Computer Algebra
Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.5-Hyperbolic-secant/179-6.5.3-Hyperbolic-secant-functions

Nasser M. Abbasi

September 5, 2023   Compiled on September 5, 2023 at 3:00pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [201]. This is test number [179].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (201)	0.00 (0)
Mathematica	95.52 (192)	4.48 (9)
Fricas	91.04 (183)	8.96 (18)
Maple	69.65 (140)	30.35 (61)
Giac	57.71 (116)	42.29 (85)
Mupad	46.77 (94)	53.23 (107)
Maxima	44.78 (90)	55.22 (111)
Sympy	6.97 (14)	93.03 (187)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

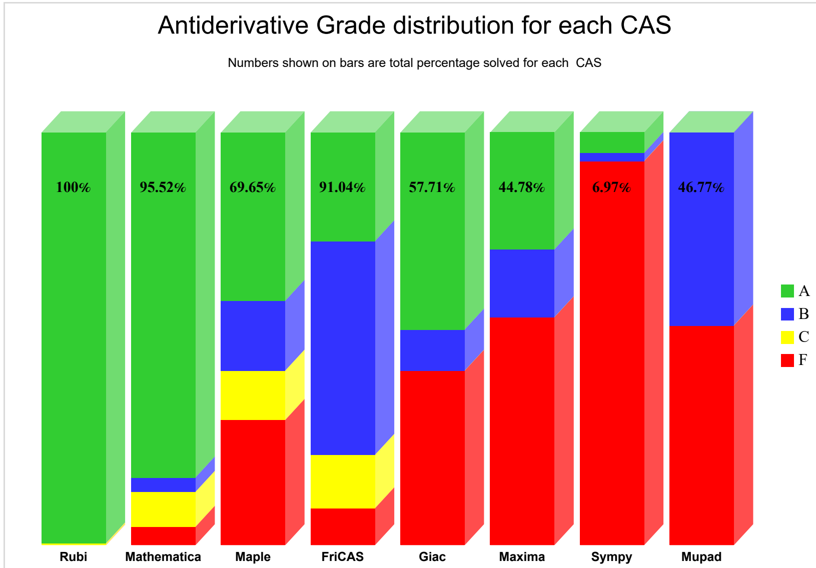
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

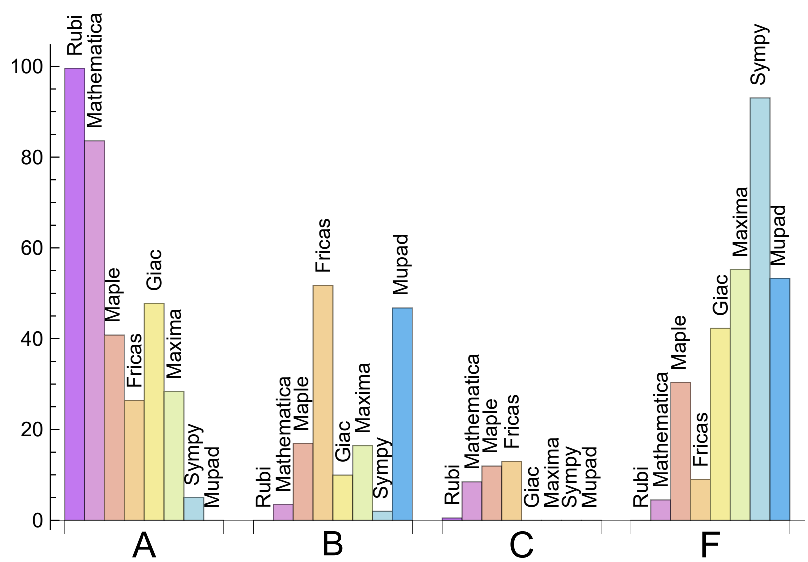
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.502	0.000	0.498	0.000
Mathematica	83.582	3.483	8.458	4.478
Giac	47.761	9.950	0.000	42.289
Maple	40.796	16.915	11.940	30.348
Maxima	28.358	16.418	0.000	55.224
Fricas	26.368	51.741	12.935	8.955
Sympy	4.975	1.990	0.000	93.035
Mupad	0.000	46.766	0.000	53.234

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	9	88.89	11.11	0.00
Fricas	18	88.89	11.11	0.00
Maple	61	100.00	0.00	0.00
Giac	85	67.06	29.41	3.53
Mupad	107	0.00	100.00	0.00
Maxima	111	81.98	0.00	18.02
Sympy	187	95.72	4.28	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.08
Maxima	0.24
Giac	0.29
Fricas	0.34
Mathematica	0.56
Mupad	2.07
Maple	8.11
Sympy	8.88

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	73.35	1.05	58.00	0.98
Giac	76.62	1.31	56.00	1.21
Sympy	78.71	1.43	42.50	1.19
Maxima	88.78	1.71	62.00	1.56
Rubi	93.02	1.01	66.00	1.00
Maple	102.94	1.70	77.50	1.24
Mupad	154.05	2.38	75.50	2.14
Fricas	1233.08	9.78	231.00	4.99

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

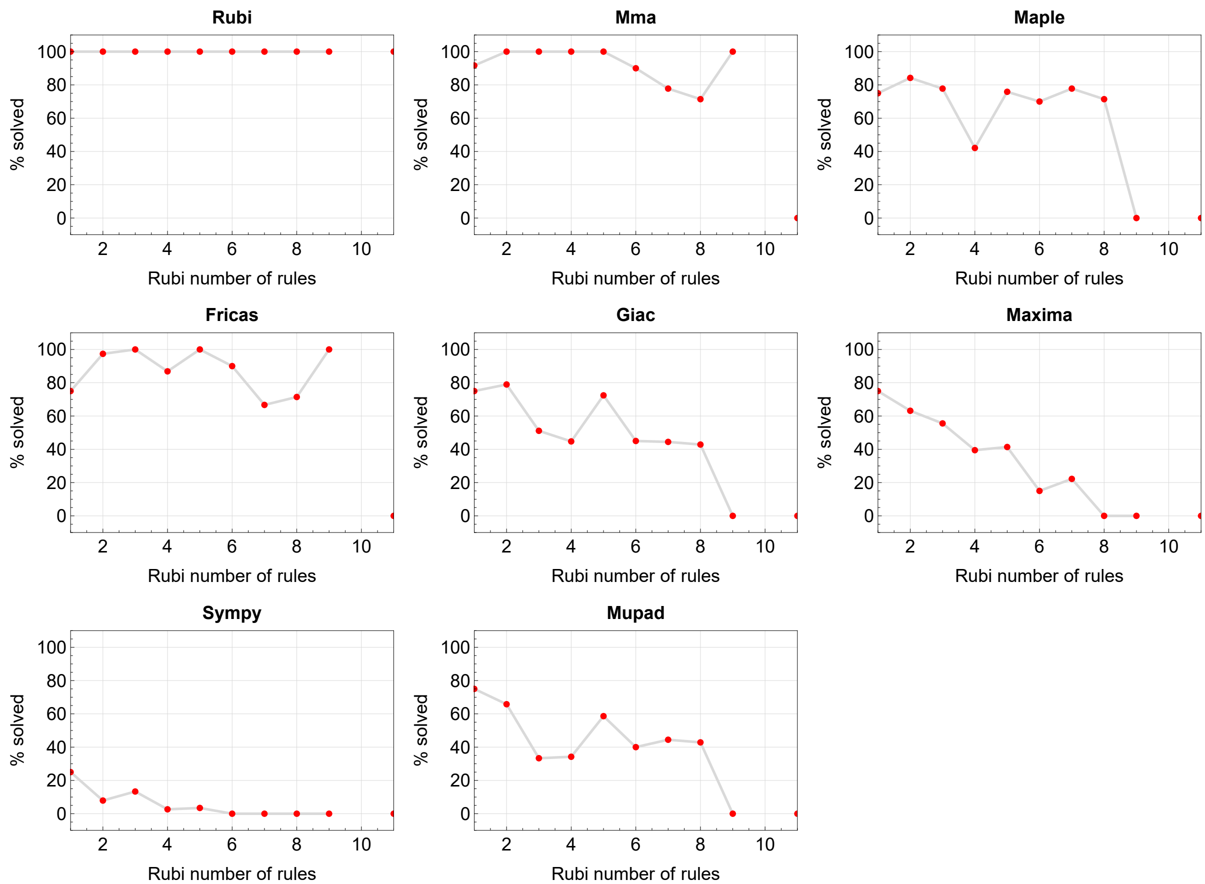


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

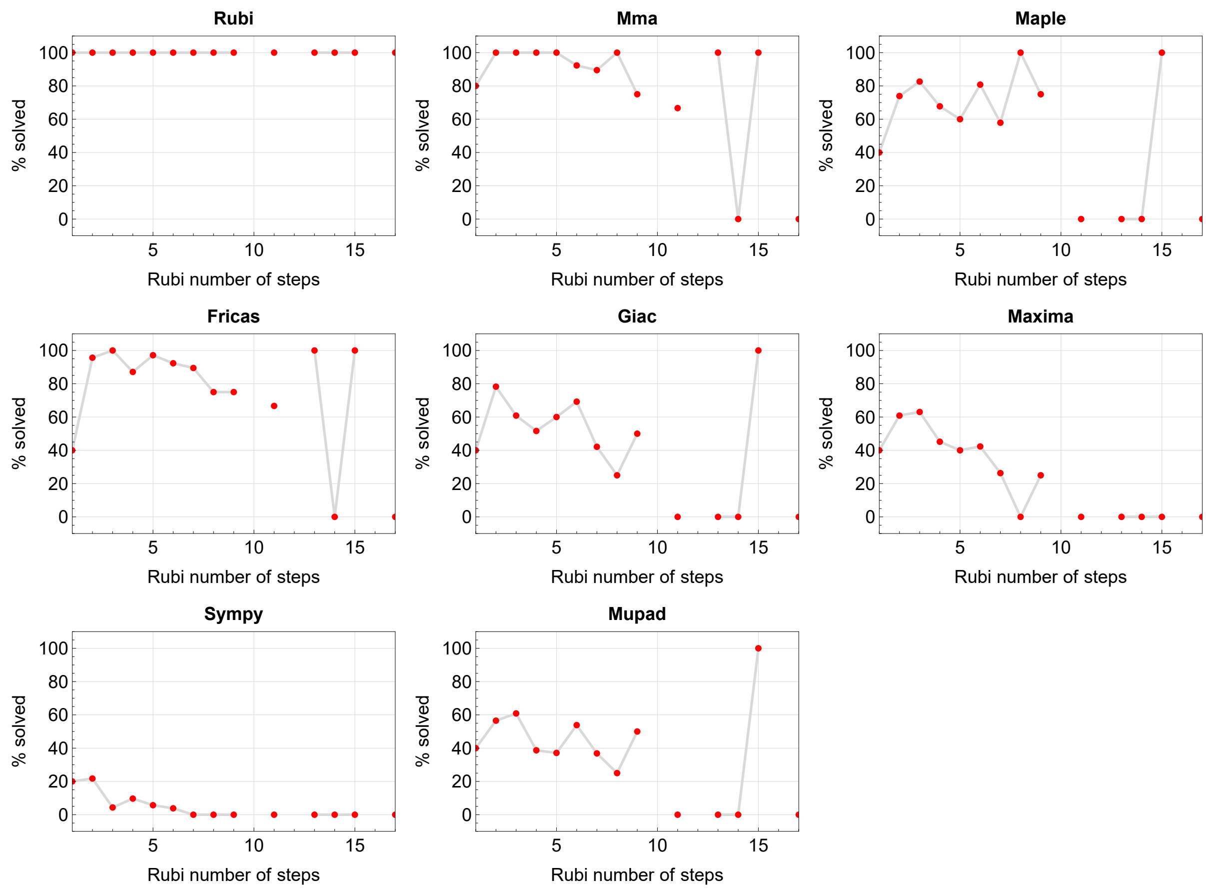


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

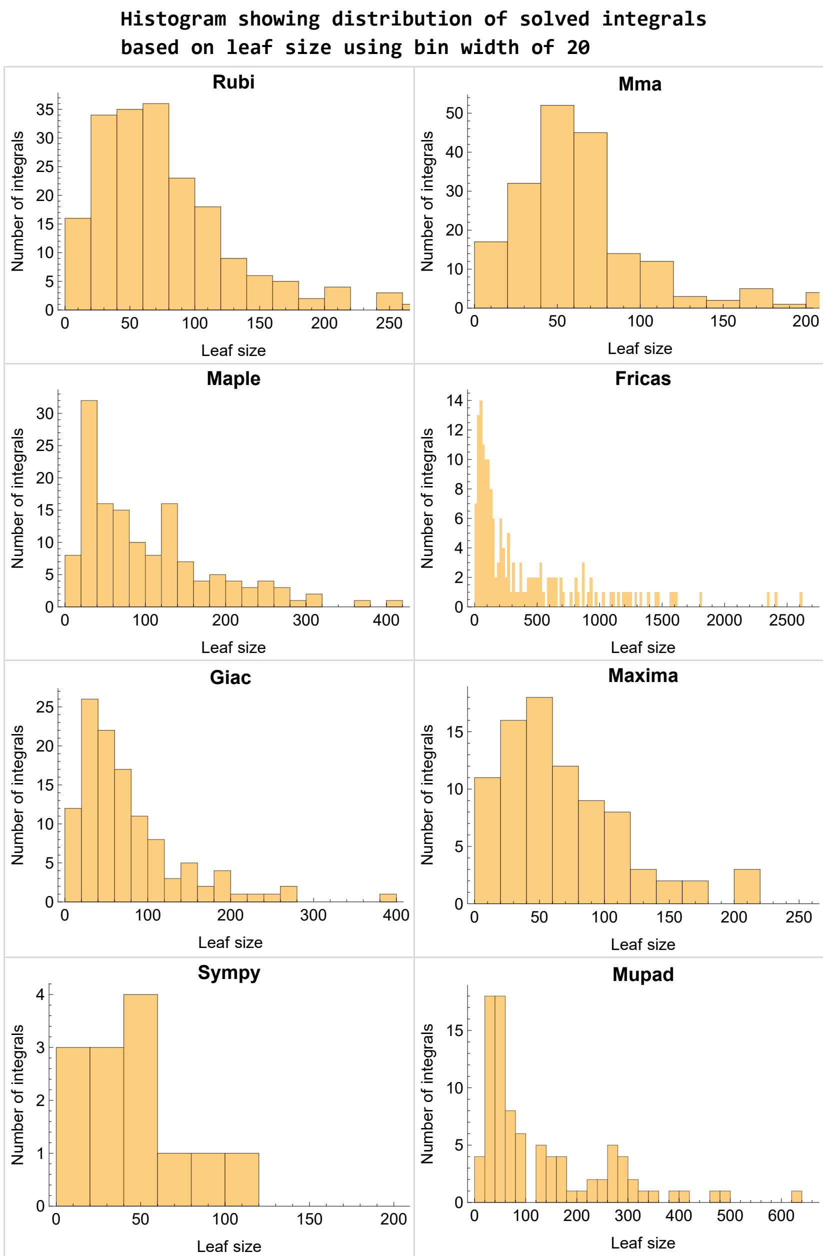


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

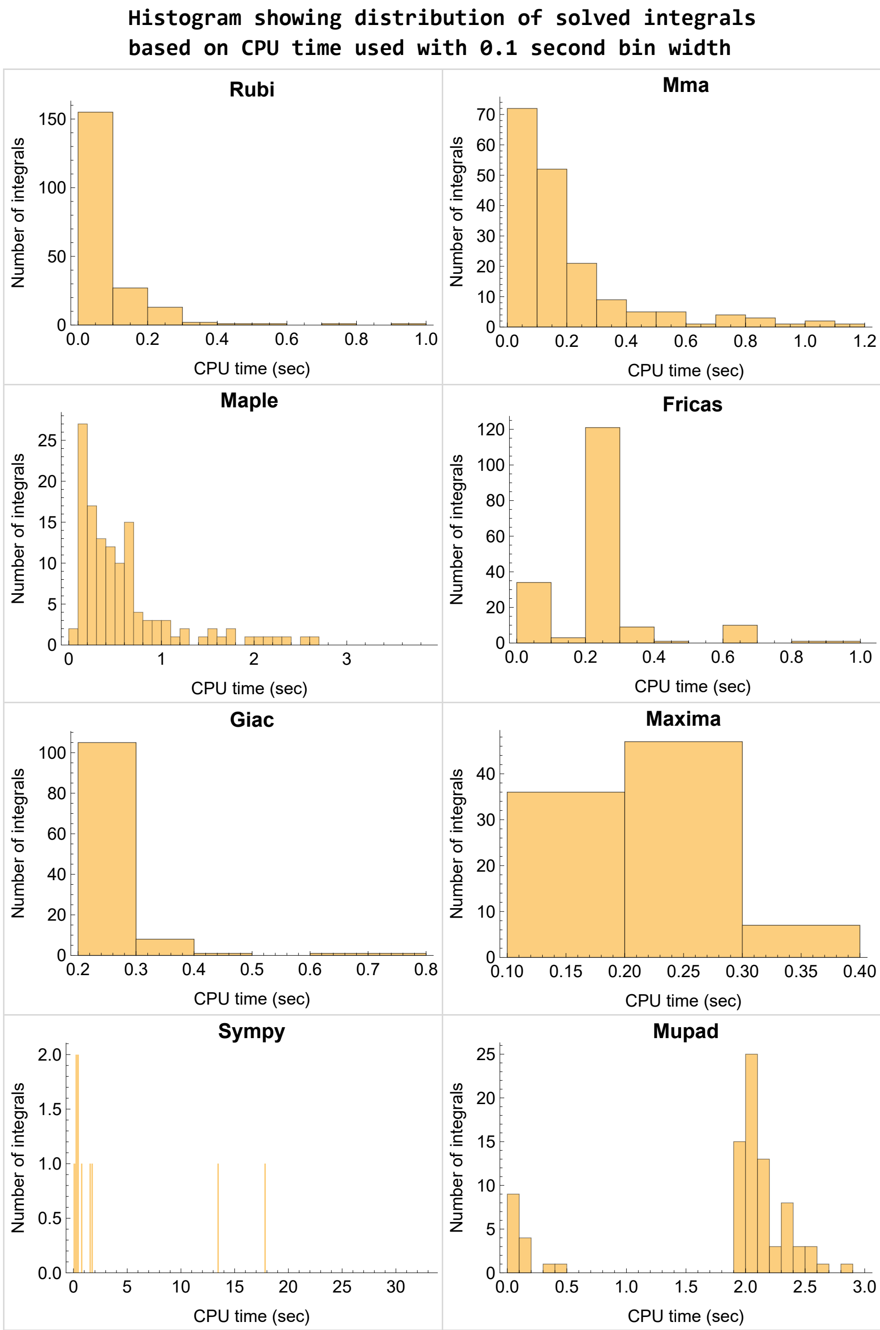


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

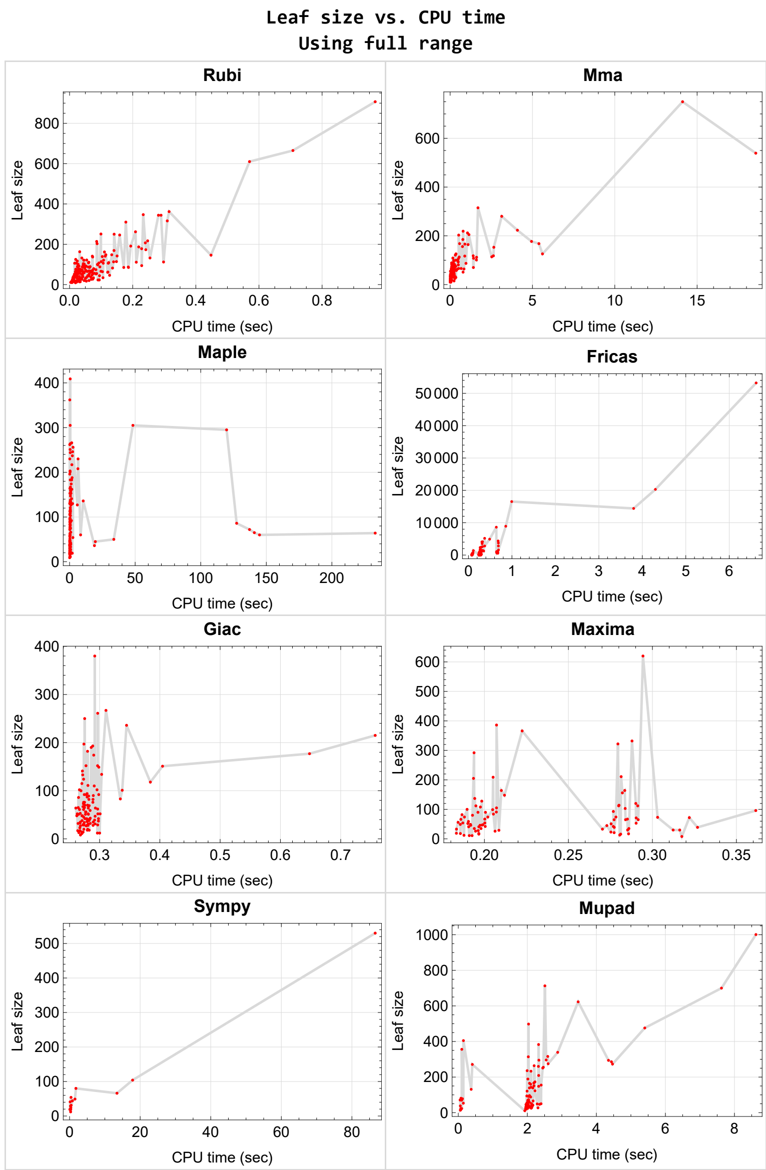


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {190}

Maple {151, 152, 153, 154, 155, 156, 157}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
  x, aa = expr.operator(), expr.operands()
  if x is None:
     return 1
  else:
     return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
 # 1.7 is a fudge factor since it is low side from actual leaf count
 leafCount = round(1.7*count_ops(anti))

 except Exception as ee:
      leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

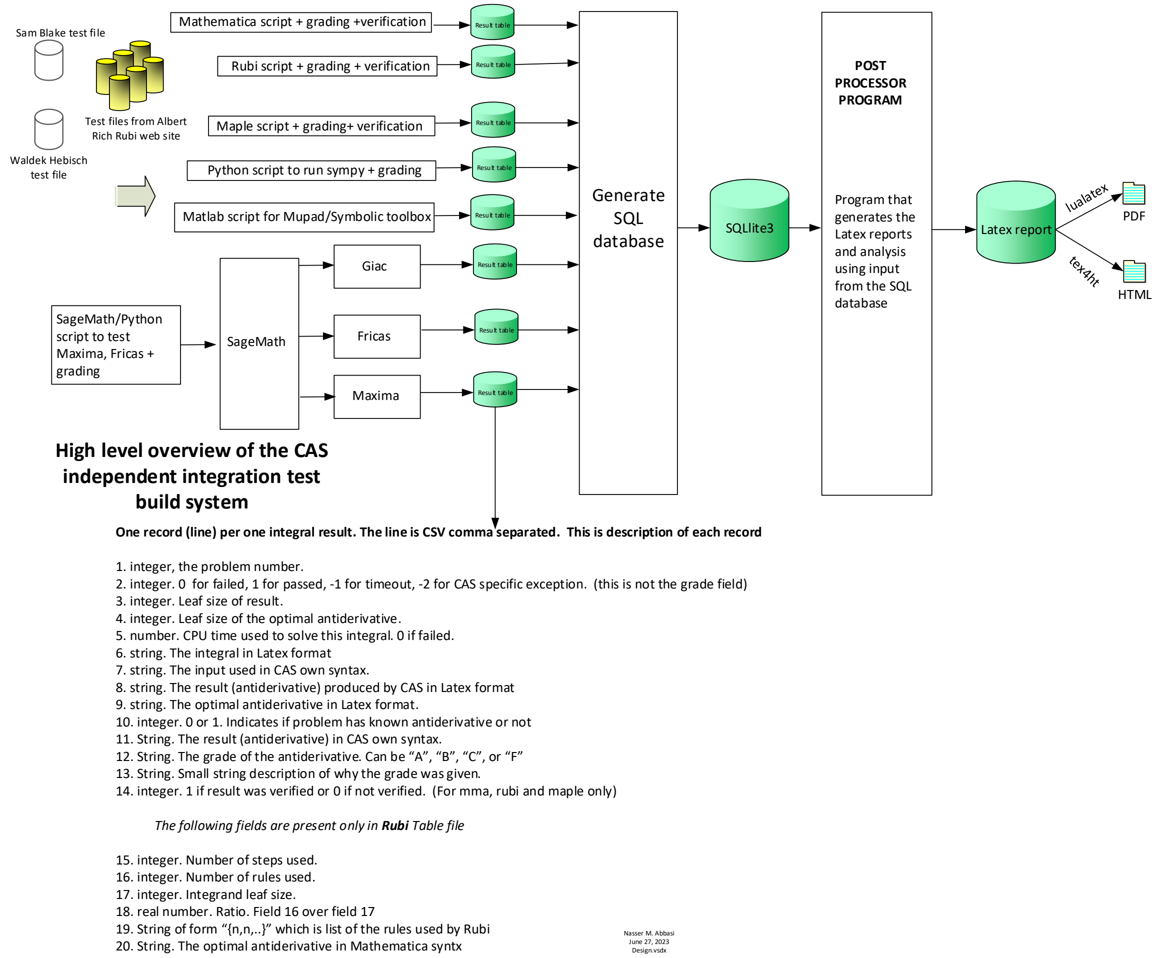
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade { }

C grade { 186 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 140, 151, 152, 153, 154, 155, 156, 157, 159, 161, 163, 164, 165, 167, 169, 171, 173, 175, 178, 179, 182, 183, 184, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

B grade { 27, 85, 86, 132, 185, 187, 188 }

C grade { 142, 143, 144, 145, 146, 158, 160, 162, 166, 168, 170, 172, 174, 176, 177, 180, 181 }

F normal fail { 130, 131, 139, 141, 147, 148, 149, 150 }

F(-1) timedout fail { 138 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 45, 46, 47, 52, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 93, 98, 99, 100, 101, 102, 103, 104, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 124, 127, 135, 144, 159, 161, 167, 169, 171, 173, 177, 178, 186, 191, 192, 193, 194, 195, 197, 200 }

B grade { 9, 11, 12, 13, 14, 19, 28, 29, 30, 31, 35, 36, 37, 38, 48, 49, 50, 51, 53, 60, 61, 95, 96, 97, 105, 106, 113, 115, 117, 164, 196, 198, 199, 201 }

C grade { 24, 25, 26, 27, 32, 33, 34, 107, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 166, 168, 170, 172, 174, 176 }

F normal fail { 15, 16, 17, 18, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 163, 165, 175, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 27, 28, 29, 30, 31, 34, 52, 53, 54, 64, 70, 71, 72, 73, 76, 77, 90, 91, 99, 100, 107, 108, 110, 118, 119, 120, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 177, 178, 179, 180, 181, 191 }

B grade { 2, 3, 4, 5, 6, 7, 8, 24, 25, 26, 32, 33, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 95, 96, 97, 98, 101, 102, 103, 104, 105, 106, 109, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 151, 152, 153, 163, 171, 186, 187, 188, 189, 190, 192, 193, 194, 195 }

C grade { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 39, 40, 41, 42, 43, 44, 196, 197, 198, 199, 200, 201 }

F normal fail { 23, 94, 130, 131, 138, 139, 140, 141, 147, 148, 149, 176, 182, 183, 184, 185 }

F(-1) timedout fail { 132, 150 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 48, 49, 50, 51, 52, 54, 56, 64, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 88, 89, 90, 107, 108, 109, 110, 111, 117, 119, 120, 122, 152, 153, 154, 155, 156, 157, 159, 171, 179, 191, 192 }

B grade { 3, 4, 5, 6, 7, 8, 24, 25, 45, 46, 47, 53, 55, 57, 58, 59, 61, 63, 87, 103, 104, 105, 106, 112, 113, 115, 124, 151, 167, 186, 187, 188, 194 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 78, 79, 80, 81, 82, 83, 84, 85, 86, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 193, 195, 196, 197, 198, 199, 200, 201 }

F(-1) timedout fail { }

F(-2) exception fail { 60, 62, 65, 67, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 114, 116, 118, 121, 123 }

Giac

A grade { 1, 2, 4, 6, 7, 8, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 123, 151, 152, 153, 154, 155, 156, 157, 187, 188, 191, 192, 194, 195 }

B grade { 3, 5, 26, 58, 59, 66, 79, 80, 81, 82, 83, 85, 86, 106, 113, 115, 117, 124, 186, 193 }

C grade { }

F normal fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 39, 40, 41, 42, 43, 44, 94, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 158, 159, 161, 182, 183, 184, 185, 189, 190 }

F(-1) timedout fail { 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 196, 197, 198, 199, 200, 201 }

F(-2) exception fail { 84, 160, 162 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 28, 35, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 135, 144, 151, 152, 153, 159, 167, 171, 179, 186, 187, 188, 191, 192, 193, 194, 195 }

C grade { }

F normal fail { }

F(-1) timedout fail { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 49, 50, 51, 78, 79, 80, 81, 82, 83, 84, 85, 86, 93, 94, 125, 126, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 189, 190, 196, 197, 198, 199, 200, 201 }

F(-2) exception fail { }

Sympy

A grade { 28, 29, 30, 31, 35, 36, 37, 38, 90, 191 }

B grade { 1, 108, 119, 157 }

C grade { }

F normal fail { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 32, 33, 34, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 197, 198, 199, 200 }

F(-1) timedout fail { 15, 24, 45, 151, 152, 153, 196, 201 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	19	19	12	23
N.S.	1	1.00	1.00	1.09	1.00	1.73	1.73	1.09	2.09
time (sec)	N/A	0.003	0.005	0.248	0.191	0.253	0.333	0.300	0.075

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	18	41	0	18	18
N.S.	1	1.00	1.00	1.10	1.80	4.10	0.00	1.80	1.80
time (sec)	N/A	0.007	0.007	0.579	0.186	0.251	0.000	0.280	0.075

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	65	267	0	76	81
N.S.	1	1.00	1.00	0.79	1.91	7.85	0.00	2.24	2.38
time (sec)	N/A	0.014	0.015	0.670	0.284	0.258	0.000	0.289	0.078

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	90	164	0	31	31
N.S.	1	1.00	1.00	0.88	3.46	6.31	0.00	1.19	1.19
time (sec)	N/A	0.009	0.009	0.627	0.196	0.245	0.000	0.281	2.031

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	41	112	812	0	102	189
N.S.	1	1.00	1.00	0.75	2.04	14.76	0.00	1.85	3.44
time (sec)	N/A	0.023	0.017	0.839	0.291	0.243	0.000	0.267	2.016

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	205	344	0	42	42
N.S.	1	1.00	1.00	0.80	5.00	8.39	0.00	1.02	1.02
time (sec)	N/A	0.013	0.015	0.700	0.193	0.256	0.000	0.279	2.024

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	49	116	0	18	30
N.S.	1	1.00	1.00	0.89	2.58	6.11	0.00	0.95	1.58
time (sec)	N/A	0.008	0.007	0.637	0.192	0.255	0.000	0.283	1.999

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	27	137	280	0	30	30
N.S.	1	1.00	1.00	0.77	3.91	8.00	0.00	0.86	0.86
time (sec)	N/A	0.011	0.008	0.766	0.194	0.270	0.000	0.273	2.072

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	217	0	190	0	0	0
N.S.	1	1.00	0.77	3.29	0.00	2.88	0.00	0.00	0.00
time (sec)	N/A	0.026	0.103	1.743	0.000	0.081	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	49	103	0	96	0	0	0
N.S.	1	1.00	0.79	1.66	0.00	1.55	0.00	0.00	0.00
time (sec)	N/A	0.025	0.058	0.642	0.000	0.077	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	24	0	0	0
N.S.	1	1.00	1.00	3.38	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.016	0.042	0.469	0.000	0.080	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	135	0	150	0	0	0
N.S.	1	1.00	1.00	3.38	0.00	3.75	0.00	0.00	0.00
time (sec)	N/A	0.016	0.050	0.794	0.000	0.083	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	53	174	0	223	0	0	0
N.S.	1	1.00	0.80	2.64	0.00	3.38	0.00	0.00	0.00
time (sec)	N/A	0.026	0.060	1.229	0.000	0.083	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	59	188	0	370	0	0	0
N.S.	1	1.00	0.89	2.85	0.00	5.61	0.00	0.00	0.00
time (sec)	N/A	0.025	0.085	1.915	0.000	0.081	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	102	102	68	0	0	478	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.046	0.252	0.000	0.000	0.087	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	56	0	0	215	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	2.91	0.00	0.00	0.00
time (sec)	N/A	0.028	0.097	0.000	0.000	0.080	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	52	0	0	107	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.032	0.063	0.000	0.000	0.083	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	27	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.017	0.041	0.000	0.000	0.072	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	244	0	154	0	0	0
N.S.	1	1.00	1.00	5.81	0.00	3.67	0.00	0.00	0.00
time (sec)	N/A	0.017	0.050	0.514	0.000	0.086	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	231	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	3.04	0.00	0.00	0.00
time (sec)	N/A	0.029	0.092	0.000	0.000	0.087	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	64	0	0	379	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	4.99	0.00	0.00	0.00
time (sec)	N/A	0.029	0.112	0.000	0.000	0.089	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	70	0	0	483	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	4.64	0.00	0.00	0.00
time (sec)	N/A	0.043	0.165	0.000	0.000	0.103	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	60	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.029	0.076	0.000	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	81	230	156	1604	0	124	0
N.S.	1	1.00	0.90	2.56	1.73	17.82	0.00	1.38	0.00
time (sec)	N/A	0.025	0.100	6.479	0.282	0.267	0.000	0.273	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	208	112	812	0	102	0
N.S.	1	1.00	1.00	3.20	1.72	12.49	0.00	1.57	0.00
time (sec)	N/A	0.018	0.070	6.337	0.280	0.261	0.000	0.296	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	46	183	65	267	0	76	0
N.S.	1	1.00	1.15	4.58	1.62	6.68	0.00	1.90	0.00
time (sec)	N/A	0.014	0.046	0.515	0.277	0.274	0.000	0.273	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	29	130	11	19	0	12	0
N.S.	1	1.00	2.64	11.82	1.00	1.73	0.00	1.09	0.00
time (sec)	N/A	0.009	0.022	0.492	0.193	0.247	0.000	0.296	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	97	26	10	29	23	53
N.S.	1	1.00	1.00	4.41	1.18	0.45	1.32	1.05	2.41
time (sec)	N/A	0.011	0.066	0.464	0.195	0.255	0.255	0.267	0.146

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	201	54	32	54	48	0
N.S.	1	1.00	0.86	3.94	1.06	0.63	1.06	0.94	0.00
time (sec)	N/A	0.015	0.082	0.456	0.187	0.254	0.407	0.262	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	47	305	82	66	80	70	0
N.S.	1	1.00	0.62	4.01	1.08	0.87	1.05	0.92	0.00
time (sec)	N/A	0.019	0.093	0.470	0.187	0.268	1.795	0.280	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	57	409	100	108	104	92	0
N.S.	1	1.00	0.56	4.05	0.99	1.07	1.03	0.91	0.00
time (sec)	N/A	0.027	0.148	0.484	0.190	0.259	17.860	0.299	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	38	127	72	1082	0	65	0
N.S.	1	1.00	0.58	1.95	1.11	16.65	0.00	1.00	0.00
time (sec)	N/A	0.026	0.042	5.907	0.322	0.280	0.000	0.275	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	24	106	39	310	0	48	0
N.S.	1	1.00	0.52	2.30	0.85	6.74	0.00	1.04	0.00
time (sec)	N/A	0.019	0.037	0.137	0.327	0.270	0.000	0.271	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	72	8	145	0	8	0
N.S.	1	1.00	0.64	2.88	0.32	5.80	0.00	0.32	0.00
time (sec)	N/A	0.013	0.008	0.178	0.318	0.273	0.000	0.268	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	58	17	79	12	14	33
N.S.	1	1.00	1.00	4.46	1.31	6.08	0.92	1.08	2.54
time (sec)	N/A	0.023	0.032	0.160	0.286	0.251	0.300	0.273	1.959

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	25	130	35	277	31	29	0
N.S.	1	1.00	0.69	3.61	0.97	7.69	0.86	0.81	0.00
time (sec)	N/A	0.015	0.021	0.134	0.286	0.256	0.455	0.297	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	33	196	53	580	49	41	0
N.S.	1	1.00	0.60	3.56	0.96	10.55	0.89	0.75	0.00
time (sec)	N/A	0.022	0.034	0.127	0.290	0.260	1.533	0.284	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	39	262	71	970	66	53	0
N.S.	1	1.00	0.53	3.54	0.96	13.11	0.89	0.72	0.00
time (sec)	N/A	0.032	0.046	0.136	0.290	0.284	13.430	0.268	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	1382	0	0	0
N.S.	1	1.00	0.52	0.00	0.00	11.42	0.00	0.00	0.00
time (sec)	N/A	0.045	0.135	0.000	0.000	0.113	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	47	0	0	391	0	0	0
N.S.	1	1.00	0.68	0.00	0.00	5.67	0.00	0.00	0.00
time (sec)	N/A	0.033	0.060	0.000	0.000	0.090	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	46	46	36	0	0	60	0	0	0
N.S.	1	1.00	0.78	0.00	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.022	0.042	0.000	0.000	0.076	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	48	38	0	0	126	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	2.62	0.00	0.00	0.00
time (sec)	N/A	0.023	0.066	0.000	0.000	0.081	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	47	0	0	407	0	0	0
N.S.	1	1.00	0.61	0.00	0.00	5.29	0.00	0.00	0.00
time (sec)	N/A	0.032	0.123	0.000	0.000	0.085	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	63	0	0	718	0	0	0
N.S.	1	1.00	0.52	0.00	0.00	5.93	0.00	0.00	0.00
time (sec)	N/A	0.045	0.122	0.000	0.000	0.105	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	54	72	620	2804	0	51	498
N.S.	1	1.00	0.33	0.44	3.80	17.20	0.00	0.31	3.06
time (sec)	N/A	0.032	0.231	137.222	0.294	0.373	0.000	0.276	2.035

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	42	60	322	1475	0	39	356
N.S.	1	1.00	0.36	0.51	2.75	12.61	0.00	0.33	3.04
time (sec)	N/A	0.025	0.126	144.832	0.279	0.280	0.000	0.292	0.101

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	30	46	120	516	0	27	46
N.S.	1	1.00	0.49	0.75	1.97	8.46	0.00	0.44	0.75
time (sec)	N/A	0.018	0.069	0.169	0.290	0.258	0.000	0.278	1.961

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	13	81	0	13	71
N.S.	1	1.00	1.00	1.93	0.87	5.40	0.00	0.87	4.73
time (sec)	N/A	0.014	0.009	0.188	0.281	0.240	0.000	0.266	0.052

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	23	89	30	253	0	28	0
N.S.	1	1.00	0.64	2.47	0.83	7.03	0.00	0.78	0.00
time (sec)	N/A	0.013	0.049	0.165	0.285	0.259	0.000	0.289	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	38	230	65	1141	0	52	0
N.S.	1	1.00	0.44	2.67	0.76	13.27	0.00	0.60	0.00
time (sec)	N/A	0.025	0.067	0.175	0.292	0.261	0.000	0.263	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	55	362	103	2600	0	76	0
N.S.	1	1.00	0.42	2.74	0.78	19.70	0.00	0.58	0.00
time (sec)	N/A	0.039	0.107	0.155	0.284	0.321	0.000	0.275	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	28	60	54	36	0	42	59
N.S.	1	1.00	0.64	1.36	1.23	0.82	0.00	0.95	1.34
time (sec)	N/A	0.095	0.265	8.395	0.200	0.244	0.000	0.298	2.007

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	54	46	30	0	37	53
N.S.	1	1.00	1.00	2.35	2.00	1.30	0.00	1.61	2.30
time (sec)	N/A	0.090	0.055	2.526	0.197	0.245	0.000	0.280	1.985

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	16	42	42	14	0	28	41
N.S.	1	1.00	0.59	1.56	1.56	0.52	0.00	1.04	1.52
time (sec)	N/A	0.073	0.046	0.825	0.200	0.253	0.000	0.281	1.971

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	23	35	50	0	32	15
N.S.	1	1.00	0.94	1.35	2.06	2.94	0.00	1.88	0.88
time (sec)	N/A	0.053	0.022	0.249	0.198	0.246	0.000	0.285	0.065

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	44	20	48	103	0	52	51
N.S.	1	1.00	1.33	0.61	1.45	3.12	0.00	1.58	1.55
time (sec)	N/A	0.076	0.056	0.362	0.199	0.240	0.000	0.281	2.121

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	90	71	0	31	91
N.S.	1	1.00	1.09	1.00	3.91	3.09	0.00	1.35	3.96
time (sec)	N/A	0.094	0.293	0.330	0.201	0.255	0.000	0.292	2.076

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	59	38	99	630	0	90	121
N.S.	1	1.00	1.28	0.83	2.15	13.70	0.00	1.96	2.63
time (sec)	N/A	0.127	0.185	0.477	0.205	0.249	0.000	0.290	1.993

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	39	292	219	0	59	236
N.S.	1	1.00	1.15	1.15	8.59	6.44	0.00	1.74	6.94
time (sec)	N/A	0.106	0.230	0.892	0.194	0.233	0.000	0.284	1.994

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	219	305	0	1812	0	197	275
N.S.	1	1.00	1.66	2.31	0.00	13.73	0.00	1.49	2.08
time (sec)	N/A	0.255	0.788	48.288	0.000	0.274	0.000	0.274	2.604

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	66	136	128	490	0	87	123
N.S.	1	1.00	1.08	2.23	2.10	8.03	0.00	1.43	2.02
time (sec)	N/A	0.121	0.215	10.408	0.198	0.259	0.000	0.279	2.222

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	130	0	536	0	100	173
N.S.	1	1.00	0.93	1.59	0.00	6.54	0.00	1.22	2.11
time (sec)	N/A	0.136	0.179	2.280	0.000	0.281	0.000	0.269	2.221

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	31	46	78	0	34	20
N.S.	1	1.00	0.95	1.55	2.30	3.90	0.00	1.70	1.00
time (sec)	N/A	0.062	0.012	0.495	0.192	0.250	0.000	0.275	1.970

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	48	59	58	0	65	148
N.S.	1	1.00	0.94	0.91	1.11	1.09	0.00	1.23	2.79
time (sec)	N/A	0.084	0.124	0.305	0.191	0.260	0.000	0.265	2.328

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	75	77	0	452	0	64	151
N.S.	1	1.00	1.14	1.17	0.00	6.85	0.00	0.97	2.29
time (sec)	N/A	0.099	0.340	0.393	0.000	0.259	0.000	0.261	2.192

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	93	82	148	828	0	174	255
N.S.	1	1.00	1.09	0.96	1.74	9.74	0.00	2.05	3.00
time (sec)	N/A	0.172	0.491	0.598	0.212	0.282	0.000	0.290	2.464

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	156	127	0	2340	0	149	295
N.S.	1	1.00	1.41	1.14	0.00	21.08	0.00	1.34	2.66
time (sec)	N/A	0.211	0.748	1.099	0.000	0.269	0.000	0.299	2.335

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	63	43	80	139	0	86	88
N.S.	1	1.00	0.94	0.64	1.19	2.07	0.00	1.28	1.31
time (sec)	N/A	0.066	0.426	0.328	0.193	0.244	0.000	0.265	2.097

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	35	66	100	0	70	70
N.S.	1	1.00	0.98	0.65	1.22	1.85	0.00	1.30	1.30
time (sec)	N/A	0.063	0.069	0.262	0.186	0.249	0.000	0.276	2.015

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	45	30	56	70	0	51	52
N.S.	1	1.00	1.10	0.73	1.37	1.71	0.00	1.24	1.27
time (sec)	N/A	0.057	0.045	0.230	0.184	0.240	0.000	0.265	1.991

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	32	20	41	47	0	35	34
N.S.	1	1.00	1.23	0.77	1.58	1.81	0.00	1.35	1.31
time (sec)	N/A	0.038	0.262	0.185	0.191	0.241	0.000	0.268	1.997

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	9	12	14	0	11	11
N.S.	1	1.00	0.91	0.82	1.09	1.27	0.00	1.00	1.00
time (sec)	N/A	0.020	0.012	0.073	0.188	0.284	0.000	0.271	1.926

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	19	23	29	0	20	31
N.S.	1	1.00	1.15	0.95	1.15	1.45	0.00	1.00	1.55
time (sec)	N/A	0.046	0.042	0.125	0.275	0.271	0.000	0.280	2.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	33	45	127	0	36	58
N.S.	1	1.00	1.15	1.27	1.73	4.88	0.00	1.38	2.23
time (sec)	N/A	0.069	0.095	0.267	0.273	0.255	0.000	0.279	2.042

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	46	73	325	0	48	73
N.S.	1	1.00	0.91	1.02	1.62	7.22	0.00	1.07	1.62
time (sec)	N/A	0.058	0.122	0.311	0.303	0.264	0.000	0.271	2.036

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	58	23	33	48	0	29	24
N.S.	1	1.00	2.00	0.79	1.14	1.66	0.00	1.00	0.83
time (sec)	N/A	0.011	0.374	0.184	0.183	0.270	0.000	0.272	1.949

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	59	25	35	50	0	29	24
N.S.	1	1.00	1.97	0.83	1.17	1.67	0.00	0.97	0.80
time (sec)	N/A	0.013	0.365	0.177	0.194	0.276	0.000	0.274	1.964

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	99	0	0	924	0	151	0
N.S.	1	1.00	1.01	0.00	0.00	9.43	0.00	1.54	0.00
time (sec)	N/A	0.089	0.459	0.000	0.000	0.293	0.000	0.404	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	75	0	0	697	0	118	0
N.S.	1	1.00	1.14	0.00	0.00	10.56	0.00	1.79	0.00
time (sec)	N/A	0.030	0.192	0.000	0.000	0.268	0.000	0.384	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	60	0	0	637	0	83	0
N.S.	1	1.00	1.62	0.00	0.00	17.22	0.00	2.24	0.00
time (sec)	N/A	0.014	0.125	0.000	0.000	0.270	0.000	0.334	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	118	0	0	868	0	177	0
N.S.	1	1.00	1.39	0.00	0.00	10.21	0.00	2.08	0.00
time (sec)	N/A	0.063	1.399	0.000	0.000	0.282	0.000	0.648	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	114	114	177	0	0	1190	0	236	0
N.S.	1	1.00	1.55	0.00	0.00	10.44	0.00	2.07	0.00
time (sec)	N/A	0.100	4.942	0.000	0.000	0.296	0.000	0.345	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	38	38	70	0	0	642	0	101	0
N.S.	1	1.00	1.84	0.00	0.00	16.89	0.00	2.66	0.00
time (sec)	N/A	0.018	1.411	0.000	0.000	0.284	0.000	0.338	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	118	0	0	871	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	10.01	0.00	0.00	0.00
time (sec)	N/A	0.059	2.627	0.000	0.000	0.286	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	39	0	0	233	0	52	0
N.S.	1	1.00	2.05	0.00	0.00	12.26	0.00	2.74	0.00
time (sec)	N/A	0.013	0.045	0.000	0.000	0.247	0.000	0.297	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	51	0	0	235	0	69	0
N.S.	1	1.00	2.43	0.00	0.00	11.19	0.00	3.29	0.00
time (sec)	N/A	0.014	0.785	0.000	0.000	0.279	0.000	0.281	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	78	92	211	1028	0	141	233
N.S.	1	1.00	0.73	0.86	1.97	9.61	0.00	1.32	2.18
time (sec)	N/A	0.093	0.269	1.414	0.281	0.267	0.000	0.272	2.105

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	55	66	114	521	0	92	165
N.S.	1	1.00	0.75	0.90	1.56	7.14	0.00	1.26	2.26
time (sec)	N/A	0.039	0.142	0.906	0.280	0.275	0.000	0.275	2.063

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	34	41	157	0	43	70
N.S.	1	1.00	0.97	1.03	1.24	4.76	0.00	1.30	2.12
time (sec)	N/A	0.023	0.073	0.665	0.197	0.276	0.000	0.284	0.098

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	26	24	17	38
N.S.	1	1.00	1.00	1.06	1.00	1.62	1.50	1.06	2.38
time (sec)	N/A	0.007	0.005	0.124	0.197	0.253	0.362	0.267	0.059

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	60	83	0	270	0	56	131
N.S.	1	1.00	1.02	1.41	0.00	4.58	0.00	0.95	2.22
time (sec)	N/A	0.046	0.200	0.208	0.000	0.290	0.000	0.277	0.373

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	203	164	0	1207	0	134	296
N.S.	1	1.00	1.86	1.50	0.00	11.07	0.00	1.23	2.72
time (sec)	N/A	0.124	0.519	0.279	0.000	0.289	0.000	0.303	2.546

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	205	251	0	4125	0	261	0
N.S.	1	1.00	1.18	1.45	0.00	23.84	0.00	1.51	0.00
time (sec)	N/A	0.241	1.141	0.400	0.000	0.323	0.000	0.297	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	5.392	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	126	264	0	2402	0	182	251
N.S.	1	1.00	0.86	1.81	0.00	16.45	0.00	1.25	1.72
time (sec)	N/A	0.448	0.353	0.567	0.000	0.288	0.000	0.280	2.446

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	99	203	0	1562	0	133	209
N.S.	1	1.00	0.88	1.81	0.00	13.95	0.00	1.19	1.87
time (sec)	N/A	0.298	0.215	0.375	0.000	0.281	0.000	0.272	2.329

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	78	153	0	860	0	92	167
N.S.	1	1.00	0.92	1.80	0.00	10.12	0.00	1.08	1.96
time (sec)	N/A	0.187	0.161	0.269	0.000	0.277	0.000	0.278	2.189

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	94	0	430	0	62	139
N.S.	1	1.00	0.92	1.52	0.00	6.94	0.00	1.00	2.24
time (sec)	N/A	0.068	0.216	0.225	0.000	0.262	0.000	0.297	2.160

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	41	36	0	165	0	32	43
N.S.	1	1.00	0.98	0.86	0.00	3.93	0.00	0.76	1.02
time (sec)	N/A	0.041	0.036	0.087	0.000	0.251	0.000	0.264	1.983

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	219	0	45	286
N.S.	1	1.00	1.00	0.94	0.00	4.06	0.00	0.83	5.30
time (sec)	N/A	0.074	0.105	0.228	0.000	0.297	0.000	0.274	4.437

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	63	73	0	504	0	61	294
N.S.	1	1.00	0.98	1.14	0.00	7.88	0.00	0.95	4.59
time (sec)	N/A	0.104	0.175	0.360	0.000	0.285	0.000	0.267	4.351

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	82	109	0	1444	0	89	476
N.S.	1	1.00	0.94	1.25	0.00	16.60	0.00	1.02	5.47
time (sec)	N/A	0.186	0.275	0.567	0.000	0.327	0.000	0.282	5.402

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	60	75	93	686	0	69	143
N.S.	1	1.00	1.25	1.56	1.94	14.29	0.00	1.44	2.98
time (sec)	N/A	0.071	0.388	0.669	0.277	0.271	0.000	0.283	2.109

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	26	74	437	0	61	96
N.S.	1	1.00	1.06	0.72	2.06	12.14	0.00	1.69	2.67
time (sec)	N/A	0.040	0.081	0.435	0.278	0.270	0.000	0.273	2.062

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	41	59	51	210	0	42	67
N.S.	1	1.00	1.32	1.90	1.65	6.77	0.00	1.35	2.16
time (sec)	N/A	0.050	0.224	0.332	0.275	0.253	0.000	0.276	2.038

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	10	34	33	85	0	35	33
N.S.	1	1.00	0.71	2.43	2.36	6.07	0.00	2.50	2.36
time (sec)	N/A	0.034	0.041	0.234	0.270	0.253	0.000	0.289	2.099

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	15	31	16	14	0	14	25
N.S.	1	1.00	1.07	2.21	1.14	1.00	0.00	1.00	1.79
time (sec)	N/A	0.031	0.236	0.170	0.281	0.272	0.000	0.271	2.113

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	12	17	18	16	19	17	14
N.S.	1	1.00	1.33	1.89	2.00	1.78	2.11	1.89	1.56
time (sec)	N/A	0.019	0.004	0.139	0.197	0.239	0.080	0.264	0.060

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	44	38	52	136	0	56	65
N.S.	1	1.00	1.10	0.95	1.30	3.40	0.00	1.40	1.62
time (sec)	N/A	0.042	0.053	0.161	0.190	0.259	0.000	0.277	2.014

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	33	36	47	46	0	40	94
N.S.	1	1.00	0.87	0.95	1.24	1.21	0.00	1.05	2.47
time (sec)	N/A	0.067	0.307	0.178	0.199	0.254	0.000	0.282	1.985

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	66	56	108	773	0	94	160
N.S.	1	1.00	0.97	0.82	1.59	11.37	0.00	1.38	2.35
time (sec)	N/A	0.061	0.166	0.207	0.198	0.267	0.000	0.279	2.096

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	69	63	105	151	0	64	264
N.S.	1	1.00	1.25	1.15	1.91	2.75	0.00	1.16	4.80
time (sec)	N/A	0.086	0.271	0.271	0.207	0.253	0.000	0.281	2.198

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	121	246	332	4077	0	267	316
N.S.	1	1.00	1.00	2.03	2.74	33.69	0.00	2.21	2.61
time (sec)	N/A	0.108	0.314	2.313	0.288	0.309	0.000	0.311	2.592

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	185	266	0	4914	0	250	1001
N.S.	1	1.00	0.99	1.42	0.00	26.28	0.00	1.34	5.35
time (sec)	N/A	0.219	0.771	1.637	0.000	0.487	0.000	0.275	8.623

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	76	158	164	1280	0	152	155
N.S.	1	1.00	1.06	2.19	2.28	17.78	0.00	2.11	2.15
time (sec)	N/A	0.073	0.197	0.941	0.284	0.287	0.000	0.297	2.401

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	113	152	0	1254	0	111	700
N.S.	1	1.00	1.20	1.62	0.00	13.34	0.00	1.18	7.45
time (sec)	N/A	0.228	0.551	0.732	0.000	0.350	0.000	0.281	7.625

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	75	67	200	0	73	260
N.S.	1	1.00	1.09	2.14	1.91	5.71	0.00	2.09	7.43
time (sec)	N/A	0.053	0.109	0.390	0.285	0.290	0.000	0.273	2.321

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	84	0	193	0	52	273
N.S.	1	1.00	1.00	1.35	0.00	3.11	0.00	0.84	4.40
time (sec)	N/A	0.122	0.132	0.299	0.000	0.295	0.000	0.301	4.468

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	11	21	26	27	41	19	23
N.S.	1	1.00	0.58	1.11	1.37	1.42	2.16	1.00	1.21
time (sec)	N/A	0.024	0.050	0.185	0.206	0.273	0.162	0.280	0.099

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	63	78	67	81	0	67	271
N.S.	1	1.00	0.95	1.18	1.02	1.23	0.00	1.02	4.11
time (sec)	N/A	0.072	0.182	0.331	0.200	0.280	0.000	0.272	0.407

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	81	104	0	646	0	82	383
N.S.	1	1.00	0.71	0.91	0.00	5.67	0.00	0.72	3.36
time (sec)	N/A	0.149	0.496	0.457	0.000	0.271	0.000	0.289	2.326

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	113	164	1222	0	193	339
N.S.	1	1.00	0.95	1.00	1.45	10.81	0.00	1.71	3.00
time (sec)	N/A	0.139	0.506	0.694	0.210	0.301	0.000	0.289	2.879

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	166	153	0	3530	0	190	713
N.S.	1	1.00	0.80	0.74	0.00	17.05	0.00	0.92	3.44
time (sec)	N/A	0.240	0.893	1.082	0.000	0.315	0.000	0.286	2.509

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	164	162	366	5181	0	380	623
N.S.	1	1.00	0.92	0.91	2.06	29.11	0.00	2.13	3.50
time (sec)	N/A	0.228	1.089	1.702	0.222	0.372	0.000	0.292	3.472

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	169	169	153	0	0	4363	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	25.82	0.00	0.00	0.00
time (sec)	N/A	0.141	2.652	0.000	0.000	0.683	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	87	0	0	1589	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	15.89	0.00	0.00	0.00
time (sec)	N/A	0.088	0.961	0.000	0.000	0.691	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	50	43	0	605	0	0	47
N.S.	1	1.00	0.98	0.84	0.00	11.86	0.00	0.00	0.92
time (sec)	N/A	0.038	0.233	0.175	0.000	0.653	0.000	0.000	2.357

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	100	0	0	8620	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	81.32	0.00	0.00	0.00
time (sec)	N/A	0.122	0.348	0.000	0.000	0.641	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	217	217	223	0	0	16532	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	76.18	0.00	0.00	0.00
time (sec)	N/A	0.247	4.086	0.000	0.000	0.995	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.282	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	246	246	539	0	0	0	0	0	0
N.S.	1	1.00	2.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.159	18.564	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	112	0	0	2813	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	19.01	0.00	0.00	0.00
time (sec)	N/A	0.115	1.605	0.000	0.000	0.688	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	66	0	0	925	0	0	0
N.S.	1	1.00	0.84	0.00	0.00	11.71	0.00	0.00	0.00
time (sec)	N/A	0.077	0.667	0.000	0.000	0.675	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	0	558	0	0	27
N.S.	1	1.00	1.00	0.84	0.00	18.00	0.00	0.00	0.87
time (sec)	N/A	0.034	0.181	0.208	0.000	0.676	0.000	0.000	2.309

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	100	0	0	8908	0	0	0
N.S.	1	1.00	0.94	0.00	0.00	84.04	0.00	0.00	0.00
time (sec)	N/A	0.107	0.441	0.000	0.000	0.858	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	262	262	280	0	0	20300	0	0	0
N.S.	1	1.00	1.07	0.00	0.00	77.48	0.00	0.00	0.00
time (sec)	N/A	0.209	3.132	0.000	0.000	4.301	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	610	610	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.570	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	310	310	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	106	106	168	0	0	0	0	0	0
N.S.	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.016	0.574	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	117	0	0	3745	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	25.30	0.00	0.00	0.00
time (sec)	N/A	0.134	0.887	0.000	0.000	0.688	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	66	0	0	1107	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	12.58	0.00	0.00	0.00
time (sec)	N/A	0.096	0.514	0.000	0.000	0.649	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	43	46	0	917	0	0	50
N.S.	1	1.00	0.80	0.85	0.00	16.98	0.00	0.00	0.93
time (sec)	N/A	0.042	0.251	0.169	0.000	0.644	0.000	0.000	2.398

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	142	142	212	0	0	14412	0	0	0
N.S.	1	1.00	1.49	0.00	0.00	101.49	0.00	0.00	0.00
time (sec)	N/A	0.150	1.053	0.000	0.000	3.800	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	316	316	315	0	0	53212	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	168.39	0.00	0.00	0.00
time (sec)	N/A	0.310	1.690	0.000	0.000	6.619	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	907	907	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.968	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	344	344	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.290	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	347	347	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	665	665	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.708	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	191	191	84	86	386	589	0	64	405
N.S.	1	1.00	0.44	0.45	2.02	3.08	0.00	0.34	2.12
time (sec)	N/A	0.194	0.105	127.386	0.207	0.290	0.000	0.293	0.149

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	72	65	209	315	0	51	91
N.S.	1	1.00	0.51	0.46	1.48	2.23	0.00	0.36	0.65
time (sec)	N/A	0.113	0.086	140.894	0.205	0.272	0.000	0.281	2.033

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	56	56	44	38	84	120	0	38	78
N.S.	1	1.00	0.79	0.68	1.50	2.14	0.00	0.68	1.39
time (sec)	N/A	0.079	0.073	0.358	0.205	0.254	0.000	0.271	0.121

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	44	44	42	29	21	42	0	20	0
N.S.	1	1.00	0.95	0.66	0.48	0.95	0.00	0.45	0.00
time (sec)	N/A	0.061	0.049	0.378	0.277	0.257	0.000	0.274	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	74	48	60	29	66	0	33	0
N.S.	1	1.00	0.65	0.81	0.39	0.89	0.00	0.45	0.00
time (sec)	N/A	0.083	0.064	0.428	0.209	0.270	0.000	0.268	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	162	162	81	75	74	126	0	82	0
N.S.	1	1.00	0.50	0.46	0.46	0.78	0.00	0.51	0.00
time (sec)	N/A	0.110	0.121	0.636	0.202	0.255	0.000	0.282	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	250	250	109	88	112	218	530	110	0
N.S.	1	1.00	0.44	0.35	0.45	0.87	2.12	0.44	0.00
time (sec)	N/A	0.141	0.164	0.672	0.195	0.262	86.597	0.291	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	77	130	0	81	0	0	0
N.S.	1	1.00	0.71	1.20	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.060	0.217	0.917	0.000	0.079	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	44	39	30	48	0	0	42
N.S.	1	1.00	1.57	1.39	1.07	1.71	0.00	0.00	1.50
time (sec)	N/A	0.028	0.060	0.161	0.312	0.255	0.000	0.000	2.125

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	65	134	0	120	0	0	0
N.S.	1	1.00	0.32	0.66	0.00	0.59	0.00	0.00	0.00
time (sec)	N/A	0.087	0.143	0.630	0.000	0.081	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	77	97	0	90	0	0	0
N.S.	1	1.00	1.15	1.45	0.00	1.34	0.00	0.00	0.00
time (sec)	N/A	0.038	0.166	0.190	0.000	0.266	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	58	114	0	71	0	0	0
N.S.	1	1.00	0.67	1.31	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.040	0.187	0.569	0.000	0.079	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	59	59	75	0	0	100	0	0	0
N.S.	1	1.00	1.27	0.00	0.00	1.69	0.00	0.00	0.00
time (sec)	N/A	0.028	0.138	0.000	0.000	0.265	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	167	0	27	0	0	0
N.S.	1	1.00	1.00	4.64	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.020	0.106	0.673	0.000	0.081	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	55	0	0	57	0	0	0
N.S.	1	1.00	1.38	0.00	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.031	0.160	0.000	0.000	0.257	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	59	134	0	94	0	0	0
N.S.	1	1.00	0.43	0.98	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.073	0.150	0.659	0.000	0.081	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	33	38	42	37	0	0	58
N.S.	1	1.00	1.43	1.65	1.83	1.61	0.00	0.00	2.52
time (sec)	N/A	0.026	0.047	0.149	0.277	0.263	0.000	0.000	2.062

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	65	117	0	67	0	0	0
N.S.	1	1.00	0.81	1.46	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.048	0.129	0.597	0.000	0.083	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	98	121	0	109	0	0	0
N.S.	1	1.00	0.80	0.99	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.054	0.234	0.546	0.000	0.272	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	77	138	0	89	0	0	0
N.S.	1	1.00	0.55	0.98	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.072	0.214	0.609	0.000	0.083	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	44	47	30	56	0	0	42
N.S.	1	1.00	1.57	1.68	1.07	2.00	0.00	0.00	1.50
time (sec)	N/A	0.027	0.058	0.160	0.316	0.252	0.000	0.000	2.161

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	65	147	0	129	0	0	0
N.S.	1	1.00	0.26	0.59	0.00	0.51	0.00	0.00	0.00
time (sec)	N/A	0.099	0.146	0.668	0.000	0.084	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	90	113	0	101	0	0	0
N.S.	1	1.00	0.98	1.23	0.00	1.10	0.00	0.00	0.00
time (sec)	N/A	0.044	0.190	0.224	0.000	0.285	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	61	129	0	79	0	0	0
N.S.	1	1.00	0.55	1.16	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.061	0.135	0.590	0.000	0.083	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	88	0	0	109	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.050	0.191	0.000	0.000	0.251	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	65	159	0	0	0	0	0
N.S.	1	1.00	0.30	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.086	0.196	0.682	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	64	131	0	106	0	0	0
N.S.	1	1.00	0.70	1.42	0.00	1.15	0.00	0.00	0.00
time (sec)	N/A	0.030	0.128	0.225	0.000	0.270	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	45	127	0	82	0	0	0
N.S.	1	1.00	0.80	2.27	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.033	0.160	1.009	0.000	0.086	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	32	0	39	28	0	0	28
N.S.	1	1.00	1.28	0.00	1.56	1.12	0.00	0.00	1.12
time (sec)	N/A	0.029	0.044	0.000	0.277	0.276	0.000	0.000	2.126

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	65	0	0	56	0	0	0
N.S.	1	1.00	0.71	0.00	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.052	0.143	0.000	0.000	0.079	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	51	0	0	93	0	0	0
N.S.	1	1.00	0.77	0.00	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.041	0.144	0.000	0.000	0.253	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	64	0	0	0	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.841	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	126	0	0	0	0	0	0
N.S.	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	5.608	0.000	0.000	0.000	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	70	101	0	0	0	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	1.595	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	69	69	750	0	0	0	0	0	0
N.S.	1	1.00	10.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.052	14.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	139	29	50	96	189	0	215	66
N.S.	1	3.48	0.72	1.25	2.40	4.72	0.00	5.38	1.65
time (sec)	N/A	0.104	0.345	33.731	0.362	0.250	0.000	0.757	2.168

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	B	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	62	0	74	48	0	38	49
N.S.	1	1.00	2.48	0.00	2.96	1.92	0.00	1.52	1.96
time (sec)	N/A	0.028	0.223	0.000	0.188	0.260	0.000	0.283	2.285

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	B	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	25	25	64	0	49	49	0	37	36
N.S.	1	1.00	2.56	0.00	1.96	1.96	0.00	1.48	1.44
time (sec)	N/A	0.032	0.125	0.000	0.186	0.260	0.000	0.298	2.141

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	114	0	0	474	0	0	0
N.S.	1	1.00	1.28	0.00	0.00	5.33	0.00	0.00	0.00
time (sec)	N/A	0.065	2.527	0.000	0.000	0.266	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	65	65	108	0	0	538	0	0	0
N.S.	1	1.00	1.66	0.00	0.00	8.28	0.00	0.00	0.00
time (sec)	N/A	0.056	1.420	0.000	0.000	0.267	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	34	44	27	41
N.S.	1	1.00	1.00	1.05	1.00	1.79	2.32	1.42	2.16
time (sec)	N/A	0.012	0.069	1.155	0.183	0.287	0.799	0.268	2.039

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	28	70	0	28	24
N.S.	1	1.00	1.00	1.06	1.56	3.89	0.00	1.56	1.33
time (sec)	N/A	0.020	0.103	2.072	0.196	0.258	0.000	0.279	2.010

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	45	0	452	0	115	139
N.S.	1	1.00	1.00	0.82	0.00	8.22	0.00	2.09	2.53
time (sec)	N/A	0.029	0.071	19.636	0.000	0.266	0.000	0.271	2.060

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	91	272	0	47	55
N.S.	1	1.00	1.00	0.86	2.17	6.48	0.00	1.12	1.31
time (sec)	N/A	0.025	0.069	18.950	0.207	0.264	0.000	0.293	2.013

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	64	0	1326	0	152	314
N.S.	1	1.00	1.00	0.72	0.00	14.90	0.00	1.71	3.53
time (sec)	N/A	0.041	0.087	233.028	0.000	0.276	0.000	0.277	2.028

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	74	295	0	315	0	0	0
N.S.	1	1.00	0.76	3.04	0.00	3.25	0.00	0.00	0.00
time (sec)	N/A	0.044	0.201	119.731	0.000	0.093	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	72	141	0	159	0	0	0
N.S.	1	1.00	0.77	1.52	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.044	0.106	1.547	0.000	0.080	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	39	0	0	0
N.S.	1	1.00	1.00	3.16	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.032	0.072	1.223	0.000	0.076	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	58	183	0	248	0	0	0
N.S.	1	1.00	1.00	3.16	0.00	4.28	0.00	0.00	0.00
time (sec)	N/A	0.031	0.084	1.586	0.000	0.087	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	76	237	0	370	0	0	0
N.S.	1	1.00	0.78	2.44	0.00	3.81	0.00	0.00	0.00
time (sec)	N/A	0.043	0.126	2.119	0.000	0.089	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	87	256	0	602	0	0	0
N.S.	1	1.00	0.90	2.64	0.00	6.21	0.00	0.00	0.00
time (sec)	N/A	0.045	0.151	2.645	0.000	0.096	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [177] had the largest ratio of [.6363999999999999966]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	2	1.00	8	0.250
4	A	2	1	1.00	8	0.125
5	A	3	2	1.00	8	0.250
6	A	2	1	1.00	8	0.125
7	A	2	1	1.00	6	0.167
8	A	2	1	1.00	6	0.167
9	A	3	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	2	2	1.00	10	0.200
12	A	2	2	1.00	10	0.200
13	A	3	3	1.00	10	0.300
14	A	3	3	1.00	10	0.300
15	A	4	3	1.00	12	0.250
16	A	3	3	1.00	12	0.250
17	A	3	3	1.00	12	0.250
18	A	2	2	1.00	12	0.167
19	A	2	2	1.00	12	0.167
20	A	3	3	1.00	12	0.250
21	A	3	3	1.00	12	0.250
22	A	4	3	1.00	12	0.250
23	A	2	2	1.00	10	0.200
24	A	5	3	1.00	12	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	3	1.00	12	0.250
26	A	3	3	1.00	12	0.250
27	A	2	2	1.00	12	0.167
28	A	2	2	1.00	12	0.167
29	A	3	3	1.00	12	0.250
30	A	4	3	1.00	12	0.250
31	A	5	3	1.00	12	0.250
32	A	5	4	1.00	10	0.400
33	A	4	4	1.00	10	0.400
34	A	3	3	1.00	10	0.300
35	A	2	2	1.00	10	0.200
36	A	3	3	1.00	10	0.300
37	A	4	3	1.00	10	0.300
38	A	5	3	1.00	10	0.300
39	A	7	4	1.00	10	0.400
40	A	5	4	1.00	10	0.400
41	A	4	4	1.00	10	0.400
42	A	4	4	1.00	10	0.400
43	A	5	4	1.00	10	0.400
44	A	7	4	1.00	10	0.400
45	A	3	2	1.00	10	0.200
46	A	3	2	1.00	10	0.200
47	A	3	2	1.00	10	0.200
48	A	3	3	1.00	10	0.300
49	A	3	3	1.00	10	0.300
50	A	5	3	1.00	10	0.300
51	A	7	3	1.00	10	0.300
52	A	7	7	1.00	13	0.538
53	A	6	5	1.00	13	0.385
54	A	5	5	1.00	13	0.385
55	A	5	4	1.00	11	0.364
56	A	6	6	1.00	11	0.546
57	A	6	5	1.00	13	0.385
58	A	7	7	1.00	13	0.538
59	A	7	6	1.00	13	0.462

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	6	5	1.00	13	0.385
61	A	5	4	1.00	13	0.308
62	A	5	5	1.00	13	0.385
63	A	5	4	1.00	11	0.364
64	A	4	3	1.00	11	0.273
65	A	5	5	1.00	13	0.385
66	A	6	5	1.00	13	0.385
67	A	6	5	1.00	13	0.385
68	A	7	5	1.00	13	0.385
69	A	6	5	1.00	13	0.385
70	A	5	5	1.00	13	0.385
71	A	4	4	1.00	11	0.364
72	A	1	1	1.00	11	0.091
73	A	3	3	1.00	13	0.231
74	A	4	4	1.00	13	0.308
75	A	6	6	1.00	13	0.462
76	A	2	2	1.00	12	0.167
77	A	2	2	1.00	13	0.154
78	A	5	5	1.00	14	0.357
79	A	4	4	1.00	14	0.286
80	A	2	2	1.00	14	0.143
81	A	5	4	1.00	14	0.286
82	A	6	5	1.00	14	0.357
83	A	2	2	1.00	15	0.133
84	A	5	4	1.00	15	0.267
85	A	2	2	1.00	10	0.200
86	A	2	2	1.00	10	0.200
87	A	6	5	1.00	12	0.417
88	A	5	4	1.00	12	0.333
89	A	4	4	1.00	12	0.333
90	A	2	1	1.00	10	0.100
91	A	3	3	1.00	12	0.250
92	A	5	5	1.00	12	0.417
93	A	6	6	1.00	12	0.500
94	A	1	1	1.00	14	0.071

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	8	6	1.00	13	0.462
96	A	7	6	1.00	13	0.462
97	A	6	6	1.00	13	0.462
98	A	5	5	1.00	11	0.454
99	A	3	3	1.00	11	0.273
100	A	5	5	1.00	13	0.385
101	A	6	6	1.00	13	0.462
102	A	7	7	1.00	13	0.538
103	A	5	3	1.00	13	0.231
104	A	3	2	1.00	13	0.154
105	A	4	3	1.00	13	0.231
106	A	3	2	1.00	13	0.154
107	A	3	2	1.00	13	0.154
108	A	2	2	1.00	11	0.182
109	A	3	2	1.00	11	0.182
110	A	4	3	1.00	13	0.231
111	A	3	2	1.00	13	0.154
112	A	5	3	1.00	13	0.231
113	A	3	2	1.00	13	0.154
114	A	15	8	1.00	13	0.615
115	A	3	2	1.00	13	0.154
116	A	6	6	1.00	13	0.462
117	A	3	2	1.00	13	0.154
118	A	7	7	1.00	13	0.538
119	A	4	4	1.00	11	0.364
120	A	3	2	1.00	11	0.182
121	A	9	8	1.00	13	0.615
122	A	3	2	1.00	13	0.154
123	A	15	8	1.00	13	0.615
124	A	3	2	1.00	13	0.154
125	A	5	4	1.00	23	0.174
126	A	5	4	1.00	23	0.174
127	A	4	4	1.00	21	0.190
128	A	7	5	1.00	21	0.238
129	A	13	9	1.00	23	0.391

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	7	7	1.00	23	0.304
131	A	1	1	1.00	14	0.071
132	A	5	4	1.00	23	0.174
133	A	5	4	1.00	23	0.174
134	A	5	4	1.00	23	0.174
135	A	3	3	1.00	21	0.143
136	A	7	5	1.00	21	0.238
137	A	11	6	1.00	23	0.261
138	A	11	8	1.00	23	0.348
139	A	6	6	1.00	23	0.261
140	A	1	1	1.00	14	0.071
141	A	9	8	1.00	23	0.348
142	A	5	4	1.00	23	0.174
143	A	5	4	1.00	23	0.174
144	A	4	4	1.00	21	0.190
145	A	7	4	1.00	21	0.190
146	A	11	5	1.00	23	0.217
147	A	17	11	1.00	23	0.478
148	A	7	7	1.00	23	0.304
149	A	6	6	1.00	14	0.429
150	A	14	11	1.00	23	0.478
151	A	6	5	1.00	25	0.200
152	A	6	5	1.00	25	0.200
153	A	4	4	1.00	25	0.160
154	A	4	4	1.00	25	0.160
155	A	5	4	1.00	25	0.160
156	A	6	5	1.00	25	0.200
157	A	6	5	1.00	25	0.200
158	A	6	6	1.00	15	0.400
159	A	3	3	1.00	15	0.200
160	A	8	8	1.00	15	0.533
161	A	6	6	1.00	15	0.400
162	A	5	5	1.00	13	0.385
163	A	6	6	1.00	11	0.546
164	A	3	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	5	5	1.00	15	0.333
166	A	6	6	1.00	15	0.400
167	A	3	3	1.00	15	0.200
168	A	5	5	1.00	15	0.333
169	A	8	7	1.00	15	0.467
170	A	7	6	1.00	15	0.400
171	A	3	3	1.00	15	0.200
172	A	9	8	1.00	15	0.533
173	A	7	6	1.00	15	0.400
174	A	6	5	1.00	15	0.333
175	A	7	6	1.00	15	0.400
176	A	8	7	1.00	13	0.538
177	A	7	7	1.00	11	0.636
178	A	4	3	1.00	15	0.200
179	A	3	3	1.00	15	0.200
180	A	5	5	1.00	15	0.333
181	A	6	6	1.00	15	0.400
182	A	4	4	1.00	11	0.364
183	A	4	4	1.00	13	0.308
184	A	4	4	1.00	13	0.308
185	A	4	4	1.00	13	0.308
186	C	9	4	3.48	44	0.091
187	A	3	3	1.00	15	0.200
188	A	4	4	1.00	15	0.267
189	A	3	3	1.00	20	0.150
190	A	3	3	1.00	21	0.143
191	A	2	1	1.00	15	0.067
192	A	3	2	1.00	17	0.118
193	A	3	2	1.00	17	0.118
194	A	3	1	1.00	17	0.059
195	A	4	2	1.00	17	0.118
196	A	4	3	1.00	19	0.158
197	A	4	3	1.00	19	0.158
198	A	3	2	1.00	19	0.105
199	A	3	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	4	3	1.00	19	0.158
201	A	4	3	1.00	19	0.158

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 11

$$\int \operatorname{sech}(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}$$

```
[Out] arctan(sinh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3855}

$$\int \operatorname{sech}(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}$$

```
[In] Int[Sech[a + b*x],x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\text{integral}=\frac{\arctan(\sinh(a+bx))}{b}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{b}$$

```
[In] Integrate[Sech[a + b*x],x]
[Out] ArcTan[Sinh[a + b*x]]/b
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\arctan(\sinh(bx+a))}{b}$	12
default	$\frac{\arctan(\sinh(bx+a))}{b}$	12
risch	$\frac{i\ln(\mathrm{e}^{bx+a}+i)}{b}-\frac{i\ln(\mathrm{e}^{bx+a}-i)}{b}$	34
parallelrisch	$-\frac{i\left(\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)\right)}{b}$	36

```
[In] int(sech(b*x+a),x,method=_RETURNVERBOSE)
[Out] arctan(sinh(b*x+a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \operatorname{sech}(a+bx)\,dx=\frac{2\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

```
[In] integrate(sech(b*x+a),x, algorithm="fricas")
[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \operatorname{sech}(a+bx)\,dx = \begin{cases} \frac{2\operatorname{atan}\left(\tanh\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for } b\neq 0 \\ x\operatorname{sech}(a) & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(b*x+a),x)
[Out] Piecewise((2*atan(tanh(a/2 + b*x/2))/b, Ne(b, 0)), (x*sech(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}(a+bx)\,dx = \frac{\arctan(\sinh(bx+a))}{b}$$

```
[In] integrate(sech(b*x+a),x, algorithm="maxima")
[Out] arctan(sinh(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \operatorname{sech}(a+bx)\,dx = \frac{2\arctan\left(e^{(bx+a)}\right)}{b}$$

```
[In] integrate(sech(b*x+a),x, algorithm="giac")
[Out] 2*arctan(e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \operatorname{sech}(a+bx)\,dx = \frac{2\,\operatorname{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}$$

```
[In] int(1/cosh(a + b*x),x)
[Out] (2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2)
```

3.2 $\int \operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 10

$$\int \operatorname{sech}^2(a+bx)\,dx=\frac{\tanh(a+bx)}{b}$$

```
[Out] tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3852, 8}

$$\int \operatorname{sech}^2(a+bx)\,dx=\frac{\tanh(a+bx)}{b}$$

```
[In] Int[Sech[a + b*x]^2,x]
```

```
[Out] Tanh[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}(\int 1\,dx, x, -i\tanh(a+bx))}{b}\\ &= \frac{\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(a+bx)\,dx = \frac{\tanh(a+bx)}{b}$$

```
[In] Integrate[Sech[a + b*x]^2,x]
[Out] Tanh[a + b*x]/b
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\tanh(bx+a)}{b}$	11
default	$\frac{\tanh(bx+a)}{b}$	11
risch	$-\frac{2}{b\left(1+e^{2bx+2a}\right)}$	19
parallelrisch	$\frac{2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)}$	30

```
[In] int(sech(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] tanh(b*x+a)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(10) = 20$.

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.10

$$\int \operatorname{sech}^2(a+bx)\,dx = -\frac{2}{b\cosh{(bx+a)}^2 + 2\,b\cosh{(bx+a)}\sinh{(bx+a)} + b\sinh{(bx+a)}^2 + b}$$

```
[In] integrate(sech(b*x+a)^2,x, algorithm="fricas")
[Out] -2/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2
 + b)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)\,dx = \int \operatorname{sech}^2{(a+bx)}\,dx$$

```
[In] integrate(sech(b*x+a)**2,x)
[Out] Integral(sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\,dx = \frac{2}{b(e^{(-2\,bx-2\,a)}+1)}$$

```
[In] integrate(sech(b*x+a)^2,x, algorithm="maxima")
[Out] 2/(b*(e^(-2*b*x - 2*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\,dx = -\frac{2}{b(e^{(2\,bx+2\,a)}+1)}$$

```
[In] integrate(sech(b*x+a)^2,x, algorithm="giac")
[Out] -2/(b*(e^(2*b*x + 2*a) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \operatorname{sech}^2(a+bx)\,dx=-\frac{2}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/cosh(a + b*x)^2,x)
[Out] -2/(b*(exp(2*a + 2*b*x) + 1))
```

3.3 $\int \operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \operatorname{sech}^3(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[Out] 1/2*arctan(sinh(b*x+a))/b+1/2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3855}

$$\int \operatorname{sech}^3(a+bx)\,dx=\frac{\arctan(\sinh(a+bx))}{2b}+\frac{\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

```
[In] Int[Sech[a + b*x]^3,x]
```

```
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{1}{2}\int \operatorname{sech}(a+bx)\,dx\\ &= \frac{\arctan(\sinh(a+bx))}{2b} + \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{\arctan(\sinh(a+bx))}{2b} + \frac{\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

```
[In] Integrate[Sech[a + b*x]^3,x]
[Out] ArcTan[Sinh[a + b*x]]/(2*b) + (Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	27
default	$\frac{\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	27
risch	$\frac{e^{bx+a}\left(e^{2bx+2a}-1\right)}{b\left(1+e^{2bx+2a}\right)^2}+\frac{i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{i\ln\left(e^{bx+a}-i\right)}{2b}$	68
parallelrisch	$\frac{i(-1-\cosh(2bx+2a))\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i\right)+i(1+\cosh(2bx+2a))\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+i\right)+2\sinh(bx+a)}{2b(1+\cosh(2bx+2a))}$	84

```
[In] int(sech(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(30) = 60$.

Time = 0.26 (sec) , antiderivative size = 267, normalized size of antiderivative = 7.85

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx+a)\sinh(b\right.}{b\cosh(bx+a)^4 + 4\,b\cosh(bx}$$

```
[In] integrate(sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^3(a+bx)\,dx = \int \operatorname{sech}^3(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**3,x)
```

```
[Out] Integral(sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \operatorname{sech}^3(a+bx)\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

```
[In] integrate(sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{\pi + \frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 2\arctan\left(\frac{1}{2}\left(e^{(2bx+2a)}-1\right)e^{(-bx-a)}\right)}{4b}$$

```
[In] integrate(sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.38

$$\int \operatorname{sech}^3(a+bx)\,dx = \frac{\operatorname{atan}\left(\frac{e^{bx}\,e^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}} - \frac{2\,e^{a+bx}}{b\,\left(2\,e^{2a+2bx}+e^{4a+4bx}+1\right)} + \frac{e^{a+bx}}{b\,\left(e^{2a+2bx}+1\right)}$$

```
[In] int(1/cosh(a + b*x)^3,x)
```

```
[Out] atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2*
exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) + exp(a + b*x)/(b*(exp(2*a + 2*b*
x) + 1))
```

3.4 $\int \operatorname{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \operatorname{sech}^4(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

```
[Out] tanh(b*x+a)/b-1/3*tanh(b*x+a)^3/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3852}

$$\int \operatorname{sech}^4(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}$$

```
[In] Int[Sech[a + b*x]^4,x]
```

```
[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{i\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-i\tanh(a+bx)\right)}{b}\\&=\frac{\tanh(a+bx)}{b}-\frac{\tanh^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^4(a+bx)\,dx = \frac{\tanh(a+bx)}{b} - \frac{\tanh^3(a+bx)}{3b}$$

```
[In] Integrate[Sech[a + b*x]^4,x]
[Out] Tanh[a + b*x]/b - Tanh[a + b*x]^3/(3*b)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	23
default	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	23
risch	$-\frac{4(3\,\mathrm{e}^{2bx+2a}+1)}{3b(1+\mathrm{e}^{2bx+2a})^3}$	32
parallelrisch	$\frac{6\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5+4\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3+6\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{3b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^3}$	59

```
[In] int(sech(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/b*(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(24) = 48.

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 6.31

$$\int \operatorname{sech}^4(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh(bx+a)^5+5\,b\cosh(bx+a)\sinh(bx+a)^4+b\sinh(bx+a)^5+3\,b\cosh(bx+a)^3+\left(10\,b\cosh\right.\right.}$$

```
[In] integrate(sech(b*x+a)^4,x, algorithm="fricas")
[Out] -8/3*(2*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^5 + 5*b*cosh(b*x +
a)*sinh(b*x + a)^4 + b*sinh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + (10*b*cosh(b
```

```
*x + a)^2 + 3*b)*sinh(b*x + a)^3 + (10*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a
))*sinh(b*x + a)^2 + 4*b*cosh(b*x + a) + (5*b*cosh(b*x + a)^4 + 9*b*cosh(b*
x + a)^2 + 2*b)*sinh(b*x + a))
```

Sympy [F]

$$\int \operatorname{sech}^4(a+bx)\,dx = \int \operatorname{sech}^4(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**4,x)
[Out] Integral(sech(a + b*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(24) = 48$.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.46

$$\int \operatorname{sech}^4(a+bx)\,dx = \frac{4\,e^{(-2\,bx-2\,a)}}{b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)} + \frac{4}{3\,b(3\,e^{(-2\,bx-2\,a)}+3\,e^{(-4\,bx-4\,a)}+e^{(-6\,bx-6\,a)}+1)}$$

```
[In] integrate(sech(b*x+a)^4,x, algorithm="maxima")
[Out] 4*e^(-2*b*x - 2*a)/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1)) + 4/3/(b*(3*e^(-2*b*x - 2*a) + 3*e^(-4*b*x - 4*a) + e^(-6*b*x
- 6*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \operatorname{sech}^4(a+bx)\,dx = -\frac{4\left(3\,e^{(2\,bx+2\,a)}+1\right)}{3\,b(e^{(2\,bx+2\,a)}+1)^3}$$

```
[In] integrate(sech(b*x+a)^4,x, algorithm="giac")
[Out] -4/3*(3*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \operatorname{sech}^4(a+bx)\,dx=-\frac{4\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{3\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^3}$$

```
[In] int(1/cosh(a + b*x)^4,x)
[Out] -(4*(3*exp(2*a + 2*b*x) + 1))/(3*b*(exp(2*a + 2*b*x) + 1)^3)
```

3.5 $\int \operatorname{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int \operatorname{sech}^5(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}+\frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b}+\frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

```
[Out] 3/8*arctan(sinh(b*x+a))/b+3/8*sech(b*x+a)*tanh(b*x+a)/b+1/4*sech(b*x+a)^3*t
anh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3855}

$$\int \operatorname{sech}^5(a+bx)\,dx=\frac{3\arctan(\sinh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^3(a+bx)}{4b}+\frac{3\tanh(a+bx)\operatorname{sech}(a+bx)}{8b}$$

```
[In] Int[Sech[a + b*x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) + (
Sech[a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b} + \frac{3}{4}\int \operatorname{sech}^3(a+bx)\,dx \\
&= \frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b} + \frac{3}{8}\int \operatorname{sech}(a+bx)\,dx \\
&= \frac{3\arctan(\sinh(a+bx))}{8b} + \frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^5(a+bx)\,dx = \frac{3\arctan(\sinh(a+bx))}{8b} + \frac{3\operatorname{sech}(a+bx)\tanh(a+bx)}{8b} + \frac{\operatorname{sech}^3(a+bx)\tanh(a+bx)}{4b}$$

```
[In] Integrate[Sech[a + b*x]^5,x]
[Out] (3*ArcTan[Sinh[a + b*x]])/(8*b) + (3*Sech[a + b*x]*Tanh[a + b*x])/(8*b) + (
Sech[a + b*x]^3*Tanh[a + b*x])/(4*b)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$
default	$\frac{\left(\frac{\operatorname{sech}(bx+a)^3}{4}+\frac{3\operatorname{sech}(bx+a)}{8}\right)\tanh(bx+a)+\frac{3\arctan\left(e^{bx+a}\right)}{4}}{b}$
risch	$\frac{e^{bx+a}\left(3\,e^{6bx+6a}+11\,e^{4bx+4a}-11\,e^{2bx+2a}-3\right)}{4b\left(1+e^{2bx+2a}\right)^4}+\frac{3i\ln\left(e^{bx+a}+i\right)}{8b}-\frac{3i\ln\left(e^{bx+a}-i\right)}{8b}$
parallelrisch	$\frac{3i(-3-\cosh(4bx+4a)-4\cosh(2bx+2a))\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-i\right)+3i(3+\cosh(4bx+4a)+4\cosh(2bx+2a))\ln\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+\right.}{8b(3+\cosh(4bx+4a)+4\cosh(2bx+2a))}$

[In] int(sech(b*x+a)^5,x,method=_RETURNVERBOSE)

[Out] 1/b*((1/4*sech(b*x+a)^3+3/8*sech(b*x+a))*tanh(b*x+a)+3/4*arctan(exp(b*x+a))
)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(49) = 98$.

Time = 0.24 (sec) , antiderivative size = 812, normalized size of antiderivative = 14.76

$$\int \operatorname{sech}^5(a+bx)\,dx = \text{Too large to display}$$

[In] integrate(sech(b*x+a)^5,x, algorithm="fricas")

[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b

```
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^5(a+bx)\,dx = \int \operatorname{sech}^5(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**5,x)
[Out] Integral(sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.04

$$\int \operatorname{sech}^5(a+bx)\,dx = -\frac{3\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{3\,e^{(-bx-a)} + 11\,e^{(-3\,bx-3\,a)} - 11\,e^{(-5\,bx-5\,a)} - 3\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1)}$$

```
[In] integrate(sech(b*x+a)^5,x, algorithm="maxima")
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \operatorname{sech}^5(a+bx)\,dx = \frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+20\,e^{(bx+a)}-20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{16\,b}$$

```
[In] integrate(sech(b*x+a)^5,x, algorithm="giac")
[Out] 1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-
b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x +
 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.44

$$\int \operatorname{sech}^5(a+bx)\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{4\,\sqrt{b^2}} + \frac{\mathrm{e}^{a+b\,x}}{2\,b\,\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+\mathrm{e}^{4\,a+4\,b\,x}+1\right)} - \frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(3\,\mathrm{e}^{2\,a+2\,b\,x}+3\,\mathrm{e}^{4\,a+4\,b\,x}+\mathrm{e}^{6\,a+6\,b\,x}+1\right)} - \frac{4\,\mathrm{e}^{3\,a+3\,b\,x}}{b\,\left(4\,\mathrm{e}^{2\,a+2\,b\,x}+6\,\mathrm{e}^{4\,a+4\,b\,x}+4\,\mathrm{e}^{6\,a+6\,b\,x}+\mathrm{e}^{8\,a+8\,b\,x}+1\right)} + \frac{3\,\mathrm{e}^{a+b\,x}}{4\,b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

```
[In] int(1/cosh(a + b*x)^5,x)
```

```
[Out] (3*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(4*(b^2)^(1/2)) + exp(a + b*x)/(2
*b*(2*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (2*exp(a + b*x))/(b*(3*ex
p(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) - (4*exp(3*a +
 3*b*x))/(b*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 4*exp(6*a + 6*b*x) +
 exp(8*a + 8*b*x) + 1)) + (3*exp(a + b*x))/(4*b*(exp(2*a + 2*b*x) + 1))
```

3.6 $\int \operatorname{sech}^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \operatorname{sech}^6(a+bx)\,dx=\frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}$$

```
[Out] tanh(b*x+a)/b-2/3*tanh(b*x+a)^3/b+1/5*tanh(b*x+a)^5/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3852}

$$\int \operatorname{sech}^6(a+bx)\,dx=\frac{\tanh^5(a+bx)}{5b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh(a+bx)}{b}$$

```
[In] Int[Sech[a + b*x]^6,x]
```

```
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{i\text{Subst}\left(\int\left(1+2x^2+x^4\right)\,dx,x,-i\tanh(a+bx)\right)}{b}\\&=\frac{\tanh(a+bx)}{b}-\frac{2\tanh^3(a+bx)}{3b}+\frac{\tanh^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^6(a+bx)\,dx = \frac{\tanh(a+bx)}{b} - \frac{2\tanh^3(a+bx)}{3b} + \frac{\tanh^5(a+bx)}{5b}$$

```
[In] Integrate[Sech[a + b*x]^6,x]
[Out] Tanh[a + b*x]/b - (2*Tanh[a + b*x]^3)/(3*b) + Tanh[a + b*x]^5/(5*b)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(bx+a)^4}{5}+\frac{4\operatorname{sech}(bx+a)^2}{15}\right)\tanh(bx+a)}{b}$	33
default	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(bx+a)^4}{5}+\frac{4\operatorname{sech}(bx+a)^2}{15}\right)\tanh(bx+a)}{b}$	33
risch	$-\frac{16\left(10\,e^{4bx+4a}+5\,e^{2bx+2a}+1\right)}{15b\left(1+e^{2bx+2a}\right)^5}$	43
parallelrisch	$\frac{\frac{8\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3}{3}+\frac{8\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^7}{3}+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^9+\frac{116\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5}{15}+2\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^5}$	85

```
[In] int(sech(b*x+a)^6,x,method=_RETURNVERBOSE)
[Out] 1/b*(8/15+1/5*sech(b*x+a)^4+4/15*sech(b*x+a)^2)*tanh(b*x+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(37) = 74.

Time = 0.26 (sec) , antiderivative size = 344, normalized size of antiderivative = 8.39

$$\int \operatorname{sech}^6(a+bx)\,dx =$$

$$-\frac{}{15\left(b\cosh(bx+a)^8+8\,b\cosh(bx+a)\sinh(bx+a)^7+b\sinh(bx+a)^8+5\,b\cosh(bx+a)^6+\left(28\,b\cosh\right.\right.}$$

```
[In] integrate(sech(b*x+a)^6,x, algorithm="fricas")
[Out] -16/15*(11*cosh(b*x + a)^2 + 18*cosh(b*x + a)*sinh(b*x + a) + 11*sinh(b*x +
 a)^2 + 5)/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)^7 + b*sinh(
```

```
b*x + a)^8 + 5*b*cosh(b*x + a)^6 + (28*b*cosh(b*x + a)^2 + 5*b)*sinh(b*x +
a)^6 + 2*(28*b*cosh(b*x + a)^3 + 15*b*cosh(b*x + a))*sinh(b*x + a)^5 + 10*b
*cosh(b*x + a)^4 + 5*(14*b*cosh(b*x + a)^4 + 15*b*cosh(b*x + a)^2 + 2*b)*si
nh(b*x + a)^4 + 4*(14*b*cosh(b*x + a)^5 + 25*b*cosh(b*x + a)^3 + 10*b*cosh(
b*x + a))*sinh(b*x + a)^3 + 11*b*cosh(b*x + a)^2 + (28*b*cosh(b*x + a)^6 +
75*b*cosh(b*x + a)^4 + 60*b*cosh(b*x + a)^2 + 11*b)*sinh(b*x + a)^2 + 2*(4*
b*cosh(b*x + a)^7 + 15*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 9*b*cosh(
b*x + a))*sinh(b*x + a) + 5*b)
```

Sympy [F]

$$\int \operatorname{sech}^6(a+bx)\,dx = \int \operatorname{sech}^6(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**6,x)
[Out] Integral(sech(a + b*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(37) = 74$.

Time = 0.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 5.00

$$\begin{aligned}\int \operatorname{sech}^6(a+bx)\,dx &= \frac{16\,e^{(-2\,bx-2\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} \\ &\quad + \frac{32\,e^{(-4\,bx-4\,a)}}{3\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)} \\ &\quad + \frac{16}{15\,b(5\,e^{(-2\,bx-2\,a)}+10\,e^{(-4\,bx-4\,a)}+10\,e^{(-6\,bx-6\,a)}+5\,e^{(-8\,bx-8\,a)}+e^{(-10\,bx-10\,a)}+1)}\end{aligned}$$

```
[In] integrate(sech(b*x+a)^6,x, algorithm="maxima")
[Out] 16/3*e^(-2*b*x - 2*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(
-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 32/3*e^(-4*
b*x - 4*a)/(b*(5*e^(-2*b*x - 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*
a) + 5*e^(-8*b*x - 8*a) + e^(-10*b*x - 10*a) + 1)) + 16/15/(b*(5*e^(-2*b*x
- 2*a) + 10*e^(-4*b*x - 4*a) + 10*e^(-6*b*x - 6*a) + 5*e^(-8*b*x - 8*a) + e
^(-10*b*x - 10*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \operatorname{sech}^6(a+bx)\,dx = -\frac{16\left(10\,e^{(4\,bx+4\,a)}+5\,e^{(2\,bx+2\,a)}+1\right)}{15\,b{\left(e^{(2\,bx+2\,a)}+1\right)}^5}$$

```
[In] integrate(sech(b*x+a)^6,x, algorithm="giac")
[Out] -16/15*(10*e^(4*b*x + 4*a) + 5*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1
)^5)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \operatorname{sech}^6(a+bx)\,dx = -\frac{16\left(5\,\mathrm{e}^{2\,a+2\,b\,x}+10\,\mathrm{e}^{4\,a+4\,b\,x}+1\right)}{15\,b\,{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^5}$$

```
[In] int(1/cosh(a + b*x)^6,x)
[Out] -(16*(5*exp(2*a + 2*b*x) + 10*exp(4*a + 4*b*x) + 1))/(15*b*(exp(2*a + 2*b*x
) + 1)^5)
```

3.7 $\int \text{sech}^4(7x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 19

$$\int \text{sech}^4(7x)\,dx = \frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

```
[Out] 1/7*tanh(7*x)-1/21*tanh(7*x)^3
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852}

$$\int \text{sech}^4(7x)\,dx = \frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

```
[In] Int[Sech[7*x]^4,x]
```

```
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{7}i\text{Subst}\left(\int (1+x^2)\ dx, x, -i\tanh(7x)\right) \\ &= \frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^4(7x)\,dx = \frac{1}{7}\tanh(7x) - \frac{1}{21}\tanh^3(7x)$$

```
[In] Integrate[Sech[7*x]^4,x]
[Out] Tanh[7*x]/7 - Tanh[7*x]^3/21
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(7x)^2}{3}\right)\tanh(7x)}{7}$	17
default	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(7x)^2}{3}\right)\tanh(7x)}{7}$	17
risch	$-\frac{4(3\,\mathrm{e}^{14x}+1)}{21(\mathrm{e}^{14x}+1)^3}$	19
parallelrisch	$\frac{6\tanh\left(\frac{7x}{2}\right)^5+4\tanh\left(\frac{7x}{2}\right)^3+6\tanh\left(\frac{7x}{2}\right)}{21\left(1+\tanh\left(\frac{7x}{2}\right)^2\right)^3}$	36

```
[In] int(sech(7*x)^4,x,method=_RETURNVERBOSE)
[Out] 1/7*(2/3+1/3*sech(7*x)^2)*tanh(7*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 6.11

$$\int \operatorname{sech}^4(7x)\,dx =$$

$$-\frac{8\,(2\,\cosh(7\,x)}{21\,\left(\cosh(7\,x)^5+5\,\cosh(7\,x)\sinh(7\,x)^4+\sinh(7\,x)^5+\left(10\,\cosh(7\,x)^2+3\right)\sinh(7\,x)^3+3\,\cosh(7\,x)^3+\right.}$$

```
[In] integrate(sech(7*x)^4,x, algorithm="fricas")
[Out] -8/21*(2*cosh(7*x) + sinh(7*x))/(cosh(7*x)^5 + 5*cosh(7*x)*sinh(7*x)^4 + si
nh(7*x)^5 + (10*cosh(7*x)^2 + 3)*sinh(7*x)^3 + 3*cosh(7*x)^3 + (10*cosh(7*x
)^3 + 9*cosh(7*x))*sinh(7*x)^2 + (5*cosh(7*x)^4 + 9*cosh(7*x)^2 + 2)*sinh(7
*x) + 4*cosh(7*x))
```

Sympy [F]

$$\int \operatorname{sech}^4(7x)\,dx = \int \operatorname{sech}^4(7x)\,dx$$

```
[In] integrate(sech(7*x)**4,x)
```

```
[Out] Integral(sech(7*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(15) = 30$.

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int \operatorname{sech}^4(7x)\,dx = \frac{4\,e^{(-14\,x)}}{7\,(3\,e^{(-14\,x)} + 3\,e^{(-28\,x)} + e^{(-42\,x)} + 1)} + \frac{4}{21\,(3\,e^{(-14\,x)} + 3\,e^{(-28\,x)} + e^{(-42\,x)} + 1)}$$

```
[In] integrate(sech(7*x)^4,x, algorithm="maxima")
```

```
[Out] 4/7*e^(-14*x)/(3*e^(-14*x) + 3*e^(-28*x) + e^(-42*x) + 1) + 4/21/(3*e^(-14*
x) + 3*e^(-28*x) + e^(-42*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \operatorname{sech}^4(7x)\,dx = -\frac{4\,(3\,e^{(14\,x)} + 1)}{21\,(e^{(14\,x)} + 1)^3}$$

```
[In] integrate(sech(7*x)^4,x, algorithm="giac")
```

```
[Out] -4/21*(3*e^(14*x) + 1)/(e^(14*x) + 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \operatorname{sech}^4(7x)\,dx = -\frac{2\left(3\,e^{14\,x}-3\,e^{28\,x}-e^{42\,x}+1\right)}{21\,\left(e^{14\,x}+1\right)^3}$$

```
[In] int(1/cosh(7*x)^4,x)
[Out] -(2*(3*exp(14*x) - 3*exp(28*x) - exp(42*x) + 1))/(21*(exp(14*x) + 1)^3)
```

3.8 $\int \text{sech}^6(\pi x)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \text{sech}^6(\pi x)\,dx = \frac{\tanh(\pi x)}{\pi} - \frac{2\tanh^3(\pi x)}{3\pi} + \frac{\tanh^5(\pi x)}{5\pi}$$

```
[Out] tanh(Pi*x)/Pi-2/3*tanh(Pi*x)^3/Pi+1/5*tanh(Pi*x)^5/Pi
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3852}

$$\int \text{sech}^6(\pi x)\,dx = \frac{\tanh^5(\pi x)}{5\pi} - \frac{2\tanh^3(\pi x)}{3\pi} + \frac{\tanh(\pi x)}{\pi}$$

```
[In] Int[Sech[Pi*x]^6,x]
```

```
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{i\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(\pi x)\right)}{\pi} \\ &= \frac{\tanh(\pi x)}{\pi} - \frac{2\tanh^3(\pi x)}{3\pi} + \frac{\tanh^5(\pi x)}{5\pi}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^6(\pi x)\,dx = \frac{\tanh(\pi x)}{\pi} - \frac{2\tanh^3(\pi x)}{3\pi} + \frac{\tanh^5(\pi x)}{5\pi}$$

```
[In] Integrate[Sech[Pi*x]^6,x]
[Out] Tanh[Pi*x]/Pi - (2*Tanh[Pi*x]^3)/(3*Pi) + Tanh[Pi*x]^5/(5*Pi)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(\pi x)^4}{5}+\frac{4\operatorname{sech}(\pi x)^2}{15}\right)\tanh(\pi x)}{\pi}$	27
default	$\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(\pi x)^4}{5}+\frac{4\operatorname{sech}(\pi x)^2}{15}\right)\tanh(\pi x)}{\pi}$	27
risch	$-\frac{16(10\,e^{4\pi x}+5\,e^{2\pi x}+1)}{15\pi(e^{2\pi x}+1)^5}$	31
parallelrisch	$\frac{2\tanh\left(\frac{\pi x}{2}\right)+2\tanh\left(\frac{\pi x}{2}\right)^9+\frac{116\tanh\left(\frac{\pi x}{2}\right)^5}{15}+\frac{8\tanh\left(\frac{\pi x}{2}\right)^3}{3}+\frac{8\tanh\left(\frac{\pi x}{2}\right)^7}{3}}{\pi\left(1+\tanh\left(\frac{\pi x}{2}\right)^2\right)^5}$	61

```
[In] int(sech(Pi*x)^6,x,method=_RETURNVERBOSE)
[Out] 1/Pi*(8/15+1/5*sech(Pi*x)^4+4/15*sech(Pi*x)^2)*tanh(Pi*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(31) = 62.

Time = 0.27 (sec) , antiderivative size = 280, normalized size of antiderivative = 8.00

$$\int \operatorname{sech}^6(\pi x)\,dx =$$

$$-\frac{}{15\left(5\pi+\pi\cosh(\pi x)^8+8\pi\cosh(\pi x)\sinh(\pi x)^7+\pi\sinh(\pi x)^8+5\pi\cosh(\pi x)^6+\left(5\pi+28\pi\cosh(\pi x)\right.\right.}$$

```
[In] integrate(sech(pi*x)^6,x, algorithm="fricas")
[Out] -16/15*(11*cosh(pi*x)^2 + 18*cosh(pi*x)*sinh(pi*x) + 11*sinh(pi*x)^2 + 5)/(
5*pi + pi*cosh(pi*x)^8 + 8*pi*cosh(pi*x)*sinh(pi*x)^7 + pi*sinh(pi*x)^8 + 5
```

```
*pi*cosh(pi*x)^6 + (5*pi + 28*pi*cosh(pi*x)^2)*sinh(pi*x)^6 + 2*(28*pi*cosh
(pi*x)^3 + 15*pi*cosh(pi*x))*sinh(pi*x)^5 + 10*pi*cosh(pi*x)^4 + 5*(2*pi +
14*pi*cosh(pi*x)^4 + 15*pi*cosh(pi*x)^2)*sinh(pi*x)^4 + 4*(14*pi*cosh(pi*x)
^5 + 25*pi*cosh(pi*x)^3 + 10*pi*cosh(pi*x))*sinh(pi*x)^3 + 11*pi*cosh(pi*x)
^2 + (11*pi + 28*pi*cosh(pi*x)^6 + 75*pi*cosh(pi*x)^4 + 60*pi*cosh(pi*x)^2)
*sinh(pi*x)^2 + 2*(4*pi*cosh(pi*x)^7 + 15*pi*cosh(pi*x)^5 + 20*pi*cosh(pi*x
)^3 + 9*pi*cosh(pi*x))*sinh(pi*x))
```

Sympy [F]

$$\int \operatorname{sech}^6(\pi x)\,dx = \int \operatorname{sech}^6(\pi x)\,dx$$

```
[In] integrate(sech(pi*x)**6,x)
```

```
[Out] Integral(sech(pi*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(31) = 62$.

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.91

$$\int \operatorname{sech}^6(\pi x)\,dx = \frac{16\,e^{(-2\,\pi x)}}{3\,\pi(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1)} + \frac{32\,e^{(-4\,\pi x)}}{3\,\pi(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1)} + \frac{16}{15\,\pi(5\,e^{(-2\,\pi x)} + 10\,e^{(-4\,\pi x)} + 10\,e^{(-6\,\pi x)} + 5\,e^{(-8\,\pi x)} + e^{(-10\,\pi x)} + 1)}$$

```
[In] integrate(sech(pi*x)^6,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*pi*x)/(pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e
^(-8*pi*x) + e^(-10*pi*x) + 1)) + 32/3*e^(-4*pi*x)/(pi*(5*e^(-2*pi*x) + 10*
e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-10*pi*x) + 1)) + 16/15/(
pi*(5*e^(-2*pi*x) + 10*e^(-4*pi*x) + 10*e^(-6*pi*x) + 5*e^(-8*pi*x) + e^(-1
0*pi*x) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \operatorname{sech}^6(\pi x)\,dx = -\frac{16\left(10\,e^{(4\,\pi x)} + 5\,e^{(2\,\pi x)} + 1\right)}{15\,\pi\left(e^{(2\,\pi x)} + 1\right)^5}$$

```
[In] integrate(sech(pi*x)^6,x, algorithm="giac")
[Out] -16/15*(10*e^(4*pi*x) + 5*e^(2*pi*x) + 1)/(pi*(e^(2*pi*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \operatorname{sech}^6(\pi x)\,dx = -\frac{16\left(5\,e^{2\,\Pi\,x} + 10\,e^{4\,\Pi\,x} + 1\right)}{15\,\Pi\left(e^{2\,\Pi\,x} + 1\right)^5}$$

```
[In] int(1/cosh(Pi*x)^6,x)
[Out] -(16*(5*exp(2*Pi*x) + 10*exp(4*Pi*x) + 1))/(15*Pi*(exp(2*Pi*x) + 1)^5)
```

3.9 $\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx=-\frac{2i\sqrt{\cosh(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b}+\frac{2\operatorname{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}$$

```
[Out] 2/3*sech(b*x+a)^(3/2)*sinh(b*x+a)/b-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3853, 3856, 2720}

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx=\frac{2\sinh(a+bx)\operatorname{sech}^{\frac{3}{2}}(a+bx)}{3b}-\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\,\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

```
[In] Int[Sech[a + b*x]^(5/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sech[a + b*x]^(3/2)*Sinh[a + b*x])/(3*b)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}+\frac{1}{3}\int\sqrt{\text{sech}(a+bx)}\,dx\\ &= \frac{2\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}+\frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\right)\int\frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{3b}+\frac{2\text{sech}^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{3b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int\text{sech}^{\frac{5}{2}}(a+bx)\,dx=\frac{2\text{sech}^{\frac{3}{2}}(a+bx)\left(-i\cosh^{\frac{3}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sinh(a+bx)\right)}{3b}$$

```
[In] Integrate[Sech[a + b*x]^(5/2),x]
[Out] (2*Sech[a + b*x]^(3/2)*((-I)*Cosh[a + b*x]^(3/2)*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[a + b*x]))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(82) = 164.

Time = 1.74 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.29

method	result
default	$\frac{2\left(2\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\ \operatorname{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+\sqrt{-\mathrm{si}}\right.}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\left(-1+2\cosh\right.}$

```
[In] int(sech(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(2*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*El
lipticF(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+2*cosh(1/2*b*x+1
/2*a)*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x
+1/2*a)^2-1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2)))*((-1+2*cosh(1/2*
b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/
2*b*x+1/2*a)^2)^(1/2)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(3/2)/sinh(1/2*b*x+1/2*a
)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.88

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \frac{2\left(\sqrt{2}\left(\cosh\left(bx+a\right)^{2}+2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}-1\right)\sqrt{\frac{\cosh(bx+a)+\sinh(bx}{\cosh(bx+a)^{2}+2\cosh(bx+a)\sinh(bx+}}\right.}{3\left(b\cosh\left(b\right.\right.}$$

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x +
a)^2 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*
x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + (sqrt(2)*cosh(b*x + a)^2 + 2
*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2 + sqrt(2))*w
eierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^
2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**(5/2),x)
[Out] Integral(sech(a + b*x)**(5/2), x)
```

Maxima [F]

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(sech(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate(sech(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(sech(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{\frac{5}{2}}(a+bx)\,dx = \int \left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2}\,dx$$

```
[In] int((1/cosh(a + b*x))^(5/2),x)
[Out] int((1/cosh(a + b*x))^(5/2), x)
```

3.10 $\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b} + \frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b}$$

```
[Out] 2*sinh(b*x+a)*sech(b*x+a)^(1/2)/b+2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/
2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sec
h(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3853, 3856, 2719}

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \frac{2\sinh(a+bx)\sqrt{\operatorname{sech}(a+bx)}}{b} + \frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[In] Int[Sech[a + b*x]^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x]
])/b + (2*Sqrt[Sech[a + b*x]]*Sinh[a + b*x])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b} - \int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx \\
&= \frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b} - \left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx \\
&= \frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b} + \frac{2\sqrt{\operatorname{sech}(a+bx)}\sinh(a+bx)}{b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \frac{2\sqrt{\operatorname{sech}(a+bx)}\left(i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sinh(a+bx)\right)}{b}$$

```
[In] Integrate[Sech[a + b*x]^(3/2),x]
[Out] (2*Sqrt[Sech[a + b*x]]*(I*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]
 + Sinh[a + b*x]))/b
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.66

method	result	size
default	$\frac{4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+2\,\mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,b}$	103

```
[In] int(sech(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+EllipticE(cosh(1/2*b*x+1/2*a
),2^(1/2))*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2
))/sinh(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bx+a)+\sinh(bx+a)}{\cosh(bx+a)^2+2\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2+1}}(\cosh(bx+a)+\sinh(bx+a))+\sqrt{2}\text{weierstrassZeta}(\cdots\right.}{b}$$

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b
*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1))*(cosh(b*x + a) + sinh(b*x + a
)) + sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*x + a
) + sinh(b*x + a))))/b
```

Sympy [F]

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx$$

```
[In] integrate(sech(b*x+a)**(3/2),x)
```

```
[Out] Integral(sech(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(sech(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \operatorname{sech}(bx+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate(sech(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sech(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx = \int \left(\frac{1}{\cosh(a+b\,x)}\right)^{3/2}\,dx$$

```
[In] int((1/cosh(a + b*x))^(3/2),x)
[Out] int((1/cosh(a + b*x))^(3/2), x)
```

3.11 $\int \sqrt{\text{sech}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \sqrt{\text{sech}(a+bx)}\,dx = -\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\text{sech}(a+bx)}}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2
*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3856, 2720}

$$\int \sqrt{\text{sech}(a+bx)}\,dx = -\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\text{sech}(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

```
[In] Int[Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = -\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}$$

```
[In] Integrate[Sqrt[Sech[a + b*x]],x]
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(62) = 124.

