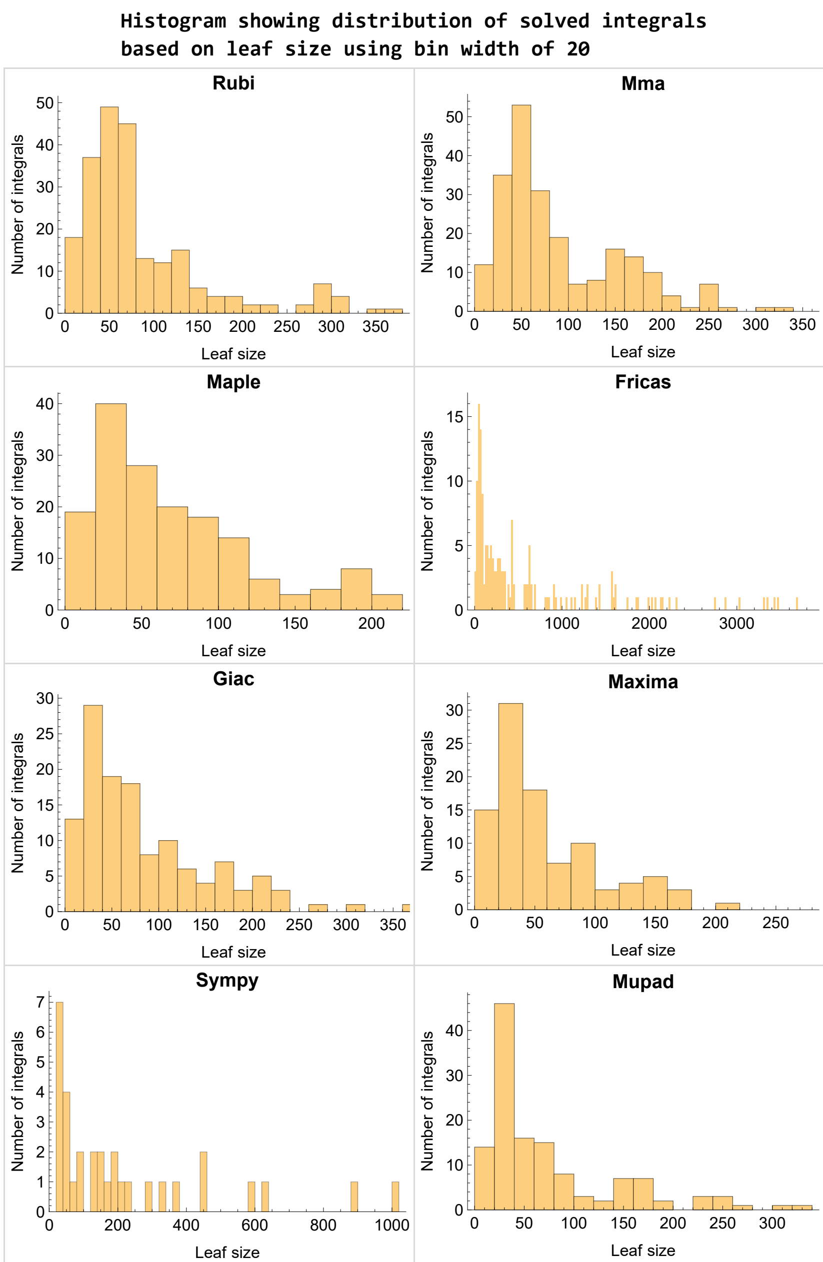


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

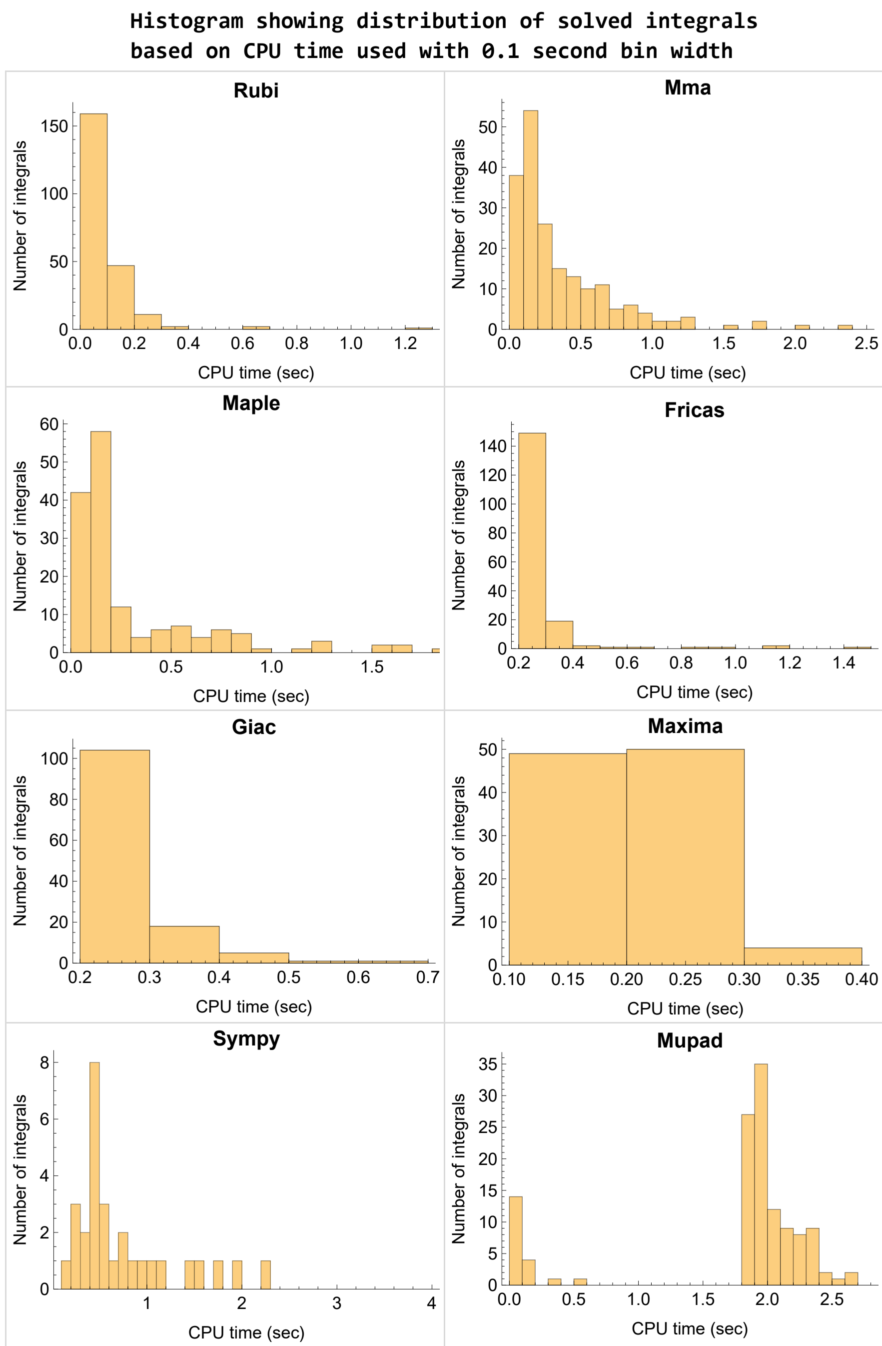


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

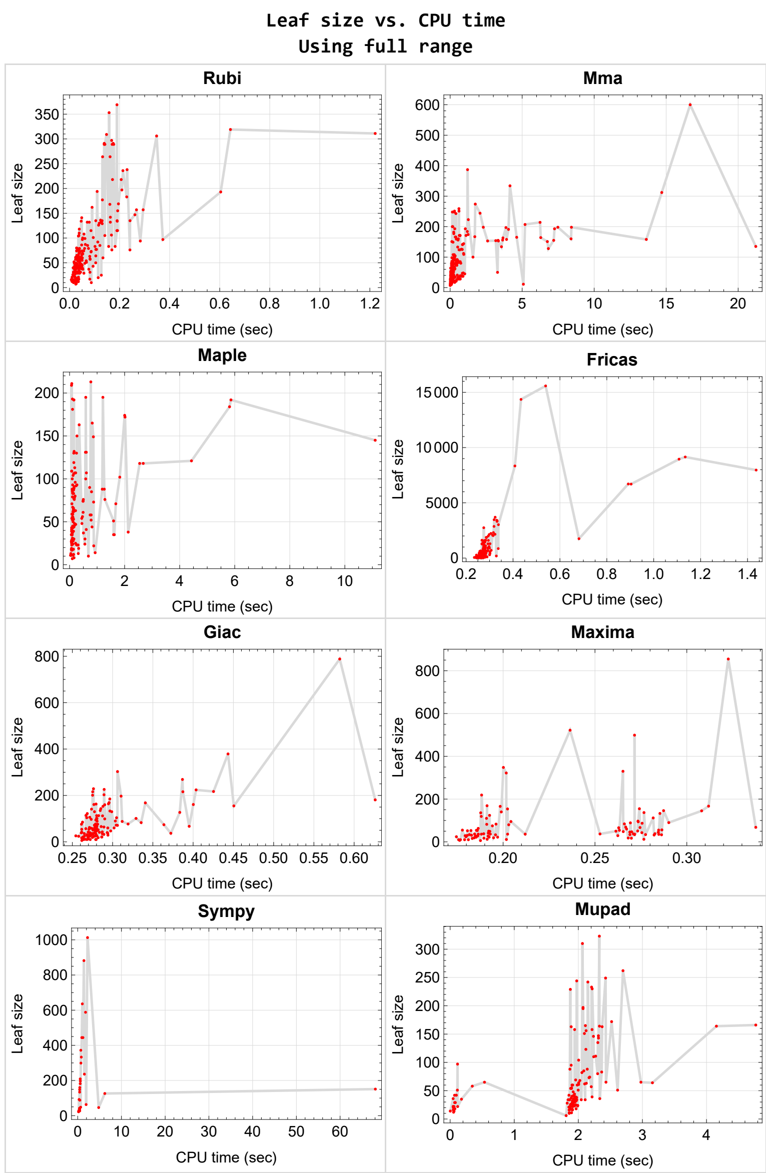


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{220, 224}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {168, 169, 172, 173, 174, 175, 176, 197, 198}

Maple {211, 212, 214, 215, 216}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

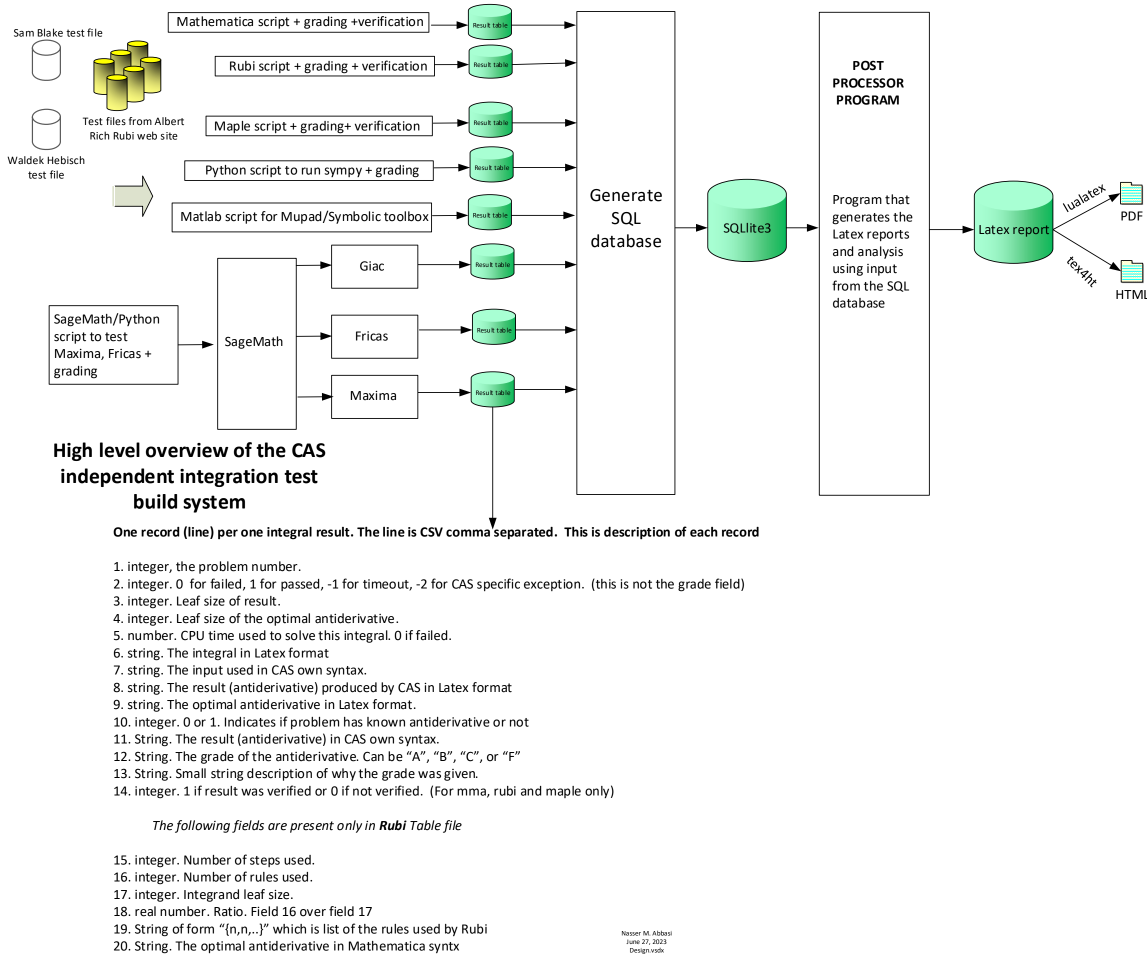
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 153, 155, 157, 158, 165, 166, 167, 169, 170, 171, 172, 173, 181, 184, 185, 186, 187, 189, 190, 191, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

B grade { 95, 151, 168, 174, 175, 176, 177, 178, 179, 180, 182, 183, 197 }

C grade { 33, 34, 40, 41, 61, 62, 63, 64, 74, 75, 76, 87, 113, 129, 130, 131, 135, 152, 154, 156, 159, 160, 161, 162, 163, 164, 188, 192, 212 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 19, 20, 21, 29, 30, 31, 32, 33, 37, 38, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 155, 157, 158, 159, 160, 161, 162, 164, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 210, 217, 218, 219, 221, 222, 223 }

B grade { 34, 35, 36, 95, 104, 113, 152, 154, 213 }

C grade { 111, 156, 163, 211, 212, 214, 215, 216 }

F normal fail { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 208, 209 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 9, 10, 46, 81, 85, 86, 91, 92, 93, 101, 107, 108, 118, 122, 143, 144, 145, 146, 151, 152, 153, 155, 156, 158, 163, 164, 213, 214 }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 47, 48, 49, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 82, 83, 84, 87, 88, 89, 90, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 147, 148, 149, 150, 154, 157, 159, 160, 161, 162, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 215, 216 }

C grade { 217, 218, 219, 221, 222, 223 }

F normal fail { 15, 16, 17, 28, 39, 50, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198 }

F(-1) timedout fail { }

F(-2) exception fail { 51, 52, 53, 54, 55, 56, 57, 58, 59, 60 }

Maxima

A grade { 18, 19, 20, 21, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 64, 65, 66, 67, 68, 69, 80, 81, 82, 85, 86, 89, 90, 91, 92, 93, 94, 97, 99, 102, 105, 106, 107, 108, 109, 110, 114, 116, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 211, 212, 213, 214, 215, 216 }

B grade { 61, 62, 63, 77, 78, 79, 83, 84, 95, 96, 104, 111, 112, 191, 192, 193 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 70, 71, 72, 73, 74, 75, 76, 87, 88, 113, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 217, 218, 219, 221, 222, 223 }

F(-1) timedout fail { }

F(-2) exception fail { 98, 100, 101, 103, 115, 117, 118, 120 }

Giac

A grade { 18, 19, 21, 40, 41, 42, 43, 61, 62, 63, 64, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 188, 192, 211, 212, 213, 214, 215, 216 }

B grade { 1, 2, 3, 4, 11, 12, 20, 29, 30, 70, 71, 72, 73, 74, 75, 76, 95, 102, 104, 113, 119, 121, 132, 133, 134, 135, 136, 137, 138, 139, 150, 155, 181, 191, 193 }

C grade { }

F normal fail { 15, 16, 17, 22, 23, 24, 25, 26, 27, 28, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 205, 206, 210, 217, 218, 219, 221, 222, 223 }

F(-1) timedout fail { 199, 200, 201, 202, 203, 204, 207, 208, 209 }

F(-2) exception fail { 5, 6, 7, 8, 9, 10, 13, 14, 31, 32, 87, 88 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 181, 188, 191, 192, 193, 199, 200, 201, 202, 203, 204 }

C grade { }

F normal fail { }

F(-1) timedout fail { 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 113, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 189, 190, 194, 195, 196, 197, 198, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223 }

F(-2) exception fail { }

Sympy

A grade { 61, 62, 63, 64, 85, 86 }

B grade { 33, 34, 65, 66, 67, 68, 69, 77, 78, 79, 80, 81, 127, 128, 129, 130, 131, 144, 145, 146, 147, 148, 149, 155, 162, 181 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 71, 72, 73, 74, 75, 76, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 201, 202, 203, 205, 206, 207, 208, 209, 210, 213, 214, 215, 217, 218, 219, 221, 222, 223 }

F(-1) timedout fail { 1, 55, 70, 189, 190, 198, 199, 200, 204, 211, 212, 216 }

F(-2) exception fail { 82, 83, 84, 191, 192, 193 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	80	80	0	1574	0	379	83
N.S.	1	1.00	0.82	0.82	0.00	16.23	0.00	3.91	0.86
time (sec)	N/A	0.051	0.388	0.200	0.000	0.282	0.000	0.443	2.364

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	63	0	988	0	224	62
N.S.	1	1.00	0.85	0.81	0.00	12.67	0.00	2.87	0.79
time (sec)	N/A	0.037	0.169	0.155	0.000	0.274	0.000	0.404	2.098

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	62	0	637	0	168	61
N.S.	1	1.00	0.88	0.83	0.00	8.49	0.00	2.24	0.81
time (sec)	N/A	0.036	0.079	0.123	0.000	0.272	0.000	0.341	2.005

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	52	47	0	594	0	101	41
N.S.	1	1.00	0.90	0.81	0.00	10.24	0.00	1.74	0.71
time (sec)	N/A	0.025	0.043	0.197	0.000	0.276	0.000	0.329	1.919

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	46	0	598	0	0	38
N.S.	1	1.00	0.86	0.81	0.00	10.49	0.00	0.00	0.67
time (sec)	N/A	0.024	0.041	0.184	0.000	0.287	0.000	0.000	1.974

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	74	65	0	923	0	0	64
N.S.	1	1.00	0.95	0.83	0.00	11.83	0.00	0.00	0.82
time (sec)	N/A	0.039	0.093	0.142	0.000	0.282	0.000	0.000	2.006

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	64	0	1428	0	0	63
N.S.	1	1.00	0.99	0.81	0.00	18.08	0.00	0.00	0.80
time (sec)	N/A	0.040	0.118	0.140	0.000	0.277	0.000	0.000	2.125

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	90	83	0	2132	0	0	80
N.S.	1	1.00	0.90	0.83	0.00	21.32	0.00	0.00	0.80
time (sec)	N/A	0.052	0.283	0.146	0.000	0.298	0.000	0.000	2.301

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	193	209	0	292	0	0	249
N.S.	1	1.00	0.82	0.89	0.00	1.24	0.00	0.00	1.06
time (sec)	N/A	0.213	0.319	0.070	0.000	0.300	0.000	0.000	2.425

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	149	181	0	310	0	0	233
N.S.	1	1.00	0.68	0.83	0.00	1.42	0.00	0.00	1.07
time (sec)	N/A	0.206	0.199	0.108	0.000	0.265	0.000	0.000	2.207

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	108	109	0	291	0	217	146
N.S.	1	1.00	0.82	0.83	0.00	2.20	0.00	1.64	1.11
time (sec)	N/A	0.079	0.204	0.080	0.000	0.267	0.000	0.425	2.236

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	98	109	0	1598	0	216	147
N.S.	1	1.00	0.74	0.83	0.00	12.11	0.00	1.64	1.11
time (sec)	N/A	0.071	0.123	0.074	0.000	0.295	0.000	0.387	2.313

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	177	193	0	356	0	0	197
N.S.	1	1.00	0.81	0.89	0.00	1.63	0.00	0.00	0.90
time (sec)	N/A	0.172	0.254	0.104	0.000	0.261	0.000	0.000	2.074

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	243	211	0	3348	0	0	165
N.S.	1	1.00	1.02	0.89	0.00	14.07	0.00	0.00	0.69
time (sec)	N/A	0.230	0.401	0.080	0.000	0.332	0.000	0.000	2.110

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	0.133	0.000	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	49	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	49	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.111	0.000	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	53	97	823	0	90	0
N.S.	1	1.00	0.92	0.87	1.59	13.49	0.00	1.48	0.00
time (sec)	N/A	0.028	0.134	0.145	0.273	0.280	0.000	0.303	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	45	54	125	0	54	0
N.S.	1	1.00	1.26	1.45	1.74	4.03	0.00	1.74	0.00
time (sec)	N/A	0.016	0.049	0.111	0.263	0.287	0.000	0.278	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	52	34	128	0	60	30
N.S.	1	1.00	1.00	1.68	1.10	4.13	0.00	1.94	0.97
time (sec)	N/A	0.016	0.068	0.107	0.269	0.257	0.000	0.294	1.947

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	48	79	84	817	0	104	0
N.S.	1	1.00	0.74	1.22	1.29	12.57	0.00	1.60	0.00
time (sec)	N/A	0.029	0.152	0.103	0.266	0.272	0.000	0.305	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	166	0	0	1994	0	0	0
N.S.	1	1.00	0.56	0.00	0.00	6.71	0.00	0.00	0.00
time (sec)	N/A	0.168	0.357	0.000	0.000	0.294	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	200	0	0	2037	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	7.05	0.00	0.00	0.00
time (sec)	N/A	0.138	0.151	0.000	0.000	0.315	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	151	0	0	1618	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	6.13	0.00	0.00	0.00
time (sec)	N/A	0.162	0.117	0.000	0.000	0.283	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	182	0	0	8338	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	31.58	0.00	0.00	0.00
time (sec)	N/A	0.132	0.248	0.000	0.000	0.408	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	248	0	0	2066	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	7.15	0.00	0.00	0.00
time (sec)	N/A	0.170	0.279	0.000	0.000	0.313	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	309	309	251	0	0	14359	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	46.47	0.00	0.00	0.00
time (sec)	N/A	0.148	0.614	0.000	0.000	0.434	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	82	107	0	2152	0	788	0
N.S.	1	1.00	0.61	0.80	0.00	16.06	0.00	5.88	0.00
time (sec)	N/A	0.046	0.556	0.193	0.000	0.323	0.000	0.582	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	63	89	0	633	0	269	0
N.S.	1	1.00	0.61	0.86	0.00	6.09	0.00	2.59	0.00
time (sec)	N/A	0.035	0.094	0.161	0.000	0.289	0.000	0.387	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	80	91	0	907	0	0	0
N.S.	1	1.00	0.76	0.87	0.00	8.64	0.00	0.00	0.00
time (sec)	N/A	0.037	0.089	0.170	0.000	0.300	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	90	106	0	3022	0	0	0
N.S.	1	1.00	0.64	0.75	0.00	21.43	0.00	0.00	0.00
time (sec)	N/A	0.049	0.217	0.150	0.000	0.338	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	43	130	87	1046	151	0	0
N.S.	1	1.00	0.58	1.76	1.18	14.14	2.04	0.00	0.00
time (sec)	N/A	0.028	0.083	0.153	0.276	0.298	68.065	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	119	34	392	126	0	0
N.S.	1	1.00	0.82	2.38	0.68	7.84	2.52	0.00	0.00
time (sec)	N/A	0.019	0.031	0.154	0.278	0.264	6.192	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	192	51	148	0	0	0
N.S.	1	1.00	1.26	6.19	1.65	4.77	0.00	0.00	0.00
time (sec)	N/A	0.015	0.032	0.154	0.273	0.264	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	192	32	187	0	0	0
N.S.	1	1.00	1.00	6.19	1.03	6.03	0.00	0.00	0.00
time (sec)	N/A	0.016	0.034	0.157	0.282	0.255	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	40	89	37	287	0	0	0
N.S.	1	1.00	0.80	1.78	0.74	5.74	0.00	0.00	0.00
time (sec)	N/A	0.020	0.072	0.147	0.271	0.262	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	51	132	89	1579	0	0	0
N.S.	1	1.00	0.64	1.65	1.11	19.74	0.00	0.00	0.00
time (sec)	N/A	0.028	0.097	0.165	0.269	0.278	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	53	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.112	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	43	77	137	3421	0	77	0
N.S.	1	1.00	0.39	0.70	1.25	31.10	0.00	0.70	0.00
time (sec)	N/A	0.038	0.073	0.156	0.277	0.331	0.000	0.319	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	41	55	34	415	0	27	0
N.S.	1	1.00	0.82	1.10	0.68	8.30	0.00	0.54	0.00
time (sec)	N/A	0.021	0.037	0.113	0.272	0.258	0.000	0.269	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	40	59	36	422	0	32	0
N.S.	1	1.00	0.80	1.18	0.72	8.44	0.00	0.64	0.00
time (sec)	N/A	0.019	0.066	0.116	0.272	0.275	0.000	0.289	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	68	84	155	3473	0	83	0
N.S.	1	1.00	0.58	0.71	1.31	29.43	0.00	0.70	0.00
time (sec)	N/A	0.039	0.253	0.109	0.274	0.319	0.000	0.335	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	353	353	244	0	0	2864	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	8.11	0.00	0.00	0.00
time (sec)	N/A	0.158	2.076	0.000	0.000	0.319	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	166	0	0	618	0	0	0
N.S.	1	1.00	0.57	0.00	0.00	2.12	0.00	0.00	0.00
time (sec)	N/A	0.175	0.396	0.000	0.000	0.278	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	200	0	0	288	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.182	0.000	0.000	0.272	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	289	289	248	0	0	3316	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	11.47	0.00	0.00	0.00
time (sec)	N/A	0.177	0.154	0.000	0.000	0.332	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	291	291	251	0	0	1159	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	0.138	0.302	0.000	0.000	0.278	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	369	369	274	0	0	15579	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	42.22	0.00	0.00	0.00
time (sec)	N/A	0.189	1.745	0.000	0.000	0.539	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	59	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.130	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.104	0.000	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.115	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	59	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.038	0.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	61	0	0	0	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.127	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.036	0.113	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.033	0.105	0.000	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	58	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.034	0.116	0.000	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	60	61	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.124	0.000	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	61	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	94	31	140	448	48	41	88
N.S.	1	1.00	2.29	0.76	3.41	10.93	1.17	1.00	2.15
time (sec)	N/A	0.032	0.270	0.105	0.199	0.255	0.669	0.284	1.868

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	84	25	95	273	37	35	60
N.S.	1	1.00	2.71	0.81	3.06	8.81	1.19	1.13	1.94
time (sec)	N/A	0.023	0.198	0.077	0.204	0.244	0.453	0.279	1.902

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	61	19	55	142	31	29	36
N.S.	1	1.00	2.65	0.83	2.39	6.17	1.35	1.26	1.57
time (sec)	N/A	0.017	0.161	0.073	0.178	0.244	0.355	0.269	0.047

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	28	13	19	53	22	21	20
N.S.	1	1.00	2.15	1.00	1.46	4.08	1.69	1.62	1.54
time (sec)	N/A	0.009	0.015	0.047	0.184	0.235	0.193	0.286	0.050

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	11	10	26	27	10	14
N.S.	1	1.00	0.88	0.69	0.62	1.62	1.69	0.62	0.88
time (sec)	N/A	0.007	0.115	0.050	0.181	0.244	0.258	0.264	0.061

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	28	17	16	52	88	18	16
N.S.	1	1.00	1.08	0.65	0.62	2.00	3.38	0.69	0.62
time (sec)	N/A	0.013	0.701	0.074	0.183	0.250	0.484	0.264	0.049

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	23	22	86	182	24	22
N.S.	1	1.00	0.92	0.64	0.61	2.39	5.06	0.67	0.61
time (sec)	N/A	0.022	0.292	0.079	0.185	0.242	0.567	0.277	0.061

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	35	29	28	121	299	30	28
N.S.	1	1.00	0.76	0.63	0.61	2.63	6.50	0.65	0.61
time (sec)	N/A	0.029	0.352	0.084	0.186	0.257	0.733	0.274	1.826

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	41	35	34	159	444	36	34
N.S.	1	1.00	0.73	0.62	0.61	2.84	7.93	0.64	0.61
time (sec)	N/A	0.039	0.496	0.096	0.182	0.265	0.929	0.274	1.865

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	47	43	0	438	0	160	44
N.S.	1	1.00	0.82	0.75	0.00	7.68	0.00	2.81	0.77
time (sec)	N/A	0.036	0.848	0.119	0.000	0.263	0.000	0.290	1.941

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	0	259	0	112	54
N.S.	1	1.00	0.87	0.78	0.00	5.76	0.00	2.49	1.20
time (sec)	N/A	0.026	0.705	0.086	0.000	0.265	0.000	0.288	1.880

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	0	131	0	63	26
N.S.	1	1.00	1.00	0.82	0.00	3.97	0.00	1.91	0.79
time (sec)	N/A	0.020	0.553	0.063	0.000	0.255	0.000	0.278	1.881

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	0	50	0	37	16
N.S.	1	1.00	1.00	0.81	0.00	2.38	0.00	1.76	0.76
time (sec)	N/A	0.010	0.418	0.093	0.000	0.255	0.000	0.276	1.909

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	26	27	0	85	0	66	26
N.S.	1	1.00	0.81	0.84	0.00	2.66	0.00	2.06	0.81
time (sec)	N/A	0.018	0.450	0.084	0.000	0.260	0.000	0.286	1.894

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	35	0	168	0	113	32
N.S.	1	1.00	0.57	0.71	0.00	3.43	0.00	2.31	0.65
time (sec)	N/A	0.027	0.557	0.077	0.000	0.266	0.000	0.280	1.907

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	28	43	0	266	0	161	40
N.S.	1	1.00	0.46	0.70	0.00	4.36	0.00	2.64	0.66
time (sec)	N/A	0.036	0.629	0.075	0.000	0.265	0.000	0.277	1.899

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	141	150	348	2748	588	226	244
N.S.	1	1.00	0.99	1.06	2.45	19.35	4.14	1.59	1.72
time (sec)	N/A	0.163	0.852	0.269	0.200	0.275	1.792	0.290	1.975

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	109	113	219	1396	444	153	158
N.S.	1	1.00	1.08	1.12	2.17	13.82	4.40	1.51	1.56
time (sec)	N/A	0.095	0.946	0.187	0.188	0.272	1.198	0.288	1.942

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	86	87	136	654	333	99	97
N.S.	1	1.00	1.25	1.26	1.97	9.48	4.83	1.43	1.41
time (sec)	N/A	0.051	0.425	0.146	0.188	0.262	0.830	0.290	0.115

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	65	59	49	205	209	57	51
N.S.	1	1.00	1.71	1.55	1.29	5.39	5.50	1.50	1.34
time (sec)	N/A	0.018	0.132	0.085	0.193	0.258	0.574	0.280	0.113

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	64	50	52	62	236	62	55
N.S.	1	1.00	1.28	1.00	1.04	1.24	4.72	1.24	1.10
time (sec)	N/A	0.041	0.112	0.126	0.181	0.249	1.527	0.271	1.894

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	100	93	124	426	0	130	104
N.S.	1	1.00	1.18	1.09	1.46	5.01	0.00	1.53	1.22
time (sec)	N/A	0.072	1.585	0.112	0.193	0.264	0.000	0.299	2.005

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	134	130	322	1431	0	203	195
N.S.	1	1.00	1.04	1.01	2.50	11.09	0.00	1.57	1.51
time (sec)	N/A	0.130	3.563	0.252	0.202	0.280	0.000	0.289	2.074

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	214	163	522	3698	0	303	310
N.S.	1	1.00	1.27	0.96	3.09	21.88	0.00	1.79	1.83
time (sec)	N/A	0.195	6.244	0.349	0.236	0.324	0.000	0.306	2.064

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	28	28	49	42	29	22
N.S.	1	1.00	1.71	0.90	0.90	1.58	1.35	0.94	0.71
time (sec)	N/A	0.032	0.045	0.080	0.185	0.266	0.414	0.294	0.047

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	53	28	29	48	42	24	22
N.S.	1	1.00	1.71	0.90	0.94	1.55	1.35	0.77	0.71
time (sec)	N/A	0.032	0.043	0.072	0.179	0.255	0.425	0.258	0.047

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	128	63	0	2231	0	0	151
N.S.	1	1.00	1.73	0.85	0.00	30.15	0.00	0.00	2.04
time (sec)	N/A	0.057	6.820	0.217	0.000	0.306	0.000	0.000	2.095

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	62	0	2307	0	0	242
N.S.	1	1.00	1.00	0.84	0.00	31.18	0.00	0.00	3.27
time (sec)	N/A	0.050	0.175	0.154	0.000	0.316	0.000	0.000	2.149

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	35	36	93	0	42	34
N.S.	1	1.00	0.70	0.58	0.60	1.55	0.00	0.70	0.57
time (sec)	N/A	0.045	0.238	1.625	0.193	0.244	0.000	0.270	2.046

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	36	30	33	60	0	31	29
N.S.	1	1.00	1.24	1.03	1.14	2.07	0.00	1.07	1.00
time (sec)	N/A	0.033	0.233	0.560	0.196	0.250	0.000	0.268	1.928

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	30	23	22	50	0	30	22
N.S.	1	1.00	0.79	0.61	0.58	1.32	0.00	0.79	0.58
time (sec)	N/A	0.036	0.191	0.267	0.197	0.247	0.000	0.268	1.913

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	17	25	0	19	17
N.S.	1	1.00	1.11	0.95	0.89	1.32	0.00	1.00	0.89
time (sec)	N/A	0.024	0.154	0.141	0.190	0.246	0.000	0.275	1.883

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	6	9	0	6	6
N.S.	1	1.00	0.70	0.70	0.60	0.90	0.00	0.60	0.60
time (sec)	N/A	0.016	0.004	0.099	0.188	0.263	0.000	0.262	1.810

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	10	8	7	18	0	12	11
N.S.	1	1.00	1.43	1.14	1.00	2.57	0.00	1.71	1.57
time (sec)	N/A	0.025	0.010	0.161	0.176	0.247	0.000	0.269	1.858

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	21	23	31	77	0	26	29
N.S.	1	1.00	2.62	2.88	3.88	9.62	0.00	3.25	3.62
time (sec)	N/A	0.027	0.322	0.309	0.183	0.255	0.000	0.279	0.081

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	41	55	0	10	16
N.S.	1	1.00	1.00	0.91	3.73	5.00	0.00	0.91	1.45
time (sec)	N/A	0.026	5.085	0.684	0.192	0.241	0.000	0.272	1.856

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	156	192	166	1279	0	229	143
N.S.	1	1.00	1.01	1.24	1.07	8.25	0.00	1.48	0.92
time (sec)	N/A	0.192	0.548	5.862	0.198	0.267	0.000	0.276	2.315

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	171	172	0	1859	0	163	172
N.S.	1	1.00	1.28	1.28	0.00	13.87	0.00	1.22	1.28
time (sec)	N/A	0.191	1.101	2.014	0.000	0.287	0.000	0.279	2.519

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	75	107	83	331	0	114	85
N.S.	1	1.00	0.82	1.16	0.90	3.60	0.00	1.24	0.92
time (sec)	N/A	0.113	0.458	0.611	0.196	0.259	0.000	0.267	2.085

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	93	0	431	0	72	156
N.S.	1	1.00	1.10	1.27	0.00	5.90	0.00	0.99	2.14
time (sec)	N/A	0.086	0.738	0.266	0.000	0.285	0.000	0.304	2.121

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	46	39	0	147	0	35	35
N.S.	1	1.00	1.21	1.03	0.00	3.87	0.00	0.92	0.92
time (sec)	N/A	0.027	0.046	0.142	0.000	0.252	0.000	0.287	0.177

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	43	0	46	51
N.S.	1	1.00	1.67	1.08	1.00	3.58	0.00	3.83	4.25
time (sec)	N/A	0.035	0.332	0.316	0.191	0.260	0.000	0.278	1.998

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	75	85	0	384	0	85	230
N.S.	1	1.00	1.32	1.49	0.00	6.74	0.00	1.49	4.04
time (sec)	N/A	0.082	0.410	0.797	0.000	0.273	0.000	0.263	2.216

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	50	102	110	434	0	106	88
N.S.	1	1.00	1.25	2.55	2.75	10.85	0.00	2.65	2.20
time (sec)	N/A	0.051	3.290	1.824	0.191	0.272	0.000	0.272	2.139

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	42	35	36	92	0	42	34
N.S.	1	1.00	0.70	0.58	0.60	1.53	0.00	0.70	0.57
time (sec)	N/A	0.051	0.231	1.599	0.202	0.257	0.000	0.262	2.107

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	34	24	27	56	0	25	23
N.S.	1	1.00	1.36	0.96	1.08	2.24	0.00	1.00	0.92
time (sec)	N/A	0.127	0.036	0.555	0.192	0.245	0.000	0.265	1.989

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	23	22	51	0	30	22
N.S.	1	1.00	0.63	0.61	0.58	1.34	0.00	0.79	0.58
time (sec)	N/A	0.047	0.022	0.255	0.192	0.249	0.000	0.273	0.117

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	12	11	23	0	11	11
N.S.	1	1.00	1.12	0.71	0.65	1.35	0.00	0.65	0.65
time (sec)	N/A	0.081	0.019	0.146	0.202	0.252	0.000	0.266	1.894

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	16	19	12	23	0	10	10
N.S.	1	1.00	1.60	1.90	1.20	2.30	0.00	1.00	1.00
time (sec)	N/A	0.087	0.116	0.330	0.277	0.262	0.000	0.280	1.858

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	10	22	18	78	0	27	21
N.S.	1	1.00	0.67	1.47	1.20	5.20	0.00	1.80	1.40
time (sec)	N/A	0.029	0.085	0.878	0.268	0.257	0.000	0.280	1.853

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	38	33	140	0	25	22
N.S.	1	1.00	0.85	1.90	1.65	7.00	0.00	1.25	1.10
time (sec)	N/A	0.114	0.316	2.130	0.286	0.251	0.000	0.272	1.877

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	14	14	75	84	0	18	18
N.S.	1	1.00	0.82	0.82	4.41	4.94	0.00	1.06	1.06
time (sec)	N/A	0.037	0.092	0.929	0.194	0.242	0.000	0.282	0.073

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	135	48	0	231	0	122	0
N.S.	1	1.00	6.43	2.29	0.00	11.00	0.00	5.81	0.00
time (sec)	N/A	0.035	21.225	0.179	0.000	0.268	0.000	0.269	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	144	184	154	1229	0	216	135
N.S.	1	1.00	0.98	1.25	1.05	8.36	0.00	1.47	0.92
time (sec)	N/A	0.261	0.937	5.811	0.202	0.271	0.000	0.275	2.305

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	167	174	0	1873	0	164	262
N.S.	1	1.00	1.24	1.29	0.00	13.87	0.00	1.21	1.94
time (sec)	N/A	0.190	1.712	2.002	0.000	0.286	0.000	0.295	2.697

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	73	100	80	334	0	104	82
N.S.	1	1.00	0.86	1.18	0.94	3.93	0.00	1.22	0.96
time (sec)	N/A	0.121	0.463	0.586	0.203	0.262	0.000	0.268	2.056

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	92	0	431	0	71	158
N.S.	1	1.00	1.10	1.28	0.00	5.99	0.00	0.99	2.19
time (sec)	N/A	0.086	0.333	0.241	0.000	0.302	0.000	0.278	2.214

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	60	54	0	200	0	48	164
N.S.	1	1.00	1.20	1.08	0.00	4.00	0.00	0.96	3.28
time (sec)	N/A	0.104	0.240	0.441	0.000	0.280	0.000	0.280	4.155

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	19	51	46	117	0	76	323
N.S.	1	1.00	0.66	1.76	1.59	4.03	0.00	2.62	11.14
time (sec)	N/A	0.040	0.152	1.598	0.269	0.277	0.000	0.281	2.328

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	121	0	856	0	102	166
N.S.	1	1.00	1.02	1.46	0.00	10.31	0.00	1.23	2.00
time (sec)	N/A	0.182	0.699	4.428	0.000	0.337	0.000	0.280	4.769

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	60	145	133	909	0	201	123
N.S.	1	1.00	0.76	1.84	1.68	11.51	0.00	2.54	1.56
time (sec)	N/A	0.076	0.308	11.103	0.285	0.292	0.000	0.275	2.118

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	38	41	38	41	0	26	65
N.S.	1	1.00	1.23	1.32	1.23	1.32	0.00	0.84	2.10
time (sec)	N/A	0.082	0.173	0.606	0.283	0.265	0.000	0.254	0.536

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	44	55	571	0	47	69
N.S.	1	1.00	0.93	1.02	1.28	13.28	0.00	1.09	1.60
time (sec)	N/A	0.094	0.250	0.159	0.285	0.260	0.000	0.298	1.958

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	30	43	354	0	39	35
N.S.	1	1.00	0.86	0.81	1.16	9.57	0.00	1.05	0.95
time (sec)	N/A	0.077	0.191	0.155	0.276	0.276	0.000	0.261	1.925

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	34	30	29	186	0	35	29
N.S.	1	1.00	1.17	1.03	1.00	6.41	0.00	1.21	1.00
time (sec)	N/A	0.057	0.179	0.134	0.262	0.329	0.000	0.272	1.905

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	31	18	17	73	0	17	17
N.S.	1	1.00	1.63	0.95	0.89	3.84	0.00	0.89	0.89
time (sec)	N/A	0.033	0.097	0.103	0.270	0.281	0.000	0.262	1.906

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	11	10	26	27	10	14
N.S.	1	1.00	0.88	0.69	0.62	1.62	1.69	0.62	0.88
time (sec)	N/A	0.007	0.010	0.035	0.179	0.274	0.252	0.268	0.002

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	11	10	26	27	10	12
N.S.	1	1.00	1.12	0.69	0.62	1.62	1.69	0.62	0.75
time (sec)	N/A	0.015	0.049	0.049	0.177	0.264	0.253	0.279	0.053

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	47	18	24	73	92	18	21
N.S.	1	1.00	2.47	0.95	1.26	3.84	4.84	0.95	1.11
time (sec)	N/A	0.029	0.185	0.065	0.180	0.268	0.315	0.270	1.858

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	47	28	38	196	160	36	21
N.S.	1	1.00	1.52	0.90	1.23	6.32	5.16	1.16	0.68
time (sec)	N/A	0.044	0.195	0.134	0.189	0.264	0.472	0.265	1.877

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	54	30	54	357	197	40	29
N.S.	1	1.00	1.46	0.81	1.46	9.65	5.32	1.08	0.78
time (sec)	N/A	0.053	0.210	0.152	0.185	0.255	0.595	0.272	0.070

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	39	35	0	259	0	135	34
N.S.	1	1.00	0.87	0.78	0.00	5.76	0.00	3.00	0.76
time (sec)	N/A	0.042	0.794	0.064	0.000	0.263	0.000	0.285	1.956

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	0	131	0	71	25
N.S.	1	1.00	1.00	0.81	0.00	4.09	0.00	2.22	0.78
time (sec)	N/A	0.030	0.579	0.074	0.000	0.268	0.000	0.306	1.926

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	85	0	64	24
N.S.	1	1.00	1.00	0.83	0.00	2.83	0.00	2.13	0.80
time (sec)	N/A	0.031	0.537	0.082	0.000	0.255	0.000	0.289	1.991

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	36	35	0	166	0	89	32
N.S.	1	1.00	0.73	0.71	0.00	3.39	0.00	1.82	0.65
time (sec)	N/A	0.042	0.653	0.073	0.000	0.282	0.000	0.281	1.967

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	35	0	436	0	197	34
N.S.	1	1.00	1.00	0.78	0.00	9.69	0.00	4.38	0.76
time (sec)	N/A	0.050	1.013	0.081	0.000	0.277	0.000	0.311	1.976

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	26	0	242	0	133	25
N.S.	1	1.00	1.09	0.76	0.00	7.12	0.00	3.91	0.74
time (sec)	N/A	0.038	0.808	0.084	0.000	0.257	0.000	0.297	1.961

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	35	0	189	0	88	36
N.S.	1	1.00	0.88	0.83	0.00	4.50	0.00	2.10	0.86
time (sec)	N/A	0.046	0.892	0.087	0.000	0.264	0.000	0.312	1.951

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	48	35	0	166	0	113	31
N.S.	1	1.00	0.98	0.71	0.00	3.39	0.00	2.31	0.63
time (sec)	N/A	0.061	0.912	0.086	0.000	0.269	0.000	0.283	1.943

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	112	107	146	1294	0	141	163
N.S.	1	1.00	1.15	1.10	1.51	13.34	0.00	1.45	1.68
time (sec)	N/A	0.373	0.931	0.222	0.287	0.293	0.000	0.274	2.372

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	91	78	94	637	0	97	111
N.S.	1	1.00	1.20	1.03	1.24	8.38	0.00	1.28	1.46
time (sec)	N/A	0.241	0.443	0.178	0.274	0.285	0.000	0.273	2.270

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	74	66	67	264	0	74	73
N.S.	1	1.00	1.23	1.10	1.12	4.40	0.00	1.23	1.22
time (sec)	N/A	0.133	0.406	0.140	0.275	0.275	0.000	0.278	2.167

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	66	44	50	73	0	57	58
N.S.	1	1.00	1.29	0.86	0.98	1.43	0.00	1.12	1.14
time (sec)	N/A	0.067	0.270	0.122	0.261	0.262	0.000	0.267	0.345

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	49	38	37	42	148	43	42
N.S.	1	1.00	1.26	0.97	0.95	1.08	3.79	1.10	1.08
time (sec)	N/A	0.041	0.213	0.101	0.184	0.250	0.444	0.287	0.097

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	50	39	36	43	134	43	42
N.S.	1	1.00	1.28	1.00	0.92	1.10	3.44	1.10	1.08
time (sec)	N/A	0.052	0.088	0.072	0.186	0.257	0.442	0.279	0.069

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	56	63	76	372	59	57
N.S.	1	1.00	0.94	0.89	1.00	1.21	5.90	0.94	0.90
time (sec)	N/A	0.081	0.142	0.109	0.186	0.271	0.712	0.273	2.202

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	67	82	271	636	76	74
N.S.	1	1.00	1.02	1.05	1.28	4.23	9.94	1.19	1.16
time (sec)	N/A	0.105	0.266	0.113	0.191	0.271	1.054	0.261	2.178

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	77	76	119	648	882	100	110
N.S.	1	1.00	1.01	1.00	1.57	8.53	11.61	1.32	1.45
time (sec)	N/A	0.169	0.437	0.171	0.188	0.286	1.418	0.281	2.240

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	92	96	169	1299	1013	143	164
N.S.	1	1.00	0.98	1.02	1.80	13.82	10.78	1.52	1.74
time (sec)	N/A	0.283	0.491	0.204	0.191	0.302	2.245	0.285	2.329

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	49	73	68	184	0	169	68
N.S.	1	1.00	0.91	1.35	1.26	3.41	0.00	3.13	1.26
time (sec)	N/A	0.067	0.183	0.873	0.338	0.264	0.000	0.296	2.024

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	64	24	36	28	0	24	23
N.S.	1	1.00	2.13	0.80	1.20	0.93	0.00	0.80	0.77
time (sec)	N/A	0.031	0.032	0.120	0.212	0.249	0.000	0.269	1.941

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	64	83	48	62	0	54	39
N.S.	1	1.00	1.42	1.84	1.07	1.38	0.00	1.20	0.87
time (sec)	N/A	0.034	0.274	0.113	0.265	0.260	0.000	0.271	1.959

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	26	37	36	33	0	37	25
N.S.	1	1.00	1.13	1.61	1.57	1.43	0.00	1.61	1.09
time (sec)	N/A	0.020	0.213	0.102	0.188	0.252	0.000	0.294	1.922

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	58	71	45	58	0	51	36
N.S.	1	1.00	1.45	1.78	1.12	1.45	0.00	1.28	0.90
time (sec)	N/A	0.014	0.212	0.085	0.285	0.244	0.000	0.276	1.933

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	21	11	10	18	27	21	18
N.S.	1	1.00	1.75	0.92	0.83	1.50	2.25	1.75	1.50
time (sec)	N/A	0.010	0.038	0.122	0.176	0.252	0.464	0.263	1.965

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	62	88	47	54	0	52	37
N.S.	1	1.00	1.51	2.15	1.15	1.32	0.00	1.27	0.90
time (sec)	N/A	0.025	0.203	0.093	0.270	0.261	0.000	0.282	1.889

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	35	30	38	0	33	25
N.S.	1	1.00	1.29	1.67	1.43	1.81	0.00	1.57	1.19
time (sec)	N/A	0.022	0.187	0.092	0.180	0.265	0.000	0.276	1.876

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	86	41	53	61	0	40	40
N.S.	1	1.00	1.83	0.87	1.13	1.30	0.00	0.85	0.85
time (sec)	N/A	0.047	0.121	0.074	0.182	0.271	0.000	0.293	1.942

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	154	100	66	104	0	72	60
N.S.	1	1.00	2.26	1.47	0.97	1.53	0.00	1.06	0.88
time (sec)	N/A	0.053	3.167	0.097	0.263	0.289	0.000	0.277	1.983

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	163	54	53	74	0	54	42
N.S.	1	1.00	3.98	1.32	1.29	1.80	0.00	1.32	1.02
time (sec)	N/A	0.036	3.669	0.081	0.186	0.279	0.000	0.262	1.854

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	153	86	60	97	0	66	54
N.S.	1	1.00	2.55	1.43	1.00	1.62	0.00	1.10	0.90
time (sec)	N/A	0.031	2.601	0.097	0.267	0.266	0.000	0.280	1.893

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	28	18	19	28	63	21	28
N.S.	1	1.00	2.00	1.29	1.36	2.00	4.50	1.50	2.00
time (sec)	N/A	0.020	0.067	0.087	0.195	0.271	1.921	0.287	1.911

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	153	104	69	97	0	77	60
N.S.	1	1.00	1.78	1.21	0.80	1.13	0.00	0.90	0.70
time (sec)	N/A	0.048	3.350	0.127	0.267	0.269	0.000	0.279	1.912

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	155	55	50	82	0	57	48
N.S.	1	1.00	2.58	0.92	0.83	1.37	0.00	0.95	0.80
time (sec)	N/A	0.043	3.329	0.123	0.192	0.259	0.000	0.275	1.920

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	46	0	0	0	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	77	0	0	0	0	0	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.060	0.241	0.000	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	177	177	108	0	0	0	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.131	0.284	0.000	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	259	0	0	0	0	0	0
N.S.	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.610	0.000	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	126	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.061	0.785	0.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	83	0	0	0	0	0	0
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.661	0.000	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	125	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.841	0.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	142	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.879	0.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	223	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	1.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.629	0.000	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.682	0.000	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	61	61	171	0	0	0	0	0	0
N.S.	1	1.00	2.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	0.692	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	198	0	0	0	0	0	0
N.S.	1	1.00	3.41	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.049	7.481	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	62	207	0	0	0	0	0	0
N.S.	1	1.00	3.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	5.200	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	54	54	193	0	0	0	0	0	0
N.S.	1	1.00	3.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.042	7.248	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	52	52	198	0	0	0	0	0	0
N.S.	1	1.00	3.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	8.422	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	40	25	24	76	46	74	34
N.S.	1	1.00	1.60	1.00	0.96	3.04	1.84	2.96	1.36
time (sec)	N/A	0.016	0.076	0.217	0.175	0.261	4.792	0.364	1.856

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	197	0	0	0	0	0	0
N.S.	1	1.00	3.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	3.876	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	55	55	191	0	0	0	0	0	0
N.S.	1	1.00	3.47	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.050	4.051	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	132	155	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	7.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	165	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.124	4.621	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	130	130	151	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	6.751	0.000	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	126	160	0	0	0	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	8.395	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	49	47	37	72	0	37	34
N.S.	1	1.00	1.75	1.68	1.32	2.57	0.00	1.32	1.21
time (sec)	N/A	0.024	0.169	0.223	0.253	0.274	0.000	0.372	1.840

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	134	134	158	0	0	0	0	0	0
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.127	3.917	0.000	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	156	0	0	0	0	0	0
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	3.645	0.000	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	52	56	330	572	0	127	95
N.S.	1	1.00	1.21	1.30	7.67	13.30	0.00	2.95	2.21
time (sec)	N/A	0.031	0.239	0.468	0.265	0.301	0.000	0.383	1.889

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	44	499	171	0	67	163
N.S.	1	1.00	0.98	0.98	11.09	3.80	0.00	1.49	3.62
time (sec)	N/A	0.031	0.124	0.815	0.272	0.275	0.000	0.395	1.888

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	71	855	1576	0	161	229
N.S.	1	1.00	1.02	1.08	12.95	23.88	0.00	2.44	3.47
time (sec)	N/A	0.049	0.304	1.680	0.323	0.277	0.000	0.400	1.876

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	158	0	0	0	0	0	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.063	13.623	0.000	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	168	312	0	0	0	0	0	0
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	14.696	0.000	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	306	306	600	0	0	0	0	0	0
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	16.670	0.000	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	115	115	387	0	0	0	0	0	0
N.S.	1	1.00	3.37	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	1.207	0.000	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	135	135	174	0	0	0	0	0	0
N.S.	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.105	1.250	0.000	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	62	76	0	626	0	0	65
N.S.	1	1.00	0.85	1.04	0.00	8.58	0.00	0.00	0.89
time (sec)	N/A	0.041	0.333	1.283	0.000	0.268	0.000	0.000	2.977

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	57	74	0	334	0	0	51
N.S.	1	1.00	0.81	1.06	0.00	4.77	0.00	0.00	0.73
time (sec)	N/A	0.040	0.166	0.487	0.000	0.262	0.000	0.000	2.613

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	43	61	0	305	0	0	39
N.S.	1	1.00	0.90	1.27	0.00	6.35	0.00	0.00	0.81
time (sec)	N/A	0.030	0.093	0.492	0.000	0.281	0.000	0.000	2.215

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	37	0	303	0	0	36
N.S.	1	1.00	1.00	0.79	0.00	6.45	0.00	0.00	0.77
time (sec)	N/A	0.031	0.125	0.504	0.000	0.282	0.000	0.000	2.335

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	97	76	0	625	0	0	65
N.S.	1	1.00	1.37	1.07	0.00	8.80	0.00	0.00	0.92
time (sec)	N/A	0.039	0.179	0.500	0.000	0.273	0.000	0.000	2.431

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	101	74	0	1104	0	0	64
N.S.	1	1.00	1.40	1.03	0.00	15.33	0.00	0.00	0.89
time (sec)	N/A	0.039	0.272	0.494	0.000	0.283	0.000	0.000	3.156

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	198	149	0	8951	0	0	0
N.S.	1	1.00	1.47	1.10	0.00	66.30	0.00	0.00	0.00
time (sec)	N/A	0.242	2.301	0.865	0.000	1.108	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	160	90	0	6695	0	0	0
N.S.	1	1.00	1.52	0.86	0.00	63.76	0.00	0.00	0.00
time (sec)	N/A	0.141	0.569	0.729	0.000	0.904	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	98	52	0	1752	0	0	0
N.S.	1	1.00	1.69	0.90	0.00	30.21	0.00	0.00	0.00
time (sec)	N/A	0.082	0.313	0.783	0.000	0.681	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	159	0	0	6705	0	0	0
N.S.	1	1.00	1.50	0.00	0.00	63.25	0.00	0.00	0.00
time (sec)	N/A	0.167	0.427	0.000	0.000	0.891	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	183	183	193	0	0	9148	0	0	0
N.S.	1	1.00	1.05	0.00	0.00	49.99	0.00	0.00	0.00
time (sec)	N/A	0.228	1.054	0.000	0.000	1.134	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	184	165	0	7964	0	0	0
N.S.	1	1.00	1.39	1.25	0.00	60.33	0.00	0.00	0.00
time (sec)	N/A	0.163	1.191	0.824	0.000	1.437	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	319	319	164	195	167	1617	0	181	0
N.S.	1	1.00	0.51	0.61	0.52	5.07	0.00	0.57	0.00
time (sec)	N/A	0.643	6.290	1.210	0.312	0.267	0.000	0.626	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	197	197	334	131	112	613	0	155	0
N.S.	1	1.00	1.70	0.66	0.57	3.11	0.00	0.79	0.00
time (sec)	N/A	0.209	4.155	0.577	0.282	0.258	0.000	0.451	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	213	56	70	0	94	0
N.S.	1	1.00	0.61	2.57	0.67	0.84	0.00	1.13	0.00
time (sec)	N/A	0.102	0.070	0.772	0.286	0.261	0.000	0.279	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	83	83	51	48	35	53	0	60	0
N.S.	1	1.00	0.61	0.58	0.42	0.64	0.00	0.72	0.00
time (sec)	N/A	0.156	0.139	0.448	0.285	0.265	0.000	0.302	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	193	104	131	90	458	0	130	0
N.S.	1	1.00	0.54	0.68	0.47	2.37	0.00	0.67	0.00
time (sec)	N/A	0.604	0.349	0.608	0.290	0.266	0.000	0.277	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	311	311	133	195	145	1226	0	185	0
N.S.	1	1.00	0.43	0.63	0.47	3.94	0.00	0.59	0.00
time (sec)	N/A	1.221	0.523	0.588	0.308	0.275	0.000	0.296	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	696	0	0	0
N.S.	1	1.00	0.79	0.75	0.00	4.43	0.00	0.00	0.00
time (sec)	N/A	0.294	0.615	2.676	0.000	0.293	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	88	0	230	0	0	0
N.S.	1	1.00	0.77	0.77	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.189	0.500	1.261	0.000	0.273	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	59	58	0	216	0	0	0
N.S.	1	1.00	0.77	0.75	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	0.108	0.189	0.784	0.000	0.265	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	7	9	9	8	9	11
N.S.	1	1.00	1.29	1.00	1.29	1.29	1.14	1.29	1.57
time (sec)	N/A	0.058	3.119	0.275	0.403	0.250	4.123	0.354	2.353

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	118	0	698	0	0	0
N.S.	1	1.00	0.79	0.75	0.00	4.45	0.00	0.00	0.00
time (sec)	N/A	0.267	0.675	2.546	0.000	0.307	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	88	88	0	230	0	0	0
N.S.	1	1.00	0.77	0.77	0.00	2.00	0.00	0.00	0.00
time (sec)	N/A	0.192	0.546	1.188	0.000	0.267	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	62	58	0	216	0	0	0
N.S.	1	1.00	0.81	0.75	0.00	2.81	0.00	0.00	0.00
time (sec)	N/A	0.105	0.169	0.747	0.000	0.263	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	7	9	7	9	9	8	9	11
N.S.	1	1.00	1.29	1.00	1.29	1.29	1.14	1.29	1.57
time (sec)	N/A	0.058	6.181	0.253	0.636	0.258	3.951	0.374	2.312

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [109] had the largest ratio of [.7778000000000000047]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.00	12	0.500
2	A	6	6	1.00	12	0.500
3	A	6	6	1.00	12	0.500
4	A	5	5	1.00	12	0.417
5	A	5	5	1.00	12	0.417
6	A	6	6	1.00	12	0.500
7	A	6	6	1.00	12	0.500
8	A	7	6	1.00	12	0.500
9	A	13	9	1.00	12	0.750
10	A	12	8	1.00	12	0.667
11	A	9	9	1.00	12	0.750
12	A	9	9	1.00	12	0.750
13	A	12	8	1.00	12	0.667
14	A	13	9	1.00	12	0.750
15	A	2	2	1.00	8	0.250
16	A	2	2	1.00	10	0.200
17	A	3	3	1.00	12	0.250
18	A	3	3	1.00	14	0.214
19	A	2	2	1.00	14	0.143
20	A	2	2	1.00	14	0.143
21	A	3	3	1.00	14	0.214
22	A	14	10	1.00	14	0.714
23	A	14	10	1.00	14	0.714
24	A	13	9	1.00	14	0.643

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	13	9	1.00	14	0.643
26	A	14	10	1.00	14	0.714
27	A	14	10	1.00	14	0.714
28	A	3	3	1.00	12	0.250
29	A	8	7	1.00	14	0.500
30	A	7	7	1.00	14	0.500
31	A	7	7	1.00	14	0.500
32	A	8	7	1.00	14	0.500
33	A	4	3	1.00	14	0.214
34	A	3	3	1.00	14	0.214
35	A	2	2	1.00	14	0.143
36	A	2	2	1.00	14	0.143
37	A	3	3	1.00	14	0.214
38	A	4	3	1.00	14	0.214
39	A	3	3	1.00	12	0.250
40	A	5	3	1.00	14	0.214
41	A	3	3	1.00	14	0.214
42	A	3	3	1.00	14	0.214
43	A	5	3	1.00	14	0.214
44	A	16	10	1.00	14	0.714
45	A	14	10	1.00	14	0.714
46	A	14	10	1.00	14	0.714
47	A	14	10	1.00	14	0.714
48	A	14	10	1.00	14	0.714
49	A	16	10	1.00	14	0.714
50	A	3	3	1.00	12	0.250
51	A	3	3	1.00	14	0.214
52	A	3	3	1.00	14	0.214
53	A	3	3	1.00	14	0.214
54	A	3	3	1.00	14	0.214
55	A	3	3	1.00	14	0.214
56	A	3	3	1.00	14	0.214
57	A	3	3	1.00	14	0.214
58	A	3	3	1.00	14	0.214
59	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	3	3	1.00	14	0.214
61	A	5	3	1.00	6	0.500
62	A	4	3	1.00	6	0.500
63	A	3	3	1.00	6	0.500
64	A	2	2	1.00	6	0.333
65	A	2	2	1.00	6	0.333
66	A	3	2	1.00	6	0.333
67	A	4	2	1.00	6	0.333
68	A	5	2	1.00	6	0.333
69	A	6	2	1.00	6	0.333
70	A	5	3	1.00	8	0.375
71	A	4	3	1.00	8	0.375
72	A	3	3	1.00	8	0.375
73	A	2	2	1.00	8	0.250
74	A	3	3	1.00	8	0.375
75	A	4	3	1.00	8	0.375
76	A	5	3	1.00	8	0.375
77	A	5	4	1.00	12	0.333
78	A	4	4	1.00	12	0.333
79	A	3	3	1.00	12	0.250
80	A	2	2	1.00	12	0.167
81	A	2	2	1.00	12	0.167
82	A	3	3	1.00	12	0.250
83	A	4	4	1.00	12	0.333
84	A	5	4	1.00	12	0.333
85	A	2	2	1.00	12	0.167
86	A	2	2	1.00	12	0.167
87	A	5	4	1.00	14	0.286
88	A	5	4	1.00	14	0.286
89	A	4	3	1.00	11	0.273
90	A	3	2	1.00	11	0.182
91	A	4	3	1.00	11	0.273
92	A	2	2	1.00	9	0.222
93	A	1	1	1.00	9	0.111
94	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	2	2	1.00	11	0.182
96	A	2	1	1.00	11	0.091
97	A	5	4	1.00	13	0.308
98	A	9	6	1.00	13	0.462
99	A	4	3	1.00	13	0.231
100	A	5	5	1.00	11	0.454
101	A	2	2	1.00	11	0.182
102	A	2	2	1.00	13	0.154
103	A	5	5	1.00	13	0.385
104	A	3	2	1.00	13	0.154
105	A	5	4	1.00	11	0.364
106	A	9	7	1.00	11	0.636
107	A	5	4	1.00	11	0.364
108	A	8	6	1.00	9	0.667
109	A	8	7	1.00	9	0.778
110	A	3	2	1.00	11	0.182
111	A	8	7	1.00	11	0.636
112	A	4	3	1.00	11	0.273
113	A	4	4	1.00	13	0.308
114	A	5	3	1.00	13	0.231
115	A	10	9	1.00	13	0.692
116	A	4	3	1.00	13	0.231
117	A	6	6	1.00	11	0.546
118	A	6	5	1.00	11	0.454
119	A	3	2	1.00	13	0.154
120	A	9	7	1.00	13	0.538
121	A	3	2	1.00	13	0.154
122	A	6	5	1.00	13	0.385
123	A	6	4	1.00	11	0.364
124	A	5	4	1.00	11	0.364
125	A	4	4	1.00	11	0.364
126	A	4	4	1.00	9	0.444
127	A	2	2	1.00	6	0.333
128	A	2	2	1.00	9	0.222
129	A	3	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	3	3	1.00	11	0.273
131	A	4	4	1.00	11	0.364
132	A	4	4	1.00	11	0.364
133	A	3	3	1.00	11	0.273
134	A	3	3	1.00	11	0.273
135	A	4	4	1.00	11	0.364
136	A	4	4	1.00	13	0.308
137	A	3	3	1.00	13	0.231
138	A	4	4	1.00	13	0.308
139	A	4	4	1.00	13	0.308
140	A	6	6	1.00	13	0.462
141	A	5	5	1.00	13	0.385
142	A	4	4	1.00	13	0.308
143	A	3	3	1.00	11	0.273
144	A	2	2	1.00	8	0.250
145	A	2	2	1.00	11	0.182
146	A	4	4	1.00	13	0.308
147	A	5	5	1.00	13	0.385
148	A	6	6	1.00	13	0.462
149	A	7	7	1.00	13	0.538
150	A	3	3	1.00	14	0.214
151	A	4	3	1.00	11	0.273
152	A	5	5	1.00	11	0.454
153	A	4	4	1.00	9	0.444
154	A	5	5	1.00	7	0.714
155	A	2	1	1.00	11	0.091
156	A	5	5	1.00	11	0.454
157	A	4	4	1.00	11	0.364
158	A	4	3	1.00	13	0.231
159	A	6	6	1.00	13	0.462
160	A	5	5	1.00	11	0.454
161	A	7	6	1.00	9	0.667
162	A	3	2	1.00	13	0.154
163	A	6	6	1.00	13	0.462
164	A	5	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	3	1.00	13	0.231
166	A	4	4	1.00	15	0.267
167	A	5	5	1.00	15	0.333
168	A	3	3	1.00	9	0.333
169	A	3	3	1.00	15	0.200
170	A	2	2	1.00	11	0.182
171	A	4	4	1.00	11	0.364
172	A	5	5	1.00	11	0.454
173	A	5	5	1.00	11	0.454
174	A	3	3	1.00	7	0.429
175	A	3	3	1.00	9	0.333
176	A	3	3	1.00	9	0.333
177	A	4	4	1.00	17	0.235
178	A	4	4	1.00	17	0.235
179	A	4	4	1.00	15	0.267
180	A	4	4	1.00	13	0.308
181	A	2	1	1.00	17	0.059
182	A	4	4	1.00	17	0.235
183	A	4	4	1.00	17	0.235
184	A	5	5	1.00	19	0.263
185	A	5	5	1.00	19	0.263
186	A	5	5	1.00	17	0.294
187	A	5	5	1.00	15	0.333
188	A	3	2	1.00	19	0.105
189	A	5	5	1.00	19	0.263
190	A	5	5	1.00	19	0.263
191	A	3	2	1.00	17	0.118
192	A	4	2	1.00	17	0.118
193	A	4	2	1.00	17	0.118
194	A	4	4	1.00	19	0.210
195	A	5	5	1.00	21	0.238
196	A	6	6	1.00	21	0.286
197	A	4	4	1.00	15	0.267
198	A	4	4	1.00	21	0.190
199	A	7	6	1.00	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	7	6	1.00	19	0.316
201	A	6	5	1.00	19	0.263
202	A	6	5	1.00	19	0.263
203	A	7	6	1.00	19	0.316
204	A	7	6	1.00	19	0.316
205	A	8	7	1.00	23	0.304
206	A	7	6	1.00	23	0.261
207	A	4	4	1.00	21	0.190
208	A	8	5	1.00	21	0.238
209	A	11	6	1.00	23	0.261
210	A	8	7	1.00	21	0.333
211	A	9	7	1.00	25	0.280
212	A	8	7	1.00	25	0.280
213	A	4	4	1.00	25	0.160
214	A	4	4	1.00	25	0.160
215	A	8	7	1.00	25	0.280
216	A	9	7	1.00	25	0.280
217	A	19	5	1.00	9	0.556
218	A	13	5	1.00	9	0.556
219	A	9	4	1.00	7	0.571
220	N/A	0	0	1.00	7	0.000
221	A	19	5	1.00	9	0.556
222	A	13	5	1.00	9	0.556
223	A	9	4	1.00	7	0.571
224	N/A	0	0	1.00	7	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (b\coth(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int (b\coth(c+dx))^{7/2}\,dx=\frac{b^{7/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

```
[Out] b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2/5*b*(b*coth(d*x+c))^(5/2)/d-2*b^3*(b*coth(d*x+c))^(
1/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\int (b\coth(c+dx))^{7/2}\,dx=\frac{b^{7/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}+\frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}-\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

```
[In] Int[(b*Coth[c + d*x])^(7/2),x]
```

```
[Out] (b^(7/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(7/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b^3*Sqrt[b*Coth[c + d*x]])/d - (2*b*(b*Cot
h[c + d*x])^(5/2))/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^2\int(b\coth(c+dx))^{3/2}\,dx\\ &= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d}-\frac{2b(b\coth(c+dx))^{5/2}}{5d}+b^4\int\frac{1}{\sqrt{b\coth(c+dx)}}\,dx\end{aligned}$$

$$= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d} - \frac{b^5\text{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d} - \frac{(2b^5)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d} + \frac{b^4\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} + \frac{b^4\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{7/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{7/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b^3\sqrt{b\coth(c+dx)}}{d} - \frac{2b(b\coth(c+dx))^{5/2}}{5d}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int (b\coth(c+dx))^{7/2}\,dx = -\frac{(b\coth(c+dx))^{7/2}\left(-\arctan\left(\sqrt{\coth(c+dx)}\right) - \text{arctanh}\left(\sqrt{\coth(c+dx)}\right) + 2\sqrt{\coth(c+dx)} + \frac{2}{5}\coth^{\frac{5}{2}}(\ldots\right)}{d\coth^{\frac{7}{2}}(c+dx)}$$

```
[In] Integrate[(b*Coth[c + d*x])^(7/2),x]
[Out] -(((b*Coth[c + d*x])^(7/2)*(-ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Cot
h[c + d*x]]] + 2*Sqrt[Coth[c + d*x]] + (2*Coth[c + d*x]^(5/2))/5))/(d*Coth[
c + d*x]^(7/2)))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{b^{\frac{7}{2}} \arctan\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b(b \coth(dx+c))^{\frac{5}{2}}}{5d}-\frac{2b^{3}\sqrt{b \coth(dx+c)}}{d}$	80
default	$\frac{b^{\frac{7}{2}} \arctan\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}+\frac{b^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b \coth(dx+c)}}{\sqrt{b}}\right)}{d}-\frac{2b(b \coth(dx+c))^{\frac{5}{2}}}{5d}-\frac{2b^{3}\sqrt{b \coth(dx+c)}}{d}$	80

```
[In] int((b*coth(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(7/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2/5*b*(b*coth(d*x+c))^(5/2)/d-2*b^3*(b*coth(d*x+c))^(
1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(79) = 158.

Time = 0.28 (sec) , antiderivative size = 1574, normalized size of antiderivative = 16.23

$$\int (b\coth(c+dx))^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3
*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 -
 b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*(b^3*
cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4
 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x +
 c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b)
*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x
 + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4
+ 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*c
osh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(3*b^3*cosh(d*x + c)^
4 + 12*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*co
sh(d*x + c)^2 + 3*b^3 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 +
 4*(3*b^3*cosh(d*x + c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh
(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
```

```
 c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 -
d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)
+ d), 1/20*(10*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 - b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b))
+ 5*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*sinh(d
*x + c)^4 - 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)^2 - b^3)*s
inh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*
sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + si
nh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 +
 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d
*x + c)/sinh(d*x + c)) - b) - 16*(3*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x +
 c)*sinh(d*x + c)^3 + 3*b^3*sinh(d*x + c)^4 - 4*b^3*cosh(d*x + c)^2 + 3*b^3
 + 2*(9*b^3*cosh(d*x + c)^2 - 2*b^3)*sinh(d*x + c)^2 + 4*(3*b^3*cosh(d*x +
c)^3 - 2*b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(b*cosh(d*x + c)/sinh(d*x +
c)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 +
4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)]
```

Sympy [F(-1)]

Timed out.

$$\int (b\coth(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*coth(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (b\coth(c+dx))^{7/2}\,dx = \int (b\coth(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c))^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(79) = 158$.

Time = 0.44 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.91

$$\int (b\coth(c+dx))^{7/2}\,dx =$$

$$-\frac{10\,b^{\frac{7}{2}}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)+5\,b^{\frac{7}{2}}\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right.}{}$$

```
[In] integrate((b*coth(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] -1/10*(10*b^(7/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
 - b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) + 5*b^(7/2)*log(abs(-sqrt(b)*e^(2*
d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) - 16*(5
*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4*b^4*sgn(e^(2*d*x
 + 2*c) - 1) - 10*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3
*b^(9/2)*sgn(e^(2*d*x + 2*c) - 1) + 20*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^
(4*d*x + 4*c) - b))^2*b^5*sgn(e^(2*d*x + 2*c) - 1) - 10*(sqrt(b)*e^(2*d*x +
 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))*b^(11/2)*sgn(e^(2*d*x + 2*c) - 1) + 3*
b^6*sgn(e^(2*d*x + 2*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x +
4*c) - b) - sqrt(b))^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int (b\coth(c+dx))^{7/2}\,dx = \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b^3\,\sqrt{b\coth\left(c+d\,x\right)}}{d} - \frac{2\,b\,(b\coth(c+d\,x))^{5/2}}{5\,d} - \frac{b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}\,1\mathrm{i}}{\sqrt{b}}\right)\,1\mathrm{i}}{d}$$

```
[In] int((b*coth(c + d*x))^(7/2),x)
```

```
[Out] (b^(7/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b^3*(b*coth(c + d*x)
)^(1/2))/d - (2*b*(b*coth(c + d*x))^(5/2))/(5*d) - (b^(7/2)*atan(((b*coth(c
 + d*x))^(1/2)*1i)/b^(1/2))*1i)/d
```

3.2 $\int (b\coth(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int (b\coth(c+dx))^{5/2}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

```
[Out] -b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3554, 3557, 335, 304, 209, 212}

$$\int (b\coth(c+dx))^{5/2}\,dx = -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

```
[In] Int[(b*Coth[c + d*x])^(5/2),x]
```

```
[Out] -((b^(5/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (b^(5/2)*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*(b*Coth[c + d*x])^(3/2))/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} + b^2\int \sqrt{b\coth(c+dx)}\,dx \\
&= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} - \frac{b^3\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d} \\
&= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}
\end{aligned}$$

$$= -\frac{2b(b\coth(c+dx))^{3/2}}{3d} + \frac{b^3\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} - \frac{b^3\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{b^{5/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{5/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(c+dx))^{3/2}}{3d}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int (b\coth(c+dx))^{5/2}\,dx = -\frac{(b\coth(c+dx))^{5/2}\left(\arctan\left(\sqrt{\coth(c+dx)}\right) - \text{arctanh}\left(\sqrt{\coth(c+dx)}\right) + \frac{2}{3}\coth^{\frac{3}{2}}(c+dx)\right)}{d\coth^{\frac{5}{2}}(c+dx)}$$

```
[In] Integrate[(b*Coth[c + d*x])^(5/2),x]
[Out] -(((b*Coth[c + d*x])^(5/2)*(ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Coth
[c + d*x]]] + (2*Coth[c + d*x]^(3/2))/3))/(d*Coth[c + d*x]^(5/2)))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} + \frac{b^{\frac{5}{2}}\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(dx+c))^{\frac{3}{2}}}{3d}$	63
default	$-\frac{b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} + \frac{b^{\frac{5}{2}}\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} - \frac{2b(b\coth(dx+c))^{\frac{3}{2}}}{3d}$	63

```
[In] int((b*coth(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -b^(5/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(5/2)*arctanh((b*coth(d*
x+c))^(1/2)/b^(1/2))/d-2/3*b*(b*coth(d*x+c))^(3/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(62) = 124.

Time = 0.27 (sec) , antiderivative size = 988, normalized size of antiderivative = 12.67

$$\int (b\coth(c+dx))^{5/2} dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*si
nh(d*x + c)^2 - b^2)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/
(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
b)) - 3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 - b^2)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*s
inh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4))
 + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*
x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2
*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d), -1/12*(6*(b^2*cosh
(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2
)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 3*(b^2*c
osh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 -
b^2)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)
^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)) - b) + 8*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x +
 c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(b*cosh(d*x + c)/sinh(d*
x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2 - d)]
```

Sympy [F]

$$\int (b\coth(c+dx))^{5/2}\,dx = \int (b\coth{(c+dx)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c))**(5/2),x)
[Out] Integral((b*coth(c + d*x))**(5/2), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{5/2}\,dx = \int (b\coth{(dx+c)})^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(62) = 124$.

Time = 0.40 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.87

$$\int (b\coth(c+dx))^{5/2}\,dx = \frac{6\,b^{\frac{5}{2}}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)-3\,b^{\frac{5}{2}}\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,d}}\right.\right.}{6}$$

```
[In] integrate((b*coth(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/6*(6*b^(5/2)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) - 3*b^(5/2)*log(abs(-sqrt(b)*e^(2*d*x
 + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) + 8*(3*(sq
rt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*b^3*sgn(e^(2*d*x + 2
*c) - 1) + b^4*sgn(e^(2*d*x + 2*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*
e^(4*d*x + 4*c) - b) - sqrt(b))^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int (b\coth(c+dx))^{5/2}\,dx = \frac{b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b\,(b\coth(c+d\,x))^{3/2}}{3\,d}$$

```
[In] int((b*coth(c + d*x))^(5/2),x)
```

```
[Out] (b^(5/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (b^(5/2)*atan((b*coth(
c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))^(3/2))/(3*d)
```

3.3 $\int (b\coth(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 75

$$\int (b\coth(c+dx))^{3/2}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\coth(c+dx)}}{d}$$

```
[Out] b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\int (b\coth(c+dx))^{3/2}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\coth(c+dx)}}{d}$$

```
[In] Int[(b*Coth[c + d*x])^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d + (b^(3/2)*ArcTanh[Sqrt[b
*Coth[c + d*x]]/Sqrt[b]])/d - (2*b*Sqrt[b*Coth[c + d*x]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2b\sqrt{b\coth(c+dx)}}{d} + b^2\int\frac{1}{\sqrt{b\coth(c+dx)}}\,dx\\ &= -\frac{2b\sqrt{b\coth(c+dx)}}{d} - \frac{b^3\text{Subst}\left(\int\frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}\\ &= -\frac{2b\sqrt{b\coth(c+dx)}}{d} - \frac{(2b^3)\,\text{Subst}\left(\int\frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\end{aligned}$$

$$= -\frac{2b\sqrt{b\coth(c+dx)}}{d} + \frac{b^2\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} + \frac{b^2\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{3/2}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\coth(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int (b\coth(c+dx))^{3/2}\,dx = -\frac{\left(-\arctan\left(\sqrt{\coth(c+dx)}\right) - \text{arctanh}\left(\sqrt{\coth(c+dx)}\right) + 2\sqrt{\coth(c+dx)}\right)(b\coth(c+dx))^{3/2}}{d\coth^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[(b*Coth[c + d*x])^(3/2),x]
[Out] -(((-ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Coth[c + d*x]]] + 2*Sqrt[Co
th[c + d*x]])*(b*Coth[c + d*x])^(3/2))/(d*Coth[c + d*x]^(3/2)))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} + \frac{b^{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\coth(dx+c)}}{d}$	62
default	$\frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} + \frac{b^{\frac{3}{2}}\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d} - \frac{2b\sqrt{b\coth(dx+c)}}{d}$	62

```
[In] int((b*coth(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] b^(3/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d+b^(3/2)*arctanh((b*coth(d*x
+c))^(1/2)/b^(1/2))/d-2*b*(b*coth(d*x+c))^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(61) = 122.

Time = 0.27 (sec) , antiderivative size = 637, normalized size of antiderivative = 8.49

$$\int (b\coth(c+dx))^{3/2}\,dx = \left[-\frac{2\sqrt{-b}b\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}b\log\left(-\right.}{}\right.$$

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*b*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt
(-b)*b*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*b*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/d, 1/4*(2*b^(3/2)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) + b^(3/2)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^
3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))
*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*b*sqrt(b*cosh(d*x + c
)/sinh(d*x + c)))/d]
```

Sympy [F]

$$\int (b\coth(c+dx))^{3/2}\,dx = \int (b\coth(c+dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{3/2}\,dx = \int (b\coth(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(61) = 122$.

Time = 0.34 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.24

$$\int (b\coth(c+dx))^{3/2}\,dx = -\frac{\left(2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(2\,dx+2\,c)}-1\right)+\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)}{2\,d}$$

```
[In] integrate((b*coth(d*x+c))^(3/2),x, algorithm="giac")
[Out] -1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b))*sgn(e^(2*d*x + 2*c) - 1) + sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x
+ 2*c) + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(2*d*x + 2*c) - 1) - 8*b*sgn(e
^(2*d*x + 2*c) - 1)/(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b)
- sqrt(b)))*b/d
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.81

$$\int (b\coth(c+dx))^{3/2}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d} - \frac{2\,b\,\sqrt{b\coth\left(c+d\,x\right)}}{d} + \frac{b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{d}$$

```
[In] int((b*coth(c + d*x))^(3/2),x)
[Out] (b^(3/2)*atan((b*coth(c + d*x))^(1/2)/b^(1/2)))/d - (2*b*(b*coth(c + d*x))^
(1/2))/d + (b^(3/2)*atanh((b*coth(c + d*x))^(1/2)/b^(1/2)))/d
```

3.4 $\int \sqrt{b \coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \sqrt{b \coth(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3557, 335, 304, 209, 212}

$$\int \sqrt{b \coth(c+dx)}\,dx = \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}$$

```
[In] Int[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[b]*ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]])/d) + (Sqrt[b]*ArcTanh[Sqr
t[b*Coth[c + d*x]]/Sqrt[b]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d} \\
&= -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} \\
&= \frac{b\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} \\
&= -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \sqrt{b\coth(c+dx)}\,dx$$
$$= -\frac{\left(\arctan\left(\sqrt{\coth(c+dx)}\right)-\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\right)\sqrt{b\coth(c+dx)}}{d\sqrt{\coth(c+dx)}}$$

```
[In] Integrate[Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] -(((ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[b*Coth
[c + d*x]])/(d*Sqrt[Coth[c + d*x]]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}$	47
default	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\sqrt{b}}{d}$	47

```
[In] int((b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^(1/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))*b^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(46) = 92$.

Time = 0.28 (sec) , antiderivative size = 594, normalized size of antiderivative = 10.24

$$\int \sqrt{b\coth(c+dx)}\,dx$$
$$= \left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4b\ldots}{\ldots}\right.}{\ldots}\right.$$
$$-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{b}\log\left(2b\cosh(dx+c)^4+8b\cosh(dx\ldots\right.}{\ldots}$$

```
[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(b)*arct
an(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(b)*log(2*b*cosh(d
*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d
*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(co
sh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh
(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
 cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
- b))/d]
```

Sympy [F]

$$\int \sqrt{b \coth(c+dx)}\,dx = \int \sqrt{b \coth{(c+dx)}}\,dx$$

[In] integrate((b*coth(d*x+c))**(1/2),x)

[Out] Integral(sqrt(b*coth(c + d*x)), x)

Maxima [F]

$$\int \sqrt{b \coth(c+dx)}\,dx = \int \sqrt{b \coth{(dx+c)}}\,dx$$

[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*coth(d*x + c)), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.74

$$\int \sqrt{b \coth(c+dx)}\,dx = \frac{\left(2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)-\sqrt{b}\log\left(\left|-\sqrt{b}e^{(2\,dx+2\,c)}+\sqrt{be^{(4\,dx+4\,c)}-b}\right|\right)\right)\mathrm{sgn}\left(e^{(2\,dx+2\,c)}-\right.}{2\,d}$$

[In] integrate((b*coth(d*x+c))^(1/2),x, algorithm="giac")

[Out] 1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))/sqrt(b)) - sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + sqrt(b*e^(4*d*x + 4*c) - b))))*sgn(e^(2*d*x + 2*c) - 1)/d

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \sqrt{b \coth (c+dx)}\, dx = -\frac{\sqrt{b} \left(\operatorname{atan}\left(\frac{\sqrt{b \coth (c+d x)}}{\sqrt{b}}\right) - \operatorname{atanh}\left(\frac{\sqrt{b \coth (c+d x)}}{\sqrt{b}}\right)\right)}{d}$$

```
[In] int((b*coth(c + d*x))^(1/2),x)
[Out] -(b^(1/2)*(atan((b*coth(c + d*x))^(1/2)/b^(1/2)) - atanh((b*coth(c + d*x))^
(1/2)/b^(1/2))))/d
```

3.5 $\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3557, 335, 218, 212, 209}

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}$$

```
[In] Int[1/Sqrt[b*Coth[c + d*x]],x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(Sqrt[b]*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(Sqrt[b]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{d}\\
&= \frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \frac{\left(\arctan\left(\sqrt{\coth(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\right)\sqrt{\coth(c+dx)}}{d\sqrt{b\coth(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]],x]
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]])*Sqrt[Coth[c +
 d*x]])/(d*Sqrt[b*Coth[c + d*x]])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}$	46
default	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{d\sqrt{b}}$	46

```
[In] int(1/(b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/d/b^(1/2)+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/d/b^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(45) = 90$.

Time = 0.29 (sec) , antiderivative size = 598, normalized size of antiderivative = 10.49

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)+\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4}{}\right.}{}\right.$$

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/(b*d), 1/4*(2*sqrt(b)*a
rctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + sqrt(b)*log(2*b*cos
h(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*
(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*c
osh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^
3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c
)) - b))/(b*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*coth(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth{(dx+c)}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*coth(d*x + c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)+\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{\sqrt{b}\,d}$$

```
[In] int(1/(b*coth(c + d*x))^(1/2),x)
```

```
[Out] (atan((b*coth(c + d*x))^(1/2)/b^(1/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(1
/2)))/(b^(1/2)*d)
```

3.6 $\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\coth(c+dx)}}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\coth(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x])^(-3/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(3/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(3/2)*d) - 2/(b*d*Sqrt[b*Coth[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2}{bd\sqrt{b\coth(c+dx)}} + \frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^2}$$

$$= -\frac{2}{bd\sqrt{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\coth(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd}$$

$$= -\frac{2}{bd\sqrt{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd}$$

$$= -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{3/2}d} - \frac{2}{bd\sqrt{b\coth(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \frac{-2 - \arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2(c+dx)} + \operatorname{arctanh}\left(\sqrt[4]{\coth^2(c+dx)}\right)}{bd\sqrt{b\coth(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x])^(-3/2),x]
```

```
[Out] (-2 - ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4) + ArcTanh[(Co
th[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4))/(b*d*Sqrt[b*Coth[c + d*x]])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}-\frac{2}{bd\sqrt{b\coth(dx+c)}}$	65
default	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{3}{2}}d}-\frac{2}{bd\sqrt{b\coth(dx+c)}}$	65

```
[In] int(1/(b*coth(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(3/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(3/2)/d-2/b/d/(b*coth(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(64) = 128$.

Time = 0.28 (sec) , antiderivative size = 923, normalized size of antiderivative = 11.83

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c
)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*s
inh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2
- cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sq
rt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^
2*d*sinh(d*x + c)^2 + b^2*d)]
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))**(3/2),x)
[Out] Integral((b*coth(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{3/2}\,d} - \frac{2}{b\,d\,\sqrt{b\coth\left(c+d\,x\right)}}$$

```
[In] int(1/(b*coth(c + d*x))^(3/2),x)
[Out] atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(3/2)*d) - atan((b*coth(c + d*x))
^(1/2)/b^(1/2))/(b^(3/2)*d) - 2/(b*d*(b*coth(c + d*x))^(1/2))
```

3.7 $\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 79

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3555, 3557, 335, 218, 212, 209}

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

```
[In] Int[(b*Coth[c + d*x])^(-5/2),x]
```

```
[Out] ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(5/2)*d) + ArcTanh[Sqrt[b*Coth[c +
 d*x]]/Sqrt[b]]/(b^(5/2)*d) - 2/(3*b*d*(b*Coth[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{3bd(b\coth(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{b\coth(c+dx)}}\,dx}{b^2} \\
&= -\frac{2}{3bd(b\coth(c+dx))^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{bd} \\
&= -\frac{2}{3bd(b\coth(c+dx))^{3/2}} - \frac{2\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{bd}
\end{aligned}$$

$$= -\frac{2}{3bd(b\coth(c+dx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{b^2d} + \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{b^2d}$$

$$= \frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{5/2}d} - \frac{2}{3bd(b\coth(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{-2+3\arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\coth^2(c+dx)^{3/4}+3\operatorname{arctanh}\left(\sqrt[4]{\coth^2(c+dx}\right.}{3bd(b\coth(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Coth[c + d*x])^(-5/2),x]
```

```
[Out] (-2 + 3*ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4) + 3*ArcTanh
[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4))/(3*b*d*(b*Coth[c + d*x])
^(3/2))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}-\frac{2}{3bd(b\coth(dx+c))^{\frac{3}{2}}}$	64
default	$\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{5}{2}}d}-\frac{2}{3bd(b\coth(dx+c))^{\frac{3}{2}}}$	64

```
[In] int(1/(b*coth(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(5/2)/d+arctanh((b*coth(d*x+c))^(1/
2)/b^(1/2))/b^(5/2)/d-2/3/b/d/(b*coth(d*x+c))^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(63) = 126$.

Time = 0.28 (sec) , antiderivative size = 1428, normalized size of antiderivative = 18.08

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*
cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sin
h(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2
+ 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1
)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*
b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
inh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x +
c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x +
c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d
*x + c))*sinh(d*x + c) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cos
h(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4 + 2*b^3*d*cosh(d*x + c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*s
inh(d*x + c)^2 + 4*(b^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x +
 c)), 1/12*(6*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(b
)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 3*(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x +
 c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c
)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x +
 c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*
x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqr
t(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*(cosh(d*x + c)^4 + 4*cosh(d*x + c
)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x +
c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c
```

```
) + 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*
d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4 + 2*b^3*d*cosh(d*x
+ c)^2 + b^3*d + 2*(3*b^3*d*cosh(d*x + c)^2 + b^3*d)*sinh(d*x + c)^2 + 4*(b
^3*d*cosh(d*x + c)^3 + b^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))**(5/2),x)
[Out] Integral((b*coth(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(-5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c))^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.80

$$\int \frac{1}{(b\coth(c+dx))^{5/2}}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d} - \frac{2}{3\,b\,d\,(b\coth\left(c+d\,x\right))^{3/2}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{5/2}\,d}$$

```
[In] int(1/(b*coth(c + d*x))^(5/2),x)
```

```
[Out] atan((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d) - 2/(3*b*d*(b*coth(c + d*
x))^(3/2)) + atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(5/2)*d)
```

3.8 $\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 100

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}}$$

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+c)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} - \frac{2}{5bd(b\coth(c+dx))^{5/2}}$$

```
[In] Int[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] -(ArcTan[Sqrt[b*Coth[c + d*x]]/Sqrt[b]]/(b^(7/2)*d)) + ArcTanh[Sqrt[b*Coth[
c + d*x]]/Sqrt[b]]/(b^(7/2)*d) - 2/(5*b*d*(b*Coth[c + d*x])^(5/2)) - 2/(b^3
*d*Sqrt[b*Coth[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{2}{5bd(b\coth(c+dx))^{5/2}} + \frac{\int \frac{1}{(b\coth(c+dx))^{3/2}}\,dx}{b^2}$$

$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} + \frac{\int \sqrt{b\coth(c+dx)}\,dx}{b^4}$$

$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-b^2+x^4}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{b^3d}$$

$$= -\frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{b^3d} - \frac{\text{Subst}\left(\int \frac{1}{b+x^2}\,dx, x, \sqrt{b\coth(c+dx)}\right)}{b^3d}$$

$$= -\frac{\arctan\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} + \frac{\text{arctanh}\left(\frac{\sqrt{b\coth(c+dx)}}{\sqrt{b}}\right)}{b^{7/2}d} - \frac{2}{5bd(b\coth(c+dx))^{5/2}} - \frac{2}{b^3d\sqrt{b\coth(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \frac{5\text{arctanh}\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2(c+dx)} - 5\left(2+\arctan\left(\sqrt[4]{\coth^2(c+dx}\right.\right.}{5b^3d\sqrt{b\coth(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x])^(-7/2),x]
```

```
[Out] (5*ArcTanh[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4) - 5*(2 + ArcTan
[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4)) - 2*Tanh[c + d*x]^2)/(5*
b^3*d*Sqrt[b*Coth[c + d*x]])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}+\frac{\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}-\frac{2}{5bd(b\coth(dx+c))^{\frac{5}{2}}}-\frac{2}{b^3d\sqrt{b\coth(dx+c)}}$	83
default	$-\frac{\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}+\frac{\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)}{b^{\frac{7}{2}}d}-\frac{2}{5bd(b\coth(dx+c))^{\frac{5}{2}}}-\frac{2}{b^3d\sqrt{b\coth(dx+c)}}$	83

```
[In] int(1/(b*coth(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((b*coth(d*x+c))^(1/2)/b^(1/2))/b^(7/2)/d+arctanh((b*coth(d*x+c))^(1
/2)/b^(1/2))/b^(7/2)/d-2/5/b/d/(b*coth(d*x+c))^(5/2)-2/b^3/d/(b*coth(d*x+c)
)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(82) = 164.

Time = 0.30 (sec) , antiderivative size = 2132, normalized size of antiderivative = 21.32

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] [-1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x +
c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5
*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4
+ 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x
+ c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arc
tan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqr
t(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 5*(cosh(d*x + c)^6 + 6*cosh
(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x +
c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*
x + c))*sinh(d*x + c) + 1)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh
(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sin
h(d*x + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 +
 4*(15*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)
^4 + 6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d
*x + c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*co
sh(d*x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*
sinh(d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d
*cosh(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^4 + 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 +
 3*(5*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c
)^2 + 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), -1/20*(10*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x
+ c)^5 + sinh(d*x + c)^6 + 3*(5*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*co
```

```
sh(d*x + c)^4 + 4*(5*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3
*(5*cosh(d*x + c)^4 + 6*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x +
 c)^2 + 6*(cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 5*
(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + 3*(5
*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + 3*cosh(d*x + c)^4 + 4*(5*cosh(d*x +
 c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 + 6*cosh(d*
x + c)^2 + 1)*sinh(d*x + c)^2 + 3*cosh(d*x + c)^2 + 6*(cosh(d*x + c)^5 + 2*
cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*log(2*b*cosh(d*
x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cos
h(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(
d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 -
cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) -
 b) + 16*(3*cosh(d*x + c)^6 + 18*cosh(d*x + c)*sinh(d*x + c)^5 + 3*sinh(d*x
 + c)^6 + (45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^4 + cosh(d*x + c)^4 + 4*(1
5*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + (45*cosh(d*x + c)^4 +
6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(9*cosh(d*x +
c)^5 + 2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt(b*cosh(d*
x + c)/sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)*sinh(
d*x + c)^5 + b^4*d*sinh(d*x + c)^6 + 3*b^4*d*cosh(d*x + c)^4 + 3*b^4*d*cosh
(d*x + c)^2 + b^4*d + 3*(5*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^4 +
 4*(5*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5
*b^4*d*cosh(d*x + c)^4 + 6*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 +
 6*(b^4*d*cosh(d*x + c)^5 + 2*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*x + c))*
sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\coth{(c+dx)})^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))**(7/2),x)
```

```
[Out] Integral((b*coth(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\coth(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(-7/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c))^(7/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{1}{(b\coth(c+dx))^{7/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\operatorname{atan}\left(\frac{\sqrt{b\coth(c+d\,x)}}{\sqrt{b}}\right)}{b^{7/2}\,d} - \frac{\frac{2}{5\,b}+\frac{2\coth(c+d\,x)^2}{b}}{d\,(b\coth(c+d\,x))^{5/2}}$$

```
[In] int(1/(b*coth(c + d*x))^(7/2),x)
[Out] atanh((b*coth(c + d*x))^(1/2)/b^(1/2))/(b^(7/2)*d) - atan((b*coth(c + d*x))
^(1/2)/b^(1/2))/(b^(7/2)*d) - (2/(5*b) + (2*coth(c + d*x)^2)/b)/(d*(b*coth(
c + d*x))^(5/2))
```

3.9 $\int (b\coth(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 236

$$\int (b\coth(c+dx))^{4/3}\,dx = -\frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}$$

```
[Out] b^(4/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d-3*b*(b*coth(d*x+c))^(1/3)/
d-1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3
))/d+1/4*b^(4/3)*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(
2/3))/d-1/2*b^(4/3)*arctan(1/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))
*3^(1/2)/d+1/2*b^(4/3)*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/
2))*3^(1/2)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$\int (b\coth(c+dx))^{4/3}\,dx = -\frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2d} + \frac{b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d}$$

```
[In] Int[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(4/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt
[3]])/d + (Sqrt[3]*b^(4/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))
/Sqrt[3]])/(2*d) + (b^(4/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (
3*b*(b*Coth[c + d*x])^(1/3))/d - (b^(4/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c +
 d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d) + (b^(4/3)*Log[b^(2/3) + b^(
1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)])/(4*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
```

```
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + b^2\int\frac{1}{(b\coth(c+dx))^{2/3}}\,dx$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^3\text{Subst}\left(\int\frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{(3b^3)\,\text{Subst}\left(\int\frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= -\frac{3b\sqrt[3]{b\coth(c+dx)}}{d} + \frac{b^{4/3}\text{Subst}\left(\int\frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{4/3}\text{Subst}\left(\int\frac{\sqrt[3]{b}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{5/3}\text{Subst}\left(\int\frac{1}{b^{2/3}-x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{4/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\text{Subst}\left(\int\frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} + \frac{b^{4/3}\text{Subst}\left(\int\frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} + \frac{(3b^{5/3})\,\text{Subst}\left(\int\frac{1}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} + \frac{(3b^{5/3})\,\text{Subst}\left(\int\frac{1}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d}$$

$$= \frac{b^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{(3b^{4/3})\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2d} - \frac{(3b^{4/3})\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2d}$$

$$= -\frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{4/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{3b\sqrt[3]{b\coth(c+dx)}}{d} - \frac{b^{4/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{4/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.82

$$\int(b\coth(c+dx))^{4/3}\,dx =$$

$$-\frac{b\sqrt[3]{b\coth(c+dx)}\left(6\sqrt[6]{\coth^2(c+dx)}+\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)-(-1)\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x])^(4/3),x]
```

```
[Out] -1/2*(b*(b*Coth[c + d*x])^(1/3)*(6*(Coth[c + d*x]^2)^(1/6) + Log[1 - (Coth[
c + d*x]^2)^(1/6)] - Log[1 + (Coth[c + d*x]^2)^(1/6)] - (-1)^(2/3)*Log[1 -
(-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(2/3)*Log[1 + (-1)^(1/3)*(Coth[c
```

```
 + d*x]^2)^(1/6)] - (-1)^(1/3)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]
+ (-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]))/(d*(Coth[c + d*x
]^2)^(1/6))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{3b(b\coth(dx+c))^{\frac{1}{3}}}{d}+\frac{b^{\frac{4}{3}}\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d}-\frac{b^{\frac{4}{3}}\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4d}+\frac{b^{\frac{4}{3}}\sqrt{3}}{}$
default	$-\frac{3b(b\coth(dx+c))^{\frac{1}{3}}}{d}+\frac{b^{\frac{4}{3}}\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d}-\frac{b^{\frac{4}{3}}\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4d}+\frac{b^{\frac{4}{3}}\sqrt{3}}{}$

```
[In] int((b*coth(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3*b*(b*coth(d*x+c))^(1/3)/d+1/2/d*b^(4/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3)
)-1/4*b^(4/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3
))/d+1/2/d*b^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1
/3)-1))-1/2/d*b^(4/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*b^(4/3)*ln(b^(2
/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/2*b^(4/3)*arct
an(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.24

$$\int (b\coth(c+dx))^{4/3}\,dx =$$

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\,b\arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)-2\sqrt{3}b^{\frac{4}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-b)^{\frac{1}{3}}\,b\log}{}$$

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*b*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(4/3)*arctan(-1/3*(s
qrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-
b)^(1/3)*b*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
```

```
) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(4/3)*log(b^(2/3) - b^(1/3)*
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/
3)) - 2*(-b)^(1/3)*b*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
) - 2*b^(4/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*b*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d
```

Sympy [F]

$$\int (b\coth(c+dx))^{4/3}\,dx = \int (b\coth{(c+dx)})^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c))**(4/3),x)
[Out] Integral((b*coth(c + d*x))**(4/3), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{4/3}\,dx = \int (b\coth{(dx+c)})^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(4/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\coth(c+dx))^{4/3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*coth(d*x+c))^(4/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fract
ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free E
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.06

$$\int (b\coth(c+dx))^{4/3}\,dx = -\frac{3\,b\,(b\coth(c+d\,x))^{1/3}}{d} - \frac{b^{4/3}\,\mathrm{atan}\left(\frac{(b\coth(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d}$$
$$-\frac{b^{4/3}\,\ln\left(\frac{486\,b^{37/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d^4}-\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$-\frac{b^{4/3}\,\ln\left(\frac{486\,b^{37/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{d^4}-\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d}$$
$$+\frac{b^{4/3}\,\ln\left(\frac{972\,b^{37/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d^4}+\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$
$$+\frac{b^{4/3}\,\ln\left(\frac{972\,b^{37/3}\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d^4}+\frac{486\,b^{12}\,(b\coth(c+d\,x))^{1/3}}{d^4}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

```
[In] int((b*coth(c + d*x))^(4/3),x)
```

```
[Out] (b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/4 - 1/4))/d^4 + (486*b^12*(b*coth(
c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(4/3)*atan(((b*coth(c
+ d*x))^(1/3)*1i)/b^(1/3))*1i)/d - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*1i)
/2 - 1/2))/d^4 - (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 -
1/2))/(2*d) - (b^(4/3)*log((486*b^(37/3)*((3^(1/2)*1i)/2 + 1/2))/d^4 - (486
*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) - (3*b*(b
*coth(c + d*x))^(1/3))/d + (b^(4/3)*log((972*b^(37/3)*((3^(1/2)*1i)/4 + 1/4
))/d^4 + (486*b^12*(b*coth(c + d*x))^(1/3))/d^4)*((3^(1/2)*1i)/4 + 1/4))/d
```

3.10 $\int (b\coth(c+dx))^{2/3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 218

$$\int (b\coth(c+dx))^{2/3}\,dx = \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}$$

```
[Out] b^(2/3)*arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/d-1/4*b^(2/3)*ln(b^(2/3)-b^(
1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/4*b^(2/3)*ln(b^(2/3)+
b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/d+1/2*b^(2/3)*arctan(1
/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d-1/2*b^(2/3)*arcta
n(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3557, 335, 302, 648, 632, 210, 642, 212}

$$\int (b\coth(c+dx))^{2/3}\,dx = \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2d} + \frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}$$

```
[In] Int[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])
/(2*d) - (Sqrt[3]*b^(2/3)*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/
Sqrt[3]])/(2*d) + (b^(2/3)*ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)])/d - (b
^(2/3)*Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2
/3)])/(4*d) + (b^(2/3)*Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*C
oth[c + d*x])^(2/3)])/(4*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
```

```
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{b\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d} + \frac{b\text{Subst}\left(\int \frac{1}{b^{2/3}-x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$= \frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} + \frac{b^{2/3}\text{Subst}\left(\int \frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} - \frac{(3b)\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d} - \frac{(3b)\text{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4d}$$

$$= \frac{b^{2/3}\text{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2d} + \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2d}$$

$$= \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt{3}b^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2d} + \frac{b^{2/3}\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{d} - \frac{b^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d} + \frac{b^{2/3}\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.68

$$\int (b\coth(c+dx))^{2/3}\,dx = \frac{(b\coth(c+dx))^{2/3}\left(2\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) - 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \cdots\right.}{\cdots}$$

```
[In] Integrate[(b*Coth[c + d*x])^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x])^(2/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt
[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[Co
th[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] + L
og[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2/3
))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{6b^{\frac{1}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{12b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{6b^{\frac{1}{3}}}-\cdots\right.}{d}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{6b^{\frac{1}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{12b^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right)}{6b^{\frac{1}{3}}}-\cdots\right.}{d}$

```
[In] int((b*coth(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*(1/6/b^(1/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))-1/12/b^(1/3)*ln(b^(2/
3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))+1/6*3^(1/2)/b^(1/3)
*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))-1/6/b^(1/3)*ln((b*
coth(d*x+c))^(1/3)+b^(1/3))+1/12/b^(1/3)*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))
^(1/3)+(b*coth(d*x+c))^(2/3))+1/6*3^(1/2)/b^(1/3)*arctan(1/3*3^(1/2)*(2*(b*
coth(d*x+c))^(1/3)/b^(1/3)-1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.42

$$\int (b\coth(c+dx))^{2/3}\,dx =$$

$$-\frac{2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b^2)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+2\sqrt{3}(b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(b^2)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3b}\right)+(-\cdots}{}$$

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b^2)^(1/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 2*sqrt(3)*(b^2)^(1/3)*arctan(
-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/
3))/b) + (-b^2)^(1/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^
(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + (b^2)^(1/3)
*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*
```

```
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(1/3)*log(b*(b*cosh(d*x +
 c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) - 2*(b^2)^(1/3)*log(b*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/d
```

Sympy [F]

$$\int (b\coth(c+dx))^{2/3}\,dx = \int (b\coth{(c+dx)})^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c))**(2/3),x)
[Out] Integral((b*coth(c + d*x))**(2/3), x)
```

Maxima [F]

$$\int (b\coth(c+dx))^{2/3}\,dx = \int (b\coth{(dx+c)})^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(2/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int (b\coth(c+dx))^{2/3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*coth(d*x+c))^(2/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fract
ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free E
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.07

$$\int (b\coth(c+dx))^{2/3}\,dx = -\frac{b^{2/3}\,\mathrm{atan}\left(\frac{(b\coth(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{d} - \frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{972\,b^{26/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} - \frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}-\frac{972\,b^{26/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{1944\,b^{26/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d} + \frac{b^{2/3}\,\ln\left(\frac{972\,b^9}{d^3}+\frac{1944\,b^{26/3}\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\coth(c+d\,x))^{1/3}}{d^3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

```
[In] int((b*coth(c + d*x))^(2/3),x)
```

```
[Out] (b^(2/3)*log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth(
c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/4 - 1/4))/d - (b^(2/3)*log((972*b^9)/d^
3 - (972*b^(26/3)*((3^(1/2)*1i)/2 - 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*((3^
(1/2)*1i)/2 - 1/2))/(2*d) - (b^(2/3)*log((972*b^9)/d^3 - (972*b^(26/3)*((3^
(1/2)*1i)/2 + 1/2)*(b*coth(c + d*x))^(1/3))/d^3)*((3^(1/2)*1i)/2 + 1/2))/(2
*d) - (b^(2/3)*atan(((b*coth(c + d*x))^(1/3)*1i)/b^(1/3))*1i)/d + (b^(2/3)*
log((972*b^9)/d^3 + (1944*b^(26/3)*((3^(1/2)*1i)/4 + 1/4)*(b*coth(c + d*x))
^(1/3))/d^3)*((3^(1/2)*1i)/4 + 1/4))/d
```

3.11 $\int \sqrt[3]{b\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 132

$$\int \sqrt[3]{b\coth(c+dx)}\,dx=-\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4d}$$

```
[Out] -1/2*b^(1/3)*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/d+1/4*b^(1/3)*ln(b^(4/3)+b^(
2/3)*(b*coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/d-1/2*b^(1/3)*arctan(1/3*
(b^(2/3)+2*(b*coth(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3557, 335, 281, 298, 31, 648, 631, 210, 642}

$$\int \sqrt[3]{b\coth(c+dx)}\,dx=-\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2d}-\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}+\frac{\sqrt[3]{b}\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4d}$$

```
[In] Int[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b^(1/3)*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]
*b^(2/3))])/d - (b^(1/3)*Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)])/(2*d) + (b
^(1/3)*Log[b^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4
/3)])/(4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{\sqrt[3]{x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(3b)\text{Subst}\left(\int \frac{x^3}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}\\
&= -\frac{(3b)\text{Subst}\left(\int \frac{x}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}\\
&= -\frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}\\
&\quad+\frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{-b^{2/3}+x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}\\
&= -\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}\\
&\quad+\frac{\sqrt[3]{b}\text{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{4d}\\
&\quad-\frac{(3b)\text{Subst}\left(\int \frac{1}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{4d}\\
&= -\frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d}\\
&\quad+\frac{\sqrt[3]{b}\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4d}\\
&\quad+\frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}\right)}{2d}
\end{aligned}
$$

$$= -\frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2d} - \frac{\sqrt[3]{b}\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2d} + \frac{\sqrt[3]{b}\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4d}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.82

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = -\frac{(b\coth(c+dx))^{4/3}\left(\log\left(1-\sqrt[3]{\coth^2(c+dx)}\right)-\sqrt[3]{-1}\log\left(1+\sqrt[3]{-1}\sqrt[3]{\coth^2(c+dx)}\right)+(-1)^{2/3}\log\right.}{2bd\coth^2(c+dx)^{2/3}}$$

```
[In] Integrate[(b*Coth[c + d*x])^(1/3),x]
```

```
[Out] -1/2*((b*Coth[c + d*x])^(4/3)*(Log[1 - (Coth[c + d*x]^2)^(1/3)] - (-1)^(1/3
)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/3)] + (-1)^(2/3)*Log[1 - (-1)^(2/
3)*(Coth[c + d*x]^2)^(1/3)]))/(b*d*(Coth[c + d*x]^2)^(2/3))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{1}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^2\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c)}{\left(b^2\right)^{\frac{1}{3}}}\right.}{3}\right.}{6\left(b^2\right)^{\frac{1}{3}}}\right.}{d}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-\left(b^2\right)^{\frac{1}{3}}\right)}{6\left(b^2\right)^{\frac{1}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+\left(b^2\right)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+\left(b^2\right)^{\frac{2}{3}}\right)}{12\left(b^2\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c)}{\left(b^2\right)^{\frac{1}{3}}}\right.}{3}\right.}{6\left(b^2\right)^{\frac{1}{3}}}\right.}{d}$

```
[In] int((b*coth(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*(1/6/(b^2)^(1/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))-1/12/(b^2)^(1
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
+1/6*3^(1/2)/(b^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+c))^
(2/3)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(99) = 198$.

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.20

$$\int \sqrt[3]{b\coth(c+dx)}\,dx =$$

$$-\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}(-b)^{\frac{1}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}}{3b}\right)-2(-b)^{\frac{1}{3}}\log\left(-(-b)^{\frac{2}{3}}+\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{2}{3}}\right)+(-b)^{\frac{1}{3}}\mathrm{lo}}{}$$

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(-b)^(1/3)*(b
*cosh(d*x + c)/sinh(d*x + c))^(2/3))/b) - 2*(-b)^(1/3)*log(-(-b)^(2/3) + (b
*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(1/3)*log(((cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x
```

```
+ c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)))/d
```

Sympy [F]

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = \int \sqrt[3]{b\coth{(c+dx)}}\,dx$$

```
[In] integrate((b*coth(d*x+c))**(1/3),x)
[Out] Integral((b*coth(c + d*x))**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = \int (b\coth{(dx+c)})^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(1/3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(99) = 198.

Time = 0.43 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.64

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = -\frac{b\left(\frac{2\sqrt{3}|b|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3\,|b|^{\frac{2}{3}}}\right)}{b^2} - \frac{|b|^{\frac{4}{3}}\log\left(\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2\,dx+2\,c)}+b\right)\left(\frac{be^{(2\,dx+2\,c)}+b}{e^{(2\,dx+2\,c)}-1}\right)^{\frac{1}{3}}}{e^{(2\,dx+2\,c)}-1}\right)}{b^2} + \cdots\right)}{4\,d}$$

```
[In] integrate((b*coth(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] -1/4*b*(2*sqrt(3)*abs(b)^(4/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) +
b)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 - abs(b)^
(4/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3
) + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d
*x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 + 2*abs(b)^(4/3)*log(abs((
(b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/
d
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \sqrt[3]{b\coth(c+dx)}\,dx = \frac{(-b)^{1/3}\ln\left(81\,(-b)^{16/3}\,(b\coth(c+d\,x))^{2/3}-81\,b^6\right)}{2\,d} - \frac{(-b)^{1/3}\ln\left(-\frac{81\,b^6}{d^4}-\frac{81\,(-b)^{16/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(b\coth(c+d\,x))^{2/3}}{d^4}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,d} + \frac{(-b)^{1/3}\ln\left(-\frac{81\,b^6}{d^4}+\frac{162\,(-b)^{16/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)\,(b\coth(c+d\,x))^{2/3}}{d^4}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{d}$$

```
[In] int((b*coth(c + d*x))^(1/3),x)
```

```
[Out] ((-b)^(1/3)*log(81*(-b)^(16/3)*(b*coth(c + d*x))^(2/3) - 81*b^6))/(2*d) - (
(-b)^(1/3)*log(- (81*b^6)/d^4 - (81*(-b)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(b*c
oth(c + d*x))^(2/3))/d^4)*((3^(1/2)*1i)/2 + 1/2))/(2*d) + ((-b)^(1/3)*log((
162*(-b)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(b*coth(c + d*x))^(2/3))/d^4 - (81*b
^6)/d^4)*((3^(1/2)*1i)/4 - 1/4))/d
```

3.12 $\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 132

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

```
[Out] -1/2*ln(b^(2/3)-(b*coth(d*x+c))^(2/3))/b^(1/3)/d+1/4*ln(b^(4/3)+b^(2/3)*(b*
coth(d*x+c))^(2/3)+(b*coth(d*x+c))^(4/3))/b^(1/3)/d+1/2*arctan(1/3*(b^(2/3)
+2*(b*coth(d*x+c))^(2/3))/b^(2/3)*3^(1/2))*3^(1/2)/b^(1/3)/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3557, 335, 281, 206, 31, 648, 631, 210, 642}

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{b^{2/3}+2(b\coth(c+dx))^{2/3}}{\sqrt{3}b^{2/3}}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{2/3}(b\coth(c+dx))^{2/3}+b^{4/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

```
[In] Int[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(b^(2/3) + 2*(b*Coth[c + d*x])^(2/3))/(Sqrt[3]*b^(2/3))])/(
2*b^(1/3)*d) - Log[b^(2/3) - (b*Coth[c + d*x])^(2/3)]/(2*b^(1/3)*d) + Log[b
```

```
^(4/3) + b^(2/3)*(b*Coth[c + d*x])^(2/3) + (b*Coth[c + d*x])^(4/3)]/(4*b^(1
/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(3b)\text{Subst}\left(\int \frac{x}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}\\
&= -\frac{(3b)\text{Subst}\left(\int \frac{1}{-b^2+x^3}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-b^{2/3}+x}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}\\
&\quad -\frac{\text{Subst}\left(\int \frac{-2b^{2/3}-x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}\\
&= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}+\frac{\text{Subst}\left(\int \frac{b^{2/3}+2x}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{4\sqrt[3]{b}d}\\
&\quad +\frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{b^{4/3}+b^{2/3}x+x^2}\,dx, x, (b\coth(c+dx))^{2/3}\right)}{4d}\\
&= -\frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d}\\
&\quad +\frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}\\
&\quad -\frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}\right)}{2\sqrt[3]{b}d}
\end{aligned}
$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2(b\coth(c+dx))^{2/3}}{b^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{b}d} - \frac{\log\left(b^{2/3}-(b\coth(c+dx))^{2/3}\right)}{2\sqrt[3]{b}d} + \frac{\log\left(b^{4/3}+b^{2/3}(b\coth(c+dx))^{2/3}+(b\coth(c+dx))^{4/3}\right)}{4\sqrt[3]{b}d}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\sqrt[3]{\coth(c+dx)}\left(2\sqrt{3}\arctan\left(\frac{1+2\coth^{\frac{2}{3}}(c+dx)}{\sqrt{3}}\right)-2\log\left(1-\coth^{\frac{2}{3}}(c+dx)\right)+\log\left(1+\coth^{\frac{2}{3}}(c+dx)+\mathrm{c}\right.\right.}{4d\sqrt[3]{b\coth(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x])^(-1/3),x]
```

```
[Out] (Coth[c + d*x]^(1/3)*(2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(2/3))/Sqrt[3]]
 - 2*Log[1 - Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(2/3) + Coth[c +
d*x]^(4/3)]))/(4*d*(b*Coth[c + d*x])^(1/3))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+(b^2)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$
default	$-\frac{3b\left(\frac{\ln\left((b\coth(dx+c))^{\frac{2}{3}}-(b^2)^{\frac{1}{3}}\right)}{6(b^2)^{\frac{2}{3}}}-\frac{\ln\left((b\coth(dx+c))^{\frac{4}{3}}+(b^2)^{\frac{1}{3}}(b\coth(dx+c))^{\frac{2}{3}}+(b^2)^{\frac{2}{3}}\right)}{12(b^2)^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(b\coth(dx+c))^{\frac{2}{3}}}{(b^2)^{\frac{1}{3}}}\right.}{3}\right.}{6(b^2)^{\frac{2}{3}}}\right.}{d}$

```
[In] int(1/(b*coth(d*x+c))^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d*b*(1/6/(b^2)^(2/3)*ln((b*coth(d*x+c))^(2/3)-(b^2)^(1/3))-1/12/(b^2)^(2
/3)*ln((b*coth(d*x+c))^(4/3)+(b^2)^(1/3)*(b*coth(d*x+c))^(2/3)+(b^2)^(2/3))
-1/6/(b^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b^2)^(1/3)*(b*coth(d*x+c))^
(2/3)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. $2(99) = 198$.

Time = 0.29 (sec) , antiderivative size = 1598, normalized size of antiderivative = 12.11

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log((3*b*cosh(d*x + c)^4 + 12*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + 3*b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(9*b*
cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 3*(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c
)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c)
 + 1)*(-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - sqrt(3)*((cosh(d*x
 + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x
```

```
 + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cos
h(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(
2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3) - 2
*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)
^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)/b) + 4*(3*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 3*b)/(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(-b)^(2/3)*log(-(-b)^(2/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*co
sh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*s
inh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c)/sinh
(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 - 1)))/(b*d), 1/4*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan((2*sqrt
(3)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
 + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(2/3)*sqrt(-(-b)^(1/3)/b) + sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*
x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) - 4*sqrt(3)*(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - b)*(
b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b))/(9*b*cosh(d*x + c
)^4 + 36*b*cosh(d*x + c)*sinh(d*x + c)^3 + 9*b*sinh(d*x + c)^4 + 14*b*cosh(
d*x + c)^2 + 2*(27*b*cosh(d*x + c)^2 + 7*b)*sinh(d*x + c)^2 + 4*(9*b*cosh(d
*x + c)^3 + 7*b*cosh(d*x + c))*sinh(d*x + c) + 9*b)) - 2*(-b)^(2/3)*log(-(-
b)^(2/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (-b)^(2/3)*log(((cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)
*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) + (b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b*cosh(d*x + c
)/sinh(d*x + c))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)))/(b*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx$$

[In] integrate(1/(b*coth(d*x+c))**(1/3),x)

[Out] Integral((b*coth(c + d*x))**(-1/3), x)

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \int \frac{1}{(b\coth(dx+c))^{\frac{1}{3}}}\,dx$$

[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(-1/3), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(99) = 198$.

Time = 0.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.64

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{b\left(\frac{2\sqrt{3}|b|^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}+|b|^{\frac{2}{3}}\right)}{3|b|^{\frac{2}{3}}}\right)}{b^2} + \frac{|b|^{\frac{2}{3}}\log\left(\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{2}{3}}|b|^{\frac{2}{3}}+|b|^{\frac{4}{3}}+\frac{\left(be^{(2dx+2c)}+b\right)\left(\frac{be^{(2dx+2c)}+b}{e^{(2dx+2c)}-1}\right)^{\frac{1}{3}}}{e^{(2dx+2c)}-1}\right)}{b^2} - \frac{2}{}\right.}{4d}$$

[In] integrate(1/(b*coth(d*x+c))^(1/3),x, algorithm="giac")

```
[Out] 1/4*b*(2*sqrt(3)*abs(b)^(2/3)*arctan(1/3*sqrt(3)*(2*((b*e^(2*d*x + 2*c) + b
)/(e^(2*d*x + 2*c) - 1))^(2/3) + abs(b)^(2/3))/abs(b)^(2/3))/b^2 + abs(b)^(
2/3)*log(((b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3)*abs(b)^(2/3)
 + abs(b)^(4/3) + (b*e^(2*d*x + 2*c) + b)*((b*e^(2*d*x + 2*c) + b)/(e^(2*d*
x + 2*c) - 1))^(1/3)/(e^(2*d*x + 2*c) - 1))/b^2 - 2*abs(b)^(2/3)*log(abs(((
b*e^(2*d*x + 2*c) + b)/(e^(2*d*x + 2*c) - 1))^(2/3) - abs(b)^(2/3)))/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{b\coth(c+dx)}}\,dx = \frac{\ln\left(162\,(-b)^{11/3}+162\,b^3\,(b\coth(c+d\,x))^{2/3}\right)}{2\,(-b)^{1/3}\,d} + \frac{\ln\left(\frac{81\,(-b)^{11/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}+\frac{162\,b^3\,(b\coth(c+d\,x))^{2/3}}{d^3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,(-b)^{1/3}\,d} - \frac{\ln\left(\frac{81\,(-b)^{11/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{d^3}-\frac{162\,b^3\,(b\coth(c+d\,x))^{2/3}}{d^3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,(-b)^{1/3}\,d}$$

```
[In] int(1/(b*coth(c + d*x))^(1/3),x)
```

```
[Out] log(162*(-b)^(11/3) + 162*b^3*(b*coth(c + d*x))^(2/3))/(2*(-b)^(1/3)*d) + (
log((81*(-b)^(11/3)*(3^(1/2)*1i - 1))/d^3 + (162*b^3*(b*coth(c + d*x))^(2/3
))/d^3)*(3^(1/2)*1i - 1))/(4*(-b)^(1/3)*d) - (log((81*(-b)^(11/3)*(3^(1/2)*
1i + 1))/d^3 - (162*b^3*(b*coth(c + d*x))^(2/3))/d^3)*(3^(1/2)*1i + 1))/(4*
(-b)^(1/3)*d)
```

3.13 $\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 218

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

```
[Out] arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(2/3)/d-1/4*ln(b^(2/3)-b^(1/3)*(b*
coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d+1/4*ln(b^(2/3)+b^(1/3)*
(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d-1/2*arctan(1/3*(1-2*
(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d+1/2*arctan(1/3*(1
+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3557, 335, 216, 648, 632, 210, 642, 212}

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

```
[In] Int[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(b
^(2/3)*d) + (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[
3]])/(2*b^(2/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(2/3)*d) -
 Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(
4*b^(2/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth[c + d*x])^(1/3) + (b*Coth[c +
d*x])^(2/3)]/(4*b^(2/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
```

```
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{b\text{Subst}\left(\int \frac{1}{x^{2/3}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}$$

$$= -\frac{(3b)\text{Subst}\left(\int \frac{1}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{d}$$

$$
\begin{aligned}
&= \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{2/3}d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{2/3}d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{\sqrt[3]{b}d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4b^{2/3}d} \\
&\quad + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4b^{2/3}d} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4\sqrt[3]{b}d} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4\sqrt[3]{b}d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} \\
&\quad - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d} \\
&\quad + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d} \\
&\quad - \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{2/3}d}
\end{aligned}
$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{2/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{2/3}d} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{2/3}d}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.81

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = -\frac{\sqrt[3]{b\coth(c+dx)}\left(\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)+\sqrt[3]{-1}\left(-\sqrt[3]{-1}\log\left(1-\sqrt[3]{-}\right.\right.\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x])^(-2/3),x]
```

```
[Out] -1/2*((b*Coth[c + d*x])^(1/3)*(Log[1 - (Coth[c + d*x]^2)^(1/6)] - Log[1 + (
Coth[c + d*x]^2)^(1/6)] + (-1)^(1/3)*(-((-1)^(1/3)*Log[1 - (-1)^(1/3)*(Coth
[c + d*x]^2)^(1/6)]) + (-1)^(1/3)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6
)] - Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)] + Log[1 + (-1)^(2/3)*(Coth
[c + d*x]^2)^(1/6)])))/(b*d*(Coth[c + d*x]^2)^(1/6))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2d\,b^{\frac{2}{3}}}+\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}d}+\frac{\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right.}{2b^{\frac{2}{3}}d}$
default	$-\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}-b^{\frac{1}{3}}\right)}{2d\,b^{\frac{2}{3}}}+\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4b^{\frac{2}{3}}d}+\frac{\arctan\left(\frac{\left(1+\frac{2(b\coth(dx+c))^{\frac{1}{3}}}{b^{\frac{1}{3}}}\right)\sqrt{3}}{3}\right.}{2b^{\frac{2}{3}}d}$

```
[In] int(1/(b*coth(d*x+c))^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d/b^(2/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*ln(b^(2/3)+b^(1/3)*(b*
coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(2/3)/d+1/2*arctan(1/3*(1+2*(b*
coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)/d+1/2/d/b^(2/3)*ln((b*
coth(d*x+c))^(1/3)+b^(1/3))-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b
*coth(d*x+c))^(2/3))/b^(2/3)/d+1/2/d/b^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*
(b*coth(d*x+c))^(1/3)/b^(1/3)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(166) = 332$.

Time = 0.26 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.63

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \frac{2\sqrt{3}b\sqrt{-(-b^2)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{3}(-b^2)^{\frac{1}{3}}b\sqrt{-(-b^2)^{\frac{1}{3}}}-2\sqrt{3}(-b^2)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}\sqrt{-(-b^2)^{\frac{1}{3}}}}{3b^2}\right)}{}$$

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sq
rt(-(-b^2)^(1/3)) - 2*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3)*sqrt(-(-b^2)^(1/3)))/b^2) + 2*sqrt(3)*(b^2)^(1/6)*b*arctan(-1/3*sqrt(
3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b^2) + (-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
 (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - (b^
2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2
)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(-b^2)^(2/3)*log(b*(b*co
sh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 2*(b^2)^(2/3)*log(b*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3)))/(b^2*d)
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\coth{(c+dx)})^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))**(2/3),x)
[Out] Integral((b*coth(c + d*x))**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \int \frac{1}{(b\coth{(dx+c)})^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^(-2/3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c))^(2/3),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fract
ion free Error: Bad Argument ValueMinimal poly. in rootof must be fraction
free E
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.90

$$\int \frac{1}{(b\coth(c+dx))^{2/3}}\,dx = \frac{\operatorname{atanh}\left(\frac{(b\coth(c+d\,x))^{1/3}}{b^{1/3}}\right)}{b^{2/3}\,d} - \frac{\operatorname{atan}\left(\frac{b^{10/3}\,(b\coth(c+d\,x))^{1/3}\,243\mathrm{i}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}} - \frac{243\,\sqrt{3}\,b^{10/3}\,(b\coth(c+d\,x))^{1/3}}{-243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{2/3}\,d} - \frac{\operatorname{atan}\left(\frac{b^{10/3}\,(b\coth(c+d\,x))^{1/3}\,243\mathrm{i}}{243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}} + \frac{243\,\sqrt{3}\,b^{10/3}\,(b\coth(c+d\,x))^{1/3}}{243\,b^{11/3}+\sqrt{3}\,b^{11/3}\,243\mathrm{i}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{2/3}\,d}$$

```
[In] int(1/(b*coth(c + d*x))^(2/3),x)
```

```
[Out] atanh((b*coth(c + d*x))^(1/3)/b^(1/3))/(b^(2/3)*d) - (atan((b^(10/3)*(b*cot
h(c + d*x))^(1/3)*243i)/(3^(1/2)*b^(11/3)*243i - 243*b^(11/3)) - (243*3^(1/
2)*b^(10/3)*(b*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i - 243*b^(11/3))
)*(3^(1/2)*1i + 1)*1i)/(2*b^(2/3)*d) - (atan((b^(10/3)*(b*coth(c + d*x))^(1
/3)*243i)/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)) + (243*3^(1/2)*b^(10/3)*(b
*coth(c + d*x))^(1/3))/(3^(1/2)*b^(11/3)*243i + 243*b^(11/3)))*(3^(1/2)*1i
- 1)*1i)/(2*b^(2/3)*d)
```

3.14 $\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 238

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}$$

```
[Out] arctanh((b*coth(d*x+c))^(1/3)/b^(1/3))/b^(4/3)/d-3/b/d/(b*coth(d*x+c))^(1/3
)-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/
3)/d+1/4*ln(b^(2/3)+b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^
(4/3)/d+1/2*arctan(1/3*(1-2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)
/b^(4/3)/d-1/2*arctan(1/3*(1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1
/2)/b^(4/3)/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{2b^{4/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x])^(-4/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]])/(2*b^(4
/3)*d) - (Sqrt[3]*ArcTan[(1 + (2*(b*Coth[c + d*x])^(1/3))/b^(1/3))/Sqrt[3]]
)/(2*b^(4/3)*d) + ArcTanh[(b*Coth[c + d*x])^(1/3)/b^(1/3)]/(b^(4/3)*d) - 3/
(b*d*(b*Coth[c + d*x])^(1/3)) - Log[b^(2/3) - b^(1/3)*(b*Coth[c + d*x])^(1/
3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d) + Log[b^(2/3) + b^(1/3)*(b*Coth
[c + d*x])^(1/3) + (b*Coth[c + d*x])^(2/3)]/(4*b^(4/3)*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
```

```
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\int (b\coth(c+dx))^{2/3}\,dx}{b^2}$$

$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{x^{2/3}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{bd}$$

$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{3\text{Subst}\left(\int \frac{x^4}{-b^2+x^6}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{bd}$$

$$= -\frac{3}{bd\sqrt[3]{b\coth(c+dx)}} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}-\frac{x}{2}}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{4/3}d} + \frac{\text{Subst}\left(\int \frac{-\frac{\sqrt[3]{b}}{2}+\frac{x}{2}}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{b^{4/3}d} + \frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{bd}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4b^{4/3}d} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2x}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4b^{4/3}d} - \frac{3\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4bd} - \frac{3\text{Subst}\left(\int \frac{1}{b^{2/3}+\sqrt[3]{b}x+x^2}\,dx, x, \sqrt[3]{b\coth(c+dx)}\right)}{4bd}$$

$$= \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d} - \frac{3\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d} + \frac{3\mathrm{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{2b^{4/3}d}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} - \frac{\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{2b^{4/3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{b\coth(c+dx)}}{\sqrt[3]{b}}\right)}{b^{4/3}d} - \frac{3}{bd\sqrt[3]{b\coth(c+dx)}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\sqrt[3]{b\coth(c+dx)}+(b\coth(c+dx))^{2/3}\right)}{4b^{4/3}d}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.02

$$\int\frac{1}{(b\coth(c+dx))^{4/3}}\,dx = -\frac{6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth^2(c+dx)}\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)+\sqrt[3]{-1}\sqrt[6]{c}}{}$$

```
[In] Integrate[(b*Coth[c + d*x])^(-4/3),x]
```

```
[Out] -1/2*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)^(1/6)] - (Coth[
c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^(1/3)*(Coth[c + d
*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - (-1)^(1/3)*(Coth
```

[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)] - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)])/(b*d*(b*Coth[c + d*x])^(1/3))

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{3}{bd(b\coth(dx+c))^{\frac{1}{3}}}+\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d\,b^{\frac{4}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4b^{\frac{4}{3}}d}-\frac{\sqrt{3}\,\mathrm{arctan}}{}$
default	$-\frac{3}{bd(b\coth(dx+c))^{\frac{1}{3}}}+\frac{\ln\left((b\coth(dx+c))^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{2d\,b^{\frac{4}{3}}}-\frac{\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}(b\coth(dx+c))^{\frac{1}{3}}+(b\coth(dx+c))^{\frac{2}{3}}\right)}{4b^{\frac{4}{3}}d}-\frac{\sqrt{3}\,\mathrm{arctan}}{}$

```
[In] int(1/(b*coth(d*x+c))^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/b/d/(b*coth(d*x+c))^(1/3)+1/2/d/b^(4/3)*ln((b*coth(d*x+c))^(1/3)+b^(1/3)
)-1/4*ln(b^(2/3)-b^(1/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/
3)/d-1/2/d/b^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*coth(d*x+c))^(1/3)/b^(1
/3)-1))-1/2/d/b^(4/3)*ln((b*coth(d*x+c))^(1/3)-b^(1/3))+1/4*ln(b^(2/3)+b^(1
/3)*(b*coth(d*x+c))^(1/3)+(b*coth(d*x+c))^(2/3))/b^(4/3)/d-1/2*arctan(1/3*(
1+2*(b*coth(d*x+c))^(1/3)/b^(1/3))*3^(1/2))*3^(1/2)/b^(4/3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(184) = 368$.

Time = 0.33 (sec) , antiderivative size = 3348, normalized size of antiderivative = 14.07

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
```

```
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*
sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x
 + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-
b)^(1/3)/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*c
osh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*s
qrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*
sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*
sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1
/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-
b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x +
 c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log(
(-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 +
```

```
2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) +
(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x +
 c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2
/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*s
inh(d*x + c)^2 + b^2*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c
)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*
x + c)^2 + 6*b*cosh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3
)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))*sqrt((-b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(
d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*c
osh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3))
- (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b
^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cos
h(d*x + c)/sinh(d*x + c))^(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x +
 c)/sinh(d*x + c))^(1/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) - 12*(cosh(d*x + c)^2 + 2*cosh(d*
x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))
^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*
d*sinh(d*x + c)^2 + b^2*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)/b)*arctan(-
1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cosh(d*x + c)/s
inh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - (cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1
/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))
^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)
+ (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*co
sh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/s
inh(d*x + c))^(1/3)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3))/b^(1/3))/b^(1/3) + 12*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d
*x + c))^(2/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c
) + b^2*d*sinh(d*x + c)^2 + b^2*d)]
```

Sympy [F]

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth(c+dx))^{\frac{4}{3}}}\,dx$$

[In] integrate(1/(b*coth(d*x+c))**(4/3),x)

[Out] Integral((b*coth(c + d*x))**(-4/3), x)

Maxima [F]

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth(dx+c))^{\frac{4}{3}}}\,dx$$

[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c))^(-4/3), x)

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/(b*coth(d*x+c))^(4/3),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Minimal poly. in rootof must be fraction free Error: Bad Argument ValueMinimal poly. in rootof must be fraction free E

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{1}{(b\coth(c+dx))^{4/3}}\,dx = -\frac{3}{b\,d\,(b\coth(c+d\,x))^{1/3}} - \frac{\mathrm{atan}\left(\frac{(b\coth(c+d\,x))^{1/3}\,1\mathrm{i}}{b^{1/3}}\right)\,1\mathrm{i}}{b^{4/3}\,d} - \frac{\mathrm{atan}\left(\frac{b^9\,d^4\,(b\coth(c+d\,x))^{1/3}\,486\mathrm{i}}{243\,b^{28/3}\,d^4-\sqrt{3}\,b^{28/3}\,d^4\,243\mathrm{i}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{4/3}\,d} + \frac{\mathrm{atan}\left(\frac{b^9\,d^4\,(b\coth(c+d\,x))^{1/3}\,486\mathrm{i}}{243\,b^{28/3}\,d^4+\sqrt{3}\,b^{28/3}\,d^4\,243\mathrm{i}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b^{4/3}\,d}$$

```
[In] int(1/(b*coth(c + d*x))^(4/3),x)
```

```
[Out] (atan((b^9*d^4*(b*coth(c + d*x))^(1/3)*486i)/(243*b^(28/3)*d^4 + 3^(1/2)*b^
(28/3)*d^4*243i))*(3^(1/2)*1i - 1)*1i)/(2*b^(4/3)*d) - (atan(((b*coth(c + d
*x))^(1/3)*1i)/b^(1/3))*1i)/(b^(4/3)*d) - (atan((b^9*d^4*(b*coth(c + d*x))^
(1/3)*486i)/(243*b^(28/3)*d^4 - 3^(1/2)*b^(28/3)*d^4*243i))*(3^(1/2)*1i + 1
)*1i)/(2*b^(4/3)*d) - 3/(b*d*(b*coth(c + d*x))^(1/3))
```

3.15 $\int \coth^n(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 43

$$\int \coth^n(a+bx)\,dx = \frac{\coth^{1+n}(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\coth^2(a+bx)\right)}{b(1+n)}$$

```
[Out] coth(b*x+a)^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(b*x+a)^2)/b/(1+
n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3557, 371}

$$\int \coth^n(a+bx)\,dx = \frac{\coth^{n+1}(a+bx)\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\coth^2(a+bx)\right)}{b(n+1)}$$

```
[In] Int[Coth[a + b*x]^n,x]
```

```
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
b*x]^2])/(b*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{x^n}{-1+x^2}\,dx, x, \coth(a+bx)\right)}{b}\\ &= \frac{\coth^{1+n}(a+bx)\,\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \coth^2(a+bx)\right)}{b(1+n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \coth^n(a+bx)\,dx = \frac{\coth^{1+n}(a+bx)\,\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \coth^2(a+bx)\right)}{b(1+n)}$$

```
[In] Integrate[Coth[a + b*x]^n,x]
[Out] (Coth[a + b*x]^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[a +
b*x]^2])/(b*(1 + n))
```

Maple [F]

$$\int \coth(bx+a)^n\,dx$$

```
[In] int(coth(b*x+a)^n,x)
[Out] int(coth(b*x+a)^n,x)
```

Fricas [F]

$$\int \coth^n(a+bx)\,dx = \int \coth(bx+a)^n\,dx$$

```
[In] integrate(coth(b*x+a)^n,x, algorithm="fricas")
[Out] integral(coth(b*x + a)^n, x)
```

Sympy [F]

$$\int \coth^n(a+bx)\,dx = \int \coth^n{(a+bx)}\,dx$$

```
[In] integrate(coth(b*x+a)**n,x)
[Out] Integral(coth(a + b*x)**n, x)
```

Maxima [F]

$$\int \coth^n(a+bx)\,dx = \int \coth{(bx+a)}^n\,dx$$

```
[In] integrate(coth(b*x+a)^n,x, algorithm="maxima")
[Out] integrate(coth(b*x + a)^n, x)
```

Giac [F]

$$\int \coth^n(a+bx)\,dx = \int \coth{(bx+a)}^n\,dx$$

```
[In] integrate(coth(b*x+a)^n,x, algorithm="giac")
[Out] integrate(coth(b*x + a)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^n(a+bx)\,dx = \int \coth(a+b\,x)^n\,dx$$

```
[In] int(coth(a + b*x)^n,x)
[Out] int(coth(a + b*x)^n, x)
```

3.16 $\int (b\coth(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int (b\coth(c+dx))^n\,dx = \frac{(b\coth(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(1,\frac{1+n}{2},\frac{3+n}{2},\coth^2(c+dx)\right)}{bd(1+n)}$$

```
[Out] (b*coth(d*x+c))^(1+n)*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],coth(d*x+c)^2)/b
/d/(1+n)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3557, 371}

$$\int (b\coth(c+dx))^n\,dx = \frac{(b\coth(c+dx))^{n+1}\,\text{Hypergeometric2F1}\left(1,\frac{n+1}{2},\frac{n+3}{2},\coth^2(c+dx)\right)}{bd(n+1)}$$

```
[In] Int[(b*Coth[c + d*x])^n,x]
```

```
[Out] ((b*Coth[c + d*x])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, Coth[
c + d*x]^2])/(b*d*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = -\frac{b\text{Subst}\left(\int \frac{x^n}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}$$
$$= \frac{(b\coth(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \coth^2(c+dx)\right)}{bd(1+n)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int (b\coth(c+dx))^n\,dx$$
$$= \frac{\coth(c+dx)(b\coth(c+dx))^n\,\text{Hypergeometric2F1}\left(1, \frac{1+n}{2}, \frac{3+n}{2}, \coth^2(c+dx)\right)}{d(1+n)}$$

```
[In] Integrate[(b*Coth[c + d*x])^n,x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x])^n*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/
2, Coth[c + d*x]^2])/(d*(1 + n))
```

Maple [F]

$$\int (b\coth(dx+c))^n\,dx$$

```
[In] int((b*coth(d*x+c))^n,x)
[Out] int((b*coth(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth(dx+c))^n\,dx$$

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth{(c+dx)})^n\,\,dx$$

```
[In] integrate((b*coth(d*x+c))**n,x)
[Out] Integral((b*coth(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth{(dx+c)})^n\,\,dx$$

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c))^n, x)
```

Giac [F]

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth{(dx+c)})^n\,\,dx$$

```
[In] integrate((b*coth(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*coth(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth(c+dx))^n\,dx = \int (b\coth(c+d\,x))^n\,dx$$

```
[In] int((b*coth(c + d*x))^n,x)
[Out] int((b*coth(c + d*x))^n, x)
```

3.17 $\int \left(b\coth^2(c+dx)\right)^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \left(b\coth^2(c+dx)\right)^n \, dx = \frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+2n),\frac{1}{2}(3+2n),\coth^2(c+dx)\right)}{d(1+2n)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^2)^n*hypergeom([1, 1/2+n],[3/2+n],coth(d*x+c)^2)
/d/(1+2*n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3739, 3557, 371}

$$\int \left(b\coth^2(c+dx)\right)^n \, dx = \frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(2n+1),\frac{1}{2}(2n+3),\coth^2(c+dx)\right)}{d(2n+1)}$$

```
[In] Int[(b*Coth[c + d*x]^2)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, (1 + 2*n)/2, (3 +
 2*n)/2, Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\int \coth^{2n}(c+dx)\,dx\\ &= -\frac{\left(\coth^{-2n}(c+dx)\left(b\coth^2(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{2n}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}\\ &= \frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+2n), \frac{1}{2}(3+2n), \coth^2(c+dx)\right)}{d(1+2n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^2(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}+n, \frac{3}{2}+n, \coth^2(c+dx)\right)}{d(1+2n)}$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^n,x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^2)^n*Hypergeometric2F1[1, 1/2 + n, 3/2 + n,
 Coth[c + d*x]^2])/(d*(1 + 2*n))
```

Maple [F]

$$\int \left(\coth \left(dx+c\right)^{2} b\right)^{n} dx$$

```
[In] int((coth(d*x+c)^2*b)^n,x)
[Out] int((coth(d*x+c)^2*b)^n,x)
```

Fricas [F]

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth\left(dx+c\right)^{2}\right)^{n}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c)^2)^n, x)
```

Sympy [F]

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth^{2}\left(c+dx\right)\right)^{n}\,dx$$

```
[In] integrate((b*coth(d*x+c)**2)**n,x)
[Out] Integral((b*coth(c + d*x)**2)**n, x)
```

Maxima [F]

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth\left(dx+c\right)^{2}\right)^{n}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

Giac [F]

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^n\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^n,x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^2(c+dx)\right)^n\,dx = \int \left(b\coth(c+d\,x)^2\right)^n\,dx$$

```
[In] int((b*coth(c + d*x)^2)^n,x)
[Out] int((b*coth(c + d*x)^2)^n, x)
```

3.18 $\int \left(b \coth^2(c+dx)\right)^{3/2} dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \left(b \coth^2(c+dx)\right)^{3/2} dx = -\frac{b \coth(c+dx)\sqrt{b \coth^2(c+dx)}}{2d} + \frac{b\sqrt{b \coth^2(c+dx)} \log(\sinh(c+dx)) \tanh(c+dx)}{d}$$

```
[Out] -1/2*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/2)/d+b*ln(sinh(d*x+c))*(b*coth(d*x+
c)^2)^(1/2)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \left(b \coth^2(c+dx)\right)^{3/2} dx = \frac{b \tanh(c+dx)\sqrt{b \coth^2(c+dx)} \log(\sinh(c+dx))}{d} - \frac{b \coth(c+dx)\sqrt{b \coth^2(c+dx)}}{2d}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(3/2),x]
```

```
[Out] -1/2*(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^2])/d + (b*Sqrt[b*Coth[c + d*x]^
2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(b\sqrt{b\coth^2(c+dx)}\tanh(c+dx)\right)\int\coth^3(c+dx)\,dx\\ &= -\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}+\left(b\sqrt{b\coth^2(c+dx)}\tanh(c+dx)\right)\int\coth(c+dx)\,dx\\ &= -\frac{b\coth(c+dx)\sqrt{b\coth^2(c+dx)}}{2d}+\frac{b\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int\left(b\coth^2(c+dx)\right)^{3/2}\,dx = -\frac{\left(b\coth^2(c+dx)\right)^{3/2}\left(\coth^2(c+dx)-2\log(\cosh(c+dx))-2\log(\tanh(c+dx))\right)\tanh^3(c+dx)}{2d}$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(3/2),x]
[Out] -1/2*((b*Coth[c + d*x]^2)^(3/2)*(Coth[c + d*x]^2 - 2*Log[Cosh[c + d*x]] - 2
*Log[Tanh[c + d*x]])*Tanh[c + d*x]^3)/d
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

method	result
derivativedivides	$-\frac{\left(\coth(dx+c)^2 b\right)^{\frac{3}{2}} \left(\coth(dx+c)^2+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)\right)}{2d\coth(dx+c)^3}$
default	$-\frac{\left(\coth(dx+c)^2 b\right)^{\frac{3}{2}} \left(\coth(dx+c)^2+\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)\right)}{2d\coth(dx+c)^3}$
risch	$\frac{b\sqrt{\frac{\left(e^{2dx+2c}+1\right)^2 b}{\left(e^{2dx+2c}-1\right)^2}}\left(-e^{4dx+4c}dx+e^{4dx+4c}\ln\left(e^{2dx+2c}-1\right)-2\,e^{4dx+4c}c+2\,e^{2dx+2c}dx-2\,e^{2dx+2c}\ln\left(e^{2dx+2c}-1\right)+4\,e^{2dx+2}\right.}{\left(e^{2dx+2c}+1\right)\left(e^{2dx+2c}-1\right)d}$

```
[In] int((coth(d*x+c)^2*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(coth(d*x+c)^2*b)^(3/2)*(coth(d*x+c)^2+ln(coth(d*x+c)-1)+ln(coth(d*x
+c)+1))/coth(d*x+c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(55) = 110$.