Time = 0.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.38

method	result	size
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\text{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,b}$	135

```
[In] int(sech(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^
4+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sinh(
1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \frac{2\sqrt{2}\text{weierstrassPInverse}(-4,0,\cosh(bx+a)+\sinh(bx+a))}{b}$$

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\operatorname{sech}(a+bx)}\,dx$$

```
[In] integrate(sech(b*x+a)**(1/2),x)
[Out] Integral(sqrt(sech(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\operatorname{sech}(bx+a)}\,dx$$

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sech(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\operatorname{sech}(bx+a)}\,dx$$

```
[In] integrate(sech(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sech(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{sech}(a+bx)}\,dx = \int \sqrt{\frac{1}{\cosh\left(a+b\,x\right)}}\,dx$$

```
[In] int((1/cosh(a + b*x))^(1/2),x)
[Out] int((1/cosh(a + b*x))^(1/2), x)
```

3.12 $\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 40

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}} \, dx = -\frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cosh(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2
*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)*sech(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3856, 2719}

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}} \, dx = -\frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

```
[In] Int[1/Sqrt[Sech[a + b*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a + b*x
]])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int\sqrt{\cosh(a+bx)}\,dx\\ &= -\frac{2i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int\frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}}$$

```
[In] Integrate[1/Sqrt[Sech[a + b*x]],x]
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/(b*Sqrt[Cosh[a + b*x]]*Sqrt[Sech[a +
 b*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(62) = 124.

Time = 0.79 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.38

method	result
default	$-\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\operatorname{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,b}$
risch	$\frac{\sqrt{2}}{b\sqrt{\frac{e^{bx+a}}{1+e^{2bx+2a}}}}+\frac{\left(-\frac{2\left(1+e^{2bx+2a}\right)}{\sqrt{\left(1+e^{2bx+2a}\right)e^{bx+a}}}+\frac{i\sqrt{-i\left(e^{bx+a}+i\right)}\sqrt{2}\sqrt{i\left(e^{bx+a}-i\right)}\sqrt{ie^{bx+a}}\left(-2i\operatorname{EllipticE}\left(\sqrt{-i\left(e^{bx+a}+i\right)},\frac{\sqrt{2}}{2}\right)+i\operatorname{EllipticF}\left(\cdots\right.\right.}{\sqrt{e^{3bx+3a}+e^{bx+a}}}\cdots}{b\sqrt{\frac{e^{bx+a}}{1+e^{2bx+2a}}}\left(1+e^{2bx+2a}\right)}$

```
[In] int(1/sech(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*
x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*x
+1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh
(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 3.75

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = -\frac{\sqrt{2}\left(\cosh\left(bx+a\right)^{2}+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^{2}+1\right)\sqrt{\frac{\cosh(bx+a)+\sinh(bx+}{\cosh(bx+a)^{2}+2\,\cosh(bx+a)\sinh(bx+a}}}{}$$

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^
2 + 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 2*(sqrt(2)*cosh(b*x + a) + sqrt
(2)*sinh(b*x + a))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b
*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(a+bx\right)}}\,dx$$

```
[In] integrate(1/sech(b*x+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sech(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(bx+a\right)}}\,dx$$

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}(bx+a)}}\,dx$$

```
[In] integrate(1/sech(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(sech(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cosh(a+b\,x)}}}\,dx$$

```
[In] int(1/(1/cosh(a + b*x))^(1/2),x)
[Out] int(1/(1/cosh(a + b*x))^(1/2), x)
```

3.13 $\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)} dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)} dx = -\frac{2i\sqrt{\cosh(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b} + \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}$$

```
[Out] 2/3*sinh(b*x+a)/b/sech(b*x+a)^(1/2)-2/3*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticF(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)} dx = \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}} - \frac{2i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

```
[In] Int[Sech[a + b*x]^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(3*b*Sqrt[Sech[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}+\frac{1}{3}\int\sqrt{\operatorname{sech}(a+bx)}\,dx\\
&= \frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}+\frac{1}{3}\left(\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int\frac{1}{\sqrt{\cosh(a+bx)}}\,dx\\
&= -\frac{2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)\sqrt{\operatorname{sech}(a+bx)}}{3b}+\frac{2\sinh(a+bx)}{3b\sqrt{\operatorname{sech}(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int\frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{\sqrt{\operatorname{sech}(a+bx)}\left(-2i\sqrt{\cosh(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sinh(2(a+bx))\right)}{3b}$$

```
[In] Integrate[Sech[a + b*x]^(-3/2),x]
[Out] (Sqrt[Sech[a + b*x]]*((-2*I)*Sqrt[Cosh[a + b*x]]*EllipticF[(I/2)*(a + b*x),
 2] + Sinh[2*(a + b*x)]))/(3*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.

Time = 1.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.64

method	result
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-6\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\text{Ellipti}}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,b}$

```
[In] int(1/sech(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*
b*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos
h(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh(1
/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1
/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.38

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \frac{\sqrt{2}\left(\cosh\left(bx+a\right)^{4}+4\,\cosh\left(bx+a\right)^{3}\sinh\left(bx+a\right)+6\,\cosh\left(bx+a\right)^{2}\sinh\left(bx+a\right)^{2}+4\,\cosh\left(bx+a\right)\sin\right.}{}$$

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(b*x + a)^4 + 4*cosh(b*x + a)^3*sinh(b*x + a) + 6*cosh(b*
x + a)^2*sinh(b*x + a)^2 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^
4 - 1)*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)) + 4*(sqrt(2)*cosh(b*x + a)^2 + 2*
sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt(2)*sinh(b*x + a)^2)*weierstrassP
Inverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cos
h(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/sech(b*x+a)**(3/2),x)
[Out] Integral(sech(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sech(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(sech(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(1/cosh(a + b*x))^(3/2),x)
[Out] int(1/(1/cosh(a + b*x))^(3/2), x)
```

3.14 $\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 66

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = -\frac{6i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)\sqrt{\operatorname{sech}(a+bx)}}{5b}+\frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/5*sinh(b*x+a)/b/sech(b*x+a)^(3/2)-6/5*I*(cosh(1/2*a+1/2*b*x)^2)^(1/2)/cos
h(1/2*a+1/2*b*x)*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))*cosh(b*x+a)^(1/2)
*sech(b*x+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3854, 3856, 2719}

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}-\frac{6i\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

```
[In] Int[Sech[a + b*x]^(-5/2),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x), 2]*Sqrt[Sech[a +
 b*x]])/b + (2*Sinh[a + b*x])/(5*b*Sech[a + b*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{3}{5}\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx \\ &= \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)} + \frac{1}{5}\left(3\sqrt{\cosh(a+bx)}\sqrt{\operatorname{sech}(a+bx)}\right)\int \sqrt{\cosh(a+bx)}\,dx \\ &= -\frac{6i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|\,2\right)\sqrt{\operatorname{sech}(a+bx)}}{5b} + \frac{2\sinh(a+bx)}{5b\operatorname{sech}^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\operatorname{sech}(a+bx)}\left(-12i\sqrt{\cosh(a+bx)}E\left(\frac{1}{2}i(a+bx)\middle|\,2\right)+\sinh(a+bx)+\sinh(3(a+bx))\right)}{10b}$$

```
[In] Integrate[Sech[a + b*x]^(-5/2),x]
[Out] (Sqrt[Sech[a + b*x]]*((-12*I)*Sqrt[Cosh[a + b*x]]*EllipticE[(I/2)*(a + b*x)
, 2] + Sinh[a + b*x] + Sinh[3*(a + b*x)]))/(10*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(82) = 164.

Time = 1.92 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.85

method	result
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\left(8\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-16\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+10\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-3\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\cos}}{5\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-1+2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)}}$

```
[In] int(1/sech(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((-1+2*cosh(1/2*b*x+1/2*a)^2)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*
b*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/2
*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*
b*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/
2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(-1+2*cosh(1/2*b*x+1/2*a)^2)^(1/2
)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 370, normalized size of antiderivative = 5.61

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{2}\big(\cosh\left(bx+a\right)^{6}+6\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+\sinh\left(bx+a\right)^{6}+\left(15\,\cosh\left(bx+a\right)^{2}-11\right)\sinh\left(bx\right.}{}$$

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*x + a)^6 + 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x
 + a)^6 + (15*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^4 - 11*cosh(b*x + a)^4 +
4*(5*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x + a)^3 + (15*cosh(b*x + a
)^4 - 66*cosh(b*x + a)^2 - 13)*sinh(b*x + a)^2 - 13*cosh(b*x + a)^2 + 2*(3*
cosh(b*x + a)^5 - 22*cosh(b*x + a)^3 - 13*cosh(b*x + a))*sinh(b*x + a) - 1)
*sqrt((cosh(b*x + a) + sinh(b*x + a))/(cosh(b*x + a)^2 + 2*cosh(b*x + a)*si
nh(b*x + a) + sinh(b*x + a)^2 + 1)) - 24*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(
2)*cosh(b*x + a)^2*sinh(b*x + a) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2
+ sqrt(2)*sinh(b*x + a)^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cosh(b*x + a) + sinh(b*x + a))))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2
*sinh(b*x + a) + 3*b*cosh(b*x + a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx$$

```
[In] integrate(1/sech(b*x+a)**(5/2),x)
[Out] Integral(sech(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\operatorname{sech}(bx+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/sech(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(sech(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(1/cosh(a + b*x))^(5/2),x)
[Out] int(1/(1/cosh(a + b*x))^(5/2), x)
```

3.15 $\int (b\text{sech}(c+dx))^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{6b^3\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d}$$

```
[Out] 2/5*b*(b*sech(d*x+c))^(5/2)*sinh(d*x+c)/d+6/5*I*b^4*(cosh(1/2*d*x+1/2*c)^2)
^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/d/cosh(
d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)+6/5*b^3*sinh(d*x+c)*(b*sech(d*x+c))^(1/2
)/d
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3856, 2719}

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{6b^3\sinh(c+dx)\sqrt{b\text{sech}(c+dx)}}{5d} + \frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{5/2}}{5d}$$

```
[In] Int[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (((6*I)/5)*b^4*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b
*Sech[c + d*x]]) + (6*b^3*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/(5*d) + (2*b
*(b*Sech[c + d*x])^(5/2)*Sinh[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} + \frac{1}{5}\left(3b^2\right)\int (b\text{sech}(c+dx))^{3/2}\,dx \\
&= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} \\
&\quad - \frac{1}{5}\left(3b^4\right)\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx \\
&= \frac{6b^3\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{5d} + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d} \\
&\quad - \frac{\left(3b^4\right)\int \sqrt{\cosh(c+dx)}\,dx}{5\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} \\
&= \frac{6ib^4E\left(\frac{1}{2}i(c+dx)\middle|\,2\right)}{5d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{6b^3\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{5d} \\
&\quad + \frac{2b(b\text{sech}(c+dx))^{5/2}\sinh(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \frac{b^2(b\text{sech}(c+dx))^{3/2}\left(6i\cosh^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}i(c+dx)\middle|\,2\right)+3\sinh(2(c+dx))+2\tanh(c+dx)\right)}{5d}$$

```
[In] Integrate[(b*Sech[c + d*x])^(7/2),x]
```

```
[Out] (b^2*(b*Sech[c + d*x])^(3/2)*((6*I)*Cosh[c + d*x]^(3/2)*EllipticE[(I/2)*(c
+ d*x), 2] + 3*Sinh[2*(c + d*x)] + 2*Tanh[c + d*x]))/(5*d)
```

Maple [F]

$$\int (b\ \text{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] int((b*sech(d*x+c))^(7/2),x)
```

```
[Out] int((b*sech(d*x+c))^(7/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.69

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \frac{2\left(3\sqrt{2}(b^3\cosh(dx+c)^4+4\,b^3\cosh(dx+c)\sinh(dx+c)^3+b^3\sinh(dx+c)^4+2\,b^3\cosh(d\right.}{}$$

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(3*sqrt(2)*(b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x + c)^3 +
 b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(d*x + c)
^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(
d*x + c) + sinh(d*x + c))) + sqrt(2)*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d
*x + c)*sinh(d*x + c)^4 + 3*b^3*sinh(d*x + c)^5 + 8*b^3*cosh(d*x + c)^3 + b
^3*cosh(d*x + c) + 2*(15*b^3*cosh(d*x + c)^2 + 4*b^3)*sinh(d*x + c)^3 + 6*(
5*b^3*cosh(d*x + c)^3 + 4*b^3*cosh(d*x + c))*sinh(d*x + c)^2 + (15*b^3*cosh
(d*x + c)^4 + 24*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c))*sqrt((b*cosh(d*x
```

```
 + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c
)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) +
d)
```

Sympy [F(-1)]

Timed out.

$$\int (b\mathrm{sech}(c+dx))^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((b*sech(d*x+c))**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int (b\mathrm{sech}(c+dx))^{7/2}\,dx = \int (b\,\mathrm{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

Giac [F]

$$\int (b\mathrm{sech}(c+dx))^{7/2}\,dx = \int (b\,\mathrm{sech}\,(dx+c))^{\frac{7}{2}}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{sech}(c+dx))^{7/2}\,dx = \int \left(\frac{b}{\cosh(c+d\,x)}\right)^{7/2}\,dx$$

```
[In] int((b/cosh(c + d*x))^(7/2),x)
[Out] int((b/cosh(c + d*x))^(7/2), x)
```

3.16 $\int (b\text{sech}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 74

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = -\frac{2ib^2\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3d} + \frac{2b(b\text{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}$$

```
[Out] 2/3*b*(b*sech(d*x+c))^(3/2)*sinh(d*x+c)/d-2/3*I*b^2*(cosh(1/2*d*x+1/2*c)^2)
^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*
x+c)^(1/2)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3856, 2720}

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \frac{2b\sinh(c+dx)(b\text{sech}(c+dx))^{3/2}}{3d} - \frac{2ib^2\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3d}$$

```
[In] Int[(b*Sech[c + d*x])^(5/2),x]
```

```
[Out] (((-2*I)/3)*b^2*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Se
ch[c + d*x]])/d + (2*b*(b*Sech[c + d*x])^(3/2)*Sinh[c + d*x])/(3*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2b(b\operatorname{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}+\frac{1}{3}b^2\int\sqrt{b\operatorname{sech}(c+dx)}\,dx\\
&= \frac{2b(b\operatorname{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}\\
&\quad+\frac{1}{3}\left(b^2\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}\right)\int\frac{1}{\sqrt{\cosh(c+dx)}}\,dx\\
&= -\frac{2ib^2\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\operatorname{sech}(c+dx)}}{3d}\\
&\quad+\frac{2b(b\operatorname{sech}(c+dx))^{3/2}\sinh(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$
\int(b\operatorname{sech}(c+dx))^{5/2}\,dx=\frac{2b^2\sqrt{b\operatorname{sech}(c+dx)}\left(-i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)+\tanh(c+dx)\right)}{3d}
$$

```
[In] Integrate[(b*Sech[c + d*x])^(5/2),x]
[Out] (2*b^2*Sqrt[b*Sech[c + d*x]]*((-I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c +
 d*x), 2] + Tanh[c + d*x]))/(3*d)
```

Maple [F]

$$\int (b\,\text{sech}\,(dx+c))^{\frac{5}{2}}\,dx$$

[In] int((b*sech(d*x+c))^(5/2),x)

[Out] int((b*sech(d*x+c))^(5/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.91

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \frac{2\left(\sqrt{2}(b^2\cosh(dx+c)^2 + 2\,b^2\cosh(dx+c)\sinh(dx+c) + b^2\sinh(dx+c)^2 + b^2)\sqrt{b}\text{weierstra}\right.}{}$$

[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 2/3*(sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c)) + sqrt(2)*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - b^2)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)

Sympy [F]

$$\int (b\text{sech}(c+dx))^{5/2}\,dx = \int (b\,\text{sech}\,(c+dx))^{\frac{5}{2}}\,dx$$

[In] integrate((b*sech(d*x+c))**(5/2),x)

[Out] Integral((b*sech(c + d*x))**(5/2), x)

Maxima [F]

$$\int (b\mathrm{sech}(c+dx))^{5/2}\,dx = \int (b\,\mathrm{sech}\,(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

Giac [F]

$$\int (b\mathrm{sech}(c+dx))^{5/2}\,dx = \int (b\,\mathrm{sech}\,(dx+c))^{\frac{5}{2}}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\mathrm{sech}(c+dx))^{5/2}\,dx = \int \left(\frac{b}{\cosh\left(c+d\,x\right)}\right)^{5/2}dx$$

```
[In] int((b/cosh(c + d*x))^(5/2),x)
[Out] int((b/cosh(c + d*x))^(5/2), x)
```

3.17 $\int (b\operatorname{sech}(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 70

$$\int (b\operatorname{sech}(c+dx))^{3/2}\,dx = \frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}} + \frac{2b\sqrt{b\operatorname{sech}(c+dx)}\sinh(c+dx)}{d}$$

```
[Out] 2*I*b^2*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(
1/2*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)+2*b*sinh(
d*x+c)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3853, 3856, 2719}

$$\int (b\operatorname{sech}(c+dx))^{3/2}\,dx = \frac{2b\sinh(c+dx)\sqrt{b\operatorname{sech}(c+dx)}}{d} + \frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\operatorname{sech}(c+dx)}}$$

```
[In] Int[(b*Sech[c + d*x])^(3/2),x]
```

```
[Out] ((2*I)*b^2*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sec
h[c + d*x]]) + (2*b*Sqrt[b*Sech[c + d*x]]*Sinh[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2b\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{d} - b^2\int\frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx\\
&= \frac{2b\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{d} - \frac{b^2\int\sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}\\
&= \frac{2ib^2E\left(\frac{1}{2}i(c+dx)\middle|\,2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{2b\sqrt{b\text{sech}(c+dx)}\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \frac{2b\sqrt{b\text{sech}(c+dx)}\left(i\sqrt{\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)\middle|\,2\right)+\sinh(c+dx)\right)}{d}$$

```
[In] Integrate[(b*Sech[c + d*x])^(3/2),x]
[Out] (2*b*Sqrt[b*Sech[c + d*x]]*(I*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x)
, 2] + Sinh[c + d*x]))/d
```

Maple [F]

$$\int (b\ \text{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] int((b*sech(d*x+c))^(3/2),x)
[Out] int((b*sech(d*x+c))^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.53

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \frac{2\left(\sqrt{2}b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cosh(dx+c)+\sinh(dx+c)))+\sqrt{}\right.}{d}$$

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(d
*x + c) + sinh(d*x + c))) + sqrt(2)*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*si
nh(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F]

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \int (b\,\text{sech}\,(c+dx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \int (b\,\text{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

Giac [F]

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \int (b\,\text{sech}\,(dx+c))^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\text{sech}(c+dx))^{3/2}\,dx = \int \left(\frac{b}{\cosh\,(c+d\,x)}\right)^{3/2}\,dx$$

```
[In] int((b/cosh(c + d*x))^(3/2),x)
[Out] int((b/cosh(c + d*x))^(3/2), x)
```

3.18 $\int \sqrt{b\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = -\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2
*d*x+1/2*c),2^(1/2))*cosh(d*x+c)^(1/2)*(b*sech(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3856, 2720}

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = -\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

```
[In] Int[Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \left(\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,dx$$
$$= -\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = -\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[b*Sech[c + d*x]],x]
[Out] ((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c + d
*x]])/d
```

Maple [F]

$$\int \sqrt{b\ \text{sech}\left(dx+c\right)}dx$$

```
[In] int((b*sech(d*x+c))^(1/2),x)
[Out] int((b*sech(d*x+c))^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \frac{2\sqrt{2}\sqrt{b}\text{weierstrassPInverse}(-4,0,\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{d}$$

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))
/d
```

Sympy [F]

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{b\operatorname{sech}\left(c+dx\right)}\,dx$$

```
[In] integrate((b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{b\operatorname{sech}\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

Giac [F]

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{b\operatorname{sech}\left(dx+c\right)}\,dx$$

```
[In] integrate((b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{b\text{sech}(c+dx)}\,dx = \int \sqrt{\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

```
[In] int((b/cosh(c + d*x))^(1/2),x)
[Out] int((b/cosh(c + d*x))^(1/2), x)
```

3.19 $\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = -\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

```
[Out] -2*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2
*d*x+1/2*c),2^(1/2))/d/cosh(d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3856, 2719}

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = -\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

```
[In] Int[1/Sqrt[b*Sech[c + d*x]],x]
```

```
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\int \sqrt{\cosh(c+dx)}\,dx}{\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} \\ &= -\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = -\frac{2iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

```
[In] Integrate[1/Sqrt[b*Sech[c + d*x]],x]
[Out] ((-2*I)*EllipticE[(I/2)*(c + d*x), 2])/(d*Sqrt[Cosh[c + d*x]]*Sqrt[b*Sech[c
 + d*x]])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(64) = 128.

Time = 0.51 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.81

method	result
risch	$\frac{\sqrt{2}}{d\sqrt{\frac{b\,\mathrm{e}^{dx+c}}{\mathrm{e}^{2dx+2c}+1}}}+\frac{\left(-\frac{2\left(b\,\mathrm{e}^{2dx+2c}+b\right)}{b\sqrt{\mathrm{e}^{dx+c}\left(b\,\mathrm{e}^{2dx+2c}+b\right)}}+\frac{i\sqrt{-i\left(\mathrm{e}^{dx+c}+i\right)}\,\sqrt{2}\,\sqrt{i\left(\mathrm{e}^{dx+c}-i\right)}\,\sqrt{i\mathrm{e}^{dx+c}}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(\mathrm{e}^{dx+c}+i\right)},\frac{\sqrt{2}}{2}\right)+i\,\mathrm{EllipticF}\right.}{\sqrt{\mathrm{e}^{3dx+3c}b+\mathrm{e}^{dx+c}b}}\right.}{d\sqrt{\frac{b\,\mathrm{e}^{dx+c}}{\mathrm{e}^{2dx+2c}+1}}\,\left(\mathrm{e}^{2dx+2c}+1\right)}$

```
[In] int(1/(b*sech(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c)^2+1))^(1/2)+1/d*(-2*(b*exp(d*x+c)^2+b
)/b/(exp(d*x+c)*(b*exp(d*x+c)^2+b))^(1/2)+I*(-I*(exp(d*x+c)+I))^(1/2)*2^(1/
2)*(I*(exp(d*x+c)-I))^(1/2)*(I*exp(d*x+c))^(1/2)/(exp(d*x+c)^3*b+exp(d*x+c)
*b)^(1/2)*(-2*I*EllipticE((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))+I*Elliptic
F((-I*(exp(d*x+c)+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/(b*exp(d*x+c)/(exp(d*x+c
)^2+1))^(1/2)*(b*exp(d*x+c)*(exp(d*x+c)^2+1))^(1/2)/(exp(d*x+c)^2+1)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.67

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = -\frac{2\sqrt{2}\sqrt{b}(\cosh(dx+c)+\sinh(dx+c))\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cosh(dx+c)}{}$$

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*sqrt(b)*(cosh(d*x + c) + sinh(d*x + c))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))) + sqrt(2)*(cosh(
d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)*sqrt((b*c
osh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)))/(b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(c+dx)}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sech(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(1/(b/cosh(c + d*x))^(1/2),x)
[Out] int(1/(b/cosh(c + d*x))^(1/2), x)
```

3.20 $\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d} + \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}$$

```
[Out] 2/3*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(1/2)-2/3*I*(cosh(1/2*d*x+1/2*c)^2)^(1/
2)/cosh(1/2*d*x+1/2*c)*EllipticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*x+c)
^(1/2)*(b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}} - \frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d}$$

```
[In] Int[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech[c
 + d*x]])/(b^2*d) + (2*Sinh[c + d*x])/(3*b*d*Sqrt[b*Sech[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}} + \frac{\int \sqrt{b\text{sech}(c+dx)}\,dx}{3b^2} \\
&= \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}} + \frac{\left(\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int \frac{1}{\sqrt{\cosh(c+dx)}}\,dx}{3b^2} \\
&= -\frac{2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{3b^2d} + \frac{2\sinh(c+dx)}{3bd\sqrt{b\text{sech}(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \frac{\text{sech}^2(c+dx)\left(-2i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)+\sinh(2(c+dx))\right)}{3d(b\text{sech}(c+dx))^{3/2}}$$

```
[In] Integrate[(b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Sech[c + d*x]^2*((-2*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]
+ Sinh[2*(c + d*x)]))/(3*d*(b*Sech[c + d*x])^(3/2))
```

Maple [F]

$$\int \frac{1}{\left(b\,\mathrm{sech}\left(dx+c\right)\right)^{\frac{3}{2}}}dx$$

[In] int(1/(b*sech(d*x+c))^(3/2),x)

[Out] int(1/(b*sech(d*x+c))^(3/2),x)

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.04

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{3/2}}\,dx = \frac{4\sqrt{2}\left(\cosh\left(dx+c\right)^2+2\cosh\left(dx+c\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2\right)\sqrt{b}\mathrm{weierstr}}{}$$

[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="fricas")

[Out] 1/6*(4*sqrt(2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c)) + sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 - 1)*sqrt((b*cosh(d*x + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)

Sympy [F]

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(b\,\mathrm{sech}\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(b*sech(d*x+c))**(3/2),x)

[Out] Integral((b*sech(c + d*x))**(-3/2), x)

Maxima [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(b/cosh(c + d*x))^(3/2),x)
[Out] int(1/(b/cosh(c + d*x))^(3/2), x)
```

3.21 $\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = -\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}$$

```
[Out] 2/5*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(3/2)-6/5*I*(cosh(1/2*d*x+1/2*c)^2)^(1/
2)/cosh(1/2*d*x+1/2*c)*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cosh(
d*x+c)^(1/2)/(b*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3854, 3856, 2719}

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}} - \frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}}$$

```
[In] Int[(b*Sech[c + d*x])^(-5/2),x]
```

```
[Out] (((-6*I)/5)*EllipticE[(I/2)*(c + d*x), 2])/(b^2*d*Sqrt[Cosh[c + d*x]]*Sqrt[
b*Sech[c + d*x]]) + (2*Sinh[c + d*x])/(5*b*d*(b*Sech[c + d*x])^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}} + \frac{3\int \frac{1}{\sqrt{b\text{sech}(c+dx)}}\,dx}{5b^2} \\ &= \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}} + \frac{3\int \sqrt{\cosh(c+dx)}\,dx}{5b^2\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} \\ &= -\frac{6iE\left(\frac{1}{2}i(c+dx)\middle|2\right)}{5b^2d\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}} + \frac{2\sinh(c+dx)}{5bd(b\text{sech}(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \frac{\sqrt{b\text{sech}(c+dx)}\left(-12i\sqrt{\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)\middle|2\right) + \sinh(c+dx) + \sinh(3(c$$

$$10b^3d$$

```
[In] Integrate[(b*Sech[c + d*x])^(-5/2),x]
[Out] (Sqrt[b*Sech[c + d*x]]*((-12*I)*Sqrt[Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*
x), 2] + Sinh[c + d*x] + Sinh[3*(c + d*x)]))/(10*b^3*d)
```

Maple [F]

$$\int \frac{1}{(b\ \text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] int(1/(b*sech(d*x+c))^(5/2),x)
[Out] int(1/(b*sech(d*x+c))^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 379, normalized size of antiderivative = 4.99

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = -\frac{24\sqrt{2}\left(\cosh(dx+c)^3 + 3\cosh(dx+c)^2\sinh(dx+c) + 3\cosh(dx+c)\sinh(dx+c)^2 + \sinh(dx+c)^3\right)\sqrt{}}{}$$

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="fricas")
[Out] -1/20*(24*sqrt(2)*(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*co
sh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)*sqrt(b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c))) - sqrt(2)*(co
sh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6 + (15*cos
h(d*x + c)^2 - 11)*sinh(d*x + c)^4 - 11*cosh(d*x + c)^4 + 4*(5*cosh(d*x + c
)^3 - 11*cosh(d*x + c))*sinh(d*x + c)^3 + (15*cosh(d*x + c)^4 - 66*cosh(d*x
 + c)^2 - 13)*sinh(d*x + c)^2 - 13*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c)^5 -
 22*cosh(d*x + c)^3 - 13*cosh(d*x + c))*sinh(d*x + c) - 1)*sqrt((b*cosh(d*x
 + c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)))/(b^3*d*cosh(d*x + c)^3 + 3*b^3*d*cosh(d*x + c)^2*si
nh(d*x + c) + 3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + b^3*d*sinh(d*x + c)^3
)
```

Sympy [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(c+dx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))**(5/2),x)
[Out] Integral((b*sech(c + d*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{sech}(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cosh(c+d\,x)}\right)^{5/2}}\,dx$$

```
[In] int(1/(b/cosh(c + d*x))^(5/2),x)
[Out] int(1/(b/cosh(c + d*x))^(5/2), x)
```

3.22 $\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 104

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = -\frac{10i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{21b^4d} + \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}}$$

```
[Out] 2/7*sinh(d*x+c)/b/d/(b*sech(d*x+c))^(5/2)+10/21*sinh(d*x+c)/b^3/d/(b*sech(d
*x+c))^(1/2)-10/21*I*(cosh(1/2*d*x+1/2*c)^2)^(1/2)/cosh(1/2*d*x+1/2*c)*Elli
pticF(I*sinh(1/2*d*x+1/2*c),2^(1/2))*cosh(d*x+c)^(1/2)*(b*sech(d*x+c))^(1/2
)/b^4/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = -\frac{10i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{21b^4d} + \frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}} + \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}$$

```
[In] Int[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (((-10*I)/21)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*x), 2]*Sqrt[b*Sech
[c + d*x]])/(b^4*d) + (2*Sinh[c + d*x])/(7*b*d*(b*Sech[c + d*x])^(5/2)) + (
10*Sinh[c + d*x])/(21*b^3*d*Sqrt[b*Sech[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{5\int\frac{1}{(b\text{sech}(c+dx))^{3/2}}\,dx}{7b^2}\\
&= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}}+\frac{5\int\sqrt{b\text{sech}(c+dx)}\,dx}{21b^4}\\
&= \frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}}\\
&\quad+\frac{\left(5\sqrt{\cosh(c+dx)}\sqrt{b\text{sech}(c+dx)}\right)\int\frac{1}{\sqrt{\cosh(c+dx)}}\,dx}{21b^4}\\
&= -\frac{10i\sqrt{\cosh(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)\sqrt{b\text{sech}(c+dx)}}{21b^4d}\\
&\quad+\frac{2\sinh(c+dx)}{7bd(b\text{sech}(c+dx))^{5/2}}+\frac{10\sinh(c+dx)}{21b^3d\sqrt{b\text{sech}(c+dx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{7/2}}\,dx = \frac{\sqrt{b\mathrm{sech}(c+dx)}\Big(-40i\sqrt{\cosh(c+dx)}\,\mathrm{EllipticF}\left(\frac{1}{2}i(c+dx),2\right)+26\sinh(2(c+d}{84b^4d}$$

```
[In] Integrate[(b*Sech[c + d*x])^(-7/2),x]
```

```
[Out] (Sqrt[b*Sech[c + d*x]]*((-40*I)*Sqrt[Cosh[c + d*x]]*EllipticF[(I/2)*(c + d*
x), 2] + 26*Sinh[2*(c + d*x)] + 3*Sinh[4*(c + d*x)]))/(84*b^4*d)
```

Maple [F]

$$\int \frac{1}{(b\ \mathrm{sech}\,(dx+c))^{\frac{7}{2}}}dx$$

```
[In] int(1/(b*sech(d*x+c))^(7/2),x)
```

```
[Out] int(1/(b*sech(d*x+c))^(7/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.64

$$\int \frac{1}{(b\mathrm{sech}(c+dx))^{7/2}}\,dx = \frac{80\sqrt{2}\left(\cosh\left(dx+c\right)^4+4\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)+6\cosh\left(dx+c\right)^2\sinh\left(d\right.\right.}{}$$

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/168*(80*sqrt(2)*(cosh(d*x + c)^4 + 4*cosh(d*x + c)^3*sinh(d*x + c) + 6*co
sh(d*x + c)^2*sinh(d*x + c)^2 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
+ c)^4)*sqrt(b)*weierstrassPInverse(-4, 0, cosh(d*x + c) + sinh(d*x + c)) +
 sqrt(2)*(3*cosh(d*x + c)^8 + 24*cosh(d*x + c)*sinh(d*x + c)^7 + 3*sinh(d*x
 + c)^8 + 2*(42*cosh(d*x + c)^2 + 13)*sinh(d*x + c)^6 + 26*cosh(d*x + c)^6
+ 12*(14*cosh(d*x + c)^3 + 13*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(7*cosh(d
*x + c)^4 + 13*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(21*cosh(d*x + c)^5 + 6
5*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 2*(42*cosh(d*x + c)^6 + 195*cosh(d*x +
 c)^4 - 13)*sinh(d*x + c)^2 - 26*cosh(d*x + c)^2 + 4*(6*cosh(d*x + c)^7 + 3
9*cosh(d*x + c)^5 - 13*cosh(d*x + c))*sinh(d*x + c) - 3)*sqrt((b*cosh(d*x +
 c) + b*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + s
inh(d*x + c)^2 + 1)))/(b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(d*x + c)^3*sinh
(d*x + c) + 6*b^4*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^4*d*cosh(d*x + c)
*sinh(d*x + c)^3 + b^4*d*sinh(d*x + c)^4)
```

Sympy [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(c+dx))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))**(7/2),x)
[Out] Integral((b*sech(c + d*x))**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{(b\,\text{sech}\,(dx+c))^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/(b*sech(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(b\text{sech}(c+dx))^{7/2}}\,dx = \int \frac{1}{\left(\frac{b}{\cosh(c+d\,x)}\right)^{7/2}}\,dx$$

```
[In] int(1/(b/cosh(c + d*x))^(7/2),x)
[Out] int(1/(b/cosh(c + d*x))^(7/2), x)
```

3.23 $\int (b\,\text{sech}(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 75

$$\int (b\,\text{sech}(c+dx))^n\,dx = -\frac{b\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cosh^2(c+dx)\right)(b\,\text{sech}(c+dx))^{-1+n}\sinh(c+dx)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

```
[Out] -b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cosh(d*x+c)^2)*(b*sech(d*x+c))^(-
1+n)*sinh(d*x+c)/d/(1-n)/(-sinh(d*x+c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3857, 2722}

$$\int (b\,\text{sech}(c+dx))^n\,dx = -\frac{b\sinh(c+dx)(b\,\text{sech}(c+dx))^{n-1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cosh^2(c+dx)\right)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

```
[In] Int[(b*Sech[c + d*x])^n,x]
```

```
[Out] -((b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cosh[c + d*x]^2]*(b*Sech[
c + d*x])^(-1 + n)*Sinh[c + d*x])/(d*(1 - n)*Sqrt[-Sinh[c + d*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
```

```
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\text{integral} = \left(\frac{\cosh(c+dx)}{b}\right)^n (b\text{sech}(c+dx))^n \int \left(\frac{\cosh(c+dx)}{b}\right)^{-n} dx$$

$$= -\frac{\cosh(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3-n}{2},\cosh^2(c+dx)\right)(b\text{sech}(c+dx))^n \sinh(c+dx)}{d(1-n)\sqrt{-\sinh^2(c+dx)}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int (b\text{sech}(c+dx))^n\,dx = -\frac{\coth(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\text{sech}^2(c+dx)\right)(b\text{sech}(c+dx))^n\sqrt{\tanh^2(c+dx)}}{dn}$$

```
[In] Integrate[(b*Sech[c + d*x])^n,x]
[Out] -((Coth[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sech[c + d*x]^2]*(b
*Sech[c + d*x])^n*Sqrt[Tanh[c + d*x]^2])/(d*n))
```

Maple [F]

$$\int (b\ \text{sech}\,(dx+c))^n\,dx$$

```
[In] int((b*sech(d*x+c))^n,x)
[Out] int((b*sech(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\text{sech}(c+dx))^n \, dx = \int (b\,\text{sech}\,(dx+c))^n \, dx$$

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sech(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\text{sech}(c+dx))^n \, dx = \int (b\,\text{sech}\,(c+dx))^n \, dx$$

```
[In] integrate((b*sech(d*x+c))**n,x)
[Out] Integral((b*sech(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\text{sech}(c+dx))^n \, dx = \int (b\,\text{sech}\,(dx+c))^n \, dx$$

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c))^n, x)
```

Giac [F]

$$\int (b\text{sech}(c+dx))^n \, dx = \int (b\,\text{sech}\,(dx+c))^n \, dx$$

```
[In] integrate((b*sech(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sech(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\,\mathrm{sech}(c+dx))^n\,dx=\int {\left(\frac{b}{\cosh \left(c+d\,x\right)}\right)}^n \,d x$$

```
[In] int((b/cosh(c + d*x))^n,x)
```

```
[Out] int((b/cosh(c + d*x))^n, x)
```

3.24 $\int \operatorname{sech}^2(a+bx)^{7/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx=\frac{5\arcsin(\tanh(a+bx))}{16b}+\frac{5\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{16b}+\frac{5\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b}+\frac{\operatorname{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b}$$

```
[Out] 5/16*arcsin(tanh(b*x+a))/b+5/24*(sech(b*x+a)^2)^(3/2)*tanh(b*x+a)/b+1/6*(se
ch(b*x+a)^2)^(5/2)*tanh(b*x+a)/b+5/16*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 201, 222}

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx=\frac{5\arcsin(\tanh(a+bx))}{16b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)^{5/2}}{6b}+\frac{5\tanh(a+bx)\operatorname{sech}^2(a+bx)^{3/2}}{24b}+\frac{5\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{16b}$$

```
[In] Int[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (5*ArcSin[Tanh[a + b*x]])/(16*b) + (5*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/
(16*b) + (5*(Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(24*b) + ((Sech[a + b*x]
^2)^(5/2)*Tanh[a + b*x])/(6*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1-x^2)^{5/2}\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\text{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\text{Subst}\left(\int (1-x^2)^{3/2}\,dx, x, \tanh(a+bx)\right)}{6b} \\
&= \frac{5\text{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\text{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} \\
&\quad + \frac{5\text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{8b} \\
&= \frac{5\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{16b} + \frac{5\text{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} \\
&\quad + \frac{\text{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b} + \frac{5\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{16b} \\
&= \frac{5\arcsin(\tanh(a+bx))}{16b} + \frac{5\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{16b} \\
&\quad + \frac{5\text{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{24b} + \frac{\text{sech}^2(a+bx)^{5/2}\tanh(a+bx)}{6b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \frac{\operatorname{sech}(a+bx)\left(15\arctan(\sinh(a+bx))+15\operatorname{sech}(a+bx)\tanh(a+bx)+10\operatorname{sech}^3(a+bx)\tanh(a+bx)\right.}{48b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[In] Integrate[(Sech[a + b*x]^2)^(7/2),x]
```

```
[Out] (Sech[a + b*x]*(15*ArcTan[Sinh[a + b*x]] + 15*Sech[a + b*x]*Tanh[a + b*x] +
 10*Sech[a + b*x]^3*Tanh[a + b*x] + 8*Sech[a + b*x]^5*Tanh[a + b*x]))/(48*b
*Sqrt[Sech[a + b*x]^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.48 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.56

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}\,(15\,e^{10bx+10a}+85\,e^{8bx+8a}+198\,e^{6bx+6a}-198\,e^{4bx+4a}-85\,e^{2bx+2a}-15)}{24(1+e^{2bx+2a})^5 b}+\frac{5i\ln(e^{bx}+ie^{-a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}\,(1+e}{16b}$

```
[In] int((sech(b*x+a)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/(1+exp(2*b*x+2*a))^5*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(15
*exp(10*b*x+10*a)+85*exp(8*b*x+8*a)+198*exp(6*b*x+6*a)-198*exp(4*b*x+4*a)-8
5*exp(2*b*x+2*a)-15)/b+5/16*I*ln(exp(b*x)+I*exp(-a))/b*(1/(1+exp(2*b*x+2*a)
)^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)-5/16*I*ln(exp(b*x)
-I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*
a))*exp(-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1604 vs. $2(76) = 152$.

Time = 0.27 (sec) , antiderivative size = 1604, normalized size of antiderivative = 17.82

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \text{Too large to display}$$

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/24*(15*cosh(b*x + a)^11 + 165*cosh(b*x + a)*sinh(b*x + a)^10 + 15*sinh(b*
x + a)^11 + 5*(165*cosh(b*x + a)^2 + 17)*sinh(b*x + a)^9 + 85*cosh(b*x + a)
^9 + 45*(55*cosh(b*x + a)^3 + 17*cosh(b*x + a))*sinh(b*x + a)^8 + 18*(275*c
osh(b*x + a)^4 + 170*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^7 + 198*cosh(b*x +
 a)^7 + 42*(165*cosh(b*x + a)^5 + 170*cosh(b*x + a)^3 + 33*cosh(b*x + a))*s
inh(b*x + a)^6 + 18*(385*cosh(b*x + a)^6 + 595*cosh(b*x + a)^4 + 231*cosh(b
*x + a)^2 - 11)*sinh(b*x + a)^5 - 198*cosh(b*x + a)^5 + 90*(55*cosh(b*x + a
)^7 + 119*cosh(b*x + a)^5 + 77*cosh(b*x + a)^3 - 11*cosh(b*x + a))*sinh(b*x
 + a)^4 + 5*(495*cosh(b*x + a)^8 + 1428*cosh(b*x + a)^6 + 1386*cosh(b*x + a
)^4 - 396*cosh(b*x + a)^2 - 17)*sinh(b*x + a)^3 - 85*cosh(b*x + a)^3 + 3*(2
75*cosh(b*x + a)^9 + 1020*cosh(b*x + a)^7 + 1386*cosh(b*x + a)^5 - 660*cosh
(b*x + a)^3 - 85*cosh(b*x + a))*sinh(b*x + a)^2 + 15*(cosh(b*x + a)^12 + 12
*cosh(b*x + a)*sinh(b*x + a)^11 + sinh(b*x + a)^12 + 6*(11*cosh(b*x + a)^2
+ 1)*sinh(b*x + a)^10 + 6*cosh(b*x + a)^10 + 20*(11*cosh(b*x + a)^3 + 3*cos
h(b*x + a))*sinh(b*x + a)^9 + 15*(33*cosh(b*x + a)^4 + 18*cosh(b*x + a)^2 +
 1)*sinh(b*x + a)^8 + 15*cosh(b*x + a)^8 + 24*(33*cosh(b*x + a)^5 + 30*cosh
(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a)^7 + 4*(231*cosh(b*x + a)^6 + 3
15*cosh(b*x + a)^4 + 105*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^6 + 20*cosh(b*x
 + a)^6 + 24*(33*cosh(b*x + a)^7 + 63*cosh(b*x + a)^5 + 35*cosh(b*x + a)^3
+ 5*cosh(b*x + a))*sinh(b*x + a)^5 + 15*(33*cosh(b*x + a)^8 + 84*cosh(b*x +
 a)^6 + 70*cosh(b*x + a)^4 + 20*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^4 + 15*c
osh(b*x + a)^4 + 20*(11*cosh(b*x + a)^9 + 36*cosh(b*x + a)^7 + 42*cosh(b*x
+ a)^5 + 20*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 6*(11*cosh
(b*x + a)^10 + 45*cosh(b*x + a)^8 + 70*cosh(b*x + a)^6 + 50*cosh(b*x + a)^4
 + 15*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 6*cosh(b*x + a)^2 + 12*(cosh(b
*x + a)^11 + 5*cosh(b*x + a)^9 + 10*cosh(b*x + a)^7 + 10*cosh(b*x + a)^5 +
5*cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a)
+ sinh(b*x + a)) + 3*(55*cosh(b*x + a)^10 + 255*cosh(b*x + a)^8 + 462*cosh(
b*x + a)^6 - 330*cosh(b*x + a)^4 - 85*cosh(b*x + a)^2 - 5)*sinh(b*x + a) -
15*cosh(b*x + a))/(b*cosh(b*x + a)^12 + 12*b*cosh(b*x + a)*sinh(b*x + a)^11
 + b*sinh(b*x + a)^12 + 6*b*cosh(b*x + a)^10 + 6*(11*b*cosh(b*x + a)^2 + b)
*sinh(b*x + a)^10 + 20*(11*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x
+ a)^9 + 15*b*cosh(b*x + a)^8 + 15*(33*b*cosh(b*x + a)^4 + 18*b*cosh(b*x +
a)^2 + b)*sinh(b*x + a)^8 + 24*(33*b*cosh(b*x + a)^5 + 30*b*cosh(b*x + a)^3
 + 5*b*cosh(b*x + a))*sinh(b*x + a)^7 + 20*b*cosh(b*x + a)^6 + 4*(231*b*cos
h(b*x + a)^6 + 315*b*cosh(b*x + a)^4 + 105*b*cosh(b*x + a)^2 + 5*b)*sinh(b*
x + a)^6 + 24*(33*b*cosh(b*x + a)^7 + 63*b*cosh(b*x + a)^5 + 35*b*cosh(b*x
+ a)^3 + 5*b*cosh(b*x + a))*sinh(b*x + a)^5 + 15*b*cosh(b*x + a)^4 + 15*(33
*b*cosh(b*x + a)^8 + 84*b*cosh(b*x + a)^6 + 70*b*cosh(b*x + a)^4 + 20*b*cos
h(b*x + a)^2 + b)*sinh(b*x + a)^4 + 20*(11*b*cosh(b*x + a)^9 + 36*b*cosh(b*
x + a)^7 + 42*b*cosh(b*x + a)^5 + 20*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))
*sinh(b*x + a)^3 + 6*b*cosh(b*x + a)^2 + 6*(11*b*cosh(b*x + a)^10 + 45*b*co
sh(b*x + a)^8 + 70*b*cosh(b*x + a)^6 + 50*b*cosh(b*x + a)^4 + 15*b*cosh(b*x
 + a)^2 + b)*sinh(b*x + a)^2 + 12*(b*cosh(b*x + a)^11 + 5*b*cosh(b*x + a)^9
 + 10*b*cosh(b*x + a)^7 + 10*b*cosh(b*x + a)^5 + 5*b*cosh(b*x + a)^3 + b*co
```

```
sh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate((sech(b*x+a)**2)**(7/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(76) = 152$.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.73

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = -\frac{5\arctan\left(e^{(-bx-a)}\right)}{8\,b} + \frac{15\,e^{(-bx-a)} + 85\,e^{(-3\,bx-3\,a)} + 198\,e^{(-5\,bx-5\,a)} - 198\,e^{(-7\,bx-7\,a)} - 85\,e^{(-9\,bx-9\,a)} - 15\,e^{(-11\,bx-11\,a)}}{24\,b(6\,e^{(-2\,bx-2\,a)} + 15\,e^{(-4\,bx-4\,a)} + 20\,e^{(-6\,bx-6\,a)} + 15\,e^{(-8\,bx-8\,a)} + 6\,e^{(-10\,bx-10\,a)} + e^{(-12\,bx-12\,a)} + 1)}$$

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
[Out] -5/8*arctan(e^(-b*x - a))/b + 1/24*(15*e^(-b*x - a) + 85*e^(-3*b*x - 3*a) +
 198*e^(-5*b*x - 5*a) - 198*e^(-7*b*x - 7*a) - 85*e^(-9*b*x - 9*a) - 15*e^(
-11*b*x - 11*a))/(b*(6*e^(-2*b*x - 2*a) + 15*e^(-4*b*x - 4*a) + 20*e^(-6*b*
x - 6*a) + 15*e^(-8*b*x - 8*a) + 6*e^(-10*b*x - 10*a) + e^(-12*b*x - 12*a)
+ 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.38

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx = \frac{15\,\pi + \frac{4\left(15\left(e^{(bx+a)}-e^{(-bx-a)}\right)^5+160\left(e^{(bx+a)}-e^{(-bx-a)}\right)^3+528\,e^{(bx+a)}-528\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4\right)^3} + 30\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a}\right.\right.}{96\,b}$$

```
[In] integrate((sech(b*x+a)^2)^(7/2),x, algorithm="giac")
[Out] 1/96*(15*pi + 4*(15*(e^(b*x + a) - e^(-b*x - a))^5 + 160*(e^(b*x + a) - e^(
-b*x - a))^3 + 528*e^(b*x + a) - 528*e^(-b*x - a))/((e^(b*x + a) - e^(-b*x
- a))^2 + 4)^3 + 30*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+bx)^{7/2}\,dx=\int \left(\frac{1}{\cosh{(a+b\,x)}^2}\right)^{7/2}\,dx$$

```
[In] int((1/cosh(a + b*x)^2)^(7/2),x)
[Out] int((1/cosh(a + b*x)^2)^(7/2), x)
```

3.25 $\int \operatorname{sech}^2(a+bx)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx=\frac{3\arcsin(\tanh(a+bx))}{8b}+\frac{3\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{8b}+\frac{\operatorname{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b}$$

```
[Out] 3/8*arcsin(tanh(b*x+a))/b+1/4*(sech(b*x+a)^2)^(3/2)*tanh(b*x+a)/b+3/8*(sech
(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 201, 222}

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx=\frac{3\arcsin(\tanh(a+bx))}{8b}+\frac{\tanh(a+bx)\operatorname{sech}^2(a+bx)^{3/2}}{4b}+\frac{3\tanh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{8b}$$

```
[In] Int[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] (3*ArcSin[Tanh[a + b*x]])/(8*b) + (3*Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(
8*b) + ((Sech[a + b*x]^2)^(3/2)*Tanh[a + b*x])/(4*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (1-x^2)^{3/2}\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\text{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} + \frac{3\text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{4b} \\
&= \frac{3\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{8b} + \frac{\text{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{8b} \\
&= \frac{3\arcsin(\tanh(a+bx))}{8b} + \frac{3\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{8b} + \frac{\text{sech}^2(a+bx)^{3/2}\tanh(a+bx)}{4b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$
\int \text{sech}^2(a+bx)^{5/2}\,dx = \frac{\text{sech}(a+bx)\left(3\arctan(\sinh(a+bx)) + 3\text{sech}(a+bx)\tanh(a+bx) + 2\text{sech}^3(a+bx)\tanh(a+b
}{8b\sqrt{\text{sech}^2(a+bx)}}
$$

```
[In] Integrate[(Sech[a + b*x]^2)^(5/2),x]
```

```
[Out] (Sech[a + b*x]*(3*ArcTan[Sinh[a + b*x]] + 3*Sech[a + b*x]*Tanh[a + b*x] + 2
*Sech[a + b*x]^3*Tanh[a + b*x]))/(8*b*Sqrt[Sech[a + b*x]^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 208, normalized size of antiderivative = 3.20

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\,\left(3\,e^{6bx+6a}+11\,e^{4bx+4a}-11\,e^{2bx+2a}-3\right)}{4\left(1+e^{2bx+2a}\right)^3 b}+\frac{3i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\,\left(1+e^{2bx+2a}\right)e^{-bx-a}}{8b}-\frac{3i\ln\left(e^{bx}-i\right.}{}$

```
[In] int((sech(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(1+exp(2*b*x+2*a))^3*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(3*e
xp(6*b*x+6*a)+11*exp(4*b*x+4*a)-11*exp(2*b*x+2*a)-3)/b+3/8*I*ln(exp(b*x)+I*
exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))
*exp(-b*x-a)-3/8*I*ln(exp(b*x)-I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b
*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 812 vs. $2(55) = 110$.

Time = 0.26 (sec) , antiderivative size = 812, normalized size of antiderivative = 12.49

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*x + a)^7 + 21*cosh(b*x + a)*sinh(b*x + a)^6 + 3*sinh(b*x + a)
^7 + (63*cosh(b*x + a)^2 + 11)*sinh(b*x + a)^5 + 11*cosh(b*x + a)^5 + 5*(21
*cosh(b*x + a)^3 + 11*cosh(b*x + a))*sinh(b*x + a)^4 + (105*cosh(b*x + a)^4
 + 110*cosh(b*x + a)^2 - 11)*sinh(b*x + a)^3 - 11*cosh(b*x + a)^3 + (63*cos
h(b*x + a)^5 + 110*cosh(b*x + a)^3 - 33*cosh(b*x + a))*sinh(b*x + a)^2 + 3*
(cosh(b*x + a)^8 + 8*cosh(b*x + a)*sinh(b*x + a)^7 + sinh(b*x + a)^8 + 4*(7
*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^6 + 4*cosh(b*x + a)^6 + 8*(7*cosh(b*x +
 a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^5 + 2*(35*cosh(b*x + a)^4 + 30*cosh(
b*x + a)^2 + 3)*sinh(b*x + a)^4 + 6*cosh(b*x + a)^4 + 8*(7*cosh(b*x + a)^5
+ 10*cosh(b*x + a)^3 + 3*cosh(b*x + a))*sinh(b*x + a)^3 + 4*(7*cosh(b*x + a
)^6 + 15*cosh(b*x + a)^4 + 9*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 4*cosh(
b*x + a)^2 + 8*(cosh(b*x + a)^7 + 3*cosh(b*x + a)^5 + 3*cosh(b*x + a)^3 + c
osh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) + (2
1*cosh(b*x + a)^6 + 55*cosh(b*x + a)^4 - 33*cosh(b*x + a)^2 - 3)*sinh(b*x +
 a) - 3*cosh(b*x + a))/(b*cosh(b*x + a)^8 + 8*b*cosh(b*x + a)*sinh(b*x + a)
^7 + b*sinh(b*x + a)^8 + 4*b*cosh(b*x + a)^6 + 4*(7*b*cosh(b*x + a)^2 + b)*
sinh(b*x + a)^6 + 8*(7*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)
```

```
^5 + 6*b*cosh(b*x + a)^4 + 2*(35*b*cosh(b*x + a)^4 + 30*b*cosh(b*x + a)^2 +
 3*b)*sinh(b*x + a)^4 + 8*(7*b*cosh(b*x + a)^5 + 10*b*cosh(b*x + a)^3 + 3*b
*cosh(b*x + a))*sinh(b*x + a)^3 + 4*b*cosh(b*x + a)^2 + 4*(7*b*cosh(b*x + a
)^6 + 15*b*cosh(b*x + a)^4 + 9*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 8*(
b*cosh(b*x + a)^7 + 3*b*cosh(b*x + a)^5 + 3*b*cosh(b*x + a)^3 + b*cosh(b*x
+ a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \int \left(\operatorname{sech}^2(a+bx)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((sech(b*x+a)**2)**(5/2),x)
[Out] Integral((sech(a + b*x)**2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.72

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = -\frac{3\arctan\left(e^{(-bx-a)}\right)}{4\,b} + \frac{3\,e^{(-bx-a)} + 11\,e^{(-3\,bx-3\,a)} - 11\,e^{(-5\,bx-5\,a)} - 3\,e^{(-7\,bx-7\,a)}}{4\,b(4\,e^{(-2\,bx-2\,a)} + 6\,e^{(-4\,bx-4\,a)} + 4\,e^{(-6\,bx-6\,a)} + e^{(-8\,bx-8\,a)} + 1)}$$

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
[Out] -3/4*arctan(e^(-b*x - a))/b + 1/4*(3*e^(-b*x - a) + 11*e^(-3*b*x - 3*a) - 1
1*e^(-5*b*x - 5*a) - 3*e^(-7*b*x - 7*a))/(b*(4*e^(-2*b*x - 2*a) + 6*e^(-4*b
*x - 4*a) + 4*e^(-6*b*x - 6*a) + e^(-8*b*x - 8*a) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.57

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx = \frac{3\,\pi + \frac{4\left(3\left(e^{(bx+a)} - e^{(-bx-a)}\right)^3 + 20\,e^{(bx+a)} - 20\,e^{(-bx-a)}\right)}{\left(\left(e^{(bx+a)} - e^{(-bx-a)}\right)^2 + 4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)} - 1\right)e^{(-bx-a)}\right)}{16\,b}$$

```
[In] integrate((sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*pi + 4*(3*(e^(b*x + a) - e^(-b*x - a))^3 + 20*e^(b*x + a) - 20*e^(-
b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 + 4)^2 + 6*arctan(1/2*(e^(2*b*x +
 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+bx)^{5/2}\,dx=\int \left(\frac{1}{\cosh\left(a+b\,x\right)^2}\right)^{5/2}\,dx$$

```
[In] int((1/cosh(a + b*x)^2)^(5/2),x)
```

```
[Out] int((1/cosh(a + b*x)^2)^(5/2), x)
```

3.26 $\int \text{sech}^2(a+bx)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 40

$$\int \text{sech}^2(a+bx)^{3/2}\,dx=\frac{\arcsin(\tanh(a+bx))}{2b}+\frac{\sqrt{\text{sech}^2(a+bx)}\tanh(a+bx)}{2b}$$

```
[Out] 1/2*arcsin(tanh(b*x+a))/b+1/2*(sech(b*x+a)^2)^(1/2)*tanh(b*x+a)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 201, 222}

$$\int \text{sech}^2(a+bx)^{3/2}\,dx=\frac{\arcsin(\tanh(a+bx))}{2b}+\frac{\tanh(a+bx)\sqrt{\text{sech}^2(a+bx)}}{2b}$$

```
[In] Int[(Sech[a + b*x]^2)^(3/2),x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/(2*b) + (Sqrt[Sech[a + b*x]^2]*Tanh[a + b*x])/(2*b)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{1-x^2}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{2b} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{2b}\\ &= \frac{\arcsin(\tanh(a+bx))}{2b} + \frac{\sqrt{\operatorname{sech}^2(a+bx)}\tanh(a+bx)}{2b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \frac{\operatorname{sech}(a+bx)(\arctan(\sinh(a+bx)) + \operatorname{sech}(a+bx)\tanh(a+bx))}{2b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[In] Integrate[(Sech[a + b*x]^2)^(3/2),x]
[Out] (Sech[a + b*x]*(ArcTan[Sinh[a + b*x]] + Sech[a + b*x]*Tanh[a + b*x]))/(2*b*
Sqrt[Sech[a + b*x]^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 4.58

method	result
risch	$\frac{\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}\,(e^{2bx+2a}-1)}{(1+e^{2bx+2a})b} + \frac{i\ln(e^{bx}+ie^{-a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}\,(1+e^{2bx+2a})e^{-bx-a}}{2b} - \frac{i\ln(e^{bx}-ie^{-a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}\,(1+e^{2b\cdots}}{2b}$

```
[In] int((sech(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(exp(2*b
*x+2*a)-1)/b+1/2*I*ln(exp(b*x)+I*exp(-a))/b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b
*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x-a)-1/2*I*ln(exp(b*x)-I*exp(-a))/
b*(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*(1+exp(2*b*x+2*a))*exp(-b*x
-a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 6.68

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 + \sinh(bx+a)^3 + \left(\cosh(bx+a)^4 + 4\cosh(bx \cdots\right.}{b\cosh(bx+a)^4 + 4 \cdots}$$

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 + sinh(b*x + a)^3 + (cos
h(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*(3*cos
h(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3
+ cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a)) +
 (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x + a))/(b*cosh(b*x + a)^4
+ 4*b*cosh(b*x + a)*sinh(b*x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)
^2 + 2*(3*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b
*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \int \left(\operatorname{sech}^2(a+bx)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Integral((sech(a + b*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = -\frac{\arctan\left(e^{(-bx-a)}\right)}{b} + \frac{e^{(-bx-a)} - e^{(-3\,bx-3\,a)}}{b(2\,e^{(-2\,bx-2\,a)} + e^{(-4\,bx-4\,a)} + 1)}$$

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
[Out] -arctan(e^(-b*x - a))/b + (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x
 - 2*a) + e^(-4*b*x - 4*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \frac{\pi + \frac{4\left(e^{(bx+a)}-e^{(-bx-a)}\right)}{\left(e^{(bx+a)}-e^{(-bx-a)}\right)^2+4} + 2\arctan\left(\frac{1}{2}\left(e^{(2\,bx+2\,a)}-1\right)e^{(-bx-a)}\right)}{4\,b}$$

```
[In] integrate((sech(b*x+a)^2)^(3/2),x, algorithm="giac")
[Out] 1/4*(pi + 4*(e^(b*x + a) - e^(-b*x - a))/((e^(b*x + a) - e^(-b*x - a))^2 +
4) + 2*arctan(1/2*(e^(2*b*x + 2*a) - 1)*e^(-b*x - a)))/b
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a+bx)^{3/2}\,dx = \int \left(\frac{1}{\cosh\left(a+b\,x\right)^2}\right)^{3/2} dx$$

```
[In] int((1/cosh(a + b*x)^2)^(3/2),x)
[Out] int((1/cosh(a + b*x)^2)^(3/2), x)
```

3.27 $\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx=\frac{\arcsin(\tanh(a+bx))}{b}$$

```
[Out] arcsin(tanh(b*x+a))/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4207, 222}

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx=\frac{\arcsin(\tanh(a+bx))}{b}$$

```
[In] Int[Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] ArcSin[Tanh[a + b*x]]/b
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\arcsin(\tanh(a+bx))}{b}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(11) = 22.

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{\arctan(\sinh(a+bx))\cosh(a+bx)\sqrt{\operatorname{sech}^2(a+bx)}}{b}$$

```
[In] Integrate[Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] (ArcTan[Sinh[a + b*x]]*Cosh[a + b*x]*Sqrt[Sech[a + b*x]^2])/b
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 130, normalized size of antiderivative = 11.82

method	result	size
risch	$\frac{i\ln\left(e^{bx}+ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\left(1+e^{2bx+2a}\right)e^{-bx-a}}{b}-\frac{i\ln\left(e^{bx}-ie^{-a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}\left(1+e^{2bx+2a}\right)e^{-bx-a}}{b}$	130

```
[In] int((sech(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*ln(exp(b*x)+I*exp(-a))/b*(1+exp(2*b*x+2*a))*(1/(1+exp(2*b*x+2*a))^2*exp(2
*b*x+2*a))^(1/2)*exp(-b*x-a)-I*ln(exp(b*x)-I*exp(-a))/b*(1+exp(2*b*x+2*a))*
(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-b*x-a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{2\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)\right)}{b}$$

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] 2*arctan(cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \int \sqrt{\operatorname{sech}^2\left(a+bx\right)}\,dx$$

```
[In] integrate((sech(b*x+a)**2)**(1/2),x)
[Out] Integral(sqrt(sech(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{\arctan\left(\sinh\left(bx+a\right)\right)}{b}$$

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] arctan(sinh(b*x + a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \frac{2\arctan\left(e^{(bx+a)}\right)}{b}$$

```
[In] integrate((sech(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] 2*arctan(e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\operatorname{sech}^2(a+bx)}\,dx = \int \sqrt{\frac{1}{\cosh\left(a+b\,x\right)^2}}\,dx$$

```
[In] int((1/cosh(a + b*x)^2)^(1/2),x)
[Out] int((1/cosh(a + b*x)^2)^(1/2), x)
```

3.28 $\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[Out] tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4207, 197}

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[In] Int[1/Sqrt[Sech[a + b*x]^2],x]
```

```
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{b}$$
$$= \frac{\tanh(a+bx)}{b\sqrt{\text{sech}^2(a+bx)}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\text{sech}^2(a+bx)}}\,dx = \frac{\tanh(a+bx)}{b\sqrt{\text{sech}^2(a+bx)}}$$

```
[In] Integrate[1/Sqrt[Sech[a + b*x]^2],x]
[Out] Tanh[a + b*x]/(b*Sqrt[Sech[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(20) = 40.

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 4.41

method	result	size
risch	$\frac{e^{2bx+2a}}{2b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}} - \frac{1}{2b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}$	97

```
[In] int(1/(sech(b*x+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(
2*b*x+2*a)-1/2/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{\sinh(bx+a)}{b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="fricas")
[Out] sinh(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \begin{cases} \frac{\tanh(a+bx)}{b\sqrt{\operatorname{sech}^2(a+bx)}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{\operatorname{sech}^2(a)}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(sech(b*x+a)**2)**(1/2),x)
[Out] Piecewise((tanh(a + b*x)/(b*sqrt(sech(a + b*x)**2)), Ne(b, 0)), (x/sqrt(sec
h(a)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{e^{(bx+a)} - e^{(-bx-a)}}{2\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(e^(b*x + a) - e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{\operatorname{sech}^2(a+bx)}}\,dx = \frac{\mathrm{e}^{-2\,a-2\,b\,x}\,\left(\mathrm{e}^{4\,a+4\,b\,x}-1\right)\,\sqrt{\frac{4\,\mathrm{e}^{2\,a+2\,b\,x}}{{\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}^2}}}{4\,b}$$

```
[In] int(1/(1/cosh(a + b*x)^2)^(1/2),x)
[Out] (exp(- 2*a - 2*b*x)*(exp(4*a + 4*b*x) - 1)*((4*exp(2*a + 2*b*x))/(exp(2*a +
 2*b*x) + 1)^2)^(1/2))/(4*b)
```

3.29 $\int \frac{1}{\mathbf{sech}^2(a+bx)^{3/2}} dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}} dx = \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[Out] 1/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+2/3*tanh(b*x+a)/b/(sech(b*x+a)^2)^(
1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 198, 197}

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}} dx = \frac{2\tanh(a+bx)}{3b\sqrt{\operatorname{sech}^2(a+bx)}} + \frac{\tanh(a+bx)}{3b\operatorname{sech}^2(a+bx)^{3/2}}$$

```
[In] Int[(Sech[a + b*x]^2)^(-3/2),x]
```

```
[Out] Tanh[a + b*x]/(3*b*(Sech[a + b*x]^2)^(3/2)) + (2*Tanh[a + b*x])/(3*b*Sqrt[S
ech[a + b*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{b}\\ &= \frac{\tanh(a+bx)}{3b\text{sech}^2(a+bx)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{3b}\\ &= \frac{\tanh(a+bx)}{3b\text{sech}^2(a+bx)^{3/2}} + \frac{2\tanh(a+bx)}{3b\sqrt{\text{sech}^2(a+bx)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{1}{\text{sech}^2(a+bx)^{3/2}}\,dx = \frac{3\text{sech}^2(a+bx)\tanh(a+bx) + \tanh^3(a+bx)}{3b\text{sech}^2(a+bx)^{3/2}}$$

```
[In] Integrate[(Sech[a + b*x]^2)^(-3/2),x]
[Out] (3*Sech[a + b*x]^2*Tanh[a + b*x] + Tanh[a + b*x]^3)/(3*b*(Sech[a + b*x]^2)^
(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(43) = 86.

Time = 0.46 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.94

method	result
risch	$\frac{e^{4bx+4a}}{24b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}} + \frac{3\,e^{2bx+2a}}{8b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}} - \frac{3}{8b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}} - \frac{e^{-2bx-2}}{24b(1+e^{2bx+2a})\sqrt{}}$

```
[In] int(1/(sech(b*x+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp
(4*b*x+4*a)+3/8/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a)
)^(1/2)*exp(2*b*x+2*a)-3/8/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp
(2*b*x+2*a))^(1/2)-1/24/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*
b*x+2*a))^(1/2)*exp(-2*b*x-2*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \frac{\sinh\left(bx+a\right)^3+3\left(\cosh\left(bx+a\right)^2+3\right)\sinh\left(bx+a\right)}{12\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \begin{cases} -\frac{2\tanh^3(a+bx)}{3b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{3}{2}}}+\frac{\tanh(a+bx)}{b\left(\operatorname{sech}^2(a+bx)\right)^{\frac{3}{2}}} & \text{for } b\neq 0 \\ \frac{x}{\left(\operatorname{sech}^2(a)\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(sech(b*x+a)**2)**(3/2),x)
```

```
[Out] Piecewise((-2*tanh(a + b*x)**3/(3*b*(sech(a + b*x)**2)**(3/2)) + tanh(a + b
*x)/(b*(sech(a + b*x)**2)**(3/2)), Ne(b, 0)), (x/(sech(a)**2)**(3/2), True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{3\,e^{(-bx-a)}}{8\,b}-\frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(-
3*b*x - 3*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = -\frac{\left(9\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-e^{(3\,bx+3\,a)}-9\,e^{(bx+a)}}{24\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(3/2),x, algorithm="giac")
[Out] -1/24*((9*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - e^(3*b*x + 3*a) - 9*e^(b*
x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{3/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(1/cosh(a + b*x)^2)^(3/2),x)
[Out] int(1/(1/cosh(a + b*x)^2)^(3/2), x)
```

3.30 $\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}} dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}} dx = \frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{8\tanh(a+bx)}{15b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[Out] 1/5*tanh(b*x+a)/b/(sech(b*x+a)^2)^(5/2)+4/15*tanh(b*x+a)/b/(sech(b*x+a)^2)^
(3/2)+8/15*tanh(b*x+a)/b/(sech(b*x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 198, 197}

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}} dx = \frac{8\tanh(a+bx)}{15b\sqrt{\operatorname{sech}^2(a+bx)}} + \frac{4\tanh(a+bx)}{15b\operatorname{sech}^2(a+bx)^{3/2}} + \frac{\tanh(a+bx)}{5b\operatorname{sech}^2(a+bx)^{5/2}}$$

```
[In] Int[(Sech[a + b*x]^2)^(-5/2),x]
```

```
[Out] Tanh[a + b*x]/(5*b*(Sech[a + b*x]^2)^(5/2)) + (4*Tanh[a + b*x])/(15*b*(Sech
[a + b*x]^2)^(3/2)) + (8*Tanh[a + b*x])/(15*b*Sqrt[Sech[a + b*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{5b} \\
&= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}} + \frac{8\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{15b} \\
&= \frac{\tanh(a+bx)}{5b\text{sech}^2(a+bx)^{5/2}} + \frac{4\tanh(a+bx)}{15b\text{sech}^2(a+bx)^{3/2}} + \frac{8\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.62

$$
\int \frac{1}{\text{sech}^2(a+bx)^{5/2}}\,dx = \frac{\left(15+10\sinh^2(a+bx)+3\sinh^4(a+bx)\right)\tanh(a+bx)}{15b\sqrt{\text{sech}^2(a+bx)}}
$$

```
[In] Integrate[(Sech[a + b*x]^2)^(-5/2),x]
[Out] ((15 + 10*Sinh[a + b*x]^2 + 3*Sinh[a + b*x]^4)*Tanh[a + b*x])/(15*b*Sqrt[Se
ch[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(64) = 128.

Time = 0.47 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.01

method	result
risch	$\frac{e^{6bx+6a}}{160b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}+\frac{5\,e^{4bx+4a}}{96b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}+\frac{5\,e^{2bx+2a}}{16b(1+e^{2bx+2a})\sqrt{\frac{e^{2bx+2a}}{(1+e^{2bx+2a})^2}}}-\frac{5}{16b(1+e^{2bx+2a})\sqrt{}}$

```
[In] int(1/(sech(b*x+a)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*ex
p(6*b*x+6*a)+5/96/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*
a))^(1/2)*exp(4*b*x+4*a)+5/16/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*
exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-5/16/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*
b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)-5/96/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x
+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-1/160/b/(1+exp(2*b*x+2*a))/(
1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-4*b*x-4*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.87

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \frac{3\,\sinh\left(bx+a\right)^5+5\left(6\,\cosh\left(bx+a\right)^2+5\right)\sinh\left(bx+a\right)^3+15\left(\cosh\left(bx+a\right)^4+5\,c\right.}{240\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*(
cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 1.79 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \begin{cases} \frac{8\tanh^5(a+bx)}{15b(\operatorname{sech}^2(a+bx))^{\frac{5}{2}}}-\frac{4\tanh^3(a+bx)}{3b(\operatorname{sech}^2(a+bx))^{\frac{5}{2}}}+\frac{\tanh(a+bx)}{b(\operatorname{sech}^2(a+bx))^{\frac{5}{2}}} & \text{for } b\neq 0 \\ \frac{x}{(\operatorname{sech}^2(a))^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(sech(b*x+a)**2)**(5/2),x)
```

```
[Out] Piecewise((8*tanh(a + b*x)**5/(15*b*(sech(a + b*x)**2)**(5/2)) - 4*tanh(a +
 b*x)**3/(3*b*(sech(a + b*x)**2)**(5/2)) + tanh(a + b*x)/(b*(sech(a + b*x)*
*2)**(5/2)), Ne(b, 0)), (x/(sech(a)**2)**(5/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/1
6*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = -\frac{\left(150\,e^{(4\,bx+4\,a)} + 25\,e^{(2\,bx+2\,a)} + 3\right)e^{(-5\,bx-5\,a)} - 3\,e^{(5\,bx+5\,a)} - 25\,e^{(3\,bx+3\,a)} - 150\,e^{(bx+a)}}{480\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*b*x + 4*a) + 25*e^(2*b*x + 2*a) + 3)*e^(-5*b*x - 5*a) - 3
*e^(5*b*x + 5*a) - 25*e^(3*b*x + 3*a) - 150*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{5/2}}\,dx = \int \frac{1}{{\left(\frac{1}{{\mathrm{cosh}\left(a+b\,x\right)}^2}\right)}^{5/2}}\,dx$$

```
[In] int(1/(1/cosh(a + b*x)^2)^(5/2),x)
```

```
[Out] int(1/(1/cosh(a + b*x)^2)^(5/2), x)
```

3.31 $\int \frac{1}{\mathbf{sech}^2(a+bx)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{\mathrm{sech}^2(a+bx)^{7/2}}\,dx = \frac{\tanh(a+bx)}{7b\,\mathrm{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\,\mathrm{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\,\mathrm{sech}^2(a+bx)^{3/2}} + \frac{16\tanh(a+bx)}{35b\sqrt{\mathrm{sech}^2(a+bx)}}$$

```
[Out] 1/7*tanh(b*x+a)/b/(sech(b*x+a)^2)^(7/2)+6/35*tanh(b*x+a)/b/(sech(b*x+a)^2)^
(5/2)+8/35*tanh(b*x+a)/b/(sech(b*x+a)^2)^(3/2)+16/35*tanh(b*x+a)/b/(sech(b*
x+a)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4207, 198, 197}

$$\int \frac{1}{\mathrm{sech}^2(a+bx)^{7/2}}\,dx = \frac{16\tanh(a+bx)}{35b\sqrt{\mathrm{sech}^2(a+bx)}} + \frac{8\tanh(a+bx)}{35b\,\mathrm{sech}^2(a+bx)^{3/2}} + \frac{6\tanh(a+bx)}{35b\,\mathrm{sech}^2(a+bx)^{5/2}} + \frac{\tanh(a+bx)}{7b\,\mathrm{sech}^2(a+bx)^{7/2}}$$

```
[In] Int[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] Tanh[a + b*x]/(7*b*(Sech[a + b*x]^2)^(7/2)) + (6*Tanh[a + b*x])/(35*b*(Sech
[a + b*x]^2)^(5/2)) + (8*Tanh[a + b*x])/(35*b*(Sech[a + b*x]^2)^(3/2)) + (1
6*Tanh[a + b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^{9/2}}\,dx, x, \tanh(a+bx)\right)}{b} \\
&= \frac{\tanh(a+bx)}{7b\text{sech}^2(a+bx)^{7/2}} + \frac{6\text{Subst}\left(\int \frac{1}{(1-x^2)^{7/2}}\,dx, x, \tanh(a+bx)\right)}{7b} \\
&= \frac{\tanh(a+bx)}{7b\text{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{5/2}} + \frac{24\text{Subst}\left(\int \frac{1}{(1-x^2)^{5/2}}\,dx, x, \tanh(a+bx)\right)}{35b} \\
&= \frac{\tanh(a+bx)}{7b\text{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{3/2}} \\
&\quad + \frac{16\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}}\,dx, x, \tanh(a+bx)\right)}{35b} \\
&= \frac{\tanh(a+bx)}{7b\text{sech}^2(a+bx)^{7/2}} + \frac{6\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{5/2}} + \frac{8\tanh(a+bx)}{35b\text{sech}^2(a+bx)^{3/2}} + \frac{16\tanh(a+bx)}{35b\sqrt{\text{sech}^2(a+bx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.56

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{\left(35+35\sinh^2(a+bx)+21\sinh^4(a+bx)+5\sinh^6(a+bx)\right)\tanh(a+bx)}{35b\sqrt{\operatorname{sech}^2(a+bx)}}$$

```
[In] Integrate[(Sech[a + b*x]^2)^(-7/2),x]
```

```
[Out] ((35 + 35*Sinh[a + b*x]^2 + 21*Sinh[a + b*x]^4 + 5*Sinh[a + b*x]^6)*Tanh[a
+ b*x])/(35*b*Sqrt[Sech[a + b*x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(85) = 170.

Time = 0.48 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.05

method	result
risch	$\frac{e^{8bx+8a}}{896b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{7\,e^{6bx+6a}}{640b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{7\,e^{4bx+4a}}{128b\left(1+e^{2bx+2a}\right)\sqrt{\frac{e^{2bx+2a}}{\left(1+e^{2bx+2a}\right)^2}}}+\frac{35\,e^{2}}{128b(1+e^{2bx+2a}}$

```
[In] int(1/(sech(b*x+a)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/896/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*ex
p(8*b*x+8*a)+7/640/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2
*a))^(1/2)*exp(6*b*x+6*a)+7/128/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^
2*exp(2*b*x+2*a))^(1/2)*exp(4*b*x+4*a)+35/128/b/(1+exp(2*b*x+2*a))/(1/(1+ex
p(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(2*b*x+2*a)-35/128/b/(1+exp(2*b*x+
2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)-7/128/b/(1+exp(2*b*x+2*
a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-2*b*x-2*a)-7/640/b/(
1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/2)*exp(-4*b*x-
4*a)-1/896/b/(1+exp(2*b*x+2*a))/(1/(1+exp(2*b*x+2*a))^2*exp(2*b*x+2*a))^(1/
2)*exp(-6*b*x-6*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{5\,\sinh\left(bx+a\right)^{7} + 7\,\left(15\,\cosh\left(bx+a\right)^{2} + 7\right)\sinh\left(bx+a\right)^{5} + 35\,\left(5\,\cosh\left(bx+a\right)^{4} \cdots\right.}{\cdots}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/2240*(5*sinh(b*x + a)^7 + 7*(15*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^5 + 35
*(5*cosh(b*x + a)^4 + 14*cosh(b*x + a)^2 + 7)*sinh(b*x + a)^3 + 35*(cosh(b*
x + a)^6 + 7*cosh(b*x + a)^4 + 21*cosh(b*x + a)^2 + 35)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 17.86 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \begin{cases} -\frac{16\tanh^{7}(a+bx)}{35b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{7}{2}}} + \frac{8\tanh^{5}(a+bx)}{5b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{7}{2}}} - \frac{2\tanh^{3}(a+bx)}{b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{7}{2}}} + \frac{\tanh(a+bx)}{b\left(\operatorname{sech}^{2}(a+bx)\right)^{\frac{7}{2}}} & \text{for } b \neq 0 \\ \frac{x}{\left(\operatorname{sech}^{2}(a)\right)^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

```
[In] integrate(1/(sech(b*x+a)**2)**(7/2),x)
```

```
[Out] Piecewise((-16*tanh(a + b*x)**7/(35*b*(sech(a + b*x)**2)**(7/2)) + 8*tanh(a
 + b*x)**5/(5*b*(sech(a + b*x)**2)**(7/2)) - 2*tanh(a + b*x)**3/(b*(sech(a
+ b*x)**2)**(7/2)) + tanh(a + b*x)/(b*(sech(a + b*x)**2)**(7/2)), Ne(b, 0))
, (x/(sech(a)**2)**(7/2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.99

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \frac{\left(49\,e^{(-2\,bx-2\,a)} + 245\,e^{(-4\,bx-4\,a)} + 1225\,e^{(-6\,bx-6\,a)} + 5\right)e^{(7\,bx+7\,a)}}{4480\,b} - \frac{1225\,e^{(-bx-a)} + 245\,e^{(-3\,bx-3\,a)} + 49\,e^{(-5\,bx-5\,a)} + 5\,e^{(-7\,bx-7\,a)}}{4480\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/4480*(49*e^(-2*b*x - 2*a) + 245*e^(-4*b*x - 4*a) + 1225*e^(-6*b*x - 6*a)
+ 5)*e^(7*b*x + 7*a)/b - 1/4480*(1225*e^(-b*x - a) + 245*e^(-3*b*x - 3*a) +
 49*e^(-5*b*x - 5*a) + 5*e^(-7*b*x - 7*a))/b
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = -\frac{\left(1225\,e^{(6\,bx+6\,a)}+245\,e^{(4\,bx+4\,a)}+49\,e^{(2\,bx+2\,a)}+5\right)e^{(-7\,bx-7\,a)}-5\,e^{(7\,bx+7\,a)}-49\,e^{(5\,bx+5\,a)}-245\,e^{(3\,bx+3\,a)}-}{4480\,b}$$

```
[In] integrate(1/(sech(b*x+a)^2)^(7/2),x, algorithm="giac")
[Out] -1/4480*((1225*e^(6*b*x + 6*a) + 245*e^(4*b*x + 4*a) + 49*e^(2*b*x + 2*a) +
 5)*e^(-7*b*x - 7*a) - 5*e^(7*b*x + 7*a) - 49*e^(5*b*x + 5*a) - 245*e^(3*b*
x + 3*a) - 1225*e^(b*x + a))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^2(a+bx)^{7/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cosh(a+b\,x)^2}\right)^{7/2}}\,dx$$

```
[In] int(1/(1/cosh(a + b*x)^2)^(7/2),x)
[Out] int(1/(1/cosh(a + b*x)^2)^(7/2), x)
```

3.32 $\int \left(a\text{sech}^2(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 65

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \frac{3}{8}a^{5/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right) + \frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x) + \frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x)$$

```
[Out] 3/8*a^(5/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+1/4*a*(a*sech(x)^2)
^(3/2)*tanh(x)+3/8*a^2*(a*sech(x)^2)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4207, 201, 223, 209}

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \frac{3}{8}a^{5/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right) + \frac{3}{8}a^2\tanh(x)\sqrt{a\text{sech}^2(x)} + \frac{1}{4}a\tanh(x)\left(a\text{sech}^2(x)\right)^{3/2}$$

```
[In] Int[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (3*a^(5/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/8 + (3*a^2*Sqrt[a*S
ech[x]^2]*Tanh[x])/8 + (a*(a*Sech[x]^2)^(3/2)*Tanh[x])/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\text{Subst}\left(\int \left(a-ax^2\right)^{3/2}\,dx,x,\tanh(x)\right)\\
&= \frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x)+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \sqrt{a-ax^2}\,dx,x,\tanh(x)\right)\\
&= \frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x)\\
&\quad+\frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x)+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx,x,\tanh(x)\right)\\
&= \frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x)\\
&\quad+\frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x)+\frac{1}{8}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx,x,\frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)\\
&= \frac{3}{8}a^{5/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)+\frac{3}{8}a^2\sqrt{a\text{sech}^2(x)}\tanh(x)+\frac{1}{4}a\left(a\text{sech}^2(x)\right)^{3/2}\tanh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.58

$$\int \left(a\operatorname{sech}^2(x)\right)^{5/2}\,dx=\frac{1}{8}\cosh(x)\left(a\operatorname{sech}^2(x)\right)^{5/2}\left(3\arctan(\sinh(x))\cosh^4(x)+2\sinh(x)+3\cosh^2(x)\sinh(x)\right)$$

```
[In] Integrate[(a*Sech[x]^2)^(5/2),x]
```

```
[Out] (Cosh[x]*(a*Sech[x]^2)^(5/2)*(3*ArcTan[Sinh[x]]*Cosh[x]^4 + 2*Sinh[x] + 3*C
osh[x]^2*Sinh[x]))/8
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.91 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.95

method	result
risch	$\frac{a^2\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,(3\,e^{6x}+11\,e^{4x}-11\,e^{2x}-3)}{4(1+e^{2x})^3}+\frac{3ia^2e^{-x}(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,\ln(e^x+i)}{8}-\frac{3ia^2e^{-x}(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,\ln(e^x-i)}{8}$

```
[In] int((sech(x)^2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2/(1+exp(2*x))^3*(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)*(3*exp(6*x)+11*exp
(4*x)-11*exp(2*x)-3)+3/8*I*a^2*exp(-x)*(1+exp(2*x))*(exp(2*x)*a/(1+exp(2*x)
)^2)^(1/2)*ln(exp(x)+I)-3/8*I*a^2*exp(-x)*(1+exp(2*x))*(exp(2*x)*a/(1+exp(2
*x))^2)^(1/2)*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1082 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 1082, normalized size of antiderivative = 16.65

$$\int \left(a\operatorname{sech}^2(x)\right)^{5/2}\,dx=\text{Too large to display}$$

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*a^2*cosh(x)^7 + 3*(a^2*e^(2*x) + a^2)*sinh(x)^7 + 11*a^2*cosh(x)^5 +
 21*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^6 + (63*a^2*cosh(x)^2 + 11*
a^2 + (63*a^2*cosh(x)^2 + 11*a^2)*e^(2*x))*sinh(x)^5 - 11*a^2*cosh(x)^3 + 5
*(21*a^2*cosh(x)^3 + 11*a^2*cosh(x) + (21*a^2*cosh(x)^3 + 11*a^2*cosh(x))*e
^(2*x))*sinh(x)^4 + (105*a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2 + (105*
a^2*cosh(x)^4 + 110*a^2*cosh(x)^2 - 11*a^2)*e^(2*x))*sinh(x)^3 - 3*a^2*cosh
```

```
(x) + (63*a^2*cosh(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x) + (63*a^2*cosh
(x)^5 + 110*a^2*cosh(x)^3 - 33*a^2*cosh(x))*e^(2*x))*sinh(x)^2 + 3*(a^2*cos
h(x)^8 + (a^2*e^(2*x) + a^2)*sinh(x)^8 + 4*a^2*cosh(x)^6 + 8*(a^2*cosh(x)*e
^(2*x) + a^2*cosh(x))*sinh(x)^7 + 4*(7*a^2*cosh(x)^2 + a^2 + (7*a^2*cosh(x)
^2 + a^2)*e^(2*x))*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 + 3*a^2
*cosh(x) + (7*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 2*(35*a^2
*cosh(x)^4 + 30*a^2*cosh(x)^2 + 3*a^2 + (35*a^2*cosh(x)^4 + 30*a^2*cosh(x)^
2 + 3*a^2)*e^(2*x))*sinh(x)^4 + 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 + 10*a
^2*cosh(x)^3 + 3*a^2*cosh(x) + (7*a^2*cosh(x)^5 + 10*a^2*cosh(x)^3 + 3*a^2*
cosh(x))*e^(2*x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2
*cosh(x)^2 + a^2 + (7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 +
a^2)*e^(2*x))*sinh(x)^2 + a^2 + (a^2*cosh(x)^8 + 4*a^2*cosh(x)^6 + 6*a^2*co
sh(x)^4 + 4*a^2*cosh(x)^2 + a^2)*e^(2*x) + 8*(a^2*cosh(x)^7 + 3*a^2*cosh(x)
^5 + 3*a^2*cosh(x)^3 + a^2*cosh(x) + (a^2*cosh(x)^7 + 3*a^2*cosh(x)^5 + 3*a
^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))*arctan(cosh(x) + sinh(x)) + (
3*a^2*cosh(x)^7 + 11*a^2*cosh(x)^5 - 11*a^2*cosh(x)^3 - 3*a^2*cosh(x))*e^(2
*x) + (21*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2 + (21
*a^2*cosh(x)^6 + 55*a^2*cosh(x)^4 - 33*a^2*cosh(x)^2 - 3*a^2)*e^(2*x))*sinh
(x))*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(8*cosh(x)*e^x*sinh(x)^7 + e^x*s
inh(x)^8 + 4*(7*cosh(x)^2 + 1)*e^x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*
e^x*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*co
sh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cos
h(x)^4 + 9*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*co
sh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4
*cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int \left(a\mathrm{sech}^2(x)\right)^{5/2} \, dx = \int \left(a\operatorname{sech}^{2}\left(x\right)\right)^{\frac{5}{2}}\, dx$$

```
[In] integrate((a*sech(x)**2)**(5/2),x)
[Out] Integral((a*sech(x)**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \left(a\mathrm{sech}^2(x)\right)^{5/2} \, dx = \frac{3}{4}\, a^{\frac{5}{2}} \arctan\left(e^{x}\right) + \frac{3\, a^{\frac{5}{2}} e^{(7\, x)} + 11\, a^{\frac{5}{2}} e^{(5\, x)} - 11\, a^{\frac{5}{2}} e^{(3\, x)} - 3\, a^{\frac{5}{2}} e^{x}}{4\left(e^{(8\, x)} + 4\, e^{(6\, x)} + 6\, e^{(4\, x)} + 4\, e^{(2\, x)} + 1\right)}$$

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/4*a^(5/2)*arctan(e^x) + 1/4*(3*a^(5/2)*e^(7*x) + 11*a^(5/2)*e^(5*x) - 11*
a^(5/2)*e^(3*x) - 3*a^(5/2)*e^x)/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*
x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \frac{1}{16}\left(3\pi - \frac{4\left(3\left(e^{(-x)} - e^x\right)^3 + 20\,e^{(-x)} - 20\,e^x\right)}{\left(\left(e^{(-x)} - e^x\right)^2 + 4\right)^2} + 6\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)\right)$$

```
[In] integrate((a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(3*pi - 4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2
+ 4)^2 + 6*arctan(1/2*(e^(2*x) - 1)*e^(-x)))*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{sech}^2(x)\right)^{5/2} dx = \int \left(\frac{a}{\cosh(x)^2}\right)^{5/2} dx$$

```
[In] int((a/cosh(x)^2)^(5/2),x)
```

```
[Out] int((a/cosh(x)^2)^(5/2), x)
```

3.33 $\int \left(a\,\mathrm{sech}^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \left(a\,\mathrm{sech}^2(x)\right)^{3/2}\,dx=\frac{1}{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}\right)+\frac{1}{2}a\sqrt{a\,\mathrm{sech}^2(x)}\tanh(x)$$

```
[Out] 1/2*a^(3/2)*arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))+1/2*a*(a*sech(x)^2)
^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4207, 201, 223, 209}

$$\int \left(a\,\mathrm{sech}^2(x)\right)^{3/2}\,dx=\frac{1}{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\,\mathrm{sech}^2(x)}}\right)+\frac{1}{2}a\tanh(x)\sqrt{a\,\mathrm{sech}^2(x)}$$

```
[In] Int[(a*Sech[x]^2)^(3/2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]])/2 + (a*Sqrt[a*Sech[x]
^2]*Tanh[x])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\text{Subst}\left(\int \sqrt{a-ax^2}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}a\sqrt{a\text{sech}^2(x)}\tanh(x) + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x, \tanh(x)\right)\\
&= \frac{1}{2}a\sqrt{a\text{sech}^2(x)}\tanh(x) + \frac{1}{2}a^2\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)\\
&= \frac{1}{2}a^{3/2}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right) + \frac{1}{2}a\sqrt{a\text{sech}^2(x)}\tanh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx = \frac{1}{2}a\sqrt{a\text{sech}^2(x)}(\arctan(\sinh(x))\cosh(x)+\tanh(x))$$

```
[In] Integrate[(a*Sech[x]^2)^(3/2),x]
[Out] (a*Sqrt[a*Sech[x]^2]*(ArcTan[Sinh[x]]*Cosh[x] + Tanh[x]))/2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.30

method	result	size
risch	$\frac{a\sqrt{\frac{e^{2x}a}{\left(1+e^{2x}\right)^{2}}}\,\left(e^{2x}-1\right)}{1+e^{2x}}+\frac{i a\,e^{-x}\left(1+e^{2x}\right)\sqrt{\frac{e^{2x}a}{\left(1+e^{2x}\right)^{2}}}\,\ln\left(e^{x}+i\right)}{2}-\frac{i a\,e^{-x}\left(1+e^{2x}\right)\sqrt{\frac{e^{2x}a}{\left(1+e^{2x}\right)^{2}}}\,\ln\left(e^{x}-i\right)}{2}$	106

```
[In] int((sech(x)^2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] a/(1+exp(2*x))*(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)*(exp(2*x)-1)+1/2*I*a*exp(-
x)*(1+exp(2*x))*(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)*ln(exp(x)+I)-1/2*I*a*exp(
-x)*(1+exp(2*x))*(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)*ln(exp(x)-I)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(34) = 68$.

Time = 0.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 6.74

$$\int \left(a\mathrm{sech}^2(x)\right)^{3/2}\,dx = \frac{\left(a\cosh\left(x\right)^{3}+\left(ae^{(2\,x)}+a\right)\sinh\left(x\right)^{3}+3\left(a\cosh\left(x\right)e^{(2\,x)}+a\cosh\left(x\right)\right)\sinh\left(x\right)^{2}+\left(a\,c\right.}{}$$

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*cosh(x)^3 + (a*e^(2*x) + a)*sinh(x)^3 + 3*(a*cosh(x)*e^(2*x) + a*cosh(x)
)*sinh(x)^2 + (a*cosh(x)^4 + (a*e^(2*x) + a)*sinh(x)^4 + 4*(a*cosh(x)*e^(2*
x) + a*cosh(x))*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 + (3*a*cosh(x)
^2 + a)*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^4 + 2*a*cosh(x)^2 + a)*e^(2*x)
+ 4*(a*cosh(x)^3 + a*cosh(x) + (a*cosh(x)^3 + a*cosh(x))*e^(2*x))*sinh(x) +
 a)*arctan(cosh(x) + sinh(x)) - a*cosh(x) + (a*cosh(x)^3 - a*cosh(x))*e^(2*
x) + (3*a*cosh(x)^2 + (3*a*cosh(x)^2 - a)*e^(2*x) - a)*sinh(x))*sqrt(a/(e^(
4*x) + 2*e^(2*x) + 1))*e^x/(4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*
cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(
x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx = \int \left(a\,\text{sech}^2\left(x\right)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sech(x)**2)**(3/2),x)
[Out] Integral((a*sech(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx = a^{\frac{3}{2}}\arctan\left(e^{x}\right) + \frac{a^{\frac{3}{2}}e^{(3\,x)} - a^{\frac{3}{2}}e^{x}}{e^{(4\,x)} + 2\,e^{(2\,x)} + 1}$$

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="maxima")
[Out] a^(3/2)*arctan(e^x) + (a^(3/2)*e^(3*x) - a^(3/2)*e^x)/(e^(4*x) + 2*e^(2*x)
+ 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \left(a\text{sech}^2(x)\right)^{3/2}\,dx = \frac{1}{4}\left(\pi - \frac{4\left(e^{(-x)} - e^{x}\right)}{\left(e^{(-x)} - e^{x}\right)^{2} + 4} + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,x)} - 1\right)e^{(-x)}\right)\right)a^{\frac{3}{2}}$$

```
[In] integrate((a*sech(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/4*(pi - 4*(e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 2*arctan(1/2*(e^(2*x) -
 1)*e^(-x)))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\operatorname{sech}^2(x)\right)^{3/2} \, dx = \int \left(\frac{a}{\cosh\left(x\right)^2}\right)^{3/2} \,dx$$

```
[In] int((a/cosh(x)^2)^(3/2),x)
[Out] int((a/cosh(x)^2)^(3/2), x)
```

3.34 $\int \sqrt{a\operatorname{sech}^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \sqrt{a\operatorname{sech}^2(x)}\,dx = \sqrt{a}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right)$$

```
[Out] arctan(a^(1/2)*tanh(x)/(a*sech(x)^2)^(1/2))*a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4207, 223, 209}

$$\int \sqrt{a\operatorname{sech}^2(x)}\,dx = \sqrt{a}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\operatorname{sech}^2(x)}}\right)$$

```
[In] Int[Sqrt[a*Sech[x]^2],x]
```

```
[Out] Sqrt[a]*ArcTan[(Sqrt[a]*Tanh[x])/Sqrt[a*Sech[x]^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\text{Subst}\left(\int \frac{1}{\sqrt{a-ax^2}}\,dx, x, \tanh(x)\right)\\ &= a\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)\\ &= \sqrt{a}\arctan\left(\frac{\sqrt{a}\tanh(x)}{\sqrt{a\text{sech}^2(x)}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.64

$$\int \sqrt{a\text{sech}^2(x)}\,dx = \arctan(\sinh(x))\cosh(x)\sqrt{a\text{sech}^2(x)}$$

```
[In] Integrate[Sqrt[a*Sech[x]^2],x]
[Out] ArcTan[Sinh[x]]*Cosh[x]*Sqrt[a*Sech[x]^2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.88

method	result	size
risch	$i\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x+i)-i\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,e^{-x}(1+e^{2x})\ln(e^x-i)$	72

```
[In] int((sech(x)^2*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] I*(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(2*x))*ln(exp(x)+I)-I*(ex
p(2*x)*a/(1+exp(2*x))^2)^(1/2)*exp(-x)*(1+exp(2*x))*ln(exp(x)-I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 5.80

$$\int \sqrt{a\text{sech}^2(x)}\,dx = \left[\sqrt{-a}\log\left(\frac{2\,a\cosh(x)\,e^x\sinh(x) + ae^x\sinh(x)^2 + 2\left(\cosh(x)\,e^{(2\,x)} + \left(e^{(2\,x)}+1\right)\sinh(x) + \cosh(x)\right)\sqrt{}}{2\,\cosh(x)\,e^x\sinh(x) + e^x\sinh(x)^2 + \left(\cosh(x)^2+1\right)}\right.\right.$$
$$\left. + \sinh(x))\right]$$

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="fricas")
[Out] [sqrt(-a)*log((2*a*cosh(x)*e^x*sinh(x) + a*e^x*sinh(x)^2 + 2*(cosh(x)*e^(2*
x) + (e^(2*x) + 1)*sinh(x) + cosh(x))*sqrt(-a)*sqrt(a/(e^(4*x) + 2*e^(2*x)
+ 1))*e^x + (a*cosh(x)^2 - a)*e^x)/(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 +
 (cosh(x)^2 + 1)*e^x)), 2*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*a
rctan(cosh(x) + sinh(x))]
```

Sympy [F]

$$\int \sqrt{a\text{sech}^2(x)}\,dx = \int \sqrt{a\,\text{sech}^2(x)}\,dx$$

```
[In] integrate((a*sech(x)**2)**(1/2),x)
[Out] Integral(sqrt(a*sech(x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.32

$$\int \sqrt{a\text{sech}^2(x)}\,dx = 2\,\sqrt{a}\arctan(e^x)$$

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(a)*arctan(e^x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.32

$$\int \sqrt{a\text{sech}^2(x)}\,dx = 2\sqrt{a}\arctan\left(e^x\right)$$

```
[In] integrate((a*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(a)*arctan(e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\text{sech}^2(x)}\,dx = \int \sqrt{\frac{a}{\cosh(x)^2}}\,dx$$

```
[In] int((a/cosh(x)^2)^(1/2),x)
[Out] int((a/cosh(x)^2)^(1/2), x)
```

3.35 $\int \frac{1}{\sqrt{a\textbf{sech}^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[Out] tanh(x)/(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4207, 197}

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Int[1/Sqrt[a*Sech[x]^2],x]
```

```
[Out] Tanh[x]/Sqrt[a*Sech[x]^2]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\text{integral} = a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\,dx, x, \tanh(x)\right)$$
$$= \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{a\text{sech}^2(x)}}$$

```
[In] Integrate[1/Sqrt[a*Sech[x]^2],x]
[Out] Tanh[x]/Sqrt[a*Sech[x]^2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(11) = 22.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.46

method	result	size
risch	$\frac{e^{2x}}{2\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,(1+e^{2x})}-\frac{1}{2(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}$	58

```
[In] int(1/(sech(x)^2*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)/(1+exp(2*x))*exp(2*x)-1/2/(1+exp(2*x)
)/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 6.08

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\left(\left(e^{(2x)}+1\right)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2-1\right)e^{(2x)}+2\left(\cosh(x)e^{(2x)}+\cosh(x)\right)\sinh(x)-1\right)\sqrt{\frac{a}{e^{(4x)}+\cdots}}\cdots}{2\left(a\cosh(x)e^x+ae^x\sinh(x)\right)}$$

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 - 1)*e^(2*x) + 2*(cos
h(x)*e^(2*x) + cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/
(a*cosh(x)*e^x + a*e^x*sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{a\,\text{sech}^2(x)}}$$

```
[In] integrate(1/(a*sech(x)**2)**(1/2),x)
```

```
[Out] tanh(x)/sqrt(a*sech(x)**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{a\text{sech}^2(x)}}\,dx = -\frac{e^{(-x)}}{2\sqrt{a}}+\frac{e^x}{2\sqrt{a}}$$

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/sqrt(a) + 1/2*e^x/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = -\frac{e^{(-x)} - e^x}{2\sqrt{a}}$$

```
[In] integrate(1/(a*sech(x)^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(e^(-x) - e^x)/sqrt(a)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int \frac{1}{\sqrt{a\operatorname{sech}^2(x)}}\,dx = -\frac{\left(\frac{e^{-2x}}{2} - \frac{e^{2x}}{2}\right)\sqrt{\frac{1}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^2}}}{2\sqrt{a}}$$

```
[In] int(1/(a/cosh(x)^2)^(1/2),x)
[Out] -((exp(-2*x)/2 - exp(2*x)/2)*(1/(exp(-x)/2 + exp(x)/2)^2)^(1/2))/(2*a^(1/2)
)
```

3.36 $\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{\tanh(x)}{3\left(a\text{sech}^2(x)\right)^{3/2}}+\frac{2\tanh(x)}{3a\sqrt{a\text{sech}^2(x)}}$$

```
[Out] 1/3*tanh(x)/(a*sech(x)^2)^(3/2)+2/3*tanh(x)/a/(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx=\frac{2\tanh(x)}{3a\sqrt{a\text{sech}^2(x)}}+\frac{\tanh(x)}{3\left(a\text{sech}^2(x)\right)^{3/2}}$$

```
[In] Int[(a*Sech[x]^2)^(-3/2),x]
```

```
[Out] Tanh[x]/(3*(a*Sech[x]^2)^(3/2)) + (2*Tanh[x])/(3*a*Sqrt[a*Sech[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{5/2}}\,dx, x, \tanh(x)\right) \\
&= \frac{\tanh(x)}{3\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\,dx, x, \tanh(x)\right) \\
&= \frac{\tanh(x)}{3\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{2\tanh(x)}{3a\sqrt{a\operatorname{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{3/2}}\,dx = \frac{\left(3+\sinh^2(x)\right)\tanh(x)}{3a\sqrt{a\operatorname{sech}^2(x)}}$$

```
[In] Integrate[(a*Sech[x]^2)^(-3/2),x]
[Out] ((3 + Sinh[x]^2)*Tanh[x])/(3*a*Sqrt[a*Sech[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(28) = 56.

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.61

method	result	size
risch	$\frac{e^{4x}}{24a(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}+\frac{3\,e^{2x}}{8a(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}-\frac{3}{8\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,a(1+e^{2x})}-\frac{e^{-2x}}{24a(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}$	130

```
[In] int(1/(sech(x)^2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a*exp(4*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)+3/8/a*exp(2*
x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)-3/8/(exp(2*x)*a/(1+exp(2*
x))^2)^(1/2)/a/(1+exp(2*x))-1/24/a*exp(-2*x)/(1+exp(2*x))/(exp(2*x)*a/(1+ex
p(2*x))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 7.69

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = \frac{\left(\left(e^{(2\,x)}+1\right)\sinh\left(x\right)^6+\cosh\left(x\right)^6+6\left(\cosh\left(x\right)e^{(2\,x)}+\cosh\left(x\right)\right)\sinh\left(x\right)^5+3\left(5\cos\right.\right.}{}$$

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((e^(2*x) + 1)*sinh(x)^6 + cosh(x)^6 + 6*(cosh(x)*e^(2*x) + cosh(x))*s
inh(x)^5 + 3*(5*cosh(x)^2 + (5*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^4 + 9*co
sh(x)^4 + 4*(5*cosh(x)^3 + (5*cosh(x)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*s
inh(x)^3 + 3*(5*cosh(x)^4 + 18*cosh(x)^2 + (5*cosh(x)^4 + 18*cosh(x)^2 - 3)
*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^6 + 9*cosh(x)^4 - 9*cosh(x
)^2 - 1)*e^(2*x) + 6*(cosh(x)^5 + 6*cosh(x)^3 + (cosh(x)^5 + 6*cosh(x)^3 -
3*cosh(x))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(4*x) + 2*e^(2*x) +
1))*e^x/(a^2*cosh(x)^3*e^x + 3*a^2*cosh(x)^2*e^x*sinh(x) + 3*a^2*cosh(x)*e^
x*sinh(x)^2 + a^2*e^x*sinh(x)^3)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = -\frac{2\tanh^3\left(x\right)}{3\left(a\,\text{sech}^2\left(x\right)\right)^{\frac{3}{2}}}+\frac{\tanh\left(x\right)}{\left(a\,\text{sech}^2\left(x\right)\right)^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*sech(x)**2)**(3/2),x)
```

```
[Out] -2*tanh(x)**3/(3*(a*sech(x)**2)**(3/2)) + tanh(x)/(a*sech(x)**2)**(3/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = \frac{e^{(3\,x)}}{24\,a^{\frac{3}{2}}} - \frac{3\,e^{(-x)}}{8\,a^{\frac{3}{2}}} - \frac{e^{(-3\,x)}}{24\,a^{\frac{3}{2}}} + \frac{3\,e^{x}}{8\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="maxima")
[Out] 1/24*e^(3*x)/a^(3/2) - 3/8*e^(-x)/a^(3/2) - 1/24*e^(-3*x)/a^(3/2) + 3/8*e^x
/a^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = -\frac{\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)} - e^{(3\,x)} - 9\,e^{x}}{24\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*sech(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{3/2}}\,dx$$

```
[In] int(1/(a/cosh(x)^2)^(3/2),x)
[Out] int(1/(a/cosh(x)^2)^(3/2), x)
```

3.37 $\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}} dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}} dx = \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}$$

```
[Out] 1/5*tanh(x)/(a*sech(x)^2)^(5/2)+4/15*tanh(x)/a/(a*sech(x)^2)^(3/2)+8/15*tan
h(x)/a^2/(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}} dx = \frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}}$$

```
[In] Int[(a*Sech[x]^2)^(-5/2),x]
```

```
[Out] Tanh[x]/(5*(a*Sech[x]^2)^(5/2)) + (4*Tanh[x])/(15*a*(a*Sech[x]^2)^(3/2)) +
(8*Tanh[x])/(15*a^2*Sqrt[a*Sech[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{7/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{(a-ax^2)^{5/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{8\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\,dx, x, \tanh(x)\right)}{15a}\\
&= \frac{\tanh(x)}{5\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{4\tanh(x)}{15a\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{8\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = \frac{\left(15+10\sinh^2(x)+3\sinh^4(x)\right)\tanh(x)}{15a^2\sqrt{a\text{sech}^2(x)}}$$

```
[In] Integrate[(a*Sech[x]^2)^(-5/2),x]
[Out] ((15 + 10*Sinh[x]^2 + 3*Sinh[x]^4)*Tanh[x])/(15*a^2*Sqrt[a*Sech[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(43) = 86.

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.56

method	result
risch	$\frac{e^{6x}}{160a^2(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}+\frac{5\,e^{4x}}{96a^2(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}+\frac{5\,e^{2x}}{16a^2(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}-\frac{5}{16\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}\,(1+e^{2x})a^2}-\frac{5}{96a^2(1+e^{2}}$

```
[In] int(1/(sech(x)^2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/160/a^2*exp(6*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)+5/96/a^2*
exp(4*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)+5/16/a^2*exp(2*x)/(
1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)-5/16/(exp(2*x)*a/(1+exp(2*x))
^2)^(1/2)/(1+exp(2*x))/a^2-5/96/a^2*exp(-2*x)/(1+exp(2*x))/(exp(2*x)*a/(1+e
xp(2*x))^2)^(1/2)-1/160/a^2*exp(-4*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))
^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(43) = 86.