Time = 0.28 (sec) , antiderivative size = 823, normalized size of antiderivative = 13.49

$$\int \left(b\coth^2(c+dx)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d*x*cosh(d*x + c)^4 - (b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x + c)^4 -
4*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sinh(d*x + c)
^3 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2 + 2*(3*b*d*x*cosh(d*x + c)^2 - b
*d*x - (3*b*d*x*cosh(d*x + c)^2 - b*d*x + b)*e^(2*d*x + 2*c) + b)*sinh(d*x
+ c)^2 - (b*d*x*cosh(d*x + c)^4 + b*d*x - 2*(b*d*x - b)*cosh(d*x + c)^2)*e^
(2*d*x + 2*c) - (b*cosh(d*x + c)^4 - (b*e^(2*d*x + 2*c) - b)*sinh(d*x + c)^
4 - 4*(b*cosh(d*x + c)*e^(2*d*x + 2*c) - b*cosh(d*x + c))*sinh(d*x + c)^3 -
 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - (3*b*cosh(d*x + c)^2 - b)*e
^(2*d*x + 2*c) - b)*sinh(d*x + c)^2 - (b*cosh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + b)*e^(2*d*x + 2*c) + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c) - (b*cosh
(d*x + c)^3 - b*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + b)*log(2*si
nh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b*d*x*cosh(d*x + c)^3 - (
b*d*x - b)*cosh(d*x + c) - (b*d*x*cosh(d*x + c)^3 - (b*d*x - b)*cosh(d*x +
c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x
+ 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^4 +
 (d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^4 + 4*(d*cosh(d*x + c)*e^(2*d*x + 2*
```

```
c) + d*cosh(d*x + c))*sinh(d*x + c)^3 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d
*x + c)^2 + (3*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^2
+ (d*cosh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + 4*(d*cosh
(d*x + c)^3 - d*cosh(d*x + c) + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx=\int \left(b\coth^2{(c+dx)}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**2)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.59

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx=-\frac{(dx+c)b^{\frac{3}{2}}}{d}-\frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{b^{\frac{3}{2}}\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}$$

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
[Out] -(d*x + c)*b^(3/2)/d - b^(3/2)*log(e^(-d*x - c) + 1)/d - b^(3/2)*log(e^(-d*
x - c) - 1)/d - 2*b^(3/2)*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d
*x - 4*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.48

$$\int \left(b\coth^2(c+dx)\right)^{3/2}\,dx=-\frac{\left((dx+c)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)+\frac{2\,e^{(2\,dx+2\,c)}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}\right)b^{\frac{3}{2}}}{d}$$

```
[In] integrate((b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1) + 2*e^(2*d*x + 2*c)*sgn(e^(4*d*x + 4*c) - 1)/(e^(2*d*x +
 2*c) - 1)^2)*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(b \coth^2(c+dx)\right)^{3/2} \, dx = \int \left(b \coth(c+d\,x)^2\right)^{3/2} dx$$

```
[In] int((b*coth(c + d*x)^2)^(3/2),x)
[Out] int((b*coth(c + d*x)^2)^(3/2), x)
```

3.19 $\int \sqrt{b\coth^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \sqrt{b\coth^2(c+dx)}\,dx = \frac{\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))\tanh(c+dx)}{d}$$

```
[Out] ln(sinh(d*x+c))*(b*coth(d*x+c)^2)^(1/2)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3739, 3556}

$$\int \sqrt{b\coth^2(c+dx)}\,dx = \frac{\tanh(c+dx)\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))}{d}$$

```
[In] Int[Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Sqrt[b*Coth[c + d*x]^2]*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\sqrt{b\coth^2(c+dx)}\tanh(c+dx)\right)\int \coth(c+dx)\,dx$$
$$= \frac{\sqrt{b\coth^2(c+dx)}\log(\sinh(c+dx))\tanh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \sqrt{b\coth^2(c+dx)}\,dx$$
$$= \frac{\sqrt{b\coth^2(c+dx)}(\log(\cosh(c+dx))+\log(\tanh(c+dx)))\tanh(c+dx)}{d}$$

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^2],x]
[Out] (Sqrt[b*Coth[c + d*x]^2]*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c +
 d*x])/d
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.45

method	result
derivativedivides	$-\frac{\sqrt{\coth(dx+c)^2 b}\,(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1))}{2d\coth(dx+c)}$
default	$-\frac{\sqrt{\coth(dx+c)^2 b}\,(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1))}{2d\coth(dx+c)}$
risch	$\frac{\sqrt{\frac{(e^{2dx+2c}+1)^2 b}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)x}{e^{2dx+2c}+1} - \frac{2\sqrt{\frac{(e^{2dx+2c}+1)^2 b}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)(dx+c)}{(e^{2dx+2c}+1)d} + \frac{\sqrt{\frac{(e^{2dx+2c}+1)^2 b}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)\ln(e^{2dx}}{(e^{2dx+2c}+1)d}$

```
[In] int((coth(d*x+c)^2*b)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(coth(d*x+c)^2*b)^(1/2)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1))/coth(d
*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.03

$$\int \sqrt{b\coth^2(c+dx)}\,dx = -\frac{\left(dxe^{(2\,dx+2\,c)} - dx - \left(e^{(2\,dx+2\,c)} - 1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{de^{(2\,dx+2\,c)}+d}$$

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(d*e^(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \sqrt{b\coth^2(c+dx)}\,dx = \int \sqrt{b\coth^2{(c+dx)}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{b\coth^2(c+dx)}\,dx = -\frac{(dx+c)\sqrt{b}}{d} - \frac{\sqrt{b}\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\sqrt{b}\log\left(e^{(-dx-c)}-1\right)}{d}$$

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(d*x + c)*sqrt(b)/d - sqrt(b)*log(e^(-d*x - c) + 1)/d - sqrt(b)*log(e^(-d*
x - c) - 1)/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \sqrt{b\coth^2(c+dx)}\,dx = -\frac{\left((dx+c)\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)-\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)\right)\sqrt{b}}{d}$$

```
[In] integrate((b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] -((d*x + c)*sgn(e^(4*d*x + 4*c) - 1) - log(abs(e^(2*d*x + 2*c) - 1))*sgn(e^
(4*d*x + 4*c) - 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\coth^2(c+dx)}\,dx = \int \sqrt{b\,\mathrm{coth}\left(c+d\,x\right)^2}\,dx$$

```
[In] int((b*coth(c + d*x)^2)^(1/2),x)
[Out] int((b*coth(c + d*x)^2)^(1/2), x)
```

3.20 $\int \frac{1}{\sqrt{b \coth^2(c+dx)}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt{b \coth^2(c+dx)}} \, dx = \frac{\coth(c+dx) \log(\cosh(c+dx))}{d\sqrt{b \coth^2(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3739, 3556}

$$\int \frac{1}{\sqrt{b \coth^2(c+dx)}} \, dx = \frac{\coth(c+dx) \log(\cosh(c+dx))}{d\sqrt{b \coth^2(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^2],x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\coth(c+dx)\int\tanh(c+dx)\,dx}{\sqrt{b\coth^2(c+dx)}}$$
$$= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int\frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt{b\coth^2(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^2],x]
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*Sqrt[b*Coth[c + d*x]^2])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

method	result	size
derivativedivides	$-\frac{\coth(dx+c)(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)-2\ln(\coth(dx+c)))}{2d\sqrt{\coth(dx+c)^2b}}$	52
default	$-\frac{\coth(dx+c)(\ln(\coth(dx+c)-1)+\ln(\coth(dx+c)+1)-2\ln(\coth(dx+c)))}{2d\sqrt{\coth(dx+c)^2b}}$	52
risch	$\frac{(e^{2dx+2c}+1)x}{\sqrt{\frac{(e^{2dx+2c}+1)^2b}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)}-\frac{2(e^{2dx+2c}+1)(dx+c)}{\sqrt{\frac{(e^{2dx+2c}+1)^2b}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)d}+\frac{(e^{2dx+2c}+1)\ln(e^{2dx+2c}+1)}{\sqrt{\frac{(e^{2dx+2c}+1)^2b}{(e^{2dx+2c}-1)^2}}\,(e^{2dx+2c}-1)d}$	192

```
[In] int(1/(coth(d*x+c)^2*b)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)-1)+ln(coth(d*x+c)+1)-2*ln(coth(d*x+c)))/
(coth(d*x+c)^2*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 4.13

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = -\frac{\left(dxe^{(2\,dx+2\,c)} - dx - \left(e^{(2\,dx+2\,c)} - 1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\sqrt{\frac{be^{(4\,dx+4\,c)}+2\,be^{(2\,dx+2\,c)}+b}{e^{(4\,dx+4\,c)}-2\,e^{(2\,dx+2\,c)}+1}}}{bde^{(2\,dx+2\,c)} + bd}$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="fricas")
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*cosh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*sqrt((b*e^(4*d*x + 4*c) + 2*b*e^(2*d*x + 2*c
) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b*d*e^(2*d*x + 2*c) + b*
d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth^2\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/2),x)
[Out] Integral(1/sqrt(b*coth(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = -\frac{dx+c}{\sqrt{b}d} - \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{\sqrt{b}d}$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="maxima")
[Out] -(d*x + c)/(sqrt(b)*d) - log(e^(-2*d*x - 2*c) + 1)/(sqrt(b)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = -\frac{\frac{dx+c}{\sqrt{b}\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)} - \frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{d}$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/2),x, algorithm="giac")
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(
sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\coth^2(c+dx)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\coth(c+d\,x)}{\sqrt{b\coth(c+d\,x)^2}}\right)}{\sqrt{b}\,d}$$

```
[In] int(1/(b*coth(c + d*x)^2)^(1/2),x)
[Out] atanh((b^(1/2)*coth(c + d*x))/(b*coth(c + d*x)^2)^(1/2))/(b^(1/2)*d)
```

3.21 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/b/d/(b*coth(d*x+c)^2)^(1/2)-1/2*tanh(d*x+c)/b/d
/(b*coth(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 3556}

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(-3/2),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(b*d*Sqrt[b*Coth[c + d*x]^2]) - Tanh[c +
 d*x]/(2*b*d*Sqrt[b*Coth[c + d*x]^2])
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth(c+dx)\int\tanh^3(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}\\
&= -\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}+\frac{\coth(c+dx)\int\tanh(c+dx)\,dx}{b\sqrt{b\coth^2(c+dx)}}\\
&= \frac{\coth(c+dx)\log(\cosh(c+dx))}{bd\sqrt{b\coth^2(c+dx)}}-\frac{\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int\frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\frac{2\coth(c+dx)\log(\cosh(c+dx))-\tanh(c+dx)}{2bd\sqrt{b\coth^2(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-3/2),x]
[Out] (2*Coth[c + d*x]*Log[Cosh[c + d*x]] - Tanh[c + d*x])/(2*b*d*Sqrt[b*Coth[c +
 d*x]^2])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

method	result
derivativedivides	$-\frac{\coth(dx+c)\left(\ln(\coth(dx+c)+1)\coth(dx+c)^2+\ln(\coth(dx+c)-1)\coth(dx+c)^2-2\ln(\coth(dx+c))\coth(dx+c)^2+1\right)}{2d\left(\coth(dx+c)^2 b\right)^{\frac{3}{2}}}$
default	$-\frac{\coth(dx+c)\left(\ln(\coth(dx+c)+1)\coth(dx+c)^2+\ln(\coth(dx+c)-1)\coth(dx+c)^2-2\ln(\coth(dx+c))\coth(dx+c)^2+1\right)}{2d\left(\coth(dx+c)^2 b\right)^{\frac{3}{2}}}$
risch	$\frac{-e^{4dx+4c}dx+e^{4dx+4c}\ln\left(e^{2dx+2c}+1\right)-2\,e^{4dx+4c}c-2\,e^{2dx+2c}dx+2\,e^{2dx+2c}\ln\left(e^{2dx+2c}+1\right)-4\,e^{2dx+2c}c-dx+2\,e^{2dx+2c}}{b\left(e^{2dx+2c}+1\right)\left(e^{2dx+2c}-1\right)\sqrt{\frac{\left(e^{2dx+2c}+1\right)^2 b}{\left(e^{2dx+2c}-1\right)^2}}\,d}$

```
[In] int(1/(coth(d*x+c)^2*b)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)+1)*coth(d*x+c)^2+ln(coth(d*x+c)-1)*coth(
d*x+c)^2-2*ln(coth(d*x+c))*coth(d*x+c)^2+1)/(coth(d*x+c)^2*b)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(59) = 118$.

Time = 0.27 (sec) , antiderivative size = 817, normalized size of antiderivative = 12.57

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^4 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^4 - 4*(d*x
*cosh(d*x + c)*e^(2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(d*
x - 1)*cosh(d*x + c)^2 + 2*(3*d*x*cosh(d*x + c)^2 + d*x - (3*d*x*cosh(d*x +
 c)^2 + d*x - 1)*e^(2*d*x + 2*c) - 1)*sinh(d*x + c)^2 + d*x - (d*x*cosh(d*x
 + c)^4 + 2*(d*x - 1)*cosh(d*x + c)^2 + d*x)*e^(2*d*x + 2*c) + ((e^(2*d*x +
 2*c) - 1)*sinh(d*x + c)^4 - cosh(d*x + c)^4 + 4*(cosh(d*x + c)*e^(2*d*x +
2*c) - cosh(d*x + c))*sinh(d*x + c)^3 - 2*(3*cosh(d*x + c)^2 - (3*cosh(d*x
+ c)^2 + 1)*e^(2*d*x + 2*c) + 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + (cos
h(d*x + c)^4 + 2*cosh(d*x + c)^2 + 1)*e^(2*d*x + 2*c) - 4*(cosh(d*x + c)^3
- (cosh(d*x + c)^3 + cosh(d*x + c))*e^(2*d*x + 2*c) + cosh(d*x + c))*sinh(d
*x + c) - 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(d*x*
cosh(d*x + c)^3 + (d*x - 1)*cosh(d*x + c) - (d*x*cosh(d*x + c)^3 + (d*x - 1
)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))*sqrt((b*e^(4*d*x + 4*c) +
2*b*e^(2*d*x + 2*c) + b)/(e^(4*d*x + 4*c) - 2*e^(2*d*x + 2*c) + 1))/(b^2*d*
cosh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + (b^2*d*e^(2*d*x + 2*c) + b^2*d)
```

```
*sinh(d*x + c)^4 + 4*(b^2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x
+ c))*sinh(d*x + c)^3 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d + (3*b^2
*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(
d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 4*(b^2*d*co
sh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**2)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=-\frac{2\sqrt{b}e^{(-2\,dx-2\,c)}}{(2\,b^2e^{(-2\,dx-2\,c)}+b^2e^{(-4\,dx-4\,c)}+b^2)d}-\frac{dx+c}{b^{\frac{3}{2}}d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{3}{2}}d}$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="maxima")
[Out] -2*sqrt(b)*e^(-2*d*x - 2*c)/((2*b^2*e^(-2*d*x - 2*c) + b^2*e^(-4*d*x - 4*c)
 + b^2)*d) - (d*x + c)/(b^(3/2)*d) - log(e^(-2*d*x - 2*c) + 1)/(b^(3/2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx=-\frac{\frac{dx+c}{\sqrt{b}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{\log\left(e^{(2\,dx+2\,c)}+1\right)}{\sqrt{b}\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}-\frac{2\,e^{(2\,dx+2\,c)}}{\sqrt{b}\left(e^{(2\,dx+2\,c)}+1\right)^2\mathrm{sgn}\left(e^{(4\,dx+4\,c)}-1\right)}}{bd}$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -((d*x + c)/(sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - log(e^(2*d*x + 2*c) + 1)/(
sqrt(b)*sgn(e^(4*d*x + 4*c) - 1)) - 2*e^(2*d*x + 2*c)/(sqrt(b)*(e^(2*d*x +
2*c) + 1)^2*sgn(e^(4*d*x + 4*c) - 1)))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{3/2}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^2)^(3/2),x)
```

```
[Out] int(1/(b*coth(c + d*x)^2)^(3/2), x)
```

3.22 $\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 297

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} + \frac{b\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)} + \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$

```
[Out] b*arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)-3/
5*b*coth(d*x+c)*(b*coth(d*x+c)^2)^(1/3)/d-1/4*b*(b*coth(d*x+c)^2)^(1/3)*ln(
1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(2/3)+1/4*b*(b*coth(d*
x+c)^2)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(2/3)
+1/2*b*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(1/3)*
3^(1/2)/d/coth(d*x+c)^(2/3)-1/2*b*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2
))*(b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}b\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} + \frac{b\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)} + \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] (Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^
(1/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x]^(
1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (b*Ar
cTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3
)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^2)^(1/3))/(5*d) - (b*(b*Coth[c + d
*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[
c + d*x]^(2/3)) + (b*(b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c + d*x]^(1/3)
+ Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int\coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}\\
&= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}+\frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\int\coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}\\
&= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}\\
&\quad-\frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\operatorname{Subst}\left(\int\frac{x^{2/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d\coth^{\frac{2}{3}}(c+dx)}\\
&= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}\\
&\quad-\frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\operatorname{Subst}\left(\int\frac{x^4}{-1+x^6}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}
\end{aligned}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{b\,\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d} - \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)} + \frac{\left(b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)} - \frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)} - \frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{b\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}$$
$$- \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$
$$+ \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$
$$+ \frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{2}{3}}(c+dx)}$$
$$+ \frac{\left(3b\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{2}{3}}(c+dx)}$$

$$= \frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}$$
$$- \frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)}$$
$$+ \frac{b\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)}$$
$$- \frac{3b\coth(c+dx)\sqrt[3]{b\coth^2(c+dx)}}{5d}$$
$$- \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$
$$+ \frac{b\sqrt[3]{b\coth^2(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.56

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx = -\frac{\left(b\coth^2(c+dx)\right)^{4/3}\left(-20\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)+12\coth^{\frac{5}{3}}(c+dx)-5\left(2\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+}}{\sqrt{3}}\right.\right.\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(4/3),x]
```

```
[Out] -1/20*((b*Coth[c + d*x]^2)^(4/3)*(-20*ArcTanh[Coth[c + d*x]^(1/3)] + 12*Cot
h[c + d*x]^(5/3) - 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]
 - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c +
 d*x]^(1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c +
 d*x]^(2/3)])))/(d*Coth[c + d*x]^(8/3))
```

Maple [F]

$$\int \left(\coth(dx+c)^2 b\right)^{\frac{4}{3}} dx$$

```
[In] int((coth(d*x+c)^2*b)^(4/3),x)
```

```
[Out] int((coth(d*x+c)^2*b)^(4/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1994 vs. 2(245) = 490.

Time = 0.29 (sec) , antiderivative size = 1994, normalized size of antiderivative = 6.71

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*(-b)^(1/3)*arctan(1/3*(sqrt(3)
*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*si
nh(d*x + c)^2 + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d
*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2
 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) +
sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) - 10*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*
```

```
sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - sqrt(3)*b)*b^(1/3)*arctan(-1/3*
(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt
(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c
)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2/3)*((b*cosh(d*x +
 c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/
3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*si
nh(d*x + c)^2 + b)) + 5*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x +
c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x +
c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6
*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d
*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cos
h(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x
+ c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x
 + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x
+ c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + s
inh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) + 5*(b
*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)
*b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4
*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (cosh(d*x +
 c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2
 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*((b*cosh(
d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1)
)^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c
)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(co
sh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*s
inh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d
*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + c
osh(d*x + c))*sinh(d*x + c) + 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*(-b)^(1/3)*log(((cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (cosh(d
*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d
*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))
^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
+ 1)) - 10*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 - b)*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(b*cosh(d*x + c)^2 + 2*
```

```
b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(d
*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F]

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx=\int \left(b\coth^2\left(c+dx\right)\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**2)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(4/3), x)
```

Maxima [F]

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx=\int \left(b\coth\left(dx+c\right)^2\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

Giac [F]

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx=\int \left(b\coth\left(dx+c\right)^2\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^2(c+dx)\right)^{4/3}\,dx=\int \left(b\coth(c+d\,x)^2\right)^{4/3}\,dx$$

```
[In] int((b*coth(c + d*x)^2)^(4/3),x)
[Out] int((b*coth(c + d*x)^2)^(4/3), x)
```

3.23 $\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 289

$$\begin{aligned}\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx =& -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} \\ &+\frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} \\ &+\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} \\ &-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} \\ &+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} \\ &-\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d}\end{aligned}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(2/3)/d/coth(d*x+c)^(4/3)-1/4*
(b*coth(d*x+c)^2)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(4/3)+1/4*(b*coth(d*x+c)^2)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(4/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^2)^(2/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(2/3)*3^(1/2)/d/coth(d*x+c)^(4/3)
-3*(b*coth(d*x+c)^2)^(2/3)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$\int \left(b\coth^2(c+dx)\right)^{2/3} dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\tanh(c+dx)\left(b\coth^2(c+dx)\right)^{2/3}}{d} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^
2)^(2/3))/(d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(2/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTa
nh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(2/3))/(d*Coth[c + d*x]^(4/3))
- ((b*Coth[c + d*x]^2)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2
/3)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^2)^(2/3)*Log[1 + Coth[c
 + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Cot
h[c + d*x]^2)^(2/3)*Tanh[c + d*x])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} - \frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$=\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)}-\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d}-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\text{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$

$$=\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)}-\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}+\frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}-\frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d}-\frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{4}{3}}(c+dx)}-\frac{\left(3\left(b\coth^2(c+dx)\right)^{2/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^2(c+dx)\right)^{2/3}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} + \frac{\left(b\coth^2(c+dx)\right)^{2/3}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\left(b\coth^2(c+dx)\right)^{2/3}\tanh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.69

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx = -\frac{\left(b\coth^2(c+dx)\right)^{2/3}\left(6\sqrt[6]{\coth^2(c+dx)}+\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)-(-1$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(2/3),x]
```

```
[Out] -1/2*((b*Coth[c + d*x]^2)^(2/3)*(6*(Coth[c + d*x]^2)^(1/6) + Log[1 - (Coth[
c + d*x]^2)^(1/6)] - Log[1 + (Coth[c + d*x]^2)^(1/6)] - (-1)^(2/3)*Log[1 -
(-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(2/3)*Log[1 + (-1)^(1/3)*(Coth[c
 + d*x]^2)^(1/6)] - (-1)^(1/3)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]
+ (-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)])*Tanh[c + d*x])/(d
*(Coth[c + d*x]^2)^(1/6))
```

Maple [F]

$$\int \left(\coth \left(dx+c\right)^{2} b\right)^{\frac{2}{3}}dx$$

```
[In] int((coth(d*x+c)^2*b)^(2/3),x)
```

```
[Out] int((coth(d*x+c)^2*b)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(239) = 478$.

Time = 0.32 (sec) , antiderivative size = 2037, normalized size of antiderivative = 7.05

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*cosh
(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x
+ c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
 + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*((b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(
3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 + b)) + 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x +
 c) + sqrt(3)*sinh(d*x + c)^2 + sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b
*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh
(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x
 + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*((b*cosh(d*x + c)^2
+ b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + s
qrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + b)) + (-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(-b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 -
b*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^
2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*
cosh(d*x + c))*sinh(d*x + c) + b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d
*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(
```

```
d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d
*x + c) + 1)) + (b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*log(-((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*
x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^
3 + sinh(d*x + c)^4 - 1)*(b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^
2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b
*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2
+ b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2
 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*c
osh(d*x + c))*sinh(d*x + c) + b)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x
 + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*
x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x
 + c) + 1)) - 2*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 + 1)*log(-((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh
(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*(b^2)^(1
/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*log(((b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d
*x + c)^2 + 1) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x
+ c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)
^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(d*cosh(d*x + c)^2
+ 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [F]

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx = \int \left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**2)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(2/3), x)
```

Maxima [F]

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

Giac [F]

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^2(c+dx)\right)^{2/3}\,dx = \int \left(b\,\mathrm{coth}(c+d\,x)^2\right)^{2/3}\,dx$$

```
[In] int((b*coth(c + d*x)^2)^(2/3),x)
[Out] int((b*coth(c + d*x)^2)^(2/3), x)
```

3.24 $\int \sqrt[3]{b\coth^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 264

$$\begin{aligned}\int \sqrt[3]{b\coth^2(c+dx)}\,dx =& \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} \\ &- \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} \\ &+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} \\ &- \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)} \\ &+ \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}\end{aligned}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^2)^(1/3)/d/coth(d*x+c)^(2/3)-1/4*
(b*coth(d*x+c)^2)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(2/3)+1/4*(b*coth(d*x+c)^2)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(2/3)+1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)-1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^2)^(1/3)*3^(1/2)/d/coth(d*x+c)^(2/3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3739, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)} + \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{2}{3}}(c+dx)}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1
/3))/(2*d*Coth[c + d*x]^(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^2)^(1/3))/(2*d*Coth[c + d*x]^(2/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^2)^(1/3))/(d*Coth[c + d*x]^(2/3)) - (
(b*Coth[c + d*x]^2)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(2/3)) + ((b*Coth[c + d*x]^2)^(1/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{b\coth^2(c+dx)}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{2}{3}}(c+dx)}\\
&= -\frac{\sqrt[3]{b\coth^2(c+dx)}\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\coth^{\frac{2}{3}}(c+dx)}\\
&= -\frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}\\
&= \frac{\sqrt[3]{b\coth^2(c+dx)}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}\\
&\quad+\frac{\sqrt[3]{b\coth^2(c+dx)}\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}\\
&\quad+\frac{\sqrt[3]{b\coth^2(c+dx)}\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{2}{3}}(c+dx)}\\
&= \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)}\\
&\quad-\frac{\sqrt[3]{b\coth^2(c+dx)}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)}\\
&\quad+\frac{\sqrt[3]{b\coth^2(c+dx)}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)}\\
&\quad-\frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)}\\
&\quad-\frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{2}{3}}(c+dx)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad - \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad + \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad + \frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad + \frac{\left(3\sqrt[3]{b\coth^2(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{2}{3}}(c+dx)} \\
&= \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad - \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^2(c+dx)}}{2d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^2(c+dx)}}{d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad - \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)} \\
&\quad + \frac{\sqrt[3]{b\coth^2(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{2}{3}}(c+dx)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.57

$$
\int\sqrt[3]{b\coth^2(c+dx)}\,dx
$$

$$
= \frac{\sqrt[3]{b\coth^2(c+dx)}\left(2\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)-2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)+4\operatorname{arctanh}\left(\sqrt[3]{\right.}\right.}{4d\coth^{\frac{2}{3}}}
$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^2)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sq
rt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] + 4*ArcTanh[
Coth[c + d*x]^(1/3)] - Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)] +
 Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)]))/(4*d*Coth[c + d*x]^(2
/3))
```

Maple [F]

$$\int \left(\coth\left(dx+c\right)^{2} b\right)^{\frac{1}{3}} dx$$

```
[In] int((coth(d*x+c)^2*b)^(1/3),x)
```

```
[Out] int((coth(d*x+c)^2*b)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1618 vs. $2(216) = 432$.

Time = 0.28 (sec) , antiderivative size = 1618, normalized size of antiderivative = 6.13

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3
)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 + 2*(sqrt(3)*co
sh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x +
c)^2 - sqrt(3))*(-b)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(co
sh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2*sqrt(3)*b^
(1/3)*arctan(-1/3*(sqrt(3)*b*cosh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c) + sqrt(3)*b*sinh(d*x + c)^2 - 2*(sqrt(3)*cosh(d*x + c)^2 + 2*sq
rt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*b^(2
/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
x + c)^2 - 1))^(1/3) + sqrt(3)*b)/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si
nh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (-b)^(1/3)*log(((cosh(d*x + c)^4 +
4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 +
1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c)
)*sinh(d*x + c) + 1)*(-b)^(2/3) - (cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4 - 1)*(-b)^(1/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c
)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (cosh(d*x + c)^4
+ 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2
- 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x +
```

```
c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d
*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)
*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c
)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)) + b^(1/3)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 +
sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x +
c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*b^(2/3) - (co
sh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*
x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*b^(1/3)*(
(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c
)^2 - 1))^(1/3) + (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*((b*cosh(d*x + c)
^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3))
/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(
3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x +
c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b)^(1/3)*log(((cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(1/3) + (co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*co
sh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 -
 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c
)^2 + 1)) - 2*b^(1/3)*log(((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2 + 1)*b^(1/3) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)
/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F]

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \int \sqrt[3]{b\coth^2\left(c+dx\right)}\,dx$$

```
[In] integrate((b*coth(d*x+c)**2)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^2\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\coth^2(c+dx)}\,dx = \int \left(b\coth(c+d\,x)^2\right)^{1/3}\,dx$$

```
[In] int((b*coth(c + d*x)^2)^(1/3),x)
[Out] int((b*coth(c + d*x)^2)^(1/3), x)
```

3.25 $\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 264

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(2/3)/d/(b*coth(d*x+c)^2)^(1/3)-1/4*
coth(d*x+c)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c
)^2)^(1/3)+1/4*coth(d*x+c)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/
d/(b*coth(d*x+c)^2)^(1/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*c
oth(d*x+c)^(2/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3739, 3557, 335, 216, 648, 632, 210, 642, 212}

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = -\frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/
3))/(d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*d*(b*Coth[c + d*x]^2)^(1/3)) + (ArcTa
nh[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(d*(b*Coth[c + d*x]^2)^(1/3))
- (Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/
(4*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth[c + d*x
]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
```

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\coth^{\frac{2}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx,x,\coth(c+dx)\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{d\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx =$$

$$-\frac{\coth(c+dx)\left(\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)+\sqrt[3]{-1}\left(-\sqrt[3]{-1}\log\left(1-\sqrt[3]{-\right.\right.\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-1/3),x]
```

```
[Out] -1/2*(Coth[c + d*x]*(Log[1 - (Coth[c + d*x]^2)^(1/6)] - Log[1 + (Coth[c + d
*x]^2)^(1/6)] + (-1)^(1/3)*(-((-1)^(1/3)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^
2)^(1/6)]) + (-1)^(1/3)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - Log[1
 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)] + Log[1 + (-1)^(2/3)*(Coth[c + d*x]^
2)^(1/6)])))/(d*(Coth[c + d*x]^2)^(1/6)*(b*Coth[c + d*x]^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(\coth\left(dx+c\right)^{2} b\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(coth(d*x+c)^2*b)^(1/3),x)
```

```
[Out] int(1/(coth(d*x+c)^2*b)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1594 vs. 2(216) = 432.

Time = 0.41 (sec) , antiderivative size = 8338, normalized size of antiderivative = 31.58

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth^2{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**2)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\left(b\coth{(dx+c)}^2\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\left(b\coth{(dx+c)}^2\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\coth^2(c+dx)}}\,dx = \int \frac{1}{\left(b\coth{(c+d\,x)}^2\right)^{1/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^2)^(1/3),x)
[Out] int(1/(b*coth(c + d*x)^2)^(1/3), x)
```

3.26 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 289

$$\begin{aligned}\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = & -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} \\ & +\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}} \\ & -\frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}} \\ & +\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} \\ & -\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}} \\ & +\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}\end{aligned}$$

```
[Out] -3*coth(d*x+c)/d/(b*coth(d*x+c)^2)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*
x+c)^(4/3)/d/(b*coth(d*x+c)^2)^(2/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*x+c)
^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^2)^(2/3)+1/4*coth(d*x+c)^(4/3)*l
n(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^2)^(2/3)+1/2*arct
an(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/d/(b*coth
(d*x+c)^2)^(2/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c
)^(4/3)*3^(1/2)/d/(b*coth(d*x+c)^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^2)^(2/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^2)^
(2/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^2)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^2)^(2/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^2)^(2
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^2)^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$

$$
\begin{aligned}
&= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad-\frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad+\frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad-\frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad-\frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}+\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad+\frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}\\
&\quad+\frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}
\end{aligned}
$$

$$= -\frac{3\coth(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}} + \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}$$
$$- \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\left(b\coth^2(c+dx)\right)^{2/3}}$$
$$+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\left(b\coth^2(c+dx)\right)^{2/3}}$$
$$- \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}$$
$$+ \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^2(c+dx)\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx =$$
$$-\frac{\coth(c+dx)\left(6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth^2(c+dx)}\log\left(1+\sqrt[6]{\coth^2(c+dx\right.\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-2/3),x]
```

```
[Out] -1/2*(Coth[c + d*x]*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)^
(1/6)] - (Coth[c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^(1
/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - (
-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6
)] + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2
)^(1/6)] - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Coth[c +
d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^2)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(\coth\left(dx+c\right)^{2} b\right)^{\frac{2}{3}}}dx$$

[In] int(1/(coth(d*x+c)^2*b)^(2/3),x)

[Out] int(1/(coth(d*x+c)^2*b)^(2/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2066 vs. $2(239) = 478$.

Time = 0.31 (sec) , antiderivative size = 2066, normalized size of antiderivative = 7.15

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \text{Too large to display}$$

[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="fricas")

[Out] 1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b^2)^(1/3))*arctan(1/3*(2*sqrt(3)*(-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-(-b^2)^(1/3))*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(-b^2)^(1/3)*sqrt(-(-b^2)^(1/3)))/(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)) + 2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/6)*arctan(1/3*sqrt(3)*(b^2)^(1/6)*(2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*(b^2)^(1/3))/(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)) + (-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(((cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(-b^2)^(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(1/3) + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(

```
d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
+ 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - (b^2)^(2/3)*(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-(
(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*(b^2)^
(2/3)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(
d*x + c)^2 - 1))^(1/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b
)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) +
 b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*
x + c)^2 - 1))^(2/3) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*
sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b
)*(b^2)^(1/3))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*
x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)) - 2*(-b^2)^(2/3)*(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(-
((-b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)
^2 + sinh(d*x + c)^2 - 1))^(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2 + 1)) + 2*(b^2)^(2/3)*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*log(((b^2)^(2/3)*(cosh(d*x + c
)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1) + (b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*((b*cosh(d*
x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2 + sinh(d*x + c)^2 - 1))^
(1/3))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
 1)) - 12*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*((b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + b)/(cosh(d*x + c)^2
+ sinh(d*x + c)^2 - 1))^(1/3))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x +
c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2 + b^2*d)
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth^2\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**2)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**2)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{2/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^2)^(2/3),x)
[Out] int(1/(b*coth(c + d*x)^2)^(2/3), x)
```

3.27 $\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 309

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*coth(d*x+c)^(2/3)/b/d/(b*coth(d*x+c)^2)^(1/3)-1/
4*coth(d*x+c)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d
*x+c)^2)^(1/3)+1/4*coth(d*x+c)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/
3))/b/d/(b*coth(d*x+c)^2)^(1/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1
/2))*coth(d*x+c)^(2/3)*3^(1/2)/b/d/(b*coth(d*x+c)^2)^(1/3)+1/2*arctan(1/3*(
1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(2/3)*3^(1/2)/b/d/(b*coth(d*x+c
)^2)^(1/3)-3/5*tanh(d*x+c)/b/d/(b*coth(d*x+c)^2)^(1/3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 216, 648, 632, 210, 642, 212}

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = -\frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\coth^{\frac{2}{3}}(c+dx)\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^2)^(-4/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/
3))/(b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^
(1/3))/Sqrt[3]]*Coth[c + d*x]^(2/3))/(2*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (A
rcTanh[Coth[c + d*x]^(1/3)]*Coth[c + d*x]^(2/3))/(b*d*(b*Coth[c + d*x]^2)^(
1/3)) - (Coth[c + d*x]^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2
/3)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) + (Coth[c + d*x]^(2/3)*Log[1 + Coth
[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^2)^(1/3)) -
 (3*Tanh[c + d*x])/(5*b*d*(b*Coth[c + d*x]^2)^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth^{\frac{2}{3}}(c+dx)\int\frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}} \\
&= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\int\frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^2(c+dx)}} \\
&= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}-\frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int\frac{1}{x^{2/3}(-1+x^2)}\,dx,x,\coth(c+dx)\right)}{bd\sqrt[3]{b\coth^2(c+dx)}} \\
&= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}-\frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\text{Subst}\left(\int\frac{1}{-1+x^6}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}} \\
&= -\frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}+\frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}} \\
&\quad+\frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int\frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}} \\
&\quad+\frac{\coth^{\frac{2}{3}}(c+dx)\text{Subst}\left(\int\frac{1+\frac{x}{2}}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^2(c+dx)}}
\end{aligned}$$

$$= \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$- \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$+ \frac{\coth^{\frac{2}{3}}(c+dx)\operatorname{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$+ \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$+ \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$- \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$+ \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$- \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$- \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{\coth(c+dx)}\right)}{2bd\sqrt[3]{b\coth^2(c+dx)}}$$
$$- \frac{\left(3\coth^{\frac{2}{3}}(c+dx)\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{\coth(c+dx)}\right)}{2bd\sqrt[3]{b\coth^2(c+dx)}}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{2}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{2}{3}}(c+dx)}{bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}} + \frac{\coth^{\frac{2}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^2(c+dx)}} - \frac{3\tanh(c+dx)}{5bd\sqrt[3]{b\coth^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = -\frac{\coth(c+dx)\left(6+5\coth^2(c+dx)^{5/6}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-5\coth^2(c+dx)^{5/6}\log\left(1+\sqrt[6]{\coth^2(c\right.\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x]^2)^(-4/3),x]
```

```
[Out] -1/10*(Coth[c + d*x]*(6 + 5*(Coth[c + d*x]^2)^(5/6)*Log[1 - (Coth[c + d*x]^
2)^(1/6)] - 5*(Coth[c + d*x]^2)^(5/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] - 5*
(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/
6)] + 5*(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x
]^2)^(1/6)] - 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(2/3)*(Coth
[c + d*x]^2)^(1/6)] + 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-1)^(2/
3)*(Coth[c + d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^2)^(4/3))
```

Maple [F]

$$\int \frac{1}{\left(\coth \left(dx+c\right)^{2} b\right)^{\frac{4}{3}}}dx$$

[In] int(1/(coth(d*x+c)^2*b)^(4/3),x)

[Out] int(1/(coth(d*x+c)^2*b)^(4/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3099 vs. 2(257) = 514.

Time = 0.43 (sec) , antiderivative size = 14359, normalized size of antiderivative = 46.47

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\, dx = \text{Too large to display}$$

[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\, dx = \int \frac{1}{\left(b\coth^{2}\left(c+dx\right)\right)^{\frac{4}{3}}}\, dx$$

[In] integrate(1/(b*coth(d*x+c)**2)**(4/3),x)

[Out] Integral((b*coth(c + d*x)**2)**(-4/3), x)

Maxima [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\, dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^{2}\right)^{\frac{4}{3}}}\, dx$$

[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)

Giac [F]

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^2\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^2)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^2)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^2(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^2\right)^{4/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^2)^(4/3),x)
[Out] int(1/(b*coth(c + d*x)^2)^(4/3), x)
```

3.28 $\int \left(b\coth^3(c+dx)\right)^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 55

$$\int \left(b\coth^3(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3n),\frac{3(1+n)}{2},\coth^2(c+dx)\right)}{d(1+3n)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^3)^n*hypergeom([1, 1/2+3/2*n],[3/2+3/2*n],coth(d
*x+c)^2)/d/(1+3*n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3739, 3557, 371}

$$\int \left(b\coth^3(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(3n+1),\frac{3(n+1)}{2},\coth^2(c+dx)\right)}{d(3n+1)}$$

```
[In] Int[(b*Coth[c + d*x]^3)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\int\coth^{3n}(c+dx)\,dx\\ &= -\frac{\left(\coth^{-3n}(c+dx)\left(b\coth^3(c+dx)\right)^n\right)\text{Subst}\left(\int\frac{x^{3n}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\ &= \frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3n),\frac{3(1+n)}{2},\coth^2(c+dx)\right)}{d(1+3n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int\left(b\coth^3(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^3(c+dx)\right)^n\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+3n),\frac{3(1+n)}{2},\coth^2(c+dx)\right)}{d(1+3n)}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^n,x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^3)^n*Hypergeometric2F1[1, (1 + 3*n)/2, (3*(
1 + n))/2, Coth[c + d*x]^2])/(d*(1 + 3*n))
```

Maple [F]

$$\int \left(b \coth \left(dx + c\right)^{3}\right)^{n} dx$$

[In] int((b*coth(d*x+c)^3)^n,x)

[Out] int((b*coth(d*x+c)^3)^n,x)

Fricas [F]

$$\int \left(b \coth^3(c + dx)\right)^n \, dx = \int \left(b \coth \left(dx + c\right)^{3}\right)^{n} \, dx$$

[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="fricas")

[Out] integral((b*coth(d*x + c)^3)^n, x)

Sympy [F]

$$\int \left(b \coth^3(c + dx)\right)^n \, dx = \int \left(b \coth^{3}\left(c + dx\right)\right)^{n} \, dx$$

[In] integrate((b*coth(d*x+c)**3)**n,x)

[Out] Integral((b*coth(c + d*x)**3)**n, x)

Maxima [F]

$$\int \left(b \coth^3(c + dx)\right)^n \, dx = \int \left(b \coth \left(dx + c\right)^{3}\right)^{n} \, dx$$

[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^3)^n, x)

Giac [F]

$$\int \left(b\coth^3(c+dx)\right)^n \, dx = \int \left(b\coth\left(dx+c\right)^3\right)^n \, dx$$

```
[In] integrate((b*coth(d*x+c)^3)^n,x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^3)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^n \, dx = \int \left(b\coth(c+d\,x)^3\right)^n \, dx$$

```
[In] int((b*coth(c + d*x)^3)^n,x)
[Out] int((b*coth(c + d*x)^3)^n, x)
```

3.29 $\int (b\coth^3(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 134

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx = -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{b\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{b\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}$$

```
[Out] -2/3*b*(b*coth(d*x+c)^3)^(1/2)/d-b*arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c)
^3)^(1/2)/d/coth(d*x+c)^(3/2)+b*arctanh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3
)^(1/2)/d/coth(d*x+c)^(3/2)-2/7*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3739, 3554, 3557, 335, 304, 209, 212}

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx = -\frac{b\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{b\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} - \frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(3/2),x]
```

```
[Out] (-2*b*Sqrt[b*Coth[c + d*x]^3])/(3*d) - (b*ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[
b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) + (b*ArcTanh[Sqrt[Coth[c + d*x]
]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2)) - (2*b*Coth[c + d*x]^2*
Sqrt[b*Coth[c + d*x]^3])/(7*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int\coth^{\frac{9}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}\\
&= -\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}+\frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int\coth^{\frac{5}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)}\\
&= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}\\
&\quad+\frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\int\sqrt{\coth(c+dx)}\,dx}{\coth^{\frac{3}{2}}(c+dx)}\\
&= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}\\
&\quad-\frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\int\frac{\sqrt{x}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d\coth^{\frac{3}{2}}(c+dx)}\\
&= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}\\
&\quad-\frac{\left(2b\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\int\frac{x^2}{-1+x^4}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}\\
&= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d}-\frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}\\
&\quad+\frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x^2}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}\\
&\quad-\frac{\left(b\sqrt{b\coth^3(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

$$= -\frac{2b\sqrt{b\coth^3(c+dx)}}{3d} - \frac{b\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{b\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} - \frac{2b\coth^2(c+dx)\sqrt{b\coth^3(c+dx)}}{7d}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.61

$$\int \left(b\coth^3(c+dx)\right)^{3/2} dx = -\frac{\left(b\coth^3(c+dx)\right)^{3/2}\left(\arctan\left(\sqrt{\coth(c+dx)}\right) - \text{arctanh}\left(\sqrt{\coth(c+dx)}\right) + \frac{2}{3}\coth^{\frac{3}{2}}(c+dx) + \frac{2}{7}\coth^{\frac{7}{2}}(\right.}{d\coth^{\frac{9}{2}}(c+dx)}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(3/2),x]
[Out] -(((b*Coth[c + d*x]^3)^(3/2)*(ArcTan[Sqrt[Coth[c + d*x]]] - ArcTanh[Sqrt[Co
th[c + d*x]]] + (2*Coth[c + d*x]^(3/2))/3 + (2*Coth[c + d*x]^(7/2))/7))/(d*
Coth[c + d*x]^(9/2)))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.80

method	result
derivativedivides	$-\frac{\left(b\coth(dx+c)^3\right)^{\frac{3}{2}}\left(-21b^{\frac{7}{2}}\,\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+21b^{\frac{7}{2}}\,\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+6(b\coth(dx+c))^{\frac{7}{2}}+14b^2(b\,\text{cot}\right.}{21d\coth(dx+c)^3(b\coth(dx+c))^{\frac{3}{2}}b^2}$
default	$-\frac{\left(b\coth(dx+c)^3\right)^{\frac{3}{2}}\left(-21b^{\frac{7}{2}}\,\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+21b^{\frac{7}{2}}\,\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)+6(b\coth(dx+c))^{\frac{7}{2}}+14b^2(b\,\text{cot}\right.}{21d\coth(dx+c)^3(b\coth(dx+c))^{\frac{3}{2}}b^2}$

```
[In] int((b*coth(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/21/d*(b*coth(d*x+c)^3)^(3/2)*(-21*b^(7/2)*arctanh((b*coth(d*x+c))^(1/2)/
b^(1/2))+21*b^(7/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2))+6*(b*coth(d*x+c))
^(7/2)+14*b^2*(b*coth(d*x+c))^(3/2))/coth(d*x+c)^3/(b*coth(d*x+c))^(3/2)/b^
2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(114) = 228$.

Time = 0.32 (sec) , antiderivative size = 2152, normalized size of antiderivative = 16.06

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(
d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c
)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cos
h(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x +
 c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(
d*x + c) - b)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
 c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh
(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 2
1*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^
6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(
5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c
)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6
*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)
- b)*sqrt(-b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) +
 6*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cos
h(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x
 + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*b*cos
h(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d*x + c)^6 + b
*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4 + 4*(25*b*cos
h(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (75*b
*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(15*b*cosh(
d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + 5*b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*s
inh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x
+ c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c))*sinh(d*x + c) - d), -1/84*(42*(b*cosh(d*x + c)^6 + 6*b*c
osh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*
(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*co
sh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4
 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*co
sh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*arctan(sqrt(b)*
```

```
sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*
sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - 21*(b*cosh(d*x + c)^6 + 6*b*cosh(
d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b
*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d
*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d
*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*sqrt(b)*log(2*b*cosh(d*x +
c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x +
c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*
x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x
+ c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh
(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b)
+ 16*(5*b*cosh(d*x + c)^6 + 30*b*cosh(d*x + c)*sinh(d*x + c)^5 + 5*b*sinh(d
*x + c)^6 + b*cosh(d*x + c)^4 + (75*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^4
+ 4*(25*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x +
 c)^2 + (75*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 +
2*(15*b*cosh(d*x + c)^5 + 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
 c) + 5*b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(d*cosh(d*x + c)^6 + 6*d*co
sh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(
5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4
- 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cos
h(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)]
```

Sympy [F]

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx=\int \left(b\coth^3{\left(c+dx\right)}\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**3)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(b\coth^3(c+dx)\right)^{3/2}\,dx=\int \left(b\coth\left(dx+c\right)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^3)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(114) = 228$.

Time = 0.58 (sec) , antiderivative size = 788, normalized size of antiderivative = 5.88

$$\int \left(b\coth^3(c+dx)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/42*(42*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c)
- b))/sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c)
- 1)*sgn(e^(4*d*x + 4*c) - 1) - 21*sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c)
 + sqrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) +
3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) + 16*(21*(sqrt(b)*e^(2*d*x
+ 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^6*b*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x
+ 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) - 42*(sqrt(b)*e^(2
*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^5*b^(3/2)*sgn(e^(6*d*x + 6*c) -
3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) + 119*(
sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^4*b^2*sgn(e^(6*d*x +
 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)
 - 56*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^3*b^(5/2)*sgn
(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x
+ 4*c) - 1) + 63*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b))^2*
b^3*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^
(4*d*x + 4*c) - 1) - 14*(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))*b^(7/2)*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
1)*sgn(e^(4*d*x + 4*c) - 1) + 5*b^4*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c)
 + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1))/(sqrt(b)*e^(2*d*x + 2*c
) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b))^7)*b/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^{3/2} dx = \int \left(b\coth(c+d\,x)^3\right)^{3/2} dx$$

```
[In] int((b*coth(c + d*x)^3)^(3/2),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(3/2), x)
```

3.30 $\int \sqrt{b\coth^3(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} - \frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d}$$

```
[Out] arctan(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)+arcta
nh(coth(d*x+c)^(1/2))*(b*coth(d*x+c)^3)^(1/2)/d/coth(d*x+c)^(3/2)-2*(b*coth
(d*x+c)^3)^(1/2)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3739, 3554, 3557, 335, 218, 212, 209}

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} - \frac{2\tanh(c+dx)\sqrt{b\coth^3(c+dx)}}{d}$$

```
[In] Int[Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (ArcTan[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2
)) + (ArcTanh[Sqrt[Coth[c + d*x]]]*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x
]^(3/2)) - (2*Sqrt[b*Coth[c + d*x]^3]*Tanh[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{b\coth^3(c+dx)}\int \coth^{\frac{3}{2}}(c+dx)\,dx}{\coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d}+\frac{\sqrt{b\coth^3(c+dx)}\int \frac{1}{\sqrt{\coth(c+dx)}}\,dx}{\coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d}-\frac{\sqrt{b\coth^3(c+dx)}\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx,x,\coth(c+dx)\right)}{d\coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} \\
&\quad -\frac{\left(2\sqrt{b\coth^3(c+dx)}\right)\mathrm{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)} \\
&= -\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d} \\
&\quad +\frac{\sqrt{b\coth^3(c+dx)}\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)} \\
&\quad +\frac{\sqrt{b\coth^3(c+dx)}\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\coth(c+dx)}\right)}{d\coth^{\frac{3}{2}}(c+dx)} \\
&= \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)} \\
&\quad +\frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}-\frac{2\sqrt{b\coth^3(c+dx)}\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \frac{\left(\arctan\left(\sqrt{\coth(c+dx)}\right)+\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)-2\sqrt{\coth(c+dx)}\right)\sqrt{b\coth^3(c+dx)}}{d\coth^{\frac{3}{2}}(c+dx)}$$

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^3],x]
[Out] ((ArcTan[Sqrt[Coth[c + d*x]]] + ArcTanh[Sqrt[Coth[c + d*x]]] - 2*Sqrt[Coth[
c + d*x]])*Sqrt[b*Coth[c + d*x]^3])/(d*Coth[c + d*x]^(3/2))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{\sqrt{b\coth(dx+c)^3}\left(2\sqrt{b\coth(dx+c)}-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)-\sqrt{b}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\right)}{d\coth(dx+c)\sqrt{b\coth(dx+c)}}$	89
default	$-\frac{\sqrt{b\coth(dx+c)^3}\left(2\sqrt{b\coth(dx+c)}-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)-\sqrt{b}\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)\right)}{d\coth(dx+c)\sqrt{b\coth(dx+c)}}$	89

```
[In] int((b*coth(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/d*(b*coth(d*x+c)^3)^(1/2)*(2*(b*coth(d*x+c))^(1/2)-b^(1/2)*arctanh((b*co
th(d*x+c))^(1/2)/b^(1/2))-b^(1/2)*arctan((b*coth(d*x+c))^(1/2)/b^(1/2)))/co
th(d*x+c)/(b*coth(d*x+c))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. 2(90) = 180.

Time = 0.29 (sec) , antiderivative size = 633, normalized size of antiderivative = 6.09

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \left[-\frac{2\sqrt{-b}\arctan\left(\frac{\left(\cosh(dx+c)^2+2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2\right)\sqrt{-b}\sqrt{\frac{b\cosh(dx+c)}{\sinh(dx+c)}}}{b\cosh(dx+c)^2+2b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+b}\right)-\sqrt{-b}\log\left(-\frac{b\cosh(dx+c)^4+4b\,\cdots}{\cdots}\right.}{\cdots}\right.$$

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) - sqrt(-
b)*log(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x
+ c)^4 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 - 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*sqrt(b*cosh(d*x + c)
/sinh(d*x + c)))/d, 1/4*(2*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh
(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*
x + c)^2 + b)) + sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh
(d*x + c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d
*x + c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh
(d*x + c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - c
osh(d*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - b) - 8*sqrt(b*cosh(d*x + c)/sinh(d
*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{b\coth^3(c+dx)}\,dx=\int \sqrt{b\coth^3{(c+dx)}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(b*coth(c + d*x)**3), x)
```

Maxima [F]

$$\int \sqrt{b\coth^3(c+dx)}\,dx=\int \sqrt{b\coth{(dx+c)}^3}\,dx$$

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*coth(d*x + c)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(90) = 180$.

Time = 0.39 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.59

$$\int \sqrt{b\coth^3(c+dx)}\,dx =$$
$$-\frac{2\sqrt{b}\arctan\left(-\frac{\sqrt{b}e^{(2\,dx+2\,c)}-\sqrt{be^{(4\,dx+4\,c)}-b}}{\sqrt{b}}\right)\operatorname{sgn}\left(e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1\right)\operatorname{sgn}\left(e^{(4\,dx+4\,c)}-1\right)$$

```
[In] integrate((b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*sqrt(b)*arctan(-(sqrt(b)*e^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) -
 b))/sqrt(b))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) -
 1)*sgn(e^(4*d*x + 4*c) - 1) + sqrt(b)*log(abs(-sqrt(b)*e^(2*d*x + 2*c) + s
qrt(b*e^(4*d*x + 4*c) - b)))*sgn(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^
(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1) - 8*b*sgn(e^(6*d*x + 6*c) - 3*e
^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1)*sgn(e^(4*d*x + 4*c) - 1)/(sqrt(b)*e
^(2*d*x + 2*c) - sqrt(b*e^(4*d*x + 4*c) - b) - sqrt(b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\coth^3(c+dx)}\,dx = \int \sqrt{b\,\mathrm{coth}\left(c+d\,x\right)^3}\,dx$$

```
[In] int((b*coth(c + d*x)^3)^(1/2),x)
```

```
[Out] int((b*coth(c + d*x)^3)^(1/2), x)
```

3.31 $\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 105

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}}$$

```
[Out] -2*coth(d*x+c)/d/(b*coth(d*x+c)^3)^(1/2)-arctan(coth(d*x+c)^(1/2))*coth(d*x
+c)^(3/2)/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c)^
(3/2)/d/(b*coth(d*x+c)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3739, 3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = -\frac{\coth^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}} - \frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] (-2*Coth[c + d*x])/(d*Sqrt[b*Coth[c + d*x]^3]) - (ArcTan[Sqrt[Coth[c + d*x]
]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c
+ d*x]]]*Coth[c + d*x]^(3/2))/(d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{3}{2}}(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \sqrt{\coth(c+dx)}\,dx}{\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\coth(c+dx)}{d\sqrt{b\coth^3(c+dx)}} - \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}} + \frac{\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{d\sqrt{b\coth^3(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = -\frac{\coth(c+dx)\left(2+\arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2(c+dx)}-\operatorname{arctanh}\left(\sqrt[4]{\coth^2(c+dx)}\right)\sqrt[4]{\coth^2(c+dx)}\right)}{d\sqrt{b\coth^3(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^3],x]
```

```
[Out] -((Coth[c + d*x]*(2 + ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/
4) - ArcTanh[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(1/4)))/(d*Sqrt[b*C
oth[c + d*x]^3]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\coth(dx+c)\left(-2b^{\frac{5}{2}}+\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^2\sqrt{b\coth(dx+c)}-\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^2\sqrt{b\coth(dx+c)}\right)}{d\sqrt{b\coth(dx+c)^3}\,b^{\frac{5}{2}}}$	91
default	$\frac{\coth(dx+c)\left(-2b^{\frac{5}{2}}+\operatorname{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^2\sqrt{b\coth(dx+c)}-\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^2\sqrt{b\coth(dx+c)}\right)}{d\sqrt{b\coth(dx+c)^3}\,b^{\frac{5}{2}}}$	91

```
[In] int(1/(b*coth(d*x+c)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*coth(d*x+c)*(-2*b^(5/2)+arctanh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*c
oth(d*x+c))^(1/2)-arctan((b*coth(d*x+c))^(1/2)/b^(1/2))*b^2*(b*coth(d*x+c))
^(1/2))/(b*coth(d*x+c)^3)^(1/2)/b^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(91) = 182.

Time = 0.30 (sec) , antiderivative size = 907, normalized size of antiderivative = 8.64

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*sqrt(-b)*arctan((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + si
nh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + (cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt(-b)*log
(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x + c
)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1
)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 + 4*
cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(
d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 8*(cosh(d*x + c)^2 + 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x
 + c)))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh
(d*x + c)^2 + b*d), -1/4*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c
) + sinh(d*x + c)^2 + 1)*sqrt(b)*arctan(sqrt(b)*sqrt(b*cosh(d*x + c)/sinh(d
*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x
+ c) + 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x +
c)^3 + 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x
+ c)^3 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d
*x + c)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sq
rt(b*cosh(d*x + c)/sinh(d*x + c)) - b) + 8*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))
)/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x +
 c)^2 + b*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth^3{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/2),x)
[Out] Integral(1/sqrt(b*coth(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth{(dx+c)}^3}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*coth(d*x + c)^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^3)^(1/2),x)
[Out] int(1/(b*coth(c + d*x)^3)^(1/2), x)
```

3.32 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 141

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} + \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}$$

```
[Out] -2/3/b/d/(b*coth(d*x+c)^3)^(1/2)+arctan(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/2
)/b/d/(b*coth(d*x+c)^3)^(1/2)+arctanh(coth(d*x+c)^(1/2))*coth(d*x+c)^(3/2)/
b/d/(b*coth(d*x+c)^3)^(1/2)-2/7*tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used

= {3739, 3555, 3557, 335, 218, 212, 209}

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \frac{\coth^{\frac{3}{2}}(c+dx)\arctan\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\mathrm{arctanh}\left(\sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}} - \frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(-3/2),x]
```

```
[Out] -2/(3*b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTan[Sqrt[Coth[c + d*x]]]*Coth[c +
d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) + (ArcTanh[Sqrt[Coth[c + d*x]]]*C
oth[c + d*x]^(3/2))/(b*d*Sqrt[b*Coth[c + d*x]^3]) - (2*Tanh[c + d*x]^2)/(7*
b*d*Sqrt[b*Coth[c + d*x]^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{9}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\coth^{\frac{5}{2}}(c+dx)}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\int \frac{1}{\sqrt{\coth(c+dx)}}\,dx}{b\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{bd\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} - \frac{\left(2\coth^{\frac{3}{2}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\coth^{\frac{3}{2}}(c+dx)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth(c+dx)}\right)}{bd\sqrt{b\coth^3(c+dx)}}$$

$$= -\frac{2}{3bd\sqrt{b\coth^3(c+dx)}} + \frac{\arctan\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} + \frac{\text{arctanh}\left(\sqrt{\coth(c+dx)}\right)\coth^{\frac{3}{2}}(c+dx)}{bd\sqrt{b\coth^3(c+dx)}} - \frac{2\tanh^2(c+dx)}{7bd\sqrt{b\coth^3(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \frac{-14 + 21\arctan\left(\sqrt[4]{\coth^2(c+dx)}\right)\coth^2(c+dx)^{3/4} + 21\text{arctanh}\left(\sqrt[4]{\coth^2(c+\right.}{21bd\sqrt{b\coth^3(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-3/2),x]
[Out] (-14 + 21*ArcTan[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4) + 21*ArcT
anh[(Coth[c + d*x]^2)^(1/4)]*(Coth[c + d*x]^2)^(3/4) - 6*Tanh[c + d*x]^2)/(
21*b*d*Sqrt[b*Coth[c + d*x]^3])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\coth(dx+c)\left(-14b^{\frac{15}{2}}\coth(dx+c)^2-6b^{\frac{15}{2}}+21\,\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^4(b\coth(dx+c))^{\frac{7}{2}}+21\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^4\right.}{21d\,b^{\frac{15}{2}}\left(b\coth(dx+c)^3\right)^{\frac{3}{2}}}$
default	$\frac{\coth(dx+c)\left(-14b^{\frac{15}{2}}\coth(dx+c)^2-6b^{\frac{15}{2}}+21\,\text{arctanh}\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^4(b\coth(dx+c))^{\frac{7}{2}}+21\arctan\left(\frac{\sqrt{b\coth(dx+c)}}{\sqrt{b}}\right)b^4\right.}{21d\,b^{\frac{15}{2}}\left(b\coth(dx+c)^3\right)^{\frac{3}{2}}}$

```
[In] int(1/(b*coth(d*x+c)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/d*coth(d*x+c)/b^(15/2)*(-14*b^(15/2)*coth(d*x+c)^2-6*b^(15/2)+21*arcta
nh((b*coth(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2)+21*arctan((b*co
th(d*x+c))^(1/2)/b^(1/2))*b^4*(b*coth(d*x+c))^(7/2))/(b*coth(d*x+c)^3)^(3/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1486 vs. 2(121) = 242.

Time = 0.34 (sec) , antiderivative size = 3022, normalized size of antiderivative = 21.43

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 + sinh(d*x +
c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7
*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4
 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(
d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*c
osh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^
2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x
 + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*arctan((cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(-b)*sqrt(b*cosh(d
*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
 + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x
 + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*c
osh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*
x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^
2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*
x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-b)*l
og(-(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)^3*sinh(d*x + c) + 6*b*cosh(d*x +
 c)^2*sinh(d*x + c)^2 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)*sqrt(-b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)) - 2*b)/(cosh(d*x + c)^4 +
4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)) + 16*(5*cosh(d*x + c)^8 + 40
*cosh(d*x + c)*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2
- 3)*sinh(d*x + c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)
*sinh(d*x + c)^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x
```

```
+ c)^3 + cosh(d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d
*x + c)^4 + 6*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*
(10*cosh(d*x + c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x +
c))*sinh(d*x + c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x
 + c)^8 + 8*b^2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4
*b^2*d*cosh(d*x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)
^2 + b^2*d)*sinh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c
)^4 + 30*b^2*d*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh
(d*x + c)^5 + 10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x +
c)^3 + b^2*d + 4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^
2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3
*b^2*d*cosh(d*x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sin
h(d*x + c)), 1/84*(42*(cosh(d*x + c)^8 + 8*cosh(d*x + c)*sinh(d*x + c)^7 +
sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^6 + 4*cosh(d*x +
c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*cosh
(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x + c)^4 + 6*cosh(d*x + c)^4 +
 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 + 9*cosh(d*x + c)^2 + 1)*sin
h(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x + c)^7 + 3*cosh(d*x + c)^5 +
 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(b)*arctan(sqrt(
b)*sqrt(b*cosh(d*x + c)/sinh(d*x + c))/(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)) + 21*(cosh(d*x + c)^8 + 8*cosh(d
*x + c)*sinh(d*x + c)^7 + sinh(d*x + c)^8 + 4*(7*cosh(d*x + c)^2 + 1)*sinh(
d*x + c)^6 + 4*cosh(d*x + c)^6 + 8*(7*cosh(d*x + c)^3 + 3*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*cosh(d*x + c)^4 + 30*cosh(d*x + c)^2 + 3)*sinh(d*x +
c)^4 + 6*cosh(d*x + c)^4 + 8*(7*cosh(d*x + c)^5 + 10*cosh(d*x + c)^3 + 3*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(7*cosh(d*x + c)^6 + 15*cosh(d*x + c)^4 +
9*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 4*cosh(d*x + c)^2 + 8*(cosh(d*x +
c)^7 + 3*cosh(d*x + c)^5 + 3*cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c)
 + 1)*sqrt(b)*log(2*b*cosh(d*x + c)^4 + 8*b*cosh(d*x + c)^3*sinh(d*x + c) +
 12*b*cosh(d*x + c)^2*sinh(d*x + c)^2 + 8*b*cosh(d*x + c)*sinh(d*x + c)^3 +
 2*b*sinh(d*x + c)^4 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3
 + sinh(d*x + c)^4 + (6*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - cosh(d*x + c
)^2 + 2*(2*cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c))*sqrt(b)*sqrt(b*c
osh(d*x + c)/sinh(d*x + c)) - b) - 16*(5*cosh(d*x + c)^8 + 40*cosh(d*x + c)
*sinh(d*x + c)^7 + 5*sinh(d*x + c)^8 + 2*(70*cosh(d*x + c)^2 - 3)*sinh(d*x
+ c)^6 - 6*cosh(d*x + c)^6 + 4*(70*cosh(d*x + c)^3 - 9*cosh(d*x + c))*sinh(
d*x + c)^5 + 2*(175*cosh(d*x + c)^4 - 45*cosh(d*x + c)^2 + 1)*sinh(d*x + c)
^4 + 2*cosh(d*x + c)^4 + 8*(35*cosh(d*x + c)^5 - 15*cosh(d*x + c)^3 + cosh(
d*x + c))*sinh(d*x + c)^3 + 2*(70*cosh(d*x + c)^6 - 45*cosh(d*x + c)^4 + 6*
cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*cosh(d*x + c)^2 + 4*(10*cosh(d*x +
 c)^7 - 9*cosh(d*x + c)^5 + 2*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sinh(d*x +
 c) + 5)*sqrt(b*cosh(d*x + c)/sinh(d*x + c)))/(b^2*d*cosh(d*x + c)^8 + 8*b^
2*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*d*sinh(d*x + c)^8 + 4*b^2*d*cosh(d*
```

```
x + c)^6 + 6*b^2*d*cosh(d*x + c)^4 + 4*(7*b^2*d*cosh(d*x + c)^2 + b^2*d)*si
nh(d*x + c)^6 + 8*(7*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*
x + c)^5 + 4*b^2*d*cosh(d*x + c)^2 + 2*(35*b^2*d*cosh(d*x + c)^4 + 30*b^2*d
*cosh(d*x + c)^2 + 3*b^2*d)*sinh(d*x + c)^4 + 8*(7*b^2*d*cosh(d*x + c)^5 +
10*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*d +
 4*(7*b^2*d*cosh(d*x + c)^6 + 15*b^2*d*cosh(d*x + c)^4 + 9*b^2*d*cosh(d*x +
 c)^2 + b^2*d)*sinh(d*x + c)^2 + 8*(b^2*d*cosh(d*x + c)^7 + 3*b^2*d*cosh(d*
x + c)^5 + 3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**3)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^3)^(-3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{3/2}}\,dx=\int \frac{1}{{\left(b\,\mathrm{coth}\left(c+d\,x\right)^3\right)}^{3/2}} \,d x$$

```
[In] int(1/(b*coth(c + d*x)^3)^(3/2),x)
[Out] int(1/(b*coth(c + d*x)^3)^(3/2), x)
```

3.33 $\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d} - \frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + bx\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)$$

```
[Out] -b*(b*coth(d*x+c)^3)^(1/3)/d-1/3*b*coth(d*x+c)^2*(b*coth(d*x+c)^3)^(1/3)/d+
b*x*(b*coth(d*x+c)^3)^(1/3)*tanh(d*x+c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \left(b\coth^3(c+dx)\right)^{4/3}\,dx = -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d} - \frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d} + bx\tanh(c+dx)\sqrt[3]{b\coth^3(c+dx)}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(4/3),x]
```

```
[Out] -((b*(b*Coth[c + d*x]^3)^(1/3))/d) - (b*Coth[c + d*x]^2*(b*Coth[c + d*x]^3)
^(1/3))/(3*d) + b*x*(b*Coth[c + d*x]^3)^(1/3)*Tanh[c + d*x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(b\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)\right)\int\coth^4(c+dx)\,dx\\
&= -\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+\left(b\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)\right)\int\coth^2(c+dx)\,dx\\
&= -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+\left(b\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)\right)\int 1\,dx\\
&= -\frac{b\sqrt[3]{b\coth^3(c+dx)}}{d}-\frac{b\coth^2(c+dx)\sqrt[3]{b\coth^3(c+dx)}}{3d}+bx\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.58

$$\int \left(b\coth^3(c+dx)\right)^{4/3} dx = -\frac{\left(b\coth^3(c+dx)\right)^{4/3} \text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{3d}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(4/3),x]
[Out] -1/3*((b*Coth[c + d*x]^3)^(4/3)*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d
*x]^2]*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.76

method	result	size
risch	$-\frac{b\left(\frac{b\left(e^{2dx+2c}+1\right)^{3}}{\left(e^{2dx+2c}-1\right)^{3}}\right)^{\frac{1}{3}}\left(-3\,e^{6dx+6c}dx+9\,e^{4dx+4c}dx-9\,e^{2dx+2c}dx+3dx+12\,e^{4dx+4c}-12\,e^{2dx+2c}+8\right)}{3\left(e^{2dx+2c}+1\right)\left(e^{2dx+2c}-1\right)^{2}d}$	130

```
[In] int((b*coth(d*x+c)^3)^(4/3),x,method=_RETURNVERBOSE)
[Out] -1/3*b*(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)*(-3*exp(6*d*x+6*
c)*d*x+9*exp(4*d*x+4*c)*d*x-9*exp(2*d*x+2*c)*d*x+3*d*x+12*exp(4*d*x+4*c)-12
*exp(2*d*x+2*c)+8)/(exp(2*d*x+2*c)+1)/(exp(2*d*x+2*c)-1)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. 2(66) = 132.

Time = 0.30 (sec) , antiderivative size = 1046, normalized size of antiderivative = 14.14

$$\int \left(b\coth^3(c+dx)\right)^{4/3} dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
[Out] -1/3*(3*b*d*x*cosh(d*x + c)^6 - 3*(b*d*x*e^(2*d*x + 2*c) - b*d*x)*sinh(d*x
+ c)^6 - 18*(b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) - b*d*x*cosh(d*x + c))*sin
h(d*x + c)^5 - 3*(3*b*d*x + 4*b)*cosh(d*x + c)^4 + 3*(15*b*d*x*cosh(d*x + c
)^2 - 3*b*d*x - (15*b*d*x*cosh(d*x + c)^2 - 3*b*d*x - 4*b)*e^(2*d*x + 2*c)
```

```
- 4*b)*sinh(d*x + c)^4 + 12*(5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh
(d*x + c) - (5*b*d*x*cosh(d*x + c)^3 - (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c)^3 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 3*(15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2
- (15*b*d*x*cosh(d*x + c)^4 + 3*b*d*x - 6*(3*b*d*x + 4*b)*cosh(d*x + c)^2 +
 4*b)*e^(2*d*x + 2*c) + 4*b)*sinh(d*x + c)^2 - (3*b*d*x*cosh(d*x + c)^6 - 3
*(3*b*d*x + 4*b)*cosh(d*x + c)^4 - 3*b*d*x + 3*(3*b*d*x + 4*b)*cosh(d*x + c
)^2 - 8*b)*e^(2*d*x + 2*c) + 6*(3*b*d*x*cosh(d*x + c)^5 - 2*(3*b*d*x + 4*b)
*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c) - (3*b*d*x*cosh(d*x + c)^5
 - 2*(3*b*d*x + 4*b)*cosh(d*x + c)^3 + (3*b*d*x + 4*b)*cosh(d*x + c))*e^(2*
d*x + 2*c))*sinh(d*x + c) - 8*b)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c)
+ 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*
x + 2*c) - 1))^(1/3)/(d*cosh(d*x + c)^6 + (d*e^(2*d*x + 2*c) + d)*sinh(d*x
+ c)^6 + 6*(d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c
)^5 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + (5*d*cosh(d*x + c)^2 -
 d)*e^(2*d*x + 2*c) - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cos
h(d*x + c) + (5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
+ c)^2 + (5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) +
d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*d*cosh(d*
x + c)^2 - d)*e^(2*d*x + 2*c) + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3
+ d*cosh(d*x + c) + (d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + d*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(66) = 132$.

Time = 68.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int (b\coth^3(c+dx))^{4/3}\,dx = \begin{cases} x\left(b\coth^3(c)\right)^{\frac{4}{3}} & \text{for } d = 0 \\ -\frac{\left(b\coth^3\left(dx+\log\left(-e^{-dx}\right)\right)\right)^{\frac{4}{3}}\log\left(-e^{-dx}\right)}{d} & \text{for } c = \log\left(-e^{-dx}\right) \\ x\left(b\coth^3\left(dx+\log\left(e^{-dx}\right)\right)\right)^{\frac{4}{3}} & \text{for } c = \log\left(e^{-dx}\right) \\ x\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh^4(c+dx) - \frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh^3(c+dx)}{d} - \frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{4}{3}}\tanh(c+dx)}{3d} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*coth(d*x+c)**3)**(4/3),x)
```

```
[Out] Piecewise((x*(b*coth(c)**3)**(4/3), Eq(d, 0)), (-(b*coth(d*x + log(-exp(-d*
x)))**3)**(4/3)*log(-exp(-d*x))/d, Eq(c, log(-exp(-d*x)))), (x*(b*coth(d*x
+ log(exp(-d*x)))**3)**(4/3), Eq(c, log(exp(-d*x)))), (x*(b/tanh(c + d*x)**
```

```
3)**(4/3)*tanh(c + d*x)**4 - (b/tanh(c + d*x)**3)**(4/3)*tanh(c + d*x)**3/d
 - (b/tanh(c + d*x)**3)**(4/3)*tanh(c + d*x)/(3*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.18

$$\int \left(b\coth^3(c+dx)\right)^{4/3} dx = \frac{(dx+c)b^{\frac{4}{3}}}{d} - \frac{4\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)} - 3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)} - 2\,b^{\frac{4}{3}}\right)}{3\,d(3\,e^{(-2\,dx-2\,c)} - 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} - 1)}$$

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
[Out] (d*x + c)*b^(4/3)/d - 4/3*(3*b^(4/3)*e^(-2*d*x - 2*c) - 3*b^(4/3)*e^(-4*d*x
 - 4*c) - 2*b^(4/3))/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1))
```

Giac [F]

$$\int \left(b\coth^3(c+dx)\right)^{4/3} dx = \int \left(b\coth\left(dx+c\right)^3\right)^{\frac{4}{3}} dx$$

```
[In] integrate((b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^3)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^{4/3} dx = \int \left(b\coth(c+d\,x)^3\right)^{4/3} dx$$

```
[In] int((b*coth(c + d*x)^3)^(4/3),x)
[Out] int((b*coth(c + d*x)^3)^(4/3), x)
```

3.34 $\int \left(b \coth^3(c+dx)\right)^{2/3} dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \left(b \coth^3(c+dx)\right)^{2/3} dx = -\frac{\left(b \coth^3(c+dx)\right)^{2/3} \tanh(c+dx)}{d} + x\left(b \coth^3(c+dx)\right)^{2/3} \tanh^2(c+dx)$$

```
[Out] -(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^3)^(2/3)*tanh(d*x+c
)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \left(b \coth^3(c+dx)\right)^{2/3} dx = x \tanh^2(c+dx) \left(b \coth^3(c+dx)\right)^{2/3} - \frac{\tanh(c+dx) \left(b \coth^3(c+dx)\right)^{2/3}}{d}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(2/3),x]
```

```
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Tanh[c + d*x])/d) + x*(b*Coth[c + d*x]^3)^(2/3
)*Tanh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int\coth^2(c+dx)\,dx\\ &= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+\left(\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\right)\int 1\,dx\\ &= -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\tanh(c+dx)}{d}+x\left(b\coth^3(c+dx)\right)^{2/3}\tanh^2(c+dx)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int\left(b\coth^3(c+dx)\right)^{2/3}\,dx = -\frac{\left(b\coth^3(c+dx)\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{d}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(2/3),x]
[Out] -(((b*Coth[c + d*x]^3)^(2/3)*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^
2]*Tanh[c + d*x])/d)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(46) = 92$.

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

method	result	size
risch	$\frac{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^2 x}{\left(e^{2dx+2c}+1\right)^2}-\frac{2\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)}{\left(e^{2dx+2c}+1\right)^2 d}$	119

```
[In] int((b*coth(d*x+c)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] (b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(exp(2*d*x+2*c)+1)^2*(e
xp(2*d*x+2*c)-1)^2*x-2*(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/
(exp(2*d*x+2*c)+1)^2*(exp(2*d*x+2*c)-1)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 392, normalized size of antiderivative = 7.84

$$\int \left(b\coth^3(c+dx)\right)^{2/3} dx = \frac{\left(dx\cosh\left(dx+c\right)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^2-dx+\left(dx\cosh\left(dx+\right.\right.}{d\cosh\left(dx+c\right)^2+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx+\right.}$$

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x +
2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2
/3)/(d*cosh(d*x + c)^2 + (d*e^(4*d*x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh
(d*x + c)^2 + (d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^
2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x
+ c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(44) = 88$.

Time = 6.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.52

$$\int (b\coth^3(c+dx))^{2/3}\,dx = \begin{cases} x\left(b\coth^3(c)\right)^{\frac{2}{3}} & \text{for } d=0 \\ -\frac{\left(b\coth^3(dx+\log(-e^{-dx}))\right)^{\frac{2}{3}}\log(-e^{-dx})}{d} & \text{for } c=\log(-e^{-dx}) \\ x\left(b\coth^3(dx+\log(e^{-dx}))\right)^{\frac{2}{3}} & \text{for } c=\log(e^{-dx}) \\ x\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{2}{3}}\tanh^2(c+dx) - \frac{\left(\frac{b}{\tanh^3(c+dx)}\right)^{\frac{2}{3}}\tanh(c+dx)}{d} & \text{otherwise} \end{cases}$$

```
[In] integrate((b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Piecewise((x*(b*coth(c)**3)**(2/3), Eq(d, 0)), (-(b*coth(d*x + log(-exp(-d*
x)))**3)**(2/3)*log(-exp(-d*x))/d, Eq(c, log(-exp(-d*x)))), (x*(b*coth(d*x
+ log(exp(-d*x)))**3)**(2/3), Eq(c, log(exp(-d*x)))), (x*(b/tanh(c + d*x)**
3)**(2/3)*tanh(c + d*x)**2 - (b/tanh(c + d*x)**3)**(2/3)*tanh(c + d*x)/d, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int (b\coth^3(c+dx))^{2/3}\,dx = \frac{(dx+c)b^{\frac{2}{3}}}{d} + \frac{2\,b^{\frac{2}{3}}}{d(e^{(-2\,dx-2\,c)}-1)}$$

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(2/3)/d + 2*b^(2/3)/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [F]

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx=\int \left(b\coth\left(dx+c\right)^3\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^3)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^3(c+dx)\right)^{2/3}\,dx=\int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^3\right)}^{2/3}\,dx$$

```
[In] int((b*coth(c + d*x)^3)^(2/3),x)
[Out] int((b*coth(c + d*x)^3)^(2/3), x)
```

3.35 $\int \sqrt[3]{b \coth^3(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \sqrt[3]{b \coth^3(c+dx)}\,dx = \frac{\sqrt[3]{b \coth^3(c+dx)} \log(\sinh(c+dx)) \tanh(c+dx)}{d}$$

```
[Out] (b*coth(d*x+c)^3)^(1/3)*ln(sinh(d*x+c))*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3739, 3556}

$$\int \sqrt[3]{b \coth^3(c+dx)}\,dx = \frac{\tanh(c+dx) \sqrt[3]{b \coth^3(c+dx)} \log(\sinh(c+dx))}{d}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(1/3),x]
```

```
[Out] ((b*Coth[c + d*x]^3)^(1/3)*Log[Sinh[c + d*x]]*Tanh[c + d*x])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt[3]{b\coth^3(c+dx)}\tanh(c+dx)\right)\int\coth(c+dx)\,dx\\ &= \frac{\sqrt[3]{b\coth^3(c+dx)}\log(\sinh(c+dx))\tanh(c+dx)}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int\sqrt[3]{b\coth^3(c+dx)}\,dx = \frac{\sqrt[3]{b\coth^3(c+dx)}(\log(\cosh(c+dx))+\log(\tanh(c+dx)))\tanh(c+dx)}{d}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(1/3),x]
[Out] ((b*Coth[c + d*x]^3)^(1/3)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]])*Tanh[c
 + d*x])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(29) = 58.

Time = 0.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.19

method	result
risch	$\frac{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)x}{e^{2dx+2c}+1}-\frac{2\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)(dx+c)}{\left(e^{2dx+2c}+1\right)d}+\frac{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)\ln\left(e^{2dx+2c}-\right.}{\left(e^{2dx+2c}+1\right)d}$

```
[In] int((b*coth(d*x+c)^3)^(1/3),x,method=_RETURNVERBOSE)
[Out] (b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)+1)*(exp
(2*d*x+2*c)-1)*x-2*(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp
(2*d*x+2*c)+1)*(exp(2*d*x+2*c)-1)/d*(d*x+c)+(b*(exp(2*d*x+2*c)+1)^3/(exp(2*
```

```
d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)+1)*(exp(2*d*x+2*c)-1)/d*ln(exp(2*d*x+2
*c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.77

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = -\frac{\left(dxe^{(2\,dx+2\,c)} - dx - \left(e^{(2\,dx+2\,c)} - 1\right)\log\left(\frac{2\,\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c)}+3\,be^{(4\,dx+4\,c)}+3\,be^{(2\,dx+2\,c)}+b}{e^{(6\,dx+6\,c)}-3\,e^{(4\,dx+4\,c)}+3\,e^{(2\,dx+2\,c)}-1}\right)^{\frac{1}{3}}}{de^{(2\,dx+2\,c)}+d}$$

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(2*d*x + 2*c) - d*x - (e^(2*d*x + 2*c) - 1)*log(2*sinh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))))*((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) +
3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x
+ 2*c) - 1))^(1/3)/(d*e^(2*d*x + 2*c) + d)
```

Sympy [F]

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \int \sqrt[3]{b\coth^3(c+dx)}\,dx$$

```
[In] integrate((b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \frac{(dx+c)b^{\frac{1}{3}}}{d} + \frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{b^{\frac{1}{3}}\log\left(e^{(-dx-c)}-1\right)}{d}$$

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(1/3)/d + b^(1/3)*log(e^(-d*x - c) + 1)/d + b^(1/3)*log(e^(-d*x
 - c) - 1)/d
```

Giac [F]

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^3\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^3)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\coth^3(c+dx)}\,dx = \int \left(b\coth(c+d\,x)^3\right)^{1/3}\,dx$$

```
[In] int((b*coth(c + d*x)^3)^(1/3),x)
[Out] int((b*coth(c + d*x)^3)^(1/3), x)
```

3.36 $\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] coth(d*x+c)*ln(cosh(d*x+c))/d/(b*coth(d*x+c)^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3739, 3556}

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(-1/3),x]
```

```
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
```

```
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\coth(c+dx)\int\tanh(c+dx)\,dx}{\sqrt[3]{b\coth^3(c+dx)}}$$
$$= \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \frac{\coth(c+dx)\log(\cosh(c+dx))}{d\sqrt[3]{b\coth^3(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-1/3),x]
[Out] (Coth[c + d*x]*Log[Cosh[c + d*x]])/(d*(b*Coth[c + d*x]^3)^(1/3))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(29) = 58$.

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 6.19

method	result	size
risch	$\frac{\left(e^{2dx+2c}+1\right)x}{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)} - \frac{2\left(e^{2dx+2c}+1\right)(dx+c)}{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)d} + \frac{\left(e^{2dx+2c}+1\right)\ln\left(e^{2dx+2c}+1\right)}{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{1}{3}}\left(e^{2dx+2c}-1\right)d}$	192

```
[In] int(1/(b*coth(d*x+c)^3)^(1/3),x,method=_RETURNVERBOSE)
[Out] 1/(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(e
xp(2*d*x+2*c)+1)*x-2/(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/(e
xp(2*d*x+2*c)-1)*(exp(2*d*x+2*c)+1)/d*(d*x+c)+1/(b*(exp(2*d*x+2*c)+1)^3/(ex
p(2*d*x+2*c)-1)^3)^(1/3)/(exp(2*d*x+2*c)-1)*(exp(2*d*x+2*c)+1)/d*ln(exp(2*d
*x+2*c)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 6.03

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx =$$
$$-\frac{\left(dxe^{(4\,dx+4\,c)} - 2\,dxe^{(2\,dx+2\,c)} + dx - \left(e^{(4\,dx+4\,c)} - 2\,e^{(2\,dx+2\,c)} + 1\right)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)\right)\left(\frac{be^{(6\,dx+6\,c)}+}{e^{(6\,dx+6\,c)}-}\right.}{bde^{(4\,dx+4\,c)} + 2\,bde^{(2\,dx+2\,c)} + bd}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="fricas")
```

```
[Out] -(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x - (e^(4*d*x + 4*c) - 2*
e^(2*d*x + 2*c) + 1)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))*
((b*e^(6*d*x + 6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*
d*x + 6*c) - 3*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b*d*e^(4*d*
x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth^3{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**3)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-1/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \frac{dx+c}{b^{\frac{1}{3}}d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^{\frac{1}{3}}d}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="maxima")
```

```
[Out] (d*x + c)/(b^(1/3)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^(1/3)*d)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^3)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\coth^3(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{1/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^3)^(1/3),x)
[Out] int(1/(b*coth(c + d*x)^3)^(1/3), x)
```

3.37 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx=-\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}+\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}$$

```
[Out] -coth(d*x+c)/d/(b*coth(d*x+c)^3)^(2/3)+x*coth(d*x+c)^2/(b*coth(d*x+c)^3)^(2
/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx=\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}-\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(-2/3),x]
```

```
[Out] -(Coth[c + d*x]/(d*(b*Coth[c + d*x]^3)^(2/3))) + (x*Coth[c + d*x]^2)/(b*Cot
h[c + d*x]^3)^(2/3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{\left(b\coth^3(c+dx)\right)^{2/3}}\\ &= -\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}+\frac{\coth^2(c+dx)\int 1\,dx}{\left(b\coth^3(c+dx)\right)^{2/3}}\\ &= -\frac{\coth(c+dx)}{d\left(b\coth^3(c+dx)\right)^{2/3}}+\frac{x\coth^2(c+dx)}{\left(b\coth^3(c+dx)\right)^{2/3}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \frac{\coth(c+dx)(-1+\operatorname{arctanh}(\tanh(c+dx))\coth(c+dx))}{d\left(b\coth^3(c+dx)\right)^{2/3}}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-2/3),x]
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*(b*Coth[c +
d*x]^3)^(2/3))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.78

method	result	size
risch	$\frac{e^{4dx+4c}dx+2\,e^{2dx+2c}dx+dx+2\,e^{2dx+2c}+2}{\left(\frac{b\left(e^{2dx+2c}+1\right)^3}{\left(e^{2dx+2c}-1\right)^3}\right)^{\frac{2}{3}}\left(e^{2dx+2c}-1\right)^2 d}$	89

```
[In] int(1/(b*coth(d*x+c)^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] (exp(4*d*x+4*c)*d*x+2*exp(2*d*x+2*c)*d*x+d*x+2*exp(2*d*x+2*c)+2)/(b*(exp(2*
d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(2/3)/(exp(2*d*x+2*c)-1)^2/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(46) = 92$.

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.74

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx =$$

$$-\frac{\left(dx\cosh\left(dx+c\right)^2-\left(dxe^{(2\,dx+2\,c)}-dx\right)\sinh\left(dx+c\right)^2+dx-\left(dx\cosh\left(dx+c\right)^2+dx+2\right)e^{(2\,dx+2\,c)}-2}{bd\cosh\left(dx+c\right)^2+\left(bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^2+bd+\left(bd\cosh\left(dx+c\right)^2+\right.}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -(d*x*cosh(d*x + c)^2 - (d*x*e^(2*d*x + 2*c) - d*x)*sinh(d*x + c)^2 + d*x -
 (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) - 2*(d*x*cosh(d*x + c)*e^(
2*d*x + 2*c) - d*x*cosh(d*x + c))*sinh(d*x + c) + 2)*((b*e^(6*d*x + 6*c) +
3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3*e^(4*d*
x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(1/3)/(b*d*cosh(d*x + c)^2 + (b*d*e^(2*d
*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(
d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**3)**(2/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**3)**(-2/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \frac{dx+c}{b^{\frac{2}{3}}d} - \frac{2}{\left(b^{\frac{2}{3}}e^{(-2\,dx-2\,c)}+b^{\frac{2}{3}}\right)d}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="maxima")
[Out] (d*x + c)/(b^(2/3)*d) - 2/((b^(2/3)*e^(-2*d*x - 2*c) + b^(2/3))*d)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^3)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{2/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^3)^(2/3),x)
[Out] int(1/(b*coth(c + d*x)^3)^(2/3), x)
```

3.38 $\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} + \frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

```
[Out] -1/b/d/(b*coth(d*x+c)^3)^(1/3)+x*coth(d*x+c)/b/(b*coth(d*x+c)^3)^(1/3)-1/3*
tanh(d*x+c)^2/b/d/(b*coth(d*x+c)^3)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}} - \frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}} - \frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^3)^(-4/3),x]
```

```
[Out] -(1/(b*d*(b*Coth[c + d*x]^3)^(1/3))) + (x*Coth[c + d*x])/(b*(b*Coth[c + d*x
]^3)^(1/3)) - Tanh[c + d*x]^2/(3*b*d*(b*Coth[c + d*x]^3)^(1/3))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}\\
&= -\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}+\frac{\coth(c+dx)\int \tanh^2(c+dx)\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}\\
&= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}+\frac{\coth(c+dx)\int 1\,dx}{b\sqrt[3]{b\coth^3(c+dx)}}\\
&= -\frac{1}{bd\sqrt[3]{b\coth^3(c+dx)}}+\frac{x\coth(c+dx)}{b\sqrt[3]{b\coth^3(c+dx)}}-\frac{\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.64

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \frac{-3+3\text{arctanh}(\tanh(c+dx))\coth(c+dx)-\tanh^2(c+dx)}{3bd\sqrt[3]{b\coth^3(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x]^3)^(-4/3),x]
[Out] (-3 + 3*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x] - Tanh[c + d*x]^2)/(3*b*d*(b*C
oth[c + d*x]^3)^(1/3))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{3\,{\mathrm e}^{6 d x +6 c} d x +9\,{\mathrm e}^{4 d x +4 c} d x +9\,{\mathrm e}^{2 d x +2 c} d x +3 d x +12\,{\mathrm e}^{4 d x +4 c}+12\,{\mathrm e}^{2 d x +2 c}+8}{3 b \left({\mathrm e}^{2 d x +2 c}+1\right)^{2} \left({\mathrm e}^{2 d x +2 c}-1\right) \left(\frac{b \left({\mathrm e}^{2 d x +2 c}+1\right)^{3}}{\left({\mathrm e}^{2 d x +2 c}-1\right)^{3}}\right)^{\frac{1}{3}} d}$	132

```
[In] int(1/(b*coth(d*x+c)^3)^(4/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3*exp(6*d*x+6*c)*d*x+9*exp(4*d*x+4*c)*d*x+9*exp(2*d*x+2*c)*d*x+3*d*x+1
2*exp(4*d*x+4*c)+12*exp(2*d*x+2*c)+8)/b/(exp(2*d*x+2*c)+1)^2/(exp(2*d*x+2*c
)-1)/(b*(exp(2*d*x+2*c)+1)^3/(exp(2*d*x+2*c)-1)^3)^(1/3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(72) = 144$.

Time = 0.28 (sec) , antiderivative size = 1579, normalized size of antiderivative = 19.74

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(3*d*x*cosh(d*x + c)^6 + 3*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c)
 + d*x)*sinh(d*x + c)^6 + 18*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d*x*cos
h(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(3*d*x
+ 4)*cosh(d*x + c)^4 + 3*(15*d*x*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x
 + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^2 + 3*d*x +
4)*e^(2*d*x + 2*c) + 4)*sinh(d*x + c)^4 + 12*(5*d*x*cosh(d*x + c)^3 + (3*d*
x + 4)*cosh(d*x + c) + (5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*
e^(4*d*x + 4*c) - 2*(5*d*x*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(
2*d*x + 2*c))*sinh(d*x + c)^3 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*(15*d*x*c
osh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + (15*d*x*cosh(d*x +
 c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(4*d*x + 4*c) - 2*(15*
d*x*cosh(d*x + c)^4 + 6*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 4)*e^(2*d*x +
 2*c) + 4)*sinh(d*x + c)^2 + 3*d*x + (3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)
*cosh(d*x + c)^4 + 3*(3*d*x + 4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(4*d*x + 4*
c) - 2*(3*d*x*cosh(d*x + c)^6 + 3*(3*d*x + 4)*cosh(d*x + c)^4 + 3*(3*d*x +
4)*cosh(d*x + c)^2 + 3*d*x + 8)*e^(2*d*x + 2*c) + 6*(3*d*x*cosh(d*x + c)^5
+ 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c) + (3*d*x*cosh(d
*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3*d*x + 4)*cosh(d*x + c))*e^(4
*d*x + 4*c) - 2*(3*d*x*cosh(d*x + c)^5 + 2*(3*d*x + 4)*cosh(d*x + c)^3 + (3
*d*x + 4)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) + 8)*((b*e^(6*d*x +
```

```
6*c) + 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) + b)/(e^(6*d*x + 6*c) - 3
*e^(4*d*x + 4*c) + 3*e^(2*d*x + 2*c) - 1))^(2/3)/(b^2*d*cosh(d*x + c)^6 + 3
*b^2*d*cosh(d*x + c)^4 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c) +
 b^2*d)*sinh(d*x + c)^6 + 6*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^2*d*
cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*b^
2*d*cosh(d*x + c)^2 + 3*(5*b^2*d*cosh(d*x + c)^2 + b^2*d + (5*b^2*d*cosh(d*
x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(
2*d*x + 2*c))*sinh(d*x + c)^4 + 4*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d
*x + c) + (5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c)
 + 2*(5*b^2*d*cosh(d*x + c)^3 + 3*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^3 + b^2*d + 3*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2
 + b^2*d + (5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4
*d*x + 4*c) + 2*(5*b^2*d*cosh(d*x + c)^4 + 6*b^2*d*cosh(d*x + c)^2 + b^2*d)
*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^6 + 3*b^2*d*cosh(d
*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cos
h(d*x + c)^6 + 3*b^2*d*cosh(d*x + c)^4 + 3*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c) + 6*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x + c)^3 + b^2*d*c
osh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^5 + 2*b^2*d*cosh(d*x
 + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth^3\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**3)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**3)**(-4/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = -\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{3\left(3\,b^{\frac{4}{3}}e^{(-2\,dx-2\,c)}+3\,b^{\frac{4}{3}}e^{(-4\,dx-4\,c)}+b^{\frac{4}{3}}e^{(-6\,dx-6\,c)}+b^{\frac{4}{3}}\right)d}+\frac{dx+c}{b^{\frac{4}{3}}d}$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/((3*b^(4/3)*e^(-2*d*x -
2*c) + 3*b^(4/3)*e^(-4*d*x - 4*c) + b^(4/3)*e^(-6*d*x - 6*c) + b^(4/3))*d)
+ (d*x + c)/(b^(4/3)*d)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^3\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^3)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*coth(d*x + c)^3)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^3(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^3\right)^{4/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^3)^(4/3),x)
```

```
[Out] int(1/(b*coth(c + d*x)^3)^(4/3), x)
```

3.39 $\int \left(b\coth^4(c+dx)\right)^n\,dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+4n),\frac{1}{2}(3+4n),\coth^2(c+dx)\right)}{d(1+4n)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^4)^n*hypergeom([1, 1/2+2*n],[3/2+2*n],coth(d*x+c
)^2)/d/(1+4*n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3739, 3557, 371}

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n \text{Hypergeometric2F1}\left(1,\frac{1}{2}(4n+1),\frac{1}{2}(4n+3),\coth^2(c+dx)\right)}{d(4n+1)}$$

```
[In] Int[(b*Coth[c + d*x]^4)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, (1 + 4*n)/2, (3 +
 4*n)/2, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\int \coth^{4n}(c+dx)\,dx\\ &= -\frac{\left(\coth^{-4n}(c+dx)\left(b\coth^4(c+dx)\right)^n\right)\text{Subst}\left(\int \frac{x^{4n}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}\\ &= \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+4n), \frac{1}{2}(3+4n), \coth^2(c+dx)\right)}{d(1+4n)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \left(b\coth^4(c+dx)\right)^n\,dx = \frac{\coth(c+dx)\left(b\coth^4(c+dx)\right)^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}+2n, \frac{3}{2}+2n, \coth^2(c+dx)\right)}{d(1+4n)}$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^n,x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^4)^n*Hypergeometric2F1[1, 1/2 + 2*n, 3/2 +
2*n, Coth[c + d*x]^2])/(d*(1 + 4*n))
```

Maple [F]

$$\int \left(b\coth\left(dx+c\right)^{4}\right)^{n}dx$$

```
[In] int((b*coth(d*x+c)^4)^n,x)
[Out] int((b*coth(d*x+c)^4)^n,x)
```

Fricas [F]

$$\int \left(b\coth^{4}(c+dx)\right)^{n}\,dx = \int \left(b\coth\left(dx+c\right)^{4}\right)^{n}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c)^4)^n, x)
```

Sympy [F]

$$\int \left(b\coth^{4}(c+dx)\right)^{n}\,dx = \int \left(b\coth^{4}\left(c+dx\right)\right)^{n}\,dx$$

```
[In] integrate((b*coth(d*x+c)**4)**n,x)
[Out] Integral((b*coth(c + d*x)**4)**n, x)
```

Maxima [F]

$$\int \left(b\coth^{4}(c+dx)\right)^{n}\,dx = \int \left(b\coth\left(dx+c\right)^{4}\right)^{n}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

Giac [F]

$$\int \left(b\coth^4(c+dx)\right)^n\,dx=\int \left(b\coth\left(dx+c\right)^4\right)^n\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^n,x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^n\,dx=\int \left(b\coth(c+d\,x)^4\right)^n\,dx$$

```
[In] int((b*coth(c + d*x)^4)^n,x)
[Out] int((b*coth(c + d*x)^4)^n, x)
```

3.40 $\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx = -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b\coth^4(c+dx)}\tanh(c+dx)}{d} + bx\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)$$

```
[Out] -1/3*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/2)/d-1/5*b*coth(d*x+c)^3*(b*coth(d*
x+c)^4)^(1/2)/d-b*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+b*x*(b*coth(d*x+c)^
4)^(1/2)*tanh(d*x+c)^2
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx = -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} + bx\tanh^2(c+dx)\sqrt{b\coth^4(c+dx)} - \frac{b\tanh(c+dx)\sqrt{b\coth^4(c+dx)}}{d}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(3/2),x]
```

```
[Out] -1/3*(b*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^4])/d - (b*Coth[c + d*x]^3*Sqrt[
b*Coth[c + d*x]^4])/(5*d) - (b*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d + b
*x*Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(b\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int \coth^6(c+dx)\,dx \\
&= -\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d}+\left(b\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int \coth^4(c+dx)\,dx \\
&= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} \\
&\quad +\left(b\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int \coth^2(c+dx)\,dx \\
&= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d}-\frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} \\
&\quad -\frac{b\sqrt{b\coth^4(c+dx)}\tanh(c+dx)}{d}+\left(b\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int 1\,dx
\end{aligned}$$

$$= -\frac{b\coth(c+dx)\sqrt{b\coth^4(c+dx)}}{3d} - \frac{b\coth^3(c+dx)\sqrt{b\coth^4(c+dx)}}{5d} - \frac{b\sqrt{b\coth^4(c+dx)}\tanh(c+dx)}{d} + bx\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.39

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = -\frac{\left(b\coth^4(c+dx)\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{5d}$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(3/2),x]
[Out] -1/5*((b*Coth[c + d*x]^4)^(3/2)*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d
*x]^2]*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{\left(b\coth(dx+c)^4\right)^{\frac{3}{2}}\left(6\coth(dx+c)^5+10\coth(dx+c)^3+15\ln(\coth(dx+c)-1)-15\ln(\coth(dx+c)+1)+30\coth(dx+c)\right)}{30d\coth(dx+c)^6}$
default	$-\frac{\left(b\coth(dx+c)^4\right)^{\frac{3}{2}}\left(6\coth(dx+c)^5+10\coth(dx+c)^3+15\ln(\coth(dx+c)-1)-15\ln(\coth(dx+c)+1)+30\coth(dx+c)\right)}{30d\coth(dx+c)^6}$
risch	$\frac{b\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,x}{\left(e^{2dx+2c}+1\right)^2}-\frac{2b\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\left(45e^{8dx+8c}-90e^{6dx+6c}+140e^{4dx+4c}-70e^{2dx+2c}+23\right)}{15\left(e^{2dx+2c}+1\right)^2\left(e^{2dx+2c}-1\right)^3d}$

```
[In] int((b*coth(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/30/d*(b*coth(d*x+c)^4)^(3/2)*(6*coth(d*x+c)^5+10*coth(d*x+c)^3+15*ln(cot
h(d*x+c)-1)-15*ln(coth(d*x+c)+1)+30*coth(d*x+c))/coth(d*x+c)^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3421 vs. $2(98) = 196$.

Time = 0.33 (sec) , antiderivative size = 3421, normalized size of antiderivative = 31.10

$$\int \left(b\coth^4(c+dx)\right)^{3/2} dx = \text{Too large to display}$$

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*b*d*x*cosh(d*x + c)^10 + 15*(b*d*x*e^(4*d*x + 4*c) - 2*b*d*x*e^(2*
d*x + 2*c) + b*d*x)*sinh(d*x + c)^10 + 150*(b*d*x*cosh(d*x + c)*e^(4*d*x +
4*c) - 2*b*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + b*d*x*cosh(d*x + c))*sinh(d*
x + c)^9 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 15*(45*b*d*x*cosh(d*x + c)^
2 - 5*b*d*x + (45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(4*d*x + 4*c) -
2*(45*b*d*x*cosh(d*x + c)^2 - 5*b*d*x - 6*b)*e^(2*d*x + 2*c) - 6*b)*sinh(d*
x + c)^8 + 120*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c) +
(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c)
- 2*(15*b*d*x*cosh(d*x + c)^3 - (5*b*d*x + 6*b)*cosh(d*x + c))*e^(2*d*x + 2
*c))*sinh(d*x + c)^7 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 30*(105*b*d*x*c
osh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (105*b*d*x*
cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + 6*b)*e^(4*
d*x + 4*c) - 2*(105*b*d*x*cosh(d*x + c)^4 + 5*b*d*x - 14*(5*b*d*x + 6*b)*co
sh(d*x + c)^2 + 6*b)*e^(2*d*x + 2*c) + 6*b)*sinh(d*x + c)^6 + 60*(63*b*d*x*
cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*co
sh(d*x + c) + (63*b*d*x*cosh(d*x + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^
3 + 3*(5*b*d*x + 6*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(63*b*d*x*cosh(d*x
 + c)^5 - 14*(5*b*d*x + 6*b)*cosh(d*x + c)^3 + 3*(5*b*d*x + 6*b)*cosh(d*x +
 c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 - 10*(15*b*d*x + 28*b)*cosh(d*x + c)^
4 + 10*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 1
5*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2 + (315*b*d*x*cosh(d*x + c)^6 -
 105*(5*b*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d
*x + c)^2 - 28*b)*e^(4*d*x + 4*c) - 2*(315*b*d*x*cosh(d*x + c)^6 - 105*(5*b
*d*x + 6*b)*cosh(d*x + c)^4 - 15*b*d*x + 45*(5*b*d*x + 6*b)*cosh(d*x + c)^2
 - 28*b)*e^(2*d*x + 2*c) - 28*b)*sinh(d*x + c)^4 + 40*(45*b*d*x*cosh(d*x +
c)^7 - 21*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c
)^3 - (15*b*d*x + 28*b)*cosh(d*x + c) + (45*b*d*x*cosh(d*x + c)^7 - 21*(5*b
*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15*b*d*
x + 28*b)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(45*b*d*x*cosh(d*x + c)^7 - 21
*(5*b*d*x + 6*b)*cosh(d*x + c)^5 + 15*(5*b*d*x + 6*b)*cosh(d*x + c)^3 - (15
*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^3 - 15*b*d*x +
 5*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 5*(135*b*d*x*cosh(d*x + c)^8 - 84*(5
*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*b*d
*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + (135*b*d*x*cosh(d*x + c)^8 - 84
*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x + 6*b)*cosh(d*x + c)^4 + 15*
```

```
b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 + 28*b)*e^(4*d*x + 4*c) - 2*(1
35*b*d*x*cosh(d*x + c)^8 - 84*(5*b*d*x + 6*b)*cosh(d*x + c)^6 + 90*(5*b*d*x
 + 6*b)*cosh(d*x + c)^4 + 15*b*d*x - 12*(15*b*d*x + 28*b)*cosh(d*x + c)^2 +
 28*b)*e^(2*d*x + 2*c) + 28*b)*sinh(d*x + c)^2 + (15*b*d*x*cosh(d*x + c)^10
 - 15*(5*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6
- 10*(15*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cos
h(d*x + c)^2 - 46*b)*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^10 - 15*(5
*b*d*x + 6*b)*cosh(d*x + c)^8 + 30*(5*b*d*x + 6*b)*cosh(d*x + c)^6 - 10*(15
*b*d*x + 28*b)*cosh(d*x + c)^4 - 15*b*d*x + 5*(15*b*d*x + 28*b)*cosh(d*x +
c)^2 - 46*b)*e^(2*d*x + 2*c) + 10*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x +
 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x +
28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c) + (15*b*d*x*cosh(d*
x + c)^9 - 12*(5*b*d*x + 6*b)*cosh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x
 + c)^5 - 4*(15*b*d*x + 28*b)*cosh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x
+ c))*e^(4*d*x + 4*c) - 2*(15*b*d*x*cosh(d*x + c)^9 - 12*(5*b*d*x + 6*b)*co
sh(d*x + c)^7 + 18*(5*b*d*x + 6*b)*cosh(d*x + c)^5 - 4*(15*b*d*x + 28*b)*co
sh(d*x + c)^3 + (15*b*d*x + 28*b)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x
+ c) - 46*b)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*x +
 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) + 6*e
^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^10 + (d*e^(4*d*x
+ 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^10 + 10*(d*cosh(d*x + c)*e^
(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x + c))*sinh(d
*x + c)^9 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + (9*d*cosh(d*x +
c)^2 - d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d
)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d
*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c)^3 - d*c
osh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*
(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^4 - 14*d
*cosh(d*x + c)^2 + d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^4 - 14*d*cosh
(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x +
c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c) + (63*d*cosh(d*x + c)^5 -
70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(63*d*cosh(d
*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^5 - 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 - 35*d*cosh(
d*x + c)^4 + 15*d*cosh(d*x + c)^2 + (21*d*cosh(d*x + c)^6 - 35*d*cosh(d*x +
 c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) + 2*(21*d*cosh(d*x + c)^6
 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) - d)*si
nh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*
x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*d*cosh(d*x + c
)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*e^(2*d*x
 + 2*c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 - 2
8*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + (9*d*co
sh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x
 + c)^2 + d)*e^(4*d*x + 4*c) + 2*(9*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^
```

```
6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*e^(2*d*x + 2*c) + d)*s
inh(d*x + c)^2 + (d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x
+ c)^6 - 10*d*cosh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(4*d*x + 4*c) +
2*(d*cosh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 10*d*cosh(d*x + c)^6 - 10*d*c
osh(d*x + c)^4 + 5*d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 10*(d*cosh(d*x
+ c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 +
d*cosh(d*x + c) + (d*cosh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x +
 c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(d*cosh(
d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c) - d)
```

Sympy [F]

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx = \int \left(b\coth^4\left(c+dx\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**4)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx = \frac{(dx+c)b^{\frac{3}{2}}}{d} - \frac{2\left(70\,b^{\frac{3}{2}}e^{(-2\,dx-2\,c)} - 140\,b^{\frac{3}{2}}e^{(-4\,dx-4\,c)} + 90\,b^{\frac{3}{2}}e^{(-6\,dx-6\,c)} - 45\,b^{\frac{3}{2}}e^{(-8\,dx-8\,c)} - 23\,b^{\frac{3}{2}}\right)}{15\,d(5\,e^{(-2\,dx-2\,c)} - 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} - 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} - 1)}$$

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] (d*x + c)*b^(3/2)/d - 2/15*(70*b^(3/2)*e^(-2*d*x - 2*c) - 140*b^(3/2)*e^(-4
*d*x - 4*c) + 90*b^(3/2)*e^(-6*d*x - 6*c) - 45*b^(3/2)*e^(-8*d*x - 8*c) - 2
3*b^(3/2))/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*
c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.70

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx = \frac{\left(15\,dx + 15\,c - \frac{2\left(45\,e^{(8\,dx+8\,c)} - 90\,e^{(6\,dx+6\,c)} + 140\,e^{(4\,dx+4\,c)} - 70\,e^{(2\,dx+2\,c)} + 23\right)}{\left(e^{(2\,dx+2\,c)} - 1\right)^5}\right)b^{\frac{3}{2}}}{15\,d}$$

```
[In] integrate((b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
[Out] 1/15*(15*d*x + 15*c - 2*(45*e^(8*d*x + 8*c) - 90*e^(6*d*x + 6*c) + 140*e^(4
*d*x + 4*c) - 70*e^(2*d*x + 2*c) + 23)/(e^(2*d*x + 2*c) - 1)^5)*b^(3/2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^{3/2}\,dx = \int \left(b\coth(c+d\,x)^4\right)^{3/2}\,dx$$

```
[In] int((b*coth(c + d*x)^4)^(3/2),x)
[Out] int((b*coth(c + d*x)^4)^(3/2), x)
```

3.41 $\int \sqrt{b \coth^4(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \sqrt{b \coth^4(c+dx)}\,dx = -\frac{\sqrt{b \coth^4(c+dx)}\tanh(c+dx)}{d} + x\sqrt{b \coth^4(c+dx)}\tanh^2(c+dx)$$

```
[Out] -(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c)/d+x*(b*coth(d*x+c)^4)^(1/2)*tanh(d*x+c
)^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \sqrt{b \coth^4(c+dx)}\,dx = x\tanh^2(c+dx)\sqrt{b \coth^4(c+dx)} - \frac{\tanh(c+dx)\sqrt{b \coth^4(c+dx)}}{d}$$

```
[In] Int[Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Tanh[c + d*x])/d) + x*Sqrt[b*Coth[c + d*x]^4]*Ta
nh[c + d*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int\coth^2(c+dx)\,dx\\ &= -\frac{\sqrt{b\coth^4(c+dx)}\tanh(c+dx)}{d}+\left(\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\right)\int 1\,dx\\ &= -\frac{\sqrt{b\coth^4(c+dx)}\tanh(c+dx)}{d}+x\sqrt{b\coth^4(c+dx)}\tanh^2(c+dx)\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int\sqrt{b\coth^4(c+dx)}\,dx = -\frac{\sqrt{b\coth^4(c+dx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)\tanh(c+dx)}{d}$$

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^4],x]
[Out] -((Sqrt[b*Coth[c + d*x]^4]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2]
*Tanh[c + d*x])/d)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\frac{\sqrt{b \coth(dx+c)^4}\,(2\coth(dx+c)+\ln(\coth(dx+c)-1)-\ln(\coth(dx+c)+1))}{2d\coth(dx+c)^2}$	55
default	$-\frac{\sqrt{b \coth(dx+c)^4}\,(2\coth(dx+c)+\ln(\coth(dx+c)-1)-\ln(\coth(dx+c)+1))}{2d\coth(dx+c)^2}$	55
risch	$\frac{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,\left(e^{2dx+2c}-1\right)^2 x}{\left(e^{2dx+2c}+1\right)^2}-\frac{2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,\left(e^{2dx+2c}-1\right)}{\left(e^{2dx+2c}+1\right)^2 d}$	119

```
[In] int((b*coth(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/d*(b*coth(d*x+c)^4)^(1/2)*(2*coth(d*x+c)+ln(coth(d*x+c)-1)-ln(coth(d*x
+c)+1))/coth(d*x+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 415, normalized size of antiderivative = 8.30

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \frac{\left(dx\cosh\left(dx+c\right)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^2-dx+\left(dx\cosh\left(dx+c\right)^2-dx-\right.}{d\cosh\left(dx+c\right)^2+\left(de^{(4\,dx+4\,c)}+2\,de^{(2\,dx+2\,c)}+d\right)\sinh\left(dx+c\right)^2+}$$

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 - d*x + (d*x*cosh(d*x + c)^2 - d*x - 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 - d*x - 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) - 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(d*cosh(d*x + c)^2 + (d*e^(4*d*
x + 4*c) + 2*d*e^(2*d*x + 2*c) + d)*sinh(d*x + c)^2 + (d*cosh(d*x + c)^2 -
d)*e^(4*d*x + 4*c) + 2*(d*cosh(d*x + c)^2 - d)*e^(2*d*x + 2*c) + 2*(d*cosh(
d*x + c)*e^(4*d*x + 4*c) + 2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + d*cosh(d*x +
 c))*sinh(d*x + c) - d)
```

Sympy [F]

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \int \sqrt{b\coth^4\left(c+dx\right)}\,dx$$

```
[In] integrate((b*coth(d*x+c)**4)**(1/2),x)
[Out] Integral(sqrt(b*coth(c + d*x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \frac{(dx+c)\sqrt{b}}{d} + \frac{2\sqrt{b}}{d(e^{(-2\,dx-2\,c)}-1)}$$

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] (d*x + c)*sqrt(b)/d + 2*sqrt(b)/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.54

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \frac{\left(dx+c-\frac{2}{e^{(2\,dx+2\,c)}-1}\right)\sqrt{b}}{d}$$

```
[In] integrate((b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] (d*x + c - 2/(e^(2*d*x + 2*c) - 1))*sqrt(b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\coth^4(c+dx)}\,dx = \int \sqrt{b\,{\mathrm{coth}\left(c+d\,x\right)}^4}\,dx$$

```
[In] int((b*coth(c + d*x)^4)^(1/2),x)
[Out] int((b*coth(c + d*x)^4)^(1/2), x)
```

3.42 $\int \frac{1}{\sqrt{b \coth^4(c+dx)}} dx$

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{1}{\sqrt{b \coth^4(c+dx)}} dx = -\frac{\coth(c+dx)}{d\sqrt{b \coth^4(c+dx)}} + \frac{x \coth^2(c+dx)}{\sqrt{b \coth^4(c+dx)}}$$

```
[Out] -coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/(b*coth(d*x+c)^4)^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\sqrt{b \coth^4(c+dx)}} dx = \frac{x \coth^2(c+dx)}{\sqrt{b \coth^4(c+dx)}} - \frac{\coth(c+dx)}{d\sqrt{b \coth^4(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] -(Coth[c + d*x]/(d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/Sqrt[b*C
oth[c + d*x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{\sqrt{b\coth^4(c+dx)}}\\ &= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}+\frac{\coth^2(c+dx)\int 1\,dx}{\sqrt{b\coth^4(c+dx)}}\\ &= -\frac{\coth(c+dx)}{d\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{\sqrt{b\coth^4(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \frac{\coth(c+dx)(-1+\operatorname{arctanh}(\tanh(c+dx))\coth(c+dx))}{d\sqrt{b\coth^4(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^4],x]
```

```
[Out] (Coth[c + d*x]*(-1 + ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]))/(d*Sqrt[b*Coth[
c + d*x]^4])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$-\frac{\coth(dx+c)(\ln(\coth(dx+c)-1)\coth(dx+c)-\ln(\coth(dx+c)+1)\coth(dx+c)+2)}{2d\sqrt{b\coth(dx+c)^4}}$	59
default	$-\frac{\coth(dx+c)(\ln(\coth(dx+c)-1)\coth(dx+c)-\ln(\coth(dx+c)+1)\coth(dx+c)+2)}{2d\sqrt{b\coth(dx+c)^4}}$	59
risch	$\frac{e^{4dx+4c}dx+2\,e^{2dx+2c}dx+dx+2\,e^{2dx+2c}+2}{\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\left(e^{2dx+2c}-1\right)^2 d}$	89

```
[In] int(1/(b*coth(d*x+c)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*coth(d*x+c)*(ln(coth(d*x+c)-1)*coth(d*x+c)-ln(coth(d*x+c)+1)*coth(d*
x+c)+2)/(b*coth(d*x+c)^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 422, normalized size of antiderivative = 8.44

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx$$
$$= \frac{\left(dx\cosh\left(dx+c\right)^2+\left(dxe^{(4\,dx+4\,c)}-2\,dxe^{(2\,dx+2\,c)}+dx\right)\sinh\left(dx+c\right)^2+dx+\left(dx\cosh\left(dx+c\right)^2+dx+\right.}{bd\cosh\left(dx+c\right)^2+\left(bde^{(4\,dx+4\,c)}+2\,bde^{(2\,dx+2\,c)}+bd\right)\sinh\left(dx+c\right)^2+bd}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] (d*x*cosh(d*x + c)^2 + (d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x + 2*c) + d*x)*
sinh(d*x + c)^2 + d*x + (d*x*cosh(d*x + c)^2 + d*x + 2)*e^(4*d*x + 4*c) - 2
*(d*x*cosh(d*x + c)^2 + d*x + 2)*e^(2*d*x + 2*c) + 2*(d*x*cosh(d*x + c)*e^(
4*d*x + 4*c) - 2*d*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sin
h(d*x + c) + 2)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e^(4*d*
x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6*c) +
6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b*d*cosh(d*x + c)^2 + (b*d*e^(
4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2*c) + b*d)*sinh(d*x + c)^2 + b*d + (b*d*co
sh(d*x + c)^2 + b*d)*e^(4*d*x + 4*c) + 2*(b*d*cosh(d*x + c)^2 + b*d)*e^(2*d
*x + 2*c) + 2*(b*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b*d*cosh(d*x + c)*e^(2
*d*x + 2*c) + b*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth^4{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/2),x)
[Out] Integral(1/sqrt(b*coth(c + d*x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \frac{dx+c}{\sqrt{b}d} - \frac{2\sqrt{b}}{(be^{(-2\,dx-2\,c)}+b)d}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] (d*x + c)/(sqrt(b)*d) - 2*sqrt(b)/((b*e^(-2*d*x - 2*c) + b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \frac{\frac{dx+c}{\sqrt{b}} + \frac{2}{\sqrt{b}(e^{(2\,dx+2\,c)}+1)}}{d}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] ((d*x + c)/sqrt(b) + 2/(sqrt(b)*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^4}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^4)^(1/2),x)
[Out] int(1/(b*coth(c + d*x)^4)^(1/2), x)
```

3.43 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 118

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}$$

```
[Out] -coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)+x*coth(d*x+c)^2/b/(b*coth(d*x+c)^4
)^(1/2)-1/3*tanh(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/2)-1/5*tanh(d*x+c)^3/b/d/(
b*coth(d*x+c)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3739, 3554, 8}

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}} + \frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}} - \frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}} - \frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(-3/2),x]
```

```
[Out] -(Coth[c + d*x]/(b*d*Sqrt[b*Coth[c + d*x]^4])) + (x*Coth[c + d*x]^2)/(b*Sqr
t[b*Coth[c + d*x]^4]) - Tanh[c + d*x]/(3*b*d*Sqrt[b*Coth[c + d*x]^4]) - Tan
h[c + d*x]^3/(5*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth^2(c+dx)\int \tanh^6(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}\\
&= -\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}+\frac{\coth^2(c+dx)\int \tanh^4(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}\\
&= -\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}+\frac{\coth^2(c+dx)\int \tanh^2(c+dx)\,dx}{b\sqrt{b\coth^4(c+dx)}}\\
&= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}+\frac{\coth^2(c+dx)\int 1\,dx}{b\sqrt{b\coth^4(c+dx)}}\\
&= -\frac{\coth(c+dx)}{bd\sqrt{b\coth^4(c+dx)}}+\frac{x\coth^2(c+dx)}{b\sqrt{b\coth^4(c+dx)}}-\frac{\tanh(c+dx)}{3bd\sqrt{b\coth^4(c+dx)}}-\frac{\tanh^3(c+dx)}{5bd\sqrt{b\coth^4(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \frac{-15\coth(c+dx)+15\text{arctanh}(\tanh(c+dx))\coth^2(c+dx)-5\tanh(c+dx)-}{15bd\sqrt{b\coth^4(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-3/2),x]
```

```
[Out] (-15*Coth[c + d*x] + 15*ArcTanh[Tanh[c + d*x]]*Coth[c + d*x]^2 - 5*Tanh[c +
 d*x] - 3*Tanh[c + d*x]^3)/(15*b*d*Sqrt[b*Coth[c + d*x]^4])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

method	result
derivativedivides	$-\frac{\coth(dx+c)\left(15\ln(\coth(dx+c)-1)\coth(dx+c)^5-15\ln(\coth(dx+c)+1)\coth(dx+c)^5+30\coth(dx+c)^4+10\coth(dx+c)^2\right.}{30d\left(b\coth(dx+c)^4\right)^{\frac{3}{2}}}$
default	$-\frac{\coth(dx+c)\left(15\ln(\coth(dx+c)-1)\coth(dx+c)^5-15\ln(\coth(dx+c)+1)\coth(dx+c)^5+30\coth(dx+c)^4+10\coth(dx+c)^2\right.}{30d\left(b\coth(dx+c)^4\right)^{\frac{3}{2}}}$
risch	$\frac{\left(e^{2dx+2c}+1\right)^2 x}{b\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}}+\frac{6\,e^{8dx+8c}+12\,e^{6dx+6c}+\frac{56\,e^{4dx+4c}}{3}+\frac{28\,e^{2dx+2c}}{3}+\frac{46}{15}}{b\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}-1\right)^2\sqrt{\frac{b\left(e^{2dx+2c}+1\right)^4}{\left(e^{2dx+2c}-1\right)^4}}\,d}$

```
[In] int(1/(b*coth(d*x+c)^4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30/d*coth(d*x+c)*(15*ln(coth(d*x+c)-1)*coth(d*x+c)^5-15*ln(coth(d*x+c)+1
)*coth(d*x+c)^5+30*coth(d*x+c)^4+10*coth(d*x+c)^2+6)/(b*coth(d*x+c)^4)^(3/2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3473 vs. 2(106) = 212.

Time = 0.32 (sec) , antiderivative size = 3473, normalized size of antiderivative = 29.43

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*d*x*cosh(d*x + c)^10 + 15*(d*x*e^(4*d*x + 4*c) - 2*d*x*e^(2*d*x +
2*c) + d*x)*sinh(d*x + c)^10 + 150*(d*x*cosh(d*x + c)*e^(4*d*x + 4*c) - 2*d
```

*x*cosh(d*x + c)*e^(2*d*x + 2*c) + d*x*cosh(d*x + c))*sinh(d*x + c)^9 + 15*
(5*d*x + 6)*cosh(d*x + c)^8 + 15*(45*d*x*cosh(d*x + c)^2 + 5*d*x + (45*d*x*
cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d*x + 4*c) - 2*(45*d*x*cosh(d*x + c)^2 +
5*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^8 + 120*(15*d*x*cosh(d*x + c)
^3 + (5*d*x + 6)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh
(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^3 + (5*d*x + 6)*cosh(d
*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^7 + 30*(5*d*x + 6)*cosh(d*x + c)^6
+ 30*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + (1
05*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5*d*x + 6)*e^(4*d
*x + 4*c) - 2*(105*d*x*cosh(d*x + c)^4 + 14*(5*d*x + 6)*cosh(d*x + c)^2 + 5
*d*x + 6)*e^(2*d*x + 2*c) + 6)*sinh(d*x + c)^6 + 60*(63*d*x*cosh(d*x + c)^5
 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x + c) + (63*d*x*c
osh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x + c)^3 + 3*(5*d*x + 6)*cosh(d*x +
c))*e^(4*d*x + 4*c) - 2*(63*d*x*cosh(d*x + c)^5 + 14*(5*d*x + 6)*cosh(d*x +
 c)^3 + 3*(5*d*x + 6)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 10*
(15*d*x + 28)*cosh(d*x + c)^4 + 10*(315*d*x*cosh(d*x + c)^6 + 105*(5*d*x +
6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + (315*d*x*cos
h(d*x + c)^6 + 105*(5*d*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x +
c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(315*d*x*cosh(d*x + c)^6 + 105*(5*d
*x + 6)*cosh(d*x + c)^4 + 45*(5*d*x + 6)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(
2*d*x + 2*c) + 28)*sinh(d*x + c)^4 + 40*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x
 + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh
(d*x + c) + (45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(
5*d*x + 6)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) -
 2*(45*d*x*cosh(d*x + c)^7 + 21*(5*d*x + 6)*cosh(d*x + c)^5 + 15*(5*d*x + 6
)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x
+ c)^3 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 5*(135*d*x*cosh(d*x + c)^8 + 84*
(5*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x +
 28)*cosh(d*x + c)^2 + 15*d*x + (135*d*x*cosh(d*x + c)^8 + 84*(5*d*x + 6)*c
osh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 28)*cosh(d*x
 + c)^2 + 15*d*x + 28)*e^(4*d*x + 4*c) - 2*(135*d*x*cosh(d*x + c)^8 + 84*(5
*d*x + 6)*cosh(d*x + c)^6 + 90*(5*d*x + 6)*cosh(d*x + c)^4 + 12*(15*d*x + 2
8)*cosh(d*x + c)^2 + 15*d*x + 28)*e^(2*d*x + 2*c) + 28)*sinh(d*x + c)^2 + 1
5*d*x + (15*d*x*cosh(d*x + c)^10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d
*x + 6)*cosh(d*x + c)^6 + 10*(15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28
)*cosh(d*x + c)^2 + 15*d*x + 46)*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c)^
10 + 15*(5*d*x + 6)*cosh(d*x + c)^8 + 30*(5*d*x + 6)*cosh(d*x + c)^6 + 10*(
15*d*x + 28)*cosh(d*x + c)^4 + 5*(15*d*x + 28)*cosh(d*x + c)^2 + 15*d*x + 4
6)*e^(2*d*x + 2*c) + 10*(15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh(d*x +
 c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c)^3 +
(15*d*x + 28)*cosh(d*x + c) + (15*d*x*cosh(d*x + c)^9 + 12*(5*d*x + 6)*cosh
(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(15*d*x + 28)*cosh(d*x + c
)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(4*d*x + 4*c) - 2*(15*d*x*cosh(d*x + c
)^9 + 12*(5*d*x + 6)*cosh(d*x + c)^7 + 18*(5*d*x + 6)*cosh(d*x + c)^5 + 4*(
15*d*x + 28)*cosh(d*x + c)^3 + (15*d*x + 28)*cosh(d*x + c))*e^(2*d*x + 2*c)

```
)*sinh(d*x + c) + 46)*sqrt((b*e^(8*d*x + 8*c) + 4*b*e^(6*d*x + 6*c) + 6*b*e
^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + b)/(e^(8*d*x + 8*c) - 4*e^(6*d*x + 6
*c) + 6*e^(4*d*x + 4*c) - 4*e^(2*d*x + 2*c) + 1))/(b^2*d*cosh(d*x + c)^10 +
 5*b^2*d*cosh(d*x + c)^8 + (b^2*d*e^(4*d*x + 4*c) + 2*b^2*d*e^(2*d*x + 2*c)
 + b^2*d)*sinh(d*x + c)^10 + 10*(b^2*d*cosh(d*x + c)*e^(4*d*x + 4*c) + 2*b^
2*d*cosh(d*x + c)*e^(2*d*x + 2*c) + b^2*d*cosh(d*x + c))*sinh(d*x + c)^9 +
10*b^2*d*cosh(d*x + c)^6 + 5*(9*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2*d*co
sh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c)^2 + b^2*d
)*e^(2*d*x + 2*c))*sinh(d*x + c)^8 + 40*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c) + (3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(4*d*x + 4*
c) + 2*(3*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sin
h(d*x + c)^7 + 10*b^2*d*cosh(d*x + c)^4 + 10*(21*b^2*d*cosh(d*x + c)^4 + 14
*b^2*d*cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^4 + 14*b^2*d*cosh(
d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(d*x + c)^4 + 14*b^2*
d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^6 + 4*(63*b^2*d*c
osh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c) + (63*b^
2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15*b^2*d*cosh(d*x + c))*e^
(4*d*x + 4*c) + 2*(63*b^2*d*cosh(d*x + c)^5 + 70*b^2*d*cosh(d*x + c)^3 + 15
*b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c)^5 + 5*b^2*d*cosh(d*x +
 c)^2 + 10*(21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*
cosh(d*x + c)^2 + b^2*d + (21*b^2*d*cosh(d*x + c)^6 + 35*b^2*d*cosh(d*x + c
)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(21*b^2*d*cosh(
d*x + c)^6 + 35*b^2*d*cosh(d*x + c)^4 + 15*b^2*d*cosh(d*x + c)^2 + b^2*d)*e
^(2*d*x + 2*c))*sinh(d*x + c)^4 + 40*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cos
h(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (3*b^2*d*cos
h(d*x + c)^7 + 7*b^2*d*cosh(d*x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*co
sh(d*x + c))*e^(4*d*x + 4*c) + 2*(3*b^2*d*cosh(d*x + c)^7 + 7*b^2*d*cosh(d*
x + c)^5 + 5*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*e^(2*d*x + 2*c))*
sinh(d*x + c)^3 + b^2*d + 5*(9*b^2*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x +
c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d*x + c)^2 + b^2*d + (9*b^2
*d*cosh(d*x + c)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 +
12*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(9*b^2*d*cosh(d*x + c
)^8 + 28*b^2*d*cosh(d*x + c)^6 + 30*b^2*d*cosh(d*x + c)^4 + 12*b^2*d*cosh(d
*x + c)^2 + b^2*d)*e^(2*d*x + 2*c))*sinh(d*x + c)^2 + (b^2*d*cosh(d*x + c)^
10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d*cosh(d*x
 + c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(
d*x + c)^10 + 5*b^2*d*cosh(d*x + c)^8 + 10*b^2*d*cosh(d*x + c)^6 + 10*b^2*d
*cosh(d*x + c)^4 + 5*b^2*d*cosh(d*x + c)^2 + b^2*d)*e^(2*d*x + 2*c) + 10*(b
^2*d*cosh(d*x + c)^9 + 4*b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 +
4*b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c) + (b^2*d*cosh(d*x + c)^9 + 4*
b^2*d*cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 +
 b^2*d*cosh(d*x + c))*e^(4*d*x + 4*c) + 2*(b^2*d*cosh(d*x + c)^9 + 4*b^2*d*
cosh(d*x + c)^7 + 6*b^2*d*cosh(d*x + c)^5 + 4*b^2*d*cosh(d*x + c)^3 + b^2*d
*cosh(d*x + c))*e^(2*d*x + 2*c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth^4{(c+dx)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**4)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.31

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = -\frac{2\left(70\sqrt{b}e^{(-2\,dx-2\,c)}+140\sqrt{b}e^{(-4\,dx-4\,c)}+90\sqrt{b}e^{(-6\,dx-6\,c)}+45\sqrt{b}e^{(-8\,dx-8\,c)}+23\sqrt{b}\right)}{15\left(5\,b^2e^{(-2\,dx-2\,c)}+10\,b^2e^{(-4\,dx-4\,c)}+10\,b^2e^{(-6\,dx-6\,c)}+5\,b^2e^{(-8\,dx-8\,c)}+b^2e^{(-10\,dx-10\,c)}+b^2\right)d} + \frac{dx+c}{b^{\frac{3}{2}}d}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="maxima")
[Out] -2/15*(70*sqrt(b)*e^(-2*d*x - 2*c) + 140*sqrt(b)*e^(-4*d*x - 4*c) + 90*sqrt
(b)*e^(-6*d*x - 6*c) + 45*sqrt(b)*e^(-8*d*x - 8*c) + 23*sqrt(b))/((5*b^2*e^
(-2*d*x - 2*c) + 10*b^2*e^(-4*d*x - 4*c) + 10*b^2*e^(-6*d*x - 6*c) + 5*b^2*
e^(-8*d*x - 8*c) + b^2*e^(-10*d*x - 10*c) + b^2)*d) + (d*x + c)/(b^(3/2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \frac{\frac{15\,(dx+c)}{\sqrt{b}} + \frac{2\,\left(45\,e^{(8\,dx+8\,c)}+90\,e^{(6\,dx+6\,c)}+140\,e^{(4\,dx+4\,c)}+70\,e^{(2\,dx+2\,c)}+23\right)}{\sqrt{b}\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,bd}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(3/2),x, algorithm="giac")
[Out] 1/15*(15*(d*x + c)/sqrt(b) + 2*(45*e^(8*d*x + 8*c) + 90*e^(6*d*x + 6*c) + 1
40*e^(4*d*x + 4*c) + 70*e^(2*d*x + 2*c) + 23)/(sqrt(b)*(e^(2*d*x + 2*c) + 1
)^5))/(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{3/2}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^4)^(3/2),x)
[Out] int(1/(b*coth(c + d*x)^4)^(3/2), x)
```

3.44 $\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 353

$$\begin{aligned}\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx = &-\frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}\\
&+\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}\\
&+\frac{b\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)}\\
&-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}\\
&-\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}\\
&+\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}\\
&-\frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}\end{aligned}$$

```
[Out] b*arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)-3/
7*b*coth(d*x+c)*(b*coth(d*x+c)^4)^(1/3)/d-3/13*b*coth(d*x+c)^3*(b*coth(d*x+
c)^4)^(1/3)/d-1/4*b*(b*coth(d*x+c)^4)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x
+c)^(2/3))/d/coth(d*x+c)^(4/3)+1/4*b*(b*coth(d*x+c)^4)^(1/3)*ln(1+coth(d*x+
```

```
c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*x+c)^(4/3)-1/2*b*arctan(1/3*(1-2*coth(
d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+
1/2*b*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3
^(1/2)/d/coth(d*x+c)^(4/3)-3*b*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$\begin{aligned}\int \left(b\coth^4(c+dx)\right)^{4/3}dx = &-\frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}\\&+\frac{\sqrt{3}b\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)}\\&+\frac{b\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)}-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}\\&-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}-\frac{3b\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{d}\\&-\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}\\&+\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}\end{aligned}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] -1/2*(Sqrt[3]*b*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x
]^4)^(1/3))/(d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*Coth[c + d*x
]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (b
*ArcTanh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(
4/3)) - (3*b*Coth[c + d*x]*(b*Coth[c + d*x]^4)^(1/3))/(7*d) - (3*b*Coth[c +
 d*x]^3*(b*Coth[c + d*x]^4)^(1/3))/(13*d) - (b*(b*Coth[c + d*x]^4)^(1/3)*Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3))
 + (b*(b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]
^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*b*(b*Coth[c + d*x]^4)^(1/3)*Tanh[c
+ d*x])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{16}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{10}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} \\
&\quad + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)} \\
&= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} \\
&\quad - \frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\coth^{\frac{4}{3}}(c+dx)}
\end{aligned}
$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} - \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx, x, \coth(c+dx)\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} - \frac{\left(3b\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)} + \frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}$$

$$=\frac{b\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)}-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}$$
$$-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}-\frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}$$
$$-\frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\operatorname{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{\left(b\sqrt[3]{b\coth^4(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{\left(3b\sqrt[3]{b\coth^4(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{\left(3b\sqrt[3]{b\coth^4(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$

$$=\frac{b\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)}$$
$$-\frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d}-\frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d}$$
$$-\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$-\frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}$$
$$-\frac{\left(3b\sqrt[3]{b\coth^4(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{4}{3}}(c+dx)}$$
$$-\frac{\left(3b\sqrt[3]{b\coth^4(c+dx)}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{b\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\coth(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{7d} - \frac{3b\coth^3(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{13d} - \frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} + \frac{b\sqrt[3]{b\coth^4(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} - \frac{3b\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.69

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx = -\frac{b\sqrt[3]{b\coth^4(c+dx)}\left(42\coth^4(c+dx)\sqrt[6]{\coth^2(c+dx)}+13\left(42\sqrt[6]{\coth^2(c+dx)}+6\coth^2(c+dx)^{7/6}+7\log\right.\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(4/3),x]
```

```
[Out] -1/182*(b*(b*Coth[c + d*x]^4)^(1/3)*(42*Coth[c + d*x]^4*(Coth[c + d*x]^2)^(
1/6) + 13*(42*(Coth[c + d*x]^2)^(1/6) + 6*(Coth[c + d*x]^2)^(7/6) + 7*Log[1
 - (Coth[c + d*x]^2)^(1/6)] - 7*Log[1 + (Coth[c + d*x]^2)^(1/6)] - 7*(-1)^(
2/3)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + 7*(-1)^(2/3)*Log[1 + (-1
)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - 7*(-1)^(1/3)*Log[1 - (-1)^(2/3)*(Coth[c
+ d*x]^2)^(1/6)] + 7*(-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]
))*Tanh[c + d*x])/(d*(Coth[c + d*x]^2)^(1/6))
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{4}\right)^{\frac{4}{3}}dx$$

[In] int((b*coth(d*x+c)^4)^(4/3),x)

[Out] int((b*coth(d*x+c)^4)^(4/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2864 vs. $2(295) = 590$.

Time = 0.32 (sec) , antiderivative size = 2864, normalized size of antiderivative = 8.11

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx = \text{Too large to display}$$

[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")

```
[Out] -1/364*(182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*sinh(d*x
 + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4*(7*sq
rt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b*cosh(
d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b*cosh(
d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x + c)^2
 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10*sqrt(
3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*sqrt(
3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*cosh(d
*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2 + 8*(s
qrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*cosh(d
*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*(-b)^(1/3)*
arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b) - 182*(sqrt(3)*b*cosh(d*x + c)^8 + 8*sqrt(3)*b*cosh(d*x + c)*si
nh(d*x + c)^7 + sqrt(3)*b*sinh(d*x + c)^8 - 4*sqrt(3)*b*cosh(d*x + c)^6 + 4
*(7*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^6 + 8*(7*sqrt(3)*b
*cosh(d*x + c)^3 - 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*sqrt(3)*b
*cosh(d*x + c)^4 + 2*(35*sqrt(3)*b*cosh(d*x + c)^4 - 30*sqrt(3)*b*cosh(d*x
+ c)^2 + 3*sqrt(3)*b)*sinh(d*x + c)^4 + 8*(7*sqrt(3)*b*cosh(d*x + c)^5 - 10
*sqrt(3)*b*cosh(d*x + c)^3 + 3*sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4
*sqrt(3)*b*cosh(d*x + c)^2 + 4*(7*sqrt(3)*b*cosh(d*x + c)^6 - 15*sqrt(3)*b*
cosh(d*x + c)^4 + 9*sqrt(3)*b*cosh(d*x + c)^2 - sqrt(3)*b)*sinh(d*x + c)^2
+ 8*(sqrt(3)*b*cosh(d*x + c)^7 - 3*sqrt(3)*b*cosh(d*x + c)^5 + 3*sqrt(3)*b*
cosh(d*x + c)^3 - sqrt(3)*b*cosh(d*x + c))*sinh(d*x + c) + sqrt(3)*b)*b^(1/
3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c
))^(1/3))/b) + 91*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 +
b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(
d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 +
```

```
6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)
*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 -
 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cos
h(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))
*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)
/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 91*(b*cosh
(d*x + c)^8 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*c
osh(d*x + c)^6 + 4*(7*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(
d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(
35*b*cosh(d*x + c)^4 - 30*b*cosh(d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b
*cosh(d*x + c)^5 - 10*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^
3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9
*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d
*x + c)^5 + 3*b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/
3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(2/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*
x + c)/sinh(d*x + c))^(1/3)) - 182*(b*cosh(d*x + c)^8 + 8*b*cosh(d*x + c)*s
inh(d*x + c)^7 + b*sinh(d*x + c)^8 - 4*b*cosh(d*x + c)^6 + 4*(7*b*cosh(d*x
+ c)^2 - b)*sinh(d*x + c)^6 + 8*(7*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*s
inh(d*x + c)^5 + 6*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - 30*b*cosh(
d*x + c)^2 + 3*b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - 10*b*cosh(d*x
+ c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^3 - 4*b*cosh(d*x + c)^2 + 4*(7*b*
cosh(d*x + c)^6 - 15*b*cosh(d*x + c)^4 + 9*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^2 + 8*(b*cosh(d*x + c)^7 - 3*b*cosh(d*x + c)^5 + 3*b*cosh(d*x + c)^3 -
 b*cosh(d*x + c))*sinh(d*x + c) + b)*b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)
/sinh(d*x + c))^(1/3)) + 12*(111*b*cosh(d*x + c)^8 + 888*b*cosh(d*x + c)*si
nh(d*x + c)^7 + 111*b*sinh(d*x + c)^8 - 336*b*cosh(d*x + c)^6 + 84*(37*b*co
sh(d*x + c)^2 - 4*b)*sinh(d*x + c)^6 + 168*(37*b*cosh(d*x + c)^3 - 12*b*cos
h(d*x + c))*sinh(d*x + c)^5 + 562*b*cosh(d*x + c)^4 + 2*(3885*b*cosh(d*x +
c)^4 - 2520*b*cosh(d*x + c)^2 + 281*b)*sinh(d*x + c)^4 + 8*(777*b*cosh(d*x
+ c)^5 - 840*b*cosh(d*x + c)^3 + 281*b*cosh(d*x + c))*sinh(d*x + c)^3 - 336
*b*cosh(d*x + c)^2 + 12*(259*b*cosh(d*x + c)^6 - 420*b*cosh(d*x + c)^4 + 28
1*b*cosh(d*x + c)^2 - 28*b)*sinh(d*x + c)^2 + 8*(111*b*cosh(d*x + c)^7 - 25
2*b*cosh(d*x + c)^5 + 281*b*cosh(d*x + c)^3 - 84*b*cosh(d*x + c))*sinh(d*x
+ c) + 111*b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(d*cosh(d*x + c)^8 + 8
*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6
```

```
+ 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*
d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x +
 c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^
5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*
x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c
)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d
*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx=\int \left(b\coth^4{(c+dx)}\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**4)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(4/3), x)
```

Maxima [F]

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx=\int \left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

Giac [F]

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx=\int \left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^{4/3}\,dx = \int \left(b\coth(c+d\,x)^4\right)^{4/3}dx$$

```
[In] int((b*coth(c + d*x)^4)^(4/3),x)
[Out] int((b*coth(c + d*x)^4)^(4/3), x)
```

3.45 $\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 291

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} - \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{8}{3}}(c+dx)} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(2/3)/d/coth(d*x+c)^(8/3)-1/4*
(b*coth(d*x+c)^4)^(2/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(8/3)+1/4*(b*coth(d*x+c)^4)^(2/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(8/3)+1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^4)^(2/3)*3^(1/2)/d/coth(d*x+c)^(8/3)-1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(2/3)*3^(1/2)/d/coth(d*x+c)^(8/3)
-3/5*(b*coth(d*x+c)^4)^(2/3)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3554, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} - \frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\tanh(c+dx)\left(b\coth^4(c+dx)\right)^{2/3}}{5d} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{8}{3}}(c+dx)}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(2/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2
/3))/(2*d*Coth[c + d*x]^(8/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3)
)/Sqrt[3]]*(b*Coth[c + d*x]^4)^(2/3))/(2*d*Coth[c + d*x]^(8/3)) + (ArcTanh[
Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(2/3))/(d*Coth[c + d*x]^(8/3)) - (
(b*Coth[c + d*x]^4)^(2/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)
])/(4*d*Coth[c + d*x]^(8/3)) + ((b*Coth[c + d*x]^4)^(2/3)*Log[1 + Coth[c +
d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(8/3)) - (3*(b*Coth[c
 + d*x]^4)^(2/3)*Tanh[c + d*x])/(5*d)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{8}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}\\
&= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\coth^{\frac{8}{3}}(c+dx)}\\
&= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}\\
&\quad-\frac{\left(b\coth^4(c+dx)\right)^{2/3}\operatorname{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d\coth^{\frac{8}{3}}(c+dx)}\\
&= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}\\
&\quad-\frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\operatorname{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{8}{3}}(c+dx)}\\
&= -\frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}\\
&\quad+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{8}{3}}(c+dx)}\\
&\quad+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\operatorname{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{8}{3}}(c+dx)}\\
&\quad+\frac{\left(b\coth^4(c+dx)\right)^{2/3}\operatorname{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{8}{3}}(c+dx)}
\end{aligned}
$$

$$= \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\operatorname{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{8}{3}}(c+dx)} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\operatorname{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\operatorname{Subst}\left(\int\frac{1}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\operatorname{Subst}\left(\int\frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{8}{3}}(c+dx)}$$

$$= \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{8}{3}}(c+dx)} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d} + \frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{8}{3}}(c+dx)} + \frac{\left(3\left(b\coth^4(c+dx)\right)^{2/3}\right)\operatorname{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{8}{3}}(c+dx)}$$

$$= \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} - \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{2d\coth^{\frac{8}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\left(b\coth^4(c+dx)\right)^{2/3}}{d\coth^{\frac{8}{3}}(c+dx)} - \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{8}{3}}(c+dx)} + \frac{\left(b\coth^4(c+dx)\right)^{2/3}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{8}{3}}(c+dx)} - \frac{3\left(b\coth^4(c+dx)\right)^{2/3}\tanh(c+dx)}{5d}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.57

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \frac{\left(b\coth^4(c+dx)\right)^{2/3}\left(20\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right) - 12\coth^{\frac{5}{3}}(c+dx) + 5\left(2\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right) \cdots\right.\right.}{\cdots}$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(2/3),x]
```

```
[Out] ((b*Coth[c + d*x]^4)^(2/3)*(20*ArcTanh[Coth[c + d*x]^(1/3)] - 12*Coth[c + d
*x]^(5/3) + 5*(2*Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - 2*Sq
rt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]] - Log[1 - Coth[c + d*x]^(
1/3) + Coth[c + d*x]^(2/3)] + Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])))/(20*d*Coth[c + d*x]^(8/3))
```

Maple [F]

$$\int \left(b\coth \left(dx+c\right)^{4}\right)^{\frac{2}{3}}dx$$

```
[In] int((b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int((b*coth(d*x+c)^4)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. 2(239) = 478.

Time = 0.28 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.12

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx =$$

$$-\frac{10\left(\sqrt{3}\cosh\left(dx+c\right)^{2}+2\sqrt{3}\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sqrt{3}\sinh\left(dx+c\right)^{2}-\sqrt{3}\right)\left(-b^{2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-}{}\right.}{}$$

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/20*(10*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c)
+ sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b -
2*sqrt(3)*(-b^2)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 10*(sqrt
(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*sinh(
d*x + c)^2 - sqrt(3))*(b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*(b^2)^
(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + 5*(-b^2)^(1/3)*(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh
(d*x + c)/sinh(d*x + c))^(2/3) - (-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x
+ c)/sinh(d*x + c))^(1/3)) + 5*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c
))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3
)) - 10*(-b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3
)) - 10*(b^2)^(1/3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 - 1)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b^2)^(2/3))
 + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 +
1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x
 + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 - d)
```

Sympy [F]

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \int \left(b\coth^4\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**4)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(2/3), x)
```

Maxima [F]

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

Giac [F]

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(b\coth^4(c+dx)\right)^{2/3}\,dx = \int \left(b\,\mathrm{coth}(c+d\,x)^4\right)^{2/3}\,dx$$

```
[In] int((b*coth(c + d*x)^4)^(2/3),x)
[Out] int((b*coth(c + d*x)^4)^(2/3), x)
```

3.46 $\int \sqrt[3]{b\coth^4(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 289

$$\begin{aligned}\int \sqrt[3]{b\coth^4(c+dx)}\,dx =& -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} \\ &+ \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} \\ &+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} \\ &- \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} \\ &+ \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} \\ &- \frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}\end{aligned}$$

```
[Out] arctanh(coth(d*x+c)^(1/3))*(b*coth(d*x+c)^4)^(1/3)/d/coth(d*x+c)^(4/3)-1/4*
(b*coth(d*x+c)^4)^(1/3)*ln(1-coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/coth(d*
x+c)^(4/3)+1/4*(b*coth(d*x+c)^4)^(1/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(
2/3))/d/coth(d*x+c)^(4/3)-1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*(
b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)+1/2*arctan(1/3*(1+2*coth
(d*x+c)^(1/3))*3^(1/2))*(b*coth(d*x+c)^4)^(1/3)*3^(1/2)/d/coth(d*x+c)^(4/3)
-3*(b*coth(d*x+c)^4)^(1/3)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3554, 3557, 335, 216, 648, 632, 210, 642, 212}

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\tanh(c+dx)\sqrt[3]{b\coth^4(c+dx)}}{d} - \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(1 - 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*(b*Coth[c + d*x]^
4)^(1/3))/(d*Coth[c + d*x]^(4/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1
/3))/Sqrt[3]]*(b*Coth[c + d*x]^4)^(1/3))/(2*d*Coth[c + d*x]^(4/3)) + (ArcTa
nh[Coth[c + d*x]^(1/3)]*(b*Coth[c + d*x]^4)^(1/3))/(d*Coth[c + d*x]^(4/3))
- ((b*Coth[c + d*x]^4)^(1/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2
/3)])/(4*d*Coth[c + d*x]^(4/3)) + ((b*Coth[c + d*x]^4)^(1/3)*Log[1 + Coth[c
 + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*Coth[c + d*x]^(4/3)) - (3*(b*Cot
h[c + d*x]^4)^(1/3)*Tanh[c + d*x])/d
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

```
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt[3]{b\coth^4(c+dx)}\int \coth^{\frac{4}{3}}(c+dx)\,dx}{\coth^{\frac{4}{3}}(c+dx)}\\
&= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}+\frac{\sqrt[3]{b\coth^4(c+dx)}\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\coth^{\frac{4}{3}}(c+dx)}\\
&= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}\\
&\quad-\frac{\sqrt[3]{b\coth^4(c+dx)}\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx,x,\coth(c+dx)\right)}{d\coth^{\frac{4}{3}}(c+dx)}\\
&= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}\\
&\quad-\frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}\\
&= -\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}\\
&\quad+\frac{\sqrt[3]{b\coth^4(c+dx)}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}\\
&\quad+\frac{\sqrt[3]{b\coth^4(c+dx)}\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}\\
&\quad+\frac{\sqrt[3]{b\coth^4(c+dx)}\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\coth^{\frac{4}{3}}(c+dx)}
\end{aligned}
$$

$$=\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)}-\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}$$
$$-\frac{\sqrt[3]{b\coth^4(c+dx)}\text{Subst}\left(\int\frac{-1+2x}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{\sqrt[3]{b\coth^4(c+dx)}\text{Subst}\left(\int\frac{1+2x}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int\frac{1}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$

$$=\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)}$$
$$-\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$+\frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)}$$
$$-\frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}$$
$$-\frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,-1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{4}{3}}(c+dx)}$$
$$-\frac{\left(3\sqrt[3]{b\coth^4(c+dx)}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\coth^{\frac{4}{3}}(c+dx)}$$

$$= -\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\sqrt[3]{b\coth^4(c+dx)}}{2d\coth^{\frac{4}{3}}(c+dx)} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\sqrt[3]{b\coth^4(c+dx)}}{d\coth^{\frac{4}{3}}(c+dx)} - \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} + \frac{\sqrt[3]{b\coth^4(c+dx)}\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\coth^{\frac{4}{3}}(c+dx)} - \frac{3\sqrt[3]{b\coth^4(c+dx)}\tanh(c+dx)}{d}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.69

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = -\frac{\sqrt[3]{b\coth^4(c+dx)}\left(6\sqrt[6]{\coth^2(c+dx)}+\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\log\left(1+\sqrt[6]{\coth^2(c+dx)}\right)-(-$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(1/3),x]
```

```
[Out] -1/2*((b*Coth[c + d*x]^4)^(1/3)*(6*(Coth[c + d*x]^2)^(1/6) + Log[1 - (Coth[
c + d*x]^2)^(1/6)] - Log[1 + (Coth[c + d*x]^2)^(1/6)] - (-1)^(2/3)*Log[1 -
(-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] + (-1)^(2/3)*Log[1 + (-1)^(1/3)*(Coth[c
 + d*x]^2)^(1/6)] - (-1)^(1/3)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)]
+ (-1)^(1/3)*Log[1 + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)])*Tanh[c + d*x])/(d
*(Coth[c + d*x]^2)^(1/6))
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{4}\right)^{\frac{1}{3}}dx$$

[In] int((b*coth(d*x+c)^4)^(1/3),x)

[Out] int((b*coth(d*x+c)^4)^(1/3),x)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{b \coth^4(c+dx)}\,dx = -\frac{2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}b+2\sqrt{3}(-b)^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3\,b}\right)-2\sqrt{3}b^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}b-2\sqrt{3}b^{\frac{2}{3}}\left(\frac{b\cosh(dx+c)}{\sinh(dx+c)}\right)^{\frac{1}{3}}}{3\,b}\right)+(-b)^{\frac{1}{3}}\log$$

[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")

[Out] -1/4*(2*sqrt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b + 2*sqrt(3)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) - 2*sqrt(3)*b^(1/3)*arctan(-1/3*(sqrt(3)*b - 2*sqrt(3)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b) + (-b)^(1/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + b^(1/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - 2*(-b)^(1/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*b^(1/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 12*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/d

Sympy [F]

$$\int \sqrt[3]{b \coth^4(c+dx)}\,dx = \int \sqrt[3]{b \coth^{4}{\left(c+dx \right)}}\,dx$$

[In] integrate((b*coth(d*x+c)**4)**(1/3),x)

[Out] Integral((b*coth(c + d*x)**4)**(1/3), x)

Maxima [F]

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \int \left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\coth^4(c+dx)}\,dx = \int {\left(b\,{\mathrm{coth}\left(c+d\,x\right)}^4\right)}^{1/3}\,dx$$

```
[In] int((b*coth(c + d*x)^4)^(1/3),x)
[Out] int((b*coth(c + d*x)^4)^(1/3), x)
```

3.47 $\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 289

$$\begin{aligned}\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = &-\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} \\ &+\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}} \\ &-\frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}} \\ &+\frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} \\ &-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^4(c+dx)}} \\ &+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}\end{aligned}$$

```
[Out] -3*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(d*
x+c)^(4/3)/d/(b*coth(d*x+c)^4)^(1/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*x+c)
^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(1/3)+1/4*coth(d*x+c)^(4/3)*l
n(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(1/3)+1/2*arct
```

```
an(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/d/(b*coth
(d*x+c)^4)^(1/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c
)^(4/3)*3^(1/2)/d/(b*coth(d*x+c)^4)^(1/3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3739, 3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(-1/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2*C
oth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*d*(b*Coth[c + d*x]^4)^
(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]
^(4/3))/(2*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*Cot
h[c + d*x]^(4/3))/(d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4/3)*Log[
1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^(1
/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/
3)])/(4*d*(b*Coth[c + d*x]^4)^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{\sqrt[3]{b\coth^4(c+dx)}}\\
&= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^4(c+dx)}}\\
&= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d\sqrt[3]{b\coth^4(c+dx)}}\\
&= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}-\frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}}\\
&= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}}\\
&\quad+\frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}}\\
&\quad+\frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\sqrt[3]{b\coth^4(c+dx)}}
\end{aligned}
$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}$$
$$- \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}$$
$$+ \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}$$
$$- \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}$$
$$- \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}$$
$$- \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}$$
$$+ \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}$$
$$+ \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\sqrt[3]{b\coth^4(c+dx)}}$$
$$+ \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\sqrt[3]{b\coth^4(c+dx)}}$$

$$
= -\frac{3\coth(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}}
- \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2d\sqrt[3]{b\coth^4(c+dx)}}
+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{d\sqrt[3]{b\coth^4(c+dx)}}
- \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}
+ \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\sqrt[3]{b\coth^4(c+dx)}}
$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.86

$$
\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = -\frac{\coth(c+dx)\left(6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth^2(c+dx)}\log\left(1+\sqrt[6]{\coth^2(c+\right.\right.}{}
$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-1/3),x]
```

```
[Out] -1/2*(Coth[c + d*x]*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)^
(1/6)] - (Coth[c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^(1
/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - (
-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6
)] + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2
)^(1/6)] - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Coth[c +
d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^4)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{4}\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(b*coth(d*x+c)^4)^(1/3),x)
[Out] int(1/(b*coth(d*x+c)^4)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 585 vs. $2(239) = 478$.