Time = 0.26 (sec) , antiderivative size = 580, normalized size of antiderivative = 10.55

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/480*(3*(e^(2*x) + 1)*sinh(x)^10 + 3*cosh(x)^10 + 30*(cosh(x)*e^(2*x) + co
sh(x))*sinh(x)^9 + 5*(27*cosh(x)^2 + (27*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x
)^8 + 25*cosh(x)^8 + 40*(9*cosh(x)^3 + (9*cosh(x)^3 + 5*cosh(x))*e^(2*x) +
5*cosh(x))*sinh(x)^7 + 10*(63*cosh(x)^4 + 70*cosh(x)^2 + (63*cosh(x)^4 + 70
*cosh(x)^2 + 15)*e^(2*x) + 15)*sinh(x)^6 + 150*cosh(x)^6 + 4*(189*cosh(x)^5
 + 350*cosh(x)^3 + (189*cosh(x)^5 + 350*cosh(x)^3 + 225*cosh(x))*e^(2*x) +
225*cosh(x))*sinh(x)^5 + 10*(63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 +
 (63*cosh(x)^6 + 175*cosh(x)^4 + 225*cosh(x)^2 - 15)*e^(2*x) - 15)*sinh(x)^
4 - 150*cosh(x)^4 + 40*(9*cosh(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 + (9*cosh
(x)^7 + 35*cosh(x)^5 + 75*cosh(x)^3 - 15*cosh(x))*e^(2*x) - 15*cosh(x))*sin
h(x)^3 + 5*(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 +
(27*cosh(x)^8 + 140*cosh(x)^6 + 450*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(2*x)
- 5)*sinh(x)^2 - 25*cosh(x)^2 + (3*cosh(x)^10 + 25*cosh(x)^8 + 150*cosh(x)^
6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 3)*e^(2*x) + 10*(3*cosh(x)^9 + 20*cosh(x
)^7 + 90*cosh(x)^5 - 60*cosh(x)^3 + (3*cosh(x)^9 + 20*cosh(x)^7 + 90*cosh(x
)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))*sinh(x) - 3)*sqrt(a/(e
```

```
^(4*x) + 2*e^(2*x) + 1))*e^x/(a^3*cosh(x)^5*e^x + 5*a^3*cosh(x)^4*e^x*sinh(
x) + 10*a^3*cosh(x)^3*e^x*sinh(x)^2 + 10*a^3*cosh(x)^2*e^x*sinh(x)^3 + 5*a^
3*cosh(x)*e^x*sinh(x)^4 + a^3*e^x*sinh(x)^5)
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = \frac{8\tanh^5(x)}{15\left(a\operatorname{sech}^2(x)\right)^{\frac{5}{2}}} - \frac{4\tanh^3(x)}{3\left(a\operatorname{sech}^2(x)\right)^{\frac{5}{2}}} + \frac{\tanh(x)}{\left(a\operatorname{sech}^2(x)\right)^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*sech(x)**2)**(5/2),x)
```

```
[Out] 8*tanh(x)**5/(15*(a*sech(x)**2)**(5/2)) - 4*tanh(x)**3/(3*(a*sech(x)**2)**(
5/2)) + tanh(x)/(a*sech(x)**2)**(5/2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = \frac{e^{(5x)}}{160a^{\frac{5}{2}}} + \frac{5e^{(3x)}}{96a^{\frac{5}{2}}} - \frac{5e^{(-x)}}{16a^{\frac{5}{2}}} - \frac{5e^{(-3x)}}{96a^{\frac{5}{2}}} - \frac{e^{(-5x)}}{160a^{\frac{5}{2}}} + \frac{5e^{x}}{16a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*x)/a^(5/2) + 5/96*e^(3*x)/a^(5/2) - 5/16*e^(-x)/a^(5/2) - 5/96*e
^(-3*x)/a^(5/2) - 1/160*e^(-5*x)/a^(5/2) + 5/16*e^x/a^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{5/2}}\,dx = -\frac{\left(150e^{(4x)} + 25e^{(2x)} + 3\right)e^{(-5x)} - 3e^{(5x)} - 25e^{(3x)} - 150e^{x}}{480a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*sech(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/480*((150*e^(4*x) + 25*e^(2*x) + 3)*e^(-5*x) - 3*e^(5*x) - 25*e^(3*x) -
150*e^x)/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^2}\right)^{5/2}}\,dx$$

```
[In] int(1/(a/cosh(x)^2)^(5/2),x)
[Out] int(1/(a/cosh(x)^2)^(5/2), x)
```

3.38 $\int \frac{1}{\left(a\mathbf{sech}^2(x)\right)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{16\tanh(x)}{35a^3\sqrt{a\operatorname{sech}^2(x)}}$$

```
[Out] 1/7*tanh(x)/(a*sech(x)^2)^(7/2)+6/35*tanh(x)/a/(a*sech(x)^2)^(5/2)+8/35*tan
h(x)/a^2/(a*sech(x)^2)^(3/2)+16/35*tanh(x)/a^3/(a*sech(x)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4207, 198, 197}

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = \frac{16\tanh(x)}{35a^3\sqrt{a\operatorname{sech}^2(x)}} + \frac{8\tanh(x)}{35a^2\left(a\operatorname{sech}^2(x)\right)^{3/2}} + \frac{6\tanh(x)}{35a\left(a\operatorname{sech}^2(x)\right)^{5/2}} + \frac{\tanh(x)}{7\left(a\operatorname{sech}^2(x)\right)^{7/2}}$$

```
[In] Int[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] Tanh[x]/(7*(a*Sech[x]^2)^(7/2)) + (6*Tanh[x])/(35*a*(a*Sech[x]^2)^(5/2)) +
(8*Tanh[x])/(35*a^2*(a*Sech[x]^2)^(3/2)) + (16*Tanh[x])/(35*a^3*Sqrt[a*Sech
[x]^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 4207

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Dist[b*(ff/f), Subst[Int[(b + b*ff^2*x^2)^(p - 1),
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\text{Subst}\left(\int \frac{1}{(a-ax^2)^{9/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6}{7}\text{Subst}\left(\int \frac{1}{(a-ax^2)^{7/2}}\,dx, x, \tanh(x)\right)\\
&= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{24\text{Subst}\left(\int \frac{1}{(a-ax^2)^{5/2}}\,dx, x, \tanh(x)\right)}{35a}\\
&= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\text{sech}^2(x)\right)^{3/2}}\\
&\quad + \frac{16\text{Subst}\left(\int \frac{1}{(a-ax^2)^{3/2}}\,dx, x, \tanh(x)\right)}{35a^2}\\
&= \frac{\tanh(x)}{7\left(a\text{sech}^2(x)\right)^{7/2}} + \frac{6\tanh(x)}{35a\left(a\text{sech}^2(x)\right)^{5/2}} + \frac{8\tanh(x)}{35a^2\left(a\text{sech}^2(x)\right)^{3/2}} + \frac{16\tanh(x)}{35a^3\sqrt{a\text{sech}^2(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = \frac{\left(35+35\sinh^2(x)+21\sinh^4(x)+5\sinh^6(x)\right)\tanh(x)}{35a^3\sqrt{a\operatorname{sech}^2(x)}}$$

```
[In] Integrate[(a*Sech[x]^2)^(-7/2),x]
```

```
[Out] ((35 + 35*Sinh[x]^2 + 21*Sinh[x]^4 + 5*Sinh[x]^6)*Tanh[x])/(35*a^3*Sqrt[a*S
ech[x]^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(58) = 116.

Time = 0.14 (sec) , antiderivative size = 262, normalized size of antiderivative = 3.54

method	result
risch	$\frac{e^{8x}}{896a^3(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}+\frac{7\,e^{6x}}{640a^3(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}+\frac{7\,e^{4x}}{128a^3(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}+\frac{35\,e^{2x}}{128a^3(1+e^{2x})\sqrt{\frac{e^{2x}a}{(1+e^{2x})^2}}}-\frac{}{128\sqrt{\frac{e}{(1+}}}$

```
[In] int(1/(sech(x)^2*a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/896/a^3*exp(8*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)+7/640/a^3
*exp(6*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)+7/128/a^3*exp(4*x)
/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)+35/128/a^3*exp(2*x)/(1+exp(
2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)-35/128/(exp(2*x)*a/(1+exp(2*x))^2)^
(1/2)/(1+exp(2*x))/a^3-7/128/a^3*exp(-2*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(
2*x))^2)^(1/2)-7/640/a^3*exp(-4*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)
^(1/2)-1/896/a^3*exp(-6*x)/(1+exp(2*x))/(exp(2*x)*a/(1+exp(2*x))^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 970 vs. 2(58) = 116.

Time = 0.28 (sec) , antiderivative size = 970, normalized size of antiderivative = 13.11

$$\int \frac{1}{\left(a\operatorname{sech}^2(x)\right)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/4480*(5*(e^(2*x) + 1)*sinh(x)^14 + 5*cosh(x)^14 + 70*(cosh(x)*e^(2*x) + c
osh(x))*sinh(x)^13 + 7*(65*cosh(x)^2 + (65*cosh(x)^2 + 7)*e^(2*x) + 7)*sinh
```

```
(x)^12 + 49*cosh(x)^12 + 28*(65*cosh(x)^3 + (65*cosh(x)^3 + 21*cosh(x))*e^(
2*x) + 21*cosh(x))*sinh(x)^11 + 7*(715*cosh(x)^4 + 462*cosh(x)^2 + (715*cos
h(x)^4 + 462*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^10 + 245*cosh(x)^10 + 70
*(143*cosh(x)^5 + 154*cosh(x)^3 + (143*cosh(x)^5 + 154*cosh(x)^3 + 35*cosh(
x))*e^(2*x) + 35*cosh(x))*sinh(x)^9 + 35*(429*cosh(x)^6 + 693*cosh(x)^4 + 3
15*cosh(x)^2 + (429*cosh(x)^6 + 693*cosh(x)^4 + 315*cosh(x)^2 + 35)*e^(2*x)
 + 35)*sinh(x)^8 + 1225*cosh(x)^8 + 8*(2145*cosh(x)^7 + 4851*cosh(x)^5 + 36
75*cosh(x)^3 + (2145*cosh(x)^7 + 4851*cosh(x)^5 + 3675*cosh(x)^3 + 1225*cos
h(x))*e^(2*x) + 1225*cosh(x))*sinh(x)^7 + 7*(2145*cosh(x)^8 + 6468*cosh(x)^
6 + 7350*cosh(x)^4 + 4900*cosh(x)^2 + (2145*cosh(x)^8 + 6468*cosh(x)^6 + 73
50*cosh(x)^4 + 4900*cosh(x)^2 - 175)*e^(2*x) - 175)*sinh(x)^6 - 1225*cosh(x
)^6 + 14*(715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3
+ (715*cosh(x)^9 + 2772*cosh(x)^7 + 4410*cosh(x)^5 + 4900*cosh(x)^3 - 525*c
osh(x))*e^(2*x) - 525*cosh(x))*sinh(x)^5 + 35*(143*cosh(x)^10 + 693*cosh(x)
^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 + (143*cosh(x)^10 + 69
3*cosh(x)^8 + 1470*cosh(x)^6 + 2450*cosh(x)^4 - 525*cosh(x)^2 - 7)*e^(2*x)
- 7)*sinh(x)^4 - 245*cosh(x)^4 + 140*(13*cosh(x)^11 + 77*cosh(x)^9 + 210*co
sh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 + (13*cosh(x)^11 + 77*cosh(x)^9 + 2
10*cosh(x)^7 + 490*cosh(x)^5 - 175*cosh(x)^3 - 7*cosh(x))*e^(2*x) - 7*cosh(
x))*sinh(x)^3 + 7*(65*cosh(x)^12 + 462*cosh(x)^10 + 1575*cosh(x)^8 + 4900*c
osh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 + (65*cosh(x)^12 + 462*cosh(x)^10
 + 1575*cosh(x)^8 + 4900*cosh(x)^6 - 2625*cosh(x)^4 - 210*cosh(x)^2 - 7)*e^
(2*x) - 7)*sinh(x)^2 - 49*cosh(x)^2 + (5*cosh(x)^14 + 49*cosh(x)^12 + 245*c
osh(x)^10 + 1225*cosh(x)^8 - 1225*cosh(x)^6 - 245*cosh(x)^4 - 49*cosh(x)^2
- 5)*e^(2*x) + 14*(5*cosh(x)^13 + 42*cosh(x)^11 + 175*cosh(x)^9 + 700*cosh(
x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 + (5*cosh(x)^13 + 42*cosh(x)^11 + 175*c
osh(x)^9 + 700*cosh(x)^7 - 525*cosh(x)^5 - 70*cosh(x)^3 - 7*cosh(x))*e^(2*x
) - 7*cosh(x))*sinh(x) - 5)*sqrt(a/(e^(4*x) + 2*e^(2*x) + 1))*e^x/(a^4*cosh
(x)^7*e^x + 7*a^4*cosh(x)^6*e^x*sinh(x) + 21*a^4*cosh(x)^5*e^x*sinh(x)^2 +
35*a^4*cosh(x)^4*e^x*sinh(x)^3 + 35*a^4*cosh(x)^3*e^x*sinh(x)^4 + 21*a^4*co
sh(x)^2*e^x*sinh(x)^5 + 7*a^4*cosh(x)*e^x*sinh(x)^6 + a^4*e^x*sinh(x)^7)
```

Sympy [A] (verification not implemented)

Time = 13.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a\text{sech}^2(x)\right)^{7/2}}\,dx = -\frac{16\tanh^7(x)}{35\left(a\,\text{sech}^2(x)\right)^{\frac{7}{2}}}+\frac{8\tanh^5(x)}{5\left(a\,\text{sech}^2(x)\right)^{\frac{7}{2}}}-\frac{2\tanh^3(x)}{\left(a\,\text{sech}^2(x)\right)^{\frac{7}{2}}}+\frac{\tanh(x)}{\left(a\,\text{sech}^2(x)\right)^{\frac{7}{2}}}$$

```
[In] integrate(1/(a*sech(x)**2)**(7/2),x)
```

```
[Out] -16*tanh(x)**7/(35*(a*sech(x)**2)**(7/2)) + 8*tanh(x)**5/(5*(a*sech(x)**2)*
*(7/2)) - 2*tanh(x)**3/(a*sech(x)**2)**(7/2) + tanh(x)/(a*sech(x)**2)**(7/2
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{7/2}}\,dx = \frac{e^{(7\,x)}}{896\,a^{\frac{7}{2}}} + \frac{7\,e^{(5\,x)}}{640\,a^{\frac{7}{2}}} + \frac{7\,e^{(3\,x)}}{128\,a^{\frac{7}{2}}} - \frac{35\,e^{(-x)}}{128\,a^{\frac{7}{2}}} - \frac{7\,e^{(-3\,x)}}{128\,a^{\frac{7}{2}}} - \frac{7\,e^{(-5\,x)}}{640\,a^{\frac{7}{2}}} - \frac{e^{(-7\,x)}}{896\,a^{\frac{7}{2}}} + \frac{35\,e^{x}}{128\,a^{\frac{7}{2}}}$$

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/896*e^(7*x)/a^(7/2) + 7/640*e^(5*x)/a^(7/2) + 7/128*e^(3*x)/a^(7/2) - 35/
128*e^(-x)/a^(7/2) - 7/128*e^(-3*x)/a^(7/2) - 7/640*e^(-5*x)/a^(7/2) - 1/89
6*e^(-7*x)/a^(7/2) + 35/128*e^x/a^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{7/2}}\,dx = -\frac{\left(1225\,e^{(6\,x)} + 245\,e^{(4\,x)} + 49\,e^{(2\,x)} + 5\right)e^{(-7\,x)} - 5\,e^{(7\,x)} - 49\,e^{(5\,x)} - 245\,e^{(3\,x)} - 1225\,e^{x}}{4480\,a^{\frac{7}{2}}}$$

```
[In] integrate(1/(a*sech(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4480*((1225*e^(6*x) + 245*e^(4*x) + 49*e^(2*x) + 5)*e^(-7*x) - 5*e^(7*x)
 - 49*e^(5*x) - 245*e^(3*x) - 1225*e^x)/a^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\mathrm{sech}^2(x)\right)^{7/2}}\,dx = \int \frac{1}{{\left(\frac{a}{{\mathrm{cosh}\left(x\right)}^2}\right)}^{7/2}} \,d x$$

```
[In] int(1/(a/cosh(x)^2)^(7/2),x)
```

```
[Out] int(1/(a/cosh(x)^2)^(7/2), x)
```

3.39 $\int \left(a\text{sech}^3(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 121

$$\begin{aligned}\int \left(a\text{sech}^3(x)\right)^{5/2} dx &= \frac{154}{195} i a^2 \cosh^{\frac{3}{2}}(x) E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} \\ &+ \frac{154}{195} a^2 \cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585} a^2 \sqrt{a\text{sech}^3(x)}\tanh(x) \\ &+ \frac{22}{117} a^2 \text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13} a^2 \text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x)\end{aligned}$$

```
[Out] 154/195*I*a^2*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*s
inh(1/2*x),2^(1/2))*(a*sech(x)^3)^(1/2)+154/195*a^2*cosh(x)*sinh(x)*(a*sech
(x)^3)^(1/2)+154/585*a^2*(a*sech(x)^3)^(1/2)*tanh(x)+22/117*a^2*sech(x)^2*(
a*sech(x)^3)^(1/2)*tanh(x)+2/13*a^2*sech(x)^4*(a*sech(x)^3)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3853, 3856, 2719}

$$\begin{aligned}\int \left(a\text{sech}^3(x)\right)^{5/2} dx &= \frac{154}{585} a^2 \tanh(x)\sqrt{a\text{sech}^3(x)} \\ &+ \frac{2}{13} a^2 \tanh(x)\text{sech}^4(x)\sqrt{a\text{sech}^3(x)} + \frac{22}{117} a^2 \tanh(x)\text{sech}^2(x)\sqrt{a\text{sech}^3(x)} \\ &+ \frac{154}{195} i a^2 \cosh^{\frac{3}{2}}(x) E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{154}{195} a^2 \sinh(x)\cosh(x)\sqrt{a\text{sech}^3(x)}\end{aligned}$$

```
[In] Int[(a*Sech[x]^3)^(5/2),x]
```

```
[Out] ((154*I)/195)*a^2*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (
154*a^2*Cosh[x]*Sqrt[a*Sech[x]^3]*Sinh[x])/195 + (154*a^2*Sqrt[a*Sech[x]^3]
*Tanh[x])/585 + (22*a^2*Sech[x]^2*Sqrt[a*Sech[x]^3]*Tanh[x])/117 + (2*a^2*S
ech[x]^4*Sqrt[a*Sech[x]^3]*Tanh[x])/13
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(a^2\sqrt{a\operatorname{sech}^3(x)}\right)\int \operatorname{sech}^{\frac{15}{2}}(x)\,dx}{\operatorname{sech}^{\frac{3}{2}}(x)}\\
&= \frac{2}{13}a^2\operatorname{sech}^4(x)\sqrt{a\operatorname{sech}^3(x)}\tanh(x)+\frac{\left(11a^2\sqrt{a\operatorname{sech}^3(x)}\right)\int \operatorname{sech}^{\frac{11}{2}}(x)\,dx}{13\operatorname{sech}^{\frac{3}{2}}(x)}\\
&= \frac{22}{117}a^2\operatorname{sech}^2(x)\sqrt{a\operatorname{sech}^3(x)}\tanh(x)+\frac{2}{13}a^2\operatorname{sech}^4(x)\sqrt{a\operatorname{sech}^3(x)}\tanh(x)\\
&\quad+\frac{\left(77a^2\sqrt{a\operatorname{sech}^3(x)}\right)\int \operatorname{sech}^{\frac{7}{2}}(x)\,dx}{117\operatorname{sech}^{\frac{3}{2}}(x)}
\end{aligned}
$$

$$= \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(77a^2\sqrt{a\text{sech}^3(x)}\right)\int\text{sech}^{\frac{3}{2}}(x)\,dx}{195\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x) - \frac{\left(77a^2\sqrt{a\text{sech}^3(x)}\right)\int\frac{1}{\sqrt{\text{sech}(x)}}\,dx}{195\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x) - \frac{1}{195}\left(77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}\right)\int\sqrt{\cosh(x)}\,dx$$

$$= \frac{154}{195}ia^2\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\text{sech}^3(x)} + \frac{154}{195}a^2\cosh(x)\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{154}{585}a^2\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{22}{117}a^2\text{sech}^2(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{2}{13}a^2\text{sech}^4(x)\sqrt{a\text{sech}^3(x)}\tanh(x)$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int\left(a\text{sech}^3(x)\right)^{5/2}\,dx = \frac{2}{585}a\text{sech}(x)\left(a\text{sech}^3(x)\right)^{3/2}\left(231i\cosh^{\frac{11}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 55\cosh(x)\sinh(x) + 77\cosh$$

```
[In] Integrate[(a*Sech[x]^3)^(5/2),x]
```

```
[Out] (2*a*Sech[x]*(a*Sech[x]^3)^(3/2)*((231*I)*Cosh[x]^(11/2)*EllipticE[(I/2)*x,
 2] + 55*Cosh[x]*Sinh[x] + 77*Cosh[x]^3*Sinh[x] + 231*Cosh[x]^5*Sinh[x] + 4
5*Tanh[x]))/585
```

Maple [F]

$$\int \left(a\operatorname{sech}\left(x\right)^{3}\right)^{\frac{5}{2}}dx$$

```
[In] int((a*sech(x)^3)^(5/2),x)
```

```
[Out] int((a*sech(x)^3)^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 1382, normalized size of antiderivative = 11.42

$$\int \left(a\text{sech}^3(x)\right)^{5/2} dx = \text{Too large to display}$$

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(231*sqrt(2)*(a^2*cosh(x)^12 + 12*a^2*cosh(x)*sinh(x)^11 + a^2*sinh(x
)^12 + 6*a^2*cosh(x)^10 + 6*(11*a^2*cosh(x)^2 + a^2)*sinh(x)^10 + 15*a^2*co
sh(x)^8 + 20*(11*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^9 + 15*(33*a^2*cosh
(x)^4 + 18*a^2*cosh(x)^2 + a^2)*sinh(x)^8 + 20*a^2*cosh(x)^6 + 24*(33*a^2*c
osh(x)^5 + 30*a^2*cosh(x)^3 + 5*a^2*cosh(x))*sinh(x)^7 + 4*(231*a^2*cosh(x)
^6 + 315*a^2*cosh(x)^4 + 105*a^2*cosh(x)^2 + 5*a^2)*sinh(x)^6 + 15*a^2*cosh
(x)^4 + 24*(33*a^2*cosh(x)^7 + 63*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 5*a^2*
cosh(x))*sinh(x)^5 + 15*(33*a^2*cosh(x)^8 + 84*a^2*cosh(x)^6 + 70*a^2*cosh(
x)^4 + 20*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 6*a^2*cosh(x)^2 + 20*(11*a^2*cos
h(x)^9 + 36*a^2*cosh(x)^7 + 42*a^2*cosh(x)^5 + 20*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 6*(11*a^2*cosh(x)^10 + 45*a^2*cosh(x)^8 + 70*a^2*cosh(x)^
6 + 50*a^2*cosh(x)^4 + 15*a^2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 + 12*(a^2*co
sh(x)^11 + 5*a^2*cosh(x)^9 + 10*a^2*cosh(x)^7 + 10*a^2*cosh(x)^5 + 5*a^2*co
sh(x)^3 + a^2*cosh(x))*sinh(x))*sqrt(a)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cosh(x) + sinh(x))) + sqrt(2)*(231*a^2*cosh(x)^13 + 3003*a^2
*cosh(x)*sinh(x)^12 + 231*a^2*sinh(x)^13 + 1540*a^2*cosh(x)^11 + 154*(117*a
^2*cosh(x)^2 + 10*a^2)*sinh(x)^11 + 4367*a^2*cosh(x)^9 + 1694*(39*a^2*cosh(
x)^3 + 10*a^2*cosh(x))*sinh(x)^10 + 11*(15015*a^2*cosh(x)^4 + 7700*a^2*cosh
(x)^2 + 397*a^2)*sinh(x)^9 + 6808*a^2*cosh(x)^7 + 33*(9009*a^2*cosh(x)^5 +
7700*a^2*cosh(x)^3 + 1191*a^2*cosh(x))*sinh(x)^8 + 4*(99099*a^2*cosh(x)^6 +
 127050*a^2*cosh(x)^4 + 39303*a^2*cosh(x)^2 + 1702*a^2)*sinh(x)^7 + 1277*a^
2*cosh(x)^5 + 28*(14157*a^2*cosh(x)^7 + 25410*a^2*cosh(x)^5 + 13101*a^2*cos
h(x)^3 + 1702*a^2*cosh(x))*sinh(x)^6 + (297297*a^2*cosh(x)^8 + 711480*a^2*c
osh(x)^6 + 550242*a^2*cosh(x)^4 + 142968*a^2*cosh(x)^2 + 1277*a^2)*sinh(x)^
5 + 484*a^2*cosh(x)^3 + (165165*a^2*cosh(x)^9 + 508200*a^2*cosh(x)^7 + 5502
42*a^2*cosh(x)^5 + 238280*a^2*cosh(x)^3 + 6385*a^2*cosh(x))*sinh(x)^4 + 2*(
33033*a^2*cosh(x)^10 + 127050*a^2*cosh(x)^8 + 183414*a^2*cosh(x)^6 + 119140
```

```
*a^2*cosh(x)^4 + 6385*a^2*cosh(x)^2 + 242*a^2)*sinh(x)^3 + 77*a^2*cosh(x) +
 2*(9009*a^2*cosh(x)^11 + 42350*a^2*cosh(x)^9 + 78606*a^2*cosh(x)^7 + 71484
*a^2*cosh(x)^5 + 6385*a^2*cosh(x)^3 + 726*a^2*cosh(x))*sinh(x)^2 + (3003*a^
2*cosh(x)^12 + 16940*a^2*cosh(x)^10 + 39303*a^2*cosh(x)^8 + 47656*a^2*cosh(
x)^6 + 6385*a^2*cosh(x)^4 + 1452*a^2*cosh(x)^2 + 77*a^2)*sinh(x))*sqrt((a*c
osh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(cosh
(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(x)^2 + 1)*sinh(x)^
10 + 6*cosh(x)^10 + 20*(11*cosh(x)^3 + 3*cosh(x))*sinh(x)^9 + 15*(33*cosh(x
)^4 + 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33*cosh(x)^5 + 30*co
sh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 + 315*cosh(x)^4 + 105*cos
h(x)^2 + 5)*sinh(x)^6 + 20*cosh(x)^6 + 24*(33*cosh(x)^7 + 63*cosh(x)^5 + 35
*cosh(x)^3 + 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 + 84*cosh(x)^6 + 70*co
sh(x)^4 + 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20*(11*cosh(x)^9 + 3
6*cosh(x)^7 + 42*cosh(x)^5 + 20*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 6*(11*co
sh(x)^10 + 45*cosh(x)^8 + 70*cosh(x)^6 + 50*cosh(x)^4 + 15*cosh(x)^2 + 1)*s
inh(x)^2 + 6*cosh(x)^2 + 12*(cosh(x)^11 + 5*cosh(x)^9 + 10*cosh(x)^7 + 10*c
osh(x)^5 + 5*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(a\text{sech}^3(x)\right)^{5/2}\,dx=\int \left(a\,\text{sech}^3\left(x\right)\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sech(x)**3)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**3)**(5/2), x)
```

Maxima [F]

$$\int \left(a\text{sech}^3(x)\right)^{5/2}\,dx=\int \left(a\,\text{sech}\left(x\right)^{3}\right)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\mathrm{sech}^3(x)\right)^{5/2} dx = \int \left(a\,\mathrm{sech}\left(x\right)^3\right)^{\frac{5}{2}} dx$$

```
[In] integrate((a*sech(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\mathrm{sech}^3(x)\right)^{5/2} dx = \int \left(\frac{a}{\cosh\left(x\right)^3}\right)^{5/2} dx$$

```
[In] int((a/cosh(x)^3)^(5/2),x)
[Out] int((a/cosh(x)^3)^(5/2), x)
```

3.40 $\int \left(a\,\text{sech}^3(x)\right)^{3/2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \left(a\,\text{sech}^3(x)\right)^{3/2} \, dx = -\frac{10}{21} i a \cosh^{\frac{3}{2}}(x) \operatorname{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{a\,\text{sech}^3(x)} + \frac{10}{21} a\sqrt{a\,\text{sech}^3(x)}\sinh(x) + \frac{2}{7} a\,\text{sech}(x)\sqrt{a\,\text{sech}^3(x)}\tanh(x)$$

```
[Out] -10/21*I*a*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh
(1/2*x),2^(1/2))*(a*sech(x)^3)^(1/2)+10/21*a*sinh(x)*(a*sech(x)^3)^(1/2)+2/
7*a*sech(x)*(a*sech(x)^3)^(1/2)*tanh(x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3853, 3856, 2720}

$$\int \left(a\,\text{sech}^3(x)\right)^{3/2} \, dx = \frac{10}{21} a \sinh(x)\sqrt{a\,\text{sech}^3(x)} + \frac{2}{7} a \tanh(x)\text{sech}(x)\sqrt{a\,\text{sech}^3(x)} - \frac{10}{21} i a \cosh^{\frac{3}{2}}(x) \operatorname{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{a\,\text{sech}^3(x)}$$

```
[In] Int[(a*Sech[x]^3)^(3/2),x]
```

```
[Out] ((-10*I)/21)*a*Cosh[x]^(3/2)*EllipticF[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + (10*
a*Sqrt[a*Sech[x]^3]*Sinh[x])/21 + (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*Tanh[x])/7
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{9}{2}}(x)\,dx}{\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int \text{sech}^{\frac{5}{2}}(x)\,dx}{7\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\sinh(x)+\frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tanh(x)+\frac{\left(5a\sqrt{a\text{sech}^3(x)}\right)\int\sqrt{\text{sech}(x)}\,dx}{21\text{sech}^{\frac{3}{2}}(x)}$$

$$= \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tanh(x) + \frac{1}{21}\left(5a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}\right)\int\frac{1}{\sqrt{\cosh(x)}}\,dx$$

$$= -\frac{10}{21}ia\cosh^{\frac{3}{2}}(x)\,\text{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{a\text{sech}^3(x)} + \frac{10}{21}a\sqrt{a\text{sech}^3(x)}\sinh(x) + \frac{2}{7}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\tanh(x)$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.68

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx=\frac{2}{21}a\text{sech}(x)\sqrt{a\text{sech}^3(x)}\left(-5i\cosh^{\frac{5}{2}}(x)\,\text{EllipticF}\left(\frac{ix}{2},2\right)+5\cosh(x)\sinh(x)+3\tanh(x)\right)$$

```
[In] Integrate[(a*Sech[x]^3)^(3/2),x]
[Out] (2*a*Sech[x]*Sqrt[a*Sech[x]^3]*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2]
+ 5*Cosh[x]*Sinh[x] + 3*Tanh[x]))/21
```

Maple [F]

$$\int \left(a\,\text{sech}\left(x\right)^{3}\right)^{\frac{3}{2}}dx$$

```
[In] int((a*sech(x)^3)^(3/2),x)
[Out] int((a*sech(x)^3)^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 391, normalized size of antiderivative = 5.67

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx=\frac{2\left(5\sqrt{2}\left(a\cosh\left(x\right)^{6}+6\,a\cosh\left(x\right)\sinh\left(x\right)^{5}+a\sinh\left(x\right)^{6}+3\,a\cosh\left(x\right)^{4}+3\,\left(5\,a\cosh\right.\right.\right.}{}$$

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="fricas")
[Out] 2/21*(5*sqrt(2)*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*co
sh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))
*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^
2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)*sqrt(a)*weiers
trassPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(5*a*cosh(x)^6 + 30*a*cos
h(x)*sinh(x)^5 + 5*a*sinh(x)^6 + 17*a*cosh(x)^4 + (75*a*cosh(x)^2 + 17*a)*s
inh(x)^4 + 4*(25*a*cosh(x)^3 + 17*a*cosh(x))*sinh(x)^3 - 17*a*cosh(x)^2 + (
75*a*cosh(x)^4 + 102*a*cosh(x)^2 - 17*a)*sinh(x)^2 + 2*(15*a*cosh(x)^5 + 34
*a*cosh(x)^3 - 17*a*cosh(x))*sinh(x) - 5*a)*sqrt((a*cosh(x) + a*sinh(x))/(c
osh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(cosh(x)^6 + 6*cosh(x)*sinh
(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh
(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2
+ 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx = \int \left(a\,\text{sech}^3\,(x)\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sech(x)**3)**(3/2),x)
[Out] Integral((a*sech(x)**3)**(3/2), x)
```

Maxima [F]

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx = \int \left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx = \int \left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a*sech(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\text{sech}^3(x)\right)^{3/2}\,dx = \int \left(\frac{a}{\cosh\,(x)^3}\right)^{3/2}\,dx$$

```
[In] int((a/cosh(x)^3)^(3/2),x)
[Out] int((a/cosh(x)^3)^(3/2), x)
```

3.41 $\int \sqrt{a \operatorname{sech}^3(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \sqrt{a \operatorname{sech}^3(x)}\,dx = 2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a \operatorname{sech}^3(x)}+2\cosh(x)\sqrt{a \operatorname{sech}^3(x)}\sinh(x)$$

```
[Out] 2*I*cosh(x)^(3/2)*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x)
,2^(1/2))*(a*sech(x)^3)^(1/2)+2*cosh(x)*sinh(x)*(a*sech(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3853, 3856, 2719}

$$\int \sqrt{a \operatorname{sech}^3(x)}\,dx = 2\sinh(x)\cosh(x)\sqrt{a \operatorname{sech}^3(x)}+2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a \operatorname{sech}^3(x)}$$

```
[In] Int[Sqrt[a*Sech[x]^3],x]
```

```
[Out] (2*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2]*Sqrt[a*Sech[x]^3] + 2*Cosh[x]*Sqr
t[a*Sech[x]^3]*Sinh[x]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a\operatorname{sech}^3(x)}\int \operatorname{sech}^{\frac{3}{2}}(x)\,dx}{\operatorname{sech}^{\frac{3}{2}}(x)}\\
&= 2\cosh(x)\sqrt{a\operatorname{sech}^3(x)}\sinh(x) - \frac{\sqrt{a\operatorname{sech}^3(x)}\int \frac{1}{\sqrt{\operatorname{sech}(x)}}\,dx}{\operatorname{sech}^{\frac{3}{2}}(x)}\\
&= 2\cosh(x)\sqrt{a\operatorname{sech}^3(x)}\sinh(x) - \left(\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}\right)\int \sqrt{\cosh(x)}\,dx\\
&= 2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\operatorname{sech}^3(x)} + 2\cosh(x)\sqrt{a\operatorname{sech}^3(x)}\sinh(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \sqrt{a\operatorname{sech}^3(x)}\,dx = 2\cosh(x)\sqrt{a\operatorname{sech}^3(x)}\left(i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)+\sinh(x)\right)$$

```
[In] Integrate[Sqrt[a*Sech[x]^3],x]
[Out] 2*Cosh[x]*Sqrt[a*Sech[x]^3]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x
])
```

Maple [F]

$$\int \sqrt{a\,\mathrm{sech}\,(x)^3}dx$$

```
[In] int((a*sech(x)^3)^(1/2),x)
[Out] int((a*sech(x)^3)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\begin{aligned}&\int \sqrt{a\mathrm{sech}^3(x)}\,dx\\&= 2\sqrt{2}\sqrt{\frac{a\cosh(x)+a\sinh(x)}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}}(\cosh(x)+\sinh(x))\\&\quad+2\sqrt{2}\sqrt{a}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x)))\end{aligned}$$

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(2)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sin
h(x)^2 + 1))*(cosh(x) + sinh(x)) + 2*sqrt(2)*sqrt(a)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cosh(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{a\mathrm{sech}^3(x)}\,dx = \int \sqrt{a\,\mathrm{sech}^3\,(x)}\,dx$$

```
[In] integrate((a*sech(x)**3)**(1/2),x)
[Out] Integral(sqrt(a*sech(x)**3), x)
```

Maxima [F]

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^3}\,dx$$

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*sech(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \int \sqrt{a\,\text{sech}\,(x)^3}\,dx$$

```
[In] integrate((a*sech(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*sech(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\text{sech}^3(x)}\,dx = \int \sqrt{\frac{a}{\cosh\,(x)^3}}\,dx$$

```
[In] int((a/cosh(x)^3)^(1/2),x)
[Out] int((a/cosh(x)^3)^(1/2), x)
```

3.42 $\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}+\frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}$$

```
[Out] -2/3*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))/c
osh(x)^(3/2)/(a*sech(x)^3)^(1/2)+2/3*tanh(x)/(a*sech(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3854, 3856, 2720}

$$\int \frac{1}{\sqrt{a\text{sech}^3(x)}}\,dx = \frac{2\tanh(x)}{3\sqrt{a\text{sech}^3(x)}}-\frac{2i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}$$

```
[In] Int[1/Sqrt[a*Sech[x]^3],x]
```

```
[Out] (((-2*I)/3)*EllipticF[(I/2)*x, 2])/(Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) + (2*T
anh[x])/(3*Sqrt[a*Sech[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^{\frac{3}{2}}(x)\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\operatorname{sech}^3(x)}} \\
&= \frac{2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}} + \frac{\operatorname{sech}^{\frac{3}{2}}(x)\int \sqrt{\operatorname{sech}(x)}\,dx}{3\sqrt{a\operatorname{sech}^3(x)}} \\
&= \frac{2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}} + \frac{\int \frac{1}{\sqrt{\cosh(x)}}\,dx}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}} \\
&= -\frac{2i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{a\operatorname{sech}^3(x)}}\,dx = \frac{-\frac{2i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{\cosh^{\frac{3}{2}}(x)} + 2\tanh(x)}{3\sqrt{a\operatorname{sech}^3(x)}}$$

```
[In] Integrate[1/Sqrt[a*Sech[x]^3],x]
[Out] (((-2*I)*EllipticF[(I/2)*x, 2])/Cosh[x]^(3/2) + 2*Tanh[x])/(3*Sqrt[a*Sech[x
]^3])
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,\mathrm{sech}\,(x)^{3}}}dx$$

```
[In] int(1/(a*sech(x)^3)^(1/2),x)
[Out] int(1/(a*sech(x)^3)^(1/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.62

$$\int \frac{1}{\sqrt{a\mathrm{sech}^3(x)}}\,dx = \frac{4\sqrt{2}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2\right)\sqrt{a}\mathrm{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x)) + \sqrt{2}}{6\left(a\cosh(x)^2\right.}$$

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="fricas")
[Out] 1/6*(4*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(a)*weierstr
assPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(cosh(x)^4 + 4*cosh(x)^3*si
nh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)*sqrt((
a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(a
*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\mathrm{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\mathrm{sech}^{3}\,(x)}}\,dx$$

```
[In] integrate(1/(a*sech(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*sech(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\mathrm{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\mathrm{sech}\,(x)^3}}\,dx$$

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\mathrm{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{a\,\mathrm{sech}\,(x)^3}}\,dx$$

```
[In] integrate(1/(a*sech(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*sech(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\mathrm{sech}^3(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\cosh(x)^3}}}\,dx$$

```
[In] int(1/(a/cosh(x)^3)^(1/2),x)
[Out] int(1/(a/cosh(x)^3)^(1/2), x)
```

3.43 $\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 77

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = -\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}+\frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}}+\frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\text{sech}^3(x)}}$$

```
[Out] -14/15*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))
/a/cosh(x)^(3/2)/(a*sech(x)^3)^(1/2)+14/45*sinh(x)/a/(a*sech(x)^3)^(1/2)+2/
9*cosh(x)^2*sinh(x)/a/(a*sech(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3854, 3856, 2719}

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \frac{14\sinh(x)}{45a\sqrt{a\text{sech}^3(x)}}-\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}}+\frac{2\sinh(x)\cosh^2(x)}{9a\sqrt{a\text{sech}^3(x)}}$$

```
[In] Int[(a*Sech[x]^3)^(-3/2),x]
```

```
[Out] (((-14*I)/15)*EllipticE[(I/2)*x, 2])/(a*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3]) +
(14*Sinh[x])/(45*a*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^2*Sinh[x])/(9*a*Sqrt[a*S
ech[x]^3])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^{\frac{3}{2}}(x)\int \frac{1}{\operatorname{sech}^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\operatorname{sech}^3(x)}} \\
&= \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}} + \frac{\left(7\operatorname{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\operatorname{sech}^{\frac{5}{2}}(x)}\,dx}{9a\sqrt{a\operatorname{sech}^3(x)}} \\
&= \frac{14\sinh(x)}{45a\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}} + \frac{\left(7\operatorname{sech}^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\operatorname{sech}(x)}}\,dx}{15a\sqrt{a\operatorname{sech}^3(x)}} \\
&= \frac{14\sinh(x)}{45a\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}} + \frac{7\int \sqrt{\cosh(x)}\,dx}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}} \\
&= -\frac{14iE\left(\frac{ix}{2}\middle|2\right)}{15a\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}} + \frac{14\sinh(x)}{45a\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^2(x)\sinh(x)}{9a\sqrt{a\operatorname{sech}^3(x)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx=\frac{-\frac{84iE\left(\frac{ix}{2}|2\right)}{\cosh^{\frac{3}{2}}(x)}+33\sinh(x)+5\sinh(3x)}{90a\sqrt{a\text{sech}^3(x)}}$$

```
[In] Integrate[(a*Sech[x]^3)^(-3/2),x]
[Out] (((-84*I)*EllipticE[(I/2)*x, 2])/Cosh[x]^(3/2) + 33*Sinh[x] + 5*Sinh[3*x])/
(90*a*Sqrt[a*Sech[x]^3])
```

Maple [F]

$$\int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a*sech(x)^3)^(3/2),x)
[Out] int(1/(a*sech(x)^3)^(3/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 5.29

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx=$$

$$-\frac{672\sqrt{2}\left(\cosh(x)^5+5\cosh(x)^4\sinh(x)+10\cosh(x)^3\sinh(x)^2+10\cosh(x)^2\sinh(x)^3+5\cosh(x)\sinh\right.}{}$$

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="fricas")
[Out] -1/720*(672*sqrt(2)*(cosh(x)^5 + 5*cosh(x)^4*sinh(x) + 10*cosh(x)^3*sinh(x)
^2 + 10*cosh(x)^2*sinh(x)^3 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5)*sqrt(a)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) - sqrt(2)
*(5*cosh(x)^10 + 50*cosh(x)*sinh(x)^9 + 5*sinh(x)^10 + (225*cosh(x)^2 + 43)
*sinh(x)^8 + 43*cosh(x)^8 + 8*(75*cosh(x)^3 + 43*cosh(x))*sinh(x)^7 + 2*(52
5*cosh(x)^4 + 602*cosh(x)^2 - 149)*sinh(x)^6 - 298*cosh(x)^6 + 4*(315*cosh(
x)^5 + 602*cosh(x)^3 - 447*cosh(x))*sinh(x)^5 + 2*(525*cosh(x)^6 + 1505*cos
h(x)^4 - 2235*cosh(x)^2 - 187)*sinh(x)^4 - 374*cosh(x)^4 + 8*(75*cosh(x)^7
+ 301*cosh(x)^5 - 745*cosh(x)^3 - 187*cosh(x))*sinh(x)^3 + (225*cosh(x)^8 +
 1204*cosh(x)^6 - 4470*cosh(x)^4 - 2244*cosh(x)^2 - 43)*sinh(x)^2 - 43*cosh
```

```
(x)^2 + 2*(25*cosh(x)^9 + 172*cosh(x)^7 - 894*cosh(x)^5 - 748*cosh(x)^3 - 4
3*cosh(x))*sinh(x) - 5)*sqrt((a*cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2 + 1)))/(a^2*cosh(x)^5 + 5*a^2*cosh(x)^4*sinh(x) + 10*a
^2*cosh(x)^3*sinh(x)^2 + 10*a^2*cosh(x)^2*sinh(x)^3 + 5*a^2*cosh(x)*sinh(x)
^4 + a^2*sinh(x)^5)
```

Sympy [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\operatorname{sech}^3{(x)}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)**3)**(3/2),x)
[Out] Integral((a*sech(x)**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\operatorname{sech}\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sech(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\operatorname{sech}\left(x\right)^{3}\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{3/2}}\,dx$$

```
[In] int(1/(a/cosh(x)^3)^(3/2),x)
[Out] int(1/(a/cosh(x)^3)^(3/2), x)
```

3.44 $\int \frac{1}{\left(a\mathbf{sech}^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx=-\frac{26i\operatorname{EllipticF}\left(\frac{ix}{2},2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}+\frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\tanh(x)}{77a^2\sqrt{a\operatorname{sech}^3(x)}}$$

```
[Out] -26/77*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/2))
/a^2/cosh(x)^(3/2)/(a*sech(x)^3)^(1/2)+78/385*cosh(x)*sinh(x)/a^2/(a*sech(x
)^3)^(1/2)+26/165*cosh(x)^3*sinh(x)/a^2/(a*sech(x)^3)^(1/2)+2/15*cosh(x)^5*
sinh(x)/a^2/(a*sech(x)^3)^(1/2)+26/77*tanh(x)/a^2/(a*sech(x)^3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4208, 3854, 3856, 2720}

$$\int \frac{1}{\left(a\operatorname{sech}^3(x)\right)^{5/2}}\,dx=\frac{26\tanh(x)}{77a^2\sqrt{a\operatorname{sech}^3(x)}}-\frac{26i\operatorname{EllipticF}\left(\frac{ix}{2},2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\operatorname{sech}^3(x)}}+\frac{2\sinh(x)\cosh^5(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{26\sinh(x)\cosh^3(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}}+\frac{78\sinh(x)\cosh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}}$$

```
[In] Int[(a*Sech[x]^3)^(-5/2),x]
```

```
[Out] (((-26*I)/77)*EllipticF[(I/2)*x, 2])/(a^2*Cosh[x]^(3/2)*Sqrt[a*Sech[x]^3])
+ (78*Cosh[x]*Sinh[x])/(385*a^2*Sqrt[a*Sech[x]^3]) + (26*Cosh[x]^3*Sinh[x])
```

```
/(165*a^2*Sqrt[a*Sech[x]^3]) + (2*Cosh[x]^5*Sinh[x])/(15*a^2*Sqrt[a*Sech[x]
^3]) + (26*Tanh[x])/(77*a^2*Sqrt[a*Sech[x]^3])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\operatorname{sech}^{\frac{3}{2}}(x)\int\frac{1}{\operatorname{sech}^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\operatorname{sech}^3(x)}}$$

$$= \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}} + \frac{\left(13\operatorname{sech}^{\frac{3}{2}}(x)\right)\int\frac{1}{\operatorname{sech}^{\frac{11}{2}}(x)}\,dx}{15a^2\sqrt{a\operatorname{sech}^3(x)}}$$

$$= \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}} + \frac{\left(39\operatorname{sech}^{\frac{3}{2}}(x)\right)\int\frac{1}{\operatorname{sech}^{\frac{7}{2}}(x)}\,dx}{55a^2\sqrt{a\operatorname{sech}^3(x)}}$$

$$= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\operatorname{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\operatorname{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\operatorname{sech}^3(x)}} + \frac{\left(39\operatorname{sech}^{\frac{3}{2}}(x)\right)\int\frac{1}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx}{77a^2\sqrt{a\operatorname{sech}^3(x)}}$$

$$
\begin{aligned}
&= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} \\
&\quad + \frac{26\tanh(x)}{77a^2\sqrt{a\text{sech}^3(x)}} + \frac{\left(13\text{sech}^{\frac{3}{2}}(x)\right)\int\sqrt{\text{sech}(x)}\,dx}{77a^2\sqrt{a\text{sech}^3(x)}} \\
&= \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} \\
&\quad + \frac{26\tanh(x)}{77a^2\sqrt{a\text{sech}^3(x)}} + \frac{13\int\frac{1}{\sqrt{\cosh(x)}}\,dx}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} \\
&= -\frac{26i\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{77a^2\cosh^{\frac{3}{2}}(x)\sqrt{a\text{sech}^3(x)}} + \frac{78\cosh(x)\sinh(x)}{385a^2\sqrt{a\text{sech}^3(x)}} \\
&\quad + \frac{26\cosh^3(x)\sinh(x)}{165a^2\sqrt{a\text{sech}^3(x)}} + \frac{2\cosh^5(x)\sinh(x)}{15a^2\sqrt{a\text{sech}^3(x)}} + \frac{26\tanh(x)}{77a^2\sqrt{a\text{sech}^3(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$
\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \frac{\cosh(x)\sqrt{a\text{sech}^3(x)}\left(-24960i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right) + 19122\sinh(2x) + 4406\sinh
}{73920a^3}
$$

```
[In] Integrate[(a*Sech[x]^3)^(-5/2),x]
[Out] (Cosh[x]*Sqrt[a*Sech[x]^3]*((-24960*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]
+ 19122*Sinh[2*x] + 4406*Sinh[4*x] + 826*Sinh[6*x] + 77*Sinh[8*x]))/(73920*
a^3)
```

Maple [F]

$$
\int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{5}{2}}}dx
$$

```
[In] int(1/(a*sech(x)^3)^(5/2),x)
[Out] int(1/(a*sech(x)^3)^(5/2),x)
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 718, normalized size of antiderivative = 5.93

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/147840*(49920*sqrt(2)*(cosh(x)^8 + 8*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sin
h(x)^2 + 56*cosh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 + 56*cosh(x)^3*sin
h(x)^5 + 28*cosh(x)^2*sinh(x)^6 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8)*sqrt(a)*
weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + sqrt(2)*(77*cosh(x)^16 + 12
32*cosh(x)*sinh(x)^15 + 77*sinh(x)^16 + 14*(660*cosh(x)^2 + 59)*sinh(x)^14
+ 826*cosh(x)^14 + 196*(220*cosh(x)^3 + 59*cosh(x))*sinh(x)^13 + 2*(70070*c
osh(x)^4 + 37583*cosh(x)^2 + 2203)*sinh(x)^12 + 4406*cosh(x)^12 + 8*(42042*
cosh(x)^5 + 37583*cosh(x)^3 + 6609*cosh(x))*sinh(x)^11 + 2*(308308*cosh(x)^
6 + 413413*cosh(x)^4 + 145398*cosh(x)^2 + 9561)*sinh(x)^10 + 19122*cosh(x)^
10 + 4*(220220*cosh(x)^7 + 413413*cosh(x)^5 + 242330*cosh(x)^3 + 47805*cosh
(x))*sinh(x)^9 + 6*(165165*cosh(x)^8 + 413413*cosh(x)^6 + 363495*cosh(x)^4
+ 143415*cosh(x)^2)*sinh(x)^8 + 16*(55055*cosh(x)^9 + 177177*cosh(x)^7 + 21
8097*cosh(x)^5 + 143415*cosh(x)^3)*sinh(x)^7 + 2*(308308*cosh(x)^10 + 12402
39*cosh(x)^8 + 2035572*cosh(x)^6 + 2007810*cosh(x)^4 - 9561)*sinh(x)^6 - 19
122*cosh(x)^6 + 4*(84084*cosh(x)^11 + 413413*cosh(x)^9 + 872388*cosh(x)^7 +
 1204686*cosh(x)^5 - 28683*cosh(x))*sinh(x)^5 + 2*(70070*cosh(x)^12 + 41341
3*cosh(x)^10 + 1090485*cosh(x)^8 + 2007810*cosh(x)^6 - 143415*cosh(x)^2 - 2
203)*sinh(x)^4 - 4406*cosh(x)^4 + 8*(5390*cosh(x)^13 + 37583*cosh(x)^11 + 1
21165*cosh(x)^9 + 286830*cosh(x)^7 - 47805*cosh(x)^3 - 2203*cosh(x))*sinh(x
)^3 + 2*(4620*cosh(x)^14 + 37583*cosh(x)^12 + 145398*cosh(x)^10 + 430245*co
sh(x)^8 - 143415*cosh(x)^4 - 13218*cosh(x)^2 - 413)*sinh(x)^2 - 826*cosh(x)
^2 + 4*(308*cosh(x)^15 + 2891*cosh(x)^13 + 13218*cosh(x)^11 + 47805*cosh(x)
^9 - 28683*cosh(x)^5 - 4406*cosh(x)^3 - 413*cosh(x))*sinh(x) - 77)*sqrt((a*
cosh(x) + a*sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)))/(a^3
*cosh(x)^8 + 8*a^3*cosh(x)^7*sinh(x) + 28*a^3*cosh(x)^6*sinh(x)^2 + 56*a^3*
cosh(x)^5*sinh(x)^3 + 70*a^3*cosh(x)^4*sinh(x)^4 + 56*a^3*cosh(x)^3*sinh(x)
^5 + 28*a^3*cosh(x)^2*sinh(x)^6 + 8*a^3*cosh(x)*sinh(x)^7 + a^3*sinh(x)^8)
```

Sympy [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}^3\,(x)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)**3)**(5/2),x)
[Out] Integral((a*sech(x)**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}\,(x)^3\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)^3)^(5/2),x, algorithm="giac")
[Out] integrate((a*sech(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\text{sech}^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(\frac{a}{\cosh(x)^3}\right)^{5/2}}\,dx$$

```
[In] int(1/(a/cosh(x)^3)^(5/2),x)
[Out] int(1/(a/cosh(x)^3)^(5/2), x)
```

3.45 $\int \left(a\text{sech}^4(x)\right)^{7/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 163

$$\begin{aligned}\int \left(a\text{sech}^4(x)\right)^{7/2} dx &= a^3\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)\\&\quad -2a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh(x)+3a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^3(x)\\&\quad -\frac{20}{7}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^5(x)+\frac{5}{3}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^7(x)\\&\quad -\frac{6}{11}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^9(x)+\frac{1}{13}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^{11}(x)\end{aligned}$$

```
[Out] a^3*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-2*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)
*tanh(x)+3*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3-20/7*a^3*sinh(x)^2*(
a*sech(x)^4)^(1/2)*tanh(x)^5+5/3*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^
7-6/11*a^3*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^9+1/13*a^3*sinh(x)^2*(a*se
ch(x)^4)^(1/2)*tanh(x)^11
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4208, 3852}

$$\begin{aligned}\int \left(a\text{sech}^4(x)\right)^{7/2} dx &= a^3\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)}\\&\quad +\frac{1}{13}a^3\sinh^2(x)\tanh^{11}(x)\sqrt{a\text{sech}^4(x)}-\frac{6}{11}a^3\sinh^2(x)\tanh^9(x)\sqrt{a\text{sech}^4(x)}\\&\quad +\frac{5}{3}a^3\sinh^2(x)\tanh^7(x)\sqrt{a\text{sech}^4(x)}-\frac{20}{7}a^3\sinh^2(x)\tanh^5(x)\sqrt{a\text{sech}^4(x)}\\&\quad +3a^3\sinh^2(x)\tanh^3(x)\sqrt{a\text{sech}^4(x)}-2a^3\sinh^2(x)\tanh(x)\sqrt{a\text{sech}^4(x)}\end{aligned}$$

```
[In] Int[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] a^3*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - 2*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*T
anh[x] + 3*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3 - (20*a^3*Sqrt[a*Sech[
x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (5*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^7
)/3 - (6*a^3*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^9)/11 + (a^3*Sqrt[a*Sech[x
]^4]*Sinh[x]^2*Tanh[x]^11)/13
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(a^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^{14}(x)\,dx\\
&= \left(ia^3\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+6x^2+15x^4+20x^6+15x^8+6x^{10}+x^{12}\right)\,dx, x, -i\tanh(x)\right)\\
&= a^3\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)-2a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh(x)\\
&\quad+3a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^3(x)-\frac{20}{7}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^5(x)\\
&\quad+\frac{5}{3}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^7(x)-\frac{6}{11}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^9(x)\\
&\quad+\frac{1}{13}a^3\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^{11}(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \left(a\mathrm{sech}^4(x)\right)^{7/2} dx = \frac{\cosh(x)(2048 + 2380\cosh(2x) + 1093\cosh(4x) + 378\cosh(6x) + 92\cosh(8x) + 14\mathrm{c}}{6006}$$

```
[In] Integrate[(a*Sech[x]^4)^(7/2),x]
```

```
[Out] (Cosh[x]*(2048 + 2380*Cosh[2*x] + 1093*Cosh[4*x] + 378*Cosh[6*x] + 92*Cosh[
8*x] + 14*Cosh[10*x] + Cosh[12*x])*(a*Sech[x]^4)^(7/2)*Sinh[x])/6006
```

Maple [A] (verified)

Time = 137.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.44

method	result	size
risch	$-\frac{2048a^3\mathrm{e}^{-2x}\sqrt{\frac{\mathrm{e}^{4x}a}{(1+\mathrm{e}^{2x})^4}}\,(1716\,\mathrm{e}^{12x}+1287\,\mathrm{e}^{10x}+715\,\mathrm{e}^{8x}+286\,\mathrm{e}^{6x}+78\,\mathrm{e}^{4x}+13\,\mathrm{e}^{2x}+1)}{3003(1+\mathrm{e}^{2x})^{11}}$	72

```
[In] int((sech(x)^4*a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2048/3003*a^3*exp(-2*x)/(1+exp(2*x))^11*(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)*
(1716*exp(12*x)+1287*exp(10*x)+715*exp(8*x)+286*exp(6*x)+78*exp(4*x)+13*exp
(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2804 vs. 2(141) = 282.

Time = 0.37 (sec) , antiderivative size = 2804, normalized size of antiderivative = 17.20

$$\int \left(a\mathrm{sech}^4(x)\right)^{7/2} dx = \text{Too large to display}$$

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="fricas")
```

```
[Out] -2048/3003*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 1716*(a^3*e^(4*x) +
 2*a^3*e^(2*x) + a^3)*sinh(x)^12 + 20592*(a^3*cosh(x)*e^(4*x) + 2*a^3*cosh(
x)*e^(2*x) + a^3*cosh(x))*sinh(x)^11 + 715*a^3*cosh(x)^8 + 1287*(88*a^3*cos
h(x)^2 + a^3 + (88*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(88*a^3*cosh(x)^2 + a^3
)*e^(2*x))*sinh(x)^10 + 4290*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x) + (88*a^3*co
sh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(88*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(
2*x))*sinh(x)^9 + 286*a^3*cosh(x)^6 + 715*(1188*a^3*cosh(x)^4 + 81*a^3*cosh
(x)^2 + a^3 + (1188*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(11
```

88*a^3*cosh(x)^4 + 81*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^8 + 1144*(1188*
a^3*cosh(x)^5 + 135*a^3*cosh(x)^3 + 5*a^3*cosh(x) + (1188*a^3*cosh(x)^5 + 1
35*a^3*cosh(x)^3 + 5*a^3*cosh(x))*e^(4*x) + 2*(1188*a^3*cosh(x)^5 + 135*a^3
*cosh(x)^3 + 5*a^3*cosh(x))*e^(2*x))*sinh(x)^7 + 78*a^3*cosh(x)^4 + 286*(55
44*a^3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3 + (5544*a^3*c
osh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(5544*a^
3*cosh(x)^6 + 945*a^3*cosh(x)^4 + 70*a^3*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^
6 + 572*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^3*
cosh(x) + (2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cosh(x)^3 + 3*a^
3*cosh(x))*e^(4*x) + 2*(2376*a^3*cosh(x)^7 + 567*a^3*cosh(x)^5 + 70*a^3*cos
h(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 13*a^3*cosh(x)^2 + 26*(32670*a
^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2
 + 3*a^3 + (32670*a^3*cosh(x)^8 + 10395*a^3*cosh(x)^6 + 1925*a^3*cosh(x)^4
+ 165*a^3*cosh(x)^2 + 3*a^3)*e^(4*x) + 2*(32670*a^3*cosh(x)^8 + 10395*a^3*c
osh(x)^6 + 1925*a^3*cosh(x)^4 + 165*a^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)
^4 + 104*(3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*
a^3*cosh(x)^3 + 3*a^3*cosh(x) + (3630*a^3*cosh(x)^9 + 1485*a^3*cosh(x)^7 +
385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(4*x) + 2*(3630*a^3
*cosh(x)^9 + 1485*a^3*cosh(x)^7 + 385*a^3*cosh(x)^5 + 55*a^3*cosh(x)^3 + 3*
a^3*cosh(x))*e^(2*x))*sinh(x)^3 + a^3 + 13*(8712*a^3*cosh(x)^10 + 4455*a^3*
cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3
 + (8712*a^3*cosh(x)^10 + 4455*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3
*cosh(x)^4 + 36*a^3*cosh(x)^2 + a^3)*e^(4*x) + 2*(8712*a^3*cosh(x)^10 + 445
5*a^3*cosh(x)^8 + 1540*a^3*cosh(x)^6 + 330*a^3*cosh(x)^4 + 36*a^3*cosh(x)^2
 + a^3)*e^(2*x))*sinh(x)^2 + (1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 7
15*a^3*cosh(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2
+ a^3)*e^(4*x) + 2*(1716*a^3*cosh(x)^12 + 1287*a^3*cosh(x)^10 + 715*a^3*cos
h(x)^8 + 286*a^3*cosh(x)^6 + 78*a^3*cosh(x)^4 + 13*a^3*cosh(x)^2 + a^3)*e^(
2*x) + 26*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*
a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x) + (792*a^3*cosh(x)^11 + 495*
a^3*cosh(x)^9 + 220*a^3*cosh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a
^3*cosh(x))*e^(4*x) + 2*(792*a^3*cosh(x)^11 + 495*a^3*cosh(x)^9 + 220*a^3*c
osh(x)^7 + 66*a^3*cosh(x)^5 + 12*a^3*cosh(x)^3 + a^3*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(26*
cosh(x)*e^(2*x)*sinh(x)^25 + e^(2*x)*sinh(x)^26 + 13*(25*cosh(x)^2 + 1)*e^(
2*x)*sinh(x)^24 + 104*(25*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^23 + 26*(5
75*cosh(x)^4 + 138*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^22 + 572*(115*cosh(x)^5 +
 46*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^21 + 286*(805*cosh(x)^6 + 483*co
sh(x)^4 + 63*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^20 + 1144*(575*cosh(x)^7 + 483*
cosh(x)^5 + 105*cosh(x)^3 + 5*cosh(x))*e^(2*x)*sinh(x)^19 + 143*(10925*cosh
(x)^8 + 12236*cosh(x)^6 + 3990*cosh(x)^4 + 380*cosh(x)^2 + 5)*e^(2*x)*sinh(
x)^18 + 286*(10925*cosh(x)^9 + 15732*cosh(x)^7 + 7182*cosh(x)^5 + 1140*cosh
(x)^3 + 45*cosh(x))*e^(2*x)*sinh(x)^17 + 143*(37145*cosh(x)^10 + 66861*cosh
(x)^8 + 40698*cosh(x)^6 + 9690*cosh(x)^4 + 765*cosh(x)^2 + 9)*e^(2*x)*sinh(
x)^16 + 208*(37145*cosh(x)^11 + 81719*cosh(x)^9 + 63954*cosh(x)^7 + 21318*c

osh(x)^5 + 2805*cosh(x)^3 + 99*cosh(x))*e^(2*x)*sinh(x)^15 + 52*(185725*cos
h(x)^12 + 490314*cosh(x)^10 + 479655*cosh(x)^8 + 213180*cosh(x)^6 + 42075*c
osh(x)^4 + 2970*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^14 + 8*(1300075*cosh(x)^13
+ 4056234*cosh(x)^11 + 4849845*cosh(x)^9 + 2771340*cosh(x)^7 + 765765*cosh(
x)^5 + 90090*cosh(x)^3 + 3003*cosh(x))*e^(2*x)*sinh(x)^13 + 52*(185725*cosh
(x)^14 + 676039*cosh(x)^12 + 969969*cosh(x)^10 + 692835*cosh(x)^8 + 255255*
cosh(x)^6 + 45045*cosh(x)^4 + 3003*cosh(x)^2 + 33)*e^(2*x)*sinh(x)^12 + 208
*(37145*cosh(x)^15 + 156009*cosh(x)^13 + 264537*cosh(x)^11 + 230945*cosh(x)
^9 + 109395*cosh(x)^7 + 27027*cosh(x)^5 + 3003*cosh(x)^3 + 99*cosh(x))*e^(2
*x)*sinh(x)^11 + 143*(37145*cosh(x)^16 + 178296*cosh(x)^14 + 352716*cosh(x)
^12 + 369512*cosh(x)^10 + 218790*cosh(x)^8 + 72072*cosh(x)^6 + 12012*cosh(x
)^4 + 792*cosh(x)^2 + 9)*e^(2*x)*sinh(x)^10 + 286*(10925*cosh(x)^17 + 59432
*cosh(x)^15 + 135660*cosh(x)^13 + 167960*cosh(x)^11 + 121550*cosh(x)^9 + 51
480*cosh(x)^7 + 12012*cosh(x)^5 + 1320*cosh(x)^3 + 45*cosh(x))*e^(2*x)*sinh
(x)^9 + 143*(10925*cosh(x)^18 + 66861*cosh(x)^16 + 174420*cosh(x)^14 + 2519
40*cosh(x)^12 + 218790*cosh(x)^10 + 115830*cosh(x)^8 + 36036*cosh(x)^6 + 59
40*cosh(x)^4 + 405*cosh(x)^2 + 5)*e^(2*x)*sinh(x)^8 + 1144*(575*cosh(x)^19
+ 3933*cosh(x)^17 + 11628*cosh(x)^15 + 19380*cosh(x)^13 + 19890*cosh(x)^11
+ 12870*cosh(x)^9 + 5148*cosh(x)^7 + 1188*cosh(x)^5 + 135*cosh(x)^3 + 5*cos
h(x))*e^(2*x)*sinh(x)^7 + 286*(805*cosh(x)^20 + 6118*cosh(x)^18 + 20349*cos
h(x)^16 + 38760*cosh(x)^14 + 46410*cosh(x)^12 + 36036*cosh(x)^10 + 18018*co
sh(x)^8 + 5544*cosh(x)^6 + 945*cosh(x)^4 + 70*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 572*(115*cosh(x)^21 + 966*cosh(x)^19 + 3591*cosh(x)^17 + 7752*cosh(x)
^15 + 10710*cosh(x)^13 + 9828*cosh(x)^11 + 6006*cosh(x)^9 + 2376*cosh(x)^7
+ 567*cosh(x)^5 + 70*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^5 + 26*(575*cos
h(x)^22 + 5313*cosh(x)^20 + 21945*cosh(x)^18 + 53295*cosh(x)^16 + 84150*cos
h(x)^14 + 90090*cosh(x)^12 + 66066*cosh(x)^10 + 32670*cosh(x)^8 + 10395*cos
h(x)^6 + 1925*cosh(x)^4 + 165*cosh(x)^2 + 3)*e^(2*x)*sinh(x)^4 + 104*(25*co
sh(x)^23 + 253*cosh(x)^21 + 1155*cosh(x)^19 + 3135*cosh(x)^17 + 5610*cosh(x
)^15 + 6930*cosh(x)^13 + 6006*cosh(x)^11 + 3630*cosh(x)^9 + 1485*cosh(x)^7
+ 385*cosh(x)^5 + 55*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 13*(25*cosh
(x)^24 + 276*cosh(x)^22 + 1386*cosh(x)^20 + 4180*cosh(x)^18 + 8415*cosh(x)^
16 + 11880*cosh(x)^14 + 12012*cosh(x)^12 + 8712*cosh(x)^10 + 4455*cosh(x)^8
 + 1540*cosh(x)^6 + 330*cosh(x)^4 + 36*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 2
6*(cosh(x)^25 + 12*cosh(x)^23 + 66*cosh(x)^21 + 220*cosh(x)^19 + 495*cosh(x
)^17 + 792*cosh(x)^15 + 924*cosh(x)^13 + 792*cosh(x)^11 + 495*cosh(x)^9 + 2
20*cosh(x)^7 + 66*cosh(x)^5 + 12*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (co
sh(x)^26 + 13*cosh(x)^24 + 78*cosh(x)^22 + 286*cosh(x)^20 + 715*cosh(x)^18
+ 1287*cosh(x)^16 + 1716*cosh(x)^14 + 1716*cosh(x)^12 + 1287*cosh(x)^10 + 7
15*cosh(x)^8 + 286*cosh(x)^6 + 78*cosh(x)^4 + 13*cosh(x)^2 + 1)*e^(2*x))

Sympy [F(-1)]

Timed out.

$$\int \left(a\text{sech}^4(x)\right)^{7/2} dx = \text{Timed out}$$

```
[In] integrate((a*sech(x)**4)**(7/2),x)
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(141) = 282.

Time = 0.29 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.80

$$\int \left(a\text{sech}^4(x)\right)^{7/2} dx = \text{Too large to display}$$

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="maxima")
[Out] 2048/231*a^(7/2)*e^(-2*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) + 715*e
^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(-16*x)
 + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-26*x)
+ 1) + 4096/77*a^(7/2)*e^(-4*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-6*x) +
 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 1287*e^(
-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x) + e^(-
26*x) + 1) + 4096/21*a^(7/2)*e^(-6*x)/(13*e^(-2*x) + 78*e^(-4*x) + 286*e^(-
6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*x) + 12
87*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(-24*x)
+ e^(-26*x) + 1) + 10240/21*a^(7/2)*e^(-8*x)/(13*e^(-2*x) + 78*e^(-4*x) + 2
86*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e^(-14*
x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) + 13*e^(
-24*x) + e^(-26*x) + 1) + 6144/7*a^(7/2)*e^(-10*x)/(13*e^(-2*x) + 78*e^(-4*
x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1716*e
^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*x) +
13*e^(-24*x) + e^(-26*x) + 1) + 8192/7*a^(7/2)*e^(-12*x)/(13*e^(-2*x) + 78*
e^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) +
1716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22
*x) + 13*e^(-24*x) + e^(-26*x) + 1) + 2048/3003*a^(7/2)/(13*e^(-2*x) + 78*e
^(-4*x) + 286*e^(-6*x) + 715*e^(-8*x) + 1287*e^(-10*x) + 1716*e^(-12*x) + 1
716*e^(-14*x) + 1287*e^(-16*x) + 715*e^(-18*x) + 286*e^(-20*x) + 78*e^(-22*
x) + 13*e^(-24*x) + e^(-26*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx = -\frac{2048\,a^{\frac{7}{2}}\left(1716\,e^{(12\,x)} + 1287\,e^{(10\,x)} + 715\,e^{(8\,x)} + 286\,e^{(6\,x)} + 78\,e^{(4\,x)} + 13\,e^{(2\,x)} + 1\right)}{3003\left(e^{(2\,x)} + 1\right)^{13}}$$

```
[In] integrate((a*sech(x)^4)^(7/2),x, algorithm="giac")
[Out] -2048/3003*a^(7/2)*(1716*e^(12*x) + 1287*e^(10*x) + 715*e^(8*x) + 286*e^(6*
x) + 78*e^(4*x) + 13*e^(2*x) + 1)/(e^(2*x) + 1)^13
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.06

$$\int \left(a\text{sech}^4(x)\right)^{7/2}\,dx = \frac{1536\,a^3\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{\left(e^{2\,x}+1\right)^8\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} - \frac{2048\,a^3\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{7\left(e^{2\,x}+1\right)^7\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} - \frac{10240\,a^3\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{3\left(e^{2\,x}+1\right)^9\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} + \frac{4096\,a^3\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{\left(e^{2\,x}+1\right)^{10}\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} - \frac{30720\,a^3\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{11\left(e^{2\,x}+1\right)^{11}\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} + \frac{1024\,a^3\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{\left(e^{2\,x}+1\right)^{12}\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} - \frac{2048\,a^3\sqrt{\frac{a}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\left(4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1\right)}{13\left(e^{2\,x}+1\right)^{13}\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}$$

```
[In] int((a/cosh(x)^4)^(7/2),x)
```

```
[Out] (1536*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*e
xp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6*x
))) - (2048*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) + 2*exp(4*x) +
 exp(6*x))) - (10240*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6
*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^9*(exp(2*x) + 2*e
xp(4*x) + exp(6*x))) + (4096*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(
2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^10*(exp(2*x
) + 2*exp(4*x) + exp(6*x))) - (30720*a^3*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)
*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(11*(exp(2*x) + 1)^
11*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*a^3*(a/(exp(-x)/2 + exp(x)/2
)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x
) + 1)^12*(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*a^3*(a/(exp(-x)/2 + e
xp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(1
3*(exp(2*x) + 1)^13*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

3.46 $\int \left(a\operatorname{sech}^4(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \left(a\operatorname{sech}^4(x)\right)^{5/2} dx = a^2\cosh(x)\sqrt{a\operatorname{sech}^4(x)}\sinh(x) - \frac{4}{3}a^2\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x)\tanh(x) + \frac{6}{5}a^2\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x)\tanh^3(x) - \frac{4}{7}a^2\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x)\tanh^5(x) + \frac{1}{9}a^2\sqrt{a\operatorname{sech}^4(x)}\sinh^2(x)\tanh^7(x)$$

```
[Out] a^2*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-4/3*a^2*sinh(x)^2*(a*sech(x)^4)^(1/
2)*tanh(x)+6/5*a^2*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3-4/7*a^2*sinh(x)^
2*(a*sech(x)^4)^(1/2)*tanh(x)^5+1/9*a^2*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(
x)^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4208, 3852}

$$\int \left(a\operatorname{sech}^4(x)\right)^{5/2} dx = a^2\sinh(x)\cosh(x)\sqrt{a\operatorname{sech}^4(x)} + \frac{1}{9}a^2\sinh^2(x)\tanh^7(x)\sqrt{a\operatorname{sech}^4(x)} - \frac{4}{7}a^2\sinh^2(x)\tanh^5(x)\sqrt{a\operatorname{sech}^4(x)} + \frac{6}{5}a^2\sinh^2(x)\tanh^3(x)\sqrt{a\operatorname{sech}^4(x)} - \frac{4}{3}a^2\sinh^2(x)\tanh(x)\sqrt{a\operatorname{sech}^4(x)}$$

```
[In] Int[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] a^2*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (4*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*
Tanh[x])/3 + (6*a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5 - (4*a^2*Sqrt[
a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^5)/7 + (a^2*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x]^7)/9
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(a^2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^{10}(x)\,dx \\
&= \left(ia^2\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -i\tanh(x)\right) \\
&= a^2\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)-\frac{4}{3}a^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh(x) \\
&\quad+\frac{6}{5}a^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^3(x)-\frac{4}{7}a^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^5(x) \\
&\quad+\frac{1}{9}a^2\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^7(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.36

$$\begin{aligned}
\int \left(a\text{sech}^4(x)\right)^{5/2}\,dx &= \frac{1}{315}\cosh(x)(128+130\cosh(2x) \\
&\quad+46\cosh(4x)+10\cosh(6x)+\cosh(8x))\left(a\text{sech}^4(x)\right)^{5/2}\sinh(x)
\end{aligned}$$

```
[In] Integrate[(a*Sech[x]^4)^(5/2),x]
```

```
[Out] (Cosh[x]*(128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*(a
*Sech[x]^4)^(5/2)*Sinh[x])/315
```

Maple [A] (verified)

Time = 144.83 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256 a^{2} {\mathrm e}^{-2 x} \sqrt{\frac{{\mathrm e}^{4 x} a}{\left(1+{\mathrm e}^{2 x}\right)^{4}}}\, \left(126 \,{\mathrm e}^{8 x}+84 \,{\mathrm e}^{6 x}+36 \,{\mathrm e}^{4 x}+9 \,{\mathrm e}^{2 x}+1\right)}{315 \left(1+{\mathrm e}^{2 x}\right)^{7}}$	60

```
[In] int((sech(x)^4*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -256/315*a^2*exp(-2*x)/(1+exp(2*x))^7*(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)*(12
6*exp(8*x)+84*exp(6*x)+36*exp(4*x)+9*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1475 vs. 2(99) = 198.

Time = 0.28 (sec) , antiderivative size = 1475, normalized size of antiderivative = 12.61

$$\int \left(a\text{sech}^4(x)\right)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -256/315*(126*a^2*cosh(x)^8 + 126*(a^2*e^(4*x) + 2*a^2*e^(2*x) + a^2)*sinh(
x)^8 + 84*a^2*cosh(x)^6 + 1008*(a^2*cosh(x)*e^(4*x) + 2*a^2*cosh(x)*e^(2*x)
 + a^2*cosh(x))*sinh(x)^7 + 84*(42*a^2*cosh(x)^2 + a^2 + (42*a^2*cosh(x)^2
+ a^2)*e^(4*x) + 2*(42*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 36*a^2*cos
h(x)^4 + 504*(14*a^2*cosh(x)^3 + a^2*cosh(x) + (14*a^2*cosh(x)^3 + a^2*cosh
(x))*e^(4*x) + 2*(14*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 36*(
245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2 + (245*a^2*cosh(x)^4 + 35*a^2*co
sh(x)^2 + a^2)*e^(4*x) + 2*(245*a^2*cosh(x)^4 + 35*a^2*cosh(x)^2 + a^2)*e^(
2*x))*sinh(x)^4 + 9*a^2*cosh(x)^2 + 48*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^
3 + 3*a^2*cosh(x) + (147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
e^(4*x) + 2*(147*a^2*cosh(x)^5 + 35*a^2*cosh(x)^3 + 3*a^2*cosh(x))*e^(2*x))
*sinh(x)^3 + 9*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 +
a^2 + (392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^(4
*x) + 2*(392*a^2*cosh(x)^6 + 140*a^2*cosh(x)^4 + 24*a^2*cosh(x)^2 + a^2)*e^
(2*x))*sinh(x)^2 + a^2 + (126*a^2*cosh(x)^8 + 84*a^2*cosh(x)^6 + 36*a^2*cos
h(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(126*a^2*cosh(x)^8 + 84*a^2*cos
h(x)^6 + 36*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 + a^2)*e^(2*x) + 18*(56*a^2*cos
h(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x) + (56*a^2*cosh(x)
^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(56*a^2*
cosh(x)^7 + 28*a^2*cosh(x)^5 + 8*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh
(x))*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(18*
cosh(x)*e^(2*x)*sinh(x)^17 + e^(2*x)*sinh(x)^18 + 9*(17*cosh(x)^2 + 1)*e^(2
```

```
*x)*sinh(x)^16 + 48*(17*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^15 + 36*(85*
cosh(x)^4 + 30*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^14 + 504*(17*cosh(x)^5 + 10*c
osh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^13 + 84*(221*cosh(x)^6 + 195*cosh(x)^4
+ 39*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^12 + 144*(221*cosh(x)^7 + 273*cosh(x)^5
 + 91*cosh(x)^3 + 7*cosh(x))*e^(2*x)*sinh(x)^11 + 18*(2431*cosh(x)^8 + 4004
*cosh(x)^6 + 2002*cosh(x)^4 + 308*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^10 + 4*(12
155*cosh(x)^9 + 25740*cosh(x)^7 + 18018*cosh(x)^5 + 4620*cosh(x)^3 + 315*co
sh(x))*e^(2*x)*sinh(x)^9 + 18*(2431*cosh(x)^10 + 6435*cosh(x)^8 + 6006*cosh
(x)^6 + 2310*cosh(x)^4 + 315*cosh(x)^2 + 7)*e^(2*x)*sinh(x)^8 + 144*(221*co
sh(x)^11 + 715*cosh(x)^9 + 858*cosh(x)^7 + 462*cosh(x)^5 + 105*cosh(x)^3 +
7*cosh(x))*e^(2*x)*sinh(x)^7 + 84*(221*cosh(x)^12 + 858*cosh(x)^10 + 1287*c
osh(x)^8 + 924*cosh(x)^6 + 315*cosh(x)^4 + 42*cosh(x)^2 + 1)*e^(2*x)*sinh(x
)^6 + 504*(17*cosh(x)^13 + 78*cosh(x)^11 + 143*cosh(x)^9 + 132*cosh(x)^7 +
63*cosh(x)^5 + 14*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^5 + 36*(85*cosh(x)^1
4 + 455*cosh(x)^12 + 1001*cosh(x)^10 + 1155*cosh(x)^8 + 735*cosh(x)^6 + 245
*cosh(x)^4 + 35*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 48*(17*cosh(x)^15 + 105*
cosh(x)^13 + 273*cosh(x)^11 + 385*cosh(x)^9 + 315*cosh(x)^7 + 147*cosh(x)^5
 + 35*cosh(x)^3 + 3*cosh(x))*e^(2*x)*sinh(x)^3 + 9*(17*cosh(x)^16 + 120*cos
h(x)^14 + 364*cosh(x)^12 + 616*cosh(x)^10 + 630*cosh(x)^8 + 392*cosh(x)^6 +
 140*cosh(x)^4 + 24*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 18*(cosh(x)^17 + 8*c
osh(x)^15 + 28*cosh(x)^13 + 56*cosh(x)^11 + 70*cosh(x)^9 + 56*cosh(x)^7 + 2
8*cosh(x)^5 + 8*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)^18 + 9*cosh
(x)^16 + 36*cosh(x)^14 + 84*cosh(x)^12 + 126*cosh(x)^10 + 126*cosh(x)^8 + 8
4*cosh(x)^6 + 36*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int \left(a\mathrm{sech}^4(x)\right)^{5/2} \, dx = \int \left(a \operatorname{sech}^{4}{\left(x \right)}\right)^{\frac{5}{2}}\, dx$$

```
[In] integrate((a*sech(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(99) = 198$.

Time = 0.28 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.75

$$\int \left(a\mathrm{sech}^4(x)\right)^{5/2}\,dx = \frac{256\,a^{\frac{5}{2}}e^{(-2\,x)}}{35\,(9\,e^{(-2\,x)}+36\,e^{(-4\,x)}+84\,e^{(-6\,x)}+126\,e^{(-8\,x)}+126\,e^{(-10\,x)}+84\,e^{(-12\,x)}+36\,e^{(-14}}$$
$$+\frac{1024\,a^{\frac{5}{2}}e^{(-4\,x)}}{35\,(9\,e^{(-2\,x)}+36\,e^{(-4\,x)}+84\,e^{(-6\,x)}+126\,e^{(-8\,x)}+126\,e^{(-10\,x)}+84\,e^{(-12\,x)}+36\,e^{(-14\,x)}+9\,e^{(-16\,x)}+e^{(-18\,x}}$$
$$+\frac{1024\,a^{\frac{5}{2}}e^{(-6\,x)}}{15\,(9\,e^{(-2\,x)}+36\,e^{(-4\,x)}+84\,e^{(-6\,x)}+126\,e^{(-8\,x)}+126\,e^{(-10\,x)}+84\,e^{(-12\,x)}+36\,e^{(-14\,x)}+9\,e^{(-16\,x)}+e^{(-18\,x}}$$
$$+\frac{512\,a^{\frac{5}{2}}e^{(-8\,x)}}{5\,(9\,e^{(-2\,x)}+36\,e^{(-4\,x)}+84\,e^{(-6\,x)}+126\,e^{(-8\,x)}+126\,e^{(-10\,x)}+84\,e^{(-12\,x)}+36\,e^{(-14\,x)}+9\,e^{(-16\,x)}+e^{(-18\,x)}}$$
$$+\frac{256\,a^{\frac{5}{2}}}{315\,(9\,e^{(-2\,x)}+36\,e^{(-4\,x)}+84\,e^{(-6\,x)}+126\,e^{(-8\,x)}+126\,e^{(-10\,x)}+84\,e^{(-12\,x)}+36\,e^{(-14\,x)}+9\,e^{(-16\,x)}+e^{(-18}}$$

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 256/35*a^(5/2)*e^(-2*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) + 126*e^(-8
*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) + e^(-18*x)
 + 1) + 1024/35*a^(5/2)*e^(-4*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^(-6*x) +
126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(-16*x) +
e^(-18*x) + 1) + 1024/15*a^(5/2)*e^(-6*x)/(9*e^(-2*x) + 36*e^(-4*x) + 84*e^
(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x) + 9*e^(
-16*x) + e^(-18*x) + 1) + 512/5*a^(5/2)*e^(-8*x)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1) + 256/315*a^(5/2)/(9*e^(-2*x) + 36*e^(-4*x)
+ 84*e^(-6*x) + 126*e^(-8*x) + 126*e^(-10*x) + 84*e^(-12*x) + 36*e^(-14*x)
+ 9*e^(-16*x) + e^(-18*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \left(a\mathrm{sech}^4(x)\right)^{5/2}\,dx = -\frac{256\,a^{\frac{5}{2}}\left(126\,e^{(8\,x)}+84\,e^{(6\,x)}+36\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)}{315\,\left(e^{(2\,x)}+1\right)^9}$$

```
[In] integrate((a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -256/315*a^(5/2)*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(e
^(2*x) + 1)^9
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.04

$$\int \left(a\mathrm{sech}^4(x)\right)^{5/2}\,dx = \frac{256\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{3\,{\left({\mathrm{e}}^{2\,x}+1\right)}^6\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} - \frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{5\,{\left({\mathrm{e}}^{2\,x}+1\right)}^5\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} - \frac{768\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{7\,{\left({\mathrm{e}}^{2\,x}+1\right)}^7\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} + \frac{64\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{{\left({\mathrm{e}}^{2\,x}+1\right)}^8\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)} - \frac{128\,a^2\,\sqrt{\frac{a}{{\left(\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^x}{2}\right)}^4}}\,\left(4\,{\mathrm{e}}^{2\,x}+6\,{\mathrm{e}}^{4\,x}+4\,{\mathrm{e}}^{6\,x}+{\mathrm{e}}^{8\,x}+1\right)}{9\,{\left({\mathrm{e}}^{2\,x}+1\right)}^9\,\left({\mathrm{e}}^{2\,x}+2\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}\right)}$$

```
[In] int((a/cosh(x)^4)^(5/2),x)
```

```
[Out] (256*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x) + 4*ex
p(6*x) + exp(8*x) + 1))/(3*(exp(2*x) + 1)^6*(exp(2*x) + 2*exp(4*x) + exp(6*
x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1))/(5*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*x) +
 exp(6*x))) - (768*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x) + 6*e
xp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(7*(exp(2*x) + 1)^7*(exp(2*x) + 2*exp
(4*x) + exp(6*x))) + (64*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp(2*x)
 + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/((exp(2*x) + 1)^8*(exp(2*x) + 2
*exp(4*x) + exp(6*x))) - (128*a^2*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(4*exp
(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))/(9*(exp(2*x) + 1)^9*(exp(2
*x) + 2*exp(4*x) + exp(6*x)))
```

3.47 $\int \left(a\text{sech}^4(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \left(a\text{sech}^4(x)\right)^{3/2}\,dx = a\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) - \frac{2}{3}a\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh(x) + \frac{1}{5}a\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^3(x)$$

```
[Out] a*cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)-2/3*a*sinh(x)^2*(a*sech(x)^4)^(1/2)*t
anh(x)+1/5*a*sinh(x)^2*(a*sech(x)^4)^(1/2)*tanh(x)^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4208, 3852}

$$\int \left(a\text{sech}^4(x)\right)^{3/2}\,dx = a\sinh(x)\cosh(x)\sqrt{a\text{sech}^4(x)} + \frac{1}{5}a\sinh^2(x)\tanh^3(x)\sqrt{a\text{sech}^4(x)} - \frac{2}{3}a\sinh^2(x)\tanh(x)\sqrt{a\text{sech}^4(x)}$$

```
[In] Int[(a*Sech[x]^4)^(3/2),x]
```

```
[Out] a*Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x] - (2*a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh
[x])/3 + (a*Sqrt[a*Sech[x]^4]*Sinh[x]^2*Tanh[x]^3)/5
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(a\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^6(x)\,dx\\ &= \left(ia\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -i\tanh(x)\right)\\ &= a\cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)-\frac{2}{3}a\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh(x)\\ &\quad+\frac{1}{5}a\sqrt{a\text{sech}^4(x)}\sinh^2(x)\tanh^3(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.49

$$\int \left(a\text{sech}^4(x)\right)^{3/2}\,dx=\frac{1}{15}\cosh(x)(8+6\cosh(2x)+\cosh(4x))\left(a\text{sech}^4(x)\right)^{3/2}\sinh(x)$$

```
[In] Integrate[(a*Sech[x]^4)^(3/2),x]
[Out] (Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*(a*Sech[x]^4)^(3/2)*Sinh[x])/15
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{16a\,e^{-2x}\sqrt{\frac{e^{4x}a}{\left(1+e^{2x}\right)^4}}\,\left(10\,e^{4x}+5\,e^{2x}+1\right)}{15\left(1+e^{2x}\right)^3}$	46

```
[In] int((sech(x)^4*a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -16/15*a*exp(-2*x)/(1+exp(2*x))^3*(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)*(10*exp
(4*x)+5*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(51) = 102.

Time = 0.26 (sec) , antiderivative size = 516, normalized size of antiderivative = 8.46

$$\int \left(a\text{sech}^4(x)\right)^{3/2} dx =$$

$$-\frac{16 \left(10 \, a \cosh\left(x\right)^{4} + 1\right.}{15 \left(10 \, \cosh\left(x\right) e^{\left(2 \, x\right)} \sinh\left(x\right)^{9} + e^{\left(2 \, x\right)} \sinh\left(x\right)^{10} + 5 \, \left(9 \, \cosh\left(x\right)^{2} + 1\right) e^{\left(2 \, x\right)} \sinh\left(x\right)^{8} + 40 \, \left(3 \, \cosh\left(x\right)^{3} + \cos\right.\right.}$$

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -16/15*(10*a*cosh(x)^4 + 10*(a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^4 + 40*(a
*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x)^3 + 5*a*cosh(x)
^2 + 5*(12*a*cosh(x)^2 + (12*a*cosh(x)^2 + a)*e^(4*x) + 2*(12*a*cosh(x)^2 +
 a)*e^(2*x) + a)*sinh(x)^2 + (10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(4*x) +
 2*(10*a*cosh(x)^4 + 5*a*cosh(x)^2 + a)*e^(2*x) + 10*(4*a*cosh(x)^3 + a*cos
h(x) + (4*a*cosh(x)^3 + a*cosh(x))*e^(4*x) + 2*(4*a*cosh(x)^3 + a*cosh(x))*
e^(2*x))*sinh(x) + a)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) +
 1))*e^(2*x)/(10*cosh(x)*e^(2*x)*sinh(x)^9 + e^(2*x)*sinh(x)^10 + 5*(9*cosh
(x)^2 + 1)*e^(2*x)*sinh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^7
 + 10*(21*cosh(x)^4 + 14*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^6 + 4*(63*cosh(x)^5
 + 70*cosh(x)^3 + 15*cosh(x))*e^(2*x)*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cos
h(x)^4 + 15*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^
5 + 5*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^
6 + 30*cosh(x)^4 + 12*cosh(x)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(cosh(x)^9 + 4*
cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) + (cosh(x)
^10 + 5*cosh(x)^8 + 10*cosh(x)^6 + 10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int \left(a\text{sech}^4(x)\right)^{3/2} dx = \int \left(a \operatorname{sech}^{4}\left(x\right)\right)^{\frac{3}{2}}\, dx$$

```
[In] integrate((a*sech(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(51) = 102.

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.97

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{3/2}\,dx = \frac{16\,a^{\frac{3}{2}}e^{(-2\,x)}}{3\,\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)} + \frac{32\,a^{\frac{3}{2}}e^{(-4\,x)}}{3\,\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)} + \frac{16\,a^{\frac{3}{2}}}{15\,\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}$$

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 16/3*a^(3/2)*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x)
+ e^(-10*x) + 1) + 32/3*a^(3/2)*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(
-6*x) + 5*e^(-8*x) + e^(-10*x) + 1) + 16/15*a^(3/2)/(5*e^(-2*x) + 10*e^(-4*
x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.44

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{3/2}\,dx = -\frac{16\,a^{\frac{3}{2}}\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\,\left(e^{(2\,x)}+1\right)^5}$$

```
[In] integrate((a*sech(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] -16/15*a^(3/2)*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \left(a\,\mathrm{sech}^4(x)\right)^{3/2}\,dx = -\frac{4\,a\,\mathrm{e}^{-2\,x}\,\sqrt{\frac{a}{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)^4}}\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\,\left(\mathrm{e}^{2\,x}+1\right)^3}$$

```
[In] int((a/cosh(x)^4)^(3/2),x)
```

```
[Out] -(4*a*exp(-2*x)*(a/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*x
) + 1))/(15*(exp(2*x) + 1)^3)
```

3.48 $\int \sqrt{a \operatorname{sech}^4(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \sqrt{a \operatorname{sech}^4(x)}\,dx = \cosh(x)\sqrt{a \operatorname{sech}^4(x)}\sinh(x)$$

```
[Out] cosh(x)*sinh(x)*(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4208, 3852, 8}

$$\int \sqrt{a \operatorname{sech}^4(x)}\,dx = \sinh(x)\cosh(x)\sqrt{a \operatorname{sech}^4(x)}$$

```
[In] Int[Sqrt[a*Sech[x]^4],x]
```

```
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \left(\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\int \text{sech}^2(x)\,dx \\ &= \left(i\cosh^2(x)\sqrt{a\text{sech}^4(x)}\right)\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right) \\ &= \cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \cosh(x)\sqrt{a\text{sech}^4(x)}\sinh(x)$$

```
[In] Integrate[Sqrt[a*Sech[x]^4],x]
[Out] Cosh[x]*Sqrt[a*Sech[x]^4]*Sinh[x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

method	result	size
risch	$-2\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}\,e^{-2x}(1+e^{2x})$	29

```
[In] int((sech(x)^4*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)*exp(-2*x)*(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 5.40

$$\int \sqrt{a\text{sech}^4(x)}\,dx = -\frac{2\sqrt{\frac{a}{e^{(8x)}+4e^{(6x)}+6e^{(4x)}+4e^{(2x)}+1}}\left(e^{(4x)}+2e^{(2x)}+1\right)e^{(2x)}}{2\cosh(x)e^{(2x)}\sinh(x)+e^{(2x)}\sinh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}}$$

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*(e^(4*x) + 2*e
^(2*x) + 1)*e^(2*x)/(2*cosh(x)*e^(2*x)*sinh(x) + e^(2*x)*sinh(x)^2 + (cosh(
x)^2 + 1)*e^(2*x))
```

Sympy [F]

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \int \sqrt{a\,\text{sech}^4(x)}\,dx$$

```
[In] integrate((a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sqrt{a\text{sech}^4(x)}\,dx = \frac{2\sqrt{a}}{e^{(-2x)}+1}$$

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \sqrt{a\text{sech}^4(x)}\,dx = -\frac{2\sqrt{a}}{e^{(2x)}+1}$$

```
[In] integrate((a*sech(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 4.73

$$\int \sqrt{a\text{sech}^4(x)}\,dx = -\frac{\sqrt{a}\,\sqrt{\frac{1}{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)^4}}\,\left(2\,e^{2\,x}+3\,e^{4\,x}+2\,e^{6\,x}+\frac{e^{8\,x}}{2}+\frac{1}{2}\right)}{\left(e^{2\,x}+1\right)\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}$$

```
[In] int((a/cosh(x)^4)^(1/2),x)
```

```
[Out] -(a^(1/2)*(1/(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(2*exp(2*x) + 3*exp(4*x) + 2*e
xp(6*x) + exp(8*x)/2 + 1/2))/((exp(2*x) + 1)*(exp(2*x) + 2*exp(4*x) + exp(6
*x)))
```

3.49 $\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx=\frac{x\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\text{sech}^4(x)}}$$

```
[Out] 1/2*x*sech(x)^2/(a*sech(x)^4)^(1/2)+1/2*tanh(x)/(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4208, 2715, 8}

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx=\frac{x\text{sech}^2(x)}{2\sqrt{a\text{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\text{sech}^4(x)}}$$

```
[In] Int[1/Sqrt[a*Sech[x]^4],x]
```

```
[Out] (x*Sech[x]^2)/(2*Sqrt[a*Sech[x]^4]) + Tanh[x]/(2*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\operatorname{sech}^2(x)\int\cosh^2(x)\,dx}{\sqrt{a\operatorname{sech}^4(x)}}\\ &= \frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\operatorname{sech}^2(x)\int 1\,dx}{2\sqrt{a\operatorname{sech}^4(x)}}\\ &= \frac{x\operatorname{sech}^2(x)}{2\sqrt{a\operatorname{sech}^4(x)}}+\frac{\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int\frac{1}{\sqrt{a\operatorname{sech}^4(x)}}\,dx=\frac{x\operatorname{sech}^2(x)+\tanh(x)}{2\sqrt{a\operatorname{sech}^4(x)}}$$

```
[In] Integrate[1/Sqrt[a*Sech[x]^4],x]
[Out] (x*Sech[x]^2 + Tanh[x])/(2*Sqrt[a*Sech[x]^4])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(28) = 56.

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.47

method	result	size
risch	$\frac{e^{2x}x}{2\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}\,(1+e^{2x})^2}+\frac{e^{4x}}{8\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}\,(1+e^{2x})^2}-\frac{1}{8(1+e^{2x})^2\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}}$	89

```
[In] int(1/(sech(x)^4*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(2*x)*x+1/8/(exp(4*
x)*a/(1+exp(2*x))^4)^(1/2)/(1+exp(2*x))^2*exp(4*x)-1/8/(1+exp(2*x))^2/(exp(
4*x)*a/(1+exp(2*x))^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 7.03

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \frac{\left(\left(e^{(4\,x)}+2\,e^{(2\,x)}+1\right)\sinh\left(x\right)^4+\cosh\left(x\right)^4+4\left(\cosh\left(x\right)e^{(4\,x)}+2\,\cosh\left(x\right)e^{(2\,x)}+\cosh\left(x\right)\right)\sinh\left(x\right)^3+4\,x\right.}{}$$

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^4 + cosh(x)^4 + 4*(cosh(x)*e^(4*x) +
 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^3 + 4*x*cosh(x)^2 + 2*(3*cosh(x)^2 +
(3*cosh(x)^2 + 2*x)*e^(4*x) + 2*(3*cosh(x)^2 + 2*x)*e^(2*x) + 2*x)*sinh(x)^
2 + (cosh(x)^4 + 4*x*cosh(x)^2 - 1)*e^(4*x) + 2*(cosh(x)^4 + 4*x*cosh(x)^2
- 1)*e^(2*x) + 4*(cosh(x)^3 + 2*x*cosh(x) + (cosh(x)^3 + 2*x*cosh(x))*e^(4*
x) + 2*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*
e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1))*e^(2*x)/(a*cosh(x)^2*e^(2*x) + 2*a*co
sh(x)*e^(2*x)*sinh(x) + a*e^(2*x)*sinh(x)^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{1}{\sqrt{a\,\text{sech}^4\left(x\right)}}\,dx$$

```
[In] integrate(1/(a*sech(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sech(x)**4), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{\left(\sqrt{a}e^{(-4\,x)}-\sqrt{a}\right)e^{(2\,x)}}{8\,a}+\frac{x}{2\,\sqrt{a}}$$

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x)/a + 1/2*x/sqrt(a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = -\frac{\left(2\,e^{(2\,x)}+1\right)e^{(-2\,x)}-4\,x-e^{(2\,x)}}{8\,\sqrt{a}}$$

```
[In] integrate(1/(a*sech(x)^4)^(1/2),x, algorithm="giac")
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\text{sech}^4(x)}}\,dx = \int \frac{1}{\sqrt{\frac{a}{\cosh(x)^4}}}\,dx$$

```
[In] int(1/(a/cosh(x)^4)^(1/2),x)
[Out] int(1/(a/cosh(x)^4)^(1/2), x)
```

3.50 $\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx=\frac{5x\operatorname{sech}^2(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}}+\frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}$$

```
[Out] 5/16*x*sech(x)^2/a/(a*sech(x)^4)^(1/2)+5/24*cosh(x)*sinh(x)/a/(a*sech(x)^4)
^(1/2)+1/6*cosh(x)^3*sinh(x)/a/(a*sech(x)^4)^(1/2)+5/16*tanh(x)/a/(a*sech(x
)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4208, 2715, 8}

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx=\frac{5x\operatorname{sech}^2(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}+\frac{\sinh(x)\cosh^3(x)}{6a\sqrt{a\operatorname{sech}^4(x)}}+\frac{5\sinh(x)\cosh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}}$$

```
[In] Int[(a*Sech[x]^4)^(-3/2),x]
```

```
[Out] (5*x*Sech[x]^2)/(16*a*Sqrt[a*Sech[x]^4]) + (5*Cosh[x]*Sinh[x])/(24*a*Sqrt[a
*Sech[x]^4]) + (Cosh[x]^3*Sinh[x])/(6*a*Sqrt[a*Sech[x]^4]) + (5*Tanh[x])/(1
6*a*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^2(x)\int \cosh^6(x)\,dx}{a\sqrt{a\operatorname{sech}^4(x)}}\\
&= \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(5\operatorname{sech}^2(x)\right)\int \cosh^4(x)\,dx}{6a\sqrt{a\operatorname{sech}^4(x)}}\\
&= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(5\operatorname{sech}^2(x)\right)\int \cosh^2(x)\,dx}{8a\sqrt{a\operatorname{sech}^4(x)}}\\
&= \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(5\operatorname{sech}^2(x)\right)\int 1\,dx}{16a\sqrt{a\operatorname{sech}^4(x)}}\\
&= \frac{5x\operatorname{sech}^2(x)}{16a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\cosh(x)\sinh(x)}{24a\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^3(x)\sinh(x)}{6a\sqrt{a\operatorname{sech}^4(x)}} + \frac{5\tanh(x)}{16a\sqrt{a\operatorname{sech}^4(x)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx = \frac{\operatorname{sech}^6(x)(60x + 45\sinh(2x) + 9\sinh(4x) + \sinh(6x))}{192\left(a\operatorname{sech}^4(x)\right)^{3/2}}$$

```
[In] Integrate[(a*Sech[x]^4)^(-3/2),x]
[Out] (Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6*x]))/(192*(a*Sech[x]
^4)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(70) = 140$.

Time = 0.18 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.67

method	result
risch	$\frac{5\,\mathrm{e}^{2x}x}{16a(1+\mathrm{e}^{2x})^2\sqrt{\frac{\mathrm{e}^{4x}a}{(1+\mathrm{e}^{2x})^4}}}+\frac{\mathrm{e}^{8x}}{384a(1+\mathrm{e}^{2x})^2\sqrt{\frac{\mathrm{e}^{4x}a}{(1+\mathrm{e}^{2x})^4}}}+\frac{3\,\mathrm{e}^{6x}}{128a(1+\mathrm{e}^{2x})^2\sqrt{\frac{\mathrm{e}^{4x}a}{(1+\mathrm{e}^{2x})^4}}}+\frac{15\,\mathrm{e}^{4x}}{128a(1+\mathrm{e}^{2x})^2\sqrt{\frac{\mathrm{e}^{4x}a}{(1+\mathrm{e}^{2x})^4}}}-\frac{}{128\sqrt{\frac{}{(1\cdots}}}$

```
[In] int(1/(sech(x)^4*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/16/a*exp(2*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)*x+1/384/a*
exp(8*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)+3/128/a*exp(6*x)/
(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)+15/128/a*exp(4*x)/(1+exp(2
*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)-15/128/(exp(4*x)*a/(1+exp(2*x))^4)
^(1/2)/(1+exp(2*x))^2/a-3/128/a*exp(-2*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp
(2*x))^4)^(1/2)-1/384/a*exp(-4*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4
)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(70) = 140$.

Time = 0.26 (sec) , antiderivative size = 1141, normalized size of antiderivative = 13.27

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/384*((e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^12 + cosh(x)^12 + 12*(cosh(x)*e^(4
*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^11 + 3*(22*cosh(x)^2 + (22*cosh(
x)^2 + 3)*e^(4*x) + 2*(22*cosh(x)^2 + 3)*e^(2*x) + 3)*sinh(x)^10 + 9*cosh(x
)^10 + 10*(22*cosh(x)^3 + (22*cosh(x)^3 + 9*cosh(x))*e^(4*x) + 2*(22*cosh(x
)^3 + 9*cosh(x))*e^(2*x) + 9*cosh(x))*sinh(x)^9 + 45*(11*cosh(x)^4 + 9*cosh
(x)^2 + (11*cosh(x)^4 + 9*cosh(x)^2 + 1)*e^(4*x) + 2*(11*cosh(x)^4 + 9*cosh
(x)^2 + 1)*e^(2*x) + 1)*sinh(x)^8 + 45*cosh(x)^8 + 72*(11*cosh(x)^5 + 15*co
sh(x)^3 + (11*cosh(x)^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(4*x) + 2*(11*cosh(x)
^5 + 15*cosh(x)^3 + 5*cosh(x))*e^(2*x) + 5*cosh(x))*sinh(x)^7 + 120*x*cosh(
x)^6 + 6*(154*cosh(x)^6 + 315*cosh(x)^4 + 210*cosh(x)^2 + (154*cosh(x)^6 +
315*cosh(x)^4 + 210*cosh(x)^2 + 20*x)*e^(4*x) + 2*(154*cosh(x)^6 + 315*cosh
(x)^4 + 210*cosh(x)^2 + 20*x)*e^(2*x) + 20*x)*sinh(x)^6 + 36*(22*cosh(x)^7
+ 63*cosh(x)^5 + 70*cosh(x)^3 + 20*x*cosh(x) + (22*cosh(x)^7 + 63*cosh(x)^5
 + 70*cosh(x)^3 + 20*x*cosh(x))*e^(4*x) + 2*(22*cosh(x)^7 + 63*cosh(x)^5 +
70*cosh(x)^3 + 20*x*cosh(x))*e^(2*x))*sinh(x)^5 + 45*(11*cosh(x)^8 + 42*cos
```

```
h(x)^6 + 70*cosh(x)^4 + 40*x*cosh(x)^2 + (11*cosh(x)^8 + 42*cosh(x)^6 + 70*
cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(4*x) + 2*(11*cosh(x)^8 + 42*cosh(x)^6 +
70*cosh(x)^4 + 40*x*cosh(x)^2 - 1)*e^(2*x) - 1)*sinh(x)^4 - 45*cosh(x)^4 +
20*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 + (11*cos
h(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*cosh(x))*e^(4*x
) + 2*(11*cosh(x)^9 + 54*cosh(x)^7 + 126*cosh(x)^5 + 120*x*cosh(x)^3 - 9*co
sh(x))*e^(2*x) - 9*cosh(x))*sinh(x)^3 + 3*(22*cosh(x)^10 + 135*cosh(x)^8 +
420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 + (22*cosh(x)^10 + 135*cosh(
x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2 - 3)*e^(4*x) + 2*(22*
cosh(x)^10 + 135*cosh(x)^8 + 420*cosh(x)^6 + 600*x*cosh(x)^4 - 90*cosh(x)^2
 - 3)*e^(2*x) - 3)*sinh(x)^2 - 9*cosh(x)^2 + (cosh(x)^12 + 9*cosh(x)^10 + 4
5*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4 - 9*cosh(x)^2 - 1)*e^(4*x) + 2
*(cosh(x)^12 + 9*cosh(x)^10 + 45*cosh(x)^8 + 120*x*cosh(x)^6 - 45*cosh(x)^4
 - 9*cosh(x)^2 - 1)*e^(2*x) + 6*(2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(x)^7
 + 120*x*cosh(x)^5 - 30*cosh(x)^3 + (2*cosh(x)^11 + 15*cosh(x)^9 + 60*cosh(
x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x))*e^(4*x) + 2*(2*cosh(x)^1
1 + 15*cosh(x)^9 + 60*cosh(x)^7 + 120*x*cosh(x)^5 - 30*cosh(x)^3 - 3*cosh(x
))*e^(2*x) - 3*cosh(x))*sinh(x) - 1)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x
) + 4*e^(2*x) + 1))*e^(2*x)/(a^2*cosh(x)^6*e^(2*x) + 6*a^2*cosh(x)^5*e^(2*x
)*sinh(x) + 15*a^2*cosh(x)^4*e^(2*x)*sinh(x)^2 + 20*a^2*cosh(x)^3*e^(2*x)*s
inh(x)^3 + 15*a^2*cosh(x)^2*e^(2*x)*sinh(x)^4 + 6*a^2*cosh(x)*e^(2*x)*sinh(
x)^5 + a^2*e^(2*x)*sinh(x)^6)
```

Sympy [F]

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\,\mathrm{sech}^4\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)**4)**(3/2),x)
[Out] Integral((a*sech(x)**4)**(-3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{3/2}}\,dx = \frac{\left(9\,\sqrt{a}e^{(-2\,x)} + 45\,\sqrt{a}e^{(-4\,x)} - 45\,\sqrt{a}e^{(-8\,x)} - 9\,\sqrt{a}e^{(-10\,x)} - \sqrt{a}e^{(-12\,x)} + \sqrt{a}\right)e^{(6\,x)}}{384\,a^2} + \frac{5\,x}{16\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/384*(9*sqrt(a)*e^(-2*x) + 45*sqrt(a)*e^(-4*x) - 45*sqrt(a)*e^(-8*x) - 9*s
qrt(a)*e^(-10*x) - sqrt(a)*e^(-12*x) + sqrt(a))*e^(6*x)/a^2 + 5/16*x/a^(3/2
)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{3/2}}\,dx = -\frac{\left(110\,e^{(6\,x)}+45\,e^{(4\,x)}+9\,e^{(2\,x)}+1\right)e^{(-6\,x)}-120\,x-e^{(6\,x)}-9\,e^{(4\,x)}-45\,e^{(2\,x)}}{384\,a^{\frac{3}{2}}}$$

```
[In] integrate(1/(a*sech(x)^4)^(3/2),x, algorithm="giac")
[Out] -1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6*
x) - 9*e^(4*x) - 45*e^(2*x))/a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{3/2}}\,dx = \int \frac{1}{{\left(\frac{a}{{\mathrm{cosh}\left(x\right)}^4}\right)}^{3/2}}\,dx$$

```
[In] int(1/(a/cosh(x)^4)^(3/2),x)
[Out] int(1/(a/cosh(x)^4)^(3/2), x)
```

3.51 $\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx=\frac{63x\text{sech}^2(x)}{256a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\text{sech}^4(x)}}+\frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\text{sech}^4(x)}}+\frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\text{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\text{sech}^4(x)}}$$

```
[Out] 63/256*x*sech(x)^2/a^2/(a*sech(x)^4)^(1/2)+21/128*cosh(x)*sinh(x)/a^2/(a*se
ch(x)^4)^(1/2)+21/160*cosh(x)^3*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+9/80*cosh(x
)^5*sinh(x)/a^2/(a*sech(x)^4)^(1/2)+1/10*cosh(x)^7*sinh(x)/a^2/(a*sech(x)^4
)^(1/2)+63/256*tanh(x)/a^2/(a*sech(x)^4)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4208, 2715, 8}

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx=\frac{63x\text{sech}^2(x)}{256a^2\sqrt{a\text{sech}^4(x)}}+\frac{63\tanh(x)}{256a^2\sqrt{a\text{sech}^4(x)}}+\frac{\sinh(x)\cosh^7(x)}{10a^2\sqrt{a\text{sech}^4(x)}}+\frac{9\sinh(x)\cosh^5(x)}{80a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\sinh(x)\cosh^3(x)}{160a^2\sqrt{a\text{sech}^4(x)}}+\frac{21\sinh(x)\cosh(x)}{128a^2\sqrt{a\text{sech}^4(x)}}$$

```
[In] Int[(a*Sech[x]^4)^(-5/2),x]
```

```
[Out] (63*x*Sech[x]^2)/(256*a^2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]*Sinh[x])/(128*a^
2*Sqrt[a*Sech[x]^4]) + (21*Cosh[x]^3*Sinh[x])/(160*a^2*Sqrt[a*Sech[x]^4]) +
```

```
(9*Cosh[x]^5*Sinh[x])/(80*a^2*Sqrt[a*Sech[x]^4]) + (Cosh[x]^7*Sinh[x])/(10
*a^2*Sqrt[a*Sech[x]^4]) + (63*Tanh[x])/(256*a^2*Sqrt[a*Sech[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4208

```
Int[((b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> Dist[b^
IntPart[p]*((b*(c*Sec[e + f*x])^n)^FracPart[p]/(c*Sec[e + f*x])^(n*FracPart
[p])), Int[(c*Sec[e + f*x])^(n*p), x], x] /; FreeQ[{b, c, e, f, n, p}, x] &
&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}^2(x)\int \cosh^{10}(x)\,dx}{a^2\sqrt{a\operatorname{sech}^4(x)}} \\
&= \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(9\operatorname{sech}^2(x)\right)\int \cosh^8(x)\,dx}{10a^2\sqrt{a\operatorname{sech}^4(x)}} \\
&= \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(63\operatorname{sech}^2(x)\right)\int \cosh^6(x)\,dx}{80a^2\sqrt{a\operatorname{sech}^4(x)}} \\
&= \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(21\operatorname{sech}^2(x)\right)\int \cosh^4(x)\,dx}{32a^2\sqrt{a\operatorname{sech}^4(x)}} \\
&= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} \\
&\quad + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(63\operatorname{sech}^2(x)\right)\int \cosh^2(x)\,dx}{128a^2\sqrt{a\operatorname{sech}^4(x)}} \\
&= \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\operatorname{sech}^4(x)}} \\
&\quad + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{63\tanh(x)}{256a^2\sqrt{a\operatorname{sech}^4(x)}} + \frac{\left(63\operatorname{sech}^2(x)\right)\int 1\,dx}{256a^2\sqrt{a\operatorname{sech}^4(x)}}
\end{aligned}
$$

$$= \frac{63x\mathrm{sech}^2(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}} + \frac{21\cosh(x)\sinh(x)}{128a^2\sqrt{a\mathrm{sech}^4(x)}} + \frac{21\cosh^3(x)\sinh(x)}{160a^2\sqrt{a\mathrm{sech}^4(x)}} + \frac{9\cosh^5(x)\sinh(x)}{80a^2\sqrt{a\mathrm{sech}^4(x)}} + \frac{\cosh^7(x)\sinh(x)}{10a^2\sqrt{a\mathrm{sech}^4(x)}} + \frac{63\tanh(x)}{256a^2\sqrt{a\mathrm{sech}^4(x)}}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.42

$$\int \frac{1}{\left(a\mathrm{sech}^4(x)\right)^{5/2}}\,dx = \frac{\cosh^2(x)\sqrt{a\mathrm{sech}^4(x)}(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) + 25\sinh}{10240a^3}$$

```
[In] Integrate[(a*Sech[x]^4)^(-5/2),x]
[Out] (Cosh[x]^2*Sqrt[a*Sech[x]^4]*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x] + 150
*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/(10240*a^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(108) = 216.