Time = 0.33 (sec) , antiderivative size = 3316, normalized size of antiderivative = 11.47

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="fricas")
[Out] [1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh
(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*cosh(d*x
+ c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(
d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x +
c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)
+ (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2
- b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-b)^(1/3)
/b) + b) + sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + b)*sqrt(-1/b^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c
)/sinh(d*x + c))^(2/3)*sqrt(-1/b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x
+ c)*sinh(d*x + c) - b*sinh(d*x + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)
) + (sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(
d*x + c)^2 - b)*sqrt(-1/b^(2/3)) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^
(1/3) - 3*b)/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
c)^2)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^
(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3))
```

```
+ 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)
*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 12*(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x
 + c)/sinh(d*x + c))^(2/3))/(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh
(d*x + c) + b*d*sinh(d*x + c)^2 + b*d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2
+ 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)
/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*sqrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cos
h(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b)^(1/3)/b)) - sqrt(3)*(b*cosh(d*x
+ c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sqrt(-1/b
^(2/3))*log(-(2*sqrt(3)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*b^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3)*sqrt(-1/
b^(2/3)) - b*cosh(d*x + c)^2 - 2*b*cosh(d*x + c)*sinh(d*x + c) - b*sinh(d*x
 + c)^2 - sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*
sinh(d*x + c)^2 - b)*b^(1/3)*sqrt(-1/b^(2/3)) + (sqrt(3)*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-1/b^(2/3)
) + 3*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
1)*b^(2/3))*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - 3*b)/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - (cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3)
 - (-b)^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh
(d*x + c))^(2/3)) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh
(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x +
c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2
*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/
3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d
*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(
d*x + c)/sinh(d*x + c))^(1/3)) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*
d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2
 + b*d), 1/4*(sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c)
+ b*sinh(d*x + c)^2 + b)*sqrt((-b)^(1/3)/b)*log(3*b*cosh(d*x + c)^2 + 6*b*c
osh(d*x + c)*sinh(d*x + c) + 3*b*sinh(d*x + c)^2 - 3*(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x +
c)/sinh(d*x + c))^(1/3) - sqrt(3)*(2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sin
h(d*x + c) + sinh(d*x + c)^2 - 1)*(-b)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c)
)^(2/3) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 - b)*(-b)^(1/3) - (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 - b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))*sqrt((-
b)^(1/3)/b) + b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) - (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/
3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(
d*x + c))^(2/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c
```

```
))^(1/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 2
*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b^(1/3))/b^(1/3) - 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*d*cos
h(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2 + b*
d), -1/4*(2*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) +
b*sinh(d*x + c)^2 + b)*sqrt(-(-b)^(1/3)/b)*arctan(-1/3*sqrt(3)*(-b)^(1/3)*s
qrt(-(-b)^(1/3)/b) + 2/3*sqrt(3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt
(-(-b)^(1/3)/b)) - (cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(2/3) - (-b)^(1/3)*(b*cosh(d*x + c)/sin
h(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(2/3)) + (cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*b^(2/3)*log(b^(2/
3) - b^(1/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) + (b*cosh(d*x + c)/sinh(
d*x + c))^(2/3)) + 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 1)*(-b)^(2/3)*log((-b)^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c
))^(1/3)) - 2*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)*b^(2/3)*log(b^(1/3) + (b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) + 2
*sqrt(3)*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + b)*arctan(-1/3*sqrt(3)*(b^(1/3) - 2*(b*cosh(d*x + c)/sinh(d*x + c)
)^(1/3))/b^(1/3))/b^(1/3) + 12*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 1)*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3))/(b*d*cos
h(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2 + b*
d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b\coth^4{(c+dx)}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**4)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**4)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b\coth^4(c+dx)}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{1/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^4)^(1/3),x)
[Out] int(1/(b*coth(c + d*x)^4)^(1/3), x)
```

3.48 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 291

$$
\begin{aligned}
\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx =& -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} \\
&- \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} \\
&+ \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} \\
&+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} \\
&- \frac{\coth^{\frac{8}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}} \\
&+ \frac{\coth^{\frac{8}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}
\end{aligned}
$$

```
[Out] -3/5*coth(d*x+c)/d/(b*coth(d*x+c)^4)^(2/3)+arctanh(coth(d*x+c)^(1/3))*coth(
d*x+c)^(8/3)/d/(b*coth(d*x+c)^4)^(2/3)-1/4*coth(d*x+c)^(8/3)*ln(1-coth(d*x+
c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(2/3)+1/4*coth(d*x+c)^(8/3)
*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/d/(b*coth(d*x+c)^4)^(2/3)-1/2*ar
ctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(8/3)*3^(1/2)/d/(b*co
th(d*x+c)^4)^(2/3)+1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x
+c)^(8/3)*3^(1/2)/d/(b*coth(d*x+c)^4)^(2/3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 216, 648, 632, 210, 642, 212}

$$\begin{aligned}\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx =& -\frac{\sqrt{3}\coth^{\frac{8}{3}}(c+dx)\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} \\ &+\frac{\sqrt{3}\coth^{\frac{8}{3}}(c+dx)\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} \\ &+\frac{\coth^{\frac{8}{3}}(c+dx)\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} \\ &-\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}} \\ &+\frac{\coth^{\frac{8}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}\end{aligned}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(5*d*(b*Coth[c + d*x]^4)^(2/3)) - (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(8/3))/(2*d*(b*Coth[c + d*x]^4
)^(2/3)) + (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*
x]^(8/3))/(2*d*(b*Coth[c + d*x]^4)^(2/3)) + (ArcTanh[Coth[c + d*x]^(1/3)]*C
oth[c + d*x]^(8/3))/(d*(b*Coth[c + d*x]^4)^(2/3)) - (Coth[c + d*x]^(8/3)*Lo
g[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*d*(b*Coth[c + d*x]^4)^
(2/3)) + (Coth[c + d*x]^(8/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c + d*x]^(
2/3)])/(4*d*(b*Coth[c + d*x]^4)^(2/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))*I
nt[1/(r^2 - s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 2)/4}], x],
 x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{8}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}\\
&= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\int \frac{1}{\coth^{\frac{2}{3}}(c+dx)}\,dx}{\left(b\coth^4(c+dx)\right)^{2/3}}\\
&= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{x^{2/3}(-1+x^2)}\,dx,x,\coth(c+dx)\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}\\
&= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}-\frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-1+x^6}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}\\
&= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}}+\frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}\\
&\quad+\frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}\\
&\quad+\frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx,x,\sqrt[3]{\coth(c+dx)}\right)}{d\left(b\coth^4(c+dx)\right)^{2/3}}
\end{aligned}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\coth^{\frac{8}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}} + \frac{\coth^{\frac{8}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\left(3\coth^{\frac{8}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{\coth(c+dx)}\right)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}$$

$$= -\frac{3\coth(c+dx)}{5d\left(b\coth^4(c+dx)\right)^{2/3}} - \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}$$
$$+ \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{8}{3}}(c+dx)}{2d\left(b\coth^4(c+dx)\right)^{2/3}}$$
$$+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{8}{3}}(c+dx)}{d\left(b\coth^4(c+dx)\right)^{2/3}}$$
$$- \frac{\coth^{\frac{8}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}$$
$$+ \frac{\coth^{\frac{8}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4d\left(b\coth^4(c+dx)\right)^{2/3}}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx =$$
$$-\frac{\coth(c+dx)\left(6+5\coth^2(c+dx)^{5/6}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-5\coth^2(c+dx)^{5/6}\log\left(1+\sqrt[6]{\coth^2(c\right.\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-2/3),x]
```

```
[Out] -1/10*(Coth[c + d*x]*(6 + 5*(Coth[c + d*x]^2)^(5/6)*Log[1 - (Coth[c + d*x]^
2)^(1/6)] - 5*(Coth[c + d*x]^2)^(5/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] - 5*
(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/
6)] + 5*(-1)^(2/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x
]^2)^(1/6)] - 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 - (-1)^(2/3)*(Coth
[c + d*x]^2)^(1/6)] + 5*(-1)^(1/3)*(Coth[c + d*x]^2)^(5/6)*Log[1 + (-1)^(2/
3)*(Coth[c + d*x]^2)^(1/6)]))/(d*(b*Coth[c + d*x]^4)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(dx+c\right)^{4}\right)^{\frac{2}{3}}}dx$$

```
[In] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^4)^(2/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1159 vs. $2(239) = 478$.

Time = 0.28 (sec) , antiderivative size = 1159, normalized size of antiderivative = 3.98

$$\int \frac{1}{\left(b \coth^4(c+dx)\right)^{2/3}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/20*(10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt
(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*sqrt(-(-b^2)^(1/3)) - 2
*sqrt(3)*(-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)*sqrt(-(-b^2)^(1
/3)))/b^2) + 10*sqrt(3)*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) +
b)*(b^2)^(1/6)*arctan(-1/3*sqrt(3)*(b^2)^(1/6)*((b^2)^(1/3)*b - 2*(b^2)^(2/
3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/b^2) + 5*(cosh(d*x + c)^4 + 4*cos
h(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*si
nh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sin
h(d*x + c) + 1)*(-b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))^(2/3) -
(-b^2)^(1/3)*b + (-b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3)) - 5*(c
osh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*c
osh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^
3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/si
nh(d*x + c))^(2/3) + (b^2)^(1/3)*b - (b^2)^(2/3)*(b*cosh(d*x + c)/sinh(d*x
+ c))^(1/3)) - 10*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh
(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2
 + 4*(cosh(d*x + c)^3 + cosh(d*x + c))*sinh(d*x + c) + 1)*(-b^2)^(2/3)*log(
b*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3) - (-b^2)^(2/3)) + 10*(cosh(d*x + c)
^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)
^2 + 1)*sinh(d*x + c)^2 + 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + cosh(d*x
 + c))*sinh(d*x + c) + 1)*(b^2)^(2/3)*log(b*(b*cosh(d*x + c)/sinh(d*x + c))
^(1/3) + (b^2)^(2/3)) - 12*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
```

```
+ c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c)
 + b)*(b*cosh(d*x + c)/sinh(d*x + c))^(1/3))/(b^2*d*cosh(d*x + c)^4 + 4*b^2
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x
 + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(
b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth^4\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**4)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**4)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{2/3}}\,dx=\int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{2/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^4)^(2/3),x)
[Out] int(1/(b*coth(c + d*x)^4)^(2/3), x)
```

3.49 $\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 369

$$\begin{aligned}
\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx &= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} \\
&+ \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} \\
&- \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}} \\
&+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} \\
&- \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}} \\
&+ \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}} \\
&- \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}
\end{aligned}$$

```
[Out] -3*coth(d*x+c)/b/d/(b*coth(d*x+c)^4)^(1/3)+arctanh(coth(d*x+c)^(1/3))*coth(
d*x+c)^(4/3)/b/d/(b*coth(d*x+c)^4)^(1/3)-1/4*coth(d*x+c)^(4/3)*ln(1-coth(d*
```

```
x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^4)^(1/3)+1/4*coth(d*x+c)^(
4/3)*ln(1+coth(d*x+c)^(1/3)+coth(d*x+c)^(2/3))/b/d/(b*coth(d*x+c)^4)^(1/3)+
1/2*arctan(1/3*(1-2*coth(d*x+c)^(1/3))*3^(1/2))*coth(d*x+c)^(4/3)*3^(1/2)/b
/d/(b*coth(d*x+c)^4)^(1/3)-1/2*arctan(1/3*(1+2*coth(d*x+c)^(1/3))*3^(1/2))*
coth(d*x+c)^(4/3)*3^(1/2)/b/d/(b*coth(d*x+c)^4)^(1/3)-3/7*tanh(d*x+c)/b/d/(
b*coth(d*x+c)^4)^(1/3)-3/13*tanh(d*x+c)^3/b/d/(b*coth(d*x+c)^4)^(1/3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3739, 3555, 3557, 335, 302, 648, 632, 210, 642, 212}

$$\begin{aligned}\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx &= \frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^4(c+dx)}}\\ &-\frac{\sqrt{3}\coth^{\frac{4}{3}}(c+dx)\arctan\left(\frac{2\sqrt[3]{\coth(c+dx)}+1}{\sqrt{3}}\right)}{2bd\sqrt[3]{b\coth^4(c+dx)}}\\ &+\frac{\coth^{\frac{4}{3}}(c+dx)\text{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}\\ &-\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}}\\ &-\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)-\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}\\ &+\frac{\coth^{\frac{4}{3}}(c+dx)\log\left(\coth^{\frac{2}{3}}(c+dx)+\sqrt[3]{\coth(c+dx)}+1\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}\end{aligned}$$

```
[In] Int[(b*Coth[c + d*x]^4)^(-4/3),x]
```

```
[Out] (-3*Coth[c + d*x])/(b*d*(b*Coth[c + d*x]^4)^(1/3)) + (Sqrt[3]*ArcTan[(1 - 2
*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c + d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]
^4)^(1/3)) - (Sqrt[3]*ArcTan[(1 + 2*Coth[c + d*x]^(1/3))/Sqrt[3]]*Coth[c +
d*x]^(4/3))/(2*b*d*(b*Coth[c + d*x]^4)^(1/3)) + (ArcTanh[Coth[c + d*x]^(1/3
)]*Coth[c + d*x]^(4/3))/(b*d*(b*Coth[c + d*x]^4)^(1/3)) - (Coth[c + d*x]^(4
/3)*Log[1 - Coth[c + d*x]^(1/3) + Coth[c + d*x]^(2/3)])/(4*b*d*(b*Coth[c +
d*x]^4)^(1/3)) + (Coth[c + d*x]^(4/3)*Log[1 + Coth[c + d*x]^(1/3) + Coth[c
+ d*x]^(2/3)])/(4*b*d*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x])/(7*b*d
```

```
*(b*Coth[c + d*x]^4)^(1/3)) - (3*Tanh[c + d*x]^3)/(13*b*d*(b*Coth[c + d*x]^
4)^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))*Int[1/(r
^2 - s^2*x^2), x] + Dist[2*(r^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}]
, x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m,
 n - 1] && NegQ[a/b]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3739

```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Tan
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{16}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}} \\
&= -\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{10}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}} \\
&= -\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\int \frac{1}{\coth^{\frac{4}{3}}(c+dx)}\,dx}{b\sqrt[3]{b\coth^4(c+dx)}} \\
&= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}-\frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} \\
&\quad -\frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}+\frac{\coth^{\frac{4}{3}}(c+dx)\int \coth^{\frac{2}{3}}(c+dx)\,dx}{b\sqrt[3]{b\coth^4(c+dx)}}
\end{aligned}
$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{2/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{x^4}{-1+x^6}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-\frac{1}{2}-\frac{x}{2}}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-\frac{1}{2}+\frac{x}{2}}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{bd\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{\coth(c+dx)}\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[3]{\coth(c+dx)}\right)}{2bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\left(3\coth^{\frac{4}{3}}(c+dx)\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{\coth(c+dx)}\right)}{2bd\sqrt[3]{b\coth^4(c+dx)}}$$

$$= -\frac{3\coth(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}} + \frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}}$$
$$- \frac{\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{\coth(c+dx)}}{\sqrt{3}}\right)\coth^{\frac{4}{3}}(c+dx)}{2bd\sqrt[3]{b\coth^4(c+dx)}}$$
$$+ \frac{\operatorname{arctanh}\left(\sqrt[3]{\coth(c+dx)}\right)\coth^{\frac{4}{3}}(c+dx)}{bd\sqrt[3]{b\coth^4(c+dx)}}$$
$$- \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1-\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}$$
$$+ \frac{\coth^{\frac{4}{3}}(c+dx)\log\left(1+\sqrt[3]{\coth(c+dx)}+\coth^{\frac{2}{3}}(c+dx)\right)}{4bd\sqrt[3]{b\coth^4(c+dx)}}$$
$$- \frac{3\tanh(c+dx)}{7bd\sqrt[3]{b\coth^4(c+dx)}} - \frac{3\tanh^3(c+dx)}{13bd\sqrt[3]{b\coth^4(c+dx)}}$$

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \frac{-91\coth(c+dx)\left(6+\sqrt[6]{\coth^2(c+dx)}\log\left(1-\sqrt[6]{\coth^2(c+dx)}\right)-\sqrt[6]{\coth^2}\right.}{}$$

```
[In] Integrate[(b*Coth[c + d*x]^4)^(-4/3),x]
```

```
[Out] (-91*Coth[c + d*x]*(6 + (Coth[c + d*x]^2)^(1/6)*Log[1 - (Coth[c + d*x]^2)^(
1/6)] - (Coth[c + d*x]^2)^(1/6)*Log[1 + (Coth[c + d*x]^2)^(1/6)] + (-1)^(1/
3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)] - (-
1)^(1/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(1/3)*(Coth[c + d*x]^2)^(1/6)
] + (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 - (-1)^(2/3)*(Coth[c + d*x]^2)
^(1/6)] - (-1)^(2/3)*(Coth[c + d*x]^2)^(1/6)*Log[1 + (-1)^(2/3)*(Coth[c + d
*x]^2)^(1/6)]) - 6*Tanh[c + d*x]*(13 + 7*Tanh[c + d*x]^2))/(182*b*d*(b*Coth
[c + d*x]^4)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x +c \right)^{4}\right)^{\frac{4}{3}}}d x$$

[In] int(1/(b*coth(d*x+c)^4)^(4/3),x)

[Out] int(1/(b*coth(d*x+c)^4)^(4/3),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3650 vs. $2(311) = 622$.

Time = 0.54 (sec) , antiderivative size = 15579, normalized size of antiderivative = 42.22

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \text{Too large to display}$$

[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth^{4}\left(c+dx\right)\right)^{\frac{4}{3}}}\,dx$$

[In] integrate(1/(b*coth(d*x+c)**4)**(4/3),x)

[Out] Integral((b*coth(c + d*x)**4)**(-4/3), x)

Maxima [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b \coth \left(d x + c\right)^{4}\right)^{\frac{4}{3}}}\,dx$$

[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="maxima")

[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)

Giac [F]

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^4\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^4)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^4)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b\coth^4(c+dx)\right)^{4/3}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^4\right)^{4/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^4)^(4/3),x)
[Out] int(1/(b*coth(c + d*x)^4)^(4/3), x)
```

3.50 $\int (b \coth^m(c+dx))^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (b \coth^m(c+dx))^n \, dx = \frac{\coth(c+dx) (b \coth^m(c+dx))^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1+mn), \frac{1}{2}(3+mn), \coth^2(c+dx)\right)}{d(1+mn)}$$

```
[Out] coth(d*x+c)*(b*coth(d*x+c)^m)^n*hypergeom([1, 1/2*m*n+1/2],[1/2*m*n+3/2],co
th(d*x+c)^2)/d/(m*n+1)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3740, 3557, 371}

$$\int (b \coth^m(c+dx))^n \, dx = \frac{\coth(c+dx) (b \coth^m(c+dx))^n \text{Hypergeometric2F1}\left(1, \frac{1}{2}(mn+1), \frac{1}{2}(mn+3), \coth^2(c+dx)\right)}{d(mn+1)}$$

```
[In] Int[(b*Coth[c + d*x]^m)^n,x]
```

```
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\right)\int\coth^{mn}(c+dx)\,dx\\ &= -\frac{\left(\coth^{-mn}(c+dx)\,(b\coth^m(c+dx))^n\right)\text{Subst}\left(\int\frac{x^{mn}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{d}\\ &= \frac{\coth(c+dx)\,(b\coth^m(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\coth^2(c+dx)\right)}{d(1+mn)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int(b\coth^m(c+dx))^n\ dx$$
$$=\frac{\coth(c+dx)\,(b\coth^m(c+dx))^n\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\coth^2(c+dx)\right)}{d(1+mn)}$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^n,x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^n*Hypergeometric2F1[1, (1 + m*n)/2, (3 +
 m*n)/2, Coth[c + d*x]^2])/(d*(1 + m*n))
```

Maple [F]

$$\int (b \coth (dx + c)^m)^n \, dx$$

```
[In] int((b*coth(d*x+c)^m)^n,x)
[Out] int((b*coth(d*x+c)^m)^n,x)
```

Fricas [F]

$$\int (b \coth^m(c + dx))^n \, dx = \int (b \coth (dx + c)^m)^n \, dx$$

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="fricas")
[Out] integral((b*coth(d*x + c)^m)^n, x)
```

Sympy [F]

$$\int (b \coth^m(c + dx))^n \, dx = \int (b \coth^m (c + dx))^n \, dx$$

```
[In] integrate((b*coth(d*x+c)**m)**n,x)
[Out] Integral((b*coth(c + d*x)**m)**n, x)
```

Maxima [F]

$$\int (b \coth^m(c + dx))^n \, dx = \int (b \coth (dx + c)^m)^n \, dx$$

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

Giac [F]

$$\int (b\coth^m(c+dx))^n \, dx = \int (b\coth{(dx+c)}^m)^n \, dx$$

```
[In] integrate((b*coth(d*x+c)^m)^n,x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^n \, dx = \int (b\coth(c+d\,x)^m)^n \, dx$$

```
[In] int((b*coth(c + d*x)^m)^n,x)
[Out] int((b*coth(c + d*x)^m)^n, x)
```

3.51 $\int (b\coth^m(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 63

$$\int (b\coth^m(c+dx))^{3/2}\,dx=\frac{2b\coth^{1+m}(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\coth^2(c+dx)\right)}{d(2+3m)}$$

```
[Out] 2*b*coth(d*x+c)^(1+m)*hypergeom([1, 1/2+3/4*m],[3/2+3/4*m],coth(d*x+c)^2)*(
b*coth(d*x+c)^m)^(1/2)/d/(2+3*m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int (b\coth^m(c+dx))^{3/2}\,dx=\frac{2b\coth^{m+1}(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(3m+2),\frac{3(m+2)}{4},\coth^2(c+dx)\right)}{d(3m+2)}$$

```
[In] Int[(b*Coth[c + d*x]^m)^(3/2),x]
```

```
[Out] (2*b*Coth[c + d*x]^(1 + m)*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2
+ 3*m)/4, (3*(2 + m))/4, Coth[c + d*x]^2])/(d*(2 + 3*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\int \coth^{\frac{3m}{2}}(c+dx)\,dx \\
&= -\frac{\left(b\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{3m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d} \\
&= \frac{2b\coth^{1+m}(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1, \frac{1}{4}(2+3m), \frac{3(2+m)}{4}, \coth^2(c+dx)\right)}{d(2+3m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$
\int (b\coth^m(c+dx))^{3/2}\,dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^{3/2}\,\text{Hypergeometric2F1}\left(1, \frac{1}{4}(2+3m), \frac{3(2+m)}{4}, \coth^2(c+dx)\right)}{d\left(1+\frac{3m}{2}\right)}
$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(3/2),x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^(3/2)*Hypergeometric2F1[1, (2 + 3*m)/4,
(3*(2 + m))/4, Coth[c + d*x]^2])/(d*(1 + (3*m)/2))
```

Maple [F]

$$\int \left(b \coth \left(dx+c\right)^{m}\right)^{\frac{3}{2}}dx$$

```
[In] int((b*coth(d*x+c)^m)^(3/2),x)
[Out] int((b*coth(d*x+c)^m)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b \coth^m(c+dx))^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (b \coth^m(c+dx))^{3/2} \, dx = \int \left(b \coth^{m}\left(c+dx\right)\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((b*coth(d*x+c)**m)**(3/2),x)
[Out] Integral((b*coth(c + d*x)**m)**(3/2), x)
```

Maxima [F]

$$\int (b \coth^m(c+dx))^{3/2} \, dx = \int \left(b \coth \left(dx+c\right)^{m}\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

Giac [F]

$$\int (b \coth^m(c+dx))^{3/2} \, dx = \int (b \coth (dx+c)^m)^{\frac{3}{2}} \, dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b \coth^m(c+dx))^{3/2} \, dx = \int (b \coth(c+d\,x)^m)^{3/2} \, dx$$

```
[In] int((b*coth(c + d*x)^m)^(3/2),x)
[Out] int((b*coth(c + d*x)^m)^(3/2), x)
```

3.52 $\int \sqrt{b \coth^m(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{4},\frac{6+m}{4},\coth^2(c+dx)\right)}{d(2+m)}$$

```
[Out] 2*coth(d*x+c)*hypergeom([1, 1/2+1/4*m],[3/2+1/4*m],coth(d*x+c)^2)*(b*coth(d
*x+c)^m)^(1/2)/d/(2+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{4},\frac{m+6}{4},\coth^2(c+dx)\right)}{d(m+2)}$$

```
[In] Int[Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\int \coth^{\frac{m}{2}}(c+dx)\,dx \\
&= -\frac{\left(\coth^{-\frac{m}{2}}(c+dx)\sqrt{b\coth^m(c+dx)}\right)\operatorname{Subst}\left(\int \frac{x^{m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d} \\
&= \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1, \frac{2+m}{4}, \frac{6+m}{4}, \coth^2(c+dx)\right)}{d(2+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$
\int \sqrt{b\coth^m(c+dx)}\,dx = \frac{2\coth(c+dx)\sqrt{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1, \frac{2+m}{4}, \frac{6+m}{4}, \coth^2(c+dx)\right)}{d(2+m)}
$$

```
[In] Integrate[Sqrt[b*Coth[c + d*x]^m],x]
[Out] (2*Coth[c + d*x]*Sqrt[b*Coth[c + d*x]^m]*Hypergeometric2F1[1, (2 + m)/4, (6
 + m)/4, Coth[c + d*x]^2])/(d*(2 + m))
```

Maple [F]

$$\int \sqrt{b \coth{(dx+c)}^m}dx$$

```
[In] int((b*coth(d*x+c)^m)^(1/2),x)
[Out] int((b*coth(d*x+c)^m)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{b \coth^m(c+dx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{b \coth^m(c+dx)}\, dx = \int \sqrt{b \coth^m{(c+dx)}}\, dx$$

```
[In] integrate((b*coth(d*x+c)**m)**(1/2),x)
[Out] Integral(sqrt(b*coth(c + d*x)**m), x)
```

Maxima [F]

$$\int \sqrt{b \coth^m(c+dx)}\, dx = \int \sqrt{b \coth{(dx+c)}^m}\, dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

Giac [F]

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \int \sqrt{b \coth(dx+c)^m}\,dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*coth(d*x + c)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b \coth^m(c+dx)}\,dx = \int \sqrt{b \coth(c+d\,x)^m}\,dx$$

```
[In] int((b*coth(c + d*x)^m)^(1/2),x)
[Out] int((b*coth(c + d*x)^m)^(1/2), x)
```

3.53 $\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \frac{2\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{4},\frac{6-m}{4},\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

```
[Out] 2*coth(d*x+c)*hypergeom([1, 1/2-1/4*m],[3/2-1/4*m],coth(d*x+c)^2)/d/(2-m)/(
b*coth(d*x+c)^m)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \frac{2\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{4},\frac{6-m}{4},\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Coth[c + d*x]^m],x]
```

```
[Out] (2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^2
])/(d*(2 - m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{m}{2}}(c+dx)\,dx}{\sqrt{b\coth^m(c+dx)}}\\
&= -\frac{\coth^{\frac{m}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt{b\coth^m(c+dx)}}\\
&= \frac{2\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{4},\frac{6-m}{4},\coth^2(c+dx)\right)}{d(2-m)\sqrt{b\coth^m(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = -\frac{2\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{2-m}{4},\frac{6-m}{4},\coth^2(c+dx)\right)}{d(-2+m)\sqrt{b\coth^m(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Coth[c + d*x]^m],x]
[Out] (-2*Coth[c + d*x]*Hypergeometric2F1[1, (2 - m)/4, (6 - m)/4, Coth[c + d*x]^
2])/(d*(-2 + m)*Sqrt[b*Coth[c + d*x]^m])
```

Maple [F]

$$\int \frac{1}{\sqrt{b \coth{(dx+c)}^{m}}}dx$$

```
[In] int(1/(b*coth(d*x+c)^m)^(1/2),x)
[Out] int(1/(b*coth(d*x+c)^m)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{b \coth^{m}(c+dx)}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{b \coth^{m}(c+dx)}}\, dx = \int \frac{1}{\sqrt{b \coth^{m}{(c+dx)}}}\, dx$$

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/2),x)
[Out] Integral(1/sqrt(b*coth(c + d*x)**m), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b \coth^{m}(c+dx)}}\, dx = \int \frac{1}{\sqrt{b \coth{(dx+c)}^{m}}}\, dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(dx+c\right)^m}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*coth(d*x + c)^m), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\coth^m(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth\left(c+d\,x\right)^m}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^m)^(1/2),x)
[Out] int(1/(b*coth(c + d*x)^m)^(1/2), x)
```

3.54 $\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx=\frac{2\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

```
[Out] 2*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-3/4*m],[3/2-3/4*m],coth(d*x+c)^2)/b/d
/(2-3*m)/(b*coth(d*x+c)^m)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx=\frac{2\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^m)^(-3/2),x]
```

```
[Out] (2*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (2 - 3*m)/4, (3*(2 - m))/4, C
oth[c + d*x]^2])/(b*d*(2 - 3*m)*Sqrt[b*Coth[c + d*x]^m])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \frac{\coth^{\frac{m}{2}}(c+dx)\int \coth^{-\frac{3m}{2}}(c+dx)\,dx}{b\sqrt{b\coth^m(c+dx)}}$$

$$= -\frac{\coth^{\frac{m}{2}}(c+dx)\text{Subst}\left(\int \frac{x^{-3m/2}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{bd\sqrt{b\coth^m(c+dx)}}$$

$$= \frac{2\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1, \frac{1}{4}(2-3m), \frac{3(2-m)}{4}, \coth^2(c+dx)\right)}{bd(2-3m)\sqrt{b\coth^m(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(1, \frac{1}{4}(2-3m), -\frac{3}{4}(-2+m), \coth^2(c+dx)\right)}{d\left(1-\frac{3m}{2}\right)(b\coth^m(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-3/2),x]
[Out] (Coth[c + d*x]*Hypergeometric2F1[1, (2 - 3*m)/4, (-3*(-2 + m))/4, Coth[c +
d*x]^2])/(d*(1 - (3*m)/2)*(b*Coth[c + d*x]^m)^(3/2))
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x+c\right)^{m}\right)^{\frac{3}{2}}}d x$$

```
[In] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b \coth^{m}{\left(c + d x \right)}\right)^{\frac{3}{2}}}\, dx$$

```
[In] integrate(1/(b*coth(d*x+c)**m)**(3/2),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \int { \frac{1}{\left(b \coth \left(d x + c\right)^{m}\right)^{\frac{3}{2}}} \,d x }$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(3/2),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\coth^m(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\coth\left(c+d\,x\right)^m\right)^{3/2}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^m)^(3/2),x)
[Out] int(1/(b*coth(c + d*x)^m)^(3/2), x)
```

3.55 $\int (b\coth^m(c+dx))^{4/3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \frac{3b\coth^{1+m}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3+4m),\frac{1}{6}(9+4m),\coth^2(c+\right.}{d(3+4m)}$$

```
[Out] 3*b*coth(d*x+c)^(1+m)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+2/3*m],[3/2
+2/3*m],coth(d*x+c)^2)/d/(3+4*m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3740, 3557, 371}

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \frac{3b\coth^{m+1}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(4m+3),\frac{1}{6}(4m+9),\coth^2(c+\right.}{d(4m+3)}$$

```
[In] Int[(b*Coth[c + d*x]^m)^(4/3),x]
```

```
[Out] (3*b*Coth[c + d*x]^(1 + m)*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (
3 + 4*m)/6, (9 + 4*m)/6, Coth[c + d*x]^2])/(d*(3 + 4*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\int \coth^{\frac{4m}{3}}(c+dx)\,dx$$

$$= -\frac{\left(b\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{4m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$

$$= \frac{3b\coth^{1+m}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1, \frac{1}{6}(3+4m), \frac{1}{6}(9+4m), \coth^2(c+dx\right)}{d(3+4m)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int (b\coth^m(c+dx))^{4/3}\,dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^{4/3}\,\text{Hypergeometric2F1}\left(1, \frac{1}{6}(3+4m), \frac{1}{6}(9+4m), \coth^2(c+d\right)}{d\left(1+\frac{4m}{3}\right)}$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(4/3),x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^(4/3)*Hypergeometric2F1[1, (3 + 4*m)/6,
(9 + 4*m)/6, Coth[c + d*x]^2])/(d*(1 + (4*m)/3))
```

Maple [F]

$$\int \left(b \coth \left(d x +c \right)^{m}\right)^{\frac{4}{3}}d x$$

```
[In] int((b*coth(d*x+c)^m)^(4/3),x)
[Out] int((b*coth(d*x+c)^m)^(4/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (b\coth^m(c+dx))^{4/3}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^{4/3}\,dx=\text{Timed out}$$

```
[In] integrate((b*coth(d*x+c)**m)**(4/3),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\coth^m(c+dx))^{4/3}\,dx=\int \left(b \coth \left(d x +c \right)^{m}\right)^{\frac{4}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

Giac [F]

$$\int (b\coth^m(c+dx))^{4/3}\ dx = \int (b\coth{(dx+c)}^m)^{\frac{4}{3}}\ dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^{4/3}\ dx = \int (b\coth(c+d\,x)^m)^{4/3}\,dx$$

```
[In] int((b*coth(c + d*x)^m)^(4/3),x)
[Out] int((b*coth(c + d*x)^m)^(4/3), x)
```

3.56 $\int (b \coth^m(c+dx))^{2/3} dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int (b \coth^m(c+dx))^{2/3} dx = \frac{3\coth(c+dx)(b\coth^m(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3+2m),\frac{1}{6}(9+2m),\coth^2(c+d$$
$$d(3+2m)$$

```
[Out] 3*coth(d*x+c)*(b*coth(d*x+c)^m)^(2/3)*hypergeom([1, 1/2+1/3*m],[3/2+1/3*m],
coth(d*x+c)^2)/d/(3+2*m)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int (b \coth^m(c+dx))^{2/3} dx = \frac{3\coth(c+dx)(b\coth^m(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(2m+3),\frac{1}{6}(2m+9),\coth^2(c+d$$
$$d(2m+3)$$

```
[In] Int[(b*Coth[c + d*x]^m)^(2/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6
, (9 + 2*m)/6, Coth[c + d*x]^2])/(d*(3 + 2*m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\right)\int \coth^{\frac{2m}{3}}(c+dx)\,dx \\
&= -\frac{\left(\coth^{-\frac{2m}{3}}(c+dx)\,(b\coth^m(c+dx))^{2/3}\right)\text{Subst}\left(\int \frac{x^{2m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d} \\
&= \frac{3\coth(c+dx)\,(b\coth^m(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(1, \frac{1}{6}(3+2m), \frac{1}{6}(9+2m), \coth^2(c+dx)\right)}{d(3+2m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int (b\coth^m(c+dx))^{2/3}\,dx = \frac{\coth(c+dx)\,(b\coth^m(c+dx))^{2/3}\,\text{Hypergeometric2F1}\left(1, \frac{1}{6}(3+2m), \frac{1}{6}(9+2m), \coth^2(c+dx)\right)}{d\left(1+\frac{2m}{3}\right)}$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(2/3),x]
[Out] (Coth[c + d*x]*(b*Coth[c + d*x]^m)^(2/3)*Hypergeometric2F1[1, (3 + 2*m)/6,
(9 + 2*m)/6, Coth[c + d*x]^2])/(d*(1 + (2*m)/3))
```

Maple [F]

$$\int \left(b\coth\left(dx+c\right)^{m}\right)^{\frac{2}{3}}dx$$

```
[In] int((b*coth(d*x+c)^m)^(2/3),x)
[Out] int((b*coth(d*x+c)^m)^(2/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(b\coth^{m}(c+dx)\right)^{2/3} dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(b\coth^{m}(c+dx)\right)^{2/3} dx = \int \left(b\coth^{m}\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**m)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(2/3), x)
```

Maxima [F]

$$\int \left(b\coth^{m}(c+dx)\right)^{2/3} dx = \int \left(b\coth\left(dx+c\right)^{m}\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

Giac [F]

$$\int (b\coth^m(c+dx))^{2/3}\, dx = \int (b\coth{(dx+c)}^m)^{\frac{2}{3}}\, dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\coth^m(c+dx))^{2/3}\, dx = \int (b\coth(c+d\,x)^m)^{2/3}\, dx$$

```
[In] int((b*coth(c + d*x)^m)^(2/3),x)
[Out] int((b*coth(c + d*x)^m)^(2/3), x)
```

3.57 $\int \sqrt[3]{b \coth^m(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \sqrt[3]{b \coth^m(c+dx)}\,dx = \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{6},\frac{9+m}{6},\coth^2(c+dx)\right)}{d(3+m)}$$

```
[Out] 3*coth(d*x+c)*(b*coth(d*x+c)^m)^(1/3)*hypergeom([1, 1/2+1/6*m],[3/2+1/6*m],
coth(d*x+c)^2)/d/(3+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \sqrt[3]{b \coth^m(c+dx)}\,dx = \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{6},\frac{m+9}{6},\coth^2(c+dx)\right)}{d(m+3)}$$

```
[In] Int[(b*Coth[c + d*x]^m)^(1/3),x]
```

```
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\text{integral} = \left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\int \coth^{\frac{m}{3}}(c+dx)\,dx$$

$$= -\frac{\left(\coth^{-\frac{m}{3}}(c+dx)\sqrt[3]{b\coth^m(c+dx)}\right)\text{Subst}\left(\int \frac{x^{m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d}$$

$$= \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1, \frac{3+m}{6}, \frac{9+m}{6}, \coth^2(c+dx)\right)}{d(3+m)}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \frac{3\coth(c+dx)\sqrt[3]{b\coth^m(c+dx)}\,\text{Hypergeometric2F1}\left(1, \frac{3+m}{6}, \frac{9+m}{6}, \coth^2(c+dx)\right)}{d(3+m)}$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(1/3),x]
[Out] (3*Coth[c + d*x]*(b*Coth[c + d*x]^m)^(1/3)*Hypergeometric2F1[1, (3 + m)/6,
(9 + m)/6, Coth[c + d*x]^2])/(d*(3 + m))
```

Maple [F]

$$\int (b\coth(dx+c)^m)^{\frac{1}{3}}\,dx$$

```
[In] int((b*coth(d*x+c)^m)^(1/3),x)
[Out] int((b*coth(d*x+c)^m)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int \sqrt[3]{b\coth^m{(c+dx)}}\,dx$$

```
[In] integrate((b*coth(d*x+c)**m)**(1/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int (b\coth(dx+c)^m)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int (b\coth{(dx+c)}^m)^{\frac{1}{3}}\,dx$$

```
[In] integrate((b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{b\coth^m(c+dx)}\,dx = \int (b\coth(c+d\,x)^m)^{1/3}\,dx$$

```
[In] int((b*coth(c + d*x)^m)^(1/3),x)
[Out] int((b*coth(c + d*x)^m)^(1/3), x)
```

3.58 $\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = \frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{3-m}{6},\frac{9-m}{6},\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[Out] 3*coth(d*x+c)*hypergeom([1, 1/2-1/6*m],[3/2-1/6*m],coth(d*x+c)^2)/d/(3-m)/(
b*coth(d*x+c)^m)^(1/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = \frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{3-m}{6},\frac{9-m}{6},\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^m)^(-1/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^2
])/(d*(3 - m)*(b*Coth[c + d*x]^m)^(1/3))
```

<u>Rule 371</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\coth^{\frac{m}{3}}(c+dx)\int \coth^{-\frac{m}{3}}(c+dx)\,dx}{\sqrt[3]{b\coth^m(c+dx)}}\\ &= -\frac{\coth^{\frac{m}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{-m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\sqrt[3]{b\coth^m(c+dx)}}\\ &= \frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{3-m}{6},\frac{9-m}{6},\coth^2(c+dx)\right)}{d(3-m)\sqrt[3]{b\coth^m(c+dx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt[3]{b\coth^m(c+dx)}}\,dx = -\frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{3-m}{6},\frac{9-m}{6},\coth^2(c+dx)\right)}{d(-3+m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-1/3),x]
[Out] (-3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - m)/6, (9 - m)/6, Coth[c + d*x]^
2])/(d*(-3 + m)*(b*Coth[c + d*x]^m)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(b \coth\left(dx+c\right)^{m}\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

```
[Out] int(1/(b*coth(d*x+c)^m)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt[3]{b \coth^{m}(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{b \coth^{m}(c+dx)}}\,dx = \int \frac{1}{\sqrt[3]{b \coth^{m}\left(c+dx\right)}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**m)**(1/3),x)
```

```
[Out] Integral((b*coth(c + d*x)**m)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{b \coth^{m}(c+dx)}}\,dx = \int \frac{1}{\left(b \coth\left(dx+c\right)^{m}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}}\,dx = \int \frac{1}{\left(b \coth\left(dx+c\right)^m\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(1/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(-1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{b \coth^m(c+dx)}}\,dx = \int \frac{1}{\left(b \coth\left(c+d\,x\right)^m\right)^{1/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^m)^(1/3),x)
[Out] int(1/(b*coth(c + d*x)^m)^(1/3), x)
```

3.59 $\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-2m),\frac{1}{6}(9-2m),\coth^2(c+dx)\right)}{d(3-2m)\,(b\coth^m(c+dx))^{2/3}}$$

```
[Out] 3*coth(d*x+c)*hypergeom([1, 1/2-1/3*m],[3/2-1/3*m],coth(d*x+c)^2)/d/(3-2*m)
/(b*coth(d*x+c)^m)^(2/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-2m),\frac{1}{6}(9-2m),\coth^2(c+dx)\right)}{d(3-2m)\,(b\coth^m(c+dx))^{2/3}}$$

```
[In] Int[(b*Coth[c + d*x]^m)^(-2/3),x]
```

```
[Out] (3*Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*
x]^2])/(d*(3 - 2*m)*(b*Coth[c + d*x]^m)^(2/3))
```

<u>Rule 371</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth^{\frac{2m}{3}}(c+dx)\int \coth^{-\frac{2m}{3}}(c+dx)\,dx}{(b\coth^m(c+dx))^{2/3}} \\
&= -\frac{\coth^{\frac{2m}{3}}(c+dx)\text{Subst}\left(\int \frac{x^{-2m/3}}{-1+x^2}\,dx, x, \coth(c+dx)\right)}{d\,(b\coth^m(c+dx))^{2/3}} \\
&= \frac{3\coth(c+dx)\,\text{Hypergeometric2F1}\left(1, \frac{1}{6}(3-2m), \frac{1}{6}(9-2m), \coth^2(c+dx)\right)}{d(3-2m)\,(b\coth^m(c+dx))^{2/3}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(1, \frac{1}{6}(3-2m), \frac{1}{6}(9-2m), \coth^2(c+dx)\right)}{d\left(1-\frac{2m}{3}\right)(b\coth^m(c+dx))^{2/3}}$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-2/3),x]
[Out] (Coth[c + d*x]*Hypergeometric2F1[1, (3 - 2*m)/6, (9 - 2*m)/6, Coth[c + d*x]
^2])/(d*(1 - (2*m)/3)*(b*Coth[c + d*x]^m)^(2/3))
```

Maple [F]

$$\int \frac{1}{\left(b \coth \left(d x+c\right)^{m}\right)^{\frac{2}{3}}}d x$$

```
[In] int(1/(b*coth(d*x+c)^m)^(2/3),x)
[Out] int(1/(b*coth(d*x+c)^m)^(2/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b \coth^{m}{\left(c + d x \right)}\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**m)**(2/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b \coth \left(d x + c\right)^{m}\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b\coth\left(dx+c\right)^m\right)^{\frac{2}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(2/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(-2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\coth^m(c+dx))^{2/3}}\,dx = \int \frac{1}{\left(b\,\mathrm{coth}\left(c+d\,x\right)^m\right)^{2/3}}\,dx$$

```
[In] int(1/(b*coth(c + d*x)^m)^(2/3),x)
[Out] int(1/(b*coth(c + d*x)^m)^(2/3), x)
```

3.60 $\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \frac{3\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-4m),\frac{1}{6}(9-4m),\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[Out] 3*coth(d*x+c)^(1-m)*hypergeom([1, 1/2-2/3*m],[3/2-2/3*m],coth(d*x+c)^2)/b/d
/(3-4*m)/(b*coth(d*x+c)^m)^(1/3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3740, 3557, 371}

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \frac{3\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-4m),\frac{1}{6}(9-4m),\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}$$

```
[In] Int[(b*Coth[c + d*x]^m)^(-4/3),x]
```

```
[Out] (3*Coth[c + d*x]^(1 - m)*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Cot
h[c + d*x]^2])/(b*d*(3 - 4*m)*(b*Coth[c + d*x]^m)^(1/3))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 3740

```
Int[(u_.)*((b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Tan[e + f*x])^n)^FracPart[p]/(c*Tan[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Tan[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth^{\frac{m}{3}}(c+dx)\int\coth^{-\frac{4m}{3}}(c+dx)\,dx}{b\sqrt[3]{b\coth^m(c+dx)}}\\
&= -\frac{\coth^{\frac{m}{3}}(c+dx)\text{Subst}\left(\int\frac{x^{-4m/3}}{-1+x^2}\,dx,x,\coth(c+dx)\right)}{bd\sqrt[3]{b\coth^m(c+dx)}}\\
&= \frac{3\coth^{1-m}(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-4m),\frac{1}{6}(9-4m),\coth^2(c+dx)\right)}{bd(3-4m)\sqrt[3]{b\coth^m(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$
\int\frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx=\frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(1,\frac{1}{6}(3-4m),\frac{1}{6}(9-4m),\coth^2(c+dx)\right)}{d\left(1-\frac{4m}{3}\right)(b\coth^m(c+dx))^{4/3}}
$$

```
[In] Integrate[(b*Coth[c + d*x]^m)^(-4/3),x]
[Out] (Coth[c + d*x]*Hypergeometric2F1[1, (3 - 4*m)/6, (9 - 4*m)/6, Coth[c + d*x]
^2])/(d*(1 - (4*m)/3)*(b*Coth[c + d*x]^m)^(4/3))
```

Maple [F]

$$\int \frac{1}{(b\coth(dx+c)^{m})^{\frac{4}{3}}}dx$$

```
[In] int(1/(b*coth(d*x+c)^m)^(4/3),x)
[Out] int(1/(b*coth(d*x+c)^m)^(4/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth^m(c+dx))^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)**m)**(4/3),x)
[Out] Integral((b*coth(c + d*x)**m)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(b\coth^m(c+dx))^{4/3}}\,dx = \int \frac{1}{(b\coth(dx+c)^{m})^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="maxima")
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{\left(b \coth^m(c+dx)\right)^{4/3}}\, dx = \int \frac{1}{\left(b \coth\left(dx+c\right)^m\right)^{\frac{4}{3}}}\, dx$$

```
[In] integrate(1/(b*coth(d*x+c)^m)^(4/3),x, algorithm="giac")
[Out] integrate((b*coth(d*x + c)^m)^(-4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(b \coth^m(c+dx)\right)^{4/3}}\, dx = \int \frac{1}{\left(b \coth\left(c+d\,x\right)^m\right)^{4/3}}\, dx$$

```
[In] int(1/(b*coth(c + d*x)^m)^(4/3),x)
[Out] int(1/(b*coth(c + d*x)^m)^(4/3), x)
```

3.61 $\int (1+\coth(x))^5 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 41

$$\int (1+\coth(x))^5 \, dx = 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\log(\sinh(x))$$

```
[Out] 16*x-8*coth(x)-2*(1+coth(x))^2-2/3*(1+coth(x))^3-1/4*(1+coth(x))^4+16*ln(si
nh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3559, 3558, 3556}

$$\int (1+\coth(x))^5 \, dx = 16x - \frac{1}{4}(\coth(x)+1)^4 - \frac{2}{3}(\coth(x)+1)^3 - 2(\coth(x)+1)^2 - 8\coth(x) + 16\log(\sinh(x))$$

```
[In] Int[(1 + Coth[x])^5,x]
```

```
[Out] 16*x - 8*Coth[x] - 2*(1 + Coth[x])^2 - (2*(1 + Coth[x])^3)/3 - (1 + Coth[x]
)^4/4 + 16*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{4}(1+\coth(x))^4 + 2\int (1+\coth(x))^4\,dx \\
&= -\frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 4\int (1+\coth(x))^3\,dx \\
&= -2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 8\int (1+\coth(x))^2\,dx \\
&= 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\int \coth(x)\,dx \\
&= 16x - 8\coth(x) - 2(1+\coth(x))^2 - \frac{2}{3}(1+\coth(x))^3 - \frac{1}{4}(1+\coth(x))^4 + 16\log(\sinh(x))
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.29

$$\int (1+\coth(x))^5\,dx = \frac{(1+\coth(x))^5 \sinh(x) \left(-3\cosh^4(x) - 20\cosh^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, 1, -\frac{1}{2}, \tanh^2(x)\right)\sinh(x) - 6\right.}{}$$

```
[In] Integrate[(1 + Coth[x])^5,x]
```

```
[Out] ((1 + Coth[x])^5*Sinh[x]*(-3*Cosh[x]^4 - 20*Cosh[x]^3*Hypergeometric2F1[-3/
2, 1, -1/2, Tanh[x]^2]*Sinh[x] - 66*Cosh[x]^2*Sinh[x]^2 - 120*Cosh[x]*Hyper
geometric2F1[-1/2, 1, 1/2, Tanh[x]^2]*Sinh[x]^3 + 12*(x + 16*Log[Cosh[x]] +
 16*Log[Tanh[x]])*Sinh[x]^4))/(12*(Cosh[x] + Sinh[x])^5)
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result
derivativedivides	$-\frac{\coth(x)^4}{4}-\frac{5\coth(x)^3}{3}-\frac{11\coth(x)^2}{2}-15\coth(x)-16\ln(\coth(x)-1)$
default	$-\frac{\coth(x)^4}{4}-\frac{5\coth(x)^3}{3}-\frac{11\coth(x)^2}{2}-15\coth(x)-16\ln(\coth(x)-1)$
parallelrisch	$-\frac{\coth(x)^4}{4}+16\ln(\tanh(x))-16\ln(1-\tanh(x))-15\coth(x)-\frac{11\coth(x)^2}{2}-\frac{5\coth(x)^3}{3}$
risch	$-\frac{4(48\,e^{6x}-108\,e^{4x}+88\,e^{2x}-25)}{3(e^{2x}-1)^4}+16\ln(e^{2x}-1)$
parts	$x-\frac{\coth(x)^4}{4}-\frac{11\coth(x)^2}{2}-13\ln(\coth(x)-1)+2\ln(1+\coth(x))-15\coth(x)-\frac{5\coth(}{3}$

```
[In] int((1+coth(x))^5,x,method=_RETURNVERBOSE)
[Out] -1/4*coth(x)^4-5/3*coth(x)^3-11/2*coth(x)^2-15*coth(x)-16*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 448, normalized size of antiderivative = 10.93

$$\int (1+\coth(x))^5\,dx = -\frac{4\left(48\cosh(x)^6+288\cosh(x)\sinh(x)^5+48\sinh(x)^6+36\left(20\cosh(x)^2-3\right)\sinh(x)^4-108\cosh(x)^4\right.}{}$$

```
[In] integrate((1+coth(x))^5,x, algorithm="fricas")
[Out] -4/3*(48*cosh(x)^6 + 288*cosh(x)*sinh(x)^5 + 48*sinh(x)^6 + 36*(20*cosh(x)^
2 - 3)*sinh(x)^4 - 108*cosh(x)^4 + 48*(20*cosh(x)^3 - 9*cosh(x))*sinh(x)^3
+ 8*(90*cosh(x)^4 - 81*cosh(x)^2 + 11)*sinh(x)^2 + 88*cosh(x)^2 - 12*(cosh(
x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*
cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*co
sh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*co
sh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)
^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*cosh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh
(x) + 1)*log(2*sinh(x)/(cosh(x) - sinh(x))) + 16*(18*cosh(x)^5 - 27*cosh(x)
^3 + 11*cosh(x))*sinh(x) - 25)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8
 + 4*(7*cosh(x)^2 - 1)*sinh(x)^6 - 4*cosh(x)^6 + 8*(7*cosh(x)^3 - 3*cosh(x)
)*sinh(x)^5 + 2*(35*cosh(x)^4 - 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 +
 8*(7*cosh(x)^5 - 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 15
*cosh(x)^4 + 9*cosh(x)^2 - 1)*sinh(x)^2 - 4*cosh(x)^2 + 8*(cosh(x)^7 - 3*co
sh(x)^5 + 3*cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int (1+\coth(x))^5\,dx = 32x - 16\log(\tanh(x)+1) + 16\log(\tanh(x)) - \frac{15}{\tanh(x)} - \frac{11}{2\tanh^2(x)} - \frac{5}{3\tanh^3(x)} - \frac{1}{4\tanh^4(x)}$$

```
[In] integrate((1+coth(x))**5,x)
```

```
[Out] 32*x - 16*log(tanh(x) + 1) + 16*log(tanh(x)) - 15/tanh(x) - 11/(2*tanh(x)**
2) - 5/(3*tanh(x)**3) - 1/(4*tanh(x)**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(37) = 74.

Time = 0.20 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.41

$$\int (1+\coth(x))^5\,dx = 27\,x - \frac{20\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 2\right)}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} + \frac{4\left(e^{(-2\,x)} - e^{(-4\,x)} + e^{(-6\,x)}\right)}{4\,e^{(-2\,x)} - 6\,e^{(-4\,x)} + 4\,e^{(-6\,x)} - e^{(-8\,x)} - 1} + \frac{20\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{20}{e^{(-2\,x)} - 1} + 11\,\log\left(e^{(-x)} + 1\right) + 11\,\log\left(e^{(-x)} - 1\right) + 5\,\log(\sinh(x))$$

```
[In] integrate((1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 27*x - 20/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*
x) - 1) + 4*(e^(-2*x) - e^(-4*x) + e^(-6*x))/(4*e^(-2*x) - 6*e^(-4*x) + 4*e
^(-6*x) - e^(-8*x) - 1) + 20*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 20/(e^(
-2*x) - 1) + 11*log(e^(-x) + 1) + 11*log(e^(-x) - 1) + 5*log(sinh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int (1+\coth(x))^5\,dx = -\frac{4\left(48\,e^{(6\,x)} - 108\,e^{(4\,x)} + 88\,e^{(2\,x)} - 25\right)}{3\left(e^{(2\,x)} - 1\right)^4} + 16\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate((1+coth(x))^5,x, algorithm="giac")
```

```
[Out] -4/3*(48*e^(6*x) - 108*e^(4*x) + 88*e^(2*x) - 25)/(e^(2*x) - 1)^4 + 16*log(
abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

$$\int (1+\coth(x))^5\,dx = 16\ln\left(e^{2x}-1\right)-\frac{64}{3\left(3e^{2x}-3e^{4x}+e^{6x}-1\right)} -\frac{48}{e^{4x}-2e^{2x}+1}-\frac{4}{6e^{4x}-4e^{2x}-4e^{6x}+e^{8x}+1}-\frac{64}{e^{2x}-1}$$

```
[In] int((coth(x) + 1)^5,x)
```

```
[Out] 16*log(exp(2*x) - 1) - 64/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 48
/(exp(4*x) - 2*exp(2*x) + 1) - 4/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + ex
p(8*x) + 1) - 64/(exp(2*x) - 1)
```

3.62 $\int (1+\coth(x))^4\,dx$

Optimal result

Integrand size = 6, antiderivative size = 31

$$\int (1+\coth(x))^4\,dx = 8x-4\coth(x)-(1+\coth(x))^2-\frac{1}{3}(1+\coth(x))^3+8\log(\sinh(x))$$

```
[Out] 8*x-4*coth(x)-(1+coth(x))^2-1/3*(1+coth(x))^3+8*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3559, 3558, 3556}

$$\int (1+\coth(x))^4\,dx = 8x-\frac{1}{3}(\coth(x)+1)^3-(\coth(x)+1)^2-4\coth(x)+8\log(\sinh(x))$$

```
[In] Int[(1 + Coth[x])^4,x]
```

```
[Out] 8*x - 4*Coth[x] - (1 + Coth[x])^2 - (1 + Coth[x])^3/3 + 8*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3}(1+\coth(x))^3 + 2\int (1+\coth(x))^3\,dx \\
&= -(1+\coth(x))^2 - \frac{1}{3}(1+\coth(x))^3 + 4\int (1+\coth(x))^2\,dx \\
&= 8x - 4\coth(x) - (1+\coth(x))^2 - \frac{1}{3}(1+\coth(x))^3 + 8\int \coth(x)\,dx \\
&= 8x - 4\coth(x) - (1+\coth(x))^2 - \frac{1}{3}(1+\coth(x))^3 + 8\log(\sinh(x))
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.71

$$\int (1+\coth(x))^4\,dx$$
$$= \frac{(1+\coth(x))^4\sinh(x)\left(-\cosh^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(x)\right)+3\sinh(x)\left(-2\cosh^2(x)-6\right.\right.}{3(\cosh(x)}$$

```
[In] Integrate[(1 + Coth[x])^4,x]
```

```
[Out] ((1 + Coth[x])^4*Sinh[x]*(-(Cosh[x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh
[x]^2]) + 3*Sinh[x]*(-2*Cosh[x]^2 - 6*Cosh[x]*Hypergeometric2F1[-1/2, 1, 1/
2, Tanh[x]^2]*Sinh[x] + (x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]])*Sinh[x]^2)))/
(3*(Cosh[x] + Sinh[x])^4)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{\coth(x)^3}{3}-2\coth(x)^2-7\coth(x)-8\ln(\coth(x)-1)$
default	$-\frac{\coth(x)^3}{3}-2\coth(x)^2-7\coth(x)-8\ln(\coth(x)-1)$
parallelrisch	$-\frac{\coth(x)^3}{3}+8\ln(\tanh(x))-8\ln(1-\tanh(x))-7\coth(x)-2\coth(x)^2$
risch	$-\frac{4(18\,e^{4x}-27\,e^{2x}+11)}{3(e^{2x}-1)^3}+8\ln(e^{2x}-1)$
parts	$x-\frac{\coth(x)^3}{3}-7\coth(x)-\frac{11\ln(\coth(x)-1)}{2}+\frac{3\ln(1+\coth(x))}{2}-2\coth(x)^2+4\ln(\sinh(x))$

```
[In] int((1+coth(x))^4,x,method=_RETURNVERBOSE)
[Out] -1/3*coth(x)^3-2*coth(x)^2-7*coth(x)-8*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(29) = 58.

Time = 0.24 (sec) , antiderivative size = 273, normalized size of antiderivative = 8.81

$$\int(1+\coth(x))^4\,dx = -\frac{4\left(18\cosh(x)^4+72\cosh(x)\sinh(x)^3+18\sinh(x)^4+27\left(4\cosh(x)^2-1\right)\sinh(x)^2-27\cosh(x)^2-\right.}{3\left(\cosh(x)^6+6\,c\right.}$$

```
[In] integrate((1+coth(x))^4,x, algorithm="fricas")
[Out] -4/3*(18*cosh(x)^4 + 72*cosh(x)*sinh(x)^3 + 18*sinh(x)^4 + 27*(4*cosh(x)^2
- 1)*sinh(x)^2 - 27*cosh(x)^2 - 6*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x
)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh
(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2
+ 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)*log(2*sinh(x)/(cosh(x)
 - sinh(x))) + 18*(4*cosh(x)^3 - 3*cosh(x))*sinh(x) + 11)/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*
sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1
)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int (1+\coth(x))^4\,dx = 16x - 8\log(\tanh(x)+1) + 8\log(\tanh(x)) - \frac{7}{\tanh(x)} - \frac{2}{\tanh^2(x)} - \frac{1}{3\tanh^3(x)}$$

```
[In] integrate((1+coth(x))**4,x)
[Out] 16*x - 8*log(tanh(x) + 1) + 8*log(tanh(x)) - 7/tanh(x) - 2/tanh(x)**2 - 1/(
3*tanh(x)**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(29) = 58.

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.06

$$\int (1+\coth(x))^4\,dx = 12\,x - \frac{4\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} - 2\right)}{3\left(3\,e^{(-2\,x)} - 3\,e^{(-4\,x)} + e^{(-6\,x)} - 1\right)} + \frac{8\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{12}{e^{(-2\,x)} - 1} + 4\,\log\left(e^{(-x)} + 1\right) + 4\,\log\left(e^{(-x)} - 1\right) + 4\,\log\left(\sinh(x)\right)$$

```
[In] integrate((1+coth(x))^4,x, algorithm="maxima")
[Out] 12*x - 4/3*(3*e^(-2*x) - 3*e^(-4*x) - 2)/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x
) - 1) + 8*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 12/(e^(-2*x) - 1) + 4*log
(e^(-x) + 1) + 4*log(e^(-x) - 1) + 4*log(sinh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int (1+\coth(x))^4\,dx = -\frac{4\left(18\,e^{(4\,x)} - 27\,e^{(2\,x)} + 11\right)}{3\left(e^{(2\,x)} - 1\right)^3} + 8\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate((1+coth(x))^4,x, algorithm="giac")
[Out] -4/3*(18*e^(4*x) - 27*e^(2*x) + 11)/(e^(2*x) - 1)^3 + 8*log(abs(e^(2*x) - 1
))
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int (1+\coth(x))^4\,dx = 8\,\ln\left(e^{2\,x}-1\right)-\frac{8}{3\,\left(3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1\right)}-\frac{12}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{24}{e^{2\,x}-1}$$

```
[In] int((coth(x) + 1)^4,x)
```

```
[Out] 8*log(exp(2*x) - 1) - 8/(3*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - 12/(
exp(4*x) - 2*exp(2*x) + 1) - 24/(exp(2*x) - 1)
```

3.63 $\int (1+\coth(x))^3\,dx$

Optimal result

Integrand size = 6, antiderivative size = 23

$$\int (1+\coth(x))^3\,dx = 4x-2\coth(x)-\frac{1}{2}(1+\coth(x))^2+4\log(\sinh(x))$$

```
[Out] 4*x-2*coth(x)-1/2*(1+coth(x))^2+4*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3559, 3558, 3556}

$$\int (1+\coth(x))^3\,dx = 4x-\frac{1}{2}(\coth(x)+1)^2-2\coth(x)+4\log(\sinh(x))$$

```
[In] Int[(1 + Coth[x])^3,x]
```

```
[Out] 4*x - 2*Coth[x] - (1 + Coth[x])^2/2 + 4*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{2}(1+\coth(x))^2 + 2\int (1+\coth(x))^2\,dx \\
&= 4x - 2\coth(x) - \frac{1}{2}(1+\coth(x))^2 + 4\int \coth(x)\,dx \\
&= 4x - 2\coth(x) - \frac{1}{2}(1+\coth(x))^2 + 4\log(\sinh(x))
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\begin{aligned}
\int (1+\coth(x))^3\,dx = \frac{1}{4}\text{csch}^2(x)\Big(&-1-2x-8\log(\cosh(x))-8\log(\tanh(x)) \\
&+\cosh(2x)(-1+2x+8\log(\cosh(x))+8\log(\tanh(x))) \\
&-6\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right)\sinh(2x)\Big)
\end{aligned}$$

```
[In] Integrate[(1 + Coth[x])^3,x]
[Out] (Csch[x]^2*(-1 - 2*x - 8*Log[Cosh[x]] - 8*Log[Tanh[x]] + Cosh[2*x]*(-1 + 2*
x + 8*Log[Cosh[x]] + 8*Log[Tanh[x]]) - 6*Hypergeometric2F1[-1/2, 1, 1/2, Ta
nh[x]^2]*Sinh[2*x]))/4
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{\coth(x)^2}{2}-3\coth(x)-4\ln(\coth(x)-1)$	19
default	$-\frac{\coth(x)^2}{2}-3\coth(x)-4\ln(\coth(x)-1)$	19
parallelrisch	$4\ln(\tanh(x))-4\ln(1-\tanh(x))-3\coth(x)-\frac{\coth(x)^2}{2}$	26
risch	$-\frac{2(4\,\mathrm{e}^{2x}-3)}{(\mathrm{e}^{2x}-1)^2}+4\ln(\mathrm{e}^{2x}-1)$	29
parts	$x-\frac{\coth(x)^2}{2}-2\ln(\coth(x)-1)+\ln(1+\coth(x))-3\coth(x)+3\ln(\sinh(x))$	30

```
[In] int((1+coth(x))^3,x,method=_RETURNVERBOSE)
[Out] -1/2*coth(x)^2-3*coth(x)-4*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(21) = 42.

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.17

$$\int(1+\coth(x))^3\,dx = -\frac{2\left(4\cosh(x)^2-2\left(\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-1\right)\sinh(x)^2-2\cosh(\right.\right.}{\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-1\right)}$$

```
[In] integrate((1+coth(x))^3,x, algorithm="fricas")
[Out] -2*(4*cosh(x)^2 - 2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*
log(2*sinh(x)/(cosh(x) - sinh(x))) + 8*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(
cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2
 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int(1+\coth(x))^3\,dx = 8x-4\log(\tanh(x)+1)+4\log(\tanh(x))-\frac{3}{\tanh(x)}-\frac{1}{2\tanh^2(x)}$$

```
[In] integrate((1+coth(x))**3,x)
[Out] 8*x - 4*log(tanh(x) + 1) + 4*log(tanh(x)) - 3/tanh(x) - 1/(2*tanh(x)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(21) = 42$.

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int (1+\coth(x))^3\,dx = 5\,x + \frac{2\,e^{(-2\,x)}}{2\,e^{(-2\,x)} - e^{(-4\,x)} - 1} + \frac{6}{e^{(-2\,x)} - 1} + \log\left(e^{(-x)} + 1\right) + \log\left(e^{(-x)} - 1\right) + 3\,\log\left(\sinh\left(x\right)\right)$$

```
[In] integrate((1+coth(x))^3,x, algorithm="maxima")
[Out] 5*x + 2*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) + 6/(e^(-2*x) - 1) + log(e^(-x
) + 1) + log(e^(-x) - 1) + 3*log(sinh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int (1+\coth(x))^3\,dx = -\frac{2\,\left(4\,e^{(2\,x)} - 3\right)}{\left(e^{(2\,x)} - 1\right)^2} + 4\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate((1+coth(x))^3,x, algorithm="giac")
[Out] -2*(4*e^(2*x) - 3)/(e^(2*x) - 1)^2 + 4*log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int (1+\coth(x))^3\,dx = 4\,\ln\left(\mathrm{e}^{2\,x} - 1\right) - \frac{2}{\mathrm{e}^{4\,x} - 2\,\mathrm{e}^{2\,x} + 1} - \frac{8}{\mathrm{e}^{2\,x} - 1}$$

```
[In] int((coth(x) + 1)^3,x)
[Out] 4*log(exp(2*x) - 1) - 2/(exp(4*x) - 2*exp(2*x) + 1) - 8/(exp(2*x) - 1)
```

3.64 $\int (1+\coth(x))^2\,dx$

Optimal result

Integrand size = 6, antiderivative size = 13

$$\int (1+\coth(x))^2\,dx = 2x-\coth(x)+2\log(\sinh(x))$$

```
[Out] 2*x-coth(x)+2*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3558, 3556}

$$\int (1+\coth(x))^2\,dx = 2x-\coth(x)+2\log(\sinh(x))$$

```
[In] Int[(1 + Coth[x])^2,x]
```

```
[Out] 2*x - Coth[x] + 2*Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2x-\coth(x)+2\int\coth(x)\,dx\\ &= 2x-\coth(x)+2\log(\sinh(x))\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.15

$$\int(1+\coth(x))^2\,dx=x-\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right)+2\log(\cosh(x))+2\log(\tanh(x))$$

```
[In] Integrate[(1 + Coth[x])^2,x]
```

```
[Out] x - Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x]^2] + 2*Log[Cosh[x]] + 2
*Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\coth\left(x\right)-2\ln\left(\coth\left(x\right)-1\right)$	13
default	$-\coth\left(x\right)-2\ln\left(\coth\left(x\right)-1\right)$	13
risch	$-\frac{2}{{\mathrm e}^{2x}-1}+2\ln\left({\mathrm e}^{2x}-1\right)$	21
parallelrisch	$\frac{-1+2\ln(\tanh(x))\tanh(x)-2\ln(1-\tanh(x))\tanh(x)}{\tanh(x)}$	26
parts	$x-\coth\left(x\right)-\frac{\ln(\coth(x)-1)}{2}+\frac{\ln(1+\coth(x))}{2}+2\ln\left(\sinh\left(x\right)\right)$	26

```
[In] int((1+coth(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -coth(x)-2*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(13) = 26$.

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 4.08

$$\int (1+\coth(x))^2\,dx = \frac{2\left(\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)-1\right)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1}$$

```
[In] integrate((1+coth(x))^2,x, algorithm="fricas")
[Out] 2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sinh(x)/(cosh(x) -
 sinh(x))) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int (1+\coth(x))^2\,dx = 4x - 2\log(\tanh(x)+1) + 2\log(\tanh(x)) - \frac{1}{\tanh(x)}$$

```
[In] integrate((1+coth(x))**2,x)
[Out] 4*x - 2*log(tanh(x) + 1) + 2*log(tanh(x)) - 1/tanh(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (1+\coth(x))^2\,dx = 2\,x + \frac{2}{e^{(-2\,x)}-1} + 2\,\log(\sinh(x))$$

```
[In] integrate((1+coth(x))^2,x, algorithm="maxima")
[Out] 2*x + 2/(e^(-2*x) - 1) + 2*log(sinh(x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int (1+\coth(x))^2 \, dx = -\frac{2}{e^{(2x)}-1} + 2 \log\left(\left|e^{(2x)}-1\right|\right)$$

```
[In] integrate((1+coth(x))^2,x, algorithm="giac")
[Out] -2/(e^(2*x) - 1) + 2*log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int (1+\coth(x))^2 \, dx = 2 \ln\left(\mathrm{e}^{2x}-1\right) - \frac{2}{\mathrm{e}^{2x}-1}$$

```
[In] int((coth(x) + 1)^2,x)
[Out] 2*log(exp(2*x) - 1) - 2/(exp(2*x) - 1)
```

3.65 $\int \frac{1}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2(1+\coth(x))}$$

```
[Out] 1/2*x-1/2/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3560, 8}

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2(\coth(x)+1)}$$

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2(1+\coth(x))} + \frac{\int 1\,dx}{2}\\ &= \frac{x}{2} - \frac{1}{2(1+\coth(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\left(\operatorname{arctanh}(\tanh(x)) + \frac{1}{1+\tanh(x)}\right)$$

```
[In] Integrate[(1 + Coth[x])^(-1),x]
[Out] (ArcTanh[Tanh[x]] + (1 + Tanh[x])^(-1))/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}+\frac{e^{-2x}}{4}$	11
parallelrisch	$\frac{\tanh(x)x+x+1}{2+2\tanh(x)}$	17
derivativedivides	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24
default	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24

```
[In] int(1/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{(2\,x+1)\cosh{(x)} + (2\,x-1)\sinh{(x)}}{4\,(\cosh{(x)} + \sinh{(x)})}$$

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x\tanh{(x)}}{2\tanh{(x)}+2} + \frac{x}{2\tanh{(x)}+2} + \frac{1}{2\tanh{(x)}+2}$$

```
[In] integrate(1/(1+coth(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
[Out] 1/2*x + 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2\,\left(\coth\left(x\right)+1\right)}$$

```
[In] int(1/(coth(x) + 1),x)
[Out] x/2 - 1/(2*(coth(x) + 1))
```

3.66 $\int \frac{1}{(1+\coth(x))^2} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 26

$$\int \frac{1}{(1+\coth(x))^2} \, dx = \frac{x}{4} - \frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}$$

```
[Out] 1/4*x-1/4/(1+coth(x))^2-1/4/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\int \frac{1}{(1+\coth(x))^2} \, dx = \frac{x}{4} - \frac{1}{4(\coth(x)+1)} - \frac{1}{4(\coth(x)+1)^2}$$

```
[In] Int[(1 + Coth[x])^(-2),x]
```

```
[Out] x/4 - 1/(4*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{4(1+\coth(x))^2} + \frac{1}{2}\int \frac{1}{1+\coth(x)}\,dx$$
$$= -\frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))} + \frac{\int 1\,dx}{4}$$
$$= \frac{x}{4} - \frac{1}{4(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{1}{4}\text{arctanh}(\tanh(x)) - \frac{1}{4(1+\tanh(x))^2} + \frac{3}{4(1+\tanh(x))}$$

```
[In] Integrate[(1 + Coth[x])^(-2),x]
[Out] ArcTanh[Tanh[x]]/4 - 1/(4*(1 + Tanh[x])^2) + 3/(4*(1 + Tanh[x]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{x}{4}+\frac{{\rm e}^{-2x}}{4}-\frac{{\rm e}^{-4x}}{16}$	17
parallelrisch	$\frac{\tanh(x)^2 x+(2x+3)\tanh(x)+x+2}{4(1+\tanh(x))^2}$	26
derivativedivides	$-\frac{1}{4(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}+\frac{\ln(1+\coth(x))}{8}-\frac{\ln(\coth(x)-1)}{8}$	32
default	$-\frac{1}{4(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}+\frac{\ln(1+\coth(x))}{8}-\frac{\ln(\coth(x)-1)}{8}$	32

```
[In] int(1/(1+coth(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x+1/4*exp(-2*x)-1/16*exp(-4*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{(4\,x-1)\cosh{(x)}^2 + 2\,(4\,x+1)\cosh{(x)}\sinh{(x)} + (4\,x-1)\sinh{(x)}^2 + 4}{16\,\left(\cosh{(x)}^2 + 2\,\cosh{(x)}\sinh{(x)} + \sinh{(x)}^2\right)}$$

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="fricas")
[Out] 1/16*((4*x - 1)*cosh(x)^2 + 2*(4*x + 1)*cosh(x)*sinh(x) + (4*x - 1)*sinh(x)
^2 + 4)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(20) = 40$.

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.38

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{x\tanh^2{(x)}}{4\tanh^2{(x)} + 8\tanh{(x)} + 4} + \frac{2x\tanh{(x)}}{4\tanh^2{(x)} + 8\tanh{(x)} + 4} + \frac{x}{4\tanh^2{(x)} + 8\tanh{(x)} + 4} + \frac{3\tanh{(x)}}{4\tanh^2{(x)} + 8\tanh{(x)} + 4} + \frac{2}{4\tanh^2{(x)} + 8\tanh{(x)} + 4}$$

```
[In] integrate(1/(1+coth(x))**2,x)
[Out] x*tanh(x)**2/(4*tanh(x)**2 + 8*tanh(x) + 4) + 2*x*tanh(x)/(4*tanh(x)**2 + 8
*tanh(x) + 4) + x/(4*tanh(x)**2 + 8*tanh(x) + 4) + 3*tanh(x)/(4*tanh(x)**2
+ 8*tanh(x) + 4) + 2/(4*tanh(x)**2 + 8*tanh(x) + 4)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{1}{4}\,x + \frac{1}{4}\,e^{(-2\,x)} - \frac{1}{16}\,e^{(-4\,x)}$$

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="maxima")
[Out] 1/4*x + 1/4*e^(-2*x) - 1/16*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{1}{16}\left(4\,e^{(2\,x)}-1\right)e^{(-4\,x)}+\frac{1}{4}\,x$$

```
[In] integrate(1/(1+coth(x))^2,x, algorithm="giac")
[Out] 1/16*(4*e^(2*x) - 1)*e^(-4*x) + 1/4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{(1+\coth(x))^2}\,dx = \frac{x}{4}+\frac{\mathrm{e}^{-2\,x}}{4}-\frac{\mathrm{e}^{-4\,x}}{16}$$

```
[In] int(1/(coth(x) + 1)^2,x)
[Out] x/4 + exp(-2*x)/4 - exp(-4*x)/16
```

3.67 $\int \frac{1}{(1+\coth(x))^3} dx$

Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \frac{1}{(1+\coth(x))^3} dx = \frac{x}{8} - \frac{1}{6(1+\coth(x))^3} - \frac{1}{8(1+\coth(x))^2} - \frac{1}{8(1+\coth(x))}$$

```
[Out] 1/8*x-1/6/(1+coth(x))^3-1/8/(1+coth(x))^2-1/8/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\int \frac{1}{(1+\coth(x))^3} dx = \frac{x}{8} - \frac{1}{8(\coth(x)+1)} - \frac{1}{8(\coth(x)+1)^2} - \frac{1}{6(\coth(x)+1)^3}$$

```
[In] Int[(1 + Coth[x])^(-3),x]
```

```
[Out] x/8 - 1/(6*(1 + Coth[x])^3) - 1/(8*(1 + Coth[x])^2) - 1/(8*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{6(1+\coth(x))^3}+\frac{1}{2}\int\frac{1}{(1+\coth(x))^2}\,dx\\
&= -\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}+\frac{1}{4}\int\frac{1}{1+\coth(x)}\,dx\\
&= -\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}+\frac{\int 1\,dx}{8}\\
&= \frac{x}{8}-\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$
\int\frac{1}{(1+\coth(x))^3}\,dx=\frac{10+27\tanh(x)+21\tanh^2(x)+3\text{arctanh}(\tanh(x))(1+\tanh(x))^3}{24(1+\tanh(x))^3}
$$

```
[In] Integrate[(1 + Coth[x])^(-3),x]
[Out] (10 + 27*Tanh[x] + 21*Tanh[x]^2 + 3*ArcTanh[Tanh[x]]*(1 + Tanh[x])^3)/(24*(
1 + Tanh[x])^3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

method	result	size
risch	$\frac{x}{8}+\frac{3\,\mathrm{e}^{-2x}}{16}-\frac{3\,\mathrm{e}^{-4x}}{32}+\frac{\mathrm{e}^{-6x}}{48}$	23
parallelrisch	$\frac{3\tanh(x)^3x+(9x+21)\tanh(x)^2+(9x+27)\tanh(x)+3x+10}{24(1+\tanh(x))^3}$	39
derivativedivides	$-\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}+\frac{\ln(1+\coth(x))}{16}-\frac{\ln(\coth(x)-1)}{16}$	40
default	$-\frac{1}{6(1+\coth(x))^3}-\frac{1}{8(1+\coth(x))^2}-\frac{1}{8(1+\coth(x))}+\frac{\ln(1+\coth(x))}{16}-\frac{\ln(\coth(x)-1)}{16}$	40

```
[In] int(1/(1+coth(x))^3,x,method=_RETURNVERBOSE)
[Out] 1/8*x+3/16*exp(-2*x)-3/32*exp(-4*x)+1/48*exp(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(28) = 56$.

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \frac{1}{(1+\coth(x))^3}\,dx$$
$$= \frac{2\,(6\,x+1)\cosh\left(x\right)^3 + 6\,(6\,x+1)\cosh\left(x\right)\sinh\left(x\right)^2 + 2\,(6\,x-1)\sinh\left(x\right)^3 + 3\,\left(2\,(6\,x-1)\cosh\left(x\right)^2 + 9\right)\sin}{96\,\left(\cosh\left(x\right)^3 + 3\,\cosh\left(x\right)^2\sinh\left(x\right) + 3\,\cosh\left(x\right)\sinh\left(x\right)^2 + \sinh\left(x\right)^3\right)}$$

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="fricas")
```

```
[Out] 1/96*(2*(6*x + 1)*cosh(x)^3 + 6*(6*x + 1)*cosh(x)*sinh(x)^2 + 2*(6*x - 1)*s
inh(x)^3 + 3*(2*(6*x - 1)*cosh(x)^2 + 9)*sinh(x) + 9*cosh(x))/(cosh(x)^3 +
3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(31) = 62$.

Time = 0.57 (sec) , antiderivative size = 182, normalized size of antiderivative = 5.06

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{3x\tanh^3(x)}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24} + \frac{9x\tanh^2(x)}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24} + \frac{9x\tanh(x)}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24} + \frac{3x}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24} + \frac{21\tanh^2(x)}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24} + \frac{27\tanh(x)}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24} + \frac{10}{24\tanh^3(x)+72\tanh^2(x)+72\tanh(x)+24}$$

```
[In] integrate(1/(1+coth(x))**3,x)
```

```
[Out] 3*x*tanh(x)**3/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh
(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 9*x*tanh(x)/(24*
tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24) + 3*x/(24*tanh(x)**3 + 72*tan
h(x)**2 + 72*tanh(x) + 24) + 21*tanh(x)**2/(24*tanh(x)**3 + 72*tanh(x)**2 +
 72*tanh(x) + 24) + 27*tanh(x)/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x)
+ 24) + 10/(24*tanh(x)**3 + 72*tanh(x)**2 + 72*tanh(x) + 24)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{1}{8}\,x + \frac{3}{16}\,e^{(-2\,x)} - \frac{3}{32}\,e^{(-4\,x)} + \frac{1}{48}\,e^{(-6\,x)}$$

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="maxima")
[Out] 1/8*x + 3/16*e^(-2*x) - 3/32*e^(-4*x) + 1/48*e^(-6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{1}{96}\,\left(18\,e^{(4\,x)} - 9\,e^{(2\,x)} + 2\right)e^{(-6\,x)} + \frac{1}{8}\,x$$

```
[In] integrate(1/(1+coth(x))^3,x, algorithm="giac")
[Out] 1/96*(18*e^(4*x) - 9*e^(2*x) + 2)*e^(-6*x) + 1/8*x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^3}\,dx = \frac{x}{8} + \frac{3\,\mathrm{e}^{-2\,x}}{16} - \frac{3\,\mathrm{e}^{-4\,x}}{32} + \frac{\mathrm{e}^{-6\,x}}{48}$$

```
[In] int(1/(coth(x) + 1)^3,x)
[Out] x/8 + (3*exp(-2*x))/16 - (3*exp(-4*x))/32 + exp(-6*x)/48
```

3.68 $\int \frac{1}{(1+\coth(x))^4} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 46

$$\int \frac{1}{(1+\coth(x))^4} \, dx = \frac{x}{16} - \frac{1}{8(1+\coth(x))^4} - \frac{1}{12(1+\coth(x))^3} - \frac{1}{16(1+\coth(x))^2} - \frac{1}{16(1+\coth(x))}$$

```
[Out] 1/16*x-1/8/(1+coth(x))^4-1/12/(1+coth(x))^3-1/16/(1+coth(x))^2-1/16/(1+coth
(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\int \frac{1}{(1+\coth(x))^4} \, dx = \frac{x}{16} - \frac{1}{16(\coth(x)+1)} - \frac{1}{16(\coth(x)+1)^2} - \frac{1}{12(\coth(x)+1)^3} - \frac{1}{8(\coth(x)+1)^4}$$

```
[In] Int[(1 + Coth[x])^(-4),x]
```

```
[Out] x/16 - 1/(8*(1 + Coth[x])^4) - 1/(12*(1 + Coth[x])^3) - 1/(16*(1 + Coth[x])
^2) - 1/(16*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{8(1+\coth(x))^4}+\frac{1}{2}\int\frac{1}{(1+\coth(x))^3}\,dx\\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}+\frac{1}{4}\int\frac{1}{(1+\coth(x))^2}\,dx\\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}+\frac{1}{8}\int\frac{1}{1+\coth(x)}\,dx\\
&= -\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int\frac{1}{(1+\coth(x))^4}\,dx=\frac{1}{48}\left(3\text{arctanh}(\tanh(x))+\frac{16+61\tanh(x)+84\tanh^2(x)+45\tanh^3(x)}{(1+\tanh(x))^4}\right)$$

```
[In] Integrate[(1 + Coth[x])^(-4),x]
[Out] (3*ArcTanh[Tanh[x]] + (16 + 61*Tanh[x] + 84*Tanh[x]^2 + 45*Tanh[x]^3)/(1 +
Tanh[x])^4)/48
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result
risch	$\frac{x}{16}+\frac{e^{-2x}}{8}-\frac{3\,e^{-4x}}{32}+\frac{e^{-6x}}{24}-\frac{e^{-8x}}{128}$
derivativedivides	$-\frac{\ln(\coth(x)-1)}{32}-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\ln(1+\coth(x))}{32}$
default	$-\frac{\ln(\coth(x)-1)}{32}-\frac{1}{8(1+\coth(x))^4}-\frac{1}{12(1+\coth(x))^3}-\frac{1}{16(1+\coth(x))^2}-\frac{1}{16(1+\coth(x))}+\frac{\ln(1+\coth(x))}{32}$
parallelrisch	$\frac{3\tanh(x)^4x+(12x+45)\tanh(x)^3+(18x+84)\tanh(x)^2+(12x+61)\tanh(x)+3x+16}{48(1+\tanh(x))^4}$

```
[In] int(1/(1+coth(x))^4,x,method=_RETURNVERBOSE)
[Out] 1/16*x+1/8*exp(-2*x)-3/32*exp(-4*x)+1/24*exp(-6*x)-1/128*exp(-8*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.63

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{3\,(8\,x-1)\cosh(x)^4+12\,(8\,x+1)\cosh(x)\sinh(x)^3+3\,(8\,x-1)\sinh(x)^4+2\,\left(9\,(8\,x-1)\cosh(x)^2+32\right)}{384\,\left(\cosh(x)^4+4\,\cosh(x)^3\sinh(x)+6\,\cosh(x)^2\sinh(x)^2+\right.}$$

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="fricas")
[Out] 1/384*(3*(8*x - 1)*cosh(x)^4 + 12*(8*x + 1)*cosh(x)*sinh(x)^3 + 3*(8*x - 1)
*sinh(x)^4 + 2*(9*(8*x - 1)*cosh(x)^2 + 32)*sinh(x)^2 + 64*cosh(x)^2 + 4*(3
*(8*x + 1)*cosh(x)^3 + 16*cosh(x))*sinh(x) - 36)/(cosh(x)^4 + 4*cosh(x)^3*s
inh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(41) = 82$.

Time = 0.73 (sec) , antiderivative size = 299, normalized size of antiderivative = 6.50

$$\begin{aligned}\int \frac{1}{(1+\coth(x))^4}\,dx &= \frac{3x\tanh^4(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{12x\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{18x\tanh^2(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{12x\tanh(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{3x}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{45\tanh^3(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{84\tanh^2(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{61\tanh(x)}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\\ &+\frac{16}{48\tanh^4(x)+192\tanh^3(x)+288\tanh^2(x)+192\tanh(x)+48}\end{aligned}$$

```
[In] integrate(1/(1+coth(x))**4,x)
```

```
[Out] 3*x*tanh(x)**4/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(
x) + 48) + 12*x*tanh(x)**3/(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2
 + 192*tanh(x) + 48) + 18*x*tanh(x)**2/(48*tanh(x)**4 + 192*tanh(x)**3 + 28
8*tanh(x)**2 + 192*tanh(x) + 48) + 12*x*tanh(x)/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 3*x/(48*tanh(x)**4 + 192*tanh(x
)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 45*tanh(x)**3/(48*tanh(x)**4 +
192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 84*tanh(x)**2/(48*tan
h(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 61*tanh(x)/
(48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48) + 16/(
48*tanh(x)**4 + 192*tanh(x)**3 + 288*tanh(x)**2 + 192*tanh(x) + 48)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{1}{16}\,x + \frac{1}{8}\,e^{(-2\,x)} - \frac{3}{32}\,e^{(-4\,x)} + \frac{1}{24}\,e^{(-6\,x)} - \frac{1}{128}\,e^{(-8\,x)}$$

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="maxima")
```

```
[Out] 1/16*x + 1/8*e^(-2*x) - 3/32*e^(-4*x) + 1/24*e^(-6*x) - 1/128*e^(-8*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{1}{384}\,\left(48\,e^{(6\,x)} - 36\,e^{(4\,x)} + 16\,e^{(2\,x)} - 3\right)e^{(-8\,x)} + \frac{1}{16}\,x$$

```
[In] integrate(1/(1+coth(x))^4,x, algorithm="giac")
```

```
[Out] 1/384*(48*e^(6*x) - 36*e^(4*x) + 16*e^(2*x) - 3)*e^(-8*x) + 1/16*x
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^4}\,dx = \frac{x}{16} + \frac{e^{-2x}}{8} - \frac{3\,e^{-4x}}{32} + \frac{e^{-6x}}{24} - \frac{e^{-8x}}{128}$$

```
[In] int(1/(coth(x) + 1)^4,x)
[Out] x/16 + exp(-2*x)/8 - (3*exp(-4*x))/32 + exp(-6*x)/24 - exp(-8*x)/128
```

3.69 $\int \frac{1}{(1+\coth(x))^5}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 56

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{x}{32} - \frac{1}{10(1+\coth(x))^5} - \frac{1}{16(1+\coth(x))^4} - \frac{1}{24(1+\coth(x))^3} - \frac{1}{32(1+\coth(x))^2} - \frac{1}{32(1+\coth(x))}$$

```
[Out] 1/32*x-1/10/(1+coth(x))^5-1/16/(1+coth(x))^4-1/24/(1+coth(x))^3-1/32/(1+cot
h(x))^2-1/32/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3560, 8}

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{x}{32} - \frac{1}{32(\coth(x)+1)} - \frac{1}{32(\coth(x)+1)^2} - \frac{1}{24(\coth(x)+1)^3} - \frac{1}{16(\coth(x)+1)^4} - \frac{1}{10(\coth(x)+1)^5}$$

```
[In] Int[(1 + Coth[x])^(-5),x]
```

```
[Out] x/32 - 1/(10*(1 + Coth[x])^5) - 1/(16*(1 + Coth[x])^4) - 1/(24*(1 + Coth[x]
)^3) - 1/(32*(1 + Coth[x])^2) - 1/(32*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{10(1+\coth(x))^5}+\frac{1}{2}\int \frac{1}{(1+\coth(x))^4}\,dx \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}+\frac{1}{4}\int \frac{1}{(1+\coth(x))^3}\,dx \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3}+\frac{1}{8}\int \frac{1}{(1+\coth(x))^2}\,dx \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3} \\
&\quad -\frac{1}{32(1+\coth(x))^2}+\frac{1}{16}\int \frac{1}{1+\coth(x)}\,dx \\
&= -\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4}-\frac{1}{24(1+\coth(x))^3} \\
&\quad -\frac{1}{32(1+\coth(x))^2}-\frac{1}{32(1+\coth(x))}+\frac{\int 1\,dx}{32} \\
&= \frac{x}{32}-\frac{1}{10(1+\coth(x))^5}-\frac{1}{16(1+\coth(x))^4} \\
&\quad -\frac{1}{24(1+\coth(x))^3}-\frac{1}{32(1+\coth(x))^2}-\frac{1}{32(1+\coth(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$
\begin{aligned}
&\int \frac{1}{(1+\coth(x))^5}\,dx \\
&= \frac{1}{480}\Bigg(15\text{arctanh}(\tanh(x)) \\
&\qquad + \frac{128+625\tanh(x)+1205\tanh^2(x)+1125\tanh^3(x)+465\tanh^4(x)}{(1+\tanh(x))^5}\Bigg)
\end{aligned}
$$

```
[In] Integrate[(1 + Coth[x])^(-5),x]
```

```
[Out] (15*ArcTanh[Tanh[x]] + (128 + 625*Tanh[x] + 1205*Tanh[x]^2 + 1125*Tanh[x]^3
 + 465*Tanh[x]^4)/(1 + Tanh[x])^5)/480
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

method	result
risch	$\frac{x}{32}+\frac{5\,{\mathrm e}^{-2 x}}{64}-\frac{5\,{\mathrm e}^{-4 x}}{64}+\frac{5\,{\mathrm e}^{-6 x}}{96}-\frac{5\,{\mathrm e}^{-8 x}}{256}+\frac{{\mathrm e}^{-10 x}}{320}$
derivativedivides	$-\frac{1}{10(1+\coth(x))^{5}}-\frac{1}{16(1+\coth(x))^{4}}-\frac{1}{24(1+\coth(x))^{3}}-\frac{1}{32(1+\coth(x))^{2}}-\frac{1}{32(1+\coth(x))}+\frac{\ln(1+\coth(x))}{64}$
default	$-\frac{1}{10(1+\coth(x))^{5}}-\frac{1}{16(1+\coth(x))^{4}}-\frac{1}{24(1+\coth(x))^{3}}-\frac{1}{32(1+\coth(x))^{2}}-\frac{1}{32(1+\coth(x))}+\frac{\ln(1+\coth(x))}{64}$
parallelrisch	$\frac{15\tanh(x)^{5}x+(75x+465)\tanh(x)^{4}+(150x+1125)\tanh(x)^{3}+(150x+1205)\tanh(x)^{2}+(75x+625)\tanh(x)+15x+128}{480(1+\tanh(x))^{5}}$

```
[In] int(1/(1+coth(x))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*x+5/64*exp(-2*x)-5/64*exp(-4*x)+5/96*exp(-6*x)-5/256*exp(-8*x)+1/320*e
xp(-10*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.84

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{12\,(10\,x+1)\cosh(x)^5+60\,(10\,x+1)\cosh(x)\sinh(x)^4+12\,(10\,x-1)\sinh(x)^5+15\,\big(8\,(10\,x-1)\cosh(\ldots}{3840\,\big(\cosh(x)^5+5\,\cosh(x)^4\sin\ldots}$$

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="fricas")
```

```
[Out] 1/3840*(12*(10*x + 1)*cosh(x)^5 + 60*(10*x + 1)*cosh(x)*sinh(x)^4 + 12*(10*
x - 1)*sinh(x)^5 + 15*(8*(10*x - 1)*cosh(x)^2 + 25)*sinh(x)^3 + 225*cosh(x)
^3 + 15*(8*(10*x + 1)*cosh(x)^3 + 45*cosh(x))*sinh(x)^2 + 5*(12*(10*x - 1)*
cosh(x)^4 + 225*cosh(x)^2 - 100)*sinh(x) - 100*cosh(x))/(cosh(x)^5 + 5*cosh
(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)
*sinh(x)^4 + sinh(x)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(51) = 102.

Time = 0.93 (sec) , antiderivative size = 444, normalized size of antiderivative = 7.93

$$\int \frac{1}{(1+\coth(x))^5}\,dx$$
$$= \frac{15x\tanh^5(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{75x\tanh^4(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{150x\tanh^3(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{150x\tanh^2(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{75x\tanh(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{15x}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{465\tanh^4(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{1125\tanh^3(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{1205\tanh^2(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{625\tanh(x)}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$
$$+\frac{128}{480\tanh^5(x)+2400\tanh^4(x)+4800\tanh^3(x)+4800\tanh^2(x)+2400\tanh(x)+480}$$

```
[In] integrate(1/(1+coth(x))**5,x)
```

```
[Out] 15*x*tanh(x)**5/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*
tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tanh(x)**4/(480*tanh(x)**5 + 2400*t
anh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 150*x
*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh
(x)**2 + 2400*tanh(x) + 480) + 150*x*tanh(x)**2/(480*tanh(x)**5 + 2400*tanh
(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 75*x*tan
h(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2
+ 2400*tanh(x) + 480) + 15*x/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(
x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 465*tanh(x)**4/(480*tanh(x)
**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) +
```

```
480) + 1125*tanh(x)**3/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3
+ 4800*tanh(x)**2 + 2400*tanh(x) + 480) + 1205*tanh(x)**2/(480*tanh(x)**5 +
 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
+ 625*tanh(x)/(480*tanh(x)**5 + 2400*tanh(x)**4 + 4800*tanh(x)**3 + 4800*ta
nh(x)**2 + 2400*tanh(x) + 480) + 128/(480*tanh(x)**5 + 2400*tanh(x)**4 + 48
00*tanh(x)**3 + 4800*tanh(x)**2 + 2400*tanh(x) + 480)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{1}{32}\,x + \frac{5}{64}\,e^{(-2\,x)} - \frac{5}{64}\,e^{(-4\,x)} + \frac{5}{96}\,e^{(-6\,x)} - \frac{5}{256}\,e^{(-8\,x)} + \frac{1}{320}\,e^{(-10\,x)}$$

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="maxima")
```

```
[Out] 1/32*x + 5/64*e^(-2*x) - 5/64*e^(-4*x) + 5/96*e^(-6*x) - 5/256*e^(-8*x) + 1
/320*e^(-10*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{1}{3840}\left(300\,e^{(8\,x)} - 300\,e^{(6\,x)} + 200\,e^{(4\,x)} - 75\,e^{(2\,x)} + 12\right)e^{(-10\,x)} + \frac{1}{32}\,x$$

```
[In] integrate(1/(1+coth(x))^5,x, algorithm="giac")
```

```
[Out] 1/3840*(300*e^(8*x) - 300*e^(6*x) + 200*e^(4*x) - 75*e^(2*x) + 12)*e^(-10*x
) + 1/32*x
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1+\coth(x))^5}\,dx = \frac{x}{32} + \frac{5\,\mathrm{e}^{-2\,x}}{64} - \frac{5\,\mathrm{e}^{-4\,x}}{64} + \frac{5\,\mathrm{e}^{-6\,x}}{96} - \frac{5\,\mathrm{e}^{-8\,x}}{256} + \frac{\mathrm{e}^{-10\,x}}{320}$$

```
[In] int(1/(coth(x) + 1)^5,x)
```

```
[Out] x/32 + (5*exp(-2*x))/64 - (5*exp(-4*x))/64 + (5*exp(-6*x))/96 - (5*exp(-8*x
))/256 + exp(-10*x)/320
```

3.70 $\int (1+\coth(x))^{7/2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 57

$$\int (1+\coth(x))^{7/2} \, dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2}$$

```
[Out] -4/3*(1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)+8*arctanh(1/2*(1+coth(x))^(1/2
)*2^(1/2))*2^(1/2)-8*(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3559, 3561, 212}

$$\int (1+\coth(x))^{7/2} \, dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - \frac{2}{5}(\coth(x)+1)^{5/2} - \frac{4}{3}(\coth(x)+1)^{3/2} - 8\sqrt{\coth(x)+1}$$

```
[In] Int[(1 + Coth[x])^(7/2),x]
```

```
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 8*Sqrt[1 + Coth[x]] - (4*(1
+ Coth[x])^(3/2))/3 - (2*(1 + Coth[x])^(5/2))/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{5}(1+\coth(x))^{5/2} + 2\int (1+\coth(x))^{5/2}\,dx \\
&= -\frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2} + 4\int (1+\coth(x))^{3/2}\,dx \\
&= -8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2} + 8\int \sqrt{1+\coth(x)}\,dx \\
&= -8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} \\
&\quad - \frac{2}{5}(1+\coth(x))^{5/2} + 16\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 8\sqrt{1+\coth(x)} - \frac{4}{3}(1+\coth(x))^{3/2} - \frac{2}{5}(1+\coth(x))^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int (1+\coth(x))^{7/2}\,dx = 8\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - \frac{2}{15}\sqrt{1+\coth(x)}\left(73+16\coth(x)+3\coth^2(x)\right)$$

```
[In] Integrate[(1 + Coth[x])^(7/2),x]
[Out] 8*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*Sqrt[1 + Coth[x]]*(73 + 1
6*Coth[x] + 3*Coth[x]^2))/15
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{4(1+\coth(x))^{\frac{3}{2}}}{3}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+8\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-8\sqrt{1+\coth\left(x\right)}$	43
default	$-\frac{4(1+\coth(x))^{\frac{3}{2}}}{3}-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+8\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-8\sqrt{1+\coth\left(x\right)}$	43

```
[In] int((1+coth(x))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3*(1+coth(x))^(3/2)-2/5*(1+coth(x))^(5/2)+8*arctanh(1/2*(1+coth(x))^(1/2
)*2^(1/2))*2^(1/2)-8*(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(42) = 84$.

Time = 0.26 (sec) , antiderivative size = 438, normalized size of antiderivative = 7.68

$$\int (1+\coth(x))^{7/2}\,dx =$$

$$-\frac{4\left(2\sqrt{2}\left(23\sqrt{2}\cosh\left(x\right)^{5}+115\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{4}+23\sqrt{2}\sinh\left(x\right)^{5}+5\left(46\sqrt{2}\cosh\left(x\right)^{2}-7\sqrt{2}\right)\sinh\left(x\right.\right.\right.}{}$$

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="fricas")
```

```
[Out] -4/15*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^5 + 115*sqrt(2)*cosh(x)*sinh(x)^4 + 23
*sqrt(2)*sinh(x)^5 + 5*(46*sqrt(2)*cosh(x)^2 - 7*sqrt(2))*sinh(x)^3 - 35*sq
rt(2)*cosh(x)^3 + 5*(46*sqrt(2)*cosh(x)^3 - 21*sqrt(2)*cosh(x))*sinh(x)^2 +
 5*(23*sqrt(2)*cosh(x)^4 - 21*sqrt(2)*cosh(x)^2 + 3*sqrt(2))*sinh(x) + 15*s
qrt(2)*cosh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^6 +
 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 -
 sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt
(2)*cosh(x))*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqr
t(2))*sinh(x)^2 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*co
sh(x)^3 + sqrt(2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(c
osh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) +
2*sinh(x)^2 - 1))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(
x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 +
 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 -
 2*cosh(x)^3 + cosh(x))*sinh(x) - 1)
```

Sympy [F(-1)]

Timed out.

$$\int (1+\coth(x))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((1+coth(x))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (1+\coth(x))^{7/2}\,dx = \int (\coth(x)+1)^{\frac{7}{2}}\,dx$$

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="maxima")
[Out] integrate((coth(x) + 1)^(7/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(42) = 84.

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.81

$$\int (1+\coth(x))^{7/2}\,dx =$$

$$-\frac{4}{15}\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4+135\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3+170\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\right.}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^5}\right.$$

$$\left.-1\right)$$

```
[In] integrate((1+coth(x))^(7/2),x, algorithm="giac")
[Out] -4/15*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 135*(sqrt(e^(4
*x) - e^(2*x)) - e^(2*x))^3 + 170*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 1
00*sqrt(e^(4*x) - e^(2*x)) - 100*e^(2*x) + 23)/(sqrt(e^(4*x) - e^(2*x)) - e
^(2*x) + 1)^5 + 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn
(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int (1+\coth(x))^{7/2}\,dx = -8\,\sqrt{\coth\left(x\right)+1}-\frac{4\left(\coth(x)+1\right)^{3/2}}{3}-\frac{2\left(\coth(x)+1\right)^{5/2}}{5}-\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{\coth\left(x\right)+1}\,1\mathrm{i}}{2}\right)\,8\mathrm{i}$$

```
[In] int((coth(x) + 1)^(7/2),x)
```

```
[Out] - 2^(1/2)*atan((2^(1/2)*(coth(x) + 1)^(1/2)*1i)/2)*8i - 8*(coth(x) + 1)^(1/
2) - (4*(coth(x) + 1)^(3/2))/3 - (2*(coth(x) + 1)^(5/2))/5
```

3.71 $\int (1+\coth(x))^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 45

$$\int (1+\coth(x))^{5/2}\,dx = 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 4\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2}$$

```
[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(
1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3559, 3561, 212}

$$\int (1+\coth(x))^{5/2}\,dx = 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - \frac{2}{3}(\coth(x)+1)^{3/2} - 4\sqrt{\coth(x)+1}$$

```
[In] Int[(1 + Coth[x])^(5/2),x]
```

```
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 4*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2}{3}(1+\coth(x))^{3/2}+2\int(1+\coth(x))^{3/2}\,dx\\ &= -4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+4\int\sqrt{1+\coth(x)}\,dx\\ &= -4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+8\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= 4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-4\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int(1+\coth(x))^{5/2}\,dx=4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}\sqrt{1+\coth(x)}(7+\coth(x))$$

```
[In] Integrate[(1 + Coth[x])^(5/2),x]
[Out] 4*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*Sqrt[1 + Coth[x]]*(7 + Co
th[x]))/3
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+4\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-4\sqrt{1+\coth\left(x\right)}$	35
default	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+4\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-4\sqrt{1+\coth\left(x\right)}$	35

```
[In] int((1+coth(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-4*(
1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 5.76

$$\int(1+\coth(x))^{5/2}\,dx = -\frac{2\left(2\sqrt{2}(4\sqrt{2}\cosh\left(x\right)^{3}+12\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{2}+4\sqrt{2}\sinh\left(x\right)^{3}+3\left(4\sqrt{2}\cosh\left(x\right)^{2}-\sqrt{2}\right)\sinh\left(x\right)-3\right.}{}$$

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^3 + 12*sqrt(2)*cosh(x)*sinh(x)^2 + 4*sqr
t(2)*sinh(x)^3 + 3*(4*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int (1+\coth(x))^{5/2}\,dx = \int (\coth(x)+1)^{\frac{5}{2}}\,dx$$

```
[In] integrate((1+coth(x))**(5/2),x)
[Out] Integral((coth(x) + 1)**(5/2), x)
```

Maxima [F]

$$\int (1+\coth(x))^{5/2}\,dx = \int (\coth(x)+1)^{\frac{5}{2}}\,dx$$

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="maxima")
[Out] integrate((coth(x) + 1)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.49

$$\int (1+\coth(x))^{5/2}\,dx = -\frac{2}{3}\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+9\sqrt{e^{(4x)}-e^{(2x)}}-9e^{(2x)}+4\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x}\right.\right.\right.$$

$$-1)$$

```
[In] integrate((1+coth(x))^(5/2),x, algorithm="giac")
[Out] -2/3*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 9*sqrt(e^(4*x) -
 e^(2*x)) - 9*e^(2*x) + 4)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3 + 3*lo
g(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int (1+\coth(x))^{5/2}\,dx = \sqrt{8}\,\ln\left(-2\sqrt{8}\,\sqrt{\coth\left(x\right)+1}-8\right) - \frac{2\left(\coth(x)+1\right)^{3/2}}{3} - 2\sqrt{2}\,\ln\left(4\sqrt{2}\,\sqrt{\coth\left(x\right)+1}-8\right) - 4\sqrt{\coth\left(x\right)+1}$$

```
[In] int((coth(x) + 1)^(5/2),x)
```

```
[Out] 8^(1/2)*log(- 2*8^(1/2)*(coth(x) + 1)^(1/2) - 8) - (2*(coth(x) + 1)^(3/2))/
3 - 2*2^(1/2)*log(4*2^(1/2)*(coth(x) + 1)^(1/2) - 8) - 4*(coth(x) + 1)^(1/2
)
```

3.72 $\int (1+\coth(x))^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int (1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}$$

```
[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3559, 3561, 212}

$$\int (1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\coth(x)+1}$$

```
[In] Int[(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
```

```
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -2\sqrt{1+\coth(x)} + 2\int \sqrt{1+\coth(x)}\,dx \\ &= -2\sqrt{1+\coth(x)} + 4\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\ &= 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int (1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}$$

```
[In] Integrate[(1 + Coth[x])^(3/2),x]
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	27
default	$2\,\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	27

```
[In] int((1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.97

$$\int (1+\coth(x))^{3/2}\,dx =$$
$$-\frac{2\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2-\right.}{\cosh(x)^2+2\cosh(x)\mathrm{si}}$$

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sin
h(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2
 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int (1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\,dx$$

```
[In] integrate((1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int (1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\,dx$$

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int (1+\coth(x))^{3/2}\,dx = -\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}+\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)\mathrm{sgn}\left(e^{(2x)}-1\right)$$

```
[In] integrate((1+coth(x))^(3/2),x, algorithm="giac")
[Out] -sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1) + log(abs(2*sqrt(e^(4*x
) - e^(2*x)) - 2*e^(2*x) + 1)))*sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int (1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right)-2\sqrt{\coth(x)+1}$$

```
[In] int((coth(x) + 1)^(3/2),x)
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

3.73 $\int \sqrt{1+\coth(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 21

$$\int \sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)$$

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3561, 212}

$$\int \sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)$$

```
[In] Int[Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
```

```
b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right)$$
$$= \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)$$

```
[In] Integrate[Sqrt[1 + Coth[x]],x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	17
default	$\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	17

```
[In] int((1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \sqrt{1+\coth(x)}\,dx = \frac{1}{2}\sqrt{2}\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x)) + 2\cosh(x)^2 + 4\cosh(x)\sinh(x) + 2\sinh(x)^2 - 1\right)$$

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1)
```

Sympy [F]

$$\int \sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\,dx$$

```
[In] integrate((1+coth(x))**(1/2),x)
[Out] Integral(sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\,dx$$

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \sqrt{1+\coth(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)$$

```
[In] integrate((1+coth(x))^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x
) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \sqrt{1+\coth(x)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right)$$

```
[In] int((coth(x) + 1)^(1/2),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2)
```

3.74 $\int \frac{1}{\sqrt{1+\coth(x)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 32

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3560, 3561, 212}

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{\coth(x)+1}}$$

```
[In] Int[1/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
```

```
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{\sqrt{1+\coth(x)}}+\frac{1}{2}\int\sqrt{1+\coth(x)}\,dx\\ &= -\frac{1}{\sqrt{1+\coth(x)}}+\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int\frac{1}{\sqrt{1+\coth(x)}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\coth(x))\right)}{\sqrt{1+\coth(x)}}$$

```
[In] Integrate[1/Sqrt[1 + Coth[x]],x]
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (1 + Coth[x])/2]/Sqrt[1 + Coth[x]])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth(x)}}$	27
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth(x)}}$	27

```
[In] int(1/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(26) = 52$.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.66

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx$$
$$= \frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(\cdots\right.\right.}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) - 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \int \frac{1}{\sqrt{\coth\left(x\right)+1}}\,dx$$

```
[In] integrate(1/(1+coth(x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \int \frac{1}{\sqrt{\coth\left(x\right)+1}}\,dx$$

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(26) = 52$.

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}} - \log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

```
[In] integrate(1/(1+coth(x))^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) - log(abs(2*sqrt(e^(4*x)
 - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2} - \frac{1}{\sqrt{\coth(x)+1}}$$

```
[In] int(1/(coth(x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 1/(coth(x) + 1)^(1/2)
```

3.75 $\int \frac{1}{(1+\coth(x))^{3/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 49

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}}$$

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1
/2/(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3560, 3561, 212}

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{2\sqrt{\coth(x)+1}} - \frac{1}{3(\coth(x)+1)^{3/2}}$$

```
[In] Int[(1 + Coth[x])^(-3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{3(1+\coth(x))^{3/2}} + \frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx \\
&= -\frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}} + \frac{1}{4}\int \sqrt{1+\coth(x)}\,dx \\
&= -\frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$
\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{1}{2}(1+\coth(x))\right)}{3(1+\coth(x))^{3/2}}
$$

```
[In] Integrate[(1 + Coth[x])^(-3/2),x]
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (1 + Coth[x])/2]/(1 + Coth[x])^(3/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35
default	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35

```
[In] int(1/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1
/2/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.43

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx =$$
$$-\frac{2\sqrt{2}\left(4\sqrt{2}\cosh(x)^2+8\sqrt{2}\cosh(x)\sinh(x)+4\sqrt{2}\sinh(x)^2-\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\right.}{24\,(\cos}$$

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(4*sqrt(2)*cosh(x)^2 + 8*sqrt(2)*cosh(x)*sinh(x) + 4*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \int \frac{1}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+coth(x))**(3/2),x)
```

```
[Out] Integral((coth(x) + 1)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \int \frac{1}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((coth(x) + 1)^(-3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.31

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2+3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3}-3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2\right.\right.}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

```
[In] integrate(1/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 3*sqrt(e^(4*x) -
 e^(2*x)) - 3*e^(2*x) + 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3 - 3*log(ab
s(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{1}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4} - \frac{\frac{\coth(x)}{2}+\frac{5}{6}}{(\coth(x)+1)^{3/2}}$$

```
[In] int(1/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 5/6)/(cot
h(x) + 1)^(3/2)
```

3.76 $\int \frac{1}{(1+\coth(x))^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 61

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{5(1+\coth(x))^{5/2}} - \frac{1}{6(1+\coth(x))^{3/2}} - \frac{1}{4\sqrt{1+\coth(x)}}$$

```
[Out] -1/5/(1+coth(x))^(5/2)-1/6/(1+coth(x))^(3/2)+1/8*arctanh(1/2*(1+coth(x))^(1
/2)*2^(1/2))*2^(1/2)-1/4/(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3560, 3561, 212}

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{4\sqrt{\coth(x)+1}} - \frac{1}{6(\coth(x)+1)^{3/2}} - \frac{1}{5(\coth(x)+1)^{5/2}}$$

```
[In] Int[(1 + Coth[x])^(-5/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(4*Sqrt[2]) - 1/(5*(1 + Coth[x])^(5/2))
- 1/(6*(1 + Coth[x])^(3/2)) - 1/(4*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{5(1+\coth(x))^{5/2}} + \frac{1}{2}\int \frac{1}{(1+\coth(x))^{3/2}}\,dx \\
&= -\frac{1}{5(1+\coth(x))^{5/2}} - \frac{1}{6(1+\coth(x))^{3/2}} + \frac{1}{4}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx \\
&= -\frac{1}{5(1+\coth(x))^{5/2}} - \frac{1}{6(1+\coth(x))^{3/2}} - \frac{1}{4\sqrt{1+\coth(x)}} + \frac{1}{8}\int \sqrt{1+\coth(x)}\,dx \\
&= -\frac{1}{5(1+\coth(x))^{5/2}} - \frac{1}{6(1+\coth(x))^{3/2}} - \frac{1}{4\sqrt{1+\coth(x)}} \\
&\quad + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= \frac{\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{5(1+\coth(x))^{5/2}} - \frac{1}{6(1+\coth(x))^{3/2}} - \frac{1}{4\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.46

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2}, 1, -\frac{3}{2}, \frac{1}{2}(1+\coth(x))\right)}{5(1+\coth(x))^{5/2}}$$

```
[In] Integrate[(1 + Coth[x])^(-5/2),x]
[Out] -1/5*Hypergeometric2F1[-5/2, 1, -3/2, (1 + Coth[x])/2]/(1 + Coth[x])^(5/2)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{1}{5(1+\coth(x))^{\frac{5}{2}}}-\frac{1}{6(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{4\sqrt{1+\coth(x)}}$	43
default	$-\frac{1}{5(1+\coth(x))^{\frac{5}{2}}}-\frac{1}{6(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{8}-\frac{1}{4\sqrt{1+\coth(x)}}$	43

```
[In] int(1/(1+coth(x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/(1+coth(x))^(5/2)-1/6/(1+coth(x))^(3/2)+1/8*arctanh(1/2*(1+coth(x))^(1
/2)*2^(1/2))*2^(1/2)-1/4/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(42) = 84.

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 4.36

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx =$$

$$-\frac{2\sqrt{2}\big(23\sqrt{2}\cosh(x)^4+92\sqrt{2}\cosh(x)\sinh(x)^3+23\sqrt{2}\sinh(x)^4+\big(138\sqrt{2}\cosh(x)^2-11\sqrt{2}\big)\sinh(x)^2-$$

```
[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/240*(2*sqrt(2)*(23*sqrt(2)*cosh(x)^4 + 92*sqrt(2)*cosh(x)*sinh(x)^3 + 23
*sqrt(2)*sinh(x)^4 + (138*sqrt(2)*cosh(x)^2 - 11*sqrt(2))*sinh(x)^2 - 11*sq
rt(2)*cosh(x)^2 + 2*(46*sqrt(2)*cosh(x)^3 - 11*sqrt(2)*cosh(x))*sinh(x) + 3
*sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 15*(sqrt(2)*cosh(x)^5 + 5*sqr
t(2)*cosh(x)^4*sinh(x) + 10*sqrt(2)*cosh(x)^3*sinh(x)^2 + 10*sqrt(2)*cosh(x
)^2*sinh(x)^3 + 5*sqrt(2)*cosh(x)*sinh(x)^4 + sqrt(2)*sinh(x)^5)*log(2*sqrt
(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4
*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*
cosh(x)^3*sinh(x)^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x
)^5)
```

Sympy [F]

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \int \frac{1}{(\coth{(x)} + 1)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(1+coth(x))**(5/2),x)

[Out] Integral((coth(x) + 1)**(-5/2), x)

Maxima [F]

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \int \frac{1}{(\coth{(x)} + 1)^{\frac{5}{2}}}\,dx$$

[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="maxima")

[Out] integrate((coth(x) + 1)^(-5/2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.64

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{\sqrt{2}\left(\frac{2\left(45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{4}+45\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{3}+35\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{2}+15\sqrt{e^{(4x)}}\cdots\right.}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^{5}}\cdots\right.}{240\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

[In] integrate(1/(1+coth(x))^(5/2),x, algorithm="giac")

[Out] 1/240*sqrt(2)*(2*(45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4 + 45*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3 + 35*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 + 15*sqrt(e^(4*x) - e^(2*x)) - 15*e^(2*x) + 3)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^5 - 15*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{1}{(1+\coth(x))^{5/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{8} - \frac{\frac{\coth(x)}{6}+\frac{(\coth(x)+1)^2}{4}+\frac{11}{30}}{(\coth(x)+1)^{5/2}}$$

```
[In] int(1/(coth(x) + 1)^(5/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/8 - (coth(x)/6 + (coth(x)
+ 1)^2/4 + 11/30)/(coth(x) + 1)^(5/2)
```

3.77 $\int (a+b\coth(c+dx))^5\,dx$

Optimal result

Integrand size = 12, antiderivative size = 142

$$\int (a+b\coth(c+dx))^5\,dx = a\left(a^4+10a^2b^2+5b^4\right)x - \frac{4ab^2(a^2+b^2)\coth(c+dx)}{d} - \frac{b(3a^2+b^2)\,(a+b\coth(c+dx))^2}{2d} - \frac{2ab(a+b\coth(c+dx))^3}{3d} - \frac{b(a+b\coth(c+dx))^4}{4d} + \frac{b(5a^4+10a^2b^2+b^4)\log(\sinh(c+dx))}{d}$$

```
[Out] a*(a^4+10*a^2*b^2+5*b^4)*x-4*a*b^2*(a^2+b^2)*coth(d*x+c)/d-1/2*b*(3*a^2+b^2
)*(a+b*coth(d*x+c))^2/d-2/3*a*b*(a+b*coth(d*x+c))^3/d-1/4*b*(a+b*coth(d*x+c
))^4/d+b*(5*a^4+10*a^2*b^2+b^4)*ln(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3563, 3609, 3606, 3556}

$$\int (a+b\coth(c+dx))^5\,dx = -\frac{b(3a^2+b^2)\,(a+b\coth(c+dx))^2}{2d} - \frac{4ab^2(a^2+b^2)\coth(c+dx)}{d} + \frac{b(5a^4+10a^2b^2+b^4)\log(\sinh(c+dx))}{d} + ax\left(a^4+10a^2b^2+5b^4\right) - \frac{b(a+b\coth(c+dx))^4}{4d} - \frac{2ab(a+b\coth(c+dx))^3}{3d}$$

```
[In] Int[(a + b*Coth[c + d*x])^5,x]
```

```
[Out] a*(a^4 + 10*a^2*b^2 + 5*b^4)*x - (4*a*b^2*(a^2 + b^2)*Coth[c + d*x])/d - (b
*(3*a^2 + b^2)*(a + b*Coth[c + d*x])^2)/(2*d) - (2*a*b*(a + b*Coth[c + d*x]
)^3)/(3*d) - (b*(a + b*Coth[c + d*x])^4)/(4*d) + (b*(5*a^4 + 10*a^2*b^2 + b
^4)*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b(a+b\coth(c+dx))^4}{4d}+\int(a+b\coth(c+dx))^3\left(a^2+b^2+2ab\coth(c+dx)\right)\,dx\\
&= -\frac{2ab(a+b\coth(c+dx))^3}{3d}-\frac{b(a+b\coth(c+dx))^4}{4d}\\
&\quad+\int(a+b\coth(c+dx))^2\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\coth(c+dx)\right)\,dx\\
&= -\frac{b(3a^2+b^2)\,(a+b\coth(c+dx))^2}{2d}-\frac{2ab(a+b\coth(c+dx))^3}{3d}\\
&\quad-\frac{b(a+b\coth(c+dx))^4}{4d}+\int(a+b\coth(c+dx))\left(a^4+6a^2b^2+b^4\right.\\
&\qquad\left.+4ab\left(a^2+b^2\right)\coth(c+dx)\right)\,dx
\end{aligned}$$

$$= a(a^4 + 10a^2b^2 + 5b^4)\,x - \frac{4ab^2(a^2 + b^2)\coth(c + dx)}{d} - \frac{b(3a^2 + b^2)\,(a + b\coth(c + dx))^2}{2d} - \frac{2ab(a + b\coth(c + dx))^3}{3d} - \frac{b(a + b\coth(c + dx))^4}{4d} + \left(b(5a^4 + 10a^2b^2 + b^4)\right)\int \coth(c + dx)\,dx$$

$$= a(a^4 + 10a^2b^2 + 5b^4)\,x - \frac{4ab^2(a^2 + b^2)\coth(c + dx)}{d} - \frac{b(3a^2 + b^2)\,(a + b\coth(c + dx))^2}{2d} - \frac{2ab(a + b\coth(c + dx))^3}{3d} - \frac{b(a + b\coth(c + dx))^4}{4d} + \frac{b(5a^4 + 10a^2b^2 + b^4)\log(\sinh(c + dx))}{d}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int (a + b\coth(c + dx))^5\,dx = -\frac{60ab^2(2a^2 + b^2)\coth(c + dx) + 6b^3(10a^2 + b^2)\coth^2(c + dx) + 20ab^4\coth^3(c + dx) + 3b^5\coth^4(c + dx\cdots}{\cdots}$$

```
[In] Integrate[(a + b*Coth[c + d*x])^5,x]
[Out] -1/12*(60*a*b^2*(2*a^2 + b^2)*Coth[c + d*x] + 6*b^3*(10*a^2 + b^2)*Coth[c +
 d*x]^2 + 20*a*b^4*Coth[c + d*x]^3 + 3*b^5*Coth[c + d*x]^4 + 6*(a + b)^5*Lo
g[1 - Tanh[c + d*x]] - 12*b*(5*a^4 + 10*a^2*b^2 + b^4)*Log[Tanh[c + d*x]] -
 6*(a - b)^5*Log[1 + Tanh[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

method	result
parallelrisch	$\frac{\left(-60a^4b-120a^2b^3-12b^5\right)\ln(1-\tanh(dx+c))+\left(60a^4b+120a^2b^3+12b^5\right)\ln(\tanh(dx+c))-3b^5\coth(dx+c)^4-20a\,b^4\coth(\cdots}{12d}$
derivativedivides	$\frac{-10a^3b^2\coth(dx+c)-5a\,b^4\coth(dx+c)-\frac{5a\,b^4\coth(dx+c)^3}{3}-5a^2b^3\coth(dx+c)^2-\frac{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5a\,b^4+b^5\right)\ln\cdots}{2}\cdots}{d}$
default	$\frac{-10a^3b^2\coth(dx+c)-5a\,b^4\coth(dx+c)-\frac{5a\,b^4\coth(dx+c)^3}{3}-5a^2b^3\coth(dx+c)^2-\frac{\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5a\,b^4+b^5\right)\ln\cdots}{2}\cdots}{d}$
parts	$a^5x+\frac{b^5\left(-\frac{\coth(dx+c)^4}{4}-\frac{\coth(dx+c)^2}{2}-\frac{\ln(\coth(dx+c)-1)}{2}-\frac{\ln(\coth(dx+c)+1)}{2}\right)}{d}+\frac{5a\,b^4\left(-\frac{\coth(dx+c)^3}{3}-\coth(dx+c)-\cdots\right.}{\cdots}$
risch	$a^5x-5b\,a^4x+10a^3b^2x-10b^3a^2x+5a\,b^4x-b^5x-\frac{10b\,a^4c}{d}-\frac{20b^3a^2c}{d}-\frac{2b^5c}{d}-\frac{4b^2\left(15a^3e^{6dx}\cdots\right.}{\cdots}$

```
[In] int((a+b*coth(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((-60*a^4*b-120*a^2*b^3-12*b^5)*ln(1-tanh(d*x+c))+(60*a^4*b+120*a^2*b^
3+12*b^5)*ln(tanh(d*x+c))-3*b^5*coth(d*x+c)^4-20*a*b^4*coth(d*x+c)^3+(-60*a
^2*b^3-6*b^5)*coth(d*x+c)^2+(-120*a^3*b^2-60*a*b^4)*coth(d*x+c)+12*d*x*(a-b
)^5)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2748 vs. $2(136) = 272$.

Time = 0.28 (sec) , antiderivative size = 2748, normalized size of antiderivative = 19.35

$$\int (a + b\coth(c + dx))^5 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d
*x + c)^8 + 24*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*
x*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^
3 + 5*a*b^4 - b^5)*d*x*sinh(d*x + c)^8 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^
4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*co
sh(d*x + c)^6 - 12*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 - 7*(a^5 - 5*a^4*
b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^2 + (a^5 - 5
*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*sinh(d*x + c)^6 + 24
*(7*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x
+ c)^3 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3
*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 60
*a^3*b^2 + 40*a*b^4 + 6*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^4
 + 6*(35*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh
(d*x + c)^4 + 30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b
 + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x - 30*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*(7*(a^5 - 5*a^4*b + 10*a^3*b^2
 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^5 - 10*(5*a^3*b^2 + 5*a^2*
b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 -
b^5)*d*x)*cosh(d*x + c)^3 + (30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3
*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x +
c))*sinh(d*x + c)^3 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
- b^5)*d*x - 4*(45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5 + 3*(a^5 - 5*a^4
*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2 + 4*(21*
(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)
^6 - 45*a^3*b^2 - 15*a^2*b^3 - 25*a*b^4 - 3*b^5 - 45*(5*a^3*b^2 + 5*a^2*b^3
 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
```

```
)*d*x)*cosh(d*x + c)^4 - 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b
^4 - b^5)*d*x + 9*(30*a^3*b^2 + 20*a^2*b^3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*
a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 3*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^8 + 8*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a^4*b + 10*a^2*b^3 +
 b^5)*sinh(d*x + c)^8 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 4*
(5*a^4*b + 10*a^2*b^3 + b^5 - 7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^3 - 3*
(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*a^4*b + 10*
a^2*b^3 + b^5 + 6*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 2*(15*a^4*
b + 30*a^2*b^3 + 3*b^5 + 35*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 -
30*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*
a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 - 10*(5*a^4*b + 10*a^2*b^3 + b^5)
*cosh(d*x + c)^3 + 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 4*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 4*(7*(5*a^4*b + 10
*a^2*b^3 + b^5)*cosh(d*x + c)^6 - 5*a^4*b - 10*a^2*b^3 - b^5 - 15*(5*a^4*b
+ 10*a^2*b^3 + b^5)*cosh(d*x + c)^4 + 9*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^
7 - 3*(5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c)^5 + 3*(5*a^4*b + 10*a^2*b^
3 + b^5)*cosh(d*x + c)^3 - (5*a^4*b + 10*a^2*b^3 + b^5)*cosh(d*x + c))*sinh
(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(3*(a^5
 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x*cosh(d*x + c)^7 -
 9*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5 + (a^5 - 5*a^4*b + 10*a^3*b^2 - 1
0*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x + c)^5 + 3*(30*a^3*b^2 + 20*a^2*b^
3 + 20*a*b^4 + 2*b^5 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4
 - b^5)*d*x)*cosh(d*x + c)^3 - (45*a^3*b^2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5
+ 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*d*x)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^
7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*s
inh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*
cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)
^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d
*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x +
 c))*sinh(d*x + c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(133) = 266$.

Time = 1.79 (sec) , antiderivative size = 588, normalized size of antiderivative = 4.14

$$\int (a + b\coth(c + dx))^5 \, dx$$

$$= \begin{cases} x(a + b\coth(c))^5 \\ -\frac{a^5\log(-e^{-dx})}{d} - \frac{5a^4b\log(-e^{-dx})\coth(dx+\log(-e^{-dx}))}{d} - \frac{10a^3b^2\log(-e^{-dx})\coth^2(dx+\log(-e^{-dx}))}{d} - \frac{10a^2b^3\log(-e^{-dx})\coth^3}{d} \\ a^5x + 5a^4bx\coth(dx + \log(e^{-dx})) + 10a^3b^2x\coth^2(dx + \log(e^{-dx})) + 10a^2b^3x\coth^3(dx + \log(e^{-dx})) \\ a^5x + 5a^4bx - \frac{5a^4b\log(\tanh(c+dx)+1)}{d} + \frac{5a^4b\log(\tanh(c+dx))}{d} + 10a^3b^2x - \frac{10a^3b^2}{d\tanh(c+dx)} + 10a^2b^3x - \frac{10a^2b^3\log(\tanh}{d} \end{cases}$$

```
[In] integrate((a+b*coth(d*x+c))**5,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**5, Eq(d, 0)), (-a**5*log(-exp(-d*x))/d - 5*a*
*4*b*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))/d - 10*a**3*b**2*log(-exp(
-d*x))*coth(d*x + log(-exp(-d*x)))**2/d - 10*a**2*b**3*log(-exp(-d*x))*coth
(d*x + log(-exp(-d*x)))**3/d - 5*a*b**4*log(-exp(-d*x))*coth(d*x + log(-exp
(-d*x)))**4/d - b**5*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))**5/d, Eq(c
, log(-exp(-d*x)))), (a**5*x + 5*a**4*b*x*coth(d*x + log(exp(-d*x))) + 10*a
**3*b**2*x*coth(d*x + log(exp(-d*x)))**2 + 10*a**2*b**3*x*coth(d*x + log(ex
p(-d*x)))**3 + 5*a*b**4*x*coth(d*x + log(exp(-d*x)))**4 + b**5*x*coth(d*x +
 log(exp(-d*x)))**5, Eq(c, log(exp(-d*x)))), (a**5*x + 5*a**4*b*x - 5*a**4*
b*log(tanh(c + d*x) + 1)/d + 5*a**4*b*log(tanh(c + d*x))/d + 10*a**3*b**2*x
 - 10*a**3*b**2/(d*tanh(c + d*x)) + 10*a**2*b**3*x - 10*a**2*b**3*log(tanh(
c + d*x) + 1)/d + 10*a**2*b**3*log(tanh(c + d*x))/d - 5*a**2*b**3/(d*tanh(c
 + d*x)**2) + 5*a*b**4*x - 5*a*b**4/(d*tanh(c + d*x)) - 5*a*b**4/(3*d*tanh(
c + d*x)**3) + b**5*x - b**5*log(tanh(c + d*x) + 1)/d + b**5*log(tanh(c + d
*x))/d - b**5/(2*d*tanh(c + d*x)**2) - b**5/(4*d*tanh(c + d*x)**4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(136) = 272$.

Time = 0.20 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.45

$$\int (a + b\coth(c + dx))^5\,dx$$
$$= \frac{5}{3}ab^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2\right)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right)$$
$$+b^5\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} - e^{(-4dx-4c)} + e^{(-6dx-6c}}{d(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} - e^{(-}}\right.$$
$$+10a^2b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right)$$
$$+ 10a^3b^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right) + a^5x + \frac{5a^4b\log(\sinh(dx + c))}{d}$$

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 5/3*a*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d
*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^5*(
x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 10*a^2*b^3*
(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
- 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 10*a^3*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^5*x + 5*a^4*b*log(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.59

$$\int (a + b\coth(c + dx))^5\,dx$$
$$= \frac{3\left(a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5\right)(dx + c) + 3\left(5a^4b + 10a^2b^3 + b^5\right)\log\left(\left|e^{(2dx+2c)} - 1\right|\right) + \frac{4(}{}}{3}$$

```
[In] integrate((a+b*coth(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*(d*x + c)
+ 3*(5*a^4*b + 10*a^2*b^3 + b^5)*log(abs(e^(2*d*x + 2*c) - 1)) + 4*(15*a^3*
b^2 + 10*a*b^4 - 3*(5*a^3*b^2 + 5*a^2*b^3 + 5*a*b^4 + b^5)*e^(6*d*x + 6*c)
```

```
+ 3*(15*a^3*b^2 + 10*a^2*b^3 + 10*a*b^4 + b^5)*e^(4*d*x + 4*c) - (45*a^3*b^
2 + 15*a^2*b^3 + 25*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4
)/d
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.72

$$\int (a+b\coth(c+dx))^5\,dx = x\,(a-b)^5 - \frac{4\,(5\,a^3\,b^2+5\,a^2\,b^3+5\,a\,b^4+b^5)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} + \frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,\left(5\,a^4\,b+10\,a^2\,b^3+b^5\right)}{d} - \frac{4\,(5\,a^2\,b^3+5\,a\,b^4+2\,b^5)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{4\,b^5}{d\,\left(6\,\mathrm{e}^{4\,c+4\,d\,x}-4\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)} - \frac{8\,(3\,b^5+5\,a\,b^4)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$

```
[In] int((a + b*coth(c + d*x))^5,x)
```

```
[Out] x*(a - b)^5 - (4*(5*a*b^4 + b^5 + 5*a^2*b^3 + 5*a^3*b^2))/(d*(exp(2*c + 2*d
*x) - 1)) + (log(exp(2*c)*exp(2*d*x) - 1)*(5*a^4*b + b^5 + 10*a^2*b^3))/d -
 (4*(5*a*b^4 + 2*b^5 + 5*a^2*b^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x
) + 1)) - (4*b^5)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c +
 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*(5*a*b^4 + 3*b^5))/(3*d*(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

3.78 $\int (a+b\coth(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int (a+b\coth(c+dx))^4\,dx = \left(a^4+6a^2b^2+b^4\right)x-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}-\frac{ab(a+b\coth(c+dx))^2}{d}-\frac{b(a+b\coth(c+dx))^3}{3d}+\frac{4ab(a^2+b^2)\log(\sinh(c+dx))}{d}$$

```
[Out] (a^4+6*a^2*b^2+b^4)*x-b^2*(3*a^2+b^2)*coth(d*x+c)/d-a*b*(a+b*coth(d*x+c))^2
/d-1/3*b*(a+b*coth(d*x+c))^3/d+4*a*b*(a^2+b^2)*ln(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3563, 3609, 3606, 3556}

$$\int (a+b\coth(c+dx))^4\,dx = -\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}+\frac{4ab(a^2+b^2)\log(\sinh(c+dx))}{d}+x\left(a^4+6a^2b^2+b^4\right)-\frac{b(a+b\coth(c+dx))^3}{3d}-\frac{ab(a+b\coth(c+dx))^2}{d}$$

```
[In] Int[(a + b*Coth[c + d*x])^4,x]
```

```
[Out] (a^4 + 6*a^2*b^2 + b^4)*x - (b^2*(3*a^2 + b^2)*Coth[c + d*x])/d - (a*b*(a +
 b*Coth[c + d*x])^2)/d - (b*(a + b*Coth[c + d*x])^3)/(3*d) + (4*a*b*(a^2 +
b^2)*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{b(a+b\coth(c+dx))^3}{3d}+\int (a+b\coth(c+dx))^2\left(a^2+b^2+2ab\coth(c+dx)\right)\,dx\\
&= -\frac{ab(a+b\coth(c+dx))^2}{d}-\frac{b(a+b\coth(c+dx))^3}{3d}\\
&\quad+\int (a+b\coth(c+dx))\left(a\left(a^2+3b^2\right)+b\left(3a^2+b^2\right)\coth(c+dx)\right)\,dx\\
&= \left(a^4+6a^2b^2+b^4\right)x-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}-\frac{ab(a+b\coth(c+dx))^2}{d}\\
&\quad-\frac{b(a+b\coth(c+dx))^3}{3d}+\left(4ab\left(a^2+b^2\right)\right)\int \coth(c+dx)\,dx\\
&= \left(a^4+6a^2b^2+b^4\right)x-\frac{b^2(3a^2+b^2)\coth(c+dx)}{d}-\frac{ab(a+b\coth(c+dx))^2}{d}\\
&\quad-\frac{b(a+b\coth(c+dx))^3}{3d}+\frac{4ab(a^2+b^2)\log(\sinh(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int (a + b\coth(c + dx))^4\,dx = -\frac{6b^2(6a^2 + b^2)\coth(c + dx) + 12ab^3\coth^2(c + dx) + 2b^4\coth^3(c + dx) + 3(a + b)^4\log(1 - \tanh(c + dx)) \ldots}{6d}$$

[In] Integrate[(a + b*Coth[c + d*x])^4,x]

[Out] -1/6*(6*b^2*(6*a^2 + b^2)*Coth[c + d*x] + 12*a*b^3*Coth[c + d*x]^2 + 2*b^4*Coth[c + d*x]^3 + 3*(a + b)^4*Log[1 - Tanh[c + d*x]] - 24*a*b*(a^2 + b^2)*Log[Tanh[c + d*x]] - 3*(a - b)^4*Log[1 + Tanh[c + d*x]])/d

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.12

method	result
parallelrisch	$\frac{12(-a^3b-a\,b^3)\ln(1-\tanh(dx+c))+12(a^3b+a\,b^3)\ln(\tanh(dx+c))-b^4\coth(dx+c)^3-6a\,b^3\coth(dx+c)^2+3(-6a^2b^2-b^4 \ldots}{3d}$
derivativedivides	$\frac{-\frac{b^4\coth(dx+c)^3}{3}-2a\,b^3\coth(dx+c)^2-6a^2b^2\coth(dx+c)-b^4\coth(dx+c)-\frac{(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)\ln(\coth(dx+c)-1)}{2}+ \ldots}{d}$
default	$\frac{-\frac{b^4\coth(dx+c)^3}{3}-2a\,b^3\coth(dx+c)^2-6a^2b^2\coth(dx+c)-b^4\coth(dx+c)-\frac{(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)\ln(\coth(dx+c)-1)}{2}+ \ldots}{d}$
parts	$x\,a^4+\frac{b^4\left(-\frac{\coth(dx+c)^3}{3}-\coth(dx+c)-\frac{\ln(\coth(dx+c)-1)}{2}+\frac{\ln(\coth(dx+c)+1)}{2}\right)}{d}+\frac{4a^3b\ln(\sinh(dx+c))}{d}+\frac{6a^2b^2\left(- \ldots\right.}{}$
risch	$x\,a^4-4b\,a^3x+6a^2b^2x-4a\,b^3x+b^4x-\frac{8a^3bc}{d}-\frac{8a\,b^3c}{d}-\frac{4b^2\left(9a^2e^{4dx+4c}+6ab\,e^{4dx+4c}+3\,e^{4dx+4c}b \ldots\right.}{3d \ldots}$

[In] int((a+b*coth(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/3*(12*(-a^3*b-a*b^3)*ln(1-tanh(d*x+c))+12*(a^3*b+a*b^3)*ln(tanh(d*x+c))-b^4*coth(d*x+c)^3-6*a*b^3*coth(d*x+c)^2+3*(-6*a^2*b^2-b^4)*coth(d*x+c)+3*d*x*(a-b)^4)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. 2(99) = 198.

Time = 0.27 (sec) , antiderivative size = 1396, normalized size of antiderivative = 13.82

$$\int (a+b\coth(c+dx))^4\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^6 + 18
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c
)^5 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*sinh(d*x + c)^6 - 3
*(12*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)^4 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4
)*d*x*cosh(d*x + c)^2 - 12*a^2*b^2 - 8*a*b^3 - 4*b^4 - 3*(a^4 - 4*a^3*b + 6
*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*sinh(d*x + c)^4 - 36*a^2*b^2 - 8*b^4 + 12*(5
*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 - (12*a^2*
b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b
^4)*d*x + 3*(24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 -
4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2 + 3*(15*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*
a*b^3 + b^4)*d*x*cosh(d*x + c)^4 + 24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x - 6*(12*a^2*b^2 + 8*a*b^3 + 4*b^4
+ 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 12*((a^3*b + a*b^3)*cosh(d*x + c)^6 + 6*(a^3*b + a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^3*b + a*b^3)*sinh(d*x + c)^6 - 3*(a^3*b + a*b^
3)*cosh(d*x + c)^4 - 3*(a^3*b + a*b^3 - 5*(a^3*b + a*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 - a^3*b - a*b^3 + 4*(5*(a^3*b + a*b^3)*cosh(d*x + c)^3 - 3*
(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^3*b + a*b^3)*cosh(d*x
 + c)^2 + 3*(5*(a^3*b + a*b^3)*cosh(d*x + c)^4 + a^3*b + a*b^3 - 6*(a^3*b +
 a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((a^3*b + a*b^3)*cosh(d*x + c)
^5 - 2*(a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*cosh(d*x + c))*sin
h(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x*cosh(d*x + c)^5 - 2*(12*a^2*b^
2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*d*x)*co
sh(d*x + c)^3 + (24*a^2*b^2 + 8*a*b^3 + 4*b^4 + 3*(a^4 - 4*a^3*b + 6*a^2*b^
2 - 4*a*b^3 + b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 +
6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4
 + 3*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x +
 c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(92) = 184.

Time = 1.20 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.40

$$\int (a+b\coth(c+dx))^4\,dx$$
$$= \begin{cases} x(a+b\coth(c))^4 \\ -\frac{a^4\log(-e^{-dx})}{d} - \frac{4a^3b\log(-e^{-dx})\coth(dx+\log(-e^{-dx}))}{d} - \frac{6a^2b^2\log(-e^{-dx})\coth^2(dx+\log(-e^{-dx}))}{d} - \frac{4ab^3\log(-e^{-dx})\coth^3}{d} \\ a^4x + 4a^3bx\coth\left(dx+\log\left(e^{-dx}\right)\right) + 6a^2b^2x\coth^2\left(dx+\log\left(e^{-dx}\right)\right) + 4ab^3x\coth^3\left(dx+\log\left(e^{-dx}\right)\right) + \\ a^4x + 4a^3bx - \frac{4a^3b\log(\tanh(c+dx)+1)}{d} + \frac{4a^3b\log(\tanh(c+dx))}{d} + 6a^2b^2x - \frac{6a^2b^2}{d\tanh(c+dx)} + 4ab^3x - \frac{4ab^3\log(\tanh(c+d}{d} \end{cases}$$

```
[In] integrate((a+b*coth(d*x+c))**4,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**4, Eq(d, 0)), (-a**4*log(-exp(-d*x))/d - 4*a*
*3*b*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))/d - 6*a**2*b**2*log(-exp(-
d*x))*coth(d*x + log(-exp(-d*x)))**2/d - 4*a*b**3*log(-exp(-d*x))*coth(d*x
+ log(-exp(-d*x)))**3/d - b**4*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))*
*4/d, Eq(c, log(-exp(-d*x)))), (a**4*x + 4*a**3*b*x*coth(d*x + log(exp(-d*x
))) + 6*a**2*b**2*x*coth(d*x + log(exp(-d*x)))**2 + 4*a*b**3*x*coth(d*x + l
og(exp(-d*x)))**3 + b**4*x*coth(d*x + log(exp(-d*x)))**4, Eq(c, log(exp(-d*
x)))), (a**4*x + 4*a**3*b*x - 4*a**3*b*log(tanh(c + d*x) + 1)/d + 4*a**3*b*
log(tanh(c + d*x))/d + 6*a**2*b**2*x - 6*a**2*b**2/(d*tanh(c + d*x)) + 4*a*
b**3*x - 4*a*b**3*log(tanh(c + d*x) + 1)/d + 4*a*b**3*log(tanh(c + d*x))/d
- 2*a*b**3/(d*tanh(c + d*x)**2) + b**4*x - b**4/(d*tanh(c + d*x)) - b**4/(3
*d*tanh(c + d*x)**3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(99) = 198.

Time = 0.19 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.17

$$\int (a+b\coth(c+dx))^4\,dx$$
$$= \frac{1}{3}b^4\left(3x + \frac{3c}{d} - \frac{4\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2\right)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right)$$
$$+ 4ab^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right)$$
$$+ 6a^2b^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right) + a^4x + \frac{4a^3b\log(\sinh(dx+c))}{d}$$

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b^3
*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x
 - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 6*a^2*b^2*(x + c
/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^4*x + 4*a^3*b*log(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.51

$$\int (a+b\coth(c+dx))^4\,dx$$
$$=\frac{3\left(a^4-4a^3b+6a^2b^2-4ab^3+b^4\right)(dx+c)+12\left(a^3b+ab^3\right)\log\left(\left|e^{(2dx+2c)}-1\right|\right)-\frac{4\left(9a^2b^2+2b^4+3\left(3a^2b^2+2ab^3\right.\right.}{}}{3d}$$

```
[In] integrate((a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)*(d*x + c) + 12*(a^3*b +
a*b^3)*log(abs(e^(2*d*x + 2*c) - 1)) - 4*(9*a^2*b^2 + 2*b^4 + 3*(3*a^2*b^2
+ 2*a*b^3 + b^4)*e^(4*d*x + 4*c) - 3*(6*a^2*b^2 + 2*a*b^3 + b^4)*e^(2*d*x +
 2*c))/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.56

$$\begin{aligned}\int (a+b\coth(c+dx))^4\,dx = x\,(a-b)^4 &- \frac{4\,(3\,a^2\,b^2+2\,a\,b^3+b^4)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} \\ &- \frac{4\,(b^4+2\,a\,b^3)}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)} \\ &+ \frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,\left(4\,a^3\,b+4\,a\,b^3\right)}{d} \\ &- \frac{8\,b^4}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}\end{aligned}$$

```
[In] int((a + b*coth(c + d*x))^4,x)
```

```
[Out] x*(a - b)^4 - (4*(2*a*b^3 + b^4 + 3*a^2*b^2))/(d*(exp(2*c + 2*d*x) - 1)) -
(4*(2*a*b^3 + b^4))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(
exp(2*c)*exp(2*d*x) - 1)*(4*a*b^3 + 4*a^3*b))/d - (8*b^4)/(3*d*(3*exp(2*c +
 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
```

3.79 $\int (a+b\coth(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int (a+b\coth(c+dx))^3\,dx = a\left(a^2+3b^2\right)x - \frac{2ab^2\coth(c+dx)}{d} - \frac{b(a+b\coth(c+dx))^2}{2d} + \frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d}$$

```
[Out] a*(a^2+3*b^2)*x-2*a*b^2*coth(d*x+c)/d-1/2*b*(a+b*coth(d*x+c))^2/d+b*(3*a^2+
b^2)*ln(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3563, 3606, 3556}

$$\int (a+b\coth(c+dx))^3\,dx = \frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d} + ax\left(a^2+3b^2\right) - \frac{2ab^2\coth(c+dx)}{d} - \frac{b(a+b\coth(c+dx))^2}{2d}$$

```
[In] Int[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] a*(a^2 + 3*b^2)*x - (2*a*b^2*Coth[c + d*x])/d - (b*(a + b*Coth[c + d*x])^2)
/(2*d) + (b*(3*a^2 + b^2)*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{b(a+b\coth(c+dx))^2}{2d} + \int (a+b\coth(c+dx))\left(a^2+b^2+2ab\coth(c+dx)\right)\,dx\\ &= a\left(a^2+3b^2\right)x - \frac{2ab^2\coth(c+dx)}{d} - \frac{b(a+b\coth(c+dx))^2}{2d} + \left(b\left(3a^2+b^2\right)\right)\int \coth(c+dx)\,dx\\ &= a\left(a^2+3b^2\right)x - \frac{2ab^2\coth(c+dx)}{d} - \frac{b(a+b\coth(c+dx))^2}{2d} + \frac{b(3a^2+b^2)\log(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int (a+b\coth(c+dx))^3\,dx = -\frac{6ab^2\coth(c+dx)+b^3\coth^2(c+dx)+(a+b)^3\log(1-\tanh(c+dx))-2b(3a^2+b^2)\log(\tanh(c+dx))}{2d}$$

```
[In] Integrate[(a + b*Coth[c + d*x])^3,x]
```

```
[Out] -1/2*(6*a*b^2*Coth[c + d*x] + b^3*Coth[c + d*x]^2 + (a + b)^3*Log[1 - Tanh[
c + d*x]] - 2*b*(3*a^2 + b^2)*Log[Tanh[c + d*x]] - (a - b)^3*Log[1 + Tanh[c
 + d*x]])/d
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

method	result
parallelrisch	$\frac{(-6a^2b-2b^3)\ln(1-\tanh(dx+c))+(6a^2b+2b^3)\ln(\tanh(dx+c))-b^3\coth(dx+c)^2-6\coth(dx+c)ab^2+2dx(a-b)^3}{2d}$
derivativedivides	$\frac{-\frac{b^3\coth(dx+c)^2}{2}-3\coth(dx+c)ab^2-\frac{(a^3+3a^2b+3ab^2+b^3)\ln(\coth(dx+c)-1)}{2}+\frac{(a^3-3a^2b+3ab^2-b^3)\ln(\coth(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b^3\coth(dx+c)^2}{2}-3\coth(dx+c)ab^2-\frac{(a^3+3a^2b+3ab^2+b^3)\ln(\coth(dx+c)-1)}{2}+\frac{(a^3-3a^2b+3ab^2-b^3)\ln(\coth(dx+c)+1)}{2}}{d}$
parts	$a^3x+\frac{b^3\left(-\frac{\coth(dx+c)^2}{2}-\frac{\ln(\coth(dx+c)-1)}{2}-\frac{\ln(\coth(dx+c)+1)}{2}\right)}{d}+\frac{3a^2b\ln(\sinh(dx+c))}{d}+\frac{3ab^2\left(-\coth(dx+c)-\frac{\ln}{}\right.}{}$
risch	$a^3x-3ba^2x+3ab^2x-b^3x-\frac{6bca^2}{d}-\frac{2b^3c}{d}-\frac{2b^2(3e^{2dx+2c}a+be^{2dx+2c}-3a)}{d(e^{2dx+2c}-1)^2}+\frac{3b\ln(e^{2dx+2c}-1)a^2}{d}$

[In] int((a+b*coth(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/2*((-6*a^2*b-2*b^3)*ln(1-tanh(d*x+c))+(6*a^2*b+2*b^3)*ln(tanh(d*x+c))-b^3 *coth(d*x+c)^2-6*coth(d*x+c)*a*b^2+2*d*x*(a-b)^3)/d

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(67) = 134.