Time = 0.16 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.74

method	result
risch	$\frac{63\,e^{2x}x}{256a^2(1+e^{2x})^2\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}} + \frac{e^{12x}}{10240a^2(1+e^{2x})^2\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}} + \frac{5\,e^{10x}}{4096a^2(1+e^{2x})^2\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}} + \frac{15\,e^{8x}}{2048a^2(1+e^{2x})^2\sqrt{\frac{e^{4x}a}{(1+e^{2x})^4}}} +$

```
[In] int(1/(sech(x)^4*a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 63/256/a^2*exp(2*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)*x+1/10
240/a^2*exp(12*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)+5/4096/a
^2*exp(10*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)+15/2048/a^2*e
xp(8*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)+15/512/a^2*exp(6*x
)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)+105/1024/a^2*exp(4*x)/(1
+exp(2*x))^2/(exp(4*x)*a/(1+exp(2*x))^4)^(1/2)-105/1024/(exp(4*x)*a/(1+exp(
2*x))^4)^(1/2)/(1+exp(2*x))^2/a^2-15/512/a^2*exp(-2*x)/(1+exp(2*x))^2/(exp(
4*x)*a/(1+exp(2*x))^4)^(1/2)-15/2048/a^2*exp(-4*x)/(1+exp(2*x))^2/(exp(4*x)
*a/(1+exp(2*x))^4)^(1/2)-5/4096/a^2*exp(-6*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1
+exp(2*x))^4)^(1/2)-1/10240/a^2*exp(-8*x)/(1+exp(2*x))^2/(exp(4*x)*a/(1+exp
(2*x))^4)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. 2(108) = 216.

Time = 0.32 (sec) , antiderivative size = 2600, normalized size of antiderivative = 19.70

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20480*(2*(e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^20 + 2*cosh(x)^20 + 40*(cosh(x
)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^19 + 5*(76*cosh(x)^2 + (76
*cosh(x)^2 + 5)*e^(4*x) + 2*(76*cosh(x)^2 + 5)*e^(2*x) + 5)*sinh(x)^18 + 25
*cosh(x)^18 + 30*(76*cosh(x)^3 + (76*cosh(x)^3 + 15*cosh(x))*e^(4*x) + 2*(7
6*cosh(x)^3 + 15*cosh(x))*e^(2*x) + 15*cosh(x))*sinh(x)^17 + 15*(646*cosh(x
)^4 + 255*cosh(x)^2 + (646*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(4*x) + 2*(646
*cosh(x)^4 + 255*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^16 + 150*cosh(x)^16
 + 48*(646*cosh(x)^5 + 425*cosh(x)^3 + (646*cosh(x)^5 + 425*cosh(x)^3 + 50*c
osh(x))*e^(4*x) + 2*(646*cosh(x)^5 + 425*cosh(x)^3 + 50*cosh(x))*e^(2*x) +
50*cosh(x))*sinh(x)^15 + 60*(1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^
2 + (1292*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(4*x) + 2*(129
2*cosh(x)^6 + 1275*cosh(x)^4 + 300*cosh(x)^2 + 10)*e^(2*x) + 10)*sinh(x)^14
 + 600*cosh(x)^14 + 120*(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 +
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(4*x) + 2*
(1292*cosh(x)^7 + 1785*cosh(x)^5 + 700*cosh(x)^3 + 70*cosh(x))*e^(2*x) + 70
*cosh(x))*sinh(x)^13 + 60*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4
 + 910*cosh(x)^2 + (4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)^4 + 910*
cosh(x)^2 + 35)*e^(4*x) + 2*(4199*cosh(x)^8 + 7735*cosh(x)^6 + 4550*cosh(x)
^4 + 910*cosh(x)^2 + 35)*e^(2*x) + 35)*sinh(x)^12 + 2100*cosh(x)^12 + 80*(4
199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + (4199*co
sh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)^3 + 315*cosh(x))*e
^(4*x) + 2*(4199*cosh(x)^9 + 9945*cosh(x)^7 + 8190*cosh(x)^5 + 2730*cosh(x)
^3 + 315*cosh(x))*e^(2*x) + 315*cosh(x))*sinh(x)^11 + 5040*x*cosh(x)^10 + 2
*(184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^
4 + 69300*cosh(x)^2 + (184756*cosh(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x
)^6 + 300300*cosh(x)^4 + 69300*cosh(x)^2 + 2520*x)*e^(4*x) + 2*(184756*cosh
(x)^10 + 546975*cosh(x)^8 + 600600*cosh(x)^6 + 300300*cosh(x)^4 + 69300*cos
h(x)^2 + 2520*x)*e^(2*x) + 2520*x)*sinh(x)^10 + 20*(16796*cosh(x)^11 + 6077
5*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 2520*x*
cosh(x) + (16796*cosh(x)^11 + 60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cos
h(x)^5 + 23100*cosh(x)^3 + 2520*x*cosh(x))*e^(4*x) + 2*(16796*cosh(x)^11 +
60775*cosh(x)^9 + 85800*cosh(x)^7 + 60060*cosh(x)^5 + 23100*cosh(x)^3 + 252
0*x*cosh(x))*e^(2*x))*sinh(x)^9 + 30*(8398*cosh(x)^12 + 36465*cosh(x)^10 +
64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*cosh(x)^2 + (8
398*cosh(x)^12 + 36465*cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 346
```

50*cosh(x)^4 + 7560*x*cosh(x)^2 - 70)*e^(4*x) + 2*(8398*cosh(x)^12 + 36465*
cosh(x)^10 + 64350*cosh(x)^8 + 60060*cosh(x)^6 + 34650*cosh(x)^4 + 7560*x*c
osh(x)^2 - 70)*e^(2*x) - 70)*sinh(x)^8 - 2100*cosh(x)^8 + 240*(646*cosh(x)^
13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*cosh(x)^5 + 2
520*x*cosh(x)^3 + (646*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580
*cosh(x)^7 + 6930*cosh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(4*x) + 2*(6
46*cosh(x)^13 + 3315*cosh(x)^11 + 7150*cosh(x)^9 + 8580*cosh(x)^7 + 6930*co
sh(x)^5 + 2520*x*cosh(x)^3 - 70*cosh(x))*e^(2*x) - 70*cosh(x))*sinh(x)^7 +
60*(1292*cosh(x)^14 + 7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8
+ 32340*cosh(x)^6 + 17640*x*cosh(x)^4 - 980*cosh(x)^2 + (1292*cosh(x)^14 +
7735*cosh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17
640*x*cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(4*x) + 2*(1292*cosh(x)^14 + 7735*c
osh(x)^12 + 20020*cosh(x)^10 + 30030*cosh(x)^8 + 32340*cosh(x)^6 + 17640*x*
cosh(x)^4 - 980*cosh(x)^2 - 10)*e^(2*x) - 10)*sinh(x)^6 - 600*cosh(x)^6 + 2
4*(1292*cosh(x)^15 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 +
 69300*cosh(x)^7 + 52920*x*cosh(x)^5 - 4900*cosh(x)^3 + (1292*cosh(x)^15 +
8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7 + 52
920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(4*x) + 2*(1292*cosh(x)^1
5 + 8925*cosh(x)^13 + 27300*cosh(x)^11 + 50050*cosh(x)^9 + 69300*cosh(x)^7
+ 52920*x*cosh(x)^5 - 4900*cosh(x)^3 - 150*cosh(x))*e^(2*x) - 150*cosh(x))*
sinh(x)^5 + 30*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*
cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cos
h(x)^2 + (323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 20020*cosh(x
)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*cosh(x)^2
 - 5)*e^(4*x) + 2*(323*cosh(x)^16 + 2550*cosh(x)^14 + 9100*cosh(x)^12 + 200
20*cosh(x)^10 + 34650*cosh(x)^8 + 35280*x*cosh(x)^6 - 4900*cosh(x)^4 - 300*
cosh(x)^2 - 5)*e^(2*x) - 5)*sinh(x)^4 - 150*cosh(x)^4 + 120*(19*cosh(x)^17
+ 170*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040
*x*cosh(x)^7 - 980*cosh(x)^5 - 100*cosh(x)^3 + (19*cosh(x)^17 + 170*cosh(x)
^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040*x*cosh(x)^7
- 980*cosh(x)^5 - 100*cosh(x)^3 - 5*cosh(x))*e^(4*x) + 2*(19*cosh(x)^17 + 1
70*cosh(x)^15 + 700*cosh(x)^13 + 1820*cosh(x)^11 + 3850*cosh(x)^9 + 5040*x*
cosh(x)^7 - 980*cosh(x)^5 - 100*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cosh(x))
sinh(x)^3 + 5(76*cosh(x)^18 + 765*cosh(x)^16 + 3600*cosh(x)^14 + 10920*co
sh(x)^12 + 27720*cosh(x)^10 + 45360*x*cosh(x)^8 - 11760*cosh(x)^6 - 1800*co
sh(x)^4 - 180*cosh(x)^2 + (76*cosh(x)^18 + 765*cosh(x)^16 + 3600*cosh(x)^14
 + 10920*cosh(x)^12 + 27720*cosh(x)^10 + 45360*x*cosh(x)^8 - 11760*cosh(x)^
6 - 1800*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(4*x) + 2*(76*cosh(x)^18 + 765*co
sh(x)^16 + 3600*cosh(x)^14 + 10920*cosh(x)^12 + 27720*cosh(x)^10 + 45360*x*
cosh(x)^8 - 11760*cosh(x)^6 - 1800*cosh(x)^4 - 180*cosh(x)^2 - 5)*e^(2*x) -
 5)*sinh(x)^2 - 25*cosh(x)^2 + (2*cosh(x)^20 + 25*cosh(x)^18 + 150*cosh(x)^
16 + 600*cosh(x)^14 + 2100*cosh(x)^12 + 5040*x*cosh(x)^10 - 2100*cosh(x)^8
- 600*cosh(x)^6 - 150*cosh(x)^4 - 25*cosh(x)^2 - 2)*e^(4*x) + 2*(2*cosh(x)^
20 + 25*cosh(x)^18 + 150*cosh(x)^16 + 600*cosh(x)^14 + 2100*cosh(x)^12 + 50
40*x*cosh(x)^10 - 2100*cosh(x)^8 - 600*cosh(x)^6 - 150*cosh(x)^4 - 25*cosh(

```
x)^2 - 2)*e^(2*x) + 10*(4*cosh(x)^19 + 45*cosh(x)^17 + 240*cosh(x)^15 + 840
*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 - 1680*cosh(x)^7 - 360*cos
h(x)^5 - 60*cosh(x)^3 + (4*cosh(x)^19 + 45*cosh(x)^17 + 240*cosh(x)^15 + 84
0*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 - 1680*cosh(x)^7 - 360*co
sh(x)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(4*x) + 2*(4*cosh(x)^19 + 45*cosh(x)^
17 + 240*cosh(x)^15 + 840*cosh(x)^13 + 2520*cosh(x)^11 + 5040*x*cosh(x)^9 -
 1680*cosh(x)^7 - 360*cosh(x)^5 - 60*cosh(x)^3 - 5*cosh(x))*e^(2*x) - 5*cos
h(x))*sinh(x) - 2)*sqrt(a/(e^(8*x) + 4*e^(6*x) + 6*e^(4*x) + 4*e^(2*x) + 1)
)*e^(2*x)/(a^3*cosh(x)^10*e^(2*x) + 10*a^3*cosh(x)^9*e^(2*x)*sinh(x) + 45*a
^3*cosh(x)^8*e^(2*x)*sinh(x)^2 + 120*a^3*cosh(x)^7*e^(2*x)*sinh(x)^3 + 210*
a^3*cosh(x)^6*e^(2*x)*sinh(x)^4 + 252*a^3*cosh(x)^5*e^(2*x)*sinh(x)^5 + 210
*a^3*cosh(x)^4*e^(2*x)*sinh(x)^6 + 120*a^3*cosh(x)^3*e^(2*x)*sinh(x)^7 + 45
*a^3*cosh(x)^2*e^(2*x)*sinh(x)^8 + 10*a^3*cosh(x)*e^(2*x)*sinh(x)^9 + a^3*e
^(2*x)*sinh(x)^10)
```

Sympy [F]

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\,\text{sech}^4\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a*sech(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sech(x)**4)**(-5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.78

$$\int \frac{1}{\left(a\text{sech}^4(x)\right)^{5/2}}\,dx = \frac{\left(25\,\sqrt{a}e^{(-2\,x)} + 150\,\sqrt{a}e^{(-4\,x)} + 600\,\sqrt{a}e^{(-6\,x)} + 2100\,\sqrt{a}e^{(-8\,x)} - 2100\,\sqrt{a}e^{(-12\,x)} - \cdots\right.}{20480\,a^3 \cdots} + \frac{63\,x}{256\,a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20480*(25*sqrt(a)*e^(-2*x) + 150*sqrt(a)*e^(-4*x) + 600*sqrt(a)*e^(-6*x)
+ 2100*sqrt(a)*e^(-8*x) - 2100*sqrt(a)*e^(-12*x) - 600*sqrt(a)*e^(-14*x) -
150*sqrt(a)*e^(-16*x) - 25*sqrt(a)*e^(-18*x) - 2*sqrt(a)*e^(-20*x) + 2*sqrt
(a))*e^(10*x)/a^3 + 63/256*x/a^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{5/2}}\,dx = -\frac{\left(5754\,e^{(10\,x)} + 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} + 150\,e^{(4\,x)} + 25\,e^{(2\,x)} + 2\right)e^{(-10\,x)} - 5040\,x - 2\,e^{(10\,x)} - 25\,e^{(8\,x)} - 150\,e}{20480\,a^{\frac{5}{2}}}$$

```
[In] integrate(1/(a*sech(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/20480*((5754*e^(10*x) + 2100*e^(8*x) + 600*e^(6*x) + 150*e^(4*x) + 25*e^
(2*x) + 2)*e^(-10*x) - 5040*x - 2*e^(10*x) - 25*e^(8*x) - 150*e^(6*x) - 600
*e^(4*x) - 2100*e^(2*x))/a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\operatorname{sech}^4(x)\right)^{5/2}}\,dx = \int \frac{1}{{\left(\frac{a}{{\cosh\left(x\right)}^4}\right)}^{5/2}}\,dx$$

```
[In] int(1/(a/cosh(x)^4)^(5/2),x)
```

```
[Out] int(1/(a/cosh(x)^4)^(5/2), x)
```

3.52 $\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx = -\frac{x}{8a}-\frac{\cosh(x)\sinh(x)}{8a}+\frac{\cosh^3(x)\sinh(x)}{4a}-\frac{\sinh^3(x)}{3a}$$

```
[Out] -1/8*x/a-1/8*cosh(x)*sinh(x)/a+1/4*cosh(x)^3*sinh(x)/a-1/3*sinh(x)^3/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3957, 2918, 2644, 30, 2648, 2715, 8}

$$\int \frac{\sinh^4(x)}{a+a\text{sech}(x)}\,dx = -\frac{x}{8a}-\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)\cosh^3(x)}{4a}-\frac{\sinh(x)\cosh(x)}{8a}$$

```
[In] Int[Sinh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] -1/8*x/a - (Cosh[x]*Sinh[x])/(8*a) + (Cosh[x]^3*Sinh[x])/(4*a) - Sinh[x]^3/
(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cosh(x)\sinh^4(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\int \cosh(x)\sinh^2(x)\,dx}{a}+\frac{\int \cosh^2(x)\sinh^2(x)\,dx}{a} \\
&= \frac{\cosh^3(x)\sinh(x)}{4a}-\frac{i\text{Subst}\left(\int x^2\,dx,x,i\sinh(x)\right)}{a}-\frac{\int \cosh^2(x)\,dx}{4a}
\end{aligned}$$

$$= -\frac{\cosh(x)\sinh(x)}{8a} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{\sinh^3(x)}{3a} - \frac{\int 1\,dx}{8a}$$

$$= -\frac{x}{8a} - \frac{\cosh(x)\sinh(x)}{8a} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{\sinh^3(x)}{3a}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{\sinh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{24\sinh(x) - 8\sinh(3x) + 3(-4x + \sinh(4x))}{96a}$$

```
[In] Integrate[Sinh[x]^4/(a + a*Sech[x]),x]
[Out] (24*Sinh[x] - 8*Sinh[3*x] + 3*(-4*x + Sinh[4*x]))/(96*a)
```

Maple [A] (verified)

Time = 8.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{x}{8a} + \frac{e^{4x}}{64a} - \frac{e^{3x}}{24a} + \frac{e^{x}}{8a} - \frac{e^{-x}}{8a} + \frac{e^{-3x}}{24a} - \frac{e^{-4x}}{64a}$
default	$\frac{\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^4} + \frac{5}{6\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} + \frac{7}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{16}{128\tanh\left(\frac{x}{2}\right)-128} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8} - \frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^4} + \frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} - \cdots 8\left(\tanh\right.}{a}$

```
[In] int(sinh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] -1/8*x/a+1/64/a*exp(4*x)-1/24/a*exp(3*x)+1/8/a*exp(x)-1/8/a*exp(-x)+1/24/a*
exp(-3*x)-1/64/a*exp(-4*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\sinh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{(3\cosh(x) - 2)\sinh(x)^3 + 3\left(\cosh(x)^3 - 2\cosh(x)^2 + 2\right)\sinh(x) - 3x}{24a}$$

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/24*((3*cosh(x) - 2)*sinh(x)^3 + 3*(cosh(x)^3 - 2*cosh(x)^2 + 2)*sinh(x) -
 3*x)/a
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\sinh^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sinh(x)**4/(a+a*sech(x)),x)
[Out] Integral(sinh(x)**4/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{\sinh^4(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\left(8\,e^{(-x)}-24\,e^{(-3\,x)}-3\right)e^{(4\,x)}}{192\,a}-\frac{x}{8\,a}-\frac{24\,e^{(-x)}-8\,e^{(-3\,x)}+3\,e^{(-4\,x)}}{192\,a}$$

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
[Out] -1/192*(8*e^(-x) - 24*e^(-3*x) - 3)*e^(4*x)/a - 1/8*x/a - 1/192*(24*e^(-x)
- 8*e^(-3*x) + 3*e^(-4*x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^4(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\left(24\,e^{(3\,x)}-8\,e^{x}+3\right)e^{(-4\,x)}+24\,x-3\,e^{(4\,x)}+8\,e^{(3\,x)}-24\,e^{x}}{192\,a}$$

```
[In] integrate(sinh(x)^4/(a+a*sech(x)),x, algorithm="giac")
[Out] -1/192*((24*e^(3*x) - 8*e^x + 3)*e^(-4*x) + 24*x - 3*e^(4*x) + 8*e^(3*x) -
24*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \frac{\sinh^4(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}-\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}-\frac{x}{8\,a}+\frac{\mathrm{e}^{x}}{8\,a}$$

```
[In] int(sinh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) - exp(-x)/(8*a) - exp(3*x)/(24*a) - exp(-4*x)/(64*a) + exp
(4*x)/(64*a) - x/(8*a) + exp(x)/(8*a)
```

3.53 $\int \frac{\sinh^3(x)}{a+a\textbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx = \frac{\cosh^3(x)}{3a} - \frac{\sinh^2(x)}{2a}$$

```
[Out] 1/3*cosh(x)^3/a-1/2*sinh(x)^2/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2914, 2644, 30, 2645}

$$\int \frac{\sinh^3(x)}{a+a\text{sech}(x)}\,dx = \frac{\cosh^3(x)}{3a} - \frac{\sinh^2(x)}{2a}$$

```
[In] Int[Sinh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] Cosh[x]^3/(3*a) - Sinh[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cosh(x)\sinh^3(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\int \cosh(x)\sinh(x)\,dx}{a}+\frac{\int \cosh^2(x)\sinh(x)\,dx}{a} \\
&= \frac{\text{Subst}\left(\int x\,dx,x,i\sinh(x)\right)}{a}+\frac{\text{Subst}\left(\int x^2\,dx,x,\cosh(x)\right)}{a} \\
&= \frac{\cosh^3(x)}{3a}-\frac{\sinh^2(x)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{-7+3\cosh(x)-3\cosh(2x)+\cosh(3x)}{12a}$$

```
[In] Integrate[Sinh[x]^3/(a + a*Sech[x]),x]
[Out] (-7 + 3*Cosh[x] - 3*Cosh[2*x] + Cosh[3*x])/(12*a)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(19) = 38.

Time = 2.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

method	result	size
risch	$\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}+\frac{e^{x}}{8a}+\frac{e^{-x}}{8a}-\frac{e^{-2x}}{8a}+\frac{e^{-3x}}{24a}$	54
default	$\frac{-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{8}{8\tanh\left(\frac{x}{2}\right)+8}}{a}$	67

```
[In] int(sinh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/24/a*exp(3*x)-1/8/a*exp(2*x)+1/8/a*exp(x)+1/8/a*exp(-x)-1/8/a*exp(-2*x)+1
/24/a*exp(-3*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{\sinh^3(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\cosh(x)^3+3(\cosh(x)-1)\sinh(x)^2-3\cosh(x)^2+3\cosh(x)}{12a}$$

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/12*(cosh(x)^3 + 3*(cosh(x) - 1)*sinh(x)^2 - 3*cosh(x)^2 + 3*cosh(x))/a
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\sinh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sinh(x)**3/(a+a*sech(x)),x)
[Out] Integral(sinh(x)**3/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(19) = 38.

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^3(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\left(3\,e^{(-x)} - 3\,e^{(-2\,x)} - 1\right)e^{(3\,x)}}{24\,a} + \frac{3\,e^{(-x)} - 3\,e^{(-2\,x)} + e^{(-3\,x)}}{24\,a}$$

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
[Out] -1/24*(3*e^(-x) - 3*e^(-2*x) - 1)*e^(3*x)/a + 1/24*(3*e^(-x) - 3*e^(-2*x) +
 e^(-3*x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\sinh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\left(3\,e^{(2\,x)} - 3\,e^{x} + 1\right)e^{(-3\,x)} + e^{(3\,x)} - 3\,e^{(2\,x)} + 3\,e^{x}}{24\,a}$$

```
[In] integrate(sinh(x)^3/(a+a*sech(x)),x, algorithm="giac")
[Out] 1/24*((3*e^(2*x) - 3*e^x + 1)*e^(-3*x) + e^(3*x) - 3*e^(2*x) + 3*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{\sinh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{e^{-x}}{8\,a} - \frac{e^{-2\,x}}{8\,a} - \frac{e^{2\,x}}{8\,a} + \frac{e^{-3\,x}}{24\,a} + \frac{e^{3\,x}}{24\,a} + \frac{e^{x}}{8\,a}$$

```
[In] int(sinh(x)^3/(a + a/cosh(x)),x)
[Out] exp(-x)/(8*a) - exp(-2*x)/(8*a) - exp(2*x)/(8*a) + exp(-3*x)/(24*a) + exp(3
*x)/(24*a) + exp(x)/(8*a)
```

3.54 $\int \frac{\sinh^2(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\sinh^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}$$

```
[Out] 1/2*x/a-sinh(x)/a+1/2*cosh(x)*sinh(x)/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2918, 2717, 2715, 8}

$$\int \frac{\sinh^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[In] Int[Sinh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/(2*a) - Sinh[x]/a + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cosh(x)\sinh^2(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\int \cosh(x)\,dx}{a}+\frac{\int \cosh^2(x)\,dx}{a} \\
&= -\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}+\frac{\int 1\,dx}{2a} \\
&= \frac{x}{2a}-\frac{\sinh(x)}{a}+\frac{\cosh(x)\sinh(x)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx=\frac{x+(-2+\cosh(x))\sinh(x)}{2a}$$

```
[In] Integrate[Sinh[x]^2/(a + a*Sech[x]),x]
[Out] (x + (-2 + Cosh[x])*Sinh[x])/(2*a)
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

method	result	size
risch	$\frac{x}{2a}+\frac{e^{2x}}{8a}-\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}-\frac{e^{-2x}}{8a}$	42
default	$\frac{-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}}{a}$	65

```
[In] int(sinh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/2*x/a+1/8/a*exp(2*x)-1/2/a*exp(x)+1/2/a*exp(-x)-1/8/a*exp(-2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx=\frac{(\cosh(x)-2)\sinh(x)+x}{2\,a}$$

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/2*((cosh(x) - 2)*sinh(x) + x)/a
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a+a\text{sech}(x)}\,dx=\frac{\int \frac{\sinh^2(x)}{\text{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sinh(x)**2/(a+a*sech(x)),x)
[Out] Integral(sinh(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int \frac{\sinh^2(x)}{a + a\text{sech}(x)}\, dx = -\frac{\left(4\, e^{(-x)} - 1\right)e^{(2\, x)}}{8\, a} + \frac{x}{2\, a} + \frac{4\, e^{(-x)} - e^{(-2\, x)}}{8\, a}$$

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] -1/8*(4*e^(-x) - 1)*e^(2*x)/a + 1/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{\sinh^2(x)}{a + a\text{sech}(x)}\, dx = \frac{(4\, e^{x} - 1)e^{(-2\, x)} + 4\, x + e^{(2\, x)} - 4\, e^{x}}{8\, a}$$

```
[In] integrate(sinh(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] 1/8*((4*e^x - 1)*e^(-2*x) + 4*x + e^(2*x) - 4*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{\sinh^2(x)}{a + a\text{sech}(x)}\, dx = \frac{\mathrm{e}^{-x}}{2\, a} - \frac{\mathrm{e}^{-2\, x}}{8\, a} + \frac{\mathrm{e}^{2\, x}}{8\, a} + \frac{x}{2\, a} - \frac{\mathrm{e}^{x}}{2\, a}$$

```
[In] int(sinh(x)^2/(a + a/cosh(x)),x)
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + x/(2*a) - exp(x)/(2*a)
```

3.55 $\int \frac{\sinh(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{\sinh(x)}{a+a\text{sech}(x)}\,dx=\frac{\cosh(x)}{a}-\frac{\log(1+\cosh(x))}{a}$$

```
[Out] cosh(x)/a-ln(1+cosh(x))/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sinh(x)}{a+a\text{sech}(x)}\,dx=\frac{\cosh(x)}{a}-\frac{\log(\cosh(x)+1)}{a}$$

```
[In] Int[Sinh[x]/(a + a*Sech[x]),x]
```

```
[Out] Cosh[x]/a - Log[1 + Cosh[x]]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\cosh(x)\sinh(x)}{-a-a\cosh(x)}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cosh(x)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cosh(x)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)\,dx, x, -a\cosh(x)\right)}{a^2} \\
&= \frac{\cosh(x)}{a} - \frac{\log(1+\cosh(x))}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\sinh(x)}{a+a\text{sech}(x)}\,dx = \frac{\cosh(x)-2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

```
[In] Integrate[Sinh[x]/(a + a*Sech[x]),x]
[Out] (Cosh[x] - 2*Log[Cosh[x/2]])/a
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$-\frac{\ln(1+\operatorname{sech}(x))-\frac{1}{\operatorname{sech}(x)}-\ln(\operatorname{sech}(x))}{a}$	23
default	$-\frac{\ln(1+\operatorname{sech}(x))-\frac{1}{\operatorname{sech}(x)}-\ln(\operatorname{sech}(x))}{a}$	23
risch	$\frac{x}{a}+\frac{e^{x}}{2a}+\frac{e^{-x}}{2a}-\frac{2\ln(e^{x}+1)}{a}$	33

```
[In] int(sinh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] -1/a*(ln(1+sech(x))-1/sech(x)-ln(sech(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.94

$$\int \frac{\sinh(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2\,x\cosh(x)+\cosh(x)^{2}-4\left(\cosh(x)+\sinh(x)\right)\log\left(\cosh(x)+\sinh(x)+1\right)+2\left(x+\cosh(x)\right)\sinh(x)+}{2\left(a\cosh(x)+a\sinh(x)\right)}$$

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/2*(2*x*cosh(x) + cosh(x)^2 - 4*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
+ 1) + 2*(x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(a*cosh(x) + a*sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\sinh(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sinh(x)/(a+a*sech(x)),x)
[Out] Integral(sinh(x)/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{\sinh(x)}{a + a\text{sech}(x)}\,dx = -\frac{x}{a} + \frac{e^{(-x)}}{2\,a} + \frac{e^{x}}{2\,a} - \frac{2\,\log\left(e^{(-x)} + 1\right)}{a}$$

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="maxima")
[Out] -x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^(-x) + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int \frac{\sinh(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{e^{(-x)}}{2\,a} + \frac{e^{x}}{2\,a} - \frac{2\,\log\left(e^{x} + 1\right)}{a}$$

```
[In] integrate(sinh(x)/(a+a*sech(x)),x, algorithm="giac")
[Out] x/a + 1/2*e^(-x)/a + 1/2*e^x/a - 2*log(e^x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{\sinh(x)}{a + a\text{sech}(x)}\,dx = -\frac{\ln\left(\cosh(x) + 1\right) - \cosh(x)}{a}$$

```
[In] int(sinh(x)/(a + a/cosh(x)),x)
[Out] -(log(cosh(x) + 1) - cosh(x))/a
```

3.56 $\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}+\frac{\operatorname{csch}^2(x)}{2a}$$

```
[Out] -1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3957, 2785, 2686, 30, 2691, 3855}

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\operatorname{arctanh}(\cosh(x))}{2a}+\frac{\operatorname{csch}^2(x)}{2a}-\frac{\coth(x)\operatorname{csch}(x)}{2a}$$

```
[In] Int[Csch[x]/(a + a*Sech[x]),x]
[Out] -1/2*ArcTanh[Cosh[x]]/a - (Coth[x]*Csch[x])/(2*a) + Csch[x]^2/(2*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2785

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\coth(x)}{-a-a\cosh(x)}\,dx \\
&= \frac{\int \coth^2(x)\text{csch}(x)\,dx}{a} - \frac{\int \coth(x)\text{csch}^2(x)\,dx}{a} \\
&= -\frac{\coth(x)\text{csch}(x)}{2a} + \frac{\int \text{csch}(x)\,dx}{2a} - \frac{\text{Subst}(\int x\,dx, x, -i\text{csch}(x))}{a} \\
&= -\frac{\text{arctanh}(\cosh(x))}{2a} - \frac{\coth(x)\text{csch}(x)}{2a} + \frac{\text{csch}^2(x)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\left(1+2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\operatorname{sech}(x)}{2a(1+\operatorname{sech}(x))}$$

```
[In] Integrate[Csch[x]/(a + a*Sech[x]),x]
[Out] -1/2*((1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))*Sech[x])/(a*(1
+ Sech[x]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^2}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$-\frac{e^x}{\left(e^x+1\right)^2 a}+\frac{\ln(e^x-1)}{2a}-\frac{\ln(e^x+1)}{2a}$	35

```
[In] int(csch(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(27) = 54.

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.12

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\left(\cosh(x)^2+2\left(\cosh(x)+1\right)\sinh(x)+\sinh(x)^2+2\cosh(x)+1\right)\log\left(\cosh(x)+\sinh(x)+1\right)-\left(\cosh\right.}{2\left(a\cosh(x)^2+a\sinh(x)^2+2a\right.}$$

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="fricas")
[Out] -1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log
(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2
+ 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*cos
h(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\int \frac{\operatorname{csch}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(csch(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \frac{\operatorname{csch}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{4\,a}+\frac{e^{(-x)}+e^{x}-2}{4\,a(e^{(-x)}+e^{x}+2)}$$

```
[In] integrate(csch(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
e^x - 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{csch}(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{1}{a\,\left(e^{2\,x} + 2\,e^{x} + 1\right)} - \frac{1}{a\,\left(e^{x} + 1\right)} - \frac{\operatorname{atan}\left(\frac{e^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

```
[In] int(1/(sinh(x)*(a + a/cosh(x))),x)
```

```
[Out] 1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)^
(1/2))/a)/(-a^2)^(1/2)
```

3.57 $\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^3(x)}{3a}$$

```
[Out] -1/3*coth(x)^3/a+1/3*csch(x)^3/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2918, 2686, 30, 2687}

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\operatorname{csch}^3(x)}{3a}-\frac{\coth^3(x)}{3a}$$

```
[In] Int[Csch[x]^2/(a + a*Sech[x]),x]
```

```
[Out] -1/3*Coth[x]^3/a + Csch[x]^3/(3*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\coth(x)\text{csch}(x)}{-a-a\cosh(x)}\,dx\\
&= \frac{\int \coth^2(x)\text{csch}^2(x)\,dx}{a}-\frac{\int \coth(x)\text{csch}^3(x)\,dx}{a}\\
&= -\frac{i\text{Subst}\left(\int x^2\,dx,x,i\coth(x)\right)}{a}-\frac{i\text{Subst}\left(\int x^2\,dx,x,-i\text{csch}(x)\right)}{a}\\
&= -\frac{\coth^3(x)}{3a}+\frac{\text{csch}^3(x)}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\text{csch}^2(x)}{a+a\text{sech}(x)}\,dx = -\frac{(3+2\cosh(x)+\cosh(2x))\text{csch}(x)}{6a(1+\cosh(x))}$$

```
[In] Integrate[Csch[x]^2/(a + a*Sech[x]),x]
[Out] -1/6*((3 + 2*Cosh[x] + Cosh[2*x])*Csch[x])/(a*(1 + Cosh[x]))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	23
risch	$-\frac{2\left(3\,\mathrm{e}^{2x}+2\,\mathrm{e}^{x}+1\right)}{3\left(\mathrm{e}^{x}+1\right)^{3}a\left(\mathrm{e}^{x}-1\right)}$	30

```
[In] int(csch(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/4/a*(-1/3*tanh(1/2*x)^3-1/tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.09

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{4\,(2\,\cosh{(x)}+\sinh{(x)}+1)}{3\,\left(a\cosh{(x)}^{3}+a\sinh{(x)}^{3}+2\,a\cosh{(x)}^{2}+(3\,a\cosh{(x)}+2\,a)\sinh{(x)}^{2}-a\cosh{(x)}+\left(3\,a\cosh{(x)}^{2}\right.\right.}$$

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] -4/3*(2*cosh(x) + sinh(x) + 1)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 +
 (3*a*cosh(x) + 2*a)*sinh(x)^2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) +
 a)*sinh(x) - 2*a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{csch}^2{(x)}}{\operatorname{sech}{(x)}+1}\,dx}{a}$$

```
[In] integrate(csch(x)**2/(a+a*sech(x)),x)
[Out] Integral(csch(x)**2/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(19) = 38$.

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.91

$$\int \frac{\operatorname{csch}^2(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{4\,e^{(-x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)} - \frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} - \frac{2}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)}$$

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] -4/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a*
e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) -
a*e^(-4*x) + a)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\operatorname{csch}^2(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{1}{2\,a(e^x - 1)} + \frac{3\,e^{(2\,x)} + 1}{6\,a(e^x + 1)^3}$$

```
[In] integrate(csch(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] -1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 1)/(a*(e^x + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.96

$$\int \frac{\operatorname{csch}^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\frac{e^{2\,x}}{6\,a} + \frac{1}{6\,a} - \frac{e^x}{3\,a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^x + 1} - \frac{\frac{1}{6\,a} - \frac{e^x}{6\,a}}{e^{2\,x} + 2\,e^x + 1} - \frac{1}{2\,a\,\left(e^x - 1\right)} + \frac{1}{6\,a\,\left(e^x + 1\right)}$$

```
[In] int(1/(sinh(x)^2*(a + a/cosh(x))),x)
[Out] (exp(2*x)/(6*a) + 1/(6*a) - exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp(x)
 + 1) - (1/(6*a) - exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(x)
 - 1)) + 1/(6*a*(exp(x) + 1))
```

3.58 $\int \frac{\text{csch}^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\text{csch}^3(x)}{a+a\text{sech}(x)}\,dx=\frac{\text{arctanh}(\cosh(x))}{8a}-\frac{\coth(x)\text{csch}(x)}{8a}-\frac{\coth(x)\text{csch}^3(x)}{4a}+\frac{\text{csch}^4(x)}{4a}$$

```
[Out] 1/8*arctanh(cosh(x))/a-1/8*coth(x)*csch(x)/a-1/4*coth(x)*csch(x)^3/a+1/4*cs
ch(x)^4/a
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3957, 2914, 2686, 30, 2691, 3853, 3855}

$$\int \frac{\text{csch}^3(x)}{a+a\text{sech}(x)}\,dx=\frac{\text{arctanh}(\cosh(x))}{8a}+\frac{\text{csch}^4(x)}{4a}-\frac{\coth(x)\text{csch}^3(x)}{4a}-\frac{\coth(x)\text{csch}(x)}{8a}$$

```
[In] Int[Csch[x]^3/(a + a*Sech[x]),x]
```

```
[Out] ArcTanh[Cosh[x]]/(8*a) - (Coth[x]*Csch[x])/(8*a) - (Coth[x]*Csch[x]^3)/(4*a
) + Csch[x]^4/(4*a)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2914

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\coth(x)\text{csch}^2(x)}{-a-a\cosh(x)}\,dx \\
&= \frac{\int \coth^2(x)\text{csch}^3(x)\,dx}{a} - \frac{\int \coth(x)\text{csch}^4(x)\,dx}{a}
\end{aligned}
$$

$$= -\frac{\coth(x)\text{csch}^3(x)}{4a} + \frac{\int \text{csch}^3(x)\,dx}{4a} + \frac{\text{Subst}\left(\int x^3\,dx, x, -i\text{csch}(x)\right)}{a}$$

$$= -\frac{\coth(x)\text{csch}(x)}{8a} - \frac{\coth(x)\text{csch}^3(x)}{4a} + \frac{\text{csch}^4(x)}{4a} - \frac{\int \text{csch}(x)\,dx}{8a}$$

$$= \frac{\text{arctanh}(\cosh(x))}{8a} - \frac{\coth(x)\text{csch}(x)}{8a} - \frac{\coth(x)\text{csch}^3(x)}{4a} + \frac{\text{csch}^4(x)}{4a}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{\text{csch}^3(x)}{a + a\text{sech}(x)}\,dx = \frac{\cosh^2\left(\frac{x}{2}\right)\left(-2\text{csch}^2\left(\frac{x}{2}\right) + 4\log\left(\cosh\left(\frac{x}{2}\right)\right) - 4\log\left(\sinh\left(\frac{x}{2}\right)\right) + \text{sech}^4\left(\frac{x}{2}\right)\right)\text{sech}(x)}{16(a + a\text{sech}(x))}$$

```
[In] Integrate[Csch[x]^3/(a + a*Sech[x]),x]
[Out] (Cosh[x/2]^2*(-2*Csch[x/2]^2 + 4*Log[Cosh[x/2]] - 4*Log[Sinh[x/2]] + Sech[x
/2]^4)*Sech[x])/(16*(a + a*Sech[x]))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^4}{4} - \frac{\tanh\left(\frac{x}{2}\right)^2}{2} - \ln\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{1}{2\tanh\left(\frac{x}{2}\right)^2}}{8a}$	38
risch	$-\frac{e^x\left(e^{4x}+2e^{3x}+10e^{2x}+2e^x+1\right)}{4(e^x+1)^4 a(e^x-1)^2} + \frac{\ln(e^x+1)}{8a} - \frac{\ln(e^x-1)}{8a}$	63

```
[In] int(csch(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/8/a*(1/4*tanh(1/2*x)^4-1/2*tanh(1/2*x)^2-ln(tanh(1/2*x))-1/2/tanh(1/2*x)^
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(38) = 76$.

Time = 0.25 (sec) , antiderivative size = 630, normalized size of antiderivative = 13.70

$$\int \frac{\operatorname{csch}^3(x)}{a + a\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*cosh(x)^5 + 2*(5*cosh(x) + 2)*sinh(x)^4 + 2*sinh(x)^5 + 4*cosh(x)^4
 + 4*(5*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x)^3 + 20*cosh(x)^3 + 4*(5*cosh(x)^
3 + 6*cosh(x)^2 + 15*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 - (cosh(x)^6 + 2*
(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*co
sh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) -
 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 -
12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*co
sh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) +
 sinh(x) + 1) + (cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*co
sh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh
(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4
+ 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3
*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)
 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(5*cosh(x)^4 + 8*cosh(x)^3
 + 30*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 2*cosh(x))/(a*cosh(x)^6 + a*sinh
(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4 + (15*a
*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*cosh(x)^3
 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*cosh(x)^4
 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*a*cosh(
x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^2 - a*c
osh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{csch}^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(csch(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(csch(x)**3/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(38) = 76$.

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{csch}^3(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{e^{(-x)} + 2\,e^{(-2\,x)} + 10\,e^{(-3\,x)} + 2\,e^{(-4\,x)} + e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} + \frac{\log\left(e^{(-x)} + 1\right)}{8\,a} - \frac{\log\left(e^{(-x)} - 1\right)}{8\,a}$$

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="maxima")
[Out] -1/4*(e^(-x) + 2*e^(-2*x) + 10*e^(-3*x) + 2*e^(-4*x) + e^(-5*x))/(2*a*e^(-x
) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x) + a)
 + 1/8*log(e^(-x) + 1)/a - 1/8*log(e^(-x) - 1)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(38) = 76$.

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{\operatorname{csch}^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\log\left(e^{(-x)} + e^x + 2\right)}{16\,a} - \frac{\log\left(e^{(-x)} + e^x - 2\right)}{16\,a} + \frac{e^{(-x)} + e^x - 6}{16\,a(e^{(-x)} + e^x - 2)} - \frac{3\left(e^{(-x)} + e^x\right)^2 + 12\,e^{(-x)} + 12\,e^x - 4}{32\,a(e^{(-x)} + e^x + 2)^2}$$

```
[In] integrate(csch(x)^3/(a+a*sech(x)),x, algorithm="giac")
[Out] 1/16*log(e^(-x) + e^x + 2)/a - 1/16*log(e^(-x) + e^x - 2)/a + 1/16*(e^(-x)
+ e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(3*(e^(-x) + e^x)^2 + 12*e^(-x) +
12*e^x - 4)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.63

$$\int \frac{\operatorname{csch}^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{1}{2\,a\,\left(\mathrm{e}^{2\,x} + 2\,\mathrm{e}^{x} + 1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{2\,x} - 2\,\mathrm{e}^{x} + 1\right)} + \frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x} + 4\,\mathrm{e}^{3\,x} + \mathrm{e}^{4\,x} + 4\,\mathrm{e}^{x} + 1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{x} - 1\right)} + \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}} - \frac{1}{a\,\left(3\,\mathrm{e}^{2\,x} + \mathrm{e}^{3\,x} + 3\,\mathrm{e}^{x} + 1\right)}$$

```
[In] int(1/(sinh(x)^3*(a + a/cosh(x))),x)
```

```
[Out] 1/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(
2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) -
 1)) + atan((exp(x)*(-a^2)^(1/2))/a)/(4*(-a^2)^(1/2)) - 1/(a*(3*exp(2*x) +
exp(3*x) + 3*exp(x) + 1))
```

3.59 $\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\coth^3(x)}{3a}-\frac{\coth^5(x)}{5a}+\frac{\operatorname{csch}^5(x)}{5a}$$

```
[Out] 1/3*coth(x)^3/a-1/5*coth(x)^5/a+1/5*csch(x)^5/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3957, 2918, 2686, 30, 2687, 14}

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\coth^5(x)}{5a}+\frac{\coth^3(x)}{3a}+\frac{\operatorname{csch}^5(x)}{5a}$$

```
[In] Int[Csch[x]^4/(a + a*Sech[x]),x]
[Out] Coth[x]^3/(3*a) - Coth[x]^5/(5*a) + Csch[x]^5/(5*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2687

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{\coth(x)\text{csch}^3(x)}{-a-a\cosh(x)}\,dx\\
&= \frac{\int \coth^2(x)\text{csch}^4(x)\,dx}{a}-\frac{\int \coth(x)\text{csch}^5(x)\,dx}{a}\\
&= \frac{i\text{Subst}\left(\int x^4\,dx,x,-i\text{csch}(x)\right)}{a}+\frac{i\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,i\coth(x)\right)}{a}\\
&= \frac{\text{csch}^5(x)}{5a}+\frac{i\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,i\coth(x)\right)}{a}\\
&= \frac{\coth^3(x)}{3a}-\frac{\coth^5(x)}{5a}+\frac{\text{csch}^5(x)}{5a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{(-15-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x))\operatorname{csch}^3(x)}{60a(1+\cosh(x))}$$

```
[In] Integrate[Csch[x]^4/(a + a*Sech[x]),x]
[Out] ((-15 - 6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(60*a
*(1 + Cosh[x]))
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^5}{5}+\frac{2\tanh\left(\frac{x}{2}\right)^3}{3}+\frac{2}{\tanh\left(\frac{x}{2}\right)}-\frac{1}{3\tanh\left(\frac{x}{2}\right)^3}}{16a}$	39
risch	$-\frac{4(15\,\mathrm{e}^{4x}+6\,\mathrm{e}^{3x}+2\,\mathrm{e}^{2x}-2\,\mathrm{e}^{x}-1)}{15(\mathrm{e}^{x}+1)^5 a(\mathrm{e}^{x}-1)^3}$	42

```
[In] int(csch(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/16/a*(-1/5*tanh(1/2*x)^5+2/3*tanh(1/2*x)^3+2/tanh(1/2*x)-1/3/tanh(1/2*x)^
3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(28) = 56$.

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 6.44

$$\int \frac{\operatorname{csch}^4(x)}{a+a\operatorname{sech}(x)}\,dx=$$

$$-\frac{}{15\left(a\cosh(x)^6+a\sinh(x)^6+2\,a\cosh(x)^5+2\,(3\,a\cosh(x)+a)\sinh(x)^5-2\,a\cosh(x)^4+\left(15\,a\cosh\right.\right.}$$

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="fricas")
[Out] -8/15*(7*cosh(x)^2 + 4*(4*cosh(x) + 1)*sinh(x) + 7*sinh(x)^2 + 2*cosh(x) +
1)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)
^5 - 2*a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - 2*a)*sinh(x)^4 - 6*a*
cosh(x)^3 + 2*(10*a*cosh(x)^3 + 10*a*cosh(x)^2 - 4*a*cosh(x) - 3*a)*sinh(x)
^3 - a*cosh(x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 12*a*cosh(x)^2 - 18*a
```

```
*cosh(x) - a)*sinh(x)^2 + 4*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 -
4*a*cosh(x)^3 - 9*a*cosh(x)^2 + a*cosh(x) + 4*a)*sinh(x) + 2*a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\, dx = \frac{\int \frac{\operatorname{csch}^4(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

```
[In] integrate(csch(x)**4/(a+a*sech(x)),x)
[Out] Integral(csch(x)**4/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(28) = 56$.

Time = 0.19 (sec) , antiderivative size = 292, normalized size of antiderivative = 8.59

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\, dx &= \frac{8\, e^{(-x)}}{15\,(2\, ae^{(-x)} - 2\, ae^{(-2\,x)} - 6\, ae^{(-3\,x)} + 6\, ae^{(-5\,x)} + 2\, ae^{(-6\,x)} - 2\, ae^{(-7\,x)} - ae^{(-8\,x)} + a)} \\ &\quad - \frac{8\, e^{(-2\,x)}}{15\,(2\, ae^{(-x)} - 2\, ae^{(-2\,x)} - 6\, ae^{(-3\,x)} + 6\, ae^{(-5\,x)} + 2\, ae^{(-6\,x)} - 2\, ae^{(-7\,x)} - ae^{(-8\,x)} + a)} \\ &\quad - \frac{8\, e^{(-3\,x)}}{5\,(2\, ae^{(-x)} - 2\, ae^{(-2\,x)} - 6\, ae^{(-3\,x)} + 6\, ae^{(-5\,x)} + 2\, ae^{(-6\,x)} - 2\, ae^{(-7\,x)} - ae^{(-8\,x)} + a)} \\ &\quad - \frac{4\, e^{(-4\,x)}}{2\, ae^{(-x)} - 2\, ae^{(-2\,x)} - 6\, ae^{(-3\,x)} + 6\, ae^{(-5\,x)} + 2\, ae^{(-6\,x)} - 2\, ae^{(-7\,x)} - ae^{(-8\,x)} + a} \\ &\quad + \frac{4}{15\,(2\, ae^{(-x)} - 2\, ae^{(-2\,x)} - 6\, ae^{(-3\,x)} + 6\, ae^{(-5\,x)} + 2\, ae^{(-6\,x)} - 2\, ae^{(-7\,x)} - ae^{(-8\,x)} + a)}\end{aligned}$$

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="maxima")
[Out] 8/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*
e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 8/15*e^(-2*x)/(2*a*e^(-x) - 2*a
*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e
^(-8*x) + a) - 8/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a
*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 4*e^(-4*x)/(2*a
*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e
^(-7*x) - a*e^(-8*x) + a) + 4/15/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x)
+ 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{3\,e^{(2\,x)} - 12\,e^{x} + 5}{24\,a(e^{x} - 1)^3} - \frac{15\,e^{(4\,x)} + 60\,e^{(3\,x)} + 10\,e^{(2\,x)} + 20\,e^{x} + 7}{120\,a(e^{x} + 1)^5}$$

```
[In] integrate(csch(x)^4/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(3*e^(2*x) - 12*e^x + 5)/(a*(e^x - 1)^3) - 1/120*(15*e^(4*x) + 60*e^(3
*x) + 10*e^(2*x) + 20*e^x + 7)/(a*(e^x + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 236, normalized size of antiderivative = 6.94

$$\begin{aligned}\int \frac{\operatorname{csch}^4(x)}{a + a\operatorname{sech}(x)}\,dx &= \frac{1}{6\,a\,\left(3\,e^{2\,x} - e^{3\,x} - 3\,e^{x} + 1\right)} - \frac{\frac{3\,e^{2\,x}}{40\,a} + \frac{e^{3\,x}}{40\,a} + \frac{1}{40\,a} - \frac{e^{x}}{8\,a}}{6\,e^{2\,x} + 4\,e^{3\,x} + e^{4\,x} + 4\,e^{x} + 1} \\ &\quad - \frac{\frac{e^{2\,x}}{40\,a} - \frac{1}{24\,a} + \frac{e^{x}}{20\,a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^{x} + 1} - \frac{\frac{e^{3\,x}}{10\,a} - \frac{e^{2\,x}}{4\,a} + \frac{e^{4\,x}}{40\,a} + \frac{1}{40\,a} + \frac{e^{x}}{10\,a}}{10\,e^{2\,x} + 10\,e^{3\,x} + 5\,e^{4\,x} + e^{5\,x} + 5\,e^{x} + 1} \\ &\quad - \frac{1}{4\,a\,\left(e^{2\,x} - 2\,e^{x} + 1\right)} + \frac{1}{8\,a\,\left(e^{x} - 1\right)} - \frac{1}{20\,a\,\left(e^{x} + 1\right)}\end{aligned}$$

```
[In] int(1/(sinh(x)^4*(a + a/cosh(x))),x)
```

```
[Out] 1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(40*a) + exp
(3*x)/(40*a) + 1/(40*a) - exp(x)/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*x)
 + 4*exp(x) + 1) - (exp(2*x)/(40*a) - 1/(24*a) + exp(x)/(20*a))/(3*exp(2*x)
 + exp(3*x) + 3*exp(x) + 1) - (exp(3*x)/(10*a) - exp(2*x)/(4*a) + exp(4*x)/
(40*a) + 1/(40*a) + exp(x)/(10*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x)
+ exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(8*a*(ex
p(x) - 1)) - 1/(20*a*(exp(x) + 1))
```

3.60 $\int \frac{\sinh^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 132

$$\int \frac{\sinh^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5} + \frac{(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b-3a\cosh(x))\sinh^3(x)}{12a^2}$$

```
[Out] 1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctan((a-b)
^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^5+1/8*(8*b*(a^2-b^2)-a*(3*a^2-4*b^2)*cosh
(x))*sinh(x)/a^4-1/12*(4*b-3*a*cosh(x))*sinh(x)^3/a^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2944, 2814, 2738, 211}

$$\int \frac{\sinh^4(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{2b(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5} - \frac{\sinh^3(x)(4b-3a\cosh(x))}{12a^2} + \frac{\sinh(x)\left(8b(a^2-b^2)-a(3a^2-4b^2)\cosh(x)\right)}{8a^4} + \frac{x(3a^4-12a^2b^2+8b^4)}{8a^5}$$

```
[In] Int[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/a^5 + ((8*b*(a^2 - b^2) - a*(
3*a^2 - 4*b^2)*Cosh[x])*Sinh[x])/(8*a^4) - ((4*b - 3*a*Cosh[x])*Sinh[x]^3)/
(12*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\cosh(x)\sinh^4(x)}{-b - a\cosh(x)}\,dx$$

$$= -\frac{(4b - 3a\cosh(x))\sinh^3(x)}{12a^2} + \frac{\int \frac{(-ab+(3a^2-4b^2)\cosh(x))\sinh^2(x)}{-b-a\cosh(x)}\,dx}{4a^2}$$

$$= \frac{(8b(a^2-b^2) - a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b - 3a\cosh(x))\sinh^3(x)}{12a^2} - \frac{\int \frac{-ab(5a^2-4b^2)+(3a^4-12a^2b^2+8b^4)\cosh(x)}{-b-a\cosh(x)}\,dx}{8a^4}$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2) - a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b - 3a\cosh(x))\sinh^3(x)}{12a^2} + \frac{\left(b(a^2-b^2)^2\right)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^5}$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} + \frac{(8b(a^2-b^2) - a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b - 3a\cosh(x))\sinh^3(x)}{12a^2} + \frac{\left(2b(a^2-b^2)^2\right)\text{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^5}$$

$$= \frac{(3a^4-12a^2b^2+8b^4)\,x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5} + \frac{(8b(a^2-b^2) - a(3a^2-4b^2)\cosh(x))\sinh(x)}{8a^4} - \frac{(4b - 3a\cosh(x))\sinh^3(x)}{12a^2}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.66

$$\int \frac{\sinh^4(x)}{a + b\text{sech}(x)}\,dx$$

$$= \frac{36a^4x - 144a^2b^2x + 96b^4x + \frac{192a^4b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - \frac{384a^2b^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{192b^5\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{96a^5}$$

```
[In] Integrate[Sinh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (36*a^4*x - 144*a^2*b^2*x + 96*b^4*x + (192*a^4*b*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - (384*a^2*b^3*ArcTan[((-a + b)*Tanh[x
/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (192*b^5*ArcTan[((-a + b)*Tanh[x/2
])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 24*a*b*(5*a^2 - 4*b^2)*Sinh[x] - 24*
a^2*(a^2 - b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(116) = 232$.

Time = 48.29 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.31

method	result
risch	$\frac{3x}{8a}-\frac{3x\,b^{2}}{2a^{3}}+\frac{x\,b^{4}}{a^{5}}+\frac{\mathrm{e}^{4x}}{64a}-\frac{b\,\mathrm{e}^{3x}}{24a^{2}}-\frac{\mathrm{e}^{2x}}{8a}+\frac{\mathrm{e}^{2x}b^{2}}{8a^{3}}+\frac{5b\,\mathrm{e}^{x}}{8a^{2}}-\frac{b^{3}\mathrm{e}^{x}}{2a^{4}}-\frac{5b\,\mathrm{e}^{-x}}{8a^{2}}+\frac{b^{3}\mathrm{e}^{-x}}{2a^{4}}+\frac{\mathrm{e}^{-2x}}{8a}-\frac{\mathrm{e}^{-2x}b^{2}}{8a^{3}}+\frac{b\,\mathrm{e}^{-3x}}{24a^{2}}$
default	$\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{-3a-2b}{6a^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{a^{2}-4ab-4b^{2}}{8a^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{\left(-3a^{4}+12a^{2}b^{2}-8b^{4}\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a^{5}}-\frac{3a^{3}+8a^{2}b-4a\,b^{2}-8b}{8a^{4}\left(\tanh\left(\frac{x}{2}\right)-1\right)}$

```
[In] int(sinh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x/a-3/2*x/a^3*b^2+x/a^5*b^4+1/64/a*exp(x)^4-1/24/a^2*b*exp(x)^3-1/8/a*e
xp(x)^2+1/8/a^3*exp(x)^2*b^2+5/8*b/a^2*exp(x)-1/2*b^3/a^4*exp(x)-5/8*b/a^2/
exp(x)+1/2*b^3/a^4/exp(x)+1/8/a/exp(x)^2-1/8/a^3/exp(x)^2*b^2+1/24/a^2*b/ex
p(x)^3-1/64/a/exp(x)^4+(-a^2+b^2)^(1/2)*b/a^3*ln(exp(x)-((-a^2+b^2)^(1/2)-b
)/a)-(-a^2+b^2)^(1/2)*b^3/a^5*ln(exp(x)-((-a^2+b^2)^(1/2)-b)/a)-(-a^2+b^2)^
(1/2)*b/a^3*ln(exp(x)+(b+(-a^2+b^2)^(1/2))/a)+(-a^2+b^2)^(1/2)*b^3/a^5*ln(e
xp(x)+(b+(-a^2+b^2)^(1/2))/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(115) = 230$.

Time = 0.27 (sec) , antiderivative size = 1812, normalized size of antiderivative = 13.73

$$\int \frac{\sinh^4(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*co
sh(x) - a^3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)
^2 - 14*a^3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b
^2 + 8*b^4)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh
(x)^3 - 7*a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*
sinh(x)^5 + 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 -
180*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a
^3*b - 4*a*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)
^3 + 8*(21*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a
^4 - a^2*b^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5
*a^3*b - 4*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*
(7*a^4*cosh(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^
4 - 2*a^2*b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b -
```

```
 4*a*b^3)*cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 - 192*((a^2*
b - b^3)*cosh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*co
sh(x)^2*sinh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(
x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
 a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x
) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
 + b)*sinh(x) + a)) + 8*(3*a^4*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^
2*b^2)*cosh(x)^5 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*
b - 4*a*b^3)*cosh(x)^4 + a^3*b - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 -
 a^2*b^2)*cosh(x))*sinh(x))/(a^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^
5*cosh(x)^2*sinh(x)^2 + 4*a^5*cosh(x)*sinh(x)^3 + a^5*sinh(x)^4), 1/192*(3*
a^4*cosh(x)^8 + 3*a^4*sinh(x)^8 - 8*a^3*b*cosh(x)^7 + 8*(3*a^4*cosh(x) - a^
3*b)*sinh(x)^7 - 24*(a^4 - a^2*b^2)*cosh(x)^6 + 4*(21*a^4*cosh(x)^2 - 14*a^
3*b*cosh(x) - 6*a^4 + 6*a^2*b^2)*sinh(x)^6 + 24*(3*a^4 - 12*a^2*b^2 + 8*b^4
)*x*cosh(x)^4 + 24*(5*a^3*b - 4*a*b^3)*cosh(x)^5 + 24*(7*a^4*cosh(x)^3 - 7*
a^3*b*cosh(x)^2 + 5*a^3*b - 4*a*b^3 - 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^5
+ 8*a^3*b*cosh(x) + 2*(105*a^4*cosh(x)^4 - 140*a^3*b*cosh(x)^3 - 180*(a^4 -
 a^2*b^2)*cosh(x)^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x + 60*(5*a^3*b - 4*a
*b^3)*cosh(x))*sinh(x)^4 - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*cosh(x)^3 + 8*(21
*a^4*cosh(x)^5 - 35*a^3*b*cosh(x)^4 - 15*a^3*b + 12*a*b^3 - 60*(a^4 - a^2*b
^2)*cosh(x)^3 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x) + 30*(5*a^3*b - 4
*a*b^3)*cosh(x)^2)*sinh(x)^3 + 24*(a^4 - a^2*b^2)*cosh(x)^2 + 12*(7*a^4*cos
h(x)^6 - 14*a^3*b*cosh(x)^5 - 30*(a^4 - a^2*b^2)*cosh(x)^4 + 2*a^4 - 2*a^2*
b^2 + 12*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x*cosh(x)^2 + 20*(5*a^3*b - 4*a*b^3)*
cosh(x)^3 - 6*(5*a^3*b - 4*a*b^3)*cosh(x))*sinh(x)^2 + 384*((a^2*b - b^3)*c
osh(x)^4 + 4*(a^2*b - b^3)*cosh(x)^3*sinh(x) + 6*(a^2*b - b^3)*cosh(x)^2*si
nh(x)^2 + 4*(a^2*b - b^3)*cosh(x)*sinh(x)^3 + (a^2*b - b^3)*sinh(x)^4)*sqrt
(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + 8*(3*a^4
*cosh(x)^7 - 7*a^3*b*cosh(x)^6 - 18*(a^4 - a^2*b^2)*cosh(x)^5 + 12*(3*a^4 -
 12*a^2*b^2 + 8*b^4)*x*cosh(x)^3 + 15*(5*a^3*b - 4*a*b^3)*cosh(x)^4 + a^3*b
 - 9*(5*a^3*b - 4*a*b^3)*cosh(x)^2 + 6*(a^4 - a^2*b^2)*cosh(x))*sinh(x))/(a
^5*cosh(x)^4 + 4*a^5*cosh(x)^3*sinh(x) + 6*a^5*cosh(x)^2*sinh(x)^2 + 4*a^5*
cosh(x)*sinh(x)^3 + a^5*sinh(x)^4)]
```

Sympy [F]

$$\int \frac{\sinh^4(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\sinh^4{(x)}}{a + b\operatorname{sech}{(x)}}\,dx$$

```
[In] integrate(sinh(x)**4/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^4(x)}{a + b\text{sech}(x)}\, dx = \text{Exception raised: ValueError}$$

[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{\sinh^4(x)}{a + b\text{sech}(x)}\, dx =& \frac{3\,a^3e^{(4\,x)} - 8\,a^2be^{(3\,x)} - 24\,a^3e^{(2\,x)} + 24\,ab^2e^{(2\,x)} + 120\,a^2be^x - 96\,b^3e^x}{192\,a^4} \\ &+ \frac{(3\,a^4 - 12\,a^2b^2 + 8\,b^4)x}{8\,a^5} \\ &+ \frac{\left(8\,a^3be^x - 3\,a^4 - 24\,(5\,a^3b - 4\,ab^3)e^{(3\,x)} + 24\,(a^4 - a^2b^2)e^{(2\,x)}\right)e^{(-4\,x)}}{192\,a^5} \\ &- \frac{2\,(a^4b - 2\,a^2b^3 + b^5)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2 - b^2}a^5}\end{aligned}$$

[In] integrate(sinh(x)^4/(a+b*sech(x)),x, algorithm="giac")

[Out] 1/192*(3*a^3*e^(4*x) - 8*a^2*b*e^(3*x) - 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x)
+ 120*a^2*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 - 12*a^2*b^2 + 8*b^4)*x/a^5
+ 1/192*(8*a^3*b*e^x - 3*a^4 - 24*(5*a^3*b - 4*a*b^3)*e^(3*x) + 24*(a^4 - a
^2*b^2)*e^(2*x))*e^(-4*x)/a^5 - 2*(a^4*b - 2*a^2*b^3 + b^5)*arctan((a*e^x +
b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5)

Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.08

$$\int \frac{\sinh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{4x}}{64a}-\frac{e^{-4x}}{64a}+\frac{x\,(3a^4-12a^2b^2+8b^4)}{8a^5}-\frac{e^{-x}\,(5a^2b-4b^3)}{8a^4}+\frac{e^{-2x}\,(a^2-b^2)}{8a^3}-\frac{e^{2x}\,(a^2-b^2)}{8a^3}+\frac{b\,e^{-3x}}{24a^2}-\frac{b\,e^{3x}}{24a^2}+\frac{e^{x}\,(5a^2b-4b^3)}{8a^4}+\frac{b\ln\left(\frac{2e^x\,(a^4b-2a^2b^3+b^5)}{a^6}-\frac{2b\,(a+b)^{3/2}\,(a+b\,e^x)\,(b-a)^{3/2}}{a^6}\right)(a+b)^{3/2}\,(b-a)^{3/2}}{a^5}-\frac{b\ln\left(\frac{2e^x\,(a^4b-2a^2b^3+b^5)}{a^6}+\frac{2b\,(a+b)^{3/2}\,(a+b\,e^x)\,(b-a)^{3/2}}{a^6}\right)(a+b)^{3/2}\,(b-a)^{3/2}}{a^5}$$

```
[In] int(sinh(x)^4/(a + b/cosh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 - 12*a^2*b^2))/(8*a^
5) - (exp(-x)*(5*a^2*b - 4*b^3))/(8*a^4) + (exp(-2*x)*(a^2 - b^2))/(8*a^3)
- (exp(2*x)*(a^2 - b^2))/(8*a^3) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(2
4*a^2) + (exp(x)*(5*a^2*b - 4*b^3))/(8*a^4) + (b*log((2*exp(x)*(a^4*b + b^5
 - 2*a^2*b^3))/a^6 - (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/a^6)*
(a + b)^(3/2)*(b - a)^(3/2))/a^5 - (b*log((2*exp(x)*(a^4*b + b^5 - 2*a^2*b^
3))/a^6 + (2*b*(a + b)^(3/2)*(a + b*exp(x))*(b - a)^(3/2))/a^6)*(a + b)^(3/
2)*(b - a)^(3/2))/a^5
```

3.61 $\int \frac{\sinh^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\sinh^3(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{(a^2-b^2)\cosh(x)}{a^3} - \frac{b\cosh^2(x)}{2a^2} + \frac{\cosh^3(x)}{3a} + \frac{b(a^2-b^2)\log(b+a\cosh(x))}{a^4}$$

```
[Out] -(a^2-b^2)*cosh(x)/a^3-1/2*b*cosh(x)^2/a^2+1/3*cosh(x)^3/a+b*(a^2-b^2)*ln(b
+a*cosh(x))/a^4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3957, 2916, 12, 786}

$$\int \frac{\sinh^3(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{b\cosh^2(x)}{2a^2} + \frac{b(a^2-b^2)\log(a\cosh(x)+b)}{a^4} - \frac{(a^2-b^2)\cosh(x)}{a^3} + \frac{\cosh^3(x)}{3a}$$

```
[In] Int[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -(((a^2 - b^2)*Cosh[x])/a^3) - (b*Cosh[x]^2)/(2*a^2) + Cosh[x]^3/(3*a) + (b
*(a^2 - b^2)*Log[b + a*Cosh[x]])/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cosh(x)\sinh^3(x)}{-b-a\cosh(x)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a^3}\\
&= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)\,dx, x, -a\cosh(x)\right)}{a^4}\\
&= -\frac{(a^2-b^2)\cosh(x)}{a^3}-\frac{b\cosh^2(x)}{2a^2}+\frac{\cosh^3(x)}{3a}+\frac{b(a^2-b^2)\log(b+a\cosh(x))}{a^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)} dx$$
$$= \frac{(-9a^3 + 12ab^2)\cosh(x) - 3a^2b\cosh(2x) + a^3\cosh(3x) + 12a^2b\log(b + a\cosh(x)) - 12b^3\log(b + a\cosh(x))}{12a^4}$$

```
[In] Integrate[Sinh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cosh[x] - 3*a^2*b*Cosh[2*x] + a^3*Cosh[3*x] + 12*a^2*b
*Log[b + a*Cosh[x]] - 12*b^3*Log[b + a*Cosh[x]])/(12*a^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(57) = 114.

Time = 10.41 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.23

method	result
risch	$-\frac{bx}{a^2} + \frac{b^3x}{a^4} + \frac{e^{3x}}{24a} - \frac{b\,e^{2x}}{8a^2} - \frac{3\,e^{x}}{8a} + \frac{e^{x}b^2}{2a^3} - \frac{3\,e^{-x}}{8a} + \frac{e^{-x}b^2}{2a^3} - \frac{b\,e^{-2x}}{8a^2} + \frac{e^{-3x}}{24a} + \frac{b\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a^2} - \frac{b^3\ln\left(e^{2x}+2\right.}{a^4}$
default	$-\frac{a^2-ab-2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)+1\right)} - \frac{b(a^2-b^2)\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^4} - \frac{a+b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} + \frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)+1\right)^3} + \frac{b(a^3-a^2b-a\,b^2+b^3)\ln\left(\tanh\left(\frac{x}{2}\right.\right.}{a^4(a-b)}$

```
[In] int(sinh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b*x/a^2+1/a^4*b^3*x+1/24/a*exp(3*x)-1/8/a^2*b*exp(2*x)-3/8/a*exp(x)+1/2/a^
3*exp(x)*b^2-3/8/a*exp(-x)+1/2/a^3*exp(-x)*b^2-1/8/a^2*b*exp(-2*x)+1/24/a*e
xp(-3*x)+1/a^2*b*ln(exp(2*x)+2*b/a*exp(x)+1)-1/a^4*b^3*ln(exp(2*x)+2*b/a*ex
p(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. 2(57) = 114.

Time = 0.26 (sec) , antiderivative size = 490, normalized size of antiderivative = 8.03

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)} dx$$
$$= \frac{a^3\cosh(x)^6 + a^3\sinh(x)^6 - 3a^2b\cosh(x)^5 + 3(2a^3\cosh(x) - a^2b)\sinh(x)^5 - 24(a^2b - b^3)x\cosh(x)^3 - }{}$$

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(a^3*cosh(x)^6 + a^3*sinh(x)^6 - 3*a^2*b*cosh(x)^5 + 3*(2*a^3*cosh(x)
- a^2*b)*sinh(x)^5 - 24*(a^2*b - b^3)*x*cosh(x)^3 - 3*(3*a^3 - 4*a*b^2)*cos
h(x)^4 + 3*(5*a^3*cosh(x)^2 - 5*a^2*b*cosh(x) - 3*a^3 + 4*a*b^2)*sinh(x)^4
- 3*a^2*b*cosh(x) + 2*(10*a^3*cosh(x)^3 - 15*a^2*b*cosh(x)^2 - 12*(a^2*b -
b^3)*x - 6*(3*a^3 - 4*a*b^2)*cosh(x))*sinh(x)^3 + a^3 - 3*(3*a^3 - 4*a*b^2)
*cosh(x)^2 + 3*(5*a^3*cosh(x)^4 - 10*a^2*b*cosh(x)^3 - 3*a^3 + 4*a*b^2 - 24
*(a^2*b - b^3)*x*cosh(x) - 6*(3*a^3 - 4*a*b^2)*cosh(x)^2)*sinh(x)^2 + 24*((
a^2*b - b^3)*cosh(x)^3 + 3*(a^2*b - b^3)*cosh(x)^2*sinh(x) + 3*(a^2*b - b^3
)*cosh(x)*sinh(x)^2 + (a^2*b - b^3)*sinh(x)^3)*log(2*(a*cosh(x) + b)/(cosh(
x) - sinh(x))) + 3*(2*a^3*cosh(x)^5 - 5*a^2*b*cosh(x)^4 - 24*(a^2*b - b^3)*
x*cosh(x)^2 - 4*(3*a^3 - 4*a*b^2)*cosh(x)^3 - a^2*b - 2*(3*a^3 - 4*a*b^2)*c
osh(x))*sinh(x))/(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*s
inh(x)^2 + a^4*sinh(x)^3)
```

Sympy [F]

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\sinh^3{(x)}}{a + b\,\text{sech}\,(x)}\,dx$$

```
[In] integrate(sinh(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**3/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(57) = 114$.

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.10

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = -\frac{\left(3\,abe^{(-x)} - a^2 + 3\,(3\,a^2 - 4\,b^2)e^{(-2\,x)}\right)e^{(3\,x)}}{24\,a^3} - \frac{3\,abe^{(-2\,x)} - a^2e^{(-3\,x)} + 3\,(3\,a^2 - 4\,b^2)e^{(-x)}}{24\,a^3} + \frac{(a^2b - b^3)x}{a^4} + \frac{(a^2b - b^3)\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^4}$$

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-x) - a^2 + 3*(3*a^2 - 4*b^2)*e^(-2*x))*e^(3*x)/a^3 - 1/24*
(3*a*b*e^(-2*x) - a^2*e^(-3*x) + 3*(3*a^2 - 4*b^2)*e^(-x))/a^3 + (a^2*b - b
^3)*x/a^4 + (a^2*b - b^3)*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.43

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = \frac{a^2\left(e^{(-x)} + e^x\right)^3 - 3\,ab\left(e^{(-x)} + e^x\right)^2 - 12\,a^2\left(e^{(-x)} + e^x\right) + 12\,b^2\left(e^{(-x)} + e^x\right)}{24\,a^3} + \frac{(a^2b - b^3)\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^4}$$

```
[In] integrate(sinh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/24*(a^2*(e^(-x) + e^x)^3 - 3*a*b*(e^(-x) + e^x)^2 - 12*a^2*(e^(-x) + e^x)
 + 12*b^2*(e^(-x) + e^x))/a^3 + (a^2*b - b^3)*log(abs(a*(e^(-x) + e^x) + 2*
b))/a^4
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.02

$$\int \frac{\sinh^3(x)}{a + b\text{sech}(x)}\,dx = \frac{e^{-3\,x}}{24\,a} + \frac{e^{3\,x}}{24\,a} - \frac{x\,(a^2\,b - b^3)}{a^4} - \frac{e^x\,(3\,a^2 - 4\,b^2)}{8\,a^3} - \frac{b\,e^{-2\,x}}{8\,a^2} - \frac{b\,e^{2\,x}}{8\,a^2} + \frac{\ln\left(a + 2\,b\,e^x + a\,e^{2\,x}\right)\,(a^2\,b - b^3)}{a^4} - \frac{e^{-x}\,(3\,a^2 - 4\,b^2)}{8\,a^3}$$

```
[In] int(sinh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] exp(-3*x)/(24*a) + exp(3*x)/(24*a) - (x*(a^2*b - b^3))/a^4 - (exp(x)*(3*a^2
 - 4*b^2))/(8*a^3) - (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) + (log(a
+ 2*b*exp(x) + a*exp(2*x))*(a^2*b - b^3))/a^4 - (exp(-x)*(3*a^2 - 4*b^2))/(
8*a^3)
```

3.62 $\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx = -\frac{(a^2-2b^2)\,x}{2a^3}+\frac{2\sqrt{a-b}b\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}$$

```
[Out] -1/2*(a^2-2*b^2)*x/a^3-1/2*(2*b-a*cosh(x))*sinh(x)/a^2+2*b*arctan((a-b)^(1/
2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2944, 2814, 2738, 211}

$$\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{2b\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3}-\frac{\sinh(x)(2b-a\cosh(x))}{2a^2}-\frac{x(a^2-2b^2)}{2a^3}$$

```
[In] Int[Sinh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] -1/2*((a^2 - 2*b^2)*x)/a^3 + (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTan[(Sqrt[a -
b]*Tanh[x/2])/Sqrt[a + b]])/a^3 - ((2*b - a*Cosh[x])*Sinh[x])/(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2944

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Dist[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1))), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cosh(x)\sinh^2(x)}{-b-a\cosh(x)}\,dx \\
&= -\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}+\frac{\int \frac{-ab+\left(a^2-2b^2\right)\cosh(x)}{-b-a\cosh(x)}\,dx}{2a^2} \\
&= -\frac{\left(a^2-2b^2\right)x}{2a^3}-\frac{(2b-a\cosh(x))\sinh(x)}{2a^2}-\frac{\left(b\left(a^2-b^2\right)\right)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{(a^2-2b^2)\,x}{2a^3} - \frac{(2b-a\cosh(x))\sinh(x)}{2a^2} \\
&\quad - \frac{(2b(a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3} \\
&= -\frac{(a^2-2b^2)\,x}{2a^3} + \frac{2\sqrt{a-b}b\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3} - \frac{(2b-a\cosh(x))\sinh(x)}{2a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$
\int \frac{\sinh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{-2a^2x+4b^2x-8b\sqrt{a^2-b^2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)-4ab\sinh(x)+a^2\sinh(2x)}{4a^3}
$$

```
[In] Integrate[Sinh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (-2*a^2*x + 4*b^2*x - 8*b*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[
a^2 - b^2]] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{x}{2a}+\frac{x\,b^2}{a^3}+\frac{e^{2x}}{8a}-\frac{b\,e^{x}}{2a^2}+\frac{b\,e^{-x}}{2a^2}-\frac{e^{-2x}}{8a}+\frac{\sqrt{-a^2+b^2}\,b\ln\left(e^{x}+\frac{b+\sqrt{-a^2+b^2}}{a}\right)}{a^3}-\frac{\sqrt{-a^2+b^2}\,b\ln\left(e^{x}-\frac{\sqrt{-a^2+b^2}-b}{a}\right)}{a^3}$
default	$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2}+\frac{(a^2-2b^2)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^3}-\frac{-a-2b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}-\frac{-a-2b}{2a^2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{(-a^2+2b^2)\ln(}{2a}$

```
[In] int(sinh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/a+x/a^3*b^2+1/8/a*exp(x)^2-1/2*b/a^2*exp(x)+1/2*b/a^2/exp(x)-1/8/a/e
xp(x)^2+(-a^2+b^2)^(1/2)*b/a^3*ln(exp(x)+(b+(-a^2+b^2)^(1/2))/a)-(-a^2+b^2)
^(1/2)*b/a^3*ln(exp(x)-((-a^2+b^2)^(1/2)-b)/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(67) = 134$.

Time = 0.28 (sec) , antiderivative size = 536, normalized size of antiderivative = 6.54

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)}\,dx$$
$$= \left[\frac{a^2\cosh(x)^4 + a^2\sinh(x)^4 - 4\,ab\cosh(x)^3 - 4\,(a^2 - 2\,b^2)x\cosh(x)^2 + 4\,(a^2\cosh(x) - ab)\sinh(x)^3 + 4\right.$$

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*c
osh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(
x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2)*x)*sinh(x)^2 + 8*(b*cosh(x)^2 + 2*b*
cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*si
nh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*s
qrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2
*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - a^2 + 4*(a^2*cosh(x)^3 - 3*a
*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^2 + 2
*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2), 1/8*(a^2*cosh(x)^4 + a^2*sinh(x)^4 -
 4*a*b*cosh(x)^3 - 4*(a^2 - 2*b^2)*x*cosh(x)^2 + 4*(a^2*cosh(x) - a*b)*sinh
(x)^3 + 4*a*b*cosh(x) + 2*(3*a^2*cosh(x)^2 - 6*a*b*cosh(x) - 2*(a^2 - 2*b^2
)*x)*sinh(x)^2 - 16*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)*sqrt(
a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - a^2 + 4*(
a^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 2*(a^2 - 2*b^2)*x*cosh(x) + a*b)*sinh(x))
/(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\sinh^2(x)}{a + b\,\text{sech}\,(x)}\,dx$$

```
[In] integrate(sinh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sinh(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)}\,dx = \frac{ae^{(2x)} - 4\,be^x}{8\,a^2} - \frac{(a^2 - 2\,b^2)x}{2\,a^3} + \frac{(4\,abe^x - a^2)e^{(-2\,x)}}{8\,a^3} + \frac{2\,(a^2b - b^3)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2 - b^2}a^3}$$

```
[In] integrate(sinh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/8*(a*e^(2*x) - 4*b*e^x)/a^2 - 1/2*(a^2 - 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x -
a^2)*e^(-2*x)/a^3 + 2*(a^2*b - b^3)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sq
rt(a^2 - b^2)*a^3)
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \frac{\sinh^2(x)}{a + b\text{sech}(x)}\,dx = \frac{e^{2x}}{8\,a} - \frac{e^{-2x}}{8\,a} - \frac{b\,e^x}{2\,a^2} + \frac{b\,e^{-x}}{2\,a^2} - \frac{x\,(a^2 - 2\,b^2)}{2\,a^3} + \frac{b\,\ln\left(-\frac{2\,b\,e^x\,(a^2-b^2)}{a^4} - \frac{2\,b\,\sqrt{a+b}\,(a+b\,e^x)\,\sqrt{b-a}}{a^4}\right)\sqrt{a+b}\,\sqrt{b-a}}{a^3} - \frac{b\,\ln\left(\frac{2\,b\,\sqrt{a+b}\,(a+b\,e^x)\,\sqrt{b-a}}{a^4} - \frac{2\,b\,e^x\,(a^2-b^2)}{a^4}\right)\sqrt{a+b}\,\sqrt{b-a}}{a^3}$$

```
[In] int(sinh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2)
 - (x*(a^2 - 2*b^2))/(2*a^3) + (b*log(- (2*b*exp(x)*(a^2 - b^2))/a^4 - (2*b
*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4)*(a + b)^(1/2)*(b - a)^(1/
2))/a^3 - (b*log((2*b*(a + b)^(1/2)*(a + b*exp(x))*(b - a)^(1/2))/a^4 - (2*
b*exp(x)*(a^2 - b^2))/a^4)*(a + b)^(1/2)*(b - a)^(1/2))/a^3
```

3.63 $\int \frac{\sinh(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx=\frac{\cosh(x)}{a}-\frac{b\log(b+a\cosh(x))}{a^2}$$

```
[Out] cosh(x)/a-b*ln(b+a*cosh(x))/a^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3957, 2912, 12, 45}

$$\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx=\frac{\cosh(x)}{a}-\frac{b\log(a\cosh(x)+b)}{a^2}$$

```
[In] Int[Sinh[x]/(a + b*Sech[x]),x]
```

```
[Out] Cosh[x]/a - (b*Log[b + a*Cosh[x]])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\cosh(x)\sinh(x)}{-b-a\cosh(x)}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cosh(x)\right)}{a} \\
&= -\frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cosh(x)\right)}{a^2} \\
&= -\frac{\text{Subst}\left(\int \left(1-\frac{b}{b-x}\right)\,dx, x, -a\cosh(x)\right)}{a^2} \\
&= \frac{\cosh(x)}{a} - \frac{b\log(b+a\cosh(x))}{a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$
\int \frac{\sinh(x)}{a+b\text{sech}(x)}\,dx = \frac{a\cosh(x)-b\log(b+a\cosh(x))}{a^2}
$$

```
[In] Integrate[Sinh[x]/(a + b*Sech[x]),x]
[Out] (a*Cosh[x] - b*Log[b + a*Cosh[x]])/a^2
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$\frac{1}{a\,\mathrm{sech}(x)}+\frac{b\ln(\mathrm{sech}(x))}{a^2}-\frac{b\ln(a+b\,\mathrm{sech}(x))}{a^2}$	31
default	$\frac{1}{a\,\mathrm{sech}(x)}+\frac{b\ln(\mathrm{sech}(x))}{a^2}-\frac{b\ln(a+b\,\mathrm{sech}(x))}{a^2}$	31
risch	$\frac{bx}{a^2}+\frac{\mathrm{e}^{x}}{2a}+\frac{\mathrm{e}^{-x}}{2a}-\frac{b\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}+1\right)}{a^2}$	45

```
[In] int(sinh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/a/sech(x)+1/a^2*b*ln(sech(x))-1/a^2*b*ln(a+b*sech(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int \frac{\sinh(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{2\,bx\cosh\left(x\right)+a\cosh\left(x\right)^2+a\sinh\left(x\right)^2-2\left(b\cosh\left(x\right)+b\sinh\left(x\right)\right)\log\left(\frac{2\left(a\cosh(x)+b\right)}{\cosh(x)-\sinh(x)}\right)+2\left(bx+a\cosh\left(x\right)\right.}{2\left(a^2\cosh\left(x\right)+a^2\sinh\left(x\right)\right)}$$

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="fricas")
[Out] 1/2*(2*b*x*cosh(x) + a*cosh(x)^2 + a*sinh(x)^2 - 2*(b*cosh(x) + b*sinh(x))*
log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + 2*(b*x + a*cosh(x))*sinh(x) +
a)/(a^2*cosh(x) + a^2*sinh(x))
```

Sympy [F]

$$\int \frac{\sinh(x)}{a+b\mathrm{sech}(x)}\,dx = \int \frac{\sinh\left(x\right)}{a+b\,\mathrm{sech}\left(x\right)}\,dx$$

```
[In] integrate(sinh(x)/(a+b*sech(x)),x)
[Out] Integral(sinh(x)/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(20) = 40$.

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{\sinh(x)}{a + b\operatorname{sech}(x)}\,dx = -\frac{bx}{a^2} + \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a} - \frac{b\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^2}$$

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b*x/a^2 + 1/2*e^(-x)/a + 1/2*e^x/a - b*log(2*b*e^(-x) + a*e^(-2*x) + a)/a^
2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sinh(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{e^{(-x)} + e^x}{2\,a} - \frac{b\log\left(\left|a\left(e^{(-x)} + e^x\right) + 2\,b\right|\right)}{a^2}$$

```
[In] integrate(sinh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)/a - b*log(abs(a*(e^(-x) + e^x) + 2*b))/a^2
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{\cosh(x)}{a} - \frac{b\,\ln\left(b + a\cosh(x)\right)}{a^2}$$

```
[In] int(sinh(x)/(a + b/cosh(x)),x)
```

```
[Out] cosh(x)/a - (b*log(b + a*cosh(x)))/a^2
```

3.64 $\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(b+a\cosh(x))}{a^2-b^2}$$

```
[Out] 1/2*ln(1-cosh(x))/(a+b)-1/2*ln(1+cosh(x))/(a-b)+b*ln(b+a*cosh(x))/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3957, 2800, 815}

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{b\log(a\cosh(x)+b)}{a^2-b^2}+\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(\cosh(x)+1)}{2(a-b)}$$

```
[In] Int[Csch[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[1 - Cosh[x]]/(2*(a + b)) - Log[1 + Cosh[x]]/(2*(a - b)) + (b*Log[b + a*
Cosh[x]])/(a^2 - b^2)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\coth(x)}{-b-a\cosh(x)}\,dx \\
&= \text{Subst}\left(\int \frac{x}{(-b+x)\,(a^2-x^2)}\,dx, x, -a\cosh(x)\right) \\
&= \text{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)} - \frac{b}{(a-b)(a+b)(b-x)} + \frac{1}{2(a+b)(a+x)}\right)\,dx, x, -a\cosh(x)\right) \\
&= \frac{\log(1-\cosh(x))}{2(a+b)} - \frac{\log(1+\cosh(x))}{2(a-b)} + \frac{b\log(b+a\cosh(x))}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$
\int \frac{\text{csch}(x)}{a+b\text{sech}(x)}\,dx = \frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{-a+b} - \frac{b\log(b+a\cosh(x))}{-a^2+b^2} + \frac{\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a+b}
$$

```
[In] Integrate[Csch[x]/(a + b*Sech[x]),x]
[Out] Log[Cosh[x/2]]/(-a + b) - (b*Log[b + a*Cosh[x]])/(-a^2 + b^2) + Log[Sinh[x/
2]]/(a + b)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b+a+b\right)}{(a+b)(a-b)}$	48
risch	$-\frac{x}{a+b}+\frac{x}{a-b}-\frac{2xb}{a^{2}-b^{2}}+\frac{\ln(e^{x}-1)}{a+b}-\frac{\ln(e^{x}+1)}{a-b}+\frac{b\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a^{2}-b^{2}}$	87

```
[In] int(csch(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a+b)*ln(tanh(1/2*x))+b/(a+b)/(a-b)*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+
b)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{b\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] (b*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh(
x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \int \frac{\operatorname{csch}\left(x\right)}{a+b\operatorname{sech}\left(x\right)}\,dx$$

```
[In] integrate(csch(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{b\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^2-b^2}-\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
log(e^(-x) - 1)/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{ab\log\left(\left|a\left(e^{(-x)}+e^{x}\right)+2\,b\right|\right)}{a^{3}-ab^{2}} - \frac{\log\left(e^{(-x)}+e^{x}+2\right)}{2\,(a-b)} + \frac{\log\left(e^{(-x)}+e^{x}-2\right)}{2\,(a+b)}$$

```
[In] integrate(csch(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) - 1/2*log(e^(-x) + e^x +
 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.79

$$\int \frac{\operatorname{csch}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\ln\left(128\,a\,b-32\,a^2-128\,b^2+32\,a^2\,\mathrm{e}^{x}+128\,b^2\,\mathrm{e}^{x}-128\,a\,b\,\mathrm{e}^{x}\right)}{a+b} - \frac{\ln\left(-128\,a\,b-32\,a^2-128\,b^2-32\,a^2\,\mathrm{e}^{x}-128\,b^2\,\mathrm{e}^{x}-128\,a\,b\,\mathrm{e}^{x}\right)}{a-b} + \frac{b\,\ln\left(16\,a\,b^2-4\,a^3\,\mathrm{e}^{2\,x}-4\,a^3+32\,b^3\,\mathrm{e}^{x}-8\,a^2\,b\,\mathrm{e}^{x}+16\,a\,b^2\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

```
[In] int(1/(sinh(x)*(a + b/cosh(x))),x)
```

```
[Out] log(128*a*b - 32*a^2 - 128*b^2 + 32*a^2*exp(x) + 128*b^2*exp(x) - 128*a*b*e
xp(x))/(a + b) - log(- 128*a*b - 32*a^2 - 128*b^2 - 32*a^2*exp(x) - 128*b^2
*exp(x) - 128*a*b*exp(x))/(a - b) + (b*log(16*a*b^2 - 4*a^3*exp(2*x) - 4*a^
3 + 32*b^3*exp(x) - 8*a^2*b*exp(x) + 16*a*b^2*exp(2*x)))/(a^2 - b^2)
```

3.65 $\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{2ab\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}$$

```
[Out] 2*a*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)+(
b-a*cosh(x))*csch(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2945, 12, 2738, 211}

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{2ab\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}$$

```
[In] Int[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (2*a*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(3/2)*(a + b)^
(3/2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\coth(x)\operatorname{csch}(x)}{-b-a\cosh(x)}\,dx \\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2} - \frac{\int \frac{ab}{-b-a\cosh(x)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2} - \frac{(ab)\int \frac{1}{-b-a\cosh(x)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2} - \frac{(2ab)\operatorname{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2-b^2} \\
&= \frac{2ab\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} + \frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{1}{2}\left(-\frac{4ab\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{a+b}+\frac{\tanh\left(\frac{x}{2}\right)}{-a+b}\right)$$

```
[In] Integrate[Csch[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((-4*a*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) -
Coth[x/2]/(a + b) + Tanh[x/2]/(-a + b))/2
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}+\frac{2ab\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}$	77
risch	$-\frac{2(-\mathrm{e}^{x}b+a)}{(\mathrm{e}^{2x}-1)(a^2-b^2)}-\frac{ba\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}+\frac{ba\ln\left(\mathrm{e}^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)}$	165

```
[In] int(csch(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a-b)*tanh(1/2*x)+2/(a+b)/(a-b)*a*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*t
anh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(58) = 116.

Time = 0.26 (sec) , antiderivative size = 452, normalized size of antiderivative = 6.85

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \left[\frac{2a^3-2ab^2-\left(ab\cosh(x)^2+2ab\cosh(x)\sinh(x)+ab\sinh(x)^2-ab\right)\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)}{\ldots}\right.}{a^4-2a^2b^2+b^4-\left(a^4-2a^2b^2+b^4\right)\cosh(x)^2-2\left(a^4\ldots\right.}\right.$$

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3 - 2*a*b^2 - (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2
- a*b)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
```

```
- a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(
x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x
) + b)*sinh(x) + a)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x))/(
a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^3 - a
*b^2 + (a*b*cosh(x)^2 + 2*a*b*cosh(x)*sinh(x) + a*b*sinh(x)^2 - a*b)*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) - (a^2*b - b
^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*
b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2
*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx=\int \frac{\operatorname{csch}^2{(x)}}{a+b\operatorname{sech}{(x)}}\,dx$$

```
[In] integrate(csch(x)**2/(a+b*sech(x)),x)
[Out] Integral(csch(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{2\,ab\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{\frac{3}{2}}}+\frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

```
[In] integrate(csch(x)^2/(a+b*sech(x)),x, algorithm="giac")
[Out] 2*a*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(a^2 - b^2)^(3/2) + 2*(b*e^x - a)
/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.29

$$\int \frac{\operatorname{csch}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{a\,b\,\ln\left(-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}-\frac{2\,b\,\left(a+b\,\mathrm{e}^x\right)}{{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}\right)}{{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}}{\mathrm{e}^{2\,x}-1}-\frac{a\,b\,\ln\left(\frac{2\,b\,\left(a+b\,\mathrm{e}^x\right)}{{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}\right)}{{\left(a+b\right)}^{3/2}\,{\left(b-a\right)}^{3/2}}$$

```
[In] int(1/(sinh(x)^2*(a + b/cosh(x))),x)
```

```
[Out] (a*b*log(- (2*b*exp(x))/(a^2 - b^2) - (2*b*(a + b*exp(x)))/((a + b)^(3/2)*(
b - a)^(3/2))))/((a + b)^(3/2)*(b - a)^(3/2)) - ((2*a)/(a^2 - b^2) - (2*b*e
xp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (a*b*log((2*b*(a + b*exp(x)))/((a + b)
^(3/2)*(b - a)^(3/2)) - (2*b*exp(x))/(a^2 - b^2)))/((a + b)^(3/2)*(b - a)^(
3/2))
```

3.66 $\int \frac{\mathbf{csch}^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{(b-a\cosh(x))\operatorname{csch}^2(x)}{2(a^2-b^2)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(1+\cosh(x))}{4(a-b)^2}-\frac{a^2b\log(b+a\cosh(x))}{(a^2-b^2)^2}$$

```
[Out] 1/2*(b-a*cosh(x))*csch(x)^2/(a^2-b^2)-1/4*a*ln(1-cosh(x))/(a+b)^2+1/4*a*ln(
1+cosh(x))/(a-b)^2-a^2*b*ln(b+a*cosh(x))/(a^2-b^2)^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2916, 12, 837, 815}

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx=-\frac{a^2b\log(a\cosh(x)+b)}{(a^2-b^2)^2}+\frac{\operatorname{csch}^2(x)(b-a\cosh(x))}{2(a^2-b^2)}-\frac{a\log(1-\cosh(x))}{4(a+b)^2}+\frac{a\log(\cosh(x)+1)}{4(a-b)^2}$$

```
[In] Int[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((b - a*Cosh[x])*Csch[x]^2)/(2*(a^2 - b^2)) - (a*Log[1 - Cosh[x]])/(4*(a +
b)^2) + (a*Log[1 + Cosh[x]])/(4*(a - b)^2) - (a^2*b*Log[b + a*Cosh[x]])/(a^
2 - b^2)^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \frac{\coth(x)\text{csch}^2(x)}{-b-a\cosh(x)}\,dx \\
&= -\left(a^3\text{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cosh(x)\right)\right) \\
&= -\left(a^2\text{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cosh(x)\right)\right) \\
&= \frac{(b-a\cosh(x))\text{csch}^2(x)}{2\left(a^2-b^2\right)} - \frac{\text{Subst}\left(\int \frac{a^2b+a^2x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cosh(x)\right)}{2\left(a^2-b^2\right)}
\end{aligned}
$$

$$= \frac{(b-a\cosh(x))\text{csch}^2(x)}{2(a^2-b^2)} - \frac{\text{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)(a-x)} - \frac{2a^2b}{(a-b)(a+b)(b-x)} + \frac{a(a-b)}{2(a+b)(a+x)}\right) dx, x, -a\cosh(x)\right)}{2(a^2-b^2)}$$

$$= \frac{(b-a\cosh(x))\text{csch}^2(x)}{2(a^2-b^2)} - \frac{a\log(1-\cosh(x))}{4(a+b)^2} + \frac{a\log(1+\cosh(x))}{4(a-b)^2} - \frac{a^2b\log(b+a\cosh(x))}{(a^2-b^2)^2}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{\text{csch}^3(x)}{a+b\text{sech}(x)}\,dx = \frac{1}{8}\left(-\frac{\text{csch}^2\left(\frac{x}{2}\right)}{a+b} + \frac{4a\left((a+b)^2\log\left(\cosh\left(\frac{x}{2}\right)\right) - 2ab\log(b+a\cosh(x)) - (a-b)^2\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{(a-b)^2(a+b)^2} - \frac{\text{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

```
[In] Integrate[Csch[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-(Csch[x/2]^2/(a + b)) + (4*a*((a + b)^2*Log[Cosh[x/2]] - 2*a*b*Log[b + a*
Cosh[x]] - (a - b)^2*Log[Sinh[x/2]]))/((a - b)^2*(a + b)^2) - Sech[x/2]^2/(
a - b))/8
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)^2}{8a-8b} - \frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^2} - \frac{a\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2(a+b)^2} - \frac{a^2b\ln\left(\tanh\left(\frac{x}{2}\right)^2 a - \tanh\left(\frac{x}{2}\right)^2 b + a + b\right)}{(a+b)^2(a-b)^2}$
risch	$\frac{ax}{2a^2+4ab+2b^2} - \frac{xa}{2(a^2-2ab+b^2)} + \frac{2a^2bx}{a^4-2a^2b^2+b^4} - \frac{e^x\left(a\,e^{2x}-2\,e^x b+a\right)}{\left(e^{2x}-1\right)^2(a^2-b^2)} - \frac{a\ln(e^x-1)}{2(a^2+2ab+b^2)} + \frac{a\ln(e^x+1)}{2a^2-4ab+2b^2} - \frac{a^2b\ln\left(e^{2x}+\right.}{a^4-2a^2b}$

```
[In] int(csch(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*tanh(1/2*x)^2/(a-b)-1/8/(a+b)/tanh(1/2*x)^2-1/2*a/(a+b)^2*ln(tanh(1/2*x
))-a^2*b/(a+b)^2/(a-b)^2*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(80) = 160$.

Time = 0.28 (sec) , antiderivative size = 828, normalized size of antiderivative = 9.74

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\, dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b^
3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 + 2*
(a^3 - a*b^2)*cosh(x) + 2*(a^2*b*cosh(x)^4 + 4*a^2*b*cosh(x)*sinh(x)^3 + a^
2*b*sinh(x)^4 - 2*a^2*b*cosh(x)^2 + a^2*b + 2*(3*a^2*b*cosh(x)^2 - a^2*b)*s
inh(x)^2 + 4*(a^2*b*cosh(x)^3 - a^2*b*cosh(x))*sinh(x))*log(2*(a*cosh(x) +
b)/(cosh(x) - sinh(x))) - ((a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 + 2*a
^2*b + a*b^2)*cosh(x)*sinh(x)^3 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 +
 2*a^2*b + a*b^2 - 2*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 + 2*a^2*b +
 a*b^2 - 3*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 2*a^2*b
 + a*b^2)*cosh(x)^3 - (a^3 + 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x)
 + sinh(x) + 1) + ((a^3 - 2*a^2*b + a*b^2)*cosh(x)^4 + 4*(a^3 - 2*a^2*b + a
*b^2)*cosh(x)*sinh(x)^3 + (a^3 - 2*a^2*b + a*b^2)*sinh(x)^4 + a^3 - 2*a^2*b
 + a*b^2 - 2*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2 - 2*(a^3 - 2*a^2*b + a*b^2 -
 3*(a^3 - 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 2*a^2*b + a*b^2
)*cosh(x)^3 - (a^3 - 2*a^2*b + a*b^2)*cosh(x))*sinh(x))*log(cosh(x) + sinh(
x) - 1) + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh
(x))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4
)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
 b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4)*co
sh(x)^3 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\, dx = \int \frac{\operatorname{csch}^3(x)}{a + b\operatorname{sech}(x)}\, dx$$

```
[In] integrate(csch(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(csch(x)**3/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{a^2 b \log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^4-2\,a^2b^2+b^4}+\frac{a\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)} - \frac{a\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)} - \frac{ae^{(-x)}-2\,be^{(-2\,x)}+ae^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}}$$

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*a*log
(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*a*log(e^(-x) - 1)/(a^2 + 2*a*b + b^2
) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(a^2 - b^2)*e^(-2
*x) + (a^2 - b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.05

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{a^3 b\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4} + \frac{a\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)} - \frac{a\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)} - \frac{a^2b\left(e^{(-x)}+e^x\right)^2+2\,a^3\left(e^{(-x)}+e^x\right)-2\,ab^2\left(e^{(-x)}+e^x\right)-8\,a^2b+4\,b^3}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

```
[In] integrate(csch(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*a*l
og(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*a*log(e^(-x) + e^x - 2)/(a^2
 + 2*a*b + b^2) - 1/2*(a^2*b*(e^(-x) + e^x)^2 + 2*a^3*(e^(-x) + e^x) - 2*a*
b^2*(e^(-x) + e^x) - 8*a^2*b + 4*b^3)/((a^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e
^x)^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{csch}^3(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$=\frac{\frac{2\left(a^2b-b^3\right)}{\left(a^2-b^2\right)^2}+\frac{e^x\left(ab^2-a^3\right)}{\left(a^2-b^2\right)^2}}{e^{2x}-1}+\frac{\frac{2b}{a^2-b^2}-\frac{2a\,e^x}{a^2-b^2}}{e^{4x}-2e^{2x}+1}-\frac{a\ln\left(e^x-1\right)}{2a^2+4ab+2b^2}+\frac{a\ln\left(e^x+1\right)}{2a^2-4ab+2b^2}$$
$$-\frac{a^2b\ln\left(a^6e^{2x}+a^6+a^2b^4-14a^4b^2+a^2b^4e^{2x}-14a^4b^2e^{2x}+2ab^5e^x+2a^5be^x-28a^3b^3e^x\right)}{a^4-2a^2b^2+b^4}$$

```
[In] int(1/(sinh(x)^3*(a + b/cosh(x))),x)
```

```
[Out] ((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(e
xp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) - 2
*exp(2*x) + 1) - (a*log(exp(x) - 1))/(4*a*b + 2*a^2 + 2*b^2) + (a*log(exp(x
) + 1))/(2*a^2 - 4*a*b + 2*b^2) - (a^2*b*log(a^6*exp(2*x) + a^6 + a^2*b^4 -
 14*a^4*b^2 + a^2*b^4*exp(2*x) - 14*a^4*b^2*exp(2*x) + 2*a*b^5*exp(x) + 2*a
^5*b*exp(x) - 28*a^3*b^3*exp(x)))/(a^4 + b^4 - 2*a^2*b^2)
```

3.67 $\int \frac{\text{csch}^4(x)}{a+b\text{sech}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int \frac{\text{csch}^4(x)}{a+b\text{sech}(x)}\,dx=-\frac{2a^3b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}-\frac{(3a^2b-a(2a^2+b^2)\cosh(x))\,\text{csch}(x)}{3\left(a^2-b^2\right)^2}+\frac{(b-a\cosh(x))\text{csch}^3(x)}{3\left(a^2-b^2\right)}$$

```
[Out] -2*a^3*b*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2
)-1/3*(3*a^2*b-a*(2*a^2+b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+1/3*(b-a*cosh(x))
*csch(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3957, 2945, 12, 2738, 211}

$$\int \frac{\text{csch}^4(x)}{a+b\text{sech}(x)}\,dx=-\frac{2a^3b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}}+\frac{\text{csch}^3(x)(b-a\cosh(x))}{3\left(a^2-b^2\right)}-\frac{\text{csch}(x)\left(3a^2b-a(2a^2+b^2)\cosh(x)\right)}{3\left(a^2-b^2\right)^2}$$

```
[In] Int[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (-2*a^3*b*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/((a - b)^(5/2)*(a +
b)^(5/2)) - ((3*a^2*b - a*(2*a^2 + b^2)*Cosh[x])*Csch[x])/(3*(a^2 - b^2)^2)
 + ((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3957

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[(g*Cos[e + f*x])^p*((b + a*Sin[e + f*x])^m/Si
n[e + f*x]^m), x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\int \frac{\coth(x)\text{csch}^3(x)}{-b-a\cosh(x)}\,dx$$

$$= \frac{(b-a\cosh(x))\text{csch}^3(x)}{3(a^2-b^2)} - \frac{\int \frac{(ab-2a^2\cosh(x))\text{csch}^2(x)}{-b-a\cosh(x)}\,dx}{3(a^2-b^2)}$$

$$= -\frac{(3a^2b-a(2a^2+b^2)\cosh(x))\,\text{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\text{csch}^3(x)}{3(a^2-b^2)} + \frac{\int \frac{3a^3b}{-b-a\cosh(x)}\,dx}{3(a^2-b^2)^2}$$

$$= -\frac{(3a^2b-a(2a^2+b^2)\cosh(x))\,\text{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\text{csch}^3(x)}{3(a^2-b^2)} + \frac{(a^3b)\int \frac{1}{-b-a\cosh(x)}\,dx}{(a^2-b^2)^2}$$

$$= -\frac{(3a^2b - a(2a^2+b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b - a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)} + \frac{(2a^3b)\operatorname{Subst}\left(\int \frac{1}{-a-b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{(a^2-b^2)^2}$$

$$= -\frac{2a^3b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(3a^2b - a(2a^2+b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b - a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)}$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\,dx$$
$$= \frac{(b + a\cosh(x))\operatorname{sech}(x)\left(\frac{48a^3b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{2(4a+b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} + \frac{8a}{}\right.}{24(a + b\operatorname{sech}(x))}$$

```
[In] Integrate[Csch[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((b + a*Cosh[x])*Sech[x]*((48*a^3*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 -
b^2]])/(a^2 - b^2)^(5/2) + (2*(4*a + b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3
*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])
/(a - b)^2 - (2*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[x]))
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.14

method	result
default	$-\frac{\frac{a\tanh\left(\frac{x}{2}\right)^3}{3} - \frac{b\tanh\left(\frac{x}{2}\right)^3}{3} - 3a\tanh\left(\frac{x}{2}\right) + b\tanh\left(\frac{x}{2}\right)}{8(a-b)^2} - \frac{2a^3b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a+b)(a-b)}} - \frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^3} - \frac{-3a-b}{8(a+b)^2\tanh\left(\frac{x}{2}\right)}$
risch	$-\frac{2(3a^2b\,e^{5x} - 3a\,b^2e^{4x} - 10a^2b\,e^{3x} + 4b^3e^{3x} + 6a^3e^{2x} + 3a^2b\,e^x - 2a^3 - a\,b^2)}{3(e^{2x}-1)^3(a^2-b^2)^2} - \frac{b\,a^3\ln\left(e^x + \frac{b\sqrt{-a^2+b^2} + a^2 - b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)^2(a-b)^2} + \frac{b\,a^3\ln\left(e^x + \frac{b\sqrt{-}}{}\right)}{\sqrt{-a^2+b^2}\,(a}$

```
[In] int(csch(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3-3*a*tanh(1/2*x)+b*tan
h(1/2*x))-2/(a-b)^2/(a+b)^2*a^3*b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2
*x)/((a+b)*(a-b))^(1/2))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-b)/tanh
(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1129 vs. $2(98) = 196$.

Time = 0.27 (sec) , antiderivative size = 2340, normalized size of antiderivative = 21.08

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(6*(a^4*b - a^2*b^3)*cosh(x)^5 + 6*(a^4*b - a^2*b^3)*sinh(x)^5 - 4*a^
5 + 2*a^3*b^2 + 2*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^
4 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^3 - 4*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)
^2 + 6*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2
+ 12*(a^5 - a^3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*c
osh(x)^2 - (5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 + 3*(a^3*b*cosh
(x)^6 + 6*a^3*b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3
*a^3*b*cosh(x)^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a
^3*b*cosh(x)^3 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*
b*cosh(x)^2 + a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a
^3*b*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2
+ 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2
 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(
x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 6*(a^4*b - a^2*b^3)*cosh(x) + 6*(a^4
*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^
3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*
sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*
a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*
sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 -
 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)), -2/3*(3
*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 - 2*a^5 + a^3*
```

```
b^2 + a*b^4 - 3*(a^3*b^2 - a*b^4)*cosh(x)^4 - 3*(a^3*b^2 - a*b^4 - 5*(a^4*b
 - a^2*b^3)*cosh(x))*sinh(x)^4 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x)^3
- 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 + 6*(a^3*
b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 6*(a^5 - a^3*b^2)*cosh(x)^2 + 6*(a^5 - a^
3*b^2 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 - 3*(a^3*b^2 - a*b^4)*cosh(x)^2 - (5*
a^4*b - 7*a^2*b^3 + 2*b^5)*cosh(x))*sinh(x)^2 - 3*(a^3*b*cosh(x)^6 + 6*a^3*
b*cosh(x)*sinh(x)^5 + a^3*b*sinh(x)^6 - 3*a^3*b*cosh(x)^4 + 3*a^3*b*cosh(x)
^2 + 3*(5*a^3*b*cosh(x)^2 - a^3*b)*sinh(x)^4 - a^3*b + 4*(5*a^3*b*cosh(x)^3
 - 3*a^3*b*cosh(x))*sinh(x)^3 + 3*(5*a^3*b*cosh(x)^4 - 6*a^3*b*cosh(x)^2 +
a^3*b)*sinh(x)^2 + 6*(a^3*b*cosh(x)^5 - 2*a^3*b*cosh(x)^3 + a^3*b*cosh(x))*
sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2
)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(a^4*b - a^2*b^3 + 5*(a^4*b - a^2*b^3)
*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 - 2*(5*a^4*b - 7*a^2*b^3 + 2*b^5
)*cosh(x)^2 + 4*(a^5 - a^3*b^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(
x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*
a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 3*(a^6 -
3*a^4*b^2 + 3*a^2*b^4 - b^6 - 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 - 3*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 3*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 5*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^4 - 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*cosh(x)^2)*sinh(x)^2 + 6*((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^
5 - 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{a + b\operatorname{sech}(x)}\,dx$$

```
[In] integrate(csch(x)**4/(a+b*sech(x)),x)
[Out] Integral(csch(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.34

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{2\,a^3b\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{a^2-b^2}} - \frac{2\left(3\,a^2be^{(5\,x)}-3\,ab^2e^{(4\,x)}-10\,a^2be^{(3\,x)}+4\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+3\,a^2be^x-2\,a^3-ab^2\right)}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

```
[In] integrate(csch(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*b*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(
a^2 - b^2)) - 2/3*(3*a^2*b*e^(5*x) - 3*a*b^2*e^(4*x) - 10*a^2*b*e^(3*x) + 4
*b^3*e^(3*x) + 6*a^3*e^(2*x) + 3*a^2*b*e^x - 2*a^3 - a*b^2)/((a^4 - 2*a^2*b
^2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.66

$$\int \frac{\operatorname{csch}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,\mathrm{e}^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1} - \frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,\mathrm{e}^x}{3\,(a^2-b^2)}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1} + \frac{\frac{2\,a\,b^2}{(a^2-b^2)^2}-\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2-b^2)^2}}{\mathrm{e}^{2\,x}-1} + \frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2-b^2)^2}-\frac{2\,a^2\,b\,(a+b\,\mathrm{e}^x)}{(a+b)^{5/2}\,(b-a)^{5/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}} - \frac{a^3\,b\,\ln\left(\frac{2\,a^2\,b\,\mathrm{e}^x}{(a^2-b^2)^2}+\frac{2\,a^2\,b\,(a+b\,\mathrm{e}^x)}{(a+b)^{5/2}\,(b-a)^{5/2}}\right)}{(a+b)^{5/2}\,(b-a)^{5/2}}$$

```
[In] int(1/(sinh(x)^4*(a + b/cosh(x))),x)
```

```
[Out] ((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^
2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*
(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*a*b^2)/(a^2 -
b^2)^2 - (2*a^2*b*exp(x))/(a^2 - b^2)^2)/(exp(2*x) - 1) + (a^3*b*log((2*a^2
*b*exp(x))/(a^2 - b^2)^2 - (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b - a)^
(5/2))))/((a + b)^(5/2)*(b - a)^(5/2)) - (a^3*b*log((2*a^2*b*exp(x))/(a^2 -
 b^2)^2 + (2*a^2*b*(a + b*exp(x)))/((a + b)^(5/2)*(b - a)^(5/2))))/((a + b)
^(5/2)*(b - a)^(5/2))
```

3.68 $\int \frac{\cosh^4(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx=\frac{15x}{8a}-\frac{4\sinh(x)}{a}+\frac{15\cosh(x)\sinh(x)}{8a}+\frac{5\cosh^3(x)\sinh(x)}{4a}-\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)}-\frac{4\sinh^3(x)}{3a}$$

```
[Out] 15/8*x/a-4*sinh(x)/a+15/8*cosh(x)*sinh(x)/a+5/4*cosh(x)^3*sinh(x)/a-cosh(x)
^3*sinh(x)/(a+a*sech(x))-4/3*sinh(x)^3/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3904, 3872, 2715, 8, 2713}

$$\int \frac{\cosh^4(x)}{a+a\text{sech}(x)}\,dx=\frac{15x}{8a}-\frac{4\sinh^3(x)}{3a}-\frac{4\sinh(x)}{a}+\frac{5\sinh(x)\cosh^3(x)}{4a}+\frac{15\sinh(x)\cosh(x)}{8a}-\frac{\sinh(x)\cosh^3(x)}{a\text{sech}(x)+a}$$

```
[In] Int[Cosh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (15*x)/(8*a) - (4*Sinh[x])/a + (15*Cosh[x]*Sinh[x])/(8*a) + (5*Cosh[x]^3*Si
nh[x])/(4*a) - (Cosh[x]^3*Sinh[x])/(a + a*Sech[x]) - (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{\int \cosh^4(x)(-5a+4a\text{sech}(x))\,dx}{a^2} \\
&= -\frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{4\int \cosh^3(x)\,dx}{a} + \frac{5\int \cosh^4(x)\,dx}{a} \\
&= \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} \\
&\quad - \frac{(4i)\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -i\sinh(x)\right)}{a} + \frac{15\int \cosh^2(x)\,dx}{4a} \\
&= -\frac{4\sinh(x)}{a} + \frac{15\cosh(x)\sinh(x)}{8a} + \frac{5\cosh^3(x)\sinh(x)}{4a} \\
&\quad - \frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{4\sinh^3(x)}{3a} + \frac{15\int 1\,dx}{8a} \\
&= \frac{15x}{8a} - \frac{4\sinh(x)}{a} + \frac{15\cosh(x)\sinh(x)}{8a} + \frac{5\cosh^3(x)\sinh(x)}{4a} - \frac{\cosh^3(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{4\sinh^3(x)}{3a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{\cosh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{\text{sech}\left(\frac{x}{2}\right)\left(360x\cosh\left(\frac{x}{2}\right) - 360\sinh\left(\frac{x}{2}\right) - 120\sinh\left(\frac{3x}{2}\right) + 40\sinh\left(\frac{5x}{2}\right) - 5\sinh\left(\frac{7x}{2}\right) + 3\sinh\left(\frac{9x}{2}\right)\right)}{192a}$$

[In] Integrate[Cosh[x]^4/(a + a*Sech[x]),x]

[Out] (Sech[x/2]*(360*x*Cosh[x/2] - 360*Sinh[x/2] - 120*Sinh[(3*x)/2] + 40*Sinh[(5*x)/2] - 5*Sinh[(7*x)/2] + 3*Sinh[(9*x)/2]))/(192*a)

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

method	result
parallelrisch	$-\frac{\text{csch}(x)(-360x\sinh(x)+240\cosh(x)+8\cosh(4x)+160\cosh(2x)-3\cosh(5x)-45\cosh(3x)-360)}{192a}$
risch	$\frac{3\,e^{5x}-5\,e^{4x}+40\,e^{3x}-120\,e^{2x}+552+120\,e^{-x}-40\,e^{-2x}+5\,e^{-3x}+360x\,e^{x}-168\,e^{x}-3\,e^{-4x}+360x}{192(e^{x}+1)a}$
default	$\frac{-\tanh\left(\frac{x}{2}\right)-\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)+1\right)^4}+\frac{5}{6\left(\tanh\left(\frac{x}{2}\right)+1\right)^3}-\frac{15}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)^2}+\frac{25}{8\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{15\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{8}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^4}+\frac{}{6\left(\tanh\right.}}{a}$

[In] int(cosh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)

[Out] -1/192*csch(x)*(-360*x*sinh(x)+240*cosh(x)+8*cosh(4*x)+160*cosh(2*x)-3*cosh(5*x)-45*cosh(3*x)-360)/a

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(59) = 118.

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{\cosh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{3\cosh(x)^5 + (15\cosh(x) - 8)\sinh(x)^4 + 3\sinh(x)^5 - 8\cosh(x)^4 + \left(30\cosh(x)^2 - 8\cosh(x) + 35\right)\sin}{}$$

[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="fricas")

[Out] 1/192*(3*cosh(x)^5 + (15*cosh(x) - 8)*sinh(x)^4 + 3*sinh(x)^5 - 8*cosh(x)^4 + (30*cosh(x)^2 - 8*cosh(x) + 35)*sinh(x)^3 + 45*cosh(x)^3 + (30*cosh(x)^3