Time = 0.26 (sec) , antiderivative size = 654, normalized size of antiderivative = 9.48

$$\int (a+b\coth(c+dx))^3\,dx$$
$$=\frac{(a^3-3a^2b+3ab^2-b^3)dx\cosh(dx+c)^4+4(a^3-3a^2b+3ab^2-b^3)dx\cosh(dx+c)\sinh(dx+c)^3+(\cdots}{}$$

[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="fricas")

[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3 *a*b^2 - b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*sinh(d*x + c)^4 + 6*a*b^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x - 2*(3*a*b^2 + b^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 2*(3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^2 - 3*a*b^2 - b^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((3*a^2*b + b^3)*co sh(d*x + c)^4 + 4*(3*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2*b + b^3)*sinh(d*x + c)^4 + 3*a^2*b + b^3 - 2*(3*a^2*b + b^3)*cosh(d*x + c)^2 - 2*(3*a^2*b + b^3 - 3*(3*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4 *((3*a^2*b + b^3)*cosh(d*x + c)^3 - (3*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^3 - 3*a

```
^2*b + 3*a*b^2 - b^3)*d*x*cosh(d*x + c)^3 - (3*a*b^2 + b^3 + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4
*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2
+ 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*co
sh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(63) = 126$.

Time = 0.83 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.83

$$\int (a + b\coth(c + dx))^3\,dx = \begin{cases} x(a + b\coth(c))^3 \\ -\frac{a^3\log(-e^{-dx})}{d} - \frac{3a^2b\log(-e^{-dx})\coth(dx+\log(-e^{-dx}))}{d} - \frac{3ab^2\log(-e^{-dx})\coth^2(dx+\log(-e^{-dx}))}{d} - \frac{b^3\log(-e^{-dx})\coth^3(dx+\mathrm{l}}{d} \\ a^3x + 3a^2bx\coth\left(dx + \log\left(e^{-dx}\right)\right) + 3ab^2x\coth^2\left(dx + \log\left(e^{-dx}\right)\right) + b^3x\coth^3\left(dx + \log\left(e^{-dx}\right)\right) \\ a^3x + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} + \frac{3a^2b\log(\tanh(c+dx))}{d} + 3ab^2x - \frac{3ab^2}{d\tanh(c+dx)} + b^3x - \frac{b^3\log(\tanh(c+dx)+1)}{d} + \end{cases}$$

```
[In] integrate((a+b*coth(d*x+c))**3,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**3, Eq(d, 0)), (-a**3*log(-exp(-d*x))/d - 3*a*
*2*b*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))/d - 3*a*b**2*log(-exp(-d*x
))*coth(d*x + log(-exp(-d*x)))**2/d - b**3*log(-exp(-d*x))*coth(d*x + log(-
exp(-d*x)))**3/d, Eq(c, log(-exp(-d*x)))), (a**3*x + 3*a**2*b*x*coth(d*x +
log(exp(-d*x))) + 3*a*b**2*x*coth(d*x + log(exp(-d*x)))**2 + b**3*x*coth(d*
x + log(exp(-d*x)))**3, Eq(c, log(exp(-d*x)))), (a**3*x + 3*a**2*b*x - 3*a*
*2*b*log(tanh(c + d*x) + 1)/d + 3*a**2*b*log(tanh(c + d*x))/d + 3*a*b**2*x
- 3*a*b**2/(d*tanh(c + d*x)) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d + b**
3*log(tanh(c + d*x))/d - b**3/(2*d*tanh(c + d*x)**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(67) = 134$.

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.97

$$\int (a + b\coth(c + dx))^3\,dx = b^3\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)} + 1\right)}{d} + \frac{\log\left(e^{(-dx-c)} - 1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} - 1)}\right) + 3\,ab^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2\,dx-2\,c)} - 1)}\right) + a^3x + \frac{3\,a^2b\log(\sinh(dx + c))}{d}$$

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] b^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + 3*a*b^2*(x +
c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^3*x + 3*a^2*b*log(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.43

$$\int (a+b\coth(c+dx))^3\,dx = \frac{(a^3-3a^2b+3ab^2-b^3)(dx+c)+(3a^2b+b^3)\log\left(\left|e^{(2dx+2c)}-1\right|\right)+\frac{2\left(3ab^2-(3ab^2+b^3)e^{(2dx+2c)}\right)}{\left(e^{(2dx+2c)}-1\right)^2}}{d}$$

```
[In] integrate((a+b*coth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(d*x + c) + (3*a^2*b + b^3)*log(abs(e^(2*d
*x + 2*c) - 1)) + 2*(3*a*b^2 - (3*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x +
 2*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int (a+b\coth(c+dx))^3\,dx = x\,(a-b)^3-\frac{2\,(b^3+3\,a\,b^2)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,b^3}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)\,\left(3\,a^2\,b+b^3\right)}{d}$$

```
[In] int((a + b*coth(c + d*x))^3,x)
```

```
[Out] x*(a - b)^3 - (2*(3*a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) - 1)) - (2*b^3)/(d*(
exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) - 1)
*(3*a^2*b + b^3))/d
```

3.80 $\int (a+b\coth(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 38

$$\int (a+b\coth(c+dx))^2\,dx=(a^2+b^2)\,x-\frac{b^2\coth(c+dx)}{d}+\frac{2ab\log(\sinh(c+dx))}{d}$$

```
[Out] (a^2+b^2)*x-b^2*coth(d*x+c)/d+2*a*b*ln(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3558, 3556}

$$\int (a+b\coth(c+dx))^2\,dx=x(a^2+b^2)+\frac{2ab\log(\sinh(c+dx))}{d}-\frac{b^2\coth(c+dx)}{d}$$

```
[In] Int[(a + b*Coth[c + d*x])^2,x]
[Out] (a^2 + b^2)*x - (b^2*Coth[c + d*x])/d + (2*a*b*Log[Sinh[c + d*x]])/d
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\text{integral} = (a^2+b^2)\,x - \frac{b^2\coth(c+dx)}{d} + (2ab)\int \coth(c+dx)\,dx$$
$$= (a^2+b^2)\,x - \frac{b^2\coth(c+dx)}{d} + \frac{2ab\log(\sinh(c+dx))}{d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (a+b\coth(c+dx))^2\,dx$$
$$= \frac{-2b^2\coth(c+dx) - (a+b)^2\log(1-\tanh(c+dx)) + 4ab\log(\tanh(c+dx)) + (a-b)^2\log(1+\tanh(c+dx))}{2d}$$

```
[In] Integrate[(a + b*Coth[c + d*x])^2,x]
```

```
[Out] (-2*b^2*Coth[c + d*x] - (a + b)^2*Log[1 - Tanh[c + d*x]] + 4*a*b*Log[Tanh[c
 + d*x]] + (a - b)^2*Log[1 + Tanh[c + d*x]])/(2*d)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

method	result	size
parts	$a^2x+\frac{b^2\left(-\coth(dx+c)-\frac{\ln(\coth(dx+c)-1)}{2}+\frac{\ln(\coth(dx+c)+1)}{2}\right)}{d}+\frac{2ab\ln(\sinh(dx+c))}{d}$	59
derivativedivides	$\frac{-\coth(dx+c)b^2-\frac{(a^2+2ab+b^2)\ln(\coth(dx+c)-1)}{2}+\frac{(a^2-2ab+b^2)\ln(\coth(dx+c)+1)}{2}}{d}$	61
default	$\frac{-\coth(dx+c)b^2-\frac{(a^2+2ab+b^2)\ln(\coth(dx+c)-1)}{2}+\frac{(a^2-2ab+b^2)\ln(\coth(dx+c)+1)}{2}}{d}$	61
risch	$a^2x-2abx+b^2x-\frac{4abc}{d}-\frac{2b^2}{d(e^{2dx+2c}-1)}+\frac{2ab\ln(e^{2dx+2c}-1)}{d}$	65
parallelrisch	$\frac{-2\ln(1-\tanh(dx+c))\tanh(dx+c)ab+2ab\ln(\tanh(dx+c))\tanh(dx+c)+dx(a-b)^2\tanh(dx+c)-b^2}{d\tanh(dx+c)}$	73

```
[In] int((a+b*coth(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+b^2/d*(-coth(d*x+c)-1/2*ln(coth(d*x+c)-1)+1/2*ln(coth(d*x+c)+1))+2*a*
b*ln(sinh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.39

$$\int (a + b\coth(c + dx))^2\,dx$$
$$= \frac{(a^2 - 2\,ab + b^2)dx\cosh{(dx + c)}^2 + 2\,(a^2 - 2\,ab + b^2)dx\cosh{(dx + c)}\sinh{(dx + c)} + (a^2 - 2\,ab + b^2)dx\sin}{d\cosh{(dx + c)}^2}$$

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - 2*a*b + b^2)*d*x*cosh(d
*x + c)*sinh(d*x + c) + (a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^2 - (a^2 - 2*
a*b + b^2)*d*x - 2*b^2 + 2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(
d*x + c) + a*b*sinh(d*x + c)^2 - a*b)*log(2*sinh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*s
inh(d*x + c)^2 - d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(34) = 68$.

Time = 0.57 (sec) , antiderivative size = 209, normalized size of antiderivative = 5.50

$$\int (a + b\coth(c + dx))^2\,dx$$
$$= \begin{cases} x(a + b\coth{(c)})^2 & \text{for } d = 0 \\ -\frac{a^2\log{(-e^{-dx})}}{d} - \frac{2ab\log{(-e^{-dx})}\coth{(dx+\log{(-e^{-dx})})}}{d} - \frac{b^2\log{(-e^{-dx})}\coth^2{(dx+\log{(-e^{-dx})})}}{d} & \text{for } c = \log{\left(-e^{-dx}\right)} \\ a^2x + 2abx\coth{\left(dx + \log{\left(e^{-dx}\right)}\right)} + b^2x\coth^2{\left(dx + \log{\left(e^{-dx}\right)}\right)} & \text{for } c = \log{\left(e^{-dx}\right)} \\ a^2x + 2abx - \frac{2ab\log{(\tanh{(c+dx)}+1)}}{d} + \frac{2ab\log{(\tanh{(c+dx)})}}{d} + b^2x - \frac{b^2}{d\tanh{(c+dx)}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*coth(d*x+c))**2,x)
```

```
[Out] Piecewise((x*(a + b*coth(c))**2, Eq(d, 0)), (-a**2*log(-exp(-d*x))/d - 2*a*
b*log(-exp(-d*x))*coth(d*x + log(-exp(-d*x)))/d - b**2*log(-exp(-d*x))*coth
(d*x + log(-exp(-d*x)))**2/d, Eq(c, log(-exp(-d*x)))), (a**2*x + 2*a*b*x*co
th(d*x + log(exp(-d*x))) + b**2*x*coth(d*x + log(exp(-d*x)))**2, Eq(c, log(
exp(-d*x)))), (a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d + 2*a*b*lo
g(tanh(c + d*x))/d + b**2*x - b**2/(d*tanh(c + d*x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int (a+b\coth(c+dx))^2\,dx = b^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+a^2x+\frac{2\,ab\log\left(\sinh\left(dx+c\right)\right)}{d}$$

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="maxima")
[Out] b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + a^2*x + 2*a*b*log(sinh(d*x +
 c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.50

$$\int (a+b\coth(c+dx))^2\,dx = \frac{2\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+(a^2-2\,ab+b^2)(dx+c)-\frac{2\,b^2}{e^{(2\,dx+2\,c)}-1}}{d}$$

```
[In] integrate((a+b*coth(d*x+c))^2,x, algorithm="giac")
[Out] (2*a*b*log(abs(e^(2*d*x + 2*c) - 1)) + (a^2 - 2*a*b + b^2)*(d*x + c) - 2*b^
2/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int (a+b\coth(c+dx))^2\,dx = x\,(a-b)^2-\frac{2\,b^2}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}+\frac{2\,a\,b\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)}{d}$$

```
[In] int((a + b*coth(c + d*x))^2,x)
[Out] x*(a - b)^2 - (2*b^2)/(d*(exp(2*c + 2*d*x) - 1)) + (2*a*b*log(exp(2*c)*exp(
2*d*x) - 1))/d
```

3.81 $\int \frac{1}{a+b\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 50

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{ax}{a^2-b^2} - \frac{b\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] a*x/(a^2-b^2)-b*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3565, 3611}

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{ax}{a^2-b^2} - \frac{b\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\,(a^2-b^2)}$$

```
[In] Int[(a + b*Coth[c + d*x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)
*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
```

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2} \\ &= \frac{ax}{a^2-b^2} - \frac{b\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)\,d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{(-a+b)\log(1-\tanh(c+dx))+(a+b)\log(1+\tanh(c+dx))-2b\log(b+a\tanh(c+dx))}{2(a-b)(a+b)d}$$

```
[In] Integrate[(a + b*Coth[c + d*x])^(-1),x]
[Out] ((-a + b)*Log[1 - Tanh[c + d*x]] + (a + b)*Log[1 + Tanh[c + d*x]] - 2*b*Log
[b + a*Tanh[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-b\ln(b+a\tanh(dx+c))+\ln(1-\tanh(dx+c))b+dx(a+b)}{d(a^2-b^2)}$	50
derivativedivides	$\frac{-\frac{b\ln(a+b\coth(dx+c))}{(a-b)(a+b)}+\frac{\ln(\coth(dx+c)+1)}{2a-2b}-\frac{\ln(\coth(dx+c)-1)}{2a+2b}}{d}$	71
default	$\frac{-\frac{b\ln(a+b\coth(dx+c))}{(a-b)(a+b)}+\frac{\ln(\coth(dx+c)+1)}{2a-2b}-\frac{\ln(\coth(dx+c)-1)}{2a+2b}}{d}$	71
risch	$\frac{x}{a+b}+\frac{2xb}{a^2-b^2}+\frac{2bc}{d(a^2-b^2)}-\frac{b\ln\left(e^{2dx+2c}-\frac{a-b}{a+b}\right)}{d(a^2-b^2)}$	82

```
[In] int(1/(a+b*coth(d*x+c)),x,method=_RETURNVERBOSE)
[Out] (-b*ln(b+a*tanh(d*x+c))+ln(1-tanh(d*x+c))*b+d*x*(a+b))/d/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{(a+b)dx - b\log\left(\frac{2\,(b\cosh(dx+c)+a\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2-b^2)d}$$

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="fricas")
[Out] ((a + b)*d*x - b*log(2*(b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/((a^2 - b^2)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(37) = 74.

Time = 1.53 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.72

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\coth(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{b} & \text{for } a=0 \\ -\frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)-2bd}+\frac{dx}{2bd\tanh(c+dx)-2bd}-\frac{1}{2bd\tanh(c+dx)-2bd} & \text{for } a=-b \\ \frac{dx\tanh(c+dx)}{2bd\tanh(c+dx)+2bd}+\frac{dx}{2bd\tanh(c+dx)+2bd}+\frac{1}{2bd\tanh(c+dx)+2bd} & \text{for } a=b \\ \frac{x}{a+b\coth(c)} & \text{for } d=0 \\ \frac{adx}{a^2d-b^2d}-\frac{bdx}{a^2d-b^2d}+\frac{b\log(\tanh(c+dx)+1)}{a^2d-b^2d}-\frac{b\log\left(\tanh(c+dx)+\frac{b}{a}\right)}{a^2d-b^2d} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*coth(d*x+c)),x)
[Out] Piecewise((zoo*x/coth(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c
 + d*x) + 1)/d)/b, Eq(a, 0)), (-d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) - 2*
b*d) + d*x/(2*b*d*tanh(c + d*x) - 2*b*d) - 1/(2*b*d*tanh(c + d*x) - 2*b*d),
 Eq(a, -b)), (d*x*tanh(c + d*x)/(2*b*d*tanh(c + d*x) + 2*b*d) + d*x/(2*b*d*
tanh(c + d*x) + 2*b*d) + 1/(2*b*d*tanh(c + d*x) + 2*b*d), Eq(a, b)), (x/(a
+ b*coth(c)), Eq(d, 0)), (a*d*x/(a**2*d - b**2*d) - b*d*x/(a**2*d - b**2*d)
 + b*log(tanh(c + d*x) + 1)/(a**2*d - b**2*d) - b*log(tanh(c + d*x) + b/a)/
(a**2*d - b**2*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^2-b^2)d}+\frac{dx+c}{(a+b)d}$$

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2 - b^2)*d) + (d*x + c)/((a +
 b)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = -\frac{\frac{b\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^2-b^2}-\frac{dx+c}{a-b}}{d}$$

```
[In] integrate(1/(a+b*coth(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^2 - b^2) - (
d*x + c)/(a - b))/d
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\coth(c+dx)}\,dx = \frac{x}{a-b}-\frac{b\,\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)}{a^2\,d-b^2\,d}$$

```
[In] int(1/(a + b*coth(c + d*x)),x)
```

```
[Out] x/(a - b) - (b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x)))/
(a^2*d - b^2*d)
```

3.82 $\int \frac{1}{(a+b\coth(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{(a^2+b^2)\,x}{(a^2-b^2)^2} + \frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))} - \frac{2ab\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^2\,d}$$

```
[Out] (a^2+b^2)*x/(a^2-b^2)^2+b/(a^2-b^2)/d/(a+b*coth(d*x+c))-2*a*b*ln(b*cosh(d*x
+c)+a*sinh(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3564, 3612, 3611}

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{b}{d\,(a^2-b^2)\,(a+b\coth(c+dx))} - \frac{2ab\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\,(a^2-b^2)^2} + \frac{x(a^2+b^2)}{(a^2-b^2)^2}$$

```
[In] Int[(a + b*Coth[c + d*x])^(-2),x]
```

```
[Out] ((a^2 + b^2)*x)/(a^2 - b^2)^2 + b/((a^2 - b^2)*d*(a + b*Coth[c + d*x])) - (
2*a*b*Log[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
```

```
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))}+\frac{\int\frac{a-b\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{a^2-b^2}\\
&= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))}-\frac{(2iab)\int\frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{(a^2-b^2)^2}\\
&= \frac{(a^2+b^2)\,x}{(a^2-b^2)^2}+\frac{b}{(a^2-b^2)\,d(a+b\coth(c+dx))}-\frac{2ab\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^2\,d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int\frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{-\frac{\log(1-\tanh(c+dx))}{(a+b)^2}+\frac{\log(1+\tanh(c+dx))}{(a-b)^2}+\frac{2b\left(-2a^2\log(b+a\tanh(c+dx))+\frac{-a^2b+b^3}{b+a\tanh(c+dx)}\right)}{a(a^2-b^2)^2}}{2d}$$

```
[In] Integrate[(a + b*Coth[c + d*x])^(-2),x]
[Out] (-(Log[1 - Tanh[c + d*x]]/(a + b)^2) + Log[1 + Tanh[c + d*x]]/(a - b)^2 + (
2*b*(-2*a^2*Log[b + a*Tanh[c + d*x]] + (-(a^2*b) + b^3)/(b + a*Tanh[c + d*x
]))/(a*(a^2 - b^2)^2))/(2*d)
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{b}{(a-b)(a+b)(a+b\coth(dx+c))}-\frac{2ab\ln(a+b\coth(dx+c))}{(a+b)^2(a-b)^2}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^2}+\frac{\ln(\coth(dx+c)+1)}{2(a-b)^2}}{d}$
default	$\frac{\frac{b}{(a-b)(a+b)(a+b\coth(dx+c))}-\frac{2ab\ln(a+b\coth(dx+c))}{(a+b)^2(a-b)^2}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^2}+\frac{\ln(\coth(dx+c)+1)}{2(a-b)^2}}{d}$
parallelrisch	$\frac{(-2a^3b\tanh(dx+c)-2a^2b^2)\ln(b+a\tanh(dx+c))+(2a^3b\tanh(dx+c)+2a^2b^2)\ln(1-\tanh(dx+c))+(a^2dx(a+b)\tanh(dx+}{(a-b)^2(a+b)^2(b+a\tanh(dx+c))da}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{4abx}{a^4-2a^2b^2+b^4}+\frac{4abc}{d(a^4-2a^2b^2+b^4)}-\frac{2b^2}{(a-b)d(a^2+2ab+b^2)(e^{2dx+2c}a+b\,e^{2dx+2c}-a+b)}-\frac{2ab\ln\left(e^{2d}\right.}{d(a^4-2}$

```
[In] int(1/(a+b*coth(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(b/(a-b)/(a+b)/(a+b*coth(d*x+c))-2*a*b/(a+b)^2/(a-b)^2*ln(a+b*coth(d*x+
c))-1/2/(a+b)^2*ln(coth(d*x+c)-1)+1/2/(a-b)^2*ln(coth(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. 2(85) = 170.

Time = 0.26 (sec) , antiderivative size = 426, normalized size of antiderivative = 5.01

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx$$
$$= \frac{(a^3+3\,a^2b+3\,ab^2+b^3)dx\cosh{(dx+c)}^2+2\,(a^3+3\,a^2b+3\,ab^2+b^3)dx\cosh{(dx+c)}\sinh{(dx+c)}+(a^3}{(a^5+a^4b-2\,a^3b^2-2\,a^2b^3+ab^4+b^5)d\cosh{(dx+c)}^2}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="fricas")

[Out] ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + 2*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c) + (a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x*sinh(d*x + c)^2 - 2*a*b^2 + 2*b^3 - (a^3 + a^2*b - a*b^2 - b^3)*d*
x + 2*(a^2*b - a*b^2 - (a^2*b + a*b^2)*cosh(d*x + c)^2 - 2*(a^2*b + a*b^2)*
cosh(d*x + c)*sinh(d*x + c) - (a^2*b + a*b^2)*sinh(d*x + c)^2)*log(2*(b*cos
h(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))))/((a^5 + a^4
*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)^2 + 2*(a^5 + a^4*
b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c) + (a
^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*sinh(d*x + c)^2 - (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*coth(d*x+c))**2,x)
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = -\frac{2\,ab\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^4-2\,a^2b^2+b^4)d} - \frac{2\,b^2}{(a^4-2\,a^2b^2+b^4-(a^4-2\,a^3b+2\,ab^3-b^4)e^{(-2\,dx-2\,c)})d} + \frac{dx+c}{(a^2+2\,ab+b^2)d}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="maxima")
[Out] -2*a*b*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^4 - 2*a^2*b^2 + b^4)*d) -
 2*b^2/((a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*d*x
- 2*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.53

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = -\frac{\frac{2\,ab\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^4-2\,a^2b^2+b^4} - \frac{dx+c}{a^2-2\,ab+b^2} + \frac{2\,(ab^2-b^3)}{(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b)(a+b)^2(a-b)^2}}{d}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^2,x, algorithm="giac")
[Out] -(2*a*b*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(a^4 - 2*a^
2*b^2 + b^4) - (d*x + c)/(a^2 - 2*a*b + b^2) + 2*(a*b^2 - b^3)/((a*e^(2*d*x
 + 2*c) + b*e^(2*d*x + 2*c) - a + b)*(a + b)^2*(a - b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\coth(c+dx))^2}\,dx = \frac{x}{(a-b)^2} - \frac{2\,a\,b\,\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)}{d\,a^4-2\,d\,a^2\,b^2+d\,b^4} - \frac{2\,b^2}{d\,(a+b)^2\,(a-b)\,\left(b-a+e^{2\,c+2\,d\,x}\,(a+b)\right)}$$

```
[In] int(1/(a + b*coth(c + d*x))^2,x)
```

```
[Out] x/(a - b)^2 - (2*a*b*log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d
*x)))/(a^4*d + b^4*d - 2*a^2*b^2*d) - (2*b^2)/(d*(a + b)^2*(a - b)*(b - a +
 exp(2*c + 2*d*x)*(a + b)))
```

3.83 $\int \frac{1}{(a+b\coth(c+dx))^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 129

$$\int \frac{1}{(a+b\coth(c+dx))^3} dx = \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} + \frac{b}{2\,(a^2-b^2)\,d(a+b\coth(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))} - \frac{b(3a^2+b^2)\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^3\,d}$$

```
[Out] a*(a^2+3*b^2)*x/(a^2-b^2)^3+1/2*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^2+2*a*b/(a^
2-b^2)^2/d/(a+b*coth(d*x+c))-b*(3*a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/
(a^2-b^2)^3/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3564, 3610, 3612, 3611}

$$\int \frac{1}{(a+b\coth(c+dx))^3} dx = \frac{2ab}{d\,(a^2-b^2)^2\,(a+b\coth(c+dx))} + \frac{b}{2d\,(a^2-b^2)\,(a+b\coth(c+dx))^2} - \frac{b(3a^2+b^2)\log(a\sinh(c+dx)+b\cosh(c+dx))}{d\,(a^2-b^2)^3} + \frac{ax(a^2+3b^2)}{(a^2-b^2)^3}$$

```
[In] Int[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/(a^2 - b^2)^3 + b/(2*(a^2 - b^2)*d*(a + b*Coth[c + d*x]
)^2) + (2*a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])) - (b*(3*a^2 + b^2)*L
og[b*Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b}{2\left(a^2-b^2\right) d(a+b\coth(c+dx))^2}+\frac{\int \frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^2}\,dx}{a^2-b^2}\\
&= \frac{b}{2\left(a^2-b^2\right) d(a+b\coth(c+dx))^2}+\frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))}+\frac{\int \frac{a^2+b^2-2ab\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{\left(a^2-b^2\right)^2}\\
&= \frac{a(a^2+3b^2)\,x}{\left(a^2-b^2\right)^3}+\frac{b}{2\left(a^2-b^2\right) d(a+b\coth(c+dx))^2}\\
&\quad+\frac{2ab}{\left(a^2-b^2\right)^2 d(a+b\coth(c+dx))}-\frac{\left(ib(3a^2+b^2)\right)\int \frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{\left(a^2-b^2\right)^3}
\end{aligned}
$$

$$= \frac{a(a^2+3b^2)\,x}{(a^2-b^2)^3} + \frac{b}{2\,(a^2-b^2)\,d(a+b\coth(c+dx))^2} + \frac{2ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))} - \frac{b(3a^2+b^2)\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^3\,d}$$

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx =$$

$$-\frac{\frac{\log(1-\tanh(c+dx))}{(a+b)^3} - \frac{\log(1+\tanh(c+dx))}{(a-b)^3} + \frac{b\left(2(3a^2+b^2)\log(b+a\tanh(c+dx)) + \frac{b(-a^2+b^2)\left(-5a^2b+b^3+(-6a^3+2ab^2)\tanh(c+dx)\right)}{a^2(b+a\tanh(c+dx))^2}\right)}{(a^2-b^2)^3}}{2d}$$

```
[In] Integrate[(a + b*Coth[c + d*x])^(-3),x]
```

```
[Out] -1/2*(Log[1 - Tanh[c + d*x]]/(a + b)^3 - Log[1 + Tanh[c + d*x]]/(a - b)^3 +
 (b*(2*(3*a^2 + b^2)*Log[b + a*Tanh[c + d*x]] + (b*(-a^2 + b^2)*(-5*a^2*b +
 b^3 + (-6*a^3 + 2*a*b^2)*Tanh[c + d*x]))/(a^2*(b + a*Tanh[c + d*x])^2)))/(
a^2 - b^2)^3)/d
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b}{2(a-b)(a+b)(a+b\coth(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))}-\frac{b\left(3a^2+b^2\right)\ln(a+b\coth(dx+c))}{(a+b)^3(a-b)^3}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^3}+\frac{\ln(\coth(d}{2(a-}}{d}$
default	$\frac{\frac{b}{2(a-b)(a+b)(a+b\coth(dx+c))^2}+\frac{2ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))}-\frac{b\left(3a^2+b^2\right)\ln(a+b\coth(dx+c))}{(a+b)^3(a-b)^3}-\frac{\ln(\coth(dx+c)-1)}{2(a+b)^3}+\frac{\ln(\coth(d}{2(a-}}{d}$
parallelrisch	$\frac{-3\left(a^2+\frac{b^2}{3}\right)b\,a^2(b+a\tanh(dx+c))^2\ln(b+a\tanh(dx+c))+3\left(a^2+\frac{b^2}{3}\right)b\,a^2(b+a\tanh(dx+c))^2\ln(1-\tanh(dx+c))+\left(a^4d\right.}{(a-b)}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}+\frac{6b\,a^2x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{2b^3x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{6bc\,a^2}{d(a^6-3a^4b^2+3a^2b^4-b^6)}+\frac{2b^3}{d(a^6-3a^4b^2-}$

```
[In] int(1/(a+b*coth(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b/(a-b)/(a+b)/(a+b*coth(d*x+c))^2+2*a*b/(a+b)^2/(a-b)^2/(a+b*coth(
d*x+c))-b*(3*a^2+b^2)/(a+b)^3/(a-b)^3*ln(a+b*coth(d*x+c))-1/2/(a+b)^3*ln(co
th(d*x+c)-1)+1/2/(a-b)^3*ln(coth(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1431 vs. $2(127) = 254$.

Time = 0.28 (sec) , antiderivative size = 1431, normalized size of antiderivative = 11.09

$$\int \frac{1}{(a + b\coth(c + dx))^3} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] ((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c
)^4 + 4*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*x*sinh(d*x + c)^4 + 6*a^3*b^2 - 12*a^2*b^3 + 6*a*b^4 + (a^5 + a
^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*d*x - 2*(3*a^3*b^2 - a^2*b^3 -
3*a*b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x
)*cosh(d*x + c)^2 - 2*(3*a^3*b^2 - a^2*b^3 - 3*a*b^4 + b^5 - 3*(a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^2 + (a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*sinh(d*x + c)^2 - (3*
a^4*b - 6*a^3*b^2 + 4*a^2*b^3 - 2*a*b^4 + b^5 + (3*a^4*b + 6*a^3*b^2 + 4*a^
2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^4 + 4*(3*a^4*b + 6*a^3*b^2 + 4*a^2*b^3
 + 2*a*b^4 + b^5)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^4*b + 6*a^3*b^2 + 4*
a^2*b^3 + 2*a*b^4 + b^5)*sinh(d*x + c)^4 - 2*(3*a^4*b - 2*a^2*b^3 - b^5)*co
sh(d*x + c)^2 - 2*(3*a^4*b - 2*a^2*b^3 - b^5 - 3*(3*a^4*b + 6*a^3*b^2 + 4*a
^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((3*a^4*b + 6*
a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 + b^5)*cosh(d*x + c)^3 - (3*a^4*b - 2*a^2*b^3
 - b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*cosh(d*x + c) + a*sinh(d*x +
 c))/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*x*cosh(d*x + c)^3 - (3*a^3*b^2 - a^2*b^3 - 3*a*
b^4 + b^5 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*x)*co
sh(d*x + c))*sinh(d*x + c))/((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3
*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^4 + 4*(a^8 + 2*a^7*b - 2*
a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c
)*sinh(d*x + c)^3 + (a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*
a^2*b^6 - 2*a*b^7 - b^8)*d*sinh(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4
 - 4*a^2*b^6 + b^8)*d*cosh(d*x + c)^2 + 2*(3*(a^8 + 2*a^7*b - 2*a^6*b^2 - 6
*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 - b^8)*d*cosh(d*x + c)^2 - (a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sinh(d*x + c)^2 + (a^8 - 2*a^
7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 - b^8)*d + 4*
((a^8 + 2*a^7*b - 2*a^6*b^2 - 6*a^5*b^3 + 6*a^3*b^5 + 2*a^2*b^6 - 2*a*b^7 -
 b^8)*d*cosh(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d
*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*coth(d*x+c))**3,x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(127) = 254$.

Time = 0.20 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = -\frac{(3\,a^2b+b^3)\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)d} - \frac{2\left(3\,a^2b^2+3\,ab^3-(3\,a^2b^2-2\cdots\right.}{(a^7+a^6b-3\,a^5b^2-3\,a^4b^3+3\,a^3b^4+3\,a^2b^5-ab^6-b^7-2\,(a^7-a^6b-3\,a^5b^2+3\,a^4b^3+3\,a^3b^4-3\,a^2b^{5}\cdots} + \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b + b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^6 - 3*a^4*b^2 +
 3*a^2*b^4 - b^6)*d) - 2*(3*a^2*b^2 + 3*a*b^3 - (3*a^2*b^2 - 2*a*b^3 - b^4)
*e^(-2*d*x - 2*c))/((a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7 - 2*(a^7 - a^6*b - 3*a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 -
3*a^2*b^5 - a*b^6 + b^7)*e^(-2*d*x - 2*c) + (a^7 - 3*a^6*b + a^5*b^2 + 5*a^
4*b^3 - 5*a^3*b^4 - a^2*b^5 + 3*a*b^6 - b^7)*e^(-4*d*x - 4*c))*d) + (d*x +
c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.57

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = -\frac{\frac{(3\,a^2b+b^3)\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{dx+c}{a^3-3\,a^2b+3\,ab^2-b^3} + \frac{2\left((3\,a^2b^2-4\,ab^3+b^4)e^{(2\,dx+2\,c)}-\frac{3\left(a^3b^2-2\,a^2b^3+ab^4\right)}{a+b}\right)}{\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right)^2(a+b)^2(a-b)^3}}{d}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((3*a^2*b + b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b))/(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) - (d*x + c)/(a^3 - 3*a^2*b + 3*a*b^2 - b
^3) + 2*((3*a^2*b^2 - 4*a*b^3 + b^4)*e^(2*d*x + 2*c) - 3*(a^3*b^2 - 2*a^2*b
^3 + a*b^4)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a + b)^2*(a
+ b)^2*(a - b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\coth(c+dx))^3}\,dx = \frac{x}{(a-b)^3} - \frac{\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,\left(3\,a^2\,b+b^3\right)}{d\,a^6-3\,d\,a^4\,b^2+3\,d\,a^2\,b^4-d\,b^6} + \frac{2\,b^3}{d\,(a+b)^3\,(a-b)\,\left(e^{4\,c+4\,d\,x}\,(a+b)^2+(a-b)^2-2\,e^{2\,c+2\,d\,x}\,(a+b)\,(a-b)\right)} - \frac{2\,(3\,a\,b^2-b^3)}{d\,(a+b)^3\,(a-b)^2\,\left(b-a+e^{2\,c+2\,d\,x}\,(a+b)\right)}$$

```
[In] int(1/(a + b*coth(c + d*x))^3,x)
```

```
[Out] x/(a - b)^3 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*(
3*a^2*b + b^3))/(a^6*d - b^6*d + 3*a^2*b^4*d - 3*a^4*b^2*d) + (2*b^3)/(d*(a
 + b)^3*(a - b)*(exp(4*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*d*x
)*(a + b)*(a - b))) - (2*(3*a*b^2 - b^3))/(d*(a + b)^3*(a - b)^2*(b - a + e
xp(2*c + 2*d*x)*(a + b)))
```

3.84 $\int \frac{1}{(a+b\coth(c+dx))^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 169

$$\int \frac{1}{(a+b\coth(c+dx))^4} dx = \frac{(a^4+6a^2b^2+b^4)x}{(a^2-b^2)^4} + \frac{b}{3(a^2-b^2)d(a+b\coth(c+dx))^3} + \frac{ab}{(a^2-b^2)^2 d(a+b\coth(c+dx))^2} + \frac{b(3a^2+b^2)}{(a^2-b^2)^3 d(a+b\coth(c+dx))} - \frac{4ab(a^2+b^2)\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^4 d}$$

```
[Out] (a^4+6*a^2*b^2+b^4)*x/(a^2-b^2)^4+1/3*b/(a^2-b^2)/d/(a+b*coth(d*x+c))^3+a*b
/(a^2-b^2)^2/d/(a+b*coth(d*x+c))^2+b*(3*a^2+b^2)/(a^2-b^2)^3/d/(a+b*coth(d*
x+c))-4*a*b*(a^2+b^2)*ln(b*cosh(d*x+c)+a*sinh(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3564, 3610, 3612, 3611}

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \frac{b(3a^2+b^2)}{d(a^2-b^2)^3(a+b\coth(c+dx))} + \frac{ab}{d(a^2-b^2)^2(a+b\coth(c+dx))^2} + \frac{b}{3d(a^2-b^2)(a+b\coth(c+dx))^3} - \frac{4ab(a^2+b^2)\log(a\sinh(c+dx)+b\cosh(c+dx))}{d(a^2-b^2)^4} + \frac{x(a^4+6a^2b^2+b^4)}{(a^2-b^2)^4}$$

```
[In] Int[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] ((a^4 + 6*a^2*b^2 + b^4)*x)/(a^2 - b^2)^4 + b/(3*(a^2 - b^2)*d*(a + b*Coth[
c + d*x])^3) + (a*b)/((a^2 - b^2)^2*d*(a + b*Coth[c + d*x])^2) + (b*(3*a^2
+ b^2))/((a^2 - b^2)^3*d*(a + b*Coth[c + d*x])) - (4*a*b*(a^2 + b^2)*Log[b*
Cosh[c + d*x] + a*Sinh[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b}{3(a^2-b^2)\,d(a+b\coth(c+dx))^3}+\frac{\int\frac{a-b\coth(c+dx)}{(a+b\coth(c+dx))^3}\,dx}{a^2-b^2}\\
&= \frac{b}{3(a^2-b^2)\,d(a+b\coth(c+dx))^3}\\
&\quad+\frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2}+\frac{\int\frac{a^2+b^2-2ab\coth(c+dx)}{(a+b\coth(c+dx))^2}\,dx}{(a^2-b^2)^2}\\
&= \frac{b}{3(a^2-b^2)\,d(a+b\coth(c+dx))^3}+\frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2}\\
&\quad+\frac{b(3a^2+b^2)}{(a^2-b^2)^3\,d(a+b\coth(c+dx))}+\frac{\int\frac{a(a^2+3b^2)-b(3a^2+b^2)\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{(a^2-b^2)^3}\\
&= \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4}+\frac{b}{3(a^2-b^2)\,d(a+b\coth(c+dx))^3}\\
&\quad+\frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2}+\frac{b(3a^2+b^2)}{(a^2-b^2)^3\,d(a+b\coth(c+dx))}\\
&\quad-\frac{(4iab(a^2+b^2))\int\frac{-ib-ia\coth(c+dx)}{a+b\coth(c+dx)}\,dx}{(a^2-b^2)^4}\\
&= \frac{(a^4+6a^2b^2+b^4)\,x}{(a^2-b^2)^4}+\frac{b}{3(a^2-b^2)\,d(a+b\coth(c+dx))^3}+\frac{ab}{(a^2-b^2)^2\,d(a+b\coth(c+dx))^2}\\
&\quad+\frac{b(3a^2+b^2)}{(a^2-b^2)^3\,d(a+b\coth(c+dx))}-\frac{4ab(a^2+b^2)\log(b\cosh(c+dx)+a\sinh(c+dx))}{(a^2-b^2)^4\,d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 6.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.27

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = -\frac{\log(1-\tanh(c+dx))}{2(a+b)^4d}+\frac{\log(1+\tanh(c+dx))}{2(a-b)^4d} - \frac{4ab(a^2+b^2)\log(b+a\tanh(c+dx))}{(a^2-b^2)^4\,d} - \frac{b^4}{3a^3\,(a^2-b^2)\,d(b+a\tanh(c+dx))^3} + \frac{b^3(2a^2-b^2)}{a^3\,(a^2-b^2)^2\,d(b+a\tanh(c+dx))^2} - \frac{b^2(6a^4-3a^2b^2+b^4)}{a^3\,(a^2-b^2)^3\,d(b+a\tanh(c+dx))}$$

```
[In] Integrate[(a + b*Coth[c + d*x])^(-4),x]
```

```
[Out] -1/2*Log[1 - Tanh[c + d*x]]/((a + b)^4*d) + Log[1 + Tanh[c + d*x]]/(2*(a -
b)^4*d) - (4*a*b*(a^2 + b^2)*Log[b + a*Tanh[c + d*x]])/((a^2 - b^2)^4*d) -
b^4/(3*a^3*(a^2 - b^2)*d*(b + a*Tanh[c + d*x])^3) + (b^3*(2*a^2 - b^2))/(a^
3*(a^2 - b^2)^2*d*(b + a*Tanh[c + d*x])^2) - (b^2*(6*a^4 - 3*a^2*b^2 + b^4)
)/(a^3*(a^2 - b^2)^3*d*(b + a*Tanh[c + d*x]))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^4}+\frac{b}{3(a-b)(a+b)(a+b\coth(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))^2}+\frac{b(3a^2+b^2)}{(a+b)^3(a-b)^3(a+b\coth(dx+c))}-\frac{4ba(a^2+\cdots}{\cdots}}{d}$
default	$\frac{\frac{\ln(\coth(dx+c)+1)}{2(a-b)^4}+\frac{b}{3(a-b)(a+b)(a+b\coth(dx+c))^3}+\frac{ab}{(a+b)^2(a-b)^2(a+b\coth(dx+c))^2}+\frac{b(3a^2+b^2)}{(a+b)^3(a-b)^3(a+b\coth(dx+c))}-\frac{4ba(a^2+\cdots}{\cdots}}{d}$
parallelrisch	$\frac{-4b\,a^2(a^2+b^2)(b+a\tanh(dx+c))^3\ln(b+a\tanh(dx+c))+4b\,a^2(a^2+b^2)(b+a\tanh(dx+c))^3\ln(1-\tanh(dx+c))+((a^6dx+\cdots}{\cdots}$
risch	$\frac{x}{a^4+4a^3b+6a^2b^2+4a\,b^3+b^4}+\frac{8b\,a^3x}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{8b^3ax}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{8b\,a^3c}{d(a^8-4a^6b^2+6a^4b^4-\cdots}$

```
[In] int(1/(a+b*coth(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a-b)^4*ln(coth(d*x+c)+1)+1/3*b/(a-b)/(a+b)/(a+b*coth(d*x+c))^3+a*
b/(a+b)^2/(a-b)^2/(a+b*coth(d*x+c))^2+b*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(a+b*co
th(d*x+c))-4*b*a*(a^2+b^2)/(a+b)^4/(a-b)^4*ln(a+b*coth(d*x+c))-1/2/(a+b)^4*
ln(coth(d*x+c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3698 vs. $2(167) = 334$.

Time = 0.32 (sec) , antiderivative size = 3698, normalized size of antiderivative = 21.88

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="fricas")
[Out] 1/3*(3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 +
 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^6 + 18*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a
^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)*sinh(d*
x + c)^5 + 3*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
*b^5 + 7*a*b^6 + b^7)*d*x*sinh(d*x + c)^6 - 36*a^5*b^2 + 108*a^4*b^3 - 116*
a^3*b^4 + 60*a^2*b^5 - 24*a*b^6 + 8*b^7 - 3*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^
3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*
b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^4 - 3*(12*a^5*b^2 + 4*a
^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 - 15*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35
*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^2 + 3
*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 -
 b^7)*d*x)*sinh(d*x + c)^4 + 12*(5*(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3
 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x + c)^3 - (12*a^5*b
^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^
2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^3 - 3*(a^7 + a^6*b - 3*a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 + 3*a^
2*b^5 - a*b^6 - b^7)*d*x + 3*(24*a^5*b^2 - 32*a^4*b^3 - 12*a^3*b^4 + 28*a^2
*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^
4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c)^2 + 3*(24*a^5*b^2 - 32*a^4*
b^3 - 12*a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 15*(a^7 + 7*a^6*b + 21*a
^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7)*d*x*cosh(d*x
 + c)^4 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*
a*b^6 + b^7)*d*x - 6*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a*b^6 - 4*b^7
 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2*b^5 - 5*a*b
^6 - b^7)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*(a^6*b - 3*a^5*b^2 + 4
*a^4*b^3 - 4*a^3*b^4 + 3*a^2*b^5 - a*b^6 - (a^6*b + 3*a^5*b^2 + 4*a^4*b^3 +
 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^6 - 6*(a^6*b + 3*a^5*b^2 + 4*
a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)*sinh(d*x + c)^5 - (a
^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*sinh(d*x + c)
^6 + 3*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^4 + 3*(a^6*b + a^5
*b^2 - a^2*b^5 - a*b^6 - 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a
^2*b^5 + a*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(a^6*b + 3*a^5*b^2
+ 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(d*x + c)^3 - 3*(a^6*b + a
^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(a^6*b - a^5*b
^2 - a^2*b^5 + a*b^6)*cosh(d*x + c)^2 - 3*(a^6*b - a^5*b^2 - a^2*b^5 + a*b^
```

6 + 5*(a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^6)*cosh(
d*x + c)^4 - 6*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^2)*sinh(d*
x + c)^2 - 6*((a^6*b + 3*a^5*b^2 + 4*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5 + a*b^
6)*cosh(d*x + c)^5 - 2*(a^6*b + a^5*b^2 - a^2*b^5 - a*b^6)*cosh(d*x + c)^3
+ (a^6*b - a^5*b^2 - a^2*b^5 + a*b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(
b*cosh(d*x + c) + a*sinh(d*x + c))/(cosh(d*x + c) - sinh(d*x + c))) + 6*(3*
(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*b^5 + 7*a*b^
6 + b^7)*d*x*cosh(d*x + c)^5 - 2*(12*a^5*b^2 + 4*a^4*b^3 - 16*a^3*b^4 + 4*a
*b^6 - 4*b^7 + 3*(a^7 + 5*a^6*b + 9*a^5*b^2 + 5*a^4*b^3 - 5*a^3*b^4 - 9*a^2
*b^5 - 5*a*b^6 - b^7)*d*x)*cosh(d*x + c)^3 + (24*a^5*b^2 - 32*a^4*b^3 - 12*
a^3*b^4 + 28*a^2*b^5 - 12*a*b^6 + 4*b^7 + 3*(a^7 + 3*a^6*b + a^5*b^2 - 5*a^
4*b^3 - 5*a^3*b^4 + a^2*b^5 + 3*a*b^6 + b^7)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/((a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14
*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x +
 c)^6 + 6*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5
+ 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d
*x + c)*sinh(d*x + c)^5 + (a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b
^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10
+ b^11)*d*sinh(d*x + c)^6 - 3*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a
^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b
^10 - b^11)*d*cosh(d*x + c)^4 + 3*(5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^
3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9
+ 3*a*b^10 + b^11)*d*cosh(d*x + c)^2 - (a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b
^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*
b^9 - a*b^10 - b^11)*d)*sinh(d*x + c)^4 + 3*(a^11 - a^10*b - 5*a^9*b^2 + 5*
a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5
*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x + c)^2 + 4*(5*(a^11 + 3*a^10*b - a^9*b
^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b^6 - 6*a^4*b^7 - 11*a^3*
b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^3 - 3*(a^11 + a^10*b - 5*a
^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a^5*b^6 - 10*a^4*b^7 + 5*
a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(
5*(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5
*b^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^
4 - 6*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10
*a^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x +
 c)^2 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 -
10*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d)*sinh(d*
x + c)^2 - (a^11 - 3*a^10*b - a^9*b^2 + 11*a^8*b^3 - 6*a^7*b^4 - 14*a^6*b^5
 + 14*a^5*b^6 + 6*a^4*b^7 - 11*a^3*b^8 + a^2*b^9 + 3*a*b^10 - b^11)*d + 6*(
(a^11 + 3*a^10*b - a^9*b^2 - 11*a^8*b^3 - 6*a^7*b^4 + 14*a^6*b^5 + 14*a^5*b
^6 - 6*a^4*b^7 - 11*a^3*b^8 - a^2*b^9 + 3*a*b^10 + b^11)*d*cosh(d*x + c)^5
- 2*(a^11 + a^10*b - 5*a^9*b^2 - 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6*b^5 - 10*a
^5*b^6 - 10*a^4*b^7 + 5*a^3*b^8 + 5*a^2*b^9 - a*b^10 - b^11)*d*cosh(d*x + c
)^3 + (a^11 - a^10*b - 5*a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 - 10*a^6*b^5 - 10
*a^5*b^6 + 10*a^4*b^7 + 5*a^3*b^8 - 5*a^2*b^9 - a*b^10 + b^11)*d*cosh(d*x +

```
c))*sinh(d*x + c))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*coth(d*x+c))**4,x)
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(167) = 334$.

Time = 0.24 (sec) , antiderivative size = 522, normalized size of antiderivative = 3.09

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = -\frac{4\,(a^3b+ab^3)\log\left(-(a-b)e^{(-2\,dx-2\,c)}+a+b\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)d}$$

$$-\frac{}{3\,(a^{10}+2\,a^9b-3\,a^8b^2-8\,a^7b^3+2\,a^6b^4+12\,a^5b^5+2\,a^4b^6-8\,a^3b^7-3\,a^2b^8+2\,ab^9+b^{10}-3\,(a^{10}-5\,a}$$

$$+\frac{dx+c}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)d}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="maxima")
[Out] -4*(a^3*b + a*b^3)*log(-(a - b)*e^(-2*d*x - 2*c) + a + b)/((a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d) - 4/3*(9*a^4*b^2 + 18*a^3*b^3 + 11*a^2*b
^4 + 4*a*b^5 + 2*b^6 - 3*(6*a^4*b^2 + 2*a^3*b^3 - 5*a^2*b^4 - 2*a*b^5 - b^6
)*e^(-2*d*x - 2*c) + 3*(3*a^4*b^2 - 4*a^3*b^3 + b^6)*e^(-4*d*x - 4*c))/((a^
10 + 2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 2*a^6*b^4 + 12*a^5*b^5 + 2*a^4*b^6 -
 8*a^3*b^7 - 3*a^2*b^8 + 2*a*b^9 + b^10 - 3*(a^10 - 5*a^8*b^2 + 10*a^6*b^4
- 10*a^4*b^6 + 5*a^2*b^8 - b^10)*e^(-2*d*x - 2*c) + 3*(a^10 - 2*a^9*b - 3*a
^8*b^2 + 8*a^7*b^3 + 2*a^6*b^4 - 12*a^5*b^5 + 2*a^4*b^6 + 8*a^3*b^7 - 3*a^2
*b^8 - 2*a*b^9 + b^10)*e^(-4*d*x - 4*c) - (a^10 - 4*a^9*b + 3*a^8*b^2 + 8*a
^7*b^3 - 14*a^6*b^4 + 14*a^4*b^6 - 8*a^3*b^7 - 3*a^2*b^8 + 4*a*b^9 - b^10)*
e^(-6*d*x - 6*c))*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = -\frac{\frac{12\,(a^3b+ab^3)\log\left(\left|ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-a+b\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{3\,(dx+c)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{4\left(3\,(3\,a^4b^2-2\,a^3b^3-2\,a^2b^4+2\,ab^5-b^6)e^{(4\,dx+4\,c)}-3\,(\cdots\right.}{(ae^{(2\cdots}}}{3\,d}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3*b + a*b^3)*log(abs(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - a
+ b))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 3*(d*x + c)/(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + 4*(3*(3*a^4*b^2 - 2*a^3*b^3 - 2*a^2*
b^4 + 2*a*b^5 - b^6)*e^(4*d*x + 4*c) - 3*(6*a^4*b^2 - 14*a^3*b^3 + 11*a^2*b
^4 - 4*a*b^5 + b^6)*e^(2*d*x + 2*c) + (9*a^5*b^2 - 27*a^4*b^3 + 29*a^3*b^4
- 15*a^2*b^5 + 6*a*b^6 - 2*b^7)/(a + b))/((a*e^(2*d*x + 2*c) + b*e^(2*d*x +
 2*c) - a + b)^3*(a + b)^3*(a - b)^4))/d
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.83

$$\int \frac{1}{(a+b\coth(c+dx))^4}\,dx = \frac{x}{(a-b)^4}-\frac{\ln\left(b-a+a\,e^{2\,c}\,e^{2\,d\,x}+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,(4\,a^3\,b+4\,a\,b^3)}{d\,a^8-4\,d\,a^6\,b^2+6\,d\,a^4\,b^4-4\,d\,a^2\,b^6+d\,b^8}$$
$$-\frac{4\,(3\,a^2\,b^2-2\,a\,b^3+b^4)}{d\,(a+b)^4\,(a-b)^3\,\left(b-a+e^{2\,c+2\,d\,x}\,(a+b)\right)}$$
$$-\frac{8\,b^4}{3\,d\,(a+b)^4\,(a-b)\,\left(e^{6\,c+6\,d\,x}\,(a+b)^3-(a-b)^3+3\,e^{2\,c+2\,d\,x}\,(a+b)\,(a-b)^2-3\,e^{4\,c+4\,d\,x}\,(a+b)^2\,(a-b\cdots\right.}$$
$$+\frac{4\,(2\,a\,b^3-b^4)}{d\,(a+b)^4\,(a-b)^2\,\left(e^{4\,c+4\,d\,x}\,(a+b)^2+(a-b)^2-2\,e^{2\,c+2\,d\,x}\,(a+b)\,(a-b)\right)}$$

```
[In] int(1/(a + b*coth(c + d*x))^4,x)
```

```
[Out] x/(a - b)^4 - (log(b - a + a*exp(2*c)*exp(2*d*x) + b*exp(2*c)*exp(2*d*x))*(
4*a*b^3 + 4*a^3*b))/(a^8*d + b^8*d - 4*a^2*b^6*d + 6*a^4*b^4*d - 4*a^6*b^2*
d) - (4*(b^4 - 2*a*b^3 + 3*a^2*b^2))/(d*(a + b)^4*(a - b)^3*(b - a + exp(2*
c + 2*d*x)*(a + b))) - (8*b^4)/(3*d*(a + b)^4*(a - b)*(exp(6*c + 6*d*x)*(a
+ b)^3 - (a - b)^3 + 3*exp(2*c + 2*d*x)*(a + b)*(a - b)^2 - 3*exp(4*c + 4*d
*x)*(a + b)^2*(a - b))) + (4*(2*a*b^3 - b^4))/(d*(a + b)^4*(a - b)^2*(exp(4
*c + 4*d*x)*(a + b)^2 + (a - b)^2 - 2*exp(2*c + 2*d*x)*(a + b)*(a - b)))
```

3.85 $\int \frac{1}{4+6\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{x}{5}+\frac{3\log(3\cosh(c+dx)+2\sinh(c+dx))}{10d}$$

```
[Out] -1/5*x+3/10*ln(3*cosh(d*x+c)+2*sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3565, 3611}

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \frac{3\log(2\sinh(c+dx)+3\cosh(c+dx))}{10d}-\frac{x}{5}$$

```
[In] Int[(4 + 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -1/5*x + (3*Log[3*Cosh[c + d*x] + 2*Sinh[c + d*x]])/(10*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{x}{5} + \frac{3}{10}i\int \frac{-6i - 4i\coth(c+dx)}{4+6\coth(c+dx)}\,dx$$
$$= -\frac{x}{5} + \frac{3\log(3\cosh(c+dx)+2\sinh(c+dx))}{10d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{\log(1-\tanh(c+dx))}{20d} - \frac{\log(1+\tanh(c+dx))}{4d} + \frac{3\log(3+2\tanh(c+dx))}{10d}$$

```
[In] Integrate[(4 + 6*Coth[c + d*x])^(-1),x]
[Out] -1/20*Log[1 - Tanh[c + d*x]]/d - Log[1 + Tanh[c + d*x]]/(4*d) + (3*Log[3 + 2*Tanh[c + d*x]])/(10*d)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{x}{2}-\frac{3c}{5d}+\frac{3\ln\left(e^{2dx+2c}+\frac{1}{5}\right)}{10d}$	28
parallelrisch	$\frac{-5dx-3\ln(1-\tanh(dx+c))+3\ln(2\tanh(dx+c)+3)-\ln(8)}{10d}$	41
derivativedivides	$\frac{\frac{3\ln(2+3\coth(dx+c))}{5}-\frac{\ln(\coth(dx+c)+1)}{2}-\frac{\ln(\coth(dx+c)-1)}{10}}{2d}$	42
default	$\frac{\frac{3\ln(2+3\coth(dx+c))}{5}-\frac{\ln(\coth(dx+c)+1)}{2}-\frac{\ln(\coth(dx+c)-1)}{10}}{2d}$	42

```
[In] int(1/(4+6*coth(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/2*x-3/5*c/d+3/10/d*ln(exp(2*d*x+2*c)+1/5)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{5\,dx-3\,\log\left(\frac{2\,(3\,\cosh(dx+c)+2\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="fricas")
[Out] -1/10*(5*d*x - 3*log(2*(3*cosh(d*x + c) + 2*sinh(d*x + c))/(cosh(d*x + c) -
 sinh(d*x + c))))/d
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \begin{cases} \frac{x}{10}-\frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d}+\frac{3\log\left(2\tanh\left(c+dx\right)+3\right)}{10d} & \text{for } d\neq 0 \\ \frac{x}{6\coth\left(c\right)+4} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(4+6*coth(d*x+c)),x)
[Out] Piecewise((x/10 - 3*log(tanh(c + d*x) + 1)/(10*d) + 3*log(2*tanh(c + d*x) +
 3)/(10*d), Ne(d, 0)), (x/(6*coth(c) + 4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \frac{dx+c}{10\,d}+\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+5\right)}{10\,d}$$

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="maxima")
[Out] 1/10*(d*x + c)/d + 3/10*log(e^(-2*d*x - 2*c) + 5)/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = -\frac{5\,dx+5\,c-3\,\log\left(5\,e^{(2\,dx+2\,c)}+1\right)}{10\,d}$$

```
[In] integrate(1/(4+6*coth(d*x+c)),x, algorithm="giac")
[Out] -1/10*(5*d*x + 5*c - 3*log(5*e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{4+6\coth(c+dx)}\,dx = \frac{3\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+\frac{1}{5}\right)}{10\,d}-\frac{x}{2}$$

```
[In] int(1/(6*coth(c + d*x) + 4),x)
[Out] (3*log(exp(2*c)*exp(2*d*x) + 1/5))/(10*d) - x/2
```

3.86 $\int \frac{1}{4-6\coth(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{4-6\coth(c+dx)}\,dx=-\frac{x}{5}-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}$$

```
[Out] -1/5*x-3/10*ln(3*cosh(d*x+c)-2*sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3565, 3611}

$$\int \frac{1}{4-6\coth(c+dx)}\,dx=-\frac{3\log(3\cosh(c+dx)-2\sinh(c+dx))}{10d}-\frac{x}{5}$$

```
[In] Int[(4 - 6*Coth[c + d*x])^(-1),x]
```

```
[Out] -1/5*x - (3*Log[3*Cosh[c + d*x] - 2*Sinh[c + d*x]])/(10*d)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\text{integral} = -\frac{x}{5} - \frac{3}{10}i\int \frac{6i - 4i\coth(c+dx)}{4 - 6\coth(c+dx)}\,dx$$
$$= -\frac{x}{5} - \frac{3\log(3\cosh(c+dx) - 2\sinh(c+dx))}{10d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{1}{4 - 6\coth(c+dx)}\,dx = -\frac{3\log(3 - 2\tanh(c+dx))}{10d} + \frac{\log(1 - \tanh(c+dx))}{4d} + \frac{\log(1 + \tanh(c+dx))}{20d}$$

```
[In] Integrate[(4 - 6*Coth[c + d*x])^(-1),x]
[Out] (-3*Log[3 - 2*Tanh[c + d*x]])/(10*d) + Log[1 - Tanh[c + d*x]]/(4*d) + Log[1
 + Tanh[c + d*x]]/(20*d)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{x}{10} + \frac{3c}{5d} - \frac{3\ln(e^{2dx+2c}+5)}{10d}$	28
parallelrisch	$\frac{3\ln(1-\tanh(dx+c))-3\ln(2\tanh(dx+c)-3)+\ln(8)+dx}{10d}$	38
derivativedivides	$\frac{\frac{\ln(\coth(dx+c)-1)}{2}+\frac{\ln(\coth(dx+c)+1)}{10}-\frac{3\ln(-2+3\coth(dx+c))}{5}}{2d}$	42
default	$\frac{\frac{\ln(\coth(dx+c)-1)}{2}+\frac{\ln(\coth(dx+c)+1)}{10}-\frac{3\ln(-2+3\coth(dx+c))}{5}}{2d}$	42

```
[In] int(1/(4-6*coth(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/10*x+3/5*c/d-3/10/d*ln(exp(2*d*x+2*c)+5)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.55

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \frac{dx-3\,\log\left(\frac{2\,(3\,\cosh(dx+c)-2\,\sinh(dx+c))}{\cosh(dx+c)-\sinh(dx+c)}\right)}{10\,d}$$

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/10*(d*x - 3*log(2*(3*cosh(d*x + c) - 2*sinh(d*x + c))/(cosh(d*x + c) - si
nh(d*x + c))))/d
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \begin{cases} -\frac{x}{2}+\frac{3\log\left(\tanh\left(c+dx\right)+1\right)}{10d}-\frac{3\log\left(2\tanh\left(c+dx\right)-3\right)}{10d} & \text{for } d\neq 0 \\ \frac{x}{4-6\coth\left(c\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(4-6*coth(d*x+c)),x)
```

```
[Out] Piecewise((-x/2 + 3*log(tanh(c + d*x) + 1)/(10*d) - 3*log(2*tanh(c + d*x) -
 3)/(10*d), Ne(d, 0)), (x/(4 - 6*coth(c)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = -\frac{1}{2}\,x-\frac{c}{2\,d}-\frac{3\,\log\left(5\,e^{(-2\,dx-2\,c)}+1\right)}{10\,d}$$

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*c/d - 3/10*log(5*e^(-2*d*x - 2*c) + 1)/d
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \frac{dx+c-3\,\log\left(e^{(2\,dx+2\,c)}+5\right)}{10\,d}$$

```
[In] integrate(1/(4-6*coth(d*x+c)),x, algorithm="giac")
[Out] 1/10*(d*x + c - 3*log(e^(2*d*x + 2*c) + 5))/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{4-6\coth(c+dx)}\,dx = \frac{x}{10}-\frac{3\,\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+5\right)}{10\,d}$$

```
[In] int(-1/(6*coth(c + d*x) - 4),x)
[Out] x/10 - (3*log(exp(2*c)*exp(2*d*x) + 5))/(10*d)
```

3.87 $\int \sqrt{a+b\coth(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \sqrt{a+b\coth(c+dx)}\,dx = -\frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d} + \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] -arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d+arctanh((a+b*co
th(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3566, 714, 1144, 213}

$$\int \sqrt{a+b\coth(c+dx)}\,dx = \frac{\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d} - \frac{\sqrt{a-b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}$$

```
[In] Int[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]])/d) + (Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]])/d
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 714

```
Int[Sqrt[(d_) + (e_.)*(x_)]/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, S
ubst[Int[x^2/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x]
 /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{\sqrt{a+x}}{-b^2+x^2}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{x^2}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}\\
&= \frac{(a-b)\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}\\
&\quad -\frac{(a+b)\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}\\
&= -\frac{\sqrt{a-b}\,\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d}+\frac{\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.82 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int \sqrt{a+b\coth(c+dx)}\,dx = \frac{\left(-\sqrt{i(a-b)}\operatorname{arctanh}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a-b)}}\right)+\sqrt{i(a+b)}\operatorname{arctanh}\left(\frac{\sqrt{i(a+b\coth(c+dx))}}{\sqrt{i(a+b)}}\right)\right)\sqrt{a+b\coth(c+dx)}}{d\sqrt{i(a+b\coth(c+dx))}}$$

```
[In] Integrate[Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] ((-(Sqrt[I*(a - b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a - b)]])
 + Sqrt[I*(a + b)]*ArcTanh[Sqrt[I*(a + b*Coth[c + d*x])]/Sqrt[I*(a + b)]])*
Sqrt[a + b*Coth[c + d*x]])/(d*Sqrt[I*(a + b*Coth[c + d*x])])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)\sqrt{a+b}}{d}$	63
default	$-\frac{\sqrt{-a+b}\,\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)\sqrt{a+b}}{d}$	63

```
[In] int((a+b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(-a+b)^(1/2)*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))+arctanh((a+b
*coth(d*x+c))^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(62) = 124.

Time = 0.31 (sec) , antiderivative size = 2231, normalized size of antiderivative = 30.15

$$\int \sqrt{a+b\coth(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh(d*x + c
```

```
)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c
)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*x + c)^4
 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - (2*a
 + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)
^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)
+ a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)) + sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*
b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*si
nh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x
 + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 -
2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x +
 c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*s
inh(d*x + c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)) -
sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*
x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c
)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh
(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x
 + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh
(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x +
c)^3 + sinh(d*x + c)^4)))/d, -1/4*(2*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^
2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a +
b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cos
h(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh
(d*x + c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*log(2*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a
^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^
2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x +
 c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*si
nh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2
```

```
 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/d, -1/2*(sqrt(-a + b)*arctan(-(a*cos
h(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)
*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2
 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2
- b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + sqrt(-a - b)*arctan(((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x
 + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
 + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 - a^2 + b^2)))/d
]
```

Sympy [F]

$$\int \sqrt{a+b\coth(c+dx)}\,dx=\int \sqrt{a+b\coth{(c+dx)}}\,dx$$

```
[In] integrate((a+b*coth(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*coth(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\coth(c+dx)}\,dx=\int \sqrt{b\coth{(dx+c)}+a}\,dx$$

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*coth(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+b\coth(c+dx)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.04

$$\int \sqrt{a+b\coth(c+dx)}\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{b^2\sqrt{a-b}\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}+a\,b\sqrt{a-b}\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a-b}\,1\mathrm{i}}{d}+\frac{\operatorname{atan}\left(\frac{b^2\sqrt{a+b}\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}-a\,b\sqrt{a+b}\sqrt{a+b\coth(c+d\,x)}\,1\mathrm{i}}{a^2\,b-b^3}\right)\sqrt{a+b}\,1\mathrm{i}}{d}$$

```
[In] int((a + b*coth(c + d*x))^(1/2),x)
```

```
[Out] (atan((b^2*(a - b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i + a*b*(a - b)^(1/2)
*(a + b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a - b)^(1/2)*1i)/d + (atan
((b^2*(a + b)^(1/2)*(a + b*coth(c + d*x))^(1/2)*1i - a*b*(a + b)^(1/2)*(a +
 b*coth(c + d*x))^(1/2)*1i)/(a^2*b - b^3))*(a + b)^(1/2)*1i)/d
```

3.88 $\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

```
[Out] -arctanh((a+b*coth(d*x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+arctanh((a+b*co
th(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3566, 722, 1107, 213}

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}}$$

```
[In] Int[1/Sqrt[a + b*Coth[c + d*x]],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3566

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Sub
st[Int[(a + x)^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d, n}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+x}(-b^2+x^2)}\,dx, x, b\coth(c+dx)\right)}{d}\\
&= -\frac{(2b)\text{Subst}\left(\int \frac{1}{a^2-b^2-2ax^2+x^4}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{-a-b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}\\
&\quad+\frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\coth(c+dx)}\right)}{d}\\
&= -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

```
[In] Integrate[1/Sqrt[a + b*Coth[c + d*x]],x]
[Out] -(ArcTanh[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d)) + ArcTanh
[Sqrt[a + b*Coth[c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}+\frac{\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}}$	62
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}+\frac{\arctan\left(\frac{\sqrt{a+b\coth(dx+c)}}{\sqrt{-a+b}}\right)}{d\sqrt{-a+b}}$	62

```
[In] int(1/(a+b*coth(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh((a+b*coth(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)+1/d/(-a+b)^(1/2)
*arctan((a+b*coth(d*x+c))^(1/2)/(-a+b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(62) = 124.

Time = 0.32 (sec) , antiderivative size = 2307, normalized size of antiderivative = 31.18

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/4*(sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*sinh
(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b^2 + 2*((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(
d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d
*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x
+ c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c)) + (a + b)*sqrt(a - b)*log(((2*a^2 - b^2)*cosh(d*x + c)^4 +
4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 - b^2)*sinh(d*x + c)
^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)*cosh(d*x + c)^2 - 2
*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 - 2*(a*cosh(d*x + c)^
4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - (2*a - b)*cosh(
d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x + c)^2 + 2*(2*a*cosh(
d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)*sqrt(a - b)*sq
rt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) + 4*((2*a^2 - b^2)*co
sh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4
```

```
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a^2 - b^2)*d), -1/4*(
2*(a - b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
 + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cos
h(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b +
 b^2)*sinh(d*x + c)^2 - a^2 + b^2)) - (a + b)*sqrt(a - b)*log(((2*a^2 - b^2
)*cosh(d*x + c)^4 + 4*(2*a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2
- b^2)*sinh(d*x + c)^4 - 4*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(2*a^2 - b^2)
*cosh(d*x + c)^2 - 2*a^2 + 2*a*b)*sinh(d*x + c)^2 + 2*a^2 - 4*a*b + 2*b^2 -
 2*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)
^4 - (2*a - b)*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 2*a + b)*sinh(d*x +
 c)^2 + 2*(2*a*cosh(d*x + c)^3 - (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + a
 - b)*sqrt(a - b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c)) +
 4*((2*a^2 - b^2)*cosh(d*x + c)^3 - 2*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x +
 c))/(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2
*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4)))/((a
^2 - b^2)*d), -1/4*(2*(a + b)*sqrt(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a
*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt
((b*cosh(d*x + c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x +
 c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x +
c)^2 - a^2 + 2*a*b - b^2)) - sqrt(a + b)*(a - b)*log(2*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(
a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 - 4*(a^2 + a*b)*cosh(d*x + c)^2 + 4*(3*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 + 2*a^2 - b
^2 + 2*((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 - (2*a + b)*cosh(d*x + c)^2 + (6*(a + b)*cosh(d*x
+ c)^2 - 2*a - b)*sinh(d*x + c)^2 + 2*(2*(a + b)*cosh(d*x + c)^3 - (2*a + b
)*cosh(d*x + c))*sinh(d*x + c) + a)*sqrt(a + b)*sqrt((b*cosh(d*x + c) + a*s
inh(d*x + c))/sinh(d*x + c)) + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (a^
2 + a*b)*cosh(d*x + c))*sinh(d*x + c)))/((a^2 - b^2)*d), -1/2*((a + b)*sqrt
(-a + b)*arctan(-(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*s
inh(d*x + c)^2 - a + b)*sqrt(-a + b)*sqrt((b*cosh(d*x + c) + a*sinh(d*x + c
))/sinh(d*x + c))/((a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c
)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c)^2 - a^2 + 2*a*b - b^2)) + (a -
b)*sqrt(-a - b)*arctan(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*s
inh(d*x + c) + (a + b)*sinh(d*x + c)^2 - a)*sqrt(-a - b)*sqrt((b*cosh(d*x +
 c) + a*sinh(d*x + c))/sinh(d*x + c))/((a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^2 - a^2 + b^2)))/((a^2 - b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\coth{(c+dx)}}}\,dx$$

```
[In] integrate(1/(a+b*coth(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*coth(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\coth{(dx+c)}+a}}\,dx$$

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*coth(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*coth(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 242, normalized size of antiderivative = 3.27

$$\int \frac{1}{\sqrt{a+b\coth(c+dx)}}\,dx = \frac{\mathrm{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3-b\,d^3}-\frac{16\,a\,b^3\,d^3}{a\,d^3-b\,d^3}\right)\,\sqrt{a-b}}+\frac{\left(a\,d^3-b\,d^3\right)\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{b\,d^3\,\sqrt{a-b}}\right)}{d\,\sqrt{a-b}} - \frac{\mathrm{atanh}\left(\frac{16\,a\,b^2\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{\left(\frac{16\,b^4\,d^3}{a\,d^3+b\,d^3}+\frac{16\,a\,b^3\,d^3}{a\,d^3+b\,d^3}\right)\,\sqrt{a+b}}-\frac{\left(a\,d^3+b\,d^3\right)\,\sqrt{a+b\,\mathrm{coth}(c+d\,x)}}{b\,d^3\,\sqrt{a+b}}\right)}{d\,\sqrt{a+b}}$$

```
[In] int(1/(a + b*coth(c + d*x))^(1/2),x)
```

```
[Out] atanh((16*a*b^2*(a + b*coth(c + d*x))^(1/2))/((((16*b^4*d^3)/(a*d^3 - b*d^3)
 - (16*a*b^3*d^3)/(a*d^3 - b*d^3))*(a - b)^(1/2)) + ((a*d^3 - b*d^3)*(a + b
*coth(c + d*x))^(1/2))/(b*d^3*(a - b)^(1/2))))/(d*(a - b)^(1/2)) - atanh((16
*a*b^2*(a + b*coth(c + d*x))^(1/2))/((((16*b^4*d^3)/(a*d^3 + b*d^3) + (16*a*
b^3*d^3)/(a*d^3 + b*d^3))*(a + b)^(1/2)) - ((a*d^3 + b*d^3)*(a + b*coth(c +
 d*x))^(1/2))/(b*d^3*(a + b)^(1/2))))/(d*(a + b)^(1/2))
```

3.89 $\int \frac{\sinh^4(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{5x}{16}+\frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))} - \frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}$$

```
[Out] 5/16*x+1/32/(1-coth(x))^2+1/8/(1-coth(x))-1/24/(1+coth(x))^3-3/32/(1+coth(x
))^2-3/16/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3568, 46, 213}

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{5x}{16}+\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)} +\frac{1}{32(1-\coth(x))^2}-\frac{3}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

```
[In] Int[Sinh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/16 + 1/(32*(1 - Coth[x])^2) + 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x
])^3) - 3/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
```

```
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{1}{(1-x)^3(1+x)^4}\,dx, x, \coth(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}+\frac{1}{8(1+x)^4}+\frac{3}{16(1+x)^3}+\frac{3}{16(1+x)^2}\right.\right.\\
&\qquad\left.\left.-\frac{5}{16\,(-1+x^2)}\right)\,dx, x, \coth(x)\right)\\
&= \frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}\\
&\quad -\frac{3}{16(1+\coth(x))}-\frac{5}{16}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \coth(x)\right)\\
&= \frac{5x}{16}+\frac{1}{32(1-\coth(x))^2}+\frac{1}{8(1-\coth(x))}\\
&\quad -\frac{1}{24(1+\coth(x))^3}-\frac{3}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{1}{192}(60x+15\cosh(2x)-6\cosh(4x)+\cosh(6x)-45\sinh(2x)\\
+9\sinh(4x)-\sinh(6x))$$

```
[In] Integrate[Sinh[x]^4/(1 + Coth[x]),x]
[Out] (60*x + 15*Cosh[2*x] - 6*Cosh[4*x] + Cosh[6*x] - 45*Sinh[2*x] + 9*Sinh[4*x]
 - Sinh[6*x])/192
```

Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result
risch	$\frac{5x}{16}+\frac{e^{4x}}{128}-\frac{5\,e^{2x}}{64}+\frac{5\,e^{-2x}}{32}-\frac{5\,e^{-4x}}{128}+\frac{e^{-6x}}{192}$
parallelrisch	$\frac{19}{96}-\frac{\cosh(4x)}{32}+\frac{\cosh(6x)}{192}+\frac{5\cosh(2x)}{64}+\frac{3\sinh(4x)}{64}-\frac{15\sinh(2x)}{64}-\frac{\sinh(6x)}{192}-\frac{5\ln(1-\tanh(x))}{32}+\frac{5\ln(1+\tanh}{32}$
default	$\frac{1}{3(\tanh(\frac{x}{2})+1)^6}-\frac{1}{(\tanh(\frac{x}{2})+1)^5}+\frac{5}{8(\tanh(\frac{x}{2})+1)^4}+\frac{5}{12(\tanh(\frac{x}{2})+1)^3}-\frac{3}{8(\tanh(\frac{x}{2})+1)}+\frac{5\ln(\tanh(\frac{x}{2})+1)}{16}+$

```
[In] int(sinh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*x+1/128*exp(4*x)-5/64*exp(2*x)+5/32*exp(-2*x)-5/128*exp(-4*x)+1/192*ex
p(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(44) = 88.

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.55

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{5\,\cosh\left(x\right)^{5}+25\,\cosh\left(x\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}+5\,\left(2\,\cosh\left(x\right)^{2}-3\right)\sinh\left(x\right)^{3}-45\,\cosh\left(x\right)^{3}+5\,\left(10\,\cosh\right.}{384\,(\cosh\left(x\right)}$$

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + 5*(2*cosh(x)^2 - 3)
*sinh(x)^3 - 45*cosh(x)^3 + 5*(10*cosh(x)^3 - 27*cosh(x))*sinh(x)^2 + 60*(2
*x + 1)*cosh(x) + 5*(cosh(x)^4 - 9*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x)
 + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \int \frac{\sinh^4\left(x\right)}{\coth\left(x\right)+1}\,dx$$

```
[In] integrate(sinh(x)**4/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = -\frac{1}{128}\left(10\,e^{(-2\,x)}-1\right)e^{(4\,x)}+\frac{5}{16}\,x+\frac{5}{32}\,e^{(-2\,x)}-\frac{5}{128}\,e^{(-4\,x)}+\frac{1}{192}\,e^{(-6\,x)}$$

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="maxima")
[Out] -1/128*(10*e^(-2*x) - 1)*e^(4*x) + 5/16*x + 5/32*e^(-2*x) - 5/128*e^(-4*x)
+ 1/192*e^(-6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = -\frac{1}{384}\left(110\,e^{(6\,x)}-60\,e^{(4\,x)}+15\,e^{(2\,x)}-2\right)e^{(-6\,x)} +\frac{5}{16}\,x+\frac{1}{128}\,e^{(4\,x)}-\frac{5}{64}\,e^{(2\,x)}$$

```
[In] integrate(sinh(x)^4/(1+coth(x)),x, algorithm="giac")
[Out] -1/384*(110*e^(6*x) - 60*e^(4*x) + 15*e^(2*x) - 2)*e^(-6*x) + 5/16*x + 1/12
8*e^(4*x) - 5/64*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\sinh^4(x)}{1+\coth(x)}\,dx = \frac{5\,x}{16}+\frac{5\,\mathrm{e}^{-2\,x}}{32}-\frac{5\,\mathrm{e}^{2\,x}}{64}-\frac{5\,\mathrm{e}^{-4\,x}}{128}+\frac{\mathrm{e}^{4\,x}}{128}+\frac{\mathrm{e}^{-6\,x}}{192}$$

```
[In] int(sinh(x)^4/(coth(x) + 1),x)
[Out] (5*x)/16 + (5*exp(-2*x))/32 - (5*exp(2*x))/64 - (5*exp(-4*x))/128 + exp(4*x
)/128 + exp(-6*x)/192
```

3.90 $\int \frac{\sinh^3(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\sinh^3(x)}{1+\coth(x)} dx = -\frac{4\cosh(x)}{5} + \frac{4\cosh^3(x)}{15} - \frac{\sinh^3(x)}{5(1+\coth(x))}$$

```
[Out] -4/5*cosh(x)+4/15*cosh(x)^3-1/5*sinh(x)^3/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3583, 2713}

$$\int \frac{\sinh^3(x)}{1+\coth(x)} dx = \frac{4\cosh^3(x)}{15} - \frac{4\cosh(x)}{5} - \frac{\sinh^3(x)}{5(\coth(x)+1)}$$

```
[In] Int[Sinh[x]^3/(1 + Coth[x]),x]
[Out] (-4*Cosh[x])/5 + (4*Cosh[x]^3)/15 - Sinh[x]^3/(5*(1 + Coth[x]))
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
```

```
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sinh^3(x)}{5(1+\coth(x))}+\frac{4}{5}\int \sinh^3(x)\,dx \\
&= -\frac{\sinh^3(x)}{5(1+\coth(x))}-\frac{4}{5}\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cosh(x)\right) \\
&= -\frac{4\cosh(x)}{5}+\frac{4\cosh^3(x)}{15}-\frac{\sinh^3(x)}{5(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx=\frac{\operatorname{csch}(x)(-45-20\cosh(2x)+\cosh(4x)-40\sinh(2x)+4\sinh(4x))}{120(1+\coth(x))}$$

```
[In] Integrate[Sinh[x]^3/(1 + Coth[x]),x]
[Out] (Csch[x]*(-45 - 20*Cosh[2*x] + Cosh[4*x] - 40*Sinh[2*x] + 4*Sinh[4*x]))/(12
0*(1 + Coth[x]))
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result
risch	$\frac{e^{3x}}{48}-\frac{e^{x}}{4}-\frac{3\,e^{-x}}{8}+\frac{e^{-3x}}{12}-\frac{e^{-5x}}{80}$
parallelrisch	$-\frac{8}{15}-\frac{5\cosh(x)}{8}+\frac{\sinh(x)}{8}+\frac{\sinh(5x)}{80}+\frac{5\cosh(3x)}{48}-\frac{\cosh(5x)}{80}-\frac{\sinh(3x)}{16}$
default	$-\frac{2}{5\left(\tanh\left(\frac{x}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{1}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-$ 4(

```
[In] int(sinh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/48*exp(3*x)-1/4*exp(x)-3/8*exp(-x)+1/12*exp(-3*x)-1/80*exp(-5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \frac{\cosh(x)^4 + 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 10\right)\sinh(x)^2 - 20\cosh(x)^2 + 16\left(\cosh(x\right.}{120\left(\cosh(x) + \sinh(x)\right)}$$

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/120*(cosh(x)^4 + 16*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 10)*
sinh(x)^2 - 20*cosh(x)^2 + 16*(cosh(x)^3 - 5*cosh(x))*sinh(x) - 45)/(cosh(x
) + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \int \frac{\sinh^3(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(sinh(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(sinh(x)**3/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = -\frac{1}{48}\left(12\,e^{(-2\,x)} - 1\right)e^{(3\,x)} - \frac{3}{8}\,e^{(-x)} + \frac{1}{12}\,e^{(-3\,x)} - \frac{1}{80}\,e^{(-5\,x)}$$

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -1/48*(12*e^(-2*x) - 1)*e^(3*x) - 3/8*e^(-x) + 1/12*e^(-3*x) - 1/80*e^(-5*x
)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = -\frac{1}{240}\left(90\,e^{(4\,x)} - 20\,e^{(2\,x)} + 3\right)e^{(-5\,x)} + \frac{1}{48}\,e^{(3\,x)} - \frac{1}{4}\,e^{x}$$

```
[In] integrate(sinh(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] -1/240*(90*e^(4*x) - 20*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) - 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{1+\coth(x)}\,dx = \frac{e^{-3\,x}}{12} - \frac{3\,e^{-x}}{8} + \frac{e^{3\,x}}{48} - \frac{e^{-5\,x}}{80} - \frac{e^{x}}{4}$$

```
[In] int(sinh(x)^3/(coth(x) + 1),x)
[Out] exp(-3*x)/12 - (3*exp(-x))/8 + exp(3*x)/48 - exp(-5*x)/80 - exp(x)/4
```

3.91 $\int \frac{\sinh^2(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = -\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}$$

```
[Out] -3/8*x-1/8/(1-coth(x))+1/8/(1+coth(x))^2+1/4/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3568, 46, 213}

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = -\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{4(\coth(x)+1)}+\frac{1}{8(\coth(x)+1)^2}$$

```
[In] Int[Sinh[x]^2/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) + 1/(4*(1 + Coth[x])
)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{(1-x)^2(1+x)^3}\,dx, x, \coth(x)\right) \\
&= -\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\left(-1+x^2\right)}\right)dx, x, \coth(x)\right) \\
&= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \coth(x)\right) \\
&= -\frac{3x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}+\frac{1}{4(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{1}{32}(-12x-4\cosh(2x)+\cosh(4x)+8\sinh(2x)-\sinh(4x))$$

```
[In] Integrate[Sinh[x]^2/(1 + Coth[x]),x]
[Out] (-12*x - 4*Cosh[2*x] + Cosh[4*x] + 8*Sinh[2*x] - Sinh[4*x])/32
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{3x}{8}+\frac{\mathrm{e}^{2x}}{16}-\frac{3\,\mathrm{e}^{-2x}}{16}+\frac{\mathrm{e}^{-4x}}{32}$
parallelrisch	$-\frac{3x}{8}+\frac{3}{32}-\frac{\cosh(2x)}{8}+\frac{\sinh(2x)}{4}-\frac{\sinh(4x)}{32}+\frac{\cosh(4x)}{32}$
default	$\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{4}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{1}{4\tanh\left(\frac{x}{2}\right)-4}+$ 3 ln(ta

```
[In] int(sinh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -3/8*x+1/16*exp(2*x)-3/16*exp(-2*x)+1/32*exp(-4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{3\,\cosh(x)^3+9\,\cosh(x)\sinh(x)^2+\sinh(x)^3-6\,(2\,x+1)\cosh(x)+3\,\left(\cosh(x)^2-4\,x+2\right)\sinh(x)}{32\,(\cosh(x)+\sinh(x))}$$

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="fricas")
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 - 6*(2*x + 1)*cosh(x) +
 3*(cosh(x)^2 - 4*x + 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \int \frac{\sinh^2(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(sinh(x)**2/(1+coth(x)),x)
[Out] Integral(sinh(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = -\frac{3}{8}\,x + \frac{1}{16}\,e^{(2\,x)} - \frac{3}{16}\,e^{(-2\,x)} + \frac{1}{32}\,e^{(-4\,x)}$$

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] -3/8*x + 1/16*e^(2*x) - 3/16*e^(-2*x) + 1/32*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{1}{32}\,\left(9\,e^{(4\,x)} - 6\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - \frac{3}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

```
[In] integrate(sinh(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] 1/32*(9*e^(4*x) - 6*e^(2*x) + 1)*e^(-4*x) - 3/8*x + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{1+\coth(x)}\,dx = \frac{\mathrm{e}^{2\,x}}{16} - \frac{3\,\mathrm{e}^{-2\,x}}{16} - \frac{3\,x}{8} + \frac{\mathrm{e}^{-4\,x}}{32}$$

```
[In] int(sinh(x)^2/(coth(x) + 1),x)
[Out] exp(2*x)/16 - (3*exp(-2*x))/16 - (3*x)/8 + exp(-4*x)/32
```

3.92 $\int \frac{\sinh(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{\sinh(x)}{1+\coth(x)} \, dx = \frac{2\cosh(x)}{3} - \frac{\sinh(x)}{3(1+\coth(x))}$$

```
[Out] 2/3*cosh(x)-1/3*sinh(x)/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3583, 2718}

$$\int \frac{\sinh(x)}{1+\coth(x)} \, dx = \frac{2\cosh(x)}{3} - \frac{\sinh(x)}{3(\coth(x)+1)}$$

```
[In] Int[Sinh[x]/(1 + Coth[x]),x]
[Out] (2*Cosh[x])/3 - Sinh[x]/(3*(1 + Coth[x]))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3583

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[a*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(b*f*(m + 2*n))), x] + Dist[Simplify[m + n]/(a*(m + 2*n)), Int[(d*Sec[e + f
*x])^m*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, m}, x]
```

```
 && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && NeQ[m + 2*n, 0] && IntegersQ[2*m, 2*n
]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sinh(x)}{3(1+\coth(x))}+\frac{2}{3}\int \sinh(x)\,dx\\ &= \frac{2\cosh(x)}{3}-\frac{\sinh(x)}{3(1+\coth(x))}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx=\frac{1}{12}\left(9\cosh(x)-\cosh(3x)+4\sinh^3(x)\right)$$

```
[In] Integrate[Sinh[x]/(1 + Coth[x]),x]
[Out] (9*Cosh[x] - Cosh[3*x] + 4*Sinh[x]^3)/12
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{{e}^{x}}{4}+\frac{{e}^{-x}}{2}-\frac{{e}^{-3x}}{12}$	18
parallelrisch	$-\frac{\cosh(3x)}{12}+\frac{3\cosh(x)}{4}+\frac{\sinh(3x)}{12}-\frac{\sinh(x)}{4}+\frac{1}{3}$	23
default	$-\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	40

```
[In] int(sinh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*exp(x)+1/2*exp(-x)-1/12*exp(-3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{\cosh(x)^2 + 4\cosh(x)\sinh(x) + \sinh(x)^2 + 3}{6(\cosh(x) + \sinh(x))}$$

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="fricas")
[Out] 1/6*(cosh(x)^2 + 4*cosh(x)*sinh(x) + sinh(x)^2 + 3)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \int \frac{\sinh(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(sinh(x)/(1+coth(x)),x)
[Out] Integral(sinh(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{1}{2}e^{(-x)} - \frac{1}{12}e^{(-3x)} + \frac{1}{4}e^{x}$$

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*e^(-x) - 1/12*e^(-3*x) + 1/4*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\left(6e^{(2x)} - 1\right)e^{(-3x)} + \frac{1}{4}e^{x}$$

```
[In] integrate(sinh(x)/(1+coth(x)),x, algorithm="giac")
[Out] 1/12*(6*e^(2*x) - 1)*e^(-3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sinh(x)}{1+\coth(x)}\,dx = \frac{e^{-x}}{2} - \frac{e^{-3x}}{12} + \frac{e^{x}}{4}$$

```
[In] int(sinh(x)/(coth(x) + 1),x)
[Out] exp(-x)/2 - exp(-3*x)/12 + exp(x)/4
```

3.93 $\int \frac{\operatorname{csch}(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)} \, dx = -\frac{\operatorname{csch}(x)}{1+\coth(x)}$$

```
[Out] -csch(x)/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3569}

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)} \, dx = -\frac{\operatorname{csch}(x)}{\coth(x)+1}$$

```
[In] Int[Csch[x]/(1 + Coth[x]),x]
```

```
[Out] -(Csch[x]/(1 + Coth[x]))
```

Rule 3569

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[b*(d*Sec[e + f*x])^m*((a + b*Tan[e + f*x])^n/
(a*f*m)), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 + b^2, 0] && EqQ
[Simplify[m + n], 0]
```

Rubi steps

$$\text{integral} = -\frac{\operatorname{csch}(x)}{1+\coth(x)}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -\cosh(x)+\sinh(x)$$

```
[In] Integrate[Csch[x]/(1 + Coth[x]),x]
[Out] -Cosh[x] + Sinh[x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
risch	$-{\mathrm e}^{-x}$	7
gosper	$-\frac{\operatorname{csch}(x)}{1+\coth(x)}$	11
default	$-\frac{2}{\tanh\left(\frac{x}{2}\right)+1}$	11

```
[In] int(csch(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -exp(-x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -\frac{1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="fricas")
[Out] -1/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(csch(x)/(1+coth(x)),x)
[Out] Integral(csch(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -e^{(-x)}$$

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="maxima")
[Out] -e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -e^{(-x)}$$

```
[In] integrate(csch(x)/(1+coth(x)),x, algorithm="giac")
[Out] -e^(-x)
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{\operatorname{csch}(x)}{1+\coth(x)}\,dx = -\mathrm{e}^{-x}$$

```
[In] int(1/(sinh(x)*(coth(x) + 1)),x)
[Out] -exp(-x)
```

3.94 $\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 7

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)} \, dx = -\log(1+\coth(x))$$

```
[Out] -ln(1+coth(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3568, 31}

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)} \, dx = -\log(\coth(x)+1)$$

```
[In] Int[Csch[x]^2/(1 + Coth[x]),x]
```

```
[Out] -Log[1 + Coth[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
```

```
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \coth(x)\right)\\ &= -\log(1+\coth(x))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.43

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -x+\log(\cosh(x))+\log(\tanh(x))$$

```
[In] Integrate[Csch[x]^2/(1 + Coth[x]),x]
[Out] -x + Log[Cosh[x]] + Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\ln\left(1+\coth\left(x\right)\right)$	8
default	$-\ln\left(1+\coth\left(x\right)\right)$	8
risch	$-2x+\ln\left(e^{2x}-1\right)$	12

```
[In] int(csch(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -ln(1+coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -2\,x+\log\left(\frac{2\,\sinh\left(x\right)}{\cosh\left(x\right)-\sinh\left(x\right)}\right)$$

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="fricas")
[Out] -2*x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(csch(x)**2/(1+coth(x)),x)
[Out] Integral(csch(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -\log(\coth(x)+1)$$

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] -log(coth(x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = -2x+\log\left(\left|e^{(2x)}-1\right|\right)$$

```
[In] integrate(csch(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] -2*x + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^2(x)}{1+\coth(x)}\,dx = \ln\left(e^{2x}-1\right)-2x$$

```
[In] int(1/(sinh(x)^2*(coth(x) + 1)),x)
[Out] log(exp(2*x) - 1) - 2*x
```

3.95 $\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \operatorname{arctanh}(\cosh(x)) - \operatorname{csch}(x)$$

```
[Out] arctanh(cosh(x))-csch(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3582, 3855}

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \operatorname{arctanh}(\cosh(x)) - \operatorname{csch}(x)$$

```
[In] Int[Csch[x]^3/(1 + Coth[x]),x]
[Out] ArcTanh[Cosh[x]] - Csch[x]
```

Rule 3582

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)])^(n_), x_Symbol] :> Simp[d^2*(d*Sec[e + f*x])^(m - 2)*((a + b*Tan[e +
f*x])^(n + 1)/(b*f*(m + n - 1))), x] + Dist[d^2*((m - 2)/(a*(m + n - 1))),
Int[(d*Sec[e + f*x])^(m - 2)*(a + b*Tan[e + f*x])^(n + 1), x], x] /; FreeQ[
{a, b, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0] && GtQ[m, 1] &&  !ILt
Q[m + n, 0] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{csch}(x) - \int \text{csch}(x)\,dx \\ &= \text{arctanh}(\cosh(x)) - \text{csch}(x) \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. $2(8) = 16$.

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int \frac{\text{csch}^3(x)}{1+\coth(x)}\,dx = -\text{csch}(x) + \log\left(\cosh\left(\frac{x}{2}\right)\right) - \log\left(\sinh\left(\frac{x}{2}\right)\right)$$

```
[In] Integrate[Csch[x]^3/(1 + Coth[x]),x]
[Out] -Csch[x] + Log[Cosh[x/2]] - Log[Sinh[x/2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tanh\left(\frac{x}{2}\right)} - \ln\left(\tanh\left(\frac{x}{2}\right)\right)$	23
risch	$-\frac{2\,\mathrm{e}^{x}}{\mathrm{e}^{2x}-1} + \ln\left(\mathrm{e}^{x}+1\right) - \ln\left(\mathrm{e}^{x}-1\right)$	26

```
[In] int(csch(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*tanh(1/2*x)-1/2/tanh(1/2*x)-ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 9.62

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \frac{\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sin\right.}{\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh}$$

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="fricas")
[Out] ((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) + 1)
 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(cosh(x) + sinh(x) -
1) - 2*cosh(x) - 2*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}^{3}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

```
[In] integrate(csch(x)**3/(1+coth(x)),x)
[Out] Integral(csch(x)**3/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.88

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \frac{2\,e^{(-x)}}{e^{(-2\,x)}-1}+\log\left(e^{(-x)}+1\right)-\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="maxima")
[Out] 2*e^(-x)/(e^(-2*x) - 1) + log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(8) = 16$.

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = -\frac{2\,e^x}{e^{(2\,x)}-1} + \log\left(e^x+1\right) - \log\left(|e^x-1|\right)$$

```
[In] integrate(csch(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] -2*e^x/(e^(2*x) - 1) + log(e^x + 1) - log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.62

$$\int \frac{\operatorname{csch}^3(x)}{1+\coth(x)}\,dx = \ln\left(2\,\mathrm{e}^x+2\right) - \ln\left(2\,\mathrm{e}^x-2\right) - \frac{2\,\mathrm{e}^x}{\mathrm{e}^{2\,x}-1}$$

```
[In] int(1/(sinh(x)^3*(coth(x) + 1)),x)
[Out] log(2*exp(x) + 2) - log(2*exp(x) - 2) - (2*exp(x))/(exp(2*x) - 1)
```

3.96 $\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)} dx = \coth(x) - \frac{\coth^2(x)}{2}$$

```
[Out] coth(x)-1/2*coth(x)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3568}

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)} dx = \coth(x) - \frac{\coth^2(x)}{2}$$

```
[In] Int[Csch[x]^4/(1 + Coth[x]),x]
[Out] Coth[x] - Coth[x]^2/2
```

Rule 3568

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(a^(m - 2)*b*f), Subst[Int[(a - x)^(m/2 - 1)*(a + x)
^(n + m/2 - 1), x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] &&
EqQ[a^2 + b^2, 0] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int (1-x)\,dx, x, \coth(x)\right) \\ &= \coth(x) - \frac{\coth^2(x)}{2} \end{aligned}$$

Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx=\coth(x)-\frac{\operatorname{csch}^2(x)}{2}$$

```
[In] Integrate[Csch[x]^4/(1 + Coth[x]),x]
[Out] Coth[x] - Csch[x]^2/2
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\coth(x)-\frac{\coth(x)^2}{2}$	10
default	$\coth(x)-\frac{\coth(x)^2}{2}$	10
risch	$-\frac{2}{\left(e^{2x}-1\right)^2}$	11
parallelrisch	$\coth(x)-\frac{7\coth(x)^2}{12}+\frac{\operatorname{csch}(x)^2}{12}$	16

```
[In] int(csch(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] coth(x)-1/2*coth(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 5.00

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx=$$

$$-\frac{2}{\cosh(x)^4+4\cosh(x)\sinh(x)^3+\sinh(x)^4+2\left(3\cosh(x)^2-1\right)\sinh(x)^2-2\cosh(x)^2+4\left(\cosh(x)^3-\right.}$$

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="fricas")
[Out] -2/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(
x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(csch(x)**4/(1+coth(x)),x)
[Out] Integral(csch(x)**4/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(9) = 18$.

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.73

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = \frac{4\,e^{(-2\,x)}}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1} - \frac{2}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1}$$

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="maxima")
[Out] 4*e^(-2*x)/(2*e^(-2*x) - e^(-4*x) - 1) - 2/(2*e^(-2*x) - e^(-4*x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = -\frac{2}{\left(e^{(2\,x)}-1\right)^2}$$

```
[In] integrate(csch(x)^4/(1+coth(x)),x, algorithm="giac")
[Out] -2/(e^(2*x) - 1)^2
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{csch}^4(x)}{1+\coth(x)}\,dx = -\frac{2}{e^{4\,x}-2\,e^{2\,x}+1}$$

```
[In] int(1/(sinh(x)^4*(coth(x) + 1)),x)
[Out] -2/(exp(4*x) - 2*exp(2*x) + 1)
```

3.97 $\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 155

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = -\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(1+\coth(x))}{16(a-b)^3} - \frac{b^5\log(a+b\coth(x))}{(a^2-b^2)^3} - \frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)}$$

```
[Out] -1/16*(3*a^2+9*a*b+8*b^2)*ln(1-coth(x))/(a+b)^3+1/16*(3*a^2-9*a*b+8*b^2)*ln
(1+coth(x))/(a-b)^3-b^5*ln(a+b*coth(x))/(a^2-b^2)^3-1/8*(4*b^3-a*(7-3*a^2/b
^2)*b^2*coth(x))*sinh(x)^2/(a^2-b^2)^2-1/4*(b-a*coth(x))*sinh(x)^4/(a^2-b^2
)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3587, 755, 837, 815}

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = -\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(\coth(x)+1)}{16(a-b)^3} - \frac{\sinh^4(x)(b-a\coth(x))}{4(a^2-b^2)} - \frac{b^5\log(a+b\coth(x))}{(a^2-b^2)^3} - \frac{\sinh^2(x)\left(4b^3-ab^2\left(7-\frac{3a^2}{b^2}\right)\coth(x)\right)}{8(a^2-b^2)^2}$$

```
[In] Int[Sinh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -1/16*((3*a^2 + 9*a*b + 8*b^2)*Log[1 - Coth[x]])/(a + b)^3 + ((3*a^2 - 9*a*
b + 8*b^2)*Log[1 + Coth[x]])/(16*(a - b)^3) - (b^5*Log[a + b*Coth[x]])/(a^2
 - b^2)^3 - ((4*b^3 - a*(7 - (3*a^2)/b^2)*b^2*Coth[x])*Sinh[x]^2)/(8*(a^2 -
 b^2)^2) - ((b - a*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^3}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} + \frac{b\text{Subst}\left(\int \frac{-4+\frac{3a^2}{b^2}+\frac{3ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{4(a^2-b^2)}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} - \frac{b^5\text{Subst}\left(\int \frac{-\frac{3a^4-7a^2b^2+8b^4}{b^6}+\frac{a\left(7-\frac{3a^2}{b^2}\right)x}{b^4}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\coth(x)\right)}{8(a^2-b^2)^2}$$

$$= -\frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} - \frac{b^5\text{Subst}\left(\int \left(-\frac{(a-b)^2(3a^2+9ab+8b^2)}{2b^5(a+b)(b-x)} + \frac{8}{(a-b)(a+b)(a+x)} - \frac{(a+b)^2(3a^2-9ab+8b^2)}{2(a-b)b^5(b+x)}\right)\,dx, x, b\coth(x)\right)}{8(a^2-b^2)^2}$$

$$= -\frac{(3a^2+9ab+8b^2)\log(1-\coth(x))}{16(a+b)^3} + \frac{(3a^2-9ab+8b^2)\log(1+\coth(x))}{16(a-b)^3} - \frac{b^5\log(a+b\coth(x))}{(a^2-b^2)^3} - \frac{\left(4b^3-a\left(7-\frac{3a^2}{b^2}\right)b^2\coth(x)\right)\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{12a^5x-40a^3b^2x+60ab^4x+4b(a^4-4a^2b^2+3b^4)\cosh(2x)-b(a^2-b^2)^2\cosh(4x)-32b^5\log(b\cosh(x)+a}{32(a-b}$$

```
[In] Integrate[Sinh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (12*a^5*x - 40*a^3*b^2*x + 60*a*b^4*x + 4*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Cosh[
2*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*b^5*Log[b*Cosh[x] + a*Sinh[x]] - 8*a^
5*Sinh[2*x] + 24*a^3*b^2*Sinh[2*x] - 16*a*b^4*Sinh[2*x] + a^5*Sinh[4*x] - 2
*a^3*b^2*Sinh[4*x] + a*b^4*Sinh[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [A] (verified)

Time = 5.86 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.24

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{9axb}{8(a+b)^3}+\frac{x\,b^2}{(a+b)^3}+\frac{e^{4x}}{64a+64b}-\frac{e^{2x}a}{8(a+b)^2}-\frac{3\,e^{2x}b}{16(a+b)^2}+\frac{e^{-2x}a}{8(a-b)^2}-\frac{3\,e^{-2x}b}{16(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2b^5x}{a^6-3a^4b^2+3a}$
default	$-\frac{16}{(64a-64b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{64}{(128a-128b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{-a+3b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{3a-5b}{8(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(3a^2-9ab+}{}$

```
[In] int(sinh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*a^2*x/(a+b)^3+9/8*a*x/(a+b)^3*b+x/(a+b)^3*b^2+1/64/(a+b)*exp(4*x)-1/8/(
a+b)^2*exp(2*x)*a-3/16/(a+b)^2*exp(2*x)*b+1/8/(a-b)^2*exp(-2*x)*a-3/16/(a-b
)^2*exp(-2*x)*b-1/64/(a-b)*exp(-4*x)+2*b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*x-
b^5/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1279 vs. 2(147) = 294.

Time = 0.27 (sec) , antiderivative size = 1279, normalized size of antiderivative = 8.25

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 - 4*(2*a^5 - a^4*b
- 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^6 - 4*(2*a^5 - a^4*b - 6
*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5 - 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
 b^5)*cosh(x)^3 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^
5)*cosh(x))*sinh(x)^5 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 +
 2*(35*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 30*(
2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^2 + 4*(3*a
^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x)*sinh(x)^4 + 8*(7*(a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 - 10*(2*a^5 - a^4*b - 6*a^3*b^2
+ 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)^3 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4
 + 8*b^5)*x*cosh(x))*sinh(x)^3 + 4*(2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 +
 4*a*b^4 + 3*b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a
*b^4 - b^5)*cosh(x)^6 + 2*a^5 + a^4*b - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3
*b^5 - 15*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
```

```
^4 + 12*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^2)*sinh(x)^2 - 64
*(b^5*cosh(x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b
^5*cosh(x)*sinh(x)^3 + b^5*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x
) - sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^7 - 3*(2*a^5 - a^4*b - 6*a^3*b^2 + 4*a^2*b^3 + 4*a*b^4 - 3*b^5)*cosh(x)
^5 + 4*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*x*cosh(x)^3 + (2*a^5 + a^4*b
 - 6*a^3*b^2 - 4*a^2*b^3 + 4*a*b^4 + 3*b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3*sinh(x) + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^
2 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*
b^2 + 3*a^2*b^4 - b^6)*sinh(x)^4)
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \int \frac{\sinh^4{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(sinh(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(sinh(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{(3a^2+9ab+8b^2)x}{8(a^3+3a^2b+3ab^2+b^3)} - \frac{\left(4(2a+3b)e^{(-2x)}-a-b\right)e^{(4x)}}{64(a^2+2ab+b^2)} + \frac{4(2a-3b)e^{(-2x)}-(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 1
/8*(3*a^2 + 9*a*b + 8*b^2)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/64*(4*(2*a
 + 3*b)*e^(-2*x) - a - b)*e^(4*x)/(a^2 + 2*a*b + b^2) + 1/64*(4*(2*a - 3*b)
*e^(-2*x) - (a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.48

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2-9ab+8b^2)x}{8(a^3-3a^2b+3ab^2-b^3)} - \frac{\left(18a^2e^{(4x)}-54abe^{(4x)}+48b^2e^{(4x)}-8a^2e^{(2x)}+20abe^{(2x)}-12b^2e^{(2x)}+a^2-2ab+b^2\right)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)} + \frac{ae^{(4x)}+be^{(4x)}-8ae^{(2x)}-12be^{(2x)}}{64(a^2+2ab+b^2)}$$

```
[In] integrate(sinh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^5*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6) + 1/8*(3*a^2 - 9*a*b + 8*b^2)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/
64*(18*a^2*e^(4*x) - 54*a*b*e^(4*x) + 48*b^2*e^(4*x) - 8*a^2*e^(2*x) + 20*a
*b*e^(2*x) - 12*b^2*e^(2*x) + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b +
3*a*b^2 - b^3) + 1/64*(a*e^(4*x) + b*e^(4*x) - 8*a*e^(2*x) - 12*b*e^(2*x))/
(a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^4(x)}{a+b\coth(x)}\,dx = \frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}+\frac{e^{-2x}(2a-3b)}{16(a-b)^2} - \frac{b^5\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{x(3a^2-9ab+8b^2)}{8(a-b)^3}-\frac{e^{2x}(2a+3b)}{16(a+b)^2}$$

```
[In] int(sinh(x)^4/(a + b*coth(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) + (exp(-2*x)*(2*a - 3*b))/
(16*(a - b)^2) - (b^5*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*
a^2*b^4 - 3*a^4*b^2) + (x*(3*a^2 - 9*a*b + 8*b^2))/(8*(a - b)^3) - (exp(2*x
)*(2*a + 3*b))/(16*(a + b)^2)
```

3.98 $\int \frac{\sinh^3(x)}{a+b\coth(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 134

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = -\frac{b^4\operatorname{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{ab^2\cosh(x)}{(a^2-b^2)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} - \frac{b^3\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] -b^4*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)+a*b^2*c
osh(x)/(a^2-b^2)^2-a*cosh(x)/(a^2-b^2)+1/3*a*cosh(x)^3/(a^2-b^2)-b^3*sinh(x
)/(a^2-b^2)^2-1/3*b*sinh(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3592, 3567, 2713, 2718, 3590, 212}

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = -\frac{b^4\operatorname{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{b\sinh^3(x)}{3(a^2-b^2)} + \frac{a\cosh^3(x)}{3(a^2-b^2)} + \frac{ab^2\cosh(x)}{(a^2-b^2)^2} - \frac{a\cosh(x)}{a^2-b^2} - \frac{b^3\sinh(x)}{(a^2-b^2)^2}$$

```
[In] Int[Sinh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b^4*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2
)) + (a*b^2*Cosh[x])/(a^2 - b^2)^2 - (a*Cosh[x])/(a^2 - b^2) + (a*Cosh[x]^3
```

```
)/(3*(a^2 - b^2)) - (b^3*Sinh[x])/(a^2 - b^2)^2 - (b*Sinh[x]^3)/(3*(a^2 - b
^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\int (a - b\coth(x))\sinh^3(x)\,dx}{a^2 - b^2} + \frac{b^2 \int \frac{\sinh(x)}{a + b\coth(x)}\,dx}{a^2 - b^2}$$

$$= -\frac{b\sinh^3(x)}{3(a^2-b^2)} + \frac{b^2\int (a-b\coth(x))\sinh(x)\,dx}{(a^2-b^2)^2} + \frac{b^4\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{(a^2-b^2)^2} + \frac{a\int \sinh^3(x)\,dx}{a^2-b^2}$$

$$= -\frac{b^3\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)} + \frac{(ab^2)\int \sinh(x)\,dx}{(a^2-b^2)^2} - \frac{b^4\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, i(-ib-ia\coth(x))\sinh(x)\right)}{(a^2-b^2)^2} - \frac{a\text{Subst}\left(\int (1-x^2)\,dx, x, \cosh(x)\right)}{a^2-b^2}$$

$$= -\frac{b^4\text{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{ab^2\cosh(x)}{(a^2-b^2)^2} - \frac{a\cosh(x)}{a^2-b^2} + \frac{a\cosh^3(x)}{3(a^2-b^2)} - \frac{b^3\sinh(x)}{(a^2-b^2)^2} - \frac{b\sinh^3(x)}{3(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \frac{24b^4\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right) - 3a\sqrt{-a+b}(3a^3+3a^2b-7ab^2-7b^3)\cosh(x) - a(-a+b)^{3/2}(a+b)^2\operatorname{co}}{12(-a+b)^{5/2}(a+b)^3}$$

```
[In] Integrate[Sinh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (24*b^4*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] -
3*a*Sqrt[-a + b]*(3*a^3 + 3*a^2*b - 7*a*b^2 - 7*b^3)*Cosh[x] - a*(-a + b)^(
3/2)*(a + b)^2*Cosh[3*x] + 3*b*Sqrt[-a + b]*(a^3 + a^2*b - 5*a*b^2 - 5*b^3)
*Sinh[x] + b*(-a + b)^(3/2)*(a + b)^2*Sinh[3*x])/(12*(-a + b)^(5/2)*(a + b)
^3)
```

Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

method	result
risch	$\frac{e^{3x}}{24a+24b} - \frac{3\,e^{x}a}{8(a+b)^2} - \frac{5\,e^{x}b}{8(a+b)^2} - \frac{3\,e^{-x}a}{8(a-b)^2} + \frac{5\,e^{-x}b}{8(a-b)^2} + \frac{e^{-3x}}{24a-24b} + \frac{b^4\ln\left(e^{x}-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2} - \frac{b^4\ln\left(e^{x}+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}$
default	$-\frac{16}{(32a-32b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{32}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3(32a-32b)} - \frac{a-2b}{2(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)} + \frac{2b^4\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{-a^2+b^2}} - \frac{}{3(\tanh}$

```
[In] int(sinh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/(a+b)*exp(x)^3-3/8/(a+b)^2*exp(x)*a-5/8/(a+b)^2*exp(x)*b-3/8/(a-b)^2/e
xp(x)*a+5/8/(a-b)^2/exp(x)*b+1/24/(a-b)/exp(x)^3+1/(a^2-b^2)^(1/2)*b^4/(a+b
)^2/(a-b)^2*ln(exp(x)-(a-b)/(a^2-b^2)^(1/2))-1/(a^2-b^2)^(1/2)*b^4/(a+b)^2/
(a-b)^2*ln(exp(x)+(a-b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(126) = 252$.

Time = 0.29 (sec) , antiderivative size = 1859, normalized size of antiderivative = 13.87

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*
a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b
^3 + 7*a*b^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3
 + 7*a*b^4 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*cosh(x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^
5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4
 + 5*b^5)*cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 +
 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 +
6*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sin
h(x)^2 + 24*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x
)^2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cos
h(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) +
a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
 a + b)) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5
 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^3 -
 (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*sinh(x
))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*
```

```
a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*co
sh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b
 - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b -
2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 + a^5 + a^4*b - 2*a^3*b^2 -
2*a^2*b^3 + a*b^4 + b^5 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b
^4 - 5*b^5)*cosh(x)^4 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4
 - 5*b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)
*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(
x)^3 - 3*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)
)*sinh(x)^3 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*
cosh(x)^2 - 3*(3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5 - 5
*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 6*(3*a^5 -
 a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x)^2)*sinh(x)^2 + 4
8*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*
sinh(x)^3)*sqrt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a +
 b)*sinh(x))) + 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh
(x)^5 - 2*(3*a^5 - a^4*b - 10*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 - 5*b^5)*cosh(x
)^3 - (3*a^5 + a^4*b - 10*a^3*b^2 - 6*a^2*b^3 + 7*a*b^4 + 5*b^5)*cosh(x))*s
inh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)*sinh(x)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \int \frac{\sinh^3{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(sinh(x)**3/(a+b*coth(x)),x)
[Out] Integral(sinh(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = -\frac{2\,b^4\arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{\left(9\,ae^{(2\,x)}-15\,be^{(2\,x)}-a+b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}-9\,a^2e^x-24\,abe^x-15\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

```
[In] integrate(sinh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b^4*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*s
qrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 15*b*e^(2*x) - a + b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) - 9*a^2*e^
x - 24*a*b*e^x - 15*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

$$\int \frac{\sinh^3(x)}{a+b\coth(x)}\,dx = \frac{\mathrm{e}^{-3\,x}}{24\,a-24\,b} + \frac{\mathrm{e}^{3\,x}}{24\,a+24\,b} - \frac{\mathrm{e}^{-x}\,(3\,a-5\,b)}{8\,{(a-b)}^2} - \frac{\mathrm{e}^{x}\,(3\,a+5\,b)}{8\,{(a+b)}^2} - \frac{b^4\,\ln\left(2\,a^3\,b-2\,a\,b^3+a^4-b^4+\mathrm{e}^x\,{(a+b)}^{7/2}\,\sqrt{a-b}\right)}{{(a+b)}^{5/2}\,{(a-b)}^{5/2}} + \frac{b^4\,\ln\left(2\,a\,b^3-2\,a^3\,b-a^4+b^4+\mathrm{e}^x\,{(a+b)}^{7/2}\,\sqrt{a-b}\right)}{{(a+b)}^{5/2}\,{(a-b)}^{5/2}}$$

```
[In] int(sinh(x)^3/(a + b*coth(x)),x)
```

```
[Out] exp(-3*x)/(24*a - 24*b) + exp(3*x)/(24*a + 24*b) - (exp(-x)*(3*a - 5*b))/(8
*(a - b)^2) - (exp(x)*(3*a + 5*b))/(8*(a + b)^2) - (b^4*log(2*a^3*b - 2*a*b
^3 + a^4 - b^4 + exp(x)*(a + b)^(7/2)*(a - b)^(1/2)))/((a + b)^(5/2)*(a - b
)^(5/2)) + (b^4*log(2*a*b^3 - 2*a^3*b - a^4 + b^4 + exp(x)*(a + b)^(7/2)*(a
 - b)^(1/2)))/((a + b)^(5/2)*(a - b)^(5/2))
```

3.99 $\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2} - \frac{(a-2b)\log(1+\coth(x))}{4(a-b)^2} - \frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^2(x)}{2(a^2-b^2)}$$

```
[Out] 1/4*(a+2*b)*ln(1-coth(x))/(a+b)^2-1/4*(a-2*b)*ln(1+coth(x))/(a-b)^2-b^3*ln(
a+b*coth(x))/(a^2-b^2)^2-1/2*(b-a*coth(x))*sinh(x)^2/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3587, 755, 815}

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{\sinh^2(x)(b-a\coth(x))}{2(a^2-b^2)} - \frac{b^3\log(a+b\coth(x))}{(a^2-b^2)^2} + \frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2} - \frac{(a-2b)\log(\coth(x)+1)}{4(a-b)^2}$$

```
[In] Int[Sinh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((a + 2*b)*Log[1 - Coth[x]])/(4*(a + b)^2) - ((a - 2*b)*Log[1 + Coth[x]])/(
4*(a - b)^2) - (b^3*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Si
nh[x]^2)/(2*(a^2 - b^2))
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{1}{(a+x)\left(1-\frac{x^2}{b^2}\right)^2}\,dx, x, b\coth(x)\right)}{b}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{b\text{Subst}\left(\int \frac{-2+\frac{a^2}{b^2}+\frac{ax}{b^2}}{(a+x)\left(1-\frac{x^2}{b^2}\right)}\,dx, x, b\coth(x)\right)}{2\left(a^2-b^2\right)}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{b\text{Subst}\left(\int \left(\frac{(a-b)(a+2b)}{2b(a+b)(b-x)} + \frac{2b^2}{(a-b)(a+b)(a+x)} + \frac{(a-2b)(a+b)}{2(a-b)b(b+x)}\right)dx, x, b\coth(x)\right)}{2\left(a^2-b^2\right)}$$

$$= \frac{(a+2b)\log(1-\coth(x))}{4(a+b)^2} - \frac{(a-2b)\log(1+\coth(x))}{4(a-b)^2} - \frac{b^3\log(a+b\coth(x))}{\left(a^2-b^2\right)^2} - \frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \frac{-2a^3x+6ab^2x+(-a^2b+b^3)\cosh(2x)-4b^3\log(b\cosh(x)+a\sinh(x))+a(a^2-b^2)\sinh(2x)}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[Sinh[x]^2/(a + b*Coth[x]),x]
[Out] (-2*a^3*x + 6*a*b^2*x + (-(a^2*b) + b^3)*Cosh[2*x] - 4*b^3*Log[b*Cosh[x] +
a*Sinh[x]] + a*(a^2 - b^2)*Sinh[2*x])/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{ax}{2(a+b)^2}-\frac{xb}{(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}+\frac{2b^3x}{a^4-2a^2b^2+b^4}-\frac{b^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$-\frac{8}{(16a-16b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{16}{(32a-32b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(-a+2b)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2(a-b)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{1}{(32a+32b)(}$

```
[In] int(sinh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*a*x/(a+b)^2-x/(a+b)^2*b+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)+2*b^3/(
a^4-2*a^2*b^2+b^4)*x-b^3/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(87) = 174.

Time = 0.26 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.60

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x)^4}{}$$

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 - 4*(a^3 - 3*a*b^
```

```
2 - 2*b^3)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3)*x)*sinh(x)^2 - 8*(b^3*cosh
(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*log(2*(b*cosh(x) + a*sinh(x)
)/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 - 2*(a^3
- 3*a*b^2 - 2*b^3)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 +
 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^2)
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(sinh(x)**2/(a+b*coth(x)),x)
[Out] Integral(sinh(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{b^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-2a^2b^2+b^4} - \frac{(a+2b)x}{2(a^2+2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)} - \frac{e^{(-2x)}}{8(a-b)}$$

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) - 1/2*(a + 2*b)
*x/(a^2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = -\frac{b^3\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4} - \frac{(a-2b)x}{2(a^2-2ab+b^2)} + \frac{\left(2ae^{(2x)}-4be^{(2x)}-a+b\right)e^{(-2x)}}{8(a^2-2ab+b^2)} + \frac{e^{(2x)}}{8(a+b)}$$

```
[In] integrate(sinh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) - 1/2
*(a - 2*b)*x/(a^2 - 2*a*b + b^2) + 1/8*(2*a*e^(2*x) - 4*b*e^(2*x) - a + b)*
e^(-2*x)/(a^2 - 2*a*b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\coth(x)}\,dx = \frac{e^{2x}}{8a+8b} - \frac{e^{-2x}}{8a-8b} - \frac{b^3\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^4-2a^2b^2+b^4} - \frac{x\,(a-2b)}{2\,(a-b)^2}$$

```
[In] int(sinh(x)^2/(a + b*coth(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) - (b^3*log(b - a + a*exp(2*x)
+ b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2) - (x*(a - 2*b))/(2*(a - b)^2)
```

3.100 $\int \frac{\sinh(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 73

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx=-\frac{b^2\text{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}$$

```
[Out] -b^2*arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)+a*cosh(
x)/(a^2-b^2)-b*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3592, 3567, 2718, 3590, 212}

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx=-\frac{b^2\text{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\sinh(x)}{a^2-b^2}+\frac{a\cosh(x)}{a^2-b^2}$$

```
[In] Int[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b^2*ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2
)) + (a*Cosh[x])/(a^2 - b^2) - (b*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3592

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[1/(a^2 + b^2), Int[(d*Sec[e + f*x])^m*(a - b*Tan[e
+ f*x]), x], x] + Dist[b^2/(d^2*(a^2 + b^2)), Int[(d*Sec[e + f*x])^(m + 2)/
(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int (a-b\coth(x))\sinh(x)\,dx}{a^2-b^2}+\frac{b^2\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\
&= -\frac{b\sinh(x)}{a^2-b^2}+\frac{a\int \sinh(x)\,dx}{a^2-b^2}-\frac{b^2\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,i(-ib-ia\coth(x))\sinh(x)\right)}{a^2-b^2}\\
&= -\frac{b^2\text{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{a\cosh(x)}{a^2-b^2}-\frac{b\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx = \frac{a\cosh(x)}{a^2-b^2}+b\left(-\frac{2b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{3/2}(a+b)^{3/2}}+\frac{\sinh(x)}{-a^2+b^2}\right)$$

```
[In] Integrate[Sinh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*Cosh[x])/(a^2 - b^2) + b*((-2*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*S
qrt[a + b])])/((-a + b)^(3/2)*(a + b)^(3/2)) + Sinh[x]/(-a^2 + b^2))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{2b^2\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(a+b)(a-b)\sqrt{-a^2+b^2}}-\frac{8}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{8}{(8a-8b)\left(\tanh\left(\frac{x}{2}\right)+1\right)}$	93
risch	$\frac{e^{x}}{2a+2b}+\frac{e^{-x}}{2a-2b}+\frac{b^2\ln\left(e^{x}-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{b^2\ln\left(e^{x}+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$	122

```
[In] int(sinh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^2/(a+b)/(a-b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b
^2)^(1/2))-8/(8*a+8*b)/(tanh(1/2*x)-1)+8/(8*a-8*b)/(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(69) = 138$.

Time = 0.28 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.90

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx$$
$$= \left[\frac{a^3+a^2b-ab^2-b^3+(a^3-a^2b-ab^2+b^3)\cosh(x)^2+2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)+(a^3-}{2((a^4-2a}\right.$$

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 + 2
*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3 - a^2*b - a*b^2 + b^3)*
```

```
sinh(x)^2 - 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cosh
(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(
cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*
a^2*b^2 + b^4)*sinh(x)), 1/2*(a^3 + a^2*b - a*b^2 - b^3 + (a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) + (a^3
 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2
 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a^4
 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F]

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx=\int \frac{\sinh{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(sinh(x)/(a+b*coth(x)),x)
[Out] Integral(sinh(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx=\frac{2\,b^2\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^x}{2\,(a+b)}$$

```
[In] integrate(sinh(x)/(a+b*coth(x)),x, algorithm="giac")
[Out] 2*b^2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2
)) + 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.14

$$\int \frac{\sinh(x)}{a+b\coth(x)}\,dx = \frac{e^x}{2\,a+2\,b}+\frac{e^{-x}}{2\,a-2\,b}-\frac{b^2\,\ln\left(\frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}-\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a-b}}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}} + \frac{b^2\,\ln\left(\frac{2\,b^2}{(a+b)^{5/2}\,\sqrt{a-b}}+\frac{2\,b^2\,e^x}{-a^3-a^2\,b+a\,b^2+b^3}\right)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

```
[In] int(sinh(x)/(a + b*coth(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) + exp(-x)/(2*a - 2*b) - (b^2*log((2*b^2*exp(x))/(a*b^2 -
 a^2*b - a^3 + b^3) - (2*b^2)/((a + b)^(5/2)*(a - b)^(1/2))))/((a + b)^(3/2
)*(a - b)^(3/2)) + (b^2*log((2*b^2)/((a + b)^(5/2)*(a - b)^(1/2)) + (2*b^2*
exp(x))/(a*b^2 - a^2*b - a^3 + b^3)))/((a + b)^(3/2)*(a - b)^(3/2))
```

3.101 $\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1/2))/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3590, 212}

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] Int[Csch[x]/(a + b*Coth[x]),x]
```

```
[Out] -(ArcTanh[((b + a*Coth[x])*Sinh[x])/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
```

```
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{1}{a^2 - b^2 - x^2}\,dx, x, i(-ib - ia\coth(x))\sinh(x)\right)$$
$$= -\frac{i\arctan\left(\frac{(-ib-ia\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{csch}(x)}{a + b\coth(x)}\,dx = \frac{2\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}$$

```
[In] Integrate[Csch[x]/(a + b*Coth[x]),x]
[Out] (2*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/(Sqrt[-a + b]*Sqrt
[a + b])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$	39
risch	$\frac{\ln\left(e^x-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{\ln\left(e^x+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$	70

```
[In] int(csch(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] 2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.87

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = \left[\frac{\log\left(\frac{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-2\,\sqrt{a^2-b^2}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^2+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-a+b}\right)}{\sqrt{a^2-b^2}}, \frac{2\,\sqrt{-a^2+b^2}\arctan\left(\frac{\ }{(a+b)\cosh}\right.}{a^2-b^2}\right.$$

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2
*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b))/sqrt(a^2 - b^2), 2*sqrt(-a^
2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))/(a^2
- b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(csch(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = \frac{2\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

```
[In] integrate(csch(x)/(a+b*coth(x)),x, algorithm="giac")
[Out] 2*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a+b\coth(x)}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{e^x\,\sqrt{b^2-a^2}}{a-b}\right)}{\sqrt{b^2-a^2}}$$

```
[In] int(1/(sinh(x)*(a + b*coth(x))),x)
[Out] -(2*atan((exp(x)*(b^2 - a^2)^(1/2))/(a - b)))/(b^2 - a^2)^(1/2)
```

3.102 $\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=-\frac{\log(a+b\coth(x))}{b}$$

```
[Out] -ln(a+b*coth(x))/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 31}

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx=-\frac{\log(a+b\coth(x))}{b}$$

```
[In] Int[Csch[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -(Log[a + b*Coth[x]]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
```

```
] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b}\\ &= -\frac{\log(a+b\coth(x))}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = \frac{\log(\sinh(x)) - \log(b\cosh(x) + a\sinh(x))}{b}$$

```
[In] Integrate[Csch[x]^2/(a + b*Coth[x]),x]
[Out] (Log[Sinh[x]] - Log[b*Cosh[x] + a*Sinh[x]])/b
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\coth(x))}{b}$	13
default	$-\frac{\ln(a+b\coth(x))}{b}$	13
risch	$\frac{\ln(\mathrm{e}^{2x}-1)}{b}-\frac{\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{b}$	36

```
[In] int(csch(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -ln(a+b*coth(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 3.58

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\sinh(x)}{\cosh(x)-\sinh(x)}\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="fricas")
[Out] -(log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - log(2*sinh(x)/(cosh(
x) - sinh(x))))/b
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*coth(x)),x)
[Out] Integral(csch(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{\log\left(b\coth(x)+a\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="maxima")
[Out] -log(b*coth(x) + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.83

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{(a+b)\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{ab+b^2}+\frac{\log\left(\left|e^{(2\,x)}-1\right|\right)}{b}$$

```
[In] integrate(csch(x)^2/(a+b*coth(x)),x, algorithm="giac")
[Out] -(a + b)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b + b^2) + log(abs(e^(2
*x) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.25

$$\int \frac{\operatorname{csch}^2(x)}{a+b\coth(x)}\,dx = -\frac{2\,\operatorname{atan}\left(\frac{a\,e^{2\,x}\,\sqrt{-b^2}-a\,\sqrt{-b^2}+b\,e^{2\,x}\,\sqrt{-b^2}}{b^2}\right)}{\sqrt{-b^2}}$$

```
[In] int(1/(sinh(x)^2*(a + b*coth(x))),x)
[Out] -(2*atan((a*exp(2*x)*(-b^2)^(1/2) - a*(-b^2)^(1/2) + b*exp(2*x)*(-b^2)^(1/2
))/b^2))/(-b^2)^(1/2)
```

3.103 $\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx=\frac{a\operatorname{arctanh}(\cosh(x))}{b^2}-\frac{\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

```
[Out] a*arctanh(cosh(x))/b^2-csch(x)/b-arctanh((b+a*coth(x))*sinh(x)/(a^2-b^2)^(1
/2))*(a^2-b^2)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3591, 3567, 3855, 3590, 212}

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx=-\frac{\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{\sinh(x)(a\coth(x)+b)}{\sqrt{a^2-b^2}}\right)}{b^2}+\frac{a\operatorname{arctanh}(\cosh(x))}{b^2}-\frac{\operatorname{csch}(x)}{b}$$

```
[In] Int[Csch[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a*ArcTanh[Cosh[x]])/b^2 - (Sqrt[a^2 - b^2]*ArcTanh[((b + a*Coth[x])*Sinh[x
])/Sqrt[a^2 - b^2]])/b^2 - Csch[x]/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3567

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[b*((d*Sec[e + f*x])^m/(f*m)), x] + Dist[a, Int[(d*
Sec[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, m}, x] && (IntegerQ[2*m] |
| NeQ[a^2 + b^2, 0])
```

Rule 3590

```
Int[sec[(e_.) + (f_.)*(x_)]/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbo
l] :> Dist[-f^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, (b - a*Tan[e + f*
x])/Sec[e + f*x]], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3591

```
Int[((d_.)*sec[(e_.) + (f_.)*(x_)])^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[-d^2/b^2, Int[(d*Sec[e + f*x])^(m - 2)*(a - b*Tan[e
 + f*x]), x], x] + Dist[d^2*((a^2 + b^2)/b^2), Int[(d*Sec[e + f*x])^(m - 2)
/(a + b*Tan[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int (a-b\coth(x))\text{csch}(x)\,dx}{b^2}+\frac{(a^2-b^2)\int \frac{\text{csch}(x)}{a+b\coth(x)}\,dx}{b^2}\\
&= -\frac{\text{csch}(x)}{b}-\frac{a\int \text{csch}(x)\,dx}{b^2}-\frac{(a^2-b^2)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,i(-ib-ia\coth(x))\sinh(x)\right)}{b^2}\\
&= \frac{a\text{arctanh}(\cosh(x))}{b^2}-\frac{\sqrt{a^2-b^2}\text{arctanh}\left(\frac{(b+a\coth(x))\sinh(x)}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{\text{csch}(x)}{b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = -\frac{2\sqrt{-a+b}\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)+b\operatorname{csch}(x)+a\left(-\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{b^2}$$

```
[In] Integrate[Csch[x]^3/(a + b*Coth[x]),x]
[Out] -((2*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a
 + b])] + b*Csch[x] + a*(-Log[Cosh[x/2]] + Log[Sinh[x/2]]))/b^2)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{2b}+\frac{\left(4a^2-4b^2\right)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{2b^2\sqrt{-a^2+b^2}}-\frac{1}{2b\tanh\left(\frac{x}{2}\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{b^2}$	85
risch	$-\frac{2\,\mathrm{e}^{x}}{b\left(\mathrm{e}^{2x}-1\right)}+\frac{\sqrt{a^2-b^2}\,\ln\left(\mathrm{e}^{x}-\frac{\sqrt{a^2-b^2}}{a+b}\right)}{b^2}-\frac{\sqrt{a^2-b^2}\,\ln\left(\mathrm{e}^{x}+\frac{\sqrt{a^2-b^2}}{a+b}\right)}{b^2}+\frac{a\ln\left(\mathrm{e}^{x}+1\right)}{b^2}-\frac{a\ln\left(\mathrm{e}^{x}-1\right)}{b^2}$	112

```
[In] int(csch(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/2/b*tanh(1/2*x)+1/2/b^2*(4*a^2-4*b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*ta
nh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))-1/2/b/tanh(1/2*x)-a/b^2*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(53) = 106.

Time = 0.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 6.74

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \left[\frac{\sqrt{a^2-b^2}\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2-1\right)\log\left(\frac{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^2-}{(a+b)\cosh(x)^2+2(a+b)\cosh(x)\sinh(x)+(}\right.}{}\right.$$

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2
- b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*
sinh(x) + (a + b)*sinh(x)^2 - a + b)) - 2*b*cosh(x) + (a*cosh(x)^2 + 2*a*co
sh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (a*cosh(x)^2
+ 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) - 1) - 2*b*s
inh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2), (2*s
qrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(sqrt
(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - 2*b*cosh(x) + (a*cosh(x
)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x) + 1) - (
a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*log(cosh(x) + sinh(x)
- 1) - 2*b*sinh(x))/(b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
- b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^3{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(csch(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx = \frac{a\log\left(e^x+1\right)}{b^2} - \frac{a\log\left(\left|e^x-1\right|\right)}{b^2} + \frac{2\left(a^2-b^2\right)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2} - \frac{2\,e^x}{b(e^{(2\,x)}-1)}$$

```
[In] integrate(csch(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a*log(e^x + 1)/b^2 - a*log(abs(e^x - 1))/b^2 + 2*(a^2 - b^2)*arctan((a*e^x
+ b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - 2*e^x/(b*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 4.04

$$\begin{aligned}\int \frac{\operatorname{csch}^3(x)}{a+b\coth(x)}\,dx &= \frac{2\,\mathrm{e}^x}{b-b\,\mathrm{e}^{2\,x}} \\ &\quad - \frac{a\,\ln\left(32\,a\,b^2-64\,a^2\,b+32\,a^3-32\,a^3\,\mathrm{e}^x-32\,a\,b^2\,\mathrm{e}^x+64\,a^2\,b\,\mathrm{e}^x\right)}{b^2} \\ &\quad + \frac{a\,\ln\left(32\,a\,b^2-64\,a^2\,b+32\,a^3+32\,a^3\,\mathrm{e}^x+32\,a\,b^2\,\mathrm{e}^x-64\,a^2\,b\,\mathrm{e}^x\right)}{b^2} \\ &\quad + \frac{\ln\left(32\,a\,\sqrt{a^2-b^2}-32\,b\,\sqrt{a^2-b^2}-32\,a^2\,\mathrm{e}^x+32\,b^2\,\mathrm{e}^x\right)\,\sqrt{a^2-b^2}}{b^2} \\ &\quad - \frac{\ln\left(32\,a\,\sqrt{a^2-b^2}-32\,b\,\sqrt{a^2-b^2}+32\,a^2\,\mathrm{e}^x-32\,b^2\,\mathrm{e}^x\right)\,\sqrt{a^2-b^2}}{b^2}\end{aligned}$$

```
[In] int(1/(sinh(x)^3*(a + b*coth(x))),x)
```

```
[Out] (2*exp(x))/(b - b*exp(2*x)) - (a*log(32*a*b^2 - 64*a^2*b + 32*a^3 - 32*a^3*
exp(x) - 32*a*b^2*exp(x) + 64*a^2*b*exp(x)))/b^2 + (a*log(32*a*b^2 - 64*a^2
*b + 32*a^3 + 32*a^3*exp(x) + 32*a*b^2*exp(x) - 64*a^2*b*exp(x)))/b^2 + (lo
g(32*a*(a^2 - b^2)^(1/2) - 32*b*(a^2 - b^2)^(1/2) - 32*a^2*exp(x) + 32*b^2*
exp(x))*(a^2 - b^2)^(1/2))/b^2 - (log(32*a*(a^2 - b^2)^(1/2) - 32*b*(a^2 -
b^2)^(1/2) + 32*a^2*exp(x) - 32*b^2*exp(x))*(a^2 - b^2)^(1/2))/b^2
```

3.104 $\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx=\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}$$

```
[Out] a*coth(x)/b^2-1/2*coth(x)^2/b-(a^2-b^2)*ln(a+b*coth(x))/b^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3587, 711}

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx=-\frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

```
[In] Int[Csch[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - ((a^2 - b^2)*Log[a + b*Coth[x]])/b^3
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 3587

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[1/(b*f), Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2, 0
] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1-\frac{x^2}{b^2}}{a+x}\,dx, x, b\coth(x)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a}{b^2} - \frac{x}{b^2} + \frac{-a^2+b^2}{b^2(a+x)}\right)\,dx, x, b\coth(x)\right)}{b}$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{(a^2-b^2)\log(a+b\coth(x))}{b^3}$$

Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \frac{2ab\coth(x) - b^2\operatorname{csch}^2(x) + 2(a^2-b^2)\left(\log(\sinh(x)) - \log(b\cosh(x) + a\sinh(x))\right)}{2b^3}$$

```
[In] Integrate[Csch[x]^4/(a + b*Coth[x]),x]
[Out] (2*a*b*Coth[x] - b^2*Csch[x]^2 + 2*(a^2 - b^2)*(Log[Sinh[x]] - Log[b*Cosh[x
] + a*Sinh[x]]))/(2*b^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(38) = 76.

Time = 1.82 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.55

method	result
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^2 b}{2}+2a\tanh\left(\frac{x}{2}\right)}{4b^2} - \frac{1}{8b\tanh\left(\frac{x}{2}\right)^2} + \frac{(4a^2-4b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4b^3} + \frac{a}{2b^2\tanh\left(\frac{x}{2}\right)} + \frac{(-4a^2+4b^2)\ln\left(\tanh\left(\frac{x}{2}\right)^2 b+2a\tanh\left(\frac{x}{2}\right.\right.}{4b^3}$
risch	$\frac{2a\,e^{2x}-2b\,e^{2x}-2a}{(e^{2x}-1)^2 b^2} - \frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)a^2}{b^3} + \frac{\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b} + \frac{\ln(e^{2x}-1)a^2}{b^3} - \frac{\ln(e^{2x}-1)}{b}$

```
[In] int(csch(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b^2*(-1/2*tanh(1/2*x)^2*b+2*a*tanh(1/2*x))-1/8/b/tanh(1/2*x)^2+1/4/b^3*
(4*a^2-4*b^2)*ln(tanh(1/2*x))+1/2*a/b^2/tanh(1/2*x)+1/4/b^3*(-4*a^2+4*b^2)*
ln(tanh(1/2*x)^2*b+2*a*tanh(1/2*x)+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 434, normalized size of antiderivative = 10.85

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \frac{2\left(ab-b^2\right)\cosh\left(x\right)^2+4\left(ab-b^2\right)\cosh\left(x\right)\sinh\left(x\right)+2\left(ab-b^2\right)\sinh\left(x\right)^2-2\,ab-\left(\left(a^2-b^2\right)\cosh\left(x\right)^4+\cdots\right.}{\cdots}$$

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] (2*(a*b - b^2)*cosh(x)^2 + 4*(a*b - b^2)*cosh(x)*sinh(x) + 2*(a*b - b^2)*si
nh(x)^2 - 2*a*b - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3
+ (a^2 - b^2)*sinh(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x
)^2 - a^2 + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 -
b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) +
 ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*sin
h(x)^4 - 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 - a^2 + b^2)*
sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 - (a^2 - b^2)*cosh(x))*sin
h(x))*log(2*sinh(x)/(cosh(x) - sinh(x))))/(b^3*cosh(x)^4 + 4*b^3*cosh(x)*si
nh(x)^3 + b^3*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)
*sinh(x)^2 + 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{csch}^4\left(x\right)}{a+b\coth\left(x\right)}\,dx$$

```
[In] integrate(csch(x)**4/(a+b*coth(x)),x)
```

```
[Out] Integral(csch(x)**4/(a + b*coth(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(38) = 76$.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.75

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \frac{2\left((a+b)e^{(-2x)}-a\right)}{2b^2e^{(-2x)}-b^2e^{(-4x)}-b^2} - \frac{(a^2-b^2)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{b^3} + \frac{(a^2-b^2)\log\left(e^{(-x)}+1\right)}{b^3} + \frac{(a^2-b^2)\log\left(e^{(-x)}-1\right)}{b^3}$$

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] 2*((a + b)*e^(-2*x) - a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) - (a^2 - b^2
)*log(-(a - b)*e^(-2*x) + a + b)/b^3 + (a^2 - b^2)*log(e^(-x) + 1)/b^3 + (a
^2 - b^2)*log(e^(-x) - 1)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.65

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = -\frac{(a^3+a^2b-ab^2-b^3)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{ab^3+b^4} + \frac{(a^2-b^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^3} - \frac{2\left(ab-(ab-b^2)e^{(2x)}\right)}{b^3\left(e^{(2x)}-1\right)^2}$$

```
[In] integrate(csch(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3 + a^2*b - a*b^2 - b^3)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a*b^3
 + b^4) + (a^2 - b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2
*x))/(b^3*(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.20

$$\int \frac{\operatorname{csch}^4(x)}{a+b\coth(x)}\,dx = \frac{2\,(a-b)}{b^2\,\left(\mathrm{e}^{2\,x}-1\right)} - \frac{2}{b\,\left(\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1\right)} - \frac{\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)\,(a+b)\,(a-b)}{b^3} + \frac{\ln\left(\mathrm{e}^{2\,x}-1\right)\,(a+b)\,(a-b)}{b^3}$$

```
[In] int(1/(sinh(x)^4*(a + b*coth(x))),x)
```

```
[Out] (2*(a - b))/(b^2*(exp(2*x) - 1)) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) - (log
(b - a + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/b^3 + (log(exp(2*x) - 1)
*(a + b)*(a - b))/b^3
```

3.105 $\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{x}{16}+\frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))} - \frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}$$

```
[Out] 1/16*x+1/32/(1-coth(x))^2-1/8/(1-coth(x))-1/24/(1+coth(x))^3+5/32/(1+coth(x
))^2-3/16/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3597, 862, 90, 213}

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{x}{16}-\frac{1}{8(1-\coth(x))}-\frac{3}{16(\coth(x)+1)} +\frac{1}{32(1-\coth(x))^2}+\frac{5}{32(\coth(x)+1)^2}-\frac{1}{24(\coth(x)+1)^3}$$

```
[In] Int[Cosh[x]^4/(1 + Coth[x]),x]
```

```
[Out] x/16 + 1/(32*(1 - Coth[x])^2) - 1/(8*(1 - Coth[x])) - 1/(24*(1 + Coth[x])^3
) + 5/(32*(1 + Coth[x])^2) - 3/(16*(1 + Coth[x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^4}{(1+x)\left(-1+x^2\right)^3}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \frac{x^4}{(-1+x)^3(1+x)^4}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{16(-1+x)^3}+\frac{1}{8(-1+x)^2}-\frac{1}{8(1+x)^4}+\frac{5}{16(1+x)^3}-\frac{3}{16(1+x)^2}\right.\right.\\
&\qquad\left.\left.+\frac{1}{16\left(-1+x^2\right)}\right)dx, x, \coth(x)\right)\\
&= \frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}\\
&\quad-\frac{3}{16(1+\coth(x))}-\frac{1}{16}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \coth(x)\right)\\
&= \frac{x}{16}+\frac{1}{32(1-\coth(x))^2}-\frac{1}{8(1-\coth(x))}\\
&\quad-\frac{1}{24(1+\coth(x))^3}+\frac{5}{32(1+\coth(x))^2}-\frac{3}{16(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{1}{192}(12x+15\cosh(2x)+6\cosh(4x)+\cosh(6x)+3\sinh(2x) - 3\sinh(4x)-\sinh(6x))$$

```
[In] Integrate[Cosh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (12*x + 15*Cosh[2*x] + 6*Cosh[4*x] + Cosh[6*x] + 3*Sinh[2*x] - 3*Sinh[4*x]
- Sinh[6*x])/192
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x}{16}+\frac{e^{4x}}{128}+\frac{3\,e^{2x}}{64}+\frac{e^{-2x}}{32}+\frac{3\,e^{-4x}}{128}+\frac{e^{-6x}}{192}$
parallelrisch	$\frac{13}{96}+\frac{\cosh(4x)}{32}+\frac{\cosh(6x)}{192}+\frac{5\cosh(2x)}{64}-\frac{\sinh(4x)}{64}+\frac{\sinh(2x)}{64}-\frac{\sinh(6x)}{192}-\frac{\ln(1-\tanh(x))}{32}+\frac{\ln(1+\tanh(x))}{32}$
default	$\frac{1}{8(\tanh(\frac{x}{2})-1)^4}+\frac{1}{4(\tanh(\frac{x}{2})-1)^3}+\frac{3}{8(\tanh(\frac{x}{2})-1)^2}+\frac{1}{4\tanh(\frac{x}{2})-4}-\frac{\ln(\tanh(\frac{x}{2})-1)}{16}+\frac{1}{3(\tanh(\frac{x}{2})+1)^6}-$ (tanh

```
[In] int(cosh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*x+1/128*exp(4*x)+3/64*exp(2*x)+1/32*exp(-2*x)+3/128*exp(-4*x)+1/192*ex
p(-6*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{5\cosh(x)^5+25\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\cosh(x)^2+9\right)\sinh(x)^3+27\cosh(x)^3+\left(50\cosh(x)\right.}{384\,(\cosh(x)+}$$

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/384*(5*cosh(x)^5 + 25*cosh(x)*sinh(x)^4 + sinh(x)^5 + (10*cosh(x)^2 + 9)*
sinh(x)^3 + 27*cosh(x)^3 + (50*cosh(x)^3 + 81*cosh(x))*sinh(x)^2 + 12*(2*x
+ 1)*cosh(x) + (5*cosh(x)^4 + 27*cosh(x)^2 + 24*x - 12)*sinh(x))/(cosh(x) +
 sinh(x))
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \int \frac{\cosh^4(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(cosh(x)**4/(1+coth(x)),x)
[Out] Integral(cosh(x)**4/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = \frac{1}{128}\left(6\,e^{(-2\,x)}+1\right)e^{(4\,x)} + \frac{1}{16}\,x + \frac{1}{32}\,e^{(-2\,x)} + \frac{3}{128}\,e^{(-4\,x)} + \frac{1}{192}\,e^{(-6\,x)}$$

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="maxima")
[Out] 1/128*(6*e^(-2*x) + 1)*e^(4*x) + 1/16*x + 1/32*e^(-2*x) + 3/128*e^(-4*x) +
1/192*e^(-6*x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx = -\frac{1}{384}\left(22\,e^{(6\,x)} - 12\,e^{(4\,x)} - 9\,e^{(2\,x)} - 2\right)e^{(-6\,x)} + \frac{1}{16}\,x + \frac{1}{128}\,e^{(4\,x)} + \frac{3}{64}\,e^{(2\,x)}$$

```
[In] integrate(cosh(x)^4/(1+coth(x)),x, algorithm="giac")
[Out] -1/384*(22*e^(6*x) - 12*e^(4*x) - 9*e^(2*x) - 2)*e^(-6*x) + 1/16*x + 1/128*
e^(4*x) + 3/64*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\cosh^4(x)}{1+\coth(x)}\,dx=\frac{x}{16}+\frac{\mathrm{e}^{-2\,x}}{32}+\frac{3\,\mathrm{e}^{2\,x}}{64}+\frac{3\,\mathrm{e}^{-4\,x}}{128}+\frac{\mathrm{e}^{4\,x}}{128}+\frac{\mathrm{e}^{-6\,x}}{192}$$

```
[In] int(cosh(x)^4/(coth(x) + 1),x)
[Out] x/16 + exp(-2*x)/32 + (3*exp(2*x))/64 + (3*exp(-4*x))/128 + exp(4*x)/128 +
exp(-6*x)/192
```

3.106 $\int \frac{\cosh^3(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{\cosh^3(x)}{1+\coth(x)} dx = \frac{\cosh^5(x)}{5} - \frac{\sinh^3(x)}{3} - \frac{\sinh^5(x)}{5}$$

```
[Out] 1/5*cosh(x)^5-1/3*sinh(x)^3-1/5*sinh(x)^5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3599, 3187, 3186, 2645, 30, 2644, 14}

$$\int \frac{\cosh^3(x)}{1+\coth(x)} dx = -\frac{\sinh^5(x)}{5} - \frac{\sinh^3(x)}{3} + \frac{\cosh^5(x)}{5}$$

```
[In] Int[Cosh[x]^3/(1 + Coth[x]),x]
```

```
[Out] Cosh[x]^5/5 - Sinh[x]^3/3 - Sinh[x]^5/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \cosh^3(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\
&= i\int \left(-i\cosh^4(x)\sinh(x)+i\cosh^3(x)\sinh^2(x)\right)\,dx
\end{aligned}
$$

$$= \int \cosh^4(x)\sinh(x)\,dx - \int \cosh^3(x)\sinh^2(x)\,dx$$

$$= -\left(i\text{Subst}\left(\int x^2(1-x^2)\ dx, x, i\sinh(x)\right)\right) + \text{Subst}\left(\int x^4\,dx, x, \cosh(x)\right)$$

$$= \frac{\cosh^5(x)}{5} - i\text{Subst}\left(\int (x^2-x^4)\ dx, x, i\sinh(x)\right)$$

$$= \frac{\cosh^5(x)}{5} - \frac{\sinh^3(x)}{3} - \frac{\sinh^5(x)}{5}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \frac{1}{120}(\cosh(x)-\sinh(x))(20\cosh(2x)+4\cosh(4x)+10\sinh(2x)+\sinh(4x))$$

```
[In] Integrate[Cosh[x]^3/(1 + Coth[x]),x]
[Out] ((Cosh[x] - Sinh[x])*(20*Cosh[2*x] + 4*Cosh[4*x] + 10*Sinh[2*x] + Sinh[4*x]
))/120
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result
risch	$\frac{e^{3x}}{48}+\frac{e^{x}}{8}+\frac{e^{-3x}}{24}+\frac{e^{-5x}}{80}$
parallelrisch	$\frac{\cosh(3x)}{16}+\frac{\cosh(x)}{8}-\frac{\sinh(3x)}{48}+\frac{\sinh(x)}{8}-\frac{2}{15}-\frac{\sinh(5x)}{80}+\frac{\cosh(5x)}{80}$
default	$-\frac{1}{(\tanh(\frac{x}{2})+1)^4}+\frac{2}{5(\tanh(\frac{x}{2})+1)^5}+\frac{4}{3(\tanh(\frac{x}{2})+1)^3}-\frac{1}{(\tanh(\frac{x}{2})+1)^2}+\frac{3}{8(\tanh(\frac{x}{2})+1)}-\frac{1}{6(\tanh(\frac{x}{2})-1)^3}-$

```
[In] int(cosh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/48*exp(3*x)+1/8*exp(x)+1/24*exp(-3*x)+1/80*exp(-5*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(19) = 38$.

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \frac{\cosh(x)^4 + \cosh(x)\sinh(x)^3 + \sinh(x)^4 + \left(6\cosh(x)^2 + 5\right)\sinh(x)^2 + 5\cosh(x)^2 + \left(\cosh(x)^3 + 5\cosh\right.}{30\left(\cosh(x) + \sinh(x)\right)}$$

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="fricas")
[Out] 1/30*(cosh(x)^4 + cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 5)*sinh(x)
^2 + 5*cosh(x)^2 + (cosh(x)^3 + 5*cosh(x))*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \int \frac{\cosh^3(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(cosh(x)**3/(1+coth(x)),x)
[Out] Integral(cosh(x)**3/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\,dx = \frac{1}{48}\left(6\,e^{(-2\,x)}+1\right)e^{(3\,x)} + \frac{1}{24}\,e^{(-3\,x)} + \frac{1}{80}\,e^{(-5\,x)}$$

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="maxima")
[Out] 1/48*(6*e^(-2*x) + 1)*e^(3*x) + 1/24*e^(-3*x) + 1/80*e^(-5*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\, dx = \frac{1}{240}\left(10\, e^{(2\, x)} + 3\right)e^{(-5\, x)} + \frac{1}{48}\, e^{(3\, x)} + \frac{1}{8}\, e^{x}$$

```
[In] integrate(cosh(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] 1/240*(10*e^(2*x) + 3)*e^(-5*x) + 1/48*e^(3*x) + 1/8*e^x
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^3(x)}{1+\coth(x)}\, dx = \frac{e^{-3\, x}}{24} + \frac{e^{3\, x}}{48} + \frac{e^{-5\, x}}{80} + \frac{e^{x}}{8}$$

```
[In] int(cosh(x)^3/(coth(x) + 1),x)
[Out] exp(-3*x)/24 + exp(3*x)/48 + exp(-5*x)/80 + exp(x)/8
```

3.107 $\int \frac{\cosh^2(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 38

$$\int \frac{\cosh^2(x)}{1+\coth(x)} \, dx = \frac{x}{8} - \frac{1}{8(1-\coth(x))} + \frac{1}{8(1+\coth(x))^2} - \frac{1}{4(1+\coth(x))}$$

```
[Out] 1/8*x-1/8/(1-coth(x))+1/8/(1+coth(x))^2-1/4/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3597, 862, 90, 213}

$$\int \frac{\cosh^2(x)}{1+\coth(x)} \, dx = \frac{x}{8} - \frac{1}{8(1-\coth(x))} - \frac{1}{4(\coth(x)+1)} + \frac{1}{8(\coth(x)+1)^2}$$

```
[In] Int[Cosh[x]^2/(1 + Coth[x]),x]
```

```
[Out] x/8 - 1/(8*(1 - Coth[x])) + 1/(8*(1 + Coth[x])^2) - 1/(4*(1 + Coth[x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^2}{(1+x)\left(-1+x^2\right)^2}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \frac{x^2}{(-1+x)^2(1+x)^3}\,dx, x, \coth(x)\right)\\
&= -\text{Subst}\left(\int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}-\frac{1}{4(1+x)^2}+\frac{1}{8\left(-1+x^2\right)}\right)dx, x, \coth(x)\right)\\
&= -\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \coth(x)\right)\\
&= \frac{x}{8}-\frac{1}{8(1-\coth(x))}+\frac{1}{8(1+\coth(x))^2}-\frac{1}{4(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{1}{32}(4x+4\cosh(2x)+\cosh(4x)-\sinh(4x))$$

```
[In] Integrate[Cosh[x]^2/(1 + Coth[x]),x]
[Out] (4*x + 4*Cosh[2*x] + Cosh[4*x] - Sinh[4*x])/32
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

method	result
risch	$\frac{x}{8}+\frac{e^{2x}}{16}+\frac{e^{-2x}}{16}+\frac{e^{-4x}}{32}$
parallelrisch	$\frac{x}{8}-\frac{\sinh(4x)}{32}+\frac{\cosh(4x)}{32}+\frac{\cosh(2x)}{8}-\frac{5}{32}$
default	$\frac{1}{4(\tanh(\frac{x}{2})-1)^2}+\frac{1}{4\tanh(\frac{x}{2})-4}-\frac{\ln(\tanh(\frac{x}{2})-1)}{8}+\frac{1}{2(\tanh(\frac{x}{2})+1)^4}-\frac{1}{(\tanh(\frac{x}{2})+1)^3}+\frac{1}{(\tanh(\frac{x}{2})+1)^2}-\frac{1}{2(\tanh(}$

```
[In] int(cosh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/8*x+1/16*exp(2*x)+1/16*exp(-2*x)+1/32*exp(-4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{3\cosh(x)^3+9\cosh(x)\sinh(x)^2+\sinh(x)^3+2(2x+1)\cosh(x)+(3\cosh(x)^2+4x-2)\sinh(x)}{32(\cosh(x)+\sinh(x))}$$

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="fricas")
[Out] 1/32*(3*cosh(x)^3 + 9*cosh(x)*sinh(x)^2 + sinh(x)^3 + 2*(2*x + 1)*cosh(x) +
 (3*cosh(x)^2 + 4*x - 2)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \int \frac{\cosh^2(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(cosh(x)**2/(1+coth(x)),x)
[Out] Integral(cosh(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)} + \frac{1}{16}\,e^{(-2\,x)} + \frac{1}{32}\,e^{(-4\,x)}$$

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] 1/8*x + 1/16*e^(2*x) + 1/16*e^(-2*x) + 1/32*e^(-4*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = -\frac{1}{32}\left(3\,e^{(4\,x)} - 2\,e^{(2\,x)} - 1\right)e^{(-4\,x)} + \frac{1}{8}\,x + \frac{1}{16}\,e^{(2\,x)}$$

```
[In] integrate(cosh(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] -1/32*(3*e^(4*x) - 2*e^(2*x) - 1)*e^(-4*x) + 1/8*x + 1/16*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \frac{\cosh^2(x)}{1+\coth(x)}\,dx = \frac{x}{8} + \frac{e^{-2\,x}}{16} + \frac{e^{2\,x}}{16} + \frac{e^{-4\,x}}{32}$$

```
[In] int(cosh(x)^2/(coth(x) + 1),x)
[Out] x/8 + exp(-2*x)/16 + exp(2*x)/16 + exp(-4*x)/32
```

3.108 $\int \frac{\cosh(x)}{1+\coth(x)} dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{\cosh(x)}{1+\coth(x)} dx = \frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

```
[Out] 1/3*cosh(x)^3-1/3*sinh(x)^3
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3599, 3187, 3186, 2645, 30, 2644}

$$\int \frac{\cosh(x)}{1+\coth(x)} dx = \frac{\cosh^3(x)}{3} - \frac{\sinh^3(x)}{3}$$

```
[In] Int[Cosh[x]/(1 + Coth[x]),x]
```

```
[Out] Cosh[x]^3/3 - Sinh[x]^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\
&= -\int \cosh(x)\sinh(x)(-\cosh(x)+\sinh(x))\,dx\\
&= i\int \left(-i\cosh^2(x)\sinh(x)+i\cosh(x)\sinh^2(x)\right)\,dx\\
&= \int \cosh^2(x)\sinh(x)\,dx-\int \cosh(x)\sinh^2(x)\,dx\\
&= -\left(i\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)\right)+\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)\\
&= \frac{\cosh^3(x)}{3}-\frac{\sinh^3(x)}{3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\left(3\cosh(x)+\cosh(3x)-4\sinh^3(x)\right)$$

```
[In] Integrate[Cosh[x]/(1 + Coth[x]),x]
[Out] (3*Cosh[x] + Cosh[3*x] - 4*Sinh[x]^3)/12
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{e^{x}}{4}+\frac{e^{-3x}}{12}$	12
parallelrisch	$\frac{\cosh(3x)}{12}+\frac{\cosh(x)}{4}-\frac{\sinh(3x)}{12}+\frac{\sinh(x)}{4}-\frac{1}{3}$	23
default	$\frac{2}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{1}{2\tanh\left(\frac{x}{2}\right)+2}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	42

```
[In] int(cosh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*exp(x)+1/12*exp(-3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{\cosh\left(x\right)^{2}+\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}}{3\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="fricas")
[Out] 1/3*(cosh(x)^2 + cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \int \frac{\cosh{(x)}}{\coth{(x)}+1}\,dx$$

```
[In] integrate(cosh(x)/(1+coth(x)),x)
[Out] Integral(cosh(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="maxima")
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{4}\,e^{x}$$

```
[In] integrate(cosh(x)/(1+coth(x)),x, algorithm="giac")
[Out] 1/12*e^(-3*x) + 1/4*e^x
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.65

$$\int \frac{\cosh(x)}{1+\coth(x)}\,dx = \frac{\mathrm{e}^{-3\,x}}{12} + \frac{\mathrm{e}^{x}}{4}$$

```
[In] int(cosh(x)/(coth(x) + 1),x)
[Out] exp(-3*x)/12 + exp(x)/4
```

3.109 $\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \arctan(\sinh(x)) + \cosh(x) - \sinh(x)$$

```
[Out] arctan(sinh(x))+cosh(x)-sinh(x)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3599, 3187, 3186, 2718, 2672, 327, 209}

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \arctan(\sinh(x)) - \sinh(x) + \cosh(x)$$

```
[In] Int[Sech[x]/(1 + Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]] + Cosh[x] - Sinh[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(i\int \frac{\tanh(x)}{-i\cosh(x)-i\sinh(x)}\,dx\right)\\ &= -\int(-\cosh(x)+\sinh(x))\tanh(x)\,dx\end{aligned}$$

$$
\begin{aligned}
&= i\int (-i\sinh(x)+i\sinh(x)\tanh(x))\,dx\\
&= \int \sinh(x)\,dx-\int \sinh(x)\tanh(x)\,dx\\
&= \cosh(x)-\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\sinh(x)\right)\\
&= \cosh(x)-\sinh(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(x)\right)\\
&= \arctan(\sinh(x))+\cosh(x)-\sinh(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$
\int \frac{\text{sech}(x)}{1+\coth(x)}\,dx = 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\cosh(x)-\sinh(x)
$$

```
[In] Integrate[Sech[x]/(1 + Coth[x]),x]
[Out] 2*ArcTan[Tanh[x/2]] + Cosh[x] - Sinh[x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

method	result	size
default	$\frac{2}{\tanh\left(\frac{x}{2}\right)+1}+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	19
risch	$\mathrm{e}^{-x}+i\ln\left(\mathrm{e}^{x}+i\right)-i\ln\left(\mathrm{e}^{x}-i\right)$	24

```
[In] int(sech(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 2/(tanh(1/2*x)+1)+2*arctan(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1}{\cosh\left(x\right)+\sinh\left(x\right)}$$

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="fricas")
[Out] (2*(cosh(x) + sinh(x))*arctan(cosh(x) + sinh(x)) + 1)/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}\left(x\right)}{\coth\left(x\right)+1}\,dx$$

```
[In] integrate(sech(x)/(1+coth(x)),x)
[Out] Integral(sech(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = -2\arctan\left(e^{(-x)}\right)+e^{(-x)}$$

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x)) + e^(-x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{1+\coth(x)}\,dx = 2\arctan\left(e^{x}\right)+e^{(-x)}$$

```
[In] integrate(sech(x)/(1+coth(x)),x, algorithm="giac")
[Out] 2*arctan(e^x) + e^(-x)
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{sech}(x)}{1+\coth(x)}\,dx = \mathrm{e}^{-x} + 2\,\mathrm{atan}(\mathrm{e}^{x})$$

```
[In] int(1/(cosh(x)*(coth(x) + 1)),x)
[Out] exp(-x) + 2*atan(exp(x))
```

3.110 $\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = -\log(1+\coth(x)) - \log(\tanh(x)) + \tanh(x)$$

```
[Out] -ln(1+coth(x))-ln(tanh(x))+tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3597, 46}

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = \tanh(x) - \log(\tanh(x)) - \log(\coth(x)+1)$$

```
[In] Int[Sech[x]^2/(1 + Coth[x]),x]
```

```
[Out] -Log[1 + Coth[x]] - Log[Tanh[x]] + Tanh[x]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx, x, \coth(x)\right) \\
&= -\text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{1}{x} + \frac{1}{1+x}\right)\,dx, x, \coth(x)\right) \\
&= -\log(1+\coth(x)) - \log(\tanh(x)) + \tanh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{\text{sech}^2(x)}{1+\coth(x)}\,dx = -\log(1+\tanh(x)) + \tanh(x)$$

```
[In] Integrate[Sech[x]^2/(1 + Coth[x]),x]
[Out] -Log[1 + Tanh[x]] + Tanh[x]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

method	result	size
risch	$-2x - \frac{2}{1+e^{2x}} + \ln\left(1+e^{2x}\right)$	22
default	$-2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + \frac{2\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^2} + \ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)$	36

```
[In] int(sech(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -2*x-2/(1+exp(2*x))+ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 5.20

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = -\frac{2\,x\cosh\left(x\right)^2 + 4\,x\cosh\left(x\right)\sinh\left(x\right) + 2\,x\sinh\left(x\right)^2 - \left(\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2 + 1\right)\mathrm{lo}}{\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2 + 1}$$

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(2*x*cosh(x)^2 + 4*x*cosh(x)*sinh(x) + 2*x*sinh(x)^2 - (cosh(x)^2 + 2*cosh
(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*x + 2)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}^2\left(x\right)}{\coth\left(x\right)+1}\,dx$$

```
[In] integrate(sech(x)**2/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = \frac{2}{e^{(-2\,x)}+1} + \log\left(e^{(-2\,x)}+1\right)$$

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) + 1) + log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = -2\,x - \frac{e^{(2\,x)} + 3}{e^{(2\,x)} + 1} + \log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(sech(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] -2*x - (e^(2*x) + 3)/(e^(2*x) + 1) + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{\operatorname{sech}^2(x)}{1+\coth(x)}\,dx = \ln\left(\mathrm{e}^{2\,x} + 1\right) - 2\,x - \frac{2}{\mathrm{e}^{2\,x} + 1}$$

```
[In] int(1/(cosh(x)^2*(coth(x) + 1)),x)
[Out] log(exp(2*x) + 1) - 2*x - 2/(exp(2*x) + 1)
```

3.111 $\int \frac{\text{sech}^3(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\text{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{1}{2}\arctan(\sinh(x)) - \text{sech}(x) + \frac{1}{2}\text{sech}(x)\tanh(x)$$

```
[Out] -1/2*arctan(sinh(x))-sech(x)+1/2*sech(x)*tanh(x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3599, 3187, 3186, 2686, 8, 2691, 3855}

$$\int \frac{\text{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{1}{2}\arctan(\sinh(x)) - \text{sech}(x) + \frac{1}{2}\tanh(x)\text{sech}(x)$$

```
[In] Int[Sech[x]^3/(1 + Coth[x]),x]
[Out] -1/2*ArcTan[Sinh[x]] - Sech[x] + (Sech[x]*Tanh[x])/2
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 2686</u>

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3186

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

Rule 3187

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[a^p*b^p, Int[(Cos[c + d*x]^m*Sin[c + d*x]^n)/(b*Cos[c + d*x] + a*Sin[c +
d*x])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[a^2 + b^2, 0] && ILt
Q[p, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i \int \frac{\operatorname{sech}^2(x)\tanh(x)}{-i\cosh(x) - i\sinh(x)}\,dx\right) \\
&= -\int \operatorname{sech}^2(x)(-\cosh(x) + \sinh(x))\tanh(x)\,dx \\
&= i\int \left(-i\operatorname{sech}(x)\tanh(x) + i\operatorname{sech}(x)\tanh^2(x)\right)\,dx
\end{aligned}$$

$$= \int \operatorname{sech}(x)\tanh(x)\,dx - \int \operatorname{sech}(x)\tanh^2(x)\,dx$$

$$= \frac{1}{2}\operatorname{sech}(x)\tanh(x) - \frac{1}{2}\int \operatorname{sech}(x)\,dx - \operatorname{Subst}\left(\int 1\,dx, x, \operatorname{sech}(x)\right)$$

$$= -\frac{1}{2}\arctan(\sinh(x)) - \operatorname{sech}(x) + \frac{1}{2}\operatorname{sech}(x)\tanh(x)$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = \frac{1}{2}(-\arctan(\sinh(x)) + \operatorname{sech}(x)(-2+\tanh(x)))$$

```
[In] Integrate[Sech[x]^3/(1 + Coth[x]),x]
[Out] (-ArcTan[Sinh[x]] + Sech[x]*(-2 + Tanh[x]))/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

method	result	size
risch	$-\frac{e^{x}\left(e^{2x}+3\right)}{\left(1+e^{2x}\right)^{2}}+\frac{i\ln\left(e^{x}-i\right)}{2}-\frac{i\ln\left(e^{x}+i\right)}{2}$	38
default	$\frac{-\tanh\left(\frac{x}{2}\right)^{3}-2\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)-2}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{2}}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$	45

```
[In] int(sech(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -exp(x)*(exp(2*x)+3)/(1+exp(2*x))^2+1/2*I*ln(exp(x)-I)-1/2*I*ln(exp(x)+I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(16) = 32$.