```
 - 48*cosh(x)^2 + 135*cosh(x) - 160)*sinh(x)^2 + 24*(15*x - 2)*cosh(x) - 16
0*cosh(x)^2 + (15*cosh(x)^4 - 8*cosh(x)^3 + 105*cosh(x)^2 + 360*x - 160*cos
h(x) - 288)*sinh(x) + 360*x + 552)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\cosh^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(cosh(x)**4/(a+a*sech(x)),x)
[Out] Integral(cosh(x)**4/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{15\,x}{8\,a} + \frac{168\,e^{(-x)} - 48\,e^{(-2\,x)} + 8\,e^{(-3\,x)} - 3\,e^{(-4\,x)}}{192\,a} - \frac{5\,e^{(-x)} - 40\,e^{(-2\,x)} + 120\,e^{(-3\,x)} + 552\,e^{(-4\,x)} - 3}{192\,(ae^{(-4\,x)} + ae^{(-5\,x)})}$$

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
[Out] 15/8*x/a + 1/192*(168*e^(-x) - 48*e^(-2*x) + 8*e^(-3*x) - 3*e^(-4*x))/a - 1
/192*(5*e^(-x) - 40*e^(-2*x) + 120*e^(-3*x) + 552*e^(-4*x) - 3)/(a*e^(-4*x)
 + a*e^(-5*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{15\,x}{8\,a} + \frac{\left(552\,e^{(4\,x)} + 120\,e^{(3\,x)} - 40\,e^{(2\,x)} + 5\,e^{x} - 3\right)e^{(-4\,x)}}{192\,a(e^{x} + 1)} + \frac{3\,a^3e^{(4\,x)} - 8\,a^3e^{(3\,x)} + 48\,a^3e^{(2\,x)} - 168\,a^3e^{x}}{192\,a^4}$$

```
[In] integrate(cosh(x)^4/(a+a*sech(x)),x, algorithm="giac")
[Out] 15/8*x/a + 1/192*(552*e^(4*x) + 120*e^(3*x) - 40*e^(2*x) + 5*e^x - 3)*e^(-4
*x)/(a*(e^x + 1)) + 1/192*(3*a^3*e^(4*x) - 8*a^3*e^(3*x) + 48*a^3*e^(2*x) -
 168*a^3*e^x)/a^4
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{\cosh^4(x)}{a + a\mathrm{sech}(x)}\,dx = \frac{7\,\mathrm{e}^{-x}}{8\,a} - \frac{\mathrm{e}^{-2\,x}}{4\,a} + \frac{\mathrm{e}^{2\,x}}{4\,a} + \frac{\mathrm{e}^{-3\,x}}{24\,a} - \frac{\mathrm{e}^{3\,x}}{24\,a} - \frac{\mathrm{e}^{-4\,x}}{64\,a} + \frac{\mathrm{e}^{4\,x}}{64\,a} + \frac{15\,x}{8\,a} + \frac{2}{a\,\left(\mathrm{e}^{x}+1\right)} - \frac{7\,\mathrm{e}^{x}}{8\,a}$$

```
[In] int(cosh(x)^4/(a + a/cosh(x)),x)
```

```
[Out] (7*exp(-x))/(8*a) - exp(-2*x)/(4*a) + exp(2*x)/(4*a) + exp(-3*x)/(24*a) - e
xp(3*x)/(24*a) - exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (15*x)/(8*a) + 2/(a*(
exp(x) + 1)) - (7*exp(x))/(8*a)
```

3.69 $\int \frac{\cosh^3(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\cosh^3(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)}+\frac{4\sinh^3(x)}{3a}$$

```
[Out] -3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*sech(x))+
4/3*sinh(x)^3/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3904, 3872, 2713, 2715, 8}

$$\int \frac{\cosh^3(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{3x}{2a}+\frac{4\sinh^3(x)}{3a}+\frac{4\sinh(x)}{a}-\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh^2(x)}{a\operatorname{sech}(x)+a}$$

```
[In] Int[Cosh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] (-3*x)/(2*a) + (4*Sinh[x])/a - (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]^2*Sinh[
x])/(a + a*Sech[x]) + (4*Sinh[x]^3)/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{\int \cosh^3(x)(-4a+3a\operatorname{sech}(x))\,dx}{a^2} \\
&= -\frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} - \frac{3\int \cosh^2(x)\,dx}{a} + \frac{4\int \cosh^3(x)\,dx}{a} \\
&= -\frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} + \frac{(4i)\text{Subst}\left(\int (1-x^2)\,dx, x, -i\sinh(x)\right)}{a} - \frac{3\int 1\,dx}{2a} \\
&= -\frac{3x}{2a} + \frac{4\sinh(x)}{a} - \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh^2(x)\sinh(x)}{a+a\operatorname{sech}(x)} + \frac{4\sinh^3(x)}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(-36x\cosh\left(\frac{x}{2}\right)+45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)\right)}{24a}$$

```
[In] Integrate[Cosh[x]^3/(a + a*Sech[x]),x]
[Out] (Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*x
)/2] + Sinh[(7*x)/2]))/(24*a)
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{\operatorname{csch}(x)(-36x\sinh(x)-3\cosh(3x)+27\cosh(x)+\cosh(4x)+20\cosh(2x)-45)}{24a}$
risch	$\frac{e^{4x}-2\,e^{3x}+18\,e^{2x}-69-18\,e^{-x}+2\,e^{-2x}-36x\,e^{x}+21\,e^{x}-e^{-3x}-36x}{24(e^{x}+1)a}$
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right)\right.}}{a}$

```
[In] int(cosh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*csch(x)*(-36*x*sinh(x)-3*cosh(3*x)+27*cosh(x)+cosh(4*x)+20*cosh(2*x)-4
5)/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(48) = 96$.

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\cosh(x)^4+(4\cosh(x)-1)\sinh(x)^3+\sinh(x)^4-3\cosh(x)^3+\left(6\cosh(x)^2-9\cosh(x)+20\right)\sinh(x)}{24\,(a\cosh}$$

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] 1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6*
cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)^2
 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x - 69
)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\cosh^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(cosh(x)**3/(a+a*sech(x)),x)
```

```
[Out] Integral(cosh(x)**3/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^3(x)}{a + a\mathrm{sech}(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{21\,e^{(-x)} - 3\,e^{(-2\,x)} + e^{(-3\,x)}}{24\,a} - \frac{2\,e^{(-x)} - 18\,e^{(-2\,x)} - 69\,e^{(-3\,x)} - 1}{24\,\left(ae^{(-3\,x)} + ae^{(-4\,x)}\right)}$$

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 18
*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^3(x)}{a + a\mathrm{sech}(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{\left(69\,e^{(3\,x)} + 18\,e^{(2\,x)} - 2\,e^{x} + 1\right)e^{(-3\,x)}}{24\,a(e^{x} + 1)} + \frac{a^2e^{(3\,x)} - 3\,a^2e^{(2\,x)} + 21\,a^2e^{x}}{24\,a^3}$$

```
[In] integrate(cosh(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1)
) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^3(x)}{a + a\mathrm{sech}(x)}\,dx = \frac{\mathrm{e}^{-2\,x}}{8\,a} - \frac{7\,\mathrm{e}^{-x}}{8\,a} - \frac{\mathrm{e}^{2\,x}}{8\,a} - \frac{\mathrm{e}^{-3\,x}}{24\,a} + \frac{\mathrm{e}^{3\,x}}{24\,a} - \frac{3\,x}{2\,a} - \frac{2}{a\,\left(\mathrm{e}^{x} + 1\right)} + \frac{7\,\mathrm{e}^{x}}{8\,a}$$

```
[In] int(cosh(x)^3/(a + a/cosh(x)),x)
```

```
[Out] exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + e
xp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

3.70 $\int \frac{\cosh^2(x)}{a+a\textbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{\cosh^2(x)}{a+a\,\text{sech}(x)}\,dx=\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh(x)\sinh(x)}{a+a\,\text{sech}(x)}$$

```
[Out] 3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)*sinh(x)/(a+a*sech(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3904, 3872, 2715, 8, 2717}

$$\int \frac{\cosh^2(x)}{a+a\,\text{sech}(x)}\,dx=\frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\sinh(x)\cosh(x)}{2a}-\frac{\sinh(x)\cosh(x)}{a\,\text{sech}(x)+a}$$

```
[In] Int[Cosh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] (3*x)/(2*a) - (2*Sinh[x])/a + (3*Cosh[x]*Sinh[x])/(2*a) - (Cosh[x]*Sinh[x])
/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{\int \cosh^2(x)(-3a+2a\text{sech}(x))\,dx}{a^2}\\
&= -\frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)} - \frac{2\int \cosh(x)\,dx}{a} + \frac{3\int \cosh^2(x)\,dx}{a}\\
&= -\frac{2\sinh(x)}{a} + \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)} + \frac{3\int 1\,dx}{2a}\\
&= \frac{3x}{2a} - \frac{2\sinh(x)}{a} + \frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh(x)\sinh(x)}{a+a\text{sech}(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^2(x)}{a+a\text{sech}(x)}\,dx = \frac{\text{sech}\left(\frac{x}{2}\right)\left(12x\cosh\left(\frac{x}{2}\right) - 12\sinh\left(\frac{x}{2}\right) - 3\sinh\left(\frac{3x}{2}\right) + \sinh\left(\frac{5x}{2}\right)\right)}{8a}$$

```
[In] Integrate[Cosh[x]^2/(a + a*Sech[x]),x]
[Out] (Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2]
))/(8*a)
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

method	result	size
parallelrisch	$\frac{\coth(x)\cosh(2x)+(-4\cosh(x)-5)\coth(x)+6x+8\,\mathrm{csch}(x)}{4a}$	30
risch	$\frac{\mathrm{e}^{3x}-3\,\mathrm{e}^{2x}+20+3\,\mathrm{e}^{-x}+12x\,\mathrm{e}^{x}-4\,\mathrm{e}^{x}-\mathrm{e}^{-2x}+12x}{8(\mathrm{e}^{x}+1)a}$	48
default	$\frac{-\tanh\left(\frac{x}{2}\right)-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)+1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{2}+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}}{a}$	70

```
[In] int(cosh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/4*(coth(x)*cosh(2*x)+(-4*cosh(x)-5)*coth(x)+6*x+8*csch(x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{\cosh^2(x)}{a+a\mathrm{sech}(x)}\,dx = \frac{\cosh(x)^3+(3\cosh(x)-4)\sinh(x)^2+\sinh(x)^3+(12x-1)\cosh(x)-4\cosh(x)^2+\left(3\cosh(x)^2+12\ldots\right.}{8\,(a\cosh(x)+a\sinh(x)+a)}$$

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x)
 - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20)/
(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a+a\mathrm{sech}(x)}\,dx = \frac{\int \frac{\cosh^2(x)}{\mathrm{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(cosh(x)**2/(a+a*sech(x)),x)
[Out] Integral(cosh(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \frac{\cosh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{3\,x}{2\,a} + \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a} - \frac{3\,e^{(-x)} + 20\,e^{(-2\,x)} - 1}{8\,(ae^{(-2\,x)} + ae^{(-3\,x)})}$$

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(a
*e^(-2*x) + a*e^(-3*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{\cosh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{3\,x}{2\,a} + \frac{\left(20\,e^{(2\,x)} + 3\,e^{x} - 1\right)e^{(-2\,x)}}{8\,a(e^{x} + 1)} + \frac{ae^{(2\,x)} - 4\,ae^{x}}{8\,a^2}$$

```
[In] integrate(cosh(x)^2/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(2
*x) - 4*a*e^x)/a^2
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{\cosh^2(x)}{a + a\text{sech}(x)}\,dx = \frac{\mathrm{e}^{-x}}{2\,a} - \frac{\mathrm{e}^{-2\,x}}{8\,a} + \frac{\mathrm{e}^{2\,x}}{8\,a} + \frac{3\,x}{2\,a} + \frac{2}{a\,\left(\mathrm{e}^{x} + 1\right)} - \frac{\mathrm{e}^{x}}{2\,a}$$

```
[In] int(cosh(x)^2/(a + a/cosh(x)),x)
```

```
[Out] exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp(
x) + 1)) - exp(x)/(2*a)
```

3.71 $\int \frac{\cosh(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int \frac{\cosh(x)}{a+a\text{sech}(x)}\,dx = -\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a+a\text{sech}(x)}$$

```
[Out] -x/a+2*sinh(x)/a-sinh(x)/(a+a*sech(x))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3904, 3872, 2717, 8}

$$\int \frac{\cosh(x)}{a+a\text{sech}(x)}\,dx = -\frac{x}{a}+\frac{2\sinh(x)}{a}-\frac{\sinh(x)}{a\text{sech}(x)+a}$$

```
[In] Int[Cosh[x]/(a + a*Sech[x]),x]
```

```
[Out] -(x/a) + (2*Sinh[x])/a - Sinh[x]/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3904

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*(a + b*Csc[e +
f*x]))), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\sinh(x)}{a+a\text{sech}(x)} - \frac{\int \cosh(x)(-2a+a\text{sech}(x))\,dx}{a^2}\\ &= -\frac{\sinh(x)}{a+a\text{sech}(x)} - \frac{\int 1\,dx}{a} + \frac{2\int \cosh(x)\,dx}{a}\\ &= -\frac{x}{a} + \frac{2\sinh(x)}{a} - \frac{\sinh(x)}{a+a\text{sech}(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{\cosh(x)}{a+a\text{sech}(x)}\,dx = \frac{-2x+\text{sech}\left(\frac{x}{2}\right)\sinh\left(\frac{3x}{2}\right)+3\tanh\left(\frac{x}{2}\right)}{2a}$$

```
[In] Integrate[Cosh[x]/(a + a*Sech[x]),x]
[Out] (-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{\coth(x)\cosh(x)-x+\coth(x)-2\,\text{csch}(x)}{a}$	20
risch	$-\frac{x}{a}+\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}-\frac{2}{(e^{x}+1)a}$	35
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)+1}-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	46

```
[In] int(cosh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] (coth(x)*cosh(x)-x+coth(x)-2*csch(x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = -\frac{2\,x\cosh\left(x\right) - \cosh\left(x\right)^{2} + 2\,{\left(x - \cosh\left(x\right) - 1\right)}\sinh\left(x\right) - \sinh\left(x\right)^{2} + 2\,x + 5}{2\,{\left(a\cosh\left(x\right) + a\sinh\left(x\right) + a\right)}}$$

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="fricas")
[Out] -1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 + 2
*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\cosh\left(x\right)}{\operatorname{sech}\left(x\right)+1}\,dx}{a}$$

```
[In] integrate(cosh(x)/(a+a*sech(x)),x)
[Out] Integral(cosh(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = -\frac{x}{a} + \frac{5\,e^{\left(-x\right)} + 1}{2\,{\left(ae^{\left(-x\right)} + ae^{\left(-2\,x\right)}\right)}} - \frac{e^{\left(-x\right)}}{2\,a}$$

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="maxima")
[Out] -x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = -\frac{x}{a} - \frac{(5\,e^{x} + 1)e^{(-x)}}{2\,a(e^{x} + 1)} + \frac{e^{x}}{2\,a}$$

```
[In] integrate(cosh(x)/(a+a*sech(x)),x, algorithm="giac")
[Out] -x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(x)}{a + a\text{sech}(x)}\,dx = \frac{e^{x}}{2\,a} - \frac{x}{a} - \frac{2}{a\,\left(e^{x} + 1\right)} - \frac{e^{-x}}{2\,a}$$

```
[In] int(cosh(x)/(a + a/cosh(x)),x)
[Out] exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

3.72 $\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

```
[Out] tanh(x)/(a+a*sech(x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3879}

$$\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

```
[In] Int[Sech[x]/(a + a*Sech[x]),x]
```

```
[Out] Tanh[x]/(a + a*Sech[x])
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral}=\frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\tanh\left(\frac{x}{2}\right)}{a}$$

```
[In] Integrate[Sech[x]/(a + a*Sech[x]),x]
[Out] Tanh[x/2]/a
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)}{a}$	9
parallelrisch	$\frac{\tanh\left(\frac{x}{2}\right)}{a}$	9
risch	$-\frac{2}{\left(e^{x}+1\right)a}$	12

```
[In] int(sech(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*tanh(1/2*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\operatorname{sech}(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{2}{a\cosh\left(x\right)+a\sinh\left(x\right)+a}$$

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="fricas")
[Out] -2/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{sech}(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sech(x)/(a+a*sech(x)),x)
[Out] Integral(sech(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{2}{ae^{(-x)} + a}$$

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="maxima")
[Out] 2/(a*e^(-x) + a)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{2}{a(e^x + 1)}$$

```
[In] integrate(sech(x)/(a+a*sech(x)),x, algorithm="giac")
[Out] -2/(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{2}{a\,\left(e^x + 1\right)}$$

```
[In] int(1/(cosh(x)*(a + a/cosh(x))),x)
[Out] -2/(a*(exp(x) + 1))
```

3.73 $\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)} dx = \frac{\arctan(\sinh(x))}{a} - \frac{\tanh(x)}{a+a\operatorname{sech}(x)}$$

```
[Out] arctan(sinh(x))/a-tanh(x)/(a+a*sech(x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3874, 3855, 3879}

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)} dx = \frac{\arctan(\sinh(x))}{a} - \frac{\tanh(x)}{a\operatorname{sech}(x)+a}$$

```
[In] Int[Sech[x]^2/(a + a*Sech[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/a - Tanh[x]/(a + a*Sech[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \operatorname{sech}(x)\,dx}{a} - \int \frac{\operatorname{sech}(x)}{a + a\operatorname{sech}(x)}\,dx \\ &= \frac{\arctan(\sinh(x))}{a} - \frac{\tanh(x)}{a + a\operatorname{sech}(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}^2(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\arctan(\sinh(x)) + \arctan(\sinh(x))\operatorname{sech}(x) - \tanh(x)}{a + a\operatorname{sech}(x)}$$

```
[In] Integrate[Sech[x]^2/(a + a*Sech[x]),x]
[Out] (ArcTan[Sinh[x]] + ArcTan[Sinh[x]]*Sech[x] - Tanh[x])/(a + a*Sech[x])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	19
parallelrisch	$\frac{-i\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+i\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-\tanh\left(\frac{x}{2}\right)}{a}$	34
risch	$\frac{2}{(e^x+1)a}+\frac{i\ln(e^x+i)}{a}-\frac{i\ln(e^x-i)}{a}$	37

```
[In] int(sech(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(-tanh(1/2*x)+2*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2\left((\cosh(x)+\sinh(x)+1)\arctan(\cosh(x)+\sinh(x))+1\right)}{a\cosh(x)+a\sinh(x)+a}$$

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] 2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*si
nh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{sech}^2(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sech(x)**2/(a+a*sech(x)),x)
[Out] Integral(sech(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{2\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-x)}+a}$$

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] -2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2\arctan(e^x)}{a} + \frac{2}{a(e^x+1)}$$

```
[In] integrate(sech(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] 2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{sech}^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{2}{a\,\left(e^x+1\right)}+\frac{2\,\operatorname{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

```
[In] int(1/(cosh(x)^2*(a + a/cosh(x))),x)
[Out] 2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.74 $\int \frac{\text{sech}^3(x)}{a+a\text{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{\text{sech}^3(x)}{a+a\text{sech}(x)} \, dx = -\frac{\arctan(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a+a\text{sech}(x)}$$

```
[Out] -arctan(sinh(x))/a+tanh(x)/a+tanh(x)/(a+a*sech(x))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3875, 3874, 3855, 3879}

$$\int \frac{\text{sech}^3(x)}{a+a\text{sech}(x)} \, dx = -\frac{\arctan(\sinh(x))}{a}+\frac{\tanh(x)}{a}+\frac{\tanh(x)}{a\text{sech}(x)+a}$$

```
[In] Int[Sech[x]^3/(a + a*Sech[x]),x]
```

```
[Out] -(ArcTan[Sinh[x]]/a) + Tanh[x]/a + Tanh[x]/(a + a*Sech[x])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
```

```
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tanh(x)}{a} - \int \frac{\text{sech}^2(x)}{a + a\text{sech}(x)}\,dx\\ &= \frac{\tanh(x)}{a} - \frac{\int \text{sech}(x)\,dx}{a} + \int \frac{\text{sech}(x)}{a + a\text{sech}(x)}\,dx\\ &= -\frac{\arctan(\sinh(x))}{a} + \frac{\tanh(x)}{a} + \frac{\tanh(x)}{a + a\text{sech}(x)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\text{sech}^3(x)}{a + a\text{sech}(x)}\,dx = -\frac{\arctan(\sinh(x)) + \arctan(\sinh(x))\text{sech}(x) - 2\tanh(x) - \text{sech}(x)\tanh(x)}{a + a\text{sech}(x)}$$

```
[In] Integrate[Sech[x]^3/(a + a*Sech[x]),x]
[Out] -((ArcTan[Sinh[x]] + ArcTan[Sinh[x]]*Sech[x] - 2*Tanh[x] - Sech[x]*Tanh[x])
/(a + a*Sech[x]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^{2}}-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	33
parallelrisch	$\frac{i\ln\left(-i+\coth(x)-\operatorname{csch}(x)\right)-i\ln\left(i+\coth(x)-\operatorname{csch}(x)\right)+\left(-\operatorname{sech}(x)-1\right)\operatorname{csch}(x)+2\coth(x)}{a}$	45
risch	$-\frac{2\left(e^{2x}+e^{x}+2\right)}{a\left(1+e^{2x}\right)\left(e^{x}+1\right)}+\frac{i\ln\left(e^{x}-i\right)}{a}-\frac{i\ln\left(e^{x}+i\right)}{a}$	53

```
[In] int(sech(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/a*(tanh(1/2*x)+2*tanh(1/2*x)/(1+tanh(1/2*x)^2)-2*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.88

$$\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx = -\frac{2\left(\left(\cosh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)+1\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+\cosh\left(x\right)^{2}+\left(3\,\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\sinh\left(x\right)\right.\right.}{a\cosh\left(x\right)^{3}+a\sinh\left(x\right)^{3}+a\cosh\left(x\right)^{2}+\left(3\,a\cosh\left(x\right)+a\right)\sin}$$

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="fricas")
[Out] -2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*cos
h(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) +
cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x)^3
 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (3
*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{sech}^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sech(x)**3/(a+a*sech(x)),x)
[Out] Integral(sech(x)**3/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{2\left(e^{(-x)} + e^{(-2x)} + 2\right)}{ae^{(-x)} + ae^{(-2x)} + ae^{(-3x)} + a} + \frac{2\arctan\left(e^{(-x)}\right)}{a}$$

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] 2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arct
an(e^(-x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{2\arctan\left(e^{x}\right)}{a} - \frac{2\left(e^{(2x)} + e^{x} + 2\right)}{a\left(e^{(3x)} + e^{(2x)} + e^{x} + 1\right)}$$

```
[In] integrate(sech(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] -2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{\operatorname{sech}^3(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{\frac{2\,\mathrm{e}^{2\,x}}{a} + \frac{4}{a} + \frac{2\,\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x} + \mathrm{e}^{3\,x} + \mathrm{e}^{x} + 1} - \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

```
[In] int(1/(cosh(x)^3*(a + a/cosh(x))),x)
```

```
[Out] - ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
- (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.75 $\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{3\arctan(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}$$

```
[Out] 3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)/(
a+a*sech(x))
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3903, 3872, 3852, 8, 3853, 3855}

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{3\arctan(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}-\frac{\tanh(x)\operatorname{sech}^2(x)}{a\operatorname{sech}(x)+a}+\frac{3\tanh(x)\operatorname{sech}(x)}{2a}$$

```
[In] Int[Sech[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(2*a) - (2*Tanh[x])/a + (3*Sech[x]*Tanh[x])/(2*a) - (Se
ch[x]^2*Tanh[x])/(a + a*Sech[x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3903

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(f*(a +
b*Csc[e + f*x]))), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)} - \frac{\int \operatorname{sech}^2(x)(2a-3a\operatorname{sech}(x))\,dx}{a^2} \\
&= -\frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)} - \frac{2\int \operatorname{sech}^2(x)\,dx}{a} + \frac{3\int \operatorname{sech}^3(x)\,dx}{a} \\
&= \frac{3\operatorname{sech}(x)\tanh(x)}{2a} - \frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)} - \frac{(2i)\operatorname{Subst}(\int 1\,dx, x, -i\tanh(x))}{a} + \frac{3\int \operatorname{sech}(x)\,dx}{2a} \\
&= \frac{3\arctan(\sinh(x))}{2a} - \frac{2\tanh(x)}{a} + \frac{3\operatorname{sech}(x)\tanh(x)}{2a} - \frac{\operatorname{sech}^2(x)\tanh(x)}{a+a\operatorname{sech}(x)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{3\arctan(\sinh(x))+2\operatorname{sech}^3(x)\tanh\left(\frac{x}{2}\right)-6\tanh(x)+3\operatorname{sech}(x)\tanh(x)+2\tanh^3(x)}{2a}$$

[In] Integrate[Sech[x]^4/(a + a*Sech[x]),x]

[Out] (3*ArcTan[Sinh[x]] + 2*Sech[x]^3*Tanh[x/2] - 6*Tanh[x] + 3*Sech[x]*Tanh[x] + 2*Tanh[x]^3)/(2*a)

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{-3\tanh\left(\frac{x}{2}\right)^3-\tanh\left(\frac{x}{2}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	46
parallelrisch	$\frac{-3i\ln(-i+\coth(x)-\operatorname{csch}(x))+3i\ln(i+\coth(x)-\operatorname{csch}(x))+\left(-\operatorname{sech}(x)^2+2\operatorname{sech}(x)+3\right)\operatorname{csch}(x)-4\coth(x)}{2a}$	52
risch	$\frac{3\,e^{4x}+3\,e^{3x}+5\,e^{2x}+e^{x}+4}{a(1+e^{2x})^2(e^{x}+1)}+\frac{3i\ln(e^{x}+i)}{2a}-\frac{3i\ln(e^{x}-i)}{2a}$	66

[In] int(sech(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)

[Out] 1/a*(-tanh(1/2*x)+2*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+3*arctan(tanh(1/2*x)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(41) = 82.

Time = 0.26 (sec) , antiderivative size = 325, normalized size of antiderivative = 7.22

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{3\cosh(x)^4+3\,(4\cosh(x)+1)\sinh(x)^3+3\sinh(x)^4+3\cosh(x)^3+\left(18\cosh(x)^2+9\cosh(x)+5\right)\sinh}{a}$$

[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="fricas")

```
[Out] (3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (1
8*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*sin
h(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)^3
+ 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 2
*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)*sinh
(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh(x)^3
+ 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5 + a*sin
h(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3 + 2*(5*a
*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*cosh(x)^3
+ 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*cosh(x)^4 +
 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\operatorname{sech}^4(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(sech(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(sech(x)**4/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{sech}^4(x)}{a + a\operatorname{sech}(x)}\,dx = -\frac{e^{(-x)} + 5\,e^{(-2\,x)} + 3\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 4}{ae^{(-x)} + 2\,ae^{(-2\,x)} + 2\,ae^{(-3\,x)} + ae^{(-4\,x)} + ae^{(-5\,x)} + a} - \frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] -(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2*
x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{3\arctan\left(e^x\right)}{a} + \frac{e^{(3x)} + 2e^{(2x)} - e^x + 2}{a\left(e^{(2x)}+1\right)^2} + \frac{2}{a(e^x+1)}$$

```
[In] integrate(sech(x)^4/(a+a*sech(x)),x, algorithm="giac")
[Out] 3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/(
a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{\operatorname{sech}^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{2}{a\,\left(\mathrm{e}^x+1\right)} + \frac{\frac{2}{a}+\frac{\mathrm{e}^x}{a}}{\mathrm{e}^{2\,x}+1} + \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,\mathrm{e}^x}{a\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

```
[In] int(1/(cosh(x)^4*(a + a/cosh(x))),x)
[Out] 2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2
)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

3.76 $\int \frac{1}{a+a\,\mathbf{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{\tanh(c+dx)}{d(a+a\operatorname{sech}(c+dx))}$$

```
[Out] x/a-tanh(d*x+c)/d/(a+a*sech(d*x+c))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3862, 8}

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{\tanh(c+dx)}{d(a\operatorname{sech}(c+dx)+a)}$$

```
[In] Int[(a + a*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - Tanh[c + d*x]/(d*(a + a*Sech[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\tanh(c+dx)}{d(a+a\operatorname{sech}(c+dx))} + \frac{\int a\,dx}{a^2} \\ &= \frac{x}{a} - \frac{\tanh(c+dx)}{d(a+a\operatorname{sech}(c+dx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.00

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx = \frac{\operatorname{sech}\left(\frac{c}{2}\right)\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(dx\cosh\left(\frac{dx}{2}\right)+dx\cosh\left(c+\frac{dx}{2}\right)-2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

```
[In] Integrate[(a + a*Sech[c + d*x])^(-1),x]
[Out] (Sech[c/2]*Sech[(c + d*x)/2]*(d*x*Cosh[(d*x)/2] + d*x*Cosh[c + (d*x)/2] - 2
*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

method	result	size
parallelrisch	$\frac{dx-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}$	23
risch	$\frac{x}{a}+\frac{2}{da\left(e^{dx+c}+1\right)}$	25
derivativedivides	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	46
default	$\frac{-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	46

```
[In] int(1/(a+sech(d*x+c)*a),x,method=_RETURNVERBOSE)
[Out] (d*x-tanh(1/2*d*x+1/2*c))/d/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.66

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{dx\cosh\left(dx+c\right)+dx\sinh\left(dx+c\right)+dx+2}{ad\cosh\left(dx+c\right)+ad\sinh\left(dx+c\right)+ad}$$

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="fricas")
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) + d*x + 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) + a*d)
```

Sympy [F]

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{\int \frac{1}{\operatorname{sech}\left(c+dx\right)+1}\,dx}{a}$$

```
[In] integrate(1/(a+a*sech(d*x+c)),x)
[Out] Integral(1/(sech(c + d*x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{dx+c}{ad}-\frac{2}{(ae^{(-dx-c)}+a)d}$$

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="maxima")
[Out] (d*x + c)/(a*d) - 2/((a*e^(-d*x - c) + a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+a\operatorname{sech}(c+dx)}\,dx=\frac{\frac{dx+c}{a}+\frac{2}{a\left(e^{(dx+c)}+1\right)}}{d}$$

```
[In] integrate(1/(a+a*sech(d*x+c)),x, algorithm="giac")
[Out] ((d*x + c)/a + 2/(a*(e^(d*x + c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{1}{a+a\text{sech}(c+dx)}\,dx=\frac{x}{a}+\frac{2}{a\,d\,\left(\mathrm{e}^{c+d\,x}+1\right)}$$

```
[In] int(1/(a + a/cosh(c + d*x)),x)
```

```
[Out] x/a + 2/(a*d*(exp(c + d*x) + 1))
```

3.77 $\int \frac{1}{a-a\,\mathbf{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{a-a\operatorname{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\operatorname{sech}(c+dx))}$$

```
[Out] x/a-tanh(d*x+c)/d/(a-a*sech(d*x+c))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3862, 8}

$$\int \frac{1}{a-a\operatorname{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{\tanh(c+dx)}{d(a-a\operatorname{sech}(c+dx))}$$

```
[In] Int[(a - a*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - Tanh[c + d*x]/(d*(a - a*Sech[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rubi steps

$$\text{integral} = -\frac{\tanh(c+dx)}{d(a-a\text{sech}(c+dx))} + \frac{\int a\,dx}{a^2}$$
$$= \frac{x}{a} - \frac{\tanh(c+dx)}{d(a-a\text{sech}(c+dx))}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int \frac{1}{a-a\text{sech}(c+dx)}\,dx$$
$$= \frac{\text{csch}\left(\frac{c}{2}\right)\text{csch}\left(\frac{1}{2}(c+dx)\right)\left(-dx\cosh\left(\frac{dx}{2}\right)+dx\cosh\left(c+\frac{dx}{2}\right)+2\sinh\left(\frac{dx}{2}\right)\right)}{2ad}$$

```
[In] Integrate[(a - a*Sech[c + d*x])^(-1),x]
[Out] (Csch[c/2]*Csch[(c + d*x)/2]*(-(d*x*Cosh[(d*x)/2]) + d*x*Cosh[c + (d*x)/2]
+ 2*Sinh[(d*x)/2]))/(2*a*d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{x}{a}-\frac{2}{da\left(e^{dx+c}-1\right)}$	25
parallelrisch	$\frac{-1+x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)d}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)ad}$	33
derivativedivides	$\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	48
default	$\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{da}$	48

```
[In] int(1/(a-sech(d*x+c)*a),x,method=_RETURNVERBOSE)
[Out] x/a-2/d/a/(exp(d*x+c)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.67

$$\int \frac{1}{a - a\operatorname{sech}(c + dx)}\,dx = \frac{dx\cosh(dx + c) + dx\sinh(dx + c) - dx - 2}{ad\cosh(dx + c) + ad\sinh(dx + c) - ad}$$

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="fricas")
[Out] (d*x*cosh(d*x + c) + d*x*sinh(d*x + c) - d*x - 2)/(a*d*cosh(d*x + c) + a*d*
sinh(d*x + c) - a*d)
```

Sympy [F]

$$\int \frac{1}{a - a\operatorname{sech}(c + dx)}\,dx = -\frac{\int \frac{1}{\operatorname{sech}(c+dx)-1}\,dx}{a}$$

```
[In] integrate(1/(a-a*sech(d*x+c)),x)
[Out] -Integral(1/(sech(c + d*x) - 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{1}{a - a\operatorname{sech}(c + dx)}\,dx = \frac{dx + c}{ad} + \frac{2}{(ae^{(-dx-c)} - a)d}$$

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="maxima")
[Out] (d*x + c)/(a*d) + 2/((a*e^(-d*x - c) - a)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{a - a\operatorname{sech}(c + dx)}\,dx = \frac{\frac{dx+c}{a} - \frac{2}{a\left(e^{(dx+c)}-1\right)}}{d}$$

```
[In] integrate(1/(a-a*sech(d*x+c)),x, algorithm="giac")
[Out] ((d*x + c)/a - 2/(a*(e^(d*x + c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{a - a\text{sech}(c + dx)} \, dx = \frac{x}{a} - \frac{2}{a\, d\, \left(\mathrm{e}^{c+d\, x} - 1\right)}$$

```
[In] int(1/(a - a/cosh(c + d*x)),x)
[Out] x/a - 2/(a*d*(exp(c + d*x) - 1))
```

3.78 $\int (a+a\operatorname{sech}(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 98

$$\int (a+a\operatorname{sech}(c+dx))^{5/2}\,dx=\frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\operatorname{sech}(c+dx)}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\operatorname{sech}(c+dx)}}+\frac{2a^2\sqrt{a+a\operatorname{sech}(c+dx)}\tanh(c+dx)}{3d}$$

```
[Out] 2*a^(5/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+14/3*a^3*t
anh(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)+2/3*a^2*(a+a*sech(d*x+c))^(1/2)*tanh(d
*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used = {3860, 4000, 3859, 209, 3877}

$$\int (a+a\operatorname{sech}(c+dx))^{5/2}\,dx=\frac{2a^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\operatorname{sech}(c+dx)+a}}\right)}{d}+\frac{14a^3\tanh(c+dx)}{3d\sqrt{a\operatorname{sech}(c+dx)+a}}+\frac{2a^2\tanh(c+dx)\sqrt{a\operatorname{sech}(c+dx)+a}}{3d}$$

```
[In] Int[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (2*a^(5/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(14*a^3*Tanh[c + d*x])/(3*d*Sqrt[a + a*Sech[c + d*x]]) + (2*a^2*Sqrt[a + a*
Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2a^2\sqrt{a+a\text{sech}(c+dx)}\tanh(c+dx)}{3d} \\
&\quad + \frac{1}{3}(2a)\int\sqrt{a+a\text{sech}(c+dx)}\left(\frac{3a}{2}+\frac{7}{2}a\text{sech}(c+dx)\right)dx \\
&= \frac{2a^2\sqrt{a+a\text{sech}(c+dx)}\tanh(c+dx)}{3d}+a^2\int\sqrt{a+a\text{sech}(c+dx)}\,dx \\
&\quad + \frac{1}{3}(7a^2)\int\text{sech}(c+dx)\sqrt{a+a\text{sech}(c+dx)}\,dx
\end{aligned}$$

$$= \frac{14a^3 \tanh(c+dx)}{3d\sqrt{a+a\text{sech}(c+dx)}} + \frac{2a^2\sqrt{a+a\text{sech}(c+dx)}\tanh(c+dx)}{3d} + \frac{(2ia^3)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}$$

$$= \frac{2a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d} + \frac{14a^3\tanh(c+dx)}{3d\sqrt{a+a\text{sech}(c+dx)}} + \frac{2a^2\sqrt{a+a\text{sech}(c+dx)}\tanh(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01

$$\int (a+a\text{sech}(c+dx))^{5/2}\,dx = \frac{a^2\text{sech}\left(\frac{1}{2}(c+dx)\right)\text{sech}(c+dx)\sqrt{a(1+\text{sech}(c+dx))}\left(3\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh\ldots\right.}{3d}$$

```
[In] Integrate[(a + a*Sech[c + d*x])^(5/2),x]
```

```
[Out] (a^2*Sech[(c + d*x)/2]*Sech[c + d*x]*Sqrt[a*(1 + Sech[c + d*x])]*(3*Sqrt[2]
*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Cosh[c + d*x]^(3/2) - 6*Sinh[(c + d*x)/
2] + 8*Sinh[(3*(c + d*x))/2]))/(3*d)
```

Maple [F]

$$\int (a+\text{sech}(dx+c)\,a)^{\frac{5}{2}}\,dx$$

```
[In] int((a+sech(d*x+c)*a)^(5/2),x)
```

```
[Out] int((a+sech(d*x+c)*a)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(84) = 168$.

Time = 0.29 (sec) , antiderivative size = 924, normalized size of antiderivative = 9.43

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \text{Too large to display}$$

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(
d*x + c)^2 + a^2)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x
 + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 +
 (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5
 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x
 + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15
*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^
4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*
x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x
+ c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(
d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c)
+ 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 3*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(
d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a)*log((a*cosh(d*x
 + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh
(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*co
sh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d
*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(
d*x + c))) + 8*(4*a^2*cosh(d*x + c)^3 + 4*a^2*sinh(d*x + c)^3 - 3*a^2*cosh(
d*x + c)^2 + 3*a^2*cosh(d*x + c) + 3*(4*a^2*cosh(d*x + c) - a^2)*sinh(d*x +
 c)^2 - 4*a^2 + 3*(4*a^2*cosh(d*x + c)^2 - 2*a^2*cosh(d*x + c) + a^2)*sinh(
d*x + c))*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 + 1)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*si
nh(d*x + c)^2 + d)
```

Sympy [F]

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \int (a \operatorname{sech}(c + dx) + a)^{\frac{5}{2}} \, dx$$

```
[In] integrate((a+a*sech(d*x+c))**(5/2),x)
[Out] Integral((a*sech(c + d*x) + a)**(5/2), x)
```

Maxima [F]

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \int (a \operatorname{sech}(dx + c) + a)^{\frac{5}{2}} \, dx$$

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((a*sech(d*x + c) + a)^(5/2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.54

$$\int (a + a\text{sech}(c + dx))^{5/2} \, dx = \frac{\frac{6\,a^{3}\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - 3\,a^{\frac{5}{2}}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right) - \frac{4\,(4\,a^{4}-(3\,a^{4}e^{c}}{}}{3\,d}$$

```
[In] integrate((a+a*sech(d*x+c))^(5/2),x, algorithm="giac")
[Out] 1/3*(6*a^3*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt
(-a))/sqrt(-a) - 3*a^(5/2)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x +
 2*c) + a))) - 4*(4*a^4 - (3*a^4*e^c + (4*a^4*e^(d*x + 3*c) - 3*a^4*e^(2*c)
)*e^(d*x))*e^(d*x))/(a*e^(2*d*x + 2*c) + a)^(3/2))/d
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\text{sech}(c + dx))^{5/2}\, dx = \int \left(a + \frac{a}{\cosh\left(c + d\,x\right)}\right)^{5/2} dx$$

```
[In] int((a + a/cosh(c + d*x))^(5/2),x)
[Out] int((a + a/cosh(c + d*x))^(5/2), x)
```

3.79 $\int (a + a\text{sech}(c + dx))^{3/2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a + a\text{sech}(c + dx))^{3/2} \, dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d} + \frac{2a^2\tanh(c + dx)}{d\sqrt{a + a\text{sech}(c + dx)}}$$

```
[Out] 2*a^(3/2)*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d+2*a^2*tanh
(d*x+c)/d/(a+a*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3860, 21, 3859, 209}

$$\int (a + a\text{sech}(c + dx))^{3/2} \, dx = \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{d} + \frac{2a^2\tanh(c + dx)}{d\sqrt{a\text{sech}(c + dx) + a}}$$

```
[In] Int[(a + a*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*a^(3/2)*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d +
(2*a^2*Tanh[c + d*x])/(d*Sqrt[a + a*Sech[c + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3860

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[a/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integ
erQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}} + (2a)\int \frac{\frac{a}{2}+\frac{1}{2}a\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx \\
&= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}} + a\int \sqrt{a+a\text{sech}(c+dx)}\,dx \\
&= \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}} + \frac{(2ia^2)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d} \\
&= \frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d} + \frac{2a^2\tanh(c+dx)}{d\sqrt{a+a\text{sech}(c+dx)}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int (a+a\text{sech}(c+dx))^{3/2}\,dx=\frac{a\text{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\text{sech}(c+dx))}\left(\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}+2\right.}{d}$$

```
[In] Integrate[(a + a*Sech[c + d*x])^(3/2),x]
[Out] (a*Sech[(c + d*x)/2]*Sqrt[a*(1 + Sech[c + d*x])]*(Sqrt[2]*ArcSinh[Sqrt[2]*S
inh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]] + 2*Sinh[(c + d*x)/2]))/d
```

Maple [F]

$$\int (a+\text{sech}\,(dx+c)\,a)^{\frac{3}{2}}\,dx$$

```
[In] int((a+sech(d*x+c)*a)^(3/2),x)
[Out] int((a+sech(d*x+c)*a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 697, normalized size of antiderivative = 10.56

$$\int (a+a\text{sech}(c+dx))^{3/2}\,dx=\frac{a^{\frac{3}{2}}\log\left(-\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4-3\,a\cosh(dx+c)^3+(4\,a\cosh(dx+c)-3\,a)\sinh(dx+c)^3+5\,a\cosh(dx+c)^2+\left(6\,a\cosh(\right.}{}\right.}{}$$

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/2*(a^(3/2)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
```

```
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + a^(3/2)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))) + 4*(a*cosh(d*x
 + c) + a*sinh(d*x + c) - a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh
(d*x + c) + sinh(d*x + c)^2 + 1)))/d
```

Sympy [F]

$$\int (a+a\text{sech}(c+dx))^{3/2}\,dx=\int (a\,\text{sech}\,(c+dx)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sech(d*x+c))**(3/2),x)
[Out] Integral((a*sech(c + d*x) + a)**(3/2), x)
```

Maxima [F]

$$\int (a+a\text{sech}(c+dx))^{3/2}\,dx=\int (a\,\text{sech}\,(dx+c)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sech(d*x + c) + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(58) = 116$.

Time = 0.38 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.79

$$\int (a+a\text{sech}(c+dx))^{3/2}\,dx=\frac{\frac{2\,a^2\arctan\left(-\frac{\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-a^{\frac{3}{2}}\log\left(\left|-\sqrt{a}e^{(dx+c)}+\sqrt{ae^{(2\,dx+2\,c)}+a}\right|\right)+\frac{2\,(a^2e^{(dx+c)}-a^2)}{\sqrt{ae^{(2\,dx+2\,c)}+a}}}{d}$$

```
[In] integrate((a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] (2*a^2*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a)
)/sqrt(-a) - a^(3/2)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c)
+ a))) + 2*(a^2*e^(d*x + c) - a^2)/sqrt(a*e^(2*d*x + 2*c) + a))/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\operatorname{sech}(c+dx))^{3/2}\,dx=\int \left(a+\frac{a}{\cosh\left(c+d\,x\right)}\right)^{3/2}\,dx$$

```
[In] int((a + a/cosh(c + d*x))^(3/2),x)
[Out] int((a + a/cosh(c + d*x))^(3/2), x)
```

3.80 $\int \sqrt{a+a\,\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \sqrt{a+a\,\text{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\,\text{sech}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3859, 209}

$$\int \sqrt{a+a\,\text{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\text{sech}(c+dx)+a}}\right)}{d}$$

```
[In] Int[Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{(2ia)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}$$
$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \sqrt{a+a\text{sech}(c+dx)}\,dx = \frac{\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cosh(c+dx)}\text{sech}\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\text{sech}(c+dx))}}{d}$$

```
[In] Integrate[Sqrt[a + a*Sech[c + d*x]],x]
[Out] (Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[(c + d*x)/2]]*Sqrt[Cosh[c + d*x]]*Sech[(c + d
*x)/2]*Sqrt[a*(1 + Sech[c + d*x])])/d
```

Maple [F]

$$\int \sqrt{a+\text{sech}\,(dx+c)\,a}dx$$

```
[In] int((a+sech(d*x+c)*a)^(1/2),x)
[Out] int((a+sech(d*x+c)*a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(31) = 62.

Time = 0.27 (sec) , antiderivative size = 637, normalized size of antiderivative = 17.22

$$\int \sqrt{a+a\text{sech}(c+dx)}\,dx = \frac{\sqrt{a}\log\left(-\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4-3a\cosh(dx+c)^3+(4a\cosh(dx+c)-3a)\sinh(dx+c)^3+5a\cosh(dx+c)^2+\left(6a\cosh(dx+c)^2-9a\cos\right.}{}\right.}{}$$

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c
)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*
a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x +
c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x
 + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*c
osh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c
) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d
*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*co
sh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
 + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 -
9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(
d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c
)^2 + sinh(d*x + c)^3)) + sqrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 + cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) + 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x +
c) + a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

Sympy [F]

$$\int \sqrt{a+a\mathrm{sech}(c+dx)}\,dx = \int \sqrt{a\,\mathrm{sech}\,(c+dx)+a}\,dx$$

```
[In] integrate((a+a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a+a\mathrm{sech}(c+dx)}\,dx = \int \sqrt{a\,\mathrm{sech}\,(dx+c)+a}\,dx$$

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sech(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(31) = 62$.

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.24

$$\int \sqrt{a + a\operatorname{sech}(c + dx)}\, dx = \frac{\frac{2\, a \arctan\left(-\frac{\sqrt{a}e^{(dx+c)} - \sqrt{ae^{(2\, dx + 2\, c)} + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \sqrt{a}\log\left(\left|-\sqrt{a}e^{(dx+c)} + \sqrt{ae^{(2\, dx + 2\, c)} + a}\right|\right)}{d}$$

```
[In] integrate((a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))/
sqrt(-a) - sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqrt(a*e^(2*d*x + 2*c) +
a))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + a\operatorname{sech}(c + dx)}\, dx = \int \sqrt{a + \frac{a}{\cosh(c + d\, x)}}\, dx$$

```
[In] int((a + a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cosh(c + d*x))^(1/2), x)
```

3.81 $\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/
2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3861, 3859, 209, 3880}

$$\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\text{sech}(c+dx)+a}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{a+a\text{sech}(c+dx)}\,dx}{a} - \int \frac{\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx$$

$$= \frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d} - \frac{(2i)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{d}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx$$
$$= \frac{(1+e^{c+dx})\left(\sqrt{2}\text{arcsinh}\left(e^{c+dx}\right) - 2\text{arctanh}\left(\frac{-1+e^{c+dx}}{\sqrt{2}\sqrt{1+e^{2(c+dx)}}}\right) - \sqrt{2}\text{arctanh}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2(c+dx)}}\sqrt{a(1+\text{sech}(c+dx))}}$$

```
[In] Integrate[1/Sqrt[a + a*Sech[c + d*x]],x]
```

```
[Out] ((1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(-1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] - Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a*(1 + Sech[c + d*x
])])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+\operatorname{sech}\left(dx+c\right)a}}dx$$

```
[In] int(1/(a+sech(d*x+c)*a)^(1/2),x)
```

```
[Out] int(1/(a+sech(d*x+c)*a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 868, normalized size of antiderivative = 10.21

$$\int \frac{1}{\sqrt{a+a\text{sech}(c+dx)}}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) - 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x +
 c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) - 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)) + sqrt(a)*log(-(a*cosh(d*x +
 c)^4 + a*sinh(d*x + c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)
*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*
x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*si
nh(d*x + c)^4 + sinh(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 -
 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
 c)^3 - 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh
(d*x + c)^2 + (5*cosh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2
- 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a
/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) -
 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh
(d*x + c) - 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*
sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(
a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh
```

```
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a + a\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\operatorname{sech}(c+dx)+a}}\,dx$$

```
[In] integrate(1/(a+a*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a + a\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\operatorname{sech}(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sech(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(70) = 140$.

Time = 0.65 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{a + a\text{sech}(c+dx)}}\,dx = \frac{\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}e^{(-dx)}+\sqrt{a}e^{c}-\sqrt{ae^{(-2dx)}+ae^{(2c)}}\right)e^{(-c)}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{\log\left(\left|-\sqrt{a}e^{(-dx)}+\sqrt{a}e^{c}+\sqrt{ae^{(-2dx)}+ae^{(2c)}}\right|\right)}{\sqrt{a}} - \frac{\log\left(\left|-\sqrt{a}e^{(-dx)}-\sqrt{a}\right.\right.}{}}{d}$$

```
[In] integrate(1/(a+a*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] (2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*e^(-d*x) + sqrt(a)*e^c - sqrt(a*e^(
-2*d*x) + a*e^(2*c)))*e^(-c)/sqrt(-a))/sqrt(-a) + log(abs(-sqrt(a)*e^(-d*x)
 + sqrt(a)*e^c + sqrt(a*e^(-2*d*x) + a*e^(2*c))))/sqrt(a) - log(abs(-sqrt(a
)*e^(-d*x) - sqrt(a)*e^c + sqrt(a*e^(-2*d*x) + a*e^(2*c))))/sqrt(a) + log(a
bs(-sqrt(a)*e^(-d*x) + sqrt(a*e^(-2*d*x) + a*e^(2*c))))/sqrt(a))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a + a\operatorname{sech}(c + dx)}}\, dx = \int \frac{1}{\sqrt{a + \frac{a}{\cosh(c+d\,x)}}}\, dx$$

```
[In] int(1/(a + a/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + a/cosh(c + d*x))^(1/2), x)
```

3.82 $\int \frac{1}{(a+a\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 114

$$\int \frac{1}{(a+a\,\mathrm{sech}(c+dx))^{3/2}}\,dx = \frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\,\mathrm{sech}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\,\mathrm{sech}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tanh(c+dx)}{2d(a+a\,\mathrm{sech}(c+dx))^{3/2}}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a+a*sech(d*x+c))^(1/2))/a^(3/2)/d-5/4*arctan
h(1/2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a+a*sech(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2
)-1/2*tanh(d*x+c)/d/(a+a*sech(d*x+c))^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3862, 4005, 3859, 209, 3880}

$$\int \frac{1}{(a+a\,\mathrm{sech}(c+dx))^{3/2}}\,dx = \frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a\,\mathrm{sech}(c+dx)+a}}\right)}{a^{3/2}d} - \frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a\,\mathrm{sech}(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tanh(c+dx)}{2d(a\,\mathrm{sech}(c+dx)+a)^{3/2}}$$

```
[In] Int[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a + a*Sech[c + d*x]]])/(a^(3/2)*d)
- (5*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sech[c + d*x]])])/
(2*Sqrt[2]*a^(3/2)*d) - Tanh[c + d*x]/(2*d*(a + a*Sech[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3862

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Dist[1/(a^2*(2*n + 1))
, Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Inte
gerQ[2*n]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} - \frac{\int \frac{-2a+\frac{1}{2}a\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} + \frac{\int \sqrt{a+a\text{sech}(c+dx)}\,dx}{a^2} - \frac{5\int \frac{\text{sech}(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\,dx}{4a}$$

$$= -\frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}} + \frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{ad} - \frac{(5i)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{ia\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{2ad}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a+a\text{sech}(c+dx)}}\right)}{a^{3/2}d} - \frac{5\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a+a\text{sech}(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\tanh(c+dx)}{2d(a+a\text{sech}(c+dx))^{3/2}}$$

Mathematica [A] (verified)

Time = 4.94 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx = \frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\text{sech}(c+dx)\left(4\left(1+e^{c+dx}\right)\text{arcsinh}\left(e^{c+dx}\right)+5\sqrt{2}\left(1+e^{c+}\right.\right.}{2d\sqrt{1}}$$

```
[In] Integrate[(a + a*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (Cosh[(c + d*x)/2]^2*Sech[c + d*x]*(4*(1 + E^(c + d*x))*ArcSinh[E^(c + d*x)
] + 5*Sqrt[2]*(1 + E^(c + d*x))*ArcTanh[(1 - E^(c + d*x))/(Sqrt[2]*Sqrt[1 +
 E^(2*(c + d*x))])] - 4*(1 + E^(c + d*x))*ArcTanh[Sqrt[1 + E^(2*(c + d*x))]
] - 2*Sqrt[1 + E^(2*(c + d*x))]*Tanh[(c + d*x)/2]))/(2*d*Sqrt[1 + E^(2*(c +
 d*x))]*(a*(1 + Sech[c + d*x]))^(3/2))
```

Maple [F]

$$\int \frac{1}{(a+\text{sech}(dx+c)\,a)^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+sech(d*x+c)*a)^(3/2),x)
```

```
[Out] int(1/(a+sech(d*x+c)*a)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(93) = 186.

Time = 0.30 (sec) , antiderivative size = 1190, normalized size of antiderivative = 10.44

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(5*sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(3*a*cosh(d*x + c)^2 + 3*a
*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(
d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cos
h(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 2*a*cosh(
d*x + c) + 2*(3*a*cosh(d*x + c) - a)*sinh(d*x + c) + 3*a)/(cosh(d*x + c)^2
+ 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) +
 1)) + 4*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x
+ c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*log(-(a*cosh(d*x + c)^4 + a*sinh(d*x
+ c)^4 - 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^3 +
5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 - 9*a*cosh(d*x + c) + 5*a)*sinh(
d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) - 3)*sinh(d*x + c)^4 + sin
h(d*x + c)^5 - 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 - 12*cosh(d*x + c) +
 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x + c)^3 - 18*cosh(d*x
 + c)^2 + 15*cosh(d*x + c) - 7)*sinh(d*x + c)^2 - 7*cosh(d*x + c)^2 + (5*co
sh(d*x + c)^4 - 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 - 14*cosh(d*x + c)
+ 4)*sinh(d*x + c) + 4*cosh(d*x + c) - 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - 4*a*cosh(d*x + c)
+ (4*a*cosh(d*x + c)^3 - 9*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) - 4*a)*si
nh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + 4*(cosh(d*x + c)^2 + 2*(
cosh(d*x + c) + 1)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*s
qrt(a)*log((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*c
osh(d*x + c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*
cosh(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*s
qrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2 + 1)) + a*cosh(d*x + c) + (2*a*cosh(d*x + c) + a)*sinh(d*x + c) + a)/
(cosh(d*x + c) + sinh(d*x + c))) - 4*(cosh(d*x + c)^3 + (3*cosh(d*x + c) -
1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2
 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(a/(cosh(d*x
 + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)))/(a^2*d*cos
h(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2*d + 2*(a
^2*d*cosh(d*x + c) + a^2*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+a\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\,\text{sech}\,(c+dx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sech(d*x+c))**(3/2),x)
```

```
[Out] Integral((a*sech(c + d*x) + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+a\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\,\mathrm{sech}\,(dx+c)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a*sech(d*x + c) + a)^(-3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(93) = 186.

Time = 0.34 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.07

$$\int \frac{1}{(a+a\mathrm{sech}(c+dx))^{3/2}}\,dx = -\frac{\frac{5\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}+\sqrt{a}\right)}{2\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2\left(3\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}\right)^3+\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}\right)^2\sqrt{a}-\left(\sqrt{ae}\right.\right.}{\left(\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c)}+a}\right)^2+2\left(\sqrt{a}e^{(dx+c)}-\sqrt{ae^{(2\,dx+2\,c}}\right.\right.}}{2\,d}$$

```
[In] integrate(1/(a+a*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] -1/2*(5*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x
+ 2*c) + a) + sqrt(a))/sqrt(-a))/(sqrt(-a)*a) + 2*(3*(sqrt(a)*e^(d*x + c) -
 sqrt(a*e^(2*d*x + 2*c) + a))^3 + (sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x +
2*c) + a))^2*sqrt(a) - (sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))*
a + a^(3/2))/(((sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))^2 + 2*(s
qrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))*sqrt(a) - a)^2*a))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+\frac{a}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + a/cosh(c + d*x))^(3/2),x)
[Out] int(1/(a + a/cosh(c + d*x))^(3/2), x)
```

3.83 $\int \sqrt{a - a\,\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \sqrt{a-a\,\text{sech}(c+dx)}\,dx=\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))*a^(1/2)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3859, 209}

$$\int \sqrt{a-a\,\text{sech}(c+dx)}\,dx=\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\,\text{sech}(c+dx)}}\right)}{d}$$

```
[In] Int[Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{(2ia)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}$$

$$= \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}$$

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \sqrt{a-a\text{sech}(c+dx)}\,dx = \frac{\sqrt{1+e^{2(c+dx)}}\left(\operatorname{arcsinh}\left(e^{c+dx}\right)+\operatorname{arctanh}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)\sqrt{a-a\text{sech}(c+dx)}}{d\left(-1+e^{c+dx}\right)}$$

```
[In] Integrate[Sqrt[a - a*Sech[c + d*x]],x]
[Out] (Sqrt[1 + E^(2*(c + d*x))]*(ArcSinh[E^(c + d*x)] + ArcTanh[Sqrt[1 + E^(2*(c
 + d*x))]])*Sqrt[a - a*Sech[c + d*x]])/(d*(-1 + E^(c + d*x)))
```

Maple [F]

$$\int \sqrt{a-\text{sech}\,(dx+c)\,a}dx$$

```
[In] int((a-sech(d*x+c)*a)^(1/2),x)
[Out] int((a-sech(d*x+c)*a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(32) = 64.

Time = 0.28 (sec) , antiderivative size = 642, normalized size of antiderivative = 16.89

$$\int \sqrt{a-a\text{sech}(c+dx)}\,dx = \frac{\sqrt{a}\log\left(\frac{a\cosh(dx+c)^4+a\sinh(dx+c)^4+3\,a\cosh(dx+c)^3+(4\,a\cosh(dx+c)+3\,a)\sinh(dx+c)^3+5\,a\cosh(dx+c)^2+\left(6\,a\cosh(dx+c)^2+9\,a\cosh(\right.}{}\right.}{}$$

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a)*log((a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)
^3 + (4*a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a
*cosh(d*x + c)^2 + 9*a*cosh(d*x + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c
)^5 + (5*cosh(d*x + c) + 3)*sinh(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x
+ c)^4 + (10*cosh(d*x + c)^2 + 12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*co
sh(d*x + c)^3 + (10*cosh(d*x + c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c)
 + 7)*sinh(d*x + c)^2 + 7*cosh(d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*
x + c)^3 + 15*cosh(d*x + c)^2 + 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cos
h(d*x + c) + 4)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x
+ c) + sinh(d*x + c)^2 + 1)) + 4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9
*a*cosh(d*x + c)^2 + 10*a*cosh(d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d
*x + c)^3 + 3*cosh(d*x + c)^2*sinh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)
^2 + sinh(d*x + c)^3)) + sqrt(a)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^
2 + (cosh(d*x + c)^3 + (3*cosh(d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c
)^3 - cosh(d*x + c)^2 + (3*cosh(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x
+ c) + cosh(d*x + c) - 1)*sqrt(a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x +
c) - a)*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x + c))))/d
```

Sympy [F]

$$\int \sqrt{a - a\operatorname{sech}(c + dx)}\, dx = \int \sqrt{-a\operatorname{sech}(c + dx) + a}\, dx$$

```
[In] integrate((a-a*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(-a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \sqrt{a - a\operatorname{sech}(c + dx)}\, dx = \int \sqrt{-a\operatorname{sech}(dx + c) + a}\, dx$$

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*sech(d*x + c) + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(32) = 64$.

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.66

$$\int \sqrt{a - a\text{sech}(c + dx)}\, dx = -\frac{\frac{2\,a\arctan\left(-\frac{\sqrt{a}e^{(dx+c)} - \sqrt{ae^{(2\,dx+2\,c)}+a}}{\sqrt{-a}}\right)\text{sgn}\left(e^{(dx+c)}-1\right)}{\sqrt{-a}} + \sqrt{a}\log\left(\left|-\sqrt{a}e^{(dx+c)} + \sqrt{ae^{(2\,dx+2\,c)} + a}\right|\right)\text{sgn}\left(e^{(dx+c)} -\right.}{d}$$

```
[In] integrate((a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*arctan(-(sqrt(a)*e^(d*x + c) - sqrt(a*e^(2*d*x + 2*c) + a))/sqrt(-a))
*sgn(e^(d*x + c) - 1)/sqrt(-a) + sqrt(a)*log(abs(-sqrt(a)*e^(d*x + c) + sqr
t(a*e^(2*d*x + 2*c) + a)))*sgn(e^(d*x + c) - 1))/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a - a\text{sech}(c + dx)}\, dx = \int \sqrt{a - \frac{a}{\cosh\left(c + d\,x\right)}}\, dx$$

```
[In] int((a - a/cosh(c + d*x))^(1/2),x)
```

```
[Out] int((a - a/cosh(c + d*x))^(1/2), x)
```

3.84 $\int \frac{1}{\sqrt{a-a\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{\sqrt{a-a\operatorname{sech}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[Out] 2*arctanh(a^(1/2)*tanh(d*x+c)/(a-a*sech(d*x+c))^(1/2))/d/a^(1/2)-arctanh(1/
2*a^(1/2)*tanh(d*x+c)*2^(1/2)/(a-a*sech(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3861, 3859, 209, 3880}

$$\int \frac{1}{\sqrt{a-a\operatorname{sech}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\operatorname{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/Sqrt[a - a*Sech[c + d*x]]])/(Sqrt[a]*d)
- (Sqrt[2]*ArcTanh[(Sqrt[a]*Tanh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sech[c + d*x
]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3861

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, In
t[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*
Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \sqrt{a - a\text{sech}(c+dx)}\,dx}{a} + \int \frac{\text{sech}(c+dx)}{\sqrt{a - a\text{sech}(c+dx)}}\,dx$$

$$= -\frac{(2i)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d} + \frac{(2i)\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, \frac{ia\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{d}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}d} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\tanh(c+dx)}{\sqrt{2}\sqrt{a-a\text{sech}(c+dx)}}\right)}{\sqrt{a}d}$$

Mathematica [A] (verified)

Time = 2.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{a - a\text{sech}(c+dx)}}\,dx$$
$$= \frac{(-1+e^{c+dx})\left(\sqrt{2}\text{arcsinh}(e^{c+dx}) - 2\text{arctanh}\left(\frac{1+e^{c+dx}}{\sqrt{2}\sqrt{1+e^{2(c+dx)}}}\right) + \sqrt{2}\text{arctanh}\left(\sqrt{1+e^{2(c+dx)}}\right)\right)}{\sqrt{2}d\sqrt{1+e^{2(c+dx)}}\sqrt{a-a\text{sech}(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a - a*Sech[c + d*x]],x]
```

```
[Out] ((-1 + E^(c + d*x))*(Sqrt[2]*ArcSinh[E^(c + d*x)] - 2*ArcTanh[(1 + E^(c + d
*x))/(Sqrt[2]*Sqrt[1 + E^(2*(c + d*x))])] + Sqrt[2]*ArcTanh[Sqrt[1 + E^(2*(
c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^(2*(c + d*x))]*Sqrt[a - a*Sech[c + d*x]
])
```

Maple [F]

$$\int \frac{1}{\sqrt{a - \operatorname{sech}(dx + c)\, a}}dx$$

```
[In] int(1/(a-sech(d*x+c)*a)^(1/2),x)
```

```
[Out] int(1/(a-sech(d*x+c)*a)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(72) = 144$.

Time = 0.29 (sec) , antiderivative size = 871, normalized size of antiderivative = 10.01

$$\int \frac{1}{\sqrt{a - a\operatorname{sech}(c + dx)}}\, dx = \text{Too large to display}$$

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(a)*log(-(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 1)*sinh
(d*x + c) + 3*sinh(d*x + c)^2 - 2*sqrt(2)*(cosh(d*x + c)^3 + (3*cosh(d*x +
c) + 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + cosh(d*x + c)^2 + (3*cosh(d*x +
 c)^2 + 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) + 1)*sqrt(a/(cos
h(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1))/sqrt(a
) + 2*cosh(d*x + c) + 3)/(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x
+ c) + sinh(d*x + c)^2 - 2*cosh(d*x + c) + 1)) + sqrt(a)*log((a*cosh(d*x +
c)^4 + a*sinh(d*x + c)^4 + 3*a*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + 3*a)*
sinh(d*x + c)^3 + 5*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 9*a*cosh(d*x
 + c) + 5*a)*sinh(d*x + c)^2 + (cosh(d*x + c)^5 + (5*cosh(d*x + c) + 3)*sin
h(d*x + c)^4 + sinh(d*x + c)^5 + 3*cosh(d*x + c)^4 + (10*cosh(d*x + c)^2 +
12*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 5*cosh(d*x + c)^3 + (10*cosh(d*x +
c)^3 + 18*cosh(d*x + c)^2 + 15*cosh(d*x + c) + 7)*sinh(d*x + c)^2 + 7*cosh(
d*x + c)^2 + (5*cosh(d*x + c)^4 + 12*cosh(d*x + c)^3 + 15*cosh(d*x + c)^2 +
 14*cosh(d*x + c) + 4)*sinh(d*x + c) + 4*cosh(d*x + c) + 4)*sqrt(a)*sqrt(a/
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)) +
4*a*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 9*a*cosh(d*x + c)^2 + 10*a*cosh(
d*x + c) + 4*a)*sinh(d*x + c) + 4*a)/(cosh(d*x + c)^3 + 3*cosh(d*x + c)^2*s
inh(d*x + c) + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3)) + sqrt(a
)*log(-(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + (cosh(d*x + c)^3 + (3*cosh(
d*x + c) - 1)*sinh(d*x + c)^2 + sinh(d*x + c)^3 - cosh(d*x + c)^2 + (3*cosh
```

```
(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sinh(d*x + c) + cosh(d*x + c) - 1)*sqrt(
a)*sqrt(a/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^
2 + 1)) - a*cosh(d*x + c) + (2*a*cosh(d*x + c) - a)*sinh(d*x + c) + a)/(cos
h(d*x + c) + sinh(d*x + c))))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}} \, dx = \int \frac{1}{\sqrt{-a\,\text{sech}\,(c + dx) + a}} \, dx$$

```
[In] integrate(1/(a-a*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(-a*sech(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}} \, dx = \int \frac{1}{\sqrt{-a\,\text{sech}\,(dx + c) + a}} \, dx$$

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-a*sech(d*x + c) + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/(a-a*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a - a\text{sech}(c + dx)}}\,dx = \int \frac{1}{\sqrt{a - \frac{a}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(1/(a - a/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a - a/cosh(c + d*x))^(1/2), x)
```

3.85 $\int \sqrt{3+3\text{sech}(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int \sqrt{3+3\text{sech}(x)}\,dx = 2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+\text{sech}(x)}}\right)$$

```
[Out] 2*arctanh(tanh(x)/(1+sech(x))^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3859, 209}

$$\int \sqrt{3+3\text{sech}(x)}\,dx = 2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{\text{sech}(x)+1}}\right)$$

```
[In] Int[Sqrt[3 + 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 + Sech[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, -\frac{3i\tanh(x)}{\sqrt{3+3\text{sech}(x)}}\right)\\ &= 2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{1+\text{sech}(x)}}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \sqrt{3+3\text{sech}(x)}\,dx = \sqrt{6}\text{arcsinh}\left(\sqrt{2}\sinh\left(\frac{x}{2}\right)\right)\sqrt{\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\sqrt{1+\text{sech}(x)}$$

```
[In] Integrate[Sqrt[3 + 3*Sech[x]],x]
[Out] Sqrt[6]*ArcSinh[Sqrt[2]*Sinh[x/2]]*Sqrt[Cosh[x]]*Sech[x/2]*Sqrt[1 + Sech[x]
]
```

Maple [F]

$$\int \sqrt{3+3\,\text{sech}\,(x)}dx$$

```
[In] int((3+3*sech(x))^(1/2),x)
[Out] int((3+3*sech(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(15) = 30$.

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 12.26

$$\int \sqrt{3+3\text{sech}(x)}\,dx$$
$$= \frac{1}{2}\sqrt{3}\log\left(-\frac{\cosh(x)^4 + (4\cosh(x)-3)\sinh(x)^3 + \sinh(x)^4 - 3\cosh(x)^3 + (6\cosh(x)^2 - 9\cosh(x) +}{}\right.$$
$$+\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x)+1) + \cosh(x)^2 + (2\cosh(x)+1)\sinh(x) + \sinh(x)^2}{\cosh(x)+\sinh(x)}\right.$$

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(-(cosh(x)^4 + (4*cosh(x) - 3)*sinh(x)^3 + sinh(x)^4 - 3*cos
h(x)^3 + (6*cosh(x)^2 - 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(
cosh(x) - 1)*sinh(x)^2 + sinh(x)^3 - 3*cosh(x)^2 + (3*cosh(x)^2 - 6*cosh(x)
 + 4)*sinh(x) + 4*cosh(x) - 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x
)^2 + (4*cosh(x)^3 - 9*cosh(x)^2 + 10*cosh(x) - 4)*sinh(x) - 4*cosh(x) + 4)
/(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2
*sqrt(3)*log((sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
+ 1) + cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{3+3\mathrm{sech}(x)}\,dx=\sqrt{3}\int\sqrt{\mathrm{sech}\,(x)+1}\,dx$$

```
[In] integrate((3+3*sech(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(sech(x) + 1), x)
```

Maxima [F]

$$\int \sqrt{3+3\mathrm{sech}(x)}\,dx=\int\sqrt{3\,\mathrm{sech}\,(x)+3}\,dx$$

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*sech(x) + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(15)=30$.

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \sqrt{3+3\mathrm{sech}(x)}\,dx = -\sqrt{3}\left(\log\left(\sqrt{e^{(2\,x)}+1}-e^{x}+1\right)+\log\left(\sqrt{e^{(2\,x)}+1}-e^{x}\right)-\log\left(-\sqrt{e^{(2\,x)}+1}+e^{x}+1\right)\right)$$

```
[In] integrate((3+3*sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1) + log(sqrt(e^(2*x) + 1) - e^x) -
 log(-sqrt(e^(2*x) + 1) + e^x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3+3\operatorname{sech}(x)}\,dx = \int \sqrt{\frac{3}{\cosh(x)}+3}\,dx$$

```
[In] int((3/cosh(x) + 3)^(1/2),x)
[Out] int((3/cosh(x) + 3)^(1/2), x)
```

3.86 $\int \sqrt{3-3\text{sech}(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 21

$$\int \sqrt{3-3\text{sech}(x)}\,dx = 2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

```
[Out] 2*arctanh(tanh(x)/(1-sech(x))^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3859, 209}

$$\int \sqrt{3-3\text{sech}(x)}\,dx = 2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

```
[In] Int[Sqrt[3 - 3*Sech[x]],x]
```

```
[Out] 2*Sqrt[3]*ArcTanh[Tanh[x]/Sqrt[1 - Sech[x]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
```

```
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(6i\text{Subst}\left(\int \frac{1}{3+x^2}\,dx, x, \frac{3i\tanh(x)}{\sqrt{3-3\text{sech}(x)}}\right)\right)$$
$$= 2\sqrt{3}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{1-\text{sech}(x)}}\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \sqrt{3-3\text{sech}(x)}\,dx = \frac{\sqrt{3}\sqrt{1+e^{2x}}\left(\text{arcsinh}(e^x)+\text{arctanh}\left(\sqrt{1+e^{2x}}\right)\right)\sqrt{1-\text{sech}(x)}}{-1+e^x}$$

```
[In] Integrate[Sqrt[3 - 3*Sech[x]],x]
[Out] (Sqrt[3]*Sqrt[1 + E^(2*x)]*(ArcSinh[E^x] + ArcTanh[Sqrt[1 + E^(2*x)]])*Sqrt
[1 - Sech[x]])/(-1 + E^x)
```

Maple [F]

$$\int \sqrt{3-3\,\text{sech}\,(x)}dx$$

```
[In] int((3-3*sech(x))^(1/2),x)
[Out] int((3-3*sech(x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 11.19

$$\int \sqrt{3-3\text{sech}(x)}\,dx$$
$$= \frac{1}{2}\sqrt{3}\log\left(\frac{\cosh(x)^4 + (4\cosh(x)+3)\sinh(x)^3 + \sinh(x)^4 + 3\cosh(x)^3 + \left(6\cosh(x)^2 + 9\cosh(x) + 5\right)}{}\right.$$
$$+ \frac{1}{2}\sqrt{3}\log\left(-\frac{\sqrt{2}\sqrt{\frac{\cosh(x)}{\cosh(x)-\sinh(x)}}(\cosh(x)+\sinh(x)-1) + \cosh(x)^2 + (2\cosh(x)-1)\sinh(x) + \sinh(x}{\cosh(x)+\sinh(x)}\right.$$

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log((cosh(x)^4 + (4*cosh(x) + 3)*sinh(x)^3 + sinh(x)^4 + 3*cosh
(x)^3 + (6*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + sqrt(2)*(cosh(x)^3 + 3*(c
osh(x) + 1)*sinh(x)^2 + sinh(x)^3 + 3*cosh(x)^2 + (3*cosh(x)^2 + 6*cosh(x)
+ 4)*sinh(x) + 4*cosh(x) + 4)*sqrt(cosh(x)/(cosh(x) - sinh(x))) + 5*cosh(x)
^2 + (4*cosh(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 4)*sinh(x) + 4*cosh(x) + 4)/
(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)) + 1/2*
sqrt(3)*log(-(sqrt(2)*sqrt(cosh(x)/(cosh(x) - sinh(x)))*(cosh(x) + sinh(x)
- 1) + cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(cosh
(x) + sinh(x)))
```

Sympy [F]

$$\int \sqrt{3-3\mathrm{sech}(x)}\,dx = \sqrt{3}\int \sqrt{1-\mathrm{sech}\,(x)}\,dx$$

```
[In] integrate((3-3*sech(x))**(1/2),x)
```

```
[Out] sqrt(3)*Integral(sqrt(1 - sech(x)), x)
```

Maxima [F]

$$\int \sqrt{3-3\mathrm{sech}(x)}\,dx = \int \sqrt{-3\,\mathrm{sech}\,(x)+3}\,dx$$

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-3*sech(x) + 3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.29

$$\int \sqrt{3-3\mathrm{sech}(x)}\,dx = \sqrt{3}\left(\log\left(\sqrt{e^{(2x)}+1}-e^x+1\right)\mathrm{sgn}(e^x-1)-\log\left(\sqrt{e^{(2x)}+1}-e^x\right)\mathrm{sgn}(e^x-1)-\log\left(-\sqrt{e^{(2x)}+1}+e\right.\right.$$

```
[In] integrate((3-3*sech(x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(3)*(log(sqrt(e^(2*x) + 1) - e^x + 1)*sgn(e^x - 1) - log(sqrt(e^(2*x) +
 1) - e^x)*sgn(e^x - 1) - log(-sqrt(e^(2*x) + 1) + e^x + 1)*sgn(e^x - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-3\operatorname{sech}(x)}\,dx=\int \sqrt{3-\frac{3}{\cosh\left(x\right)}}\,dx$$

```
[In] int((3 - 3/cosh(x))^(1/2),x)
[Out] int((3 - 3/cosh(x))^(1/2), x)
```

3.87 $\int (a+b\text{sech}(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 107

$$\int (a+b\text{sech}(c+dx))^4\,dx = a^4x+\frac{2ab(2a^2+b^2)\arctan(\sinh(c+dx))}{d}+\frac{b^2(17a^2+2b^2)\tanh(c+dx)}{3d}+\frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d}$$

```
[Out] a^4*x+2*a*b*(2*a^2+b^2)*arctan(sinh(d*x+c))/d+1/3*b^2*(17*a^2+2*b^2)*tanh(d
*x+c)/d+4/3*a*b^3*sech(d*x+c)*tanh(d*x+c)/d+1/3*b^2*(a+b*sech(d*x+c))^2*tan
h(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3867, 4133, 3855, 3852, 8}

$$\int (a+b\text{sech}(c+dx))^4\,dx = a^4x+\frac{2ab(2a^2+b^2)\arctan(\sinh(c+dx))}{d}+\frac{b^2(17a^2+2b^2)\tanh(c+dx)}{3d}+\frac{4ab^3\tanh(c+dx)\text{sech}(c+dx)}{3d}+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))^2}{3d}$$

```
[In] Int[(a + b*Sech[c + d*x])^4,x]
```

```
[Out] a^4*x + (2*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/d + (b^2*(17*a^2 + 2*b^
2)*Tanh[c + d*x])/(3*d) + (4*a*b^3*Sech[c + d*x]*Tanh[c + d*x])/(3*d) + (b^
2*(a + b*Sech[c + d*x])^2*Tanh[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d} \\
&\quad+\frac{1}{3}\int(a+b\text{sech}(c+dx))\left(3a^3+b\left(9a^2+2b^2\right)\text{sech}(c+dx)+8ab^2\text{sech}^2(c+dx)\right)\,dx \\
&=\frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d} \\
&\quad+\frac{1}{6}\int\left(6a^4+12ab\left(2a^2+b^2\right)\text{sech}(c+dx)+2b^2\left(17a^2+2b^2\right)\text{sech}^2(c+dx)\right)\,dx \\
&=a^4x+\frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d}+\frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d} \\
&\quad+\left(2ab\left(2a^2+b^2\right)\right)\int\text{sech}(c+dx)\,dx+\frac{1}{3}\left(b^2\left(17a^2+2b^2\right)\right)\int\text{sech}^2(c+dx)\,dx
\end{aligned}
$$

$$= a^4x + \frac{2ab(2a^2+b^2)\arctan(\sinh(c+dx))}{d} + \frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d} + \frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d} + \frac{(ib^2(17a^2+2b^2))\,\text{Subst}(\int 1\,dx, x, -i\tanh(c+dx))}{3d}$$

$$= a^4x + \frac{2ab(2a^2+b^2)\arctan(\sinh(c+dx))}{d} + \frac{b^2(17a^2+2b^2)\tanh(c+dx)}{3d} + \frac{4ab^3\text{sech}(c+dx)\tanh(c+dx)}{3d} + \frac{b^2(a+b\text{sech}(c+dx))^2\tanh(c+dx)}{3d}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.73

$$\int (a+b\text{sech}(c+dx))^4\,dx$$
$$= \frac{3a^4dx + 6ab(2a^2+b^2)\arctan(\sinh(c+dx)) + 3b^2(6a^2+b^2+2ab\text{sech}(c+dx))\tanh(c+dx) - b^4\tanh^3(c}{3d}$$

```
[In] Integrate[(a + b*Sech[c + d*x])^4,x]
[Out] (3*a^4*d*x + 6*a*b*(2*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + 3*b^2*(6*a^2 + b^2
 + 2*a*b*Sech[c + d*x])*Tanh[c + d*x] - b^4*Tanh[c + d*x]^3)/(3*d)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{a^4(dx+c)+8a^3b\arctan(e^{dx+c})+6a^2b^2\tanh(dx+c)+4a\,b^3\left(\frac{\text{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^4\left(\frac{2}{3}+\frac{\text{sech}(dx+c)^2}{3}\right.}{d}$
default	$\frac{a^4(dx+c)+8a^3b\arctan(e^{dx+c})+6a^2b^2\tanh(dx+c)+4a\,b^3\left(\frac{\text{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)+b^4\left(\frac{2}{3}+\frac{\text{sech}(dx+c)^2}{3}\right.}{d}$
parts	$x\,a^4+\frac{b^4\left(\frac{2}{3}+\frac{\text{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}+\frac{4a\,b^3\left(\frac{\text{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}+\frac{6a^2b^2\tanh(dx+c)}{d}+$
risch	$x\,a^4-\frac{4b^2(-3ab\,e^{5dx+5c}+9a^2e^{4dx+4c}+18a^2e^{2dx+2c}+3\,e^{2dx+2c}b^2+3\,e^{dx+c}ab+9a^2+b^2)}{3d(e^{2dx+2c}+1)^3}+\frac{4ia^3b\ln(e^{dx+c}+i)}{d}+$
parallelrisch	$\frac{-36i\left(\frac{\cosh(3dx+3c)}{3}+\cosh(dx+c)\right)ba\left(a^2+\frac{b^2}{2}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+36i\left(\frac{\cosh(3dx+3c)}{3}+\cosh(dx+c)\right)ba\left(a^2+\frac{b^2}{2}\right)\ln(\text{t}}{3d(\cosh}$

```
[In] int((a+b*sech(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)+8*a^3*b*arctan(exp(d*x+c))+6*a^2*b^2*tanh(d*x+c)+4*a*b^3*(
1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^4*(2/3+1/3*sech(d*x+c)^2)
*tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(101) = 202$.

Time = 0.27 (sec) , antiderivative size = 1028, normalized size of antiderivative = 9.61

$$\int (a + b\text{sech}(c + dx))^4 \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^4*d*x*cosh(d*x + c)^6 + 3*a^4*d*x*sinh(d*x + c)^6 + 12*a*b^3*cosh(
d*x + c)^5 + 3*a^4*d*x + 6*(3*a^4*d*x*cosh(d*x + c) + 2*a*b^3)*sinh(d*x + c
)^5 - 12*a*b^3*cosh(d*x + c) + 9*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^4 + 3*
(15*a^4*d*x*cosh(d*x + c)^2 + 3*a^4*d*x + 20*a*b^3*cosh(d*x + c) - 12*a^2*b
^2)*sinh(d*x + c)^4 - 36*a^2*b^2 - 4*b^4 + 12*(5*a^4*d*x*cosh(d*x + c)^3 +
10*a*b^3*cosh(d*x + c)^2 + 3*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c)^2 + 3*(15*a^4*d*x
*cosh(d*x + c)^4 + 40*a*b^3*cosh(d*x + c)^3 + 3*a^4*d*x - 24*a^2*b^2 - 4*b^
4 + 18*(a^4*d*x - 4*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((2*a^3*
b + a*b^3)*cosh(d*x + c)^6 + 6*(2*a^3*b + a*b^3)*cosh(d*x + c)*sinh(d*x + c
)^5 + (2*a^3*b + a*b^3)*sinh(d*x + c)^6 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c)
^4 + 3*(2*a^3*b + a*b^3 + 5*(2*a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 2*a^3*b + a*b^3 + 4*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + 3*(2*a^3*b
 + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d*x + c
)^2 + 3*(5*(2*a^3*b + a*b^3)*cosh(d*x + c)^4 + 2*a^3*b + a*b^3 + 6*(2*a^3*b
 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 6*((2*a^3*b + a*b^3)*cosh(d*x
+ c)^5 + 2*(2*a^3*b + a*b^3)*cosh(d*x + c)^3 + (2*a^3*b + a*b^3)*cosh(d*x +
 c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*(3*a^4*d*x*co
sh(d*x + c)^5 + 10*a*b^3*cosh(d*x + c)^4 - 2*a*b^3 + 6*(a^4*d*x - 4*a^2*b^2
)*cosh(d*x + c)^3 + (3*a^4*d*x - 24*a^2*b^2 - 4*b^4)*cosh(d*x + c))*sinh(d*
x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x
 + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4
 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d
*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)
^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

Sympy [F]

$$\int (a+b\text{sech}(c+dx))^4\,dx=\int (a+b\,\text{sech}\,(c+dx))^4\;dx$$

```
[In] integrate((a+b*sech(d*x+c))**4,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(101) = 202$.

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.97

$$\begin{aligned}
&\int (a+b\text{sech}(c+dx))^4\,dx\\
&=a^4x-4\,ab^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\
&\quad+\frac{4}{3}\,b^4\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)\\
&\quad+\frac{4\,a^3b\arctan\left(\sinh\left(dx+c\right)\right)}{d}+\frac{12\,a^2b^2}{d(e^{(-2\,dx-2\,c)}+1)}
\end{aligned}$$

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] a^4*x - 4*a*b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))
/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 4/3*b^4*(3*e^(-2*d*x -
2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) +
 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) +
4*a^3*b*arctan(sinh(d*x + c))/d + 12*a^2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.32

$$\begin{aligned}
&\int (a+b\text{sech}(c+dx))^4\,dx\\
&=\frac{3\,(dx+c)a^4+12\,(2\,a^3b+ab^3)\arctan\left(e^{(dx+c)}\right)+\frac{4\,(3\,ab^3e^{(5\,dx+5\,c)}-9\,a^2b^2e^{(4\,dx+4\,c)}-18\,a^2b^2e^{(2\,dx+2\,c)}-3\,b^4e^{(2\,dx+2\,c)}-3\,a\cdots}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}
\end{aligned}$$

```
[In] integrate((a+b*sech(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 + 12*(2*a^3*b + a*b^3)*arctan(e^(d*x + c)) + 4*(3*a*b^
3*e^(5*d*x + 5*c) - 9*a^2*b^2*e^(4*d*x + 4*c) - 18*a^2*b^2*e^(2*d*x + 2*c)
- 3*b^4*e^(2*d*x + 2*c) - 3*a*b^3*e^(d*x + c) - 9*a^2*b^2 - b^4)/(e^(2*d*x
+ 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.18

$$\int (a+b\mathrm{sech}(c+dx))^4\,dx = a^4\,x - \frac{\frac{12\,a^2\,b^2}{d}-\frac{4\,a\,b^3\,\mathrm{e}^{c+d\,x}}{d}}{\mathrm{e}^{2\,c+2\,d\,x}+1} - \frac{\frac{4\,b^4}{d}+\frac{8\,a\,b^3\,\mathrm{e}^{c+d\,x}}{d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1} + \frac{8\,b^4}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} + \frac{4\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,b^3\,\sqrt{d^2}+2\,a^3\,b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6}}\right)\,\sqrt{4\,a^6\,b^2+4\,a^4\,b^4+a^2\,b^6}}{\sqrt{d^2}}$$

```
[In] int((a + b/cosh(c + d*x))^4,x)
```

```
[Out] a^4*x - ((12*a^2*b^2)/d - (4*a*b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1)
- ((4*b^4)/d + (8*a*b^3*exp(c + d*x))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*
d*x) + 1) + (8*b^4)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c
 + 6*d*x) + 1)) + (4*atan((exp(d*x)*exp(c)*(a*b^3*(d^2)^(1/2) + 2*a^3*b*(d^
2)^(1/2)))/(d*(a^2*b^6 + 4*a^4*b^4 + 4*a^6*b^2)^(1/2)))*(a^2*b^6 + 4*a^4*b^
4 + 4*a^6*b^2)^(1/2))/(d^2)^(1/2)
```

3.88 $\int (a+b\text{sech}(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int (a+b\text{sech}(c+dx))^3\,dx = a^3x+\frac{b(6a^2+b^2)\arctan(\sinh(c+dx))}{2d}+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2(a+b\text{sech}(c+dx))\tanh(c+dx)}{2d}$$

```
[Out] a^3*x+1/2*b*(6*a^2+b^2)*arctan(sinh(d*x+c))/d+5/2*a*b^2*tanh(d*x+c)/d+1/2*b
^2*(a+b*sech(d*x+c))*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3867, 3855, 3852, 8}

$$\int (a+b\text{sech}(c+dx))^3\,dx = a^3x+\frac{b(6a^2+b^2)\arctan(\sinh(c+dx))}{2d}+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2\tanh(c+dx)(a+b\text{sech}(c+dx))}{2d}$$

```
[In] Int[(a + b*Sech[c + d*x])^3,x]
```

```
[Out] a^3*x + (b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]])/(2*d) + (5*a*b^2*Tanh[c + d
*x])/(2*d) + (b^2*(a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3867

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Dist[1/(n - 1)
, Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a
^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2(a+b\,\text{sech}(c+dx))\tanh(c+dx)}{2d} \\
&\quad+\frac{1}{2}\int\left(2a^3+b\left(6a^2+b^2\right)\text{sech}(c+dx)+5ab^2\text{sech}^2(c+dx)\right)\,dx \\
&= a^3x+\frac{b^2(a+b\,\text{sech}(c+dx))\tanh(c+dx)}{2d} \\
&\quad+\frac{1}{2}\left(5ab^2\right)\int\text{sech}^2(c+dx)\,dx+\frac{1}{2}\left(b\left(6a^2+b^2\right)\right)\int\text{sech}(c+dx)\,dx \\
&= a^3x+\frac{b(6a^2+b^2)\arctan(\sinh(c+dx))}{2d}+\frac{b^2(a+b\,\text{sech}(c+dx))\tanh(c+dx)}{2d} \\
&\quad+\frac{(5iab^2)\,\text{Subst}(\int 1\,dx,x,-i\tanh(c+dx))}{2d} \\
&= a^3x+\frac{b(6a^2+b^2)\arctan(\sinh(c+dx))}{2d} \\
&\quad+\frac{5ab^2\tanh(c+dx)}{2d}+\frac{b^2(a+b\,\text{sech}(c+dx))\tanh(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int (a+b\operatorname{sech}(c+dx))^3\,dx = \frac{2a^3dx+b(6a^2+b^2)\arctan(\sinh(c+dx))+b^2(6a+b\operatorname{sech}(c+dx))\tanh(c+dx)}{2d}$$

[In] Integrate[(a + b*Sech[c + d*x])^3,x]

[Out] (2*a^3*d*x + b*(6*a^2 + b^2)*ArcTan[Sinh[c + d*x]] + b^2*(6*a + b*Sech[c + d*x])*Tanh[c + d*x])/(2*d)

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{a^3(dx+c)+6a^2b\arctan(e^{dx+c})+3ab^2\tanh(dx+c)+b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}$
default	$\frac{a^3(dx+c)+6a^2b\arctan(e^{dx+c})+3ab^2\tanh(dx+c)+b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}$
parts	$a^3x+\frac{b^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan(e^{dx+c})\right)}{d}+\frac{3ab^2\tanh(dx+c)}{d}+\frac{3a^2b\arctan(\sinh(dx+c))}{d}$
parallelrisch	$\frac{-3i(1+\cosh(2dx+2c))b\left(a^2+\frac{b^2}{6}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)+3i(1+\cosh(2dx+2c))b\left(a^2+\frac{b^2}{6}\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)+a^3dx\cosh}{d(1+\cosh(2dx+2c))}$
risch	$a^3x-\frac{b^2(-e^{3dx+3c}b+6e^{2dx+2c}a+e^{dx+c}b+6a)}{d(e^{2dx+2c}+1)^2}+\frac{3ib\ln(e^{dx+c}+i)a^2}{d}+\frac{ib^3\ln(e^{dx+c}+i)}{2d}-\frac{3ib\ln(e^{dx+c}-i)a^2}{d}-$

[In] int((a+b*sech(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^3*(d*x+c)+6*a^2*b*arctan(exp(d*x+c))+3*a*b^2*tanh(d*x+c)+b^3*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 521, normalized size of antiderivative = 7.14

$$\int (a+b\operatorname{sech}(c+dx))^3\,dx = \frac{a^3dx\cosh(dx+c)^4+a^3dx\sinh(dx+c)^4+b^3\cosh(dx+c)^3+a^3dx-b^3\cosh(dx+c)+(4a^3dx\cosh(dx}{}$$

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] (a^3*d*x*cosh(d*x + c)^4 + a^3*d*x*sinh(d*x + c)^4 + b^3*cosh(d*x + c)^3 +
a^3*d*x - b^3*cosh(d*x + c) + (4*a^3*d*x*cosh(d*x + c) + b^3)*sinh(d*x + c)
^3 - 6*a*b^2 + 2*(a^3*d*x - 3*a*b^2)*cosh(d*x + c)^2 + (6*a^3*d*x*cosh(d*x
+ c)^2 + 2*a^3*d*x + 3*b^3*cosh(d*x + c) - 6*a*b^2)*sinh(d*x + c)^2 + ((6*a
^2*b + b^3)*cosh(d*x + c)^4 + 4*(6*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c)
^3 + (6*a^2*b + b^3)*sinh(d*x + c)^4 + 6*a^2*b + b^3 + 2*(6*a^2*b + b^3)*co
sh(d*x + c)^2 + 2*(6*a^2*b + b^3 + 3*(6*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 4*((6*a^2*b + b^3)*cosh(d*x + c)^3 + (6*a^2*b + b^3)*cosh(d*x
+ c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + (4*a^3*d*x*cos
h(d*x + c)^3 + 3*b^3*cosh(d*x + c)^2 - b^3 + 4*(a^3*d*x - 3*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^
3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*s
inh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int (a + b\mathrm{sech}(c + dx))^3\,dx = \int (a + b\,\mathrm{sech}\,(c + dx))^3\,dx$$

```
[In] integrate((a+b*sech(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.56

$$\int (a + b\mathrm{sech}(c + dx))^3\,dx = a^3x - b^3\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)} - e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{3\,a^2b\arctan\left(\sinh\left(dx + c\right)\right)}{d} + \frac{6\,ab^2}{d(e^{(-2\,dx-2\,c)} + 1)}$$

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] a^3*x - b^3*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*
(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 3*a^2*b*arctan(sinh(d*x + c
))/d + 6*a*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.26

$$\int (a + b\text{sech}(c + dx))^3 \, dx = \frac{(dx + c)a^3 + (6\,a^2b + b^3)\arctan\left(e^{(dx+c)}\right) + \frac{b^3e^{(3\,dx+3\,c)} - 6\,ab^2e^{(2\,dx+2\,c)} - b^3e^{(dx+c)} - 6\,ab^2}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

```
[In] integrate((a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^3 + (6*a^2*b + b^3)*arctan(e^(d*x + c)) + (b^3*e^(3*d*x + 3*c)
 - 6*a*b^2*e^(2*d*x + 2*c) - b^3*e^(d*x + c) - 6*a*b^2)/(e^(2*d*x + 2*c) +
1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.26

$$\int (a + b\text{sech}(c + dx))^3 \, dx = a^3\,x - \frac{\frac{6\,a\,b^2}{d} - \frac{b^3\,e^{c+d\,x}}{d}}{e^{2\,c+2\,d\,x} + 1} + \frac{\text{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(b^3\,\sqrt{d^2}+6\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}\right)\,\sqrt{36\,a^4\,b^2+12\,a^2\,b^4+b^6}}{\sqrt{d^2}} - \frac{2\,b^3\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x} + e^{4\,c+4\,d\,x} + 1\right)}$$

```
[In] int((a + b/cosh(c + d*x))^3,x)
```

```
[Out] a^3*x - ((6*a*b^2)/d - (b^3*exp(c + d*x))/d)/(exp(2*c + 2*d*x) + 1) + (atan
((exp(d*x)*exp(c)*(b^3*(d^2)^(1/2) + 6*a^2*b*(d^2)^(1/2)))/(d*(b^6 + 12*a^2
*b^4 + 36*a^4*b^2)^(1/2)))*(b^6 + 12*a^2*b^4 + 36*a^4*b^2)^(1/2))/(d^2)^(1/
2) - (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

3.89 $\int (a+b\text{sech}(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int (a+b\text{sech}(c+dx))^2\,dx=a^2x+\frac{2ab\arctan(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

```
[Out] a^2*x+2*a*b*arctan(sinh(d*x+c))/d+b^2*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3858, 3855, 3852, 8}

$$\int (a+b\text{sech}(c+dx))^2\,dx=a^2x+\frac{2ab\arctan(\sinh(c+dx))}{d}+\frac{b^2\tanh(c+dx)}{d}$$

```
[In] Int[(a + b*Sech[c + d*x])^2,x]
```

```
[Out] a^2*x + (2*a*b*ArcTan[Sinh[c + d*x]])/d + (b^2*Tanh[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a^2x + (2ab)\int \operatorname{sech}(c+dx)\,dx + b^2\int \operatorname{sech}^2(c+dx)\,dx \\
&= a^2x + \frac{2ab\arctan(\sinh(c+dx))}{d} + \frac{(ib^2)\,\text{Subst}\left(\int 1\,dx, x, -i\tanh(c+dx)\right)}{d} \\
&= a^2x + \frac{2ab\arctan(\sinh(c+dx))}{d} + \frac{b^2\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a+b\operatorname{sech}(c+dx))^2\,dx = \frac{a(adx+2b\arctan(\sinh(c+dx)))+b^2\tanh(c+dx)}{d}$$

```
[In] Integrate[(a + b*Sech[c + d*x])^2,x]
[Out] (a*(a*d*x + 2*b*ArcTan[Sinh[c + d*x]]) + b^2*Tanh[c + d*x])/d
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

method	result	s
parts	$a^2x+\frac{2ab\arctan(\sinh(dx+c))}{d}+\frac{b^2\tanh(dx+c)}{d}$	3
derivativedivides	$\frac{a^2(dx+c)+4ab\arctan(e^{dx+c})+b^2\tanh(dx+c)}{d}$	3
default	$\frac{a^2(dx+c)+4ab\arctan(e^{dx+c})+b^2\tanh(dx+c)}{d}$	3
risch	$a^2x-\frac{2b^2}{d(e^{2dx+2c}+1)}+\frac{2iba\ln(e^{dx+c}+i)}{d}-\frac{2iba\ln(e^{dx+c}-i)}{d}$	6
parallelrisch	$\frac{-2i\cosh(dx+c)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)ab+2i\cosh(dx+c)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)ab+a^2dx\cosh(dx+c)+b^2\sinh(dx+c)}{d\cosh(dx+c)}$	8

```
[In] int((a+b*sech(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+2*a*b*arctan(sinh(d*x+c))/d+b^2*tanh(d*x+c)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(33) = 66$.

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 4.76

$$\int (a + b\text{sech}(c + dx))^2 \, dx$$
$$= \frac{a^2dx\cosh\left(dx+c\right)^2 + 2\,a^2dx\cosh\left(dx+c\right)\sinh\left(dx+c\right) + a^2dx\sinh\left(dx+c\right)^2 + a^2dx - 2\,b^2 + 4\left(ab\cosh\left(c\right.\right.}{d\cosh\left(dx+c\right)^2 + 2\,d\cosh\left(dx+c\right)\text{s}}$$

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="fricas")
[Out] (a^2*d*x*cosh(d*x + c)^2 + 2*a^2*d*x*cosh(d*x + c)*sinh(d*x + c) + a^2*d*x*
sinh(d*x + c)^2 + a^2*d*x - 2*b^2 + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + a*b)*arctan(cosh(d*x + c) + sin
h(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(
d*x + c)^2 + d)
```

Sympy [F]

$$\int (a + b\text{sech}(c + dx))^2 \, dx = \int \left(a + b\,\text{sech}\left(c + dx\right)\right)^2 \, dx$$

```
[In] integrate((a+b*sech(d*x+c))**2,x)
[Out] Integral((a + b*sech(c + d*x))**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.24

$$\int (a + b\text{sech}(c + dx))^2 \, dx = a^2x + \frac{2\,ab\arctan\left(\sinh\left(dx+c\right)\right)}{d} + \frac{2\,b^2}{d(e^{(-2\,dx-2\,c)} + 1)}$$

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="maxima")
[Out] a^2*x + 2*a*b*arctan(sinh(d*x + c))/d + 2*b^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int (a + b\text{sech}(c + dx))^2 \, dx = \frac{(dx + c)a^2 + 4\,ab\arctan\left(e^{(dx+c)}\right) - \frac{2\,b^2}{e^{(2\,dx+2\,c)}+1}}{d}$$

```
[In] integrate((a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a^2 + 4*a*b*arctan(e^(d*x + c)) - 2*b^2/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int (a + b\text{sech}(c + dx))^2 \, dx = a^2\,x - \frac{2\,b^2}{d\,\left(e^{2\,c+2\,d\,x} + 1\right)} + \frac{4\,\text{atan}\left(\frac{a\,b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}$$

```
[In] int((a + b/cosh(c + d*x))^2,x)
```

```
[Out] a^2*x - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) + (4*atan((a*b*exp(d*x)*exp(c)*(
d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2)
```

3.90 $\int (a+b\,\mathbf{sech}(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int (a+b\operatorname{sech}(c+dx))\,dx=ax+\frac{b\arctan(\sinh(c+dx))}{d}$$

```
[Out] a*x+b*arctan(sinh(d*x+c))/d
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3855}

$$\int (a+b\operatorname{sech}(c+dx))\,dx=ax+\frac{b\arctan(\sinh(c+dx))}{d}$$

```
[In] Int[a + b*Sech[c + d*x],x]
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=ax+b\int \operatorname{sech}(c+dx)\,dx\\&=ax+\frac{b\arctan(\sinh(c+dx))}{d}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + b\operatorname{sech}(c + dx))\,dx = ax + \frac{b\arctan(\sinh(c + dx))}{d}$$

```
[In] Integrate[a + b*Sech[c + d*x],x]
[Out] a*x + (b*ArcTan[Sinh[c + d*x]])/d
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$ax + \frac{b\arctan(\sinh(dx+c))}{d}$	17
parts	$ax + \frac{b\arctan(\sinh(dx+c))}{d}$	17
derivativedivides	$\frac{a(dx+c)+b\arctan(\sinh(dx+c))}{d}$	22
risch	$ax + \frac{ib\ln(\mathrm{e}^{dx+c}+i)}{d} - \frac{ib\ln(\mathrm{e}^{dx+c}-i)}{d}$	39
parallelrisch	$-\frac{ib\left(\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)\right)}{d} + ax$	41

```
[In] int(a+b*sech(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*x+b*arctan(sinh(d*x+c))/d
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int (a + b\operatorname{sech}(c + dx))\,dx = \frac{adx + 2\,b\arctan\left(\cosh\left(dx + c\right) + \sinh\left(dx + c\right)\right)}{d}$$

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="fricas")
[Out] (a*d*x + 2*b*arctan(cosh(d*x + c) + sinh(d*x + c)))/d
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int (a + b\text{sech}(c + dx))\, dx = ax + b \left(\begin{cases} \frac{2\,\text{atan}\left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} & \text{for } d \neq 0 \\ x\,\text{sech}\,(c) & \text{otherwise} \end{cases} \right)$$

```
[In] integrate(a+b*sech(d*x+c),x)
[Out] a*x + b*Piecewise((2*atan(tanh(c/2 + d*x/2))/d, Ne(d, 0)), (x*sech(c), True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (a + b\text{sech}(c + dx))\, dx = ax + \frac{b \arctan\left(\sinh\left(dx + c\right)\right)}{d}$$

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="maxima")
[Out] a*x + b*arctan(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (a + b\text{sech}(c + dx))\, dx = ax + \frac{2\, b \arctan\left(e^{(dx+c)}\right)}{d}$$

```
[In] integrate(a+b*sech(d*x+c),x, algorithm="giac")
[Out] a*x + 2*b*arctan(e^(d*x + c))/d
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.38

$$\int (a + b\text{sech}(c + dx))\, dx = a\, x + \frac{2\, \text{atan}\left(\frac{b\, e^{d\, x}\, e^{c}\, \sqrt{d^2}}{d\, \sqrt{b^2}}\right)\, \sqrt{b^2}}{\sqrt{d^2}}$$

```
[In] int(a + b/cosh(c + d*x),x)
```

```
[Out] a*x + (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))
/(d^2)^(1/2)
```

3.91 $\int \frac{1}{a+b\,\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{1}{a+b\text{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}$$

```
[Out] x/a-2*b*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/d/(a-b)^(1/2)
/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3868, 2738, 214}

$$\int \frac{1}{a+b\text{sech}(c+dx)}\,dx=\frac{x}{a}-\frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[(a + b*Sech[c + d*x])^(-1),x]
```

```
[Out] x/a - (2*b*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(a*Sqrt[a -
 b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{a} - \frac{\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a}\\
&= \frac{x}{a} + \frac{(2i)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad}\\
&= \frac{x}{a} - \frac{2b\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$
\int \frac{1}{a+b\,\text{sech}(c+dx)}\,dx = \frac{\frac{c}{d}+x+\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}d}}{a}
$$

```
[In] Integrate[(a + b*Sech[c + d*x])^(-1),x]
[Out] (c/d + x + (2*b*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(Sqrt
[a^2 - b^2]*d))/a
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.41

method	result	size
derivativedivides	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	83
default	$\frac{\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a}-\frac{2b\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$	83
risch	$\frac{x}{a}-\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,da}+\frac{b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,da}$	136

```
[In] int(1/(a+b*sech(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*ln(1+tanh(1/2*d*x+1/2*c))-1/a*ln(tanh(1/2*d*x+1/2*c)-1)-2*b/a/((a+
b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 4.58

$$\int \frac{1}{a+b\mathrm{sech}(c+dx)}\,dx$$

$$=\left[\frac{(a^2-b^2)dx-\sqrt{-a^2+b^2}b\log\left(\frac{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)-a^2+2\,b^2+2\,(a^2\cosh(dx+c)+ab)\sinh(dx+c)+2\,\sqrt{}}{a\cosh(dx+c)^2+a\sinh(dx+c)^2+2\,b\cosh(dx+c)+2\,(a\cosh(dx+c)+b)\sin}\right.}{(a^3-ab^2)d}\right.$$

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="fricas")
```

```
[Out] [((a^2 - b^2)*d*x - sqrt(-a^2 + b^2)*b*log((a^2*cosh(d*x + c)^2 + a^2*sinh(
d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c) + b
))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d
*x + c) + b)*sinh(d*x + c) + a)))/((a^3 - a*b^2)*d), ((a^2 - b^2)*d*x + 2*s
qrt(a^2 - b^2)*b*arctan(-(a*cosh(d*x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 -
 b^2)))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{a+b\mathrm{sech}(c+dx)}\,dx = \int \frac{1}{a+b\operatorname{sech}(c+dx)}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c)),x)
[Out] Integral(1/(a + b*sech(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\mathrm{sech}(c+dx)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{1}{a+b\mathrm{sech}(c+dx)}\,dx = -\frac{\frac{2\,b\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a} - \frac{dx+c}{a}}{d}$$

```
[In] integrate(1/(a+b*sech(d*x+c)),x, algorithm="giac")
[Out] -(2*b*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a) - (d*
x + c)/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.22

$$\int \frac{1}{a + b\text{sech}(c + dx)}\, dx = \frac{x}{a} + \frac{b\,\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} - \frac{2\,b\,\left(a+b\,e^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b\,\ln\left(\frac{2\,b\,e^{c+d\,x}}{a^2} + \frac{2\,b\,\left(a+b\,e^{c+d\,x}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(1/(a + b/cosh(c + d*x)),x)
```

```
[Out] x/a + (b*log((2*b*exp(c + d*x))/a^2 - (2*b*(a + b*exp(c + d*x)))/(a^2*(a +
b)^(1/2)*(b - a)^(1/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2)) - (b*log((2*b*e
xp(c + d*x))/a^2 + (2*b*(a + b*exp(c + d*x)))/(a^2*(a + b)^(1/2)*(b - a)^(1
/2))))/(a*d*(a + b)^(1/2)*(b - a)^(1/2))
```

3.92 $\int \frac{1}{(a+b\text{sech}(c+dx))^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 109

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2} dx = \frac{x}{a^2} - \frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b^2\tanh(c+dx)}{a(a^2-b^2)\,d(a+b\text{sech}(c+dx))}$$

```
[Out] x/a^2-2*b*(2*a^2-b^2)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c)
)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3870, 4004, 3916, 2738, 214}

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2} dx = -\frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}} + \frac{b^2\tanh(c+dx)}{ad(a^2-b^2)(a+b\text{sech}(c+dx))} + \frac{x}{a^2}$$

```
[In] Int[(a + b*Sech[c + d*x])^(-2),x]
```

```
[Out] x/a^2 - (2*b*(2*a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a +
b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*(a + b*Sech[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))} - \frac{\int \frac{-a^2+b^2+ab\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx}{a\,(a^2-b^2)} \\
&= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))} - \frac{(b(2a^2-b^2))\int \frac{\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx}{a^2\,(a^2-b^2)} \\
&= \frac{x}{a^2} + \frac{b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d(a+b\text{sech}(c+dx))} - \frac{(2a^2-b^2)\int \frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{a^2\,(a^2-b^2)}
\end{aligned}$$

$$=\frac{x}{a^2}+\frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))}+\frac{(2i(2a^2-b^2))\,\text{Subst}\left(\int\frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^2\left(a^2-b^2\right)d}$$

$$=\frac{x}{a^2}-\frac{2b(2a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.86

$$\int\frac{1}{(a+b\text{sech}(c+dx))^2}\,dx$$
$$=\frac{a\left(\left(a^2-b^2\right)^{3/2}(c+dx)+\left(4a^2b-2b^3\right)\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)\cosh(c+dx)+b\left(\left(a^2-b^2\right)^{3/2}(c+dx\right.}{a^2(a-b)(a+b)\sqrt{a^2-b^2}d(b+a\cosh(c}$$

```
[In] Integrate[(a + b*Sech[c + d*x])^(-2),x]
```

```
[Out] (a*((a^2 - b^2)^(3/2)*(c + d*x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(
c + d*x)/2])/Sqrt[a^2 - b^2]])*Cosh[c + d*x] + b*((a^2 - b^2)^(3/2)*(c + d*
x) + (4*a^2*b - 2*b^3)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqrt[a^2 - b^2]]
 + a*b*Sqrt[a^2 - b^2]*Sinh[c + d*x]))/(a^2*(a - b)*(a + b)*Sqrt[a^2 - b^2]
*d*(b + a*Cosh[c + d*x]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

method	result
derivativedivides	$\frac{-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^2}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-}{d}$
default	$\frac{-\frac{2b\left(-\frac{ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)}+\frac{\left(2a^2-b^2\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\right)}{a^2}+\frac{\ln\left(1+\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-}{d}$
risch	$\frac{x}{a^2}-\frac{2b^2\left(e^{dx+c}b+a\right)}{d\,a^2(a^2-b^2)\left(e^{2dx+2c}a+2\,e^{dx+c}b+a\right)}-\frac{2b\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}+\frac{b^3\ln\left(e^{dx+c}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right.}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d\,a^2}$

```
[In] int(1/(a+b*sech(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^2*b*(-a*b/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2*a-
tanh(1/2*d*x+1/2*c)^2*b+a+b)+(2*a^2-b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*ar
ctan((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+1/a^2*ln(1+tanh(1/2*d*
x+1/2*c))-1/a^2*ln(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(100) = 200$.