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 7.00

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + \left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x\right.\right.}{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 +}$$

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x
)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(cosh(x)^2 + 1)*s
inh(x) + 3*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cos
h(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(sech(x)**3/(1+coth(x)),x)
```

```
[Out] Integral(sech(x)**3/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{e^{(-x)} + 3\,e^{(-3\,x)}}{2\,e^{(-2\,x)} + e^{(-4\,x)} + 1} + \arctan\left(e^{(-x)}\right)$$

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 3*e^(-3*x))/(2*e^(-2*x) + e^(-4*x) + 1) + arctan(e^(-x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\frac{e^{(3\,x)} + 3\,e^x}{\left(e^{(2\,x)} + 1\right)^2} - \arctan\left(e^x\right)$$

```
[In] integrate(sech(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] -(e^(3*x) + 3*e^x)/(e^(2*x) + 1)^2 - arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{sech}^3(x)}{1+\coth(x)}\,dx = -\operatorname{atan}(\mathrm{e}^x) - \frac{1}{2\cosh\left(x\right)} - \frac{\mathrm{e}^{-x}}{2\cosh\left(x\right)^2}$$

```
[In] int(1/(cosh(x)^3*(coth(x) + 1)),x)
[Out] - atan(exp(x)) - 1/(2*cosh(x)) - exp(-x)/(2*cosh(x)^2)
```

3.112 $\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)} \, dx = \frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}$$

```
[Out] 1/2*tanh(x)^2-1/3*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3597, 862, 45}

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)} \, dx = \frac{\tanh^2(x)}{2} - \frac{\tanh^3(x)}{3}$$

```
[In] Int[Sech[x]^4/(1 + Coth[x]),x]
```

```
[Out] Tanh[x]^2/2 - Tanh[x]^3/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
```

```
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{-1+x^2}{x^4(1+x)}\,dx, x, \coth(x)\right)\\ &= -\text{Subst}\left(\int \frac{-1+x}{x^4}\,dx, x, \coth(x)\right)\\ &= -\text{Subst}\left(\int \left(-\frac{1}{x^4}+\frac{1}{x^3}\right)\,dx, x, \coth(x)\right)\\ &= \frac{\tanh^2(x)}{2}-\frac{\tanh^3(x)}{3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = \frac{1}{6}(-2+3\coth(x))\tanh^3(x)$$

```
[In] Integrate[Sech[x]^4/(1 + Coth[x]),x]
[Out] ((-2 + 3*Coth[x])*Tanh[x]^3)/6
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{1}{3\coth(x)^3}+\frac{1}{2\coth(x)^2}$	14
default	$-\frac{1}{3\coth(x)^3}+\frac{1}{2\coth(x)^2}$	14
risch	$-\frac{2(3\,e^{2x}-1)}{3(1+e^{2x})^3}$	19
parallelrisch	$\frac{11}{18}+\frac{(-3+2\tanh(x))\operatorname{sech}(x)^2}{6}-\frac{\tanh(x)}{3}$	19

```
[In] int(sech(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -1/3/coth(x)^3+1/2/coth(x)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.94

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{4 (\cosh(x) + 2 \sinh(x)}{3 (\cosh(x)^5 + 5 \cosh(x) \sinh(x)^4 + \sinh(x)^5 + (10 \cosh(x)^2 + 3) \sinh(x)^3 + 3 \cosh(x)^3 + (10 \cosh(x}$$

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="fricas")
[Out] -4/3*(cosh(x) + 2*sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(sech(x)**4/(1+coth(x)),x)
[Out] Integral(sech(x)**4/(coth(x) + 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 4.41

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{2\,e^{(-2\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} - \frac{4\,e^{(-4\,x)}}{3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1} - \frac{2}{3\,(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1)}$$

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="maxima")
[Out] -2*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) - 4*e^(-4*x)/(3*e^(-2*
x) + 3*e^(-4*x) + e^(-6*x) + 1) - 2/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{2\left(3\,e^{(2\,x)}-1\right)}{3\,\left(e^{(2\,x)}+1\right)^3}$$

```
[In] integrate(sech(x)^4/(1+coth(x)),x, algorithm="giac")
[Out] -2/3*(3*e^(2*x) - 1)/(e^(2*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^4(x)}{1+\coth(x)}\,dx = -\frac{2\,(3\,e^{2\,x}-1)}{3\,(e^{2\,x}+1)^3}$$

```
[In] int(1/(cosh(x)^4*(coth(x) + 1)),x)
[Out] -(2*(3*exp(2*x) - 1))/(3*(exp(2*x) + 1)^3)
```

3.113 $\int \sqrt{1+\coth(x)}\,\mathrm{sech}^2(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \sqrt{1+\coth(x)}\,\mathrm{sech}^2(x)\,dx = \operatorname{arctanh}\left(\sqrt{1+\coth(x)}\right) + \sqrt{1+\coth(x)}\tanh(x)$$

```
[Out] arctanh((1+coth(x))^(1/2))+(1+coth(x))^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3597, 43, 65, 213}

$$\int \sqrt{1+\coth(x)}\,\mathrm{sech}^2(x)\,dx = \operatorname{arctanh}\left(\sqrt{\coth(x)+1}\right) + \tanh(x)\sqrt{\coth(x)+1}$$

```
[In] Int[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]] + Sqrt[1 + Coth[x]]*Tanh[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \coth(x)\right) \\
&= \sqrt{1+\coth(x)}\tanh(x) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \coth(x)\right) \\
&= \sqrt{1+\coth(x)}\tanh(x) - \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= \operatorname{arctanh}\left(\sqrt{1+\coth(x)}\right) + \sqrt{1+\coth(x)}\tanh(x)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.43

$$
\begin{aligned}
\int \sqrt{1+\coth(x)}\operatorname{sech}^2(x)\,dx &= \frac{1}{2}\sqrt{1+\coth(x)}\left(\frac{(1-i)\arctan\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{i(1+\coth(x))}\right)}{\sqrt{i(1+\coth(x))}}\right. \\
&\quad + \frac{2\left(-2\operatorname{arctanh}\left(\sqrt{\tanh\left(\frac{x}{2}\right)}\right)+\sqrt{2}\operatorname{arctanh}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{\tanh\left(\frac{x}{2}\right)}}\right)\right)\cosh^2\left(\frac{x}{2}\right)\operatorname{csch}(x)\sqrt{\tanh\left(\frac{x}{2}\right)}\left(1+\tanh\left(\frac{x}{2}\right)\right)}{1+\coth(x)} \\
&\quad \left. + 2\tanh(x)\right)
\end{aligned}
$$

```
[In] Integrate[Sqrt[1 + Coth[x]]*Sech[x]^2,x]
```

```
[Out] (Sqrt[1 + Coth[x]]*(((1 - I)*ArcTan[(1/2 + I/2)*Sqrt[I*(1 + Coth[x])]])/Sqr
t[I*(1 + Coth[x])] + (2*(-2*ArcTanh[Sqrt[Tanh[x/2]]] + Sqrt[2]*ArcTanh[(1 +
 Tanh[x/2])/(Sqrt[2]*Sqrt[Tanh[x/2]])])*Cosh[x/2]^2*Csch[x]*Sqrt[Tanh[x/2]]
*(1 + Tanh[x/2]))/(1 + Coth[x]) + 2*Tanh[x]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

method	result	size
derivativedivides	$\frac{1}{2\sqrt{1+\coth(x)}-2}-\frac{\ln\left(\sqrt{1+\coth(x)}-1\right)}{2}+\frac{1}{2\sqrt{1+\coth(x)}+2}+\frac{\ln\left(\sqrt{1+\coth(x)}+1\right)}{2}$	48
default	$\frac{1}{2\sqrt{1+\coth(x)}-2}-\frac{\ln\left(\sqrt{1+\coth(x)}-1\right)}{2}+\frac{1}{2\sqrt{1+\coth(x)}+2}+\frac{\ln\left(\sqrt{1+\coth(x)}+1\right)}{2}$	48

```
[In] int(sech(x)^2*(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((1+coth(x))^(1/2)-1)-1/2*ln((1+coth(x))^(1/2)-1)+1/2/((1+coth(x))^(1/2
)+1)+1/2*ln((1+coth(x))^(1/2)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 231, normalized size of antiderivative = 11.00

$$\int \sqrt{1+\coth(x)}\operatorname{sech}^2(x)\,dx = \frac{4\sqrt{2}\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}+\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)\log\left(\frac{2\cdots}{\cdots}\right.}{\cdots}$$

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
sinh(x))) + (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((2*sqrt(2)*
(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*c
osh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 - 1)/(cosh(x)^2 + 2*cosh(x)*sinh
(x) + sinh(x)^2)) - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(-(2
*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) - sinh(x
))) - 3*cosh(x)^2 - 6*cosh(x)*sinh(x) - 3*sinh(x)^2 + 1)/(cosh(x)^2 + 2*cos
h(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F]

$$\int \sqrt{1+\coth(x)}\mathrm{sech}^2(x)\,dx=\int \sqrt{\coth(x)+1}\,\mathrm{sech}^2(x)\,dx$$

```
[In] integrate(sech(x)**2*(1+coth(x))**(1/2),x)
[Out] Integral(sqrt(coth(x) + 1)*sech(x)**2, x)
```

Maxima [F]

$$\int \sqrt{1+\coth(x)}\mathrm{sech}^2(x)\,dx=\int \sqrt{\coth(x)+1}\,\mathrm{sech}(x)^2\,dx$$

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(coth(x) + 1)*sech(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 5.81

$$\int \sqrt{1+\coth(x)}\mathrm{sech}^2(x)\,dx=-\frac{1}{4}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left(\sqrt{e^{(2x)}-1}-e^x\right)^2-2\sqrt{2}+3}{\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+2\sqrt{2}+3}\right)-\frac{8\left(3\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+1\right)}{\left(\sqrt{e^{(2x)}-1}-e^x\right)^4+6\left(\sqrt{e^{(2x)}-1}-e^x\right)^2+1}\right)\mathrm{sgn}\left(e^{(2x)}-1\right)$$

```
[In] integrate(sech(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*(sqrt(2)*log(((sqrt(e^(2*x) - 1) - e^x)^2 - 2*sqrt(2) + 3)/((s
qrt(e^(2*x) - 1) - e^x)^2 + 2*sqrt(2) + 3)) - 8*(3*(sqrt(e^(2*x) - 1) - e^x
)^2 + 1)/((sqrt(e^(2*x) - 1) - e^x)^4 + 6*(sqrt(e^(2*x) - 1) - e^x)^2 + 1))
*sgn(e^(2*x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\coth(x)}\,\mathrm{sech}^2(x)\,dx = \int \frac{\sqrt{\coth(x)+1}}{\cosh(x)^2}\,dx$$

```
[In] int((coth(x) + 1)^(1/2)/cosh(x)^2,x)
[Out] int((coth(x) + 1)^(1/2)/cosh(x)^2, x)
```

3.114 $\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 147

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = -\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\coth(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\coth(x))}{(a^2-b^2)^3} - \frac{(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x))\sinh^2(x)}{8\,(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4\,(a^2-b^2)}$$

```
[Out] -1/16*a*(3*a+b)*ln(1-coth(x))/(a+b)^3+1/16*a*(3*a-b)*ln(1+coth(x))/(a-b)^3-
a^4*b*ln(a+b*coth(x))/(a^2-b^2)^3-1/8*(4*b*(2*a^2-b^2)-a*(5*a^2-b^2)*coth(x
))*sinh(x)^2/(a^2-b^2)^2-1/4*(b-a*coth(x))*sinh(x)^4/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used

$= \{3597, 1661, 815\}$

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = -\frac{\sinh^4(x)(b-a\coth(x))}{4\left(a^2-b^2\right)} - \frac{\sinh^2(x)\left(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x)\right)}{8\left(a^2-b^2\right)^2} - \frac{a^4b\log(a+b\coth(x))}{\left(a^2-b^2\right)^3} - \frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3} + \frac{a(3a-b)\log(\coth(x)+1)}{16(a-b)^3}$$

```
[In] Int[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -1/16*(a*(3*a + b)*Log[1 - Coth[x]])/(a + b)^3 + (a*(3*a - b)*Log[1 + Coth[
x]])/(16*(a - b)^3) - (a^4*b*Log[a + b*Coth[x]])/(a^2 - b^2)^3 - ((4*b*(2*a
^2 - b^2) - a*(5*a^2 - b^2)*Coth[x])*Sinh[x]^2)/(8*(a^2 - b^2)^2) - ((b - a
*Coth[x])*Sinh[x]^4)/(4*(a^2 - b^2))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\left(b\text{Subst}\left(\int \frac{x^4}{(a+x)\left(-b^2+x^2\right)^3}\,dx, x, b\coth(x)\right)\right)$$

$$= -\frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4}{a^2-b^2}-\frac{3ab^4x}{a^2-b^2}+4b^2x^2}{(a+x)(-b^2+x^2)^2}\,dx, x, b\coth(x)\right)}{4b}$$

$$= -\frac{(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x))\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4(3a^2+b^2)}{(a^2-b^2)^2}-\frac{ab^4(5a^2-b^2)x}{(a^2-b^2)^2}}{(a+x)(-b^2+x^2)}\,dx, x, b\coth(x)\right)}{8b^3}$$

$$= -\frac{(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x))\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)} - \frac{\text{Subst}\left(\int \left(-\frac{ab^3(3a+b)}{2(a+b)^3(b-x)}+\frac{8a^4b^4}{(a-b)^3(a+b)^3(a+x)}-\frac{a(3a-b)b^3}{2(a-b)^3(b+x)}\right)dx, x, b\coth(x)\right)}{8b^3}$$

$$= -\frac{a(3a+b)\log(1-\coth(x))}{16(a+b)^3} + \frac{a(3a-b)\log(1+\coth(x))}{16(a-b)^3} - \frac{a^4b\log(a+b\coth(x))}{(a^2-b^2)^3} - \frac{(4b(2a^2-b^2)-a(5a^2-b^2)\coth(x))\sinh^2(x)}{8(a^2-b^2)^2} - \frac{(b-a\coth(x))\sinh^4(x)}{4(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = \frac{12a^5x+24a^3b^2x-4ab^4x-4b(3a^4-4a^2b^2+b^4)\cosh(2x)-b(a^2-b^2)^2\cosh(4x)-32a^4b\log(b\cosh(x)+}{32(a-b)^3(a+b)^3}$$

```
[In] Integrate[Cosh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (12*a^5*x + 24*a^3*b^2*x - 4*a*b^4*x - 4*b*(3*a^4 - 4*a^2*b^2 + b^4)*Cosh[2
*x] - b*(a^2 - b^2)^2*Cosh[4*x] - 32*a^4*b*Log[b*Cosh[x] + a*Sinh[x]] + 8*a
^3*(a^2 - b^2)*Sinh[2*x] + a^5*Sinh[4*x] - 2*a^3*b^2*Sinh[4*x] + a*b^4*Sinh
[4*x])/(32*(a - b)^3*(a + b)^3)
```

Maple [A] (verified)

Time = 5.81 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.25

method	result
risch	$\frac{3a^2x}{8(a+b)^3}+\frac{axb}{8(a+b)^3}+\frac{e^{4x}}{64a+64b}+\frac{e^{2x}a}{8(a+b)^2}+\frac{e^{2x}b}{16(a+b)^2}-\frac{e^{-2x}a}{8(a-b)^2}+\frac{e^{-2x}b}{16(a-b)^2}-\frac{e^{-4x}}{64(a-b)}+\frac{2a^4bx}{a^6-3a^4b^2+3a^2b^4-b^6}-$
default	$\frac{1}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^3}-\frac{-7a-5b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}-\frac{-5a-3b}{8(a+b)^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{a(3a+b)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right.}{8(a+b)^3}$

```
[In] int(cosh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] 3/8*a^2*x/(a+b)^3+1/8*a*x/(a+b)^3*b+1/64/(a+b)*exp(4*x)+1/8/(a+b)^2*exp(2*x
)*a+1/16/(a+b)^2*exp(2*x)*b-1/8/(a-b)^2*exp(-2*x)*a+1/16/(a-b)^2*exp(-2*x)*
b-1/64/(a-b)*exp(-4*x)+2*a^4*b/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*x-a^4*b/(a^6-3
*a^4*b^2+3*a^2*b^4-b^6)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. 2(139) = 278.

Time = 0.27 (sec) , antiderivative size = 1229, normalized size of antiderivative = 8.36

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
[Out] 1/64*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^8 + 8*(a^
5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^7 + (a^5 -
 a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^8 + 4*(2*a^5 - 3*a^4*
b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^6 + 4*(2*a^5 - 3*a^4*b - 2*a^3*b^2
 + 4*a^2*b^3 - b^5 + 7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*
cosh(x)^2)*sinh(x)^6 + 8*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^4
 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(2
*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x))*sinh(x)^5 - a^5 - a^
4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 2*(35*(a^5 - a^4*b - 2*a^3*b^2
 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + 30*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*
a^2*b^3 - b^5)*cosh(x)^2 + 4*(3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x)*sinh(
x)^4 + 8*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 +
 10*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^3 + 4*(3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x))*sinh(x)^3 - 4*(2*a^5 + 3*a^4*b - 2*
a^3*b^2 - 4*a^2*b^3 + b^5)*cosh(x)^2 + 4*(7*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*cosh(x)^6 - 2*a^5 - 3*a^4*b + 2*a^3*b^2 + 4*a^2*b^3 -
b^5 + 15*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^4 + 12*(3*
a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^2)*sinh(x)^2 - 64*(a^4*b*cosh(
```

```
x)^4 + 4*a^4*b*cosh(x)^3*sinh(x) + 6*a^4*b*cosh(x)^2*sinh(x)^2 + 4*a^4*b*co
sh(x)*sinh(x)^3 + a^4*b*sinh(x)^4)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) -
 sinh(x))) + 8*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)
^7 + 3*(2*a^5 - 3*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 - b^5)*cosh(x)^5 + 4*(3*a^5
 + 8*a^4*b + 6*a^3*b^2 - a*b^4)*x*cosh(x)^3 - (2*a^5 + 3*a^4*b - 2*a^3*b^2
- 4*a^2*b^3 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^4 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3*sinh(x) + 6*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*sinh(x)^2 + 4*(a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*si
nh(x)^4)
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx=\int \frac{\cosh^4{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(cosh(x)**4/(a+b*coth(x)),x)
[Out] Integral(cosh(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx=-\frac{a^4b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2+ab)x}{8(a^3+3a^2b+3ab^2+b^3)}+\frac{(4(2a+b)e^{(-2x)}+a+b)e^{(4x)}}{64(a^2+2ab+b^2)}-\frac{4(2a-b)e^{(-2x)}+(a-b)e^{(-4x)}}{64(a^2-2ab+b^2)}$$

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
[Out] -a^4*b*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) +
 1/8*(3*a^2 + a*b)*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/64*(4*(2*a + b)*e^
(-2*x) + a + b)*e^(4*x)/(a^2 + 2*a*b + b^2) - 1/64*(4*(2*a - b)*e^(-2*x) +
(a - b)*e^(-4*x))/(a^2 - 2*a*b + b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.47

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx$$
$$= -\frac{a^4b\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{(3a^2-ab)x}{8(a^3-3a^2b+3ab^2-b^3)}$$
$$-\frac{\left(18a^2e^{(4x)}-6abe^{(4x)}+8a^2e^{(2x)}-12abe^{(2x)}+4b^2e^{(2x)}+a^2-2ab+b^2\right)e^{(-4x)}}{64(a^3-3a^2b+3ab^2-b^3)}$$
$$+\frac{ae^{(4x)}+be^{(4x)}+8ae^{(2x)}+4be^{(2x)}}{64(a^2+2ab+b^2)}$$

```
[In] integrate(cosh(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) + 1/8*(3*a^2 - a*b)*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 1/64*(18*a
^2*e^(4*x) - 6*a*b*e^(4*x) + 8*a^2*e^(2*x) - 12*a*b*e^(2*x) + 4*b^2*e^(2*x)
 + a^2 - 2*a*b + b^2)*e^(-4*x)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 1/64*(a*e^
(4*x) + b*e^(4*x) + 8*a*e^(2*x) + 4*b*e^(2*x))/(a^2 + 2*a*b + b^2)
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^4(x)}{a+b\coth(x)}\,dx = \frac{e^{4x}}{64a+64b}-\frac{e^{-4x}}{64a-64b}-\frac{e^{-2x}(2a-b)}{16(a-b)^2}+\frac{e^{2x}(2a+b)}{16(a+b)^2}$$
$$-\frac{a^4\,b\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{a\,x\,(3a-b)}{8(a-b)^3}$$

```
[In] int(cosh(x)^4/(a + b*coth(x)),x)
```

```
[Out] exp(4*x)/(64*a + 64*b) - exp(-4*x)/(64*a - 64*b) - (exp(-2*x)*(2*a - b))/(1
6*(a - b)^2) + (exp(2*x)*(2*a + b))/(16*(a + b)^2) - (a^4*b*log(b - a + a*e
xp(2*x) + b*exp(2*x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*x*(3*a - b)
)/(8*(a - b)^3)
```

3.115 $\int \frac{\cosh^3(x)}{a+b\coth(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 135

$$\int \frac{\cosh^3(x)}{a+b\coth(x)} \, dx = \frac{a^3 b \operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{a^2 b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)}$$

```
[Out] a^3*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)-a^2*b*
cosh(x)/(a^2-b^2)^2-1/3*b*cosh(x)^3/(a^2-b^2)+a*b^2*sinh(x)/(a^2-b^2)^2+a*s
inh(x)/(a^2-b^2)+1/3*a*sinh(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3599, 3188, 2713, 2645, 30, 3179, 2717, 3153, 212}

$$\int \frac{\cosh^3(x)}{a+b\coth(x)} \, dx = \frac{a\sinh^3(x)}{3(a^2-b^2)} + \frac{a\sinh(x)}{a^2-b^2} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} - \frac{a^2 b\cosh(x)}{(a^2-b^2)^2} + \frac{a^3 b \operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}$$

```
[In] Int[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (a^3*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2)
- (a^2*b*Cosh[x])/(a^2 - b^2)^2 - (b*Cosh[x]^3)/(3*(a^2 - b^2)) + (a*b^2*Si
```

```
nh[x])/(a^2 - b^2)^2 + (a*Sinh[x])/(a^2 - b^2) + (a*Sinh[x]^3)/(3*(a^2 - b^
2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i\int \frac{\cosh^3(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= \frac{a\int \cosh^3(x)\,dx}{a^2-b^2} - \frac{b\int \cosh^2(x)\sinh(x)\,dx}{a^2-b^2} + \frac{(iab)\int \frac{\cosh^2(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}\\
&= -\frac{a^2b\cosh(x)}{(a^2-b^2)^2} + \frac{(ia^3b)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{(a^2-b^2)^2} + \frac{(ab^2)\int \cosh(x)\,dx}{(a^2-b^2)^2}\\
&\quad + \frac{(ia)\text{Subst}\left(\int (1-x^2)\,dx, x, -i\sinh(x)\right)}{a^2-b^2} - \frac{b\text{Subst}\left(\int x^2\,dx, x, \cosh(x)\right)}{a^2-b^2}\\
&= -\frac{a^2b\cosh(x)}{(a^2-b^2)^2} - \frac{b\cosh^3(x)}{3(a^2-b^2)} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)}\\
&\quad - \frac{(a^3b)\,\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx, x, -a\cosh(x)-b\sinh(x)\right)}{(a^2-b^2)^2}\\
&= \frac{a^3b\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{a^2b\cosh(x)}{(a^2-b^2)^2}\\
&\quad - \frac{b\cosh^3(x)}{3(a^2-b^2)} + \frac{ab^2\sinh(x)}{(a^2-b^2)^2} + \frac{a\sinh(x)}{a^2-b^2} + \frac{a\sinh^3(x)}{3(a^2-b^2)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \frac{1}{12}\left(-\frac{24a^3b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{5/2}(a+b)^{5/2}}+\frac{3b(-5a^2+b^2)\cosh(x)}{(a-b)^2(a+b)^2}+\frac{b\cosh(3x)}{(-a+b)(a+b)}+\frac{3a(3a^2+b^2)\sinh(x)}{(a-b)^2(a+b)^2}+\frac{a^3\sinh(3x)}{(a-b)^2(a+b)^2}-\frac{ab^2\sinh(3x)}{(a-b)^2(a+b)^2}\right)$$

```
[In] Integrate[Cosh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ((-24*a^3*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)
^(5/2)*(a + b)^(5/2)) + (3*b*(-5*a^2 + b^2)*Cosh[x])/((a - b)^2*(a + b)^2)
+ (b*Cosh[3*x])/((-a + b)*(a + b)) + (3*a*(3*a^2 + b^2)*Sinh[x])/((a - b)^2
*(a + b)^2) + (a^3*Sinh[3*x])/((a - b)^2*(a + b)^2) - (a*b^2*Sinh[3*x])/((a
 - b)^2*(a + b)^2))/12
```

Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

method	result
risch	$\frac{e^{3x}}{24a+24b}+\frac{3\,e^{x}a}{8(a+b)^2}+\frac{e^{x}b}{8(a+b)^2}-\frac{3\,e^{-x}a}{8(a-b)^2}+\frac{e^{-x}b}{8(a-b)^2}-\frac{e^{-3x}}{24(a-b)}+\frac{b\,a^3\ln\left(e^{x}+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}-\frac{b\,a^3\ln\left(e^{x}-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2}$
default	$-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^3(4a-4b)}+\frac{2}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{2a-b}{2(a-b)^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{2a^3b\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(a-b)^2(a+b)^2\sqrt{-a^2+b^2}}-\frac{4}{3\left(\tanh\left(\frac{x}{2}\right)-\right.}$

```
[In] int(cosh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/(a+b)*exp(x)^3+3/8/(a+b)^2*exp(x)*a+1/8/(a+b)^2*exp(x)*b-3/8/(a-b)^2/e
xp(x)*a+1/8/(a-b)^2/exp(x)*b-1/24/(a-b)/exp(x)^3+1/(a^2-b^2)^(1/2)*b*a^3/(a
+b)^2/(a-b)^2*ln(exp(x)+(a-b)/(a^2-b^2)^(1/2))-1/(a^2-b^2)^(1/2)*b*a^3/(a+b
)^2/(a-b)^2*ln(exp(x)-(a-b)/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(127) = 254$.

Time = 0.29 (sec) , antiderivative size = 1873, normalized size of antiderivative = 13.87

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
[Out] [1/24*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a
^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5
- a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*
a^3*b^2 + 2*a^2*b^3 - a*b^4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*
b^3 - a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 -
 a*b^4 - b^5 + 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x
)^2)*sinh(x)^4 + 4*(5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*c
osh(x)^3 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x
))*sinh(x)^3 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*co
sh(x)^2 - 3*(3*a^5 + 5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 24*(a^3*b
*cosh(x)^3 + 3*a^3*b*cosh(x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*
sinh(x)^3)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(
x) + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a
 + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) +
 6*((a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^
5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a
^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^2*sinh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2
+ (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^5 - a^4*b - 2*a^
3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^6 + 6*(a^5 - a^4*b - 2*a^3*b^2 + 2
*a^2*b^3 + a*b^4 - b^5)*cosh(x)*sinh(x)^5 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^
2*b^3 + a*b^4 - b^5)*sinh(x)^6 - a^5 - a^4*b + 2*a^3*b^2 + 2*a^2*b^3 - a*b^
4 - b^5 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)
^4 + 3*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5 + 5*(a^5 - a^
4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^5
 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + 3*(3*a^5 - 5*a^
4*b - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x)^3 - 3*(3*a^5 +
5*a^4*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x)^2 - 3*(3*a^5 + 5*a^4
*b - 2*a^3*b^2 - 6*a^2*b^3 - a*b^4 + b^5 - 5*(a^5 - a^4*b - 2*a^3*b^2 + 2*a
^2*b^3 + a*b^4 - b^5)*cosh(x)^4 - 6*(3*a^5 - 5*a^4*b - 2*a^3*b^2 + 6*a^2*b^
3 - a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 - 48*(a^3*b*cosh(x)^3 + 3*a^3*b*cosh(
x)^2*sinh(x) + 3*a^3*b*cosh(x)*sinh(x)^2 + a^3*b*sinh(x)^3)*sqrt(-a^2 + b^2
```

```
)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) + 6*((a^5 -
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*cosh(x)^5 + 2*(3*a^5 - 5*a^4*b
 - 2*a^3*b^2 + 6*a^2*b^3 - a*b^4 - b^5)*cosh(x)^3 - (3*a^5 + 5*a^4*b - 2*a^
3*b^2 - 6*a^2*b^3 - a*b^4 + b^5)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2*si
nh(x) + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^2 + (a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^3)]
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(cosh(x)**3/(a+b*coth(x)),x)
[Out] Integral(cosh(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.21

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \frac{2\,a^3b\arctan\left(-\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} - \frac{\left(9\,ae^{(2\,x)}-3\,be^{(2\,x)}+a-b\right)e^{(-3\,x)}}{24\,(a^2-2\,ab+b^2)} + \frac{a^2e^{(3\,x)}+2\,abe^{(3\,x)}+b^2e^{(3\,x)}+9\,a^2e^x+12\,abe^x+3\,b^2e^x}{24\,(a^3+3\,a^2b+3\,ab^2+b^3)}$$

```
[In] integrate(cosh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] 2*a^3*b*arctan(-(a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*
sqrt(-a^2 + b^2)) - 1/24*(9*a*e^(2*x) - 3*b*e^(2*x) + a - b)*e^(-3*x)/(a^2
- 2*a*b + b^2) + 1/24*(a^2*e^(3*x) + 2*a*b*e^(3*x) + b^2*e^(3*x) + 9*a^2*e^
x + 12*a*b*e^x + 3*b^2*e^x)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

Mupad [B] (verification not implemented)

Time = 2.70 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.94

$$\int \frac{\cosh^3(x)}{a+b\coth(x)}\,dx = \frac{e^{3x}}{24a+24b} - \frac{e^{-3x}}{24a-24b} + \frac{e^{x}(3a+b)}{8(a+b)^2} - \frac{e^{-x}(3a-b)}{8(a-b)^2} + \frac{2\,\mathrm{atan}\left(\frac{a^3\,b\,e^{x}\sqrt{-a^{10}+5a^8b^2-10a^6b^4+10a^4b^6-5a^2b^8+b^{10}}}{a^5\sqrt{a^6b^2}-b^5\sqrt{a^6b^2}+2a^2b^3\sqrt{a^6b^2}-2a^3b^2\sqrt{a^6b^2}+ab^4\sqrt{a^6b^2}-a^4b\sqrt{a^6b^2}}\right)\sqrt{a^6b^2}}{\sqrt{-a^{10}+5a^8b^2-10a^6b^4+10a^4b^6-5a^2b^8+b^{10}}}$$

```
[In] int(cosh(x)^3/(a + b*coth(x)),x)
```

```
[Out] exp(3*x)/(24*a + 24*b) - exp(-3*x)/(24*a - 24*b) + (exp(x)*(3*a + b))/(8*(a
 + b)^2) - (exp(-x)*(3*a - b))/(8*(a - b)^2) + (2*atan((a^3*b*exp(x)*(b^10
- a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/(a^5*(a^6*
b^2)^(1/2) - b^5*(a^6*b^2)^(1/2) + 2*a^2*b^3*(a^6*b^2)^(1/2) - 2*a^3*b^2*(a
^6*b^2)^(1/2) + a*b^4*(a^6*b^2)^(1/2) - a^4*b*(a^6*b^2)^(1/2)))*(a^6*b^2)^(
1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

3.116 $\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = -\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(1+\coth(x))}{4(a-b)^2} -\frac{a^2b\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{(b-a\coth(x))\sinh^2(x)}{2(a^2-b^2)}$$

```
[Out] -1/4*a*ln(1-coth(x))/(a+b)^2+1/4*a*ln(1+coth(x))/(a-b)^2-a^2*b*ln(a+b*coth(
x))/(a^2-b^2)^2-1/2*(b-a*coth(x))*sinh(x)^2/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3597, 1661, 815}

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = -\frac{a^2b\log(a+b\coth(x))}{(a^2-b^2)^2}-\frac{\sinh^2(x)(b-a\coth(x))}{2(a^2-b^2)} -\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(\coth(x)+1)}{4(a-b)^2}$$

```
[In] Int[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -1/4*(a*Log[1 - Coth[x]])/(a + b)^2 + (a*Log[1 + Coth[x]])/(4*(a - b)^2) -
(a^2*b*Log[a + b*Coth[x]])/(a^2 - b^2)^2 - ((b - a*Coth[x])*Sinh[x]^2)/(2*(
a^2 - b^2))
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\text{integral} = -\left(b\text{Subst}\left(\int \frac{x^2}{(a+x)\left(-b^2+x^2\right)^2}\,dx, x, b\coth(x)\right)\right)$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^2}{a^2-b^2}-\frac{ab^2x}{a^2-b^2}}{(a+x)(-b^2+x^2)}\,dx, x, b\coth(x)\right)}{2b}$$

$$= -\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \left(-\frac{ab}{2(a+b)^2(b-x)}+\frac{2a^2b^2}{(a-b)^2(a+b)^2(a+x)}-\frac{ab}{2(a-b)^2(b+x)}\right)dx, x, b\coth(x)\right)}{2b}$$

$$= -\frac{a\log(1-\coth(x))}{4(a+b)^2}+\frac{a\log(1+\coth(x))}{4(a-b)^2}-\frac{a^2b\log(a+b\coth(x))}{\left(a^2-b^2\right)^2}-\frac{(b-a\coth(x))\sinh^2(x)}{2\left(a^2-b^2\right)}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = \frac{(-a^2b+b^3)\cosh(2x)+a(2(a^2+b^2)\,x-4ab\log(b\cosh(x)+a\sinh(x))+(a^2-b^2)\sinh(2x))}{4(a-b)^2(a+b)^2}$$

```
[In] Integrate[Cosh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] ((-(a^2*b) + b^3)*Cosh[2*x] + a*(2*(a^2 + b^2)*x - 4*a*b*Log[b*Cosh[x] + a*
Sinh[x]] + (a^2 - b^2)*Sinh[2*x]))/(4*(a - b)^2*(a + b)^2)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{e^{2x}}{8a+8b}-\frac{e^{-2x}}{8(a-b)}+\frac{2a^2bx}{a^4-2a^2b^2+b^4}-\frac{a^2b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^4-2a^2b^2+b^4}$
default	$-\frac{a^2b\ln\left(\tanh\left(\frac{x}{2}\right)^2b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{(a-b)^2(a+b)^2}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{4}{(8a+8b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2(a+b)^2}-\frac{2}{(4a-4b)(\tanh$

```
[In] int(cosh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x/(a+b)^2+1/8/(a+b)*exp(2*x)-1/8/(a-b)*exp(-2*x)+2*a^2*b/(a^4-2*a^2*b
^2+b^4)*x-a^2*b/(a^4-2*a^2*b^2+b^4)*ln(exp(2*x)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. 2(80) = 160.

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 3.93

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx = \frac{(a^3-a^2b-ab^2+b^3)\cosh(x)^4+4(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)^3+(a^3-a^2b-ab^2+b^3)\sinh(x)}{}$$

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] 1/8*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 - a^2*b - a*b^2 + b^3)*
cosh(x)*sinh(x)^3 + (a^3 - a^2*b - a*b^2 + b^3)*sinh(x)^4 + 4*(a^3 + 2*a^2*
```

```
b + a*b^2)*x*cosh(x)^2 - a^3 - a^2*b + a*b^2 + b^3 + 2*(3*(a^3 - a^2*b - a*
b^2 + b^3)*cosh(x)^2 + 2*(a^3 + 2*a^2*b + a*b^2)*x)*sinh(x)^2 - 8*(a^2*b*co
sh(x)^2 + 2*a^2*b*cosh(x)*sinh(x) + a^2*b*sinh(x)^2)*log(2*(b*cosh(x) + a*s
inh(x))/(cosh(x) - sinh(x))) + 4*((a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^3 + 2
*(a^3 + 2*a^2*b + a*b^2)*x*cosh(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) + (a^4 - 2*a^2*b^2 + b^4)*
sinh(x)^2)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx=\int \frac{\cosh^2{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(cosh(x)**2/(a+b*coth(x)),x)
[Out] Integral(cosh(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx=-\frac{a^2b\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-2a^2b^2+b^4}+\frac{ax}{2(a^2+2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}-\frac{e^{(-2x)}}{8(a-b)}$$

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
[Out] -a^2*b*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*x/(a^
2 + 2*a*b + b^2) + 1/8*e^(2*x)/(a + b) - 1/8*e^(-2*x)/(a - b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx=-\frac{a^2b\log\left(\left|-ae^{(2x)}-be^{(2x)}+a-b\right|\right)}{a^4-2a^2b^2+b^4}+\frac{ax}{2(a^2-2ab+b^2)}-\frac{\left(2ae^{(2x)}+a-b\right)e^{(-2x)}}{8(a^2-2ab+b^2)}+\frac{e^{(2x)}}{8(a+b)}$$

```
[In] integrate(cosh(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*b*log(abs(-a*e^(2*x) - b*e^(2*x) + a - b))/(a^4 - 2*a^2*b^2 + b^4) + 1
/2*a*x/(a^2 - 2*a*b + b^2) - 1/8*(2*a*e^(2*x) + a - b)*e^(-2*x)/(a^2 - 2*a*
b + b^2) + 1/8*e^(2*x)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{\cosh^2(x)}{a+b\coth(x)}\,dx=\frac{e^{2\,x}}{8\,a+8\,b}-\frac{e^{-2\,x}}{8\,a-8\,b}+\frac{a\,x}{2\,{\left(a-b\right)}^2}-\frac{a^2\,b\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^4-2\,a^2\,b^2+b^4}$$

```
[In] int(cosh(x)^2/(a + b*coth(x)),x)
```

```
[Out] exp(2*x)/(8*a + 8*b) - exp(-2*x)/(8*a - 8*b) + (a*x)/(2*(a - b)^2) - (a^2*b
*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.117 $\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=\frac{ab\,\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}$$

```
[Out] a*b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)-b*cosh(x
)/(a^2-b^2)+a*sinh(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3599, 3188, 2717, 2718, 3153, 212}

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=\frac{ab\,\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+\frac{a\sinh(x)}{a^2-b^2}-\frac{b\cosh(x)}{a^2-b^2}$$

```
[In] Int[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (a*b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
(b*Cosh[x])/(a^2 - b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\cosh(x)\sinh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= \frac{a\int \cosh(x)\,dx}{a^2-b^2}-\frac{b\int \sinh(x)\,dx}{a^2-b^2}+\frac{(iab)\int \frac{1}{-ib\cosh(x)-ia\sinh(x)}\,dx}{a^2-b^2}\\
&= -\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}-\frac{(ab)\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,-a\cosh(x)-b\sinh(x)\right)}{a^2-b^2}\\
&= \frac{ab\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}-\frac{b\cosh(x)}{a^2-b^2}+\frac{a\sinh(x)}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx = \frac{2ab\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{(-a+b)^{3/2}(a+b)^{3/2}} + \frac{b\cosh(x)}{-a^2+b^2} + \frac{a\sinh(x)}{a^2-b^2}$$

```
[In] Integrate[Cosh[x]/(a + b*Coth[x]),x]
```

```
[Out] (2*a*b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])])/((-a + b)^(3/2
)*(a + b)^(3/2)) + (b*Cosh[x])/(-a^2 + b^2) + (a*Sinh[x])/(a^2 - b^2)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

method	result	size
default	$-\frac{2ab\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{(a+b)(a-b)\sqrt{-a^2+b^2}} - \frac{4}{(4a-4b)\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{4}{(4a+4b)\left(\tanh\left(\frac{x}{2}\right)-1\right)}$	92
risch	$\frac{e^x}{2a+2b} - \frac{e^{-x}}{2(a-b)} + \frac{ba\ln\left(e^x+\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)} - \frac{ba\ln\left(e^x-\frac{a-b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$	120

```
[In] int(cosh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*b/(a+b)/(a-b)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+
b^2)^(1/2))-4/(4*a-4*b)/(tanh(1/2*x)+1)-4/(4*a+4*b)/(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.99

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx$$

$$= \left[-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2\left((a^4-2\right.}\right.$$

$$-\frac{a^3+a^2b-ab^2-b^3-(a^3-a^2b-ab^2+b^3)\cosh(x)^2-2(a^3-a^2b-ab^2+b^3)\cosh(x)\sinh(x)-(a^3}{2\left((a^4-2a^2b^2+b^4)\cosh(x)+\right.}$$

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b - a*b^2 + b^3)*cosh(x)^2 -
2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a^3 - a^2*b - a*b^2 + b^3)
*sinh(x)^2 + 2*(a*b*cosh(x) + a*b*sinh(x))*sqrt(a^2 - b^2)*log(((a + b)*cos
h(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - 2*sqrt(a^2 - b^2)*
(cosh(x) + sinh(x)) + a - b)/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 - a + b)))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)), -1/2*(a^3 + a^2*b - a*b^2 - b^3 - (a^3 - a^2*b -
a*b^2 + b^3)*cosh(x)^2 - 2*(a^3 - a^2*b - a*b^2 + b^3)*cosh(x)*sinh(x) - (a
^3 - a^2*b - a*b^2 + b^3)*sinh(x)^2 + 4*(a*b*cosh(x) + a*b*sinh(x))*sqrt(-a
^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))))/((a
^4 - 2*a^2*b^2 + b^4)*cosh(x) + (a^4 - 2*a^2*b^2 + b^4)*sinh(x))]
```

Sympy [F]

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=\int \frac{\cosh{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(cosh(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=-\frac{2\,ab\arctan\left(\frac{ae^{x}+be^{x}}{\sqrt{-a^{2}+b^{2}}}\right)}{(a^{2}-b^{2})\sqrt{-a^{2}+b^{2}}}-\frac{e^{(-x)}}{2\,(a-b)}+\frac{e^{x}}{2\,(a+b)}$$

```
[In] integrate(cosh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*a*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^
2)) - 1/2*e^(-x)/(a - b) + 1/2*e^x/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\int \frac{\cosh(x)}{a+b\coth(x)}\,dx=\frac{e^x}{2\,a+2\,b}-\frac{e^{-x}}{2\,a-2\,b}+\frac{2\,\mathrm{atan}\left(\frac{a\,b\,e^x\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}{a^3\,\sqrt{a^2\,b^2}+b^3\,\sqrt{a^2\,b^2}-a\,b^2\,\sqrt{a^2\,b^2}-a^2\,b\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

```
[In] int(cosh(x)/(a + b*coth(x)),x)
```

```
[Out] exp(x)/(2*a + 2*b) - exp(-x)/(2*a - 2*b) + (2*atan((a*b*exp(x)*(b^6 - a^6 -
 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(a^3*(a^2*b^2)^(1/2) + b^3*(a^2*b^2)^(1/2) -
 a*b^2*(a^2*b^2)^(1/2) - a^2*b*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(b^6 - a^
6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
```

3.118 $\int \frac{\mathrm{sech}(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{\mathrm{sech}(x)}{a+b\coth(x)}\,dx=\frac{\arctan(\sinh(x))}{a}+\frac{b\,\mathrm{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[Out] arctan(sinh(x))/a+b*arctanh((a*cosh(x)+b*sinh(x))/(a^2-b^2)^(1/2))/a/(a^2-b
^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3599, 3189, 3855, 3153, 212}

$$\int \frac{\mathrm{sech}(x)}{a+b\coth(x)}\,dx=\frac{b\,\mathrm{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}+\frac{\arctan(\sinh(x))}{a}$$

```
[In] Int[Sech[x]/(a + b*Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a + (b*ArcTanh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/(a
*Sqrt[a^2 - b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= -\int \left(-\frac{\operatorname{sech}(x)}{a}+\frac{ib}{a(ib\cosh(x)+ia\sinh(x))}\right)dx\\
&= \frac{\int \operatorname{sech}(x)\,dx}{a}-\frac{(ib)\int \frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\
&= \frac{\arctan(\sinh(x))}{a}+\frac{b\text{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{a}\\
&= \frac{\arctan(\sinh(x))}{a}+\frac{b\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.20

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx=\frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{b\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)}{a}$$

```
[In] Integrate[Sech[x]/(a + b*Coth[x]),x]
[Out] (2*(ArcTan[Tanh[x/2]] - (b*ArcTan[(a + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a +
b])])/(Sqrt[-a + b]*Sqrt[a + b])))/a
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{2b\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{a\sqrt{-a^{2}+b^{2}}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	54
risch	$\frac{b\ln\left(e^{x}+\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}-\frac{b\ln\left(e^{x}-\frac{a-b}{\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}+\frac{i\ln(e^{x}+i)}{a}-\frac{i\ln(e^{x}-i)}{a}$	102

```
[In] int(sech(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -2*b/a/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))+
2/a*arctan(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 200, normalized size of antiderivative = 4.00

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$$
$$=\left[\frac{\sqrt{a^{2}-b^{2}}b\log\left(\frac{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}+2\sqrt{a^{2}-b^{2}}(\cosh(x)+\sinh(x))+a-b}{(a+b)\cosh(x)^{2}+2\,(a+b)\cosh(x)\sinh(x)+(a+b)\sinh(x)^{2}-a+b}\right)+2\left(a^{2}-b^{2}\right)\arctan}{a^{3}-ab^{2}}\right.$$
$$\left.-\frac{2\left(\sqrt{-a^{2}+b^{2}}b\arctan\left(\frac{\sqrt{-a^{2}+b^{2}}}{(a+b)\cosh(x)+(a+b)\sinh(x)}\right)-\left(a^{2}-b^{2}\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)}{a^{3}-ab^{2}}\right]$$

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [(sqrt(a^2 - b^2)*b*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a
 + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a + b)*c
osh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + 2*(a^2
 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2), -2*(sqrt(-a^2 + b^2)*b*ar
ctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x))) - (a^2 - b^2)*ar
ctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx=\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(sech(x)/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx=-\frac{2\,b\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a}+\frac{2\arctan\left(e^x\right)}{a}$$

```
[In] integrate(sech(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -2*b*arctan((a*e^x + b*e^x)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arct
an(e^x)/a
```

Mupad [B] (verification not implemented)

Time = 4.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 3.28

$$\int \frac{\operatorname{sech}(x)}{a+b\coth(x)}\,dx = \frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x+32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}} - \frac{b\,\ln\left(32\,a\,b^2\,\mathrm{e}^x+32\,a^2\,b\,\mathrm{e}^x-32\,a\,b\,\sqrt{a^2-b^2}\right)}{a\,\sqrt{a^2-b^2}} + \frac{\ln\left(32\,a\,b\,\mathrm{e}^x-32\,a^2\,\mathrm{e}^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a} - \frac{\ln\left(32\,a^2\,\mathrm{e}^x-32\,a\,b\,\mathrm{e}^x+a\,b\,32\mathrm{i}-a^2\,32\mathrm{i}\right)\,1\mathrm{i}}{a}$$

```
[In] int(1/(cosh(x)*(a + b*coth(x))),x)
```

```
[Out] (log(a*b*32i - a^2*32i - 32*a^2*exp(x) + 32*a*b*exp(x))*1i)/a - (log(a*b*32
i - a^2*32i + 32*a^2*exp(x) - 32*a*b*exp(x))*1i)/a - (b*log(32*a*b^2*exp(x)
 + 32*a^2*b*exp(x) - 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) + (b*
log(32*a*b^2*exp(x) + 32*a^2*b*exp(x) + 32*a*b*(a^2 - b^2)^(1/2)))/(a*(a^2
- b^2)^(1/2))
```

3.119 $\int \frac{\text{sech}^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 29

$$\int \frac{\text{sech}^2(x)}{a+b\coth(x)}\,dx=-\frac{b\log(a+b\coth(x))}{a^2}-\frac{b\log(\tanh(x))}{a^2}+\frac{\tanh(x)}{a}$$

```
[Out] -b*ln(a+b*coth(x))/a^2-b*ln(tanh(x))/a^2+tanh(x)/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3597, 46}

$$\int \frac{\text{sech}^2(x)}{a+b\coth(x)}\,dx=-\frac{b\log(\tanh(x))}{a^2}-\frac{b\log(a+b\coth(x))}{a^2}+\frac{\tanh(x)}{a}$$

```
[In] Int[Sech[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((b*Log[a + b*Coth[x]])/a^2) - (b*Log[Tanh[x]])/a^2 + Tanh[x]/a
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
```

```
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(b\text{Subst}\left(\int \frac{1}{x^2(a+x)}\,dx, x, b\coth(x)\right)\right)\\ &= -\left(b\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{1}{a^2x}+\frac{1}{a^2(a+x)}\right)dx, x, b\coth(x)\right)\right)\\ &= -\frac{b\log(a+b\coth(x))}{a^2}-\frac{b\log(\tanh(x))}{a^2}+\frac{\tanh(x)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = \frac{-b\log(b+a\tanh(x))+a\tanh(x)}{a^2}$$

```
[In] Integrate[Sech[x]^2/(a + b*Coth[x]),x]
[Out] (-(b*Log[b + a*Tanh[x]]) + a*Tanh[x])/a^2
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

method	result	size
risch	$-\frac{2}{a(1+e^{2x})}-\frac{b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2}+\frac{b\ln(1+e^{2x})}{a^2}$	51
default	$-\frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^2}-\frac{b\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{2}\right)}{a^2}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^2b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^2}$	61

```
[In] int(sech(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -2/a/(1+exp(2*x))-1/a^2*b*ln(exp(2*x)-(a-b)/(a+b))+1/a^2*b*ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(29) = 58.

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.03

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = -\frac{\left(b\cosh(x)^2+2\,b\cosh(x)\sinh(x)+b\sinh(x)^2+b\right)\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-\left(b\cosh(x)^2+2\,b\cosh(x\right.}{a^2\cosh(x)^2+2\,a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+}$$

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*log(2*(b*cosh(x) +
a*sinh(x))/(cosh(x) - sinh(x))) - (b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 + b)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*a)/(a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(sech(x)**2/(a+b*coth(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^2}+\frac{b\log\left(e^{(-2\,x)}+1\right)}{a^2}+\frac{2}{ae^{(-2\,x)}+a}$$

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/a^2 + b*log(e^(-2*x) + 1)/a^2 + 2/(a*e^(-
2*x) + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.62

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = -\frac{(ab+b^2)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^3+a^2b} + \frac{b\log\left(e^{(2x)}+1\right)}{a^2} - \frac{be^{(2x)}+2a+b}{a^2(e^{(2x)}+1)}$$

```
[In] integrate(sech(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a*b + b^2)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 + a^2*b) + b*log(
e^(2*x) + 1)/a^2 - (b*e^(2*x) + 2*a + b)/(a^2*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 323, normalized size of antiderivative = 11.14

$$\int \frac{\operatorname{sech}^2(x)}{a+b\coth(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b\,\left(a^4\,{\left(b^2\right)}^{3/2}-a^6\,\sqrt{b^2}\right)\,\left(b^6\,\sqrt{-a^4}-a\,b^5\,\sqrt{-a^4}-a^2\,b^4\,\sqrt{-a^4}+a^3\,b^3\,\sqrt{-a^4}+b^6\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}-2\,a^2\,b^4\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}+a^4\,b^2\,\mathrm{e}^{2\,x}\,\sqrt{-a^4}\right)+b^2\,\left(a^3\,(\right.}{-a^{12}\,b^4+3\,a^{10}\,b^6-3\,a^8\,b^8}\right.}{\sqrt{-a^4}} - \frac{2}{a\,\left(\mathrm{e}^{2\,x}+1\right)}$$

```
[In] int(1/(cosh(x)^2*(a + b*coth(x))),x)
```

```
[Out] (2*atan((b*(a^4*(b^2)^(3/2) - a^6*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(-
a^4)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a
^4)^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)
) + b^2*(a^3*(b^2)^(3/2) - a^5*(b^2)^(1/2))*(b^6*(-a^4)^(1/2) - a*b^5*(-a^4
)^(1/2) - a^2*b^4*(-a^4)^(1/2) + a^3*b^3*(-a^4)^(1/2) + b^6*exp(2*x)*(-a^4)
^(1/2) - 2*a^2*b^4*exp(2*x)*(-a^4)^(1/2) + a^4*b^2*exp(2*x)*(-a^4)^(1/2)))/
(a^6*b^10 - 3*a^8*b^8 + 3*a^10*b^6 - a^12*b^4))*(b^2)^(1/2))/(-a^4)^(1/2) -
 2/(a*(exp(2*x) + 1))
```

3.120 $\int \frac{\text{sech}^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\text{sech}^3(x)}{a+b\coth(x)}\,dx = \frac{\arctan(\sinh(x))}{2a} - \frac{b^2\arctan(\sinh(x))}{a^3} + \frac{b\sqrt{a^2-b^2}\,\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3} - \frac{b\,\text{sech}(x)}{a^2} + \frac{\text{sech}(x)\tanh(x)}{2a}$$

```
[Out] 1/2*arctan(sinh(x))/a-b^2*arctan(sinh(x))/a^3-b*sech(x)/a^2+b*arctanh((a*co
sh(x)+b*sinh(x))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^3+1/2*sech(x)*tanh(x)/a
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3599, 3189, 3853, 3855, 3183, 3153, 212}

$$\int \frac{\text{sech}^3(x)}{a+b\coth(x)}\,dx = -\frac{b^2\arctan(\sinh(x))}{a^3} - \frac{b\,\text{sech}(x)}{a^2} + \frac{b\sqrt{a^2-b^2}\,\text{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3} + \frac{\arctan(\sinh(x))}{2a} + \frac{\tanh(x)\text{sech}(x)}{2a}$$

```
[In] Int[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) - (b^2*ArcTan[Sinh[x]])/a^3 + (b*Sqrt[a^2 - b^2]*ArcT
anh[(a*Cosh[x] + b*Sinh[x])/Sqrt[a^2 - b^2]])/a^3 - (b*Sech[x])/a^2 + (Sech
[x]*Tanh[x])/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(i\int \frac{\operatorname{sech}^2(x)\tanh(x)}{-ib\cosh(x)-ia\sinh(x)}\,dx\right)\\
&= -\int \left(-\frac{\operatorname{sech}^3(x)}{a}+\frac{ib\operatorname{sech}^2(x)}{a(ib\cosh(x)+ia\sinh(x))}\right)\,dx\\
&= \frac{\int \operatorname{sech}^3(x)\,dx}{a}-\frac{(ib)\int \frac{\operatorname{sech}^2(x)}{ib\cosh(x)+ia\sinh(x)}\,dx}{a}\\
&= -\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}\\
&\quad -\frac{b^2\int \operatorname{sech}(x)\,dx}{a^3}-\frac{(ib(a^2-b^2))\int \frac{1}{ib\cosh(x)+ia\sinh(x)}\,dx}{a^3}\\
&= \frac{\arctan(\sinh(x))}{2a}-\frac{b^2\arctan(\sinh(x))}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}\\
&\quad +\frac{(b(a^2-b^2))\operatorname{Subst}\left(\int \frac{1}{a^2-b^2-x^2}\,dx,x,a\cosh(x)+b\sinh(x)\right)}{a^3}\\
&= \frac{\arctan(\sinh(x))}{2a}-\frac{b^2\arctan(\sinh(x))}{a^3}\\
&\quad +\frac{b\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{a\cosh(x)+b\sinh(x)}{\sqrt{a^2-b^2}}\right)}{a^3}-\frac{b\operatorname{sech}(x)}{a^2}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$
\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx
= \frac{2(a^2-2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+4b\sqrt{-a+b}\sqrt{a+b}\arctan\left(\frac{a+b\tanh\left(\frac{x}{2}\right)}{\sqrt{-a+b}\sqrt{a+b}}\right)+a\operatorname{sech}(x)(-2b+a\tanh(x))}{2a^3}
$$

```
[In] Integrate[Sech[x]^3/(a + b*Coth[x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*ArcTan[Tanh[x/2]] + 4*b*Sqrt[-a + b]*Sqrt[a + b]*ArcTan[(a
 + b*Tanh[x/2])/(Sqrt[-a + b]*Sqrt[a + b])] + a*Sech[x]*(-2*b + a*Tanh[x]))
/(2*a^3)
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46

method	result
default	$\frac{\frac{2\left(-\frac{a^2\tanh\left(\frac{x}{2}\right)^3}{2}-\tanh\left(\frac{x}{2}\right)^2ab+\frac{a^2\tanh\left(\frac{x}{2}\right)}{2}-ab\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+\left(a^2-2b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^3}-\frac{2b\left(a^2-b^2\right)\arctan\left(\frac{2b\tanh\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a^3\sqrt{-a^2+b^2}}$
risch	$\frac{\mathrm{e}^x\left(a\,\mathrm{e}^{2x}-2b\,\mathrm{e}^{2x}-a-2b\right)}{\left(1+\mathrm{e}^{2x}\right)^2a^2}+\frac{\sqrt{a^2-b^2}\,b\ln\left(\mathrm{e}^x+\frac{\sqrt{a^2-b^2}}{a+b}\right)}{a^3}-\frac{\sqrt{a^2-b^2}\,b\ln\left(\mathrm{e}^x-\frac{\sqrt{a^2-b^2}}{a+b}\right)}{a^3}+\frac{i\ln(\mathrm{e}^x+i)}{2a}-\frac{i\ln(\mathrm{e}^x+i)b^2}{a^3}-\frac{i\ln(\mathrm{e}^x-}{2a}$

```
[In] int(sech(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^3*((-1/2*a^2*tanh(1/2*x)^3-tanh(1/2*x)^2*a*b+1/2*a^2*tanh(1/2*x)-a*b)/(
1+tanh(1/2*x)^2)^2+1/2*(a^2-2*b^2)*arctan(tanh(1/2*x)))-2*b*(a^2-b^2)/a^3/(
-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tanh(1/2*x)+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. 2(75) = 150.

Time = 0.34 (sec) , antiderivative size = 856, normalized size of antiderivative = 10.31

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] [((a^2 - 2*a*b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*
b)*sinh(x)^3 + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cos
h(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh
(x) + b)*sqrt(a^2 - b^2)*log(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
 + (a + b)*sinh(x)^2 + 2*sqrt(a^2 - b^2)*(cosh(x) + sinh(x)) + a - b)/((a +
 b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a + b)) + (
(a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*b^2)
*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 + a^2
 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 - 2*b
^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x) + (
3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*co
sh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^
2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x)), ((a^2 - 2*a*
b)*cosh(x)^3 + 3*(a^2 - 2*a*b)*cosh(x)*sinh(x)^2 + (a^2 - 2*a*b)*sinh(x)^3
- 2*(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*
(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sq
rt(-a^2 + b^2)*arctan(sqrt(-a^2 + b^2)/((a + b)*cosh(x) + (a + b)*sinh(x)))
```

```
 + ((a^2 - 2*b^2)*cosh(x)^4 + 4*(a^2 - 2*b^2)*cosh(x)*sinh(x)^3 + (a^2 - 2*
b^2)*sinh(x)^4 + 2*(a^2 - 2*b^2)*cosh(x)^2 + 2*(3*(a^2 - 2*b^2)*cosh(x)^2 +
 a^2 - 2*b^2)*sinh(x)^2 + a^2 - 2*b^2 + 4*((a^2 - 2*b^2)*cosh(x)^3 + (a^2 -
 2*b^2)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2 + 2*a*b)*cosh(x)
 + (3*(a^2 - 2*a*b)*cosh(x)^2 - a^2 - 2*a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^
3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh
(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx=\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(sech(x)**3/(a+b*coth(x)),x)
[Out] Integral(sech(x)**3/(a + b*coth(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{sech}^3(x)}{a+b\coth(x)}\,dx=\frac{(a^2-2\,b^2)\arctan\left(e^x\right)}{a^3}-\frac{2\,(a^2b-b^3)\arctan\left(\frac{ae^x+be^x}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^3}+\frac{ae^{(3\,x)}-2\,be^{(3\,x)}-ae^x-2\,be^x}{a^2(e^{(2\,x)}+1)^2}$$

```
[In] integrate(sech(x)^3/(a+b*coth(x)),x, algorithm="giac")
[Out] (a^2 - 2*b^2)*arctan(e^x)/a^3 - 2*(a^2*b - b^3)*arctan((a*e^x + b*e^x)/sqrt
(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a*e^(3*x) - 2*b*e^(3*x) - a*e^x - 2
*b*e^x)/(a^2*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 4.77 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.00

$$\int \frac{\mathrm{sech}^3(x)}{a+b\coth(x)}\,dx = \frac{e^x\,(a-2\,b)}{a^2\,\left(e^{2\,x}+1\right)}+\frac{\ln\left(e^x+1\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}-b^2\,2\mathrm{i}\right)}{2\,a^3}-\frac{2\,e^x}{a\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}-\frac{\ln\left(e^x-\mathrm{i}\right)\,\left(a^2\,1\mathrm{i}-b^2\,2\mathrm{i}\right)}{2\,a^3}+\frac{b\,\ln\left(a\,e^x+b\,e^x+\sqrt{a^2-b^2}\right)\,\sqrt{(a+b)\,(a-b)}}{a^3}-\frac{b\,\ln\left(a\,e^x+b\,e^x-\sqrt{a^2-b^2}\right)\,\sqrt{(a+b)\,(a-b)}}{a^3}$$

```
[In] int(1/(cosh(x)^3*(a + b*coth(x))),x)
```

```
[Out] (log(exp(x) + 1i)*(a^2*1i - b^2*2i))/(2*a^3) - (log(exp(x) - 1i)*(a^2*1i -
b^2*2i))/(2*a^3) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1)) + (exp(x)*(a
- 2*b))/(a^2*(exp(2*x) + 1)) + (b*log(a*exp(x) + b*exp(x) + (a^2 - b^2)^(1/
2))*((a + b)*(a - b))^(1/2))/a^3 - (b*log(a*exp(x) + b*exp(x) - (a^2 - b^2)
^(1/2))*((a + b)*(a - b))^(1/2))/a^3
```

3.121 $\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = -\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4} - \frac{b(a^2-b^2)\log(\tanh(x))}{a^4} + \frac{(a^2-b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}$$

```
[Out] -b*(a^2-b^2)*ln(a+b*coth(x))/a^4-b*(a^2-b^2)*ln(tanh(x))/a^4+(a^2-b^2)*tanh
(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3597, 908}

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \frac{b\tanh^2(x)}{2a^2} - \frac{b(a^2-b^2)\log(\tanh(x))}{a^4} - \frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4} + \frac{(a^2-b^2)\tanh(x)}{a^3} - \frac{\tanh^3(x)}{3a}$$

```
[In] Int[Sech[x]^4/(a + b*Coth[x]),x]
```

```
[Out] -((b*(a^2 - b^2)*Log[a + b*Coth[x]])/a^4) - (b*(a^2 - b^2)*Log[Tanh[x]])/a^
4 + ((a^2 - b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3597

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Dist[b/f, Subst[Int[x^m*((a + x)^n/(b^2 + x^2)^(m/2 + 1)),
x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(b\text{Subst}\left(\int \frac{-b^2+x^2}{x^4(a+x)}\,dx, x, b\coth(x)\right)\right) \\
&= -\left(b\text{Subst}\left(\int \left(-\frac{b^2}{ax^4}+\frac{b^2}{a^2x^3}+\frac{a^2-b^2}{a^3x^2}+\frac{-a^2+b^2}{a^4x}+\frac{a^2-b^2}{a^4(a+x)}\right)\,dx, x, b\coth(x)\right)\right) \\
&= -\frac{b(a^2-b^2)\log(a+b\coth(x))}{a^4}-\frac{b(a^2-b^2)\log(\tanh(x))}{a^4} \\
&\quad +\frac{(a^2-b^2)\tanh(x)}{a^3}+\frac{b\tanh^2(x)}{2a^2}-\frac{\tanh^3(x)}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \frac{6b(-a^2+b^2)\log(b+a\tanh(x))+6a(a^2-b^2)\tanh(x)+3a^2b\tanh^2(x)-2a^3\tanh^3(x)}{6a^4}$$

```
[In] Integrate[Sech[x]^4/(a + b*Coth[x]),x]
[Out] (6*b*(-a^2 + b^2)*Log[b + a*Tanh[x]] + 6*a*(a^2 - b^2)*Tanh[x] + 3*a^2*b*Ta
nh[x]^2 - 2*a^3*Tanh[x]^3)/(6*a^4)
```

Maple [A] (verified)

Time = 11.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.84

method	result
risch	$-\frac{2\left(3ab\,\mathrm{e}^{4x}-3b^{2}\mathrm{e}^{4x}+6a^{2}\mathrm{e}^{2x}+3b\,\mathrm{e}^{2x}a-6b^{2}\mathrm{e}^{2x}+2a^{2}-3b^{2}\right)}{3a^{3}\left(1+\mathrm{e}^{2x}\right)^{3}}+\frac{b\ln\left(1+\mathrm{e}^{2x}\right)}{a^{2}}-\frac{b^{3}\ln\left(1+\mathrm{e}^{2x}\right)}{a^{4}}-\frac{b\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a^{2}}+\frac{b^{3}\ln\left(\mathrm{e}^{2x}-\frac{a-}{a+}\right.}{a^{4}}$
default	$-\frac{b\left(a^{2}-b^{2}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}b+2a\tanh\left(\frac{x}{2}\right)+b\right)}{a^{4}}-\frac{2\left(\frac{\left(-a^{3}+a\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{5}-a^{2}b\tanh\left(\frac{x}{2}\right)^{4}+\left(-\frac{2}{3}a^{3}+2a\,b^{2}\right)\tanh\left(\frac{x}{2}\right)^{3}-a^{2}b\tanh\left(\frac{x}{2}\right)^{2}+(-}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{3}}\right.}{a^{4}}$

[In] int(sech(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)

[Out] -2/3*(3*a*b*exp(4*x)-3*b^2*exp(4*x)+6*a^2*exp(2*x)+3*b*exp(2*x)*a-6*b^2*exp
(2*x)+2*a^2-3*b^2)/a^3/(1+exp(2*x))^3+1/a^2*b*ln(1+exp(2*x))-1/a^4*b^3*ln(1
+exp(2*x))-1/a^2*b*ln(exp(2*x)-(a-b)/(a+b))+1/a^4*b^3*ln(exp(2*x)-(a-b)/(a+
b))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. 2(75) = 150.

Time = 0.29 (sec) , antiderivative size = 909, normalized size of antiderivative = 11.51

$$\int \frac{\mathrm{sech}^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="fricas")

[Out] -1/3*(6*(a^2*b - a*b^2)*cosh(x)^4 + 24*(a^2*b - a*b^2)*cosh(x)*sinh(x)^3 +
6*(a^2*b - a*b^2)*sinh(x)^4 + 4*a^3 - 6*a*b^2 + 6*(2*a^3 + a^2*b - 2*a*b^2)
cosh(x)^2 + 6(2*a^3 + a^2*b - 2*a*b^2 + 6*(a^2*b - a*b^2)*cosh(x)^2)*sinh
(x)^2 + 3*((a^2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a
^2*b - b^3)*sinh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2
*b - b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b -
b^3)*cosh(x))*sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a
^2*b - b^3)*cosh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2
+ 6*((a^2*b - b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*co
sh(x))*sinh(x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^
2*b - b^3)*cosh(x)^6 + 6*(a^2*b - b^3)*cosh(x)*sinh(x)^5 + (a^2*b - b^3)*si
nh(x)^6 + 3*(a^2*b - b^3)*cosh(x)^4 + 3*(a^2*b - b^3 + 5*(a^2*b - b^3)*cosh
(x)^2)*sinh(x)^4 + 4*(5*(a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x))*
sinh(x)^3 + a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 + 3*(5*(a^2*b - b^3)*co
sh(x)^4 + a^2*b - b^3 + 6*(a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 6*((a^2*b -
b^3)*cosh(x)^5 + 2*(a^2*b - b^3)*cosh(x)^3 + (a^2*b - b^3)*cosh(x))*sinh(x)
)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 12*(2*(a^2*b - a*b^2)*cosh(x)^3 + (2

```
*a^3 + a^2*b - 2*a*b^2)*cosh(x))*sinh(x))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*si
nh(x)^5 + a^4*sinh(x)^6 + 3*a^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh
(x)^2 + a^4)*sinh(x)^4 + a^4 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)^
3 + 3*(5*a^4*cosh(x)^4 + 6*a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^
5 + 2*a^4*cosh(x)^3 + a^4*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(sech(x)**4/(a+b*coth(x)),x)
[Out] Integral(sech(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.68

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \frac{2\left(2a^2-3b^2+3(2a^2-ab-2b^2)e^{(-2x)}-3(ab+b^2)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}+3a^3e^{(-4x)}+a^3e^{(-6x)}+a^3\right)} - \frac{(a^2b-b^3)\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4} + \frac{(a^2b-b^3)\log\left(e^{(-2x)}+1\right)}{a^4}$$

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="maxima")
[Out] 2/3*(2*a^2 - 3*b^2 + 3*(2*a^2 - a*b - 2*b^2)*e^(-2*x) - 3*(a*b + b^2)*e^(-4
*x))/(3*a^3*e^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) - (a^2*b - b^3)
*log(-(a - b)*e^(-2*x) + a + b)/a^4 + (a^2*b - b^3)*log(e^(-2*x) + 1)/a^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(75) = 150$.

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.54

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = -\frac{(a^3b+a^2b^2-ab^3-b^4)\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^5+a^4b} + \frac{(a^2b-b^3)\log\left(e^{(2x)}+1\right)}{a^4} - \frac{11a^2be^{(6x)}-11b^3e^{(6x)}+45a^2be^{(4x)}-12ab^2e^{(4x)}-33b^3e^{(4x)}+24a^3e^{(2x)}+45a^2be^{(2x)}-24ab^2e^{(2x)}-}{6a^4\left(e^{(2x)}+1\right)^3}$$

```
[In] integrate(sech(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -(a^3*b + a^2*b^2 - a*b^3 - b^4)*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a
^5 + a^4*b) + (a^2*b - b^3)*log(e^(2*x) + 1)/a^4 - 1/6*(11*a^2*b*e^(6*x) -
11*b^3*e^(6*x) + 45*a^2*b*e^(4*x) - 12*a*b^2*e^(4*x) - 33*b^3*e^(4*x) + 24*
a^3*e^(2*x) + 45*a^2*b*e^(2*x) - 24*a*b^2*e^(2*x) - 33*b^3*e^(2*x) + 8*a^3
+ 11*a^2*b - 12*a*b^2 - 11*b^3)/(a^4*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^4(x)}{a+b\coth(x)}\,dx = \frac{8}{3\,a\,\left(3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1\right)}-\frac{2\,\left(2\,a-b\right)}{a^2\,\left(2\,e^{2\,x}+e^{4\,x}+1\right)}-\frac{2\,b\,\left(a-b\right)}{a^3\,\left(e^{2\,x}+1\right)}-\frac{b\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)\,\left(a+b\right)\,\left(a-b\right)}{a^4}+\frac{b\,\ln\left(e^{2\,x}+1\right)\,\left(a+b\right)\,\left(a-b\right)}{a^4}$$

```
[In] int(1/(cosh(x)^4*(a + b*coth(x))),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (2*(2*a - b))/(a^2*(2*ex
p(2*x) + exp(4*x) + 1)) - (2*b*(a - b))/(a^3*(exp(2*x) + 1)) - (b*log(b - a
 + a*exp(2*x) + b*exp(2*x))*(a + b)*(a - b))/a^4 + (b*log(exp(2*x) + 1)*(a
+ b)*(a - b))/a^4
```

3.122 $\int \frac{\text{sech}(x)}{i+2\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\text{sech}(x)}{i+2\coth(x)}\,dx = -i\arctan(\sinh(x)) - \frac{2\text{arctanh}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -I*arctan(sinh(x))-2/5*arctanh(1/5*(cosh(x)-2*I*sinh(x))*5^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3599, 3189, 3855, 3153, 212}

$$\int \frac{\text{sech}(x)}{i+2\coth(x)}\,dx = -i\arctan(\sinh(x)) - \frac{2\text{arctanh}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[In] Int[Sech[x]/(I + 2*Coth[x]),x]
```

```
[Out] (-I)*ArcTan[Sinh[x]] - (2*ArcTanh[(Cosh[x] - (2*I)*Sinh[x])/Sqrt[5]])/Sqrt[
5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3599

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Int[Sin[e + f*x]^m*((a*Cos[e + f*x] + b*Sin[e + f*x])^n/C
os[e + f*x]^n), x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && ILtQ
[n, 0] && ((LtQ[m, 5] && GtQ[n, -4]) || (EqQ[m, 5] && EqQ[n, -1]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(i\int \frac{\tanh(x)}{-2i\cosh(x)+\sinh(x)}\,dx\right)\\
&= -\int \left(i\operatorname{sech}(x)-\frac{2i}{2\cosh(x)+i\sinh(x)}\right)\,dx\\
&= -(i\int \operatorname{sech}(x)\,dx)+2i\int \frac{1}{2\cosh(x)+i\sinh(x)}\,dx\\
&= -i\arctan(\sinh(x))-2\text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,\cosh(x)-2i\sinh(x)\right)\\
&= -i\arctan(\sinh(x))-\frac{2\text{arctanh}\left(\frac{\cosh(x)-2i\sinh(x)}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = -2i\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{4\operatorname{arctanh}\left(\frac{1-2i\tanh\left(\frac{x}{2}\right)}{\sqrt{5}}\right)}{\sqrt{5}}$$

[In] Integrate[Sech[x]/(I + 2*Coth[x]),x]

[Out] (-2*I)*ArcTan[Tanh[x/2]] - (4*ArcTanh[(1 - (2*I)*Tanh[x/2])/Sqrt[5]])/Sqrt[
5]

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{4i\sqrt{5}\,\arctan\left(\frac{\left(2\tanh\left(\frac{x}{2}\right)+i\right)\sqrt{5}}{5}\right)}{5}-\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)$	41
risch	$\ln\left(e^{x}+i\right)-\ln\left(e^{x}-i\right)+\frac{2\sqrt{5}\,\ln\left(e^{x}-\frac{2i\sqrt{5}}{5}-\frac{\sqrt{5}}{5}\right)}{5}-\frac{2\sqrt{5}\,\ln\left(e^{x}+\frac{2i\sqrt{5}}{5}+\frac{\sqrt{5}}{5}\right)}{5}$	56

[In] int(sech(x)/(I+2*coth(x)),x,method=_RETURNVERBOSE)

[Out] 4/5*I*5^(1/2)*arctan(1/5*(2*tanh(1/2*x)+I)*5^(1/2))-ln(tanh(1/2*x)-I)+ln(ta
nh(1/2*x)+I)

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = -\frac{2}{5}\sqrt{5}\log\left(\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right) + \frac{2}{5}\sqrt{5}\log\left(-\left(\frac{2}{5}i+\frac{1}{5}\right)\sqrt{5}+e^{x}\right) + \log\left(e^{x}+i\right) - \log\left(e^{x}-i\right)$$

[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="fricas")

[Out] -2/5*sqrt(5)*log((2/5*I + 1/5)*sqrt(5) + e^x) + 2/5*sqrt(5)*log(-(2/5*I + 1
/5)*sqrt(5) + e^x) + log(e^x + I) - log(e^x - I)

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx=\int \frac{\operatorname{sech}(x)}{2\coth(x)+i}\,dx$$

```
[In] integrate(sech(x)/(I+2*coth(x)),x)
[Out] Integral(sech(x)/(2*coth(x) + I), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx=\frac{2}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-(2i+1)\,e^{(-x)}}{\sqrt{5}+(2i+1)\,e^{(-x)}}\right)+2i\,\arctan\left(e^{(-x)}\right)$$

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="maxima")
[Out] 2/5*sqrt(5)*log(-(sqrt(5) - (2*I + 1)*e^(-x))/(sqrt(5) + (2*I + 1)*e^(-x)))
 + 2*I*arctan(e^(-x))
```

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx=\frac{4}{5}i\sqrt{5}\arctan\left(\left(\frac{1}{5}i+\frac{2}{5}\right)\sqrt{5}e^{x}\right)+\log\left(e^{x}+i\right)-\log\left(e^{x}-i\right)$$

```
[In] integrate(sech(x)/(I+2*coth(x)),x, algorithm="giac")
[Out] 4/5*I*sqrt(5)*arctan((1/5*I + 2/5)*sqrt(5)*e^x) + log(e^x + I) - log(e^x -
I)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{\operatorname{sech}(x)}{i+2\coth(x)}\,dx = \ln\left(e^x\,(32+64\mathrm{i})-64+32\mathrm{i}\right)-\ln\left(e^x\,(32+64\mathrm{i})+64-32\mathrm{i}\right) - \frac{2\sqrt{5}\,\ln\left(e^x\left(-\frac{256}{5}+\frac{192}{5}\mathrm{i}\right)+\sqrt{5}\left(-\frac{128}{5}-\frac{64}{5}\mathrm{i}\right)\right)}{5} + \frac{2\sqrt{5}\,\ln\left(e^x\left(-\frac{256}{5}+\frac{192}{5}\mathrm{i}\right)+\sqrt{5}\left(\frac{128}{5}+\frac{64}{5}\mathrm{i}\right)\right)}{5}$$

```
[In] int(1/(cosh(x)*(2*coth(x) + 1i)),x)
```

```
[Out] log(exp(x)*(32 + 64i) - (64 - 32i)) - log(exp(x)*(32 + 64i) + (64 - 32i)) -
 (2*5^(1/2)*log(- exp(x)*(256/5 - 192i/5) - 5^(1/2)*(128/5 + 64i/5)))/5 + (
2*5^(1/2)*log(5^(1/2)*(128/5 + 64i/5) - exp(x)*(256/5 - 192i/5)))/5
```

3.123 $\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx=\frac{5x}{2}-2\log(\cosh(x))-\frac{5\tanh(x)}{2}+\tanh^2(x)-\frac{5\tanh^3(x)}{6}+\frac{\tanh^3(x)}{2(1+\coth(x))}$$

```
[Out] 5/2*x-2*ln(cosh(x))-5/2*tanh(x)+tanh(x)^2-5/6*tanh(x)^3+1/2*tanh(x)^3/(1+co
th(x))
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3633, 3610, 3612, 3556}

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx=\frac{5x}{2}-\frac{5\tanh^3(x)}{6}+\tanh^2(x)-\frac{5\tanh(x)}{2}-2\log(\cosh(x))+\frac{\tanh^3(x)}{2(\coth(x)+1)}$$

```
[In] Int[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*x)/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 - (5*Tanh[x]^3)/6 + Ta
nh[x]^3/(2*(1 + Coth[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\tanh^3(x)}{2(1+\coth(x))} - \frac{1}{2}\int(-5+4\coth(x))\tanh^4(x)\,dx \\
&= -\frac{5}{6}\tanh^3(x) + \frac{\tanh^3(x)}{2(1+\coth(x))} - \frac{1}{2}i\int(-4i+5i\coth(x))\tanh^3(x)\,dx \\
&= \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))} + \frac{1}{2}\int(5-4\coth(x))\tanh^2(x)\,dx \\
&= -\frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))} + \frac{1}{2}i\int(4i-5i\coth(x))\tanh(x)\,dx \\
&= \frac{5x}{2} - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))} - 2\int\tanh(x)\,dx \\
&= \frac{5x}{2} - 2\log(\cosh(x)) - \frac{5\tanh(x)}{2} + \tanh^2(x) - \frac{5\tanh^3(x)}{6} + \frac{\tanh^3(x)}{2(1+\coth(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{5}{2}\operatorname{arctanh}(\tanh(x)) - 2\log(\cosh(x)) - \frac{5\tanh(x)}{2} + \tanh^2(x) + \left(-\frac{5}{6} + \frac{1}{2+2\coth(x)}\right)\tanh^3(x)$$

```
[In] Integrate[Tanh[x]^4/(1 + Coth[x]),x]
```

```
[Out] (5*ArcTanh[Tanh[x]])/2 - 2*Log[Cosh[x]] - (5*Tanh[x])/2 + Tanh[x]^2 + (-5/6
 + (2 + 2*Coth[x])^(-1))*Tanh[x]^3
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result
risch	$\frac{9x}{2} + \frac{e^{-2x}}{4} + \frac{4\,e^{4x}+6\,e^{2x}+\frac{14}{3}}{\left(1+e^{2x}\right)^3} - 2\ln\left(1+e^{2x}\right)$
parallelrisch	$\frac{(12\tanh(x)+12)\ln(1-\tanh(x))-2\tanh(x)^4+\tanh(x)^3+27\tanh(x)x-9\tanh(x)^2+27x+15}{6+6\tanh(x)}$
default	$\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - \frac{1}{\tanh\left(\frac{x}{2}\right)+1} + \frac{9\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2} - \frac{4\left(\tanh\left(\frac{x}{2}\right)^5 - \frac{\tanh\left(\frac{x}{2}\right)^4}{2} + \frac{8\tanh\left(\frac{x}{2}\right)^3}{3} - \frac{\tanh\left(\frac{x}{2}\right)^2}{2} + \tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3} - 2$

```
[In] int(tanh(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 9/2*x+1/4*exp(-2*x)+2/3*(6*exp(4*x)+9*exp(2*x)+7)/(1+exp(2*x))^3-2*ln(1+exp
(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(35) = 70$.

Time = 0.26 (sec) , antiderivative size = 571, normalized size of antiderivative = 13.28

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/12*(54*x*cosh(x)^8 + 432*x*cosh(x)*sinh(x)^7 + 54*x*sinh(x)^8 + 3*(54*x +
 17)*cosh(x)^6 + 3*(504*x*cosh(x)^2 + 54*x + 17)*sinh(x)^6 + 18*(168*x*cosh
(x)^3 + (54*x + 17)*cosh(x))*sinh(x)^5 + 81*(2*x + 1)*cosh(x)^4 + 9*(420*x*
```

```
cosh(x)^4 + 5*(54*x + 17)*cosh(x)^2 + 18*x + 9)*sinh(x)^4 + 12*(252*x*cosh(
x)^5 + 5*(54*x + 17)*cosh(x)^3 + 27*(2*x + 1)*cosh(x))*sinh(x)^3 + (54*x +
65)*cosh(x)^2 + (1512*x*cosh(x)^6 + 45*(54*x + 17)*cosh(x)^4 + 486*(2*x + 1
)*cosh(x)^2 + 54*x + 65)*sinh(x)^2 - 24*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 +
sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^3 +
9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*cosh
(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cosh(x)
^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh(x)^
7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) -
sinh(x))) + 2*(216*x*cosh(x)^7 + 9*(54*x + 17)*cosh(x)^5 + 162*(2*x + 1)*co
sh(x)^3 + (54*x + 65)*cosh(x))*sinh(x) + 3)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^
7 + sinh(x)^8 + (28*cosh(x)^2 + 3)*sinh(x)^6 + 3*cosh(x)^6 + 2*(28*cosh(x)^
3 + 9*cosh(x))*sinh(x)^5 + (70*cosh(x)^4 + 45*cosh(x)^2 + 3)*sinh(x)^4 + 3*
cosh(x)^4 + 4*(14*cosh(x)^5 + 15*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + (28*cos
h(x)^6 + 45*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(4*cosh
(x)^7 + 9*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \int \frac{\tanh^4{(x)}}{\coth{(x)}+1}\,dx$$

```
[In] integrate(tanh(x)**4/(1+coth(x)),x)
[Out] Integral(tanh(x)**4/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x - \frac{2\,\left(15\,e^{(-2\,x)} + 12\,e^{(-4\,x)} + 7\right)}{3\,\left(3\,e^{(-2\,x)} + 3\,e^{(-4\,x)} + e^{(-6\,x)} + 1\right)} + \frac{1}{4}\,e^{(-2\,x)} - 2\,\log\left(e^{(-2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x - 2/3*(15*e^(-2*x) + 12*e^(-4*x) + 7)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-
6*x) + 1) + 1/4*e^(-2*x) - 2*log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{9}{2}\,x + \frac{\left(51\,e^{(6\,x)} + 81\,e^{(4\,x)} + 65\,e^{(2\,x)} + 3\right)e^{(-2\,x)}}{12\,\left(e^{(2\,x)} + 1\right)^3} - 2\,\log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)^4/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 9/2*x + 1/12*(51*e^(6*x) + 81*e^(4*x) + 65*e^(2*x) + 3)*e^(-2*x)/(e^(2*x) +
 1)^3 - 2*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{\tanh^4(x)}{1+\coth(x)}\,dx = \frac{9\,x}{2} - 2\,\ln\left(\mathrm{e}^{2\,x}+1\right) + \frac{\mathrm{e}^{-2\,x}}{4} + \frac{8}{3\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)} - \frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + \frac{4}{\mathrm{e}^{2\,x}+1}$$

```
[In] int(tanh(x)^4/(coth(x) + 1),x)
```

```
[Out] (9*x)/2 - 2*log(exp(2*x) + 1) + exp(-2*x)/4 + 8/(3*(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1)) - 2/(2*exp(2*x) + exp(4*x) + 1) + 4/(exp(2*x) + 1)
```

3.124 $\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx=-\frac{3x}{2}+2\log(\cosh(x))+\frac{3\tanh(x)}{2}-\tanh^2(x)+\frac{\tanh^2(x)}{2(1+\coth(x))}$$

```
[Out] -3/2*x+2*ln(cosh(x))+3/2*tanh(x)-tanh(x)^2+1/2*tanh(x)^2/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3633, 3610, 3612, 3556}

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx=-\frac{3x}{2}-\tanh^2(x)+\frac{3\tanh(x)}{2}+2\log(\cosh(x))+\frac{\tanh^2(x)}{2(\coth(x)+1)}$$

```
[In] Int[Tanh[x]^3/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/2 + 2*Log[Cosh[x]] + (3*Tanh[x])/2 - Tanh[x]^2 + Tanh[x]^2/(2*(1 + C
oth[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
```

```
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tanh^2(x)}{2(1+\coth(x))} - \frac{1}{2}\int(-4+3\coth(x))\tanh^3(x)\,dx \\
&= -\tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} - \frac{1}{2}i\int(-3i+4i\coth(x))\tanh^2(x)\,dx \\
&= \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} + \frac{1}{2}\int(4-3\coth(x))\tanh(x)\,dx \\
&= -\frac{3x}{2} + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))} + 2\int\tanh(x)\,dx \\
&= -\frac{3x}{2} + 2\log(\cosh(x)) + \frac{3\tanh(x)}{2} - \tanh^2(x) + \frac{\tanh^2(x)}{2(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \frac{1}{2}\left(-3\text{arctanh}(\tanh(x)) + 4\log(\cosh(x)) + 3\tanh(x) + \left(-2+\frac{1}{1+\coth(x)}\right)\tanh^2(x)\right)$$

[In] Integrate[Tanh[x]^3/(1 + Coth[x]),x]

[Out] (-3*ArcTanh[Tanh[x]] + 4*Log[Cosh[x]] + 3*Tanh[x] + (-2 + (1 + Coth[x])^(-1
))*Tanh[x]^2)/2

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{7x}{2}-\frac{e^{-2x}}{4}-\frac{2}{(1+e^{2x})^2}+2\ln(1+e^{2x})$
parallelrisch	$\frac{(-4\tanh(x)-4)\ln(1-\tanh(x))-\tanh(x)^3-7\tanh(x)x+\tanh(x)^2-7x-3}{2+2\tanh(x)}$
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{2}+\frac{2\tanh(\frac{x}{2})^3-2\tanh(\frac{x}{2})^2+2\tanh(\frac{x}{2})}{(1+\tanh(\frac{x}{2})^2)^2}+2\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)-\frac{1}{(\tanh(\frac{x}{2})+1)^2}+\frac{1}{\tanh(\frac{x}{2})+}$

[In] int(tanh(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)

[Out] -7/2*x-1/4*exp(-2*x)-2/(1+exp(2*x))^2+2*ln(1+exp(2*x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(31) = 62.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 9.57

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = -\frac{14\,x\cosh(x)^6+84\,x\cosh(x)\sinh(x)^5+14\,x\sinh(x)^6+(28\,x+1)\cosh(x)^4+\left(210\,x\cosh(x)^2+28\,x+\right.}{}$$

[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="fricas")

[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 + (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 + 28*x + 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 + (

```
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 +
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh
(x)^3 + 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 + (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) + 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 + 2)*sinh(x)^4 + 2*cosh(x)^4 + 4*(5*cosh(x)^3 + 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \int \frac{\tanh^3(x)}{\coth(x)+1}\,dx$$

```
[In] integrate(tanh(x)**3/(1+coth(x)),x)
[Out] Integral(tanh(x)**3/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{2\left(2\,e^{(-2\,x)}+1\right)}{2\,e^{(-2\,x)}+e^{(-4\,x)}+1} - \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 2*(2*e^(-2*x) + 1)/(2*e^(-2*x) + e^(-4*x) + 1) - 1/4*e^(-2*x) + 2*l
og(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = -\frac{7}{2}\,x - \frac{\left(e^{(4\,x)}+10\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}+1\right)^2} + 2\,\log\left(e^{(2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^3/(1+coth(x)),x, algorithm="giac")
[Out] -7/2*x - 1/4*(e^(4*x) + 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1)^2 + 2*log(e^
(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\tanh^3(x)}{1+\coth(x)}\,dx = 2\,\ln\left(\mathrm{e}^{2\,x}+1\right) - \frac{7\,x}{2} - \frac{\mathrm{e}^{-2\,x}}{4} - \frac{2}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}$$

```
[In] int(tanh(x)^3/(coth(x) + 1),x)
[Out] 2*log(exp(2*x) + 1) - (7*x)/2 - exp(-2*x)/4 - 2/(2*exp(2*x) + exp(4*x) + 1)
```

3.125 $\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{3x}{2} - \log(\cosh(x)) - \frac{3\tanh(x)}{2} + \frac{\tanh(x)}{2(1+\coth(x))}$$

```
[Out] 3/2*x-ln(cosh(x))-3/2*tanh(x)+1/2*tanh(x)/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3633, 3610, 3612, 3556}

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{3x}{2} - \frac{3\tanh(x)}{2} - \log(\cosh(x)) + \frac{\tanh(x)}{2(\coth(x)+1)}$$

```
[In] Int[Tanh[x]^2/(1 + Coth[x]),x]
[Out] (3*x)/2 - Log[Cosh[x]] - (3*Tanh[x])/2 + Tanh[x]/(2*(1 + Coth[x]))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
```

```
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3633

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-a)*((c + d*Tan[e + f*x])^(n + 1)/(2*f*(b*
c - a*d)*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a*(b*c - a*d)), Int[(c + d*
Tan[e + f*x])^n*Simp[b*c + a*d*(n - 1) - b*d*n*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &
& NeQ[c^2 + d^2, 0] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\tanh(x)}{2(1+\coth(x))} - \frac{1}{2}\int (-3+2\coth(x))\tanh^2(x)\,dx \\
&= -\frac{3\tanh(x)}{2} + \frac{\tanh(x)}{2(1+\coth(x))} - \frac{1}{2}i\int (-2i+3i\coth(x))\tanh(x)\,dx \\
&= \frac{3x}{2} - \frac{3\tanh(x)}{2} + \frac{\tanh(x)}{2(1+\coth(x))} - \int \tanh(x)\,dx \\
&= \frac{3x}{2} - \log(\cosh(x)) - \frac{3\tanh(x)}{2} + \frac{\tanh(x)}{2(1+\coth(x))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{1}{4}\left(-\log(1-\tanh(x)) + 5\log(1+\tanh(x)) + \left(-6+\frac{2}{1+\coth(x)}\right)\tanh(x)\right)$$

```
[In] Integrate[Tanh[x]^2/(1 + Coth[x]),x]
[Out] (-Log[1 - Tanh[x]] + 5*Log[1 + Tanh[x]] + (-6 + 2/(1 + Coth[x]))*Tanh[x])/4
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result
risch	$\frac{5x}{2}+\frac{{\mathrm e}^{-2x}}{4}+\frac{2}{1+{\mathrm e}^{2x}}-\ln\left(1+{\mathrm e}^{2x}\right)$
parallelrisch	$\frac{(2+2\tanh(x))\ln(1-\tanh(x))+5\tanh(x)x-2\tanh(x)^{2}+5x+3}{2+2\tanh(x)}$
default	$-\frac{2\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}-\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}+\frac{5\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}$

```
[In] int(tanh(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 5/2*x+1/4*exp(-2*x)+2/(1+exp(2*x))-ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(23) = 46.

Time = 0.33 (sec) , antiderivative size = 186, normalized size of antiderivative = 6.41

$$\int\frac{\tanh^{2}(x)}{1+\coth(x)}\,dx$$
$$=\frac{10\,x\cosh\left(x\right)^{4}+40\,x\cosh\left(x\right)\sinh\left(x\right)^{3}+10\,x\sinh\left(x\right)^{4}+\left(10\,x+9\right)\cosh\left(x\right)^{2}+\left(60\,x\cosh\left(x\right)^{2}+10\,x+9\right.}{4\left(\cosh\left(x\right.\right.}$$

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 + (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 + 10*x + 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*co
sh(x)^3 + cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 + (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + c
osh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \int \frac{\tanh^2{(x)}}{\coth{(x)}+1}\,dx$$

```
[In] integrate(tanh(x)**2/(1+coth(x)),x)
[Out] Integral(tanh(x)**2/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x - \frac{2}{e^{(-2\,x)}+1} + \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x - 2/(e^(-2*x) + 1) + 1/4*e^(-2*x) - log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{5}{2}\,x + \frac{\left(9\,e^{(2\,x)}+1\right)e^{(-2\,x)}}{4\left(e^{(2\,x)}+1\right)} - \log\left(e^{(2\,x)}+1\right)$$

```
[In] integrate(tanh(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] 5/2*x + 1/4*(9*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) + 1) - log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{1+\coth(x)}\,dx = \frac{5\,x}{2} - \ln\left(\mathrm{e}^{2\,x}+1\right) + \frac{\mathrm{e}^{-2\,x}}{4} + \frac{2}{\mathrm{e}^{2\,x}+1}$$

```
[In] int(tanh(x)^2/(coth(x) + 1),x)
[Out] (5*x)/2 - log(exp(2*x) + 1) + exp(-2*x)/4 + 2/(exp(2*x) + 1)
```

3.126 $\int \frac{\tanh(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{\tanh(x)}{1+\coth(x)} \, dx = -\frac{x}{2} + \frac{1}{2(1+\coth(x))} + \log(\cosh(x))$$

```
[Out] -1/2*x+1/2/(1+coth(x))+ln(cosh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3632, 3560, 8, 3556}

$$\int \frac{\tanh(x)}{1+\coth(x)} \, dx = -\frac{x}{2} + \frac{1}{2(\coth(x)+1)} + \log(\cosh(x))$$

```
[In] Int[Tanh[x]/(1 + Coth[x]),x]
[Out] -1/2*x + 1/(2*(1 + Coth[x])) + Log[Cosh[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3632

```
Int[1/(((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*Tan[e + f*x]),
x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*Tan[e + f*x]), x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] && NeQ[c^2 +
 d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{1}{1+\coth(x)}\,dx + \int \tanh(x)\,dx \\
&= \frac{1}{2(1+\coth(x))} + \log(\cosh(x)) - \frac{\int 1\,dx}{2} \\
&= -\frac{x}{2} + \frac{1}{2(1+\coth(x))} + \log(\cosh(x))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \frac{1}{2(1+\coth(x))} - \frac{1}{4}\log(1-\tanh(x)) - \frac{3}{4}\log(1+\tanh(x))$$

```
[In] Integrate[Tanh[x]/(1 + Coth[x]),x]
[Out] 1/(2*(1 + Coth[x])) - Log[1 - Tanh[x]]/4 - (3*Log[1 + Tanh[x]])/4
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{3x}{2}-\frac{e^{-2x}}{4}+\ln\left(1+e^{2x}\right)$	18
parallelrisch	$\frac{(-2-2\tanh(x))\ln(1-\tanh(x))-3\tanh(x)x-3x-1}{2+2\tanh(x)}$	34
default	$-\frac{\ln(\tanh(\frac{x}{2})-1)}{2}+\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)-\frac{1}{(\tanh(\frac{x}{2})+1)^{2}}+\frac{1}{\tanh(\frac{x}{2})+1}-\frac{3\ln(\tanh(\frac{x}{2})+1)}{2}$	47

```
[In] int(tanh(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x-1/4*exp(-2*x)+ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(15) = 30.

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = -\frac{6\,x\cosh\left(x\right)^{2}+12\,x\cosh\left(x\right)\sinh\left(x\right)+6\,x\sinh\left(x\right)^{2}-4\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)\log}{4\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}\right)}$$

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \int \frac{\tanh\left(x\right)}{\coth\left(x\right)+1}\,dx$$

```
[In] integrate(tanh(x)/(1+coth(x)),x)
```

```
[Out] Integral(tanh(x)/(coth(x) + 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x-\frac{1}{4}\,e^{\left(-2\,x\right)}+\log\left(e^{\left(-2\,x\right)}+1\right)$$

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*e^(-2*x) + log(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = -\frac{3}{2}\,x - \frac{1}{4}\,e^{(-2\,x)} + \log\left(e^{(2\,x)} + 1\right)$$

```
[In] integrate(tanh(x)/(1+coth(x)),x, algorithm="giac")
[Out] -3/2*x - 1/4*e^(-2*x) + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh(x)}{1+\coth(x)}\,dx = \ln\left(e^{2\,x} + 1\right) - \frac{3\,x}{2} - \frac{e^{-2\,x}}{4}$$

```
[In] int(tanh(x)/(coth(x) + 1),x)
[Out] log(exp(2*x) + 1) - (3*x)/2 - exp(-2*x)/4
```

3.127 $\int \frac{1}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 16

$$\int \frac{1}{1+\coth(x)} \, dx = \frac{x}{2} - \frac{1}{2(1+\coth(x))}$$

```
[Out] 1/2*x-1/2/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3560, 8}

$$\int \frac{1}{1+\coth(x)} \, dx = \frac{x}{2} - \frac{1}{2(\coth(x)+1)}$$

```
[In] Int[(1 + Coth[x])^(-1),x]
```

```
[Out] x/2 - 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{2(1+\coth(x))}+\frac{\int 1\,dx}{2}\\
&= \frac{x}{2}-\frac{1}{2(1+\coth(x))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$
\int \frac{1}{1+\coth(x)}\,dx=\frac{1}{2}\left(\operatorname{arctanh}(\tanh(x))+\frac{1}{1+\tanh(x)}\right)
$$

```
[In] Integrate[(1 + Coth[x])^(-1),x]
[Out] (ArcTanh[Tanh[x]] + (1 + Tanh[x])^(-1))/2
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}+\frac{{\mathrm e}^{-2 x}}{4}$	11
parallelrisch	$\frac{\tanh(x) x+x+1}{2+2\tanh(x)}$	17
derivativedivides	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24
default	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}+\frac{\ln(1+\coth(x))}{4}$	24

```
[In] int(1/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x+1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{(2\,x+1)\cosh\left(x\right)+(2\,x-1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(1/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*((2*x + 1)*cosh(x) + (2*x - 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2} + \frac{x}{2\tanh\left(x\right)+2} + \frac{1}{2\tanh\left(x\right)+2}$$

```
[In] integrate(1/(1+coth(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(1/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x + 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(1/(1+coth(x)),x, algorithm="giac")
[Out] 1/2*x + 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+\coth(x)}\,dx = \frac{x}{2} - \frac{1}{2\,\left(\coth\left(x\right)+1\right)}$$

```
[In] int(1/(coth(x) + 1),x)
[Out] x/2 - 1/(2*(coth(x) + 1))
```

3.128 $\int \frac{\coth(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{x}{2}+\frac{1}{2(1+\coth(x))}$$

```
[Out] 1/2*x+1/2/(1+coth(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3607, 8}

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{x}{2}+\frac{1}{2(\coth(x)+1)}$$

```
[In] Int[Coth[x]/(1 + Coth[x]),x]
```

```
[Out] x/2 + 1/(2*(1 + Coth[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2(1+\coth(x))} + \frac{\int 1\,dx}{2}$$
$$= \frac{x}{2} + \frac{1}{2(1+\coth(x))}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{1}{2}\text{arctanh}(\tanh(x)) - \frac{1}{2(1+\tanh(x))}$$

```
[In] Integrate[Coth[x]/(1 + Coth[x]),x]
[Out] ArcTanh[Tanh[x]]/2 - 1/(2*(1 + Tanh[x]))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{x}{2}-\frac{{\mathrm e}^{-2 x}}{4}$	11
parallelrisch	$\frac{\tanh \left(x \right) x +x -1}{2+2 \tanh \left(x \right)}$	17
derivativedivides	$\frac{1}{2+2 \coth \left(x \right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}$	24
default	$\frac{1}{2+2 \coth \left(x \right)}+\frac{\ln \left(1+\coth \left(x \right)\right)}{4}-\frac{\ln \left(\coth \left(x \right)-1\right)}{4}$	24

```
[In] int(coth(x)/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x-1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{(2\,x-1)\cosh\left(x\right)+(2\,x+1)\sinh\left(x\right)}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="fricas")
[Out] 1/4*((2*x - 1)*cosh(x) + (2*x + 1)*sinh(x))/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2}+\frac{x}{2\tanh\left(x\right)+2}-\frac{1}{2\tanh\left(x\right)+2}$$

```
[In] integrate(coth(x)/(1+coth(x)),x)
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\coth(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x-\frac{1}{4}\,e^{(-2\,x)}$$

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="maxima")
[Out] 1/2*x - 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\coth(x)}{1+\coth(x)}\, dx = \frac{1}{2}\, x - \frac{1}{4}\, e^{(-2\, x)}$$

```
[In] integrate(coth(x)/(1+coth(x)),x, algorithm="giac")
[Out] 1/2*x - 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\coth(x)}{1+\coth(x)}\, dx = \frac{x}{2} + \frac{1}{2\, \left(\coth\left(x\right)+1\right)}$$

```
[In] int(coth(x)/(coth(x) + 1),x)
[Out] x/2 + 1/(2*(coth(x) + 1))
```

3.129 $\int \frac{\coth^2(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx=-\frac{x}{2}-\frac{1}{2(1+\coth(x))}+\log(\sinh(x))$$

```
[Out] -1/2*x-1/2/(1+coth(x))+ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3621, 3556}

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx=-\frac{x}{2}-\frac{1}{2(\coth(x)+1)}+\log(\sinh(x))$$

```
[In] Int[Coth[x]^2/(1 + Coth[x]),x]
[Out] -1/2*x - 1/(2*(1 + Coth[x])) + Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{1}{2(1+\coth(x))} - \frac{1}{2}\int (1-2\coth(x))\,dx \\ &= -\frac{x}{2} - \frac{1}{2(1+\coth(x))} + \int \coth(x)\,dx \\ &= -\frac{x}{2} - \frac{1}{2(1+\coth(x))} + \log(\sinh(x))\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = -\frac{1}{2}\coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + \frac{1}{2}\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(x)\right) + \log(\cosh(x)) + \log(\tanh(x))$$

```
[In] Integrate[Coth[x]^2/(1 + Coth[x]),x]
[Out] -1/2*Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + (Coth[x]*Hypergeometric2F1[-
1/2, 1, 1/2, Tanh[x]^2])/2 + Log[Cosh[x]] + Log[Tanh[x]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{3x}{2}+\frac{e^{-2x}}{4}+\ln\left(e^{2x}-1\right)$	18
derivativedivides	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{3\ln(1+\coth(x))}{4}$	24
default	$-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{3\ln(1+\coth(x))}{4}$	24
parallelrisch	$\frac{(-2-2\tanh(x))\ln(1-\tanh(x))+(2+2\tanh(x))\ln(\tanh(x))-3\tanh(x)x-3x+1}{2+2\tanh(x)}$	44

```
[In] int(coth(x)^2/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -3/2*x+1/4*exp(-2*x)+ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(15) = 30$.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.84

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx =$$
$$-\frac{6\,x\cosh\left(x\right)^{2} + 12\,x\cosh\left(x\right)\sinh\left(x\right) + 6\,x\sinh\left(x\right)^{2} - 4\,\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)\log}{4\,\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right)}$$

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x*cosh(x)^2 + 12*x*cosh(x)*sinh(x) + 6*x*sinh(x)^2 - 4*(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x))) - 1)/(co
sh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\begin{aligned}\int \frac{\coth^2(x)}{1+\coth(x)}\,dx &= \frac{x\tanh\left(x\right)}{2\tanh\left(x\right)+2} + \frac{x}{2\tanh\left(x\right)+2} - \frac{2\log\left(\tanh\left(x\right)+1\right)\tanh\left(x\right)}{2\tanh\left(x\right)+2}\\ &\quad - \frac{2\log\left(\tanh\left(x\right)+1\right)}{2\tanh\left(x\right)+2} + \frac{2\log\left(\tanh\left(x\right)\right)\tanh\left(x\right)}{2\tanh\left(x\right)+2}\\ &\quad + \frac{2\log\left(\tanh\left(x\right)\right)}{2\tanh\left(x\right)+2} + \frac{1}{2\tanh\left(x\right)+2}\end{aligned}$$

```
[In] integrate(coth(x)**2/(1+coth(x)),x)
```

```
[Out] x*tanh(x)/(2*tanh(x) + 2) + x/(2*tanh(x) + 2) - 2*log(tanh(x) + 1)*tanh(x)/
(2*tanh(x) + 2) - 2*log(tanh(x) + 1)/(2*tanh(x) + 2) + 2*log(tanh(x))*tanh(
x)/(2*tanh(x) + 2) + 2*log(tanh(x))/(2*tanh(x) + 2) + 1/(2*tanh(x) + 2)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{1}{4}\,e^{\left(-2\,x\right)} + \log\left(e^{\left(-x\right)} + 1\right) + \log\left(e^{\left(-x\right)} - 1\right)$$

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*e^(-2*x) + log(e^(-x) + 1) + log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = -\frac{3}{2}\,x + \frac{1}{4}\,e^{(-2\,x)} + \log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(coth(x)^2/(1+coth(x)),x, algorithm="giac")
[Out] -3/2*x + 1/4*e^(-2*x) + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\coth^2(x)}{1+\coth(x)}\,dx = \frac{x}{2} - \ln\left(\coth(x) + 1\right) - \frac{1}{2\,\left(\coth\left(x\right) + 1\right)}$$

```
[In] int(coth(x)^2/(coth(x) + 1),x)
[Out] x/2 - log(coth(x) + 1) - 1/(2*(coth(x) + 1))
```

3.130 $\int \frac{\coth^3(x)}{1+\coth(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \frac{\coth^3(x)}{1+\coth(x)} \, dx = \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth^2(x)}{2(1+\coth(x))} - \log(\sinh(x))$$

```
[Out] 3/2*x-3/2*coth(x)+1/2*coth(x)^2/(1+coth(x))-ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3631, 3606, 3556}

$$\int \frac{\coth^3(x)}{1+\coth(x)} \, dx = \frac{3x}{2} + \frac{\coth^2(x)}{2(\coth(x)+1)} - \frac{3\coth(x)}{2} - \log(\sinh(x))$$

```
[In] Int[Coth[x]^3/(1 + Coth[x]),x]
[Out] (3*x)/2 - (3*Coth[x])/2 + Coth[x]^2/(2*(1 + Coth[x])) - Log[Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\coth^2(x)}{2(1+\coth(x))} - \frac{1}{2}\int (2-3\coth(x))\coth(x)\,dx \\
&= \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth^2(x)}{2(1+\coth(x))} - \int \coth(x)\,dx \\
&= \frac{3x}{2} - \frac{3\coth(x)}{2} + \frac{\coth^2(x)}{2(1+\coth(x))} - \log(\sinh(x))
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx = \frac{1}{2}\left(\coth^2(x) + \frac{\coth^4(x)}{1+\coth(x)} - \coth^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(x)\right) - 2(\log(\cosh(x)) + \log(\tanh(x)))\right)$$

```
[In] Integrate[Coth[x]^3/(1 + Coth[x]),x]
[Out] (Coth[x]^2 + Coth[x]^4/(1 + Coth[x]) - Coth[x]^3*Hypergeometric2F1[-3/2, 1,
 -1/2, Tanh[x]^2] - 2*(Log[Cosh[x]] + Log[Tanh[x]]))/2
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\coth(x)-\frac{\ln(\coth(x)-1)}{4}+\frac{1}{2+2\coth(x)}+\frac{5\ln(1+\coth(x))}{4}$	28
default	$-\coth(x)-\frac{\ln(\coth(x)-1)}{4}+\frac{1}{2+2\coth(x)}+\frac{5\ln(1+\coth(x))}{4}$	28
risch	$\frac{5x}{2}-\frac{e^{-2x}}{4}-\frac{2}{e^{2x}-1}-\ln(e^{2x}-1)$	30
parallelrisch	$\frac{(2+2\tanh(x))\ln(1-\tanh(x))+(-2-2\tanh(x))\ln(\tanh(x))+5\tanh(x)x+5x-2\coth(x)-3}{2+2\tanh(x)}$	48

```
[In] int(coth(x)^3/(1+coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -coth(x)-1/4*ln(coth(x)-1)+1/2/(1+coth(x))+5/4*ln(1+coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 6.32

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx$$
$$=\frac{10\,x\cosh(x)^4+40\,x\cosh(x)\sinh(x)^3+10\,x\sinh(x)^4-(10\,x+9)\cosh(x)^2+\left(60\,x\cosh(x)^2-10\,x-9\right.}{4\left(\cosh(\right.}$$

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x*cosh(x)^4 + 40*x*cosh(x)*sinh(x)^3 + 10*x*sinh(x)^4 - (10*x + 9)*
cosh(x)^2 + (60*x*cosh(x)^2 - 10*x - 9)*sinh(x)^2 - 4*(cosh(x)^4 + 4*cosh(x
)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*co
sh(x)^3 - cosh(x))*sinh(x))*log(2*sinh(x)/(cosh(x) - sinh(x))) + 2*(20*x*co
sh(x)^3 - (10*x + 9)*cosh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3
 + sinh(x)^4 + (6*cosh(x)^2 - 1)*sinh(x)^2 - cosh(x)^2 + 2*(2*cosh(x)^3 - c
osh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(27) = 54$.

Time = 0.47 (sec) , antiderivative size = 160, normalized size of antiderivative = 5.16

$$\int \frac{\coth^3(x)}{1+\coth(x)} \, dx = \frac{x\tanh^2{(x)}}{2\tanh^2{(x)}+2\tanh{(x)}} + \frac{x\tanh{(x)}}{2\tanh^2{(x)}+2\tanh{(x)}} + \frac{2\log{(\tanh{(x)}+1)}\tanh^2{(x)}}{2\tanh^2{(x)}+2\tanh{(x)}} + \frac{2\log{(\tanh{(x)}+1)}\tanh{(x)}}{2\tanh^2{(x)}+2\tanh{(x)}} - \frac{2\log{(\tanh{(x)})}\tanh^2{(x)}}{2\tanh^2{(x)}+2\tanh{(x)}} - \frac{2\log{(\tanh{(x)})}\tanh{(x)}}{2\tanh^2{(x)}+2\tanh{(x)}} - \frac{3\tanh{(x)}}{2\tanh^2{(x)}+2\tanh{(x)}} - \frac{2}{2\tanh^2{(x)}+2\tanh{(x)}}$$

```
[In] integrate(coth(x)**3/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + x*tanh(x)/(2*tanh(x)**2 + 2*tanh(
x)) + 2*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**2 + 2*tanh(x)) + 2*log(tanh
(x) + 1)*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*
tanh(x)**2 + 2*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*tanh(x)**2 + 2*tanh(x))
 - 3*tanh(x)/(2*tanh(x)**2 + 2*tanh(x)) - 2/(2*tanh(x)**2 + 2*tanh(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int \frac{\coth^3(x)}{1+\coth(x)} \, dx = \frac{1}{2}\,x + \frac{2}{e^{(-2\,x)}-1} - \frac{1}{4}\,e^{(-2\,x)} - \log\left(e^{(-x)}+1\right) - \log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2/(e^(-2*x) - 1) - 1/4*e^(-2*x) - log(e^(-x) + 1) - log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\coth^3(x)}{1+\coth(x)} \, dx = \frac{5}{2}\,x - \frac{\left(9\,e^{(2\,x)}-1\right)e^{(-2\,x)}}{4\,\left(e^{(2\,x)}-1\right)} - \log\left(\left|e^{(2\,x)}-1\right|\right)$$

```
[In] integrate(coth(x)^3/(1+coth(x)),x, algorithm="giac")
```

```
[Out] 5/2*x - 1/4*(9*e^(2*x) - 1)*e^(-2*x)/(e^(2*x) - 1) - log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \frac{\coth^3(x)}{1+\coth(x)}\,dx = \frac{x}{2}+\ln\left(\coth(x)+1\right)-\coth(x)+\frac{1}{2\,\left(\coth\left(x\right)+1\right)}$$

```
[In] int(coth(x)^3/(coth(x) + 1),x)
```

```
[Out] x/2 + log(coth(x) + 1) - coth(x) + 1/(2*(coth(x) + 1))
```

3.131 $\int \frac{\coth^4(x)}{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = -\frac{3x}{2}+\frac{3\coth(x)}{2}-\coth^2(x)+\frac{\coth^3(x)}{2(1+\coth(x))}+2\log(\sinh(x))$$

```
[Out] -3/2*x+3/2*coth(x)-coth(x)^2+1/2*coth(x)^3/(1+coth(x))+2*ln(sinh(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3631, 3609, 3606, 3556}

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = -\frac{3x}{2}+\frac{\coth^3(x)}{2(\coth(x)+1)}-\coth^2(x)+\frac{3\coth(x)}{2}+2\log(\sinh(x))$$

```
[In] Int[Coth[x]^4/(1 + Coth[x]),x]
```

```
[Out] (-3*x)/2 + (3*Coth[x])/2 - Coth[x]^2 + Coth[x]^3/(2*(1 + Coth[x])) + 2*Log[
Sinh[x]]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
```

```
f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3631

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((c + d*Tan[e + f*x])^(n - 1)/(
2*a*f*(a + b*Tan[e + f*x]))), x] + Dist[1/(2*a^2), Int[(c + d*Tan[e + f*x])
^(n - 2)*Simp[a*c^2 + a*d^2*(n - 1) - b*c*d*n - d*(a*c*(n - 2) + b*d*n)*Tan
[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth^3(x)}{2(1+\coth(x))} - \frac{1}{2}\int (3-4\coth(x))\coth^2(x)\,dx \\
&= -\coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + \frac{1}{2}i\int (-4i+3i\coth(x))\coth(x)\,dx \\
&= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + 2\int \coth(x)\,dx \\
&= -\frac{3x}{2} + \frac{3\coth(x)}{2} - \coth^2(x) + \frac{\coth^3(x)}{2(1+\coth(x))} + 2\log(\sinh(x))
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$
\begin{aligned}
\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{1}{2}\Bigg(&-2\coth^2(x) - \coth^4(x) + \frac{\coth^5(x)}{1+\coth(x)} \\
&+ \coth^3(x)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(x)\right) \\
&+ 4(\log(\cosh(x)) + \log(\tanh(x)))\Bigg)
\end{aligned}
$$

```
[In] Integrate[Coth[x]^4/(1 + Coth[x]),x]
[Out] (-2*Coth[x]^2 - Coth[x]^4 + Coth[x]^5/(1 + Coth[x]) + Coth[x]^3*Hypergeomet
ric2F1[-3/2, 1, -1/2, Tanh[x]^2] + 4*(Log[Cosh[x]] + Log[Tanh[x]]))/2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{7x}{2}+\frac{e^{-2x}}{4}-\frac{2}{(e^{2x}-1)^2}+2\ln\left(e^{2x}-1\right)$	30
derivativedivides	$-\frac{\coth(x)^2}{2}+\coth\left(x\right)-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{7\ln(1+\coth(x))}{4}$	32
default	$-\frac{\coth(x)^2}{2}+\coth\left(x\right)-\frac{\ln(\coth(x)-1)}{4}-\frac{1}{2(1+\coth(x))}-\frac{7\ln(1+\coth(x))}{4}$	32
parallelrisch	$\frac{(-4\tanh(x)-4)\ln(1-\tanh(x))+(4\tanh(x)+4)\ln(\tanh(x))-7\tanh(x)x-\coth(x)^2-7x+\coth(x)+3}{2+2\tanh(x)}$	52

```
[In] int(coth(x)^4/(1+coth(x)),x,method=_RETURNVERBOSE)
[Out] -7/2*x+1/4*exp(-2*x)-2/(exp(2*x)-1)^2+2*ln(exp(2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 357, normalized size of antiderivative = 9.65

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx =$$
$$-\frac{14\,x\cosh\left(x\right)^6+84\,x\cosh\left(x\right)\sinh\left(x\right)^5+14\,x\sinh\left(x\right)^6-\left(28\,x+1\right)\cosh\left(x\right)^4+\left(210\,x\cosh\left(x\right)^2-28\,x-\right.}{}$$

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="fricas")
[Out] -1/4*(14*x*cosh(x)^6 + 84*x*cosh(x)*sinh(x)^5 + 14*x*sinh(x)^6 - (28*x + 1)
*cosh(x)^4 + (210*x*cosh(x)^2 - 28*x - 1)*sinh(x)^4 + 4*(70*x*cosh(x)^3 - (
28*x + 1)*cosh(x))*sinh(x)^3 + 2*(7*x + 5)*cosh(x)^2 + 2*(105*x*cosh(x)^4 -
 3*(28*x + 1)*cosh(x)^2 + 7*x + 5)*sinh(x)^2 - 8*(cosh(x)^6 + 6*cosh(x)*sin
h(x)^5 + sinh(x)^6 + (15*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh
(x)^3 - 2*cosh(x))*sinh(x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2
+ cosh(x)^2 + 2*(3*cosh(x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))*log(2*sinh(x
)/(cosh(x) - sinh(x))) + 4*(21*x*cosh(x)^5 - (28*x + 1)*cosh(x)^3 + (7*x +
5)*cosh(x))*sinh(x) - 1)/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + (15
*cosh(x)^2 - 2)*sinh(x)^4 - 2*cosh(x)^4 + 4*(5*cosh(x)^3 - 2*cosh(x))*sinh(
x)^3 + (15*cosh(x)^4 - 12*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(3*cosh(
x)^5 - 4*cosh(x)^3 + cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(34) = 68.

Time = 0.59 (sec) , antiderivative size = 197, normalized size of antiderivative = 5.32

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{x\tanh^3(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{x\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} - \frac{4\log(\tanh(x)+1)\tanh^3(x)}{2\tanh^3(x)+2\tanh^2(x)} - \frac{4\log(\tanh(x)+1)\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{4\log(\tanh(x))\tanh^3(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{4\log(\tanh(x))\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{3\tanh^2(x)}{2\tanh^3(x)+2\tanh^2(x)} + \frac{\tanh(x)}{2\tanh^3(x)+2\tanh^2(x)} - \frac{1}{2\tanh^3(x)+2\tanh^2(x)}$$

```
[In] integrate(coth(x)**4/(1+coth(x)),x)
```

```
[Out] x*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + x*tanh(x)**2/(2*tanh(x)**3 + 2
*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2)
- 4*log(tanh(x) + 1)*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(
x))*tanh(x)**3/(2*tanh(x)**3 + 2*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**2/(2
*tanh(x)**3 + 2*tanh(x)**2) + 3*tanh(x)**2/(2*tanh(x)**3 + 2*tanh(x)**2) +
tanh(x)/(2*tanh(x)**3 + 2*tanh(x)**2) - 1/(2*tanh(x)**3 + 2*tanh(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{1}{2}\,x + \frac{2\left(2\,e^{(-2\,x)}-1\right)}{2\,e^{(-2\,x)}-e^{(-4\,x)}-1} + \frac{1}{4}\,e^{(-2\,x)} + 2\,\log\left(e^{(-x)}+1\right) + 2\,\log\left(e^{(-x)}-1\right)$$

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x + 2*(2*e^(-2*x) - 1)/(2*e^(-2*x) - e^(-4*x) - 1) + 1/4*e^(-2*x) + 2*l
og(e^(-x) + 1) + 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = -\frac{7}{2}\,x + \frac{\left(e^{(4\,x)} - 10\,e^{(2\,x)} + 1\right)e^{(-2\,x)}}{4\,\left(e^{(2\,x)} - 1\right)^2} + 2\,\log\left(\left|e^{(2\,x)} - 1\right|\right)$$

```
[In] integrate(coth(x)^4/(1+coth(x)),x, algorithm="giac")
[Out] -7/2*x + 1/4*(e^(4*x) - 10*e^(2*x) + 1)*e^(-2*x)/(e^(2*x) - 1)^2 + 2*log(ab
s(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\coth^4(x)}{1+\coth(x)}\,dx = \frac{x}{2} - 2\,\ln\left(\coth(x) + 1\right) + \coth(x) - \frac{\coth(x)^2}{2} - \frac{1}{2\,\left(\coth\left(x\right) + 1\right)}$$

```
[In] int(coth(x)^4/(coth(x) + 1),x)
[Out] x/2 - 2*log(coth(x) + 1) + coth(x) - coth(x)^2/2 - 1/(2*(coth(x) + 1))
```

3.132 $\int \coth(x)(1+\coth(x))^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{3}(1+\coth(x))^{3/2}$$

```
[Out] -2/3*(1+coth(x))^(3/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3608, 3559, 3561, 212}

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - \frac{2}{3}(\coth(x)+1)^{3/2} - 2\sqrt{\coth(x)+1}$$

```
[In] Int[Coth[x]*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{2}{3}(1+\coth(x))^{3/2}+\int(1+\coth(x))^{3/2}\,dx\\
&= -2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+2\int\sqrt{1+\coth(x)}\,dx\\
&= -2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}+4\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\
&= 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}-\frac{2}{3}(1+\coth(x))^{3/2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int\coth(x)(1+\coth(x))^{3/2}\,dx=2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}\sqrt{1+\coth(x)}(4+\coth(x))$$

```
[In] Integrate[Coth[x]*(1 + Coth[x])^(3/2),x]
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*Sqrt[1 + Coth[x]]*(4 + Co
th[x]))/3
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	35
default	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+2\,\mathrm{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	35

```
[In] int(coth(x)*(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1+coth(x))^(3/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 5.76

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh\left(x\right)^3+15\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^2+5\sqrt{2}\sinh\left(x\right)^3+3\left(5\sqrt{2}\cosh\left(x\right)^2-\sqrt{2}\right)\sinh\left(x\right)-3\sqrt{}\right.}{}$$

```
[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="fricas")
[Out] -1/3*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^3 + 15*sqrt(2)*cosh(x)*sinh(x)^2 + 5*sqr
t(2)*sinh(x)^3 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x) - 3*sqrt(2)*cosh
(x))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*c
osh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sin
h(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 +
 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x
)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\coth(x)\,dx$$

[In] integrate(coth(x)*(1+coth(x))**(3/2),x)

[Out] Integral((coth(x) + 1)**(3/2)*coth(x), x)

Maxima [F]

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\coth(x)\,dx$$

[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="maxima")

[Out] integrate((coth(x) + 1)^(3/2)*coth(x), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.00

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = -\frac{1}{3}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{2\left(9\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)}-\right.\right.}{\left(\sqrt{}\right.}\right.$$

[In] integrate(coth(x)*(1+coth(x))^(3/2),x, algorithm="giac")

[Out] -1/3*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*x) - 1) + 2*(9*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 12*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + 5*sgn(e^(2*x) - 1))/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \coth(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right) - 2\sqrt{\coth(x)+1} - \frac{2\,(\coth(x)+1)^{3/2}}{3}$$

```
[In] int(coth(x)*(coth(x) + 1)^(3/2),x)
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
(2*(coth(x) + 1)^(3/2))/3
```

3.133 $\int \coth(x)\sqrt{1+\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 32

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)}$$

```
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3608, 3561, 212}

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - 2\sqrt{\coth(x)+1}$$

```
[In] Int[Coth[x]*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
```

```
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3608

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Dist
[(b*c + a*d)/b, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2, 0] &&  !LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -2\sqrt{1+\coth(x)}+\int\sqrt{1+\coth(x)}\,dx\\ &= -2\sqrt{1+\coth(x)}+2\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int\coth(x)\sqrt{1+\coth(x)}\,dx=\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-2\sqrt{1+\coth(x)}$$

```
[In] Integrate[Coth[x]*Sqrt[1 + Coth[x]],x]
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	26
default	$\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	26

```
[In] int(coth(x)*(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.09

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = -\frac{4\sqrt{2}\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh\left(x\right)^{2}+2\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+\sqrt{2}\sinh\left(x\right)^{2}\right.}{2\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right.\right.}$$

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(2)*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt(sinh(x)/(cosh(x) -
 sinh(x))) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) +
sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^2 +
 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [F]

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth\left(x\right)+1}\coth\left(x\right)dx$$

```
[In] integrate(coth(x)*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth\left(x\right)+1}\coth\left(x\right)\,dx$$

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.22

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = -\frac{1}{2}\sqrt{2}\left(\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{4\operatorname{sgn}\left(e^{(2x)}-1\right)}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}+1}\right)$$

```
[In] integrate(coth(x)*(1+coth(x))^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*(log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(2*
x) - 1) + 4*sgn(e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \coth(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)-2\sqrt{\coth(x)+1}$$

```
[In] int(coth(x)*(coth(x) + 1)^(1/2),x)
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2)
```

3.134 $\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{\sqrt{1+\coth(x)}}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3607, 3561, 212}

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{1}{\sqrt{\coth(x)+1}}$$

```
[In] Int[Coth[x]/Sqrt[1 + Coth[x]],x]
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{\sqrt{1+\coth(x)}}+\frac{1}{2}\int\sqrt{1+\coth(x)}\,dx\\ &= \frac{1}{\sqrt{1+\coth(x)}}+\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\coth(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int\frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{\sqrt{1+\coth(x)}}$$

```
[In] Integrate[Coth[x]/Sqrt[1 + Coth[x]],x]
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + 1/Sqrt[1 + Coth[x]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1}{\sqrt{1+\coth(x)}}$	25
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{1}{\sqrt{1+\coth(x)}}$	25

```
[In] int(coth(x)/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+1/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.83

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx$$
$$= \frac{\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\log\left(2\sqrt{2}\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+2\cosh\left(x\right)^{2}+4\cosh\left(x\right)\sinh\left(\right.\right.}{4\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}$$

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="fricas")
[Out] 1/4*((sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x
) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sin
h(x)^2 - 1) + 4*sqrt(sinh(x)/(cosh(x) - sinh(x))))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth\left(x\right)}{\sqrt{\coth\left(x\right)+1}}\,dx$$

```
[In] integrate(coth(x)/(1+coth(x))**(1/2),x)
[Out] Integral(coth(x)/sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth\left(x\right)}{\sqrt{\coth\left(x\right)+1}}\,dx$$

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(24) = 48$.

Time $= 0.29$ (sec) , antiderivative size $= 64$, normalized size of antiderivative $= 2.13$

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = -\frac{\sqrt{2}\left(\frac{2}{\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}} + \log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}} - 2e^{(2x)} + 1\right|\right)\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

```
[In] integrate(coth(x)/(1+coth(x))^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(2)*(2/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x)) + log(abs(2*sqrt(e^(4*x
) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time $= 1.99$ (sec) , antiderivative size $= 24$, normalized size of antiderivative $= 0.80$

$$\int \frac{\coth(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2} + \frac{1}{\sqrt{\coth\left(x\right)+1}}$$

```
[In] int(coth(x)/(coth(x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 + 1/(coth(x) + 1)^(1/2)
```

3.135 $\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}}$$

```
[Out] 1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/
2/(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3607, 3560, 3561, 212}

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{2\sqrt{\coth(x)+1}} + \frac{1}{3(\coth(x)+1)^{3/2}}$$

```
[In] Int[Coth[x]/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) + 1/(3*(1 + Coth[x])^(3/2))
- 1/(2*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3(1+\coth(x))^{3/2}} + \frac{1}{2}\int \frac{1}{\sqrt{1+\coth(x)}}\,dx \\
&= \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}} + \frac{1}{4}\int \sqrt{1+\coth(x)}\,dx \\
&= \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} + \frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2\sqrt{1+\coth(x)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$
\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{2-3(1+\coth(x))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{1}{2}(1+\coth(x))\right)}{6(1+\coth(x))^{3/2}}
$$

```
[In] Integrate[Coth[x]/(1 + Coth[x])^(3/2),x]
[Out] (2 - 3*(1 + Coth[x])*Hypergeometric2F1[-1/2, 1, 1/2, (1 + Coth[x])/2])/(6*(
1 + Coth[x])^(3/2))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35
default	$\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}-\frac{1}{2\sqrt{1+\coth(x)}}$	35

```
[In] int(coth(x)/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/
2/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(34) = 68.

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.39

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx =$$

$$-\frac{2\sqrt{2}\left(2\sqrt{2}\cosh(x)^2+4\sqrt{2}\cosh(x)\sinh(x)+2\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-3\left(\sqrt{2}\cosh(x)^3+3\cdots\right.}{24\,(\cos\cdots}$$

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(2)*(2*sqrt(2)*cosh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x) + 2*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\coth(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(x)/(1+coth(x))**(3/2),x)
[Out] Integral(coth(x)/(coth(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\coth(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(x)/(coth(x) + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.82

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\left(3\sqrt{e^{(4x)}-e^{(2x)}}-3e^{(2x)}+1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

```
[In] integrate(coth(x)/(1+coth(x))^(3/2),x, algorithm="giac")
[Out] -1/24*sqrt(2)*(2*(3*sqrt(e^(4*x) - e^(2*x)) - 3*e^(2*x) + 1)/(sqrt(e^(4*x)
- e^(2*x)) - e^(2*x))^3 + 3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) +
 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.65

$$\int \frac{\coth(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4} - \frac{\frac{\coth(x)}{2}+\frac{1}{6}}{(\coth(x)+1)^{3/2}}$$

```
[In] int(coth(x)/(coth(x) + 1)^(3/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 - (coth(x)/2 + 1/6)/(cot
h(x) + 1)^(3/2)
```

3.136 $\int \coth^2(x)(1+\coth(x))^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2}$$

```
[Out] -2/5*(1+coth(x))^(5/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3624, 3559, 3561, 212}

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - \frac{2}{5}(\coth(x)+1)^{5/2} - 2\sqrt{\coth(x)+1}$$

```
[In] Int[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(5/2))/5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3559

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[2*a, Int[(a + b*Tan[c + d*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && GtQ[n,
 1]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2}{5}(1+\coth(x))^{5/2} + \int (1+\coth(x))^{3/2}\,dx \\
&= -2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2} + 2\int \sqrt{1+\coth(x)}\,dx \\
&= -2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2} + 4\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - 2\sqrt{1+\coth(x)} - \frac{2}{5}(1+\coth(x))^{5/2}$$

```
[In] Integrate[Coth[x]^2*(1 + Coth[x])^(3/2),x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - 2*Sqrt[1 + Coth[x]] - (2*(1
+ Coth[x])^(5/2))/5
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+2\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	35
default	$-\frac{2(1+\coth(x))^{\frac{5}{2}}}{5}+2\,\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}-2\sqrt{1+\coth\left(x\right)}$	35

```
[In] int(coth(x)^2*(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(1+coth(x))^(5/2)+2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-2*(
1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 436, normalized size of antiderivative = 9.69

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx =$$

$$-\frac{2\sqrt{2}\left(9\sqrt{2}\cosh\left(x\right)^{5}+45\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)^{4}+9\sqrt{2}\sinh\left(x\right)^{5}+10\left(9\sqrt{2}\cosh\left(x\right)^{2}-\sqrt{2}\right)\sinh\left(x\right)^{3}-10\right.}{}$$

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*sqrt(2)*(9*sqrt(2)*cosh(x)^5 + 45*sqrt(2)*cosh(x)*sinh(x)^4 + 9*sqr
t(2)*sinh(x)^5 + 10*(9*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^3 - 10*sqrt(2)*
cosh(x)^3 + 30*(3*sqrt(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x)^2 + 5*(9*sqr
t(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x) + 5*sqrt(2)*cosh(x)
)*sqrt(sinh(x)/(cosh(x) - sinh(x))) - 5*(sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh
(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(
x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*si
nh(x)^3 + 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2
 + 3*sqrt(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(
2)*cosh(x))*sinh(x) - sqrt(2))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x
)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1
))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(
x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4
```

```
 - 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 +
cosh(x))*sinh(x) - 1)
```

Sympy [F]

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\coth^2(x)\,dx$$

```
[In] integrate(coth(x)**2*(1+coth(x))**(3/2),x)
[Out] Integral((coth(x) + 1)**(3/2)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = \int (\coth(x)+1)^{\frac{3}{2}}\coth(x)^2\,dx$$

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="maxima")
[Out] integrate((coth(x) + 1)^(3/2)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(34) = 68.

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.38

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx =$$

$$-\frac{1}{5}\sqrt{2}\left(5\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\mathrm{sgn}\left(e^{(2x)}-1\right)+\frac{2\left(25\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^4\mathrm{sgn}\left(e^{(2x)}-\right.\right.}{}\right.$$

```
[In] integrate(coth(x)^2*(1+coth(x))^(3/2),x, algorithm="giac")
[Out] -1/5*sqrt(2)*(5*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 2*(25*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^4*sgn(e^(2*x) - 1) +
60*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3*sgn(e^(2*x) - 1) + 70*(sqrt(e^(4*x
) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 40*(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x))*sgn(e^(2*x) - 1) + 9*sgn(e^(2*x) - 1))/(sqrt(e^(4*x) - e^(2*x)) -
e^(2*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int \coth^2(x)(1+\coth(x))^{3/2}\,dx = 2\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\sqrt{\coth(x)+1}}{2}\right) - 2\sqrt{\coth(x)+1} - \frac{2\left(\coth(x)+1\right)^{5/2}}{5}$$

```
[In] int(coth(x)^2*(coth(x) + 1)^(3/2),x)
```

```
[Out] 2*2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - 2*(coth(x) + 1)^(1/2) -
(2*(coth(x) + 1)^(5/2))/5
```

3.137 $\int \coth^2(x)\sqrt{1+\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right) - \frac{2}{3}(1+\coth(x))^{3/2}$$

```
[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3624, 3561, 212}

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\text{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right) - \frac{2}{3}(\coth(x)+1)^{3/2}$$

```
[In] Int[Coth[x]^2*Sqrt[1 + Coth[x]],x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]] - (2*(1 + Coth[x])^(3/2))/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
```

```
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2}{3}(1+\coth(x))^{3/2}+\int\sqrt{1+\coth(x)}\,dx\\ &= -\frac{2}{3}(1+\coth(x))^{3/2}+2\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,\sqrt{1+\coth(x)}\right)\\ &= \sqrt{2}\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)-\frac{2}{3}(1+\coth(x))^{3/2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int\coth^2(x)\sqrt{1+\coth(x)}\,dx=-2\left(-\frac{\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{3}(1+\coth(x))^{3/2}\right)$$

```
[In] Integrate[Coth[x]^2*Sqrt[1 + Coth[x]],x]
[Out] -2*(-(ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2]) + (1 + Coth[x])^(3/2)/3)
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	26
default	$-\frac{2(1+\coth(x))^{\frac{3}{2}}}{3}+\text{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}$	26

```
[In] int(coth(x)^2*(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(1+coth(x))^(3/2)+arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 7.12

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx =$$

$$-\frac{8\sqrt{2}\left(\sqrt{2}\cosh(x)^3 + 3\sqrt{2}\cosh(x)^2\sinh(x) + 3\sqrt{2}\cosh(x)\sinh(x)^2 + \sqrt{2}\sinh(x)^3\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}} - 3}{}$$

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(8*sqrt(2)*(sqrt(2)*cosh(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2
)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3)*sqrt(sinh(x)/(cosh(x) - sinh(x)))
- 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 +
2*(3*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 2*sqrt(2)*cosh(x)^2 + 4*(sqrt
(2)*cosh(x)^3 - sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*sqrt(2)*sqrt(sinh
(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh
(x) + 2*sinh(x)^2 - 1))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3
*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) +
 1)
```

Sympy [F]

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\coth^2(x)\,dx$$

```
[In] integrate(coth(x)**2*(1+coth(x))**(1/2),x)
```

```
[Out] Integral(sqrt(coth(x) + 1)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \int \sqrt{\coth(x)+1}\coth(x)^2\,dx$$

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(coth(x) + 1)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(25) = 50$.