Time = 0.29 (sec) , antiderivative size = 1207, normalized size of antiderivative = 11.07

$$\int \frac{1}{(a + b\text{sech}(c + dx))^2} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-(2*a^3*b^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a
^5 - 2*a^3*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x
 + (2*a^3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)
*sinh(d*x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 +
 (2*a^3*b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*
cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x
 + c) + a*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*
cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(a^2*b^3 -
b^5 - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^
5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*s
inh(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5
*b^2 + a^3*b^4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(
d*x + c) + (a^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d
*cosh(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)), -(2*a^3*b
^2 - 2*a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x*cosh(d*x + c)^2 - (a^5 - 2*a^3
*b^2 + a*b^4)*d*x*sinh(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4)*d*x - 2*(2*a^
3*b - a*b^3 + (2*a^3*b - a*b^3)*cosh(d*x + c)^2 + (2*a^3*b - a*b^3)*sinh(d*
x + c)^2 + 2*(2*a^2*b^2 - b^4)*cosh(d*x + c) + 2*(2*a^2*b^2 - b^4 + (2*a^3*
b - a*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cosh(d*
x + c) + a*sinh(d*x + c) + b)/sqrt(a^2 - b^2)) + 2*(a^2*b^3 - b^5 - (a^4*b
- 2*a^2*b^3 + b^5)*d*x)*cosh(d*x + c) + 2*(a^2*b^3 - b^5 - (a^5 - 2*a^3*b^2
 + a*b^4)*d*x*cosh(d*x + c) - (a^4*b - 2*a^2*b^3 + b^5)*d*x)*sinh(d*x + c))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c)^2 + (a^7 - 2*a^5*b^2 + a^3*b^
4)*d*sinh(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c) + (a
^7 - 2*a^5*b^2 + a^3*b^4)*d + 2*((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cosh(d*x + c
) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx = \int \frac{1}{\left(a+b\,\text{sech}\,(c+dx)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sech(c + d*x))**(-2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int \frac{1}{(a+b\text{sech}(c+dx))^2}\,dx = -\frac{\frac{2\,(2\,a^2b-b^3)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{(a^4-a^2b^2)\sqrt{a^2-b^2}} + \frac{2\,(b^3e^{(dx+c)}+ab^2)}{(a^4-a^2b^2)(ae^{(2\,dx+2\,c)}+2\,be^{(dx+c)}+a)} - \frac{dx+c}{a^2}}{d}$$

```
[In] integrate(1/(a+b*sech(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*(2*a^2*b - b^3)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))/((a^4 - a^2
*b^2)*sqrt(a^2 - b^2)) + 2*(b^3*e^(d*x + c) + a*b^2)/((a^4 - a^2*b^2)*(a*e^
(2*d*x + 2*c) + 2*b*e^(d*x + c) + a)) - (d*x + c)/a^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.72

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^2}\,dx=\frac{\frac{2\,b^2}{d\,(a\,b^2-a^3)}+\frac{2\,b^3\,\mathrm{e}^{c+d\,x}}{a\,d\,(a\,b^2-a^3)}}{a+2\,b\,\mathrm{e}^{c+d\,x}+a\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{x}{a^2}+\frac{b\,\ln\left(\frac{2\,\mathrm{e}^{c+d\,x}\,(2\,a^2\,b-b^3)}{a^3\,(a^2-b^2)}-\frac{2\,b\,(2\,a^2-b^2)\,(a+b\,\mathrm{e}^{c+d\,x})}{a^3\,{(a+b)}^{3/2}\,{(b-a)}^{3/2}}\right)\,(2\,a^2-b^2)}{a^2\,d\,{(a+b)}^{3/2}\,{(b-a)}^{3/2}}-\frac{b\,\ln\left(\frac{2\,\mathrm{e}^{c+d\,x}\,(2\,a^2\,b-b^3)}{a^3\,(a^2-b^2)}+\frac{2\,b\,(2\,a^2-b^2)\,(a+b\,\mathrm{e}^{c+d\,x})}{a^3\,{(a+b)}^{3/2}\,{(b-a)}^{3/2}}\right)\,(2\,a^2-b^2)}{a^2\,d\,{(a+b)}^{3/2}\,{(b-a)}^{3/2}}$$

```
[In] int(1/(a + b/cosh(c + d*x))^2,x)
```

```
[Out] ((2*b^2)/(d*(a*b^2 - a^3)) + (2*b^3*exp(c + d*x))/(a*d*(a*b^2 - a^3)))/(a +
 2*b*exp(c + d*x) + a*exp(2*c + 2*d*x)) + x/a^2 + (b*log((2*exp(c + d*x)*(2
*a^2*b - b^3))/(a^3*(a^2 - b^2)) - (2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x)))
/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b
- a)^(3/2)) - (b*log((2*exp(c + d*x)*(2*a^2*b - b^3))/(a^3*(a^2 - b^2)) + (
2*b*(2*a^2 - b^2)*(a + b*exp(c + d*x)))/(a^3*(a + b)^(3/2)*(b - a)^(3/2)))*
(2*a^2 - b^2))/(a^2*d*(a + b)^(3/2)*(b - a)^(3/2))
```

3.93 $\int \frac{1}{(a+b\text{sech}(c+dx))^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 173

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = \frac{x}{a^3} - \frac{b(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{b^2\tanh(c+dx)}{2a(a^2-b^2)\,d(a+b\text{sech}(c+dx))^2} + \frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2(a^2-b^2)^2\,d(a+b\text{sech}(c+dx))}$$

```
[Out] x/a^3-b*(6*a^4-5*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a
+b*sech(d*x+c))^2+1/2*b^2*(5*a^2-2*b^2)*tanh(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*
sech(d*x+c))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3870, 4145, 4004, 3916, 2738, 214}

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = \frac{x}{a^3} + \frac{b^2(5a^2-2b^2)\tanh(c+dx)}{2a^2d(a^2-b^2)^2(a+b\text{sech}(c+dx))} + \frac{b^2\tanh(c+dx)}{2ad(a^2-b^2)(a+b\text{sech}(c+dx))^2} - \frac{b(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[In] Int[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] x/a^3 - (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tanh[(c + d*x)/2
])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (b^2*Tanh[c + d*x])/
(2*a*(a^2 - b^2)*d*(a + b*Sech[c + d*x])^2) + (b^2*(5*a^2 - 2*b^2)*Tanh[c +
 d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sech[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
```

```
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^2}-\frac{\int\frac{-2\left(a^2-b^2\right)+2ab\text{sech}(c+dx)-b^2\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}\\
&= \frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^2}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\text{sech}(c+dx))}\\
&\quad+\frac{\int\frac{2\left(a^2-b^2\right)^2-ab\left(4a^2-b^2\right)\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx}{2a^2\left(a^2-b^2\right)^2}\\
&= \frac{x}{a^3}+\frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^2}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\text{sech}(c+dx))}\\
&\quad-\frac{\left(b\left(6a^4-5a^2b^2+2b^4\right)\right)\int\frac{\text{sech}(c+dx)}{a+b\text{sech}(c+dx)}\,dx}{2a^3\left(a^2-b^2\right)^2}\\
&= \frac{x}{a^3}+\frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^2}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\text{sech}(c+dx))}\\
&\quad-\frac{\left(6a^4-5a^2b^2+2b^4\right)\int\frac{1}{1+\frac{a\cosh(c+dx)}{b}}\,dx}{2a^3\left(a^2-b^2\right)^2}\\
&= \frac{x}{a^3}+\frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^2}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\text{sech}(c+dx))}\\
&\quad+\frac{\left(i\left(6a^4-5a^2b^2+2b^4\right)\right)\text{Subst}\left(\int\frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^3\left(a^2-b^2\right)^2d}\\
&= \frac{x}{a^3}-\frac{b\left(6a^4-5a^2b^2+2b^4\right)\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}\\
&\quad+\frac{b^2\tanh(c+dx)}{2a\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^2}+\frac{b^2\left(5a^2-2b^2\right)\tanh(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\text{sech}(c+dx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = \frac{(b+a\cosh(c+dx))\operatorname{sech}^3(c+dx)\left(2(c+dx)(b+a\cosh(c+dx))^2+\frac{2b(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{(-a+b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}\right.}{2a^3d(a+b\operatorname{sech}(c+dx))^3}$$

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3),x]
```

```
[Out] ((b + a*Cosh[c + d*x])*Sech[c + d*x]^3*(2*(c + d*x)*(b + a*Cosh[c + d*x])^2
 + (2*b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*ArcTan[((-a + b)*Tanh[(c + d*x)/2])/Sqr
t[a^2 - b^2]]*(b + a*Cosh[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*Sinh[c +
d*x])/((-a + b)*(a + b)) + (3*a*b^2*(2*a^2 - b^2)*(b + a*Cosh[c + d*x])*Sin
h[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(a + b*Sech[c + d*x])^3)
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{2b\left(\frac{-\frac{(6a^2+ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)(a^2+2ab+b^2)}-\frac{(6a^2-ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}\right)}{a^3}+}{d}$
default	$\frac{-\frac{2b\left(\frac{-\frac{(6a^2+ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)(a^2+2ab+b^2)}-\frac{(6a^2-ab-2b^2)ab\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^2-2ab+b^2)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2b+a+b\right)^2}+\frac{(6a^4-5a^2b^2+2b^4)\arctan\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{2(a^4-2a^2b^2+b^4)\sqrt{(a+b)(a-b)}}\right)}{a^3}+}{d}$
risch	$\frac{x}{a^3}-\frac{b^2(7a^3b\,e^{3dx+3c}-4a\,b^3e^{3dx+3c}+6a^4e^{2dx+2c}+9a^2b^2e^{2dx+2c}-6b^4e^{2dx+2c}+17a^3b\,e^{dx+c}-8\,e^{dx+c}a\,b^3+6a^4-3a^2b^2)}{a^3d(a^2-b^2)^2\left(e^{2dx+2c}a+2\,e^{dx+c}b+a\right)^2}$

```
[In] int(1/(a+b*sech(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/a^3*((-1/2*(6*a^2+a*b-2*b^2)*a*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3-1/2*(6*a^2-a*b-2*b^2)*a*b/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c))/(tanh(1/2*d*x+1/2*c)^2*a-tanh(1/2*d*x+1/2*c)^2*b+a+b)^2+1/2*(6*a^4-5*
a^2*b^2+2*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+1/a^3*ln(1+tanh(1/2*d*x+1/2*c))-1/a^3*ln
(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2000 vs. 2(160) = 320.

Time = 0.32 (sec) , antiderivative size = 4125, normalized size of antiderivative = 23.84

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(12*a^6*b^2 - 18*a^4*b^4 + 6*a^2*b^6 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
 - a^2*b^6)*d*x*cosh(d*x + c)^4 - 2*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)
*d*x*sinh(d*x + c)^4 + 2*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a
^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + 2*(7*a^5*b^3 - 11*a^3*b^
5 + 4*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) -
 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - 2*(a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^
6 + 6*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x
 + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^
2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4
 + 5*a^2*b^6 - 2*b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + (6*a
^6*b - 5*a^4*b^3 + 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x +
 c)^4 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 -
5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 +
 (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^
6*b + 7*a^4*b^3 - 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b
^3 - 8*a^2*b^5 + 4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^
2 + 6*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*
(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4
+ 2*a*b^6 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + 3*(6*a^5*b^
2 - 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5
 + 4*b^7)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + b^2)*log((a^2*cosh(d*x
+ c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) - a^2 + 2*b^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(-a^2 + b^2)*(a*cosh(d*x + c) + a
*sinh(d*x + c) + b))/(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x
+ c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)) + 2*(17*a^5*b^3 - 25*a^3
*b^5 + 8*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x +
c) + 2*(17*a^5*b^3 - 25*a^3*b^5 + 8*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4
- a^2*b^6)*d*x*cosh(d*x + c)^3 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*
d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7)*d*x)*cosh(d*x + c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6
*b^8 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)
)*sinh(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^
4 + (a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b
```

```
- 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c)^3 + 2*(a^11 - a^9*b^2 -
3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 - 3*a^9*b^2
 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
 a^4*b^7)*d)*sinh(d*x + c)^3 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c) + 2*(3*(a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x
 + c)^2 + 6*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + (a
^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^
11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4
- a^5*b^6)*d*cosh(d*x + c)^3 + 3*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)
*d*cosh(d*x + c)^2 + (a^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d
*cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c
)), -(6*a^6*b^2 - 9*a^4*b^4 + 3*a^2*b^6 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^
2*b^6)*d*x*cosh(d*x + c)^4 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*si
nh(d*x + c)^4 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c)^3 + (7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7
 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x + c) - 4*(a^7*b -
 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*sinh(d*x + c)^3 - (a^8 - 3*a^6*b^2 + 3
*a^4*b^4 - a^2*b^6)*d*x + (6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 2*(
a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c)^2 + (6*a^
6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 6*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a
^2*b^6)*d*x*cosh(d*x + c)^2 - 2*(a^8 - a^6*b^2 - 3*a^4*b^4 + 5*a^2*b^6 - 2*
b^8)*d*x + 3*(7*a^5*b^3 - 11*a^3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a
^3*b^5 - a*b^7)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - (6*a^6*b - 5*a^4*b^3
+ 2*a^2*b^5 + (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^4 + (6*a^6*b
- 5*a^4*b^3 + 2*a^2*b^5)*sinh(d*x + c)^4 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b
^6)*cosh(d*x + c)^3 + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^6*b - 5*a^4
*b^3 + 2*a^2*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(6*a^6*b + 7*a^4*b^3 -
 8*a^2*b^5 + 4*b^7)*cosh(d*x + c)^2 + 2*(6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 +
4*b^7 + 3*(6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^2 + 6*(6*a^5*b^2
- 5*a^3*b^4 + 2*a*b^6)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(6*a^5*b^2 - 5*a^
3*b^4 + 2*a*b^6)*cosh(d*x + c) + 4*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6 + (6*a^
6*b - 5*a^4*b^3 + 2*a^2*b^5)*cosh(d*x + c)^3 + 3*(6*a^5*b^2 - 5*a^3*b^4 + 2
*a*b^6)*cosh(d*x + c)^2 + (6*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 + 4*b^7)*cosh(d*
x + c))*sinh(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*cosh(d*x + c) + a*sinh(d*
x + c) + b)/sqrt(a^2 - b^2)) + (17*a^5*b^3 - 25*a^3*b^5 + 8*a*b^7 - 4*(a^7*
b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x + c) + (17*a^5*b^3 - 25*a^
3*b^5 + 8*a*b^7 - 4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cosh(d*x +
c)^3 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x + 3*(7*a^5*b^3 - 11*a^
3*b^5 + 4*a*b^7 - 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x)*cosh(d*x +
 c)^2 + 2*(6*a^6*b^2 + 3*a^4*b^4 - 15*a^2*b^6 + 6*b^8 - 2*(a^8 - a^6*b^2 -
3*a^4*b^4 + 5*a^2*b^6 - 2*b^8)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^11 -
3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^4 + (a^11 - 3*a^9*b^2 + 3*
a^7*b^4 - a^5*b^6)*d*sinh(d*x + c)^4 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 -
a^4*b^7)*d*cosh(d*x + c)^3 + 2*(a^11 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*
a^3*b^8)*d*cosh(d*x + c)^2 + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*
```

```
cosh(d*x + c) + (a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c)
^3 + 4*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + 2*(3*(a
^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)^2 + 6*(a^10*b - 3*a^
8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c) + (a^11 - a^9*b^2 - 3*a^7*b^4
+ 5*a^5*b^6 - 2*a^3*b^8)*d)*sinh(d*x + c)^2 + (a^11 - 3*a^9*b^2 + 3*a^7*b^4
 - a^5*b^6)*d + 4*((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cosh(d*x + c)
^3 + 3*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cosh(d*x + c)^2 + (a^11
 - a^9*b^2 - 3*a^7*b^4 + 5*a^5*b^6 - 2*a^3*b^8)*d*cosh(d*x + c) + (a^10*b -
 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d)*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = \int \frac{1}{(a+b\,\text{sech}\,(c+dx))^3}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))**3,x)
[Out] Integral((a + b*sech(c + d*x))**(-3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.51

$$\int \frac{1}{(a+b\text{sech}(c+dx))^3}\,dx = -\frac{\frac{(6a^4b-5a^2b^3+2b^5)\arctan\left(\frac{ae^{(dx+c)}+b}{\sqrt{a^2-b^2}}\right)}{(a^7-2a^5b^2+a^3b^4)\sqrt{a^2-b^2}} + \frac{7a^3b^3e^{(3dx+3c)}-4ab^5e^{(3dx+3c)}+6a^4b^2e^{(2dx+2c)}+9a^2b^4e^{(2dx+2c)}-6b^6e^{(2dx+2c)}+17a^3b}{(a^7-2a^5b^2+a^3b^4)\left(ae^{(2dx+2c)}+2be^{(dx+c)}+a\right)^2}}{d}$$

```
[In] integrate(1/(a+b*sech(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -((6*a^4*b - 5*a^2*b^3 + 2*b^5)*arctan((a*e^(d*x + c) + b)/sqrt(a^2 - b^2))
/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(a^2 - b^2)) + (7*a^3*b^3*e^(3*d*x + 3*c)
 - 4*a*b^5*e^(3*d*x + 3*c) + 6*a^4*b^2*e^(2*d*x + 2*c) + 9*a^2*b^4*e^(2*d*x
 + 2*c) - 6*b^6*e^(2*d*x + 2*c) + 17*a^3*b^3*e^(d*x + c) - 8*a*b^5*e^(d*x +
 c) + 6*a^4*b^2 - 3*a^2*b^4)/((a^7 - 2*a^5*b^2 + a^3*b^4)*(a*e^(2*d*x + 2*c
) + 2*b*e^(d*x + c) + a)^2) - (d*x + c)/a^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\text{sech}(c + dx))^3} dx = \int \frac{1}{\left(a + \frac{b}{\cosh(c+d\,x)}\right)^3} dx$$

```
[In] int(1/(a + b/cosh(c + d*x))^3,x)
```

```
[Out] int(1/(a + b/cosh(c + d*x))^3, x)
```

3.94 $\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{ad}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3869}

$$\int \frac{1}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ad}$$

Mathematica [A] (verified)

Time = 5.39 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$= \frac{2b\sqrt{b+a\cosh(c+dx)}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1+\text{sech}(c+dx))}{-a+b}}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a}\sqrt{a+b}d\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(-1+\text{sech}(c+dx))}{a+b}}\sqrt{a+b\text{sech}(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\,\text{sech}\,(dx+c)}}dx$$

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(1/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

3.95 $\int \frac{\cosh^4(x)}{a+b\operatorname{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 146

$$\int \frac{\cosh^4(x)}{a+b\operatorname{sech}(x)} \, dx = \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5}-\frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}-\frac{b(2a^2+3b^2)\sinh(x)}{3a^4}+\frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3}-\frac{b\cosh^2(x)\sinh(x)}{3a^2}+\frac{\cosh^3(x)\sinh(x)}{4a}$$

```
[Out] 1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5-1/3*b*(2*a^2+3*b^2)*sinh(x)/a^4+1/8*(3*a^
2+4*b^2)*cosh(x)*sinh(x)/a^3-1/3*b*cosh(x)^2*sinh(x)/a^2+1/4*cosh(x)^3*sinh
(x)/a-2*b^5*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^5/(a-b)^(1/2)/(a+
b)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3938, 4189, 4004, 3916, 2738, 211}

$$\int \frac{\cosh^4(x)}{a+b\operatorname{sech}(x)} \, dx = -\frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)\cosh^2(x)}{3a^2}-\frac{b(2a^2+3b^2)\sinh(x)}{3a^4}+\frac{(3a^2+4b^2)\sinh(x)\cosh(x)}{8a^3}+\frac{x(3a^4+4a^2b^2+8b^4)}{8a^5}+\frac{\sinh(x)\cosh^3(x)}{4a}$$

```
[In] Int[Cosh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(a^5*Sqrt[a - b]*Sqrt[a + b]) - (b*(2*a^2 + 3*b^2)*Sinh[
x])/(3*a^4) + ((3*a^2 + 4*b^2)*Cosh[x]*Sinh[x])/(8*a^3) - (b*Cosh[x]^2*Sinh
[x])/(3*a^2) + (Cosh[x]^3*Sinh[x])/(4*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\cosh^3(x)\sinh(x)}{4a} + \frac{\int \frac{\cosh^3(x)\left(-4b+3a\,\text{sech}(x)+3b\,\text{sech}^2(x)\right)}{a+b\,\text{sech}(x)}\,dx}{4a}$$

$$= -\frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{\int \frac{\cosh^2(x)\left(-3(3a^2+4b^2)-ab\,\text{sech}(x)+8b^2\,\text{sech}^2(x)\right)}{a+b\,\text{sech}(x)}\,dx}{12a^2}$$

$$= \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} + \frac{\int \frac{\cosh(x)\left(-8b(2a^2+3b^2)+a(9a^2-4b^2)\,\text{sech}(x)+3b(3a^2+4b^2)\,\text{sech}^2(x)\right)}{a+b\,\text{sech}(x)}\,dx}{24a^3}$$

$$= -\frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{\int \frac{-3(3a^4+4a^2b^2+8b^4)-3ab(3a^2+4b^2)\,\text{sech}(x)}{a+b\,\text{sech}(x)}\,dx}{24a^4}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{b^5\int \frac{\text{sech}(x)}{a+b\,\text{sech}(x)}\,dx}{a^5}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{b^4\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^5}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a} - \frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^5}$$

$$= \frac{(3a^4+4a^2b^2+8b^4)\,x}{8a^5} - \frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\sqrt{a+b}} - \frac{b(2a^2+3b^2)\sinh(x)}{3a^4} + \frac{(3a^2+4b^2)\cosh(x)\sinh(x)}{8a^3} - \frac{b\cosh^2(x)\sinh(x)}{3a^2} + \frac{\cosh^3(x)\sinh(x)}{4a}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^4(x)}{a + b\text{sech}(x)}\,dx$$

$$= \frac{12(3a^4 + 4a^2b^2 + 8b^4)\,x + \frac{192b^5 \arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 24ab(3a^2 + 4b^2)\sinh(x) + 24a^2(a^2 + b^2)\sinh(2x) - 8}{96a^5}$$

```
[In] Integrate[Cosh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (12*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x + (192*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/Sqrt[a^2 - b^2] - 24*a*b*(3*a^2 + 4*b^2)*Sinh[x] + 24*a^2*(
a^2 + b^2)*Sinh[2*x] - 8*a^3*b*Sinh[3*x] + 3*a^4*Sinh[4*x])/(96*a^5)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(126) = 252.

Time = 0.57 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.81

method	result
risch	$\frac{3x}{8a} + \frac{x\,b^2}{2a^3} + \frac{x\,b^4}{a^5} + \frac{e^{4x}}{64a} - \frac{b\,e^{3x}}{24a^2} + \frac{e^{2x}}{8a} + \frac{e^{2x}b^2}{8a^3} - \frac{3b\,e^{x}}{8a^2} - \frac{b^3 e^{x}}{2a^4} + \frac{3b\,e^{-x}}{8a^2} + \frac{b^3 e^{-x}}{2a^4} - \frac{e^{-2x}}{8a} - \frac{e^{-2x}b^2}{8a^3} + \frac{b\,e^{-3x}}{24a^2} -$
default	$\frac{1}{4a\left(\tanh\left(\frac{x}{2}\right)-1\right)^4} - \frac{-3a-2b}{6a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{-7a^2-4ab-4b^2}{8a^3\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} + \frac{\left(-3a^4-4a^2b^2-8b^4\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8a^5} - \frac{-5a^3-8a^2b-4a\,b^2-8b^3}{8a^4\left(\tanh\left(\frac{x}{2}\right)-1\right)}$

```
[In] int(cosh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x/a+1/2*x/a^3*b^2+x/a^5*b^4+1/64/a*exp(x)^4-1/24/a^2*b*exp(x)^3+1/8/a*e
xp(x)^2+1/8/a^3*exp(x)^2*b^2-3/8*b/a^2*exp(x)-1/2*b^3/a^4*exp(x)+3/8*b/a^2/
exp(x)+1/2*b^3/a^4/exp(x)-1/8/a/exp(x)^2-1/8/a^3/exp(x)^2*b^2+1/24/a^2*b/ex
p(x)^3-1/64/a/exp(x)^4-1/(-a^2+b^2)^(1/2)*b^5/a^5*ln(exp(x)+(b*(-a^2+b^2)^(
1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/a)+1/(-a^2+b^2)^(1/2)*b^5/a^5*ln(exp(x)+(b*(
-a^2+b^2)^(1/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. 2(126) = 252.

Time = 0.29 (sec) , antiderivative size = 2402, normalized size of antiderivative = 16.45

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx=\text{Too large to display}$$

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 - 8*(a^5*
b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh(x))*si
nh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(a^6 - a
^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6 + 3*a^4
*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*a^5*b +
 a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*(a^6 -
a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh
(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3*b^3)*c
osh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4
- 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 24*(3*a^
5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b^5 + 21
*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^6 - a^2
*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x) - 30*(
3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^4)*cosh
(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^5*b - a
^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b^2 + 4*
a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 +
 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 - 192*(b^5*cosh(x)^4 +
4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*sinh(x)
^3 + b^5*sinh(x)^4)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2
*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 +
b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x)
+ 2*(a*cosh(x) + b)*sinh(x) + a)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6
 - a^4*b^2)*cosh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 1
8*(a^6 - a^2*b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*co
sh(x)^3 - 15*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3
 - 4*a*b^5)*cosh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2
)*cosh(x)^4 + 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(
x)^2*sinh(x)^2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh
(x)^4), 1/192*(3*(a^6 - a^4*b^2)*cosh(x)^8 + 3*(a^6 - a^4*b^2)*sinh(x)^8 -
8*(a^5*b - a^3*b^3)*cosh(x)^7 - 8*(a^5*b - a^3*b^3 - 3*(a^6 - a^4*b^2)*cosh
(x))*sinh(x)^7 + 24*(a^6 - a^2*b^4)*cosh(x)^6 + 4*(6*a^6 - 6*a^2*b^4 + 21*(
a^6 - a^4*b^2)*cosh(x)^2 - 14*(a^5*b - a^3*b^3)*cosh(x))*sinh(x)^6 - 3*a^6
+ 3*a^4*b^2 + 24*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^4 - 24*(3*
a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^5 - 24*(3*a^5*b + a^3*b^3 - 4*a*b^5 - 7*
```

```
(a^6 - a^4*b^2)*cosh(x)^3 + 7*(a^5*b - a^3*b^3)*cosh(x)^2 - 6*(a^6 - a^2*b^
4)*cosh(x))*sinh(x)^5 + 2*(105*(a^6 - a^4*b^2)*cosh(x)^4 - 140*(a^5*b - a^3
*b^3)*cosh(x)^3 + 180*(a^6 - a^2*b^4)*cosh(x)^2 + 12*(3*a^6 + a^4*b^2 + 4*a
^2*b^4 - 8*b^6)*x - 60*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^4 + 2
4*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^3 + 8*(9*a^5*b + 3*a^3*b^3 - 12*a*b
^5 + 21*(a^6 - a^4*b^2)*cosh(x)^5 - 35*(a^5*b - a^3*b^3)*cosh(x)^4 + 60*(a^
6 - a^2*b^4)*cosh(x)^3 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)
 - 30*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^2)*sinh(x)^3 - 24*(a^6 - a^2*b^
4)*cosh(x)^2 + 12*(7*(a^6 - a^4*b^2)*cosh(x)^6 - 2*a^6 + 2*a^2*b^4 - 14*(a^
5*b - a^3*b^3)*cosh(x)^5 + 30*(a^6 - a^2*b^4)*cosh(x)^4 + 12*(3*a^6 + a^4*b
^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^2 - 20*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh
(x)^3 + 6*(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x))*sinh(x)^2 + 384*(b^5*cosh(
x)^4 + 4*b^5*cosh(x)^3*sinh(x) + 6*b^5*cosh(x)^2*sinh(x)^2 + 4*b^5*cosh(x)*
sinh(x)^3 + b^5*sinh(x)^4)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) + 8*(a^5*b - a^3*b^3)*cosh(x) + 8*(3*(a^6 - a^4*b^2)*c
osh(x)^7 - 7*(a^5*b - a^3*b^3)*cosh(x)^6 + a^5*b - a^3*b^3 + 18*(a^6 - a^2*
b^4)*cosh(x)^5 + 12*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x*cosh(x)^3 - 15*
(3*a^5*b + a^3*b^3 - 4*a*b^5)*cosh(x)^4 + 9*(3*a^5*b + a^3*b^3 - 4*a*b^5)*c
osh(x)^2 - 6*(a^6 - a^2*b^4)*cosh(x))*sinh(x))/((a^7 - a^5*b^2)*cosh(x)^4 +
 4*(a^7 - a^5*b^2)*cosh(x)^3*sinh(x) + 6*(a^7 - a^5*b^2)*cosh(x)^2*sinh(x)^
2 + 4*(a^7 - a^5*b^2)*cosh(x)*sinh(x)^3 + (a^7 - a^5*b^2)*sinh(x)^4)]
```

Sympy [F]

$$\int \frac{\cosh^4(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\cosh^4{(x)}}{a + b\operatorname{sech}{(x)}}\,dx$$

```
[In] integrate(cosh(x)**4/(a+b*sech(x)),x)
[Out] Integral(cosh(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^4(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx = -\frac{2\,b^5\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^5} + \frac{3\,a^3e^{(4\,x)}-8\,a^2be^{(3\,x)}+24\,a^3e^{(2\,x)}+24\,ab^2e^{(2\,x)}-72\,a^2be^x-96\,b^3e^x}{192\,a^4} + \frac{(3\,a^4+4\,a^2b^2+8\,b^4)x}{8\,a^5} + \frac{\left(8\,a^3be^x-3\,a^4+24\,(3\,a^3b+4\,ab^3)e^{(3\,x)}-24\,(a^4+a^2b^2)e^{(2\,x)}\right)e^{(-4\,x)}}{192\,a^5}$$

```
[In] integrate(cosh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^5) + 1/192*(3
*a^3*e^(4*x) - 8*a^2*b*e^(3*x) + 24*a^3*e^(2*x) + 24*a*b^2*e^(2*x) - 72*a^2
*b*e^x - 96*b^3*e^x)/a^4 + 1/8*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x/a^5 + 1/192*(8
*a^3*b*e^x - 3*a^4 + 24*(3*a^3*b + 4*a*b^3)*e^(3*x) - 24*(a^4 + a^2*b^2)*e^
(2*x))*e^(-4*x)/a^5
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.72

$$\int \frac{\cosh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{4\,x}}{64\,a}-\frac{e^{-4\,x}}{64\,a}+\frac{x\,(3\,a^4+4\,a^2\,b^2+8\,b^4)}{8\,a^5}-\frac{e^{-2\,x}\,(a^2+b^2)}{8\,a^3}+\frac{e^{2\,x}\,(a^2+b^2)}{8\,a^3} + \frac{e^{-x}\,(3\,a^2\,b+4\,b^3)}{8\,a^4}+\frac{b\,e^{-3\,x}}{24\,a^2}-\frac{b\,e^{3\,x}}{24\,a^2}-\frac{e^{x}\,(3\,a^2\,b+4\,b^3)}{8\,a^4} + \frac{b^5\,\ln\left(\frac{2\,b^5\,e^x}{a^6}-\frac{2\,b^5\,(a+b\,e^x)}{a^6\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}-\frac{b^5\,\ln\left(\frac{2\,b^5\,e^x}{a^6}+\frac{2\,b^5\,(a+b\,e^x)}{a^6\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(cosh(x)^4/(a + b/cosh(x)),x)
```

```
[Out] exp(4*x)/(64*a) - exp(-4*x)/(64*a) + (x*(3*a^4 + 8*b^4 + 4*a^2*b^2))/(8*a^5
) - (exp(-2*x)*(a^2 + b^2))/(8*a^3) + (exp(2*x)*(a^2 + b^2))/(8*a^3) + (exp
(-x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b*exp(-3*x))/(24*a^2) - (b*exp(3*x))/(24
*a^2) - (exp(x)*(3*a^2*b + 4*b^3))/(8*a^4) + (b^5*log((2*b^5*exp(x))/a^6 -
(2*b^5*(a + b*exp(x)))/(a^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b)^(1/
2)*(b - a)^(1/2)) - (b^5*log((2*b^5*exp(x))/a^6 + (2*b^5*(a + b*exp(x)))/(a
^6*(a + b)^(1/2)*(b - a)^(1/2))))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))
```

3.96 $\int \frac{\cosh^3(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{\cosh^3(x)}{a+b\operatorname{sech}(x)}\,dx=-\frac{b(a^2+2b^2)\,x}{2a^4}+\frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}+\frac{(2a^2+3b^2)\sinh(x)}{3a^3}-\frac{b\cosh(x)\sinh(x)}{2a^2}+\frac{\cosh^2(x)\sinh(x)}{3a}$$

```
[Out] -1/2*b*(a^2+2*b^2)*x/a^4+1/3*(2*a^2+3*b^2)*sinh(x)/a^3-1/2*b*cosh(x)*sinh(x
)/a^2+1/3*cosh(x)^2*sinh(x)/a+2*b^4*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1
/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3938, 4189, 4004, 3916, 2738, 211}

$$\int \frac{\cosh^3(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}}-\frac{b\sinh(x)\cosh(x)}{2a^2}-\frac{bx(a^2+2b^2)}{2a^4}+\frac{(2a^2+3b^2)\sinh(x)}{3a^3}+\frac{\sinh(x)\cosh^2(x)}{3a}$$

```
[In] Int[Cosh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -1/2*(b*(a^2 + 2*b^2)*x)/a^4 + (2*b^4*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
 + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]) + ((2*a^2 + 3*b^2)*Sinh[x])/(3*a^3) -
 (b*Cosh[x]*Sinh[x])/(2*a^2) + (Cosh[x]^2*Sinh[x])/(3*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{\int \frac{\cosh^2(x)\left(-3b+2a\operatorname{sech}(x)+2b\operatorname{sech}^2(x)\right)}{a+b\operatorname{sech}(x)}\,dx}{3a}$$

$$= -\frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} - \frac{\int \frac{\cosh(x)\left(-2(2a^2+3b^2)-ab\operatorname{sech}(x)+3b^2\operatorname{sech}^2(x)\right)}{a+b\operatorname{sech}(x)}dx}{6a^2}$$

$$= \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{\int \frac{-3b(a^2+2b^2)-3ab^2\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}dx}{6a^3}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{b^4\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}dx}{a^4}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{b^3\int \frac{1}{1+\frac{a\cosh(x)}{b}}dx}{a^4}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a} + \frac{(2b^3)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^4}$$

$$= -\frac{b(a^2+2b^2)\,x}{2a^4} + \frac{2b^4\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}} + \frac{(2a^2+3b^2)\sinh(x)}{3a^3} - \frac{b\cosh(x)\sinh(x)}{2a^2} + \frac{\cosh^2(x)\sinh(x)}{3a}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^3(x)}{a+b\operatorname{sech}(x)}\,dx$$

$$= \frac{-6b(a^2+2b^2)\,x - \frac{24b^4\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 3a(3a^2+4b^2)\sinh(x) - 3a^2b\sinh(2x) + a^3\sinh(3x)}{12a^4}$$

```
[In] Integrate[Cosh[x]^3/(a + b*Sech[x]),x]
[Out] (-6*b*(a^2 + 2*b^2)*x - (24*b^4*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]
])/Sqrt[a^2 - b^2] + 3*a*(3*a^2 + 4*b^2)*Sinh[x] - 3*a^2*b*Sinh[2*x] + a^3*
Sinh[3*x])/(12*a^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(94) = 188$.

Time = 0.38 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.81

method	result
default	$\frac{2b^4 \arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4\sqrt{(a+b)(a-b)}} - \frac{1}{3a\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{a+b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{2a^2+ab+2b^2}{2a^3\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{b(a^2+2b^2)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^4} -$
risch	$-\frac{bx}{2a^2} - \frac{b^3x}{a^4} + \frac{e^{3x}}{24a} - \frac{b\,e^{2x}}{8a^2} + \frac{3\,e^{x}}{8a} + \frac{e^{x}b^2}{2a^3} - \frac{3\,e^{-x}}{8a} - \frac{e^{-x}b^2}{2a^3} + \frac{b\,e^{-2x}}{8a^2} - \frac{e^{-3x}}{24a} - \frac{b^4\ln\left(e^x + \frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^4} +$

```
[In] int(cosh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*b^4/a^4/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
-1/3/a/(tanh(1/2*x)-1)^3-1/2*(a+b)/a^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+2*b
^2)/a^3/(tanh(1/2*x)-1)+1/2*b*(a^2+2*b^2)/a^4*ln(tanh(1/2*x)-1)-1/3/a/(tanh
(1/2*x)+1)^3-1/2*(-a-b)/a^2/(tanh(1/2*x)+1)^2-1/2*(2*a^2+a*b+2*b^2)/a^3/(ta
nh(1/2*x)+1)-1/2*b*(a^2+2*b^2)/a^4*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 1562, normalized size of antiderivative = 13.95

$$\int \frac{\cosh^3(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(x)^6 - 3*(a^4*b - a
^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^2)*cosh(x))*sinh(x)
^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh(x)^3 + 3*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4 + 5*(a^5 - a^3*
b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 2*(10*(a^5 - a^3*
b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4*b + a^2*b^3 - 2*b^
5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3 - 3*(3*a^5 + a^3*b^
2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 - 5*(a^5 - a^3*b^2)*c
osh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b + a^2*b^3 - 2*b^5)*x*
cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(x)^2 - 24*(b^4*cosh
(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2 + b^4*sinh(x)^3)*
sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 +
 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*
sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*
```

```
sinh(x) + a)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^
5 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
 2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 +
a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 -
a^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a
^4*b^2)*sinh(x)^3), 1/24*((a^5 - a^3*b^2)*cosh(x)^6 + (a^5 - a^3*b^2)*sinh(
x)^6 - 3*(a^4*b - a^2*b^3)*cosh(x)^5 - 3*(a^4*b - a^2*b^3 - 2*(a^5 - a^3*b^
2)*cosh(x))*sinh(x)^5 - a^5 + a^3*b^2 - 12*(a^4*b + a^2*b^3 - 2*b^5)*x*cosh
(x)^3 + 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^4 + 3*(3*a^5 + a^3*b^2 - 4*a*
b^4 + 5*(a^5 - a^3*b^2)*cosh(x)^2 - 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4
+ 2*(10*(a^5 - a^3*b^2)*cosh(x)^3 - 15*(a^4*b - a^2*b^3)*cosh(x)^2 - 6*(a^4
*b + a^2*b^3 - 2*b^5)*x + 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x)^3
- 3*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2 - 3*(3*a^5 + a^3*b^2 - 4*a*b^4 -
5*(a^5 - a^3*b^2)*cosh(x)^4 + 10*(a^4*b - a^2*b^3)*cosh(x)^3 + 12*(a^4*b +
a^2*b^3 - 2*b^5)*x*cosh(x) - 6*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^2)*sinh(
x)^2 - 48*(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^
2 + b^4*sinh(x)^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt
(a^2 - b^2)) + 3*(a^4*b - a^2*b^3)*cosh(x) + 3*(2*(a^5 - a^3*b^2)*cosh(x)^5
 + a^4*b - a^2*b^3 - 5*(a^4*b - a^2*b^3)*cosh(x)^4 - 12*(a^4*b + a^2*b^3 -
2*b^5)*x*cosh(x)^2 + 4*(3*a^5 + a^3*b^2 - 4*a*b^4)*cosh(x)^3 - 2*(3*a^5 + a
^3*b^2 - 4*a*b^4)*cosh(x))*sinh(x))/((a^6 - a^4*b^2)*cosh(x)^3 + 3*(a^6 - a
^4*b^2)*cosh(x)^2*sinh(x) + 3*(a^6 - a^4*b^2)*cosh(x)*sinh(x)^2 + (a^6 - a^
4*b^2)*sinh(x)^3)]
```

Sympy [F]

$$\int \frac{\cosh^3(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\cosh^3{(x)}}{a + b\operatorname{sech}{(x)}}\,dx$$

```
[In] integrate(cosh(x)**3/(a+b*sech(x)),x)
[Out] Integral(cosh(x)**3/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{2\,b^4\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^4} + \frac{a^2e^{(3\,x)} - 3\,abe^{(2\,x)} + 9\,a^2e^x + 12\,b^2e^x}{24\,a^3} - \frac{(a^2b+2\,b^3)x}{2\,a^4} + \frac{\left(3\,a^2be^x - a^3 - 3\,(3\,a^3+4\,ab^2)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,a^4}$$

```
[In] integrate(cosh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^4*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^4) + 1/24*(a^2
*e^(3*x) - 3*a*b*e^(2*x) + 9*a^2*e^x + 12*b^2*e^x)/a^3 - 1/2*(a^2*b + 2*b^3
)*x/a^4 + 1/24*(3*a^2*b*e^x - a^3 - 3*(3*a^3 + 4*a*b^2)*e^(2*x))*e^(-3*x)/a
^4
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{e^{3\,x}}{24\,a} - \frac{e^{-3\,x}}{24\,a} - \frac{x\,(a^2\,b+2\,b^3)}{2\,a^4} + \frac{e^x\,(3\,a^2+4\,b^2)}{8\,a^3} + \frac{b\,e^{-2\,x}}{8\,a^2} - \frac{b\,e^{2\,x}}{8\,a^2} - \frac{e^{-x}\,(3\,a^2+4\,b^2)}{8\,a^3} + \frac{b^4\,\ln\left(-\frac{2\,b^4\,e^x}{a^5} - \frac{2\,b^4\,(a+b\,e^x)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b^4\,\ln\left(\frac{2\,b^4\,(a+b\,e^x)}{a^5\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{2\,b^4\,e^x}{a^5}\right)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(cosh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] exp(3*x)/(24*a) - exp(-3*x)/(24*a) - (x*(a^2*b + 2*b^3))/(2*a^4) + (exp(x)*
(3*a^2 + 4*b^2))/(8*a^3) + (b*exp(-2*x))/(8*a^2) - (b*exp(2*x))/(8*a^2) - (
exp(-x)*(3*a^2 + 4*b^2))/(8*a^3) + (b^4*log(- (2*b^4*exp(x))/a^5 - (2*b^4*(
a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)^(1/2))))/(a^4*(a + b)^(1/2)*(b -
a)^(1/2)) - (b^4*log((2*b^4*(a + b*exp(x)))/(a^5*(a + b)^(1/2)*(b - a)^(1/2
)) - (2*b^4*exp(x))/a^5))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))
```

3.97 $\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)} dx = \frac{(a^2+2b^2)\,x}{2a^3} - \frac{2b^3 \arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}$$

```
[Out] 1/2*(a^2+2*b^2)*x/a^3-b*sinh(x)/a^2+1/2*cosh(x)*sinh(x)/a-2*b^3*arctan((a-b
)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3938, 4189, 4004, 3916, 2738, 211}

$$\int \frac{\cosh^2(x)}{a+b\operatorname{sech}(x)} dx = -\frac{2b^3 \arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\sinh(x)}{a^2} + \frac{x(a^2+2b^2)}{2a^3} + \frac{\sinh(x)\cosh(x)}{2a}$$

```
[In] Int[Cosh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ((a^2 + 2*b^2)*x)/(2*a^3) - (2*b^3*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]) - (b*Sinh[x])/a^2 + (Cosh[x]*Sinh[x])/(2
*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\cosh(x)\sinh(x)}{2a} + \frac{\int \frac{\cosh(x)\left(-2b+a\operatorname{sech}(x)+b\operatorname{sech}^2(x)\right)}{a+b\operatorname{sech}(x)}\,dx}{2a} \\
&= -\frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{\int \frac{-a^2-2b^2-ab\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{2a^2}
\end{aligned}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^3\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{a^3}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{b^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^3}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a} - \frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^3}$$

$$= \frac{(a^2+2b^2)\,x}{2a^3} - \frac{2b^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\sinh(x)}{a^2} + \frac{\cosh(x)\sinh(x)}{2a}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(x)}{a+b\text{sech}(x)}\,dx = \frac{2a^2x + 4b^2x + \frac{8b^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 4ab\sinh(x) + a^2\sinh(2x)}{4a^3}$$

```
[In] Integrate[Cosh[x]^2/(a + b*Sech[x]),x]
[Out] (2*a^2*x + 4*b^2*x + (8*b^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] - 4*a*b*Sinh[x] + a^2*Sinh[2*x])/(4*a^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(71) = 142.

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

method	result
default	$\frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{-a-2b}{2a^2\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{\left(-a^2-2b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a^3} - \frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^3\sqrt{(a+b)(a-b)}} - \frac{1}{2a\left(\tanh\left(\frac{x}{2}\right)+1\right)^2} - 2a$
risch	$\frac{x}{2a} + \frac{x\,b^2}{a^3} + \frac{e^{2x}}{8a} - \frac{b\,e^{x}}{2a^2} + \frac{b\,e^{-x}}{2a^2} - \frac{e^{-2x}}{8a} - \frac{b^3\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^3} + \frac{b^3\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^3}$

```
[In] int(cosh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/2/a/(tanh(1/2*x)-1)^2-1/2*(-a-2*b)/a^2/(tanh(1/2*x)-1)+1/2/a^3*(-a^2-2*b^
2)*ln(tanh(1/2*x)-1)-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)
/((a+b)*(a-b))^(1/2))-1/2/a/(tanh(1/2*x)+1)^2-1/2*(-a-2*b)/a^2/(tanh(1/2*x)
+1)+1/2*(a^2+2*b^2)/a^3*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 860, normalized size of antiderivative = 10.12

$$\int \frac{\cosh^2(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*sinh(x)^4 - a^4 + a^2*b^2
 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*
(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3 + 2*(3*(a^4 - a^2*b^2)*
cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x
)^2 - 8*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2)*sqrt(-a^2 +
 b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*
(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b
))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a
)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cosh(x)
^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)*sin
h(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) + (a^5
 - a^3*b^2)*sinh(x)^2), 1/8*((a^4 - a^2*b^2)*cosh(x)^4 + (a^4 - a^2*b^2)*si
nh(x)^4 - a^4 + a^2*b^2 + 4*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x)^2 - 4*(a^3*b
- a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^4 - a^2*b^2)*cosh(x))*sinh(x)^3
+ 2*(3*(a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4)*x - 6*(a^3*b -
 a*b^3)*cosh(x))*sinh(x)^2 + 16*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^
3*sinh(x)^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 -
 b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^4 - a^2*b^2)*cos
h(x)^3 + 2*(a^4 + a^2*b^2 - 2*b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x)^2)
*sinh(x))/((a^5 - a^3*b^2)*cosh(x)^2 + 2*(a^5 - a^3*b^2)*cosh(x)*sinh(x) +
(a^5 - a^3*b^2)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\cosh^2(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\cosh^2(x)}{a + b\,\text{sech}\,(x)}\,dx$$

```
[In] integrate(cosh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a + b\operatorname{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^2(x)}{a + b\operatorname{sech}(x)}\,dx = -\frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^3} + \frac{ae^{(2\,x)} - 4\,be^x}{8\,a^2} + \frac{(a^2+2\,b^2)x}{2\,a^3} + \frac{(4\,abe^x - a^2)e^{(-2\,x)}}{8\,a^3}$$

```
[In] integrate(cosh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^3) + 1/8*(a*e
^(2*x) - 4*b*e^x)/a^2 + 1/2*(a^2 + 2*b^2)*x/a^3 + 1/8*(4*a*b*e^x - a^2)*e^(
-2*x)/a^3
```

Mupad [B] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.96

$$\int \frac{\cosh^2(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{e^{2\,x}}{8\,a} - \frac{e^{-2\,x}}{8\,a} - \frac{b\,e^x}{2\,a^2} + \frac{b\,e^{-x}}{2\,a^2} + \frac{x\,(a^2+2\,b^2)}{2\,a^3} + \frac{b^3\,\ln\left(\frac{2\,b^3\,e^x}{a^4} - \frac{2\,b^3\,(a+b\,e^x)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b^3\,\ln\left(\frac{2\,b^3\,e^x}{a^4} + \frac{2\,b^3\,(a+b\,e^x)}{a^4\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(cosh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] exp(2*x)/(8*a) - exp(-2*x)/(8*a) - (b*exp(x))/(2*a^2) + (b*exp(-x))/(2*a^2)
 + (x*(a^2 + 2*b^2))/(2*a^3) + (b^3*log((2*b^3*exp(x))/a^4 - (2*b^3*(a + b*
exp(x)))/(a^4*(a + b)^(1/2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^(1/
2)) - (b^3*log((2*b^3*exp(x))/a^4 + (2*b^3*(a + b*exp(x)))/(a^4*(a + b)^(1/
2)*(b - a)^(1/2))))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))
```

3.98 $\int \frac{\cosh(x)}{a+b\,\text{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 62

$$\int \frac{\cosh(x)}{a+b\,\text{sech}(x)}\,dx = -\frac{bx}{a^2}+\frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{\sinh(x)}{a}$$

```
[Out] -b*x/a^2+sinh(x)/a+2*b^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^2/(a
-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3938, 12, 3868, 2738, 211}

$$\int \frac{\cosh(x)}{a+b\,\text{sech}(x)}\,dx = \frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}-\frac{bx}{a^2}+\frac{\sinh(x)}{a}$$

```
[In] Int[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] -((b*x)/a^2) + (2*b^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a^2*Sqr
t[a - b]*Sqrt[a + b]) + Sinh[x]/a
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 3938

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n)), x] - Dis
t[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)/(a + b*Csc[e + f*x]))*Simp[b*n -
 a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sinh(x)}{a} - \frac{\int \frac{b}{a+b\operatorname{sech}(x)}\,dx}{a}\\
&= \frac{\sinh(x)}{a} - \frac{b\int \frac{1}{a+b\operatorname{sech}(x)}\,dx}{a}\\
&= -\frac{bx}{a^2} + \frac{\sinh(x)}{a} + \frac{b\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{a^2}\\
&= -\frac{bx}{a^2} + \frac{\sinh(x)}{a} + \frac{(2b)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a^2}\\
&= -\frac{bx}{a^2} + \frac{2b^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}} + \frac{\sinh(x)}{a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(x)}{a+b\text{sech}(x)}\,dx=\frac{b\left(-x-\frac{2b\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)+a\sinh(x)}{a^2}$$

```
[In] Integrate[Cosh[x]/(a + b*Sech[x]),x]
```

```
[Out] (b*(-x - (2*b*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
) + a*Sinh[x])/a^2
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)+1\right)}-\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a^2}-\frac{1}{a\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{2b^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}$	94
risch	$-\frac{bx}{a^2}+\frac{e^x}{2a}-\frac{e^{-x}}{2a}-\frac{b^2\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^2}+\frac{b^2\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,a^2}$	144

```
[In] int(cosh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/(tanh(1/2*x)+1)-1/a^2*b*ln(tanh(1/2*x)+1)-1/a/(tanh(1/2*x)-1)+1/a^2*b*
ln(tanh(1/2*x)-1)+2*b^2/a^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((
a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 430, normalized size of antiderivative = 6.94

$$\int \frac{\cosh(x)}{a+b\text{sech}(x)}\,dx$$

$$=\left[-\frac{a^3-ab^2+2\,(a^2b-b^3)x\cosh(x)-(a^3-ab^2)\cosh(x)^2-(a^3-ab^2)\sinh(x)^2+2\,(b^2\cosh(x)+b^2\sin\ldots}{\ldots}\right.$$

$$-\frac{a^3-ab^2+2\,(a^2b-b^3)x\cosh(x)-(a^3-ab^2)\cosh(x)^2-(a^3-ab^2)\sinh(x)^2+4\,(b^2\cosh(x)+b^2\sinh\ldots}{2\,((a^4-a^2b^2)\cosh(x)+(a^4\ldots}$$

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) - (a^3 - a*b^2)*cosh(x)^2 -
(a^3 - a*b^2)*sinh(x)^2 + 2*(b^2*cosh(x) + b^2*sinh(x))*sqrt(-a^2 + b^2)*lo
g((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*co
sh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + 2*(
(a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*cosh(x)
+ (a^4 - a^2*b^2)*sinh(x)), -1/2*(a^3 - a*b^2 + 2*(a^2*b - b^3)*x*cosh(x) -
 (a^3 - a*b^2)*cosh(x)^2 - (a^3 - a*b^2)*sinh(x)^2 + 4*(b^2*cosh(x) + b^2*s
inh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)
) + 2*((a^2*b - b^3)*x - (a^3 - a*b^2)*cosh(x))*sinh(x))/((a^4 - a^2*b^2)*c
osh(x) + (a^4 - a^2*b^2)*sinh(x))]
```

Sympy [F]

$$\int \frac{\cosh(x)}{a + b\mathrm{sech}(x)}\,dx = \int \frac{\cosh(x)}{a + b\operatorname{sech}(x)}\,dx$$

```
[In] integrate(cosh(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(cosh(x)/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{a + b\mathrm{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a + b\text{sech}(x)}\,dx = \frac{2\,b^2 \arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}a^2} - \frac{bx}{a^2} - \frac{e^{(-x)}}{2\,a} + \frac{e^x}{2\,a}$$

```
[In] integrate(cosh(x)/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a^2) - b*x/a^2 -
 1/2*e^(-x)/a + 1/2*e^x/a
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{\cosh(x)}{a + b\text{sech}(x)}\,dx = \frac{e^x}{2\,a} - \frac{e^{-x}}{2\,a} - \frac{b\,x}{a^2} + \frac{b^2\,\ln\left(-\frac{2\,b^2\,e^x}{a^3} - \frac{2\,b^2\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{b^2\,\ln\left(\frac{2\,b^2\,(a+b\,e^x)}{a^3\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{2\,b^2\,e^x}{a^3}\right)}{a^2\,\sqrt{a+b}\,\sqrt{b-a}}$$

```
[In] int(cosh(x)/(a + b/cosh(x)),x)
```

```
[Out] exp(x)/(2*a) - exp(-x)/(2*a) - (b*x)/a^2 + (b^2*log(- (2*b^2*exp(x))/a^3 -
(2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b - a)^(1/2))))/(a^2*(a + b)^(1/
2)*(b - a)^(1/2)) - (b^2*log((2*b^2*(a + b*exp(x)))/(a^3*(a + b)^(1/2)*(b -
 a)^(1/2)) - (2*b^2*exp(x))/a^3))/(a^2*(a + b)^(1/2)*(b - a)^(1/2))
```

3.99 $\int \frac{\text{sech}(x)}{a+b\text{sech}(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[Out] 2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3916, 2738, 211}

$$\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx=\frac{2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[Sech[x]/(a + b*Sech[x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b} \\
&= \frac{2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx = -\frac{2\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] Integrate[Sech[x]/(a + b*Sech[x]),x]
[Out] (-2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}$	36
risch	$-\frac{\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}}+\frac{\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}}$	109

```
[In] int(sech(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.93

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,(a^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{-a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)}{a^2-b^2}, -\frac{2\arctan\left(-\frac{a\cosh(x)+a\sinh(x)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right]$$

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2
 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh(x) +
a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b
)*sinh(x) + a))/(a^2 - b^2), -2*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^
2 - b^2))/sqrt(a^2 - b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx$$

```
[In] integrate(sech(x)/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] integrate(sech(x)/(a+b*sech(x)),x, algorithm="giac")
[Out] 2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/sqrt(a^2 - b^2)
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{a^2-b^2}}+\frac{a\,\mathrm{e}^x}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[In] int(1/(cosh(x)*(a + b/cosh(x))),x)
[Out] (2*atan(b/(a^2 - b^2)^(1/2) + (a*exp(x))/(a^2 - b^2)^(1/2)))/(a^2 - b^2)^(1
/2)
```

3.100 $\int \frac{\text{sech}^2(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\text{sech}^2(x)}{a+b\text{sech}(x)}\,dx=\frac{\arctan(\sinh(x))}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}$$

```
[Out] arctan(sinh(x))/b-2*a*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b/(a-b)^(
1/2)/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3874, 3855, 3916, 2738, 211}

$$\int \frac{\text{sech}^2(x)}{a+b\text{sech}(x)}\,dx=\frac{\arctan(\sinh(x))}{b}-\frac{2a\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

```
[In] Int[Sech[x]^2/(a + b*Sech[x]),x]
```

```
[Out] ArcTan[Sinh[x]]/b - (2*a*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt
[a - b]*b*Sqrt[a + b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \operatorname{sech}(x)\,dx}{b} - \frac{a\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b}\\
&= \frac{\arctan(\sinh(x))}{b} - \frac{a\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^2}\\
&= \frac{\arctan(\sinh(x))}{b} - \frac{(2a)\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^2}\\
&= \frac{\arctan(\sinh(x))}{b} - \frac{2a\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{a\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right)}{b}$$

```
[In] Integrate[Sech[x]^2/(a + b*Sech[x]),x]
[Out] (2*(ArcTan[Tanh[x/2]] + (a*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2]))/b
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}+\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b}$	51
risch	$-\frac{a\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{a\ln\left(e^{x}+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{i\ln(e^{x}+i)}{b}-\frac{i\ln(e^{x}-i)}{b}$	141

```
[In] int(sech(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] -2*a/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+2/
b*arctan(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.06

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}a\log\left(\frac{a^2\cosh(x)^2+a^2\sinh(x)^2+2\,ab\cosh(x)-a^2+2\,b^2+2\,\left(a^2\cosh(x)+ab\right)\sinh(x)+2\,\sqrt{-a^2+b^2}(a\cosh(x)+a\sinh(x)+b)}{a\cosh(x)^2+a\sinh(x)^2+2\,b\cosh(x)+2\,(a\cosh(x)+b)\sinh(x)+a}\right)}{a^2b-b^3}\right.$$

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-a^2 + b^2)*a*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) -
a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x)
 + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x)
+ b)*sinh(x) + a)) - 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)
, 2*(sqrt(a^2 - b^2)*a*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2))
 + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^2*b - b^3)]
```

Sympy [F]

$$\int \frac{\mathrm{sech}^2(x)}{a + b\mathrm{sech}(x)}\,dx = \int \frac{\mathrm{sech}^2(x)}{a + b\,\mathrm{sech}(x)}\,dx$$

```
[In] integrate(sech(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\mathrm{sech}^2(x)}{a + b\mathrm{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\mathrm{sech}^2(x)}{a + b\mathrm{sech}(x)}\,dx = -\frac{2\,a\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b} + \frac{2\,\arctan\left(e^x\right)}{b}$$

```
[In] integrate(sech(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b) + 2*arctan(e^x
)/b
```

Mupad [B] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.30

$$\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx$$
$$= \frac{a\,\ln\left(64\,a\,b^4-64\,a^3\,b^2+128\,b^5\,\mathrm{e}^x-64\,a\,b^3\,\sqrt{b^2-a^2}+32\,a^3\,b\,\sqrt{b^2-a^2}+32\,a^4\,b\,\mathrm{e}^x-128\,b^4\,\mathrm{e}^x\,\sqrt{b^2-a^2}-\right.}{b\,\sqrt{b^2-a^2}}$$
$$-\frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{b}$$
$$-\frac{a\,\ln\left(64\,a\,b^4-64\,a^3\,b^2+128\,b^5\,\mathrm{e}^x+64\,a\,b^3\,\sqrt{b^2-a^2}-32\,a^3\,b\,\sqrt{b^2-a^2}+32\,a^4\,b\,\mathrm{e}^x+128\,b^4\,\mathrm{e}^x\,\sqrt{b^2-a^2}\right.}{b\,\sqrt{b^2-a^2}}$$

```
[In] int(1/(cosh(x)^2*(a + b/cosh(x))),x)
```

```
[Out] (a*log(64*a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) - 64*a*b^3*(b^2 - a^2)^(1/2)
+ 32*a^3*b*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) - 128*b^4*exp(x)*(b^2 - a^2)
^(1/2) - 160*a^2*b^3*exp(x) + 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(b^2
 - a^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b - (a*log(64*
a*b^4 - 64*a^3*b^2 + 128*b^5*exp(x) + 64*a*b^3*(b^2 - a^2)^(1/2) - 32*a^3*b
*(b^2 - a^2)^(1/2) + 32*a^4*b*exp(x) + 128*b^4*exp(x)*(b^2 - a^2)^(1/2) - 1
60*a^2*b^3*exp(x) - 96*a^2*b^2*exp(x)*(b^2 - a^2)^(1/2)))/(b*(b^2 - a^2)^(1
/2))
```

3.101 $\int \frac{\mathbf{sech}^3(x)}{a+b\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx=-\frac{a\arctan(\sinh(x))}{b^2}+\frac{2a^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}}+\frac{\tanh(x)}{b}$$

```
[Out] -a*arctan(sinh(x))/b^2+2*a^2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^
2/(a-b)^(1/2)/(a+b)^(1/2)+tanh(x)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3875, 3874, 3855, 3916, 2738, 211}

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{2a^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2\sqrt{a-b}\sqrt{a+b}}-\frac{a\arctan(\sinh(x))}{b^2}+\frac{\tanh(x)}{b}$$

```
[In] Int[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] -((a*ArcTan[Sinh[x]])/b^2) + (2*a^2*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a +
 b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]) + Tanh[x]/b
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3875

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Simp[-Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{\tanh(x)}{b} - \frac{a\int \frac{\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx}{b}$$

$$= \frac{\tanh(x)}{b} - \frac{a\int \operatorname{sech}(x)\,dx}{b^2} + \frac{a^2\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b^2}$$

$$= -\frac{a\arctan(\sinh(x))}{b^2} + \frac{\tanh(x)}{b} + \frac{a^2\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^3}$$

$$= -\frac{a\arctan(\sinh(x))}{b^2} + \frac{\tanh(x)}{b} + \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^3}$$

$$= -\frac{a\arctan(\sinh(x))}{b^2} + \frac{2a^2\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}} + \frac{\tanh(x)}{b}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{-2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2a^2\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+b\tanh(x)}{b^2}$$

```
[In] Integrate[Sech[x]^3/(a + b*Sech[x]),x]
```

```
[Out] (-2*a*ArcTan[Tanh[x/2]] - (2*a^2*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2
]])/Sqrt[a^2 - b^2] + b*Tanh[x])/b^2
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{2a^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{2\left(-\frac{b\tanh\left(\frac{x}{2}\right)}{1+\tanh\left(\frac{x}{2}\right)^2}+a\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{b^2}$	73
risch	$-\frac{2}{b(1+e^{2x})}-\frac{a^2\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^2}+\frac{a^2\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^2}+\frac{ia\ln(e^x-i)}{b^2}-\frac{ia\ln(e^x+i)}{b^2}$	160

```
[In] int(sech(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
-2/b^2*(-b*tanh(1/2*x)/(1+tanh(1/2*x)^2)+a*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(54) = 108.

Time = 0.28 (sec) , antiderivative size = 504, normalized size of antiderivative = 7.88

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx$$

$$=\left[-\frac{2a^2b-2b^3+\left(a^2\cosh(x)^2+2a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+a^2\right)\sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2+a^2\sinh\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{2\left(a^2b-b^3+\left(a^2\cosh(x)^2+2a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2+a^2\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cosh(x)+a\sinh\cdots}{\sqrt{a^2-b^2}}\right.\cdots}{a^2b^2-b^4+\left(a^2b^2-b^4\right)\cosh(x)^2\cdots}$$

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-(2*a^2*b - 2*b^3 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
 + a^2)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(-a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + 2*(a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^2 + 2*(a^3 -
 a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(x) + sinh(x)
))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^4)*cosh(x)*s
inh(x) + (a^2*b^2 - b^4)*sinh(x)^2), -2*(a^2*b - b^3 + (a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x
) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + (a^3 - a*b^2 + (a^3 - a*b^2)*cosh(x)^
2 + 2*(a^3 - a*b^2)*cosh(x)*sinh(x) + (a^3 - a*b^2)*sinh(x)^2)*arctan(cosh(
x) + sinh(x)))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + 2*(a^2*b^2 - b^
4)*cosh(x)*sinh(x) + (a^2*b^2 - b^4)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{sech}(x)}\, dx = \int \frac{\operatorname{sech}^3{(x)}}{a + b\operatorname{sech}{(x)}}\, dx$$

```
[In] integrate(sech(x)**3/(a+b*sech(x)),x)
```

```
[Out] Integral(sech(x)**3/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{a + b\operatorname{sech}(x)}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{2\,a^2\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^2} - \frac{2\,a\arctan\left(e^x\right)}{b^2} - \frac{2}{b(e^{(2\,x)}+1)}$$

```
[In] integrate(sech(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^2) - 2*a*arcta
n(e^x)/b^2 - 2/(b*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 4.35 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.59

$$\int \frac{\operatorname{sech}^3(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{a^2\,\ln\left(64\,a^3\,b-64\,a\,b^3+32\,a^3\,\sqrt{b^2-a^2}-32\,a^4\,e^x-128\,b^4\,e^x-64\,a\,b^2\,\sqrt{b^2-a^2}-128\,b^3\,e^x\,\sqrt{b^2-a^2}+1\right.}{b^2\,\sqrt{b^2-a^2}} + \frac{a\left(\ln\left(32\,e^x-32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,e^x+32\mathrm{i}\right)\,1\mathrm{i}\right)}{b^2} - \frac{2}{b+b\,e^{2\,x}} - \frac{a^2\,\ln\left(64\,a\,b^3-64\,a^3\,b+32\,a^3\,\sqrt{b^2-a^2}+32\,a^4\,e^x+128\,b^4\,e^x-64\,a\,b^2\,\sqrt{b^2-a^2}-128\,b^3\,e^x\,\sqrt{b^2-a^2}-\right.}{b^2\,\sqrt{b^2-a^2}}$$

```
[In] int(1/(cosh(x)^3*(a + b/cosh(x))),x)
```

```
[Out] (a*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/b^2 - 2/(b + b*exp(
2*x)) + (a^2*log(64*a^3*b - 64*a*b^3 + 32*a^3*(b^2 - a^2)^(1/2) - 32*a^4*ex
p(x) - 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128*b^3*exp(x)*(b^2 -
a^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2 - a^2)^(1/2)))/(b^2*
(b^2 - a^2)^(1/2)) - (a^2*log(64*a*b^3 - 64*a^3*b + 32*a^3*(b^2 - a^2)^(1/2
) + 32*a^4*exp(x) + 128*b^4*exp(x) - 64*a*b^2*(b^2 - a^2)^(1/2) - 128*b^3*e
xp(x)*(b^2 - a^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a^2*b*exp(x)*(b^2 - a^2)^
(1/2)))/(b^2*(b^2 - a^2)^(1/2))
```

3.102 $\int \frac{\text{sech}^4(x)}{a+b\text{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\text{sech}^4(x)}{a+b\text{sech}(x)} \, dx = \frac{(2a^2+b^2)\arctan(\sinh(x))}{2b^3} - \frac{2a^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b}$$

```
[Out] 1/2*(2*a^2+b^2)*arctan(sinh(x))/b^3-2*a^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a
+b)^(1/2))/b^3/(a-b)^(1/2)/(a+b)^(1/2)-a*tanh(x)/b^2+1/2*sech(x)*tanh(x)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3936, 4167, 4083, 3855, 3916, 2738, 211}

$$\int \frac{\text{sech}^4(x)}{a+b\text{sech}(x)} \, dx = -\frac{2a^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^3\sqrt{a-b}\sqrt{a+b}} + \frac{(2a^2+b^2)\arctan(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\tanh(x)\text{sech}(x)}{2b}$$

```
[In] Int[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] ((2*a^2 + b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*a^3*ArcTan[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]) - (a*Tanh[x])/b^2 + (Sech[
x]*Tanh[x])/(2*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3936

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-d^3)*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 3)/(b*f*
(n - 2))), x] + Dist[d^3/(b*(n - 2)), Int[(d*Csc[e + f*x])^(n - 3)*(Simp[a*
(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x]/(a + b*Csc[
e + f*x])), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ
[n, 3]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{\text{sech}(x)\tanh(x)}{2b} + \frac{\int \frac{\text{sech}(x)\left(a+b\text{sech}(x)-2a\text{sech}^2(x)\right)}{a+b\text{sech}(x)}\,dx}{2b}$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} + \frac{\int \frac{\text{sech}(x)\left(ab+(2a^2+b^2)\text{sech}(x)\right)}{a+b\text{sech}(x)}\,dx}{2b^2}$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{a^3\int \frac{\text{sech}(x)}{a+b\text{sech}(x)}\,dx}{b^3} + \frac{(2a^2+b^2)\int \text{sech}(x)\,dx}{2b^3}$$

$$= \frac{(2a^2+b^2)\arctan(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{a^3\int \frac{1}{1+\frac{a\cosh(x)}{b}}\,dx}{b^4}$$

$$= \frac{(2a^2+b^2)\arctan(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b} - \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b^4}$$

$$= \frac{(2a^2+b^2)\arctan(\sinh(x))}{2b^3} - \frac{2a^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}} - \frac{a\tanh(x)}{b^2} + \frac{\text{sech}(x)\tanh(x)}{2b}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{\text{sech}^4(x)}{a+b\text{sech}(x)}\,dx = \frac{2(2a^2+b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{4a^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + b(-2a+b\text{sech}(x))\tanh(x)}{2b^3}$$

```
[In] Integrate[Sech[x]^4/(a + b*Sech[x]),x]
```

```
[Out] (2*(2*a^2 + b^2)*ArcTan[Tanh[x/2]] + (4*a^3*ArcTan[((-a + b)*Tanh[x/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + b*(-2*a + b*Sech[x])*Tanh[x])/(2*b^3)
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
default	$\frac{\frac{2\left(\left(-ab-\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-ab+\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+\left(2a^2+b^2\right)\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b^3}-\frac{2a^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}}$
risch	$\frac{e^{3x}b+2a\,e^{2x}-e^{x}b+2a}{(1+e^{2x})^2b^2}+\frac{i\ln(e^x+i)a^2}{b^3}+\frac{i\ln(e^x+i)}{2b}-\frac{i\ln(e^x-i)a^2}{b^3}-\frac{i\ln(e^x-i)}{2b}-\frac{a^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,b^3}+\frac{a^3\ln\left(\right.}{}$

```
[In] int(sech(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b^3*(((-a*b-1/2*b^2)*tanh(1/2*x)^3+(-a*b+1/2*b^2)*tanh(1/2*x))/(1+tanh(1/
2*x)^2)^2+1/2*(2*a^2+b^2)*arctan(tanh(1/2*x)))-2*a^3/b^3/((a+b)*(a-b))^(1/2
)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(73) = 146.

Time = 0.33 (sec) , antiderivative size = 1444, normalized size of antiderivative = 16.60

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3
 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cos
h(x))*sinh(x)^2 - (a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4
+ 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh
(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sin
h(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sq
rt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*
b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + ((2*a^4 - a^2*b^2 - b^4)*cosh
(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2*a^4 - a^2*b^2 - b^
4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2
+ 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*cosh(x)^2)*sinh(x)^2
 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b^2 - b^4)*cosh(x))*
sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cosh(x) - (a^2*b^2 - b
^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*sinh(x))/(a^2
*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^5)*cosh(x)*sinh(x)^
3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 -
b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b^3 - b^5)*cosh(x)^3
```

```
 + (a^2*b^3 - b^5)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 + (a^2*b^2 - b^4)*
cosh(x)^3 + (a^2*b^2 - b^4)*sinh(x)^3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^
3*b - 2*a*b^3 + 3*(a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + 2*(a^3*cosh(x)^4 + 4
*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*c
osh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))*sqrt(a
^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + ((2*a^4 -
a^2*b^2 - b^4)*cosh(x)^4 + 4*(2*a^4 - a^2*b^2 - b^4)*cosh(x)*sinh(x)^3 + (2
*a^4 - a^2*b^2 - b^4)*sinh(x)^4 + 2*a^4 - a^2*b^2 - b^4 + 2*(2*a^4 - a^2*b^
2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4 + 3*(2*a^4 - a^2*b^2 - b^4)*c
osh(x)^2)*sinh(x)^2 + 4*((2*a^4 - a^2*b^2 - b^4)*cosh(x)^3 + (2*a^4 - a^2*b
^2 - b^4)*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^2*b^2 - b^4)*cos
h(x) - (a^2*b^2 - b^4 - 3*(a^2*b^2 - b^4)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cos
h(x))*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^4 + 4*(a^2*b^3 - b^
5)*cosh(x)*sinh(x)^3 + (a^2*b^3 - b^5)*sinh(x)^4 + 2*(a^2*b^3 - b^5)*cosh(x
)^2 + 2*(a^2*b^3 - b^5 + 3*(a^2*b^3 - b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^2*b
^3 - b^5)*cosh(x)^3 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx=\int \frac{\operatorname{sech}^4{(x)}}{a+b\operatorname{sech}{(x)}}\,dx$$

```
[In] integrate(sech(x)**4/(a+b*sech(x)),x)
[Out] Integral(sech(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{2\,a^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}b^3}+\frac{(2\,a^2+b^2)\arctan\left(e^x\right)}{b^3}+\frac{be^{(3\,x)}+2\,ae^{(2\,x)}-be^x+2\,a}{b^2\left(e^{(2\,x)}+1\right)^2}$$

```
[In] integrate(sech(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*b^3) + (2*a^2 +
 b^2)*arctan(e^x)/b^3 + (b*e^(3*x) + 2*a*e^(2*x) - b*e^x + 2*a)/(b^2*(e^(2*
x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.47

$$\int \frac{\operatorname{sech}^4(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{e^x}{b+b\,e^{2\,x}}-\frac{2\,e^x}{b+2\,b\,e^{2\,x}+b\,e^{4\,x}}+\frac{2\,a}{b^2\,e^{2\,x}+b^2}-\frac{\ln\left(1+e^x\,1\mathrm{i}\right)\,1\mathrm{i}-\ln\left(e^x+1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}-\frac{a^2\left(\ln\left(1+e^x\,1\mathrm{i}\right)\,1\mathrm{i}-\ln\left(e^x+1\mathrm{i}\right)\,1\mathrm{i}\right)}{b^3}-\frac{a^3\,\ln\left(16\,a\,b^5-48\,a^5\,b-24\,a^5\,\sqrt{b^2-a^2}+32\,a^3\,b^3+24\,a^6\,e^x+32\,b^6\,e^x+16\,a\,b^4\,\sqrt{b^2-a^2}+40\,a^3\,b^2\,\sqrt{b}\right.}{b^3\,\sqrt{b^2-}}+\frac{a^3\,\ln\left(16\,a\,b^5-48\,a^5\,b+24\,a^5\,\sqrt{b^2-a^2}+32\,a^3\,b^3+24\,a^6\,e^x+32\,b^6\,e^x-16\,a\,b^4\,\sqrt{b^2-a^2}-40\,a^3\,b^2\,\sqrt{b}\right.}{b^3\,\sqrt{b^2-}}$$

```
[In] int(1/(cosh(x)^4*(a + b/cosh(x))),x)
```

```
[Out] exp(x)/(b + b*exp(2*x)) - (2*exp(x))/(b + 2*b*exp(2*x) + b*exp(4*x)) + (2*a
)/(b^2*exp(2*x) + b^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2*b
) - (a^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/b^3 - (a^3*log(16*a
*b^5 - 48*a^5*b - 24*a^5*(b^2 - a^2)^(1/2) + 32*a^3*b^3 + 24*a^6*exp(x) + 3
2*b^6*exp(x) + 16*a*b^4*(b^2 - a^2)^(1/2) + 40*a^3*b^2*(b^2 - a^2)^(1/2) +
32*b^5*exp(x)*(b^2 - a^2)^(1/2) + 56*a^2*b^4*exp(x) - 112*a^4*b^2*exp(x) +
72*a^2*b^3*exp(x)*(b^2 - a^2)^(1/2) - 72*a^4*b*exp(x)*(b^2 - a^2)^(1/2)))/(
b^3*(b^2 - a^2)^(1/2)) + (a^3*log(16*a*b^5 - 48*a^5*b + 24*a^5*(b^2 - a^2)^
(1/2) + 32*a^3*b^3 + 24*a^6*exp(x) + 32*b^6*exp(x) - 16*a*b^4*(b^2 - a^2)^(
1/2) - 40*a^3*b^2*(b^2 - a^2)^(1/2) - 32*b^5*exp(x)*(b^2 - a^2)^(1/2) + 56*
a^2*b^4*exp(x) - 112*a^4*b^2*exp(x) - 72*a^2*b^3*exp(x)*(b^2 - a^2)^(1/2) +
 72*a^4*b*exp(x)*(b^2 - a^2)^(1/2)))/(b^3*(b^2 - a^2)^(1/2))
```

3.103 $\int \frac{\tanh^6(x)}{a+a\text{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int \frac{\tanh^6(x)}{a+a\text{sech}(x)} \, dx = \frac{x}{a}-\frac{3\arctan(\sinh(x))}{8a}-\frac{(8-3\text{sech}(x))\tanh(x)}{8a} -\frac{(4-3\text{sech}(x))\tanh^3(x)}{12a}$$

```
[Out] x/a-3/8*arctan(sinh(x))/a-1/8*(8-3*sech(x))*tanh(x)/a-1/12*(4-3*sech(x))*ta
nh(x)^3/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3973, 3966, 3855}

$$\int \frac{\tanh^6(x)}{a+a\text{sech}(x)} \, dx = -\frac{3\arctan(\sinh(x))}{8a}+\frac{x}{a}-\frac{\tanh^3(x)(4-3\text{sech}(x))}{12a} -\frac{\tanh(x)(8-3\text{sech}(x))}{8a}$$

```
[In] Int[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*a) - ((8 - 3*Sech[x])*Tanh[x])/(8*a) - ((4 - 3
*Sech[x])*Tanh[x]^3)/(12*a)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int (-a+a\operatorname{sech}(x))\tanh^4(x)\,dx}{a^2}\\
&= -\frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a}-\frac{\int (-4a+3a\operatorname{sech}(x))\tanh^2(x)\,dx}{4a^2}\\
&= -\frac{(8-3\operatorname{sech}(x))\tanh(x)}{8a}-\frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a}-\frac{\int (-8a+3a\operatorname{sech}(x))\,dx}{8a^2}\\
&= \frac{x}{a}-\frac{(8-3\operatorname{sech}(x))\tanh(x)}{8a}-\frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a}-\frac{3\int \operatorname{sech}(x)\,dx}{8a}\\
&= \frac{x}{a}-\frac{3\arctan(\sinh(x))}{8a}-\frac{(8-3\operatorname{sech}(x))\tanh(x)}{8a}-\frac{(4-3\operatorname{sech}(x))\tanh^3(x)}{12a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{\tanh^6(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(6\left(4x-3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)+\left(-32+15\operatorname{sech}(x)+8\operatorname{sech}^2(x)-6\operatorname{sech}^3(x)\right)\tanh(x)\right)}{12a(1+\operatorname{sech}(x))}$$

```
[In] Integrate[Tanh[x]^6/(a + a*Sech[x]),x]
```

```
[Out] (Cosh[x/2]^2*Sech[x]*(6*(4*x - 3*ArcTan[Tanh[x/2]]) + (-32 + 15*Sech[x] + 8
*Sech[x]^2 - 6*Sech[x]^3)*Tanh[x]))/(12*a*(1 + Sech[x]))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

method	result	size
default	$\frac{\frac{2\left(-\frac{11\tanh\left(\frac{x}{2}\right)^{7}}{8}-\frac{137\tanh\left(\frac{x}{2}\right)^{5}}{24}-\frac{71\tanh\left(\frac{x}{2}\right)^{3}}{24}-\frac{5\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{4}}-\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	75
risch	$\frac{x}{a}+\frac{15\,e^{7x}+48\,e^{6x}-9\,e^{5x}+96\,e^{4x}+9\,e^{3x}+80\,e^{2x}-15\,e^{x}+32}{12(1+e^{2x})^{4}a}+\frac{3i\ln(e^{x}-i)}{8a}-\frac{3i\ln(e^{x}+i)}{8a}$	86

```
[In] int(tanh(x)^6/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 64/a*(1/32*(-11/8*tanh(1/2*x)^7-137/24*tanh(1/2*x)^5-71/24*tanh(1/2*x)^3-5/
8*tanh(1/2*x))/(1+tanh(1/2*x)^2)^4-3/256*arctan(tanh(1/2*x))+1/64*ln(tanh(1
/2*x)+1)-1/64*ln(tanh(1/2*x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(42) = 84$.

Time = 0.27 (sec) , antiderivative size = 686, normalized size of antiderivative = 14.29

$$\int \frac{\tanh^6(x)}{a + a\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="fricas")
[Out] 1/12*(12*x*cosh(x)^8 + 12*x*sinh(x)^8 + 3*(32*x*cosh(x) + 5)*sinh(x)^7 + 48
*(x + 1)*cosh(x)^6 + 15*cosh(x)^7 + 3*(112*x*cosh(x)^2 + 16*x + 35*cosh(x)
+ 16)*sinh(x)^6 + 3*(224*x*cosh(x)^3 + 96*(x + 1)*cosh(x) + 105*cosh(x)^2 -
 3)*sinh(x)^5 + 24*(3*x + 4)*cosh(x)^4 - 9*cosh(x)^5 + 3*(280*x*cosh(x)^4 +
 240*(x + 1)*cosh(x)^2 + 175*cosh(x)^3 + 24*x - 15*cosh(x) + 32)*sinh(x)^4
+ 3*(224*x*cosh(x)^5 + 320*(x + 1)*cosh(x)^3 + 175*cosh(x)^4 + 32*(3*x + 4)
*cosh(x) - 30*cosh(x)^2 + 3)*sinh(x)^3 + 16*(3*x + 5)*cosh(x)^2 + 9*cosh(x)
^3 + (336*x*cosh(x)^6 + 720*(x + 1)*cosh(x)^4 + 315*cosh(x)^5 + 144*(3*x +
4)*cosh(x)^2 - 90*cosh(x)^3 + 48*x + 27*cosh(x) + 80)*sinh(x)^2 - 9*(cosh(x
)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*c
osh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cos
h(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cos
h(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^
2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(
x) + 1)*arctan(cosh(x) + sinh(x)) + (96*x*cosh(x)^7 + 288*(x + 1)*cosh(x)^5
 + 105*cosh(x)^6 + 96*(3*x + 4)*cosh(x)^3 - 45*cosh(x)^4 + 32*(3*x + 5)*cos
h(x) + 27*cosh(x)^2 - 15)*sinh(x) + 12*x - 15*cosh(x) + 32)/(a*cosh(x)^8 +
8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4*(7*a*cosh(x)^2 + a)
```

```
*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5 + 6*a*cosh(x)^4 + 2*
(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(7*a*cosh(x)^5 + 10*a
*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4*(7*a*cosh(x)^6 + 15
*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh(x)^7 + 3*a*cosh(x)^
5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\tanh^6(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\tanh^6(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(tanh(x)**6/(a+a*sech(x)),x)
[Out] Integral(tanh(x)**6/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.94

$$\begin{aligned}&\int \frac{\tanh^6(x)}{a + a\operatorname{sech}(x)}\,dx\\&= \frac{x}{a} + \frac{15\,e^{(-x)} - 80\,e^{(-2\,x)} - 9\,e^{(-3\,x)} - 96\,e^{(-4\,x)} + 9\,e^{(-5\,x)} - 48\,e^{(-6\,x)} - 15\,e^{(-7\,x)} - 32}{12\,(4\,ae^{(-2\,x)} + 6\,ae^{(-4\,x)} + 4\,ae^{(-6\,x)} + ae^{(-8\,x)} + a)}\\&\quad + \frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}\end{aligned}$$

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 1/12*(15*e^(-x) - 80*e^(-2*x) - 9*e^(-3*x) - 96*e^(-4*x) + 9*e^(-5*x)
 - 48*e^(-6*x) - 15*e^(-7*x) - 32)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6
*x) + a*e^(-8*x) + a) + 3/4*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\begin{aligned}&\int \frac{\tanh^6(x)}{a + a\operatorname{sech}(x)}\,dx\\&= \frac{x}{a} - \frac{3\,\arctan\left(e^{x}\right)}{4\,a}\\&\quad + \frac{15\,e^{(7\,x)} + 48\,e^{(6\,x)} - 9\,e^{(5\,x)} + 96\,e^{(4\,x)} + 9\,e^{(3\,x)} + 80\,e^{(2\,x)} - 15\,e^{x} + 32}{12\,a(e^{(2\,x)} + 1)^4}\end{aligned}$$

```
[In] integrate(tanh(x)^6/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 3/4*arctan(e^x)/a + 1/12*(15*e^(7*x) + 48*e^(6*x) - 9*e^(5*x) + 96*e^
(4*x) + 9*e^(3*x) + 80*e^(2*x) - 15*e^x + 32)/(a*(e^(2*x) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.98

$$\int \frac{\tanh^6(x)}{a+a\mathrm{sech}(x)}\,dx = \frac{\frac{8}{3\,a}+\frac{6\,\mathrm{e}^{x}}{a}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1}-\frac{\frac{4}{a}+\frac{9\,\mathrm{e}^{x}}{2\,a}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{x}{a}+\frac{\frac{4}{a}+\frac{5\,\mathrm{e}^{x}}{4\,a}}{\mathrm{e}^{2\,x}+1}-\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{4\,\sqrt{a^2}}-\frac{4\,\mathrm{e}^{x}}{a\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}$$

```
[In] int(tanh(x)^6/(a + a/cosh(x)),x)
```

```
[Out] (8/(3*a) + (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (4/a +
(9*exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + x/a + (4/a + (5*exp(x))/(4*
a))/(exp(2*x) + 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2)) - (4*
exp(x))/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

3.104 $\int \frac{\tanh^5(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}+\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^2(x)}{2a}-\frac{\operatorname{sech}^3(x)}{3a}$$

```
[Out] ln(cosh(x))/a+sech(x)/a+1/2*sech(x)^2/a-1/3*sech(x)^3/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3964, 76}

$$\int \frac{\tanh^5(x)}{a+a\operatorname{sech}(x)}\,dx=-\frac{\operatorname{sech}^3(x)}{3a}+\frac{\operatorname{sech}^2(x)}{2a}+\frac{\operatorname{sech}(x)}{a}+\frac{\log(\cosh(x))}{a}$$

```
[In] Int[Tanh[x]^5/(a + a*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Sech[x]/a + Sech[x]^2/(2*a) - Sech[x]^3/(3*a)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] && !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{(a-ax)^2(a+ax)}{x^4}\,dx, x, \cosh(x)\right)}{a^4}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^4} - \frac{a^3}{x^3} - \frac{a^3}{x^2} + \frac{a^3}{x}\right)\,dx, x, \cosh(x)\right)}{a^4}\\
&= \frac{\log(\cosh(x))}{a} + \frac{\text{sech}(x)}{a} + \frac{\text{sech}^2(x)}{2a} - \frac{\text{sech}^3(x)}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\tanh^5(x)}{a + a\text{sech}(x)}\,dx = \frac{(2 + 6\cosh(2x) + 3\cosh(3x)\log(\cosh(x)) + \cosh(x)(6 + 9\log(\cosh(x))))\text{sech}^3(x)}{12a}$$

```
[In] Integrate[Tanh[x]^5/(a + a*Sech[x]),x]
[Out] ((2 + 6*Cosh[2*x] + 3*Cosh[3*x]*Log[Cosh[x]] + Cosh[x]*(6 + 9*Log[Cosh[x]])
)*Sech[x]^3)/(12*a)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$-\frac{\frac{\text{sech}(x)^3}{3} - \frac{\text{sech}(x)^2}{2} - \text{sech}(x) + \ln(\text{sech}(x))}{a}$	26
default	$-\frac{\frac{\text{sech}(x)^3}{3} - \frac{\text{sech}(x)^2}{2} - \text{sech}(x) + \ln(\text{sech}(x))}{a}$	26
risch	$-\frac{x}{a} + \frac{2\,\mathrm{e}^{x}\left(3\,\mathrm{e}^{4x} + 3\,\mathrm{e}^{3x} + 2\,\mathrm{e}^{2x} + 3\,\mathrm{e}^{x} + 3\right)}{3\left(1 + \mathrm{e}^{2x}\right)^3 a} + \frac{\ln\left(1 + \mathrm{e}^{2x}\right)}{a}$	58

```
[In] int(tanh(x)^5/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] -1/a*(1/3*sech(x)^3-1/2*sech(x)^2-sech(x)+ln(sech(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(32) = 64.

Time = 0.27 (sec) , antiderivative size = 437, normalized size of antiderivative = 12.14

$$\int \frac{\tanh^5(x)}{a+a\text{sech}(x)}\,dx = -\frac{3\,x\cosh\left(x\right)^6+3\,x\sinh\left(x\right)^6+6\,(3\,x\cosh\left(x\right)-1)\sinh\left(x\right)^5+3\,(3\,x-2)\cosh\left(x\right)^4-6\,\cosh\left(x\right)^5+3\,(15}{}$$

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*x*cosh(x)^6 + 3*x*sinh(x)^6 + 6*(3*x*cosh(x) - 1)*sinh(x)^5 + 3*(3*
x - 2)*cosh(x)^4 - 6*cosh(x)^5 + 3*(15*x*cosh(x)^2 + 3*x - 10*cosh(x) - 2)*
sinh(x)^4 + 4*(15*x*cosh(x)^3 + 3*(3*x - 2)*cosh(x) - 15*cosh(x)^2 - 1)*sin
h(x)^3 + 3*(3*x - 2)*cosh(x)^2 - 4*cosh(x)^3 + 3*(15*x*cosh(x)^4 + 6*(3*x -
 2)*cosh(x)^2 - 20*cosh(x)^3 + 3*x - 4*cosh(x) - 2)*sinh(x)^2 - 3*(cosh(x)^
6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cos
h(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)
^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sin
h(x) + 1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*x*cosh(x)^5 + 2*(3*x -
2)*cosh(x)^3 - 5*cosh(x)^4 + (3*x - 2)*cosh(x) - 2*cosh(x)^2 - 1)*sinh(x) +
 3*x - 6*cosh(x))/(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 + 3*a*
cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x
))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x
)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{a+a\text{sech}(x)}\,dx = \frac{\int \frac{\tanh^5(x)}{\text{sech}\,(x)+1}\,dx}{a}$$

```
[In] integrate(tanh(x)**5/(a+a*sech(x)),x)
```

```
[Out] Integral(tanh(x)**5/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(32) = 64$.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.06

$$\int \frac{\tanh^5(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{2\left(3\,e^{(-x)} + 3\,e^{(-2\,x)} + 2\,e^{(-3\,x)} + 3\,e^{(-4\,x)} + 3\,e^{(-5\,x)}\right)}{3\left(3\,ae^{(-2\,x)} + 3\,ae^{(-4\,x)} + ae^{(-6\,x)} + a\right)} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + 2/3*(3*e^(-x) + 3*e^(-2*x) + 2*e^(-3*x) + 3*e^(-4*x) + 3*e^(-5*x))/(3
*a*e^(-2*x) + 3*a*e^(-4*x) + a*e^(-6*x) + a) + log(e^(-2*x) + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{\tanh^5(x)}{a + a\text{sech}(x)}\,dx = \frac{\log\left(e^{(-x)} + e^x\right)}{a} - \frac{11\left(e^{(-x)} + e^x\right)^3 - 12\left(e^{(-x)} + e^x\right)^2 - 12\,e^{(-x)} - 12\,e^x + 16}{6\,a\left(e^{(-x)} + e^x\right)^3}$$

```
[In] integrate(tanh(x)^5/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] log(e^(-x) + e^x)/a - 1/6*(11*(e^(-x) + e^x)^3 - 12*(e^(-x) + e^x)^2 - 12*e
^(-x) - 12*e^x + 16)/(a*(e^(-x) + e^x)^3)
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{\tanh^5(x)}{a + a\text{sech}(x)}\,dx = \frac{\ln\left(e^{2\,x} + 1\right)}{a} - \frac{\frac{2}{a} + \frac{8\,e^x}{3\,a}}{2\,e^{2\,x} + e^{4\,x} + 1} - \frac{x}{a} + \frac{\frac{2}{a} + \frac{2\,e^x}{a}}{e^{2\,x} + 1} + \frac{8\,e^x}{3\,a\,\left(3\,e^{2\,x} + 3\,e^{4\,x} + e^{6\,x} + 1\right)}$$

```
[In] int(tanh(x)^5/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) + 1)/a - (2/a + (8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1)
- x/a + (2/a + (2*exp(x))/a)/(exp(2*x) + 1) + (8*exp(x))/(3*a*(3*exp(2*x) +
 3*exp(4*x) + exp(6*x) + 1))
```

3.105 $\int \frac{\tanh^4(x)}{a+a\text{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\tanh^4(x)}{a+a\text{sech}(x)} \, dx = \frac{x}{a} - \frac{\arctan(\sinh(x))}{2a} - \frac{(2-\text{sech}(x))\tanh(x)}{2a}$$

```
[Out] x/a-1/2*arctan(sinh(x))/a-1/2*(2-sech(x))*tanh(x)/a
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3973, 3966, 3855}

$$\int \frac{\tanh^4(x)}{a+a\text{sech}(x)} \, dx = -\frac{\arctan(\sinh(x))}{2a} + \frac{x}{a} - \frac{\tanh(x)(2-\text{sech}(x))}{2a}$$

```
[In] Int[Tanh[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/(2*a) - ((2 - Sech[x])*Tanh[x])/(2*a)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3966

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-e)*(e*Cot[c + d*x])^(m - 1)*((a*m + b*(m - 1)*Csc
[c + d*x])/(d*m*(m - 1))), x] - Dist[e^2/m, Int[(e*Cot[c + d*x])^(m - 2)*(a
```

```
*m + b*(m - 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m,
 1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int(-a+a\operatorname{sech}(x))\tanh^2(x)\,dx}{a^2}\\
&= -\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}-\frac{\int(-2a+a\operatorname{sech}(x))\,dx}{2a^2}\\
&= \frac{x}{a}-\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}-\frac{\int\operatorname{sech}(x)\,dx}{2a}\\
&= \frac{x}{a}-\frac{\arctan(\sinh(x))}{2a}-\frac{(2-\operatorname{sech}(x))\tanh(x)}{2a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int\frac{\tanh^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\cosh^2\left(\frac{x}{2}\right)\operatorname{sech}(x)\left(2\left(x-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)+(-2+\operatorname{sech}(x))\tanh(x)\right)}{a(1+\operatorname{sech}(x))}$$

```
[In] Integrate[Tanh[x]^4/(a + a*Sech[x]),x]
[Out] (Cosh[x/2]^2*Sech[x]*(2*(x - ArcTan[Tanh[x/2]]) + (-2 + Sech[x])*Tanh[x]))/
(a*(1 + Sech[x]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

method	result	size
default	$\frac{\frac{2\left(-\frac{3\tanh\left(\frac{x}{2}\right)^{3}}{2}-\frac{\tanh\left(\frac{x}{2}\right)}{2}\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{2}}-\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	59
risch	$\frac{x}{a}+\frac{e^{3x}+2\,e^{2x}-e^{x}+2}{\left(1+e^{2x}\right)^{2}a}+\frac{i\ln\left(e^{x}-i\right)}{2a}-\frac{i\ln\left(e^{x}+i\right)}{2a}$	59

```
[In] int(tanh(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a*(1/8*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2-1/16*arc
tan(tanh(1/2*x))+1/16*ln(tanh(1/2*x)+1)-1/16*ln(tanh(1/2*x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(25) = 50$.

Time = 0.25 (sec) , antiderivative size = 210, normalized size of antiderivative = 6.77

$$\int \frac{\tanh^4(x)}{a+a\text{sech}(x)}\,dx$$
$$= \frac{x\cosh\left(x\right)^{4}+x\sinh\left(x\right)^{4}+\left(4\,x\cosh\left(x\right)+1\right)\sinh\left(x\right)^{3}+2\left(x+1\right)\cosh\left(x\right)^{2}+\cosh\left(x\right)^{3}+\left(6\,x\cosh\left(x\right)^{2}+\right.}{}$$

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] (x*cosh(x)^4 + x*sinh(x)^4 + (4*x*cosh(x) + 1)*sinh(x)^3 + 2*(x + 1)*cosh(x
)^2 + cosh(x)^3 + (6*x*cosh(x)^2 + 2*x + 3*cosh(x) + 2)*sinh(x)^2 - (cosh(x
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) +
 (4*x*cosh(x)^3 + 4*(x + 1)*cosh(x) + 3*cosh(x)^2 - 1)*sinh(x) + x - cosh(x
) + 2)/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*cosh(x)^2 +
 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\tanh^4(x)}{\text{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(tanh(x)**4/(a+a*sech(x)),x)
[Out] Integral(tanh(x)**4/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{e^{(-x)} - 2\,e^{(-2\,x)} - e^{(-3\,x)} - 2}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} + \frac{\arctan\left(e^{(-x)}\right)}{a}$$

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + (e^(-x) - 2*e^(-2*x) - e^(-3*x) - 2)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
+ arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} - \frac{\arctan\left(e^{x}\right)}{a} + \frac{e^{(3\,x)} + 2\,e^{(2\,x)} - e^{x} + 2}{a{\left(e^{(2\,x)} + 1\right)}^2}$$

```
[In] integrate(tanh(x)^4/(a+a*sech(x)),x, algorithm="giac")
[Out] x/a - arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.16

$$\int \frac{\tanh^4(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{\frac{2}{a} + \frac{e^{x}}{a}}{e^{2\,x} + 1} - \frac{\operatorname{atan}\left(\frac{e^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,e^{x}}{a\,\left(2\,e^{2\,x} + e^{4\,x} + 1\right)}$$

```
[In] int(tanh(x)^4/(a + a/cosh(x)),x)
[Out] x/a + (2/a + exp(x)/a)/(exp(2*x) + 1) - atan((exp(x)*(a^2)^(1/2))/a)/(a^2)^
(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

3.106 $\int \frac{\tanh^3(x)}{a+a\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\tanh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a} + \frac{\operatorname{sech}(x)}{a}$$

```
[Out] ln(cosh(x))/a+sech(x)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3964, 45}

$$\int \frac{\tanh^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\operatorname{sech}(x)}{a} + \frac{\log(\cosh(x))}{a}$$

```
[In] Int[Tanh[x]^3/(a + a*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Sech[x]/a
```

<u>Rule 45</u>

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

<u>Rule 3964</u>

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{a-ax}{x^2}\,dx, x, \cosh(x)\right)}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a}{x^2}-\frac{a}{x}\right)\,dx, x, \cosh(x)\right)}{a^2}\\ &= \frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\tanh^3(x)}{a+a\text{sech}(x)}\,dx = \frac{\log(\cosh(x))+\text{sech}(x)}{a}$$

```
[In] Integrate[Tanh[x]^3/(a + a*Sech[x]),x]
[Out] (Log[Cosh[x]] + Sech[x])/a
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

method	result	size
risch	$-\frac{x}{a}+\frac{2\,\mathrm{e}^{x}}{a(1+\mathrm{e}^{2x})}+\frac{\ln(1+\mathrm{e}^{2x})}{a}$	34
default	$\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\ln\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)+\frac{8}{4+4\tanh\left(\frac{x}{2}\right)^{2}}}{a}$	48

```
[In] int(tanh(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] -x/a+2/a*exp(x)/(1+exp(2*x))+1/a*ln(1+exp(2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 6.07

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\,dx = -\frac{x\cosh(x)^2 + x\sinh(x)^2 - (\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1)\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) + 2\,(x\cosh(x)-1)\sinh(x) + x - 2\cosh(x)}{a\cosh(x)^2 + 2\,a\cosh(x)\sinh(x) + a\sinh(x)^2 + a}$$

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="fricas")
[Out] -(x*cosh(x)^2 + x*sinh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 +
1)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(x*cosh(x) - 1)*sinh(x) + x - 2*c
osh(x))/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{\int \frac{\tanh^3(x)}{\text{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(tanh(x)**3/(a+a*sech(x)),x)
[Out] Integral(tanh(x)**3/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(14) = 28$.

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^3(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} + \frac{2\,e^{(-x)}}{ae^{(-2\,x)} + a} + \frac{\log\left(e^{(-2\,x)} + 1\right)}{a}$$

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 2*e^(-x)/(a*e^(-2*x) + a) + log(e^(-2*x) + 1)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{\tanh^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\log\left(e^{(-x)} + e^x\right)}{a} - \frac{e^{(-x)} + e^x - 2}{a(e^{(-x)} + e^x)}$$

```
[In] integrate(tanh(x)^3/(a+a*sech(x)),x, algorithm="giac")
[Out] log(e^(-x) + e^x)/a - (e^(-x) + e^x - 2)/(a*(e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^3(x)}{a + a\operatorname{sech}(x)}\,dx = \frac{\ln\left(e^{2x} + 1\right)}{a} - \frac{x}{a} + \frac{2\,e^x}{a\,\left(e^{2x} + 1\right)}$$

```
[In] int(tanh(x)^3/(a + a/cosh(x)),x)
[Out] log(exp(2*x) + 1)/a - x/a + (2*exp(x))/(a*(exp(2*x) + 1))
```

3.107 $\int \frac{\tanh^2(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\tanh^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{a}-\frac{\arctan(\sinh(x))}{a}$$

```
[Out] x/a-arctan(sinh(x))/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3973, 3855}

$$\int \frac{\tanh^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{a}-\frac{\arctan(\sinh(x))}{a}$$

```
[In] Int[Tanh[x]^2/(a + a*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/a
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
```

```
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\int (-a + a\,\mathrm{sech}(x))\,dx}{a^2} \\ &= \frac{x}{a} - \frac{\int \mathrm{sech}(x)\,dx}{a} \\ &= \frac{x}{a} - \frac{\arctan(\sinh(x))}{a} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^2(x)}{a + a\,\mathrm{sech}(x)}\,dx = \frac{x - 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

```
[In] Integrate[Tanh[x]^2/(a + a*Sech[x]),x]
[Out] (x - 2*ArcTan[Tanh[x/2]])/a
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

method	result	size
risch	$\frac{x}{a} + \frac{i\ln(\mathrm{e}^x - i)}{a} - \frac{i\ln(\mathrm{e}^x + i)}{a}$	31
default	$\frac{-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right) - 2\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}$	32

```
[In] int(tanh(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] x/a+I/a*ln(exp(x)-I)-I/a*ln(exp(x)+I)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\, dx = \frac{x - 2 \arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}{a}$$

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] (x - 2*arctan(cosh(x) + sinh(x)))/a
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\, dx = \frac{\int \frac{\tanh^2(x)}{\operatorname{sech}(x)+1}\, dx}{a}$$

```
[In] integrate(tanh(x)**2/(a+a*sech(x)),x)
[Out] Integral(tanh(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} + \frac{2 \arctan\left(e^{(-x)}\right)}{a}$$

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 2*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} - \frac{2 \arctan\left(e^{x}\right)}{a}$$

```
[In] integrate(tanh(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] x/a - 2*arctan(e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{\tanh^2(x)}{a+a\mathrm{sech}(x)}\,dx = \frac{x}{a}-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

```
[In] int(tanh(x)^2/(a + a/cosh(x)),x)
[Out] x/a - (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

3.108 $\int \frac{\tanh(x)}{a+a\,\text{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{\tanh(x)}{a+a\text{sech}(x)}\,dx = \frac{\log(1+\cosh(x))}{a}$$

```
[Out] ln(1+cosh(x))/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3964, 31}

$$\int \frac{\tanh(x)}{a+a\text{sech}(x)}\,dx = \frac{\log(\cosh(x)+1)}{a}$$

```
[In] Int[Tanh[x]/(a + a*Sech[x]),x]
```

```
[Out] Log[1 + Cosh[x]]/a
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
```

```
[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{a+ax}\,dx, x, \cosh(x)\right)$$
$$= \frac{\log(1+\cosh(x))}{a}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\tanh(x)}{a+a\text{sech}(x)}\,dx = \frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

```
[In] Integrate[Tanh[x]/(a + a*Sech[x]),x]
[Out] (2*Log[Cosh[x/2]])/a
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

method	result	size
derivativedivides	$-\frac{-\ln(1+\text{sech}(x))+\ln(\text{sech}(x))}{a}$	17
default	$-\frac{-\ln(1+\text{sech}(x))+\ln(\text{sech}(x))}{a}$	17
risch	$-\frac{x}{a}+\frac{2\ln(e^{x}+1)}{a}$	18

```
[In] int(tanh(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] -1/a*(-ln(1+sech(x))+ln(sech(x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)}\, dx = -\frac{x - 2\,\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right)}{a}$$

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="fricas")
[Out] -(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} - \frac{\log\left(\tanh\left(x\right) + 1\right)}{a} + \frac{\log\left(\text{sech}\left(x\right) + 1\right)}{a}$$

```
[In] integrate(tanh(x)/(a+a*sech(x)),x)
[Out] x/a - log(tanh(x) + 1)/a + log(sech(x) + 1)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} + \frac{2\,\log\left(e^{(-x)} + 1\right)}{a}$$

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 2*log(e^(-x) + 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{\tanh(x)}{a + a\mathrm{sech}(x)}\, dx = -\frac{x}{a} + \frac{2 \log\left(e^{x} + 1\right)}{a}$$

```
[In] integrate(tanh(x)/(a+a*sech(x)),x, algorithm="giac")
[Out] -x/a + 2*log(e^x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \frac{\tanh(x)}{a + a\mathrm{sech}(x)}\, dx = -\frac{x - 2 \ln\left(\mathrm{e}^{x} + 1\right)}{a}$$

```
[In] int(tanh(x)/(a + a/cosh(x)),x)
[Out] -(x - 2*log(exp(x) + 1))/a
```

3.109 $\int \frac{\coth(x)}{a+a\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int \frac{\coth(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{1}{2a(1+\cosh(x))}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(1+\cosh(x))}{4a}$$

```
[Out] 1/2/a/(1+cosh(x))+1/4*ln(1-cosh(x))/a+3/4*ln(1+cosh(x))/a
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3964, 90}

$$\int \frac{\coth(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{1}{2a(\cosh(x)+1)}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(\cosh(x)+1)}{4a}$$

```
[In] Int[Coth[x]/(a + a*Sech[x]),x]
```

```
[Out] 1/(2*a*(1 + Cosh[x])) + Log[1 - Cosh[x]]/(4*a) + (3*Log[1 + Cosh[x]])/(4*a)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
```

```
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a^2\text{Subst}\left(\int \frac{x^2}{(a-ax)(a+ax)^2}\,dx, x, \cosh(x)\right)\right)\\
&= -\left(a^2\text{Subst}\left(\int \left(-\frac{1}{4a^3(-1+x)}+\frac{1}{2a^3(1+x)^2}-\frac{3}{4a^3(1+x)}\right)\,dx, x, \cosh(x)\right)\right)\\
&= \frac{1}{2a(1+\cosh(x))}+\frac{\log(1-\cosh(x))}{4a}+\frac{3\log(1+\cosh(x))}{4a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{\coth(x)}{a+a\text{sech}(x)}\,dx = \frac{\left(1+2\cosh^2\left(\frac{x}{2}\right)\left(3\log\left(\cosh\left(\frac{x}{2}\right)\right)+\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)\right)\text{sech}(x)}{2a(1+\text{sech}(x))}$$

```
[In] Integrate[Coth[x]/(a + a*Sech[x]),x]
[Out] ((1 + 2*Cosh[x/2]^2*(3*Log[Cosh[x/2]] + Log[Sinh[x/2]]))*Sech[x])/(2*a*(1 +
 Sech[x]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^2}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)-2\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-2\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2a}$	38
risch	$-\frac{x}{a}+\frac{e^x}{\left(e^x+1\right)^2 a}+\frac{3\ln(e^x+1)}{2a}+\frac{\ln(e^x-1)}{2a}$	40

```
[In] int(coth(x)/(a+a*sech(x)),x,method=_RETURNVERBOSE)
[Out] 1/2/a*(-1/2*tanh(1/2*x)^2+ln(tanh(1/2*x))-2*ln(tanh(1/2*x)+1)-2*ln(tanh(1/2
*x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(34) = 68$.

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.40

$$\int \frac{\coth(x)}{a + a\text{sech}(x)}\, dx = -\frac{2\,x\cosh\left(x\right)^{2} + 2\,x\sinh\left(x\right)^{2} + 2\,(2\,x - 1)\cosh\left(x\right) - 3\,\left(\cosh\left(x\right)^{2} + 2\,(\cosh\left(x\right) + 1)\sinh\left(x\right) + \sinh\left(x\right)^{2}\right.}{}$$

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cosh(x)^2 + 2*x*sinh(x)^2 + 2*(2*x - 1)*cosh(x) - 3*(cosh(x)^2 +
2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x)
+ 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) - 1) + 2*(2*x*cosh(x) + 2*x - 1)*sinh(x) + 2*x)/(a*cosh
(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\coth(x)}{a + a\text{sech}(x)}\, dx = \frac{\int \frac{\coth\left(x\right)}{\operatorname{sech}\left(x\right)+1}\, dx}{a}$$

```
[In] integrate(coth(x)/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{\coth(x)}{a + a\text{sech}(x)}\, dx = \frac{x}{a} + \frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} + \frac{3\,\log\left(e^{(-x)} + 1\right)}{2\,a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a}$$

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="maxima")
```

```
[Out] x/a + e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) + 3/2*log(e^(-x) + 1)/a + 1/2*lo
g(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{\coth(x)}{a + a\text{sech}(x)}\,dx = \frac{3\,\log\left(e^{(-x)} + e^{x} + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\,a} - \frac{3\,e^{(-x)} + 3\,e^{x} + 2}{4\,a(e^{(-x)} + e^{x} + 2)}$$

```
[In] integrate(coth(x)/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 3/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a - 1/4*(3*e^(-x) +
 3*e^x + 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int \frac{\coth(x)}{a + a\text{sech}(x)}\,dx = \frac{\ln\left(\mathrm{e}^{2\,x} - 1\right)}{a} - \frac{x}{a} - \frac{1}{a + 2\,a\,\mathrm{e}^{x} + a\,\mathrm{e}^{2\,x}} + \frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}} + \frac{1}{a + a\,\mathrm{e}^{x}}$$

```
[In] int(coth(x)/(a + a/cosh(x)),x)
```

```
[Out] log(exp(2*x) - 1)/a - x/a - 1/(a + 2*a*exp(x) + a*exp(2*x)) + atan((exp(x)*
(-a^2)^(1/2))/a)/(-a^2)^(1/2) + 1/(a + a*exp(x))
```

3.110 $\int \frac{\coth^2(x)}{a+a\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{\coth^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}$$

```
[Out] x/a-1/3*coth(x)*(3-2*sech(x))/a-1/3*coth(x)^3*(1-sech(x))/a
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3973, 3967, 8}

$$\int \frac{\coth^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{x}{a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}$$

```
[In] Int[Coth[x]^2/(a + a*Sech[x]),x]
[Out] x/a - (Coth[x]*(3 - 2*Sech[x]))/(3*a) - (Coth[x]^3*(1 - Sech[x]))/(3*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
```

```
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \coth^4(x)(-a+a\operatorname{sech}(x))\,dx}{a^2}\\
&= -\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}+\frac{\int \coth^2(x)(3a-2a\operatorname{sech}(x))\,dx}{3a^2}\\
&= -\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}-\frac{\int -3a\,dx}{3a^2}\\
&= \frac{x}{a}-\frac{\coth(x)(3-2\operatorname{sech}(x))}{3a}-\frac{\coth^3(x)(1-\operatorname{sech}(x))}{3a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{\coth^2(x)}{a+a\operatorname{sech}(x)}\,dx=\frac{6x-4\coth(x)-2\operatorname{csch}(x)+6x\operatorname{sech}(x)-4\tanh(x)}{6a+6a\operatorname{sech}(x)}$$

```
[In] Integrate[Coth[x]^2/(a + a*Sech[x]),x]
[Out] (6*x - 4*Coth[x] - 2*Csch[x] + 6*x*Sech[x] - 4*Tanh[x])/(6*a + 6*a*Sech[x])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{x}{a}+\frac{2\,{\mathrm e}^{3x}-\frac{10\,{\mathrm e}^{x}}{3}-\frac{8}{3}}{a\left({\mathrm e}^{x}+1\right)^{3}\left({\mathrm e}^{x}-1\right)}$	36
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}-4\tanh\left(\frac{x}{2}\right)+4\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-4\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	47

```
[In] int(coth(x)^2/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] x/a+2/3*(3*exp(3*x)-5*exp(x)-4)/a/(exp(x)+1)^3/(exp(x)-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{\coth^2(x)}{a+a\text{sech}(x)}\,dx = -\frac{2\cosh(x)^2-((3x+4)\cosh(x)+3x+4)\sinh(x)+2\sinh(x)^2+\cosh(x)}{3(a\cosh(x)+a)\sinh(x)}$$

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="fricas")
[Out] -1/3*(2*cosh(x)^2 - ((3*x + 4)*cosh(x) + 3*x + 4)*sinh(x) + 2*sinh(x)^2 + c
osh(x))/((a*cosh(x) + a)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a+a\text{sech}(x)}\,dx = \frac{\int \frac{\coth^2(x)}{\text{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(coth(x)**2/(a+a*sech(x)),x)
[Out] Integral(coth(x)**2/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{\coth^2(x)}{a+a\text{sech}(x)}\,dx = \frac{x}{a}-\frac{2\left(5e^{(-x)}-3e^{(-3x)}+4\right)}{3\left(2ae^{(-x)}-2ae^{(-3x)}-ae^{(-4x)}+a\right)}$$

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a - 2/3*(5*e^(-x) - 3*e^(-3*x) + 4)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*
x) + a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{\coth^2(x)}{a+a\text{sech}(x)}\,dx=\frac{x}{a}-\frac{1}{2\,a(e^x-1)}+\frac{15\,e^{(2\,x)}+24\,e^x+13}{6\,a(e^x+1)^3}$$

```
[In] integrate(coth(x)^2/(a+a*sech(x)),x, algorithm="giac")
[Out] x/a - 1/2/(a*(e^x - 1)) + 1/6*(15*e^(2*x) + 24*e^x + 13)/(a*(e^x + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.47

$$\int \frac{\coth^2(x)}{a+a\text{sech}(x)}\,dx=\frac{\frac{5\,e^{2\,x}}{6\,a}+\frac{5}{6\,a}+\frac{e^x}{a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^x+1}+\frac{\frac{1}{2\,a}+\frac{5\,e^x}{6\,a}}{e^{2\,x}+2\,e^x+1}+\frac{x}{a}-\frac{1}{2\,a\,\left(e^x-1\right)}+\frac{5}{6\,a\,\left(e^x+1\right)}$$

```
[In] int(coth(x)^2/(a + a/cosh(x)),x)
[Out] ((5*exp(2*x))/(6*a) + 5/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x)
 + 1) + (1/(2*a) + (5*exp(x))/(6*a))/(exp(2*x) + 2*exp(x) + 1) + x/a - 1/(2
*a*(exp(x) - 1)) + 5/(6*a*(exp(x) + 1))
```

3.111 $\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx=\frac{1}{8a(1-\cosh(x))}-\frac{1}{8a(1+\cosh(x))^2}+\frac{3}{4a(1+\cosh(x))}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(1+\cosh(x))}{16a}$$

```
[Out] 1/8/a/(1-cosh(x))-1/8/a/(1+cosh(x))^2+3/4/a/(1+cosh(x))+5/16*ln(1-cosh(x))/
a+11/16*ln(1+cosh(x))/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3964, 90}

$$\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx=\frac{1}{8a(1-\cosh(x))}+\frac{3}{4a(\cosh(x)+1)}-\frac{1}{8a(\cosh(x)+1)^2}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(\cosh(x)+1)}{16a}$$

```
[In] Int[Coth[x]^3/(a + a*Sech[x]),x]
```

```
[Out] 1/(8*a*(1 - Cosh[x])) - 1/(8*a*(1 + Cosh[x])^2) + 3/(4*a*(1 + Cosh[x])) + (
5*Log[1 - Cosh[x]])/(16*a) + (11*Log[1 + Cosh[x]])/(16*a)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
```

```
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3964

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[1/(a^(m - n - 1)*b^n*d), Subst[Int[(a - b*x)^((m - 1
)/2)*((a + b*x)^((m - 1)/2 + n)/x^(m + n)), x], x, Sin[c + d*x]], x] /; Fre
eQ[{a, b, c, d}, x] && IntegerQ[(m - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a^4\text{Subst}\left(\int \frac{x^4}{(a-ax)^2(a+ax)^3}\,dx, x, \cosh(x)\right)\\ &= a^4\text{Subst}\left(\int \left(\frac{1}{8a^5(-1+x)^2}+\frac{5}{16a^5(-1+x)}+\frac{1}{4a^5(1+x)^3}-\frac{3}{4a^5(1+x)^2}+\frac{11}{16a^5(1+x)}\right)dx, x, \cosh(x)\right)\\ &= \frac{1}{8a(1-\cosh(x))}-\frac{1}{8a(1+\cosh(x))^2}+\frac{3}{4a(1+\cosh(x))}+\frac{5\log(1-\cosh(x))}{16a}+\frac{11\log(1+\cosh(x))}{16a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{\coth^3(x)}{a+a\text{sech}(x)}\,dx = \frac{\left(12-2\coth^2\left(\frac{x}{2}\right)+4\cosh^2\left(\frac{x}{2}\right)\left(11\log\left(\cosh\left(\frac{x}{2}\right)\right)+5\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)-\text{sech}^2\left(\frac{x}{2}\right)\right)\text{sech}(x)}{16a(1+\text{sech}(x))}$$

```
[In] Integrate[Coth[x]^3/(a + a*Sech[x]),x]
[Out] ((12 - 2*Coth[x/2]^2 + 4*Cosh[x/2]^2*(11*Log[Cosh[x/2]] + 5*Log[Sinh[x/2]])
 - Sech[x/2]^2)*Sech[x])/(16*a*(1 + Sech[x]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{4}}{4}-\frac{5\tanh\left(\frac{x}{2}\right)^{2}}{2}-8\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-8\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{2\tanh\left(\frac{x}{2}\right)^{2}}+5\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$	56
risch	$-\frac{x}{a}+\frac{e^{x}\left(5\,e^{4x}-6\,e^{3x}-14\,e^{2x}-6\,e^{x}+5\right)}{4a\left(e^{x}-1\right)^{2}\left(e^{x}+1\right)^{4}}+\frac{11\ln\left(e^{x}+1\right)}{8a}+\frac{5\ln\left(e^{x}-1\right)}{8a}$	71

```
[In] int(coth(x)^3/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a*(-1/4*tanh(1/2*x)^4-5/2*tanh(1/2*x)^2-8*ln(tanh(1/2*x)+1)-8*ln(tanh(1
/2*x)-1)-1/2/tanh(1/2*x)^2+5*ln(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(56) = 112$.

Time = 0.27 (sec) , antiderivative size = 773, normalized size of antiderivative = 11.37

$$\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*x*cosh(x)^6 + 8*x*sinh(x)^6 + 2*(8*x - 5)*cosh(x)^5 + 2*(24*x*cosh(
x) + 8*x - 5)*sinh(x)^5 - 4*(2*x - 3)*cosh(x)^4 + 2*(60*x*cosh(x)^2 + 5*(8*
x - 5)*cosh(x) - 4*x + 6)*sinh(x)^4 - 4*(8*x - 7)*cosh(x)^3 + 4*(40*x*cosh(
x)^3 + 5*(8*x - 5)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) - 8*x + 7)*sinh(x)^3 - 4
*(2*x - 3)*cosh(x)^2 + 4*(30*x*cosh(x)^4 + 5*(8*x - 5)*cosh(x)^3 - 6*(2*x -
 3)*cosh(x)^2 - 3*(8*x - 7)*cosh(x) - 2*x + 3)*sinh(x)^2 + 2*(8*x - 5)*cosh
(x) - 11*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5
 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 +
 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*co
sh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x
)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*co
sh(x) + 1)*log(cosh(x) + sinh(x) + 1) - 5*(cosh(x)^6 + 2*(3*cosh(x) + 1)*si
nh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)
^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*
cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*si
nh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x
)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*
(24*x*cosh(x)^5 + 5*(8*x - 5)*cosh(x)^4 - 8*(2*x - 3)*cosh(x)^3 - 6*(8*x -
7)*cosh(x)^2 - 4*(2*x - 3)*cosh(x) + 8*x - 5)*sinh(x) + 8*x)/(a*cosh(x)^6 +
 a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 - a*cosh(x)^4
```

```
+ (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)^3 + 4*(5*a*co
sh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(x)^2 + (15*a*co
sh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a)*sinh(x)^2 + 2*
a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh(x)^3 - 6*a*cosh(x)^
2 - a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\int \frac{\coth^3(x)}{\operatorname{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(coth(x)**3/(a+a*sech(x)),x)
[Out] Integral(coth(x)**3/(sech(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.59

$$\begin{aligned}\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx &= \frac{x}{a} \\ &+ \frac{5\,e^{(-x)} - 6\,e^{(-2\,x)} - 14\,e^{(-3\,x)} - 6\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} \\ &+ \frac{11\,\log\left(e^{(-x)}+1\right)}{8\,a} + \frac{5\,\log\left(e^{(-x)}-1\right)}{8\,a}\end{aligned}$$

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a + 1/4*(5*e^(-x) - 6*e^(-2*x) - 14*e^(-3*x) - 6*e^(-4*x) + 5*e^(-5*x))/(
2*a*e^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-
6*x) + a) + 11/8*log(e^(-x) + 1)/a + 5/8*log(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\begin{aligned}\int \frac{\coth^3(x)}{a+a\operatorname{sech}(x)}\,dx &= \frac{11\,\log\left(e^{(-x)}+e^{x}+2\right)}{16\,a} + \frac{5\,\log\left(e^{(-x)}+e^{x}-2\right)}{16\,a} \\ &- \frac{5\,e^{(-x)}+5\,e^{x}-6}{16\,a(e^{(-x)}+e^{x}-2)} - \frac{33\,\left(e^{(-x)}+e^{x}\right)^2+84\,e^{(-x)}+84\,e^{x}+52}{32\,a(e^{(-x)}+e^{x}+2)^2}\end{aligned}$$

```
[In] integrate(coth(x)^3/(a+a*sech(x)),x, algorithm="giac")
```

```
[Out] 11/16*log(e^(-x) + e^x + 2)/a + 5/16*log(e^(-x) + e^x - 2)/a - 1/16*(5*e^(-
x) + 5*e^x - 6)/(a*(e^(-x) + e^x - 2)) - 1/32*(33*(e^(-x) + e^x)^2 + 84*e^(
-x) + 84*e^x + 52)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.35

$$\int \frac{\coth^3(x)}{a+a\mathrm{sech}(x)}\,dx = \frac{\ln\left(9\,\mathrm{e}^{2\,x}-9\right)}{a}-\frac{x}{a}-\frac{1}{2\,\left(a+4\,a\,\mathrm{e}^{x}+6\,a\,\mathrm{e}^{2\,x}+4\,a\,\mathrm{e}^{3\,x}+a\,\mathrm{e}^{4\,x}\right)}+\frac{1}{a+3\,a\,\mathrm{e}^{x}+3\,a\,\mathrm{e}^{2\,x}+a\,\mathrm{e}^{3\,x}}-\frac{1}{4\,\left(a-2\,a\,\mathrm{e}^{x}+a\,\mathrm{e}^{2\,x}\right)}-\frac{2}{a+2\,a\,\mathrm{e}^{x}+a\,\mathrm{e}^{2\,x}}+\frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}}+\frac{3}{2\,\left(a+a\,\mathrm{e}^{x}\right)}+\frac{1}{4\,\left(a-a\,\mathrm{e}^{x}\right)}$$

```
[In] int(coth(x)^3/(a + a/cosh(x)),x)
```

```
[Out] log(9*exp(2*x) - 9)/a - x/a - 1/(2*(a + 4*a*exp(x) + 6*a*exp(2*x) + 4*a*exp
(3*x) + a*exp(4*x))) + 1/(a + 3*a*exp(x) + 3*a*exp(2*x) + a*exp(3*x)) - 1/(
4*(a - 2*a*exp(x) + a*exp(2*x))) - 2/(a + 2*a*exp(x) + a*exp(2*x)) + (3*ata
n((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) + 3/(2*(a + a*exp(x))) + 1/(4*
(a - a*exp(x)))
```

3.112 $\int \frac{\coth^4(x)}{a+a\mathbf{sech}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{\coth^4(x)}{a+a\operatorname{sech}(x)} dx = \frac{x}{a}-\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a}-\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a}-\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}$$

```
[Out] x/a-1/15*coth(x)*(15-8*sech(x))/a-1/15*coth(x)^3*(5-4*sech(x))/a-1/5*coth(x
)^5*(1-sech(x))/a
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3973, 3967, 8}

$$\int \frac{\coth^4(x)}{a+a\operatorname{sech}(x)} dx = \frac{x}{a}-\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}-\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a}-\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a}$$

```
[In] Int[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] x/a - (Coth[x]*(15 - 8*Sech[x]))/(15*a) - (Coth[x]^3*(5 - 4*Sech[x]))/(15*a
) - (Coth[x]^5*(1 - Sech[x]))/(5*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3967

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-(e*Cot[c + d*x])^(m + 1))*((a + b*Csc[c + d*x])/(
d*e*(m + 1))), x] - Dist[1/(e^2*(m + 1)), Int[(e*Cot[c + d*x])^(m + 2)*(a*(
m + 1) + b*(m + 2)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e}, x] && Lt
Q[m, -1]
```

Rule 3973

```
Int[(cot[(c_.) + (d_.)*(x_)]*(e_.))^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (
a_))^(n_), x_Symbol] :> Dist[a^(2*n)/e^(2*n), Int[(e*Cot[c + d*x])^(m + 2*n
)/(-a + b*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[a^
2 - b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \coth^6(x)(-a+a\operatorname{sech}(x))\,dx}{a^2} \\
&= -\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}+\frac{\int \coth^4(x)(5a-4a\operatorname{sech}(x))\,dx}{5a^2} \\
&= -\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a}-\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}-\frac{\int \coth^2(x)(-15a+8a\operatorname{sech}(x))\,dx}{15a^2} \\
&= -\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a}-\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a}-\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}+\frac{\int 15a\,dx}{15a^2} \\
&= \frac{x}{a}-\frac{\coth(x)(15-8\operatorname{sech}(x))}{15a}-\frac{\coth^3(x)(5-4\operatorname{sech}(x))}{15a}-\frac{\coth^5(x)(1-\operatorname{sech}(x))}{5a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{\coth^4(x)}{a+a\operatorname{sech}(x)}\,dx = \frac{\operatorname{csch}^3(x)\operatorname{sech}(x)(-25+8\cosh(x)+16\cosh(2x)-16\cosh(3x)-23\cosh(4x)-90x\sinh(x)-30x\sinh(2x}{120a(1+\operatorname{sech}(x))}$$

```
[In] Integrate[Coth[x]^4/(a + a*Sech[x]),x]
```

```
[Out] (Csch[x]^3*Sech[x]*(-25 + 8*Cosh[x] + 16*Cosh[2*x] - 16*Cosh[3*x] - 23*Cosh
[4*x] - 90*x*Sinh[x] - 30*x*Sinh[2*x] + 30*x*Sinh[3*x] + 15*x*Sinh[4*x]))/(
120*a*(1 + Sech[x]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{5}}{5}-2\tanh\left(\frac{x}{2}\right)^{3}-16\tanh\left(\frac{x}{2}\right)+16\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)-16\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)-\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}-\frac{6}{\tanh\left(\frac{x}{2}\right)}}{16a}$	63
risch	$\frac{x}{a}+\frac{2\,e^{7x}-2\,e^{6x}-\frac{26\,e^{5x}}{3}-\frac{10\,e^{4x}}{3}+\frac{146\,e^{3x}}{15}+\frac{62\,e^{2x}}{15}-\frac{62\,e^{x}}{15}-\frac{46}{15}}{a\left(e^{x}-1\right)^{3}\left(e^{x}+1\right)^{5}}$	66

```
[In] int(coth(x)^4/(a+a*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a*(-1/5*tanh(1/2*x)^5-2*tanh(1/2*x)^3-16*tanh(1/2*x)+16*ln(tanh(1/2*x)
+1)-16*ln(tanh(1/2*x)-1)-1/3/tanh(1/2*x)^3-6/tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(47) = 94.

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.75

$$\int \frac{\coth^4(x)}{a+a\text{sech}(x)}\,dx = -\frac{23\cosh(x)^4-2\left(2\left(15x+23\right)\cosh(x)+15x+23\right)\sinh(x)^3+23\sinh(x)^4+16\cosh(x)^3+2\left(69\cosh\right.}{30\left(\left(2a\cosh(x\right.\right.}$$

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/30*(23*cosh(x)^4 - 2*(2*(15*x + 23)*cosh(x) + 15*x + 23)*sinh(x)^3 + 23*
sinh(x)^4 + 16*cosh(x)^3 + 2*(69*cosh(x)^2 + 24*cosh(x) - 8)*sinh(x)^2 - 16
*cosh(x)^2 - 2*(2*(15*x + 23)*cosh(x)^3 + 3*(15*x + 23)*cosh(x)^2 - 2*(15*x
 + 23)*cosh(x) - 45*x - 69)*sinh(x) - 8*cosh(x) + 25)/((2*a*cosh(x) + a)*si
nh(x)^3 + (2*a*cosh(x)^3 + 3*a*cosh(x)^2 - 2*a*cosh(x) - 3*a)*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a+a\text{sech}(x)}\,dx = \frac{\int \frac{\coth^4(x)}{\text{sech}(x)+1}\,dx}{a}$$

```
[In] integrate(coth(x)**4/(a+a*sech(x)),x)
```

```
[Out] Integral(coth(x)**4/(sech(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int \frac{\coth^4(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} - \frac{2\left(31\,e^{(-x)} - 31\,e^{(-2\,x)} - 73\,e^{(-3\,x)} + 25\,e^{(-4\,x)} + 65\,e^{(-5\,x)} + 15\,e^{(-6\,x)} - 15\,e^{(-7\,x)} + 23\right)}{15\left(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a\right)}$$

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="maxima")
[Out] x/a - 2/15*(31*e^(-x) - 31*e^(-2*x) - 73*e^(-3*x) + 25*e^(-4*x) + 65*e^(-5*
x) + 15*e^(-6*x) - 15*e^(-7*x) + 23)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3
*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\coth^4(x)}{a + a\text{sech}(x)}\,dx = \frac{x}{a} - \frac{21\,e^{(2\,x)} - 36\,e^{x} + 19}{24\,a(e^{x} - 1)^3} + \frac{115\,e^{(4\,x)} + 380\,e^{(3\,x)} + 530\,e^{(2\,x)} + 340\,e^{x} + 91}{40\,a(e^{x} + 1)^5}$$

```
[In] integrate(coth(x)^4/(a+a*sech(x)),x, algorithm="giac")
[Out] x/a - 1/24*(21*e^(2*x) - 36*e^x + 19)/(a*(e^x - 1)^3) + 1/40*(115*e^(4*x) +
 380*e^(3*x) + 530*e^(2*x) + 340*e^x + 91)/(a*(e^x + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 4.80

$$\int \frac{\coth^4(x)}{a + a\text{sech}(x)}\,dx = \frac{\frac{9\,e^{2\,x}}{4\,a} + \frac{3\,e^{3\,x}}{2\,a} + \frac{23\,e^{4\,x}}{40\,a} + \frac{23}{40\,a} + \frac{3\,e^{x}}{2\,a}}{10\,e^{2\,x} + 10\,e^{3\,x} + 5\,e^{4\,x} + e^{5\,x} + 5\,e^{x} + 1} + \frac{\frac{9\,e^{2\,x}}{8\,a} + \frac{23\,e^{3\,x}}{40\,a} + \frac{3}{8\,a} + \frac{9\,e^{x}}{8\,a}}{6\,e^{2\,x} + 4\,e^{3\,x} + e^{4\,x} + 4\,e^{x} + 1} + \frac{\frac{23\,e^{2\,x}}{40\,a} + \frac{3}{8\,a} + \frac{3\,e^{x}}{4\,a}}{3\,e^{2\,x} + e^{3\,x} + 3\,e^{x} + 1} + \frac{\frac{3}{8\,a} + \frac{23\,e^{x}}{40\,a}}{e^{2\,x} + 2\,e^{x} + 1} + \frac{1}{6\,a\,\left(3\,e^{2\,x} - e^{3\,x} - 3\,e^{x} + 1\right)} - \frac{1}{4\,a\,\left(e^{2\,x} - 2\,e^{x} + 1\right)} + \frac{x}{a} - \frac{7}{8\,a\,\left(e^{x} - 1\right)} + \frac{23}{40\,a\,\left(e^{x} + 1\right)}$$

```
[In] int(coth(x)^4/(a + a/cosh(x)),x)
```

```
[Out] ((9*exp(2*x))/(4*a) + (3*exp(3*x))/(2*a) + (23*exp(4*x))/(40*a) + 23/(40*a)
 + (3*exp(x))/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5
*exp(x) + 1) + ((9*exp(2*x))/(8*a) + (23*exp(3*x))/(40*a) + 3/(8*a) + (9*ex
p(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) + ((23*exp
(2*x))/(40*a) + 3/(8*a) + (3*exp(x))/(4*a))/(3*exp(2*x) + exp(3*x) + 3*exp(
x) + 1) + (3/(8*a) + (23*exp(x))/(40*a))/(exp(2*x) + 2*exp(x) + 1) + 1/(6*a
*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)
) + x/a - 7/(8*a*(exp(x) - 1)) + 23/(40*a*(exp(x) + 1))
```

3.113 $\int \frac{\tanh^7(x)}{a+b\operatorname{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 121

$$\int \frac{\tanh^7(x)}{a+b\operatorname{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}-\frac{(a^2-b^2)^3\log(a+b\operatorname{sech}(x))}{ab^6}+\frac{(a^4-3a^2b^2+3b^4)\operatorname{sech}(x)}{b^5}-\frac{a(a^2-3b^2)\operatorname{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\operatorname{sech}^3(x)}{3b^3}-\frac{a\operatorname{sech}^4(x)}{4b^2}+\frac{\operatorname{sech}^5(x)}{5b}$$

```
[Out] ln(cosh(x))/a-(a^2-b^2)^3*ln(a+b*sech(x))/a/b^6+(a^4-3*a^2*b^2+3*b^4)*sech(
x)/b^5-1/2*a*(a^2-3*b^2)*sech(x)^2/b^4+1/3*(a^2-3*b^2)*sech(x)^3/b^3-1/4*a*
sech(x)^4/b^2+1/5*sech(x)^5/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3970, 908}

$$\int \frac{\tanh^7(x)}{a+b\operatorname{sech}(x)}\,dx=-\frac{(a^2-b^2)^3\log(a+b\operatorname{sech}(x))}{ab^6}-\frac{a(a^2-3b^2)\operatorname{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\operatorname{sech}^3(x)}{3b^3}+\frac{(a^4-3a^2b^2+3b^4)\operatorname{sech}(x)}{b^5}-\frac{a\operatorname{sech}^4(x)}{4b^2}+\frac{\log(\cosh(x))}{a}+\frac{\operatorname{sech}^5(x)}{5b}$$

```
[In] Int[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a - ((a^2 - b^2)^3*Log[a + b*Sech[x]])/(a*b^6) + ((a^4 - 3*a^2
*b^2 + 3*b^4)*Sech[x])/b^5 - (a*(a^2 - 3*b^2)*Sech[x]^2)/(2*b^4) + ((a^2 -
3*b^2)*Sech[x]^3)/(3*b^3) - (a*Sech[x]^4)/(4*b^2) + Sech[x]^5/(5*b)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^3}{x(a+x)}\,dx, x, b\text{sech}(x)\right)}{b^6}$$

$$= -\frac{\text{Subst}\left(\int \left(-a^4\left(1+\frac{3b^2(-a^2+b^2)}{a^4}\right)+\frac{b^6}{ax}+a(a^2-3b^2)\,x-(a^2-3b^2)\,x^2+ax^3-x^4+\frac{(a^2-b^2)^3}{a(a+x)}\right)\,dx}{b^6}$$

$$= \frac{\log(\cosh(x))}{a}-\frac{(a^2-b^2)^3\log(a+b\text{sech}(x))}{ab^6}+\frac{(a^4-3a^2b^2+3b^4)\,\text{sech}(x)}{b^5} - \frac{a(a^2-3b^2)\,\text{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\,\text{sech}^3(x)}{3b^3}-\frac{a\text{sech}^4(x)}{4b^2}+\frac{\text{sech}^5(x)}{5b}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a}-\frac{(a^2-b^2)^3\log(a+b\text{sech}(x))}{ab^6}+\frac{(a^4-3a^2b^2+3b^4)\,\text{sech}(x)}{b^5} - \frac{a(a^2-3b^2)\,\text{sech}^2(x)}{2b^4}+\frac{(a^2-3b^2)\,\text{sech}^3(x)}{3b^3}-\frac{a\text{sech}^4(x)}{4b^2}+\frac{\text{sech}^5(x)}{5b}$$

```
[In] Integrate[Tanh[x]^7/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a - ((a^2 - b^2)^3*Log[a + b*Sech[x]])/(a*b^6) + ((a^4 - 3*a^2
*b^2 + 3*b^4)*Sech[x])/b^5 - (a*(a^2 - 3*b^2)*Sech[x]^2)/(2*b^4) + ((a^2 -
3*b^2)*Sech[x]^3)/(3*b^3) - (a*Sech[x]^4)/(4*b^2) + Sech[x]^5/(5*b)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(113) = 226$.

Time = 2.31 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.03

method	result
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{(a-b)^{3}\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-\tanh\left(\frac{x}{2}\right)^{2}b+a+b\right)}{a\,b^{6}}+\frac{\frac{8b^{3}\left(a^{2}+3ab+3b^{2}\right)}{3\left(1+\tanh\left(\frac{x}{2}\right)^{2}\right)^{3}}+\cdots}{\cdots}$
risch	$-\frac{x}{a}+\frac{2\,\mathrm{e}^{x}\left(15a^{4}\mathrm{e}^{8x}-45a^{2}b^{2}\mathrm{e}^{8x}+45b^{4}\mathrm{e}^{8x}-15a^{3}b\,\mathrm{e}^{7x}+45a\,b^{3}\mathrm{e}^{7x}+60a^{4}\mathrm{e}^{6x}-160a^{2}b^{2}\mathrm{e}^{6x}+120b^{4}\mathrm{e}^{6x}-45a^{3}b\,\mathrm{e}^{5x}+105a\,b^{3}\mathrm{e}^{5x}+90a^{4}\mathrm{e}\cdots\right)}{15b^{5}\cdots}$

```
[In] int(tanh(x)^7/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(tanh(1/2*x)+1)-1/a*ln(tanh(1/2*x)-1)-(a-b)^3*(a^3+3*a^2*b+3*a*b^2+b
^3)/a/b^6*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)+1/b^6*(8/3*b^3*(a^2+3*a*b
+3*b^2)/(1+tanh(1/2*x)^2)^3+a*(a^4-3*a^2*b^2+3*b^4)*ln(1+tanh(1/2*x)^2)+32/
5*b^5/(1+tanh(1/2*x)^2)^5-2*b^2*(a^3+2*a^2*b-2*b^3)/(1+tanh(1/2*x)^2)^2+2*b
*(a^4+a^3*b-2*a^2*b^2-2*a*b^3+b^4)/(1+tanh(1/2*x)^2)-4*b^4*(a+4*b)/(1+tanh(
1/2*x)^2)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4077 vs. $2(113) = 226$.

Time = 0.31 (sec) , antiderivative size = 4077, normalized size of antiderivative = 33.69

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx=\text{Too large to display}$$

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^6*x*cosh(x)^10 + 15*b^6*x*sinh(x)^10 - 30*(a^5*b - 3*a^3*b^3 +
3*a*b^5)*cosh(x)^9 + 30*(5*b^6*x*cosh(x) - a^5*b + 3*a^3*b^3 - 3*a*b^5)*sin
h(x)^9 + 15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^8 + 15*(45*b^6*x*cosh
(x)^2 + 5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4 - 18*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*
cosh(x))*sinh(x)^8 - 40*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^7 + 40*(45*
b^6*x*cosh(x)^3 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 - 27*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x))*sinh(x)^7
+ 15*b^6*x + 30*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^6 + 10*(315*b^6*x
*cosh(x)^4 + 15*b^6*x + 9*a^4*b^2 - 21*a^2*b^4 - 252*(a^5*b - 3*a^3*b^3 + 3
*a*b^5)*cosh(x)^3 + 42*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 - 28*(3*
a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(x)^6 - 4*(45*a^5*b - 115*a^3*b^3
 + 99*a*b^5)*cosh(x)^5 + 4*(945*b^6*x*cosh(x)^5 - 45*a^5*b + 115*a^3*b^3 -
99*a*b^5 - 945*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^4 + 210*(5*b^6*x + 2*a
^4*b^2 - 6*a^2*b^4)*cosh(x)^3 - 210*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)
```

^2 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^5 + 30*(5*b^6*x
+ 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4 + 10*(315*b^6*x*cosh(x)^6 + 15*b^6*x + 9
*a^4*b^2 - 21*a^2*b^4 - 378*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^5 + 105*(
5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^4 - 140*(3*a^5*b - 8*a^3*b^3 + 6*a
*b^5)*cosh(x)^3 + 45*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^2 - 2*(45*a^
5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x))*sinh(x)^4 - 40*(3*a^5*b - 8*a^3*b^3
+ 6*a*b^5)*cosh(x)^3 + 40*(45*b^6*x*cosh(x)^7 - 63*(a^5*b - 3*a^3*b^3 + 3*a
*b^5)*cosh(x)^6 - 3*a^5*b + 8*a^3*b^3 - 6*a*b^5 + 21*(5*b^6*x + 2*a^4*b^2 -
6*a^2*b^4)*cosh(x)^5 - 35*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^4 + 15*(
5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^3 - (45*a^5*b - 115*a^3*b^3 + 99*a
*b^5)*cosh(x)^2 + 3*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x))*sinh(x)^3 +
15*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^2 + 5*(135*b^6*x*cosh(x)^8 - 2
16*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x)^7 + 15*b^6*x + 84*(5*b^6*x + 2*a^4
*b^2 - 6*a^2*b^4)*cosh(x)^6 + 6*a^4*b^2 - 18*a^2*b^4 - 168*(3*a^5*b - 8*a^3
*b^3 + 6*a*b^5)*cosh(x)^5 + 90*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^4
- 8*(45*a^5*b - 115*a^3*b^3 + 99*a*b^5)*cosh(x)^3 + 36*(5*b^6*x + 3*a^4*b^2
- 7*a^2*b^4)*cosh(x)^2 - 24*(3*a^5*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x))*sinh(
x)^2 - 30*(a^5*b - 3*a^3*b^3 + 3*a*b^5)*cosh(x) + 15*((a^6 - 3*a^4*b^2 + 3*
a^2*b^4 - b^6)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*
sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sinh(x)^10 + 5*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 +
9*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3
*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*cosh(x))*sinh(x)^7 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 10
*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^6 +
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 4*(63*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + 15*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^5 + 10*(a^6 - 3*a^4*b^2 + 3
*a^2*b^4 - b^6)*cosh(x)^4 + 10*(21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh
(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*cosh(x)^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(
x)^4 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^7 + 7*(a^6 - 3*a^4
*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*c
osh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x))*sinh(x)^3 + 5*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2 + 5*(9*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x)^8 + 28*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos
h(x)^4 + 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^2)*sinh(x)^2 + 10*(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^9 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*cosh(x)^7 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^5 + 4*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6)*cosh(x))*sinh(x))*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - 15*
((a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^10 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
)*cosh(x)*sinh(x)^9 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*sinh(x)^10 + 5*(a^6 - 3

```
*a^4*b^2 + 3*a^2*b^4)*cosh(x)^8 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 9*(a^6 -
 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^8 + 40*(3*(a^6 - 3*a^4*b^2 + 3*a
^2*b^4)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^7 + 10*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + 10*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 +
21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 14*(a^6 - 3*a^4*b^2 + 3*a^2*b^
4)*cosh(x)^2)*sinh(x)^6 + a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 4*(63*(a^6 - 3*a^4*
b^2 + 3*a^2*b^4)*cosh(x)^5 + 70*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + 1
5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x)^5 + 10*(a^6 - 3*a^4*b^2 +
3*a^2*b^4)*cosh(x)^4 + 10*(21*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 + a^6
 - 3*a^4*b^2 + 3*a^2*b^4 + 35*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 + 15*
(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^4 + 40*(3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4)*cosh(x)^7 + 7*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 5*(a^
6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x
))*sinh(x)^3 + 5*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2 + 5*(9*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4)*cosh(x)^8 + 28*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^6 +
 a^6 - 3*a^4*b^2 + 3*a^2*b^4 + 30*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^4 +
 12*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^2)*sinh(x)^2 + 10*((a^6 - 3*a^4*b
^2 + 3*a^2*b^4)*cosh(x)^9 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^7 + 6*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x)^5 + 4*(a^6 - 3*a^4*b^2 + 3*a^2*b^4)*co
sh(x)^3 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4)*cosh(x))*sinh(x))*log(2*cosh(x)/(co
sh(x) - sinh(x))) + 10*(15*b^6*x*cosh(x)^9 - 27*(a^5*b - 3*a^3*b^3 + 3*a*b^
5)*cosh(x)^8 + 12*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*cosh(x)^7 - 28*(3*a^5*b
 - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^6 - 3*a^5*b + 9*a^3*b^3 - 9*a*b^5 + 18*(5*b
^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^5 - 2*(45*a^5*b - 115*a^3*b^3 + 99*a*
b^5)*cosh(x)^4 + 12*(5*b^6*x + 3*a^4*b^2 - 7*a^2*b^4)*cosh(x)^3 - 12*(3*a^5
*b - 8*a^3*b^3 + 6*a*b^5)*cosh(x)^2 + 3*(5*b^6*x + 2*a^4*b^2 - 6*a^2*b^4)*c
osh(x))*sinh(x))/(a*b^6*cosh(x)^10 + 10*a*b^6*cosh(x)*sinh(x)^9 + a*b^6*sin
h(x)^10 + 5*a*b^6*cosh(x)^8 + 10*a*b^6*cosh(x)^6 + 10*a*b^6*cosh(x)^4 + 5*a
*b^6*cosh(x)^2 + 5*(9*a*b^6*cosh(x)^2 + a*b^6)*sinh(x)^8 + 40*(3*a*b^6*cosh
(x)^3 + a*b^6*cosh(x))*sinh(x)^7 + a*b^6 + 10*(21*a*b^6*cosh(x)^4 + 14*a*b^
6*cosh(x)^2 + a*b^6)*sinh(x)^6 + 4*(63*a*b^6*cosh(x)^5 + 70*a*b^6*cosh(x)^3
 + 15*a*b^6*cosh(x))*sinh(x)^5 + 10*(21*a*b^6*cosh(x)^6 + 35*a*b^6*cosh(x)^
4 + 15*a*b^6*cosh(x)^2 + a*b^6)*sinh(x)^4 + 40*(3*a*b^6*cosh(x)^7 + 7*a*b^6
*cosh(x)^5 + 5*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x)^3 + 5*(9*a*b^6*cosh
(x)^8 + 28*a*b^6*cosh(x)^6 + 30*a*b^6*cosh(x)^4 + 12*a*b^6*cosh(x)^2 + a*b^
6)*sinh(x)^2 + 10*(a*b^6*cosh(x)^9 + 4*a*b^6*cosh(x)^7 + 6*a*b^6*cosh(x)^5
+ 4*a*b^6*cosh(x)^3 + a*b^6*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\tanh^7{(x)}}{a+b\,\text{sech}\,(x)}\,dx$$

```
[In] integrate(tanh(x)**7/(a+b*sech(x)),x)
[Out] Integral(tanh(x)**7/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(113) = 226.

Time = 0.29 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.74

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx = \frac{2\left(15\left(a^4-3a^2b^2+3b^4\right)e^{(-x)}-15\left(a^3b-3ab^3\right)e^{(-2x)}+20\left(3a^4-8a^2b^2+6b^4\right)e^{(-3x)}-15\left(3a^3b-7ab^3\right)\dots\right.}{15\left(5b^5e^{(-\dots}\right.} + \frac{x}{a} + \frac{\left(a^5-3a^3b^2+3ab^4\right)\log\left(e^{(-2x)}+1\right)}{b^6} - \frac{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\log\left(2be^{(-x)}+ae^{(-2x)}+a\right)}{ab^6}$$

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="maxima")
[Out] 2/15*(15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-x) - 15*(a^3*b - 3*a*b^3)*e^(-2*x) +
 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-3*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-4*x)
+ 2*(45*a^4 - 115*a^2*b^2 + 99*b^4)*e^(-5*x) - 15*(3*a^3*b - 7*a*b^3)*e^(-6
*x) + 20*(3*a^4 - 8*a^2*b^2 + 6*b^4)*e^(-7*x) - 15*(a^3*b - 3*a*b^3)*e^(-8*
x) + 15*(a^4 - 3*a^2*b^2 + 3*b^4)*e^(-9*x))/(5*b^5*e^(-2*x) + 10*b^5*e^(-4*
x) + 10*b^5*e^(-6*x) + 5*b^5*e^(-8*x) + b^5*e^(-10*x) + b^5) + x/a + (a^5 -
 3*a^3*b^2 + 3*a*b^4)*log(e^(-2*x) + 1)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4
- b^6)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(113) = 226.

Time = 0.31 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.21

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx = \frac{\left(a^5-3a^3b^2+3ab^4\right)\log\left(e^{(-x)}+e^x\right)}{b^6} - \frac{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\log\left(\left|a\left(e^{(-x)}+e^x\right)+2b\right|\right)}{ab^6} - \frac{137a^5\left(e^{(-x)}+e^x\right)^5-411a^3b^2\left(e^{(-x)}+e^x\right)^5+411ab^4\left(e^{(-x)}+e^x\right)^5-120a^4b\left(e^{(-x)}+e^x\right)^4+360a^2b^3\left(e^{(-\dots}\right.}{\dots}$$

```
[In] integrate(tanh(x)^7/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] (a^5 - 3*a^3*b^2 + 3*a*b^4)*log(e^(-x) + e^x)/b^6 - (a^6 - 3*a^4*b^2 + 3*a^
2*b^4 - b^6)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b^6) - 1/60*(137*a^5*(e^(-
x) + e^x)^5 - 411*a^3*b^2*(e^(-x) + e^x)^5 + 411*a*b^4*(e^(-x) + e^x)^5 - 1
20*a^4*b*(e^(-x) + e^x)^4 + 360*a^2*b^3*(e^(-x) + e^x)^4 - 360*b^5*(e^(-x)
+ e^x)^4 + 120*a^3*b^2*(e^(-x) + e^x)^3 - 360*a*b^4*(e^(-x) + e^x)^3 - 160*
a^2*b^3*(e^(-x) + e^x)^2 + 480*b^5*(e^(-x) + e^x)^2 + 240*a*b^4*(e^(-x) + e
^x) - 384*b^5)/(b^6*(e^(-x) + e^x)^5)
```

Mupad [B] (verification not implemented)

Time = 2.59 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.61

$$\int \frac{\tanh^7(x)}{a+b\text{sech}(x)}\,dx = \frac{\frac{8a}{b^2}-\frac{8e^x(5a^2-27b^2)}{15b^3}}{3e^{2x}+3e^{4x}+e^{6x}+1}-\frac{\frac{4a}{b^2}+\frac{64e^x}{5b}}{4e^{2x}+6e^{4x}+4e^{6x}+e^{8x}+1} + \frac{\frac{8e^x(a^2-3b^2)}{3b^3}+\frac{2(a^4-5a^2b^2)}{ab^4}}{2e^{2x}+e^{4x}+1}-\frac{x}{a}+\frac{\frac{2e^x(a^4-3a^2b^2+3b^4)}{b^5}-\frac{2(a^4-3a^2b^2)}{ab^4}}{e^{2x}+1} + \frac{32e^x}{5b(5e^{2x}+10e^{4x}+10e^{6x}+5e^{8x}+e^{10x}+1)} + \frac{\ln(e^{2x}+1)(a^5-3a^3b^2+3ab^4)}{b^6} - \frac{\ln(a+2be^x+ae^{2x})(a^6-3a^4b^2+3a^2b^4-b^6)}{ab^6}$$

```
[In] int(tanh(x)^7/(a + b/cosh(x)),x)
```

```
[Out] ((8*a)/b^2 - (8*exp(x)*(5*a^2 - 27*b^2))/(15*b^3))/(3*exp(2*x) + 3*exp(4*x)
 + exp(6*x) + 1) - ((4*a)/b^2 + (64*exp(x))/(5*b))/(4*exp(2*x) + 6*exp(4*x)
 + 4*exp(6*x) + exp(8*x) + 1) + ((8*exp(x)*(a^2 - 3*b^2))/(3*b^3) + (2*(a^4
 - 5*a^2*b^2))/(a*b^4))/(2*exp(2*x) + exp(4*x) + 1) - x/a + ((2*exp(x)*(a^4
 + 3*b^4 - 3*a^2*b^2))/b^5 - (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*x) + 1)
+ (32*exp(x))/(5*b*(5*exp(2*x) + 10*exp(4*x) + 10*exp(6*x) + 5*exp(8*x) + e
xp(10*x) + 1)) + (log(exp(2*x) + 1)*(3*a*b^4 + a^5 - 3*a^3*b^2))/b^6 - (log
(a + 2*b*exp(x) + a*exp(2*x))*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(a*b^6)
```

3.114 $\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 187

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx = \frac{x}{a}-\frac{3\arctan(\sinh(x))}{8b}-\frac{(a^2-3b^2)\arctan(\sinh(x))}{2b^3} -\frac{(a^4-3a^2b^2+3b^4)\arctan(\sinh(x))}{b^5} +\frac{2(a-b)^{5/2}(a+b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^5} +\frac{a\tanh(x)}{b^2}+\frac{a(a^2-3b^2)\tanh(x)}{b^4}-\frac{3\text{sech}(x)\tanh(x)}{8b} -\frac{(a^2-3b^2)\text{sech}(x)\tanh(x)}{2b^3}-\frac{\text{sech}^3(x)\tanh(x)}{4b}-\frac{a\tanh^3(x)}{3b^2}$$

```
[Out] x/a-3/8*arctan(sinh(x))/b-1/2*(a^2-3*b^2)*arctan(sinh(x))/b^3-(a^4-3*a^2*b^
2+3*b^4)*arctan(sinh(x))/b^5+2*(a-b)^(5/2)*(a+b)^(5/2)*arctan((a-b)^(1/2)*t
anh(1/2*x)/(a+b)^(1/2))/a/b^5+a*tanh(x)/b^2+a*(a^2-3*b^2)*tanh(x)/b^4-3/8*s
ech(x)*tanh(x)/b-1/2*(a^2-3*b^2)*sech(x)*tanh(x)/b^3-1/4*sech(x)^3*tanh(x)/
b-1/3*a*tanh(x)^3/b^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3983, 2976, 2738, 211, 3855, 3852, 8, 3853}

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx = -\frac{(a^2-3b^2)\arctan(\sinh(x))}{2b^3}+\frac{a(a^2-3b^2)\tanh(x)}{b^4} - \frac{(a^2-3b^2)\tanh(x)\text{sech}(x)}{2b^3}-\frac{(a^4-3a^2b^2+3b^4)\arctan(\sinh(x))}{b^5} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^5}-\frac{a\tanh^3(x)}{3b^2}+\frac{a\tanh(x)}{b^2} + \frac{x}{a}-\frac{3\arctan(\sinh(x))}{8b}-\frac{\tanh(x)\text{sech}^3(x)}{4b}-\frac{3\tanh(x)\text{sech}(x)}{8b}$$

```
[In] Int[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] x/a - (3*ArcTan[Sinh[x]])/(8*b) - ((a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) -
 ((a^4 - 3*a^2*b^2 + 3*b^4)*ArcTan[Sinh[x]])/b^5 + (2*(a - b)^(5/2)*(a + b)
^(5/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^5) + (a*Tanh[x])/b
^2 + (a*(a^2 - 3*b^2)*Tanh[x])/b^4 - (3*Sech[x]*Tanh[x])/(8*b) - ((a^2 - 3*
b^2)*Sech[x]*Tanh[x])/(2*b^3) - (Sech[x]^3*Tanh[x])/(4*b) - (a*Tanh[x]^3)/(
3*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
```

```
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\sinh(x)\tanh^5(x)}{b+a\cosh(x)}\,dx\\
&= -\int \left(-\frac{1}{a}-\frac{(a^2-b^2)^3}{ab^5(b+a\cosh(x))}+\frac{(a^4-3a^2b^2+3b^4)\operatorname{sech}(x)}{b^5}\right.\\
&\qquad \left.+\frac{(-a^3+3ab^2)\operatorname{sech}^2(x)}{b^4}+\frac{(a^2-3b^2)\operatorname{sech}^3(x)}{b^3}-\frac{a\operatorname{sech}^4(x)}{b^2}+\frac{\operatorname{sech}^5(x)}{b}\right)\,dx\\
&= \frac{x}{a}+\frac{a\int \operatorname{sech}^4(x)\,dx}{b^2}-\frac{\int \operatorname{sech}^5(x)\,dx}{b}\\
&\quad +\frac{(a(a^2-3b^2))\int \operatorname{sech}^2(x)\,dx}{b^4}-\frac{(a^2-3b^2)\int \operatorname{sech}^3(x)\,dx}{b^3}\\
&\quad +\frac{(a^2-b^2)^3\int \frac{1}{b+a\cosh(x)}\,dx}{ab^5}-\frac{(a^4-3a^2b^2+3b^4)\int \operatorname{sech}(x)\,dx}{b^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{a} - \frac{(a^4-3a^2b^2+3b^4)\arctan(\sinh(x))}{b^5} - \frac{(a^2-3b^2)\operatorname{sech}(x)\tanh(x)}{2b^3} \\
&\quad - \frac{\operatorname{sech}^3(x)\tanh(x)}{4b} + \frac{(ia)\text{Subst}\left(\int(1+x^2)\ dx, x, -i\tanh(x)\right)}{b^2} \\
&\quad - \frac{3\int \operatorname{sech}^3(x)\,dx}{4b} + \frac{(ia(a^2-3b^2))\,\text{Subst}\left(\int 1\,dx, x, -i\tanh(x)\right)}{b^4} \\
&\quad - \frac{(a^2-3b^2)\int\operatorname{sech}(x)\,dx}{2b^3} + \frac{\left(2(a^2-b^2)^3\right)\text{Subst}\left(\int\frac{1}{a+b-(-a+b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{ab^5} \\
&= \frac{x}{a} - \frac{(a^2-3b^2)\arctan(\sinh(x))}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\arctan(\sinh(x))}{b^5} \\
&\quad + \frac{2(a-b)^{5/2}(a+b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^5} + \frac{a\tanh(x)}{b^2} \\
&\quad + \frac{a(a^2-3b^2)\tanh(x)}{b^4} - \frac{3\operatorname{sech}(x)\tanh(x)}{8b} - \frac{(a^2-3b^2)\operatorname{sech}(x)\tanh(x)}{2b^3} \\
&\quad - \frac{\operatorname{sech}^3(x)\tanh(x)}{4b} - \frac{a\tanh^3(x)}{3b^2} - \frac{3\int\operatorname{sech}(x)\,dx}{8b} \\
&= \frac{x}{a} - \frac{3\arctan(\sinh(x))}{8b} - \frac{(a^2-3b^2)\arctan(\sinh(x))}{2b^3} - \frac{(a^4-3a^2b^2+3b^4)\arctan(\sinh(x))}{b^5} \\
&\quad + \frac{2(a-b)^{5/2}(a+b)^{5/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^5} + \frac{a\tanh(x)}{b^2} + \frac{a(a^2-3b^2)\tanh(x)}{b^4} \\
&\quad - \frac{3\operatorname{sech}(x)\tanh(x)}{8b} - \frac{(a^2-3b^2)\operatorname{sech}(x)\tanh(x)}{2b^3} - \frac{\operatorname{sech}^3(x)\tanh(x)}{4b} - \frac{a\tanh^3(x)}{3b^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$
\int \frac{\tanh^6(x)}{a+b\operatorname{sech}(x)}\,dx
$$

$$
= \frac{-12(8a^4-20a^2b^2+15b^4)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{48\left(b^5\sqrt{a^2-b^2}x-2(a^2-b^2)^3\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)}{a\sqrt{a^2-b^2}} + b(-12a^2b + \cdots}{}
$$

```
[In] Integrate[Tanh[x]^6/(a + b*Sech[x]),x]
```

```
[Out] (-12*(8*a^4 - 20*a^2*b^2 + 15*b^4)*ArcTan[Tanh[x/2]] + (48*(b^5*Sqrt[a^2 -
b^2]*x - 2*(a^2 - b^2)^3*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]]))/(a*
Sqrt[a^2 - b^2]) + b*(-12*a^2*b + 15*b^3 + 4*a*(9*a^2 - 17*b^2)*Cosh[x] + 3
*b*(-4*a^2 + 9*b^2)*Cosh[2*x] + 12*a^3*Cosh[3*x] - 28*a*b^2*Cosh[3*x])*Sech
[x]^3*Tanh[x])/(48*b^5)
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.42

method	result
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}+\frac{2(a-b)^{3}\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\,b^{5}\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}-\frac{2\left(\frac{\left(-a^{3}b-\frac{1}{2}a^{2}b^{2}+2a\,b^{3}+\frac{7}{8}b^{4}\right)\tan}{}\right.}{}$
risch	$\frac{x}{a}-\frac{12a^{2}b\,e^{7x}-27b^{3}e^{7x}+24a^{3}e^{6x}-72a\,b^{2}e^{6x}+12a^{2}b\,e^{5x}-3b^{3}e^{5x}+72a^{3}e^{4x}-168a\,b^{2}e^{4x}-12a^{2}b\,e^{3x}+3b^{3}e^{3x}+72a^{3}e^{2x}-152a\,b^{2}e^{2x}-1}{12b^{4}\left(1+e^{2x}\right)^{4}}$

```
[In] int(tanh(x)^6/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*ln(tanh(1/2*x)+1)+2/a*(a-b)^3*(a^3+3*a^2*b+3*a*b^2+b^3)/b^5/((a+b)*(a-b
))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/a*ln(tanh(1/2*x)-1
)-2/b^5*(((-a^3*b-1/2*a^2*b^2+2*a*b^3+7/8*b^4)*tanh(1/2*x)^7+(-3*a^3*b-1/2*
a^2*b^2+15/8*b^4+22/3*a*b^3)*tanh(1/2*x)^5+(1/2*a^2*b^2-15/8*b^4-3*a^3*b+22
/3*a*b^3)*tanh(1/2*x)^3+(-a^3*b+2*a*b^3+1/2*a^2*b^2-7/8*b^4)*tanh(1/2*x))/(
1+tanh(1/2*x)^2)^4+1/8*(8*a^4-20*a^2*b^2+15*b^4)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2417 vs. 2(165) = 330.

Time = 0.49 (sec) , antiderivative size = 4914, normalized size of antiderivative = 26.28

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2 - 9*a*b^4)*co
sh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^7 + 24*(2*b^5*
x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 16*b^5*x - 8*a^
4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6 + 12*b^5*x -
3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*a^3*b^2 + a*b^
4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^2 + 48*(2*b^5*x - a^4*b + 3*a^2*b^3)*c
osh(x))*sinh(x)^5 - 24*a^4*b + 56*a^2*b^3 + 24*(3*b^5*x - 3*a^4*b + 7*a^2*b
^3)*cosh(x)^4 + 3*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b + 56*a^2*b^3 -
 35*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^3 + 120*(2*b^5*x - a^4*b + 3*a^2*b^3)*cos
h(x)^2 - 5*(4*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 3*(4*a^3*b^2 - a*b^4)*c
osh(x)^3 + 3*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 - a*b^4 - 35*(4*a^3*b^2 - 9*a
*b^4)*cosh(x)^4 + 160*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^3 - 10*(4*a^3*b
^2 - a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x))*sinh(x)
^3 + 8*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x)^2 + (336*b^5*x*cosh(x)^6 +
```

```
48*b^5*x - 63*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^5 - 72*a^4*b + 152*a^2*b^3 + 36
0*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^4 - 30*(4*a^3*b^2 - a*b^4)*cosh(x)^
3 + 144*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*b^2 - a*b^4)*c
osh(x))*sinh(x)^2 + 12*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8*(a^4 - 2*a^2*
b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^8 + 4*(a^4 -
 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(
x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^2*b^2 + 3*b^4
+ 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 - 2*a^2*b^2 + b
^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b^2 + b^4)*cosh
(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2
 + b^4)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)
^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-
a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*co
sh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*((8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^8 + 8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5
- 20*a^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^6 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3
*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 +
15*a*b^4 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 - 60*a^3
*b^2 + 45*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(8*a^5
- 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 20*a^3*b^2 +
15*a*b^4)*cosh(x)^5 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a
^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(8*a^5 - 20*a^3*b^2 + 15
*a*b^4)*cosh(x)^2 + 4*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5
- 20*a^3*b^2 + 15*a*b^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 20*a^3*b
^2 + 15*a*b^4)*cosh(x)^7 + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*
(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)
*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(4*a^3*b^2 - 9*a*b^4)*cosh
(x) + (96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^6 + 144*(2*b^5
*x - a^4*b + 3*a^2*b^3)*cosh(x)^5 + 12*a^3*b^2 - 27*a*b^4 - 15*(4*a^3*b^2 -
 a*b^4)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^3 + 9*(4*a^3
*b^2 - a*b^4)*cosh(x)^2 + 16*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x))*sinh
(x))/(a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)^8 + 4*a*b
^5*cosh(x)^6 + 6*a*b^5*cosh(x)^4 + 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5*cosh(x)^2
 + a*b^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(
x)^5 + 2*(35*a*b^5*cosh(x)^4 + 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh(x)^4 + 8*
(7*a*b^5*cosh(x)^5 + 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^3 + 4*(7
```

```
*a*b^5*cosh(x)^6 + 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 + a*b^5)*sinh(x)^
2 + 8*(a*b^5*cosh(x)^7 + 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 + a*b^5*cosh
(x))*sinh(x)), 1/12*(12*b^5*x*cosh(x)^8 + 12*b^5*x*sinh(x)^8 - 3*(4*a^3*b^2
 - 9*a*b^4)*cosh(x)^7 + 3*(32*b^5*x*cosh(x) - 4*a^3*b^2 + 9*a*b^4)*sinh(x)^
7 + 24*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^6 + 3*(112*b^5*x*cosh(x)^2 + 1
6*b^5*x - 8*a^4*b + 24*a^2*b^3 - 7*(4*a^3*b^2 - 9*a*b^4)*cosh(x))*sinh(x)^6
 + 12*b^5*x - 3*(4*a^3*b^2 - a*b^4)*cosh(x)^5 + 3*(224*b^5*x*cosh(x)^3 - 4*
a^3*b^2 + a*b^4 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^2 + 48*(2*b^5*x - a^4*b
+ 3*a^2*b^3)*cosh(x))*sinh(x)^5 - 24*a^4*b + 56*a^2*b^3 + 24*(3*b^5*x - 3*a
^4*b + 7*a^2*b^3)*cosh(x)^4 + 3*(280*b^5*x*cosh(x)^4 + 24*b^5*x - 24*a^4*b
+ 56*a^2*b^3 - 35*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^3 + 120*(2*b^5*x - a^4*b +
3*a^2*b^3)*cosh(x)^2 - 5*(4*a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 3*(4*a^3*
b^2 - a*b^4)*cosh(x)^3 + 3*(224*b^5*x*cosh(x)^5 + 4*a^3*b^2 - a*b^4 - 35*(4
*a^3*b^2 - 9*a*b^4)*cosh(x)^4 + 160*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^3
 - 10*(4*a^3*b^2 - a*b^4)*cosh(x)^2 + 32*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*co
sh(x))*sinh(x)^3 + 8*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x)^2 + (336*b^5*
x*cosh(x)^6 + 48*b^5*x - 63*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^5 - 72*a^4*b + 15
2*a^2*b^3 + 360*(2*b^5*x - a^4*b + 3*a^2*b^3)*cosh(x)^4 - 30*(4*a^3*b^2 - a
*b^4)*cosh(x)^3 + 144*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^2 + 9*(4*a^3*
b^2 - a*b^4)*cosh(x))*sinh(x)^2 - 24*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^8 + 8
*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^7 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x
)^8 + 4*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 4*(a^4 - 2*a^2*b^2 + b^4 + 7*(a
^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*c
osh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^5 + 6*(a^4 - 2*a^2*b^
2 + b^4)*cosh(x)^4 + 2*(35*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*a^4 - 6*a^
2*b^2 + 3*b^4 + 30*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a
^2*b^2 + b^4 + 8*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 10*(a^4 - 2*a^2*b^2
 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x)^3 + 4*(a^4 -
 2*a^2*b^2 + b^4)*cosh(x)^2 + 4*(7*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^6 + 15*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 9*(a^4 - 2*a^2*b
^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^7 + 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + (a
^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x)
+ a*sinh(x) + b)/sqrt(a^2 - b^2)) - 3*((8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^8 + 8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)*sinh(x)^7 + (8*a^5 - 20*a
^3*b^2 + 15*a*b^4)*sinh(x)^8 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6
+ 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4 + 7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh
(x)^2)*sinh(x)^6 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^
5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^5 + 8*a^5 - 20*a^3*b^2 + 15*a*b
^4 + 6*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 2*(24*a^5 - 60*a^3*b^2 +
 45*a*b^4 + 35*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 30*(8*a^5 - 20*a
^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^4 + 8*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b
^4)*cosh(x)^5 + 10*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + 3*(8*a^5 - 2
0*a^3*b^2 + 15*a*b^4)*cosh(x))*sinh(x)^3 + 4*(8*a^5 - 20*a^3*b^2 + 15*a*b^4
)*cosh(x)^2 + 4*(7*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^6 + 8*a^5 - 20*a
```

```
^3*b^2 + 15*a*b^4 + 15*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^4 + 9*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^2)*sinh(x)^2 + 8*((8*a^5 - 20*a^3*b^2 + 1
5*a*b^4)*cosh(x)^7 + 3*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^5 + 3*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*cosh(x)^3 + (8*a^5 - 20*a^3*b^2 + 15*a*b^4)*cosh(
x))*sinh(x))*arctan(cosh(x) + sinh(x)) + 3*(4*a^3*b^2 - 9*a*b^4)*cosh(x) +
(96*b^5*x*cosh(x)^7 - 21*(4*a^3*b^2 - 9*a*b^4)*cosh(x)^6 + 144*(2*b^5*x - a
^4*b + 3*a^2*b^3)*cosh(x)^5 + 12*a^3*b^2 - 27*a*b^4 - 15*(4*a^3*b^2 - a*b^4
)*cosh(x)^4 + 96*(3*b^5*x - 3*a^4*b + 7*a^2*b^3)*cosh(x)^3 + 9*(4*a^3*b^2 -
 a*b^4)*cosh(x)^2 + 16*(6*b^5*x - 9*a^4*b + 19*a^2*b^3)*cosh(x))*sinh(x))/(
a*b^5*cosh(x)^8 + 8*a*b^5*cosh(x)*sinh(x)^7 + a*b^5*sinh(x)^8 + 4*a*b^5*cos
h(x)^6 + 6*a*b^5*cosh(x)^4 + 4*a*b^5*cosh(x)^2 + 4*(7*a*b^5*cosh(x)^2 + a*b
^5)*sinh(x)^6 + a*b^5 + 8*(7*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^5 +
 2*(35*a*b^5*cosh(x)^4 + 30*a*b^5*cosh(x)^2 + 3*a*b^5)*sinh(x)^4 + 8*(7*a*b
^5*cosh(x)^5 + 10*a*b^5*cosh(x)^3 + 3*a*b^5*cosh(x))*sinh(x)^3 + 4*(7*a*b^5
*cosh(x)^6 + 15*a*b^5*cosh(x)^4 + 9*a*b^5*cosh(x)^2 + a*b^5)*sinh(x)^2 + 8*
(a*b^5*cosh(x)^7 + 3*a*b^5*cosh(x)^5 + 3*a*b^5*cosh(x)^3 + a*b^5*cosh(x))*s
inh(x))]
```

Sympy [F]

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)}\, dx = \int \frac{\tanh^6{(x)}}{a + b\,\text{sech}\,(x)}\, dx$$

```
[In] integrate(tanh(x)**6/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**6/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^6(x)}{a + b\text{sech}(x)}\, dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.34

$$\int \frac{\tanh^6(x)}{a+b\text{sech}(x)}\,dx$$
$$= \frac{x}{a} - \frac{(8\,a^4 - 20\,a^2b^2 + 15\,b^4)\arctan\left(e^x\right)}{4\,b^5} + \frac{2\,(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}ab^5} - \frac{12\,a^2be^{(7\,x)} - 27\,b^3e^{(7\,x)} + 24\,a^3e^{(6\,x)} - 72\,ab^2e^{(6\,x)} + 12\,a^2be^{(5\,x)} - 3\,b^3e^{(5\,x)} + 72\,a^3e^{(4\,x)} - 168\,ab^2e^{(4\,x)} - \cdots}{12\,b^4(e^{(2\,x)}+1)^4}$$

```
[In] integrate(tanh(x)^6/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 1/4*(8*a^4 - 20*a^2*b^2 + 15*b^4)*arctan(e^x)/b^5 + 2*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*arctan((a*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a
*b^5) - 1/12*(12*a^2*b*e^(7*x) - 27*b^3*e^(7*x) + 24*a^3*e^(6*x) - 72*a*b^2
*e^(6*x) + 12*a^2*b*e^(5*x) - 3*b^3*e^(5*x) + 72*a^3*e^(4*x) - 168*a*b^2*e^
(4*x) - 12*a^2*b*e^(3*x) + 3*b^3*e^(3*x) + 72*a^3*e^(2*x) - 152*a*b^2*e^(2*
x) - 12*a^2*b*e^x + 27*b^3*e^x + 24*a^3 - 56*a*b^2)/(b^4*(e^(2*x) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.35

$$\int \frac{\tanh^6(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{\frac{8\,a}{3\,b^2}+\frac{6\,\mathrm{e}^x}{b}}{3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1} - \frac{\frac{\mathrm{e}^x\,(4\,a^2-9\,b^2)}{4\,b^3}+\frac{2\,(a^4-3\,a^2\,b^2)}{a\,b^4}}{\mathrm{e}^{2\,x}+1}$$

$$-\frac{\frac{4\,a}{b^2}-\frac{\mathrm{e}^x\,(4\,a^2-13\,b^2)}{2\,b^3}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{x}{a}+\frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,(a^4\,8\mathrm{i}-a^2\,b^2\,20\mathrm{i}+b^4\,15\mathrm{i})}{8\,b^5}$$

$$-\frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,(a^4\,8\mathrm{i}-a^2\,b^2\,20\mathrm{i}+b^4\,15\mathrm{i})}{8\,b^5}-\frac{4\,\mathrm{e}^x}{b\,\left(4\,\mathrm{e}^{2\,x}+6\,\mathrm{e}^{4\,x}+4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1\right)}$$

$$+\frac{\ln\left(\frac{\sqrt{-(a+b)^5\,(a-b)^5}\left(\frac{128\,a^{12}+192\,\mathrm{e}^x\,a^{11}\,b-832\,a^{10}\,b^2-1216\,\mathrm{e}^x\,a^9\,b^3+2240\,a^8\,b^4+3200\,\mathrm{e}^x\,a^7\,b^5-3160\,a^6\,b^6-4360\,\mathrm{e}^x\,a^5\,b^7+2385\,a^4\,b^8+3075\,\mathrm{e}^x\,a^3\,b^{\cdots}}{2\,a^6\,b^8}\right.}{\cdots}\right.}{\cdots}$$

$$-\frac{\ln\left(-\frac{\sqrt{-(a+b)^5\,(a-b)^5}\left(\frac{128\,a^{12}+192\,\mathrm{e}^x\,a^{11}\,b-832\,a^{10}\,b^2-1216\,\mathrm{e}^x\,a^9\,b^3+2240\,a^8\,b^4+3200\,\mathrm{e}^x\,a^7\,b^5-3160\,a^6\,b^6-4360\,\mathrm{e}^x\,a^5\,b^7+2385\,a^4\,b^8+3075\,\mathrm{e}^x\,a\cdots}{2\,a^6\,b^8}\right.}{\cdots}\right.}{\cdots}$$

```
[In] int(tanh(x)^6/(a + b/cosh(x)),x)
```

```
[Out] ((8*a)/(3*b^2) + (6*exp(x))/b)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (
(exp(x)*(4*a^2 - 9*b^2))/(4*b^3) + (2*(a^4 - 3*a^2*b^2))/(a*b^4))/(exp(2*x)
 + 1) - ((4*a)/b^2 - (exp(x)*(4*a^2 - 13*b^2))/(2*b^3))/(2*exp(2*x) + exp(4
*x) + 1) + x/a + (log(exp(x) - 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5
) - (log(exp(x) + 1i)*(a^4*8i + b^4*15i - a^2*b^2*20i))/(8*b^5) - (4*exp(x)
)/(b*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1)) + (log(((-(a +
b)^5*(a - b)^5)^(1/2)*((128*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4*b^8 -
3160*a^6*b^6 + 2240*a^8*b^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 192*a^11*b
*exp(x) + 3075*a^3*b^9*exp(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5*exp(x) -
 1216*a^9*b^3*exp(x))/(2*a^6*b^8) - ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(a^2
- b^2)*(16*a*b^4 + 16*a^5 - 32*a^3*b^2 + 32*b^5*exp(x) + 28*a^4*b*exp(x) -
57*a^2*b^3*exp(x)))/(a^6*b^2) + (32*(-(a + b)^5*(a - b)^5)^(1/2)*(3*a*b^2 -
 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b^3)))/(a*b^5)))/(a*b^5) - ((
a^2 - b^2)^3*(8*a^4 + 15*b^4 - 20*a^2*b^2)*(30*a*b^4 + 16*a^5 - 40*a^3*b^2
+ 52*b^5*exp(x) + 28*a^4*b*exp(x) - 71*a^2*b^3*exp(x)))/(2*a^6*b^12))*(-(a
```

```
+ b)^5*(a - b)^5)^(1/2))/(a*b^5) - (log(- ((-(a + b)^5*(a - b)^5)^(1/2)*((1
28*a^12 + 64*b^12 - 834*a^2*b^10 + 2385*a^4*b^8 - 3160*a^6*b^6 + 2240*a^8*b
^4 - 832*a^10*b^2 - 900*a*b^11*exp(x) + 192*a^11*b*exp(x) + 3075*a^3*b^9*ex
p(x) - 4360*a^5*b^7*exp(x) + 3200*a^7*b^5*exp(x) - 1216*a^9*b^3*exp(x))/(2*
a^6*b^8) + ((-(a + b)^5*(a - b)^5)^(1/2)*((4*(a^2 - b^2)*(16*a*b^4 + 16*a^5
 - 32*a^3*b^2 + 32*b^5*exp(x) + 28*a^4*b*exp(x) - 57*a^2*b^3*exp(x)))/(a^6*
b^2) - (32*(-(a + b)^5*(a - b)^5)^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3
*a^2*b*exp(x)))/(a^6*b^3)))/(a*b^5)))/(a*b^5) - ((a^2 - b^2)^3*(8*a^4 + 15*
b^4 - 20*a^2*b^2)*(30*a*b^4 + 16*a^5 - 40*a^3*b^2 + 52*b^5*exp(x) + 28*a^4*
b*exp(x) - 71*a^2*b^3*exp(x)))/(2*a^6*b^12))*(-(a + b)^5*(a - b)^5)^(1/2))/
(a*b^5)
```

3.115 $\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}+\frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\,\text{sech}(x)}{b^3}+\frac{a\text{sech}^2(x)}{2b^2}-\frac{\text{sech}^3(x)}{3b}$$

```
[Out] ln(cosh(x))/a+(a^2-b^2)^2*ln(a+b*sech(x))/a/b^4-(a^2-2*b^2)*sech(x)/b^3+1/2
*a*sech(x)^2/b^2-1/3*sech(x)^3/b
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3970, 908}

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx=\frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\,\text{sech}(x)}{b^3}+\frac{a\text{sech}^2(x)}{2b^2}+\frac{\log(\cosh(x))}{a}-\frac{\text{sech}^3(x)}{3b}$$

```
[In] Int[Tanh[x]^5/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((a^2 - b^2)^2*Log[a + b*Sech[x]])/(a*b^4) - ((a^2 - 2*b^2
)*Sech[x])/b^3 + (a*Sech[x]^2)/(2*b^2) - Sech[x]^3/(3*b)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)}\,dx, x, b\text{sech}(x)\right)}{b^4}$$

$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{2b^2}{a^2}\right)+\frac{b^4}{ax}-ax+x^2-\frac{(a^2-b^2)^2}{a(a+x)}\right)\,dx, x, b\text{sech}(x)\right)}{b^4}$$

$$= \frac{\log(\cosh(x))}{a}+\frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{ab^4}-\frac{(a^2-2b^2)\,\text{sech}(x)}{b^3}+\frac{a\text{sech}^2(x)}{2b^2}-\frac{\text{sech}^3(x)}{3b}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = -\frac{-\frac{b^4\log(\cosh(x))}{a}-\frac{(a^2-b^2)^2\log(a+b\text{sech}(x))}{a}+b(a^2-2b^2)\,\text{sech}(x)-\frac{1}{2}ab^2\text{sech}^2(x)+\frac{1}{3}b^3\text{sech}^3(x)}{b^4}$$

```
[In] Integrate[Tanh[x]^5/(a + b*Sech[x]),x]
[Out] -((-((b^4*Log[Cosh[x]])/a) - ((a^2 - b^2)^2*Log[a + b*Sech[x]])/a + b*(a^2
- 2*b^2)*Sech[x] - (a*b^2*Sech[x]^2)/2 + (b^3*Sech[x]^3)/3)/b^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(68) = 136.

Time = 0.94 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

method	result
default	$-\frac{-\frac{2b^2(a+2b)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+\frac{8b^3}{3\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^3}+a(a^2-2b^2)\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)+\frac{2b\left(a^2+ab-b^2\right)}{1+\tanh\left(\frac{x}{2}\right)^2}}{b^4}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{(a-b)^2(a^2+2ab+b^2}{}$
risch	$-\frac{x}{a}-\frac{2\,e^x(3a^2e^{4x}-6b^2e^{4x}-3ab\,e^{3x}+6a^2e^{2x}-8b^2e^{2x}-3b\,e^x a+3a^2-6b^2)}{3b^3(1+e^{2x})^3}-\frac{a^3\ln(1+e^{2x})}{b^4}+\frac{2a\ln(1+e^{2x})}{b^2}+\frac{a^3\ln\left(e^{2x}+\frac{2b\,e^x}{a}\right.}{b^4}$

```
[In] int(tanh(x)^5/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-2*b^2*(a+2*b)/(1+tanh(1/2*x)^2)^2+8/3*b^3/(1+tanh(1/2*x)^2)^3+a*(a
^2-2*b^2)*ln(1+tanh(1/2*x)^2)+2*b*(a^2+a*b-b^2)/(1+tanh(1/2*x)^2))-1/a*ln(t
anh(1/2*x)-1)+(a-b)^2*(a^2+2*a*b+b^2)/a/b^4*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^
2*b+a+b)-1/a*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. 2(68) = 136.

Time = 0.29 (sec) , antiderivative size = 1280, normalized size of antiderivative = 17.78

$$\int \frac{\tanh^5(x)}{a+b\operatorname{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^4*x*cosh(x)^6 + 3*b^4*x*sinh(x)^6 + 6*(a^3*b - 2*a*b^3)*cosh(x)^5
 + 6*(3*b^4*x*cosh(x) + a^3*b - 2*a*b^3)*sinh(x)^5 + 3*b^4*x + 3*(3*b^4*x -
 2*a^2*b^2)*cosh(x)^4 + 3*(15*b^4*x*cosh(x)^2 + 3*b^4*x - 2*a^2*b^2 + 10*(a
^3*b - 2*a*b^3)*cosh(x))*sinh(x)^4 + 4*(3*a^3*b - 4*a*b^3)*cosh(x)^3 + 4*(1
5*b^4*x*cosh(x)^3 + 3*a^3*b - 4*a*b^3 + 15*(a^3*b - 2*a*b^3)*cosh(x)^2 + 3*
(3*b^4*x - 2*a^2*b^2)*cosh(x))*sinh(x)^3 + 3*(3*b^4*x - 2*a^2*b^2)*cosh(x)^
2 + 3*(15*b^4*x*cosh(x)^4 + 3*b^4*x - 2*a^2*b^2 + 20*(a^3*b - 2*a*b^3)*cosh
(x)^3 + 6*(3*b^4*x - 2*a^2*b^2)*cosh(x)^2 + 4*(3*a^3*b - 4*a*b^3)*cosh(x))*
sinh(x)^2 + 6*(a^3*b - 2*a*b^3)*cosh(x) - 3*((a^4 - 2*a^2*b^2 + b^4)*cosh(x
)^6 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2 + b^4)
*sinh(x)^6 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + b^4
 + 5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^4 + a^4 - 2*a^2*b^2 + b^4 +
 4*(5*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
)*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^2 +
 b^4)*cosh(x)^4 + a^4 - 2*a^2*b^2 + b^4 + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)
^2)*sinh(x)^2 + 6*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2 +
```

```
b^4)*cosh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*cosh(x))*sinh(x))*log(2*(a*cosh(x
) + b)/(cosh(x) - sinh(x))) + 3*((a^4 - 2*a^2*b^2)*cosh(x)^6 + 6*(a^4 - 2*a
^2*b^2)*cosh(x)*sinh(x)^5 + (a^4 - 2*a^2*b^2)*sinh(x)^6 + 3*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + 3*(a^4 - 2*a^2*b^2 + 5*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^
4 + a^4 - 2*a^2*b^2 + 4*(5*(a^4 - 2*a^2*b^2)*cosh(x)^3 + 3*(a^4 - 2*a^2*b^2
)*cosh(x))*sinh(x)^3 + 3*(a^4 - 2*a^2*b^2)*cosh(x)^2 + 3*(5*(a^4 - 2*a^2*b^
2)*cosh(x)^4 + a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2)*cosh(x)^2)*sinh(x)^2 +
 6*((a^4 - 2*a^2*b^2)*cosh(x)^5 + 2*(a^4 - 2*a^2*b^2)*cosh(x)^3 + (a^4 - 2*
a^2*b^2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 6*(3*b^4*x*
cosh(x)^5 + 5*(a^3*b - 2*a*b^3)*cosh(x)^4 + a^3*b - 2*a*b^3 + 2*(3*b^4*x -
2*a^2*b^2)*cosh(x)^3 + 2*(3*a^3*b - 4*a*b^3)*cosh(x)^2 + (3*b^4*x - 2*a^2*b
^2)*cosh(x))*sinh(x))/(a*b^4*cosh(x)^6 + 6*a*b^4*cosh(x)*sinh(x)^5 + a*b^4*
sinh(x)^6 + 3*a*b^4*cosh(x)^4 + 3*a*b^4*cosh(x)^2 + a*b^4 + 3*(5*a*b^4*cosh
(x)^2 + a*b^4)*sinh(x)^4 + 4*(5*a*b^4*cosh(x)^3 + 3*a*b^4*cosh(x))*sinh(x)^
3 + 3*(5*a*b^4*cosh(x)^4 + 6*a*b^4*cosh(x)^2 + a*b^4)*sinh(x)^2 + 6*(a*b^4*
cosh(x)^5 + 2*a*b^4*cosh(x)^3 + a*b^4*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^5(x)}{a + b\,\text{sech}(x)}\,dx$$

[In] integrate(tanh(x)**5/(a+b*sech(x)),x)

[Out] Integral(tanh(x)**5/(a + b*sech(x)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(68) = 136$.

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.28

$$\int \frac{\tanh^5(x)}{a + b\text{sech}(x)}\,dx = \frac{2\left(3\,abe^{(-2\,x)} + 3\,abe^{(-4\,x)} - 3\,(a^2 - 2\,b^2)e^{(-x)} - 2\,(3\,a^2 - 4\,b^2)e^{(-3\,x)} - 3\,(a^2 - 2\,b^2)e^{(-5\,x)}\right)}{3\,(3\,b^3e^{(-2\,x)} + 3\,b^3e^{(-4\,x)} + b^3e^{(-6\,x)} + b^3)} + \frac{x}{a} - \frac{(a^3 - 2\,ab^2)\log\left(e^{(-2\,x)} + 1\right)}{b^4} + \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{ab^4}$$

[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="maxima")

```
[Out] 2/3*(3*a*b*e^(-2*x) + 3*a*b*e^(-4*x) - 3*(a^2 - 2*b^2)*e^(-x) - 2*(3*a^2 -
4*b^2)*e^(-3*x) - 3*(a^2 - 2*b^2)*e^(-5*x))/(3*b^3*e^(-2*x) + 3*b^3*e^(-4*x
) + b^3*e^(-6*x) + b^3) + x/a - (a^3 - 2*a*b^2)*log(e^(-2*x) + 1)/b^4 + (a^
4 - 2*a^2*b^2 + b^4)*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(68) = 136.

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.11

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = -\frac{(a^3-2\,ab^2)\log\left(e^{(-x)}+e^x\right)}{b^4} + \frac{(a^4-2\,a^2b^2+b^4)\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{ab^4} + \frac{11\,a^3\left(e^{(-x)}+e^x\right)^3-22\,ab^2\left(e^{(-x)}+e^x\right)^3-12\,a^2b\left(e^{(-x)}+e^x\right)^2+24\,b^3\left(e^{(-x)}+e^x\right)^2+12\,ab^2\left(e^{(-x)}+e^x\right.}{6\,b^4\left(e^{(-x)}+e^x\right)^3}$$

```
[In] integrate(tanh(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -(a^3 - 2*a*b^2)*log(e^(-x) + e^x)/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(a*
(e^(-x) + e^x) + 2*b))/(a*b^4) + 1/6*(11*a^3*(e^(-x) + e^x)^3 - 22*a*b^2*(e
^(-x) + e^x)^3 - 12*a^2*b*(e^(-x) + e^x)^2 + 24*b^3*(e^(-x) + e^x)^2 + 12*a
*b^2*(e^(-x) + e^x) - 16*b^3)/(b^4*(e^(-x) + e^x)^3)
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.15

$$\int \frac{\tanh^5(x)}{a+b\text{sech}(x)}\,dx = \frac{\frac{2\,a}{b^2}-\frac{2\,\mathrm{e}^x\,(a^2-2\,b^2)}{b^3}}{\mathrm{e}^{2\,x}+1} - \frac{x}{a} - \frac{\frac{2\,a}{b^2}+\frac{8\,\mathrm{e}^x}{3\,b}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1} + \frac{\ln\left(\mathrm{e}^{2\,x}+1\right)\,\left(2\,a\,b^2-a^3\right)}{b^4} + \frac{8\,\mathrm{e}^x}{3\,b\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)} + \frac{\ln\left(a+2\,b\,\mathrm{e}^x+a\,\mathrm{e}^{2\,x}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{a\,b^4}$$

```
[In] int(tanh(x)^5/(a + b/cosh(x)),x)
```

```
[Out] ((2*a)/b^2 - (2*exp(x)*(a^2 - 2*b^2))/b^3)/(exp(2*x) + 1) - x/a - ((2*a)/b^
2 + (8*exp(x))/(3*b))/(2*exp(2*x) + exp(4*x) + 1) + (log(exp(2*x) + 1)*(2*a
*b^2 - a^3))/b^4 + (8*exp(x))/(3*b*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)
) + (log(a + 2*b*exp(x) + a*exp(2*x))*(a^4 + b^4 - 2*a^2*b^2))/(a*b^4)
```

3.116 $\int \frac{\tanh^4(x)}{a+b\,\mathbf{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\tanh^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{x}{a}+\frac{(2a^2-3b^2)\arctan(\sinh(x))}{2b^3} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3} - \frac{a\tanh(x)}{b^2}+\frac{\mathrm{sech}(x)\tanh(x)}{2b}$$

```
[Out] x/a+1/2*(2*a^2-3*b^2)*arctan(sinh(x))/b^3-2*(a-b)^(3/2)*(a+b)^(3/2)*arctan(
(a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/b^3-a*tanh(x)/b^2+1/2*sech(x)*tanh(x
)/b
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3983, 2972, 3136, 2738, 211, 3855}

$$\int \frac{\tanh^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = \frac{(2a^2-3b^2)\arctan(\sinh(x))}{2b^3} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3} - \frac{a\tanh(x)}{b^2}+\frac{x}{a}+\frac{\tanh(x)\mathrm{sech}(x)}{2b}$$

```
[In] Int[Tanh[x]^4/(a + b*Sech[x]),x]
```

```
[Out] x/a + ((2*a^2 - 3*b^2)*ArcTan[Sinh[x]])/(2*b^3) - (2*(a - b)^(3/2)*(a + b)^
(3/2)*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*b^3) - (a*Tanh[x])/b^
2 + (Sech[x]*Tanh[x])/(2*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2972

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[Cos[e + f*x]*(a + b*
Sin[e + f*x])^(m + 1)*((d*Sin[e + f*x])^(n + 1)/(a*d*f*(n + 1))), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((
d*Sin[e + f*x])^(n + 2)/(a^2*d^2*f*(n + 1)*(n + 2))), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3136

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)])), x_Symbol] :> Simp[C*(x/(b*d)), x] + (Dist[(A*b^2 - a*b*B + a^2*C)
/(b*(b*c - a*d)), Int[1/(a + b*Sin[e + f*x]), x], x] - Dist[(c^2*C - B*c*d
+ A*d^2)/(d*(b*c - a*d)), Int[1/(c + d*Sin[e + f*x]), x], x]) /; FreeQ[{a,
b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && Ne
Q[c^2 - d^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
```

```
+ n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = \int \frac{\sinh(x)\tanh^3(x)}{b + a\cosh(x)}\,dx$$

$$= -\frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{\int \frac{(-2a^2+3b^2-ab\cosh(x)-2b^2\cosh^2(x))\operatorname{sech}(x)}{b+a\cosh(x)}\,dx}{2b^2}$$

$$= \frac{x}{a} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{(a^2-b^2)^2\int \frac{1}{b+a\cosh(x)}\,dx}{ab^3} - \frac{(-2a^2+3b^2)\int \operatorname{sech}(x)\,dx}{2b^3}$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\arctan(\sinh(x))}{2b^3} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b} - \frac{\left(2(a^2-b^2)^2\right)\operatorname{Subst}\left(\int \frac{1}{a+b-(-a+b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{ab^3}$$

$$= \frac{x}{a} + \frac{(2a^2-3b^2)\arctan(\sinh(x))}{2b^3} - \frac{2(a-b)^{3/2}(a+b)^{3/2}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab^3} - \frac{a\tanh(x)}{b^2} + \frac{\operatorname{sech}(x)\tanh(x)}{2b}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^4(x)}{a + b\operatorname{sech}(x)}\,dx$$
$$= \frac{(b+a\cosh(x))\operatorname{sech}^2(x)\left(2\left(b^3x + a(2a^2-3b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + 2(a^2-b^2)^{3/2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)\right) c}{2ab^3(a+b\operatorname{sech}(x))}$$

```
[In] Integrate[Tanh[x]^4/(a + b*Sech[x]),x]
[Out] ((b + a*Cosh[x])*Sech[x]^2*(2*(b^3*x + a*(2*a^2 - 3*b^2)*ArcTan[Tanh[x/2]]
+ 2*(a^2 - b^2)^(3/2)*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[a^2 - b^2]])*Cosh[x]
 + a*b*(-2*a*Sinh[x] + b*Tanh[x])))/(2*a*b^3*(a + b*Sech[x]))
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.62

method	result
default	$-\frac{2(a-b)^2(a^2+2ab+b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\,b^3\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\frac{2\left(\left(-ab-\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)^3+\left(-ab+\frac{1}{2}b^2\right)\tanh\left(\frac{x}{2}\right)\right)}{\left(1+\tanh\left(\frac{x}{2}\right)^2\right)^2}+(2a^2-3b}{b^3}$
risch	$\frac{x}{a}+\frac{e^{3x}b+2a\,e^{2x}-e^{x}b+2a}{(1+e^{2x})^2b^2}+\frac{i\ln(e^x+i)a^2}{b^3}-\frac{3i\ln(e^x+i)}{2b}-\frac{i\ln(e^x-i)a^2}{b^3}+\frac{3i\ln(e^x-i)}{2b}+\frac{\sqrt{-a^2+b^2}\,a\ln\left(e^x-\frac{\sqrt{-a^2+b^2}-b}{a}\right)}{b^3}$

```
[In] int(tanh(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a*(a-b)^2*(a^2+2*a*b+b^2)/b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*
x)/((a+b)*(a-b))^(1/2))-1/a*ln(tanh(1/2*x)-1)+2/b^3*(((-a*b-1/2*b^2)*tanh(1
/2*x)^3+(-a*b+1/2*b^2)*tanh(1/2*x))/(1+tanh(1/2*x)^2)^2+1/2*(2*a^2-3*b^2)*a
rctan(tanh(1/2*x)))+1/a*ln(tanh(1/2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. 2(80) = 160.

Time = 0.35 (sec) , antiderivative size = 1254, normalized size of antiderivative = 13.34

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh(
x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cosh
(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^2
 - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2)*s
inh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2
)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*s
inh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x)
 - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh
(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(
x) + b)*sinh(x) + a)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*
cosh(x)*sinh(x)^3 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^
3 - 3*a*b^2)*cosh(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2
)*sinh(x)^2 + 4*((2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*s
inh(x))*arctan(cosh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2
- a*b^2 + 4*(b^3*x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*co
sh(x)*sinh(x)^3 + a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*
```

```
cosh(x)^2 + a*b^3)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))
, (b^3*x*cosh(x)^4 + b^3*x*sinh(x)^4 + a*b^2*cosh(x)^3 + b^3*x - a*b^2*cosh
(x) + (4*b^3*x*cosh(x) + a*b^2)*sinh(x)^3 + 2*a^2*b + 2*(b^3*x + a^2*b)*cos
h(x)^2 + (6*b^3*x*cosh(x)^2 + 2*b^3*x + 3*a*b^2*cosh(x) + 2*a^2*b)*sinh(x)^
2 + 2*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^2 - b^2
)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2 + a^2 -
b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x)
)*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b
^2)) + ((2*a^3 - 3*a*b^2)*cosh(x)^4 + 4*(2*a^3 - 3*a*b^2)*cosh(x)*sinh(x)^3
 + (2*a^3 - 3*a*b^2)*sinh(x)^4 + 2*a^3 - 3*a*b^2 + 2*(2*a^3 - 3*a*b^2)*cosh
(x)^2 + 2*(2*a^3 - 3*a*b^2 + 3*(2*a^3 - 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 4*(
(2*a^3 - 3*a*b^2)*cosh(x)^3 + (2*a^3 - 3*a*b^2)*cosh(x))*sinh(x))*arctan(co
sh(x) + sinh(x)) + (4*b^3*x*cosh(x)^3 + 3*a*b^2*cosh(x)^2 - a*b^2 + 4*(b^3*
x + a^2*b)*cosh(x))*sinh(x))/(a*b^3*cosh(x)^4 + 4*a*b^3*cosh(x)*sinh(x)^3 +
 a*b^3*sinh(x)^4 + 2*a*b^3*cosh(x)^2 + a*b^3 + 2*(3*a*b^3*cosh(x)^2 + a*b^3
)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + a*b^3*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^4{(x)}}{a + b\,\text{sech}\,(x)}\,dx$$

```
[In] integrate(tanh(x)**4/(a+b*sech(x)),x)
[Out] Integral(tanh(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^4(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.18

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}\,dx = \frac{x}{a} + \frac{(2\,a^2-3\,b^2)\arctan\left(e^x\right)}{b^3} - \frac{2\,(a^4-2\,a^2b^2+b^4)\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}ab^3} + \frac{be^{(3\,x)}+2\,ae^{(2\,x)}-be^x+2\,a}{b^2\left(e^{(2\,x)}+1\right)^2}$$

```
[In] integrate(tanh(x)^4/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a + (2*a^2 - 3*b^2)*arctan(e^x)/b^3 - 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((a
*e^x + b)/sqrt(a^2 - b^2))/(sqrt(a^2 - b^2)*a*b^3) + (b*e^(3*x) + 2*a*e^(2*
x) - b*e^x + 2*a)/(b^2*(e^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 700, normalized size of antiderivative = 7.45

$$\int \frac{\tanh^4(x)}{a+b\text{sech}(x)}\,dx$$

$$= \frac{\frac{2\,a}{b^2}+\frac{\mathrm{e}^x}{b}}{\mathrm{e}^{2\,x}+1} + \frac{x}{a} - \frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,\left(a^2\,2\mathrm{i}-b^2\,3\mathrm{i}\right)}{2\,b^3} + \frac{\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,\left(a^2\,2\mathrm{i}-b^2\,3\mathrm{i}\right)}{2\,b^3} - \frac{2\,\mathrm{e}^x}{b\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

$$+ \frac{\ln\left(\frac{\left(\frac{64\,a^8+96\,\mathrm{e}^x\,a^7\,b-288\,a^6\,b^2-416\,\mathrm{e}^x\,a^5\,b^3+456\,a^4\,b^4+600\,\mathrm{e}^x\,a^3\,b^5-272\,a^2\,b^6-288\,\mathrm{e}^x\,a\,b^7+32\,b^8}{a^6\,b^4} - \frac{\left(\frac{16\,\left(a^2-b^2\right)\,\left(-4\,a^3-7\,\mathrm{e}^x\,a^2\,b+4\,a\,b^2+8\,\mathrm{e}^x\,b^3\right)}{a^6}+\cdots\right.}{\cdots}\right.}{a\,b^3}\cdots\right)}{\cdots}$$

$$- \frac{\ln\left(-\frac{\left(\frac{64\,a^8+96\,\mathrm{e}^x\,a^7\,b-288\,a^6\,b^2-416\,\mathrm{e}^x\,a^5\,b^3+456\,a^4\,b^4+600\,\mathrm{e}^x\,a^3\,b^5-272\,a^2\,b^6-288\,\mathrm{e}^x\,a\,b^7+32\,b^8}{a^6\,b^4} + \frac{\left(\frac{16\,\left(a^2-b^2\right)\,\left(-4\,a^3-7\,\mathrm{e}^x\,a^2\,b+4\,a\,b^2+8\,\mathrm{e}^x\,b^3\cdots\right.}{a^6}\right.}{\cdots}\right.}{a\,b^3}\cdots\right)}{\cdots}$$

```
[In] int(tanh(x)^4/(a + b/cosh(x)),x)
```

```
[Out] ((2*a)/b^2 + exp(x)/b)/(exp(2*x) + 1) + x/a - (log(exp(x) - 1i)*(a^2*2i - b
^2*3i))/(2*b^3) + (log(exp(x) + 1i)*(a^2*2i - b^2*3i))/(2*b^3) - (2*exp(x))
/(b*(2*exp(2*x) + exp(4*x) + 1)) + (log((((64*a^8 + 32*b^8 - 272*a^2*b^6 +
456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) + 96*a^7*b*exp(x) + 600*a^3*b^
5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) - (((16*(a^2 - b^2)*(4*a*b^2 - 4*a
^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 + (32*(-(a + b)^3*(a - b)^3)^(1/2)
*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*b))*(-(a + b)^3*(a
 - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (8*(a^2 -
b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 + 10*b^3*exp(x) - 7*a^2*b*exp(x)))/
(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3) - (log(- (((64*a^8 + 32*b^
8 - 272*a^2*b^6 + 456*a^4*b^4 - 288*a^6*b^2 - 288*a*b^7*exp(x) + 96*a^7*b*e
xp(x) + 600*a^3*b^5*exp(x) - 416*a^5*b^3*exp(x))/(a^6*b^4) + (((16*(a^2 - b
^2)*(4*a*b^2 - 4*a^3 + 8*b^3*exp(x) - 7*a^2*b*exp(x)))/a^6 - (32*(-(a + b)^
3*(a - b)^3)^(1/2)*(3*a*b^2 - 2*a^3 + 4*b^3*exp(x) - 3*a^2*b*exp(x)))/(a^6*
b))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3))*(-(a + b)^3*(a - b)^3)^(1/2))/(a
*b^3) - (8*(a^2 - b^2)^2*(2*a^2 - 3*b^2)*(6*a*b^2 - 4*a^3 + 10*b^3*exp(x) -
 7*a^2*b*exp(x)))/(a^6*b^6))*(-(a + b)^3*(a - b)^3)^(1/2))/(a*b^3)
```

3.117 $\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx=\frac{\log(\cosh(x))}{a}+\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\text{sech}(x))}{a}+\frac{\text{sech}(x)}{b}$$

```
[Out] ln(cosh(x))/a+(1-a^2/b^2)*ln(a+b*sech(x))/a+sech(x)/b
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3970, 908}

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx=\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\text{sech}(x))}{a}+\frac{\log(\cosh(x))}{a}+\frac{\text{sech}(x)}{b}$$

```
[In] Int[Tanh[x]^3/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((1 - a^2/b^2)*Log[a + b*Sech[x]])/a + Sech[x]/b
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)}\,dx, x, b\text{sech}(x)\right)}{b^2}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{b^2}{ax}+\frac{a^2-b^2}{a(a+x)}\right)\,dx, x, b\text{sech}(x)\right)}{b^2}\\ &= \frac{\log(\cosh(x))}{a}+\frac{\left(1-\frac{a^2}{b^2}\right)\log(a+b\text{sech}(x))}{a}+\frac{\text{sech}(x)}{b}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a}+\frac{\log(a+b\text{sech}(x))}{a}-\frac{a\log(a+b\text{sech}(x))}{b^2}+\frac{\text{sech}(x)}{b}$$

```
[In] Integrate[Tanh[x]^3/(a + b*Sech[x]),x]
[Out] Log[Cosh[x]]/a + Log[a + b*Sech[x]]/a - (a*Log[a + b*Sech[x]])/b^2 + Sech[x
]/b
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(35) = 70.

Time = 0.39 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

method	result	siz
risch	$-\frac{x}{a}+\frac{2\,\mathrm{e}^{x}}{b(1+\mathrm{e}^{2x})}+\frac{a\ln(1+\mathrm{e}^{2x})}{b^2}-\frac{a\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}+1\right)}{b^2}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2b\,\mathrm{e}^{x}}{a}+1\right)}{a}$	75
default	$\frac{\frac{2b}{1+\tanh\left(\frac{x}{2}\right)^2}+a\ln\left(1+\tanh\left(\frac{x}{2}\right)^2\right)}{b^2}-\frac{(a-b)(a+b)\ln\left(\tanh\left(\frac{x}{2}\right)^2a-\tanh\left(\frac{x}{2}\right)^2b+a+b\right)}{a\,b^2}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	92

```
[In] int(tanh(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

[Out] -x/a+2*exp(x)/b/(1+exp(2*x))+a/b^2*ln(1+exp(2*x))-a/b^2*ln(exp(2*x)+2*b/a*e xp(x)+1)+1/a*ln(exp(2*x)+2*b/a*exp(x)+1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(35) = 70$.

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.71

$$\int \frac{\tanh^3(x)}{a + b\text{sech}(x)}\,dx = -\frac{b^2 x \cosh(x)^2 + b^2 x \sinh(x)^2 + b^2 x - 2\,ab\cosh(x) + \left((a^2 - b^2)\cosh(x)^2 + 2\,(a^2 - b^2)\cosh(x)\sinh(x)\right.}{}$$

[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="fricas")

[Out] -(b^2*x*cosh(x)^2 + b^2*x*sinh(x)^2 + b^2*x - 2*a*b*cosh(x) + ((a^2 - b^2)* cosh(x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2 + a^2 - b ^2)*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - (a^2*cosh(x)^2 + 2*a^2*cos h(x)*sinh(x) + a^2*sinh(x)^2 + a^2)*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2* (b^2*x*cosh(x) - a*b)*sinh(x))/(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*sinh(x)^2 + a*b^2)

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\tanh^3(x)}{a + b\,\text{sech}(x)}\,dx$$

[In] integrate(tanh(x)**3/(a+b*sech(x)),x)

[Out] Integral(tanh(x)**3/(a + b*sech(x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.91

$$\int \frac{\tanh^3(x)}{a + b\text{sech}(x)}\,dx = \frac{x}{a} + \frac{2\,e^{(-x)}}{be^{(-2\,x)} + b} + \frac{a\log\left(e^{(-2\,x)} + 1\right)}{b^2} - \frac{(a^2 - b^2)\log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{ab^2}$$

[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="maxima")

[Out] x/a + 2*e^(-x)/(b*e^(-2*x) + b) + a*log(e^(-2*x) + 1)/b^2 - (a^2 - b^2)*log (2*b*e^(-x) + a*e^(-2*x) + a)/(a*b^2)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.09

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{a\log\left(e^{(-x)}+e^x\right)}{b^2} - \frac{(a^2-b^2)\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{ab^2} - \frac{a\left(e^{(-x)}+e^x\right)-2\,b}{b^2\left(e^{(-x)}+e^x\right)}$$

```
[In] integrate(tanh(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] a*log(e^(-x) + e^x)/b^2 - (a^2 - b^2)*log(abs(a*(e^(-x) + e^x) + 2*b))/(a*b
^2) - (a*(e^(-x) + e^x) - 2*b)/(b^2*(e^(-x) + e^x))
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 7.43

$$\int \frac{\tanh^3(x)}{a+b\text{sech}(x)}\,dx = \frac{2\,\mathrm{e}^x}{b+b\,\mathrm{e}^{2\,x}} - \frac{x}{a} + \frac{\ln\left(16\,a^5\,\mathrm{e}^{2\,x}+4\,a\,b^4+16\,a^5-16\,a^3\,b^2+8\,b^5\,\mathrm{e}^x-16\,a^3\,b^2\,\mathrm{e}^{2\,x}+32\,a^4\,b\,\mathrm{e}^x+4\,a\,b^4\,\mathrm{e}^{2\,x}-32\,a^2\,b^3\,\mathrm{e}^x\right)}{a} - \frac{a\,\ln\left(16\,a^5\,\mathrm{e}^{2\,x}+4\,a\,b^4+16\,a^5-16\,a^3\,b^2+8\,b^5\,\mathrm{e}^x-16\,a^3\,b^2\,\mathrm{e}^{2\,x}+32\,a^4\,b\,\mathrm{e}^x+4\,a\,b^4\,\mathrm{e}^{2\,x}-32\,a^2\,b^3\,\mathrm{e}^x\right)}{b^2} + \frac{a\,\ln\left(16\,a^6\,\mathrm{e}^{2\,x}-4\,b^6\,\mathrm{e}^{2\,x}+16\,a^6-4\,b^6+20\,a^2\,b^4-32\,a^4\,b^2+20\,a^2\,b^4\,\mathrm{e}^{2\,x}-32\,a^4\,b^2\,\mathrm{e}^{2\,x}\right)}{b^2}$$

```
[In] int(tanh(x)^3/(a + b/cosh(x)),x)
```

```
[Out] (2*exp(x))/(b + b*exp(2*x)) - x/a + log(16*a^5*exp(2*x) + 4*a*b^4 + 16*a^5
- 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4*a*b
^4*exp(2*x) - 32*a^2*b^3*exp(x))/a - (a*log(16*a^5*exp(2*x) + 4*a*b^4 + 16*
a^5 - 16*a^3*b^2 + 8*b^5*exp(x) - 16*a^3*b^2*exp(2*x) + 32*a^4*b*exp(x) + 4
*a*b^4*exp(2*x) - 32*a^2*b^3*exp(x)))/b^2 + (a*log(16*a^6*exp(2*x) - 4*b^6*
exp(2*x) + 16*a^6 - 4*b^6 + 20*a^2*b^4 - 32*a^4*b^2 + 20*a^2*b^4*exp(2*x) -
 32*a^4*b^2*exp(2*x)))/b^2
```

3.118 $\int \frac{\tanh^2(x)}{a+b\text{sech}(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)} dx = \frac{x}{a} - \frac{\arctan(\sinh(x))}{b} + \frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

```
[Out] x/a-arctan(sinh(x))/b+2*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))*(a-b)^(
1/2)*(a+b)^(1/2)/a/b
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3979, 4136, 3855, 4004, 3916, 2738, 211}

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)} dx = \frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab} + \frac{x}{a} - \frac{\arctan(\sinh(x))}{b}$$

```
[In] Int[Tanh[x]^2/(a + b*Sech[x]),x]
```

```
[Out] x/a - ArcTan[Sinh[x]]/b + (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4136

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)), x_Symbol] :> Dist[C/b, Int[Csc[e + f*x], x], x] + Dist[1/b, Int[
(A*b - a*C*Csc[e + f*x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f,
 A, C}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\int \frac{-1+\operatorname{sech}^2(x)}{a+b\operatorname{sech}(x)}\,dx \\
&= -\frac{\int \operatorname{sech}(x)\,dx}{b} - \frac{\int \frac{-b-a\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx}{b} \\
&= \frac{x}{a} - \frac{\arctan(\sinh(x))}{b} + \left(\frac{a}{b}-\frac{b}{a}\right)\int \frac{\operatorname{sech}(x)}{a+b\operatorname{sech}(x)}\,dx
\end{aligned}$$

$$= \frac{x}{a} - \frac{\arctan(\sinh(x))}{b} + \frac{\left(\frac{a}{b} - \frac{b}{a}\right) \int \frac{1}{1+\frac{a\cosh(x)}{b}} dx}{b}$$

$$= \frac{x}{a} - \frac{\arctan(\sinh(x))}{b} + \frac{\left(2\left(\frac{a}{b} - \frac{b}{a}\right)\right) \text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\left(1-\frac{a}{b}\right)x^2} dx, x, \tanh\left(\frac{x}{2}\right)\right)}{b}$$

$$= \frac{x}{a} - \frac{\arctan(\sinh(x))}{b} + \frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a + b\text{sech}(x)} dx = \frac{bx - 2a\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - 2\sqrt{a^2-b^2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{ab}$$

```
[In] Integrate[Tanh[x]^2/(a + b*Sech[x]),x]
[Out] (b*x - 2*a*ArcTan[Tanh[x/2]] - 2*Sqrt[a^2 - b^2]*ArcTan[((-a + b)*Tanh[x/2]
)/Sqrt[a^2 - b^2]])/(a*b)
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.35

method	result	size
default	$-\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{b} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a} + \frac{2(a+b)(a-b)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{ab\sqrt{(a+b)(a-b)}}$	84
risch	$\frac{x}{a} + \frac{\sqrt{-a^2+b^2}\ \ln\left(e^x + \frac{b+\sqrt{-a^2+b^2}}{a}\right)}{ba} - \frac{\sqrt{-a^2+b^2}\ \ln\left(e^x - \frac{\sqrt{-a^2+b^2}-b}{a}\right)}{ba} + \frac{i\ln(e^x-i)}{b} - \frac{i\ln(e^x+i)}{b}$	113

```
[In] int(tanh(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)
[Out] -2/b*arctan(tanh(1/2*x))-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)+2/a*(a
+b)*(a-b)/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 3.11

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx$$
$$= \left[\frac{bx - 2\,a\arctan\left(\cosh\left(x\right) + \sinh\left(x\right)\right) + \sqrt{-a^2+b^2}\log\left(\frac{a^2\cosh(x)^2 + a^2\sinh(x)^2 + 2\,ab\cosh(x) - a^2 + 2\,b^2 + 2\,\left(a^2\cosh(x) + ab\right)}{a\cosh(x)^2 + a\sinh(x)^2 + 2\,b\cosh(x) + 2\,(a\,\mathrm{co}}\right.}{ab}\right.$$

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [(b*x - 2*a*arctan(cosh(x) + sinh(x)) + sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2
 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh
(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh
(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)))/(a*b), (b*x - 2*a*ar
ctan(cosh(x) + sinh(x)) - 2*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x)
+ b)/sqrt(a^2 - b^2)))/(a*b)]
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\tanh^2\left(x\right)}{a+b\,\text{sech}\left(x\right)}\,dx$$

```
[In] integrate(tanh(x)**2/(a+b*sech(x)),x)
```

```
[Out] Integral(tanh(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^2(x)}{a+b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.84

$$\int \frac{\tanh^2(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{x}{a} - \frac{2\arctan\left(e^x\right)}{b} + \frac{2\sqrt{a^2-b^2}\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{ab}$$

```
[In] integrate(tanh(x)^2/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] x/a - 2*arctan(e^x)/b + 2*sqrt(a^2 - b^2)*arctan((a*e^x + b)/sqrt(a^2 - b^2
))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.40

$$\int \frac{\tanh^2(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{b} + \frac{\ln\left(2\,a\,b^3-2\,a^3\,b+a^3\,\sqrt{b^2-a^2}+a^4\,\mathrm{e}^x+4\,b^4\,\mathrm{e}^x-2\,a\,b^2\,\sqrt{b^2-a^2}-4\,b^3\,\mathrm{e}^x\,\sqrt{b^2-a^2}-5\,a^2\,b^2\,\mathrm{e}^x+3\,a^2\right.}{}$$

```
[In] int(tanh(x)^2/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1i)*1i - log(exp(x) + 1i)*1i)/b + (log(2*a*b^3 - 2*a^3*b + a^
3*(b^2 - a^2)^(1/2) + a^4*exp(x) + 4*b^4*exp(x) - 2*a*b^2*(b^2 - a^2)^(1/2)
 - 4*b^3*exp(x)*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) + 3*a^2*b*exp(x)*(b^2
- a^2)^(1/2))*(b^2 - a^2)^(1/2) - log(2*a*b^3 - 2*a^3*b - a^3*(b^2 - a^2)^(
1/2) + a^4*exp(x) + 4*b^4*exp(x) + 2*a*b^2*(b^2 - a^2)^(1/2) + 4*b^3*exp(x)
*(b^2 - a^2)^(1/2) - 5*a^2*b^2*exp(x) - 3*a^2*b*exp(x)*(b^2 - a^2)^(1/2))*(
b^2 - a^2)^(1/2) + b*x)/(a*b)
```

3.119 $\int \frac{\tanh(x)}{a+b\mathbf{sech}(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)} \, dx = \frac{\log(\cosh(x))}{a} + \frac{\log(a+b\text{sech}(x))}{a}$$

```
[Out] ln(cosh(x))/a+ln(a+b*sech(x))/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3970, 36, 29, 31}

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)} \, dx = \frac{\log(a+b\text{sech}(x))}{a} + \frac{\log(\cosh(x))}{a}$$

```
[In] Int[Tanh[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[a + b*Sech[x]]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{1}{x(a+x)}\,dx, x, b\text{sech}(x)\right)\\ &= -\frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, b\text{sech}(x)\right)}{a} + \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, b\text{sech}(x)\right)}{a}\\ &= \frac{\log(\cosh(x))}{a} + \frac{\log(a+b\text{sech}(x))}{a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx = \frac{\log(b+a\cosh(x))}{a}$$

```
[In] Integrate[Tanh[x]/(a + b*Sech[x]),x]
[Out] Log[b + a*Cosh[x]]/a
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-\frac{\ln(\text{sech}(x))}{a}+\frac{\ln(a+b\,\text{sech}(x))}{a}$	21
default	$-\frac{\ln(\text{sech}(x))}{a}+\frac{\ln(a+b\,\text{sech}(x))}{a}$	21
risch	$-\frac{x}{a}+\frac{\ln\left(e^{2x}+\frac{2b\,e^{x}}{a}+1\right)}{a}$	27

```
[In] int(tanh(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*ln(sech(x))+ln(a+b*sech(x))/a
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx = -\frac{x-\log\left(\frac{2\,(a\cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)}{a}$$

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="fricas")
[Out] -(x - log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(15) = 30$.