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 3.91

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = -\frac{1}{6}\sqrt{2}\left(3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\operatorname{sgn}\left(e^{(2x)}-1\right)+\frac{8\left(3\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2\operatorname{sgn}\left(e^{(2x)}\right.\right.}{}\right.$$

```
[In] integrate(coth(x)^2*(1+coth(x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*(3*log(abs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1))*sgn(e^(
2*x) - 1) + 8*(3*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2*sgn(e^(2*x) - 1) + 3
*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))*sgn(e^(2*x) - 1) + sgn(e^(2*x) - 1))/(
sqrt(e^(4*x) - e^(2*x)) - e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \coth^2(x)\sqrt{1+\coth(x)}\,dx = \sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)-\frac{2\left(\coth(x)+1\right)^{3/2}}{3}$$

```
[In] int(coth(x)^2*(coth(x) + 1)^(1/2),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2) - (2*(coth(x) + 1)^(3/2))/3
```

3.138 $\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth(x)}$$

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1
+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3624, 3560, 3561, 212}

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{\sqrt{2}}-2\sqrt{\coth(x)+1}-\frac{1}{\sqrt{\coth(x)+1}}$$

```
[In] Int[Coth[x]^2/Sqrt[1 + Coth[x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] - 1/Sqrt[1 + Coth[x]] - 2*Sqrt[1
 + Coth[x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3560

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3624

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^2, x_Symbol] :> Simp[d^2*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(
m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Simp[c^2 - d^2 + 2*c*d*Tan[e + f*
x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&  !LeQ
[m, -1] &&  !(EqQ[m, 2] && EqQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -2\sqrt{1+\coth(x)} + \int \frac{1}{\sqrt{1+\coth(x)}}\,dx \\
&= -\frac{1}{\sqrt{1+\coth(x)}} - 2\sqrt{1+\coth(x)} + \frac{1}{2}\int \sqrt{1+\coth(x)}\,dx \\
&= -\frac{1}{\sqrt{1+\coth(x)}} - 2\sqrt{1+\coth(x)} + \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{\sqrt{1+\coth(x)}} - 2\sqrt{1+\coth(x)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$
\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{-3-2\coth(x)}{\sqrt{1+\coth(x)}}
$$

```
[In] Integrate[Coth[x]^2/Sqrt[1 + Coth[x]],x]
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/Sqrt[2] + (-3 - 2*Coth[x])/Sqrt[1 + Coth
[x]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth\left(x\right)}$	35
default	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{1}{\sqrt{1+\coth(x)}}-2\sqrt{1+\coth\left(x\right)}$	35

```
[In] int(coth(x)^2/(1+coth(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)-1/(1+coth(x))^(1/2)-2*(1
+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(34) = 68.

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.50

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx =$$

$$-\frac{2\sqrt{2}\left(5\sqrt{2}\cosh\left(x\right)^2+10\sqrt{2}\cosh\left(x\right)\sinh\left(x\right)+5\sqrt{2}\sinh\left(x\right)^2-\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}-\left(\sqrt{2}\cosh\left(x\right)^3+\right.}{4}$$

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*(5*sqrt(2)*cosh(x)^2 + 10*sqrt(2)*cosh(x)*sinh(x) + 5*sqrt(
2)*sinh(x)^2 - sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) - (sqrt(2)*cosh(x
)^3 + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)*sinh(x)^3 + (3*sqrt(2)*cosh(x)^
2 - sqrt(2))*sinh(x) - sqrt(2)*cosh(x))*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x)
 - sinh(x)))*(cosh(x) + sinh(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh
(x)^2 - 1))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1
)*sinh(x) - cosh(x))
```

Sympy [F]

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth^2{(x)}}{\sqrt{\coth{(x)}+1}}\,dx$$

```
[In] integrate(coth(x)**2/(1+coth(x))**(1/2),x)
[Out] Integral(coth(x)**2/sqrt(coth(x) + 1), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \int \frac{\coth{(x)}^2}{\sqrt{\coth{(x)}+1}}\,dx$$

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)^2/sqrt(coth(x) + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(34) = 68.

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.10

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = -\frac{\frac{5\sqrt{2}e^{(2x)}}{\mathrm{sgn}\left(e^{(2x)}-1\right)} - \frac{\sqrt{2}}{\mathrm{sgn}\left(e^{(2x)}-1\right)}}{2\sqrt{e^{(4x)}-e^{(2x)}}} - \frac{\sqrt{2}\log\left(\left|4\sqrt{e^{(4x)}-e^{(2x)}}-4e^{(2x)}+2\right|\right)}{4\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

```
[In] integrate(coth(x)^2/(1+coth(x))^(1/2),x, algorithm="giac")
[Out] -1/2*(5*sqrt(2)*e^(2*x)/sgn(e^(2*x) - 1) - sqrt(2)/sgn(e^(2*x) - 1))/sqrt(e
^(4*x) - e^(2*x)) - 1/4*sqrt(2)*log(abs(4*sqrt(e^(4*x) - e^(2*x)) - 4*e^(2*
x) + 2))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{\coth^2(x)}{\sqrt{1+\coth(x)}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{2} - \frac{3}{\sqrt{\coth{(x)}+1}} - \frac{2\coth(x)}{\sqrt{\coth{(x)}+1}}$$

```
[In] int(coth(x)^2/(coth(x) + 1)^(1/2),x)
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/2 - 3/(coth(x) + 1)^(1/2)
- (2*coth(x))/(coth(x) + 1)^(1/2)
```

3.139 $\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{3(1+\coth(x))^{3/2}}+\frac{3}{2\sqrt{1+\coth(x)}}$$

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3
/2/(1+coth(x))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3621, 3607, 3561, 212}

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{\coth(x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{3}{2\sqrt{\coth(x)+1}}-\frac{1}{3(\coth(x)+1)^{3/2}}$$

```
[In] Int[Coth[x]^2/(1 + Coth[x])^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]/(2*Sqrt[2]) - 1/(3*(1 + Coth[x])^(3/2))
+ 3/(2*Sqrt[1 + Coth[x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3561

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3607

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*((a + b*Tan[e + f*x])^m/(2*a
*f*m)), x] + Dist[(b*c + a*d)/(2*a*b), Int[(a + b*Tan[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 + b^2,
 0] && LtQ[m, 0]
```

Rule 3621

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
(f_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(a*c + b*d)^2*((a + b*Tan[e + f*x])^
m/(2*a^3*f*m)), x] + Dist[1/(2*a^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[
a*c^2 - 2*b*c*d + a*d^2 - 2*b*d^2*Tan[e + f*x], x], x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LeQ[m, -1] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{1}{3(1+\coth(x))^{3/2}} - \frac{1}{2}\int \frac{1-2\coth(x)}{\sqrt{1+\coth(x)}}\,dx \\
&= -\frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}} + \frac{1}{4}\int \sqrt{1+\coth(x)}\,dx \\
&= -\frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\coth(x)}\right) \\
&= \frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{1}{3(1+\coth(x))^{3/2}} + \frac{3}{2\sqrt{1+\coth(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{14+18\coth(x)+3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}}{\sqrt{2}}\right)(1+\coth(x))^{3/2}}{12(1+\coth(x))^{3/2}}$$

```
[In] Integrate[Coth[x]^2/(1 + Coth[x])^(3/2),x]
[Out] (14 + 18*Coth[x] + 3*Sqrt[2]*ArcTanh[Sqrt[1 + Coth[x]]/Sqrt[2]]*(1 + Coth[x
])^(3/2))/(12*(1 + Coth[x])^(3/2))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{3}{2\sqrt{1+\coth(x)}}$	35
default	$-\frac{1}{3(1+\coth(x))^{\frac{3}{2}}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{1+\coth(x)}\,\sqrt{2}}{2}\right)\sqrt{2}}{4}+\frac{3}{2\sqrt{1+\coth(x)}}$	35

```
[In] int(coth(x)^2/(1+coth(x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/(1+coth(x))^(3/2)+1/4*arctanh(1/2*(1+coth(x))^(1/2)*2^(1/2))*2^(1/2)+3
/2/(1+coth(x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.39

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{2\sqrt{2}\left(8\sqrt{2}\cosh(x)^2+16\sqrt{2}\cosh(x)\sinh(x)+8\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\sqrt{\frac{\sinh(x)}{\cosh(x)-\sinh(x)}}}{}$$

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(2)*(8*sqrt(2)*cosh(x)^2 + 16*sqrt(2)*cosh(x)*sinh(x) + 8*sqrt(
2)*sinh(x)^2 + sqrt(2))*sqrt(sinh(x)/(cosh(x) - sinh(x))) + 3*(sqrt(2)*cosh
(x)^3 + 3*sqrt(2)*cosh(x)^2*sinh(x) + 3*sqrt(2)*cosh(x)*sinh(x)^2 + sqrt(2)
*sinh(x)^3)*log(2*sqrt(2)*sqrt(sinh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh
(x)) + 2*cosh(x)^2 + 4*cosh(x)*sinh(x) + 2*sinh(x)^2 - 1))/(cosh(x)^3 + 3*c
osh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\coth^2(x)}{(\coth(x)+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(x)**2/(1+coth(x))**(3/2),x)
```

```
[Out] Integral(coth(x)**2/(coth(x) + 1)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \int \frac{\coth{(x)}^2}{(\coth{(x)}+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)^2/(coth(x) + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.31

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = -\frac{\sqrt{2}\left(\frac{2\left(6\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^2-3\sqrt{e^{(4x)}-e^{(2x)}}+3e^{(2x)}-1\right)}{\left(\sqrt{e^{(4x)}-e^{(2x)}}-e^{(2x)}\right)^3}+3\log\left(\left|2\sqrt{e^{(4x)}-e^{(2x)}}-2e^{(2x)}+1\right|\right)\right)}{24\,\mathrm{sgn}\left(e^{(2x)}-1\right)}$$

```
[In] integrate(coth(x)^2/(1+coth(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*(6*(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^2 - 3*sqrt(e^(4*x)
- e^(2*x)) + 3*e^(2*x) - 1)/(sqrt(e^(4*x) - e^(2*x)) - e^(2*x))^3 + 3*log(a
bs(2*sqrt(e^(4*x) - e^(2*x)) - 2*e^(2*x) + 1)))/sgn(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{\coth^2(x)}{(1+\coth(x))^{3/2}}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{\coth(x)+1}}{2}\right)}{4} + \frac{\frac{3\coth(x)}{2}+\frac{7}{6}}{(\coth{(x)}+1)^{3/2}}$$

```
[In] int(coth(x)^2/(coth(x) + 1)^(3/2),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(coth(x) + 1)^(1/2))/2))/4 + ((3*coth(x))/2 + 7/6)/
(coth(x) + 1)^(3/2)
```

3.140 $\int \frac{\tanh^4(x)}{a+b\coth(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{\tanh^4(x)}{a+b\coth(x)} \, dx = \frac{ax}{a^2-b^2} - \frac{b(a^2+b^2)\log(\cosh(x))}{a^4} - \frac{b^5\log(b\cosh(x)+a\sinh(x))}{a^4(a^2-b^2)} - \frac{(a^2+b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}$$

```
[Out] a*x/(a^2-b^2)-b*(a^2+b^2)*ln(cosh(x))/a^4-b^5*ln(b*cosh(x)+a*sinh(x))/a^4/(
a^2-b^2)-(a^2+b^2)*tanh(x)/a^3+1/2*b*tanh(x)^2/a^2-1/3*tanh(x)^3/a
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3650, 3730, 3731, 3732, 3611, 3556}

$$\int \frac{\tanh^4(x)}{a+b\coth(x)} \, dx = \frac{ax}{a^2-b^2} + \frac{b\tanh^2(x)}{2a^2} - \frac{b(a^2+b^2)\log(\cosh(x))}{a^4} - \frac{b^5\log(a\sinh(x)+b\cosh(x))}{a^4(a^2-b^2)} - \frac{(a^2+b^2)\tanh(x)}{a^3} - \frac{\tanh^3(x)}{3a}$$

```
[In] Int[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*(a^2 + b^2)*Log[Cosh[x]])/a^4 - (b^5*Log[b*Cosh[x] +
 a*Sinh[x]])/(a^4*(a^2 - b^2)) - ((a^2 + b^2)*Tanh[x])/a^3 + (b*Tanh[x]^2)/
(2*a^2) - Tanh[x]^3/(3*a)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3731

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n
+ 1)/(f*(m + 1)*(b*c - a*d)*(a^2 + b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)
*(a^2 + b^2)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp
[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2)) - a*C*(b*c*(m + 1) + a*d*(n
+ 1)) - (m + 1)*(b*c - a*d)*(A*b - b*C)*Tan[e + f*x] - d*(A*b^2 + a^2*C)*(m
 + n + 2)*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n},
```

```
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m,
 -1] &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\tanh^3(x)}{3a} - \frac{i\int \frac{(-3ib+3ia\coth(x)+3ib\coth^2(x))\tanh^3(x)}{a+b\coth(x)}\,dx}{3a} \\
&= \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} - \frac{\int \frac{(-6(a^2+b^2)+6b^2\coth^2(x))\tanh^2(x)}{a+b\coth(x)}\,dx}{6a^2} \\
&= -\frac{(a^2+b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} \\
&\quad + \frac{i\int \frac{(6ib(a^2+b^2)-6ia^3\coth(x)-6ib(a^2+b^2)\coth^2(x))\tanh(x)}{a+b\coth(x)}\,dx}{6a^3} \\
&= \frac{ax}{a^2-b^2} - \frac{(a^2+b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a} \\
&\quad - \frac{(ib^5)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^4(a^2-b^2)} - \frac{(b(a^2+b^2))\int \tanh(x)\,dx}{a^4} \\
&= \frac{ax}{a^2-b^2} - \frac{b(a^2+b^2)\log(\cosh(x))}{a^4} - \frac{b^5\log(b\cosh(x)+a\sinh(x))}{a^4(a^2-b^2)} \\
&\quad - \frac{(a^2+b^2)\tanh(x)}{a^3} + \frac{b\tanh^2(x)}{2a^2} - \frac{\tanh^3(x)}{3a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.15

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = \frac{1}{6}\left(-\frac{3\log(1-\coth(x))}{a+b}+\frac{3\log(1+\coth(x))}{a-b}+\frac{6b^5\log(a+b\coth(x))}{a^4\left(-a^2+b^2\right)}+\frac{6b\log(\tanh(x))}{a^2}+\frac{6b^3\log(\tanh(x))}{a^4}-\frac{6(a^2+b^2)\tanh(x)}{a^3}+\frac{3b\tanh^2(x)}{a^2}-\frac{2\tanh^3(x)}{a}\right)$$

```
[In] Integrate[Tanh[x]^4/(a + b*Coth[x]),x]
```

```
[Out] ((-3*Log[1 - Coth[x]])/(a + b) + (3*Log[1 + Coth[x]])/(a - b) + (6*b^5*Log[
a + b*Coth[x]])/(a^4*(-a^2 + b^2)) + (6*b*Log[Tanh[x]])/a^2 + (6*b^3*Log[Ta
nh[x]])/a^4 - (6*(a^2 + b^2)*Tanh[x])/a^3 + (3*b*Tanh[x]^2)/a^2 - (2*Tanh[x
]^3)/a)/6
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{-6b^5\ln(b+a\tanh(x))+6\ln(1-\tanh(x))a^4b+\left(-2a^5+2a^3b^2\right)\tanh(x)^3+\left(3a^4b-3a^2b^3\right)\tanh(x)^2+\left(-6a^5+6a\,b^4\right)\tanh(x)}{6a^6-6a^4b^2}$
derivativedivides	$-\frac{b^5\ln(a+b\coth(x))}{a^4(a+b)(a-b)}+\frac{b}{2a^2\coth(x)^2}+\frac{-a^2-b^2}{a^3\coth(x)}-\frac{(a^2+b^2)b\ln(\coth(x))}{a^4}-\frac{1}{3a\coth(x)^3}-\frac{\ln(\coth(x)-1)}{2a+2b}+$
default	$-\frac{b^5\ln(a+b\coth(x))}{a^4(a+b)(a-b)}+\frac{b}{2a^2\coth(x)^2}+\frac{-a^2-b^2}{a^3\coth(x)}-\frac{(a^2+b^2)b\ln(\coth(x))}{a^4}-\frac{1}{3a\coth(x)^3}-\frac{\ln(\coth(x)-1)}{2a+2b}+$
risch	$\frac{x}{a+b}+\frac{2xb}{a^2}+\frac{2x\,b^3}{a^4}+\frac{2x\,b^5}{a^4(a^2-b^2)}+\frac{4a^2e^{4x}-2ab\,e^{4x}+2b^2e^{4x}+4a^2e^{2x}-2b\,e^{2x}a+4b^2e^{2x}+\frac{8a^2}{3}+2b^2}{a^3(1+e^{2x})^3}-\frac{b\ln(1+e^{2x}}{a^2}$

```
[In] int(tanh(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (-6*b^5*ln(b+a*tanh(x))+6*ln(1-tanh(x))*a^4*b+(-2*a^5+2*a^3*b^2)*tanh(x)^3+
(3*a^4*b-3*a^2*b^3)*tanh(x)^2+(-6*a^5+6*a*b^4)*tanh(x)+6*a^4*x*(a+b))/(6*a^
6-6*a^4*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. 2(93) = 186.

Time = 0.29 (sec) , antiderivative size = 1294, normalized size of antiderivative = 13.34

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="fricas")
[Out] 1/3*(3*(a^5 + a^4*b)*x*cosh(x)^6 + 18*(a^5 + a^4*b)*x*cosh(x)*sinh(x)^5 + 3
*(a^5 + a^4*b)*x*sinh(x)^6 + 8*a^5 - 2*a^3*b^2 - 6*a*b^4 + 3*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^4 + 3*(4*
a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 15*(a^5 + a^4*b)*x*cosh(x
)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^4 + 12*(5*(a^5 + a^4*b)*x*cosh(x)^3 + (4*a
^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)
)*sinh(x)^3 + 3*(4*a^5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)
*cosh(x)^2 + 3*(15*(a^5 + a^4*b)*x*cosh(x)^4 + 4*a^5 - 2*a^4*b + 2*a^2*b^3
- 4*a*b^4 + 6*(4*a^5 - 2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 +
 a^4*b)*x)*cosh(x)^2 + 3*(a^5 + a^4*b)*x)*sinh(x)^2 + 3*(a^5 + a^4*b)*x - 3
*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sinh(x)^6 + 3*b^5*cosh(x)^4
 + 3*b^5*cosh(x)^2 + b^5 + 3*(5*b^5*cosh(x)^2 + b^5)*sinh(x)^4 + 4*(5*b^5*c
osh(x)^3 + 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh(x)^4 + 6*b^5*cosh(x)^2
+ b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 + 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - 3*((a^4*b - b^5)*co
sh(x)^6 + 6*(a^4*b - b^5)*cosh(x)*sinh(x)^5 + (a^4*b - b^5)*sinh(x)^6 + a^4
*b - b^5 + 3*(a^4*b - b^5)*cosh(x)^4 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*cos
h(x)^2)*sinh(x)^4 + 4*(5*(a^4*b - b^5)*cosh(x)^3 + 3*(a^4*b - b^5)*cosh(x))
*sinh(x)^3 + 3*(a^4*b - b^5)*cosh(x)^2 + 3*(a^4*b - b^5 + 5*(a^4*b - b^5)*c
osh(x)^4 + 6*(a^4*b - b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b - b^5)*cosh(x)^
5 + 2*(a^4*b - b^5)*cosh(x)^3 + (a^4*b - b^5)*cosh(x))*sinh(x))*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 6*(3*(a^5 + a^4*b)*x*cosh(x)^5 + 2*(4*a^5 - 2*a^4
*b - 2*a^3*b^2 + 2*a^2*b^3 - 2*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x)^3 + (4*a^
5 - 2*a^4*b + 2*a^2*b^3 - 4*a*b^4 + 3*(a^5 + a^4*b)*x)*cosh(x))*sinh(x))/((
a^6 - a^4*b^2)*cosh(x)^6 + 6*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^5 + (a^6 - a^4
*b^2)*sinh(x)^6 + a^6 - a^4*b^2 + 3*(a^6 - a^4*b^2)*cosh(x)^4 + 3*(a^6 - a^
4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - a^4*b^2)*cosh(
x)^3 + 3*(a^6 - a^4*b^2)*cosh(x))*sinh(x)^3 + 3*(a^6 - a^4*b^2)*cosh(x)^2 +
 3*(a^6 - a^4*b^2 + 5*(a^6 - a^4*b^2)*cosh(x)^4 + 6*(a^6 - a^4*b^2)*cosh(x)
^2)*sinh(x)^2 + 6*((a^6 - a^4*b^2)*cosh(x)^5 + 2*(a^6 - a^4*b^2)*cosh(x)^3
+ (a^6 - a^4*b^2)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = \int \frac{\tanh^4{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*coth(x)),x)
[Out] Integral(tanh(x)**4/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.51

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^6-a^4b^2} - \frac{2\left(4a^2+3b^2+3(2a^2+ab+2b^2)e^{(-2x)}+3(2a^2+ab+b^2)e^{(-4x)}\right)}{3\left(3a^3e^{(-2x)}+3a^3e^{(-4x)}+a^3e^{(-6x)}+a^3\right)} + \frac{x}{a+b} - \frac{(a^2b+b^3)\log\left(e^{(-2x)}+1\right)}{a^4}$$

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="maxima")
[Out] -b^5*log(-(a - b)*e^(-2*x) + a + b)/(a^6 - a^4*b^2) - 2/3*(4*a^2 + 3*b^2 +
3*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(2*a^2 + a*b + b^2)*e^(-4*x))/(3*a^3*e
^(-2*x) + 3*a^3*e^(-4*x) + a^3*e^(-6*x) + a^3) + x/(a + b) - (a^2*b + b^3)*
log(e^(-2*x) + 1)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.45

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = -\frac{b^5\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^6-a^4b^2} + \frac{x}{a-b} - \frac{(a^2b+b^3)\log\left(e^{(2x)}+1\right)}{a^4} + \frac{2\left(4a^3+3ab^2+3(2a^3-a^2b+ab^2)e^{(4x)}+3(2a^3-a^2b+2ab^2)e^{(2x)}\right)}{3a^4\left(e^{(2x)}+1\right)^3}$$

```
[In] integrate(tanh(x)^4/(a+b*coth(x)),x, algorithm="giac")
[Out] -b^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^6 - a^4*b^2) + x/(a - b) -
(a^2*b + b^3)*log(e^(2*x) + 1)/a^4 + 2/3*(4*a^3 + 3*a*b^2 + 3*(2*a^3 - a^2*
b + a*b^2)*e^(4*x) + 3*(2*a^3 - a^2*b + 2*a*b^2)*e^(2*x))/(a^4*(e^(2*x) + 1
)^3)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.68

$$\int \frac{\tanh^4(x)}{a+b\coth(x)}\,dx = \frac{8}{3\,a\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)}+\frac{x}{a-b}-\frac{b^5\,\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^6-a^4\,b^2}-\frac{\ln\left(\mathrm{e}^{2\,x}+1\right)\,\left(a^2\,b+b^3\right)}{a^4}+\frac{2\,\left(2\,a^3+a^2\,b+b^3\right)}{a^3\,\left(a+b\right)\,\left(\mathrm{e}^{2\,x}+1\right)}-\frac{2\,\left(2\,a^2+a\,b-b^2\right)}{a^2\,\left(a+b\right)\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

```
[In] int(tanh(x)^4/(a + b*coth(x)),x)
```

```
[Out] 8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) + x/(a - b) - (b^5*log(b -
 a + a*exp(2*x) + b*exp(2*x)))/(a^6 - a^4*b^2) - (log(exp(2*x) + 1)*(a^2*b
+ b^3))/a^4 + (2*(a^2*b + 2*a^3 + b^3))/(a^3*(a + b)*(exp(2*x) + 1)) - (2*(
a*b + 2*a^2 - b^2))/(a^2*(a + b)*(2*exp(2*x) + exp(4*x) + 1))
```

3.141 $\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3} +\frac{b^4\log(b\cosh(x)+a\sinh(x))}{a^3(a^2-b^2)}+\frac{b\tanh(x)}{a^2}-\frac{\tanh^2(x)}{2a}$$

```
[Out] -b*x/(a^2-b^2)+(a^2+b^2)*ln(cosh(x))/a^3+b^4*ln(b*cosh(x)+a*sinh(x))/a^3/(a
^2-b^2)+b*tanh(x)/a^2-1/2*tanh(x)^2/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3650, 3730, 3733, 3611, 3556}

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{b\tanh(x)}{a^2}+\frac{(a^2+b^2)\log(\cosh(x))}{a^3} +\frac{b^4\log(a\sinh(x)+b\cosh(x))}{a^3(a^2-b^2)}-\frac{\tanh^2(x)}{2a}$$

```
[In] Int[Tanh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + ((a^2 + b^2)*Log[Cosh[x]])/a^3 + (b^4*Log[b*Cosh[x]
+ a*Sinh[x]])/(a^3*(a^2 - b^2)) + (b*Tanh[x])/a^2 - Tanh[x]^2/(2*a)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3730

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 - a*(b*B - a*C))*(a + b*Tan[e +
 f*x])^(m + 1)*((c + d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 +
b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 + b^2)), Int[(a + b*Tan[e + f
*x])^(m + 1)*(c + d*Tan[e + f*x])^n*Simp[A*(a*(b*c - a*d)*(m + 1) - b^2*d*(
m + n + 2)) + (b*B - a*C)*(b*c*(m + 1) + a*d*(n + 1)) - (m + 1)*(b*c - a*d)
*(A*b - a*B - b*C)*Tan[e + f*x] - d*(A*b^2 - a*(b*B - a*C))*(m + n + 2)*Tan
[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[
b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && LtQ[m, -1] &&  !
(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3733

```
Int[((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/(((a_) + (b_.)*tan[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(a*(
A*c - c*C) - b*(A*d - C*d))*(x/((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^
2 + a^2*C)/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e
 + f*x]), x], x] - Dist[(c^2*C + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d -
 c*Tan[e + f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f,
A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tanh^2(x)}{2a} - \frac{i\int \frac{(-2ib+2ia\coth(x)+2ib\coth^2(x))\tanh^2(x)}{a+b\coth(x)}\,dx}{2a}$$

$$= \frac{b\tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a} - \frac{\int \frac{(-2(a^2+b^2)+2b^2\coth^2(x))\tanh(x)}{a+b\coth(x)}dx}{2a^2}$$

$$= -\frac{bx}{a^2-b^2} + \frac{b\tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a} + \frac{(ib^4)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}dx}{a^3(a^2-b^2)} + \frac{(a^2+b^2)\int\tanh(x)\,dx}{a^3}$$

$$= -\frac{bx}{a^2-b^2} + \frac{(a^2+b^2)\log(\cosh(x))}{a^3} + \frac{b^4\log(b\cosh(x)+a\sinh(x))}{a^3(a^2-b^2)} + \frac{b\tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}dx = -\frac{\log(1-\coth(x))}{2(a+b)} - \frac{\log(1+\coth(x))}{2(a-b)} + \frac{b^4\log(a+b\coth(x))}{a^3(a^2-b^2)} - \frac{(a^2+b^2)\log(\tanh(x))}{a^3} + \frac{b\tanh(x)}{a^2} - \frac{\tanh^2(x)}{2a}$$

```
[In] Integrate[Tanh[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -1/2*Log[1 - Coth[x]]/(a + b) - Log[1 + Coth[x]]/(2*(a - b)) + (b^4*Log[a +
 b*Coth[x]])/(a^3*(a^2 - b^2)) - ((a^2 + b^2)*Log[Tanh[x]])/a^3 + (b*Tanh[x
])/a^2 - Tanh[x]^2/(2*a)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{2b^4\ln(b+a\tanh(x))-2\left(a^3\ln(1-\tanh(x))+\left(\frac{a(a-b)\tanh(x)^2}{2}-b(a-b)\tanh(x)+a^2x\right)(a+b)\right)a}{2a^5-2a^3b^2}$
derivativedivides	$-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{b^4\ln(a+b\coth(x))}{a^3(a+b)(a-b)}-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{b}{a^2\coth(x)}-\frac{(-a^2-b^2)\ln(\coth(x))}{a^3}-\frac{1}{2a\coth(x)^2}$
default	$-\frac{\ln(\coth(x)-1)}{2a+2b}+\frac{b^4\ln(a+b\coth(x))}{a^3(a+b)(a-b)}-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{b}{a^2\coth(x)}-\frac{(-a^2-b^2)\ln(\coth(x))}{a^3}-\frac{1}{2a\coth(x)^2}$
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^2}{a^3}-\frac{2x\,b^4}{a^3(a^2-b^2)}+\frac{2a\,e^{2x}-2b\,e^{2x}-2b}{(1+e^{2x})^2a^2}+\frac{b^4\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^3(a^2-b^2)}+\frac{\ln(1+e^{2x})}{a}+\frac{\ln(1+e^{2x})b^2}{a^3}$

```
[In] int(tanh(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (2*b^4*ln(b+a*tanh(x))-2*(a^3*ln(1-tanh(x))+(1/2*a*(a-b)*tanh(x)^2-b*(a-b)*
tanh(x)+a^2*x)*(a+b))*a)/(2*a^5-2*a^3*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(74) = 148$.

Time = 0.28 (sec) , antiderivative size = 637, normalized size of antiderivative = 8.38

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = -\frac{(a^4+a^3b)x\cosh(x)^4+4(a^4+a^3b)x\cosh(x)\sinh(x)^3+(a^4+a^3b)x\sinh(x)^4+2a^3b-2ab^3-2(a^4-}{}$$

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -((a^4 + a^3*b)*x*cosh(x)^4 + 4*(a^4 + a^3*b)*x*cosh(x)*sinh(x)^3 + (a^4 +
a^3*b)*x*sinh(x)^4 + 2*a^3*b - 2*a*b^3 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 -
 (a^4 + a^3*b)*x)*cosh(x)^2 - 2*(a^4 - a^3*b - a^2*b^2 + a*b^3 - 3*(a^4 + a
^3*b)*x*cosh(x)^2 - (a^4 + a^3*b)*x)*sinh(x)^2 + (a^4 + a^3*b)*x - (b^4*cos
h(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 + 2*b^4*cosh(x)^2 + b^4 +
2*(3*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + b^4*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - ((a^4 - b^4)*cosh(
x)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4
+ 2*(a^4 - b^4)*cosh(x)^2 + 2*(a^4 - b^4 + 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)
^2 + 4*((a^4 - b^4)*cosh(x)^3 + (a^4 - b^4)*cosh(x))*sinh(x))*log(2*cosh(x)
/(cosh(x) - sinh(x))) + 4*((a^4 + a^3*b)*x*cosh(x)^3 - (a^4 - a^3*b - a^2*b
^2 + a*b^3 - (a^4 + a^3*b)*x)*cosh(x))*sinh(x))/(a^5 - a^3*b^2 + (a^5 - a^3
*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cosh(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sin
h(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)
*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh
(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx$$

```
[In] integrate(tanh(x)**3/(a+b*coth(x)),x)
```

```
[Out] Integral(tanh(x)**3/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \frac{b^4\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^5-a^3b^2}+\frac{2\left((a+b)e^{(-2x)}+b\right)}{2a^2e^{(-2x)}+a^2e^{(-4x)}+a^2}+\frac{x}{a+b}+\frac{(a^2+b^2)\log\left(e^{(-2x)}+1\right)}{a^3}$$

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] b^4*log(-(a - b)*e^(-2*x) + a + b)/(a^5 - a^3*b^2) + 2*((a + b)*e^(-2*x) +
b)/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a^2) + x/(a + b) + (a^2 + b^2)*log(e^(-
2*x) + 1)/a^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \frac{b^4\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^5-a^3b^2}-\frac{x}{a-b}+\frac{(a^2+b^2)\log\left(e^{(2x)}+1\right)}{a^3}-\frac{2\left(ab-(a^2-ab)e^{(2x)}\right)}{a^3(e^{(2x)}+1)^2}$$

```
[In] integrate(tanh(x)^3/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^5 - a^3*b^2) - x/(a - b) + (
a^2 + b^2)*log(e^(2*x) + 1)/a^3 - 2*(a*b - (a^2 - a*b)*e^(2*x))/(a^3*(e^(2*
x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.46

$$\int \frac{\tanh^3(x)}{a+b\coth(x)}\,dx = \frac{\ln\left(\mathrm{e}^{2x}+1\right)\,\left(a^2+b^2\right)}{a^3}-\frac{x}{a-b}-\frac{2}{a\,\left(2\,\mathrm{e}^{2x}+\mathrm{e}^{4x}+1\right)}+\frac{b^4\,\ln\left(b-a+a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}\right)}{a^5-a^3\,b^2}+\frac{2\,\left(a^2-b^2\right)}{a^2\,\left(a+b\right)\,\left(\mathrm{e}^{2x}+1\right)}$$

```
[In] int(tanh(x)^3/(a + b*coth(x)),x)
```

```
[Out] (log(exp(2*x) + 1)*(a^2 + b^2))/a^3 - x/(a - b) - 2/(a*(2*exp(2*x) + exp(4*
x) + 1)) + (b^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^5 - a^3*b^2) + (2*
(a^2 - b^2))/(a^2*(a + b)*(exp(2*x) + 1))
```

3.142 $\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(\cosh(x))}{a^2}-\frac{b^3\log(b\cosh(x)+a\sinh(x))}{a^2(a^2-b^2)}-\frac{\tanh(x)}{a}$$

```
[Out] a*x/(a^2-b^2)-b*ln(cosh(x))/a^2-b^3*ln(b*cosh(x)+a*sinh(x))/a^2/(a^2-b^2)-t
anh(x)/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3650, 3732, 3611, 3556}

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b^3\log(a\sinh(x)+b\cosh(x))}{a^2(a^2-b^2)}-\frac{b\log(\cosh(x))}{a^2}-\frac{\tanh(x)}{a}$$

```
[In] Int[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[Cosh[x]])/a^2 - (b^3*Log[b*Cosh[x] + a*Sinh[x]])
/(a^2*(a^2 - b^2)) - Tanh[x]/a
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
```

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3650

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m + 1)*((c
+ d*Tan[e + f*x])^(n + 1)/(f*(m + 1)*(a^2 + b^2)*(b*c - a*d))), x] + Dist[1
/((m + 1)*(a^2 + b^2)*(b*c - a*d)), Int[(a + b*Tan[e + f*x])^(m + 1)*(c + d
*Tan[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) - b^2*d*(m + n + 2) - b*(b*c -
a*d)*(m + 1)*Tan[e + f*x] - b^2*d*(m + n + 2)*Tan[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
&& NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && LtQ[m, -1] && (LtQ[n, 0] || Intege
rQ[m]) &&  !(ILtQ[n, -1] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3732

```
Int[((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[(a*(A*c - c*C + B*d) + b*(B*c - A*d + C*d))*(x/
((a^2 + b^2)*(c^2 + d^2))), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/((b*c - a*d)
*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] - Dist
[(c^2*C - B*c*d + A*d^2)/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e + f*x]
)/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] &&
 NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\tanh(x)}{a} - \frac{i \int \frac{(-ib+ia\coth(x)+ib\coth^2(x))\tanh(x)}{a+b\coth(x)}\,dx}{a} \\
&= \frac{ax}{a^2-b^2} - \frac{\tanh(x)}{a} - \frac{b\int \tanh(x)\,dx}{a^2} - \frac{(ib^3)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2(a^2-b^2)} \\
&= \frac{ax}{a^2-b^2} - \frac{b\log(\cosh(x))}{a^2} - \frac{b^3\log(b\cosh(x)+a\sinh(x))}{a^2(a^2-b^2)} - \frac{\tanh(x)}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = -\frac{\log(1-\coth(x))}{2(a+b)}+\frac{\log(1+\coth(x))}{2(a-b)} - \frac{b^3\log(a+b\coth(x))}{a^2(a^2-b^2)}+\frac{b\log(\tanh(x))}{a^2}-\frac{\tanh(x)}{a}$$

```
[In] Integrate[Tanh[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -1/2*Log[1 - Coth[x]]/(a + b) + Log[1 + Coth[x]]/(2*(a - b)) - (b^3*Log[a +
 b*Coth[x]])/(a^2*(a^2 - b^2)) + (b*Log[Tanh[x]])/a^2 - Tanh[x]/a
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$\frac{\ln(1-\tanh(x))a^2b-b^3\ln(b+a\tanh(x))+a^3x+b\,a^2x-\tanh(x)a^3+\tanh(x)a\,b^2}{a^2(a^2-b^2)}$	66
derivativedivides	$-\frac{b^3\ln(a+b\coth(x))}{a^2(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{b\ln(\coth(x))}{a^2}-\frac{1}{a\coth(x)}+\frac{\ln(1+\coth(x))}{2a-2b}$	78
default	$-\frac{b^3\ln(a+b\coth(x))}{a^2(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{b\ln(\coth(x))}{a^2}-\frac{1}{a\coth(x)}+\frac{\ln(1+\coth(x))}{2a-2b}$	78
risch	$\frac{x}{a+b}+\frac{2xb}{a^2}+\frac{2x\,b^3}{a^2(a^2-b^2)}+\frac{2}{a(1+e^{2x})}-\frac{b\ln(1+e^{2x})}{a^2}-\frac{b^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2(a^2-b^2)}$	99

```
[In] int(tanh(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (ln(1-tanh(x))*a^2*b-b^3*ln(b+a*tanh(x))+a^3*x+b*a^2*x-tanh(x)*a^3+tanh(x)*
a*b^2)/a^2/(a^2-b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.40

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = \frac{(a^3+a^2b)x\cosh(x)^2+2(a^3+a^2b)x\cosh(x)\sinh(x)+(a^3+a^2b)x\sinh(x)^2+2a^3-2ab^2+(a^3+a^2b)x}{a^4}$$

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a^3 + a^2*b)*x*cosh(x)^2 + 2*(a^3 + a^2*b)*x*cosh(x)*sinh(x) + (a^3 + a^2
*b)*x*sinh(x)^2 + 2*a^3 - 2*a*b^2 + (a^3 + a^2*b)*x - (b^3*cosh(x)^2 + 2*b^
3*cosh(x)*sinh(x) + b^3*sinh(x)^2 + b^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b - b^3)
*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*log(2*cosh(x)/(cosh(x) - sinh(x
))))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)
*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx=\int \frac{\tanh^2{(x)}}{a+b\coth{(x)}}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*coth(x)),x)
[Out] Integral(tanh(x)**2/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx=-\frac{b^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^4-a^2b^2}+\frac{x}{a+b}-\frac{b\log\left(e^{(-2x)}+1\right)}{a^2}-\frac{2}{ae^{(-2x)}+a}$$

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="maxima")
[Out] -b^3*log(-(a - b)*e^(-2*x) + a + b)/(a^4 - a^2*b^2) + x/(a + b) - b*log(e^(
-2*x) + 1)/a^2 - 2/(a*e^(-2*x) + a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx=-\frac{b^3\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^4-a^2b^2}+\frac{x}{a-b}-\frac{b\log\left(e^{(2x)}+1\right)}{a^2}+\frac{2}{a(e^{(2x)}+1)}$$

```
[In] integrate(tanh(x)^2/(a+b*coth(x)),x, algorithm="giac")
[Out] -b^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^4 - a^2*b^2) + x/(a - b) -
b*log(e^(2*x) + 1)/a^2 + 2/(a*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{\tanh^2(x)}{a+b\coth(x)}\,dx = \frac{2}{a\,\left(e^{2\,x}+1\right)}+\frac{x}{a-b}-\frac{b^3\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a^4-a^2\,b^2}-\frac{b\,\ln\left(e^{2\,x}+1\right)}{a^2}$$

```
[In] int(tanh(x)^2/(a + b*coth(x)),x)
[Out] 2/(a*(exp(2*x) + 1)) + x/(a - b) - (b^3*log(b - a + a*exp(2*x) + b*exp(2*x)
))/(a^4 - a^2*b^2) - (b*log(exp(2*x) + 1))/a^2
```

3.143 $\int \frac{\tanh(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{\log(\cosh(x))}{a}+\frac{b^2\log(b\cosh(x)+a\sinh(x))}{a\,(a^2-b^2)}$$

```
[Out] -b*x/(a^2-b^2)+ln(cosh(x))/a+b^2*ln(b*cosh(x)+a*sinh(x))/a/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3652, 3611, 3556}

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{b^2\log(a\sinh(x)+b\cosh(x))}{a\,(a^2-b^2)}+\frac{\log(\cosh(x))}{a}$$

```
[In] Int[Tanh[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + Log[Cosh[x]]/a + (b^2*Log[b*Cosh[x] + a*Sinh[x]])/(a
*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
```

```
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3652

```
Int[1/(((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f
_.)*(x_)])), x_Symbol] :> Simp[(a*c - b*d)*(x/((a^2 + b^2)*(c^2 + d^2))), x
] + (Dist[b^2/((b*c - a*d)*(a^2 + b^2)), Int[(b - a*Tan[e + f*x])/(a + b*Ta
n[e + f*x]), x], x] - Dist[d^2/((b*c - a*d)*(c^2 + d^2)), Int[(d - c*Tan[e
+ f*x])/(c + d*Tan[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && Ne
Q[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{bx}{a^2-b^2}+\frac{\int \tanh(x)\,dx}{a}+\frac{(ib^2)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a\,(a^2-b^2)} \\ &= -\frac{bx}{a^2-b^2}+\frac{\log(\cosh(x))}{a}+\frac{b^2\log(b\cosh(x)+a\sinh(x))}{a\,(a^2-b^2)} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.29

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = -\frac{\log(1-\coth(x))}{2(a+b)}-\frac{\log(1+\coth(x))}{2(a-b)} + \frac{b^2\log(a+b\coth(x))}{a\,(a^2-b^2)}-\frac{\log(\tanh(x))}{a}$$

```
[In] Integrate[Tanh[x]/(a + b*Coth[x]),x]
[Out] -1/2*Log[1 - Coth[x]]/(a + b) - Log[1 + Coth[x]]/(2*(a - b)) + (b^2*Log[a +
 b*Coth[x]])/(a*(a^2 - b^2)) - Log[Tanh[x]]/a
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$\frac{b^{2}\ln\left(b+a\tanh\left(x\right)\right)-\left(a\ln\left(1-\tanh\left(x\right)\right)+\left(a+b\right)x\right)a}{a^{3}-a\,b^{2}}$	44
derivativedivides	$-\frac{\ln\left(1+\coth\left(x\right)\right)}{2a-2b}+\frac{b^{2}\ln\left(a+b\coth\left(x\right)\right)}{a\left(a+b\right)\left(a-b\right)}-\frac{\ln\left(\coth\left(x\right)-1\right)}{2a+2b}+\frac{\ln\left(\coth\left(x\right)\right)}{a}$	67
default	$-\frac{\ln\left(1+\coth\left(x\right)\right)}{2a-2b}+\frac{b^{2}\ln\left(a+b\coth\left(x\right)\right)}{a\left(a+b\right)\left(a-b\right)}-\frac{\ln\left(\coth\left(x\right)-1\right)}{2a+2b}+\frac{\ln\left(\coth\left(x\right)\right)}{a}$	67
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2x\,b^{2}}{a\left(a^{2}-b^{2}\right)}+\frac{\ln\left(1+\mathrm{e}^{2x}\right)}{a}+\frac{b^{2}\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a\left(a^{2}-b^{2}\right)}$	82

```
[In] int(tanh(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] (b^2*ln(b+a*tanh(x))-(a*ln(1-tanh(x))+(a+b)*x)*a)/(a^3-a*b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \frac{b^2\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)-(a^2+ab)x+(a^2-b^2)\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a^3-ab^2}$$

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="fricas")
[Out] (b^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a^2 + a*b)*x + (
a^2 - b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
```

Sympy [F]

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \int \frac{\tanh\left(x\right)}{a+b\coth\left(x\right)}\,dx$$

```
[In] integrate(tanh(x)/(a+b*coth(x)),x)
[Out] Integral(tanh(x)/(a + b*coth(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \frac{b^2\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^3-ab^2} + \frac{x}{a+b} + \frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] b^2*log(-(a - b)*e^(-2*x) + a + b)/(a^3 - a*b^2) + x/(a + b) + log(e^(-2*x)
 + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \frac{b^2\log\left(\left|ae^{(2\,x)}+be^{(2\,x)}-a+b\right|\right)}{a^3-ab^2} - \frac{x}{a-b} + \frac{\log\left(e^{(2\,x)}+1\right)}{a}$$

```
[In] integrate(tanh(x)/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] b^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3 - a*b^2) - x/(a - b) + log
(e^(2*x) + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(x)}{a+b\coth(x)}\,dx = \frac{\ln\left(e^{2\,x}+1\right)}{a} - \frac{x}{a-b} - \frac{b^2\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{a\,b^2-a^3}$$

```
[In] int(tanh(x)/(a + b*coth(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - x/(a - b) - (b^2*log(b - a + a*exp(2*x) + b*exp(2*x))
)/(a*b^2 - a^3)
```

3.144 $\int \frac{1}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 39

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3565, 3611}

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{ax}{a^2-b^2} - \frac{b\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}$$

```
[In] Int[(a + b*Coth[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
```

```
NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{ax}{a^2-b^2} - \frac{(ib)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2} - \frac{b\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))+(a+b)\log(1+\tanh(x))-2b\log(b+a\tanh(x))}{2(a-b)(a+b)}$$

```
[In] Integrate[(a + b*Coth[x])^(-1),x]
[Out] ((-a + b)*Log[1 - Tanh[x]] + (a + b)*Log[1 + Tanh[x]] - 2*b*Log[b + a*Tanh[
x]])/(2*(a - b)*(a + b))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
parallelrisch	$\frac{-b\ln(b+a\tanh(x))+\ln(1-\tanh(x))b+(a+b)x}{a^2-b^2}$	38
derivativedivides	$\frac{\ln(1+\coth(x))}{2a-2b} - \frac{b\ln(a+b\coth(x))}{(a-b)(a+b)} - \frac{\ln(\coth(x)-1)}{2a+2b}$	55
default	$\frac{\ln(1+\coth(x))}{2a-2b} - \frac{b\ln(a+b\coth(x))}{(a-b)(a+b)} - \frac{\ln(\coth(x)-1)}{2a+2b}$	55
risch	$\frac{x}{a+b} + \frac{2xb}{a^2-b^2} - \frac{b\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	56

```
[In] int(1/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] (-b*ln(b+a*tanh(x))+ln(1-tanh(x))*b+(a+b)*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\coth(x)}\,dx = \frac{(a+b)x - b\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(29) = 58.

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.79

$$\int \frac{1}{a+b\coth(x)}\,dx = \begin{cases} \tilde{\infty}(x-\log(\tanh(x)+1)) & \text{for } a=0 \wedge b=0 \\ \frac{x-\log(\tanh(x)+1)}{b} & \text{for } a=0 \\ -\frac{x\tanh(x)}{2b\tanh(x)-2b} + \frac{x}{2b\tanh(x)-2b} - \frac{1}{2b\tanh(x)-2b} & \text{for } a=-b \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} + \frac{1}{2b\tanh(x)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2} - \frac{bx}{a^2-b^2} + \frac{b\log(\tanh(x)+1)}{a^2-b^2} - \frac{b\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1)), Eq(a, 0) & Eq(b, 0)), ((x - log(tanh
(x) + 1))/b, Eq(a, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) -
2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) +
x/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2
) - b*x/(a**2 - b**2) + b*log(tanh(x) + 1)/(a**2 - b**2) - b*log(tanh(x) +
b/a)/(a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{1}{a + b\coth(x)}\,dx = -\frac{b\log\left(-(a-b)e^{(-2\,x)} + a + b\right)}{a^2 - b^2} + \frac{x}{a+b}$$

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="maxima")
[Out] -b*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{a + b\coth(x)}\,dx = -\frac{b\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} - a + b\right|\right)}{a^2 - b^2} + \frac{x}{a-b}$$

```
[In] integrate(1/(a+b*coth(x)),x, algorithm="giac")
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) + x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{1}{a + b\coth(x)}\,dx = \frac{x}{a-b} - \frac{b\,\ln\left(b - a + a\,\mathrm{e}^{2\,x} + b\,\mathrm{e}^{2\,x}\right)}{a^2 - b^2}$$

```
[In] int(1/(a + b*coth(x)),x)
[Out] x/(a - b) - (b*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2)
```

3.145 $\int \frac{\coth(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = -\frac{bx}{a^2-b^2}+\frac{a\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)+a*ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3612, 3611}

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = \frac{a\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}-\frac{bx}{a^2-b^2}$$

```
[In] Int[Coth[x]/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) + (a*Log[b*Cosh[x] + a*Sinh[x]])/(a^2 - b^2)
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
```

```
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{bx}{a^2-b^2}+\frac{(ia)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2}\\ &= -\frac{bx}{a^2-b^2}+\frac{a\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx = \frac{(-a+b)\log(1-\tanh(x))-(a+b)\log(1+\tanh(x))+2a\log(b+a\tanh(x))}{2(a-b)(a+b)}$$

```
[In] Integrate[Coth[x]/(a + b*Coth[x]),x]
[Out] ((-a + b)*Log[1 - Tanh[x]] - (a + b)*Log[1 + Tanh[x]] + 2*a*Log[b + a*Tanh[
x]])/(2*(a - b)*(a + b))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{a\ln(b+a\tanh(x))-a\ln(1-\tanh(x))-(a+b)x}{a^2-b^2}$	39
derivativedivides	$-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{a\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}$	55
default	$-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{a\ln(a+b\coth(x))}{(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}$	55
risch	$\frac{x}{a+b}-\frac{2ax}{a^2-b^2}+\frac{a\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	55

```
[In] int(coth(x)/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] (a*ln(b+a*tanh(x))-a*ln(1-tanh(x))-(a+b)*x)/(a^2-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx=-\frac{(a+b)x-a\log\left(\frac{2\,(b\cosh(x)+a\sinh(x))}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="fricas")
[Out] -((a + b)*x - a*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^2 -
b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(29) = 58.

Time = 0.44 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.44

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx=\begin{cases}\tilde{\infty}x & \text{for } a=0\wedge b=0\\ \frac{x}{b} & \text{for } a=0\\ \frac{x\tanh(x)}{2b\tanh(x)-2b}-\frac{x}{2b\tanh(x)-2b}-\frac{1}{2b\tanh(x)-2b} & \text{for } a=-b\\ \frac{x\tanh(x)}{2b\tanh(x)+2b}+\frac{x}{2b\tanh(x)+2b}-\frac{1}{2b\tanh(x)+2b} & \text{for } a=b\\ \frac{ax}{a^2-b^2}-\frac{a\log(\tanh(x)+1)}{a^2-b^2}+\frac{a\log\left(\tanh(x)+\frac{b}{a}\right)}{a^2-b^2}-\frac{bx}{a^2-b^2} & \text{otherwise}\end{cases}$$

```
[In] integrate(coth(x)/(a+b*coth(x)),x)
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b, Eq(a, 0)), (x*tanh(x)/(2*b*ta
nh(x) - 2*b) - x/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (
x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*
b), Eq(a, b)), (a*x/(a**2 - b**2) - a*log(tanh(x) + 1)/(a**2 - b**2) + a*lo
g(tanh(x) + b/a)/(a**2 - b**2) - b*x/(a**2 - b**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\coth(x)}{a+b\coth(x)}\,dx=\frac{a\log\left(-(a-b)e^{(-2\,x)}+a+b\right)}{a^2-b^2}+\frac{x}{a+b}$$

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="maxima")
[Out] a*log(-(a - b)*e^(-2*x) + a + b)/(a^2 - b^2) + x/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\coth(x)}{a + b\coth(x)}\, dx = \frac{a\log\left(\left|ae^{(2\,x)} + be^{(2\,x)} - a + b\right|\right)}{a^2 - b^2} - \frac{x}{a - b}$$

```
[In] integrate(coth(x)/(a+b*coth(x)),x, algorithm="giac")
[Out] a*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2 - b^2) - x/(a - b)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\coth(x)}{a + b\coth(x)}\, dx = \frac{a\,\ln\left(b - a + a\,e^{2\,x} + b\,e^{2\,x}\right)}{a^2 - b^2} - \frac{x}{a - b}$$

```
[In] int(coth(x)/(a + b*coth(x)),x)
[Out] (a*log(b - a + a*exp(2*x) + b*exp(2*x)))/(a^2 - b^2) - x/(a - b)
```

3.146 $\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx=-\frac{ax}{b^2}+\frac{a^3x}{b^2(a^2-b^2)}+\frac{\log(\sinh(x))}{b}-\frac{a^2\log(b\cosh(x)+a\sinh(x))}{b(a^2-b^2)}$$

```
[Out] -a*x/b^2+a^3*x/b^2/(a^2-b^2)+ln(sinh(x))/b-a^2*ln(b*cosh(x)+a*sinh(x))/b/(a
^2-b^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3622, 3556, 3565, 3611}

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx=-\frac{a^2\log(a\sinh(x)+b\cosh(x))}{b(a^2-b^2)}+\frac{a^3x}{b^2(a^2-b^2)}-\frac{ax}{b^2}+\frac{\log(\sinh(x))}{b}$$

```
[In] Int[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -((a*x)/b^2) + (a^3*x)/(b^2*(a^2 - b^2)) + Log[Sinh[x]]/b - (a^2*Log[b*Cosh
[x] + a*Sinh[x]])/(b*(a^2 - b^2))
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
```

```
d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3622

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*tan[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[d*(2*b*c - a*d)*(x/b^2), x] + (Dist[d^2/b, In
t[Tan[e + f*x], x], x] + Dist[(b*c - a*d)^2/b^2, Int[1/(a + b*Tan[e + f*x])
, x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{ax}{b^2} + \frac{a^2 \int \frac{1}{a+b\coth(x)}\,dx}{b^2} + \frac{\int \coth(x)\,dx}{b} \\ &= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\sinh(x))}{b} - \frac{(ia^2)\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{b(a^2-b^2)} \\ &= -\frac{ax}{b^2} + \frac{a^3x}{b^2(a^2-b^2)} + \frac{\log(\sinh(x))}{b} - \frac{a^2\log(b\cosh(x)+a\sinh(x))}{b(a^2-b^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{\log(1-\coth(x))}{2(a+b)} + \frac{\log(1+\coth(x))}{2(a-b)} - \frac{a^2\log(a+b\coth(x))}{b(a^2-b^2)}$$

```
[In] Integrate[Coth[x]^2/(a + b*Coth[x]),x]
```

```
[Out] -1/2*Log[1 - Coth[x]]/(a + b) + Log[1 + Coth[x]]/(2*(a - b)) - (a^2*Log[a +
 b*Coth[x]])/(b*(a^2 - b^2))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{-a^{2} \ln \left(b+a \tanh \left(x \right)\right)+\ln \left(1-\tanh \left(x \right)\right) b^{2}+\left(\left(a-b \right) \ln \left(\tanh \left(x \right)\right)+b x \right) \left(a+b \right)}{a^{2} b-b^{3}}$	56
derivativedivides	$\frac{\ln \left(1+\coth \left(x \right)\right)}{2 a-2 b}-\frac{\ln \left(\coth \left(x \right)-1\right)}{2 a+2 b}-\frac{a^{2} \ln \left(a+b \coth \left(x \right)\right)}{\left(a+b \right) \left(a-b \right) b}$	60
default	$\frac{\ln \left(1+\coth \left(x \right)\right)}{2 a-2 b}-\frac{\ln \left(\coth \left(x \right)-1\right)}{2 a+2 b}-\frac{a^{2} \ln \left(a+b \coth \left(x \right)\right)}{\left(a+b \right) \left(a-b \right) b}$	60
risch	$\frac{x}{a+b}+\frac{2 x \,a^{2}}{b \left(a^{2}-b^{2}\right)}-\frac{2 x}{b}-\frac{a^{2} \ln \left({\mathrm e}^{2 x}-\frac{a-b}{a+b}\right)}{b \left(a^{2}-b^{2}\right)}+\frac{\ln \left({\mathrm e}^{2 x}-1\right)}{b}$	83

```
[In] int(coth(x)^2/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (-a^2*ln(b+a*tanh(x))+ln(1-tanh(x))*b^2+((a-b)*ln(tanh(x))+b*x)*(a+b))/(a^2
*b-b^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.21

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{a^{2} \log\left(\frac{2 \,{\left(b \cosh\left(x\right) + a \sinh\left(x\right)\right)}}{\cosh\left(x\right) - \sinh\left(x\right)}\right) - {\left(a b + b^{2}\right)} x - {\left(a^{2} - b^{2}\right)} \log\left(\frac{2 \, \sinh\left(x\right)}{\cosh\left(x\right) - \sinh\left(x\right)}\right)}{a^{2} b - b^{3}}$$

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -(a^2*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) - (a*b + b^2)*x -
(a^2 - b^2)*log(2*sinh(x)/(cosh(x) - sinh(x))))/(a^2*b - b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(51) = 102$.

Time = 0.71 (sec) , antiderivative size = 372, normalized size of antiderivative = 5.90

$$\int \frac{\coth^2(x)}{a + b\coth(x)}\,dx = \begin{cases} \tilde{\infty}(x - \log(\tanh(x) + 1) + \log(\tanh(x))) \\ \frac{x - \log(\tanh(x)+1) + \log(\tanh(x))}{b} \\ \frac{3x\tanh(x)}{2b\tanh(x)-2b} - \frac{3x}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x)+1)}{2b\tanh(x)-2b} + \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)-2b} - \frac{2\log(\tanh(x))}{2b\tanh(x)-2b} - \frac{}{2b} \\ \frac{x\tanh(x)}{2b\tanh(x)+2b} + \frac{x}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)\tanh(x)}{2b\tanh(x)+2b} - \frac{2\log(\tanh(x)+1)}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x))\tanh(x)}{2b\tanh(x)+2b} + \frac{2\log(\tanh(x))}{2b\tanh(x)+2b} + \frac{}{2b} \\ \frac{x - \frac{1}{\tanh(x)}}{a} \\ -\frac{a^2\log\left(\tanh(x) + \frac{b}{a}\right)}{a^2b - b^3} + \frac{a^2\log(\tanh(x))}{a^2b-b^3} + \frac{abx}{a^2b-b^3} - \frac{b^2x}{a^2b-b^3} + \frac{b^2\log(\tanh(x)+1)}{a^2b-b^3} - \frac{b^2\log(\tanh(x))}{a^2b-b^3} \end{cases}$$

```
[In] integrate(coth(x)**2/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 ((x - log(tanh(x) + 1) + log(tanh(x)))/b, Eq(a, 0)), (3*x*tanh(x)/(2*b*tan
h(x) - 2*b) - 3*x/(2*b*tanh(x) - 2*b) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tan
h(x) - 2*b) + 2*log(tanh(x) + 1)/(2*b*tanh(x) - 2*b) + 2*log(tanh(x))*tanh(
x)/(2*b*tanh(x) - 2*b) - 2*log(tanh(x))/(2*b*tanh(x) - 2*b) - 1/(2*b*tanh(x
) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b*tanh(x) + 2*b
) - 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x) + 2*b) - 2*log(tanh(x) + 1)/(2*
b*tanh(x) + 2*b) + 2*log(tanh(x))*tanh(x)/(2*b*tanh(x) + 2*b) + 2*log(tanh(
x))/(2*b*tanh(x) + 2*b) + 1/(2*b*tanh(x) + 2*b), Eq(a, b)), ((x - 1/tanh(x)
)/a, Eq(b, 0)), (-a**2*log(tanh(x) + b/a)/(a**2*b - b**3) + a**2*log(tanh(x
))/(a**2*b - b**3) + a*b*x/(a**2*b - b**3) - b**2*x/(a**2*b - b**3) + b**2*
log(tanh(x) + 1)/(a**2*b - b**3) - b**2*log(tanh(x))/(a**2*b - b**3), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{\coth^2(x)}{a + b\coth(x)}\,dx = -\frac{a^2\log\left(-(a-b)e^{(-2x)} + a + b\right)}{a^2b - b^3} + \frac{x}{a+b} + \frac{\log\left(e^{(-x)} + 1\right)}{b} + \frac{\log\left(e^{(-x)} - 1\right)}{b}$$

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b - b^3) + x/(a + b) + log(e^(-x)
+ 1)/b + log(e^(-x) - 1)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = -\frac{a^2\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b-b^3}+\frac{x}{a-b}+\frac{\log\left(\left|e^{(2x)}-1\right|\right)}{b}$$

```
[In] integrate(coth(x)^2/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b - b^3) + x/(a - b) + lo
g(abs(e^(2*x) - 1))/b
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{\coth^2(x)}{a+b\coth(x)}\,dx = \frac{\ln\left(e^{2x}-1\right)}{b}+\frac{x}{a-b}-\frac{a^2\,\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{a^2\,b-b^3}$$

```
[In] int(coth(x)^2/(a + b*coth(x)),x)
```

```
[Out] log(exp(2*x) - 1)/b + x/(a - b) - (a^2*log(b - a + a*exp(2*x) + b*exp(2*x))
)/(a^2*b - b^3)
```

3.147 $\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx=-\frac{bx}{a^2-b^2}-\frac{\coth(x)}{b}+\frac{a^3\log(a+b\coth(x))}{b^2(a^2-b^2)}+\frac{a\log(\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)-coth(x)/b+a^3*ln(a+b*coth(x))/b^2/(a^2-b^2)+a*ln(sinh(x))/(a
^2-b^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3647, 3707, 3698, 31, 3556}

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx=-\frac{bx}{a^2-b^2}+\frac{a\log(\sinh(x))}{a^2-b^2}+\frac{a^3\log(a+b\coth(x))}{b^2(a^2-b^2)}-\frac{\coth(x)}{b}$$

```
[In] Int[Coth[x]^3/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - Coth[x]/b + (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2
)) + (a*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\coth(x)}{b} - \frac{\int \frac{-a-b\coth(x)+a\coth^2(x)}{a+b\coth(x)}\,dx}{b}\\
&= -\frac{bx}{a^2-b^2} - \frac{\coth(x)}{b} + \frac{a\int \coth(x)\,dx}{a^2-b^2} + \frac{a^3\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b\,(a^2-b^2)}\\
&= -\frac{bx}{a^2-b^2} - \frac{\coth(x)}{b} + \frac{a\log(\sinh(x))}{a^2-b^2} + \frac{a^3\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b^2\,(a^2-b^2)}\\
&= -\frac{bx}{a^2-b^2} - \frac{\coth(x)}{b} + \frac{a^3\log(a+b\coth(x))}{b^2\,(a^2-b^2)} + \frac{a\log(\sinh(x))}{a^2-b^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx=-\frac{\coth(x)}{b}-\frac{\log(1-\coth(x))}{2(a+b)}-\frac{\log(1+\coth(x))}{2(a-b)}+\frac{a^3\log(a+b\coth(x))}{b^2\left(a^2-b^2\right)}$$

```
[In] Integrate[Coth[x]^3/(a + b*Coth[x]),x]
[Out] -(Coth[x]/b) - Log[1 - Coth[x]]/(2*(a + b)) - Log[1 + Coth[x]]/(2*(a - b))
+ (a^3*Log[a + b*Coth[x]])/(b^2*(a^2 - b^2))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

method	result	siz
derivativedivides	$-\frac{\coth(x)}{b}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	67
default	$-\frac{\coth(x)}{b}+\frac{a^3\ln(a+b\coth(x))}{b^2(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}$	67
parallelrisch	$\frac{a^3\ln(b+a\tanh(x))\tanh(x)-\ln(1-\tanh(x))\tanh(x)a\,b^2-(a\tanh(x)(a-b)\ln(\tanh(x))+b(bx\tanh(x)+a-b))(a+b)}{(a^2b^2-b^4)\tanh(x)}$	80
risch	$\frac{x}{a+b}-\frac{2a^3x}{b^2(a^2-b^2)}+\frac{2ax}{b^2}-\frac{2}{b(e^{2x}-1)}-\frac{a\ln(e^{2x}-1)}{b^2}+\frac{a^3\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b^2(a^2-b^2)}$	98

```
[In] int(coth(x)^3/(a+b*coth(x)),x,method=_RETURNVERBOSE)
[Out] -coth(x)/b+1/b^2*a^3/(a+b)/(a-b)*ln(a+b*coth(x))-1/(2*a+2*b)*ln(coth(x)-1)-
1/(2*a-2*b)*ln(1+coth(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.23

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx$$
$$=\frac{(ab^2+b^3)x\cosh(x)^2+2(ab^2+b^3)x\cosh(x)\sinh(x)+(ab^2+b^3)x\sinh(x)^2+2a^2b-2b^3-(ab^2+b^3)x-}{a}$$

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="fricas")
[Out] ((a*b^2 + b^3)*x*cosh(x)^2 + 2*(a*b^2 + b^3)*x*cosh(x)*sinh(x) + (a*b^2 + b
^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3 - (a*b^2 + b^3)*x - (a^3*cosh(x)^2 + 2*a^
```

```
3*cosh(x)*sinh(x) + a^3*sinh(x)^2 - a^3)*log(2*(b*cosh(x) + a*sinh(x))/(cos
h(x) - sinh(x))) - (a^3 - a*b^2 - (a^3 - a*b^2)*cosh(x)^2 - 2*(a^3 - a*b^2)
*cosh(x)*sinh(x) - (a^3 - a*b^2)*sinh(x)^2)*log(2*sinh(x)/(cosh(x) - sinh(x
))))/(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cosh(x)^2 - 2*(a^2*b^2 - b^4)*cosh(x)
*sinh(x) - (a^2*b^2 - b^4)*sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(49) = 98$.

Time = 1.05 (sec) , antiderivative size = 636, normalized size of antiderivative = 9.94

$$\int \frac{\coth^3(x)}{a + b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)**3/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x)), Eq(a, 0) & Eq(b, 0)), ((x - 1/tanh(x))/b, E
q(a, 0)), (5*x*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 5*x*tanh(x)/(2*b
*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2
- 2*b*tanh(x)) + 2*log(tanh(x) + 1)*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x))
+ 2*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 2*log(tanh(x))
*tanh(x)/(2*b*tanh(x)**2 - 2*b*tanh(x)) - 3*tanh(x)/(2*b*tanh(x)**2 - 2*b*t
anh(x)) + 2/(2*b*tanh(x)**2 - 2*b*tanh(x)), Eq(a, -b)), (x*tanh(x)**2/(2*b*
tanh(x)**2 + 2*b*tanh(x)) + x*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*lo
g(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**2 + 2*b*tanh(x)) + 2*log(tanh(x) +
1)*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)**2/(2*b*
tanh(x)**2 + 2*b*tanh(x)) - 2*log(tanh(x))*tanh(x)/(2*b*tanh(x)**2 + 2*b*ta
nh(x)) - 3*tanh(x)/(2*b*tanh(x)**2 + 2*b*tanh(x)) - 2/(2*b*tanh(x)**2 + 2*b
*tanh(x)), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)*
*2))/a, Eq(b, 0)), (a**3*log(tanh(x) + b/a)*tanh(x)/(a**2*b**2*tanh(x) - b*
*4*tanh(x)) - a**3*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x))
- a**2*b/(a**2*b**2*tanh(x) - b**4*tanh(x)) + a*b**2*x*tanh(x)/(a**2*b**2*t
anh(x) - b**4*tanh(x)) - a*b**2*log(tanh(x) + 1)*tanh(x)/(a**2*b**2*tanh(x)
 - b**4*tanh(x)) + a*b**2*log(tanh(x))*tanh(x)/(a**2*b**2*tanh(x) - b**4*ta
nh(x)) - b**3*x*tanh(x)/(a**2*b**2*tanh(x) - b**4*tanh(x)) + b**3/(a**2*b**
2*tanh(x) - b**4*tanh(x)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = \frac{a^3\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^2-b^4}+\frac{x}{a+b} - \frac{a\log\left(e^{(-x)}+1\right)}{b^2}-\frac{a\log\left(e^{(-x)}-1\right)}{b^2}+\frac{2}{be^{(-2x)}-b}$$

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="maxima")
[Out] a^3*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^2 - b^4) + x/(a + b) - a*log(e^(-
x) + 1)/b^2 - a*log(e^(-x) - 1)/b^2 + 2/(b*e^(-2*x) - b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.19

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = \frac{a^3\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^2-b^4}-\frac{x}{a-b} - \frac{a\log\left(\left|e^{(2x)}-1\right|\right)}{b^2}-\frac{2}{b(e^{(2x)}-1)}$$

```
[In] integrate(coth(x)^3/(a+b*coth(x)),x, algorithm="giac")
[Out] a^3*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^2 - b^4) - x/(a - b) - a
*log(abs(e^(2*x) - 1))/b^2 - 2/(b*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16

$$\int \frac{\coth^3(x)}{a+b\coth(x)}\,dx = -\frac{2}{b\,\left(e^{2\,x}-1\right)}-\frac{x}{a-b}-\frac{a^3\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{b^4-a^2\,b^2}-\frac{a\,\ln\left(e^{2\,x}-1\right)}{b^2}$$

```
[In] int(coth(x)^3/(a + b*coth(x)),x)
[Out] - 2/(b*(exp(2*x) - 1)) - x/(a - b) - (a^3*log(b - a + a*exp(2*x) + b*exp(2*
x)))/(b^4 - a^2*b^2) - (a*log(exp(2*x) - 1))/b^2
```

3.148 $\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx=\frac{ax}{a^2-b^2}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}-\frac{a^4\log(a+b\coth(x))}{b^3(a^2-b^2)}-\frac{b\log(\sinh(x))}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)+a*coth(x)/b^2-1/2*coth(x)^2/b-a^4*ln(a+b*coth(x))/b^3/(a^2-b^
2)-b*ln(sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3647, 3728, 3708, 3698, 31, 3556}

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b\log(\sinh(x))}{a^2-b^2}-\frac{a^4\log(a+b\coth(x))}{b^3(a^2-b^2)}+\frac{a\coth(x)}{b^2}-\frac{\coth^2(x)}{2b}$$

```
[In] Int[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) + (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - (a^4*Log[a + b*Coth
[x]])/(b^3*(a^2 - b^2)) - (b*Log[Sinh[x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3708

```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[a*(A - C)*(x/(a^2 + b^2)), x] + (Dist[(a^2*C +
A*b^2)/(a^2 + b^2), Int[(1 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] -
 Dist[b*((A - C)/(a^2 + b^2)), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e,
 f, A, C}, x] && NeQ[a^2*C + A*b^2, 0] && NeQ[a^2 + b^2, 0] && NeQ[A, C]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rubi steps

$$\text{integral} = -\frac{\coth^2(x)}{2b} - \frac{\int \frac{\coth(x)(-2a-2b\coth(x)+2a\coth^2(x))}{a+b\coth(x)}\,dx}{2b}$$

$$= \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{\int \frac{2a^2-2(a^2+b^2)\coth^2(x)}{a+b\coth(x)}\,dx}{2b^2}$$

$$= \frac{ax}{a^2-b^2} + \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{a^4\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b^2(a^2-b^2)} - \frac{b\int \coth(x)\,dx}{a^2-b^2}$$

$$= \frac{ax}{a^2-b^2} + \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{b\log(\sinh(x))}{a^2-b^2} - \frac{a^4\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b^3(a^2-b^2)}$$

$$= \frac{ax}{a^2-b^2} + \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{a^4\log(a+b\coth(x))}{b^3(a^2-b^2)} - \frac{b\log(\sinh(x))}{a^2-b^2}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.01

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = \frac{a\coth(x)}{b^2} - \frac{\coth^2(x)}{2b} - \frac{\log(1-\coth(x))}{2(a+b)} + \frac{\log(1+\coth(x))}{2(a-b)} - \frac{a^4\log(a+b\coth(x))}{b^3(a^2-b^2)}$$

```
[In] Integrate[Coth[x]^4/(a + b*Coth[x]),x]
```

```
[Out] (a*Coth[x])/b^2 - Coth[x]^2/(2*b) - Log[1 - Coth[x]]/(2*(a + b)) + Log[1 +
Coth[x]]/(2*(a - b)) - (a^4*Log[a + b*Coth[x]])/(b^3*(a^2 - b^2))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{\coth(x)^2}{2b}+\frac{a\coth(x)}{b^2}+\frac{\ln(1+\coth(x))}{2a-2b}-\frac{a^4\ln(a+b\coth(x))}{b^3(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}$
default	$-\frac{\coth(x)^2}{2b}+\frac{a\coth(x)}{b^2}+\frac{\ln(1+\coth(x))}{2a-2b}-\frac{a^4\ln(a+b\coth(x))}{b^3(a+b)(a-b)}-\frac{\ln(\coth(x)-1)}{2a+2b}$
parallelrisch	$\frac{-2\ln(b+a\tanh(x))a^4+2\ln(1-\tanh(x))b^4+(2a^4-2b^4)\ln(\tanh(x))+2\left(-\frac{b\coth(x)^2(a-b)}{2}+a\coth(x)(a-b)+b^2x\right)b(a+b}{2a^2b^3-2b^5}$
risch	$\frac{x}{a+b}-\frac{2x\,a^2}{b^3}-\frac{2x}{b}+\frac{2x\,a^4}{b^3(a^2-b^2)}+\frac{2a\,e^{2x}-2b\,e^{2x}-2a}{(e^{2x}-1)^2b^2}+\frac{\ln(e^{2x}-1)a^2}{b^3}+\frac{\ln(e^{2x}-1)}{b}-\frac{a^4\ln\left(e^{2x}-\frac{a-b}{a+b}\right)}{b^3(a^2-b^2)}$

```
[In] int(coth(x)^4/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*coth(x)^2/b+a*coth(x)/b^2+1/(2*a-2*b)*ln(1+coth(x))-1/b^3*a^4/(a+b)/(a
-b)*ln(a+b*coth(x))-1/(2*a+2*b)*ln(coth(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 648, normalized size of antiderivative = 8.53

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = \frac{(ab^3+b^4)x\cosh(x)^4 + 4(ab^3+b^4)x\cosh(x)\sinh(x)^3 + (ab^3+b^4)x\sinh(x)^4 - 2a^3b + 2ab^3 + 2(a^3b - a^2}{}$$

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] ((a*b^3 + b^4)*x*cosh(x)^4 + 4*(a*b^3 + b^4)*x*cosh(x)*sinh(x)^3 + (a*b^3 +
 b^4)*x*sinh(x)^4 - 2*a^3*b + 2*a*b^3 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 -
(a*b^3 + b^4)*x)*cosh(x)^2 + 2*(a^3*b - a^2*b^2 - a*b^3 + b^4 + 3*(a*b^3 +
b^4)*x*cosh(x)^2 - (a*b^3 + b^4)*x)*sinh(x)^2 + (a*b^3 + b^4)*x - (a^4*cosh
(x)^4 + 4*a^4*cosh(x)*sinh(x)^3 + a^4*sinh(x)^4 - 2*a^4*cosh(x)^2 + a^4 + 2
*(3*a^4*cosh(x)^2 - a^4)*sinh(x)^2 + 4*(a^4*cosh(x)^3 - a^4*cosh(x))*sinh(x
))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + ((a^4 - b^4)*cosh(x
)^4 + 4*(a^4 - b^4)*cosh(x)*sinh(x)^3 + (a^4 - b^4)*sinh(x)^4 + a^4 - b^4 -
 2*(a^4 - b^4)*cosh(x)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(x)^2)*sinh(x)^
2 + 4*((a^4 - b^4)*cosh(x)^3 - (a^4 - b^4)*cosh(x))*sinh(x))*log(2*sinh(x)/
(cosh(x) - sinh(x))) + 4*((a*b^3 + b^4)*x*cosh(x)^3 + (a^3*b - a^2*b^2 - a*
b^3 + b^4 - (a*b^3 + b^4)*x)*cosh(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 -
b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh
(x)^4 - 2*(a^2*b^3 - b^5)*cosh(x)^2 - 2*(a^2*b^3 - b^5 - 3*(a^2*b^3 - b^5)*
cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3 - (a^2*b^3 - b^5)*cosh(
x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(61) = 122$.

Time = 1.42 (sec) , antiderivative size = 882, normalized size of antiderivative = 11.61

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)**4/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2)), Eq
(a, 0) & Eq(b, 0)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1/(2*tanh(x)**2
))/b, Eq(a, 0)), (7*x*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 7*x*ta
nh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/
```

```
(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh
(x)**3 - 2*b*tanh(x)**2) + 4*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 - 2*b*
tanh(x)**2) - 4*log(tanh(x))*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) -
 3*tanh(x)**2/(2*b*tanh(x)**3 - 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 -
 2*b*tanh(x)**2) + 1/(2*b*tanh(x)**3 - 2*b*tanh(x)**2), Eq(a, -b)), (x*tanh
(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + x*tanh(x)**2/(2*b*tanh(x)**3 + 2
*b*tanh(x)**2) - 4*log(tanh(x) + 1)*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x
)**2) - 4*log(tanh(x) + 1)*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4
*log(tanh(x))*tanh(x)**3/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 4*log(tanh(x))
*tanh(x)**2/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) + 3*tanh(x)**2/(2*b*tanh(x)**
3 + 2*b*tanh(x)**2) + tanh(x)/(2*b*tanh(x)**3 + 2*b*tanh(x)**2) - 1/(2*b*ta
nh(x)**3 + 2*b*tanh(x)**2), Eq(a, b)), ((x - 1/tanh(x) - 1/(3*tanh(x)**3))/
a, Eq(b, 0)), (-2*a**4*log(tanh(x) + b/a)*tanh(x)**2/(2*a**2*b**3*tanh(x)**
2 - 2*b**5*tanh(x)**2) + 2*a**4*log(tanh(x))*tanh(x)**2/(2*a**2*b**3*tanh(x
)**2 - 2*b**5*tanh(x)**2) + 2*a**3*b*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b*
*5*tanh(x)**2) - a**2*b**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2
*a*b**3*x*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*a*b**
3*tanh(x)/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*x*tanh(x)**
2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + 2*b**4*log(tanh(x) + 1)*ta
nh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) - 2*b**4*log(tanh(x))
*tanh(x)**2/(2*a**2*b**3*tanh(x)**2 - 2*b**5*tanh(x)**2) + b**4/(2*a**2*b**
3*tanh(x)**2 - 2*b**5*tanh(x)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = -\frac{a^4\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^3-b^5} + \frac{2\left((a+b)e^{(-2x)}-a\right)}{2\,b^2e^{(-2x)}-b^2e^{(-4x)}-b^2} + \frac{x}{a+b} + \frac{(a^2+b^2)\log\left(e^{(-x)}+1\right)}{b^3} + \frac{(a^2+b^2)\log\left(e^{(-x)}-1\right)}{b^3}$$

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] -a^4*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^3 - b^5) + 2*((a + b)*e^(-2*x) -
 a)/(2*b^2*e^(-2*x) - b^2*e^(-4*x) - b^2) + x/(a + b) + (a^2 + b^2)*log(e^(
-x) + 1)/b^3 + (a^2 + b^2)*log(e^(-x) - 1)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = -\frac{a^4\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^3-b^5}+\frac{x}{a-b}+\frac{(a^2+b^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^3}-\frac{2\left(ab-(ab-b^2)e^{(2x)}\right)}{b^3\left(e^{(2x)}-1\right)^2}$$

```
[In] integrate(coth(x)^4/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] -a^4*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^3 - b^5) + x/(a - b) +
(a^2 + b^2)*log(abs(e^(2*x) - 1))/b^3 - 2*(a*b - (a*b - b^2)*e^(2*x))/(b^3*
(e^(2*x) - 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.45

$$\int \frac{\coth^4(x)}{a+b\coth(x)}\,dx = \frac{x}{a-b}-\frac{2}{b\,\left(e^{4\,x}-2\,e^{2\,x}+1\right)}+\frac{\ln\left(e^{2\,x}-1\right)\,\left(a^2+b^2\right)}{b^3}+\frac{a^4\,\ln\left(b-a+a\,e^{2\,x}+b\,e^{2\,x}\right)}{b^5-a^2\,b^3}+\frac{2\,\left(a^2-b^2\right)}{b^2\,\left(a+b\right)\,\left(e^{2\,x}-1\right)}$$

```
[In] int(coth(x)^4/(a + b*coth(x)),x)
```

```
[Out] x/(a - b) - 2/(b*(exp(4*x) - 2*exp(2*x) + 1)) + (log(exp(2*x) - 1)*(a^2 + b
^2))/b^3 + (a^4*log(b - a + a*exp(2*x) + b*exp(2*x)))/(b^5 - a^2*b^3) + (2*
(a^2 - b^2))/(b^2*(a + b)*(exp(2*x) - 1))
```

3.149 $\int \frac{\coth^5(x)}{a+b\coth(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\coth^5(x)}{a+b\coth(x)} dx = -\frac{bx}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a^5\log(a+b\coth(x))}{b^4(a^2-b^2)} + \frac{a\log(\sinh(x))}{a^2-b^2}$$

```
[Out] -b*x/(a^2-b^2)-(a^2+b^2)*coth(x)/b^3+1/2*a*coth(x)^2/b^2-1/3*coth(x)^3/b+a^
5*ln(a+b*coth(x))/b^4/(a^2-b^2)+a*ln(sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3647, 3728, 3729, 3707, 3698, 31, 3556}

$$\int \frac{\coth^5(x)}{a+b\coth(x)} dx = -\frac{bx}{a^2-b^2} + \frac{a\log(\sinh(x))}{a^2-b^2} - \frac{(a^2+b^2)\coth(x)}{b^3} + \frac{a^5\log(a+b\coth(x))}{b^4(a^2-b^2)} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b}$$

```
[In] Int[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] -((b*x)/(a^2 - b^2)) - ((a^2 + b^2)*Coth[x])/b^3 + (a*Coth[x]^2)/(2*b^2) -
Coth[x]^3/(3*b) + (a^5*Log[a + b*Coth[x]])/(b^4*(a^2 - b^2)) + (a*Log[Sinh[
x]])/(a^2 - b^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3647

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[b^2*(a + b*Tan[e + f*x])^(m - 2)*((c
+ d*Tan[e + f*x])^(n + 1)/(d*f*(m + n - 1))), x] + Dist[1/(d*(m + n - 1)),
Int[(a + b*Tan[e + f*x])^(m - 3)*(c + d*Tan[e + f*x])^n*Simp[a^3*d*(m + n -
 1) - b^2*(b*c*(m - 2) + a*d*(1 + n)) + b*d*(m + n - 1)*(3*a^2 - b^2)*Tan[e
 + f*x] - b^2*(b*c*(m - 2) - a*d*(3*m + 2*n - 4))*Tan[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && NeQ[c^2 + d^2, 0] && IntegerQ[2*m] && GtQ[m, 2] && (GeQ[n, -1] || In
tegerQ[m]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rule 3698

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Dist[A/(b*f), Subst[Int[(a + x)^m, x], x, b*T
an[e + f*x]], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A, C]
```

Rule 3707

```
Int[((A_) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2
)/((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(a*A + b*B - a
*C)*(x/(a^2 + b^2)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)/(a^2 + b^2), Int[(1
 + Tan[e + f*x]^2)/(a + b*Tan[e + f*x]), x], x] - Dist[(A*b - a*B - b*C)/(a
^2 + b^2), Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, e, f, A, B, C}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] && NeQ[a^2 + b^2, 0] && NeQ[A*b - a*B - b*C,
0]
```

Rule 3728

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (B_.)*tan[(e_.) + (f_.)*(x_)] + (C_.)*tan[(e_.
) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[
e + f*x])^(n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a +
 b*Tan[e + f*x])^(m - 1)*(c + d*Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*
(b*c*m + a*d*(n + 1)) + d*(A*b + a*B - b*C)*(m + n + 1)*Tan[e + f*x] - (C*m
*(b*c - a*d) - b*B*d*(m + n + 1))*Tan[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] &&
```

```
NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[c
, 0] && NeQ[a, 0])))
```

Rule 3729

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*tan[(e_.)
+ (f_.)*(x_)])^(n_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 Simp[C*(a + b*Tan[e + f*x])^m*((c + d*Tan[e + f*x])^(n + 1)/(d*f*(m + n +
1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Tan[e + f*x])^(m - 1)*(c + d*
Tan[e + f*x])^n*Simp[a*A*d*(m + n + 1) - C*(b*c*m + a*d*(n + 1)) + d*(A*b -
 b*C)*(m + n + 1)*Tan[e + f*x] - C*m*(b*c - a*d)*Tan[e + f*x]^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 +
 b^2, 0] && NeQ[c^2 + d^2, 0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[
m] || (EqQ[c, 0] && NeQ[a, 0])))
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth^2(x)\left(-3a-3b\coth(x)+3a\coth^2(x)\right)}{a+b\coth(x)}\,dx}{3b} \\
&= \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{\coth(x)\left(6a^2-6\left(a^2+b^2\right)\coth^2(x)\right)}{a+b\coth(x)}\,dx}{6b^2} \\
&= -\frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} - \frac{\int \frac{-6a\left(a^2+b^2\right)-6b^3\coth(x)+6a\left(a^2+b^2\right)\coth^2(x)}{a+b\coth(x)}\,dx}{6b^3} \\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} + \frac{a\int \coth(x)\,dx}{a^2-b^2} + \frac{a^5\int \frac{1-\coth^2(x)}{a+b\coth(x)}\,dx}{b^3\left(a^2-b^2\right)} \\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} - \frac{\coth^3(x)}{3b} \\
&\quad + \frac{a\log(\sinh(x))}{a^2-b^2} + \frac{a^5\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\coth(x)\right)}{b^4\left(a^2-b^2\right)} \\
&= -\frac{bx}{a^2-b^2} - \frac{\left(a^2+b^2\right)\coth(x)}{b^3} + \frac{a\coth^2(x)}{2b^2} \\
&\quad - \frac{\coth^3(x)}{3b} + \frac{a^5\log(a+b\coth(x))}{b^4\left(a^2-b^2\right)} + \frac{a\log(\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = \frac{1}{6}\left(-\frac{6(a^2+b^2)\coth(x)}{b^3}+\frac{3a\coth^2(x)}{b^2}-\frac{2\coth^3(x)}{b} - \frac{3\log(1-\coth(x))}{a+b}-\frac{3\log(1+\coth(x))}{a-b}+\frac{6a^5\log(a+b\coth(x))}{b^4(a^2-b^2)}\right)$$

```
[In] Integrate[Coth[x]^5/(a + b*Coth[x]),x]
```

```
[Out] ((-6*(a^2 + b^2)*Coth[x])/b^3 + (3*a*Coth[x]^2)/b^2 - (2*Coth[x]^3)/b - (3*
Log[1 - Coth[x]])/(a + b) - (3*Log[1 + Coth[x]])/(a - b) + (6*a^5*Log[a + b
*Coth[x]])/(b^4*(a^2 - b^2)))/6
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

method	result
derivativedivides	$-\frac{\coth(x)^3}{3b}+\frac{a\coth(x)^2}{2b^2}-\frac{a^2\coth(x)}{b^3}-\frac{\coth(x)}{b}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{a^5\ln(a+b\coth(x))}{b^4(a+b)(a-b)}$
default	$-\frac{\coth(x)^3}{3b}+\frac{a\coth(x)^2}{2b^2}-\frac{a^2\coth(x)}{b^3}-\frac{\coth(x)}{b}-\frac{\ln(\coth(x)-1)}{2a+2b}-\frac{\ln(1+\coth(x))}{2a-2b}+\frac{a^5\ln(a+b\coth(x))}{b^4(a+b)(a-b)}$
parallelrisch	$\frac{6\ln(b+a\tanh(x))a^5-6\ln(1-\tanh(x))a\,b^4+(-6a^5+6a\,b^4)\ln(\tanh(x))+(-2a^2b^3+2b^5)\coth(x)^3+(3a^3b^2-3a\,b^4)\coth(x)}{6a^2b^4-6b^6}$
risch	$\frac{x}{a+b}+\frac{2x\,a^3}{b^4}+\frac{2ax}{b^2}-\frac{2x\,a^5}{b^4(a^2-b^2)}-\frac{2(3a^2e^{4x}-3ab\,e^{4x}+6b^2e^{4x}-6a^2e^{2x}+3b\,e^{2x}a-6b^2e^{2x}+3a^2+4b^2)}{3b^3(e^{2x}-1)^3}-\frac{a^3\ln(e^{2x}}{b^4}$

```
[In] int(coth(x)^5/(a+b*coth(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*coth(x)^3/b+1/2*a*coth(x)^2/b^2-1/b^3*a^2*coth(x)-coth(x)/b-1/(2*a+2*b
)*ln(coth(x)-1)-1/(2*a-2*b)*ln(1+coth(x))+1/b^4*a^5/(a+b)/(a-b)*ln(a+b*coth
(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(90) = 180$.

Time = 0.30 (sec) , antiderivative size = 1299, normalized size of antiderivative = 13.82

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*(a*b^4 + b^5)*x*cosh(x)^6 + 18*(a*b^4 + b^5)*x*cosh(x)*sinh(x)^5 +
3*(a*b^4 + b^5)*x*sinh(x)^6 + 6*a^4*b + 2*a^2*b^3 - 8*b^5 + 3*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^4 + 3*(2
*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 + 15*(a*b^4 + b^5)*x*cosh(
x)^2 - 3*(a*b^4 + b^5)*x)*sinh(x)^4 + 12*(5*(a*b^4 + b^5)*x*cosh(x)^3 + (2*
a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x
))*sinh(x)^3 - 3*(4*a^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x
)*cosh(x)^2 + 3*(15*(a*b^4 + b^5)*x*cosh(x)^4 - 4*a^4*b + 2*a^3*b^2 - 2*a*b
^4 + 4*b^5 + 6*(2*a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^
4 + b^5)*x)*cosh(x)^2 + 3*(a*b^4 + b^5)*x)*sinh(x)^2 - 3*(a*b^4 + b^5)*x -
3*(a^5*cosh(x)^6 + 6*a^5*cosh(x)*sinh(x)^5 + a^5*sinh(x)^6 - 3*a^5*cosh(x)^
4 + 3*a^5*cosh(x)^2 - a^5 + 3*(5*a^5*cosh(x)^2 - a^5)*sinh(x)^4 + 4*(5*a^5*
cosh(x)^3 - 3*a^5*cosh(x))*sinh(x)^3 + 3*(5*a^5*cosh(x)^4 - 6*a^5*cosh(x)^2
 + a^5)*sinh(x)^2 + 6*(a^5*cosh(x)^5 - 2*a^5*cosh(x)^3 + a^5*cosh(x))*sinh(
x))*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))) + 3*((a^5 - a*b^4)*c
osh(x)^6 + 6*(a^5 - a*b^4)*cosh(x)*sinh(x)^5 + (a^5 - a*b^4)*sinh(x)^6 - a^
5 + a*b^4 - 3*(a^5 - a*b^4)*cosh(x)^4 - 3*(a^5 - a*b^4 - 5*(a^5 - a*b^4)*co
sh(x)^2)*sinh(x)^4 + 4*(5*(a^5 - a*b^4)*cosh(x)^3 - 3*(a^5 - a*b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^5 - a*b^4)*cosh(x)^2 + 3*(a^5 - a*b^4 + 5*(a^5 - a*b^4)*
cosh(x)^4 - 6*(a^5 - a*b^4)*cosh(x)^2)*sinh(x)^2 + 6*((a^5 - a*b^4)*cosh(x)
^5 - 2*(a^5 - a*b^4)*cosh(x)^3 + (a^5 - a*b^4)*cosh(x))*sinh(x))*log(2*sinh
(x)/(cosh(x) - sinh(x))) + 6*(3*(a*b^4 + b^5)*x*cosh(x)^5 + 2*(2*a^4*b - 2*
a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x)^3 - (4*a
^4*b - 2*a^3*b^2 + 2*a*b^4 - 4*b^5 - 3*(a*b^4 + b^5)*x)*cosh(x))*sinh(x))/(
(a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^2*b^4
- b^6)*sinh(x)^6 - a^2*b^4 + b^6 - 3*(a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^2*b^4
 - b^6 - 5*(a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^2*b^4 - b^6)*cosh
(x)^3 - 3*(a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2
+ 3*(a^2*b^4 - b^6 + 5*(a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^2*b^4 - b^6)*cosh(x
)^2)*sinh(x)^2 + 6*((a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^2*b^4 - b^6)*cosh(x)^3
 + (a^2*b^4 - b^6)*cosh(x))*sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(78) = 156$.

Time = 2.25 (sec) , antiderivative size = 1013, normalized size of antiderivative = 10.78

$$\int \frac{\coth^5(x)}{a + b\coth(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)**5/(a+b*coth(x)),x)
```

```
[Out] Piecewise((zoo*(x - 1/tanh(x) - 1/(3*tanh(x)**3)), Eq(a, 0) & Eq(b, 0)), ((
x - 1/tanh(x) - 1/(3*tanh(x)**3))/b, Eq(a, 0)), (27*x*tanh(x)**4/(6*b*tanh(
```

```
x)**4 - 6*b*tanh(x)**3) - 27*x*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3)
 - 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*lo
g(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + 12*log(tanh(x
))*tanh(x)**4/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**
3/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) - 15*tanh(x)**3/(6*b*tanh(x)**4 - 6*b*t
anh(x)**3) + 9*tanh(x)**2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3) + tanh(x)/(6*b*
tanh(x)**4 - 6*b*tanh(x)**3) + 2/(6*b*tanh(x)**4 - 6*b*tanh(x)**3), Eq(a, -
b)), (3*x*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) + 3*x*tanh(x)**3/(6*
b*tanh(x)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**4/(6*b*tanh(x
)**4 + 6*b*tanh(x)**3) + 12*log(tanh(x) + 1)*tanh(x)**3/(6*b*tanh(x)**4 + 6
*b*tanh(x)**3) - 12*log(tanh(x))*tanh(x)**4/(6*b*tanh(x)**4 + 6*b*tanh(x)**
3) - 12*log(tanh(x))*tanh(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 15*tanh
(x)**3/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 9*tanh(x)**2/(6*b*tanh(x)**4 + 6
*b*tanh(x)**3) + tanh(x)/(6*b*tanh(x)**4 + 6*b*tanh(x)**3) - 2/(6*b*tanh(x)
**4 + 6*b*tanh(x)**3), Eq(a, b)), ((x - log(tanh(x) + 1) + log(tanh(x)) - 1
/(2*tanh(x)**2) - 1/(4*tanh(x)**4))/a, Eq(b, 0)), (6*a**5*log(tanh(x) + b/a
)*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**5*log(tanh
(x))*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 6*a**4*b*tan
h(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) + 3*a**3*b**2*tanh(x)/
(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 2*a**2*b**3/(6*a**2*b**4*tan
h(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**
3 - 6*b**6*tanh(x)**3) - 6*a*b**4*log(tanh(x) + 1)*tanh(x)**3/(6*a**2*b**4*
tanh(x)**3 - 6*b**6*tanh(x)**3) + 6*a*b**4*log(tanh(x))*tanh(x)**3/(6*a**2*
b**4*tanh(x)**3 - 6*b**6*tanh(x)**3) - 3*a*b**4*tanh(x)/(6*a**2*b**4*tanh(x
)**3 - 6*b**6*tanh(x)**3) - 6*b**5*x*tanh(x)**3/(6*a**2*b**4*tanh(x)**3 - 6
*b**6*tanh(x)**3) + 6*b**5*tanh(x)**2/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh
(x)**3) + 2*b**5/(6*a**2*b**4*tanh(x)**3 - 6*b**6*tanh(x)**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.80

$$\begin{aligned}\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx &= \frac{a^5\log\left(-(a-b)e^{(-2x)}+a+b\right)}{a^2b^4-b^6} \\ &\quad+\frac{2\left(3a^2+4b^2-3(2a^2+ab+2b^2)e^{(-2x)}+3(a^2+ab+2b^2)e^{(-4x)}\right)}{3\left(3b^3e^{(-2x)}-3b^3e^{(-4x)}+b^3e^{(-6x)}-b^3\right)} \\ &\quad+\frac{x}{a+b}-\frac{(a^3+ab^2)\log\left(e^{(-x)}+1\right)}{b^4}-\frac{(a^3+ab^2)\log\left(e^{(-x)}-1\right)}{b^4}\end{aligned}$$

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="maxima")
```

```
[Out] a^5*log(-(a - b)*e^(-2*x) + a + b)/(a^2*b^4 - b^6) + 2/3*(3*a^2 + 4*b^2 - 3
*(2*a^2 + a*b + 2*b^2)*e^(-2*x) + 3*(a^2 + a*b + 2*b^2)*e^(-4*x))/(3*b^3*e^
```

```
(-2*x) - 3*b^3*e^(-4*x) + b^3*e^(-6*x) - b^3) + x/(a + b) - (a^3 + a*b^2)*l
og(e^(-x) + 1)/b^4 - (a^3 + a*b^2)*log(e^(-x) - 1)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.52

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = \frac{a^5\log\left(\left|ae^{(2x)}+be^{(2x)}-a+b\right|\right)}{a^2b^4-b^6} - \frac{x}{a-b} - \frac{(a^3+ab^2)\log\left(\left|e^{(2x)}-1\right|\right)}{b^4} - \frac{2\left(3a^2b+4b^3+3\left(a^2b-ab^2+2b^3\right)e^{(4x)}-3\left(2a^2b-ab^2+2b^3\right)e^{(2x)}\right)}{3b^4\left(e^{(2x)}-1\right)^3}$$

```
[In] integrate(coth(x)^5/(a+b*coth(x)),x, algorithm="giac")
```

```
[Out] a^5*log(abs(a*e^(2*x) + b*e^(2*x) - a + b))/(a^2*b^4 - b^6) - x/(a - b) - (
a^3 + a*b^2)*log(abs(e^(2*x) - 1))/b^4 - 2/3*(3*a^2*b + 4*b^3 + 3*(a^2*b -
a*b^2 + 2*b^3)*e^(4*x) - 3*(2*a^2*b - a*b^2 + 2*b^3)*e^(2*x))/(b^4*(e^(2*x)
 - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.74

$$\int \frac{\coth^5(x)}{a+b\coth(x)}\,dx = -\frac{8}{3b\left(3e^{2x}-3e^{4x}+e^{6x}-1\right)} - \frac{x}{a-b} - \frac{a^5\ln\left(b-a+a\,e^{2x}+b\,e^{2x}\right)}{b^6-a^2b^4} - \frac{\ln\left(e^{2x}-1\right)\left(a^3+a\,b^2\right)}{b^4} - \frac{2\left(a^3+a\,b^2+2\,b^3\right)}{b^3\left(a+b\right)\left(e^{2x}-1\right)} - \frac{2\left(-a^2+a\,b+2\,b^2\right)}{b^2\left(a+b\right)\left(e^{4x}-2\,e^{2x}+1\right)}$$

```
[In] int(coth(x)^5/(a + b*coth(x)),x)
```

```
[Out] - 8/(3*b*(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1)) - x/(a - b) - (a^5*log(b
 - a + a*exp(2*x) + b*exp(2*x)))/(b^6 - a^2*b^4) - (log(exp(2*x) - 1)*(a*b^
2 + a^3))/b^4 - (2*(a*b^2 + a^3 + 2*b^3))/(b^3*(a + b)*(exp(2*x) - 1)) - (2
*(a*b - a^2 + 2*b^2))/(b^2*(a + b)*(exp(4*x) - 2*exp(2*x) + 1))
```

3.150 $\int \frac{x\mathbf{csch}^2(x)}{(a+b\coth(x))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx=-\frac{ax}{b\,(a^2-b^2)}+\frac{x}{b(a+b\coth(x))}+\frac{\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}$$

```
[Out] -a*x/b/(a^2-b^2)+x/b/(a+b*coth(x))+ln(b*cosh(x)+a*sinh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5575, 3565, 3611}

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx=-\frac{ax}{b\,(a^2-b^2)}+\frac{\log(a\sinh(x)+b\cosh(x))}{a^2-b^2}+\frac{x}{b(a+b\coth(x))}$$

```
[In] Int[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
```

```
[Out] -((a*x)/(b*(a^2 - b^2))) + x/(b*(a + b*Coth[x])) + Log[b*Cosh[x] + a*Sinh[x
]]/(a^2 - b^2)
```

Rule 3565

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 5575

```
Int[Csch[(c_.) + (d_.)*(x_)]^2*(Coth[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.
)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Coth
[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[f*(m/(b*d*(n + 1))), Int[(e +
f*x)^(m - 1)*(a + b*Coth[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{b(a+b\coth(x))} - \frac{\int \frac{1}{a+b\coth(x)}\,dx}{b} \\
&= -\frac{ax}{b\left(a^2-b^2\right)} + \frac{x}{b(a+b\coth(x))} + \frac{i\int \frac{-ib-ia\coth(x)}{a+b\coth(x)}\,dx}{a^2-b^2} \\
&= -\frac{ax}{b\left(a^2-b^2\right)} + \frac{x}{b(a+b\coth(x))} + \frac{\log(b\cosh(x)+a\sinh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$
\int \frac{x\operatorname{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{ax-b\log(b\cosh(x)+a\sinh(x))}{-a^2b+b^3} + \frac{x\sinh(x)}{b^2\cosh(x)+ab\sinh(x)}
$$

```
[In] Integrate[(x*Csch[x]^2)/(a + b*Coth[x])^2,x]
[Out] (a*x - b*Log[b*Cosh[x] + a*Sinh[x]])/(-(a^2*b) + b^3) + (x*Sinh[x])/(b^2*Co
sh[x] + a*b*Sinh[x])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

method	result	size
risch	$-\frac{2x}{a^2-b^2}-\frac{2x}{(a\,\mathrm{e}^{2x}+b\,\mathrm{e}^{2x}-a+b)(a+b)}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{a-b}{a+b}\right)}{a^2-b^2}$	73

```
[In] int(x*csch(x)^2/(a+b*coth(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/(a^2-b^2)*x-2*x/(a*exp(2*x)+b*exp(2*x)-a+b)/(a+b)+1/(a^2-b^2)*ln(exp(2*x
)-(a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(54) = 108$.

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.41

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{2\,(a+b)x\cosh(x)^2+4\,(a+b)x\cosh(x)\sinh(x)+2\,(a+b)x\sinh(x)^2-\left((a+b)\cosh(x)^2+2\,(a+b)\cos\right.}{a^3-a^2b-ab^2+b^3-\left(a^3+a^2b-ab^2-b^3\right)\cosh(x)^2-2\,\left(a^3+a^2b-ab^2-b^3\right)\cosh(}$$

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="fricas")
```

```
[Out] (2*(a + b)*x*cosh(x)^2 + 4*(a + b)*x*cosh(x)*sinh(x) + 2*(a + b)*x*sinh(x)^
2 - ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a
+ b)*log(2*(b*cosh(x) + a*sinh(x))/(cosh(x) - sinh(x))))/(a^3 - a^2*b - a*b
^2 + b^3 - (a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 - 2*(a^3 + a^2*b - a*b^2 -
 b^3)*cosh(x)*sinh(x) - (a^3 + a^2*b - a*b^2 - b^3)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \int \frac{x\,\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx$$

```
[In] integrate(x*csch(x)**2/(a+b*coth(x))**2,x)
```

```
[Out] Integral(x*csch(x)**2/(a + b*coth(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{2\,xe^{(2\,x)}}{a^2-2\,ab+b^2-(a^2-b^2)e^{(2\,x)}}+\frac{\log\left(\frac{(a+b)e^{(2\,x)}-a+b}{a+b}\right)}{a^2-b^2}$$

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="maxima")
```

```
[Out] 2*x*e^(2*x)/(a^2 - 2*a*b + b^2 - (a^2 - b^2)*e^(2*x)) + log(((a + b)*e^(2*x
) - a + b)/(a + b))/(a^2 - b^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(54) = 108.

Time = 0.30 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.13

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = -\frac{2\,axe^{(2\,x)} + 2\,bxe^{(2\,x)} - ae^{(2\,x)}\log\left(ae^{(2\,x)} + be^{(2\,x)} - a + b\right) - be^{(2\,x)}\log\left(ae^{(2\,x)} + be^{(2\,x)} - a + b\right) + a\log}{a^3e^{(2\,x)} + a^2be^{(2\,x)} - ab^2e^{(2\,x)} - b^3e^{(2\,x)} - a^3 + a^2b + a}$$

```
[In] integrate(x*csch(x)^2/(a+b*coth(x))^2,x, algorithm="giac")
```

```
[Out] -(2*a*x*e^(2*x) + 2*b*x*e^(2*x) - a*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a +
 b) - b*e^(2*x)*log(a*e^(2*x) + b*e^(2*x) - a + b) + a*log(a*e^(2*x) + b*e^
(2*x) - a + b) - b*log(a*e^(2*x) + b*e^(2*x) - a + b))/(a^3*e^(2*x) + a^2*b
*e^(2*x) - a*b^2*e^(2*x) - b^3*e^(2*x) - a^3 + a^2*b + a*b^2 - b^3)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{x\text{csch}^2(x)}{(a+b\coth(x))^2}\,dx = \frac{\ln\left(b-a+a\,\mathrm{e}^{2\,x}+b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2} - \frac{2\,x}{a^2-b^2} - \frac{2\,x}{(a+b)\,\left(b-a+\mathrm{e}^{2\,x}\,(a+b)\right)}$$

```
[In] int(x/(sinh(x)^2*(a + b*coth(x))^2),x)
```

```
[Out] log(b - a + a*exp(2*x) + b*exp(2*x))/(a^2 - b^2) - (2*x)/(a^2 - b^2) - (2*x
)/((a + b)*(b - a + exp(2*x)*(a + b)))
```

3.151 $\int x^3 \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{x^4}{4}+\frac{1}{2}e^{-2a}\log\left(1-e^{2a}x^4\right)$$

```
[Out] 1/4*x^4+1/2*ln(1-exp(2*a)*x^4)/exp(2*a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5657, 455, 45}

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{1}{2}e^{-2a}\log\left(1-e^{2a}x^4\right)+\frac{x^4}{4}$$

```
[In] Int[x^3*Coth[a + 2*Log[x]],x]
```

```
[Out] x^4/4 + Log[1 - E^(2*a)*x^4]/(2*E^(2*a))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(-1-e^{2a}x^4)}{1-e^{2a}x^4}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{-1-e^{2a}x}{1-e^{2a}x}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(1+\frac{2}{-1+e^{2a}x}\right)\,dx, x, x^4\right) \\
&= \frac{x^4}{4}+\frac{1}{2}e^{-2a}\log\left(1-e^{2a}x^4\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(30) = 60$.

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int x^3\coth(a+2\log(x))\,dx = \frac{x^4}{4} + \frac{1}{2}\cosh(2a)\log\left(-\cosh(a)+x^4\cosh(a)+\sinh(a)+x^4\sinh(a)\right) - \frac{1}{2}\log\left(-\cosh(a)+x^4\cosh(a)+\sinh(a)+x^4\sinh(a)\right)\sinh(2a)$$

```
[In] Integrate[x^3*Coth[a + 2*Log[x]],x]
[Out] x^4/4 + (Cosh[2*a]*Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]])/2 -
 (Log[-Cosh[a] + x^4*Cosh[a] + Sinh[a] + x^4*Sinh[a]]*Sinh[2*a])/2
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{x^4}{4}+\frac{e^{-2a}\ln(-1+e^{2a}x^4)}{2}$	24

```
[In] int(x^3*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4+1/2*exp(-2*a)*ln(-1+exp(2*a)*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{1}{4}\left(x^4 e^{(2a)} + 2\log\left(x^4 e^{(2a)} - 1\right)\right)e^{(-2a)}$$

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="fricas")
[Out] 1/4*(x^4*e^(2*a) + 2*log(x^4*e^(2*a) - 1))*e^(-2*a)
```

Sympy [F]

$$\int x^3 \coth(a+2\log(x))\,dx = \int x^3 \coth\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(x**3*coth(a+2*ln(x)),x)
[Out] Integral(x**3*coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{1}{4}x^4 + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a+1\right) + \frac{1}{2}e^{(-2a)}\log\left(x^2e^a-1\right)$$

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="maxima")
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(x^2*e^a + 1) + 1/2*e^(-2*a)*log(x^2*e^a - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{1}{4}\,x^4 + \frac{1}{2}\,e^{(-2\,a)}\log\left(\left|x^4e^{(2\,a)} - 1\right|\right)$$

```
[In] integrate(x^3*coth(a+2*log(x)),x, algorithm="giac")
[Out] 1/4*x^4 + 1/2*e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int x^3 \coth(a+2\log(x))\,dx = \frac{\ln\left(x^4 - \mathrm{e}^{-2\,a}\right)\,\mathrm{e}^{-2\,a}}{2} + \frac{x^4}{4}$$

```
[In] int(x^3*coth(a + 2*log(x)),x)
[Out] (log(x^4 - exp(-2*a))*exp(-2*a))/2 + x^4/4
```

3.152 $\int x^2 \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int x^2 \coth(a+2\log(x))\,dx = \frac{x^3}{3} + e^{-3a/2} \arctan\left(e^{a/2}x\right) - e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)$$

```
[Out] 1/3*x^3+arctan(exp(1/2*a)*x)/exp(3/2*a)-arctanh(exp(1/2*a)*x)/exp(3/2*a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5657, 470, 304, 209, 212}

$$\int x^2 \coth(a+2\log(x))\,dx = e^{-3a/2} \arctan\left(e^{a/2}x\right) - e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right) + \frac{x^3}{3}$$

```
[In] Int[x^2*Coth[a + 2*Log[x]],x]
[Out] x^3/3 + ArcTan[E^(a/2)*x]/E^((3*a)/2) - ArcTanh[E^(a/2)*x]/E^((3*a)/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^2(-1-e^{2a}x^4)}{1-e^{2a}x^4}\,dx \\
&= \frac{x^3}{3} - 2\int \frac{x^2}{1-e^{2a}x^4}\,dx \\
&= \frac{x^3}{3} - e^{-a}\int \frac{1}{1-e^{a}x^2}\,dx + e^{-a}\int \frac{1}{1+e^{a}x^2}\,dx \\
&= \frac{x^3}{3} + e^{-3a/2}\arctan\left(e^{a/2}x\right) - e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int x^2\coth(a+2\log(x))\,dx = \frac{1}{6}\left(2x^3 + 3\text{RootSum}\left[-\cosh(a)+\sinh(a)+\cosh(a)\#1^4 + \sinh(a)\#1^4\&, \frac{\log(x)-\log(x-\#1)}{\#1}\&\right](-\cosh(2a) + \sinh(2a))\right)$$

```
[In] Integrate[x^2*Coth[a + 2*Log[x]],x]
```

```
[Out] (2*x^3 + 3*RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Lo
g[x] - Log[x - #1])/#1 & ]*(-Cosh[2*a] + Sinh[2*a]))/6
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(35) = 70$.

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.84

method	result	size
risch	$\frac{x^{3}}{3}+\frac{\ln\left(\left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}-{\mathrm e}^{2 a} x\right)}{2 \left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}}-\frac{\ln\left(\left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}+{\mathrm e}^{2 a} x\right)}{2 \left(-{\mathrm e}^{a}\right)^{\frac{3}{2}}}+\frac{\ln\left(-\sqrt{{\mathrm e}^{a}}\, x+1\right)}{2 \left({\mathrm e}^{a}\right)^{\frac{3}{2}}}-\frac{\ln\left(\sqrt{{\mathrm e}^{a}}\, x+1\right)}{2 \left({\mathrm e}^{a}\right)^{\frac{3}{2}}}$	83

```
[In] int(x^2*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+1/2/(-exp(a))^(3/2)*ln((-exp(a))^(3/2)-exp(2*a)*x)-1/2/(-exp(a))^(3
/2)*ln((-exp(a))^(3/2)+exp(2*a)*x)+1/2/exp(a)^(3/2)*ln(-exp(a)^(1/2)*x+1)-1
/2/exp(a)^(3/2)*ln(exp(a)^(1/2)*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int x^2 \coth(a+2\log(x))\,dx = \frac{1}{6}\left(2\,x^{3}e^{(2\,a)}+6\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+3\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^{2}e^{a}-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^{2}e^{a}-1}\right)\right)e^{(-2\,a)}$$

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^3*e^(2*a) + 6*arctan(x*e^(1/2*a))*e^(1/2*a) + 3*e^(1/2*a)*log((x^2
*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-2*a)
```

Sympy [F]

$$\int x^2 \coth(a + 2\log(x))\,dx = \int x^2 \coth\left(a + 2\log\left(x\right)\right)\,dx$$

```
[In] integrate(x**2*coth(a+2*ln(x)),x)
[Out] Integral(x**2*coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int x^2 \coth(a + 2\log(x))\,dx = \frac{1}{3}\,x^3 + \arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{xe^a - e^{\left(\frac{1}{2}\,a\right)}}{xe^a + e^{\left(\frac{1}{2}\,a\right)}}\right)$$

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="maxima")
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log((x*e^a - e^(1
/2*a))/(x*e^a + e^(1/2*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int x^2 \coth(a+2\log(x))\,dx = \frac{1}{3}\,x^3 + \arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{\left|2\,xe^a - 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^a + 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right)$$

```
[In] integrate(x^2*coth(a+2*log(x)),x, algorithm="giac")
[Out] 1/3*x^3 + arctan(x*e^(1/2*a))*e^(-3/2*a) + 1/2*e^(-3/2*a)*log(abs(2*x*e^a -
 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int x^2 \coth(a + 2\log(x))\,dx = \frac{\operatorname{atan}\left(x\,(e^{2a})^{1/4}\right)}{(e^{2a})^{3/4}} - \frac{\operatorname{atanh}\left(x\,(e^{2a})^{1/4}\right)}{(e^{2a})^{3/4}} + \frac{x^3}{3}$$

```
[In] int(x^2*coth(a + 2*log(x)),x)
```

```
[Out] atan(x*exp(2*a)^(1/4))/exp(2*a)^(3/4) - atanh(x*exp(2*a)^(1/4))/exp(2*a)^(3
/4) + x^3/3
```

3.153 $\int x \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \coth(a+2\log(x))\,dx=\frac{x^2}{2}-e^{-a}\text{arctanh}\left(e^a x^2\right)$$

```
[Out] 1/2*x^2-arctanh(exp(a)*x^2)/exp(a)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5657, 470, 281, 212}

$$\int x \coth(a+2\log(x))\,dx=\frac{x^2}{2}-e^{-a}\text{arctanh}\left(e^a x^2\right)$$

```
[In] Int[x*Coth[a + 2*Log[x]],x]
```

```
[Out] x^2/2 - ArcTanh[E^a*x^2]/E^a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(-1-e^{2a}x^4)}{1-e^{2a}x^4}\,dx \\ &= \frac{x^2}{2} - 2\int \frac{x}{1-e^{2a}x^4}\,dx \\ &= \frac{x^2}{2} - \text{Subst}\left(\int \frac{1}{1-e^{2a}x^2}\,dx, x, x^2\right) \\ &= \frac{x^2}{2} - e^{-a}\text{arctanh}\left(e^a x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x\coth(a+2\log(x))\,dx = \frac{x^2}{2} + \text{arctanh}\left(x^2(\cosh(a)+\sinh(a))\right)(-\cosh(a)+\sinh(a))$$

```
[In] Integrate[x*Coth[a + 2*Log[x]],x]
[Out] x^2/2 + ArcTanh[x^2*(Cosh[a] + Sinh[a])]*(-Cosh[a] + Sinh[a])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

method	result	size
risch	$\frac{x^2}{2}+\frac{e^{-a}\ln(e^a x^2-1)}{2}-\frac{e^{-a}\ln(e^a x^2+1)}{2}$	37

```
[In] int(x*coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2+1/2/exp(a)*ln(exp(a)*x^2-1)-1/2/exp(a)*ln(exp(a)*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int x\coth(a+2\log(x))\,dx=\frac{1}{2}\left(x^2e^a-\log\left(x^2e^a+1\right)+\log\left(x^2e^a-1\right)\right)e^{(-a)}$$

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="fricas")
[Out] 1/2*(x^2*e^a - log(x^2*e^a + 1) + log(x^2*e^a - 1))*e^(-a)
```

Sympy [F]

$$\int x\coth(a+2\log(x))\,dx=\int x\coth\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(x*coth(a+2*ln(x)),x)
[Out] Integral(x*coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int x\coth(a+2\log(x))\,dx=\frac{1}{2}x^2-\frac{1}{2}e^{(-a)}\log\left(x^2e^a+1\right)+\frac{1}{2}e^{(-a)}\log\left(x^2e^a-1\right)$$

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int x\coth(a+2\log(x))\,dx = \frac{1}{2}\,x^2 - \frac{1}{2}\,e^{(-a)}\log\left(x^2e^a+1\right) + \frac{1}{2}\,e^{(-a)}\log\left(\left|x^2e^a-1\right|\right)$$

```
[In] integrate(x*coth(a+2*log(x)),x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1))
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\coth(a+2\log(x))\,dx = \frac{x^2}{2} - \frac{\operatorname{atanh}\left(x^2\sqrt{\mathrm{e}^{2a}}\right)}{\sqrt{\mathrm{e}^{2a}}}$$

```
[In] int(x*coth(a + 2*log(x)),x)
[Out] x^2/2 - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

3.154 $\int \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 40

$$\int \coth(a+2\log(x))\,dx = x - e^{-a/2}\arctan\left(e^{a/2}x\right) - e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)$$

```
[Out] x-arctan(exp(1/2*a)*x)/exp(1/2*a)-arctanh(exp(1/2*a)*x)/exp(1/2*a)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5653, 396, 218, 212, 209}

$$\int \coth(a+2\log(x))\,dx = -e^{-a/2}\arctan\left(e^{a/2}x\right) - e^{-a/2}\text{arctanh}\left(e^{a/2}x\right) + x$$

```
[In] Int[Coth[a + 2*Log[x]],x]
```

```
[Out] x - ArcTan[E^(a/2)*x]/E^(a/2) - ArcTanh[E^(a/2)*x]/E^(a/2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{-1-e^{2a}x^4}{1-e^{2a}x^4}\,dx \\ &= x-2\int \frac{1}{1-e^{2a}x^4}\,dx \\ &= x-\int \frac{1}{1-e^ax^2}\,dx-\int \frac{1}{1+e^ax^2}\,dx \\ &= x-e^{-a/2}\arctan\left(e^{a/2}x\right)-e^{-a/2}\text{arctanh}\left(e^{a/2}x\right) \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \coth(a+2\log(x))\,dx = x+\frac{1}{2}\text{RootSum}\left[-\cosh(a)+\sinh(a)+\cosh(a)\#1^4+\sinh(a)\#1^4\&,\frac{\log(x)-\log(x-\#1)}{\#1^3}\&\right](-\cosh(2a)+\sinh(2a))$$

```
[In] Integrate[Coth[a + 2*Log[x]],x]
[Out] x + (RootSum[-Cosh[a] + Sinh[a] + Cosh[a]*#1^4 + Sinh[a]*#1^4 & , (Log[x] -
 Log[x - #1])/#1^3 & ]*(-Cosh[2*a] + Sinh[2*a]))/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(32) = 64.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.78

method	result	size
risch	$x-\frac{\ln\left(x\sqrt{-\mathrm{e}^{a}}+1\right)}{2\sqrt{-\mathrm{e}^{a}}}+\frac{\ln\left(x\sqrt{-\mathrm{e}^{a}}-1\right)}{2\sqrt{-\mathrm{e}^{a}}}+\frac{\ln\left(\sqrt{\mathrm{e}^{a}}\,x-1\right)}{2\sqrt{\mathrm{e}^{a}}}-\frac{\ln\left(\sqrt{\mathrm{e}^{a}}\,x+1\right)}{2\sqrt{\mathrm{e}^{a}}}$	71

```
[In] int(coth(a+2*ln(x)),x,method=_RETURNVERBOSE)
[Out] x-1/2/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/2/(-exp(a))^(1/2)*ln(x*(-ex
p(a))^(1/2)-1)+1/2/exp(a)^(1/2)*ln(exp(a)^(1/2)*x-1)-1/2/exp(a)^(1/2)*ln(ex
p(a)^(1/2)*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \coth(a+2\log(x))\,dx$$
$$=-\frac{1}{2}\left(2\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)}-2\,xe^{a}-e^{\left(\frac{1}{2}a\right)}\log\left(\frac{x^{2}e^{a}-2\,xe^{\left(\frac{1}{2}a\right)}+1}{x^{2}e^{a}-1}\right)\right)e^{(-a)}$$

```
[In] integrate(coth(a+2*log(x)),x, algorithm="fricas")
[Out] -1/2*(2*arctan(x*e^(1/2*a))*e^(1/2*a) - 2*x*e^a - e^(1/2*a)*log((x^2*e^a -
2*x*e^(1/2*a) + 1)/(x^2*e^a - 1)))*e^(-a)
```

Sympy [F]

$$\int \coth(a+2\log(x))\,dx=\int \coth\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(coth(a+2*ln(x)),x)
[Out] Integral(coth(a + 2*log(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \coth(a+2\log(x))\,dx = -\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{1}{2}\,a\right)}\log\left(\frac{xe^{a} - e^{\left(\frac{1}{2}\,a\right)}}{xe^{a} + e^{\left(\frac{1}{2}\,a\right)}}\right) + x$$

```
[In] integrate(coth(a+2*log(x)),x, algorithm="maxima")
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log((x*e^a - e^(1/2*a))/(x
*e^a + e^(1/2*a))) + x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \coth(a+2\log(x))\,dx = -\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{1}{2}\,a\right)} + \frac{1}{2}\,e^{\left(-\frac{1}{2}\,a\right)}\log\left(\frac{\left|2\,xe^{a} - 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^{a} + 2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right) + x$$

```
[In] integrate(coth(a+2*log(x)),x, algorithm="giac")
[Out] -arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/2*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^(1/2
*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \coth(a+2\log(x))\,dx = x - \frac{\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{{\left(\mathrm{e}^{2\,a}\right)}^{1/4}} - \frac{\mathrm{atanh}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{{\left(\mathrm{e}^{2\,a}\right)}^{1/4}}$$

```
[In] int(coth(a + 2*log(x)),x)
[Out] x - atan(x*exp(2*a)^(1/4))/exp(2*a)^(1/4) - atanh(x*exp(2*a)^(1/4))/exp(2*a
)^(1/4)
```

3.155 $\int \frac{\coth(a+2\log(x))}{x}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\coth(a+2\log(x))}{x}\,dx = \frac{1}{2}\log(\sinh(a+2\log(x)))$$

```
[Out] 1/2*ln(sinh(a+2*ln(x)))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3556}

$$\int \frac{\coth(a+2\log(x))}{x}\,dx = \frac{1}{2}\log(\sinh(a+2\log(x)))$$

```
[In] Int[Coth[a + 2*Log[x]]/x,x]
```

```
[Out] Log[Sinh[a + 2*Log[x]]]/2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \coth(a+2x)\,dx, x, \log(x)\right) \\ &= \frac{1}{2}\log(\sinh(a+2\log(x))) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{\coth(a+2\log(x))}{x}\,dx = \frac{1}{2}(\log(\cosh(a+2\log(x))) + \log(\tanh(a+2\log(x))))$$

```
[In] Integrate[Coth[a + 2*Log[x]]/x,x]
[Out] (Log[Cosh[a + 2*Log[x]]] + Log[Tanh[a + 2*Log[x]]])/2
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln(\sinh(a+2\ln(x)))}{2}$	11
default	$\frac{\ln(\sinh(a+2\ln(x)))}{2}$	11
risch	$-\ln\left(x\right)+\frac{\ln\left(1-e^{2a}x^{4}\right)}{2}$	20
parallelrisch	$-\ln\left(x\right)+\ln\left(\sqrt{\tanh\left(a+2\ln\left(x\right)\right)}\right)+\ln\left(\frac{1}{\sqrt{1-\tanh(a+2\ln(x))}}\right)$	30

```
[In] int(coth(a+2*ln(x))/x,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(sinh(a+2*ln(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\coth(a+2\log(x))}{x}\,dx = \frac{1}{2}\log\left(x^{4}e^{(2a)}-1\right) - \log\left(x\right)$$

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="fricas")
[Out] 1/2*log(x^4*e^(2*a) - 1) - log(x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{\coth(a + 2\log(x))}{x}\,dx = \log(x) - \frac{\log(\tanh(a + 2\log(x)) + 1)}{2} + \frac{\log(\tanh(a + 2\log(x)))}{2}$$

```
[In] integrate(coth(a+2*ln(x))/x,x)
[Out] log(x) - log(tanh(a + 2*log(x)) + 1)/2 + log(tanh(a + 2*log(x)))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\coth(a + 2\log(x))}{x}\,dx = \frac{1}{2}\log(\sinh(a + 2\log(x)))$$

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="maxima")
[Out] 1/2*log(sinh(a + 2*log(x)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{\coth(a + 2\log(x))}{x}\,dx = -\frac{1}{4}\log(x^4) + \frac{1}{2}\log\left(\left|x^4e^{(2a)} - 1\right|\right)$$

```
[In] integrate(coth(a+2*log(x))/x,x, algorithm="giac")
[Out] -1/4*log(x^4) + 1/2*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\coth(a + 2\log(x))}{x} \, dx = \frac{\ln\left(x^4 - e^{-2\,a}\right)}{2} - \ln\left(x\right)$$

```
[In] int(coth(a + 2*log(x))/x,x)
[Out] log(x^4 - exp(-2*a))/2 - log(x)
```

3.156 $\int \frac{\coth(a+2\log(x))}{x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \frac{1}{x} + e^{a/2}\arctan\left(e^{a/2}x\right) - e^{a/2}\text{arctanh}\left(e^{a/2}x\right)$$

```
[Out] 1/x+exp(1/2*a)*arctan(exp(1/2*a)*x)-exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5657, 464, 304, 209, 212}

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = e^{a/2}\arctan\left(e^{a/2}x\right) - e^{a/2}\text{arctanh}\left(e^{a/2}x\right) + \frac{1}{x}$$

```
[In] Int[Coth[a + 2*Log[x]]/x^2,x]
```

```
[Out] x^(-1) + E^(a/2)*ArcTan[E^(a/2)*x] - E^(a/2)*ArcTanh[E^(a/2)*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{-1-e^{2a}x^4}{x^2\left(1-e^{2a}x^4\right)}\,dx \\
&= \frac{1}{x}-\left(2e^{2a}\right)\int \frac{x^2}{1-e^{2a}x^4}\,dx \\
&= \frac{1}{x}-e^a\int \frac{1}{1-e^ax^2}\,dx+e^a\int \frac{1}{1+e^ax^2}\,dx \\
&= \frac{1}{x}+e^{a/2}\arctan\left(e^{a/2}x\right)-e^{a/2}\text{arctanh}\left(e^{a/2}x\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx$$

$$= \frac{2+x\text{RootSum}\left[-\cosh(a)-\sinh(a)+\cosh(a)\#1^4-\sinh(a)\#1^4\&,\frac{\log(x)+\log\left(\frac{1}{x}-\#1\right)}{\#1^3}\&\right](\cosh(a)+\sinh(a)}{2x}$$

```
[In] Integrate[Coth[a + 2*Log[x]]/x^2,x]
[Out] (2 + x*RootSum[-Cosh[a] - Sinh[a] + Cosh[a]*#1^4 - Sinh[a]*#1^4 & , (Log[x]
 + Log[x^(-1) - #1])/#1^3 & ]*(Cosh[a] + Sinh[a])^2)/(2*x)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

method	result
risch	$\frac{1}{x}+\frac{\sqrt{-\mathrm{e}^{a}}\,\ln\left(\left(-\mathrm{e}^{a}\right)^{\frac{3}{2}}-\mathrm{e}^{2a}x\right)}{2}-\frac{\sqrt{-\mathrm{e}^{a}}\,\ln\left(-\left(-\mathrm{e}^{a}\right)^{\frac{3}{2}}-\mathrm{e}^{2a}x\right)}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{2}-\mathrm{e}^{a}\right)}\_R\ln\left(\left(-5\_R^{4}+4\,\mathrm{e}^{2a}\right)x+\_R^{3}\right)\right.}{2}$

```
[In] int(coth(a+2*ln(x))/x^2,x,method=_RETURNVERBOSE)
[Out] 1/x+1/2*(-exp(a))^(1/2)*ln((-exp(a))^(3/2)-exp(2*a)*x)-1/2*(-exp(a))^(1/2)*
ln(-(-exp(a))^(3/2)-exp(2*a)*x)+1/2*sum(_R*ln((-5*_R^4+4*exp(2*a))*x+_R^3),
_R=RootOf(_Z^2-exp(a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx=\frac{2\,x\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+xe^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)+2}{2\,x}$$

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="fricas")
[Out] 1/2*(2*x*arctan(x*e^(1/2*a))*e^(1/2*a) + x*e^(1/2*a)*log((x^2*e^a - 2*x*e^(
1/2*a) + 1)/(x^2*e^a - 1)) + 2)/x
```

Sympy [F]

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx=\int \frac{\coth\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(coth(a+2*ln(x))/x**2,x)
[Out] Integral(coth(a + 2*log(x))/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = -\arctan\left(\frac{e^{\left(-\frac{1}{2}\,a\right)}}{x}\right)e^{\left(\frac{1}{2}\,a\right)} + \frac{1}{2}\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{\frac{1}{x}-e^{\left(\frac{1}{2}\,a\right)}}{\frac{1}{x}+e^{\left(\frac{1}{2}\,a\right)}}\right) + \frac{1}{x}$$

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-1/2*a)/x)*e^(1/2*a) + 1/2*e^(1/2*a)*log((1/x - e^(1/2*a))/(1/x
+ e^(1/2*a))) + 1/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} + \frac{1}{2}\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right) + \frac{1}{x}$$

```
[In] integrate(coth(a+2*log(x))/x^2,x, algorithm="giac")
```

```
[Out] arctan(x*e^(1/2*a))*e^(1/2*a) + 1/2*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a)
)/abs(2*x*e^a + 2*e^(1/2*a))) + 1/x
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\coth(a+2\log(x))}{x^2}\,dx = \left(\mathrm{e}^{2\,a}\right)^{1/4}\mathrm{atan}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right) - \left(\mathrm{e}^{2\,a}\right)^{1/4}\mathrm{atanh}\left(x\,\left(\mathrm{e}^{2\,a}\right)^{1/4}\right) + \frac{1}{x}$$

```
[In] int(coth(a + 2*log(x))/x^2,x)
```

```
[Out] exp(2*a)^(1/4)*atan(x*exp(2*a)^(1/4)) - exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1/
4)) + 1/x
```

3.157 $\int \frac{\coth(a+2\log(x))}{x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\coth(a+2\log(x))}{x^3} \, dx = \frac{1}{2x^2} - e^a \text{arctanh}\left(e^a x^2\right)$$

```
[Out] 1/2/x^2-exp(a)*arctanh(exp(a)*x^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5657, 464, 281, 212}

$$\int \frac{\coth(a+2\log(x))}{x^3} \, dx = \frac{1}{2x^2} - e^a \text{arctanh}\left(e^a x^2\right)$$

```
[In] Int[Coth[a + 2*Log[x]]/x^3,x]
```

```
[Out] 1/(2*x^2) - E^a*ArcTanh[E^a*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{-1-e^{2a}x^4}{x^3\left(1-e^{2a}x^4\right)}\,dx \\
&= \frac{1}{2x^2}-\left(2e^{2a}\right)\int \frac{x}{1-e^{2a}x^4}\,dx \\
&= \frac{1}{2x^2}-e^{2a}\text{Subst}\left(\int \frac{1}{1-e^{2a}x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2x^2}-e^{a}\text{arctanh}\left(e^{a}x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx=\frac{1}{2x^2}-\text{arctanh}\left(\frac{\cosh(a)-\sinh(a)}{x^2}\right)(\cosh(a)+\sinh(a))$$

```
[In] Integrate[Coth[a + 2*Log[x]]/x^3,x]
[Out] 1/(2*x^2) - ArcTanh[(Cosh[a] - Sinh[a])/x^2]*(Cosh[a] + Sinh[a])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

method	result	size
risch	$\frac{1}{2x^{2}}+\frac{\mathrm{e}^{a} \ln\left(-\mathrm{e}^{a} x^{2}+1\right)}{2}-\frac{\mathrm{e}^{a} \ln\left(-\mathrm{e}^{a} x^{2}-1\right)}{2}$	35

```
[In] int(coth(a+2*ln(x))/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2/x^2+1/2*exp(a)*ln(-exp(a)*x^2+1)-1/2*exp(a)*ln(-exp(a)*x^2-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = -\frac{x^2e^a\log\left(x^2e^a+1\right)-x^2e^a\log\left(x^2e^a-1\right)-1}{2\,x^2}$$

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="fricas")
[Out] -1/2*(x^2*e^a*log(x^2*e^a + 1) - x^2*e^a*log(x^2*e^a - 1) - 1)/x^2
```

Sympy [F]

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \int \frac{\coth\left(a+2\log\left(x\right)\right)}{x^3}\,dx$$

```
[In] integrate(coth(a+2*ln(x))/x**3,x)
[Out] Integral(coth(a + 2*log(x))/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = -\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}+e^a\right)+\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}-e^a\right)+\frac{1}{2\,x^2}$$

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="maxima")
[Out] -1/2*e^a*log(1/x^2 + e^a) + 1/2*e^a*log(1/x^2 - e^a) + 1/2/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = -\frac{1}{2}\,e^a\log\left(x^2e^a+1\right)+\frac{1}{2}\,e^a\log\left(\left|x^2e^a-1\right|\right)+\frac{1}{2\,x^2}$$

```
[In] integrate(coth(a+2*log(x))/x^3,x, algorithm="giac")
[Out] -1/2*e^a*log(x^2*e^a + 1) + 1/2*e^a*log(abs(x^2*e^a - 1)) + 1/2/x^2
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\coth(a+2\log(x))}{x^3}\,dx = \frac{1}{2\,x^2}-\mathrm{atanh}\left(x^2\,\sqrt{\mathrm{e}^{2\,a}}\right)\,\sqrt{\mathrm{e}^{2\,a}}$$

```
[In] int(coth(a + 2*log(x))/x^3,x)
[Out] 1/(2*x^2) - atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2)
```

3.158 $\int x^3 \coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int x^3 \coth^2(a+2\log(x))\,dx=\frac{x^4}{4}+\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)$$

```
[Out] 1/4*x^4+1/exp(2*a)/(1-exp(2*a)*x^4)+ln(1-exp(2*a)*x^4)/exp(2*a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5657, 455, 45}

$$\int x^3 \coth^2(a+2\log(x))\,dx=\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)+\frac{x^4}{4}$$

```
[In] Int[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 + 1/(E^(2*a)*(1 - E^(2*a)*x^4)) + Log[1 - E^(2*a)*x^4]/E^(2*a)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x^3(-1-e^{2a}x^4)^2}{(1-e^{2a}x^4)^2}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{(-1-e^{2a}x)^2}{(1-e^{2a}x)^2}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(1+\frac{4}{(-1+e^{2a}x)^2}+\frac{4}{-1+e^{2a}x}\right)\,dx, x, x^4\right) \\
&= \frac{x^4}{4}+\frac{e^{-2a}}{1-e^{2a}x^4}+e^{-2a}\log\left(1-e^{2a}x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.83

$$\begin{aligned}
\int x^3\coth^2(a+2\log(x))\,dx &= \frac{x^4}{4}+\cosh(2a)\log\left(\left(-1+x^4\right)\cosh(a)+\left(1+x^4\right)\sinh(a)\right) \\
&\quad -\log\left(\left(-1+x^4\right)\cosh(a)+\left(1+x^4\right)\sinh(a)\right)\sinh(2a) \\
&\quad +\frac{-\cosh(3a)+\sinh(3a)}{(-1+x^4)\cosh(a)+(1+x^4)\sinh(a)}
\end{aligned}$$

```
[In] Integrate[x^3*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^4/4 + Cosh[2*a]*Log[(-1 + x^4)*Cosh[a] + (1 + x^4)*Sinh[a]] - Log[(-1 + x
^4)*Cosh[a] + (1 + x^4)*Sinh[a]]*Sinh[2*a] + (-Cosh[3*a] + Sinh[3*a])/((-1
+ x^4)*Cosh[a] + (1 + x^4)*Sinh[a])
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
risch	$\frac{x^4}{4} - \frac{e^{-2a}}{-1+e^{2a}x^4} + e^{-2a} \ln\left(-1+e^{2a}x^4\right)$	41

```
[In] int(x^3*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x^4-exp(-2*a)/(-1+exp(2*a)*x^4)+exp(-2*a)*ln(-1+exp(2*a)*x^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int x^3 \coth^2(a+2\log(x))\,dx = \frac{x^8e^{(4\,a)} - x^4e^{(2\,a)} + 4\left(x^4e^{(2\,a)} - 1\right)\log\left(x^4e^{(2\,a)} - 1\right) - 4}{4\left(x^4e^{(4\,a)} - e^{(2\,a)}\right)}$$

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="fricas")
[Out] 1/4*(x^8*e^(4*a) - x^4*e^(2*a) + 4*(x^4*e^(2*a) - 1)*log(x^4*e^(2*a) - 1) -
 4)/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F]

$$\int x^3 \coth^2(a+2\log(x))\,dx = \int x^3 \coth^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(x**3*coth(a+2*ln(x))**2,x)
[Out] Integral(x**3*coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int x^3 \coth^2(a+2\log(x))\,dx = \frac{1}{4}x^4 + e^{(-2\,a)}\log\left(x^2e^a + 1\right) + e^{(-2\,a)}\log\left(x^2e^a - 1\right) - \frac{1}{x^4e^{(4\,a)} - e^{(2\,a)}}$$

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="maxima")
[Out] 1/4*x^4 + e^(-2*a)*log(x^2*e^a + 1) + e^(-2*a)*log(x^2*e^a - 1) - 1/(x^4*e^
(4*a) - e^(2*a))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^3 \coth^2(a + 2\log(x))\,dx = \frac{1}{4}\,x^4 - \frac{x^4}{x^4 e^{(2\,a)} - 1} + e^{(-2\,a)} \log\left(\left|x^4 e^{(2\,a)} - 1\right|\right)$$

```
[In] integrate(x^3*coth(a+2*log(x))^2,x, algorithm="giac")
[Out] 1/4*x^4 - x^4/(x^4*e^(2*a) - 1) + e^(-2*a)*log(abs(x^4*e^(2*a) - 1))
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int x^3 \coth^2(a + 2\log(x))\,dx = \ln\left(x^4 - \mathrm{e}^{-2\,a}\right)\,\mathrm{e}^{-2\,a} - \frac{\mathrm{e}^{-2\,a}}{x^4\,\mathrm{e}^{2\,a} - 1} + \frac{x^4}{4}$$

```
[In] int(x^3*coth(a + 2*log(x))^2,x)
[Out] log(x^4 - exp(-2*a))*exp(-2*a) - exp(-2*a)/(x^4*exp(2*a) - 1) + x^4/4
```

3.159 $\int x^2 \coth^2(a + 2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int x^2 \coth^2(a + 2\log(x))\,dx = \frac{x^3}{3} + \frac{x^3}{1 - e^{2a}x^4} + \frac{3}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right) - \frac{3}{2}e^{-3a/2}\operatorname{arctanh}\left(e^{a/2}x\right)$$

```
[Out] 1/3*x^3+x^3/(1-exp(2*a)*x^4)+3/2*arctan(exp(1/2*a)*x)/exp(3/2*a)-3/2*arctan
h(exp(1/2*a)*x)/exp(3/2*a)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5657, 474, 470, 304, 209, 212}

$$\int x^2 \coth^2(a + 2\log(x))\,dx = \frac{3}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right) - \frac{3}{2}e^{-3a/2}\operatorname{arctanh}\left(e^{a/2}x\right) + \frac{x^3}{1 - e^{2a}x^4} + \frac{x^3}{3}$$

```
[In] Int[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^3/3 + x^3/(1 - E^(2*a)*x^4) + (3*ArcTan[E^(a/2)*x])/(2*E^((3*a)/2)) - (3*
ArcTanh[E^(a/2)*x])/(2*E^((3*a)/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x^2(-1-e^{2a}x^4)^2}{(1-e^{2a}x^4)^2}\,dx\\ &= \frac{x^3}{1-e^{2a}x^4}-\frac{1}{4}e^{-4a}\int \frac{x^2(8e^{4a}+4e^{6a}x^4)}{1-e^{2a}x^4}\,dx\end{aligned}$$

$$= \frac{x^3}{3} + \frac{x^3}{1 - e^{2a}x^4} - 3\int \frac{x^2}{1 - e^{2a}x^4}\,dx$$

$$= \frac{x^3}{3} + \frac{x^3}{1 - e^{2a}x^4} - \frac{1}{2}(3e^{-a})\int \frac{1}{1 - e^a x^2}\,dx + \frac{1}{2}(3e^{-a})\int \frac{1}{1 + e^a x^2}\,dx$$

$$= \frac{x^3}{3} + \frac{x^3}{1 - e^{2a}x^4} + \frac{3}{2}e^{-3a/2}\arctan\left(e^{a/2}x\right) - \frac{3}{2}e^{-3a/2}\text{arctanh}\left(e^{a/2}x\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.26

$$\int x^2 \coth^2(a + 2\log(x))\,dx$$
$$= \frac{e^{-4a}\left(-9317 - 17825e^{2a}x^4 - 4787e^{4a}x^8 + 1481e^{6a}x^{12} + 7(1331 + 1976e^{2a}x^4 - 398e^{4a}x^8 - 632e^{6a}x^{12} + 27\right.}{2688x^5}$$
$$+ \frac{16e^{2a}x^7(1 + e^{2a}x^4)^2\ {}_4F_3\left(\frac{7}{4}, 2, 2, 2; 1, 1, \frac{19}{4}; e^{2a}x^4\right)}{1155}$$

```
[In] Integrate[x^2*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-9317 - 17825*E^(2*a)*x^4 - 4787*E^(4*a)*x^8 + 1481*E^(6*a)*x^12 + 7*(1331
 + 1976*E^(2*a)*x^4 - 398*E^(4*a)*x^8 - 632*E^(6*a)*x^12 + 27*E^(8*a)*x^16)
*Hypergeometric2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(2688*E^(4*a)*x^5) + (16*E^(2
*a)*x^7*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{7/4, 2, 2, 2}, {1, 1, 19/4},
 E^(2*a)*x^4])/1155
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{x^3}{3} - \frac{x^3}{-1+e^{2a}x^4} + \frac{3\ln\left(-\sqrt{e^a}\,x+1\right)}{4(e^a)^{\frac{3}{2}}} - \frac{3\ln\left(\sqrt{e^a}\,x+1\right)}{4(e^a)^{\frac{3}{2}}} + \frac{3\ln\left((-e^a)^{\frac{3}{2}}-e^{2a}x\right)}{4(-e^a)^{\frac{3}{2}}} - \frac{3\ln\left((-e^a)^{\frac{3}{2}}+e^{2a}x\right)}{4(-e^a)^{\frac{3}{2}}}$	100

```
[In] int(x^2*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3-x^3/(-1+exp(2*a)*x^4)+3/4/exp(a)^(3/2)*ln(-exp(a)^(1/2)*x+1)-3/4/ex
p(a)^(3/2)*ln(exp(a)^(1/2)*x+1)+3/4/(-exp(a))^(3/2)*ln((-exp(a))^(3/2)-exp(
2*a)*x)-3/4/(-exp(a))^(3/2)*ln((-exp(a))^(3/2)+exp(2*a)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.53

$$\int x^2 \coth^2(a + 2\log(x))\,dx$$
$$= \frac{4\,x^7 e^{(4\,a)} - 16\,x^3 e^{(2\,a)} + 18\,\left(x^4 e^{(2\,a)} - 1\right)\arctan\left(x e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} + 9\,\left(x^4 e^{(2\,a)} - 1\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2 e^a - 2\,x e^{\left(\frac{1}{2}\,a\right)} + 1}{x^2 e^a - 1}\right)}{12\,\left(x^4 e^{(4\,a)} - e^{(2\,a)}\right)}$$

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="fricas")
[Out] 1/12*(4*x^7*e^(4*a) - 16*x^3*e^(2*a) + 18*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2
*a))*e^(1/2*a) + 9*(x^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a)
 + 1)/(x^2*e^a - 1)))/(x^4*e^(4*a) - e^(2*a))
```

Sympy [F]

$$\int x^2 \coth^2(a + 2\log(x))\,dx = \int x^2 \coth^2\left(a + 2\log\left(x\right)\right)dx$$

```
[In] integrate(x**2*coth(a+2*ln(x))**2,x)
[Out] Integral(x**2*coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int x^2 \coth^2(a + 2\log(x))\,dx = \frac{1}{3}\,x^3 - \frac{x^3}{x^4 e^{(2\,a)} - 1} + \frac{3}{2}\,\arctan\left(x e^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{3}{4}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{x e^a - e^{\left(\frac{1}{2}\,a\right)}}{x e^a + e^{\left(\frac{1}{2}\,a\right)}}\right)$$

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="maxima")
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log((x*e^a - e^(1/2*a))/(x*e^a + e^(1/2*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int x^2\coth^2(a+2\log(x))\,dx = \frac{1}{3}\,x^3 - \frac{x^3}{x^4e^{(2\,a)}-1} + \frac{3}{2}\,\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(-\frac{3}{2}\,a\right)} + \frac{3}{4}\,e^{\left(-\frac{3}{2}\,a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}\,a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}\,a\right)}\right|}\right)$$

```
[In] integrate(x^2*coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - x^3/(x^4*e^(2*a) - 1) + 3/2*arctan(x*e^(1/2*a))*e^(-3/2*a) + 3/4*
e^(-3/2*a)*log(abs(2*x*e^a - 2*e^(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a)))
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int x^2\coth^2(a+2\log(x))\,dx = \frac{3\,\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\right)}{2\,{\left(\mathrm{e}^{2\,a}\right)}^{3/4}} - \frac{x^3}{x^4\,\mathrm{e}^{2\,a}-1} + \frac{x^3}{3} + \frac{\mathrm{atan}\left(x\,{\left(\mathrm{e}^{2\,a}\right)}^{1/4}\,1\mathrm{i}\right)\,3\mathrm{i}}{2\,{\left(\mathrm{e}^{2\,a}\right)}^{3/4}}$$

```
[In] int(x^2*coth(a + 2*log(x))^2,x)
```

```
[Out] (3*atan(x*exp(2*a)^(1/4)))/(2*exp(2*a)^(3/4)) - x^3/(x^4*exp(2*a) - 1) + (a
tan(x*exp(2*a)^(1/4)*1i)*3i)/(2*exp(2*a)^(3/4)) + x^3/3
```

3.160 $\int x\coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int x\coth^2(a+2\log(x))\,dx=\frac{x^2}{2}+\frac{x^2}{1-e^{2a}x^4}-e^{-a}\text{arctanh}\left(e^ax^2\right)$$

```
[Out] 1/2*x^2+x^2/(1-exp(2*a)*x^4)-arctanh(exp(a)*x^2)/exp(a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5657, 474, 470, 281, 212}

$$\int x\coth^2(a+2\log(x))\,dx=-e^{-a}\text{arctanh}\left(e^ax^2\right)+\frac{x^2}{1-e^{2a}x^4}+\frac{x^2}{2}$$

```
[In] Int[x*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x^2/2 + x^2/(1 - E^(2*a)*x^4) - ArcTanh[E^a*x^2]/E^a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{x\left(-1-e^{2a}x^4\right)^2}{\left(1-e^{2a}x^4\right)^2}\,dx\\
&= \frac{x^2}{1-e^{2a}x^4}-\frac{1}{4}e^{-4a}\int \frac{x\left(4e^{4a}+4e^{6a}x^4\right)}{1-e^{2a}x^4}\,dx\\
&= \frac{x^2}{2}+\frac{x^2}{1-e^{2a}x^4}-2\int \frac{x}{1-e^{2a}x^4}\,dx\\
&= \frac{x^2}{2}+\frac{x^2}{1-e^{2a}x^4}-\text{Subst}\left(\int \frac{1}{1-e^{2a}x^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}+\frac{x^2}{1-e^{2a}x^4}-e^{-a}\text{arctanh}\left(e^a x^2\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.67 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.98

$$\int x\coth^2(a+2\log(x))\,dx = \frac{e^{-4a}\left(-375-713e^{2a}x^4-181e^{4a}x^8+61e^{6a}x^{12}+\frac{3(125+196e^{2a}x^4-14e^{4a}x^8-52e^{6a}x^{12}+e^{8a}x^{16})\operatorname{arctanh}\left(\sqrt{e^{2a}x^4}\right)}{\sqrt{e^{2a}x^4}}\right)}{96x^6} + \frac{2}{105}e^{2a}x^6\left(1+e^{2a}x^4\right)^2\,{}_4F_3\left(\frac{3}{2},2,2,2;1,1,\frac{9}{2};e^{2a}x^4\right)$$

```
[In] Integrate[x*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-375 - 713*E^(2*a)*x^4 - 181*E^(4*a)*x^8 + 61*E^(6*a)*x^12 + (3*(125 + 196
*E^(2*a)*x^4 - 14*E^(4*a)*x^8 - 52*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqr
t[E^(2*a)*x^4]])/Sqrt[E^(2*a)*x^4])/(96*E^(4*a)*x^6) + (2*E^(2*a)*x^6*(1 +
E^(2*a)*x^4)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, E^(2*a)*x^4])
/105
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{x^2}{2}-\frac{x^2}{-1+e^{2a}x^4}+\frac{e^{-a}\ln(e^a x^2-1)}{2}-\frac{e^{-a}\ln(e^a x^2+1)}{2}$	54

```
[In] int(x*coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-x^2/(exp(a)^2*x^4-1)+1/2/exp(a)*ln(exp(a)*x^2-1)-1/2/exp(a)*ln(exp(
a)*x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(36) = 72.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int x\coth^2(a+2\log(x))\,dx = \frac{x^6e^{(3a)}-3x^2e^a-\left(x^4e^{(2a)}-1\right)\log\left(x^2e^a+1\right)+\left(x^4e^{(2a)}-1\right)\log\left(x^2e^a-1\right)}{2\left(x^4e^{(3a)}-e^a\right)}$$

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^6*e^(3*a) - 3*x^2*e^a - (x^4*e^(2*a) - 1)*log(x^2*e^a + 1) + (x^4*e^
(2*a) - 1)*log(x^2*e^a - 1))/(x^4*e^(3*a) - e^a)
```

Sympy [F]

$$\int x\coth^2(a+2\log(x))\,dx=\int x\coth^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(x*coth(a+2*ln(x))**2,x)
[Out] Integral(x*coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int x\coth^2(a+2\log(x))\,dx=\frac{1}{2}x^2-\frac{1}{2}e^{(-a)}\log\left(x^2e^a+1\right)+\frac{1}{2}e^{(-a)}\log\left(x^2e^a-1\right)-\frac{x^2}{x^4e^{(2a)}-1}$$

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="maxima")
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(x^2*e^a - 1) - x^2/(
x^4*e^(2*a) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int x\coth^2(a+2\log(x))\,dx=\frac{1}{2}x^2-\frac{1}{2}e^{(-a)}\log\left(x^2e^a+1\right)+\frac{1}{2}e^{(-a)}\log\left(\left|x^2e^a-1\right|\right)-\frac{x^2}{x^4e^{(2a)}-1}$$

```
[In] integrate(x*coth(a+2*log(x))^2,x, algorithm="giac")
[Out] 1/2*x^2 - 1/2*e^(-a)*log(x^2*e^a + 1) + 1/2*e^(-a)*log(abs(x^2*e^a - 1)) -
x^2/(x^4*e^(2*a) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int x \coth^2(a+2\log(x))\, dx = \frac{x^2}{2} - \frac{x^2}{x^4\, e^{2\,a} - 1} - \frac{\operatorname{atanh}\left(x^2\,\sqrt{e^{2\,a}}\right)}{\sqrt{e^{2\,a}}}$$

```
[In] int(x*coth(a + 2*log(x))^2,x)
[Out] x^2/2 - x^2/(x^4*exp(2*a) - 1) - atanh(x^2*exp(2*a)^(1/2))/exp(2*a)^(1/2)
```

3.161 $\int \coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 60

$$\int \coth^2(a+2\log(x))\,dx=x+\frac{x}{1-e^{2a}x^4}-\frac{1}{2}e^{-a/2}\arctan\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)$$

```
[Out] x+x/(1-exp(2*a)*x^4)-1/2*arctan(exp(1/2*a)*x)/exp(1/2*a)-1/2*arctanh(exp(1/
2*a)*x)/exp(1/2*a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5653, 398, 294, 218, 212, 209}

$$\int \coth^2(a+2\log(x))\,dx=-\frac{1}{2}e^{-a/2}\arctan\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)+\frac{x}{1-e^{2a}x^4}+x$$

```
[In] Int[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] x + x/(1 - E^(2*a)*x^4) - ArcTan[E^(a/2)*x]/(2*E^(a/2)) - ArcTanh[E^(a/2)*x
]/(2*E^(a/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(-1-e^{2a}x^4)^2}{(1-e^{2a}x^4)^2}\,dx \\
&= \int \left(1+\frac{4e^{2a}x^4}{(1-e^{2a}x^4)^2}\right)dx \\
&= x+\left(4e^{2a}\right)\int \frac{x^4}{(1-e^{2a}x^4)^2}\,dx \\
&= x+\frac{x}{1-e^{2a}x^4}-\int \frac{1}{1-e^{2a}x^4}\,dx \\
&= x+\frac{x}{1-e^{2a}x^4}-\frac{1}{2}\int \frac{1}{1-e^{a}x^2}\,dx-\frac{1}{2}\int \frac{1}{1+e^{a}x^2}\,dx \\
&= x+\frac{x}{1-e^{2a}x^4}-\frac{1}{2}e^{-a/2}\arctan\left(e^{a/2}x\right)-\frac{1}{2}e^{-a/2}\text{arctanh}\left(e^{a/2}x\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.55

$$\int \coth^2(a+2\log(x))\,dx$$
$$= \frac{e^{-4a}\left(-3645-6769e^{2a}x^4-1483e^{4a}x^8+681e^{6a}x^{12}+5(729+1208e^{2a}x^4+102e^{4a}x^8-248e^{6a}x^{12}+e^{8a}x^{16}\right.}{640x^7}$$
$$+\frac{16}{585}e^{2a}x^5\left(1+e^{2a}x^4\right)^2 {}_4F_3\left(\frac{5}{4},2,2,2;1,1,\frac{17}{4};e^{2a}x^4\right)$$

```
[In] Integrate[Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (-3645 - 6769*E^(2*a)*x^4 - 1483*E^(4*a)*x^8 + 681*E^(6*a)*x^12 + 5*(729 +
1208*E^(2*a)*x^4 + 102*E^(4*a)*x^8 - 248*E^(6*a)*x^12 + E^(8*a)*x^16)*Hyper
geometric2F1[1/4, 1, 5/4, E^(2*a)*x^4])/(640*E^(4*a)*x^7) + (16*E^(2*a)*x^5
*(1 + E^(2*a)*x^4)^2*HypergeometricPFQ[{5/4, 2, 2, 2}, {1, 1, 17/4}, E^(2*a
)*x^4])/585
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

method	result	size
risch	$x-\frac{x}{-1+{\mathrm e}^{2 a} x^{4}}-\frac{\ln\left(x \sqrt{-{\mathrm e}^{a}}+1\right)}{4 \sqrt{-{\mathrm e}^{a}}}+\frac{\ln\left(x \sqrt{-{\mathrm e}^{a}}-1\right)}{4 \sqrt{-{\mathrm e}^{a}}}+\frac{\ln\left(\sqrt{{\mathrm e}^{a}}\, x-1\right)}{4 \sqrt{{\mathrm e}^{a}}}-\frac{\ln\left(\sqrt{{\mathrm e}^{a}}\, x+1\right)}{4 \sqrt{{\mathrm e}^{a}}}$	86

```
[In] int(coth(a+2*ln(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x-x/(exp(a)^2*x^4-1)-1/4/(-exp(a))^(1/2)*ln(x*(-exp(a))^(1/2)+1)+1/4/(-exp(
a))^(1/2)*ln(x*(-exp(a))^(1/2)-1)+1/4/exp(a)^(1/2)*ln(exp(a)^(1/2)*x-1)-1/4
/exp(a)^(1/2)*ln(exp(a)^(1/2)*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(43) = 86.

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \coth^2(a+2\log(x))\,dx$$
$$= \frac{4\,x^5e^{(3\,a)}-2\left(x^4e^{(2\,a)}-1\right)\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)}+\left(x^4e^{(2\,a)}-1\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a-2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right)-8\,xe^a}{4\left(x^4e^{(3\,a)}-e^a\right)}$$

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^5*e^(3*a) - 2*(x^4*e^(2*a) - 1)*arctan(x*e^(1/2*a))*e^(1/2*a) + (x
^4*e^(2*a) - 1)*e^(1/2*a)*log((x^2*e^a - 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
- 8*x*e^a)/(x^4*e^(3*a) - e^a)
```

Sympy [F]

$$\int \coth^2(a+2\log(x))\,dx = \int \coth^2\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(coth(a+2*ln(x))**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \coth^2(a+2\log(x))\,dx = -\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{xe^{a}-e^{\left(\frac{1}{2}a\right)}}{xe^{a}+e^{\left(\frac{1}{2}a\right)}}\right) + x - \frac{x}{x^{4}e^{(2a)}-1}$$

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log((x*e^a - e^(1/2*a)
)/(x*e^a + e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.10

$$\int \coth^2(a+2\log(x))\,dx = -\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(-\frac{1}{2}a\right)} + \frac{1}{4}e^{\left(-\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^{a}-2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^{a}+2\,e^{\left(\frac{1}{2}a\right)}\right|}\right) + x - \frac{x}{x^{4}e^{(2a)}-1}$$

```
[In] integrate(coth(a+2*log(x))^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(-1/2*a) + 1/4*e^(-1/2*a)*log(abs(2*x*e^a - 2*e^
(1/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) + x - x/(x^4*e^(2*a) - 1)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \coth^2(a+2\log(x))\,dx = x - \frac{\operatorname{atan}\left(x\,{\left(e^{2\,a}\right)}^{1/4}\right)}{2\,{\left(e^{2\,a}\right)}^{1/4}} - \frac{x}{x^4\,e^{2\,a}-1} + \frac{\operatorname{atan}\left(x\,{\left(e^{2\,a}\right)}^{1/4}\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{2\,{\left(e^{2\,a}\right)}^{1/4}}$$

```
[In] int(coth(a + 2*log(x))^2,x)
```

```
[Out] x - atan(x*exp(2*a)^(1/4))/(2*exp(2*a)^(1/4)) + (atan(x*exp(2*a)^(1/4)*1i)*
1i)/(2*exp(2*a)^(1/4)) - x/(x^4*exp(2*a) - 1)
```

3.162 $\int \frac{\coth^2(a+2\log(x))}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = -\frac{1}{2}\coth(a+2\log(x))+\log(x)$$

```
[Out] -1/2*coth(a+2*ln(x))+ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3554, 8}

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \log(x)-\frac{1}{2}\coth(a+2\log(x))$$

```
[In] Int[Coth[a + 2*Log[x]]^2/x,x]
```

```
[Out] -1/2*Coth[a + 2*Log[x]] + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \coth^2(a+2x)\,dx, x, \log(x)\right) \\ &= -\frac{1}{2}\coth(a+2\log(x)) + \text{Subst}(\int 1\,dx, x, \log(x)) \\ &= -\frac{1}{2}\coth(a+2\log(x)) + \log(x) \end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = -\frac{1}{2}\coth(a+2\log(x))\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(a+2\log(x))\right)$$

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x,x]
[Out] -1/2*(Coth[a + 2*Log[x]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a + 2*Log[x]]
^2])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

method	result	size
risch	$-\frac{1}{-1+e^{2a}x^4}+\ln(x)$	18
parallelrisch	$\frac{-1+2\ln(x)\tanh(a+2\ln(x))}{2\tanh(a+2\ln(x))}$	25
derivativedivides	$-\frac{\coth(a+2\ln(x))}{2}-\frac{\ln(\coth(a+2\ln(x))-1)}{4}+\frac{\ln(\coth(a+2\ln(x))+1)}{4}$	35
default	$-\frac{\coth(a+2\ln(x))}{2}-\frac{\ln(\coth(a+2\ln(x))-1)}{4}+\frac{\ln(\coth(a+2\ln(x))+1)}{4}$	35

```
[In] int(coth(a+2*ln(x))^2/x,x,method=_RETURNVERBOSE)
[Out] -1/(-1+exp(2*a)*x^4)+ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \frac{\left(x^4e^{(2\,a)}-1\right)\log\left(x\right)-1}{x^4e^{(2\,a)}-1}$$

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="fricas")
[Out] ((x^4*e^(2*a) - 1)*log(x) - 1)/(x^4*e^(2*a) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(12) = 24$.

Time = 1.92 (sec) , antiderivative size = 63, normalized size of antiderivative = 4.50

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \begin{cases} \frac{\log\left(x\right)}{\tanh^2\left(\log\left(-\frac{1}{x^2}\right)+2\log\left(x\right)\right)} & \text{for}\ a=\log\left(-\frac{1}{x^2}\right) \\ \frac{\log\left(x\right)}{\tanh^2\left(\log\left(\frac{1}{x^2}\right)+2\log\left(x\right)\right)} & \text{for}\ a=\log\left(\frac{1}{x^2}\right) \\ \log\left(x\right)-\frac{1}{2\tanh\left(a+2\log\left(x\right)\right)} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(a+2*ln(x))**2/x,x)
[Out] Piecewise((log(x)/tanh(log(-1/x**2) + 2*log(x))**2, Eq(a, log(-1/x**2))), (
log(x)/tanh(log(x**(-2)) + 2*log(x))**2, Eq(a, log(x**(-2)))), (log(x) - 1/
(2*tanh(a + 2*log(x))), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \frac{1}{2}\,a+\frac{1}{e^{(-2\,a-4\,\log\left(x\right))}-1}+\log\left(x\right)$$

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="maxima")
[Out] 1/2*a + 1/(e^(-2*a - 4*log(x)) - 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = -\frac{1}{x^4e^{(2\,a)}-1} + \frac{1}{4}\,\log\left(x^4\right)$$

```
[In] integrate(coth(a+2*log(x))^2/x,x, algorithm="giac")
[Out] -1/(x^4*e^(2*a) - 1) + 1/4*log(x^4)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{\coth^2(a+2\log(x))}{x}\,dx = \ln\left(x\right) - \frac{\mathrm{e}^{2\,a}\,x^4+1}{2\,\left(x^4\,\mathrm{e}^{2\,a}-1\right)}$$

```
[In] int(coth(a + 2*log(x))^2/x,x)
[Out] log(x) - (x^4*exp(2*a) + 1)/(2*(x^4*exp(2*a) - 1))
```

3.163 $\int \frac{\coth^2(a+2\log(x))}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \frac{\coth^2(a+2\log(x))}{x^2} dx = -\frac{1}{x(1-e^{2a}x^4)} + \frac{2e^{2a}x^3}{1-e^{2a}x^4} - \frac{1}{2}e^{a/2}\arctan\left(e^{a/2}x\right) + \frac{1}{2}e^{a/2}\text{arctanh}\left(e^{a/2}x\right)$$

```
[Out] -1/x/(1-exp(2*a)*x^4)+2*exp(2*a)*x^3/(1-exp(2*a)*x^4)-1/2*exp(1/2*a)*arctan
(exp(1/2*a)*x)+1/2*exp(1/2*a)*arctanh(exp(1/2*a)*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5657, 473, 468, 304, 209, 212}

$$\int \frac{\coth^2(a+2\log(x))}{x^2} dx = -\frac{1}{2}e^{a/2}\arctan\left(e^{a/2}x\right) + \frac{1}{2}e^{a/2}\text{arctanh}\left(e^{a/2}x\right) - \frac{1}{x(1-e^{2a}x^4)} + \frac{2e^{2a}x^3}{1-e^{2a}x^4}$$

```
[In] Int[Coth[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] -(1/(x*(1 - E^(2*a)*x^4))) + (2*E^(2*a)*x^3)/(1 - E^(2*a)*x^4) - (E^(a/2)*A
rcTan[E^(a/2)*x])/2 + (E^(a/2)*ArcTanh[E^(a/2)*x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(-1-e^{2a}x^4\right)^2}{x^2\left(1-e^{2a}x^4\right)^2}\,dx$$

$$= -\frac{1}{x(1-e^{2a}x^4)} + \int \frac{x^2(7e^{2a}+e^{4a}x^4)}{(1-e^{2a}x^4)^2}\,dx$$

$$= -\frac{1}{x(1-e^{2a}x^4)} + \frac{2e^{2a}x^3}{1-e^{2a}x^4} + e^{2a}\int \frac{x^2}{1-e^{2a}x^4}\,dx$$

$$= -\frac{1}{x(1-e^{2a}x^4)} + \frac{2e^{2a}x^3}{1-e^{2a}x^4} + \frac{1}{2}e^{a}\int \frac{1}{1-e^{a}x^2}\,dx - \frac{1}{2}e^{a}\int \frac{1}{1+e^{a}x^2}\,dx$$

$$= -\frac{1}{x(1-e^{2a}x^4)} + \frac{2e^{2a}x^3}{1-e^{2a}x^4} - \frac{1}{2}e^{a/2}\arctan\left(e^{a/2}x\right) + \frac{1}{2}e^{a/2}\text{arctanh}\left(e^{a/2}x\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.78

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx$$
$$= \frac{e^{-2a}\left(-343-1163e^{2a}x^4-241e^{4a}x^8+3e^{6a}x^{12}+(343+632e^{2a}x^4+362e^{4a}x^8-56e^{6a}x^{12}-e^{8a}x^{16})\,\text{Hyperge}\right.}{384x^5}$$
$$+\frac{16}{231}e^{2a}x^3\left(1+e^{2a}x^4\right)^2\,{}_4F_3\left(\frac{3}{4},2,2,2;1,1,\frac{15}{4};e^{2a}x^4\right)$$

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^2,x]
```

```
[Out] (-343 - 1163*E^(2*a)*x^4 - 241*E^(4*a)*x^8 + 3*E^(6*a)*x^12 + (343 + 632*E^
(2*a)*x^4 + 362*E^(4*a)*x^8 - 56*E^(6*a)*x^12 - E^(8*a)*x^16)*Hypergeometri
c2F1[3/4, 1, 7/4, E^(2*a)*x^4])/(384*E^(2*a)*x^5) + (16*E^(2*a)*x^3*(1 + E^
(2*a)*x^4)^2*HypergeometricPFQ[{3/4, 2, 2, 2}, {1, 1, 15/4}, E^(2*a)*x^4])/
231
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

method	result
risch	$\frac{-2\,e^{2a}x^4+1}{x(-1+e^{2a}x^4)} + \frac{\sqrt{e^{a}}\,\ln\left(-(e^{a})^{\frac{3}{2}}-e^{2a}x\right)}{4} - \frac{\sqrt{e^{a}}\,\ln\left((e^{a})^{\frac{3}{2}}-e^{2a}x\right)}{4} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^2+e^{a}\right)}\_R\ln\left(\left(-5\_R^4+4\,e^{2a}\right)x-\_R\right.\right.}{4}$

```
[In] int(coth(a+2*ln(x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-2*exp(2*a)*x^4+1)/x/(-1+exp(2*a)*x^4)+1/4*exp(a)^(1/2)*ln(-exp(a)^(3/2)-e
xp(2*a)*x)-1/4*exp(a)^(1/2)*ln(exp(a)^(3/2)-exp(2*a)*x)+1/4*sum(_R*ln((-5*_
R^4+4*exp(2*a))*x-_R^3),_R=RootOf(_Z^2+exp(a)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = -\frac{8\,x^4e^{(2\,a)} + 2\left(x^5e^{(2\,a)} - x\right)\arctan\left(xe^{\left(\frac{1}{2}\,a\right)}\right)e^{\left(\frac{1}{2}\,a\right)} - \left(x^5e^{(2\,a)} - x\right)e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{x^2e^a+2\,xe^{\left(\frac{1}{2}\,a\right)}+1}{x^2e^a-1}\right) - 4}{4\left(x^5e^{(2\,a)} - x\right)}$$

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*x^4*e^(2*a) + 2*(x^5*e^(2*a) - x)*arctan(x*e^(1/2*a))*e^(1/2*a) - (
x^5*e^(2*a) - x)*e^(1/2*a)*log((x^2*e^a + 2*x*e^(1/2*a) + 1)/(x^2*e^a - 1))
 - 4)/(x^5*e^(2*a) - x)
```

Sympy [F]

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \int \frac{\coth^2\left(a+2\log\left(x\right)\right)}{x^2}\,dx$$

```
[In] integrate(coth(a+2*ln(x))**2/x**2,x)
```

```
[Out] Integral(coth(a + 2*log(x))**2/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \frac{1}{2}\arctan\left(\frac{e^{\left(-\frac{1}{2}\,a\right)}}{x}\right)e^{\left(\frac{1}{2}\,a\right)} - \frac{1}{4}\,e^{\left(\frac{1}{2}\,a\right)}\log\left(\frac{\frac{1}{x} - e^{\left(\frac{1}{2}\,a\right)}}{\frac{1}{x} + e^{\left(\frac{1}{2}\,a\right)}}\right) - \frac{1}{x} + \frac{e^{(2\,a)}}{x\left(\frac{1}{x^4} - e^{(2\,a)}\right)}$$

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*arctan(e^(-1/2*a)/x)*e^(1/2*a) - 1/4*e^(1/2*a)*log((1/x - e^(1/2*a))/(1
/x + e^(1/2*a))) - 1/x + e^(2*a)/(x*(1/x^4 - e^(2*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = -\frac{1}{2}\arctan\left(xe^{\left(\frac{1}{2}a\right)}\right)e^{\left(\frac{1}{2}a\right)} - \frac{1}{4}e^{\left(\frac{1}{2}a\right)}\log\left(\frac{\left|2\,xe^a - 2\,e^{\left(\frac{1}{2}a\right)}\right|}{\left|2\,xe^a + 2\,e^{\left(\frac{1}{2}a\right)}\right|}\right) - \frac{2\,x^4e^{(2\,a)} - 1}{x^5e^{(2\,a)} - x}$$

```
[In] integrate(coth(a+2*log(x))^2/x^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(x*e^(1/2*a))*e^(1/2*a) - 1/4*e^(1/2*a)*log(abs(2*x*e^a - 2*e^(1
/2*a))/abs(2*x*e^a + 2*e^(1/2*a))) - (2*x^4*e^(2*a) - 1)/(x^5*e^(2*a) - x)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$\int \frac{\coth^2(a+2\log(x))}{x^2}\,dx = \frac{\left(e^{2\,a}\right)^{1/4}\,\mathrm{atanh}\left(x\,\left(e^{2\,a}\right)^{1/4}\right)}{2} - \frac{\left(e^{2\,a}\right)^{1/4}\,\mathrm{atan}\left(x\,\left(e^{2\,a}\right)^{1/4}\right)}{2} + \frac{2\,x^4\,e^{2\,a} - 1}{x - x^5\,e^{2\,a}}$$

```
[In] int(coth(a + 2*log(x))^2/x^2,x)
```

```
[Out] (exp(2*a)^(1/4)*atanh(x*exp(2*a)^(1/4)))/2 - (exp(2*a)^(1/4)*atan(x*exp(2*a
)^(1/4)))/2 + (2*x^4*exp(2*a) - 1)/(x - x^5*exp(2*a))
```

3.164 $\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx=-\frac{1}{2x^2(1-e^{2a}x^4)}+\frac{3e^{2a}x^2}{2(1-e^{2a}x^4)}+e^a\operatorname{arctanh}\left(e^ax^2\right)$$

```
[Out] -1/2/x^2/(1-exp(2*a)*x^4)+3/2*exp(2*a)*x^2/(1-exp(2*a)*x^4)+exp(a)*arctanh(
exp(a)*x^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5657, 473, 468, 281, 212}

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx=e^a\operatorname{arctanh}\left(e^ax^2\right)+\frac{3e^{2a}x^2}{2(1-e^{2a}x^4)}-\frac{1}{2x^2(1-e^{2a}x^4)}$$

```
[In] Int[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] -1/2*1/(x^2*(1 - E^(2*a)*x^4)) + (3*E^(2*a)*x^2)/(2*(1 - E^(2*a)*x^4)) + E^
a*ArcTanh[E^a*x^2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(-1-e^{2a}x^4)^2}{x^3\,(1-e^{2a}x^4)^2}\,dx \\
&= -\frac{1}{2x^2\,(1-e^{2a}x^4)} + \frac{1}{2}\int \frac{x(10e^{2a}+2e^{4a}x^4)}{(1-e^{2a}x^4)^2}\,dx \\
&= -\frac{1}{2x^2\,(1-e^{2a}x^4)} + \frac{3e^{2a}x^2}{2\,(1-e^{2a}x^4)} + \left(2e^{2a}\right)\int \frac{x}{1-e^{2a}x^4}\,dx \\
&= -\frac{1}{2x^2\,(1-e^{2a}x^4)} + \frac{3e^{2a}x^2}{2\,(1-e^{2a}x^4)} + e^{2a}\text{Subst}\left(\int \frac{1}{1-e^{2a}x^2}\,dx, x, x^2\right) \\
&= -\frac{1}{2x^2\,(1-e^{2a}x^4)} + \frac{3e^{2a}x^2}{2\,(1-e^{2a}x^4)} + e^{a}\text{arctanh}\left(e^{a}x^2\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 3.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.58

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$
$$= \frac{15\left(-77-\frac{27e^{-2a}}{x^4}-17e^{2a}x^4+e^{4a}x^8\right)-\frac{15(-27-52e^{2a}x^4-54e^{4a}x^8+4e^{6a}x^{12}+e^{8a}x^{16})\text{arctanh}\left(\sqrt{e^{2a}x^4}\right)}{(e^{2a}x^4)^{3/2}}+64(e^ax^2+e}{480x^2}$$

```
[In] Integrate[Coth[a + 2*Log[x]]^2/x^3,x]
```

```
[Out] (15*(-77 - 27/(E^(2*a)*x^4) - 17*E^(2*a)*x^4 + E^(4*a)*x^8) - (15*(-27 - 52
*E^(2*a)*x^4 - 54*E^(4*a)*x^8 + 4*E^(6*a)*x^12 + E^(8*a)*x^16)*ArcTanh[Sqrt
[E^(2*a)*x^4]])/(E^(2*a)*x^4)^(3/2) + 64*(E^a*x^2 + E^(3*a)*x^6)^2*Hypergeo
metricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, E^(2*a)*x^4])/(480*x^2)
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{-\frac{3\,\mathrm{e}^{2a}x^4}{2}+\frac{1}{2}}{x^2(-1+\mathrm{e}^{2a}x^4)}-\frac{\mathrm{e}^a\ln(\mathrm{e}^ax^2-1)}{2}+\frac{\mathrm{e}^a\ln(\mathrm{e}^ax^2+1)}{2}$	55

```
[In] int(coth(a+2*ln(x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/2*exp(a)^2*x^4+1/2)/x^2/(exp(a)^2*x^4-1)-1/2*exp(a)*ln(exp(a)*x^2-1)+1/
2*exp(a)*ln(exp(a)*x^2+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx$$
$$= -\frac{3\,x^4e^{(2\,a)}-\left(x^6e^{(3\,a)}-x^2e^a\right)\log\left(x^2e^a+1\right)+\left(x^6e^{(3\,a)}-x^2e^a\right)\log\left(x^2e^a-1\right)-1}{2\left(x^6e^{(2\,a)}-x^2\right)}$$

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4*e^(2*a) - (x^6*e^(3*a) - x^2*e^a)*log(x^2*e^a + 1) + (x^6*e^(3*
a) - x^2*e^a)*log(x^2*e^a - 1) - 1)/(x^6*e^(2*a) - x^2)
```

Sympy [F]

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \int \frac{\coth^2\left(a+2\log\left(x\right)\right)}{x^3}\,dx$$

```
[In] integrate(coth(a+2*ln(x))**2/x**3,x)
[Out] Integral(coth(a + 2*log(x))**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \frac{1}{2}\,e^a\log\left(\frac{1}{x^2}+e^a\right)-\frac{1}{2}\,e^a\log\left(\frac{1}{x^2}-e^a\right)-\frac{1}{2\,x^2}+\frac{e^{(2\,a)}}{x^2\left(\frac{1}{x^4}-e^{(2\,a)}\right)}$$

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="maxima")
[Out] 1/2*e^a*log(1/x^2 + e^a) - 1/2*e^a*log(1/x^2 - e^a) - 1/2/x^2 + e^(2*a)/(x^
2*(1/x^4 - e^(2*a)))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \frac{1}{2}\,e^a\log\left(x^2e^a+1\right)-\frac{1}{2}\,e^a\log\left(\left|x^2e^a-1\right|\right)-\frac{3\,x^4e^{(2\,a)}-1}{2\left(x^6e^{(2\,a)}-x^2\right)}$$

```
[In] integrate(coth(a+2*log(x))^2/x^3,x, algorithm="giac")
[Out] 1/2*e^a*log(x^2*e^a + 1) - 1/2*e^a*log(abs(x^2*e^a - 1)) - 1/2*(3*x^4*e^(2*
a) - 1)/(x^6*e^(2*a) - x^2)
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{\coth^2(a+2\log(x))}{x^3}\,dx = \operatorname{atanh}\left(x^2\sqrt{e^{2a}}\right)\sqrt{e^{2a}} - \frac{\frac{3x^4 e^{2a}}{2} - \frac{1}{2}}{x^6 e^{2a} - x^2}$$

```
[In] int(coth(a + 2*log(x))^2/x^3,x)
[Out] atanh(x^2*exp(2*a)^(1/2))*exp(2*a)^(1/2) - ((3*x^4*exp(2*a))/2 - 1/2)/(x^6*
exp(2*a) - x^2)
```

3.165 $\int (ex)^m \coth(a+2\log(x))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int (ex)^m \coth(a+2\log(x))\,dx = \frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],exp(
2*a)*x^4)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5657, 470, 371}

$$\int (ex)^m \coth(a+2\log(x))\,dx = \frac{(ex)^{m+1}}{e(m+1)} - \frac{2(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},e^{2a}x^4\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]],x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/4
, (5 + m)/4, E^(2*a)*x^4])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \frac{(ex)^m\left(-1-e^{2a}x^4\right)}{1-e^{2a}x^4}\,dx \\
&= \frac{(ex)^{1+m}}{e(1+m)} - 2\int \frac{(ex)^m}{1-e^{2a}x^4}\,dx \\
&= \frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{e(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int (ex)^m \coth(a+2\log(x))\,dx = -\frac{x(ex)^m\left(-1+2\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},x^4(\cosh(2a)+\sinh(2a))\right)\right)}{1+m}$$

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]],x]
```

```
[Out] -((x*(e*x)^m*(-1 + 2*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])]))/(1 + m))
```

Maple [F]

$$\int (ex)^m \coth(a + 2\ln(x))\,dx$$

```
[In] int((e*x)^m*coth(a+2*ln(x)),x)
[Out] int((e*x)^m*coth(a+2*ln(x)),x)
```

Fricas [F]

$$\int (ex)^m \coth(a + 2\log(x))\,dx = \int (ex)^m \coth(a + 2\log(x))\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="fricas")
[Out] integral((e*x)^m*coth(a + 2*log(x)), x)
```

Sympy [F]

$$\int (ex)^m \coth(a + 2\log(x))\,dx = \int (ex)^m \coth(a + 2\log(x))\,dx$$

```
[In] integrate((e*x)**m*coth(a+2*ln(x)),x)
[Out] Integral((e*x)**m*coth(a + 2*log(x)), x)
```

Maxima [F]

$$\int (ex)^m \coth(a + 2\log(x))\,dx = \int (ex)^m \coth(a + 2\log(x))\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="maxima")
[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)
```

Giac [F]

$$\int (ex)^m \coth(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x)),x, algorithm="giac")
[Out] integrate((e*x)^m*coth(a + 2*log(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth(a+2\log(x))\,dx = \int \coth(a+2\,\ln\left(x\right))\,\left(e\,x\right)^m\,dx$$

```
[In] int(coth(a + 2*log(x))*(e*x)^m,x)
[Out] int(coth(a + 2*log(x))*(e*x)^m, x)
```

3.166 $\int (ex)^m \coth^2(a+2\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int (ex)^m \coth^2(a+2\log(x))\,dx=\frac{(ex)^{1+m}}{e(1+m)}+\frac{(ex)^{1+m}}{e\left(1-e^{2a}x^4\right)}-\frac{(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{e}$$

```
[Out] (e*x)^(1+m)/e/(1+m)+(e*x)^(1+m)/e/(1-exp(2*a)*x^4)-(e*x)^(1+m)*hypergeom([1
, 1/4+1/4*m],[5/4+1/4*m],exp(2*a)*x^4)/e
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5657, 474, 470, 371}

$$\int (ex)^m \coth^2(a+2\log(x))\,dx=-\frac{(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},e^{2a}x^4\right)}{e}+\frac{(ex)^{m+1}}{e\left(1-e^{2a}x^4\right)}+\frac{(ex)^{m+1}}{e(m+1)}$$

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^2,x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) + (e*x)^(1 + m)/(e*(1 - E^(2*a)*x^4)) - ((e*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^(2*a)*x^4])/e
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(ex)^m \left(-1-e^{2a}x^4\right)^2}{\left(1-e^{2a}x^4\right)^2}\,dx \\
&= \frac{(ex)^{1+m}}{e\left(1-e^{2a}x^4\right)} - \frac{1}{4}e^{-4a}\int \frac{(ex)^m \left(4e^{4a}m+4e^{6a}x^4\right)}{1-e^{2a}x^4}\,dx \\
&= \frac{(ex)^{1+m}}{e(1+m)} + \frac{(ex)^{1+m}}{e\left(1-e^{2a}x^4\right)} - (1+m)\int \frac{(ex)^m}{1-e^{2a}x^4}\,dx \\
&= \frac{(ex)^{1+m}}{e(1+m)} + \frac{(ex)^{1+m}}{e\left(1-e^{2a}x^4\right)} - \frac{(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{e}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = -\frac{x(ex)^m\left(-1+4\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},x^4(\cosh(2a)+\sinh(2a))\right)-4\,\text{Hypergeometric2F1}\left(2,\right.\right.}{1+m}$$

[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^2,x]

[Out] -((x*(e*x)^m*(-1 + 4*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])] - 4*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Cosh[
2*a] + Sinh[2*a])]))/(1 + m))

Maple [F]

$$\int (ex)^m \coth(a+2\ln(x))^2\,dx$$

[In] int((e*x)^m*coth(a+2*ln(x))^2,x)

[Out] int((e*x)^m*coth(a+2*ln(x))^2,x)

Fricas [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int (ex)^m \coth(a+2\log(x))^2\,dx$$

[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="fricas")

[Out] integral((e*x)^m*coth(a + 2*log(x))^2, x)

Sympy [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int (ex)^m \coth^2(a+2\log(x))\,dx$$

[In] integrate((e*x)**m*coth(a+2*ln(x))**2,x)

[Out] Integral((e*x)**m*coth(a + 2*log(x))**2, x)

Maxima [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^2\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="maxima")
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

Giac [F]

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^2\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*coth(a + 2*log(x))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^2(a+2\log(x))\,dx = \int \coth(a+2\,\ln\left(x\right))^2\,\left(e\,x\right)^m\,dx$$

```
[In] int(coth(a + 2*log(x))^2*(e*x)^m,x)
[Out] int(coth(a + 2*log(x))^2*(e*x)^m, x)
```

3.167 $\int (ex)^m \coth^3(a+2\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 177

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)} - \frac{(ex)^{1+m}\left(1+e^{2a}x^4\right)^2}{4e\left(1-e^{2a}x^4\right)^2} - \frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)-e^{4a}(5+m)x^4\right)}{8e\left(1-e^{2a}x^4\right)} - \frac{\left(9+2m+m^2\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{4e(1+m)}$$

```
[Out] 1/8*(3+m)*(5+m)*(e*x)^(1+m)/e/(1+m)-1/4*(e*x)^(1+m)*(1+exp(2*a)*x^4)^2/e/(1
-exp(2*a)*x^4)^2-1/8*(e*x)^(1+m)*(exp(2*a)*(3-m)-exp(4*a)*(5+m)*x^4)/e/exp(
2*a)/(1-exp(2*a)*x^4)-1/4*(m^2+2*m+9)*(e*x)^(1+m)*hypergeom([1, 1/4+1/4*m],
[5/4+1/4*m],exp(2*a)*x^4)/e/(1+m)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {5657, 479, 591, 470, 371}

$$\int (ex)^m \coth^3(a+2\log(x))\,dx$$
$$= -\frac{(m^2+2m+9)\,(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{4},\frac{m+5}{4},e^{2a}x^4\right)}{4e(m+1)} - \frac{(e^{2a}x^4+1)^2\,(ex)^{m+1}}{4e\,(1-e^{2a}x^4)^2} - \frac{e^{-2a}(e^{2a}(3-m)-e^{4a}(m+5)x^4)\,(ex)^{m+1}}{8e\,(1-e^{2a}x^4)} + \frac{(m+3)(m+5)(ex)^{m+1}}{8e(m+1)}$$

```
[In] Int[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
```

```
[Out] ((3 + m)*(5 + m)*(e*x)^(1 + m))/(8*e*(1 + m)) - ((e*x)^(1 + m)*(1 + E^(2*a)
*x^4)^2)/(4*e*(1 - E^(2*a)*x^4)^2) - ((e*x)^(1 + m)*(E^(2*a)*(3 - m) - E^(4
*a)*(5 + m)*x^4))/(8*e*E^(2*a)*(1 - E^(2*a)*x^4)) - ((9 + 2*m + m^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, E^(2*a)*x^4])/(4*e*(1 +
m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 591

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n,
 0] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e -
 a*f])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{(ex)^m\left(-1-e^{2a}x^4\right)^3}{\left(1-e^{2a}x^4\right)^3}\,dx\\
&= -\frac{(ex)^{1+m}\left(1+e^{2a}x^4\right)^2}{4e\left(1-e^{2a}x^4\right)^2}+\frac{1}{8}e^{-2a}\int \frac{(ex)^m\left(-1-e^{2a}x^4\right)\left(2e^{2a}(3-m)-2e^{4a}(5+m)x^4\right)}{\left(1-e^{2a}x^4\right)^2}\,dx\\
&= -\frac{(ex)^{1+m}\left(1+e^{2a}x^4\right)^2}{4e\left(1-e^{2a}x^4\right)^2}-\frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)-e^{4a}(5+m)x^4\right)}{8e\left(1-e^{2a}x^4\right)}\\
&\quad+\frac{1}{32}e^{-4a}\int \frac{(ex)^m\left(-4e^{4a}(1-m)(3-m)-4e^{6a}(3+m)(5+m)x^4\right)}{1-e^{2a}x^4}\,dx\\
&= \frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)}-\frac{(ex)^{1+m}\left(1+e^{2a}x^4\right)^2}{4e\left(1-e^{2a}x^4\right)^2}\\
&\quad-\frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)-e^{4a}(5+m)x^4\right)}{8e\left(1-e^{2a}x^4\right)}+\frac{1}{4}\left(-9-2m-m^2\right)\int \frac{(ex)^m}{1-e^{2a}x^4}\,dx\\
&= \frac{(3+m)(5+m)(ex)^{1+m}}{8e(1+m)}-\frac{(ex)^{1+m}\left(1+e^{2a}x^4\right)^2}{4e\left(1-e^{2a}x^4\right)^2}\\
&\quad-\frac{e^{-2a}(ex)^{1+m}\left(e^{2a}(3-m)-e^{4a}(5+m)x^4\right)}{8e\left(1-e^{2a}x^4\right)}\\
&\quad-\frac{\left(9+2m+m^2\right)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},e^{2a}x^4\right)}{4e(1+m)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.61

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = -\frac{x(ex)^m \left(-1+6\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{4},\frac{5+m}{4},x^4(\cosh(2a)+\sinh(2a))\right)-12\,\text{Hypergeometric2F1}\right.}{1}$$

```
[In] Integrate[(e*x)^m*Coth[a + 2*Log[x]]^3,x]
[Out] -((x*(e*x)^m*(-1 + 6*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, x^4*(Cosh[2
*a] + Sinh[2*a])] - 12*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4*(Cosh
[2*a] + Sinh[2*a])] + 8*Hypergeometric2F1[3, (1 + m)/4, (5 + m)/4, x^4*(Cos
h[2*a] + Sinh[2*a])]))/(1 + m))
```

Maple [F]

$$\int (ex)^m \coth(a+2\ln(x))^3\,dx$$

```
[In] int((e*x)^m*coth(a+2*ln(x))^3,x)
[Out] int((e*x)^m*coth(a+2*ln(x))^3,x)
```

Fricas [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth(a+2\log(x))^3\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="fricas")
[Out] integral((e*x)^m*coth(a + 2*log(x))^3, x)
```

Sympy [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth^3(a+2\log(x))\,dx$$

```
[In] integrate((e*x)**m*coth(a+2*ln(x))**3,x)
[Out] Integral((e*x)**m*coth(a + 2*log(x))**3, x)
```

Maxima [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^3\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="maxima")
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

Giac [F]

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int (ex)^m \coth\left(a+2\,\log\left(x\right)\right)^3\,dx$$

```
[In] integrate((e*x)^m*coth(a+2*log(x))^3,x, algorithm="giac")
[Out] integrate((e*x)^m*coth(a + 2*log(x))^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^3(a+2\log(x))\,dx = \int \coth(a+2\,\ln\left(x\right))^3\,\left(e\,x\right)^m\,dx$$

```
[In] int(coth(a + 2*log(x))^3*(e*x)^m,x)
[Out] int(coth(a + 2*log(x))^3*(e*x)^m, x)
```

3.168 $\int \coth^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 79

$$\int \coth^p(a+b\log(x))\,dx = x\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2b},p,-p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[Out] x*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2/b,p,-p,1+1/2/b,exp(2*a)*x^(2*b),-exp
(2*a)*x^(2*b))/((1+exp(2*a)*x^(2*b))^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5653, 441, 440}

$$\int \coth^p(a+b\log(x))\,dx = x\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2b},p,-p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)$$

```
[In] Int[Coth[a + b*Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[1/(2*b), p, -p, (2 + b^(-1))/2, E^(2*a
)*x^(2*b), -(E^(2*a)*x^(2*b))])/(1 + E^(2*a)*x^(2*b))^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-1-e^{2a}x^{2b}\right)^p \left(1-e^{2a}x^{2b}\right)^{-p}\,dx \\
&= \left(\left(-1-e^{2a}x^{2b}\right)^p \left(1+e^{2a}x^{2b}\right)^{-p}\right)\int \left(1-e^{2a}x^{2b}\right)^{-p}\left(1+e^{2a}x^{2b}\right)^p\,dx \\
&= x\left(-1-e^{2a}x^{2b}\right)^p \left(1+e^{2a}x^{2b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2b},p,-p,\frac{1}{2}\left(2+\frac{1}{b}\right),e^{2a}x^{2b},-e^{2a}x^{2b}\right)
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 259 vs. $2(79) = 158$.

Time = 0.61 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.28

$$\int \coth^p(a+b\log(x))\,dx$$

$$= \frac{(1+2b)x\left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p \operatorname{AppellF1}\left(\frac{1}{2b},p,-p,1+\frac{1}{2b},\right.}{2be^{2a}px^{2b}\operatorname{AppellF1}\left(1+\frac{1}{2b},p,1-p,2+\frac{1}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)+2be^{2a}px^{2b}\operatorname{AppellF1}\left(1+\frac{1}{2b},1+p,-p,2+\frac{1}{2}\right.}$$

```
[In] Integrate[Coth[a + b*Log[x]]^p,x]
```

```
[Out] ((1 + 2*b)*x*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(2*b)))^p*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(2*b*E^(2*a)
*p*x^(2*b)*AppellF1[1 + 1/(2*b), p, 1 - p, 2 + 1/(2*b), E^(2*a)*x^(2*b), -(
E^(2*a)*x^(2*b))] + 2*b*E^(2*a)*p*x^(2*b)*AppellF1[1 + 1/(2*b), 1 + p, -p,
2 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))] + (1 + 2*b)*AppellF1[1/(2
*b), p, -p, 1 + 1/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])
```

Maple [F]

$$\int \coth\left(a+b\ln\left(x\right)\right)^{p}dx$$

```
[In] int(coth(a+b*ln(x))^p,x)
[Out] int(coth(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^{p}(a+b\log(x))\,dx = \int \coth\left(b\log\left(x\right)+a\right)^{p}\,dx$$

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int \coth^{p}(a+b\log(x))\,dx = \int \coth^{p}\left(a+b\log\left(x\right)\right)dx$$

```
[In] integrate(coth(a+b*ln(x))**p,x)
[Out] Integral(coth(a + b*log(x))**p, x)
```

Maxima [F]

$$\int \coth^{p}(a+b\log(x))\,dx = \int \coth\left(b\log\left(x\right)+a\right)^{p}\,dx$$

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(b*log(x) + a)^p, x)
```

Giac [F]

$$\int \coth^p(a + b\log(x))\,dx = \int \coth\left(b\log\left(x\right) + a\right)^p\,dx$$

```
[In] integrate(coth(a+b*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a + b\log(x))\,dx = \int \coth(a + b\,\ln\left(x\right))^p\,dx$$

```
[In] int(coth(a + b*log(x))^p,x)
[Out] int(coth(a + b*log(x))^p, x)
```

3.169 $\int (ex)^m \coth^p(a+b\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \frac{(ex)^{1+m}\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{2b},p,-p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(-1-exp(2*a)*x^(2*b))^p*AppellF1(1/2*(1+m)/b,p,-p,1+1/2*(1+m)/b
,exp(2*a)*x^(2*b),-exp(2*a)*x^(2*b))/e/(1+m)/((1+exp(2*a)*x^(2*b))^p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5657, 525, 524}

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \frac{(ex)^{m+1}\left(-e^{2a}x^{2b}-1\right)^p\left(e^{2a}x^{2b}+1\right)^{-p}\text{AppellF1}\left(\frac{m+1}{2b},p,-p,\frac{m+1}{2b}+1,e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(-1 - E^(2*a)*x^(2*b))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 +
(1 + m)/(2*b), E^(2*a)*x^(2*b), -(E^(2*a)*x^(2*b))])/(e*(1 + m)*(1 + E^(2*a
)*x^(2*b))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (ex)^m \left(-1-e^{2a}x^{2b}\right)^p \left(1-e^{2a}x^{2b}\right)^{-p}\,dx\\
&= \left(\left(-1-e^{2a}x^{2b}\right)^p \left(1+e^{2a}x^{2b}\right)^{-p}\right)\int (ex)^m \left(1-e^{2a}x^{2b}\right)^{-p}\left(1+e^{2a}x^{2b}\right)^p\,dx\\
&= \frac{(ex)^{1+m}\left(-1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2b},p,-p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{e(1+m)}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 0.78 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int (ex)^m \coth^p(a+b\log(x))\,dx\\
&= \frac{x(ex)^m\left(1-e^{2a}x^{2b}\right)^p\left(1+e^{2a}x^{2b}\right)^{-p}\left(\frac{1+e^{2a}x^{2b}}{-1+e^{2a}x^{2b}}\right)^p\operatorname{AppellF1}\left(\frac{1+m}{2b},p,-p,1+\frac{1+m}{2b},e^{2a}x^{2b},-e^{2a}x^{2b}\right)}{1+m}
\end{aligned}$$

```
[In] Integrate[(e*x)^m*Coth[a + b*Log[x]]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^(2*a)*x^(2*b))^p*((1 + E^(2*a)*x^(2*b))/(-1 + E^(2*a)*x^(
2*b)))^p*AppellF1[(1 + m)/(2*b), p, -p, 1 + (1 + m)/(2*b), E^(2*a)*x^(2*b),
 -(E^(2*a)*x^(2*b))])/((1 + m)*(1 + E^(2*a)*x^(2*b))^p)
```

Maple [F]

$$\int (ex)^m \coth\left(a+b\ln\left(x\right)\right)^p dx$$

```
[In] int((e*x)^m*coth(a+b*ln(x))^p,x)
[Out] int((e*x)^m*coth(a+b*ln(x))^p,x)
```

Fricas [F]

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int (ex)^m \coth\left(b\log\left(x\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*coth(b*log(x) + a)^p, x)
```

Sympy [F]

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int (ex)^m \coth^p\left(a+b\log\left(x\right)\right)dx$$

```
[In] integrate((e*x)**m*coth(a+b*ln(x))**p,x)
[Out] Integral((e*x)**m*coth(a + b*log(x))**p, x)
```

Maxima [F]

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int (ex)^m \coth\left(b\log\left(x\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

Giac [F]

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int (ex)^m \coth\left(b\log\left(x\right)+a\right)^p\,dx$$

```
[In] integrate((e*x)^m*coth(a+b*log(x))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*coth(b*log(x) + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^p(a+b\log(x))\,dx = \int \coth(a+b\,\ln\left(x\right))^p\,\left(e\,x\right)^m\,dx$$

```
[In] int(coth(a + b*log(x))^p*(e*x)^m,x)
[Out] int(coth(a + b*log(x))^p*(e*x)^m, x)
```

3.170 $\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx = -\frac{2^{-p}e^{-2a}(-1-e^{2a}x)^{1+p}\,\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}(1+e^{2a}x)\right)}{1+p}$$

```
[Out] -(-1-exp(2*a)*x)^(p+1)*hypergeom([p, p+1],[2+p],1/2+1/2*exp(2*a)*x)/(2^p)/e
xp(2*a)/(p+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5653, 71}

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx = -\frac{e^{-2a}2^{-p}(-e^{2a}x-1)^{p+1}\,\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}(e^{2a}x+1)\right)}{p+1}$$

```
[In] Int[Coth[a + Log[x]/2]^p,x]
```

```
[Out] -(((-1 - E^(2*a)*x)^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(2*a)
*x)/2])/(2^p*E^(2*a)*(1 + p)))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
```

```
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-1-e^{2a}x\right)^p \left(1-e^{2a}x\right)^{-p}\, dx\\ &= -\frac{2^{-p}e^{-2a}(-1-e^{2a}x)^{1+p}\,\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}(1+e^{2a}x)\right)}{1+p}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\begin{aligned}&\int \coth^p\left(a+\frac{\log(x)}{2}\right)\, dx\\ &= -\frac{2^p e^{-2a}(1+e^{2a}x)^{1-p}\left(\frac{1+e^{2a}x}{-1+e^{2a}x}\right)^{-1+p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,\frac{1}{2}-\frac{1}{2}e^{2a}x\right)}{-1+p}\end{aligned}$$

```
[In] Integrate[Coth[a + Log[x]/2]^p,x]
[Out] -((2^p*(1 + E^(2*a)*x)^(1 - p)*((1 + E^(2*a)*x)/(-1 + E^(2*a)*x))^(-1 + p)*
Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x)/2])/(E^(2*a)*(-1 + p)
))
```

Maple [F]

$$\int \coth\left(a+\frac{\ln(x)}{2}\right)^p dx$$

```
[In] int(coth(a+1/2*ln(x))^p,x)
[Out] int(coth(a+1/2*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p\left(a + \frac{\log(x)}{2}\right) dx = \int \coth\left(a + \frac{1}{2}\log(x)\right)^p dx$$

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + 1/2*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p\left(a + \frac{\log(x)}{2}\right) dx = \int \coth^p\left(a + \frac{\log(x)}{2}\right) dx$$

```
[In] integrate(coth(a+1/2*ln(x))**p,x)
[Out] Integral(coth(a + log(x)/2)**p, x)
```

Maxima [F]

$$\int \coth^p\left(a + \frac{\log(x)}{2}\right) dx = \int \coth\left(a + \frac{1}{2}\log(x)\right)^p dx$$

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a + \frac{\log(x)}{2}\right) dx = \int \coth\left(a + \frac{1}{2}\log(x)\right)^p dx$$

```
[In] integrate(coth(a+1/2*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 1/2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{2}\right)\,dx=\int \coth\left(a+\frac{\ln(x)}{2}\right)^p\,dx$$

```
[In] int(coth(a + log(x)/2)^p,x)
[Out] int(coth(a + log(x)/2)^p, x)
```

3.171 $\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 108

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx = e^{-4a}\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\left(1-e^{2a}\sqrt{x}\right)^{1-p} - \frac{2^{1-p}e^{-4a}p\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt{x}\right)\right)}{1+p}$$

```
[Out] -2^(1-p)*p*hypergeom([p, p+1],[2+p],1/2+1/2*exp(2*a)*x^(1/2))*(-1-exp(2*a)*
x^(1/2))^(p+1)/exp(4*a)/(p+1)+(-1-exp(2*a)*x^(1/2))^(p+1)*(1-exp(2*a)*x^(1/
2))^(1-p)/exp(4*a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5653, 383, 81, 71}

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx = e^{-4a}\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt{x}\right)^{1-p} - \frac{e^{-4a}2^{1-p}p\left(-e^{2a}\sqrt{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\left(e^{2a}\sqrt{x}+1\right)\right)}{p+1}$$

```
[In] Int[Coth[a + Log[x]/4]^p,x]
```

```
[Out] ((-1 - E^(2*a)*Sqrt[x])^(1 + p)*(1 - E^(2*a)*Sqrt[x])^(1 - p))/E^(4*a) - (2
^(1 - p)*p*(-1 - E^(2*a)*Sqrt[x])^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p
, (1 + E^(2*a)*Sqrt[x])/2])/(E^(4*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-1-e^{2a}\sqrt{x}\right)^p \left(1-e^{2a}\sqrt{x}\right)^{-p}\,dx\\
&= 2\text{Subst}\left(\int x\left(-1-e^{2a}x\right)^p \left(1-e^{2a}x\right)^{-p}\,dx, x, \sqrt{x}\right)\\
&= e^{-4a}\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\left(1-e^{2a}\sqrt{x}\right)^{1-p}\\
&\quad + \left(2e^{-2a}p\right)\text{Subst}\left(\int \left(-1-e^{2a}x\right)^p \left(1-e^{2a}x\right)^{-p}\,dx, x, \sqrt{x}\right)\\
&= e^{-4a}\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\left(1-e^{2a}\sqrt{x}\right)^{1-p}\\
&\quad - \frac{2^{1-p}e^{-4a}p\left(-1-e^{2a}\sqrt{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt{x}\right)\right)}{1+p}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx$$
$$=\frac{e^{-4a}\left(1+e^{2a}\sqrt{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt{x}}{-1+e^{2a}\sqrt{x}}\right)^{-1+p}\left((-1+p)\left(1+e^{2a}\sqrt{x}\right)^{1+p}-2^{1+p}p\,\text{Hypergeometric2F1}\left(1-p,-p,2\right.\right.}{-1+p}$$

```
[In] Integrate[Coth[a + Log[x]/4]^p,x]
```

```
[Out] ((1 + E^(2*a)*Sqrt[x])^(1 - p)*((1 + E^(2*a)*Sqrt[x])/(-1 + E^(2*a)*Sqrt[x]
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*Sqrt[x])^(1 + p) - 2^(1 + p)*p*Hypergeom
etric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*Sqrt[x])/2]))/(E^(4*a)*(-1 + p))
```

Maple [F]

$$\int \coth\left(a+\frac{\ln(x)}{4}\right)^p dx$$

```
[In] int(coth(a+1/4*ln(x))^p,x)
```

```
[Out] int(coth(a+1/4*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{1}{4}\log(x)\right)^p dx$$

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(a + 1/4*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth^p\left(a+\frac{\log(x)}{4}\right)dx$$

```
[In] integrate(coth(a+1/4*ln(x))**p,x)
```

```
[Out] Integral(coth(a + log(x)/4)**p, x)
```

Maxima [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{1}{4}\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{1}{4}\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(coth(a+1/4*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 1/4*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{4}\right)\,dx=\int \coth\left(a+\frac{\ln\left(x\right)}{4}\right)^{p}\,dx$$

```
[In] int(coth(a + log(x)/4)^p,x)
[Out] int(coth(a + log(x)/4)^p, x)
```

3.172 $\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 162

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx = e^{-6a}p\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x} - \frac{2^{-p}e^{-6a}(1+2p^2)\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt[3]{x}\right)\right)}{1+p}$$

```
[Out] p*(-1-exp(2*a)*x^(1/3))^(p+1)*(1-exp(2*a)*x^(1/3))^(1-p)/exp(6*a)+(-1-exp(2
*a)*x^(1/3))^(p+1)*(1-exp(2*a)*x^(1/3))^(1-p)*x^(1/3)/exp(4*a)-(2*p^2+1)*(-
1-exp(2*a)*x^(1/3))^(p+1)*hypergeom([p, p+1],[2+p],1/2+1/2*exp(2*a)*x^(1/3)
)/(2^p)/exp(6*a)/(p+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5653, 383, 92, 81, 71}

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx = -\frac{e^{-6a}2^{-p}(2p^2+1)\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\left(e^{2a}\sqrt[3]{x}+1\right)\right)}{p+1} + e^{-6a}p\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\sqrt[3]{x}\left(-e^{2a}\sqrt[3]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}$$

```
[In] Int[Coth[a + Log[x]/6]^p,x]
```

```
[Out] (p*(-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p))/E^(6*a) +
((-1 - E^(2*a)*x^(1/3))^(1 + p)*(1 - E^(2*a)*x^(1/3))^(1 - p)*x^(1/3))/E^(4
*a) - ((1 + 2*p^2)*(-1 - E^(2*a)*x^(1/3))^(1 + p)*Hypergeometric2F1[p, 1 +
p, 2 + p, (1 + E^(2*a)*x^(1/3))/2])/(2^p*E^(6*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\text{integral} = \int \left(-1 - e^{2a}\sqrt[3]{x}\right)^p \left(1 - e^{2a}\sqrt[3]{x}\right)^{-p} \, dx$$

$$= 3\text{Subst}\left(\int x^2\left(-1-e^{2a}x\right)^p\left(1-e^{2a}x\right)^{-p}\,dx,x,\sqrt[3]{x}\right)$$

$$= e^{-4a}\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x} + e^{-4a}\text{Subst}\left(\int\left(-1-e^{2a}x\right)^p\left(1-e^{2a}x\right)^{-p}\left(1+2e^{2a}px\right)\,dx,x,\sqrt[3]{x}\right)$$

$$= e^{-6a}p\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x} + \left(e^{-4a}\left(1+2p^2\right)\right)\text{Subst}\left(\int\left(-1-e^{2a}x\right)^p\left(1-e^{2a}x\right)^{-p}\,dx,x,\sqrt[3]{x}\right)$$

$$= e^{-6a}p\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}+e^{-4a}\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[3]{x}\right)^{1-p}\sqrt[3]{x} - \frac{2^{-p}e^{-6a}(1+2p^2)\left(-1-e^{2a}\sqrt[3]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt[3]{x}\right)\right)}{1+p}$$

Mathematica [A] (warning: unable to verify)

Time = 0.88 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.88

$$\int\coth^p\left(a+\frac{\log(x)}{6}\right)\,dx = \frac{e^{-6a}\left(1+e^{2a}\sqrt[3]{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt[3]{x}}{-1+e^{2a}\sqrt[3]{x}}\right)^{-1+p}\left(\left(-1+p\right)\left(1+e^{2a}\sqrt[3]{x}\right)^{1+p}\left(p+e^{2a}\sqrt[3]{x}\right)-2^p(1+2p^2)\,\text{Hypergeom}\right.}{-1+p}$$

```
[In] Integrate[Coth[a + Log[x]/6]^p,x]
```

```
[Out] ((1 + E^(2*a)*x^(1/3))^(1 - p)*((1 + E^(2*a)*x^(1/3))/(-1 + E^(2*a)*x^(1/3)
))^(-1 + p)*((-1 + p)*(1 + E^(2*a)*x^(1/3))^(1 + p)*(p + E^(2*a)*x^(1/3)) -
 2^p*(1 + 2*p^2)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^(2*a)*x^(1/3)
)/2]))/(E^(6*a)*(-1 + p))
```

Maple [F]

$$\int\coth\left(a+\frac{\ln(x)}{6}\right)^p\,dx$$

```
[In] int(coth(a+1/6*ln(x))^p,x)
```

```
[Out] int(coth(a+1/6*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth\left(a+\frac{1}{6}\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + 1/6*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth^{p}\left(a+\frac{\log\left(x\right)}{6}\right)\,dx$$

```
[In] integrate(coth(a+1/6*ln(x))**p,x)
[Out] Integral(coth(a + log(x)/6)**p, x)
```

Maxima [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth\left(a+\frac{1}{6}\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth\left(a+\frac{1}{6}\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(coth(a+1/6*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 1/6*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{6}\right)\,dx=\int \coth\left(a+\frac{\ln\left(x\right)}{6}\right)^p\,dx$$

```
[In] int(coth(a + log(x)/6)^p,x)
[Out] int(coth(a + log(x)/6)^p, x)
```

3.173 $\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 194

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\frac{1}{3}e^{-12a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}\left(3+2p^2\right)+2e^{6a}p\sqrt[4]{x}\right)+e^{-4a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x}-\frac{2^{2-p}e^{-8a}p(2+p^2)\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt[4]{x}\right)\right)}{3(1+p)}$$

```
[Out] 1/3*(-1-exp(2*a)*x^(1/4))^(p+1)*(1-exp(2*a)*x^(1/4))^(1-p)*(exp(4*a)*(2*p^2
+3)+2*exp(6*a)*p*x^(1/4))/exp(12*a)-1/3*2^(2-p)*p*(p^2+2)*(-1-exp(2*a)*x^(1
/4))^(p+1)*hypergeom([p, p+1],[2+p],1/2+1/2*exp(2*a)*x^(1/4))/exp(8*a)/(p+1
)+(-1-exp(2*a)*x^(1/4))^(p+1)*(1-exp(2*a)*x^(1/4))^(1-p)*x^(1/2)/exp(4*a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5653, 383, 102, 152, 71}

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx$$
$$=-\frac{e^{-8a}2^{2-p}p(p^2+2)\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\text{Hypergeometric2F1}\left(p,p+1,p+2,\frac{1}{2}\left(e^{2a}\sqrt[4]{x}+1\right)\right)}{3(p+1)}+\frac{1}{3}e^{-12a}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(e^{4a}\left(2p^2+3\right)+2e^{6a}p\sqrt[4]{x}\right)\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}+e^{-4a}\sqrt{x}\left(-e^{2a}\sqrt[4]{x}-1\right)^{p+1}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}$$

```
[In] Int[Coth[a + Log[x]/8]^p,x]
```

```
[Out] ((-1 - E^(2*a)*x^(1/4))^(1 + p)*(1 - E^(2*a)*x^(1/4))^(1 - p)*(E^(4*a)*(3 +
 2*p^2) + 2*E^(6*a)*p*x^(1/4)))/(3*E^(12*a)) + ((-1 - E^(2*a)*x^(1/4))^(1 +
 p)*(1 - E^(2*a)*x^(1/4))^(1 - p)*Sqrt[x])/E^(4*a) - (2^(2 - p)*p*(2 + p^2)
*(-1 - E^(2*a)*x^(1/4))^(1 + p)*Hypergeometric2F1[p, 1 + p, 2 + p, (1 + E^(
2*a)*x^(1/4))/2])/(3*E^(8*a)*(1 + p))
```

Rule 71

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-1-e^{2a}\sqrt[4]{x}\right)^p \left(1-e^{2a}\sqrt[4]{x}\right)^{-p}\,dx \\
&= 4\text{Subst}\left(\int x^3\left(-1-e^{2a}x\right)^p\left(1-e^{2a}x\right)^{-p}\,dx, x, \sqrt[4]{x}\right) \\
&= e^{-4a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x} \\
&\quad + e^{-4a}\text{Subst}\left(\int x\left(-1-e^{2a}x\right)^p\left(1-e^{2a}x\right)^{-p}\left(2+2e^{2a}px\right)\,dx, x, \sqrt[4]{x}\right) \\
&= \frac{1}{3}e^{-12a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}\left(3+2p^2\right)+2e^{6a}p\sqrt[4]{x}\right) \\
&\quad + e^{-4a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x} \\
&\quad + \frac{1}{3}\left(4e^{-6a}p\left(2+p^2\right)\right)\text{Subst}\left(\int\left(-1-e^{2a}x\right)^p\left(1-e^{2a}x\right)^{-p}\,dx, x, \sqrt[4]{x}\right) \\
&= \frac{1}{3}e^{-12a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\left(e^{4a}\left(3+2p^2\right)+2e^{6a}p\sqrt[4]{x}\right) \\
&\quad + e^{-4a}\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\left(1-e^{2a}\sqrt[4]{x}\right)^{1-p}\sqrt{x} \\
&\quad - \frac{2^{2-p}e^{-8a}p\left(2+p^2\right)\left(-1-e^{2a}\sqrt[4]{x}\right)^{1+p}\text{Hypergeometric2F1}\left(p,1+p,2+p,\frac{1}{2}\left(1+e^{2a}\sqrt[4]{x}\right)\right)}{3(1+p)}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 1.26 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.15

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)dx$$
$$= \frac{e^{-8a}\left(1+e^{2a}\sqrt[4]{x}\right)^{1-p}\left(\frac{1+e^{2a}\sqrt[4]{x}}{-1+e^{2a}\sqrt[4]{x}}\right)^{-1+p}\left(-2^{3+p}p\,\text{Hypergeometric2F1}\left(-2-p,1-p,2-p,\frac{1}{2}-\frac{1}{2}e^{2a}\sqrt[4]{x}\right)+\right.}{}$$

```
[In] Integrate[Coth[a + Log[x]/8]^p,x]
```

```
[Out] ((1 + E^(2*a)*x^(1/4))^(1 - p)*((1 + E^(2*a)*x^(1/4))/(-1 + E^(2*a)*x^(1/4)
))^(-1 + p)*(-(2^(3 + p)*p*Hypergeometric2F1[-2 - p, 1 - p, 2 - p, 1/2 - (E
^(2*a)*x^(1/4))/2]) + 2^(2 + p)*(-1 + 2*p)*Hypergeometric2F1[-1 - p, 1 - p,
 2 - p, 1/2 - (E^(2*a)*x^(1/4))/2] + (-1 + p)*(E^(4*a)*(1 + E^(2*a)*x^(1/4)
)^(1 + p)*Sqrt[x] - 2^(1 + p)*Hypergeometric2F1[1 - p, -p, 2 - p, 1/2 - (E^
(2*a)*x^(1/4))/2])))/(E^(8*a)*(-1 + p))
```

Maple [F]

$$\int \coth\left(a+\frac{\ln(x)}{8}\right)^{p} dx$$

```
[In] int(coth(a+1/8*ln(x))^p,x)
[Out] int(coth(a+1/8*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^{p}\left(a+\frac{\log(x)}{8}\right) dx = \int \coth\left(a+\frac{1}{8}\log(x)\right)^{p} dx$$

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + 1/8*log(x))^p, x)
```

Sympy [F]

$$\int \coth^{p}\left(a+\frac{\log(x)}{8}\right) dx = \int \coth^{p}\left(a+\frac{\log(x)}{8}\right) dx$$

```
[In] integrate(coth(a+1/8*ln(x))**p,x)
[Out] Integral(coth(a + log(x)/8)**p, x)
```

Maxima [F]

$$\int \coth^{p}\left(a+\frac{\log(x)}{8}\right) dx = \int \coth\left(a+\frac{1}{8}\log(x)\right)^{p} dx$$

```
[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 1/8*log(x))^p, x)
```

Giac [F]

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth\left(a+\frac{1}{8}\log\left(x\right)\right)^p\,dx$$

[In] integrate(coth(a+1/8*log(x))^p,x, algorithm="giac")

[Out] integrate(coth(a + 1/8*log(x))^p, x)

Mupad [F(-1)]

Timed out.

$$\int \coth^p\left(a+\frac{\log(x)}{8}\right)\,dx=\int \coth\left(a+\frac{\ln\left(x\right)}{8}\right)^p\,dx$$

[In] int(coth(a + log(x)/8)^p,x)

[Out] int(coth(a + log(x)/8)^p, x)

3.174 $\int \coth^p(a+\log(x))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 61

$$\int \coth^p(a+\log(x))\,dx = x\left(-1-e^{2a}x^2\right)^p \left(1+e^{2a}x^2\right)^{-p} \text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)$$

```
[Out] x*(-1-exp(2*a)*x^2)^p*AppellF1(1/2,p,-p,3/2,exp(2*a)*x^2,-exp(2*a)*x^2)/((1
+exp(2*a)*x^2)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5653, 441, 440}

$$\int \coth^p(a+\log(x))\,dx = x\left(-e^{2a}x^2-1\right)^p \left(e^{2a}x^2+1\right)^{-p} \text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)$$

```
[In] Int[Coth[a + Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^2)^p*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2*a)*x^
2)])/(1 + E^(2*a)*x^2)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (-1-e^{2a}x^2)^p (1-e^{2a}x^2)^{-p}\,dx\\ &= \left((-1-e^{2a}x^2)^p (1+e^{2a}x^2)^{-p}\right)\int (1-e^{2a}x^2)^{-p}(1+e^{2a}x^2)^p\,dx\\ &= x(-1-e^{2a}x^2)^p (1+e^{2a}x^2)^{-p}\,\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.

Time = 0.63 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \coth^p(a+\log(x))\,dx$$

$$= \frac{3x\left(\frac{1+e^{2a}x^2}{-1+e^{2a}x^2}\right)^p \text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)}{3\,\text{AppellF1}\left(\frac{1}{2},p,-p,\frac{3}{2},e^{2a}x^2,-e^{2a}x^2\right)+2e^{2a}px^2\left(\text{AppellF1}\left(\frac{3}{2},p,1-p,\frac{5}{2},e^{2a}x^2,-e^{2a}x^2\right)+\text{AppellF1}\left(\frac{3}{2},1\right.\right.}$$

```
[In] Integrate[Coth[a + Log[x]]^p,x]
```

```
[Out] (3*x*((1 + E^(2*a)*x^2)/(-1 + E^(2*a)*x^2))^p*AppellF1[1/2, p, -p, 3/2, E^(
2*a)*x^2, -(E^(2*a)*x^2)])/(3*AppellF1[1/2, p, -p, 3/2, E^(2*a)*x^2, -(E^(2
*a)*x^2)] + 2*E^(2*a)*p*x^2*(AppellF1[3/2, p, 1 - p, 5/2, E^(2*a)*x^2, -(E^
(2*a)*x^2)] + AppellF1[3/2, 1 + p, -p, 5/2, E^(2*a)*x^2, -(E^(2*a)*x^2)]))
```

Maple [F]

$$\int \coth(a + \ln(x))^p \, dx$$

```
[In] int(coth(a+ln(x))^p,x)
[Out] int(coth(a+ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p(a + \log(x)) \, dx = \int \coth(a + \log(x))^p \, dx$$

```
[In] integrate(coth(a+log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + log(x))^p, x)
```

Sympy [F]

$$\int \coth^p(a + \log(x)) \, dx = \int \coth^p(a + \log(x)) \, dx$$

```
[In] integrate(coth(a+ln(x))**p,x)
[Out] Integral(coth(a + log(x))**p, x)
```

Maxima [F]

$$\int \coth^p(a + \log(x)) \, dx = \int \coth(a + \log(x))^p \, dx$$

```
[In] integrate(coth(a+log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + log(x))^p, x)
```

Giac [F]

$$\int \coth^p(a + \log(x))\,dx = \int \coth\left(a + \log\left(x\right)\right)^p\,dx$$

```
[In] integrate(coth(a+log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a + \log(x))\,dx = \int \coth(a + \ln\left(x\right))^p\,dx$$

```
[In] int(coth(a + log(x))^p,x)
[Out] int(coth(a + log(x))^p, x)
```

3.175 $\int \coth^p(a+2\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 61

$$\int \coth^p(a+2\log(x))\,dx = x\left(-1-e^{2a}x^4\right)^p\left(1+e^{2a}x^4\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)$$

```
[Out] x*(-1-exp(2*a)*x^4)^p*AppellF1(1/4,p,-p,5/4,exp(2*a)*x^4,-exp(2*a)*x^4)/((1
+exp(2*a)*x^4)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5653, 441, 440}

$$\int \coth^p(a+2\log(x))\,dx = x\left(-e^{2a}x^4-1\right)^p\left(e^{2a}x^4+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)$$

```
[In] Int[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^4)^p*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2*a)*x^
4)])/(1 + E^(2*a)*x^4)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(-1-e^{2a}x^4\right)^p \left(1-e^{2a}x^4\right)^{-p}\,dx\\ &= \left(\left(-1-e^{2a}x^4\right)^p \left(1+e^{2a}x^4\right)^{-p}\right)\int \left(1-e^{2a}x^4\right)^{-p} \left(1+e^{2a}x^4\right)^p\,dx\\ &= x\left(-1-e^{2a}x^4\right)^p \left(1+e^{2a}x^4\right)^{-p}\,\text{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.

Time = 0.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \coth^p(a+2\log(x))\,dx$$

$$= \frac{5x\left(\frac{1+e^{2a}x^4}{-1+e^{2a}x^4}\right)^p \text{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)}{5\,\text{AppellF1}\left(\frac{1}{4},p,-p,\frac{5}{4},e^{2a}x^4,-e^{2a}x^4\right)+4e^{2a}px^4\left(\text{AppellF1}\left(\frac{5}{4},p,1-p,\frac{9}{4},e^{2a}x^4,-e^{2a}x^4\right)+\text{AppellF1}\left(\frac{5}{4},1\right.\right.}$$

```
[In] Integrate[Coth[a + 2*Log[x]]^p,x]
```

```
[Out] (5*x*((1 + E^(2*a)*x^4)/(-1 + E^(2*a)*x^4))^p*AppellF1[1/4, p, -p, 5/4, E^(
2*a)*x^4, -(E^(2*a)*x^4)])/(5*AppellF1[1/4, p, -p, 5/4, E^(2*a)*x^4, -(E^(2
*a)*x^4)] + 4*E^(2*a)*p*x^4*(AppellF1[5/4, p, 1 - p, 9/4, E^(2*a)*x^4, -(E^
(2*a)*x^4)] + AppellF1[5/4, 1 + p, -p, 9/4, E^(2*a)*x^4, -(E^(2*a)*x^4)]))
```

Maple [F]

$$\int \coth\left(a+2\ln\left(x\right)\right)^{p}dx$$

```
[In] int(coth(a+2*ln(x))^p,x)
[Out] int(coth(a+2*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + 2*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth^{p}\left(a+2\log\left(x\right)\right)dx$$

```
[In] integrate(coth(a+2*ln(x))**p,x)
[Out] Integral(coth(a + 2*log(x))**p, x)
```

Maxima [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth\left(a+2\,\log\left(x\right)\right)^{p}\,dx$$

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 2*log(x))^p, x)
```

Giac [F]

$$\int \coth^p(a+2\log(x))\,dx = \int \coth\left(a+2\,\log\left(x\right)\right)^p\,dx$$

```
[In] integrate(coth(a+2*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 2*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a+2\log(x))\,dx = \int \mathrm{coth}(a+2\,\ln\left(x\right))^p\,dx$$

```
[In] int(coth(a + 2*log(x))^p,x)
[Out] int(coth(a + 2*log(x))^p, x)
```

3.176 $\int \coth^p(a+3\log(x))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 61

$$\int \coth^p(a+3\log(x))\,dx = x\left(-1-e^{2a}x^6\right)^p\left(1+e^{2a}x^6\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)$$

```
[Out] x*(-1-exp(2*a)*x^6)^p*AppellF1(1/6,p,-p,7/6,exp(2*a)*x^6,-exp(2*a)*x^6)/((1
+exp(2*a)*x^6)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5653, 441, 440}

$$\int \coth^p(a+3\log(x))\,dx = x\left(-e^{2a}x^6-1\right)^p\left(e^{2a}x^6+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)$$

```
[In] Int[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] (x*(-1 - E^(2*a)*x^6)^p*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2*a)*x^
6)])/(1 + E^(2*a)*x^6)^p
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5653

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Int[(-1 - E^(2*
a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p, x] /; FreeQ[{a, b, d, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-1-e^{2a}x^6\right)^p \left(1-e^{2a}x^6\right)^{-p}\,dx\\
&= \left(\left(-1-e^{2a}x^6\right)^p \left(1+e^{2a}x^6\right)^{-p}\right)\int \left(1-e^{2a}x^6\right)^{-p}\left(1+e^{2a}x^6\right)^p\,dx\\
&= x\left(-1-e^{2a}x^6\right)^p \left(1+e^{2a}x^6\right)^{-p}\,\text{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)
\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 171 vs. $2(61) = 122$.

Time = 0.69 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.80

$$\int \coth^p(a+3\log(x))\,dx$$

$$= \frac{7x\left(\frac{1+e^{2a}x^6}{-1+e^{2a}x^6}\right)^p \text{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)}{7\,\text{AppellF1}\left(\frac{1}{6},p,-p,\frac{7}{6},e^{2a}x^6,-e^{2a}x^6\right)+6e^{2a}px^6\left(\text{AppellF1}\left(\frac{7}{6},p,1-p,\frac{13}{6},e^{2a}x^6,-e^{2a}x^6\right)+\text{AppellF1}\left(\frac{7}{6},\right.\right.}$$

```
[In] Integrate[Coth[a + 3*Log[x]]^p,x]
```

```
[Out] (7*x*((1 + E^(2*a)*x^6)/(-1 + E^(2*a)*x^6))^p*AppellF1[1/6, p, -p, 7/6, E^(
2*a)*x^6, -(E^(2*a)*x^6)])/(7*AppellF1[1/6, p, -p, 7/6, E^(2*a)*x^6, -(E^(2
*a)*x^6)] + 6*E^(2*a)*p*x^6*(AppellF1[7/6, p, 1 - p, 13/6, E^(2*a)*x^6, -(E
^(2*a)*x^6)] + AppellF1[7/6, 1 + p, -p, 13/6, E^(2*a)*x^6, -(E^(2*a)*x^6)])
)
```

Maple [F]

$$\int \coth \left(a+3 \ln \left(x\right)\right)^{p}d x$$

```
[In] int(coth(a+3*ln(x))^p,x)
[Out] int(coth(a+3*ln(x))^p,x)
```

Fricas [F]

$$\int \coth^p(a+3\log(x))\,dx=\int \coth \left(a+3\, \log \left(x\right)\right)^{p}\, dx$$

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="fricas")
[Out] integral(coth(a + 3*log(x))^p, x)
```

Sympy [F]

$$\int \coth^p(a+3\log(x))\,dx=\int \coth^{p}{\left(a+3\log{\left(x \right)} \right)}\,dx$$

```
[In] integrate(coth(a+3*ln(x))**p,x)
[Out] Integral(coth(a + 3*log(x))**p, x)
```

Maxima [F]

$$\int \coth^p(a+3\log(x))\,dx=\int \coth \left(a+3\, \log \left(x\right)\right)^{p}\, dx$$

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="maxima")
[Out] integrate(coth(a + 3*log(x))^p, x)
```

Giac [F]

$$\int \coth^p(a + 3\log(x))\, dx = \int \coth\left(a + 3\, \log\left(x\right)\right)^p\, dx$$

```
[In] integrate(coth(a+3*log(x))^p,x, algorithm="giac")
[Out] integrate(coth(a + 3*log(x))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(a + 3\log(x))\, dx = \int \coth(a + 3\, \ln\left(x\right))^p\, dx$$

```
[In] int(coth(a + 3*log(x))^p,x)
[Out] int(coth(a + 3*log(x))^p, x)
```

3.177 $\int x^3 \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \frac{x^4}{4} - \frac{1}{2}x^4 \text{Hypergeometric2F1}\left(1, \frac{2}{bdn}, 1 + \frac{2}{bdn}, e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/4*x^4-1/2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*
d))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5659, 5657, 470, 371}

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \frac{x^4}{4} - \frac{1}{2}x^4 \text{Hypergeometric2F1}\left(1, \frac{2}{bdn}, 1 + \frac{2}{bdn}, e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^4/4 - (x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*a*d)*(c*x^
n)^(2*b*d)])/2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int x^{-1+\frac{4}{n}}\coth(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{4}{n}}\left(-1-e^{2ad}x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^4}{4}-\frac{\left(2x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{4}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^4}{4}-\frac{1}{2}x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. $2(58) = 116$.

Time = 7.48 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.41

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = -\frac{x^4\left(2e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1,1+\frac{2}{bdn},2+\frac{2}{bdn},e^{2d(a+b\log(cx^n))}\right)+(2+bdn)\left(\coth(d(a+b\log\right.\right.}{}$$

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^4*(2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2
+ 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(Coth[d*(a + b*Log[c
*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 2/
(b*d*n), 1 + 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x
^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(8 + 4*
b*d*n))
```

Maple [F]

$$\int x^3 \coth(d(a+b\ln(c\,x^n)))\,dx$$

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \int x^3 \coth(ad+bd\log(cx^n))\,dx$$

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**3*coth(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] 1/4*x^4 - integrate(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^3/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)\,dx$$

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth(d(a+b\log(cx^n)))\,dx = \int x^3 \coth(d(a+b\ln(c\,x^n)))\,dx$$

```
[In] int(x^3*coth(d*(a + b*log(c*x^n))),x)
[Out] int(x^3*coth(d*(a + b*log(c*x^n))), x)
```

3.178 $\int x^2 \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int x^2 \coth(d(a+b\log(cx^n)))\,dx = \frac{x^3}{3} - \frac{2}{3}x^3 \text{Hypergeometric2F1}\left(1, \frac{3}{2bdn}, 1 + \frac{3}{2bdn}, e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/3*x^3-2/3*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*(c*x^n)^(
2*b*d))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5659, 5657, 470, 371}

$$\int x^2 \coth(d(a+b\log(cx^n)))\,dx = \frac{x^3}{3} - \frac{2}{3}x^3 \text{Hypergeometric2F1}\left(1, \frac{3}{2bdn}, 1 + \frac{3}{2bdn}, e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^3/3 - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*a*d)
*(c*x^n)^(2*b*d)])/3
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\coth(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}\left(-1-e^{2ad}x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^3}{3}-\frac{\left(2x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^3}{3}-\frac{2}{3}x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. $2(62) = 124$.

Time = 5.20 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.34

$$\int x^2 \coth(d(a + b\log(cx^n))) \, dx = -\frac{x^3\left(3e^{2d(a+b\log(cx^n))}\text{Hypergeometric2F1}\left(1, 1 + \frac{3}{2bdn}, 2 + \frac{3}{2bdn}, e^{2d(a+b\log(cx^n))}\right) + (3 + 2bdn)\left(\coth(d(a + \right.\right.}{}$$

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((x^3*(3*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n),
2 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(Coth[d*(a + b
*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1
[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
 b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
))/(9 + 6*b*d*n))
```

Maple [F]

$$\int x^2 \coth(d(a + b\ln(c\,x^n)))\,dx$$

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(x^2*coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int x^2 \coth(d(a + b\log(cx^n))) \, dx = \int x^2 \coth((b\log(cx^n) + a)d) \, dx$$

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int x^2 \coth\left(d(a + b\log\left(cx^n\right))\right) \,dx = \int x^2 \coth\left(ad + bd\log\left(cx^n\right)\right) dx$$

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n))),x)
[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int x^2 \coth\left(d(a + b\log\left(cx^n\right))\right) \,dx = \int x^2 \coth\left((b\log\left(cx^n\right) + a)d\right) \,dx$$

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
[Out] 1/3*x^3 - integrate(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrat
e(x^2/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int x^2 \coth\left(d(a + b\log\left(cx^n\right))\right) \,dx = \int x^2 \coth\left((b\log\left(cx^n\right) + a)d\right) \,dx$$

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth\left(d(a + b\log\left(cx^n\right))\right) \,dx = \int x^2 \,\mathrm{coth}(d\left(a + b\,\ln\left(c\,x^n\right)\right)) \,dx$$

```
[In] int(x^2*coth(d*(a + b*log(c*x^n))),x)
[Out] int(x^2*coth(d*(a + b*log(c*x^n))), x)
```

3.179 $\int x \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int x\coth(d(a+b\log(cx^n)))\,dx=\frac{x^2}{2}-x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] 1/2*x^2-x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5659, 5657, 470, 371}

$$\int x\coth(d(a+b\log(cx^n)))\,dx=\frac{x^2}{2}-x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x^2/2 - x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\coth(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}\left(-1-e^{2ad}x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^2}{2}-\frac{\left(2x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= \frac{x^2}{2}-x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 193 vs. 2(54) = 108.

Time = 7.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.57

$$\int x\coth\left(d(a+b\log\left(cx^n\right))\right)\,dx = -\frac{x^2\left(e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{bdn},2+\frac{1}{bdn},e^{2d(a+b\log(cx^n))}\right)+(1+bdn)\left(\coth\left(d(a+b\log\right.\right.\right.}{}$$

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])],x]
```

[Out] -((x^2*(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 + 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(Coth[d*(a + b*Log[c*x ^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[1, 1/(b *d*n), 1 + 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])))/(2 + 2*b* d*n))

Maple [F]

$$\int x \coth \left(d(a+b\ln \left(c\,x^{n}\right))\right) dx$$

[In] int(x*coth(d*(a+b*ln(c*x^n))),x)

[Out] int(x*coth(d*(a+b*ln(c*x^n))),x)

Fricas [F]

$$\int x \coth \left(d(a+b\log \left(cx^{n}\right))\right)\, dx = \int x \coth \left((b\log \left(cx^{n}\right)+a)d\right)\, dx$$

[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")

[Out] integral(x*coth(b*d*log(c*x^n) + a*d), x)

Sympy [F]

$$\int x \coth \left(d(a+b\log \left(cx^{n}\right))\right)\, dx = \int x \coth \left(ad+bd\log \left(cx^{n}\right)\right) dx$$

[In] integrate(x*coth(d*(a+b*ln(c*x**n))),x)

[Out] Integral(x*coth(a*d + b*d*log(c*x**n)), x)

Maxima [F]

$$\int x \coth \left(d(a+b\log \left(cx^{n}\right))\right)\, dx = \int x \coth \left((b\log \left(cx^{n}\right)+a)d\right)\, dx$$

[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")

[Out] 1/2*x^2 - integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(x/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)

Giac [F]

$$\int x \coth (d(a + b \log (cx^n))) \ dx = \int x \coth ((b \log (cx^n) + a)d) \ dx$$

```
[In] integrate(x*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(x*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth (d(a + b \log (cx^n))) \ dx = \int x \coth(d (a + b \ln (c x^n))) \ dx$$

```
[In] int(x*coth(d*(a + b*log(c*x^n))),x)
[Out] int(x*coth(d*(a + b*log(c*x^n))), x)
```

3.180 $\int \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \coth(d(a+b\log(cx^n)))\,dx = x-2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] x-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5655, 5657, 470, 371}

$$\int \coth(d(a+b\log(cx^n)))\,dx = x-2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] x - 2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n
)^(2*b*d)]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5655

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\coth(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}\left(-1-e^{2ad}x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= x-\frac{\left(2x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{n}\\
&= x-2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 198 vs. $2(52) = 104$.

Time = 8.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 3.81

$$\begin{aligned}
&\int \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx\\
&= -\frac{e^{2d(a+b\log(cx^n))}x\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bdn},2+\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right)}{1+2bdn}\\
&\quad - x\Bigg(\coth\left(d(a+b\log\left(cx^n\right))\right)-\coth\left(d(a-bn\log(x)+b\log\left(cx^n\right))\right)\\
&\quad + \text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right)\\
&\quad + \text{csch}(d(a+b\log\left(cx^n\right)))\,\text{csch}(d(a-bn\log(x)+b\log\left(cx^n\right)))\sinh(bdn\log(x))\Bigg)
\end{aligned}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] -((E^(2*d*(a + b*Log[c*x^n]))*x*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 + 1
/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])/(1 + 2*b*d*n)) - x*(Coth[d*(a + b*
Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] + Hypergeometric2F1[
1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a +
b*Log[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])
```

Maple [F]

$$\int \coth \left(d(a+b\ln \left(c\,x^{n}\right))\right) dx$$

```
[In] int(coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int \coth \left(d(a+b\log \left(cx^n\right))\right)\, dx = \int \coth \left((b\log \left(cx^n\right)+a)d\right)\, dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int \coth \left(d(a+b\log \left(cx^n\right))\right)\, dx = \int \coth \left(d(a+b\log \left(cx^n\right))\right) dx$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral(coth(d*(a + b*log(c*x**n))), x)
```

Maxima [F]

$$\int \coth \left(d(a+b\log \left(cx^n\right))\right)\, dx = \int \coth \left((b\log \left(cx^n\right)+a)d\right)\, dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] x - integrate(1/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1), x) + integrate(1/(c^(
b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int \coth(d(a + b\log(cx^n)))\, dx = \int \coth((b\log(cx^n) + a)d)\, dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(d(a + b\log(cx^n)))\, dx = \int \coth(d\,(a + b\,\ln(c\,x^n)))\, dx$$

```
[In] int(coth(d*(a + b*log(c*x^n))),x)
[Out] int(coth(d*(a + b*log(c*x^n))), x)
```

3.181 $\int \frac{\coth(d(a+b\log(cx^n)))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x} dx = \frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn}$$

```
[Out] ln(sinh(a*d+b*d*ln(c*x^n)))/b/d/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3556}

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x} dx = \frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn}$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x,x]
```

```
[Out] Log[Sinh[a*d + b*d*Log[c*x^n]]]/(b*d*n)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \coth(d(a+bx))\,dx, x, \log(cx^n)\right)}{n} \\ &= \frac{\log(\sinh(ad+bd\log(cx^n)))}{bdn} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{\coth (d(a+b\log (cx^n)))}{x} \, dx = \frac{\log (\cosh (d(a+b\log (cx^n))))+\log (\tanh (ad+bd\log (cx^n)))}{bdn}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x,x]
[Out] (Log[Cosh[d*(a + b*Log[c*x^n])]] + Log[Tanh[a*d + b*d*Log[c*x^n]]])/(b*d*n)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{\ln(\sinh(d(a+b\ln(c\,x^n))))}{nbd}$
default	$\frac{\ln(\sinh(d(a+b\ln(c\,x^n))))}{nbd}$
parallelrisch	$\frac{-bd\ln(c\,x^n)+\ln(\tanh(d(a+b\ln(c\,x^n))))-\ln(1-\tanh(d(a+b\ln(c\,x^n))))}{ndb}$
risch	$\ln(x)-\frac{2a}{bn}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\,\mathrm{csgn}(ic}{n}$

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b/d*ln(sinh(d*(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.04

$$\int \frac{\coth (d(a+b\log (cx^n)))}{x} \, dx = -\frac{bdn\log (x)-\log \left(\frac{2\sinh(bdn\log(x)+bd\log(c)+ad)}{\cosh(bdn\log(x)+bd\log(c)+ad)-\sinh(bdn\log(x)+bd\log(c)+ad)}\right)}{bdn}$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="fricas")
[Out] -(b*d*n*log(x) - log(2*sinh(b*d*n*log(x) + b*d*log(c) + a*d)/(cosh(b*d*n*lo
g(x) + b*d*log(c) + a*d) - sinh(b*d*n*log(x) + b*d*log(c) + a*d))))/(b*d*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 4.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \begin{cases} \log\left(x\right)\coth\left(ad\right) & \text{for } b=0 \\ \tilde{\infty}\log\left(x\right) & \text{for } d=0 \\ \log\left(x\right)\coth\left(ad+bd\log\left(c\right)\right) & \text{for } n=0 \\ \frac{\log\left(\sinh\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn} & \text{otherwise} \end{cases}$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x,x)
[Out] Piecewise((log(x)*coth(a*d), Eq(b, 0)), (zoo*log(x), Eq(d, 0)), (log(x)*cot
h(a*d + b*d*log(c)), Eq(n, 0)), (log(sinh(a*d + b*d*log(c*x**n)))/(b*d*n),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{\log\left(\sinh\left((b\log\left(cx^n\right)+a)d\right)\right)}{bdn}$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="maxima")
[Out] log(sinh((b*log(c*x^n) + a)*d))/(b*d*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(25) = 50$.

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.96

$$\int \frac{\coth\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \frac{\log\left(\sqrt{-2\,x^{2\,bdn}|c|^{2\,bd}\cos\left(\pi bd\mathrm{sgn}\left(c\right)-\pi bd\right)e^{(2\,ad)}+x^{4\,bdn}|c|^{4\,bd}e^{(4\,ad)}+1}\right)}{bdn} - \log\left(x\right)$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x,x, algorithm="giac")
[Out] log(sqrt(-2*x^(2*b*d*n)*abs(c)^(2*b*d)*cos(pi*b*d*sgn(c) - pi*b*d)*e^(2*a*d
) + x^(4*b*d*n)*abs(c)^(4*b*d)*e^(4*a*d) + 1))/(b*d*n) - log(x)
```

Mupad [B] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x}\,dx = \frac{\ln\left(e^{2\,a\,d}\,\left(c\,x^n\right)^{2\,b\,d}-1\right)}{b\,d\,n}-\ln\left(x\right)$$

```
[In] int(coth(d*(a + b*log(c*x^n)))/x,x)
[Out] log(exp(2*a*d)*(c*x^n)^(2*b*d) - 1)/(b*d*n) - log(x)
```

3.182 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2} dx = -\frac{1}{x} + \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{x}$$

```
[Out] -1/x+2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/
x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5659, 5657, 470, 371}

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2} dx = \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{x} - \frac{1}{x}$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] -x^(-1) + (2*Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*a*d
)*(c*x^n)^(2*b*d)])/x
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cx^n)^{\frac{1}{n}}\operatorname{Subst}\left(\int x^{-1-\frac{1}{n}}\coth(d(a+b\log(x)))\,dx,x,cx^n\right)}{nx}\\
&= \frac{(cx^n)^{\frac{1}{n}}\operatorname{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}\left(-1-e^{2ad}x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{nx}\\
&= -\frac{1}{x}-\frac{\left(2(cx^n)^{\frac{1}{n}}\right)\operatorname{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{nx}\\
&= -\frac{1}{x}+\frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{x}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(58) = 116.

Time = 3.88 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.40

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx$$
$$= \frac{\coth(d(a+b\log(cx^n))) - \coth(d(a-bn\log(x)+b\log(cx^n))) - \frac{e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2bdn},2-\frac{1}{2bdn}\right.}{-1+2bdn}}{}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^2,x]
```

```
[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^
(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*b*d
*n), E^(2*d*(a + b*Log[c*x^n]))])/(-1 + 2*b*d*n) + Hypergeometric2F1[1, -1/
2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*L
og[c*x^n])]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/x
```

Maple [F]

$$\int \frac{\coth(d(a+b\ln(c\,x^n)))}{x^2}dx$$

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^2,x)
```

Fricas [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^2}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^2, x)
```

Sympy [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth(ad+bd\log(cx^n))}{x^2}\,dx$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**2,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^2}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="maxima")
```

```
[Out] -1/x - integrate(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) + x^2), x) + integra
te(1/(c^(b*d)*x^2*e^(b*d*log(x^n) + a*d) - x^2), x)
```

Giac [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^2}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^2,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth(d\,(a+b\,\ln(c\,x^n)))}{x^2}\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))/x^2,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))/x^2, x)
```

3.183 $\int \frac{\coth(d(a+b\log(cx^n)))}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3} dx = -\frac{1}{2x^2} + \frac{\text{Hypergeometric2F1}\left(1, -\frac{1}{bdn}, 1-\frac{1}{bdn}, e^{2ad}(cx^n)^{2bd}\right)}{x^2}$$

```
[Out] -1/2/x^2+hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/x^
2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5659, 5657, 470, 371}

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3} dx = \frac{\text{Hypergeometric2F1}\left(1, -\frac{1}{bdn}, 1-\frac{1}{bdn}, e^{2ad}(cx^n)^{2bd}\right)}{x^2} - \frac{1}{2x^2}$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] -1/2*1/x^2 + Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*a*d)*(c
*x^n)^(2*b*d)]/x^2
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int x^{-1-\frac{2}{n}}\coth(d(a+b\log(x)))\,dx,x,cx^n\right)}{nx^2}\\
&= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}\left(-1-e^{2ad}x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{nx^2}\\
&= -\frac{1}{2x^2}-\frac{\left(2(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{nx^2}\\
&= -\frac{1}{2x^2}+\frac{\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{x^2}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 191 vs. $2(55) = 110$.

Time = 4.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.47

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx$$

$$= \frac{\coth(d(a+b\log(cx^n)))-\coth(d(a-bn\log(x)+b\log(cx^n)))-\frac{e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{bdn},2-\frac{1}{bdn}\right.}{-1+bdn}}{}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]/x^3,x]
```

```
[Out] (Coth[d*(a + b*Log[c*x^n])] - Coth[d*(a - b*n*Log[x] + b*Log[c*x^n])] - (E^
(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))])/(-1 + b*d*n) + Hypergeometric2F1[1, -(1/(b*d*
n)), 1 - 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + Csch[d*(a + b*Log[c*x^n])
]*Csch[d*(a - b*n*Log[x] + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/(2*x^2)
```

Maple [F]

$$\int \frac{\coth\left(d(a+b\ln\left(c\,x^{n}\right))\right)}{x^{3}}dx$$

```
[In] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))/x^3,x)
```

Fricas [F]

$$\int \frac{\coth\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx=\int \frac{\coth\left((b\log\left(cx^{n}\right)+a)d\right)}{x^{3}}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)/x^3, x)
```

Sympy [F]

$$\int \frac{\coth\left(d(a+b\log\left(cx^{n}\right))\right)}{x^{3}}\,dx=\int \frac{\coth\left(ad+bd\log\left(cx^{n}\right)\right)}{x^{3}}\,dx$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))/x**3,x)
```

```
[Out] Integral(coth(a*d + b*d*log(c*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^3}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) + x^3), x) + int
egrate(1/(c^(b*d)*x^3*e^(b*d*log(x^n) + a*d) - x^3), x)
```

Giac [F]

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)}{x^3}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))/x^3,x, algorithm="giac")
```

```
[Out] integrate(coth((b*log(c*x^n) + a)*d)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)}{x^3}\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))/x^3,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))/x^3, x)
```

3.184 $\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4 + \frac{x^4\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/4*(1+4/b/d/n)*x^4+x^4*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^4*hypergeom([1, 2/b/d/n],[1+2/b/d/n],exp(2*a*d)*(c*x^n
)^(2*b*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5659, 5657, 516, 470, 371}

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = -\frac{2x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^4\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} + \frac{1}{4}x^4\left(\frac{4}{bdn}+1\right)$$

```
[In] Int[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 4/(b*d*n))*x^4)/4 + (x^4*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2
/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int x^{-1+\frac{4}{n}}\coth^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{4}{n}}\left(-1-e^{2ad}x^{2bd}\right)^2}{\left(1-e^{2ad}x^{2bd}\right)^2}\,dx,x,cx^n\right)}{n}$$

$$=\frac{x^4\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}+\frac{\left(e^{-2ad}x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int\frac{x^{-1+\frac{4}{n}}\left(-\frac{2e^{2ad}(4-bdn)}{n}-\frac{2e^{4ad}(4+bdn)x^{2bd}}{n}\right)}{1-e^{2ad}x^{2bd}}dx,x,cx^n\right)}{2bdn}$$

$$=\frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4+\frac{x^4\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{\left(8x^4(cx^n)^{-4/n}\right)\text{Subst}\left(\int\frac{x^{-1+\frac{4}{n}}}{1-e^{2ad}x^{2bd}}dx,x,cx^n\right)}{bdn^2}$$

$$=\frac{1}{4}\left(1+\frac{4}{bdn}\right)x^4+\frac{x^4\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}-\frac{2x^4\,\text{Hypergeometric2F1}\left(1,\frac{2}{bdn},1+\frac{2}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 7.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17

$$\int x^3\coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\frac{x^4\left(-8e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{2}{bdn},2+\frac{2}{bdn},e^{2d(a+b\log(cx^n))}\right)+(2+bdn)\left(bdn-4\coth\left(d\right.\right.\right.}{4bdn(2+bdn)}$$

```
[In] Integrate[x^3*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^4*(-8*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 2/(b*d*n), 2 +
 2/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (2 + b*d*n)*(b*d*n - 4*Coth[d*(a
+ b*Log[c*x^n])] - 4*Hypergeometric2F1[1, 2/(b*d*n), 1 + 2/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(4*b*d*n*(2 + b*d*n))
```

Maple [F]

$$\int x^3\coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2dx$$

```
[In] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x^3*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(x^3*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \int x^3 \coth^2(ad+bd\log(cx^n))\,dx$$

```
[In] integrate(x**3*coth(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral(x**3*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] 1/4*(b*c^(2*b*d)*d*n*x^4*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 8)*x^4)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 4*integrate(x^3/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 4*integrate(x^3/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Giac [F]

$$\int x^3 \coth^2(d(a+b\log(cx^n)))\,dx = \int x^3 \coth((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(x^3*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate(x^3*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \coth^2\left(d(a+b\log\left(cx^n\right))\right) \, dx = \int x^3 \coth(d\left(a+b\,\ln\left(c\,x^n\right)\right))^2 \, dx$$

```
[In] int(x^3*coth(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x^3*coth(d*(a + b*log(c*x^n)))^2, x)
```

3.185 $\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 136

$$\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx = \frac{1}{3}\left(1+\frac{3}{bdn}\right)x^3 + \frac{x^3\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/3*(1+3/b/d/n)*x^3+x^3*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^3*hypergeom([1, 3/2/b/d/n],[1+3/2/b/d/n],exp(2*a*d)*(c
*x^n)^(2*b*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5659, 5657, 516, 470, 371}

$$\int x^2 \coth^2(d(a+b\log(cx^n)))\,dx = -\frac{2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x^3\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} + \frac{1}{3}x^3\left(\frac{3}{bdn}+1\right)$$

```
[In] Int[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 3/(b*d*n))*x^3)/3 + (x^3*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 +
 3/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int x^{-1+\frac{3}{n}}\coth^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}(-1-e^{2ad}x^{2bd})^2}{(1-e^{2ad}x^{2bd})^2}\,dx, x, cx^n\right)}{n}$$

$$= \frac{x^3\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} + \frac{\left(e^{-2ad}x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}\left(-\frac{2e^{2ad}(3-bdn)}{n}-\frac{2e^{4ad}(3+bdn)x^{2bd}}{n}\right)}{1-e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{2bdn}$$

$$= \frac{1}{3}\left(1+\frac{3}{bdn}\right)x^3 + \frac{x^3\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{\left(6x^3(cx^n)^{-3/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{3}{n}}}{1-e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{bdn^2}$$

$$= \frac{1}{3}\left(1+\frac{3}{bdn}\right)x^3 + \frac{x^3\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x^3\,\text{Hypergeometric2F1}\left(1,\frac{3}{2bdn},1+\frac{3}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 4.62 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.21

$$\int x^2\coth^2(d(a+b\log(cx^n)))\,dx = \frac{x^3\left(-9e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{3}{2bdn},2+\frac{3}{2bdn},e^{2d(a+b\log(cx^n))}\right)+(3+2bdn)\left(bdn-3\coth(\cdots\right.\right.}{3bdn(3+2bdn)}$$

```
[In] Integrate[x^2*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^3*(-9*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 3/(2*b*d*n), 2
 + 3/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (3 + 2*b*d*n)*(b*d*n - 3*Coth
[d*(a + b*Log[c*x^n])] - 3*Hypergeometric2F1[1, 3/(2*b*d*n), 1 + 3/(2*b*d*n
), E^(2*d*(a + b*Log[c*x^n]))])))/(3*b*d*n*(3 + 2*b*d*n))
```

Maple [F]

$$\int x^2 \coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

```
[In] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
[Out] int(x^2*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\, dx = \int x^2 \coth\left((b\log\left(cx^n\right)+a)d\right)^2\, dx$$

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(x^2*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\, dx = \int x^2 \coth^2\left(ad+bd\log\left(cx^n\right)\right) dx$$

```
[In] integrate(x**2*coth(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral(x**2*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x^2 \coth^2\left(d(a+b\log\left(cx^n\right))\right)\, dx = \int x^2 \coth\left((b\log\left(cx^n\right)+a)d\right)^2\, dx$$

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] 1/3*(b*c^(2*b*d)*d*n*x^3*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 6)*x^3)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 3*integrate(x^2/(b*c^(b*
d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 3*integrate(x^2/(b*c^(b*d)*d*n
*e^(b*d*log(x^n) + a*d) - b*d*n), x)
```

Giac [F]

$$\int x^2 \coth^2 (d(a + b\log (cx^n)))\ dx = \int x^2 \coth ((b\log (cx^n) + a)d)^2\ dx$$

```
[In] integrate(x^2*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate(x^2*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \coth^2 (d(a + b\log (cx^n)))\ dx = \int x^2 \coth(d\,(a + b\,\ln (c\,x^n)))^2\,dx$$

```
[In] int(x^2*coth(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x^2*coth(d*(a + b*log(c*x^n)))^2, x)
```

3.186 $\int x \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int x\coth^2(d(a+b\log(cx^n)))\,dx=\frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2+\frac{x^2\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}-\frac{2x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] 1/2*(1+2/b/d/n)*x^2+x^2*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*
(c*x^n)^(2*b*d))-2*x^2*hypergeom([1, 1/b/d/n],[1+1/b/d/n],exp(2*a*d)*(c*x^n
)^(2*b*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5659, 5657, 516, 470, 371}

$$\int x\coth^2(d(a+b\log(cx^n)))\,dx=-\frac{2x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}+\frac{x^2\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{1}{2}x^2\left(\frac{2}{bdn}+1\right)$$

```
[In] Int[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + 2/(b*d*n))*x^2)/2 + (x^2*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*x^2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1
/(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int x^{-1+\frac{2}{n}}\coth^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}\left(-1-e^{2ad}x^{2bd}\right)^2}{\left(1-e^{2ad}x^{2bd}\right)^2}\,dx,x,cx^n\right)}{n}$$

$$= \frac{x^2\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(e^{-2ad}x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}\left(-\frac{2e^{2ad}(2-bdn)}{n}-\frac{2e^{4ad}(2+bdn)x^{2bd}}{n}\right)}{1-e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{2bdn}$$

$$= \frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2 + \frac{x^2\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(4x^2(cx^n)^{-2/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{2}{n}}}{1-e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{bdn^2}$$

$$= \frac{1}{2}\left(1+\frac{2}{bdn}\right)x^2 + \frac{x^2\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2x^2\,\text{Hypergeometric2F1}\left(1,\frac{1}{bdn},1+\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 6.75 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int x\coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$= \frac{x^2\left(-2e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{bdn},2+\frac{1}{bdn},e^{2d(a+b\log(cx^n))}\right)+(1+bdn)\left(bdn-2\coth\left(d\right.\right.\right.}{2bdn(1+bdn)}$$

```
[In] Integrate[x*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x^2*(-2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(b*d*n), 2 +
 1/(b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + (1 + b*d*n)*(b*d*n - 2*Coth[d*(a
+ b*Log[c*x^n])] - 2*Hypergeometric2F1[1, 1/(b*d*n), 1 + 1/(b*d*n), E^(2*d*
(a + b*Log[c*x^n]))])))/(2*b*d*n*(1 + b*d*n))
```

Maple [F]

$$\int x\coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2dx$$

```
[In] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(x*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx = \int x \coth\left((b\log(cx^n)+a)d\right)^2\,dx$$

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
```

```
[Out] integral(x*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx = \int x\coth^2(ad+bd\log(cx^n))\,dx$$

```
[In] integrate(x*coth(d*(a+b*ln(c*x**n)))**2,x)
```

```
[Out] Integral(x*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx = \int x \coth\left((b\log(cx^n)+a)d\right)^2\,dx$$

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c^(2*b*d)*d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 4)*x^2)/(b*c
^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - 2*integrate(x/(b*c^(b*d)
*d*n*e^(b*d*log(x^n) + a*d) + b*d*n), x) + 2*integrate(x/(b*c^(b*d)*d*n*e^(
b*d*log(x^n) + a*d) - b*d*n), x)
```

Giac [F]

$$\int x \coth^2(d(a+b\log(cx^n)))\,dx = \int x \coth\left((b\log(cx^n)+a)d\right)^2\,dx$$

```
[In] integrate(x*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
```

```
[Out] integrate(x*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \coth^2 (d(a + b \log (cx^n))) \ dx = \int x \coth(d (a + b \ln (c x^n)))^2 \, dx$$

```
[In] int(x*coth(d*(a + b*log(c*x^n)))^2,x)
[Out] int(x*coth(d*(a + b*log(c*x^n)))^2, x)
```

3.187 $\int \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \left(1+\frac{1}{bdn}\right)x+\frac{x\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

```
[Out] (1+1/b/d/n)*x+x*(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/(1-exp(2*a*d)*(c*x^n)^
(2*b*d))-2*x*hypergeom([1, 1/2/b/d/n],[1+1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b
*d))/b/d/n
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5655, 5657, 516, 470, 371}

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = -\frac{2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn} + \frac{x\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bdn\left(1-e^{2ad}(cx^n)^{2bd}\right)}+x\left(\frac{1}{bdn}+1\right)$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (1 + 1/(b*d*n))*x + (x*(1 + E^(2*a*d)*(c*x^n)^(2*b*d)))/(b*d*n*(1 - E^(2*a*
d)*(c*x^n)^(2*b*d))) - (2*x*Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*
n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5655

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\coth^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}\left(-1-e^{2ad}x^{2bd}\right)^2}{\left(1-e^{2ad}x^{2bd}\right)^2}\,dx,x,cx^n\right)}{n}$$

$$= \frac{x\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(e^{-2ad}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}\left(-\frac{2e^{2ad}(1-bdn)}{n}-2e^{4ad}\left(bd+\frac{1}{n}\right)x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{2bdn}$$

$$= \left(1+\frac{1}{bdn}\right)x + \frac{x\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(2x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{bdn^2}$$

$$= \left(1+\frac{1}{bdn}\right)x + \frac{x\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bdn\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2x\,\text{Hypergeometric2F1}\left(1,\frac{1}{2bdn},1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdn}$$

Mathematica [A] (verified)

Time = 8.40 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.27

$$\int \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx$$
$$= \frac{x\left(-e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bdn},2+\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right)+(1+2bdn)\left(bdn-\coth\left(d(a\right.\right.\right.}{bdn(1+2bdn)}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (x*(-(E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 + 1/(2*b*d*n), 2 +
1/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))]) + (1 + 2*b*d*n)*(b*d*n - Coth[d*(
a + b*Log[c*x^n])] - Hypergeometric2F1[1, 1/(2*b*d*n), 1 + 1/(2*b*d*n), E^(
2*d*(a + b*Log[c*x^n]))])))/(b*d*n*(1 + 2*b*d*n))
```

Maple [F]

$$\int \coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2 dx$$

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth^2(d(a+b\log(cx^n)))\,dx$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral(coth(d*(a + b*log(c*x**n)))**2, x)
```

Maxima [F]

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] (b*c^(2*b*d)*d*n*x*e^(2*b*d*log(x^n) + 2*a*d) - (b*d*n + 2)*x)/(b*c^(2*b*d)
*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) - integrate(1/(b*c^(b*d)*d*n*e^(b*
d*log(x^n) + a*d) + b*d*n), x) + integrate(1/(b*c^(b*d)*d*n*e^(b*d*log(x^n)
 + a*d) - b*d*n), x)
```

Giac [F]

$$\int \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth((b\log(cx^n)+a)d)^2\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int \coth\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^2,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2, x)
```

3.188 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{\coth(ad+bd\log(cx^n))}{bdn}+\log(x)$$

```
[Out] -coth(a*d+b*d*ln(c*x^n))/b/d/n+ln(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3554, 8}

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \log(x)-\frac{\coth(ad+bd\log(cx^n))}{bdn}$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -(Coth[a*d + b*d*Log[c*x^n]]/(b*d*n)) + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{Subst}\left(\int \coth^2(d(a+bx))\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn} + \frac{\operatorname{Subst}\left(\int 1\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= -\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)}{bdn} + \log(x)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.75

$$
\int \frac{\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)}{x}\,dx = -\frac{\coth\left(ad+bd\log\left(cx^n\right)\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2\left(ad+bd\log\left(cx^n\right)\right)\right)}{bdn}
$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x,x]
```

```
[Out] -((Coth[a*d + b*d*Log[c*x^n]]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[a*d + b*
d*Log[c*x^n]]^2])/(b*d*n))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

method	result
parallelrisch	$\frac{-1+\ln(x) dbn \tanh(d(a+b\ln(c\,x^n)))}{dbn \tanh(d(a+b\ln(c\,x^n)))}$
derivativedivides	$\frac{-\coth(d(a+b\ln(c\,x^n)))-\frac{\ln(\coth(d(a+b\ln(c\,x^n)))-1)}{2}+\frac{\ln(\coth(d(a+b\ln(c\,x^n)))+1)}{2}}{nbd}$
default	$\frac{-\coth(d(a+b\ln(c\,x^n)))-\frac{\ln(\coth(d(a+b\ln(c\,x^n)))-1)}{2}+\frac{\ln(\coth(d(a+b\ln(c\,x^n)))+1)}{2}}{nbd}$
risch	$\ln(x)-\frac{2}{dbn\left(c^{2bd}(x^n)^{2bd}\mathrm{e}^{d\left(ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2-ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)-ib\pi\operatorname{csgn}(ic\,x^n)^3+ib\pi\operatorname{csgn}(ic\,x^n)^2\cdots\right.}\right.}$

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-1+ln(x)*d*b*n*tanh(d*(a+b*ln(c*x^n))))/d/b/n/tanh(d*(a+b*ln(c*x^n)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \frac{(bdn\log(x)+1)\sinh(bdn\log(x)+bd\log(c)+ad)-\cosh(bdn\log(x)+bd\log(c)+ad)}{bdn\sinh(bdn\log(x)+bd\log(c)+ad)}$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="fricas")
[Out] ((b*d*n*log(x) + 1)*sinh(b*d*n*log(x) + b*d*log(c) + a*d) - cosh(b*d*n*log(
x) + b*d*log(c) + a*d))/(b*d*n*sinh(b*d*n*log(x) + b*d*log(c) + a*d))
```

Sympy [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = \int \frac{\coth^2(ad+bd\log(cx^n))}{x}\,dx$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x,x)
[Out] Integral(coth(a*d + b*d*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x}\,dx = -\frac{2}{bc^{2\,bd}dne^{(2\,bd\log(x^n)+2\,ad)}-bdn}+\log(x)$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="maxima")
[Out] -2/(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = -\frac{2}{(c^{2\,bd}x^{2\,bdn}e^{(2\,ad)}-1)bdn} + \log\left(x\right)$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x,x, algorithm="giac")
[Out] -2/((c^(2*b*d)*x^(2*b*d*n)*e^(2*a*d) - 1)*b*d*n) + log(x)
```

Mupad [B] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x}\,dx = \ln\left(x\right) - \frac{2}{b\,d\,n\,\left(\mathrm{e}^{2\,a\,d}\,{\left(c\,x^n\right)}^{2\,b\,d}-1\right)}$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x,x)
[Out] log(x) - 2/(b*d*n*(exp(2*a*d)*(c*x^n)^(2*b*d) - 1))
```

3.189 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{1-\frac{1}{bdn}}{x}+\frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx}$$

```
[Out] (-1+1/b/d/n)/x+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x/(1-exp(2*a*d)*(c*x^n)
^(2*b*d))-2*hypergeom([1, -1/2/b/d/n],[1-1/2/b/d/n],exp(2*a*d)*(c*x^n)^(2*b
*d))/b/d/n/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5659, 5657, 516, 470, 371}

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = -\frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx} + \frac{e^{2ad}(cx^n)^{2bd}+1}{bdnx\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{1-\frac{1}{bdn}}{x}$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] -((1 - 1/(b*d*n))/x) + (1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x*(1 - E^(2*a
*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*
d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int x^{-1-\frac{1}{n}}\coth^2(d(a+b\log(x)))\,dx, x, cx^n\right)}{nx} \\
&= \frac{(cx^n)^{\frac{1}{n}}\,\text{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}\left(-1-e^{2ad}x^{2bd}\right)^2}{\left(1-e^{2ad}x^{2bd}\right)^2}\,dx, x, cx^n\right)}{nx}
\end{aligned}$$

$$= \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(e^{-2ad}(cx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}\left(\frac{2e^{2ad}(1+bdn)}{n}+\frac{2e^{4ad}(1-bdn)x^{2bd}}{n}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{2bdnx}$$

$$= -\frac{1-\frac{1}{bdn}}{x} + \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(2(cx^n)^{\frac{1}{n}}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{1}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{bdn^2x}$$

$$= -\frac{1-\frac{1}{bdn}}{x} + \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2bdn},1-\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx}$$

Mathematica [A] (verified)

Time = 3.92 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.18

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x^2}\,dx = \frac{e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2bdn},2-\frac{1}{2bdn},e^{2d(a+b\log(cx^n))}\right)-(-1+2bdn)\left(bdn+\coth\left(d(a+\right.\right.}{bdn(-1+2bdn)x}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^2,x]
```

```
[Out] (E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(2*b*d*n), 2 - 1/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + 2*b*d*n)*(b*d*n + Coth[d*(a + b
*Log[c*x^n])] + Hypergeometric2F1[1, -1/2*1/(b*d*n), 1 - 1/(2*b*d*n), E^(2*
d*(a + b*Log[c*x^n]))]))/(b*d*n*(-1 + 2*b*d*n)*x)
```

Maple [F]

$$\int \frac{\coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2}{x^2}dx$$

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^2,x)
```

Fricas [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth\left((b\log(cx^n)+a)d\right)^2}{x^2}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="fricas")
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^2, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth\left((b\log(cx^n)+a)d\right)^2}{x^2}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="maxima")
[Out] -(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 2)/(b*c^(2*b*d)*d*n*
x*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x) + integrate(1/(b*c^(b*d)*d*n*x^2*e^
(b*d*log(x^n) + a*d) + b*d*n*x^2), x) - integrate(1/(b*c^(b*d)*d*n*x^2*e^(b
*d*log(x^n) + a*d) - b*d*n*x^2), x)
```

Giac [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth\left((b\log(cx^n)+a)d\right)^2}{x^2}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^2,x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^2}\,dx = \int \frac{\coth(d\,(a+b\,\ln(c\,x^n)))^2}{x^2}\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x^2,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2/x^2, x)
```

3.190 $\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 135

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3} dx = \frac{2-bdn}{2bdnx^2} + \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}$$

```
[Out] 1/2*(-b*d*n+2)/b/d/n/x^2+(1+exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/n/x^2/(1-exp(2*
a*d)*(c*x^n)^(2*b*d))-2*hypergeom([1, -1/b/d/n],[1-1/b/d/n],exp(2*a*d)*(c*x
^n)^(2*b*d))/b/d/n/x^2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5659, 5657, 516, 470, 371}

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3} dx = -\frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2} + \frac{e^{2ad}(cx^n)^{2bd}+1}{bdnx^2\left(1-e^{2ad}(cx^n)^{2bd}\right)} + \frac{2-bdn}{2bdnx^2}$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2 - b*d*n)/(2*b*d*n*x^2) + (1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(b*d*n*x^2*(1 -
 E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/
(b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*n*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int x^{-1-\frac{2}{n}}\coth^2(d(a+b\log(x)))\,dx, x, cx^n\right)}{nx^2}$$

$$= \frac{(cx^n)^{2/n}\,\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}\left(-1-e^{2ad}x^{2bd}\right)^2}{\left(1-e^{2ad}x^{2bd}\right)^2}\,dx, x, cx^n\right)}{nx^2}$$

$$= \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(e^{-2ad}(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}\left(\frac{2e^{2ad}(2+bdn)}{n}+\frac{2e^{4ad}(2-bdn)x^{2bd}}{n}\right)}{1-e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{2bdnx^2}$$

$$= \frac{2-bdn}{2bdnx^2} + \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(4(cx^n)^{2/n}\right)\text{Subst}\left(\int \frac{x^{-1-\frac{2}{n}}}{1-e^{2ad}x^{2bd}}\,dx, x, cx^n\right)}{bdn^2x^2}$$

$$= \frac{2-bdn}{2bdnx^2} + \frac{1+e^{2ad}(cx^n)^{2bd}}{bdnx^2\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2\,\text{Hypergeometric2F1}\left(1,-\frac{1}{bdn},1-\frac{1}{bdn},e^{2ad}(cx^n)^{2bd}\right)}{bdnx^2}$$

Mathematica [A] (verified)

Time = 3.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \frac{2e^{2d(a+b\log(cx^n))}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{bdn},2-\frac{1}{bdn},e^{2d(a+b\log(cx^n))}\right)-(-1+bdn)\left(bdn+2\coth\left(d(a+\right.\right.}{2bdn(-1+bdn)x^2}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^2/x^3,x]
```

```
[Out] (2*E^(2*d*(a + b*Log[c*x^n]))*Hypergeometric2F1[1, 1 - 1/(b*d*n), 2 - 1/(b*
d*n), E^(2*d*(a + b*Log[c*x^n]))] - (-1 + b*d*n)*(b*d*n + 2*Coth[d*(a + b*L
og[c*x^n])] + 2*Hypergeometric2F1[1, -(1/(b*d*n)), 1 - 1/(b*d*n), E^(2*d*(a
 + b*Log[c*x^n]))]))/(2*b*d*n*(-1 + b*d*n)*x^2)
```

Maple [F]

$$\int \frac{\coth\left(d(a+b\ln\left(c\,x^n\right))\right)^2}{x^3}dx$$

```
[In] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^2/x^3,x)
```

Fricas [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="fricas")
[Out] integral(coth(b*d*log(c*x^n) + a*d)^2/x^3, x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**2/x**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="maxima")
[Out] -1/2*(b*c^(2*b*d)*d*n*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n + 4)/(b*c^(2*b*d)*
d*n*x^2*e^(2*b*d*log(x^n) + 2*a*d) - b*d*n*x^2) + 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) + b*d*n*x^3), x) - 2*integrate(1/(b*c^(b*d)*
d*n*x^3*e^(b*d*log(x^n) + a*d) - b*d*n*x^3), x)
```

Giac [F]

$$\int \frac{\coth^2(d(a+b\log(cx^n)))}{x^3}\,dx = \int \frac{\coth((b\log(cx^n)+a)d)^2}{x^3}\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^2/x^3,x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2\left(d(a+b\log\left(cx^n\right))\right)}{x^3}\,dx = \int \frac{\coth\left(d\left(a+b\,\ln\left(c\,x^n\right)\right)\right)^2}{x^3}\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^2/x^3,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2/x^3, x)
```

3.191 $\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx=-\frac{\coth^2(a+b\log(cx^n))}{2bn}+\frac{\log(\sinh(a+b\log(cx^n)))}{bn}$$

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n+ln(sinh(a+b*ln(c*x^n)))/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3554, 3556}

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx=\frac{\log(\sinh(a+b\log(cx^n)))}{bn}-\frac{\coth^2(a+b\log(cx^n))}{2bn}$$

```
[In] Int[Coth[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] -1/2*Coth[a + b*Log[c*x^n]]^2/(b*n) + Log[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \coth^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\coth^2(a+b\log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int \coth(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\ &= -\frac{\coth^2(a+b\log(cx^n))}{2bn} + \frac{\log(\sinh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = -\frac{\coth^2(a+b\log(cx^n)) - 2\log(\cosh(a+b\log(cx^n))) - 2\log(\tanh(a+b\log(cx^n)))}{2bn}$$

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^3/x,x]
[Out] -1/2*(Coth[a + b*Log[c*x^n]]^2 - 2*Log[Cosh[a + b*Log[c*x^n]]] - 2*Log[Tanh
[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
parallelrisch	$\frac{-2\ln(x)bn+2\ln(\tanh(a+b\ln(c\,x^n)))-2\ln(1-\tanh(a+b\ln(c\,x^n)))-\coth(a+b\ln(c\,x^n))^2}{2bn}$
risch	$\ln(x)-\frac{2a}{bn}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\,\mathrm{csgn}(ic}{n}$

```
[In] int(coth(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b*(-1/2*coth(a+b*ln(c*x^n))^2-1/2*ln(coth(a+b*ln(c*x^n))-1)-1/2*ln(coth
(a+b*ln(c*x^n))+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. 2(41) = 82.

Time = 0.30 (sec) , antiderivative size = 572, normalized size of antiderivative = 13.30

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = -\frac{bn\cosh(bn\log(x)+b\log(c)+a)^4\log(x)+4\,bn\cosh(bn\log(x)+b\log(c)+a)\log(x)\sinh(bn\log(x)}{}$$

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) + 4*b*n*cosh(b*n*log(x) + b*
log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*log(x)*sinh(b*n*
log(x) + b*log(c) + a)^4 - 2*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) +
a)^2 + b*n*log(x) + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n
*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c
) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) +
 a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c)
 + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(c)
 + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(b*n*log(x) + b*log(c
) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)))
 + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) - (b*n*log(x) - 1)*cosh(
b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*l
og(x) + b*log(c) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 - 2*b*n*cos
h(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 - b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) +
b*log(c) + a)^3 - b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*
log(c) + a))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(coth(a+b*ln(c*x**n))**3/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 7.67

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = -\frac{4\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-3}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} - \frac{3\,(2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-1)}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \frac{2\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)}-3}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} - \frac{3}{4\,(bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)}-2\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn)} + \frac{\log\left(\frac{(c^be^{(b\log(x^n)+a)}+1)e^{(-a)}}{c^b}\right)}{bn} + \frac{\log\left(\frac{(c^be^{(b\log(x^n)+a)}-1)e^{(-a)}}{c^b}\right)}{bn} - \log(x)$$

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b*c^(4*b)*n*e^(4*b*log(x^n) +
4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 3/4*(2*c^(2*b)*e^(2*b*
log(x^n) + 2*a) - 1)/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^
(2*b*log(x^n) + 2*a) + b*n) + 1/4*(2*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 3)/(b
*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) +
b*n) - 3/4/(b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 2*b*c^(2*b)*n*e^(2*b*log(x
^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n) + lo
g((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(41) = 82$.

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.95

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos(\pi b\mathrm{sgn}(c)-\pi b)\,e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1}\right)}{bn} - \frac{3\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{2\,(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1)^2bn} - \log(x)$$

```
[In] integrate(coth(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b
*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/2*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 2
*c^(2*b)*x^(2*b*n)*e^(2*a) + 3)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^2*b*n) - l
og(x)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int \frac{\coth^3(a+b\log(cx^n))}{x}\,dx = \frac{2}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}-\ln\left(x\right)-\frac{2}{b\,n-2\,b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}+\frac{\ln\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}{b\,n}$$

```
[In] int(coth(a + b*log(c*x^n))^3/x,x)
```

```
[Out] 2/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) - log(x) - 2/(b*n - 2*b*n*exp(2*a)*(c*
x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*b) - 1)/
(b*n)
```

3.192 $\int \frac{\coth^4(a+b\log(cx^n))}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{\coth^4(a+b\log(cx^n))}{x} dx = -\frac{\coth(a+b\log(cx^n))}{bn} - \frac{\coth^3(a+b\log(cx^n))}{3bn} + \log(x)$$

```
[Out] -coth(a+b*ln(c*x^n))/b/n-1/3*coth(a+b*ln(c*x^n))^3/b/n+ln(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3554, 8}

$$\int \frac{\coth^4(a+b\log(cx^n))}{x} dx = -\frac{\coth^3(a+b\log(cx^n))}{3bn} - \frac{\coth(a+b\log(cx^n))}{bn} + \log(x)$$

```
[In] Int[Coth[a + b*Log[c*x^n]]^4/x,x]
[Out] -(Coth[a + b*Log[c*x^n]]/(b*n)) - Coth[a + b*Log[c*x^n]]^3/(3*b*n) + Log[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \coth^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\text{Subst}\left(\int \coth^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn} + \frac{\text{Subst}\left(\int 1\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\coth\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\coth^3\left(a+b\log\left(cx^n\right)\right)}{3bn} + \log(x)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$
\begin{aligned}
&\int \frac{\coth^4\left(a+b\log\left(cx^n\right)\right)}{x}\,dx\\
&= -\frac{\coth^3\left(a+b\log\left(cx^n\right)\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2\left(a+b\log\left(cx^n\right)\right)\right)}{3bn}
\end{aligned}
$$

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^4/x,x]
[Out] -1/3*(Coth[a + b*Log[c*x^n]]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[a + b*
Log[c*x^n]]^2])/(b*n)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{-\coth(a+b\ln(c\,x^n))^3+3\ln(x)bn-3\coth(a+b\ln(c\,x^n))}{3bn}$
derivativedivides	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^3}{3}-\coth(a+b\ln(c\,x^n))-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}+\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^3}{3}-\coth(a+b\ln(c\,x^n))-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}+\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
risch	$\ln(x)-\frac{4\left(3(x^n)^{4b}c^{4b}e^{4a}e^{2ib\pi\ \text{csgn}(ix^n)\ \text{csgn}(ic\,x^n)^2}e^{-2ib\pi\ \text{csgn}(ix^n)\ \text{csgn}(ic\,x^n)\ \text{csgn}(ic)}e^{-2ib\pi\ \text{csgn}(ic\,x^n)^3}e^{2ib\pi\ \text{csgn}(i}\right.}{3bn\left((x^n)^{2b}c^{2b}e^{2a}e^{ib\pi\ \text{csgn}(ix^n)\ \text{csgn}(ic\,x^n)^2}e^{-ib}\right.}$

```
[In] int(coth(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
[Out] 1/3*(-coth(a+b*ln(c*x^n))^3+3*ln(x)*b*n-3*coth(a+b*ln(c*x^n)))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(43) = 86$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.80

$$\int \frac{\coth^4(a + b\log(cx^n))}{x}\,dx$$
$$= \frac{(3\,bn\log(x) + 4)\sinh(bn\log(x) + b\log(c) + a)^3 - 4\,\cosh(bn\log(x) + b\log(c) + a)^3 - 12\,\cosh(bn\log(x}{3\,\left(bn\sinh(bn\log(x) + b\log(c) + a}$$

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*n*log(x) + 4)*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*cosh(b*n*log(
x) + b*log(c) + a)^3 - 12*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + 3*((3*b*n*log(x) + 4)*cosh(b*n*log(x) + b*log(c) + a)^2
- 3*b*n*log(x) - 4)*sinh(b*n*log(x) + b*log(c) + a))/(b*n*sinh(b*n*log(x) +
 b*log(c) + a)^3 + 3*(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n
*log(x) + b*log(c) + a))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\coth^4(a + b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(coth(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(43) = 86$.

Time = 0.27 (sec) , antiderivative size = 499, normalized size of antiderivative = 11.09

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx$$
$$= -\frac{18\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)} - 27\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} + 11}{12\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)}$$
$$- \frac{6\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)} - 15\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} + 11}{12\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)}$$
$$- \frac{2\,(3\,c^{4\,b}e^{(4\,b\log(x^n)+4\,a)} - 3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} + 1)}{3\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)}$$
$$- \frac{3\,c^{2\,b}e^{(2\,b\log(x^n)+2\,a)} - 1}{2\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)}$$
$$- \frac{2}{3\,(bc^{6\,b}ne^{(6\,b\log(x^n)+6\,a)} - 3\,bc^{4\,b}ne^{(4\,b\log(x^n)+4\,a)} + 3\,bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)} - bn)} + \log(x)$$

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -1/12*(18*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 27*c^(2*b)*e^(2*b*log(x^n) + 2*a
) + 11)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n)
 + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n) - 1/12*(6*c^(4*b)*e^(
4*b*log(x^n) + 4*a) - 15*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 11)/(b*c^(6*b)*n*
e^(6*b*log(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)
*n*e^(2*b*log(x^n) + 2*a) - b*n) - 2/3*(3*c^(4*b)*e^(4*b*log(x^n) + 4*a) -
3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) -
 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a
) - b*n) - 1/2*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*c^(6*b)*n*e^(6*b*l
og(x^n) + 6*a) - 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*
b*log(x^n) + 2*a) - b*n) - 2/3/(b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) - 3*b*c^
(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) - b*n
) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.49

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = -\frac{4\,(3\,c^{4\,b}x^{4\,bn}e^{(4\,a)} - 3\,c^{2\,b}x^{2\,bn}e^{(2\,a)} + 2)}{3\,(c^{2\,b}x^{2\,bn}e^{(2\,a)} - 1)^3bn} + \log(x)$$

```
[In] integrate(coth(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

```
[Out] -4/3*(3*c^(4*b)*x^(4*b*n)*e^(4*a) - 3*c^(2*b)*x^(2*b*n)*e^(2*a) + 2)/((c^(2
*b)*x^(2*b*n)*e^(2*a) - 1)^3*b*n) + log(x)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.62

$$\int \frac{\coth^4(a+b\log(cx^n))}{x}\,dx = \ln(x) - \frac{\frac{4}{3\,b\,n}+\frac{4\,e^{4\,a}\,(c\,x^n)^{4\,b}}{3\,b\,n}}{3\,e^{2\,a}\,(c\,x^n)^{2\,b}-3\,e^{4\,a}\,(c\,x^n)^{4\,b}+e^{6\,a}\,(c\,x^n)^{6\,b}-1} - \frac{4}{3\,b\,n\,\left(e^{2\,a}\,(c\,x^n)^{2\,b}-1\right)} - \frac{4\,e^{2\,a}\,(c\,x^n)^{2\,b}}{3\,b\,n\,\left(e^{4\,a}\,(c\,x^n)^{4\,b}-2\,e^{2\,a}\,(c\,x^n)^{2\,b}+1\right)}$$

```
[In] int(coth(a + b*log(c*x^n))^4/x,x)
```

```
[Out] log(x) - (4/(3*b*n) + (4*exp(4*a)*(c*x^n)^(4*b))/(3*b*n))/(3*exp(2*a)*(c*x^
n)^(2*b) - 3*exp(4*a)*(c*x^n)^(4*b) + exp(6*a)*(c*x^n)^(6*b) - 1) - 4/(3*b*
n*(exp(2*a)*(c*x^n)^(2*b) - 1)) - (4*exp(2*a)*(c*x^n)^(2*b))/(3*b*n*(exp(4*
a)*(c*x^n)^(4*b) - 2*exp(2*a)*(c*x^n)^(2*b) + 1))
```

3.193 $\int \frac{\coth^5(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{\coth^5(a+b\log(cx^n))}{x} \, dx = -\frac{\coth^2(a+b\log(cx^n))}{2bn} - \frac{\coth^4(a+b\log(cx^n))}{4bn} + \frac{\log(\sinh(a+b\log(cx^n)))}{bn}$$

```
[Out] -1/2*coth(a+b*ln(c*x^n))^2/b/n-1/4*coth(a+b*ln(c*x^n))^4/b/n+ln(sinh(a+b*ln
(c*x^n)))/b/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3554, 3556}

$$\int \frac{\coth^5(a+b\log(cx^n))}{x} \, dx = \frac{\log(\sinh(a+b\log(cx^n)))}{bn} - \frac{\coth^4(a+b\log(cx^n))}{4bn} - \frac{\coth^2(a+b\log(cx^n))}{2bn}$$

```
[In] Int[Coth[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] -1/2*Coth[a + b*Log[c*x^n]]^2/(b*n) - Coth[a + b*Log[c*x^n]]^4/(4*b*n) + Lo
g[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \coth^5(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth^4(a+b\log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \coth^3(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth^2(a+b\log(cx^n))}{2bn} - \frac{\coth^4(a+b\log(cx^n))}{4bn} + \frac{\text{Subst}\left(\int \coth(a+bx)\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\coth^2(a+b\log(cx^n))}{2bn} - \frac{\coth^4(a+b\log(cx^n))}{4bn} + \frac{\log(\sinh(a+b\log(cx^n)))}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = -\frac{2\coth^2(a+b\log(cx^n)) + \coth^4(a+b\log(cx^n)) - 4\log(\cosh(a+b\log(cx^n))) - 4\log(\tanh(a+b\log(cx^n)))}{4bn}$$

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^5/x,x]
[Out] -1/4*(2*Coth[a + b*Log[c*x^n]]^2 + Coth[a + b*Log[c*x^n]]^4 - 4*Log[Cosh[a
+ b*Log[c*x^n]]] - 4*Log[Tanh[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^4}{4}-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
default	$\frac{-\frac{\coth(a+b\ln(c\,x^n))^4}{4}-\frac{\coth(a+b\ln(c\,x^n))^2}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))-1)}{2}-\frac{\ln(\coth(a+b\ln(c\,x^n))+1)}{2}}{nb}$
parallelrisch	$\frac{-\coth(a+b\ln(c\,x^n))^4-4\ln(x)bn+4\ln(\tanh(a+b\ln(c\,x^n)))-4\ln(1-\tanh(a+b\ln(c\,x^n)))-2\coth(a+b\ln(c\,x^n))^2}{4bn}$
risch	$\ln(x)-\frac{2a}{bn}-\frac{2\ln(c)}{n}-\frac{2\ln(x^n)}{n}-\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{n}+\frac{i\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{n}+\frac{i\pi\,\mathrm{csgn}(}{n}$

[In] int(coth(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)

[Out] 1/n/b*(-1/4*coth(a+b*ln(c*x^n))^4-1/2*coth(a+b*ln(c*x^n))^2-1/2*ln(coth(a+b*ln(c*x^n))-1)-1/2*ln(coth(a+b*ln(c*x^n))+1))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1576 vs. $2(62) = 124$.

Time = 0.28 (sec) , antiderivative size = 1576, normalized size of antiderivative = 23.88

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="fricas")

[Out] -(b*n*cosh(b*n*log(x) + b*log(c) + a)^8*log(x) + 8*b*n*cosh(b*n*log(x) + b*log(c) + a)*log(x)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*log(x)*sinh(b*n*log(x) + b*log(c) + a)^8 - 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2*log(x) - b*n*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^3*log(x) - 3*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a)^4 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4*log(x) - 30*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n*log(x) - 2)*sinh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5*log(x) - 10*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^3 + (3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 - 4*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n*log(x) + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6*log(x) - 15*(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^4 + 3*(3*b*n*log(x) - 2)*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n*log(x) + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log(x) + b*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^6 - 4*cosh(b*n*log(x) + b*log

```
(c) + a)^6 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^3 - 3*cosh(b*n*log(x) + b
*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b
*log(c) + a)^4 - 30*cosh(b*n*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x)
+ b*log(c) + a)^4 + 6*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log
(x) + b*log(c) + a)^5 - 10*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*l
og(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*lo
g(x) + b*log(c) + a)^6 - 15*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*
log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 + 8*(cosh(b*n*log(x) + b*log(c) + a)^7 - 3*cosh(
b*n*log(x) + b*log(c) + a)^5 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b
*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*log(2*sinh(
b*n*log(x) + b*log(c) + a)/(cosh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(
x) + b*log(c) + a))) + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7*log(x) - 3*
(b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c) + a)^5 + (3*b*n*log(x) - 2)*cos
h(b*n*log(x) + b*log(c) + a)^3 - (b*n*log(x) - 1)*cosh(b*n*log(x) + b*log(c
) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) +
a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a
)^7 + b*n*sinh(b*n*log(x) + b*log(c) + a)^8 - 4*b*n*cosh(b*n*log(x) + b*log
(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log
(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 8*(7*b*n*
cosh(b*n*log(x) + b*log(c) + a)^3 - 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(35*b*n*cosh(b*n*log(x) + b*log(c) +
a)^4 - 30*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) +
b*log(c) + a)^4 - 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 8*(7*b*n*cosh(b
*n*log(x) + b*log(c) + a)^5 - 10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*
b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*
(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 - 15*b*n*cosh(b*n*log(x) + b*log(c
) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 - b*n)*sinh(b*n*log(x) +
 b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*n*log(x) + b*log(c) + a)^7 - 3*b*n*c
osh(b*n*log(x) + b*log(c) + a)^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3
- b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(coth(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 855 vs. 2(62) = 124.

Time = 0.32 (sec) , antiderivative size = 855, normalized size of antiderivative = 12.95

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] -1/24*(48*c^(6*b)*e^(6*b*log(x^n) + 6*a) - 108*c^(4*b)*e^(4*b*log(x^n) + 4*
a) + 88*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) +
 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n
) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 1/24*(12*c^(6*b)*e
^(6*b*log(x^n) + 6*a) - 42*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 52*c^(2*b)*e^(2
*b*log(x^n) + 2*a) - 25)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*
n*e^(6*b*log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*
b)*n*e^(2*b*log(x^n) + 2*a) + b*n) - 5/8*(4*c^(6*b)*e^(6*b*log(x^n) + 6*a)
- 6*c^(4*b)*e^(4*b*log(x^n) + 4*a) + 4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n) - 5/12*(6*c^(4*b)*e^(4*b*log(x^n) + 4*a) - 4*c^(2*b)*e^(2*b*log(x
^n) + 2*a) + 1)/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*
log(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2
*b*log(x^n) + 2*a) + b*n) - 5/12*(4*c^(2*b)*e^(2*b*log(x^n) + 2*a) - 1)/(b*
c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a) + 6
*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a)
+ b*n) - 5/8/(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) - 4*b*c^(6*b)*n*e^(6*b*log
(x^n) + 6*a) + 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) - 4*b*c^(2*b)*n*e^(2*b*
log(x^n) + 2*a) + b*n) + log((c^b*e^(b*log(x^n) + a) + 1)*e^(-a)/c^b)/(b*n)
 + log((c^b*e^(b*log(x^n) + a) - 1)*e^(-a)/c^b)/(b*n) - log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(62) = 124.

Time = 0.40 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.44

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \frac{\log\left(\sqrt{-2\,x^{2\,bn}|c|^{2\,b}\cos\left(\pi b\mathrm{sgn}\left(c\right)-\pi b\right)e^{(2\,a)}+x^{4\,bn}|c|^{4\,b}e^{(4\,a)}+1}\right)}{bn} - \frac{25\,c^{8\,b}x^{8\,bn}e^{(8\,a)}-52\,c^{6\,b}x^{6\,bn}e^{(6\,a)}+102\,c^{4\,b}x^{4\,bn}e^{(4\,a)}-52\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+25}{12\,\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}-1\right)^{4}bn} - \log\left(x\right)$$

```
[In] integrate(coth(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] log(sqrt(-2*x^(2*b*n)*abs(c)^(2*b)*cos(pi*b*sgn(c) - pi*b)*e^(2*a) + x^(4*b
*n)*abs(c)^(4*b)*e^(4*a) + 1))/(b*n) - 1/12*(25*c^(8*b)*x^(8*b*n)*e^(8*a) -
 52*c^(6*b)*x^(6*b*n)*e^(6*a) + 102*c^(4*b)*x^(4*b*n)*e^(4*a) - 52*c^(2*b)*
x^(2*b*n)*e^(2*a) + 25)/((c^(2*b)*x^(2*b*n)*e^(2*a) - 1)^4*b*n) - log(x)
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.47

$$\int \frac{\coth^5(a+b\log(cx^n))}{x}\,dx = \frac{8}{b\,n-3\,b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+3\,b\,n\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}-b\,n\,\mathrm{e}^{6\,a}\,{\left(c\,x^n\right)}^{6\,b}} - \ln(x) + \frac{4}{b\,n-b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}} - \frac{4}{b\,n-4\,b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+6\,b\,n\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}-4\,b\,n\,\mathrm{e}^{6\,a}\,{\left(c\,x^n\right)}^{6\,b}+b\,n\,\mathrm{e}^{8\,a}\,{\left(c\,x^n\right)}^{8\,b}} - \frac{8}{b\,n-2\,b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+b\,n\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}} + \frac{\ln\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-1\right)}{b\,n}$$

```
[In] int(coth(a + b*log(c*x^n))^5/x,x)
```

```
[Out] 8/(b*n - 3*b*n*exp(2*a)*(c*x^n)^(2*b) + 3*b*n*exp(4*a)*(c*x^n)^(4*b) - b*n*
exp(6*a)*(c*x^n)^(6*b)) - log(x) + 4/(b*n - b*n*exp(2*a)*(c*x^n)^(2*b)) - 4
/(b*n - 4*b*n*exp(2*a)*(c*x^n)^(2*b) + 6*b*n*exp(4*a)*(c*x^n)^(4*b) - 4*b*n
*exp(6*a)*(c*x^n)^(6*b) + b*n*exp(8*a)*(c*x^n)^(8*b)) - 8/(b*n - 2*b*n*exp(
2*a)*(c*x^n)^(2*b) + b*n*exp(4*a)*(c*x^n)^(4*b)) + log(exp(2*a)*(c*x^n)^(2*
b) - 1)/(b*n)
```

3.194 $\int (ex)^m \coth(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (ex)^m \coth(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}}{e(1+m)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)/e/(1+m)-2*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+
m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/e/(1+m)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5659, 5657, 470, 371}

$$\int (ex)^m \coth(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{m+1}}{e(m+1)} - \frac{2(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2bdn},\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd}\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (e*x)^(1 + m)/(e*(1 + m)) - (2*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(
2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d)])/(e*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\coth(d(a+b\log(x)))\,dx,x,cx^n\right)}{en}\\
&= \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{x^{-1+\frac{1+m}{n}}\left(-1-e^{2ad}x^{2bd}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{en}\\
&= \frac{(ex)^{1+m}}{e(1+m)}-\frac{\left(2(ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int\frac{x^{-1+\frac{1+m}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{en}\\
&= \frac{(ex)^{1+m}}{e(1+m)}-\frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{e(1+m)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 13.62 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int (ex)^m \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx = \frac{x(ex)^m \left(-\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2d(a+b\log(cx^n))}\right)-\frac{e^{2ad}(1+m)(cx^n)^{2bd}\,\text{Hypergeometric2F1}\left(1,\frac{1+m+2}{2bd}\right.}{1+m+2bdn}\right.}{1+m}$$

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])],x]
```

```
[Out] (x*(e*x)^m*(-Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n),
 E^(2*d*(a + b*Log[c*x^n]))] - (E^(2*a*d)*(1 + m)*(c*x^n)^(2*b*d)*Hypergeom
etric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*
a*d)*(c*x^n)^(2*b*d)])/(1 + m + 2*b*d*n)))/(1 + m)
```

Maple [F]

$$\int (ex)^m \coth\left(d(a+b\ln\left(c\,x^n\right))\right)dx$$

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n))),x)
```

Fricas [F]

$$\int (ex)^m \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d), x)
```

Sympy [F]

$$\int (ex)^m \coth\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \coth\left(ad+bd\log\left(cx^n\right)\right)dx$$

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n))),x)
```

```
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n)), x)
```

Maxima [F]

$$\int (ex)^m \coth (d(a + b\log (cx^n)))\, dx = \int (ex)^m \coth ((b\log (cx^n) + a)d)\, dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="maxima")
```

```
[Out] e^m*x*x^m/(m + 1) - e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) + 1),
 x) + e^m*integrate(x^m/(c^(b*d)*e^(b*d*log(x^n) + a*d) - 1), x)
```

Giac [F]

$$\int (ex)^m \coth (d(a + b\log (cx^n)))\, dx = \int (ex)^m \coth ((b\log (cx^n) + a)d)\, dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n))),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth (d(a + b\log (cx^n)))\, dx = \int \coth(d\,(a + b\,\ln (c\,x^n)))\,(e\,x)^m\, dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))*(e*x)^m,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))*(e*x)^m, x)
```

3.195 $\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 168

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n}+\frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)}{bden\left(1-e^{2ad}(cx^n)^{2bd}\right)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{bden}$$

```
[Out] (b*d*n+m+1)*(e*x)^(1+m)/b/d/e/(1+m)/n+(e*x)^(1+m)*(1+exp(2*a*d)*(c*x^n)^(2*
b*d))/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-2*(e*x)^(1+m)*hypergeom([1, 1/
2*(1+m)/b/d/n],[1+1/2*(1+m)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b/d/e/n
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5659, 5657, 516, 470, 371}

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = -\frac{2(ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2bdn},\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd}\right)}{bden} + \frac{(ex)^{m+1}\left(e^{2ad}(cx^n)^{2bd}+1\right)}{bden\left(1-e^{2ad}(cx^n)^{2bd}\right)}+\frac{(ex)^{m+1}(bdn+m+1)}{bde(m+1)n}$$

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] ((1 + m + b*d*n)*(e*x)^(1 + m))/(b*d*e*(1 + m)*n) + ((e*x)^(1 + m)*(1 + E^(
2*a*d)*(c*x^n)^(2*b*d)))/(b*d*e*n*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - (2*(e*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E
^(2*a*d)*(c*x^n)^(2*b*d)])/(b*d*e*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\coth^2(d(a+b\log(x)))\,dx,x,cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-1-e^{2ad}x^{2bd}\right)^2}{\left(1-e^{2ad}x^{2bd}\right)^2}\,dx,x,cx^n\right)}{en}$$

$$= \frac{(ex)^{1+m}\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)}{bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} + \frac{\left(e^{-2ad}(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-\frac{2e^{2ad}(1+m-bdn)}{n}-\frac{2e^{4ad}(1+m+bdn)x^{2bd}}{n}\right)}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{2bden}$$

$$= \frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n} + \frac{(ex)^{1+m}\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)}{bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{\left(2(1+m)(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{bden^2}$$

$$= \frac{(1+m+bdn)(ex)^{1+m}}{bde(1+m)n} + \frac{(ex)^{1+m}\left(1+e^{2ad}\left(cx^n\right)^{2bd}\right)}{bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)} - \frac{2(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}\left(cx^n\right)^{2bd}\right)}{bden}$$

Mathematica [A] (verified)

Time = 14.70 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.86

$$\int (ex)^m\coth^2\left(d\left(a+b\log\left(cx^n\right)\right)\right)\,dx = (ex)^m\left(\frac{x}{1+m} - \frac{e^{-\frac{(1+2m)(a-bn\log(x)+b\log(cx^n))}{bn}}x^{-2m}\left(e^{\frac{(1+2m)(a+b\log(cx^n))}{bn}}(1+m+2bdn)\coth\left(d\left(a+b\log\left(cx^n\right)\right)\right)+e^{\frac{(1+2m)(a+b\log}{bn}}\right.}{}\right.$$

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^2,x]
```

```
[Out] (e*x)^m*(x/(1 + m) - (E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))*(1 + m + 2*b
*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^(((1 + 2*m)*(a + b*Log[c*x^n]))/(b*n))
```

```
*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b
*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^(((1 + 2*m + 2*b*d*n)*(a - b*n*Log[x
] + b*Log[c*x^n]))/(b*n))*(1 + m)*x^(1 + 2*m + 2*b*d*n)*Hypergeometric2F1[1
, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*n)/(2*b*d*n), E^(2*d*(a + b*L
og[c*x^n]))])/(b*d*E^(((1 + 2*m)*(a - b*n*Log[x] + b*Log[c*x^n]))/(b*n))*n*
(1 + m + 2*b*d*n)*x^(2*m)))
```

Maple [F]

$$\int (ex)^m \coth \left(d(a+b\ln \left(c\,x^n\right))\right)^2 dx$$

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^2,x)
```

Fricas [F]

$$\int (ex)^m \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="fricas")
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^2, x)
```

Sympy [F]

$$\int (ex)^m \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \coth^2\left(ad+bd\log\left(cx^n\right)\right)dx$$

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**2,x)
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^m \coth^2\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)^2\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="maxima")
[Out] -e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) + b*d*n),
x) + e^m*(m + 1)*integrate(x^m/(b*c^(b*d)*d*n*e^(b*d*log(x^n) + a*d) - b*d*
n), x) + (b*c^(2*b*d)*d*e^m*n*x*e^(2*b*d*log(x^n) + 2*a*d + m*log(x)) - (b*
d*e^m*n + 2*e^m*(m + 1))*x*x^m)/((m*n + n)*b*c^(2*b*d)*d*e^(2*b*d*log(x^n)
+ 2*a*d) - (m*n + n)*b*d)
```

Giac [F]

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth\left((b\log(cx^n)+a)d\right)^2\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^2,x, algorithm="giac")
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^2(d(a+b\log(cx^n)))\,dx = \int \coth(d\,(a+b\,\ln(c\,x^n)))^2\,(e\,x)^m\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^2*(e*x)^m, x)
```

3.196 $\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 306

$$\begin{aligned}
&\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx \\
&= \frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} - \frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}(cx^n)^{2bd}\right)^2} \\
&\quad + \frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}+\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1-e^{2ad}(cx^n)^{2bd}\right)} \\
&\quad - \frac{(1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(1+m)n^2}
\end{aligned}$$

```
[Out] 1/2*(b*d*n+m+1)*(2*b*d*n+m+1)*(e*x)^(1+m)/b^2/d^2/e/(1+m)/n^2-1/2*(e*x)^(1+
m)*(1+exp(2*a*d)*(c*x^n)^(2*b*d))^2/b/d/e/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))^
2+1/2*(e*x)^(1+m)*(exp(2*a*d)*(-2*b*d*n+m+1)/n+exp(4*a*d)*(2*b*d*n+m+1)*(c*
x^n)^(2*b*d)/n)/b^2/d^2/e/exp(2*a*d)/n/(1-exp(2*a*d)*(c*x^n)^(2*b*d))-(2*b^
2*d^2*n^2+m^2+2*m+1)*(e*x)^(1+m)*hypergeom([1, 1/2*(1+m)/b/d/n],[1+1/2*(1+m
)/b/d/n],exp(2*a*d)*(c*x^n)^(2*b*d))/b^2/d^2/e/(1+m)/n^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5659, 5657, 516, 608, 470, 371}

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx$$
$$= -\frac{(ex)^{m+1}\left(2b^2d^2n^2+m^2+2m+1\right)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2bdn},\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(m+1)n^2}$$
$$+\frac{e^{-2ad}(ex)^{m+1}\left(\frac{e^{4ad}(2bdn+m+1)(cx^n)^{2bd}}{n}+\frac{e^{2ad}(-2bdn+m+1)}{n}\right)}{2b^2d^2en\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)}$$
$$-\frac{(ex)^{m+1}\left(e^{2ad}(cx^n)^{2bd}+1\right)^2}{2bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^2}+\frac{(ex)^{m+1}(bdn+m+1)(2bdn+m+1)}{2b^2d^2e(m+1)n^2}$$

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] ((1 + m + b*d*n)*(1 + m + 2*b*d*n)*(e*x)^(1 + m))/(2*b^2*d^2*e*(1 + m)*n^2)
 - ((e*x)^(1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^2)/(2*b*d*e*n*(1 - E^(2*a
*d)*(c*x^n)^(2*b*d))^2) + ((e*x)^(1 + m)*((E^(2*a*d)*(1 + m - 2*b*d*n))/n +
 (E^(4*a*d)*(1 + m + 2*b*d*n)*(c*x^n)^(2*b*d))/n))/(2*b^2*d^2*e*E^(2*a*d)*n
*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*b*d*n), E^(2
*a*d)*(c*x^n)^(2*b*d)])/(b^2*d^2*e*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 516

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
```

```
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && LtQ
[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 608

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(
m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*b*g*n*(p + 1))), x] + Dist[1/(
a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*
(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m +
 n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && LtQ[
p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\coth^3(d(a+b\log(x)))\,dx,x,cx^n\right)}{en}$$

$$=\frac{\left((ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-1-e^{2ad}x^{2bd}\right)^3}{\left(1-e^{2ad}x^{2bd}\right)^3}\,dx,x,cx^n\right)}{en}$$

$$=-\frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}\left(cx^n\right)^{2bd}\right)^2}$$

$$+\frac{\left(e^{-2ad}(ex)^{1+m}\left(cx^n\right)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-1-e^{2ad}x^{2bd}\right)\left(-\frac{2e^{2ad}(1+m-2bdn)}{n}-\frac{2e^{4ad}(1+m+2bdn)x^{2bd}}{n}\right)}{\left(1-e^{2ad}x^{2bd}\right)^2}\,dx,x\right.}{4bden}$$

$$= -\frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}(cx^n)^{2bd}\right)^2} + \frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}+\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1-e^{2ad}(cx^n)^{2bd}\right)}$$
$$+\frac{\left(e^{-4ad}(ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}\left(-\frac{4e^{4ad}(1+m-2bdn)(1+m-bdn)}{n^2}-\frac{4e^{6ad}(1+m+bdn)(1+m+2bdn)x^{2bd}}{n^2}\right)}{1-e^{2ad}x^{2bd}}\right.}{8b^2d^2en}$$

$$= \frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} - \frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}$$
$$+\frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}+\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1-e^{2ad}(cx^n)^{2bd}\right)}$$
$$-\frac{\left((1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int \frac{x^{-1+\frac{1+m}{n}}}{1-e^{2ad}x^{2bd}}\,dx,x,cx^n\right)}{b^2d^2en^3}$$

$$= \frac{(1+m+bdn)(1+m+2bdn)(ex)^{1+m}}{2b^2d^2e(1+m)n^2} - \frac{(ex)^{1+m}\left(1+e^{2ad}(cx^n)^{2bd}\right)^2}{2bden\left(1-e^{2ad}(cx^n)^{2bd}\right)^2}$$
$$+\frac{e^{-2ad}(ex)^{1+m}\left(\frac{e^{2ad}(1+m-2bdn)}{n}+\frac{e^{4ad}(1+m+2bdn)(cx^n)^{2bd}}{n}\right)}{2b^2d^2en\left(1-e^{2ad}(cx^n)^{2bd}\right)}$$
$$-\frac{(1+2m+m^2+2b^2d^2n^2)(ex)^{1+m}\text{Hypergeometric2F1}\left(1,\frac{1+m}{2bdn},1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd}\right)}{b^2d^2e(1+m)n^2}$$

Mathematica [A] (verified)

Time = 16.67 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.96

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m\coth(d(a+b(-n\log(x)+\log(cx^n))))}{1+m}$$
$$-\frac{x(ex)^m\text{csch}^2(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n))))}{2bdn}$$
$$+\frac{(1+m)x(ex)^m\text{csch}(d(a+b(-n\log(x)+\log(cx^n))))\,\text{csch}(bdn\log(x)+d(a+b(-n\log(x)+\log(cx^n)))}{2b^2d^2n^2}$$
$$-\frac{(1+2m+m^2+2b^2d^2n^2)\,x^{-m}(ex)^m\text{csch}(d(a+b(-n\log(x)+\log(cx^n))))\left(\frac{x^{1+m}\text{csch}(d(a+b\log(cx^n)))\sinh(}{1+m}\right.}{}$$

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^3,x]
```

```
[Out] (x*(e*x)^m*Coth[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))])/(1 + m) - (x*(e*x)^m
*Csch[b*d*n*Log[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]^2)/(2*b*d*n) + (
(1 + m)*x*(e*x)^m*Csch[d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Csch[b*d*n*Log
[x] + d*(a + b*(-(n*Log[x]) + Log[c*x^n]))]*Sinh[b*d*n*Log[x]])/(2*b^2*d^2*
n^2) - ((1 + 2*m + m^2 + 2*b^2*d^2*n^2)*(e*x)^m*Csch[d*(a + b*(-(n*Log[x])
+ Log[c*x^n]))]*((x^(1 + m)*Csch[d*(a + b*Log[c*x^n])]*Sinh[b*d*n*Log[x]])/
(1 + m) + ((E^((a + 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) +
 Log[c*x^n]))/(b*n))*(1 + m + 2*b*d*n)*Coth[d*(a + b*Log[c*x^n])] + E^((a +
 2*a*m + b*(1 + m)*n*Log[x] + b*(1 + 2*m)*(-(n*Log[x]) + Log[c*x^n]))/(b*n)
)*(1 + m + 2*b*d*n)*Hypergeometric2F1[1, (1 + m)/(2*b*d*n), 1 + (1 + m)/(2*
b*d*n), E^(2*d*(a + b*Log[c*x^n]))] + E^((a*(1 + 2*m + 2*b*d*n))/(b*n) + (1
 + m + 2*b*d*n)*Log[x] + ((1 + 2*m + 2*b*d*n)*(-(n*Log[x]) + Log[c*x^n]))/n
)*(1 + m)*Hypergeometric2F1[1, (1 + m + 2*b*d*n)/(2*b*d*n), (1 + m + 4*b*d*
n)/(2*b*d*n), E^(2*d*(a + b*Log[c*x^n]))])*Sinh[d*(a + b*(-(n*Log[x]) + Log
[c*x^n]))])/(E^(((1 + 2*m)*(a + b*(-(n*Log[x]) + Log[c*x^n])))/(b*n))*(1 +
m)*(1 + m + 2*b*d*n))))/(2*b^2*d^2*n^2*x^m)
```

Maple [F]

$$\int (ex)^m \coth\left(d(a+b\ln\left(c\,x^n\right))\right)^3 dx$$

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^3,x)
```

Fricas [F]

$$\int (ex)^m \coth^3\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int (ex)^m \coth\left((b\log\left(cx^n\right)+a)d\right)^3\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^3, x)
```

Sympy [F]

$$\int (ex)^m \coth^3\left(d(a+b\log\left(cx^n\right))\right)\,dx=\int (ex)^m \coth^3\left(ad+bd\log\left(cx^n\right)\right)dx$$

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**3,x)
```

```
[Out] Integral((e*x)**m*coth(a*d + b*d*log(c*x**n))**3, x)
```

Maxima [F]

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\, dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^3\, dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="maxima")
```

```
[Out] -(2*b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d
^2*n^2*e^(b*d*log(x^n) + a*d) + b^2*d^2*n^2), x) + (2*b^2*d^2*e^m*n^2 + (m^
2 + 2*m + 1)*e^m)*integrate(1/2*x^m/(b^2*c^(b*d)*d^2*n^2*e^(b*d*log(x^n) +
a*d) - b^2*d^2*n^2), x) + (b^2*c^(4*b*d)*d^2*e^m*n^2*x*e^(4*b*d*log(x^n) +
4*a*d + m*log(x)) + (b^2*d^2*e^m*n^2 + (m^2 + 2*m + 1)*e^m)*x*x^m - (2*b^2*
c^(2*b*d)*d^2*e^m*n^2*e^(2*a*d) + 2*(m*n + n)*b*c^(2*b*d)*d*e^m*e^(2*a*d) +
 (m^2 + 2*m + 1)*c^(2*b*d)*e^m*e^(2*a*d))*x*e^(2*b*d*log(x^n) + m*log(x)))/
((m*n^2 + n^2)*b^2*c^(4*b*d)*d^2*e^(4*b*d*log(x^n) + 4*a*d) - 2*(m*n^2 + n^
2)*b^2*c^(2*b*d)*d^2*e^(2*b*d*log(x^n) + 2*a*d) + (m*n^2 + n^2)*b^2*d^2)
```

Giac [F]

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\, dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^3\, dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^3(d(a+b\log(cx^n)))\, dx = \int \coth(d\,(a+b\,\ln(c\,x^n)))^3\,(e\,x)^m\, dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m,x)
```

```
[Out] int(coth(d*(a + b*log(c*x^n)))^3*(e*x)^m, x)
```

3.197 $\int \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = x\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2bdn},p,-p,1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[Out] x*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2/b/d/n,p,-p,1+1/2/b/d/n,exp
(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/((1+exp(2*a*d)*(c*x^n)
^(2*b*d))^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5655, 5657, 525, 524}

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = x\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2bdn},p,-p,1+\frac{1}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)$$

```
[In] Int[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2
*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(1 + E^(
2*a*d)*(c*x^n)^(2*b*d))^p
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5655

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Coth[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\coth^p(d(a+b\log(x)))\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\left(-1-e^{2ad}x^{2bd}\right)^p\left(1-e^{2ad}x^{2bd}\right)^{-p}\,dx, x, cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-1/n}\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\left(1-e^{2ad}x^{2bd}\right)^{-p}\left(1+e^{2\ldots}\right.\right.}{n}$$

$$= x\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\text{AppellF1}\left(\frac{1}{2bdn}, p, -p, 1 + \frac{1}{2bdn}, e^{2ad}(cx^n)^{2bd}, -e^{2ad}(cx^n)^{2bd}\right)$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 387 vs. $2(115) = 230$.

Time = 1.21 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.37

$$\int \coth^p\left(d(a + b\log\left(cx^n\right))\right)\,dx$$

$$= \frac{(1+2bdn)x\left(\frac{1+e^{2ad}(cx^n)^{2bd}}{-1+e^{2ad}(cx^n)^{2bd}}\right.}{2bde^{2ad}np\left(cx^n\right)^{2bd}\text{AppellF1}\left(1+\frac{1}{2bdn},p,1-p,2+\frac{1}{2bdn},e^{2ad}\left(cx^n\right)^{2bd},-e^{2ad}\left(cx^n\right)^{2bd}\right)+2bde^{2ad}np\left(cx^n\right)^{2bd}}$$

```
[In] Integrate[Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((1 + 2*b*d*n)*x*((1 + E^(2*a*d)*(c*x^n)^(2*b*d))/(-1 + E^(2*a*d)*(c*x^n)^(
2*b*d)))^p*AppellF1[1/(2*b*d*n), p, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^
(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))])/(2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d
)*AppellF1[1 + 1/(2*b*d*n), p, 1 - p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2
*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + 2*b*d*E^(2*a*d)*n*p*(c*x^n)^(2*b*d)*
AppellF1[1 + 1/(2*b*d*n), 1 + p, -p, 2 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*
b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d))] + (1 + 2*b*d*n)*AppellF1[1/(2*b*d*n), p
, -p, 1 + 1/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*
d))])
```

Maple [F]

$$\int \coth\left(d(a+b\ln\left(c\,x^n\right))\right)^p dx$$

```
[In] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int(coth(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int \coth^p\left(d(a+b\log\left(cx^n\right))\right)\,dx = \int \coth\left((b\log\left(cx^n\right)+a)d\right)^p\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral(coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F]

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth^p(d(a+b\log(cx^n)))\,dx$$

```
[In] integrate(coth(d*(a+b*ln(c*x**n)))**p,x)
[Out] Integral(coth(d*(a + b*log(c*x**n)))**p, x)
```

Maxima [F]

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth((b\log(cx^n)+a)d)^p\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth((b\log(cx^n)+a)d)^p\,dx$$

```
[In] integrate(coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate(coth((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^p(d(a+b\log(cx^n)))\,dx = \int \mathrm{coth}(d\,(a+b\,\ln(c\,x^n)))^p\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^p,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^p, x)
```

3.198 $\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{1+m}\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\operatorname{AppellF1}\left(\frac{1+m}{2bdn},p,-p,1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{e(1+m)}$$

```
[Out] (e*x)^(1+m)*(-1-exp(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1(1/2*(1+m)/b/d/n,p,-p
,1+1/2*(1+m)/b/d/n,exp(2*a*d)*(c*x^n)^(2*b*d),-exp(2*a*d)*(c*x^n)^(2*b*d))/
e/(1+m)/((1+exp(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5659, 5657, 525, 524}

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \frac{(ex)^{m+1}\left(-e^{2ad}(cx^n)^{2bd}-1\right)^p\left(e^{2ad}(cx^n)^{2bd}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{m+1}{2bdn},p,-p,\frac{m+1}{2bdn}+1,e^{2ad}(cx^n)^{2bd},-e^{2ad}(cx^n)^{2bd}\right)}{e(m+1)}$$

```
[In] Int[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] ((e*x)^(1 + m)*(-1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*AppellF1[(1 + m)/(2*b*d*n
), p, -p, 1 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*
x^n)^(2*b*d))])/(e*(1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 5657

```
Int[Coth[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m_.), x_Symbol]
 :> Int[(e*x)^m*((-1 - E^(2*a*d)*x^(2*b*d))^p/(1 - E^(2*a*d)*x^(2*b*d))^p),
 x] /; FreeQ[{a, b, d, e, m, p}, x]
```

Rule 5659

```
Int[Coth[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.)*((e_.)*(x_))^(m
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Coth[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\coth^p(d(a+b\log(x)))\,dx,x,cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\left(-1-e^{2ad}x^{2bd}\right)^p\left(1-e^{2ad}x^{2bd}\right)^{-p}\,dx,x,cx^n\right)}{en}$$

$$= \frac{\left((ex)^{1+m}(cx^n)^{-\frac{1+m}{n}}\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\right)\text{Subst}\left(\int x^{-1+\frac{1+m}{n}}\left(1-e^{2ad}x^{2bd}\right.\right.}{en}$$

$$= \frac{(ex)^{1+m}\left(-1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\text{AppellF1}\left(\frac{1+m}{2bdn},p,-p,1+\frac{1+m}{2bdn},e^{2ad}(cx^n)^{2bd},\right.}{e(1+m)}$$

Mathematica [A] (warning: unable to verify)

Time = 1.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \frac{x(ex)^m\left(1-e^{2ad}(cx^n)^{2bd}\right)^p\left(1+e^{2ad}(cx^n)^{2bd}\right)^{-p}\left(\frac{1+e^{2ad}(cx^n)^{2bd}}{-1+e^{2ad}(cx^n)^{2bd}}\right)^p \text{AppellF1}\left(\frac{1+m}{2bdn},p,-p,1+\frac{1+m}{2bdn},e^{2ad}(cx$$
$$1+m$$

```
[In] Integrate[(e*x)^m*Coth[d*(a + b*Log[c*x^n])]^p,x]
```

```
[Out] (x*(e*x)^m*(1 - E^(2*a*d)*(c*x^n)^(2*b*d))^p*((1 + E^(2*a*d)*(c*x^n)^(2*b*d
))/(-1 + E^(2*a*d)*(c*x^n)^(2*b*d)))^p*AppellF1[(1 + m)/(2*b*d*n), p, -p, 1
 + (1 + m)/(2*b*d*n), E^(2*a*d)*(c*x^n)^(2*b*d), -(E^(2*a*d)*(c*x^n)^(2*b*d
))])/((1 + m)*(1 + E^(2*a*d)*(c*x^n)^(2*b*d))^p)
```

Maple [F]

$$\int (ex)^m \coth(d(a+b\ln(c\,x^n)))^p dx$$

```
[In] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

```
[Out] int((e*x)^m*coth(d*(a+b*ln(c*x^n)))^p,x)
```

Fricas [F]

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth\left((b\log(cx^n)+a)d\right)^p\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x)^m*coth(b*d*log(c*x^n) + a*d)^p, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \text{Timed out}$$

```
[In] integrate((e*x)**m*coth(d*(a+b*ln(c*x**n)))**p,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^p\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Giac [F]

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int (ex)^m \coth((b\log(cx^n)+a)d)^p\,dx$$

```
[In] integrate((e*x)^m*coth(d*(a+b*log(c*x^n)))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*coth((b*log(c*x^n) + a)*d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m \coth^p(d(a+b\log(cx^n)))\,dx = \int \coth(d\,(a+b\,\ln(c\,x^n)))^p\,(e\,x)^m\,dx$$

```
[In] int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m,x)
[Out] int(coth(d*(a + b*log(c*x^n)))^p*(e*x)^m, x)
```

3.199 $\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2/3*coth(a+b*ln(c*x^n))^(3/2)/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3554, 3557, 335, 304, 209, 212}

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

```
[In] Int[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - (2*Coth[a + b*Log[c*x^n]]^(3/2))/(3*b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \coth^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\coth^{\frac{3}{2}}(a+b\log(cx^n))}{3bn}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right) - \text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right) + \frac{2}{3}\coth^{\frac{3}{2}}(a+b\log(cx^n))}{bn}$$

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] -((ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - ArcTanh[Sqrt[Coth[a + b*Log[c*x^n
]]]] + (2*Coth[a + b*Log[c*x^n]]^(3/2))/3)/(b*n))
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-\frac{2\coth(a+b\ln(c\,x^n))^{\frac{3}{2}}}{3}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	76
default	$\frac{-\frac{2\coth(a+b\ln(c\,x^n))^{\frac{3}{2}}}{3}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	76

```
[In] int(coth(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2/3*coth(a+b*ln(c*x^n))^(3/2)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1
/2*ln(coth(a+b*ln(c*x^n))^(1/2)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 626 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 626, normalized size of antiderivative = 8.58

$$\int \frac{\coth^{\frac{5}{2}}(a + b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*(6*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - 3*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)^2 - 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x)
+ b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c
) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(
c) + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a) - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) +
a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
sinh(b*n*log(x) + b*log(c) + a)^2 + 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)
/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
+ b*n*sinh(b*n*log(x) + b*log(c) + a)^2 - b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(coth(a+b*ln(c*x**n))**(5/2)/x,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\coth(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
[Out] integrate(coth(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(coth(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

$$\int \frac{\coth^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{\operatorname{atan}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{2\coth(a+b\,\ln(c\,x^n))^{3/2}}{3\,b\,n}$$

```
[In] int(coth(a + b*log(c*x^n))^(5/2)/x,x)
[Out] atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - (2*coth(a + b*log(c*x^n))^(3/2))/(3*b*n)
```

3.200 $\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n-2*coth(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3554, 3557, 335, 218, 212, 209}

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2\sqrt{\coth(a+b\log(cx^n))}}{bn}$$

```
[In] Int[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - (2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \coth^{\frac{3}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$= -\frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$= \frac{\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int \frac{\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}{x}\,dx = \frac{\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right) + \text{arctanh}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right) - 2\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

```
[In] Integrate[Coth[a + b*Log[c*x^n]]^(3/2)/x,x]
[Out] (ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] + ArcTanh[Sqrt[Coth[a + b*Log[c*x^n]]
]] - 2*Sqrt[Coth[a + b*Log[c*x^n]]])/(b*n)
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

method	result	siz
derivativedivides	$\frac{-2\sqrt{\coth(a+b\ln(c\,x^n))}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	74
default	$\frac{-2\sqrt{\coth(a+b\ln(c\,x^n))}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	74

```
[In] int(coth(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(-2*coth(a+b*ln(c*x^n))^(1/2)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2
*ln(coth(a+b*ln(c*x^n))^(1/2)+1)+arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.77

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{4\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)}{\sinh(bn\log(x)+b\log(c)+a)}} + 2\arctan\left(-\cosh(bn\log(x)+b\log(c)+a)^2 - 2\cosh(bn\log(x)+b\log(c)\right.}{}$$

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a
)) + 2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(coth(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\coth(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(coth(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(coth(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{\coth^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\operatorname{atan}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)+\operatorname{atanh}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)-2\,\sqrt{\coth(a+b\,\ln(c\,x^n))}}{b\,n}$$

```
[In] int(coth(a + b*log(c*x^n))^(3/2)/x,x)
[Out] (atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2)) -
 2*coth(a + b*log(c*x^n))^(1/2))/(b*n)
```

3.201 $\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3557, 335, 304, 209, 212}

$$\int \frac{\sqrt{\coth(a+b\log(cx^n))}}{x}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[In] Int[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}\\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}\\
&= -\frac{\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{\coth (a+b\log (cx^n))}}{x}\,dx = -\frac{\arctan \left(\sqrt{\coth (a+b\log (cx^n))}\right)-\operatorname{arctanh}\left(\sqrt{\coth (a+b\log (cx^n))}\right)}{bn}$$

```
[In] Integrate[Sqrt[Coth[a + b*Log[c*x^n]]]/x,x]
[Out] -((ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]] - ArcTanh[Sqrt[Coth[a + b*Log[c*x^n
]]]])/(b*n))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	61
default	$\frac{-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	61

```
[In] int(coth(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b*(-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2*ln(coth(a+b*ln(c*x^n))^(1/2
)+1)-arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(44) = 88.

Time = 0.28 (sec) , antiderivative size = 305, normalized size of antiderivative = 6.35

$$\int \frac{\sqrt{\coth (a+b\log (cx^n))}}{x}\,dx = \frac{2 \arctan \left(-\cosh (bn\log (x)+b\log (c)+a)^2-2 \cosh (bn\log (x)+b\log (c)+a)\sinh (bn\log (x)+b\log (c)$$

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
[Out] 1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)
```

```
^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - log
(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*sin
h(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b*
n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*
log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx$$

```
[In] integrate(coth(a+b*ln(c*x**n))**(1/2)/x,x)
[Out] Integral(sqrt(coth(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\int \frac{\sqrt{\coth\left(b\log\left(cx^n\right)+a\right)}}{x}\,dx$$

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(coth(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx=\text{Timed out}$$

```
[In] integrate(coth(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\coth (a+b\log (cx^n))}}{x}\,dx = -\frac{\mathrm{atan}\left(\sqrt{\coth \left(a+b\,\ln \left(c\,x^n\right)\right)}\right)-\mathrm{atanh}\left(\sqrt{\coth \left(a+b\,\ln \left(c\,x^n\right)\right)}\right)}{b\,n}$$

```
[In] int(coth(a + b*log(c*x^n))^(1/2)/x,x)
[Out] -(atan(coth(a + b*log(c*x^n))^(1/2)) - atanh(coth(a + b*log(c*x^n))^(1/2)))
/(b*n)
```

3.202 $\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3557, 335, 218, 212, 209}

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

```
[In] Int[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}\\
&= \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\coth (a+b\log (cx^n))}}\,dx=\frac{\arctan \left(\sqrt{\coth (a+b\log (cx^n))}\right)}{bn}+\frac{\operatorname{arctanh}\left(\sqrt{\coth (a+b\log (cx^n))}\right)}{bn}$$

```
[In] Integrate[1/(x*Sqrt[Coth[a + b*Log[c*x^n]]]),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	37
default	$\frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)}{nb}$	37

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(arctanh(coth(a+b*ln(c*x^n))^(1/2))+arctan(coth(a+b*ln(c*x^n))^(1/2))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(43) = 86.

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 6.45

$$\int \frac{1}{x\sqrt{\coth (a+b\log (cx^n))}}\,dx = -\frac{2 \arctan \left(-\cosh (bn\log (x)+b\log (c)+a)^2-2\cosh (bn\log (x)+b\log (c)+a)\sinh (bn\log (x)+b\log$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*l
og(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a
```

```
)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a
)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*
sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + lo
g(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (cosh(b
*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n
*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(coth(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\sqrt{\coth\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(coth(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\sqrt{\coth(a+b\log(cx^n))}}\,dx = \frac{\operatorname{atan}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)+\operatorname{atanh}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)}{b\,n}$$

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(1/2)),x)
[Out] (atan(coth(a + b*log(c*x^n))^(1/2)) + atanh(coth(a + b*log(c*x^n))^(1/2)))/
(b*n)
```

3.203 $\int \frac{1}{x \coth^{\frac{3}{2}}(a+b\log(cx^n))} dx$

Optimal result

Integrand size = 19, antiderivative size = 71

$$\int \frac{1}{x \coth^{\frac{3}{2}}(a+b\log(cx^n))} dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}}$$

```
[Out] -arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b
/n-2/b/n/coth(a+b*ln(c*x^n))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3555, 3557, 335, 304, 209, 212}

$$\int \frac{1}{x \coth^{\frac{3}{2}}(a+b\log(cx^n))} dx = -\frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{bn\sqrt{\coth(a+b\log(cx^n))}}$$

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] -(ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n)) + ArcTanh[Sqrt[Coth[a + b*Log
[c*x^n]]]]/(b*n) - 2/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}} + \frac{\text{Subst}\left(\int \sqrt{\coth(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}$$

$$= -\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}} - \frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \coth\left(a+b\log\left(cx^n\right)\right)\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}} - \frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$= -\frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn}$$

$$= -\frac{\arctan\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}\right)}{bn} - \frac{2}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.37

$$\int \frac{1}{x\coth^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \frac{-2-\arctan\left(\sqrt[4]{\coth^2\left(a+b\log\left(cx^n\right)\right)}\right)\sqrt[4]{\coth^2\left(a+b\log\left(cx^n\right)\right)}+\text{arctanh}\left(\sqrt[4]{\coth^2\left(a+b\log\left(cx^n\right)\right)}\right)\sqrt[4]{}}{bn\sqrt{\coth\left(a+b\log\left(cx^n\right)\right)}}$$

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (-2 - ArcTan[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^n]]^2)^(
1/4) + ArcTanh[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^n]]^2)
^(1/4))/(b*n*Sqrt[Coth[a + b*Log[c*x^n]]])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}+1\right)}{2}-\frac{2}{\sqrt{\coth(a+b\ln(c\,x^{n}))}}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}\right)}{nb}$	76
default	$\frac{-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}-1\right)}{2}+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}+1\right)}{2}-\frac{2}{\sqrt{\coth(a+b\ln(c\,x^{n}))}}-\arctan\left(\sqrt{\coth(a+b\ln(c\,x^{n}))}\right)}{nb}$	76

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/n/b*(-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)+1/2*ln(coth(a+b*ln(c*x^n))^(1/2
)+1)-2/coth(a+b*ln(c*x^n))^(1/2)-arctan(coth(a+b*ln(c*x^n))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(65) = 130$.

Time = 0.27 (sec) , antiderivative size = 625, normalized size of antiderivative = 8.80

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
[Out] 1/2*(2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)
*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2 + (
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(co
sh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) - 4*cosh(b*
n*log(x) + b*log(c) + a)^2 - (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x)
+ b*log(c) + a)^2 + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) +
b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c)
+ a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c)
 + a))) - 8*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 - 4*sinh(b*n*log(x) + b*log(c) + a)^2 - 4*(cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + si
nh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/s
inh(b*n*log(x) + b*log(c) + a)) - 4)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) +
b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F]

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(3/2),x)
[Out] Integral(1/(x*coth(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\coth(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\coth^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\operatorname{atanh}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{\operatorname{atan}\left(\sqrt{\coth(a+b\,\ln(c\,x^n))}\right)}{b\,n} - \frac{2}{b\,n\,\sqrt{\coth(a+b\,\ln(c\,x^n))}}$$

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(3/2)),x)
[Out] atanh(coth(a + b*log(c*x^n))^(1/2))/(b*n) - atan(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(b*n*coth(a + b*log(c*x^n))^(1/2))
```

3.204 $\int \frac{1}{x \coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{1}{x \coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[Out] arctan(coth(a+b*ln(c*x^n))^(1/2))/b/n+arctanh(coth(a+b*ln(c*x^n))^(1/2))/b/
n-2/3/b/n/coth(a+b*ln(c*x^n))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3555, 3557, 335, 218, 212, 209}

$$\int \frac{1}{x \coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\operatorname{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[In] Int[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] ArcTan[Sqrt[Coth[a + b*Log[c*x^n]]]]/(b*n) + ArcTanh[Sqrt[Coth[a + b*Log[c*
x^n]]]]/(b*n) - 2/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3555

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Tan[c + d*x]
)^(n + 1)/(b*d*(n + 1)), x] - Dist[1/b^2, Int[(b*Tan[c + d*x])^(n + 2), x],
 x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1]
```

Rule 3557

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\coth^{\frac{5}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\coth(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}(-1+x^2)}\,dx, x, \coth(a+b\log(cx^n))\right)}{bn}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

$$= -\frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\coth(a+b\log(cx^n))}\right)}{bn}$$

$$= \frac{\arctan\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} + \frac{\text{arctanh}\left(\sqrt{\coth(a+b\log(cx^n))}\right)}{bn} - \frac{2}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.40

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$$

$$= \frac{-2+3\arctan\left(\sqrt[4]{\coth^2(a+b\log(cx^n))}\right)\coth^2(a+b\log(cx^n))^{3/4}+3\text{arctanh}\left(\sqrt[4]{\coth^2(a+b\log(cx^n)}\right.}{3bn\coth^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[In] Integrate[1/(x*Coth[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (-2 + 3*ArcTan[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^n]]^2)
^(3/4) + 3*ArcTanh[(Coth[a + b*Log[c*x^n]]^2)^(1/4)]*(Coth[a + b*Log[c*x^n]
]^2)^(3/4))/(3*b*n*Coth[a + b*Log[c*x^n]]^(3/2))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}-\frac{2}{3\coth(a+b\ln(c\,x^n))^{\frac{3}{2}}}}{nb}$	74
default	$\frac{\arctan\left(\sqrt{\coth(a+b\ln(c\,x^n))}\right)+\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}+1\right)}{2}-\frac{\ln\left(\sqrt{\coth(a+b\ln(c\,x^n))}-1\right)}{2}-\frac{2}{3\coth(a+b\ln(c\,x^n))^{\frac{3}{2}}}}{nb}$	74

```
[In] int(1/x/coth(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(arctan(coth(a+b*ln(c*x^n))^(1/2))+1/2*ln(coth(a+b*ln(c*x^n))^(1/2)+1
)-1/2*ln(coth(a+b*ln(c*x^n))^(1/2)-1)-2/3/coth(a+b*ln(c*x^n))^(3/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 1104, normalized size of antiderivative = 15.33

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*cosh(b*n*log(x) + b*log(c) + a)^4 + 16*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*sinh(b*n*log(x) + b*log(c) + a)^4
 + 8*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 6*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4
 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) +
 a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c)
 + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)
+ 1)*arctan(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*log(x) + b*log(c) + a)^2
 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*s
inh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 - 1)*sqr
t(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b*log(c) + a))) + 8*cos
h(b*n*log(x) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*c
osh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sin
h(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(c
osh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*sinh(b*
n*log(x) + b*log(c) + a) + 1)*log(-cosh(b*n*log(x) + b*log(c) + a)^2 - 2*co
```

```
sh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) - sinh(b*n*lo
g(x) + b*log(c) + a)^2 + (cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*lo
g(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*
log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)/sinh(b*n*log(x) + b
*log(c) + a))) + 16*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) +
b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 4*(cosh(b*n*log(x) + b*log
(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
 a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*log(x) + b*log(c)
 + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a)^2 - 2*cosh(b*n*log(x) + b*log(
c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 - cosh(b*n*log(x) + b*log(
c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*sqrt(cosh(b*n*log(x) + b*log(
c) + a)/sinh(b*n*log(x) + b*log(c) + a)) + 4)/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^4 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a)^3 + b*n*sinh(b*n*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) +
b*log(c) + a)^2 + 2*(3*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*
n*log(x) + b*log(c) + a)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3
 + b*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/coth(a+b*ln(c*x**n))**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x\coth^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\coth(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*coth(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x \coth^{\frac{5}{2}}(a + b\log(cx^n))} \, dx = \text{Timed out}$$

```
[In] integrate(1/x/coth(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 3.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{1}{x \coth^{\frac{5}{2}}(a + b\log(cx^n))} \, dx = \frac{\operatorname{atan}\left(\sqrt{\coth(a + b\ln(c\,x^n))}\right)}{b\,n} + \frac{\operatorname{atanh}\left(\sqrt{\coth(a + b\ln(c\,x^n))}\right)}{b\,n} - \frac{2}{3\,b\,n\coth(a + b\ln(c\,x^n))^{3/2}}$$

```
[In] int(1/(x*coth(a + b*log(c*x^n))^(5/2)),x)
```

```
[Out] atan(coth(a + b*log(c*x^n))^(1/2))/(b*n) + atanh(coth(a + b*log(c*x^n))^(1/
2))/(b*n) - 2/(3*b*n*coth(a + b*log(c*x^n))^(3/2))
```

3.205 $\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{(b-2c)\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}$$

```
[Out] 1/4*(b-2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)^
4)^(1/2))/c^(3/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(
a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*coth(x
)^4)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used

$= \{3782, 1265, 1667, 857, 635, 212, 738\}$

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{(b-2c)\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}$$

```
[In] Int[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ((b - 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*
Coth[x]^4])])/(4*c^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt
[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c]) - Sqr
t[a + b*Coth[x]^2 + c*Coth[x]^4]/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x^5}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)\\
&= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{\text{Subst}\left(\int \frac{\frac{b}{2}+\frac{1}{2}(b-2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)}{2c}\\
&= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}\\
&\quad -\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\\
&\quad -\frac{(b-2c)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)}{4c}
\end{aligned}
$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c} - \frac{(b-2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2c} + \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$

$$= \frac{(b-2c)\text{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4c^{3/2}} + \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2c}$$

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.47

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$$

$$= \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)\left((b-2c)(a+b+c)\text{arctanh}\left(\frac{2c+b\tanh^2(x)}{2\sqrt{c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)+2c^{3/2}\sqrt{a}\right.}{4c^{3/2}(a+b+c)\sqrt{c+b}}$$

```
[In] Integrate[Coth[x]^5/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2*((b - 2*c)*(a + b + c)*ArcTa
nh[(2*c + b*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])] + 2
*c^(3/2)*Sqrt[a + b + c]*ArcTanh[(b + 2*c + (2*a + b)*Tanh[x]^2)/(2*Sqrt[a
+ b + c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])] - 2*Sqrt[c]*(a + b + c)*Coth
[x]^2*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4]))/(4*c^(3/2)*(a + b + c)*Sqrt[c +
 b*Tanh[x]^2 + a*Tanh[x]^4])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

method	result
derivativedivides	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(x)^2+c\coth(x)^4}\right)}{2\sqrt{c}}-\frac{\sqrt{a+b\coth(x)^2+c\coth(x)^4}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(\right.}{4c^{\frac{3}{2}}}$
default	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(x)^2+c\coth(x)^4}\right)}{2\sqrt{c}}-\frac{\sqrt{a+b\coth(x)^2+c\coth(x)^4}}{2c}+\frac{b\ln\left(\frac{\frac{b}{2}+c\coth(x)^2}{\sqrt{c}}+\sqrt{a+b\coth(\right.}{4c^{\frac{3}{2}}}$

```
[In] int(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1
/2)-1/2*(a+b*coth(x)^2+c*coth(x)^4)^(1/2)/c+1/4*b/c^(3/2)*ln((1/2*b+c*coth(
x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))+1/2/(a+b+c)^(1/2)*arctanh(
1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*coth(x
)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2086 vs. 2(111) = 222.

Time = 1.11 (sec) , antiderivative size = 8951, normalized size of antiderivative = 66.30

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\text{Too large to display}$$

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$$

```
[In] integrate(coth(x)**5/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(coth(x)**5/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)^5/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^5/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^5(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth(x)^5}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

```
[In] int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)^5/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.206 $\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] -1/2*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2
))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*cot
h(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3782, 1265, 857, 635, 212, 738}

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}}$$

```
[In] Int[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x
]^4])]/Sqrt[c] + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \frac{x^3}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\\
&\quad - \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)\\
&\quad + \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)\\
&= \frac{\text{arctanh}\left(\frac{-b-2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{c}} + \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.52

$$
\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\frac{\left((a+b+c)\text{arctanh}\left(\frac{2c+b\tanh^2(x)}{2\sqrt{c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right) - \sqrt{c}\sqrt{a+b+c}\,\text{arctanh}\left(\frac{b+2c+(2a+b)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right.\right.}{2\sqrt{c}(a+b+c)\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}
$$

```
[In] Integrate[Coth[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*(((a + b + c)*ArcTanh[(2*c + b*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[c + b*Tanh[x
]^2 + a*Tanh[x]^4])] - Sqrt[c]*Sqrt[a + b + c]*ArcTanh[(b + 2*c + (2*a + b)
*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])])*Sqrt[
a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2)/(Sqrt[c]*(a + b + c)*Sqrt[c + b*T
anh[x]^2 + a*Tanh[x]^4])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^{2}}{\sqrt{c}}+\sqrt{a+b\coth(x)^{2}+c\coth(x)^{4}}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^{2}+2c\coth(x)^{2}+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^{2}+c\coth(x)^{4}}}\right)}{2\sqrt{a+b+c}}$	90
default	$-\frac{\ln\left(\frac{\frac{b}{2}+c\coth(x)^{2}}{\sqrt{c}}+\sqrt{a+b\coth(x)^{2}+c\coth(x)^{4}}\right)}{2\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^{2}+2c\coth(x)^{2}+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^{2}+c\coth(x)^{4}}}\right)}{2\sqrt{a+b+c}}$	90

```
[In] int(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*ln((1/2*b+c*coth(x)^2)/c^(1/2)+(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/c^(1
/2)+1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)
^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1522 vs. 2(85) = 170.

Time = 0.90 (sec) , antiderivative size = 6695, normalized size of antiderivative = 63.76

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\coth^3{(x)}}{\sqrt{a+b\coth^2{(x)}+c\coth^4{(x)}}}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(coth(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

```
[In] integrate(coth(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

```
[In] int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
```

```
[Out] int(coth(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.207 $\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] 1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*coth(x)^2+c*co
th(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3782, 1261, 738, 212}

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[In] Int[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{x}{(1+x^2)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)\\
&= \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)\\
&= \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.69

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2c+(2a+b)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}$$

```
[In] Integrate[Coth[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] (ArcTanh[(b + 2*c + (2*a + b)*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[c + b*Tanh
[x]^2 + a*Tanh[x]^4])]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2)/(2*Sq
rt[a + b + c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^2+2c\coth(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\right)}{2\sqrt{a+b+c}}$	52
default	$\frac{\operatorname{arctanh}\left(\frac{b\coth(x)^2+2c\coth(x)^2+2a+b}{2\sqrt{a+b+c}\,\sqrt{a+b\coth(x)^2+c\coth(x)^4}}\right)}{2\sqrt{a+b+c}}$	52

```
[In] int(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a+b+c)^(1/2)*arctanh(1/2*(b*coth(x)^2+2*c*coth(x)^2+2*a+b)/(a+b+c)^(1/
2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(48) = 96.

Time = 0.68 (sec) , antiderivative size = 1752, normalized size of antiderivative = 30.21

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

[Out] [1/4*log(((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2 + 2*(a + b)*c + c^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a
+ b)*c + c^2)*sinh(x)^8 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 +
 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x
)^6 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b
 - b*c - c^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*
cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a
^2 + a*b - b*c - c^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*sinh
(x)^4 + 8*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 +
a*b - b*c - c^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x))
sinh(x)^3 - 4(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2
+ 2*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(
3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sin
h(x)^2 + sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*cosh(x)*sinh(x)^3 +
 (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*cosh(x))*sinh(x) + a
 + b + c)*sqrt(a + b + c)*sqrt(((a + b + c)*cosh(x)^4 + (a + b + c)*sinh(x)
^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 - 2*a + 2*c)*sinh(x)^
2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^
2 + 8*((a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b
*c - c^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 2*(a + b)*c + 3*c^2)*cosh(x)^3 - (a^
2 + a*b - b*c - c^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sqrt(a + b + c), -
1/2*sqrt(-a - b - c)*arctan(sqrt(2)*((a + b + c)*cosh(x)^4 + 4*(a + b + c)*
cosh(x)*sinh(x)^3 + (a + b + c)*sinh(x)^4 - 2*(a - c)*cosh(x)^2 + 2*(3*(a +
 b + c)*cosh(x)^2 - a + c)*sinh(x)^2 + 4*((a + b + c)*cosh(x)^3 - (a - c)*c
osh(x))*sinh(x) + a + b + c)*sqrt(-a - b - c)*sqrt(((a + b + c)*cosh(x)^4 +
 (a + b + c)*sinh(x)^4 - 4*(a - c)*cosh(x)^2 + 2*(3*(a + b + c)*cosh(x)^2 -
 2*a + 2*c)*sinh(x)^2 + 3*a - b + 3*c)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6
*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^
2 + 2*(a + b)*c + c^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2
)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*sinh(x)^8 - 4
*(a^2 + a*b - b*c - c^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c
+ c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b
^2 + 2*(a + b)*c + c^2)*cosh(x)^3 - 3*(a^2 + a*b - b*c - c^2)*cosh(x))*sinh
(x)^5 + 2*(3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^4 + 2*(35*(
a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^4 - 30*(a^2 + a*b - b*c - c^
2)*cosh(x)^2 + 3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*sinh(x)^4 + 8*(
7*(a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2)*cosh(x)^5 - 10*(a^2 + a*b - b*c -
 c^2)*cosh(x)^3 + (3*a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x))*si
nh(x)^3 - 4*(a^2 + a*b - b*c - c^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2 + 2
*(a + b)*c + c^2)*cosh(x)^6 - 15*(a^2 + a*b - b*c - c^2)*cosh(x)^4 + 3*(3*a
^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^2 - a^2 - a*b + b*c + c^2
)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(a + b)*c + c^2 + 8*((a^2 + 2*a*b + b^2

```
 + 2*(a + b)*c + c^2)*cosh(x)^7 - 3*(a^2 + a*b - b*c - c^2)*cosh(x)^5 + (3*
a^2 + 2*a*b - b^2 + 2*(3*a + b)*c + 3*c^2)*cosh(x)^3 - (a^2 + a*b - b*c - c
^2)*cosh(x))*sinh(x)))/(a + b + c)]
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx = \int \frac{\coth(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx$$

```
[In] integrate(coth(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(coth(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx = \int \frac{\coth(x)}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\, dx$$

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(coth(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx = \text{Timed out}$$

```
[In] integrate(coth(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\coth(x)}{\sqrt{c\coth{(x)}^4+b\coth{(x)}^2+a}}\,dx$$

```
[In] int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(coth(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.208 $\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$

```
[Out] -1/2*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2
))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/(a+b*cot
h(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3782, 1265, 974, 738, 212}

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}} \, dx = \frac{\operatorname{arctanh}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\operatorname{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}}$$

```
[In] Int[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x
]^4])]/Sqrt[a] + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2)/(2*Sqrt[a + b + c]
*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b + c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{x\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)$$

$$
\begin{aligned}
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1-x)\sqrt{a-bx+cx^2}}+\frac{1}{x\sqrt{a-bx+cx^2}}\right)\,dx, x, -\coth^2(x)\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right) \\
&\quad + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right) \\
&= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) \\
&\quad - \text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{-2a-b+(-b-2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) \\
&= -\frac{\text{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} - \frac{\text{arctanh}\left(\frac{-2a-b+(-b-2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.50

$$
\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx =
$$

$$
-\frac{\left((a+b+c)\text{arctanh}\left(\frac{b+2a\tanh^2(x)}{2\sqrt{a}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)-\sqrt{a}\sqrt{a+b+c}\,\text{arctanh}\left(\frac{b+2c+(2a+b)\tanh^2(x)}{2\sqrt{a+b+c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)\right.}{2\sqrt{a}(a+b+c)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}
$$

```
[In] Integrate[Tanh[x]/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*(((a + b + c)*ArcTanh[(b + 2*a*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[c + b*Tanh[x
]^2 + a*Tanh[x]^4])] - Sqrt[a]*Sqrt[a + b + c]*ArcTanh[(b + 2*c + (2*a + b)
*Tanh[x]^2)/(2*Sqrt[a + b + c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])])*Coth[
x]^2*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])/(Sqrt[a]*(a + b + c)*Sqrt[a + b*C
oth[x]^2 + c*Coth[x]^4])
```

Maple [F]

$$\int \frac{\tanh (x)}{\sqrt{a + b \coth (x)^{2} + c \coth (x)^{4}}}dx$$

```
[In] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. $2(86) = 172$.

Time = 0.89 (sec) , antiderivative size = 6705, normalized size of antiderivative = 63.25

$$\int \frac{\tanh(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx = \int \frac{\tanh (x)}{\sqrt{a + b \coth^{2} (x) + c \coth^{4} (x)}}\, dx$$

```
[In] integrate(tanh(x)/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
[Out] Integral(tanh(x)/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\, dx = \int \frac{\tanh (x)}{\sqrt{c \coth (x)^{4} + b \coth (x)^{2} + a}}\, dx$$

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(x)/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\text{Timed out}$$

```
[In] integrate(tanh(x)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx=\int \frac{\tanh(x)}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

```
[In] int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.209 $\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{b\operatorname{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)}{2a}$$

```
[Out] 1/4*b*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/
2))/a^(3/2)-1/2*arctanh(1/2*(2*a+b*coth(x)^2)/a^(1/2)/(a+b*coth(x)^2+c*coth
(x)^4)^(1/2))/a^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/
2)/(a+b*coth(x)^2+c*coth(x)^4)^(1/2))/(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*co
th(x)^4)^(1/2)*tanh(x)^2/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3782, 1265, 974, 744, 738, 212}

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \frac{b\text{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}} - \frac{\text{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{\text{arctanh}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\tanh^2(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}}{2a}$$

```
[In] Int[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[x]^2 + c*Coth[x
]^4])]/Sqrt[a] + (b*ArcTanh[(2*a + b*Coth[x]^2)/(2*Sqrt[a]*Sqrt[a + b*Coth[
x]^2 + c*Coth[x]^4])])/(4*a^(3/2)) + ArcTanh[(2*a + b + (b + 2*c)*Coth[x]^2
)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])]/(2*Sqrt[a + b +
c]) - (Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2)/(2*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
```

```
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{1}{x^3\left(1+x^2\right)\sqrt{a-bx^2+cx^4}}\,dx, x, -i\coth(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x^2\sqrt{a-bx+cx^2}}-\frac{1}{x\sqrt{a-bx+cx^2}}+\frac{1}{(1+x)\sqrt{a-bx+cx^2}}\right)dx, x, -\coth^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\right)\\
&\quad+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)\\
&\quad-\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)
\end{aligned}
$$

$$= -\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)}{2a} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)}{4a}$$
$$-\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$
$$+\text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$
$$= -\frac{\text{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}}$$
$$-\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)}{2a} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2a}$$
$$= -\frac{\text{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a}} + \frac{b\text{arctanh}\left(\frac{2a+b\coth^2(x)}{2\sqrt{a}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4a^{3/2}}$$
$$+\frac{\text{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{2\sqrt{a+b+c}} - \frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)}{2a}$$

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx =$$
$$-\frac{\coth^2(x)\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}\left((2a-b)(a+b+c)\text{arctanh}\left(\frac{b+2a\tanh^2(x)}{2\sqrt{a}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)+2\sqrt{a}\left(\right.\right.}{4a^{3/2}(a+b+c)\sqrt{a+b}}$$

```
[In] Integrate[Tanh[x]^3/Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/4*(Coth[x]^2*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4]*((2*a - b)*(a + b + c)*
ArcTanh[(b + 2*a*Tanh[x]^2)/(2*Sqrt[a]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])
] + 2*Sqrt[a]*(-(a*Sqrt[a + b + c]*ArcTanh[(b + 2*c + (2*a + b)*Tanh[x]^2)/
(2*Sqrt[a + b + c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])]) + (a + b + c)*Sqr
t[c + b*Tanh[x]^2 + a*Tanh[x]^4])))/(a^(3/2)*(a + b + c)*Sqrt[a + b*Coth[x]
^2 + c*Coth[x]^4])
```

Maple [F]

$$\int \frac{\tanh(x)^3}{\sqrt{a+b\coth(x)^2+c\coth(x)^4}}dx$$

[In] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)

[Out] int(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2135 vs. 2(149) = 298.

Time = 1.13 (sec) , antiderivative size = 9148, normalized size of antiderivative = 49.99

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \text{Too large to display}$$

[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas"
)

[Out] Too large to include

Sympy [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx$$

[In] integrate(tanh(x)**3/(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)

[Out] Integral(tanh(x)**3/sqrt(a + b*coth(x)**2 + c*coth(x)**4), x)

Maxima [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{c\coth(x)^4+b\coth(x)^2+a}}\,dx$$

[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima"
)

[Out] integrate(tanh(x)^3/sqrt(c*coth(x)^4 + b*coth(x)^2 + a), x)

Giac [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx = \text{Timed out}$$

```
[In] integrate(tanh(x)^3/(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(x)}{\sqrt{a + b\coth^2(x) + c\coth^4(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{c\coth(x)^4 + b\coth(x)^2 + a}}\,dx$$

```
[In] int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(tanh(x)^3/(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.210 $\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = -\frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}} + \frac{1}{2}\sqrt{a+b+c}\operatorname{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}$$

```
[Out] -1/4*(b+2*c)*arctanh(1/2*(b+2*c*coth(x)^2)/c^(1/2)/(a+b*coth(x)^2+c*coth(x)
^4)^(1/2))/c^(1/2)+1/2*arctanh(1/2*(2*a+b+(b+2*c)*coth(x)^2)/(a+b+c)^(1/2)/
(a+b*coth(x)^2+c*coth(x)^4)^(1/2))*(a+b+c)^(1/2)-1/2*(a+b*coth(x)^2+c*coth(
x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {3782, 1261, 748, 857, 635, 212, 738}

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx$$
$$= -\frac{(b+2c)\operatorname{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}} + \frac{1}{2}\sqrt{a+b+c}\operatorname{arctanh}\left(\frac{2a+(b+2c)\coth^2(x)+b}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}$$

```
[In] Int[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/4*((b + 2*c)*ArcTanh[(b + 2*c*Coth[x]^2)/(2*Sqrt[c]*Sqrt[a + b*Coth[x]^2
 + c*Coth[x]^4])])/Sqrt[c] + (Sqrt[a + b + c]*ArcTanh[(2*a + b + (b + 2*c)*
Coth[x]^2)/(2*Sqrt[a + b + c]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4])])/2 - Sq
rt[a + b*Coth[x]^2 + c*Coth[x]^4]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
```

```
& !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 3782

```
Int[cot[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*(cot[(d_.) + (e_.)*(x_)]*(
f_.))^(n_.) + (c_.)*(cot[(d_.) + (e_.)*(x_)]*(f_.))^(n2_.))^(p_), x_Symbol]
 :> Dist[-f/e, Subst[Int[(x/f)^m*((a + b*x^n + c*x^(2*n))^p/(f^2 + x^2)), x
], x, f*Cot[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[n
2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{x\sqrt{a-bx^2+cx^4}}{1+x^2}\,dx, x, -i\coth(x)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a-bx+cx^2}}{1+x}\,dx, x, -\coth^2(x)\right)\right)$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}+\frac{1}{4}\text{Subst}\left(\int \frac{-2a-b+(b+2c)x}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}$$
$$+\frac{1}{2}(-a-b-c)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$
$$+\frac{1}{4}(b+2c)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx+cx^2}}\,dx, x, -\coth^2(x)\right)$$

$$= -\frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}$$
$$+ (a+b+c)\text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx, x, \frac{2a+b+(b+2c)\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$
$$+ \frac{1}{2}(b+2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{-b-2c\coth^2(x)}{\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$

$$= -\frac{(b+2c)\text{arctanh}\left(\frac{b+2c\coth^2(x)}{2\sqrt{c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)}{4\sqrt{c}}$$
$$+ \frac{1}{2}\sqrt{a+b+c}\,\text{arctanh}\left(\frac{2a+b+(b+2c)\coth^2(x)}{2\sqrt{a+b+c}\sqrt{a+b\coth^2(x)+c\coth^4(x)}}\right)$$
$$- \frac{1}{2}\sqrt{a+b\coth^2(x)+c\coth^4(x)}$$

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.39

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx =$$
$$-\frac{\sqrt{a+b\coth^2(x)+c\coth^4(x)}\tanh^2(x)\left((b+2c)\text{arctanh}\left(\frac{2c+b\tanh^2(x)}{2\sqrt{c}\sqrt{c+b\tanh^2(x)+a\tanh^4(x)}}\right)-2\sqrt{c}\sqrt{a+b+c}\text{ar}\right.}{4\sqrt{c}\sqrt{c+b\tanh^2(x)+a\,\text{t}}}$$

```
[In] Integrate[Coth[x]*Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4],x]
```

```
[Out] -1/4*(Sqrt[a + b*Coth[x]^2 + c*Coth[x]^4]*Tanh[x]^2*((b + 2*c)*ArcTanh[(2*c
 + b*Tanh[x]^2)/(2*Sqrt[c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])] - 2*Sqrt[c
]*Sqrt[a + b + c]*ArcTanh[(b + 2*c + (2*a + b)*Tanh[x]^2)/(2*Sqrt[a + b + c
]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])] + 2*Sqrt[c]*Coth[x]^2*Sqrt[c + b*Ta
nh[x]^2 + a*Tanh[x]^4]))/(Sqrt[c]*Sqrt[c + b*Tanh[x]^2 + a*Tanh[x]^4])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.25

method	result
derivativedivides	$-\frac{\sqrt{\left(\coth(x)^2-1\right)^2 c+(b+2c)\left(\coth(x)^2-1\right)+a+b+c}}{2}-\frac{(b+2c)\ln\left(\frac{\frac{b}{2}+c+c\left(\coth(x)^2-1\right)}{\sqrt{c}}+\sqrt{\left(\coth(x)^2-1\right)^2 c+(b+2c)}\right.}{4\sqrt{c}}$
default	$-\frac{\sqrt{\left(\coth(x)^2-1\right)^2 c+(b+2c)\left(\coth(x)^2-1\right)+a+b+c}}{2}-\frac{(b+2c)\ln\left(\frac{\frac{b}{2}+c+c\left(\coth(x)^2-1\right)}{\sqrt{c}}+\sqrt{\left(\coth(x)^2-1\right)^2 c+(b+2c)}\right.}{4\sqrt{c}}$

```
[In] int(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((coth(x)^2-1)^2*c+(b+2*c)*(coth(x)^2-1)+a+b+c)^(1/2)-1/4*(b+2*c)*ln((
1/2*b+c+c*(coth(x)^2-1))/c^(1/2)+((coth(x)^2-1)^2*c+(b+2*c)*(coth(x)^2-1)+a
+b+c)^(1/2))/c^(1/2)+1/2*(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(coth(x)^2-1
)+2*(a+b+c)^(1/2)*((coth(x)^2-1)^2*c+(b+2*c)*(coth(x)^2-1)+a+b+c)^(1/2))/(c
oth(x)^2-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. 2(108) = 216.

Time = 1.44 (sec) , antiderivative size = 7964, normalized size of antiderivative = 60.33

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = \int \sqrt{a+b\coth^2(x)+c\coth^4(x)}\coth(x)\,dx$$

```
[In] integrate(coth(x)*(a+b*coth(x)**2+c*coth(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*coth(x)**2 + c*coth(x)**4)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = \int \sqrt{c\coth{(x)}^4+b\coth{(x)}^2+a}\coth{(x)}\ dx$$

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

Giac [F]

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = \int \sqrt{c\coth{(x)}^4+b\coth{(x)}^2+a}\coth{(x)}\ dx$$

```
[In] integrate(coth(x)*(a+b*coth(x)^2+c*coth(x)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*coth(x)^4 + b*coth(x)^2 + a)*coth(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\coth^2(x)+c\coth^4(x)}\,dx = \int \coth(x)\ \sqrt{c\coth{(x)}^4+b\coth{(x)}^2+a}\,dx$$

```
[In] int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2),x)
[Out] int(coth(x)*(a + b*coth(x)^2 + c*coth(x)^4)^(1/2), x)
```

3.211 $\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 319

$$\begin{aligned}\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx &= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}\\
&-\frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4}\\
&+\frac{26e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3}\\
&-\frac{55e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{6bc\left(1-e^{2c(a+bx)}\right)^2}\\
&+\frac{25e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)}\\
&-\frac{15\operatorname{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{4bc}\end{aligned}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-4*exp(c*(b*x+a
))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^4+26/
3*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(
b*x+a)))^3-55/6*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/
c/(1-exp(2*c*(b*x+a)))^2+25/4*exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh
(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))-15/4*arctanh(exp(c*(b*x+a)))*(coth(b*c
*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6852, 2320, 398, 1828, 1171, 393, 213}

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx = -\frac{15\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc} + \frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc} + \frac{25e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{4bc\left(1-e^{2c(a+bx)}\right)} - \frac{55e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{6bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{26e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{4e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^4}$$

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (4*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^4) + (26*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c +
b*c*x])/(3*b*c*(1 - E^(2*c*(a + b*x)))^3) - (55*E^(c*(a + b*x))*Sqrt[Coth[a
*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(6*b*c*(1 - E^(2*c*(a + b*x)))^2) + (25*E
^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(4*b*c*(1 - E^(
2*c*(a + b*x)))) - (15*ArcTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*T
anh[a*c + b*c*x])/(4*b*c)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^5(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \left(1+\frac{2(1+10x^4+5x^8)}{(-1+x^2)^5}\right)\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{1+10x^4+5x^8}{(-1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{8+120x^2+40x^4+40x^6}{(-1+x^2)^4}\,dx, x, e^{c(a+bx)}\right)}{4bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3} + \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int \frac{160+480x^2+240x^4}{(-1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{24bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{55e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{6bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int\frac{240+960x^2}{(-1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{96bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{55e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{6bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{25e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)} + \frac{\left(15\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\operatorname{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,e^{c(a+bx)}\right)}{4bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{4e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^4} + \frac{26e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{3bc\left(1-e^{2c(a+bx)}\right)^3} - \frac{55e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{6bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{25e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{4bc\left(1-e^{2c(a+bx)}\right)} - \frac{15\operatorname{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{4bc}$$

Mathematica [A] (verified)

Time = 6.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.51

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx = \frac{\sqrt{\coth^2(c(a+bx))}\left(66e^{c(a+bx)} - 314e^{3c(a+bx)} + 374e^{5c(a+bx)} - 246e^{7c(a+bx)} + 24e^{9c(a+bx)} + 45(-\right.}{24bc\,(-1+e^{2}}$$

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[Coth[c*(a + b*x)]^2]*(66*E^(c*(a + b*x)) - 314*E^(3*c*(a + b*x)) + 37
4*E^(5*c*(a + b*x)) - 246*E^(7*c*(a + b*x)) + 24*E^(9*c*(a + b*x)) + 45*(-1
 + E^(2*c*(a + b*x)))^4*Log[1 - E^(c*(a + b*x))] - 45*(-1 + E^(2*c*(a + b*x
)))^4*Log[1 + E^(c*(a + b*x))])*Tanh[c*(a + b*x)])/(24*b*c*(-1 + E^(2*c*(a
+ b*x)))^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.61

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\cosh(bcx+ac)^5}{\sinh(bcx+ac)^4}-\frac{5\cosh(bcx+ac)^3}{\sinh(bcx+ac)^4}+\frac{5\cosh(bcx+ac)}{\sinh(bcx+ac)^4}+5\left(-\frac{\operatorname{csch}(bcx+ac)^3}{4}+\frac{3\operatorname{csch}(bcx+ac)}{8}\right)\coth(bcx+ac)-\frac{15\operatorname{arctan}}{}\right.}{cb}$
risch	$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)bc}-\frac{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{c(bx+a)}\left(75e^{6c(bx+a)}-115e^{4c(bx+a)}+109e^{2c(bx+a)}-21\right)}{12\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)^3cb}-\frac{15(e}{}$

[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] csgn(coth(c*(b*x+a)))/c/b*(cosh(b*c*x+a*c)^5/sinh(b*c*x+a*c)^4-5/sinh(b*c*x+a*c)^4*cosh(b*c*x+a*c)^3+5/sinh(b*c*x+a*c)^4*cosh(b*c*x+a*c)+5*(-1/4*csch(b*c*x+a*c)^3+3/8*csch(b*c*x+a*c))*coth(b*c*x+a*c)-15/4*arctanh(exp(b*c*x+a*c))+1/sinh(b*c*x+a*c)^3*cosh(b*c*x+a*c)^4-4/sinh(b*c*x+a*c)^3*cosh(b*c*x+a*c)^2+8/3/sinh(b*c*x+a*c)^3)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1617 vs. $2(281) = 562$.

Time = 0.27 (sec) , antiderivative size = 1617, normalized size of antiderivative = 5.07

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx = \text{Too large to display}$$

[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")

[Out] 1/24*(24*cosh(b*c*x + a*c)^9 + 216*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 + 24*sinh(b*c*x + a*c)^9 + 6*(144*cosh(b*c*x + a*c)^2 - 41)*sinh(b*c*x + a*c)^7 - 246*cosh(b*c*x + a*c)^7 + 42*(48*cosh(b*c*x + a*c)^3 - 41*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^6 + 2*(1512*cosh(b*c*x + a*c)^4 - 2583*cosh(b*c*x + a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 374*cosh(b*c*x + a*c)^5 + 2*(1512*cosh(b*c*x + a*c)^5 - 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^4 + 2*(1008*cosh(b*c*x + a*c)^6 - 4305*cosh(b*c*x + a*c)^4 + 1870*cosh(b*c*x + a*c)^2 - 157)*sinh(b*c*x + a*c)^3 - 314*cosh(b*c*x + a*c)^3 + 2*(432*cosh(b*c*x + a*c)^7 - 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3 - 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x + a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c)

```
 + sinh(b*c*x + a*c) + 1) + 45*(cosh(b*c*x + a*c)^8 + 8*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*c*x + a*c)^2 - 1)*si
nh(b*c*x + a*c)^6 - 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b*c*x + a*c)^3 - 3*co
sh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x + a*c)^4 - 30*cosh(
b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x + a*c)^4 + 8*(7*cosh
(b*c*x + a*c)^5 - 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 - 15*cosh(b*c*x + a*c)^4 + 9*cosh(b*c*x
 + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 4*cosh(b*c*x + a*c)^2 + 8*(cosh(b*c*x
+ a*c)^7 - 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c
))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) +
2*(108*cosh(b*c*x + a*c)^8 - 861*cosh(b*c*x + a*c)^6 + 935*cosh(b*c*x + a*c
)^4 - 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 66*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 +
 b*c*sinh(b*c*x + a*c)^8 - 4*b*c*cosh(b*c*x + a*c)^6 + 4*(7*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)^4 + 8*(7*b*c*
cosh(b*c*x + a*c)^3 - 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*
b*c*cosh(b*c*x + a*c)^4 - 30*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x +
a*c)^4 - 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x + a*c)^5 - 10*b*c*
cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*b
*c*cosh(b*c*x + a*c)^6 - 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*c*cosh(b*c*x + a*
c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x + a*c)^7 - 3*b*c*
cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 - b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx=\text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.52

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{5/2}\,dx=-\frac{15\,\log\left(e^{(bcx+ac)}+1\right)}{8\,bc}+\frac{15\,\log\left(e^{(bcx+ac)}-1\right)}{8\,bc}+\frac{12\,e^{(9\,bcx+9\,ac)}-123\,e^{(7\,bcx+7\,ac)}+187\,e^{(5\,bcx+5\,ac)}-157\,e^{(3\,bcx+3\,ac)}+33\,e^{(bcx+ac)}}{12\,bc(e^{(8\,bcx+8\,ac)}-4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}-4\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -15/8*log(e^(b*c*x + a*c) + 1)/(b*c) + 15/8*log(e^(b*c*x + a*c) - 1)/(b*c)
+ 1/12*(12*e^(9*b*c*x + 9*a*c) - 123*e^(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x +
 5*a*c) - 157*e^(3*b*c*x + 3*a*c) + 33*e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x +
8*a*c) - 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) - 4*e^(2*b*c*x + 2*a
*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.63 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.57

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = \frac{\frac{24\,e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{45\,\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} + \frac{45\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{2\,(75\,e^{(7\,bcx+7\,ac)}-115\,e^{(5\,bcx+5\,ac)}+109\,e^{(3\,bcx+3}}{\left(e^{(2\,bcx+2\,ac)}-1\right)^4\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}\right.}}{24\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(24*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 45*log(e^(b*c*x + a
*c) + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 45*log(abs(e^(b*c*x + a*c) - 1))/sg
n(e^(2*b*c*x + 2*a*c) - 1) - 2*(75*e^(7*b*c*x + 7*a*c) - 115*e^(5*b*c*x + 5
*a*c) + 109*e^(3*b*c*x + 3*a*c) - 21*e^(b*c*x + a*c))/((e^(2*b*c*x + 2*a*c)
 - 1)^4*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{5/2}\,dx = \int e^{c\,(a+b\,x)}\,\left(\coth(a\,c+b\,c\,x)^2\right)^{5/2}\,dx$$

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(5/2), x)
```

3.212 $\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 197

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx=\frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}-\frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2}+\frac{3e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)}-\frac{3\text{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*exp(c*(b*x+a
))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x+a)))^2+3*e
xp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c/(1-exp(2*c*(b*x
+a)))-3*arctanh(exp(c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b
/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used

= {6852, 2320, 398, 1172, 12, 294, 213}

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = -\frac{3\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc} + \frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc} + \frac{3e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)} - \frac{2e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc\left(1-e^{2c(a+bx)}\right)^2}$$

```
[In] Int[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*E^
(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c*(1 - E^(2*c
*(a + b*x)))^2) + (3*E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b
*c*x])/(b*c*(1 - E^(2*c*(a + b*x)))) - (3*ArcTanh[E^(c*(a + b*x))]*Sqrt[Cot
h[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth^3(ac+bcx)\,dx$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int\frac{(1+x^2)^3}{(-1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int\left(1+\frac{2(1+3x^4)}{(-1+x^2)^3}\right)\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} + \frac{\left(2\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int\frac{1+3x^4}{(-1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int\frac{12x^2}{(-1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{2bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{\left(6\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{3e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)} + \frac{\left(3\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc} - \frac{2e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)^2} + \frac{3e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc\left(1-e^{2c(a+bx)}\right)} - \frac{3\text{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.15 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx =$$

$$-\frac{e^{-5c(a+bx)}\coth^2(c(a+bx))^{3/2}\left(-21\left(252105+507305e^{2c(a+bx)}+173916e^{4c(a+bx)}-154296e^{6c(a+bx)}-7388\right.\right.}{}$$

```
[In] Integrate[E^(c*(a + b*x))*(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -1/60480*((Coth[c*(a + b*x)]^2)^(3/2)*(-21*(252105 + 507305*E^(2*c*(a + b*x
)) + 173916*E^(4*c*(a + b*x)) - 154296*E^(6*c*(a + b*x)) - 73885*E^(8*c*(a
```

```
+ b*x)) + 4887*E^(10*c*(a + b*x))) - (315*(-16807 - 28218*E^(2*c*(a + b*x))
 + 1173*E^(4*c*(a + b*x)) + 17748*E^(6*c*(a + b*x)) + 4299*E^(8*c*(a + b*x)
) - 1434*E^(10*c*(a + b*x)) + 7*E^(12*c*(a + b*x)))*ArcTanh[Sqrt[E^(2*c*(a
+ b*x))]])/Sqrt[E^(2*c*(a + b*x))] + 384*E^(8*c*(a + b*x))*(1 + E^(2*c*(a +
 b*x)))^2*(7 + 5*E^(2*c*(a + b*x)))*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1
, 1, 1, 11/2}, E^(2*c*(a + b*x))] + 256*E^(8*c*(a + b*x))*(1 + E^(2*c*(a +
b*x)))^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, E^(2*c
*(a + b*x))])*Tanh[c*(a + b*x)]^3)/(b*c*E^(5*c*(a + b*x)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\cosh(bcx+ac)^3}{\sinh(bcx+ac)^2}-\frac{3\cosh(bcx+ac)}{\sinh(bcx+ac)^2}+\frac{3\operatorname{csch}(bcx+ac)\coth(bcx+ac)}{2}-3\operatorname{arctanh}\left(e^{bcx+ac}\right)+\frac{\cosh(bcx+ac)^2}{\sinh(bcx+ac)}-\frac{2}{\sinh(bcx+ac)}\right)}{cb}$
risch	$\frac{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\left(2e^{5c(bx+a)}+3e^{4c(bx+a)}\ln\left(e^{c(bx+a)}-1\right)-3e^{4c(bx+a)}\ln\left(e^{c(bx+a)}+1\right)-10e^{3c(bx+a)}-6e^{2c(bx+a)}\ln\left(e^{c(bx+a)}-1\right)+\cdots\right.}{2\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)cb}$

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(coth(c*(b*x+a)))/c/b*(cosh(b*c*x+a*c)^3/sinh(b*c*x+a*c)^2-3/sinh(b*c*x
+a*c)^2*cosh(b*c*x+a*c)+3/2*csch(b*c*x+a*c)*coth(b*c*x+a*c)-3*arctanh(exp(b
*c*x+a*c))+1/sinh(b*c*x+a*c)*cosh(b*c*x+a*c)^2-2/sinh(b*c*x+a*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(179) = 358$.

Time = 0.26 (sec) , antiderivative size = 613, normalized size of antiderivative = 3.11

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = \frac{2\cosh(bcx+ac)^5+10\cosh(bcx+ac)\sinh(bcx+ac)^4+2\sinh(bcx+ac)^5+10\left(2\cosh(bcx\cdots\right.}{}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + 2*s
inh(b*c*x + a*c)^5 + 10*(2*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^3 - 1
0*cosh(b*c*x + a*c)^3 + 10*(2*cosh(b*c*x + a*c)^3 - 3*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x
 + a*c)^3 + sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x
```

```
+ a*c)^2 - 2*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*
c))*sinh(b*c*x + a*c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) +
 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 + sinh(b*
c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 - 1)*sinh(b*c*x + a*c)^2 - 2*cosh(b
*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 - cosh(b*c*x + a*c))*sinh(b*c*x + a*
c) + 1)*log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + 2*(5*cosh(b*c*x +
a*c)^4 - 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 4*cosh(b*c*x + a*c
))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
 b*c*sinh(b*c*x + a*c)^4 - 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*c*x
+ a*c)^2 - b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3 - b*
c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\, dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.57

$$\int e^{c(a+bx)} \coth^2(ac+bcx)^{3/2}\, dx = -\frac{3 \log\left(e^{(bcx+ac)}+1\right)}{2\, bc} + \frac{3 \log\left(e^{(bcx+ac)}-1\right)}{2\, bc} + \frac{e^{(5\, bcx+5\, ac)} - 5\, e^{(3\, bcx+3\, ac)} + 2\, e^{(bcx+ac)}}{bc(e^{(4\, bcx+4\, ac)} - 2\, e^{(2\, bcx+2\, ac)} + 1)}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(e^(b*c*x + a*c) + 1)/(b*c) + 3/2*log(e^(b*c*x + a*c) - 1)/(b*c) +
(e^(5*b*c*x + 5*a*c) - 5*e^(3*b*c*x + 3*a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(
4*b*c*x + 4*a*c) - 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = \frac{\frac{2\,e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{3\,\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} + \frac{3\,\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{2\,\left(3\,e^{(3\,bcx+3\,ac)}-e^{(bcx+ac)}\right)}{\left(e^{(2\,bcx+2\,ac)}-1\right)^2\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{2\,bc}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - 3*log(e^(b*c*x + a*c)
 + 1)/sgn(e^(2*b*c*x + 2*a*c) - 1) + 3*log(abs(e^(b*c*x + a*c) - 1))/sgn(e^
(2*b*c*x + 2*a*c) - 1) - 2*(3*e^(3*b*c*x + 3*a*c) - e^(b*c*x + a*c))/((e^(2
*b*c*x + 2*a*c) - 1)^2*sgn(e^(2*b*c*x + 2*a*c) - 1)))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\coth^2(ac+bcx)^{3/2}\,dx = \int \mathrm{e}^{c\,(a+b\,x)}\,\left(\coth(a\,c+b\,c\,x)^2\right)^{3/2}\,dx$$

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(3/2), x)
```

3.213 $\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx=\frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}-\frac{2\text{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}$$

```
[Out] exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c-2*arctanh(exp(
c*(b*x+a)))*(coth(b*c*x+a*c)^2)^(1/2)*tanh(b*c*x+a*c)/b/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 396, 212}

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx=\frac{e^{c(a+bx)}\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}-\frac{2\text{arctanh}\left(e^{c(a+bx)}\right)\tanh(ac+bcx)\sqrt{\coth^2(ac+bcx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c) - (2*Ar
cTanh[E^(c*(a + b*x))]*Sqrt[Coth[a*c + b*c*x]^2]*Tanh[a*c + b*c*x])/(b*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\int e^{c(a+bx)}\coth(ac+bcx)\,dx\\
&= \frac{\left(\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{-1-x^2}{1-x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}\\
&\quad - \frac{\left(2\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, e^{c(a+bx)}\right)}{bc}\\
&= \frac{e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}\\
&\quad - \frac{2\text{arctanh}\left(e^{c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}\tanh(ac+bcx)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \frac{\left(e^{c(a+bx)}-2\text{arctanh}\left(e^{c(a+bx)}\right)\right)\sqrt{\coth^2(c(a+bx))}\tanh(c(a+bx))}{bc}$$

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Coth[a*c + b*c*x]^2],x]
[Out] ((E^(c*(a + b*x)) - 2*ArcTanh[E^(c*(a + b*x))])*Sqrt[Coth[c*(a + b*x)]^2]*T
anh[c*(a + b*x)])/(b*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(77) = 154.

Time = 0.77 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.57

method	result
risch	$\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,e^{c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)bc}+\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,\ln\left(e^{c(bx+a)}-1\right)}{\left(1+e^{2c(bx+a)}\right)cb}-\frac{\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right.}{\left(e^{2c(bx+a)}-1\right.}}}{\left(1+e^{2c(bx+a)}\right)}$

```
[In] int(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*
c*(b*x+a))-1)^2)^(1/2)*exp(c*(b*x+a))/b/c+1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(
b*x+a))-1)*((1+exp(2*c*(b*x+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp
(c*(b*x+a))-1)-1/(1+exp(2*c*(b*x+a)))*(exp(2*c*(b*x+a))-1)*((1+exp(2*c*(b*x
+a)))^2/(exp(2*c*(b*x+a))-1)^2)^(1/2)/c/b*ln(exp(c*(b*x+a))+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \frac{\cosh(bcx+ac)-\log(\cosh(bcx+ac)+\sinh(bcx+ac)+1)+\log(\cosh(bcx+ac)+\sinh(bcx+ac)-1}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] (cosh(b*c*x + a*c) - log(cosh(b*c*x + a*c) + sinh(b*c*x + a*c) + 1) + log(c
osh(b*c*x + a*c) + sinh(b*c*x + a*c) - 1) + sinh(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = e^{ac}\int\sqrt{\coth^2(ac+bcx)}e^{bcx}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)**2)**(1/2),x)
```

```
[Out] exp(a*c)*Integral(sqrt(coth(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \frac{e^{(bcx+ac)}}{bc} - \frac{\log\left(e^{(bcx+ac)}+1\right)}{bc} + \frac{\log\left(e^{(bcx+ac)}-1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] e^(b*c*x + a*c)/(b*c) - log(e^(b*c*x + a*c) + 1)/(b*c) + log(e^(b*c*x + a*c
) - 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \frac{\frac{e^{(bcx+ac)}}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} - \frac{\log\left(e^{(bcx+ac)}+1\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)} + \frac{\log\left(\left|e^{(bcx+ac)}-1\right|\right)}{\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (e^(b*c*x + a*c)/sgn(e^(2*b*c*x + 2*a*c) - 1) - log(e^(b*c*x + a*c) + 1)/sg
n(e^(2*b*c*x + 2*a*c) - 1) + log(abs(e^(b*c*x + a*c) - 1))/sgn(e^(2*b*c*x +
 2*a*c) - 1))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sqrt{\coth^2(ac+bcx)}\,dx = \int e^{c\,(a+b\,x)}\,\sqrt{\coth\left(a\,c+b\,c\,x\right)^2}\,dx$$

```
[In] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))*(coth(a*c + b*c*x)^2)^(1/2), x)
```

3.214 $\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 83

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx=\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*arctan(exp(c
*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 396, 209}

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx=\frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}-\frac{2\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*Ar
cTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}\\
&= \frac{\coth(ac+bcx)\text{Subst}\left(\int \frac{-1+x^2}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}\\
&= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}\\
&= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = \frac{\left(e^{c(a+bx)}-2\arctan\left(e^{c(a+bx)}\right)\right)\coth(c(a+bx))}{bc\sqrt{\coth^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Coth[a*c + b*c*x]^2],x]
```

```
[Out] ((E^(c*(a + b*x)) - 2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*Sqrt
[Coth[c*(a + b*x)]^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))(\sinh(bcx+ac)-2\arctan(e^{bcx+ac})+\cosh(bcx+ac))}{cb}$	48
risch	$\frac{(1+e^{2c(bx+a)})e^{c(bx+a)}}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)bc}+\frac{i(1+e^{2c(bx+a)})\ln(e^{c(bx+a)}-i)}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)cb}-\frac{i(1+e^{2c(bx+a)})\ln(e^{c(bx+a)}+i)}{\sqrt{\frac{(1+e^{2c(bx+a)})^2}{(e^{2c(bx+a)}-1)^2}}\,(e^{2c(bx+a)}-1)cb}$	218

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(coth(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)-2*arctan(exp(b*c*x+a*c))+cosh(b*
c*x+a*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.64

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = -\frac{2\arctan\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)-\cosh\left(bcx+ac\right)-\sinh\left(bcx+ac\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - cosh(b*c*x + a*c) - sin
h(b*c*x + a*c))/(b*c)
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\coth^2\left(ac+bcx\right)}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(coth(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = -\frac{2\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(bcx+ac)}}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] -2*arctan(e^(b*c*x + a*c))/(b*c) + e^(b*c*x + a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = -\frac{2\arctan\left(e^{(bcx+ac)}\right)\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - e^{(bcx+ac)}\operatorname{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] -(2*arctan(e^(b*c*x + a*c))*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*x + a*c)*
sgn(e^(2*b*c*x + 2*a*c) - 1))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\coth^2(ac+bcx)}}\,dx = \int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\coth\left(a\,c+b\,c\,x\right)^2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(1/2), x)
```

3.215 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}} dx$

Optimal result

Integrand size = 25, antiderivative size = 193

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}} dx = \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}} - \frac{3\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-2*exp(c*(b*x+a
))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(coth(b*c*x+a*c)^2)^(1/2)+3*e
xp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))/(coth(b*c*x+a*c)^2)^
(1/2)-3*arctan(exp(c*(b*x+a)))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2
)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6852, 2320, 398, 1172, 12, 294, 209}

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}} dx = -\frac{3\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} + \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (2*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a
*c + b*c*x]^2]) + (3*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a
 + b*x)))*Sqrt[Coth[a*c + b*c*x]^2]) - (3*ArcTan[E^(c*(a + b*x))]*Coth[a*c
+ b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^3(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int\frac{(-1+x^2)^3}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int\left(1-\frac{2(1+3x^4)}{(1+x^2)^3}\right)\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\text{Subst}\left(\int\frac{1+3x^4}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)\text{Subst}\left(\int-\frac{12x^2}{(1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{2bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} - \frac{(6\coth(ac+bcx))\text{Subst}\left(\int\frac{x^2}{(1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}} - \frac{(3\coth(ac+bcx))\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{2e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{3e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}} - \frac{3\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.54

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{\left(e^{c(a+bx)}\left(2+5e^{2c(a+bx)}+e^{4c(a+bx)}\right)-3\left(1+e^{2c(a+bx)}\right)^2\arctan\left(e^{c(a+bx)}\right)\right)\coth(c(a}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(2 + 5*E^(2*c*(a + b*x)) + E^(4*c*(a + b*x))) - 3*(1 + E^
(2*c*(a + b*x)))^2*ArcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(b*c*(1 + E^
(2*c*(a + b*x)))^2*Sqrt[Coth[c*(a + b*x)]^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\sinh(bcx+ac)^2}{\cosh(bcx+ac)}+\frac{2}{\cosh(bcx+ac)}+\frac{\sinh(bcx+ac)^3}{\cosh(bcx+ac)^2}+\frac{3\sinh(bcx+ac)}{\cosh(bcx+ac)^2}-\frac{3\,\operatorname{sech}(bcx+ac)\tanh(bcx+ac)}{2}-3\arctan\left(e^{bcx+ac}\right)\right)}{cb}$
risch	$\frac{3ie^{4c(bx+a)}\ln\left(e^{c(bx+a)}-i\right)-3ie^{4c(bx+a)}\ln\left(e^{c(bx+a)}+i\right)+2\,e^{5c(bx+a)}+6ie^{2c(bx+a)}\ln\left(e^{c(bx+a)}-i\right)-6ie^{2c(bx+a)}\ln\left(e^{c(bx+a)}+i\right)+10\,e^{3c}}{2\left(1+e^{2c(bx+a)}\right)\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,cb}$

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(coth(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)^2/cosh(b*c*x+a*c)+2/cosh(b*c*x+a
*c)+sinh(b*c*x+a*c)^3/cosh(b*c*x+a*c)^2+3*sinh(b*c*x+a*c)/cosh(b*c*x+a*c)^2
-3/2*sech(b*c*x+a*c)*tanh(b*c*x+a*c)-3*arctan(exp(b*c*x+a*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(179) = 358$.

Time = 0.27 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.37

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \frac{\cosh(bcx+ac)^5 + 5\cosh(bcx+ac)\sinh(bcx+ac)^4 + \sinh(bcx+ac)^5 + 5\,(2\,c$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 + sinh(b*c*x
 + a*c)^5 + 5*(2*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^3 + 5*cosh(b*c*
x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a
*c)^2 - 3*(cosh(b*c*x + a*c)^4 + 4*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^3 +
sinh(b*c*x + a*c)^4 + 2*(3*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 2
*cosh(b*c*x + a*c)^2 + 4*(cosh(b*c*x + a*c)^3 + cosh(b*c*x + a*c))*sinh(b*c
*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + (5*cosh(b*c*
x + a*c)^4 + 15*cosh(b*c*x + a*c)^2 + 2)*sinh(b*c*x + a*c) + 2*cosh(b*c*x +
 a*c))/(b*c*cosh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)
^3 + b*c*sinh(b*c*x + a*c)^4 + 2*b*c*cosh(b*c*x + a*c)^2 + 2*(3*b*c*cosh(b*
c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 4*(b*c*cosh(b*c*x + a*c)^3
+ b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\coth^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(coth(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = -\frac{3\arctan\left(e^{(bcx+ac)}\right)}{bc} + \frac{e^{(5\,bcx+5\,ac)} + 5\,e^{(3\,bcx+3\,ac)} + 2\,e^{(bcx+ac)}}{bc\left(e^{(4\,bcx+4\,ac)} + 2\,e^{(2\,bcx+2\,ac)} + 1\right)}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(e^(b*c*x + a*c))/(b*c) + (e^(5*b*c*x + 5*a*c) + 5*e^(3*b*c*x + 3*
a*c) + 2*e^(b*c*x + a*c))/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c)
 + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx =$$
$$-\frac{\left(3\arctan\left(e^{(bcx+ac)}\right)e^{(-ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - e^{(bcx)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - \frac{3\,e^{(3\,bcx+2\,ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right)+e^{(bc}}{\left(e^{(2\,bcx+2\,ac)}+1\right)^2}\right.}{bc}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - e^(b*c*
x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (3*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2
*a*c) - 1) + e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) +
 1)^2)*e^(a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{3/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\coth\left(a\,c+b\,c\,x\right)^2\right)^{3/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(3/2), x)
```

3.216 $\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 311

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\coth^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}} - \frac{15\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{4bc\sqrt{\coth^2(ac+bcx)}}$$

```
[Out] exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)-4*exp(c*(b*x+a
))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(coth(b*c*x+a*c)^2)^(1/2)+26/
3*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(coth(b*c*x+a*c
)^2)^(1/2)-55/6*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(
coth(b*c*x+a*c)^2)^(1/2)+25/4*exp(c*(b*x+a))*coth(b*c*x+a*c)/b/c/(1+exp(2*c
*(b*x+a)))/(coth(b*c*x+a*c)^2)^(1/2)-15/4*arctan(exp(c*(b*x+a)))*coth(b*c*x
+a*c)/b/c/(coth(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {6852, 2320, 398, 1828, 1171, 393, 209}

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = -\frac{15\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{4bc\sqrt{\coth^2(ac+bcx)}} + \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} + \frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(e^{2c(a+bx)}+1\right)\sqrt{\coth^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(e^{2c(a+bx)}+1\right)^3\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^4\sqrt{\coth^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (E^(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*Sqrt[Coth[a*c + b*c*x]^2]) - (4*E^
(c*(a + b*x))*Coth[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Coth[a
*c + b*c*x]^2]) + (26*E^(c*(a + b*x))*Coth[a*c + b*c*x])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3*Sqrt[Coth[a*c + b*c*x]^2]) - (55*E^(c*(a + b*x))*Coth[a*c +
b*c*x])/(6*b*c*(1 + E^(2*c*(a + b*x)))^2*Sqrt[Coth[a*c + b*c*x]^2]) + (25*E
^(c*(a + b*x))*Coth[a*c + b*c*x])/(4*b*c*(1 + E^(2*c*(a + b*x)))*Sqrt[Coth[
a*c + b*c*x]^2]) - (15*ArcTan[E^(c*(a + b*x))]*Coth[a*c + b*c*x])/(4*b*c*Sq
rt[Coth[a*c + b*c*x]^2])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
```

```
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \frac{\coth(ac+bcx)\int e^{c(a+bx)}\tanh^5(ac+bcx)\,dx}{\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int \frac{(-1+x^2)^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{\coth(ac+bcx)\text{Subst}\left(\int\left(1-\frac{2(1+10x^4+5x^8)}{(1+x^2)^5}\right)\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{(2\coth(ac+bcx))\text{Subst}\left(\int\frac{1+10x^4+5x^8}{(1+x^2)^5}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)\text{Subst}\left(\int\frac{8-120x^2+40x^4-40x^6}{(1+x^2)^4}\,dx,x,e^{c(a+bx)}\right)}{4bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\coth^2(ac+bcx)}} - \frac{\coth(ac+bcx)\text{Subst}\left(\int\frac{160-480x^2+240x^4}{(1+x^2)^3}\,dx,x,e^{c(a+bx)}\right)}{24bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\coth^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{\coth(ac+bcx)\text{Subst}\left(\int\frac{240-960x^2}{(1+x^2)^2}\,dx,x,e^{c(a+bx)}\right)}{96bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\coth^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}} - \frac{(15\coth(ac+bcx))\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,e^{c(a+bx)}\right)}{4bc\sqrt{\coth^2(ac+bcx)}}$$

$$= \frac{e^{c(a+bx)}\coth(ac+bcx)}{bc\sqrt{\coth^2(ac+bcx)}} - \frac{4e^{c(a+bx)}\coth(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(ac+bcx)}} + \frac{26e^{c(a+bx)}\coth(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\coth^2(ac+bcx)}} - \frac{55e^{c(a+bx)}\coth(ac+bcx)}{6bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\coth^2(ac+bcx)}} + \frac{25e^{c(a+bx)}\coth(ac+bcx)}{4bc\left(1+e^{2c(a+bx)}\right)\sqrt{\coth^2(ac+bcx)}} - \frac{15\arctan\left(e^{c(a+bx)}\right)\coth(ac+bcx)}{4bc\sqrt{\coth^2(ac+bcx)}}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.43

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \frac{\left(e^{c(a+bx)}\left(33+157e^{2c(a+bx)}+187e^{4c(a+bx)}+123e^{6c(a+bx)}+12e^{8c(a+bx)}\right)-45\left(1+\right.\right.}{12bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\coth^2(c(a+bx}}$$

```
[In] Integrate[E^(c*(a + b*x))/(Coth[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((E^(c*(a + b*x))*(33 + 157*E^(2*c*(a + b*x)) + 187*E^(4*c*(a + b*x)) + 123
*E^(6*c*(a + b*x)) + 12*E^(8*c*(a + b*x))) - 45*(1 + E^(2*c*(a + b*x)))^4*A
rcTan[E^(c*(a + b*x))])*Coth[c*(a + b*x)])/(12*b*c*(1 + E^(2*c*(a + b*x)))^
4*Sqrt[Coth[c*(a + b*x)]^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.59 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.63

method	result
default	$\frac{\operatorname{csgn}(\coth(c(bx+a)))\left(\frac{\sinh(bcx+ac)^4}{\cosh(bcx+ac)^3}+\frac{4\sinh(bcx+ac)^2}{\cosh(bcx+ac)^3}+\frac{8}{3\cosh(bcx+ac)^3}+\frac{\sinh(bcx+ac)^5}{\cosh(bcx+ac)^4}+\frac{5\sinh(bcx+ac)^3}{\cosh(bcx+ac)^4}+\frac{5\sinh(bcx+ac)}{\cosh(bcx+ac)^4}-5\left(\frac{\operatorname{sech}(bcx-}{4}\right.\right.}{cb}$
risch	$\frac{\left(1+e^{2c(bx+a)}\right)e^{c(bx+a)}}{\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\left(e^{2c(bx+a)}-1\right)bc}+\frac{e^{c(bx+a)}\left(75\,e^{6c(bx+a)}+115\,e^{4c(bx+a)}+109\,e^{2c(bx+a)}+21\right)}{12\left(1+e^{2c(bx+a)}\right)^3\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+a)}\right)^2}{\left(e^{2c(bx+a)}-1\right)^2}}\,cb}+\frac{15i\left(1+e^{2c(bx+a)}\right)\ln\left(e^{c(bx+\right.}{8\left(e^{2c(bx+a)}-1\right)\sqrt{\frac{\left(1+e^{2c(bx+\right.}{\left(e^{2c(bx+a)}\right.}}}$

```
[In] int(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(coth(c*(b*x+a)))/c/b*(sinh(b*c*x+a*c)^4/cosh(b*c*x+a*c)^3+4*sinh(b*c*x
+a*c)^2/cosh(b*c*x+a*c)^3+8/3/cosh(b*c*x+a*c)^3+sinh(b*c*x+a*c)^5/cosh(b*c*
x+a*c)^4+5*sinh(b*c*x+a*c)^3/cosh(b*c*x+a*c)^4+5*sinh(b*c*x+a*c)/cosh(b*c*x
+a*c)^4-5*(1/4*sech(b*c*x+a*c)^3+3/8*sech(b*c*x+a*c))*tanh(b*c*x+a*c)-15/4*
arctan(exp(b*c*x+a*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1226 vs. 2(281) = 562.

Time = 0.28 (sec) , antiderivative size = 1226, normalized size of antiderivative = 3.94

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(12*cosh(b*c*x + a*c)^9 + 108*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^8 +
12*sinh(b*c*x + a*c)^9 + 3*(144*cosh(b*c*x + a*c)^2 + 41)*sinh(b*c*x + a*c)
^7 + 123*cosh(b*c*x + a*c)^7 + 21*(48*cosh(b*c*x + a*c)^3 + 41*cosh(b*c*x +
 a*c))*sinh(b*c*x + a*c)^6 + (1512*cosh(b*c*x + a*c)^4 + 2583*cosh(b*c*x +
a*c)^2 + 187)*sinh(b*c*x + a*c)^5 + 187*cosh(b*c*x + a*c)^5 + (1512*cosh(b*
c*x + a*c)^5 + 4305*cosh(b*c*x + a*c)^3 + 935*cosh(b*c*x + a*c))*sinh(b*c*x
 + a*c)^4 + (1008*cosh(b*c*x + a*c)^6 + 4305*cosh(b*c*x + a*c)^4 + 1870*cos
h(b*c*x + a*c)^2 + 157)*sinh(b*c*x + a*c)^3 + 157*cosh(b*c*x + a*c)^3 + (43
2*cosh(b*c*x + a*c)^7 + 2583*cosh(b*c*x + a*c)^5 + 1870*cosh(b*c*x + a*c)^3
 + 471*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2 - 45*(cosh(b*c*x + a*c)^8 + 8
*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^7 + sinh(b*c*x + a*c)^8 + 4*(7*cosh(b*
c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^6 + 4*cosh(b*c*x + a*c)^6 + 8*(7*cosh(b
*c*x + a*c)^3 + 3*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^5 + 2*(35*cosh(b*c*x
 + a*c)^4 + 30*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)^4 + 6*cosh(b*c*x
+ a*c)^4 + 8*(7*cosh(b*c*x + a*c)^5 + 10*cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x
 + a*c))*sinh(b*c*x + a*c)^3 + 4*(7*cosh(b*c*x + a*c)^6 + 15*cosh(b*c*x + a
*c)^4 + 9*cosh(b*c*x + a*c)^2 + 1)*sinh(b*c*x + a*c)^2 + 4*cosh(b*c*x + a*c
)^2 + 8*(cosh(b*c*x + a*c)^7 + 3*cosh(b*c*x + a*c)^5 + 3*cosh(b*c*x + a*c)^
3 + cosh(b*c*x + a*c))*sinh(b*c*x + a*c) + 1)*arctan(cosh(b*c*x + a*c) + si
nh(b*c*x + a*c)) + (108*cosh(b*c*x + a*c)^8 + 861*cosh(b*c*x + a*c)^6 + 935
*cosh(b*c*x + a*c)^4 + 471*cosh(b*c*x + a*c)^2 + 33)*sinh(b*c*x + a*c) + 33
*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^8 + 8*b*c*cosh(b*c*x + a*c)*sinh
(b*c*x + a*c)^7 + b*c*sinh(b*c*x + a*c)^8 + 4*b*c*cosh(b*c*x + a*c)^6 + 4*(
7*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a
*c)^4 + 8*(7*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^5 + 2*(35*b*c*cosh(b*c*x + a*c)^4 + 30*b*c*cosh(b*c*x + a*c)^2 + 3*b
*c)*sinh(b*c*x + a*c)^4 + 4*b*c*cosh(b*c*x + a*c)^2 + 8*(7*b*c*cosh(b*c*x +
 a*c)^5 + 10*b*c*cosh(b*c*x + a*c)^3 + 3*b*c*cosh(b*c*x + a*c))*sinh(b*c*x
+ a*c)^3 + 4*(7*b*c*cosh(b*c*x + a*c)^6 + 15*b*c*cosh(b*c*x + a*c)^4 + 9*b*
c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^2 + b*c + 8*(b*c*cosh(b*c*x
+ a*c)^7 + 3*b*c*cosh(b*c*x + a*c)^5 + 3*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh
(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)**2)**(5/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.47

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = -\frac{15\arctan\left(e^{(bcx+ac)}\right)}{4\,bc} + \frac{12\,e^{(9\,bcx+9\,ac)} + 123\,e^{(7\,bcx+7\,ac)} + 187\,e^{(5\,bcx+5\,ac)} + 157\,e^{(3\,bcx+3\,ac)} + 33\,e^{(bcx+ac)}}{12\,bc\left(e^{(8\,bcx+8\,ac)} + 4\,e^{(6\,bcx+6\,ac)} + 6\,e^{(4\,bcx+4\,ac)} + 4\,e^{(2\,bcx+2\,ac)} + 1\right)}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
[Out] -15/4*arctan(e^(b*c*x + a*c))/(b*c) + 1/12*(12*e^(9*b*c*x + 9*a*c) + 123*e^
(7*b*c*x + 7*a*c) + 187*e^(5*b*c*x + 5*a*c) + 157*e^(3*b*c*x + 3*a*c) + 33*
e^(b*c*x + a*c))/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4
*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.59

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = -\frac{\left(45\arctan\left(e^{(bcx+ac)}\right)e^{(-ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - 12\,e^{(bcx)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-1\right) - \frac{75\,e^{(7\,bcx+6\,ac)}\mathrm{sgn}\left(e^{(2\,bcx+2\,ac)}-\right.}{}\right.}{12\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(coth(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
[Out] -1/12*(45*arctan(e^(b*c*x + a*c))*e^(-a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) - 1
2*e^(b*c*x)*sgn(e^(2*b*c*x + 2*a*c) - 1) - (75*e^(7*b*c*x + 6*a*c)*sgn(e^(2
*b*c*x + 2*a*c) - 1) + 115*e^(5*b*c*x + 4*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1)
 + 109*e^(3*b*c*x + 2*a*c)*sgn(e^(2*b*c*x + 2*a*c) - 1) + 21*e^(b*c*x)*sgn(
e^(2*b*c*x + 2*a*c) - 1))/(e^(2*b*c*x + 2*a*c) + 1)^4)*e^(a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\coth^2(ac+bcx)^{5/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\coth\left(a\,c+b\,c\,x\right)^2\right)^{5/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2),x)
[Out] int(exp(c*(a + b*x))/(coth(a*c + b*c*x)^2)^(5/2), x)
```

3.217 $\int \sin^3(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 157

$$\begin{aligned}\int \sin^3(\coth(a+bx))\,dx = &-\frac{3\,\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{8b}\\ &-\frac{3\,\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{8b}\\ &+\frac{\text{CosIntegral}(3-3\coth(a+bx))\sin(3)}{8b}\\ &+\frac{\text{CosIntegral}(3+3\coth(a+bx))\sin(3)}{8b}\\ &-\frac{\cos(3)\text{Si}(3-3\coth(a+bx))}{8b}+\frac{3\cos(1)\text{Si}(1-\coth(a+bx))}{8b}\\ &+\frac{3\cos(1)\text{Si}(1+\coth(a+bx))}{8b}-\frac{\cos(3)\text{Si}(3+3\coth(a+bx))}{8b}\end{aligned}$$

```
[Out] 1/8*cos(3)*Si(-3+3*coth(b*x+a))/b-3/8*cos(1)*Si(-1+coth(b*x+a))/b+3/8*cos(1
)*Si(1+coth(b*x+a))/b-1/8*cos(3)*Si(3+3*coth(b*x+a))/b-3/8*Ci(1-coth(b*x+a)
)*sin(1)/b-3/8*Ci(1+coth(b*x+a))*sin(1)/b+1/8*Ci(3-3*coth(b*x+a))*sin(3)/b+
1/8*Ci(3+3*coth(b*x+a))*sin(3)/b
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used

= {6857, 3393, 3384, 3380, 3383}

$$\int \sin^3(\coth(a+bx))\,dx = \frac{\sin(3)\,\text{CosIntegral}(3-3\coth(a+bx))}{8b} + \frac{\sin(3)\,\text{CosIntegral}(3\coth(a+bx)+3)}{8b} - \frac{3\sin(1)\,\text{CosIntegral}(1-\coth(a+bx))}{8b} - \frac{3\sin(1)\,\text{CosIntegral}(\coth(a+bx)+1)}{8b} - \frac{\cos(3)\text{Si}(3-3\coth(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1-\coth(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(\coth(a+bx)+1)}{8b} - \frac{\cos(3)\text{Si}(3\coth(a+bx)+3)}{8b}$$

```
[In] Int[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-3*CosIntegral[1 - Coth[a + b*x]]*Sin[1])/(8*b) - (3*CosIntegral[1 + Coth[
a + b*x]]*Sin[1])/(8*b) + (CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3])/(8*b) +
 (CosIntegral[3 + 3*Coth[a + b*x]]*Sin[3])/(8*b) - (Cos[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) + (3*Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Cos[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) - (Cos[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\sin^3(x)}{2(-1+x)}+\frac{\sin^3(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\sin^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(-1+x)}-\frac{\sin(3x)}{4(-1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b}\\
&\quad +\frac{\text{Subst}\left(\int \left(\frac{3\sin(x)}{4(1+x)}-\frac{\sin(3x)}{4(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{\sin(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}-\frac{\text{Subst}\left(\int \frac{\sin(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}\\
&\quad -\frac{3\text{Subst}\left(\int \frac{\sin(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{3\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(3\cos(1))\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&+ \frac{(3\cos(1))\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&- \frac{\cos(3)\text{Subst}\left(\int \frac{\sin(3-3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&- \frac{\cos(3)\text{Subst}\left(\int \frac{\sin(3+3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&- \frac{(3\sin(1))\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&- \frac{(3\sin(1))\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&+ \frac{\sin(3)\text{Subst}\left(\int \frac{\cos(3-3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&+ \frac{\sin(3)\text{Subst}\left(\int \frac{\cos(3+3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&= -\frac{3\,\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{8b} - \frac{3\,\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{8b} \\
&+ \frac{\text{CosIntegral}(3-3\coth(a+bx))\sin(3)}{8b} + \frac{\text{CosIntegral}(3+3\coth(a+bx))\sin(3)}{8b} \\
&- \frac{\cos(3)\text{Si}(3-3\coth(a+bx))}{8b} + \frac{3\cos(1)\text{Si}(1-\coth(a+bx))}{8b} \\
&+ \frac{3\cos(1)\text{Si}(1+\coth(a+bx))}{8b} - \frac{\cos(3)\text{Si}(3+3\coth(a+bx))}{8b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$
\int \sin^3(\coth(a+bx))\,dx = \frac{-6\,\text{CosIntegral}(1-\coth(a+bx))\sin(1) - 6\,\text{CosIntegral}(1+\coth(a+bx))\sin(1) + 2\,\text{CosIntegral}(3-3\,\text{co}
$$

```
[In] Integrate[Sin[Coth[a + b*x]]^3,x]
```

```
[Out] (-6*CosIntegral[1 - Coth[a + b*x]]*Sin[1] - 6*CosIntegral[1 + Coth[a + b*x]
]*Sin[1] + 2*CosIntegral[3 - 3*Coth[a + b*x]]*Sin[3] + 2*CosIntegral[3 + 3*
Coth[a + b*x]]*Sin[3] - 2*Cos[3]*SinIntegral[3 - 3*Coth[a + b*x]] + 6*Cos[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Cos[1]*SinIntegral[1 + Coth[a + b*x]]
- 2*Cos[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Si}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\sin(3)}{8}-\frac{3\ \mathrm{Si}(\coth(bx+}{8}}{b}$
default	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Si}(3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\sin(3)}{8}-\frac{3\ \mathrm{Si}(\coth(bx+}{8}}{b}$
risch	$-\frac{i\mathrm{e}^{-3i}\,\mathrm{Ei}_1\left(-\frac{6i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}-6i\right)}{16b}+\frac{\pi\,\mathrm{e}^{3i}\,\mathrm{csgn}\left(\frac{\mathrm{e}^{-a}}{-\mathrm{e}^{2bx+a}+\mathrm{e}^{-a}}\right)}{16b}+\frac{\mathrm{e}^{3i}\,\mathrm{Si}\left(\frac{6\,\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)}{8b}+\frac{i\mathrm{e}^{3i}\,\mathrm{Ei}_1\left(\frac{6i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)}{16b}$

```
[In] int(sin(coth(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*Si(-3+3*coth(b*x+a))*cos(3)+1/8*Ci(-3+3*coth(b*x+a))*sin(3)-1/8*Si
(3+3*coth(b*x+a))*cos(3)+1/8*Ci(3+3*coth(b*x+a))*sin(3)-3/8*Si(coth(b*x+a)-
1)*cos(1)-3/8*Ci(coth(b*x+a)-1)*sin(1)+3/8*Si(coth(b*x+a)+1)*cos(1)-3/8*Ci(
coth(b*x+a)+1)*sin(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 696, normalized size of antiderivative = 4.43

$$\int \sin^3(\coth(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(sin(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*((-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*sin(3)^2 - 2*I*(I*cos(3)*c
os(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I)*sin(1) + I*cos(1))*cos_
integral(3*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - 3*(2*cos(3)*cos
(1)*sin(1) + I*cos(3)*sin(1)^2 + (-I*cos(1)^2 + I)*cos(3) + I*(-I*cos(1)^2
+ 2*cos(1)*sin(1) + I*sin(1)^2 + I)*sin(3))*cos_integral((cosh(b*x + a) + s
inh(b*x + a))/sinh(b*x + a)) + (-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*s
in(3)^2 - 2*I*(I*cos(3)*cos(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I
)*sin(1) + I*cos(1))*cos_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh
(b*x + a) + sinh(b*x + a)^2 - 1)) - 3*(2*cos(3)*cos(1)*sin(1) + I*cos(3)*si
n(1)^2 + (-I*cos(1)^2 + I)*cos(3) + I*(-I*cos(1)^2 + 2*cos(1)*sin(1) + I*si
n(1)^2 + I)*sin(3))*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 - 1)) - (cos(3)^2*cos(1) - (cos(1) + I*sin(1))*s
in(3)^2 + 2*I*(cos(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*s
in(1) + cos(1))*sin_integral(3*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a
)) - 3*(-2*I*cos(3)*cos(1)*sin(1) + cos(3)*sin(1)^2 - (cos(1)^2 + 1)*cos(3)
 - I*(cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*sin(3))*sin_integral((co
sh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (cos(3)^2*cos(1) - (cos(1) +
```

```
I*sin(1))*sin(3)^2 + 2*I*(cos(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(cos(
3)^2 + 1)*sin(1) + cos(1))*sin_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) - 3*(2*I*cos(3)*cos(1)*sin(1) - cos
(3)*sin(1)^2 + (cos(1)^2 + 1)*cos(3) + I*(cos(1)^2 + 2*I*cos(1)*sin(1) - si
n(1)^2 + 1)*sin(3))*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(
b*x + a) + sinh(b*x + a)^2 - 1)))/(b*cos(3)*cos(1) + I*b*cos(3)*sin(1) + I*
(b*cos(1) + I*b*sin(1))*sin(3))
```

Sympy [F]

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin^3\left(\coth\left(a+bx\right)\right)dx$$

[In] integrate(sin(coth(b*x+a))**3,x)

[Out] Integral(sin(coth(a + b*x))**3, x)

Maxima [F]

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^3\,dx$$

[In] integrate(sin(coth(b*x+a))^3,x, algorithm="maxima")

[Out] integrate(sin(coth(b*x + a))^3, x)

Giac [F]

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)^3\,dx$$

[In] integrate(sin(coth(b*x+a))^3,x, algorithm="giac")

[Out] integrate(sin(coth(b*x + a))^3, x)

Mupad [F(-1)]

Timed out.

$$\int \sin^3(\coth(a+bx))\,dx = \int \sin\left(\coth(a+b\,x)\right)^3\,dx$$

```
[In] int(sin(coth(a + b*x))^3,x)
```

```
[Out] int(sin(coth(a + b*x))^3, x)
```

3.218 $\int \sin^2(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \sin^2(\coth(a+bx))\,dx = \frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))}{4b} - \frac{\cos(2)\,\text{CosIntegral}(2+2\coth(a+bx))}{4b} - \frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2+2\coth(a+bx))}{4b}$$

```
[Out] 1/4*Ci(2-2*coth(b*x+a))*cos(2)/b-1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*ln(1-
coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b-1/4*Si(-2+2*coth(b*x+a))*sin(2)/b-1/
4*Si(2+2*coth(b*x+a))*sin(2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6857, 3393, 3384, 3380, 3383}

$$\int \sin^2(\coth(a+bx))\,dx = \frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))}{4b} - \frac{\cos(2)\,\text{CosIntegral}(2\coth(a+bx)+2)}{4b} + \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2\coth(a+bx)+2)}{4b} - \frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(\coth(a+bx)+1)}{4b}$$

```
[In] Int[Sin[Coth[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Cos[2]*CosIntegral[2 + 2
*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b*
x]]/(4*b) + (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) - (Sin[2]*SinIn
tegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{\sin^2(x)}{2(-1+x)} + \frac{\sin^2(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sin^2(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin^2(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$

$$
\begin{aligned}
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)}-\frac{\cos(2x)}{2(-1+x)}\right)\,dx,x,\coth(a+bx)\right)}{2b}\\
&\quad+\frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)}-\frac{\cos(2x)}{2(1+x)}\right)\,dx,x,\coth(a+bx)\right)}{2b}\\
&= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}\\
&\quad+\frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{4b}-\frac{\text{Subst}\left(\int \frac{\cos(2x)}{1+x}\,dx,x,\coth(a+bx)\right)}{4b}\\
&= -\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}\\
&\quad+\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{4b}\\
&\quad-\frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2+2x)}{1+x}\,dx,x,\coth(a+bx)\right)}{4b}\\
&\quad+\frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2-2x)}{-1+x}\,dx,x,\coth(a+bx)\right)}{4b}\\
&\quad-\frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2+2x)}{1+x}\,dx,x,\coth(a+bx)\right)}{4b}\\
&= \frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))}{4b}-\frac{\cos(2)\,\text{CosIntegral}(2+2\coth(a+bx))}{4b}\\
&\quad-\frac{\log(1-\coth(a+bx))}{4b}+\frac{\log(1+\coth(a+bx))}{4b}\\
&\quad+\frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b}-\frac{\sin(2)\text{Si}(2+2\coth(a+bx))}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$
\int \sin^2(\coth(a+bx))\,dx = \frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))-\cos(2)\,\text{CosIntegral}(2(1+\coth(a+bx)))-\log(1-\coth(a+bx))}{4b}
$$

```
[In] Integrate[Sin[Coth[a + b*x]]^2,x]
```

```
[Out] (Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]] - Cos[2]*CosIntegral[2*(1 + Coth[a
 + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] + Sin[2]*SinInt
egral[2 - 2*Coth[a + b*x]] - Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/(4*
b)
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{\ln(\coth(bx+a)-1)}{4}+\frac{\ln(\coth(bx+a)+1)}{4}-\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(2)}{4}-}{b}$
default	$\frac{-\frac{\ln(\coth(bx+a)-1)}{4}+\frac{\ln(\coth(bx+a)+1)}{4}-\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(2)}{4}-}{b}$
risch	$\frac{\mathrm{e}^{2i}\,\mathrm{Ei}_1\left(\frac{4i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}+4i\right)}{8b}-\frac{i\,\mathrm{csgn}\left(\frac{\mathrm{e}^{-a}}{-\mathrm{e}^{2bx+a}+\mathrm{e}^{-a}}\right)\pi\,\mathrm{e}^{-2i}}{8b}-\frac{i\,\mathrm{Si}\left(\frac{4\,\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)\mathrm{e}^{-2i}}{4b}-\frac{\mathrm{e}^{-2i}\,\mathrm{Ei}_1\left(-\frac{4i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)}{8b}$

```
[In] int(sin(coth(b*x+a))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/4*ln(coth(b*x+a)-1)+1/4*ln(coth(b*x+a)+1)-1/4*Si(-2+2*coth(b*x+a))*
sin(2)+1/4*Ci(-2+2*coth(b*x+a))*cos(2)-1/4*Si(2+2*coth(b*x+a))*sin(2)-1/4*C
i(2+2*coth(b*x+a))*cos(2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.00

$$\int \sin^2(\coth(a+bx))\,dx$$
$$= \frac{4\,bx\cos(2)+4i\,bx\sin(2)-\left(\cos(2)^2+2i\,\cos(2)\sin(2)-\sin(2)^2+1\right)\mathrm{Ci}\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\sinh(bx+a)}\right)+(\cos}{}$$

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cos(2) + 4*I*b*x*sin(2) - (cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2)
^2 + 1)*cos_integral(2*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (co
s(2)^2 + 2*I*cos(2)*sin(2) - sin(2)^2 + 1)*cos_integral(4/(cosh(b*x + a)^2
+ 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + (I*cos(2)^2 - 2*c
os(2)*sin(2) - I*sin(2)^2 - I)*sin_integral(2*(cosh(b*x + a) + sinh(b*x + a
))/sinh(b*x + a)) + (I*cos(2)^2 - 2*cos(2)*sin(2) - I*sin(2)^2 - I)*sin_int
egral(4/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2
- 1)))/(b*cos(2) + I*b*sin(2))
```

Sympy [F]

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin^2(\coth(a+bx))\,dx$$

```
[In] integrate(sin(coth(b*x+a))**2,x)
[Out] Integral(sin(coth(a + b*x))**2, x)
```

Maxima [F]

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin(\coth(bx+a))^2\,dx$$

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="maxima")
[Out] 1/2*x - 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

Giac [F]

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin(\coth(bx+a))^2\,dx$$

```
[In] integrate(sin(coth(b*x+a))^2,x, algorithm="giac")
[Out] integrate(sin(coth(b*x + a))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin^2(\coth(a+bx))\,dx = \int \sin(\coth(a+b\,x))^2\,dx$$

```
[In] int(sin(coth(a + b*x))^2,x)
[Out] int(sin(coth(a + b*x))^2, x)
```

3.219 $\int \sin(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \sin(\coth(a+bx))\,dx = -\frac{\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{2b} - \frac{\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{2b} + \frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\text{Si}(1+\coth(a+bx))}{2b}$$

```
[Out] -1/2*cos(1)*Si(-1+coth(b*x+a))/b+1/2*cos(1)*Si(1+coth(b*x+a))/b-1/2*Ci(1-co
th(b*x+a))*sin(1)/b-1/2*Ci(1+coth(b*x+a))*sin(1)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3414, 3384, 3380, 3383}

$$\int \sin(\coth(a+bx))\,dx = -\frac{\sin(1)\,\text{CosIntegral}(1-\coth(a+bx))}{2b} - \frac{\sin(1)\,\text{CosIntegral}(\coth(a+bx)+1)}{2b} + \frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\text{Si}(\coth(a+bx)+1)}{2b}$$

```
[In] Int[Sin[Coth[a + b*x]],x]
```

```
[Out] -1/2*(CosIntegral[1 - Coth[a + b*x]]*Sin[1])/b - (CosIntegral[1 + Coth[a +
b*x]]*Sin[1])/(2*b) + (Cos[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Cos[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{2(1-x)}+\frac{\sin(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\sin(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&= -\frac{\cos(1)\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&\quad + \frac{\cos(1)\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&\quad + \frac{\sin(1)\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&\quad - \frac{\sin(1)\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}
\end{aligned}$$

$$= -\frac{\text{CosIntegral}(1-\coth(a+bx))\sin(1)}{2b} - \frac{\text{CosIntegral}(1+\coth(a+bx))\sin(1)}{2b} + \frac{\cos(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\text{Si}(1+\coth(a+bx))}{2b}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \sin(\coth(a+bx))\,dx = -\frac{\text{CosIntegral}(1-\coth(a+bx))\sin(1) + \text{CosIntegral}(1+\coth(a+bx))\sin(1) - \cos(1)(\text{Si}(1-\coth(a+$$

```
[In] Integrate[Sin[Coth[a + b*x]],x]
```

```
[Out] -1/2*(CosIntegral[1 - Coth[a + b*x]]*Sin[1] + CosIntegral[1 + Coth[a + b*x]
]*Sin[1] - Cos[1]*(SinIntegral[1 - Coth[a + b*x]] + SinIntegral[1 + Coth[a
+ b*x]]))/b
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{\text{Si}(\coth(bx+a)-1)\cos(1)}{2}-\frac{\text{Ci}(\coth(bx+a)-1)\sin(1)}{2}+\frac{\text{Si}(\coth(bx+a)+1)\cos(1)}{2}-\frac{\text{Ci}(\coth(bx+a)+1)\sin(1)}{2}}{b}$
default	$\frac{-\frac{\text{Si}(\coth(bx+a)-1)\cos(1)}{2}-\frac{\text{Ci}(\coth(bx+a)-1)\sin(1)}{2}+\frac{\text{Si}(\coth(bx+a)+1)\cos(1)}{2}-\frac{\text{Ci}(\coth(bx+a)+1)\sin(1)}{2}}{b}$
risch	$-\frac{i e^{i} \text{Ei}_1\left(-\frac{2i e^{-a}}{e^{2bx+a}-e^{-a}}\right)}{4b}+\frac{i e^{-i} \text{Ei}_1\left(-\frac{2i e^{-a}}{e^{2bx+a}-e^{-a}}-2i\right)}{4b}-\frac{\text{csgn}\left(\frac{e^{-a}}{-e^{2bx+a}+e^{-a}}\right)\pi e^{-i}}{4b}-\frac{\text{Si}\left(\frac{2 e^{-a}}{e^{2bx+a}-e^{-a}}\right)e^{-i}}{2b}$

```
[In] int(sin(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/2*Si(coth(b*x+a)-1)*cos(1)-1/2*Ci(coth(b*x+a)-1)*sin(1)+1/2*Si(coth
(b*x+a)+1)*cos(1)-1/2*Ci(coth(b*x+a)+1)*sin(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.81

$$\int \sin(\coth(a+bx))\,dx$$
$$= \frac{\left(i\cos(1)^2 - 2\cos(1)\sin(1) - i\sin(1)^2 - i\right)\operatorname{Ci}\left(\frac{\cosh(bx+a)+\sinh(bx+a)}{\sinh(bx+a)}\right) + \left(i\cos(1)^2 - 2\cos(1)\sin(1) - i\,s\right.}{}$$

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/4*((I*cos(1)^2 - 2*cos(1)*sin(1) - I*sin(1)^2 - I)*cos_integral((cosh(b*x
 + a) + sinh(b*x + a))/sinh(b*x + a)) + (I*cos(1)^2 - 2*cos(1)*sin(1) - I*s
in(1)^2 - I)*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a
) + sinh(b*x + a)^2 - 1)) + (cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*s
in_integral((cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - (cos(1)^2 + 2*
I*cos(1)*sin(1) - sin(1)^2 + 1)*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/(b*cos(1) + I*b*sin(1))
```

Sympy [F]

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth\left(a+bx\right)\right)dx$$

```
[In] integrate(sin(coth(b*x+a)),x)
[Out] Integral(sin(coth(a + b*x)), x)
```

Maxima [F]

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(sin(coth(b*x + a)), x)
```

Giac [F]

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sin(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(sin(coth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sin(\coth(a+bx))\,dx = \int \sin\left(\coth(a+b\,x)\right)\,dx$$

```
[In] int(sin(coth(a + b*x)),x)
[Out] int(sin(coth(a + b*x)), x)
```

3.220 $\int \csc(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \csc(\coth(a+bx))\,dx = \frac{1}{2}\text{Int}\left(\frac{\csc(\coth(a+bx))\text{csch}^2(a+bx)}{-1+\coth(a+bx)},x\right) - \frac{1}{2}\text{Int}\left(\frac{\csc(\coth(a+bx))\text{csch}^2(a+bx)}{1+\coth(a+bx)},x\right)$$

```
[Out] 1/2*Unintegrable(csc(coth(b*x+a))*csch(b*x+a)^2/(-1+coth(b*x+a)),x)-1/2*Uni
ntegrable(csc(coth(b*x+a))*csch(b*x+a)^2/(1+coth(b*x+a)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc(\coth(a+bx))\,dx = \int \csc(\coth(a+bx))\,dx$$

```
[In] Int[Csc[Coth[a + b*x]],x]
```

```
[Out] -1/2*Defer[Subst][Defer[Int][Csc[x]/(-1 + x), x], x, Coth[a + b*x]]/b + Def
er[Subst][Defer[Int][Csc[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\csc(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\csc(x)}{2(-1+x)}+\frac{\csc(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\csc(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\csc(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}
\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx=\int \csc(\coth(a+bx))\,dx$$

```
[In] Integrate[Csc[Coth[a + b*x]],x]
[Out] Integrate[Csc[Coth[a + b*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \csc\left(\coth\left(bx+a\right)\right)dx$$

```
[In] int(csc(coth(b*x+a)),x)
[Out] int(csc(coth(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx=\int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="fricas")
[Out] integral(csc(coth(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \csc(\coth(a+bx))\,dx = \int \csc\left(\coth\left(a+bx\right)\right)dx$$

```
[In] integrate(csc(coth(b*x+a)),x)
[Out] Integral(csc(coth(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx = \int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(csc(coth(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \csc(\coth(a+bx))\,dx = \int \csc\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(csc(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(csc(coth(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \csc(\coth(a+bx))\,dx = \int \frac{1}{\sin\left(\coth\left(a+b\,x\right)\right)}\,dx$$

```
[In] int(1/sin(coth(a + b*x)),x)
[Out] int(1/sin(coth(a + b*x)), x)
```

3.221 $\int \cos^3(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 157

$$\begin{aligned}\int \cos^3(\coth(a+bx))\,dx = &-\frac{\cos(3)\,\text{CosIntegral}(3-3\coth(a+bx))}{8b}\\ &-\frac{3\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{8b}\\ &+\frac{3\cos(1)\,\text{CosIntegral}(1+\coth(a+bx))}{8b}\\ &+\frac{\cos(3)\,\text{CosIntegral}(3+3\coth(a+bx))}{8b}\\ &-\frac{\sin(3)\text{Si}(3-3\coth(a+bx))}{8b}-\frac{3\sin(1)\text{Si}(1-\coth(a+bx))}{8b}\\ &+\frac{3\sin(1)\text{Si}(1+\coth(a+bx))}{8b}+\frac{\sin(3)\text{Si}(3+3\coth(a+bx))}{8b}\end{aligned}$$

```
[Out] -3/8*Ci(1-coth(b*x+a))*cos(1)/b+3/8*Ci(1+coth(b*x+a))*cos(1)/b-1/8*Ci(3-3*c
oth(b*x+a))*cos(3)/b+1/8*Ci(3+3*coth(b*x+a))*cos(3)/b+3/8*Si(-1+coth(b*x+a)
)*sin(1)/b+3/8*Si(1+coth(b*x+a))*sin(1)/b+1/8*Si(-3+3*coth(b*x+a))*sin(3)/b
+1/8*Si(3+3*coth(b*x+a))*sin(3)/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used

$= \{6857, 3393, 3384, 3380, 3383\}$

$$\int \cos^3(\coth(a+bx))\,dx = -\frac{\cos(3)\,\text{CosIntegral}(3-3\coth(a+bx))}{8b} - \frac{3\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{8b} + \frac{3\cos(1)\,\text{CosIntegral}(\coth(a+bx)+1)}{8b} + \frac{\cos(3)\,\text{CosIntegral}(3\coth(a+bx)+3)}{8b} - \frac{\sin(3)\text{Si}(3-3\coth(a+bx))}{8b} - \frac{3\sin(1)\text{Si}(1-\coth(a+bx))}{8b} + \frac{3\sin(1)\text{Si}(\coth(a+bx)+1)}{8b} + \frac{\sin(3)\text{Si}(3\coth(a+bx)+3)}{8b}$$

```
[In] Int[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] -1/8*(Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]])/b - (3*Cos[1]*CosIntegral[1
- Coth[a + b*x]])/(8*b) + (3*Cos[1]*CosIntegral[1 + Coth[a + b*x]])/(8*b) +
 (Cos[3]*CosIntegral[3 + 3*Coth[a + b*x]])/(8*b) - (Sin[3]*SinIntegral[3 -
3*Coth[a + b*x]])/(8*b) - (3*Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(8*b) +
 (3*Sin[1]*SinIntegral[1 + Coth[a + b*x]])/(8*b) + (Sin[3]*SinIntegral[3 +
3*Coth[a + b*x]])/(8*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cos^3(x)}{2(-1+x)}+\frac{\cos^3(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^3(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(-1+x)}+\frac{\cos(3x)}{4(-1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b} \\
&\quad +\frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4(1+x)}+\frac{\cos(3x)}{4(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{2b} \\
&= -\frac{\text{Subst}\left(\int \frac{\cos(3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{\text{Subst}\left(\int \frac{\cos(3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} \\
&\quad -\frac{3\text{Subst}\left(\int \frac{\cos(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b}+\frac{3\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}
\end{aligned}$$

$$= -\frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} + \frac{(3\cos(1))\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} - \frac{\cos(3)\text{Subst}\left(\int \frac{\cos(3-3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} + \frac{\cos(3)\text{Subst}\left(\int \frac{\cos(3+3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} - \frac{(3\sin(1))\text{Subst}\left(\int \frac{\sin(1-x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} + \frac{(3\sin(1))\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b} - \frac{\sin(3)\text{Subst}\left(\int \frac{\sin(3-3x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{8b} + \frac{\sin(3)\text{Subst}\left(\int \frac{\sin(3+3x)}{1+x}\,dx, x, \coth(a+bx)\right)}{8b}$$

$$= -\frac{\cos(3)\,\text{CosIntegral}(3-3\coth(a+bx))}{8b} - \frac{3\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{8b} + \frac{3\cos(1)\,\text{CosIntegral}(1+\coth(a+bx))}{8b} + \frac{\cos(3)\,\text{CosIntegral}(3+3\coth(a+bx))}{8b} - \frac{\sin(3)\text{Si}(3-3\coth(a+bx))}{8b} - \frac{3\sin(1)\text{Si}(1-\coth(a+bx))}{8b} + \frac{3\sin(1)\text{Si}(1+\coth(a+bx))}{8b} + \frac{\sin(3)\text{Si}(3+3\coth(a+bx))}{8b}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int \cos^3(\coth(a+bx))\,dx = \frac{-2\cos(3)\,\text{CosIntegral}(3-3\coth(a+bx)) - 6\cos(1)\,\text{CosIntegral}(1-\coth(a+bx)) + 6\cos(1)\,\text{CosIntegral}$$

```
[In] Integrate[Cos[Coth[a + b*x]]^3,x]
```

```
[Out] (-2*Cos[3]*CosIntegral[3 - 3*Coth[a + b*x]] - 6*Cos[1]*CosIntegral[1 - Coth
[a + b*x]] + 6*Cos[1]*CosIntegral[1 + Coth[a + b*x]] + 2*Cos[3]*CosIntegral
[3 + 3*Coth[a + b*x]] - 2*Sin[3]*SinIntegral[3 - 3*Coth[a + b*x]] - 6*Sin[1
]*SinIntegral[1 - Coth[a + b*x]] + 6*Sin[1]*SinIntegral[1 + Coth[a + b*x]]
+ 2*Sin[3]*SinIntegral[3 + 3*Coth[a + b*x]])/(16*b)
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Ci}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Si}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\cos(3)}{8}+\frac{3\ \mathrm{Si}(\coth(bx+}{8}}{b}$
default	$\frac{\frac{\mathrm{Si}(-3+3\coth(bx+a))\sin(3)}{8}-\frac{\mathrm{Ci}(-3+3\coth(bx+a))\cos(3)}{8}+\frac{\mathrm{Si}(3+3\coth(bx+a))\sin(3)}{8}+\frac{\mathrm{Ci}(3+3\coth(bx+a))\cos(3)}{8}+\frac{3\ \mathrm{Si}(\coth(bx+}{8}}{b}$
risch	$\frac{\mathrm{e}^{-3i}\,\mathrm{Ei}_1\left(\frac{6i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}\right)}{16b}-\frac{\mathrm{e}^{3i}\,\mathrm{Ei}_1\left(\frac{6i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}+6i\right)}{16b}-\frac{\mathrm{e}^{-3i}\,\mathrm{Ei}_1\left(-\frac{6i\mathrm{e}^{-a}}{\mathrm{e}^{2bx+a}-\mathrm{e}^{-a}}-6i\right)}{16b}-\frac{i\,\mathrm{csgn}\left(\frac{\mathrm{e}^{-a}}{-\mathrm{e}^{2bx+a}+\mathrm{e}^{-a}}\right)\pi}{16b}$

```
[In] int(cos(coth(b*x+a))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/8*Si(-3+3*coth(b*x+a))*sin(3)-1/8*Ci(-3+3*coth(b*x+a))*cos(3)+1/8*Si
(3+3*coth(b*x+a))*sin(3)+1/8*Ci(3+3*coth(b*x+a))*cos(3)+3/8*Si(coth(b*x+a)-
1)*sin(1)-3/8*Ci(coth(b*x+a)-1)*cos(1)+3/8*Si(coth(b*x+a)+1)*sin(1)+3/8*Ci(
coth(b*x+a)+1)*cos(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.45

$$\int \cos^3(\coth(a+bx))\,dx = \text{Too large to display}$$

```
[In] integrate(cos(coth(b*x+a))^3,x, algorithm="fricas")
```

```
[Out] 1/16*((cos(3)^2*cos(1) - (cos(1) + I*sin(1))*sin(3)^2 + 2*I*(cos(3)*cos(1)
+ I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*sin(1) + cos(1))*cos_integral(
3*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - 3*(-2*I*cos(3)*cos(1)*si
n(1) + cos(3)*sin(1)^2 - (cos(1)^2 + 1)*cos(3) - I*(cos(1)^2 + 2*I*cos(1)*s
in(1) - sin(1)^2 + 1)*sin(3))*cos_integral((cosh(b*x + a) + sinh(b*x + a))/
sinh(b*x + a)) - (cos(3)^2*cos(1) - (cos(1) + I*sin(1))*sin(3)^2 + 2*I*(cos
(3)*cos(1) + I*cos(3)*sin(1))*sin(3) + I*(cos(3)^2 + 1)*sin(1) + cos(1))*co
s_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)) - 3*(2*I*cos(3)*cos(1)*sin(1) - cos(3)*sin(1)^2 + (cos(1)^2 + 1)
*cos(3) + I*(cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*sin(3))*cos_integ
ral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 -
1)) + (-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*sin(3)^2 - 2*I*(I*cos(3)*c
os(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I)*sin(1) + I*cos(1))*sin_
integral(3*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + 3*(2*cos(3)*cos
(1)*sin(1) + I*cos(3)*sin(1)^2 - (I*cos(1)^2 - I)*cos(3) - I*(I*cos(1)^2 -
2*cos(1)*sin(1) - I*sin(1)^2 - I)*sin(3))*sin_integral((cosh(b*x + a) + sin
h(b*x + a))/sinh(b*x + a)) + (-I*cos(3)^2*cos(1) - (-I*cos(1) + sin(1))*sin
```

```
(3)^2 - 2*I*(I*cos(3)*cos(1) - cos(3)*sin(1))*sin(3) + I*(-I*cos(3)^2 + I)*
sin(1) + I*cos(1))*sin_integral(6/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b
*x + a) + sinh(b*x + a)^2 - 1)) + 3*(2*cos(3)*cos(1)*sin(1) + I*cos(3)*sin(
1)^2 - (I*cos(1)^2 - I)*cos(3) - I*(I*cos(1)^2 - 2*cos(1)*sin(1) - I*sin(1)
^2 - I)*sin(3))*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x
+ a) + sinh(b*x + a)^2 - 1)))/(b*cos(3)*cos(1) + I*b*cos(3)*sin(1) + I*(b*c
os(1) + I*b*sin(1))*sin(3))
```

Sympy [F]

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos^3\left(\coth\left(a+bx\right)\right)dx$$

[In] integrate(cos(coth(b*x+a))**3,x)

[Out] Integral(cos(coth(a + b*x))**3, x)

Maxima [F]

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^3\,dx$$

[In] integrate(cos(coth(b*x+a))^3,x, algorithm="maxima")

[Out] integrate(cos(coth(b*x + a))^3, x)

Giac [F]

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^3\,dx$$

[In] integrate(cos(coth(b*x+a))^3,x, algorithm="giac")

[Out] integrate(cos(coth(b*x + a))^3, x)

Mupad [F(-1)]

Timed out.

$$\int \cos^3(\coth(a+bx))\,dx = \int \cos\left(\coth(a+b\,x)\right)^3\,dx$$

```
[In] int(cos(coth(a + b*x))^3,x)
[Out] int(cos(coth(a + b*x))^3, x)
```

3.222 $\int \cos^2(\coth(a+bx))\,dx$

Optimal result

Integrand size = 9, antiderivative size = 115

$$\int \cos^2(\coth(a+bx))\,dx = -\frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))}{4b} + \frac{\cos(2)\,\text{CosIntegral}(2+2\coth(a+bx))}{4b} - \frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2+2\coth(a+bx))}{4b}$$

```
[Out] -1/4*Ci(2-2*coth(b*x+a))*cos(2)/b+1/4*Ci(2+2*coth(b*x+a))*cos(2)/b-1/4*ln(1
-coth(b*x+a))/b+1/4*ln(1+coth(b*x+a))/b+1/4*Si(-2+2*coth(b*x+a))*sin(2)/b+1
/4*Si(2+2*coth(b*x+a))*sin(2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6857, 3393, 3384, 3380, 3383}

$$\int \cos^2(\coth(a+bx))\,dx = -\frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))}{4b} + \frac{\cos(2)\,\text{CosIntegral}(2\coth(a+bx)+2)}{4b} - \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2\coth(a+bx)+2)}{4b} - \frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(\coth(a+bx)+1)}{4b}$$

```
[In] Int[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] -1/4*(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]])/b + (Cos[2]*CosIntegral[2 +
2*Coth[a + b*x]])/(4*b) - Log[1 - Coth[a + b*x]]/(4*b) + Log[1 + Coth[a + b
*x]]/(4*b) - (Sin[2]*SinIntegral[2 - 2*Coth[a + b*x]])/(4*b) + (Sin[2]*SinI
ntegral[2 + 2*Coth[a + b*x]])/(4*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{\cos^2(x)}{2(-1+x)}+\frac{\cos^2(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b}\\
&= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b}+\frac{\text{Subst}\left(\int \frac{\cos^2(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}
\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(-1+x)} + \frac{\cos(2x)}{2(-1+x)}\right) dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \left(\frac{1}{2(1+x)} + \frac{\cos(2x)}{2(1+x)}\right) dx, x, \coth(a+bx)\right)}{2b}$$

$$= -\frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{-1+x} dx, x, \coth(a+bx)\right)}{4b} + \frac{\text{Subst}\left(\int \frac{\cos(2x)}{1+x} dx, x, \coth(a+bx)\right)}{4b}$$

$$= -\frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} - \frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2-2x)}{-1+x} dx, x, \coth(a+bx)\right)}{4b} + \frac{\cos(2)\text{Subst}\left(\int \frac{\cos(2+2x)}{1+x} dx, x, \coth(a+bx)\right)}{4b} - \frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2-2x)}{-1+x} dx, x, \coth(a+bx)\right)}{4b} + \frac{\sin(2)\text{Subst}\left(\int \frac{\sin(2+2x)}{1+x} dx, x, \coth(a+bx)\right)}{4b}$$

$$= -\frac{\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx))}{4b} + \frac{\cos(2)\,\text{CosIntegral}(2+2\coth(a+bx))}{4b} - \frac{\log(1-\coth(a+bx))}{4b} + \frac{\log(1+\coth(a+bx))}{4b} - \frac{\sin(2)\text{Si}(2-2\coth(a+bx))}{4b} + \frac{\sin(2)\text{Si}(2+2\coth(a+bx))}{4b}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \cos^2(\coth(a+bx))\,dx = \frac{-\cos(2)\,\text{CosIntegral}(2-2\coth(a+bx)) + \cos(2)\,\text{CosIntegral}(2(1+\coth(a+bx))) - \log(1-\coth(a+b}{4b}$$

```
[In] Integrate[Cos[Coth[a + b*x]]^2,x]
```

```
[Out] (-(Cos[2]*CosIntegral[2 - 2*Coth[a + b*x]]) + Cos[2]*CosIntegral[2*(1 + Cot
h[a + b*x])] - Log[1 - Coth[a + b*x]] + Log[1 + Coth[a + b*x]] - Sin[2]*Sin
Integral[2 - 2*Coth[a + b*x]] + Sin[2]*SinIntegral[2*(1 + Coth[a + b*x])])/
(4*b)
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(2+2\coth(bx+a))\cos(2)}{4}+\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}-\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\ln(\coth(bx+a)}{4}}{b}$
default	$\frac{\frac{\mathrm{Si}(2+2\coth(bx+a))\sin(2)}{4}+\frac{\mathrm{Ci}(2+2\coth(bx+a))\cos(2)}{4}+\frac{\mathrm{Si}(-2+2\coth(bx+a))\sin(2)}{4}-\frac{\mathrm{Ci}(-2+2\coth(bx+a))\cos(2)}{4}-\frac{\ln(\coth(bx+a)}{4}}{b}$
risch	$-\frac{e^{2i}\,\mathrm{Ei}_1\left(\frac{4ie^{-a}}{e^{2bx+a}-e^{-a}}+4i\right)}{8b}+\frac{e^{-2i}\,\mathrm{Ei}_1\left(\frac{4ie^{-a}}{e^{2bx+a}-e^{-a}}\right)}{8b}-\frac{e^{-2i}\,\mathrm{Ei}_1\left(-\frac{4ie^{-a}}{e^{2bx+a}-e^{-a}}-4i\right)}{8b}-\frac{ie^{2i}\,\mathrm{csgn}\left(\frac{e^{-a}}{-e^{2bx+a}+e^{-}}\right.}{8b}$

[In] int(cos(coth(b*x+a))^2,x,method=_RETURNVERBOSE)

[Out] 1/b*(1/4*Si(2+2*coth(b*x+a))*sin(2)+1/4*Ci(2+2*coth(b*x+a))*cos(2)+1/4*Si(-2+2*coth(b*x+a))*sin(2)-1/4*Ci(-2+2*coth(b*x+a))*cos(2)-1/4*ln(coth(b*x+a)-1)+1/4*ln(coth(b*x+a)+1))

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.00

$$\int \cos^2(\coth(a+bx))\,dx$$
$$=\frac{4\,bx\cos(2)+4i\,bx\sin(2)+\left(\cos(2)^2+2i\,\cos(2)\sin(2)-\sin(2)^2+1\right)\mathrm{Ci}\left(\frac{2\,(\cosh(bx+a)+\sinh(bx+a))}{\sinh(bx+a)}\right)-(\cos}{}$$

[In] integrate(cos(coth(b*x+a))^2,x, algorithm="fricas")

[Out] 1/8*(4*b*x*cos(2) + 4*I*b*x*sin(2) + (cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2)^2 + 1)*cos_integral(2*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) - (cos(2)^2 + 2*I*cos(2)*sin(2) - sin(2)^2 + 1)*cos_integral(4/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)) + (-I*cos(2)^2 + 2*cos(2)*sin(2) + I*sin(2)^2 + I)*sin_integral(2*(cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (-I*cos(2)^2 + 2*cos(2)*sin(2) + I*sin(2)^2 + I)*sin_integral(4/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/(b*cos(2) + I*b*sin(2))

Sympy [F]

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos^2\left(\coth\left(a+bx\right)\right)dx$$

```
[In] integrate(cos(coth(b*x+a))**2,x)
[Out] Integral(cos(coth(a + b*x))**2, x)
```

Maxima [F]

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^2\,dx$$

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="maxima")
[Out] 1/2*x + 1/2*integrate(cos(2*(e^(2*b*x + 2*a) + 1)/(e^(2*b*x + 2*a) - 1)), x
)
```

Giac [F]

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)^2\,dx$$

```
[In] integrate(cos(coth(b*x+a))^2,x, algorithm="giac")
[Out] integrate(cos(coth(b*x + a))^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos^2(\coth(a+bx))\,dx = \int \cos\left(\coth(a+b\,x)\right)^2\,dx$$

```
[In] int(cos(coth(a + b*x))^2,x)
[Out] int(cos(coth(a + b*x))^2, x)
```

3.223 $\int \cos(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 77

$$\int \cos(\coth(a+bx))\,dx = -\frac{\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\,\text{CosIntegral}(1+\coth(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\sin(1)\text{Si}(1+\coth(a+bx))}{2b}$$

```
[Out] -1/2*Ci(1-coth(b*x+a))*cos(1)/b+1/2*Ci(1+coth(b*x+a))*cos(1)/b+1/2*Si(-1+co
th(b*x+a))*sin(1)/b+1/2*Si(1+coth(b*x+a))*sin(1)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3415, 3384, 3380, 3383}

$$\int \cos(\coth(a+bx))\,dx = -\frac{\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\,\text{CosIntegral}(\coth(a+bx)+1)}{2b} - \frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\sin(1)\text{Si}(\coth(a+bx)+1)}{2b}$$

```
[In] Int[Cos[Coth[a + b*x]],x]
```

```
[Out] -1/2*(Cos[1]*CosIntegral[1 - Coth[a + b*x]])/b + (Cos[1]*CosIntegral[1 + Co
th[a + b*x]])/(2*b) - (Sin[1]*SinIntegral[1 - Coth[a + b*x]])/(2*b) + (Sin[
1]*SinIntegral[1 + Coth[a + b*x]])/(2*b)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3415

```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{\cos(x)}{2(1-x)} + \frac{\cos(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b} \\
&= \frac{\text{Subst}\left(\int \frac{\cos(x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\cos(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&= \frac{\cos(1)\text{Subst}\left(\int \frac{\cos(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&\quad + \frac{\cos(1)\text{Subst}\left(\int \frac{\cos(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&\quad + \frac{\sin(1)\text{Subst}\left(\int \frac{\sin(1-x)}{1-x}\,dx, x, \coth(a+bx)\right)}{2b} \\
&\quad + \frac{\sin(1)\text{Subst}\left(\int \frac{\sin(1+x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}
\end{aligned}$$

$$= -\frac{\cos(1)\,\text{CosIntegral}(1-\coth(a+bx))}{2b} + \frac{\cos(1)\,\text{CosIntegral}(1+\coth(a+bx))}{2b} - \frac{\sin(1)\text{Si}(1-\coth(a+bx))}{2b} + \frac{\sin(1)\text{Si}(1+\coth(a+bx))}{2b}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \cos(\coth(a+bx))\,dx = -\frac{\cos(1)\,\text{CosIntegral}(1-\coth(a+bx)) - \cos(1)\,\text{CosIntegral}(1+\coth(a+bx)) + \sin(1)\text{Si}(1-\coth(a+$$
$$2b$$

```
[In] Integrate[Cos[Coth[a + b*x]],x]
[Out] -1/2*(Cos[1]*CosIntegral[1 - Coth[a + b*x]] - Cos[1]*CosIntegral[1 + Coth[a
 + b*x]] + Sin[1]*SinIntegral[1 - Coth[a + b*x]] - Sin[1]*SinIntegral[1 + C
oth[a + b*x]])/b
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\frac{\text{Si}(\coth(bx+a)-1)\sin(1)}{2}-\frac{\text{Ci}(\coth(bx+a)-1)\cos(1)}{2}+\frac{\text{Si}(\coth(bx+a)+1)\sin(1)}{2}+\frac{\text{Ci}(\coth(bx+a)+1)\cos(1)}{2}}{b}$
default	$\frac{\frac{\text{Si}(\coth(bx+a)-1)\sin(1)}{2}-\frac{\text{Ci}(\coth(bx+a)-1)\cos(1)}{2}+\frac{\text{Si}(\coth(bx+a)+1)\sin(1)}{2}+\frac{\text{Ci}(\coth(bx+a)+1)\cos(1)}{2}}{b}$
risch	$\frac{e^{-i}\,\text{Ei}_1\left(\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}\right)}{4b}-\frac{e^{i}\,\text{Ei}_1\left(\frac{2ie^{-a}}{e^{2bx+a}-e^{-a}}+2i\right)}{4b}-\frac{i\,\text{csgn}\left(\frac{e^{-a}}{-e^{2bx+a}+e^{-a}}\right)e^{i}\pi}{4b}-\frac{i\,\text{Si}\left(\frac{2\,e^{-a}}{e^{2bx+a}-e^{-a}}\right)e^{i}}{2b}+\frac{e^{i}\,\text{Ei}}{}$

```
[In] int(cos(coth(b*x+a)),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*Si(coth(b*x+a)-1)*sin(1)-1/2*Ci(coth(b*x+a)-1)*cos(1)+1/2*Si(coth(
b*x+a)+1)*sin(1)+1/2*Ci(coth(b*x+a)+1)*cos(1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.81

$$\int \cos(\coth(a+bx))\,dx$$
$$= \frac{\left(\cos(1)^2 + 2i\cos(1)\sin(1) - \sin(1)^2 + 1\right)\operatorname{Ci}\left(\frac{\cosh(bx+a)+\sinh(bx+a)}{\sinh(bx+a)}\right) - \left(\cos(1)^2 + 2i\cos(1)\sin(1) - \sin(1\right.}{}$$

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="fricas")
[Out] 1/4*((cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1)^2 + 1)*cos_integral((cosh(b*x +
 a) + sinh(b*x + a))/sinh(b*x + a)) - (cos(1)^2 + 2*I*cos(1)*sin(1) - sin(1
)^2 + 1)*cos_integral(2/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) +
sinh(b*x + a)^2 - 1)) + (-I*cos(1)^2 + 2*cos(1)*sin(1) + I*sin(1)^2 + I)*si
n_integral((cosh(b*x + a) + sinh(b*x + a))/sinh(b*x + a)) + (-I*cos(1)^2 +
2*cos(1)*sin(1) + I*sin(1)^2 + I)*sin_integral(2/(cosh(b*x + a)^2 + 2*cosh(
b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)))/(b*cos(1) + I*b*sin(1))
```

Sympy [F]

$$\int \cos(\coth(a+bx))\,dx = \int \cos\left(\coth\left(a+bx\right)\right)dx$$

```
[In] integrate(cos(coth(b*x+a)),x)
[Out] Integral(cos(coth(a + b*x)), x)
```

Maxima [F]

$$\int \cos(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(cos(coth(b*x + a)), x)
```

Giac [F]

$$\int \cos(\coth(a+bx))\,dx = \int \cos\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(cos(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(cos(coth(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cos(\coth(a+bx))\,dx = \int \cos\left(\coth(a+b\,x)\right)\,dx$$

```
[In] int(cos(coth(a + b*x)),x)
[Out] int(cos(coth(a + b*x)), x)
```

3.224 $\int \sec(\coth(a+bx))\,dx$

Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \sec(\coth(a+bx))\,dx = \frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{-1+\coth(a+bx)},x\right) - \frac{1}{2}\text{Int}\left(\frac{\text{csch}^2(a+bx)\sec(\coth(a+bx))}{1+\coth(a+bx)},x\right)$$

```
[Out] 1/2*Unintegrable(csch(b*x+a)^2*sec(coth(b*x+a))/(-1+coth(b*x+a)),x)-1/2*Uni
ntegrable(csch(b*x+a)^2*sec(coth(b*x+a))/(1+coth(b*x+a)),x)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec(\coth(a+bx))\,dx = \int \sec(\coth(a+bx))\,dx$$

```
[In] Int[Sec[Coth[a + b*x]],x]
```

```
[Out] -1/2*Defer[Subst][Defer[Int][Sec[x]/(-1 + x), x], x, Coth[a + b*x]]/b + Def
er[Subst][Defer[Int][Sec[x]/(1 + x), x], x, Coth[a + b*x]]/(2*b)
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sec(x)}{1-x^2}\,dx, x, \coth(a+bx)\right)}{b}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{\sec(x)}{2(-1+x)} + \frac{\sec(x)}{2(1+x)}\right)\,dx, x, \coth(a+bx)\right)}{b}$$
$$= -\frac{\text{Subst}\left(\int \frac{\sec(x)}{-1+x}\,dx, x, \coth(a+bx)\right)}{2b} + \frac{\text{Subst}\left(\int \frac{\sec(x)}{1+x}\,dx, x, \coth(a+bx)\right)}{2b}$$

Mathematica [N/A]

Not integrable

Time = 6.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec(\coth(a+bx))\,dx$$

```
[In] Integrate[Sec[Coth[a + b*x]],x]
[Out] Integrate[Sec[Coth[a + b*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \sec\left(\coth\left(bx+a\right)\right)dx$$

```
[In] int(sec(coth(b*x+a)),x)
[Out] int(sec(coth(b*x+a)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="fricas")
[Out] integral(sec(coth(b*x + a)), x)
```

Sympy [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \sec(\coth(a+bx))\,dx = \int \sec\left(\coth\left(a+bx\right)\right)dx$$

```
[In] integrate(sec(coth(b*x+a)),x)
[Out] Integral(sec(coth(a + b*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="maxima")
[Out] integrate(sec(coth(b*x + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \sec(\coth(a+bx))\,dx = \int \sec\left(\coth\left(bx+a\right)\right)\,dx$$

```
[In] integrate(sec(coth(b*x+a)),x, algorithm="giac")
[Out] integrate(sec(coth(b*x + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \sec(\coth(a+bx))\,dx = \int \frac{1}{\cos\left(\coth\left(a+b\,x\right)\right)}\,dx$$

```
[In] int(1/cos(coth(a + b*x)),x)
[Out] int(1/cos(coth(a + b*x)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                   end if
               else #result contains complex but optimal is not
                   if debug then
                           print("result contains complex but optimal is not");
                   fi;
                   return "C","Result contains complex when optimal does not.";
               fi;
           else # result do not contain complex
                # this assumes optimal do not as well. No check is needed here.
               if debug then
                      print("result do not contain complex, this assumes optimal do not as well")
               fi;
               if leaf_count_result<=2*leaf_count_optimal then
                   if debug then
                       print("leaf_count_result<=2*leaf_count_optimal");
                   fi;
                   return "A"," ";
               else
                   if debug then
                       print("leaf_count_result>2*leaf_count_optimal");
                   fi;
                   return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
               fi;
           fi;
       else #ExpnType(result) > ExpnType(optimal)
           if debug then
               print("ExpnType(result) > ExpnType(optimal)");
           fi;
           return "C",cat("Result contains higher order function than in optimal. Order ",
                         convert(ExpnType_result,string)," vs. order ",
                         convert(ExpnType_optimal,string),".");
       fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```python
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.a
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```