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx = \begin{cases} \frac{\tilde{\infty}}{\operatorname{sech}(x)} & \text{for}\ a=0\wedge b=0 \\ \frac{1}{b\operatorname{sech}(x)} & \text{for}\ a=0 \\ \frac{x-\log(\tanh(x)+1)}{a} & \text{for}\ b=0 \\ \frac{x}{a}+\frac{\log\left(\frac{a}{b}+\operatorname{sech}(x)\right)}{a}-\frac{\log(\tanh(x)+1)}{a} & \text{otherwise} \end{cases}$$

```
[In] integrate(tanh(x)/(a+b*sech(x)),x)
[Out] Piecewise((zoo/sech(x), Eq(a, 0) & Eq(b, 0)), (1/(b*sech(x)), Eq(a, 0)), ((
x - log(tanh(x) + 1))/a, Eq(b, 0)), (x/a + log(a/b + sech(x))/a - log(tanh(
x) + 1)/a, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{\tanh(x)}{a+b\text{sech}(x)}\,dx = \frac{x}{a}+\frac{\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a}$$

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="maxima")
[Out] x/a + log(2*b*e^(-x) + a*e^(-2*x) + a)/a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{a + b\text{sech}(x)}\,dx = \frac{\log\left(\left|a\left(e^{(-x)} + e^{x}\right) + 2\,b\right|\right)}{a}$$

```
[In] integrate(tanh(x)/(a+b*sech(x)),x, algorithm="giac")
[Out] log(abs(a*(e^(-x) + e^x) + 2*b))/a
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{\tanh(x)}{a + b\text{sech}(x)}\,dx = -\frac{x - \ln\left(a + 2\,b\,e^{x} + a\,e^{2\,x}\right)}{a}$$

```
[In] int(tanh(x)/(a + b/cosh(x)),x)
[Out] -(x - log(a + 2*b*exp(x) + a*exp(2*x)))/a
```

3.120 $\int \frac{\coth(x)}{a+b\mathbf{sech}(x)} dx$

Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx = \frac{\log(\cosh(x))}{a}+\frac{\log(1-\text{sech}(x))}{2(a+b)}+\frac{\log(1+\text{sech}(x))}{2(a-b)}-\frac{b^2\log(a+b\text{sech}(x))}{a\,(a^2-b^2)}$$

```
[Out] ln(cosh(x))/a+1/2*ln(1-sech(x))/(a+b)+1/2*ln(1+sech(x))/(a-b)-b^2*ln(a+b*se
ch(x))/a/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3970, 908}

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx = -\frac{b^2\log(a+b\text{sech}(x))}{a\,(a^2-b^2)}+\frac{\log(1-\text{sech}(x))}{2(a+b)}+\frac{\log(\text{sech}(x)+1)}{2(a-b)}+\frac{\log(\cosh(x))}{a}$$

```
[In] Int[Coth[x]/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + Log[1 - Sech[x]]/(2*(a + b)) + Log[1 + Sech[x]]/(2*(a - b)
) - (b^2*Log[a + b*Sech[x]])/(a*(a^2 - b^2))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(b^2\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)}\,dx, x, b\text{sech}(x)\right)\right)$$

$$= -\left(b^2\text{Subst}\left(\int \left(\frac{1}{2b^2(a+b)(b-x)}+\frac{1}{ab^2x}+\frac{1}{a(a-b)(a+b)(a+x)}-\frac{1}{2(a-b)b^2(b+x)}\right)\,dx, x, b\right.\right.$$

$$= \frac{\log(\cosh(x))}{a}+\frac{\log(1-\text{sech}(x))}{2(a+b)}+\frac{\log(1+\text{sech}(x))}{2(a-b)}-\frac{b^2\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{\coth(x)}{a+b\text{sech}(x)}\,dx = \frac{1}{2}\left(\frac{2\log(\cosh(x))}{a}+\frac{\log(1-\text{sech}(x))}{a+b}+\frac{\log(1+\text{sech}(x))}{a-b}-\frac{2b^2\log(a+b\text{sech}(x))}{a^3-ab^2}\right)$$

```
[In] Integrate[Coth[x]/(a + b*Sech[x]),x]
[Out] ((2*Log[Cosh[x]])/a + Log[1 - Sech[x]]/(a + b) + Log[1 + Sech[x]]/(a - b) -
 (2*b^2*Log[a + b*Sech[x]])/(a^3 - a*b^2))/2
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{b^{2} \ln \left(\tanh \left(\frac{x}{2}\right)^{2} a-\tanh \left(\frac{x}{2}\right)^{2} b+a+b\right)}{a(a+b)(a-b)}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)}{a}-\frac{\ln \left(\tanh \left(\frac{x}{2}\right)+1\right)}{a}+\frac{\ln \left(\tanh \left(\frac{x}{2}\right)\right)}{a+b}$	78
risch	$\frac{x}{a}-\frac{x}{a+b}-\frac{x}{a-b}+\frac{2 b^{2} x}{a\left(a^{2}-b^{2}\right)}+\frac{\ln \left(\mathrm{e}^{x}-1\right)}{a+b}+\frac{\ln \left(\mathrm{e}^{x}+1\right)}{a-b}-\frac{b^{2} \ln \left(\mathrm{e}^{2 x}+\frac{2 b\,\mathrm{e}^{x}}{a}+1\right)}{a\left(a^{2}-b^{2}\right)}$	103

[In] int(coth(x)/(a+b*sech(x)),x,method=_RETURNVERBOSE)

[Out] -b^2/a/(a+b)/(a-b)*ln(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)-1/a*ln(tanh(1/2*x)-1)-1/a*ln(tanh(1/2*x)+1)+1/(a+b)*ln(tanh(1/2*x))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{\coth(x)}{a+b\mathrm{sech}(x)}\,dx = -\frac{b^{2} \log \left(\frac{2\,(a \cosh(x)+b)}{\cosh(x)-\sinh(x)}\right)+(a^{2}-b^{2})x-(a^{2}+ab) \log \left(\cosh \left(x\right)+\sinh \left(x\right)+1\right)-(a^{2}-ab) \log \left(\cosh \left(x\right)+\sinh \left(x\right)-1\right)}{a^{3}-ab^{2}}$$

[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="fricas")

[Out] -(b^2*log(2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) + (a^2 - b^2)*x - (a^2 + a*b)*log(cosh(x) + sinh(x) + 1) - (a^2 - a*b)*log(cosh(x) + sinh(x) - 1))/(a^3 - a*b^2)

Sympy [F]

$$\int \frac{\coth(x)}{a+b\mathrm{sech}(x)}\,dx = \int \frac{\coth\left(x\right)}{a+b\operatorname{sech}\left(x\right)}\,dx$$

[In] integrate(coth(x)/(a+b*sech(x)),x)

[Out] Integral(coth(x)/(a + b*sech(x)), x)

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{\coth(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{b^2\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^3-ab^2}+\frac{x}{a}+\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="maxima")
[Out] -b^2*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^3 - a*b^2) + x/a + log(e^(-x) + 1)
/(a - b) + log(e^(-x) - 1)/(a + b)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{\coth(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{b^2\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^3-ab^2}+\frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)}+\frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

```
[In] integrate(coth(x)/(a+b*sech(x)),x, algorithm="giac")
[Out] -b^2*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^3 - a*b^2) + 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 271, normalized size of antiderivative = 4.11

$$\int \frac{\coth(x)}{a+b\operatorname{sech}(x)}\,dx = \frac{\ln\left(64\,a\,b^4+32\,a^4\,b+32\,b^5+96\,a^2\,b^3+64\,a^3\,b^2+32\,b^5\,\mathrm{e}^x+64\,a\,b^4\,\mathrm{e}^x+32\,a^4\,b\,\mathrm{e}^x+96\,a^2\,b^3\,\mathrm{e}^x+64\,a^3\,b^2\,\mathrm{e}\right.}{a-b} - \frac{x}{a} + \frac{\ln\left(64\,a\,b^4-32\,a^4\,b-32\,b^5-96\,a^2\,b^3+64\,a^3\,b^2+32\,b^5\,\mathrm{e}^x-64\,a\,b^4\,\mathrm{e}^x+32\,a^4\,b\,\mathrm{e}^x+96\,a^2\,b^3\,\mathrm{e}^x-64\,a^3\,b\right.}{a+b} + \frac{b^2\,\ln\left(4\,a^5\,\mathrm{e}^{2\,x}+4\,a\,b^4+4\,a^5+4\,a^3\,b^2+8\,b^5\,\mathrm{e}^x+4\,a^3\,b^2\,\mathrm{e}^{2\,x}+8\,a^4\,b\,\mathrm{e}^x+4\,a\,b^4\,\mathrm{e}^{2\,x}+8\,a^2\,b^3\,\mathrm{e}^x\right)}{a\,b^2-a^3}$$

```
[In] int(coth(x)/(a + b/cosh(x)),x)
```

```
[Out] log(64*a*b^4 + 32*a^4*b + 32*b^5 + 96*a^2*b^3 + 64*a^3*b^2 + 32*b^5*exp(x)
+ 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(x) + 64*a^3*b^2*exp(x)
)/(a - b) - x/a + log(64*a*b^4 - 32*a^4*b - 32*b^5 - 96*a^2*b^3 + 64*a^3*b^
2 + 32*b^5*exp(x) - 64*a*b^4*exp(x) + 32*a^4*b*exp(x) + 96*a^2*b^3*exp(x) -
 64*a^3*b^2*exp(x))/(a + b) + (b^2*log(4*a^5*exp(2*x) + 4*a*b^4 + 4*a^5 + 4
*a^3*b^2 + 8*b^5*exp(x) + 4*a^3*b^2*exp(2*x) + 8*a^4*b*exp(x) + 4*a*b^4*exp
(2*x) + 8*a^2*b^3*exp(x)))/(a*b^2 - a^3)
```

3.121 $\int \frac{\coth^2(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 114

$$\int \frac{\coth^2(x)}{a+b\,\mathrm{sech}(x)}\,dx=\frac{ax}{a^2-b^2}-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{2b^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\,\mathrm{csch}(x)}{a^2-b^2}$$

```
[Out] a*x/(a^2-b^2)-b^2*x/a/(a^2-b^2)+2*b^3*arctan((a-b)^(1/2)*tanh(1/2*x)/(a+b)^
(1/2))/a/(a-b)^(3/2)/(a+b)^(3/2)-a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3983, 2981, 2686, 8, 3554, 2814, 2738, 211}

$$\int \frac{\coth^2(x)}{a+b\,\mathrm{sech}(x)}\,dx=-\frac{b^2x}{a\left(a^2-b^2\right)}+\frac{ax}{a^2-b^2}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\,\mathrm{csch}(x)}{a^2-b^2}+\frac{2b^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}}$$

```
[In] Int[Coth[x]^2/(a + b*Sech[x]),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b^2*x)/(a*(a^2 - b^2)) + (2*b^3*ArcTan[(Sqrt[a - b]*Ta
nh[x/2])/Sqrt[a + b]])/(a*(a - b)^(3/2)*(a + b)^(3/2)) - (a*Coth[x])/(a^2 -
 b^2) + (b*Csch[x])/(a^2 - b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
```

```
IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$\text{integral} = \int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx$$

$$= \frac{a\int \coth^2(x)\,dx}{a^2-b^2} - \frac{b\int \coth(x)\text{csch}(x)\,dx}{a^2-b^2} - \frac{b^2\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{a^2-b^2}$$

$$= -\frac{b^2x}{a(a^2-b^2)} - \frac{a\coth(x)}{a^2-b^2} + \frac{a\int 1\,dx}{a^2-b^2} + \frac{(ib)\text{Subst}(\int 1\,dx, x, -i\text{csch}(x))}{a^2-b^2} + \frac{b^3\int \frac{1}{b+a\cosh(x)}\,dx}{a(a^2-b^2)}$$

$$= \frac{ax}{a^2-b^2} - \frac{b^2x}{a(a^2-b^2)} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\text{csch}(x)}{a^2-b^2} + \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+b-(-a+b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{a(a^2-b^2)}$$

$$= \frac{ax}{a^2-b^2} - \frac{b^2x}{a(a^2-b^2)} + \frac{2b^3\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{3/2}(a+b)^{3/2}} - \frac{a\coth(x)}{a^2-b^2} + \frac{b\text{csch}(x)}{a^2-b^2}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.71

$$\int \frac{\coth^2(x)}{a+b\text{sech}(x)}\,dx = \frac{a^2x - b^2x + \frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - a^2\coth(x) + ab\text{csch}(x)}{a^3-ab^2}$$

```
[In] Integrate[Coth[x]^2/(a + b*Sech[x]),x]
[Out] (a^2*x - b^2*x + (2*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a
^2 - b^2] - a^2*Coth[x] + a*b*Csch[x])/(a^3 - a*b^2)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)} + \frac{2b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)a(a+b)\sqrt{(a+b)(a-b)}} + \frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a} - \frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)} - \frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	104
risch	$\frac{x}{a} - \frac{2(-e^x b+a)}{(e^{2x}-1)(a^2-b^2)} - \frac{b^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)a} + \frac{b^3\ln\left(e^x+\frac{b\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,a}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)a}$	178

[In] int(coth(x)^2/(a+b*sech(x)),x,method=_RETURNVERBOSE)

[Out] -1/2/(a-b)*tanh(1/2*x)+2/(a-b)/a/(a+b)*b^3/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a*ln(tanh(1/2*x)+1)-1/2/(a+b)/tanh(1/2*x)-1/a*ln(tanh(1/2*x)-1)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(104) = 208$.

Time = 0.27 (sec) , antiderivative size = 646, normalized size of antiderivative = 5.67

$$\int \frac{\coth^2(x)}{a + b\text{sech}(x)}\,dx$$
$$= \left[\frac{2\,a^4 - 2\,a^2b^2 - (a^4 - 2\,a^2b^2 + b^4)x\cosh{(x)}^2 - (a^4 - 2\,a^2b^2 + b^4)x\sinh{(x)}^2 - \left(b^3\cosh{(x)}^2 + 2\,b^3\cosh{(x)}\right.}{}\right.$$

[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="fricas")

[Out] [(2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^2 + b^4)*x*sinh(x)^2 - (b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 - b^3)*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sinh(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) + (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(a^3*b - a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)*sinh(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2), (2*a^4 - 2*a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2 - (a^4 - 2*a^2*b^2 + b^4)*x*sinh(x)^2 + 2*(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh(x) + b^3*sinh(x)^2 - b^3)*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) + b)/sqrt(a^2 - b^2)) + (a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(a^3*b - a*b^3 + (a^4 - 2*a^2*b^2 + b^4)*x*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a*b^4 - (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)*sinh(x) - (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^2)]

Sympy [F]

$$\int \frac{\coth^2(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\coth^2(x)}{a + b\,\text{sech}\,(x)}\,dx$$

```
[In] integrate(coth(x)**2/(a+b*sech(x)),x)
[Out] Integral(coth(x)**2/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^2(x)}{a + b\text{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.72

$$\int \frac{\coth^2(x)}{a + b\text{sech}(x)}\,dx = \frac{2\,b^3\arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^3 - ab^2)\sqrt{a^2 - b^2}} + \frac{x}{a} + \frac{2\,(be^x - a)}{(a^2 - b^2)(e^{(2\,x)} - 1)}$$

```
[In] integrate(coth(x)^2/(a+b*sech(x)),x, algorithm="giac")
[Out] 2*b^3*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^3 - a*b^2)*sqrt(a^2 - b^2)) +
 x/a + 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.36

$$\int \frac{\coth^2(x)}{a+b\text{sech}(x)}\,dx = \frac{x}{a} - \frac{\frac{2a}{a^2-b^2} - \frac{2b\,e^x}{a^2-b^2}}{e^{2x}-1}$$
$$- \frac{2\,\text{atan}\left(\left(e^x\left(\frac{2b^3}{a^3\,(a\,b^2-a^3)\,(a^2-b^2)\,\sqrt{b^6}} - \frac{2\left(a\,b^3\,\sqrt{b^6}-a^3\,b\,\sqrt{b^6}\right)}{a^2\,b^2\,(a\,b^2-a^3)\,\sqrt{a^2\,(a^2-b^2)^3}\,\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-a^2\,b^6}}\right) + \frac{2\left(a^4\,\sqrt{b^6}\right.}{a^2\,b^2\,(a\,b^2-a^3)\,\sqrt{a^2\,(a^2-b^2}}\right.\right.}{\sqrt{a^8-3\,a^6\,b^2+3\,a^4\,b^4-}}$$

```
[In] int(coth(x)^2/(a + b/cosh(x)),x)
```

```
[Out] x/a - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*at
an((exp(x)*((2*b^3)/(a^3*(a*b^2 - a^3)*(a^2 - b^2)*(b^6)^(1/2)) - (2*(a*b^3
*(b^6)^(1/2) - a^3*b*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^2)^
3)^(1/2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2))) + (2*(a^4*(b^6)^(1
/2) - a^2*b^2*(b^6)^(1/2)))/(a^2*b^2*(a*b^2 - a^3)*(a^2*(a^2 - b^2)^3)^(1/2
)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2)))*((a^4*(a^8 - a^2*b^6 + 3*
a^4*b^4 - 3*a^6*b^2)^(1/2))/2 - (a^2*b^2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6
*b^2)^(1/2))/2))*(b^6)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)^(1/2)
```

3.122 $\int \frac{\coth^3(x)}{a+b\mathbf{sech}(x)} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)} \, dx = \frac{\log(\cosh(x))}{a} + \frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2} + \frac{(2a-3b)\log(1+\text{sech}(x))}{4(a-b)^2} + \frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2} - \frac{1}{4(a+b)(1-\text{sech}(x))} - \frac{1}{4(a-b)(1+\text{sech}(x))}$$

```
[Out] ln(cosh(x))/a+1/4*(2*a+3*b)*ln(1-sech(x))/(a+b)^2+1/4*(2*a-3*b)*ln(1+sech(x
))/(a-b)^2+b^4*ln(a+b*sech(x))/a/(a^2-b^2)^2-1/4/(a+b)/(1-sech(x))-1/4/(a-b
)/(1+sech(x))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3970, 908}

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)} \, dx = \frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2} - \frac{1}{4(a+b)(1-\text{sech}(x))} - \frac{1}{4(a-b)(\text{sech}(x)+1)} + \frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2} + \frac{(2a-3b)\log(\text{sech}(x)+1)}{4(a-b)^2} + \frac{\log(\cosh(x))}{a}$$

```
[In] Int[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((2*a + 3*b)*Log[1 - Sech[x]])/(4*(a + b)^2) + ((2*a - 3*b
)*Log[1 + Sech[x]])/(4*(a - b)^2) + (b^4*Log[a + b*Sech[x]])/(a*(a^2 - b^2)
^2) - 1/(4*(a + b)*(1 - Sech[x])) - 1/(4*(a - b)*(1 + Sech[x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(b^4\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^2}\,dx, x, b\text{sech}(x)\right)\right)$$

$$= -\left(b^4\text{Subst}\left(\int \left(\frac{1}{4b^3(a+b)(b-x)^2}+\frac{2a+3b}{4b^4(a+b)^2(b-x)}+\frac{1}{ab^4x}-\frac{1}{a(a-b)^2(a+b)^2(a+x)}-\frac{1}{4(a-\right.\right.\right.$$

$$= \frac{\log(\cosh(x))}{a}+\frac{(2a+3b)\log(1-\text{sech}(x))}{4(a+b)^2}+\frac{(2a-3b)\log(1+\text{sech}(x))}{4(a-b)^2}+\frac{b^4\log(a+b\text{sech}(x))}{a\left(a^2-b^2\right)^2}-\frac{1}{4(a+b)(1-\text{sech}(x))}-\frac{1}{4(a-b)(1+\text{sech}(x))}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \frac{1}{4}\left(\frac{4\log(\cosh(x))}{a}+\frac{(2a+3b)\log(1-\text{sech}(x))}{(a+b)^2}+\frac{(2a-3b)\log(1+\text{sech}(x))}{(a-b)^2}+\frac{4b^4\log(a+b\text{sech}(x))}{a(a-b)^2(a+b)^2}+\frac{1}{(a+b)(-1+\text{sech}(x))}-\frac{1}{(a-b)(1+\text{sech}(x))}\right)$$

```
[In] Integrate[Coth[x]^3/(a + b*Sech[x]),x]
```

```
[Out] ((4*Log[Cosh[x]])/a + ((2*a + 3*b)*Log[1 - Sech[x]])/(a + b)^2 + ((2*a - 3*
b)*Log[1 + Sech[x]])/(a - b)^2 + (4*b^4*Log[a + b*Sech[x]])/(a*(a - b)^2*(a
 + b)^2) + 1/((a + b)*(-1 + Sech[x])) - 1/((a - b)*(1 + Sech[x])))/4
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\tanh(\frac{x}{2})^2}{8(a-b)} - \frac{\ln(\tanh(\frac{x}{2})-1)}{a} + \frac{b^4\ln\left(\tanh(\frac{x}{2})^2 a-\tanh(\frac{x}{2})^2 b+a+b\right)}{(a-b)^2(a+b)^2 a} - \frac{\ln(\tanh(\frac{x}{2})+1)}{a} - \frac{1}{8(a+b)\tanh(\frac{x}{2})^2} + \frac{(4a+6b)\ln}{4(a}$
risch	$\frac{x}{a} - \frac{xa}{a^2-2ab+b^2} + \frac{3xb}{2(a^2-2ab+b^2)} - \frac{xa}{a^2+2ab+b^2} - \frac{3xb}{2(a^2+2ab+b^2)} - \frac{2x\,b^4}{(a^4-2a^2b^2+b^4)a} - \frac{e^x(-b\,e^{2x}+2a\,e^x-b)}{(e^{2x}-1)^2(a^2-b^2)} + \frac{a\ln(e^x}{a^2-2a}$

```
[In] int(coth(x)^3/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*tanh(1/2*x)^2/(a-b)-1/a*ln(tanh(1/2*x)-1)+1/(a-b)^2*b^4/(a+b)^2/a*ln(t
anh(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)-1/a*ln(tanh(1/2*x)+1)-1/8/(a+b)/tanh(1/
2*x)^2+1/4/(a+b)^2*(4*a+6*b)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 1222, normalized size of antiderivative = 10.81

$$\int \frac{\coth^3(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*x*s
inh(x)^4 - 2*(a^3*b - a*b^3)*cosh(x)^3 - 2*(a^3*b - a*b^3 - 4*(a^4 - 2*a^2*
b^2 + b^4)*x*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4
)*x)*cosh(x)^2 + 2*(2*a^4 - 2*a^2*b^2 + 6*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)
^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*x - 3*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 + 2
*(a^4 - 2*a^2*b^2 + b^4)*x - 2*(a^3*b - a*b^3)*cosh(x) - 2*(b^4*cosh(x)^4 +
 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh(x)^4 - 2*b^4*cosh(x)^2 + b^4 + 2*(3*b^4
*cosh(x)^2 - b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - b^4*cosh(x))*sinh(x))*log(
2*(a*cosh(x) + b)/(cosh(x) - sinh(x))) - ((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*
b^3)*cosh(x)^4 + 4*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)*sinh(x)^3
+ (2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*sinh(x)^4 + 2*a^4 + a^3*b - 4*a^2*b
^2 - 3*a*b^3 - 2*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^2 - 2*(2*a^4
 + a^3*b - 4*a^2*b^2 - 3*a*b^3 - 3*(2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*co
sh(x)^2)*sinh(x)^2 + 4*((2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x)^3 - (
2*a^4 + a^3*b - 4*a^2*b^2 - 3*a*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x
) + 1) - ((2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^4 + 4*(2*a^4 - a^3*
b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)*sinh(x)^3 + (2*a^4 - a^3*b - 4*a^2*b^2 + 3
*a*b^3)*sinh(x)^4 + 2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 - 2*(2*a^4 - a^3*b
- 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2 - 2*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3 -
```

```
3*(2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^4 -
 a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(x)^3 - (2*a^4 - a^3*b - 4*a^2*b^2 + 3*a*
b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) - 1) + 2*(4*(a^4 - 2*a^2*b^2 +
 b^4)*x*cosh(x)^3 - a^3*b + a*b^3 - 3*(a^3*b - a*b^3)*cosh(x)^2 + 4*(a^4 -
a^2*b^2 - (a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^5 - 2*a^3*b^2 + a
*b^4 + (a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^4 + 4*(a^5 - 2*a^3*b^2 + a*b^4)*co
sh(x)*sinh(x)^3 + (a^5 - 2*a^3*b^2 + a*b^4)*sinh(x)^4 - 2*(a^5 - 2*a^3*b^2
+ a*b^4)*cosh(x)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - 3*(a^5 - 2*a^3*b^2 + a*b^
4)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - 2*a^3*b^2 + a*b^4)*cosh(x)^3 - (a^5 - 2
*a^3*b^2 + a*b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \int \frac{\coth^3(x)}{a+b\operatorname{sech}(x)}\,dx$$

```
[In] integrate(coth(x)**3/(a+b*sech(x)),x)
[Out] Integral(coth(x)**3/(a + b*sech(x)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \frac{b^4\log\left(2\,be^{(-x)}+ae^{(-2\,x)}+a\right)}{a^5-2\,a^3b^2+ab^4} + \frac{(2\,a-3\,b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)} + \frac{(2\,a+3\,b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)} + \frac{be^{(-x)}-2\,ae^{(-2\,x)}+be^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}} + \frac{x}{a}$$

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="maxima")
[Out] b^4*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^5 - 2*a^3*b^2 + a*b^4) + 1/2*(2*a -
 3*b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + 3*b)*log(e^(-x) - 1)
/(a^2 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 -
2*(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x)) + x/a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.71

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \frac{b^4\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^5-2\,a^3b^2+ab^4}+\frac{(2\,a-3\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)} + \frac{(2\,a+3\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)} - \frac{a^3\left(e^{(-x)}+e^x\right)^2-2\,ab^2\left(e^{(-x)}+e^x\right)^2-2\,a^2b\left(e^{(-x)}+e^x\right)+2\,b^3\left(e^{(-x)}+e^x\right)+4\,ab^2}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

```
[In] integrate(coth(x)^3/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] b^4*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^5 - 2*a^3*b^2 + a*b^4) + 1/4*(2*a -
 3*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + 3*b)*log(e^(-x
) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a*b^2*(e^(
-x) + e^x)^2 - 2*a^2*b*(e^(-x) + e^x) + 2*b^3*(e^(-x) + e^x) + 4*a*b^2)/((a
^4 - 2*a^2*b^2 + b^4)*((e^(-x) + e^x)^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.00

$$\int \frac{\coth^3(x)}{a+b\text{sech}(x)}\,dx = \frac{\ln\left(e^x-1\right)\,(2\,a+3\,b)}{2\,a^2+4\,a\,b+2\,b^2}-\frac{x}{a}-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1} - \frac{\frac{2\,(a^4-a^2\,b^2)}{a\,(a^2-b^2)^2}-\frac{e^x\,(a^2\,b-b^3)}{(a^2-b^2)^2}}{e^{2\,x}-1}+\frac{\ln\left(e^x+1\right)\,(2\,a-3\,b)}{2\,a^2-4\,a\,b+2\,b^2} + \frac{b^4\,\ln\left(4\,a^9\,e^{2\,x}+4\,a\,b^8+4\,a^9+7\,a^3\,b^6+14\,a^5\,b^4-17\,a^7\,b^2+8\,b^9\,e^x+7\,a^3\,b^6\,e^{2\,x}+14\,a^5\,b^4\,e^{2\,x}-17\,a^7\,b\right.}{a^5-2\,a^3\,b^2+a\,b^4}$$

```
[In] int(coth(x)^3/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1)*(2*a + 3*b))/(4*a*b + 2*a^2 + 2*b^2) - x/a - ((2*a)/(a^2 -
 b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(a^4 -
a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(exp(2*
x) - 1) + (log(exp(x) + 1)*(2*a - 3*b))/(2*a^2 - 4*a*b + 2*b^2) + (b^4*log(
4*a^9*exp(2*x) + 4*a*b^8 + 4*a^9 + 7*a^3*b^6 + 14*a^5*b^4 - 17*a^7*b^2 + 8*
b^9*exp(x) + 7*a^3*b^6*exp(2*x) + 14*a^5*b^4*exp(2*x) - 17*a^7*b^2*exp(2*x)
 + 8*a^8*b*exp(x) + 4*a*b^8*exp(2*x) + 14*a^2*b^7*exp(x) + 28*a^4*b^5*exp(x
) - 34*a^6*b^3*exp(x)))/(a*b^4 + a^5 - 2*a^3*b^2)
```

3.123 $\int \frac{\coth^4(x)}{a+b\,\mathrm{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 207

$$\int \frac{\coth^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = -\frac{ab^2x}{(a^2-b^2)^2}+\frac{b^4x}{a(a^2-b^2)^2}+\frac{ax}{a^2-b^2} - \frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}}+\frac{ab^2\coth(x)}{(a^2-b^2)^2}-\frac{a\coth(x)}{a^2-b^2} - \frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{b^3\mathrm{csch}(x)}{(a^2-b^2)^2}+\frac{b\mathrm{csch}(x)}{a^2-b^2}+\frac{b\mathrm{csch}^3(x)}{3(a^2-b^2)}$$

```
[Out] -a*b^2*x/(a^2-b^2)^2+b^4*x/a/(a^2-b^2)^2+a*x/(a^2-b^2)-2*b^5*arctan((a-b)^(
1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)^(5/2)/(a+b)^(5/2)+a*b^2*coth(x)/(a^2-
b^2)^2-a*coth(x)/(a^2-b^2)-1/3*a*coth(x)^3/(a^2-b^2)-b^3*csch(x)/(a^2-b^2)^
2+b*csch(x)/(a^2-b^2)+1/3*b*csch(x)^3/(a^2-b^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3983, 2981, 2686, 3554, 8, 2814, 2738, 211}

$$\int \frac{\coth^4(x)}{a+b\,\mathrm{sech}(x)}\,dx = -\frac{ab^2x}{(a^2-b^2)^2}+\frac{ax}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}+\frac{ab^2\coth(x)}{(a^2-b^2)^2} - \frac{a\coth(x)}{a^2-b^2}+\frac{b\mathrm{csch}^3(x)}{3(a^2-b^2)}+\frac{b\mathrm{csch}(x)}{a^2-b^2}+\frac{b^4x}{a(a^2-b^2)^2} - \frac{b^3\mathrm{csch}(x)}{(a^2-b^2)^2}-\frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}}$$

```
[In] Int[Coth[x]^4/(a + b*Sech[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 - b^2)^2) + (b^4*x)/(a*(a^2 - b^2)^2) + (a*x)/(a^2 - b^2)
- (2*b^5*ArcTan[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a +
 b)^(5/2)) + (a*b^2*Coth[x])/(a^2 - b^2)^2 - (a*Coth[x])/(a^2 - b^2) - (a*C
oth[x]^3)/(3*(a^2 - b^2)) - (b^3*Csch[x])/(a^2 - b^2)^2 + (b*Csch[x])/(a^2
- b^2) + (b*Csch[x]^3)/(3*(a^2 - b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2981

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a*(d^2/(a^2
- b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 2), x], x] + (-Dist[b
*(d/(a^2 - b^2)), Int[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n - 1), x], x] -
 Dist[a^2*(d^2/(g^2*(a^2 - b^2))), Int[(g*Cos[e + f*x])^(p + 2)*((d*Sin[e +
 f*x])^(n - 2)/(a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, d, e, f, g},
x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*n, 2*p] && LtQ[p, -1] && GtQ[n, 1]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3983

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Int[Cos[c + d*x]^m*((b + a*Sin[c + d*x])^n/Sin[c + d*x]^(m
 + n)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] &&
 IntegerQ[m] && (IntegerQ[m/2] || LeQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{\cosh(x)\coth^4(x)}{b+a\cosh(x)}\,dx \\
&= \frac{a\int \coth^4(x)\,dx}{a^2-b^2}-\frac{b\int \coth^3(x)\text{csch}(x)\,dx}{a^2-b^2}-\frac{b^2\int \frac{\cosh(x)\coth^2(x)}{b+a\cosh(x)}\,dx}{a^2-b^2} \\
&= -\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{\left(ab^2\right)\int \coth^2(x)\,dx}{\left(a^2-b^2\right)^2}+\frac{b^3\int \coth(x)\text{csch}(x)\,dx}{\left(a^2-b^2\right)^2}+\frac{b^4\int \frac{\cosh(x)}{b+a\cosh(x)}\,dx}{\left(a^2-b^2\right)^2} \\
&\quad+\frac{a\int \coth^2(x)\,dx}{a^2-b^2}-\frac{(ib)\text{Subst}\left(\int \left(-1+x^2\right)\,dx,x,-i\text{csch}(x)\right)}{a^2-b^2} \\
&= \frac{b^4x}{a\left(a^2-b^2\right)^2}+\frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2}-\frac{a\coth(x)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}+\frac{b\text{csch}(x)}{a^2-b^2}+\frac{b\text{csch}^3(x)}{3\left(a^2-b^2\right)} \\
&\quad-\frac{\left(ab^2\right)\int 1\,dx}{\left(a^2-b^2\right)^2}-\frac{\left(ib^3\right)\text{Subst}\left(\int 1\,dx,x,-i\text{csch}(x)\right)}{\left(a^2-b^2\right)^2}-\frac{b^5\int \frac{1}{b+a\cosh(x)}\,dx}{a\left(a^2-b^2\right)^2}+\frac{a\int 1\,dx}{a^2-b^2} \\
&= -\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{b^4x}{a\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}+\frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2}-\frac{a\coth(x)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)} \\
&\quad-\frac{b^3\text{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\text{csch}(x)}{a^2-b^2}+\frac{b\text{csch}^3(x)}{3\left(a^2-b^2\right)}-\frac{\left(2b^5\right)\text{Subst}\left(\int \frac{1}{a+b-(-a+b)x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)}{a\left(a^2-b^2\right)^2} \\
&= -\frac{ab^2x}{\left(a^2-b^2\right)^2}+\frac{b^4x}{a\left(a^2-b^2\right)^2}+\frac{ax}{a^2-b^2}-\frac{2b^5\arctan\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}} \\
&\quad+\frac{ab^2\coth(x)}{\left(a^2-b^2\right)^2}-\frac{a\coth(x)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{b^3\text{csch}(x)}{\left(a^2-b^2\right)^2}+\frac{b\text{csch}(x)}{a^2-b^2}+\frac{b\text{csch}^3(x)}{3\left(a^2-b^2\right)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.80

$$\int \frac{\coth^4(x)}{a+b\text{sech}(x)}\,dx = \frac{(b+a\cosh(x))\text{sech}(x)\left(\frac{24x}{a}+\frac{48b^5\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a(a^2-b^2)^{5/2}}-\frac{2(8a+11b)\coth\left(\frac{x}{2}\right)}{(a+b)^2}+\frac{8\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b}-\frac{\text{csch}^4\left(\frac{x}{2}\right)\sinh}{2(a+b)}\right.}{24(a+b\text{sech}(x))}$$

[In] Integrate[Coth[x]^4/(a + b*Sech[x]),x]

[Out] ((b + a*Cosh[x])*Sech[x]*((24*x)/a + (48*b^5*ArcTan[((-a + b)*Tanh[x/2])/Sq
rt[a^2 - b^2]])/(a*(a^2 - b^2)^(5/2)) - (2*(8*a + 11*b)*Coth[x/2])/(a + b)^
2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b) - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) -
 (16*a*Tanh[x/2])/(a - b)^2 + (22*b*Tanh[x/2])/(a - b)^2))/(24*(a + b*Sech[
x]))

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.74

method	result
default	$-\frac{\frac{a\tanh\left(\frac{x}{2}\right)^3}{3}-\frac{b\tanh\left(\frac{x}{2}\right)^3}{3}+5a\tanh\left(\frac{x}{2}\right)-7b\tanh\left(\frac{x}{2}\right)}{8(a-b)^2}-\frac{2b^5\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2a\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)}{a}-$
risch	$\frac{x}{a}-\frac{2(-3a^2b\,e^{5x}+6b^3e^{5x}+6a^3e^{4x}-9a\,b^2e^{4x}+2a^2b\,e^{3x}-8b^3e^{3x}-6a^3e^{2x}+12a\,b^2e^{2x}-3a^2b\,e^{x}+6b^3e^{x}+4a^3-7a\,b^2)}{3(a^2-b^2)^2(e^{2x}-1)^3}-\frac{b^5\ln\left(e^x+\frac{b}{}\right.}{\sqrt{-a^2+b^2}}$

[In] int(coth(x)^4/(a+b*sech(x)),x,method=_RETURNVERBOSE)

[Out] -1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3+5*a*tanh(1/2*x)-7*b*t
anh(1/2*x))-2/(a-b)^2/(a+b)^2/a*b^5/((a+b)*(a-b))^(1/2)*arctan((a-b)*tanh(1
/2*x)/((a+b)*(a-b))^(1/2))-1/a*ln(tanh(1/2*x)-1)+1/a*ln(tanh(1/2*x)+1)-1/24
/(a+b)/tanh(1/2*x)^3-1/8*(5*a+7*b)/(a+b)^2/tanh(1/2*x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1725 vs. $2(193) = 386$.

Time = 0.32 (sec) , antiderivative size = 3530, normalized size of antiderivative = 17.05

$$\int \frac{\coth^4(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b
^2 + 3*a^2*b^4 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^
5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4
*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(
4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*c
osh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3
+ 2*a*b^5)*cosh(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 -
 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*
cosh(x)^3 - 15*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*
b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)
^3 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b
^6)*x)*cosh(x)^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3
 - 6*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
)*x)*cosh(x)^2 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cosh(x))*sinh(x)^2 - 3*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(
x)^5 + b^5*sinh(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*c
osh(x)^2 - b^5)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 +
 3*(5*b^5*cosh(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 -
 2*b^5*cosh(x)^3 + b^5*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*log((a^2*cosh(x)^
2 + a^2*sinh(x)^2 + 2*a*b*cosh(x) - a^2 + 2*b^2 + 2*(a^2*cosh(x) + a*b)*sin
h(x) + 2*sqrt(-a^2 + b^2)*(a*cosh(x) + a*sinh(x) + b))/(a*cosh(x)^2 + a*sin
h(x)^2 + 2*b*cosh(x) + 2*(a*cosh(x) + b)*sinh(x) + a)) - 3*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 +
 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*
b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3
*b^3 + 4*a*b^5)*cosh(x)^2 + (4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^
4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^6 - 6*(a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*sinh(x)^5 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
 a*b^6)*sinh(x)^6 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 + 3*(
a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6)*cosh(x)^2)*sinh(x)^4 - 4*(5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x
```

```
)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)^3 - 3*(a^7 -
 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6 + 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 - 6*(a^7 - 3*a^
5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^2 - 6*((a^7 - 3*a^5*b^2 + 3*a
^3*b^4 - a*b^6)*cosh(x)^5 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)
^3 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x))*sinh(x)), -1/3*(3*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*x*sinh(x)^6 - 8*a^6 + 22*a^4*b^2 - 14*a^2*b^4 + 6*(a^5*b - 3*a^3*b^
3 + 2*a*b^5)*cosh(x)^5 + 6*(a^5*b - 3*a^3*b^3 + 2*a*b^5 + 3*(a^6 - 3*a^4*b^
2 + 3*a^2*b^4 - b^6)*x*cosh(x))*sinh(x)^5 - 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b
^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^4 - 3*(4*a^6 - 10*a^4
*b^2 + 6*a^2*b^4 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^2 + 3*(
a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 10*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos
h(x))*sinh(x)^4 - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^3 - 4*(a^5*b - 5*
a^3*b^3 + 4*a*b^5 - 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh(x)^3 - 15
*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^2 + 3*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^
4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x))*sinh(x)^3 + 3*(4*a^6
- 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)
^2 + 3*(4*a^6 - 12*a^4*b^2 + 8*a^2*b^4 + 15*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
b^6)*x*cosh(x)^4 + 20*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*cosh(x)^3 - 6*(4*a^6 -
10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x)^2
 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x - 4*(a^5*b - 5*a^3*b^3 + 4*a*b^5
)*cosh(x))*sinh(x)^2 + 6*(b^5*cosh(x)^6 + 6*b^5*cosh(x)*sinh(x)^5 + b^5*sin
h(x)^6 - 3*b^5*cosh(x)^4 + 3*b^5*cosh(x)^2 - b^5 + 3*(5*b^5*cosh(x)^2 - b^5
)*sinh(x)^4 + 4*(5*b^5*cosh(x)^3 - 3*b^5*cosh(x))*sinh(x)^3 + 3*(5*b^5*cosh
(x)^4 - 6*b^5*cosh(x)^2 + b^5)*sinh(x)^2 + 6*(b^5*cosh(x)^5 - 2*b^5*cosh(x)
^3 + b^5*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*arctan(-(a*cosh(x) + a*sinh(x) +
 b)/sqrt(a^2 - b^2)) - 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x + 6*(a^5*b -
 3*a^3*b^3 + 2*a*b^5)*cosh(x) + 6*(3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*
cosh(x)^5 + a^5*b - 3*a^3*b^3 + 2*a*b^5 + 5*(a^5*b - 3*a^3*b^3 + 2*a*b^5)*c
osh(x)^4 - 2*(4*a^6 - 10*a^4*b^2 + 6*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6)*x)*cosh(x)^3 - 2*(a^5*b - 5*a^3*b^3 + 4*a*b^5)*cosh(x)^2 + (4*a^6
 - 12*a^4*b^2 + 8*a^2*b^4 + 3*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x)*cosh(x
))*sinh(x))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*cosh(x)^6 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)*s
inh(x)^5 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sinh(x)^6 + 3*(a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^4 + 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6 - 5*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^2)*sinh(x)^4 - 4*(5*(a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*cosh(x))*sinh(x)^3 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*co
sh(x)^2 - 3*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + 5*(a^7 - 3*a^5*b^2 + 3*a
^3*b^4 - a*b^6)*cosh(x)^4 - 6*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)
^2)*sinh(x)^2 - 6*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^5 - 2*(a^7
 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cosh(x)^3 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4
- a*b^6)*cosh(x))*sinh(x))]
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a + b\mathrm{sech}(x)}\,dx = \int \frac{\coth^4{(x)}}{a + b\,\mathrm{sech}\,(x)}\,dx$$

```
[In] integrate(coth(x)**4/(a+b*sech(x)),x)
[Out] Integral(coth(x)**4/(a + b*sech(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^4(x)}{a + b\mathrm{sech}(x)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.92

$$\int \frac{\coth^4(x)}{a + b\mathrm{sech}(x)}\,dx = -\frac{2\,b^5 \arctan\left(\frac{ae^x+b}{\sqrt{a^2-b^2}}\right)}{(a^5 - 2\,a^3b^2 + ab^4)\sqrt{a^2 - b^2}} + \frac{x}{a} + \frac{2\left(3\,a^2be^{(5\,x)} - 6\,b^3e^{(5\,x)} - 6\,a^3e^{(4\,x)} + 9\,ab^2e^{(4\,x)} - 2\,a^2be^{(3\,x)} + 8\,b^3e^{(3\,x)} + 6\,a^3e^{(2\,x)} - 12\,ab^2e^{(2\,x)} + 3\,a^2be\right.}{3\,(a^4 - 2\,a^2b^2 + b^4)(e^{(2\,x)} - 1)^3}$$

```
[In] integrate(coth(x)^4/(a+b*sech(x)),x, algorithm="giac")
[Out] -2*b^5*arctan((a*e^x + b)/sqrt(a^2 - b^2))/((a^5 - 2*a^3*b^2 + a*b^4)*sqrt(
a^2 - b^2)) + x/a + 2/3*(3*a^2*b*e^(5*x) - 6*b^3*e^(5*x) - 6*a^3*e^(4*x) +
9*a*b^2*e^(4*x) - 2*a^2*b*e^(3*x) + 8*b^3*e^(3*x) + 6*a^3*e^(2*x) - 12*a*b^
2*e^(2*x) + 3*a^2*b*e^x - 6*b^3*e^x - 4*a^3 + 7*a*b^2)/((a^4 - 2*a^2*b^2 +
b^4)*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 713, normalized size of antiderivative = 3.44

$$\int \frac{\coth^4(x)}{a+b\text{sech}(x)}\,dx$$
$$= \frac{x}{a} - \frac{\frac{8\,a}{3\,(a^2-b^2)} - \frac{8\,b\,e^x}{3\,(a^2-b^2)}}{3\,e^{2\,x} - 3\,e^{4\,x} + e^{6\,x} - 1} - \frac{\frac{2\,(2\,a^4-3\,a^2\,b^2)}{a\,(a^2-b^2)^2} - \frac{2\,e^x\,(a^2\,b-2\,b^3)}{(a^2-b^2)^2}}{e^{2\,x}-1} - \frac{\frac{4\,(a^4-a^2\,b^2)}{a\,(a^2-b^2)^2} - \frac{8\,e^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{e^{4\,x} - 2\,e^{2\,x} + 1}$$
$$- \frac{2\,\text{atan}\left(\left(\left(e^x\left(\frac{2\,b^5}{a^3\,(a^2-b^2)^2\,\sqrt{b^{10}}\,(a^5-2\,a^3\,b^2+a\,b^4)} + \frac{2\left(a\,b^5\,\sqrt{b^{10}} - 2\,a^3\,b^3\,\sqrt{b^{10}} + a^5\,b\,\sqrt{b^{10}}\right)}{a^2\,b^4\,\sqrt{a^2\,(a^2-b^2)^5}\,(a^5-2\,a^3\,b^2+a\,b^4)\,\sqrt{a^{12}-5\,a^{10}\,b^2+10\,a^8\,b^4-10\,a^6\,b^6+5\,a^4\,b^8}}\right.\right.\right.\right.}{}$$

```
[In] int(coth(x)^4/(a + b/cosh(x)),x)
```

```
[Out] x/a - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(3*(a^2 - b^2)))/(3*exp(2*x) -
3*exp(4*x) + exp(6*x) - 1) - ((2*(2*a^4 - 3*a^2*b^2))/(a*(a^2 - b^2)^2) - (
2*exp(x)*(a^2*b - 2*b^3))/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((4*(a^4 - a^2*b^
2))/(a*(a^2 - b^2)^2) - (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)^2))/(exp(4*
x) - 2*exp(2*x) + 1) - (2*atan((exp(x)*((2*b^5)/(a^3*(a^2 - b^2)^2*(b^10)^(
1/2)*(a*b^4 + a^5 - 2*a^3*b^2)) + (2*(a*b^5*(b^10)^(1/2) - 2*a^3*b^3*(b^10)
^(1/2) + a^5*b*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*b^4 + a
^5 - 2*a^3*b^2)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*
a^10*b^2)^(1/2))) + (2*(a^6*(b^10)^(1/2) + a^2*b^4*(b^10)^(1/2) - 2*a^4*b^2
*(b^10)^(1/2)))/(a^2*b^4*(a^2*(a^2 - b^2)^5)^(1/2)*(a*b^4 + a^5 - 2*a^3*b^2
)*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2
)))*((a^6*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b
^2)^(1/2))/2 + (a^2*b^4*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*b^6 + 10*a^8*
b^4 - 5*a^10*b^2)^(1/2))/2 - a^4*b^2*(a^12 - a^2*b^10 + 5*a^4*b^8 - 10*a^6*
b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)))*(b^10)^(1/2))/(a^12 - a^2*b^10 + 5*a
^4*b^8 - 10*a^6*b^6 + 10*a^8*b^4 - 5*a^10*b^2)^(1/2)
```

3.124 $\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 178

$$\begin{aligned}\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx &= \frac{\log(\cosh(x))}{a}+\frac{(8a^2+21ab+15b^2)\log(1-\text{sech}(x))}{16(a+b)^3}\\ &\quad+\frac{(8a^2-21ab+15b^2)\log(1+\text{sech}(x))}{16(a-b)^3}-\frac{b^6\log(a+b\text{sech}(x))}{a\,(a^2-b^2)^3}\\ &\quad-\frac{1}{16(a+b)(1-\text{sech}(x))^2}-\frac{5a+7b}{16(a+b)^2(1-\text{sech}(x))}\\ &\quad-\frac{1}{16(a-b)(1+\text{sech}(x))^2}-\frac{5a-7b}{16(a-b)^2(1+\text{sech}(x))}\end{aligned}$$

```
[Out] ln(cosh(x))/a+1/16*(8*a^2+21*a*b+15*b^2)*ln(1-sech(x))/(a+b)^3+1/16*(8*a^2-
21*a*b+15*b^2)*ln(1+sech(x))/(a-b)^3-b^6*ln(a+b*sech(x))/a/(a^2-b^2)^3-1/16
/(a+b)/(1-sech(x))^2+1/16*(-5*a-7*b)/(a+b)^2/(1-sech(x))-1/16/(a-b)/(1+sech
(x))^2+1/16*(-5*a+7*b)/(a-b)^2/(1+sech(x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

$= \{3970, 908\}$

$$\int \frac{\coth^5(x)}{a+b\text{sech}(x)}\,dx = \frac{(8a^2+21ab+15b^2)\log(1-\text{sech}(x))}{16(a+b)^3} + \frac{(8a^2-21ab+15b^2)\log(\text{sech}(x)+1)}{16(a-b)^3} - \frac{b^6\log(a+b\text{sech}(x))}{a\,(a^2-b^2)^3} - \frac{5a+7b}{16(a+b)^2(1-\text{sech}(x))} - \frac{5a-7b}{16(a-b)^2(\text{sech}(x)+1)} - \frac{1}{16(a+b)(1-\text{sech}(x))^2} - \frac{1}{16(a-b)(\text{sech}(x)+1)^2} + \frac{\log(\cosh(x))}{a}$$

```
[In] Int[Coth[x]^5/(a + b*Sech[x]),x]
```

```
[Out] Log[Cosh[x]]/a + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sech[x]])/(16*(a + b)^3
) + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sech[x]])/(16*(a - b)^3) - (b^6*Log[
a + b*Sech[x]])/(a*(a^2 - b^2)^3) - 1/(16*(a + b)*(1 - Sech[x])^2) - (5*a +
 7*b)/(16*(a + b)^2*(1 - Sech[x])) - 1/(16*(a - b)*(1 + Sech[x])^2) - (5*a
- 7*b)/(16*(a - b)^2*(1 + Sech[x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\left(b^6\text{Subst}\left(\int \frac{1}{x(a+x)\left(b^2-x^2\right)^3}\,dx, x, b\text{sech}(x)\right)\right)$$

$$= -\left(b^6\text{Subst}\left(\int \left(\frac{1}{8b^4(a+b)(b-x)^3}+\frac{5a+7b}{16b^5(a+b)^2(b-x)^2}+\frac{8a^2+21ab+15b^2}{16b^6(a+b)^3(b-x)}+\frac{1}{ab^6x}+\frac{}{a(a-}\right.\right.\right.$$

$$= \frac{\log(\cosh(x))}{a} + \frac{(8a^2 + 21ab + 15b^2)\log(1 - \operatorname{sech}(x))}{16(a + b)^3} + \frac{(8a^2 - 21ab + 15b^2)\log(1 + \operatorname{sech}(x))}{16(a - b)^3} - \frac{b^6 \log(a + b\operatorname{sech}(x))}{a\left(a^2 - b^2\right)^3} - \frac{1}{16(a + b)(1 - \operatorname{sech}(x))^2} - \frac{5a + 7b}{16(a + b)^2(1 - \operatorname{sech}(x))} - \frac{1}{16(a - b)(1 + \operatorname{sech}(x))^2} - \frac{5a - 7b}{16(a - b)^2(1 + \operatorname{sech}(x))}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{\coth^5(x)}{a + b\operatorname{sech}(x)}\,dx = \frac{1}{16}\left(\frac{16\log(\cosh(x))}{a} + \frac{(8a^2 + 21ab + 15b^2)\log(1 - \operatorname{sech}(x))}{(a + b)^3} + \frac{(8a^2 - 21ab + 15b^2)\log(1 + \operatorname{sech}(x))}{(a - b)^3} - \frac{16b^6\log(a + b\operatorname{sech}(x))}{a(a - b)^3(a + b)^3} - \frac{1}{(a + b)(-1 + \operatorname{sech}(x))^2} + \frac{5a + 7b}{(a + b)^2(-1 + \operatorname{sech}(x))} - \frac{1}{(a - b)(1 + \operatorname{sech}(x))^2} + \frac{-5a + 7b}{(a - b)^2(1 + \operatorname{sech}(x))}\right)$$

```
[In] Integrate[Coth[x]^5/(a + b*Sech[x]),x]
```

```
[Out] ((16*Log[Cosh[x]])/a + ((8*a^2 + 21*a*b + 15*b^2)*Log[1 - Sech[x]])/(a + b)
^3 + ((8*a^2 - 21*a*b + 15*b^2)*Log[1 + Sech[x]])/(a - b)^3 - (16*b^6*Log[a
 + b*Sech[x]])/(a*(a - b)^3*(a + b)^3) - 1/((a + b)*(-1 + Sech[x])^2) + (5*
a + 7*b)/((a + b)^2*(-1 + Sech[x])) - 1/((a - b)*(1 + Sech[x])^2) + (-5*a +
 7*b)/((a - b)^2*(1 + Sech[x])))/16
```

Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.91

method	result
default	$-\frac{\left(\tanh\left(\frac{x}{2}\right)^2 a-\tanh\left(\frac{x}{2}\right)^2 b+6a-8b\right)^2}{64(a-b)^3}-\frac{1}{64(a+b)\tanh\left(\frac{x}{2}\right)^4}-\frac{6a+8b}{32(a+b)^2\tanh\left(\frac{x}{2}\right)^2}+\frac{(16a^2+42ab+30b^2)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{16(a+b)^3}-\frac{\ln(\mathrm{ta}}{}$
risch	$\frac{x}{a}-\frac{x\,a^2}{a^3+3a^2b+3a\,b^2+b^3}-\frac{21xab}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{15x\,b^2}{8(a^3+3a^2b+3a\,b^2+b^3)}-\frac{x\,a^2}{a^3-3a^2b+3a\,b^2-b^3}+\frac{21xab}{8(a^3-3a^2b+3a\,b^2-b^3)}$

```
[In] int(coth(x)^5/(a+b*sech(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(tanh(1/2*x)^2*a-tanh(1/2*x)^2*b+6*a-8*b)^2/(a-b)^3-1/64/(a+b)/tanh(1
/2*x)^4-1/32*(6*a+8*b)/(a+b)^2/tanh(1/2*x)^2+1/16/(a+b)^3*(16*a^2+42*a*b+30
*b^2)*ln(tanh(1/2*x))-1/a*ln(tanh(1/2*x)+1)-1/(a-b)^3*b^6/(a+b)^3/a*ln(tanh
(1/2*x)^2*a-tanh(1/2*x)^2*b+a+b)-1/a*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5181 vs. $2(162) = 324$.

Time = 0.37 (sec) , antiderivative size = 5181, normalized size of antiderivative = 29.11

$$\int \frac{\coth^5(x)}{a + b\text{sech}(x)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^5(x)}{a + b\text{sech}(x)}\,dx = \int \frac{\coth^5(x)}{a + b\operatorname{sech}(x)}\,dx$$

```
[In] integrate(coth(x)**5/(a+b*sech(x)),x)
[Out] Integral(coth(x)**5/(a + b*sech(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(162) = 324$.

Time = 0.22 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.06

$$\int \frac{\coth^5(x)}{a + b\text{sech}(x)}\,dx = -\frac{b^6 \log\left(2\,be^{(-x)} + ae^{(-2\,x)} + a\right)}{a^7 - 3\,a^5b^2 + 3\,a^3b^4 - ab^6} + \frac{(8\,a^2 - 21\,ab + 15\,b^2)\log\left(e^{(-x)} + 1\right)}{8\,(a^3 - 3\,a^2b + 3\,ab^2 - b^3)} + \frac{(8\,a^2 + 21\,ab + 15\,b^2)\log\left(e^{(-x)} - 1\right)}{8\,(a^3 + 3\,a^2b + 3\,ab^2 + b^3)} + \frac{(5\,a^2b - 9\,b^3)e^{(-x)} - 8\,(2\,a^3 - 3\,ab^2)e^{(-2\,x)} + (3\,a^2b + b^3)e^{(-3\,x)} + 16\,(a^3 - 2\,ab^2)e^{(-4\,x)} + (3\,a^2b + b^3)e^{(-}}{4\,(a^4 - 2\,a^2b^2 + b^4 - 4\,(a^4 - 2\,a^2b^2 + b^4)e^{(-2\,x)} + 6\,(a^4 - 2\,a^2b^2 + b^4)e^{(-4\,x)} - 4\,(a^4 - 2\,a^2b^2 -} + \frac{x}{a}$$

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="maxima")
```

```
[Out] -b^6*log(2*b*e^(-x) + a*e^(-2*x) + a)/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/8*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3) + 1/8*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*
b^2 + b^3) + 1/4*((5*a^2*b - 9*b^3)*e^(-x) - 8*(2*a^3 - 3*a*b^2)*e^(-2*x) +
 (3*a^2*b + b^3)*e^(-3*x) + 16*(a^3 - 2*a*b^2)*e^(-4*x) + (3*a^2*b + b^3)*e
^(-5*x) - 8*(2*a^3 - 3*a*b^2)*e^(-6*x) + (5*a^2*b - 9*b^3)*e^(-7*x))/(a^4 -
 2*a^2*b^2 + b^4 - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2
+ b^4)*e^(-4*x) - 4*(a^4 - 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b
^4)*e^(-8*x)) + x/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(162) = 324$.

Time = 0.29 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.13

$$\int \frac{\coth^5(x)}{a+b\operatorname{sech}(x)}\,dx = -\frac{b^6\log\left(\left|a\left(e^{(-x)}+e^x\right)+2\,b\right|\right)}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6} + \frac{\left(8\,a^2-21\,ab+15\,b^2\right)\log\left(e^{(-x)}+e^x+2\right)}{16\left(a^3-3\,a^2b+3\,ab^2-b^3\right)} + \frac{\left(8\,a^2+21\,ab+15\,b^2\right)\log\left(e^{(-x)}+e^x-2\right)}{16\left(a^3+3\,a^2b+3\,ab^2+b^3\right)} - \frac{3\,a^5\left(e^{(-x)}+e^x\right)^4-9\,a^3b^2\left(e^{(-x)}+e^x\right)^4+9\,ab^4\left(e^{(-x)}+e^x\right)^4-5\,a^4b\left(e^{(-x)}+e^x\right)^3+14\,a^2b^3\left(e^{(-x)}+e^x\right)^3 -}{}$$

```
[In] integrate(coth(x)^5/(a+b*sech(x)),x, algorithm="giac")
```

```
[Out] -b^6*log(abs(a*(e^(-x) + e^x) + 2*b))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)
 + 1/16*(8*a^2 - 21*a*b + 15*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*
a*b^2 - b^3) + 1/16*(8*a^2 + 21*a*b + 15*b^2)*log(e^(-x) + e^x - 2)/(a^3 +
3*a^2*b + 3*a*b^2 + b^3) - 1/4*(3*a^5*(e^(-x) + e^x)^4 - 9*a^3*b^2*(e^(-x)
+ e^x)^4 + 9*a*b^4*(e^(-x) + e^x)^4 - 5*a^4*b*(e^(-x) + e^x)^3 + 14*a^2*b^3
*(e^(-x) + e^x)^3 - 9*b^5*(e^(-x) + e^x)^3 - 8*a^5*(e^(-x) + e^x)^2 + 32*a^
3*b^2*(e^(-x) + e^x)^2 - 48*a*b^4*(e^(-x) + e^x)^2 + 12*a^4*b*(e^(-x) + e^x
) - 40*a^2*b^3*(e^(-x) + e^x) + 28*b^5*(e^(-x) + e^x) - 16*a^3*b^2 + 64*a*b
^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.50

$$\int \frac{\coth^5(x)}{a+b\mathrm{sech}(x)}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)\,\left(8\,a^2+21\,a\,b+15\,b^2\right)}{8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3} - \frac{\frac{2\,\left(4\,a^4-5\,a^2\,b^2\right)}{a\,{\left(a^2-b^2\right)}^2}-\frac{\mathrm{e}^x\,\left(9\,a^2\,b-13\,b^3\right)}{2\,{\left(a^2-b^2\right)}^2}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1} - \frac{\frac{2\,\left(2\,a^6-5\,a^4\,b^2+3\,a^2\,b^4\right)}{a\,{\left(a^2-b^2\right)}^3}-\frac{\mathrm{e}^x\,\left(5\,a^4\,b-14\,a^2\,b^3+9\,b^5\right)}{4\,{\left(a^2-b^2\right)}^3}}{\mathrm{e}^{2\,x}-1} - \frac{\frac{8\,\left(a^4-a^2\,b^2\right)}{a\,{\left(a^2-b^2\right)}^2}-\frac{6\,\mathrm{e}^x\,\left(a^2\,b-b^3\right)}{{\left(a^2-b^2\right)}^2}}{3\,\mathrm{e}^{2\,x}-3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}-1} - \frac{x}{a} - \frac{\frac{4\,a}{a^2-b^2}-\frac{4\,b\,\mathrm{e}^x}{a^2-b^2}}{6\,\mathrm{e}^{4\,x}-4\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{6\,x}+\mathrm{e}^{8\,x}+1} + \frac{\ln\left(\mathrm{e}^x+1\right)\,\left(8\,a^2-21\,a\,b+15\,b^2\right)}{8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3} + \frac{b^6\,\ln\left(64\,a^{13}\,\mathrm{e}^{2\,x}+64\,a\,b^{12}+64\,a^{13}+159\,a^3\,b^{10}+492\,a^5\,b^8-1214\,a^7\,b^6+1020\,a^9\,b^4-393\,a^{11}\,b^2+128\,b\right.}{}$$

```
[In] int(coth(x)^5/(a + b/cosh(x)),x)
```

```
[Out] (log(exp(x) - 1)*(21*a*b + 8*a^2 + 15*b^2))/(24*a*b^2 + 24*a^2*b + 8*a^3 +
8*b^3) - ((2*(4*a^4 - 5*a^2*b^2))/(a*(a^2 - b^2)^2) - (exp(x)*(9*a^2*b - 13
*b^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*(2*a^6 + 3*a^2*
b^4 - 5*a^4*b^2))/(a*(a^2 - b^2)^3) - (exp(x)*(5*a^4*b + 9*b^5 - 14*a^2*b^3
))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) - ((8*(a^4 - a^2*b^2))/(a*(a^2 - b^2)^
2) - (6*exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(3*exp(2*x) - 3*exp(4*x) + exp
(6*x) - 1) - x/a - ((4*a)/(a^2 - b^2) - (4*b*exp(x))/(a^2 - b^2))/(6*exp(4*
x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) + (log(exp(x) + 1)*(8*a^2 - 21
*a*b + 15*b^2))/(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) + (b^6*log(64*a^13*ex
p(2*x) + 64*a*b^12 + 64*a^13 + 159*a^3*b^10 + 492*a^5*b^8 - 1214*a^7*b^6 +
1020*a^9*b^4 - 393*a^11*b^2 + 128*b^13*exp(x) + 159*a^3*b^10*exp(2*x) + 492
*a^5*b^8*exp(2*x) - 1214*a^7*b^6*exp(2*x) + 1020*a^9*b^4*exp(2*x) - 393*a^1
1*b^2*exp(2*x) + 128*a^12*b*exp(x) + 64*a*b^12*exp(2*x) + 318*a^2*b^11*exp(
x) + 984*a^4*b^9*exp(x) - 2428*a^6*b^7*exp(x) + 2040*a^8*b^5*exp(x) - 786*a
^10*b^3*exp(x)))/(a*b^6 - a^7 - 3*a^3*b^4 + 3*a^5*b^2)
```

3.125 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} + \frac{2a(a^2-2b^2)\,(a+b\text{sech}(c+dx))^{3/2}}{3b^4d} - \frac{2(3a^2-2b^2)\,(a+b\text{sech}(c+dx))^{5/2}}{5b^4d} + \frac{6a(a+b\text{sech}(c+dx))^{7/2}}{7b^4d} - \frac{2(a+b\text{sech}(c+dx))^{9/2}}{9b^4d}$$

```
[Out] 2/3*a*(a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d-2/5*(3*a^2-2*b^2)*(a+b*sech
(d*x+c))^(5/2)/b^4/d+6/7*a*(a+b*sech(d*x+c))^(7/2)/b^4/d-2/9*(a+b*sech(d*x+
c))^(9/2)/b^4/d+2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b
*sech(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1275, 213}

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx = -\frac{2(3a^2-2b^2)\,(a+b\text{sech}(c+dx))^{5/2}}{5b^4d} + \frac{2a(a^2-2b^2)\,(a+b\text{sech}(c+dx))^{3/2}}{3b^4d} + \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2(a+b\text{sech}(c+dx))^{9/2}}{9b^4d} + \frac{6a(a+b\text{sech}(c+dx))^{7/2}}{7b^4d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d + (2*a*(a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(3/2))/(3*b^4*d)
 - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) + (6*a*(a + b*
Sech[c + d*x])^(7/2))/(7*b^4*d) - (2*(a + b*Sech[c + d*x])^(9/2))/(9*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
```

```
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}(b^2-x^2)^2}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^4d} \\
&= -\frac{2\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)^2}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4d} \\
&= -\frac{2\text{Subst}\left(\int \left(b^4-a(a^2-2b^2)x^2+(3a^2-2b^2)x^4-3ax^6+x^8+\frac{ab^4}{-a+x^2}\right)dx, x, \sqrt{a+b\text{sech}(c+dx}}{b^4d} \\
&= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}+\frac{2a(a^2-2b^2)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d} \\
&\quad -\frac{2(3a^2-2b^2)(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}+\frac{6a(a+b\text{sech}(c+dx))^{7/2}}{7b^4d} \\
&\quad -\frac{2(a+b\text{sech}(c+dx))^{9/2}}{9b^4d}-\frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} \\
&= \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} \\
&\quad +\frac{2a(a^2-2b^2)(a+b\text{sech}(c+dx))^{3/2}}{3b^4d}-\frac{2(3a^2-2b^2)(a+b\text{sech}(c+dx))^{5/2}}{5b^4d} \\
&\quad +\frac{6a(a+b\text{sech}(c+dx))^{7/2}}{7b^4d}-\frac{2(a+b\text{sech}(c+dx))^{9/2}}{9b^4d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.91

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx =$$
$$-\frac{-2\sqrt{a}b^4\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+2b^4\sqrt{a+b\text{sech}(c+dx)}-\frac{2}{3}a(a^2-2b^2)\left(a+b\text{sech}(c+dx)\right)^{3/2}+\frac{2}{5}\cdots}{b^4d}$$

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^5,x]
```

```
[Out] -((-2*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + 2*b^4*Sqrt[a
 + b*Sech[c + d*x]] - (2*a*(a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(3/2))/3 + (
2*(3*a^2 - 2*b^2)*(a + b*Sech[c + d*x])^(5/2))/5 - (6*a*(a + b*Sech[c + d*x
])^(7/2))/7 + (2*(a + b*Sech[c + d*x])^(9/2))/9)/(b^4*d))
```

Maple [F]

$$\int \sqrt{a+b\,\text{sech}\,(dx+c)}\,\tanh\,(dx+c)^5\,dx$$

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

```
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. 2(145) = 290.

Time = 0.68 (sec) , antiderivative size = 4363, normalized size of antiderivative = 25.82

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^5(c+dx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^
4*sinh(d*x + c)^8 + 4*b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4
*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*
x + c)^4 + 30*b^4*cosh(d*x + c)^2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4
*cosh(d*x + c)^5 + 10*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(7*b^4*cosh(d*x + c)^6 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x
+ c)^2 + b^4)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c
)^5 + 3*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log
(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 +
```

4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^
2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) +
 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x +
 c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*c
osh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x +
 c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (
4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*((16*a^4 - 84*a^2*b^2 - 315*b^4)*c
osh(d*x + c)^8 + (16*a^4 - 84*a^2*b^2 - 315*b^4)*sinh(d*x + c)^8 - 4*(4*a^3
*b - 21*a*b^3)*cosh(d*x + c)^7 - 4*(4*a^3*b - 21*a*b^3 - 2*(16*a^4 - 84*a^2
*b^2 - 315*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(16*a^4 - 78*a^2*b^2 - 1
89*b^4)*cosh(d*x + c)^6 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4 + 7*(16*a^4 - 84
*a^2*b^2 - 315*b^4)*cosh(d*x + c)^2 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c))
*sinh(d*x + c)^6 - 4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^5 - 4*(12*a^3*b -
53*a*b^3 - 14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^3 + 21*(4*a^3*b
 - 21*a*b^3)*cosh(d*x + c)^2 - 6*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^4 +
 2*(35*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^4 + 48*a^4 - 228*a^2*b
^2 - 721*b^4 - 70*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^3 + 30*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c)^2 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*
sinh(d*x + c)^4 + 16*a^4 - 84*a^2*b^2 - 315*b^4 - 4*(12*a^3*b - 53*a*b^3)*c
osh(d*x + c)^3 + 4*(14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^5 - 35
*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 53*a*b^3 + 20*(16*a^4 -
78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^3 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x +
 c)^2 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^2 + 4*(7*(16*a^4 - 84*a^2*
b^2 - 315*b^4)*cosh(d*x + c)^6 - 21*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^5 +
15*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^4 + 16*a^4 - 78*a^2*b^2 -
189*b^4 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^3 + 3*(48*a^4 - 228*a^2*b^
2 - 721*b^4)*cosh(d*x + c)^2 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(
d*x + c)^2 - 4*(4*a^3*b - 21*a*b^3)*cosh(d*x + c) + 4*(2*(16*a^4 - 84*a^2*b
^2 - 315*b^4)*cosh(d*x + c)^7 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^6 + 6*
(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c)^5 - 5*(12*a^3*b - 53*a*b^3)*c
osh(d*x + c)^4 - 4*a^3*b + 21*a*b^3 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*co
sh(d*x + c)^3 - 3*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(16*a^4 - 78*a^
2*b^2 - 189*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c)))/(b^4*d*cosh(d*x + c)^8 + 8*b^4*d*cosh(d*x + c)*sinh(d*x + c)
^7 + b^4*d*sinh(d*x + c)^8 + 4*b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c
)^4 + 4*b^4*d*cosh(d*x + c)^2 + 4*(7*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*
x + c)^6 + 8*(7*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c
)^5 + b^4*d + 2*(35*b^4*d*cosh(d*x + c)^4 + 30*b^4*d*cosh(d*x + c)^2 + 3*b^
4*d)*sinh(d*x + c)^4 + 8*(7*b^4*d*cosh(d*x + c)^5 + 10*b^4*d*cosh(d*x + c)^
3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*d*cosh(d*x + c)^6 + 1

5*b^4*d*cosh(d*x + c)^4 + 9*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2
+ 8*(b^4*d*cosh(d*x + c)^7 + 3*b^4*d*cosh(d*x + c)^5 + 3*b^4*d*cosh(d*x + c
)^3 + b^4*d*cosh(d*x + c))*sinh(d*x + c)), -1/315*(315*(b^4*cosh(d*x + c)^8
 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 + 4*b^4*cosh(d
*x + c)^6 + 6*b^4*cosh(d*x + c)^4 + 4*(7*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*
x + c)^6 + 4*b^4*cosh(d*x + c)^2 + 8*(7*b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*b^4*cosh(d*x + c)^4 + 30*b^4*cosh(d*x + c)^
2 + 3*b^4)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*b^4*cosh(d
*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^4*cosh(d*x + c)^6
 + 15*b^4*cosh(d*x + c)^4 + 9*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 +
8*(b^4*cosh(d*x + c)^7 + 3*b^4*cosh(d*x + c)^5 + 3*b^4*cosh(d*x + c)^3 + b^
4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh
(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*
sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a
^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b
)*sinh(d*x + c))) - 2*((16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^8 + (1
6*a^4 - 84*a^2*b^2 - 315*b^4)*sinh(d*x + c)^8 - 4*(4*a^3*b - 21*a*b^3)*cosh
(d*x + c)^7 - 4*(4*a^3*b - 21*a*b^3 - 2*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cos
h(d*x + c))*sinh(d*x + c)^7 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x +
c)^6 + 4*(16*a^4 - 78*a^2*b^2 - 189*b^4 + 7*(16*a^4 - 84*a^2*b^2 - 315*b^4)
*cosh(d*x + c)^2 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 -
4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^5 - 4*(12*a^3*b - 53*a*b^3 - 14*(16*a
^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^3 + 21*(4*a^3*b - 21*a*b^3)*cosh(d
x + c)^2 - 6(16*a^4 - 78*a^2*b^2 - 189*b^4)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^4 + 2*(35*(16*a^4 - 84
*a^2*b^2 - 315*b^4)*cosh(d*x + c)^4 + 48*a^4 - 228*a^2*b^2 - 721*b^4 - 70*(
4*a^3*b - 21*a*b^3)*cosh(d*x + c)^3 + 30*(16*a^4 - 78*a^2*b^2 - 189*b^4)*co
sh(d*x + c)^2 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 1
6*a^4 - 84*a^2*b^2 - 315*b^4 - 4*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^3 + 4*
(14*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(d*x + c)^5 - 35*(4*a^3*b - 21*a*b^
3)*cosh(d*x + c)^4 - 12*a^3*b + 53*a*b^3 + 20*(16*a^4 - 78*a^2*b^2 - 189*b^
4)*cosh(d*x + c)^3 - 10*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(48*a^4 -
 228*a^2*b^2 - 721*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^4 - 78*a^2
*b^2 - 189*b^4)*cosh(d*x + c)^2 + 4*(7*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh
(d*x + c)^6 - 21*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^5 + 15*(16*a^4 - 78*a^2
*b^2 - 189*b^4)*cosh(d*x + c)^4 + 16*a^4 - 78*a^2*b^2 - 189*b^4 - 10*(12*a^
3*b - 53*a*b^3)*cosh(d*x + c)^3 + 3*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d
x + c)^2 - 3(12*a^3*b - 53*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(4*a
^3*b - 21*a*b^3)*cosh(d*x + c) + 4*(2*(16*a^4 - 84*a^2*b^2 - 315*b^4)*cosh(
d*x + c)^7 - 7*(4*a^3*b - 21*a*b^3)*cosh(d*x + c)^6 + 6*(16*a^4 - 78*a^2*b^
2 - 189*b^4)*cosh(d*x + c)^5 - 5*(12*a^3*b - 53*a*b^3)*cosh(d*x + c)^4 - 4*
a^3*b + 21*a*b^3 + 2*(48*a^4 - 228*a^2*b^2 - 721*b^4)*cosh(d*x + c)^3 - 3*(
12*a^3*b - 53*a*b^3)*cosh(d*x + c)^2 + 2*(16*a^4 - 78*a^2*b^2 - 189*b^4)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(b^4
*d*cosh(d*x + c)^8 + 8*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*d*sinh(d*x

```
 + c)^8 + 4*b^4*d*cosh(d*x + c)^6 + 6*b^4*d*cosh(d*x + c)^4 + 4*b^4*d*cosh(
d*x + c)^2 + 4*(7*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^6 + 8*(7*b^4
*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*x + c))*sinh(d*x + c)^5 + b^4*d + 2*(35
*b^4*d*cosh(d*x + c)^4 + 30*b^4*d*cosh(d*x + c)^2 + 3*b^4*d)*sinh(d*x + c)^
4 + 8*(7*b^4*d*cosh(d*x + c)^5 + 10*b^4*d*cosh(d*x + c)^3 + 3*b^4*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*b^4*d*cosh(d*x + c)^6 + 15*b^4*d*cosh(d*x +
c)^4 + 9*b^4*d*cosh(d*x + c)^2 + b^4*d)*sinh(d*x + c)^2 + 8*(b^4*d*cosh(d*x
 + c)^7 + 3*b^4*d*cosh(d*x + c)^5 + 3*b^4*d*cosh(d*x + c)^3 + b^4*d*cosh(d*
x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \sqrt{a + b\mathrm{sech}(c + dx)} \tanh^5(c + dx)\, dx = \int \sqrt{a + b\,\mathrm{sech}\,(c + dx)} \tanh^5(c + dx)\, dx$$

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**5,x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**5, x)
```

Maxima [F]

$$\int \sqrt{a + b\mathrm{sech}(c + dx)} \tanh^5(c + dx)\, dx = \int \sqrt{b\,\mathrm{sech}\,(dx + c) + a} \tanh(dx + c)^5\, dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

Giac [F]

$$\int \sqrt{a + b\mathrm{sech}(c + dx)} \tanh^5(c + dx)\, dx = \int \sqrt{b\,\mathrm{sech}\,(dx + c) + a} \tanh(dx + c)^5\, dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^5,x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh^5(c+dx)\,dx=\int \tanh(c+d\,x)^5\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

```
[In] int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^5*(a + b/cosh(c + d*x))^(1/2), x)
```

3.126 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^2d}$$

```
[Out] -2/3*a*(a+b*sech(d*x+c))^(3/2)/b^2/d+2/5*(a+b*sech(d*x+c))^(5/2)/b^2/d+2*ar
ctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sech(d*x+c))^(1/2)/
d
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 213}

$$\int \sqrt{a+b\,\text{sech}(c+dx)}\tanh^3(c+dx)\,dx = \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{2(a+b\,\text{sech}(c+dx))^{5/2}}{5b^2d} - \frac{2a(a+b\,\text{sech}(c+dx))^{3/2}}{3b^2d} - \frac{2\sqrt{a+b\,\text{sech}(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d - (2*a*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d) + (2*(a + b*S
ech[c + d*x])^(5/2))/(5*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}(b^2-x^2)}{x}\,dx, x, b\text{sech}(c+dx)\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \frac{x^2(-a^2+b^2+2ax^2-x^4)}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \left(b^2+ax^2-x^4+\frac{ab^2}{-a+x^2}\right)\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d} \\
&= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} \\
&\quad + \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^2d} - \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} \\
&= \frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} \\
&\quad - \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} + \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx = \frac{2\left(15\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+\frac{\sqrt{a+b\text{sech}(c+dx)}\left(-2a^2-15b^2+ab\text{sech}(c+dx)+3b^2\text{sech}^2(c+dx)\right)}{b^2}\right)}{15d}$$

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^3,x]
```

```
[Out] (2*(15*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + (Sqrt[a + b*Sec
h[c + d*x]]*(-2*a^2 - 15*b^2 + a*b*Sech[c + d*x] + 3*b^2*Sech[c + d*x]^2))/
b^2))/(15*d)
```

Maple [F]

$$\int \sqrt{a + b\,\mathrm{sech}\,(dx + c)}\,\tanh\,(dx + c)^3\,dx$$

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(84) = 168$.

Time = 0.69 (sec) , antiderivative size = 1589, normalized size of antiderivative = 15.89

$$\int \sqrt{a + b\mathrm{sech}(c + dx)}\tanh^3(c + dx)\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="fricas")
[Out] [1/30*(15*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*
sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*s
inh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x
 + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*
cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cos
h(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a
*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c
)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(
d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
 + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh
(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cos
h(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x
+ c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)
^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 4*(2*a*b*cosh(d*x
+ c)^3 - (2*a^2 + 15*b^2)*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^
4 + 2*(a*b - 2*(2*a^2 + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh
(d*x + c) - 2*(2*a^2 + 9*b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*
(2*a^2 + 15*b^2)*cosh(d*x + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 -
 15*b^2 + 2*(3*a*b*cosh(d*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a
*b - 2*(2*a^2 + 9*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c)
+ b)/cosh(d*x + c)))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*
x + c)^3 + b^2*d*sinh(d*x + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b
^2*d*cosh(d*x + c)^2 + b^2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 +
b^2*d*cosh(d*x + c))*sinh(d*x + c)), -1/15*(15*(b^2*cosh(d*x + c)^4 + 4*b^2
*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*b^2*cosh(d*x + c)^
2 + 2*(3*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x
 + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a)*arctan((a*cosh(d*x + c
```

```
)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*
x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*
x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*
x + c) + a*b)*sinh(d*x + c))) - 2*(2*a*b*cosh(d*x + c)^3 - (2*a^2 + 15*b^2)
*cosh(d*x + c)^4 - (2*a^2 + 15*b^2)*sinh(d*x + c)^4 + 2*(a*b - 2*(2*a^2 + 1
5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*a*b*cosh(d*x + c) - 2*(2*a^2 + 9*
b^2)*cosh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c) - 3*(2*a^2 + 15*b^2)*cosh(d*x
 + c)^2 - 2*a^2 - 9*b^2)*sinh(d*x + c)^2 - 2*a^2 - 15*b^2 + 2*(3*a*b*cosh(d
*x + c)^2 - 2*(2*a^2 + 15*b^2)*cosh(d*x + c)^3 + a*b - 2*(2*a^2 + 9*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(b^2
*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*d*sinh(d*x
 + c)^4 + 2*b^2*d*cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*cosh(d*x + c)^2 + b^
2*d)*sinh(d*x + c)^2 + 4*(b^2*d*cosh(d*x + c)^3 + b^2*d*cosh(d*x + c))*sinh
(d*x + c))]
```

Sympy [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx=\int \sqrt{a+b\,\text{sech}\,(c+dx)}\tanh^3{(c+dx)}\,dx$$

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**3,x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**3, x)
```

Maxima [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh{(dx+c)}^3\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

Giac [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx=\int \sqrt{b\,\text{sech}\,(dx+c)+a}\tanh{(dx+c)}^3\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^3,x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^3(c+dx)\,dx=\int \tanh(c+d\,x)^3\,\sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

```
[In] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

3.127 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx=\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-2*(a+b*sech(d*x+c))^(1
/2)/d
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3970, 52, 65, 213}

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx=\frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d}-\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (2*Sqrt[a + b*Se
ch[c + d*x]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+x}}{x}\,dx, x, b\text{sech}(c+dx)\right)}{d}\\
&= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{d}\\
&= -\frac{2\sqrt{a+b\text{sech}(c+dx)}}{d} - \frac{(2a)\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}\\
&= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+b\text{sech}(c+dx)}}{d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)\,dx$$
$$=-\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+2\sqrt{a+b\text{sech}(c+dx)}}{d}$$

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x],x]
[Out] -((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + 2*Sqrt[a + b*Sec
h[c + d*x]])/d)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2\sqrt{a+b\,\text{sech}(dx+c)}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(dx+c)}}{\sqrt{a}}\right)}{d}$	43
default	$-\frac{2\sqrt{a+b\,\text{sech}(dx+c)}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(dx+c)}}{\sqrt{a}}\right)}{d}$	43

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x,method=_RETURNVERBOSE)
[Out] -1/d*(2*(a+b*sech(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*sech(d*x+c))^(1/2)/a
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(43) = 86$.

Time = 0.65 (sec) , antiderivative size = 605, normalized size of antiderivative = 11.86

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)\,dx$$
$$= \left[\frac{\sqrt{a}\log\left(-\frac{2\,a^2\cosh(dx+c)^4+2\,a^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^3+4\,(2\,a^2\cosh(dx+c)+ab)\sinh(dx+c)^3+4\,ab\cosh(dx+c)+(4\,a^2+b^2)\cosh\right.}{}\right.$$
$$-\frac{\sqrt{-a}\arctan\left(\frac{\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+b\cosh(dx+c)+(2\,a\cosh(dx+c)+b)\sinh(dx+c)+a\right)\sqrt{-a}\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}}{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+a^2+2\,(a^2\cosh(dx+c)+ab)\sinh(dx+c)}\right)+2\sqrt{\frac{a}{}}}{d}$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*c
osh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh
(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*
b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)
^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d
*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c)
+ 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(
d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh
(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x +
 c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^
2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*sqrt((a*cosh(d*x
+ c) + b)/cosh(d*x + c)))/d, -(sqrt(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(
d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*s
qrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^
2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)
*sinh(d*x + c))) + 2*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/d]
```

Sympy [F]

$$\int \sqrt{a + b\text{sech}(c + dx)} \tanh(c + dx)\, dx = \int \sqrt{a + b\,\text{sech}\,(c + dx)} \tanh\,(c + dx)\, dx$$

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c),x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{a + b\text{sech}(c + dx)} \tanh(c + dx)\, dx = \int \sqrt{b\,\text{sech}\,(dx + c) + a} \tanh\,(dx + c)\, dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

Giac [F]

$$\int \sqrt{a + b\text{sech}(c + dx)} \tanh(c + dx)\, dx = \int \sqrt{b\,\text{sech}\,(dx + c) + a} \tanh\,(dx + c)\, dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \sqrt{a + b\text{sech}(c + dx)} \tanh(c + dx)\, dx = \frac{2\sqrt{a}\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{d} - \frac{2\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{d}$$

```
[In] int(tanh(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
[Out] (2*a^(1/2)*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/d - (2*(a + b/cosh(c
 + d*x))^(1/2))/d
```

3.128 $\int \coth(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \coth(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))*(a-b)^(1/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))*(a+b)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3970, 912, 1301, 212, 213}

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[In] Int[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (Sqrt[a - b]*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/d - (Sqrt[a + b]*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a + b]])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
```

```
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{a}{b^2(a-x^2)}+\frac{a+b}{2b^2(a+b-x^2)}+\frac{-a+b}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} + \frac{(a-b)\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d} - \frac{\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\frac{-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)+\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)+\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d}$$

```
[In] Integrate[Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] -((-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] + Sqrt[a - b]*ArcT
anh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]] + Sqrt[a + b]*ArcTanh[Sqrt[a + b
*Sech[c + d*x]]/Sqrt[a + b]])/d)
```

Maple [F]

$$\int \coth(dx+c)\sqrt{a+b\,\text{sech}(dx+c)}dx$$

```
[In] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(88) = 176.

Time = 0.64 (sec) , antiderivative size = 8620, normalized size of antiderivative = 81.32

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{a+b\operatorname{sech}{(c+dx)}}\coth{(c+dx)}\,dx$$

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x), x)
```

Maxima [F]

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b\operatorname{sech}(dx+c)+a}\coth(dx+c)\,dx$$

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

Giac [F]

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b\operatorname{sech}(dx+c)+a}\coth(dx+c)\,dx$$

```
[In] integrate(coth(d*x+c)*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \coth(c+d\,x)\,\sqrt{a+\frac{b}{\cosh(c+d\,x)}}\,dx$$

```
[In] int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)*(a + b/cosh(c + d*x))^(1/2), x)
```

3.129 $\int \coth^3(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 217

$$\int \coth^3(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}d} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} - \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}d} - \frac{\coth^2(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}}{2d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))*a^(1/2)/d-a*arctanh((a+b*sech(d*
x+c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)+3/4*b*arctanh((a+b*sech(d*x+c))^(1/2
)/(a-b)^(1/2))/d/(a-b)^(1/2)-a*arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))
/d/(a+b)^(1/2)-3/4*b*arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(
1/2)-1/2*coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3970, 912, 1329, 1192, 12, 1107, 212, 1184, 213}

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} + \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d\sqrt{a-b}} - \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}} - \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4d\sqrt{a+b}} - \frac{\coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{2d}$$

```
[In] Int[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/d - (a*ArcTanh[Sqrt[
a + b*Sech[c + d*x]]/Sqrt[a - b]])/(Sqrt[a - b]*d) + (3*b*ArcTanh[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a - b]])/(4*Sqrt[a - b]*d) - (a*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a + b]])/(Sqrt[a + b]*d) - (3*b*ArcTanh[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]])/(4*Sqrt[a + b]*d) - (Coth[c + d*x]^2*Sqrt[a + b*Sech
[c + d*x]])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1329

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2)), Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^4\text{Subst}\left(\int \frac{\sqrt{a+x}}{x(b^2-x^2)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \frac{x^2}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{-a^2+b^2+ax^2}{(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} - \frac{(2ab^2)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)} - \frac{(2ab^2)\,\text{Subst}\left(\int \left(-\frac{1}{b^2(a-x^2)}+\frac{1}{2b^2(a+b-x^2)}-\frac{1}{2b^2(-a+b+x^2)}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} - \frac{\text{Subst}\left(\int \frac{6b^2(a^2-b^2)}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{4(a^2-b^2)\,d}$$

$$= \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)} - \frac{a\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} + \frac{a\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} - \frac{(3b^2)\,\text{Subst}\left(\int \frac{1}{-a^2+b^2+2ax^2-x^4}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{2d}$$

$$= \frac{2\sqrt{a}a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} + \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)} + \frac{(3b)\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{4d} - \frac{(3b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{4d}$$

$$= \frac{2\sqrt{a}a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{d} - \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} + \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4\sqrt{a-b}d} - \frac{a\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} - \frac{3b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4\sqrt{a+b}d} + \frac{b^2\sqrt{a+b\text{sech}(c+dx)}}{2d\left(a^2-b^2-2a(a+b\text{sech}(c+dx))+(a+b\text{sech}(c+dx))^2\right)}$$

Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.03

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$= \frac{\frac{b\arctan\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} + 8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right) - 4\sqrt{a-b}\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right) + \frac{b\text{arctanh}\left(\cdots\right.}{\cdots}}{4d}$$

```
[In] Integrate[Coth[c + d*x]^3*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] ((b*ArcTan[Sqrt[a + b*Sech[c + d*x]]/Sqrt[-a + b]])/Sqrt[-a + b] + 8*Sqrt[a
]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]] - 4*Sqrt[a - b]*ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]] + (b*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqr
t[a + b]])/Sqrt[a + b] - 4*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sq
rt[a + b]] + Sqrt[a + b*Sech[c + d*x]]/(-1 + Sech[c + d*x]) - Sqrt[a + b*Se
ch[c + d*x]]/(1 + Sech[c + d*x]))/(4*d)
```

Maple [F]

$$\int \coth (dx + c)^3 \sqrt{a + b \operatorname{sech} (dx + c)}dx$$

```
[In] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
[Out] int(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1399 vs. $2(179) = 358$.

Time = 0.99 (sec) , antiderivative size = 16532, normalized size of antiderivative = 76.18

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{a + b \operatorname{sech} (c + dx)} \coth^3 (c + dx) \, dx$$

```
[In] integrate(coth(d*x+c)**3*(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**3, x)
```

Maxima [F]

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b \operatorname{sech} (dx + c) + a} \coth (dx + c)^3 \, dx$$

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

Giac [F]

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth\,(dx+c)^3\;dx$$

```
[In] integrate(coth(d*x+c)^3*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \coth(c+d\,x)^3\,\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}\,dx$$

```
[In] int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^3*(a + b/cosh(c + d*x))^(1/2), x)
```

3.130 $\int \sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx = -\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{3b^2d}$$
$$-\frac{2\sqrt{a+b}(a+2b)\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(}{a-b}}}{3bd}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+d}{a-b}}}{d}$$
$$-\frac{2\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3d}$$

```
[Out] -2/3*a*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+
b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*
x+c))/(a-b))^(1/2)/b^2/d-2/3*(a+2*b)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c)
)^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+
b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*coth(d*x+c)*EllipticPi((a+
b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(
b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/d-2/3*(a+b*
sech(d*x+c))^(1/2)*tanh(d*x+c)/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3979, 4142, 4143, 4006, 3869, 3917, 4089}

$$\int \sqrt{a+b\operatorname{sech}(c+dx)}\tanh^2(c+dx)\,dx =$$

$$-\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}$$

$$-\frac{2\sqrt{a+b}(a+2b)\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a\cdots}{a\cdots}\right.}{3bd}$$

$$+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$

$$-\frac{2\tanh(c+dx)\sqrt{a+b\operatorname{sech}(c+dx)}}{3d}$$

```
[In] Int[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
```

```
[Out] (-2*a*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^2*d) - (2*Sqrt[a + b]*(a +
 2*b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]
, (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
h[c + d*x]))/(a - b))])/(3*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(
a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b)
)])/d - (2*Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4142

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^
m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[
a*A*(m + 1) + (A*b*(m + 1) + b*C*m)*Csc[e + f*x] + a*C*m*Csc[e + f*x]^2, x]
, x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m,
0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\int \sqrt{a + b\text{sech}(c + dx)}\left(-1 + \text{sech}^2(c + dx)\right)\, dx$$

$$=-\frac{2\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3d}-\frac{2}{3}\int\frac{-\frac{3a}{2}-b\text{sech}(c+dx)+\frac{1}{2}a\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$=-\frac{2\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3d}-\frac{2}{3}\int\frac{-\frac{3a}{2}+\left(-\frac{a}{2}-b\right)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$-\frac{1}{3}a\int\frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$=$$
$$-\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{3b^2d}$$
$$-\frac{2\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3d}+a\int\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$-\frac{1}{3}(-a-2b)\int\frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$=$$
$$-\frac{2a(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{3b^2d}$$
$$-\frac{2\sqrt{a+b}(a+2b)\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1}{}}}{3bd}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{d}$$
$$-\frac{2\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3d}$$

Mathematica [F]

$$\int\sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int\sqrt{a+b\text{sech}(c+dx)}\tanh^2(c+dx)\,dx$$

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2,x]
[Out] Integrate[Sqrt[a + b*Sech[c + d*x]]*Tanh[c + d*x]^2, x]
```

Maple [F]

$$\int \sqrt{a+b\,\mathrm{sech}\,(dx+c)}\,\tanh\,(dx+c)^2\,dx$$

```
[In] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
[Out] int((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x)
```

Fricas [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\tanh\,(dx+c)^2\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Sympy [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \sqrt{a+b\,\mathrm{sech}\,(c+dx)}\tanh^2\,(c+dx)\,dx$$

```
[In] integrate((a+b*sech(d*x+c))**(1/2)*tanh(d*x+c)**2,x)
[Out] Integral(sqrt(a + b*sech(c + d*x))*tanh(c + d*x)**2, x)
```

Maxima [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^2(c+dx)\,dx=\int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\tanh\,(dx+c)^2\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Giac [F]

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^2(c+dx)\,dx = \int \sqrt{b\,\mathrm{sech}\,(dx+c)+a}\tanh\,(dx+c)^2\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\mathrm{sech}(c+dx)}\tanh^2(c+dx)\,dx = \int \tanh(c+d\,x)^2\,\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}\,dx$$

```
[In] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

3.131 $\int \sqrt{a+b\text{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 125

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx = \frac{2\coth(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(1+\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}(a+bs}{\sqrt{a+b}d}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b),((a-b)
/(a+b))^(1/2))*(a+b*sech(d*x+c))*(-b*(1-sech(d*x+c))/(a+b*sech(d*x+c)))^(1/
2)*(b*(1+sech(d*x+c))/(a+b*sech(d*x+c)))^(1/2)/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3865}

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx = \frac{2\coth(c+dx)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(\text{sech}(c+dx)+1)}{a+b\text{sech}(c+dx)}}(a+b\text{sech}(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c}}\right.\right.}{d\sqrt{a+b}}$$

```
[In] Int[Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sech[c
 + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sech[c +
```

```
d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Sech[c +
 d*x]))/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{2\coth(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b}, \arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+dx)}}\right), \frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(1+\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}(a+b\text{sec}}{\sqrt{a+b}d}$$

Mathematica [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{a+b\text{sech}(c+dx)}\,dx$$

```
[In] Integrate[Sqrt[a + b*Sech[c + d*x]],x]
[Out] Integrate[Sqrt[a + b*Sech[c + d*x]], x]
```

Maple [F]

$$\int \sqrt{a+b\,\text{sech}\,(dx+c)}dx$$

```
[In] int((a+b*sech(d*x+c))^(1/2),x)
[Out] int((a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(dx+c)+a}\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{a+b\operatorname{sech}{(c+dx)}}\,dx$$

```
[In] integrate((a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{b\operatorname{sech}\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{b\operatorname{sech}\left(dx+c\right)+a}\,dx$$

```
[In] integrate((a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\text{sech}(c+dx)}\,dx=\int \sqrt{a+\frac{b}{\cosh\left(c+d\,x\right)}}\,dx$$

```
[In] int((a + b/cosh(c + d*x))^(1/2),x)
[Out] int((a + b/cosh(c + d*x))^(1/2), x)
```

3.132 $\int \coth^2(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 246

$$\int \coth^2(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}\,dx$$
$$=\frac{\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{d}$$
$$-\frac{\coth(c+dx)\sqrt{a+b\,\mathrm{sech}(c+dx)}}{d}$$
$$+\frac{2\coth(c+dx)\,\mathrm{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\right),\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\mathrm{sech}(c+dx))}{a+b\,\mathrm{sech}(c+dx)}}\sqrt{\frac{b(1+\mathrm{sech}(c+dx))}{a+b\,\mathrm{sech}(c+dx)}}(a+b}{\sqrt{a+b}d}$$

```
[Out] coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^
(1/2)/d+2*coth(d*x+c)*EllipticPi((a+b)^(1/2)/(a+b*sech(d*x+c))^(1/2),a/(a+b
),((a-b)/(a+b))^(1/2))*(a+b*sech(d*x+c))*(-b*(1-sech(d*x+c))/(a+b*sech(d*x+
c)))^(1/2)*(b*(1+sech(d*x+c))/(a+b*sech(d*x+c)))^(1/2)/d/(a+b)^(1/2)-coth(d
*x+c)*(a+b*sech(d*x+c))^(1/2)/d
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3981, 3865, 3960, 3917}

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d}$$
$$+\frac{2\coth(c+dx)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(\text{sech}(c+dx)+1)}{a+b\text{sech}(c+dx)}}(a+b\text{sech}(c+dx))\,\text{EllipticPi}\left(\frac{a}{a+b},\arcsin\left(\frac{\sqrt{a+}}{\sqrt{a+b\text{sech}}}\right.\right.}{d\sqrt{a+b}}$$
$$-\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d}$$

```
[In] Int[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Sqrt[a + b]*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sech[c + d*x]))/(a - b))])/d - (Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]]
)/d + (2*Coth[c + d*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*
Sech[c + d*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sech[c + d*x]))/(a + b*Sec
h[c + d*x]))]*Sqrt[(b*(1 + Sech[c + d*x]))/(a + b*Sech[c + d*x])]*(a + b*Se
ch[c + d*x]))/(Sqrt[a + b]*d)
```

Rule 3865

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*((a + b
*Csc[c + d*x])/(d*Rt[a + b, 2]*Cot[c + d*x]))*Sqrt[b*((1 + Csc[c + d*x])/(a
 + b*Csc[c + d*x]))]*Sqrt[(-b)*((1 - Csc[c + d*x])/(a + b*Csc[c + d*x]))]*E
llipticPi[a/(a + b), ArcSin[Rt[a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)
/(a + b)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
```

```
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rubi steps

$$\text{integral} = -\int \left(-\sqrt{a+b\text{sech}(c+dx)} - \text{csch}^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\right) dx$$

$$= \int \sqrt{a+b\text{sech}(c+dx)}\,dx + \int \text{csch}^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx$$

$$= -\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d} + \frac{2\coth(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b}, \arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+dx)}}\right), \frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(1+\text{sech}(c+dx}{a+b\text{sech}(c+dx}}}{\sqrt{a+b}d} - \frac{1}{2}b\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= \frac{\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right), \frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{d} - \frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d} + \frac{2\coth(c+dx)\,\text{EllipticPi}\left(\frac{a}{a+b}, \arcsin\left(\frac{\sqrt{a+b}}{\sqrt{a+b\text{sech}(c+dx)}}\right), \frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\text{sech}(c+dx))}{a+b\text{sech}(c+dx)}}\sqrt{\frac{b(1+\text{sech}(c+dx}{a+b\text{sech}(c+dx}}}{\sqrt{a+b}d}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 539 vs. $2(246) = 492$.

Time = 18.56 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.19

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = -\frac{\coth(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{d} + \frac{\sqrt{a+b\text{sech}(c+dx)}\left(\frac{2\sqrt{b}(a-a\cosh(c+dx))^{3/2}\sqrt{\frac{(a+b)(a+a\cosh(c+dx))}{(a-b)(a-a\cosh(c+dx))}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{b}\sqrt{a-a\cosh(c+dx)}}\right), -\frac{2b}{a-b}\right)\sinh(c+da}{a^{3/2}\sqrt{-1+\cosh(c+dx)}\sqrt{1+\cosh(c+dx)}\sqrt{-\frac{a(a+b)\cosh(c+dx)}{b(a-a\cosh(c+dx))}}\left(-\frac{a-a\cosh(c+dx)}{a}\right)^{3/2}\sqrt{\frac{a+a\cosh(c+dx)}{a}}\sqrt{\text{sech}(c}}\right.}{2d\sqrt{b+a\cos}}$$

```
[In] Integrate[Coth[c + d*x]^2*Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] -((Coth[c + d*x]*Sqrt[a + b*Sech[c + d*x]])/d) + (Sqrt[a + b*Sech[c + d*x]]
*((2*Sqrt[b]*(a - a*Cosh[c + d*x])^(3/2)*Sqrt[((a + b)*(a + a*Cosh[c + d*x]
))/((a - b)*(a - a*Cosh[c + d*x]))]*EllipticF[ArcSin[(Sqrt[a]*Sqrt[b + a*Co
sh[c + d*x]])/(Sqrt[b]*Sqrt[a - a*Cosh[c + d*x]])], (-2*b)/(a - b)]*Sinh[c
+ d*x])/(a^(3/2)*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d*x]]*Sqrt[-((a
*(a + b)*Cosh[c + d*x])/(b*(a - a*Cosh[c + d*x])))]*(-((a - a*Cosh[c + d*x]
)/a))^(3/2)*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Sech[c + d*x]]) - (4*b*(a -
a*Cosh[c + d*x])*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b + a*Cosh[c +
d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]*Sqrt[-((b*(a
+ a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a - b)))]*Sinh[c + d*x])/(Sqrt[a]*Sqr
t[a + b]*Sqrt[-1 + Cosh[c + d*x]]*Sqrt[a*Cosh[c + d*x]]*Sqrt[1 + Cosh[c + d
*x]]*Sqrt[-((a - a*Cosh[c + d*x])/a)]*Sqrt[(a + a*Cosh[c + d*x])/a]*Sqrt[Se
ch[c + d*x]]*Sqrt[-((b*(a - a*Cosh[c + d*x])*Sech[c + d*x])/(a*(a + b)))]))
)/(2*d*Sqrt[b + a*Cosh[c + d*x]]*Sqrt[Sech[c + d*x]])
```

Maple [F]

$$\int \coth\left(dx+c\right)^{2}\sqrt{a+b\,\mathrm{sech}\left(dx+c\right)}dx$$

```
[In] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \coth^2(c+dx)\sqrt{a+b\mathrm{sech}(c+dx)}\,dx=\text{Timed out}$$

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \coth^2(c+dx)\sqrt{a+b\mathrm{sech}(c+dx)}\,dx=\int \sqrt{a+b\,\mathrm{sech}\left(c+dx\right)}\coth^2\left(c+dx\right)dx$$

```
[In] integrate(coth(d*x+c)**2*(a+b*sech(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sech(c + d*x))*coth(c + d*x)**2, x)
```

Maxima [F]

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth\,(dx+c)^2\,dx$$

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Giac [F]

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \sqrt{b\,\text{sech}\,(dx+c)+a}\coth\,(dx+c)^2\,dx$$

```
[In] integrate(coth(d*x+c)^2*(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sech(d*x + c) + a)*coth(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx = \int \coth(c+d\,x)^2\,\sqrt{a+\frac{b}{\cosh\,(c+d\,x)}}\,dx$$

```
[In] int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^2*(a + b/cosh(c + d*x))^(1/2), x)
```

3.133 $\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2a(a^2-2b^2)\sqrt{a+b\operatorname{sech}(c+dx)}}{b^4d} - \frac{2(3a^2-2b^2)\left(a+b\operatorname{sech}(c+dx)\right)^{3/2}}{3b^4d} + \frac{6a(a+b\operatorname{sech}(c+dx))^{5/2}}{5b^4d} - \frac{2(a+b\operatorname{sech}(c+dx))^{7/2}}{7b^4d}$$

```
[Out] -2/3*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(3/2)/b^4/d+6/5*a*(a+b*sech(d*x+c))^(5
/2)/b^4/d-2/7*(a+b*sech(d*x+c))^(7/2)/b^4/d+2*arctanh((a+b*sech(d*x+c))^(1/
2)/a^(1/2))/d/a^(1/2)+2*a*(a^2-2*b^2)*(a+b*sech(d*x+c))^(1/2)/b^4/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1167, 213}

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{2(3a^2-2b^2)\,(a+b\text{sech}(c+dx))^{3/2}}{3b^4d} + \frac{2a(a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d} + \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2(a+b\text{sech}(c+dx))^{7/2}}{7b^4d} + \frac{6a(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}$$

```
[In] Int[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) + (2*a*(a^2 - 2*
b^2)*Sqrt[a + b*Sech[c + d*x]])/(b^4*d) - (2*(3*a^2 - 2*b^2)*(a + b*Sech[c
+ d*x])^(3/2))/(3*b^4*d) + (6*a*(a + b*Sech[c + d*x])^(5/2))/(5*b^4*d) - (2
*(a + b*Sech[c + d*x])^(7/2))/(7*b^4*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{b^4 d} \\
&= -\frac{2\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4 d} \\
&= -\frac{2\text{Subst}\left(\int \left(-a^3+2ab^2+(3a^2-2b^2)\,x^2-3ax^4+x^6+\frac{b^4}{-a+x^2}\right)dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4 d} \\
&= \frac{2a(a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4 d} - \frac{2(3a^2-2b^2)\,(a+b\text{sech}(c+dx))^{3/2}}{3b^4 d} \\
&\quad + \frac{6a(a+b\text{sech}(c+dx))^{5/2}}{5b^4 d} - \frac{2(a+b\text{sech}(c+dx))^{7/2}}{7b^4 d} \\
&\quad - \frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} \\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2a(a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4 d} \\
&\quad - \frac{2(3a^2-2b^2)\,(a+b\text{sech}(c+dx))^{3/2}}{3b^4 d} \\
&\quad + \frac{6a(a+b\text{sech}(c+dx))^{5/2}}{5b^4 d} - \frac{2(a+b\text{sech}(c+dx))^{7/2}}{7b^4 d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.76

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\left(\frac{105\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{a+b\text{sech}(c+dx)}\left(48a^3-140ab^2+(-24a^2b+70b^3)\text{sech}(c+dx)+18ab^2\text{sech}^2(c+dx)-15b^3\text{sech}\right.}{b^4}\right.}{105d}$$

```
[In] Integrate[Tanh[c + d*x]^5/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*((105*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + (Sqrt[a + b*
Sech[c + d*x]]*(48*a^3 - 140*a*b^2 + (-24*a^2*b + 70*b^3)*Sech[c + d*x] + 1
8*a*b^2*Sech[c + d*x]^2 - 15*b^3*Sech[c + d*x]^3))/b^4))/(105*d)
```

Maple [F]

$$\int \frac{\tanh (dx+c)^5}{\sqrt{a+b\,\mathrm{sech}\,(dx+c)}}dx$$

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. 2(128) = 256.

Time = 0.69 (sec) , antiderivative size = 2813, normalized size of antiderivative = 19.01

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/210*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)*sinh(d*x + c)^5 + b^
4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh(d*x + c)^2 + 3*(5*b^
4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*b^4*cosh(d*x + c)^3 +
 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(d*x + c)^4 + 6*b^4*co
sh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x + c)^5 + 2*b^4*cosh(
d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x
 + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x
 + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x
 + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sin
h(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*
x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 +
(6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*
x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b
)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*
(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh
(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2)) + 16*((12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^6 + (12*a^4 -
 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c)^5 - (12*
a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 +
 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 + (36*a^4 - 87*a^2*b^2 + 15*(12*a^
4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^3
```

```
 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^3 + 5*(1
2*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 + (15*(12*a^4 -
 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87*a^2*b^2 - 10*(12*a^3*b - 35*a*b^
3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^2 - 24*(3*a^3*b
 - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (12*a^3*b - 35*a*b^3)*cosh(d*x
 + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^5 - 5*(12*a^3*b - 35*a*b^3)*
cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12*(12*a^4 - 29*a^2*b^2)*cosh(d*x +
 c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c)^2 + 6*(12*a^4 - 29*a^2*b^2)*co
sh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b
^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^4*d*si
nh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4 + 3*a*b^4*d*cosh(d*x + c)^2 + a*b
^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4
*d*cosh(d*x + c)^3 + 3*a*b^4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*b^4*
d*cosh(d*x + c)^4 + 6*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d)*sinh(d*x + c)^2 +
6*(a*b^4*d*cosh(d*x + c)^5 + 2*a*b^4*d*cosh(d*x + c)^3 + a*b^4*d*cosh(d*x +
 c))*sinh(d*x + c)), -1/105*(105*(b^4*cosh(d*x + c)^6 + 6*b^4*cosh(d*x + c)
*sinh(d*x + c)^5 + b^4*sinh(d*x + c)^6 + 3*b^4*cosh(d*x + c)^4 + 3*b^4*cosh
(d*x + c)^2 + 3*(5*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^4 + b^4 + 4*(5*
b^4*cosh(d*x + c)^3 + 3*b^4*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^4*cosh(
d*x + c)^4 + 6*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 6*(b^4*cosh(d*x
 + c)^5 + 2*b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a
)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*co
sh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cos
h(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c
) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 8*((12*a^4 - 35*a^2
*b^2)*cosh(d*x + c)^6 + (12*a^4 - 35*a^2*b^2)*sinh(d*x + c)^6 - (12*a^3*b -
 35*a*b^3)*cosh(d*x + c)^5 - (12*a^3*b - 35*a*b^3 - 6*(12*a^4 - 35*a^2*b^2)
*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^4 +
 (36*a^4 - 87*a^2*b^2 + 15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^2 - 5*(12*a^
3*b - 35*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 12*a^4 - 35*a^2*b^2 - 8*(3
*a^3*b - 5*a*b^3)*cosh(d*x + c)^3 - 2*(12*a^3*b - 20*a*b^3 - 10*(12*a^4 - 3
5*a^2*b^2)*cosh(d*x + c)^3 + 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^2 - 6*(1
2*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(12*a^4 - 29*a^2*b^2
)*cosh(d*x + c)^2 + (15*(12*a^4 - 35*a^2*b^2)*cosh(d*x + c)^4 + 36*a^4 - 87
*a^2*b^2 - 10*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^3 + 18*(12*a^4 - 29*a^2*b
^2)*cosh(d*x + c)^2 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2
 - (12*a^3*b - 35*a*b^3)*cosh(d*x + c) + (6*(12*a^4 - 35*a^2*b^2)*cosh(d*x
+ c)^5 - 5*(12*a^3*b - 35*a*b^3)*cosh(d*x + c)^4 - 12*a^3*b + 35*a*b^3 + 12
*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c)^3 - 24*(3*a^3*b - 5*a*b^3)*cosh(d*x +
c)^2 + 6*(12*a^4 - 29*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d
*x + c) + b)/cosh(d*x + c)))/(a*b^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + a*b^4*d*sinh(d*x + c)^6 + 3*a*b^4*d*cosh(d*x + c)^4
+ 3*a*b^4*d*cosh(d*x + c)^2 + a*b^4*d + 3*(5*a*b^4*d*cosh(d*x + c)^2 + a*b^
4*d)*sinh(d*x + c)^4 + 4*(5*a*b^4*d*cosh(d*x + c)^3 + 3*a*b^4*d*cosh(d*x +
```

```
c))*sinh(d*x + c)^3 + 3*(5*a*b^4*d*cosh(d*x + c)^4 + 6*a*b^4*d*cosh(d*x + c
)^2 + a*b^4*d)*sinh(d*x + c)^2 + 6*(a*b^4*d*cosh(d*x + c)^5 + 2*a*b^4*d*cos
h(d*x + c)^3 + a*b^4*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh^5(c+dx)}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**5/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)^5}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)^5}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^5/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(c+d\,x)^5}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(1/2), x)
```

3.134 $\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d} + \frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d}$$

```
[Out] 2/3*(a+b*sech(d*x+c))^(3/2)/b^2/d+2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2)
)/d/a^(1/2)-2*a*(a+b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1167, 213}

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} + \frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2d} - \frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

```
[In] Int[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - (2*a*Sqrt[a +
b*Sech[c + d*x]])/(b^2*d) + (2*(a + b*Sech[c + d*x])^(3/2))/(3*b^2*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{b^2 d}\\
&= -\frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2 d}\\
&= -\frac{2\text{Subst}\left(\int \left(a-x^2+\frac{b^2}{-a+x^2}\right)\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2 d}\\
&= -\frac{2a\sqrt{a+b\text{sech}(c+dx)}}{b^2 d}+\frac{2(a+b\text{sech}(c+dx))^{3/2}}{3b^2 d}\\
&\quad -\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}
\end{aligned}
$$

$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{2a\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2d} + \frac{2(a+b\operatorname{sech}(c+dx))^{3/2}}{3b^2d}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \frac{2\left(\frac{3\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(-2a+b\operatorname{sech}(c+dx))\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2}\right)}{3d}$$

```
[In] Integrate[Tanh[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
[Out] (2*((3*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + ((-2*a + b*Sec
h[c + d*x])*Sqrt[a + b*Sech[c + d*x]])/b^2))/(3*d)
```

Maple [F]

$$\int \frac{\tanh(dx+c)^3}{\sqrt{a+b\ \operatorname{sech}(dx+c)}}dx$$

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(67) = 134.

Time = 0.68 (sec) , antiderivative size = 925, normalized size of antiderivative = 11.71

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/6*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh
(d*x + c)^2 + b^2)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c
)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
 c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*
cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c
) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*c
```

```
osh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^
3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a
*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(c
osh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 8*(a^2
*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*c
osh(d*x + c) - a*b)*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)
))/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2
*d*sinh(d*x + c)^2 + a*b^2*d), -1/3*(3*(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*
x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + b^2)*sqrt(-a)*arctan((a*cosh(d
*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x + c) + b)*s
inh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c))/(a^2*c
osh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 + 2*(a^2*c
osh(d*x + c) + a*b)*sinh(d*x + c))) + 4*(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x
 + c)^2 - a*b*cosh(d*x + c) + a^2 + (2*a^2*cosh(d*x + c) - a*b)*sinh(d*x +
c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a*b^2*d*cosh(d*x + c)^2 + 2
*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d)]
```

Sympy [F]

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh^3\left(c+dx\right)}{\sqrt{a+b\,\text{sech}\left(c+dx\right)}}\,dx$$

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh\left(dx+c\right)^3}{\sqrt{b\,\text{sech}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(c+d\,x)^3}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

3.135 $\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx=\frac{2\mathrm{arctanh}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3970, 65, 213}

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx=\frac{2\mathrm{arctanh}\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Int[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\,\text{sech}(c+dx)\right)}{d}\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\,\text{sech}(c+dx)}\right)}{d}\\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\,\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Integrate[Tanh[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d)
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26
default	$\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(dx+c)}}{\sqrt{a}}\right)}{d\sqrt{a}}$	26

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(25) = 50.

Time = 0.68 (sec) , antiderivative size = 558, normalized size of antiderivative = 18.00

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx$$

$$= \left[\log\left(-\frac{2\,a^2\cosh(dx+c)^4+2\,a^2\sinh(dx+c)^4+4\,ab\cosh(dx+c)^3+4\,(2\,a^2\cosh(dx+c)+ab)\sinh(dx+c)^3+4\,ab\cosh(dx+c)+(4\,a^2+b^2)\cosh(dx+c\right.\right.$$

$$\left. - \frac{\sqrt{-a}\arctan\left(\frac{\left(a\cosh(dx+c)^2+a\sinh(dx+c)^2+b\cosh(dx+c)+(2\,a\cosh(dx+c)+b)\sinh(dx+c)+a\right)\sqrt{-a}\sqrt{\frac{a\cosh(dx+c)+b}{\cosh(dx+c)}}}{a^2\cosh(dx+c)^2+a^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+a^2+2\,(a^2\cosh(dx+c)+ab)\sinh(dx+c)}\right)}{ad} \right]$$

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] [1/2*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x +
 c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c)
 + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*
x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*si
nh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3
 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*si
nh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^
2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c)
 + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2
*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cos
h(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))/(sqrt(a)*d), -sqrt(-a)*arctan(
```

```
(a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*cosh(d*x +
c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c
))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + a^2 +
 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)))/(a*d)]
```

Sympy [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(c+dx)}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\tanh(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\,\text{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

```
[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
[Out] (2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

3.136 $\int \frac{\coth(c+dx)}{\sqrt{a+b\mathbf{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))/d/(a-b)^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))/d/(a+b)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3970, 912, 1184, 212, 213}

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}}$$

```
[In] Int[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) - ArcTanh[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}(b^2-x^2)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(-\frac{1}{b^2(a-x^2)} + \frac{1}{2b^2(a+b-x^2)} - \frac{1}{2b^2(-a+b+x^2)}\right) dx, x, \sqrt{a + b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2} dx, x, \sqrt{a + b\text{sech}(c+dx)}\right)}{d} + \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2} dx, x, \sqrt{a + b\text{sech}(c+dx)}\right)}{d} + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2} dx, x, \sqrt{a + b\text{sech}(c+dx)}\right)}{d}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

$$\int \frac{\coth(c+dx)}{\sqrt{a + b\text{sech}(c+dx)}} dx = -\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{d}$$

```
[In] Integrate[Coth[c + d*x]/Sqrt[a + b*Sech[c + d*x]],x]
[Out] -(((-2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b] + ArcTanh[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]]/Sqrt[a + b])/d)
```

Maple [F]

$$\int \frac{\coth(dx+c)}{\sqrt{a + b\,\text{sech}\,(dx+c)}}dx$$

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(88) = 176$.

Time = 0.86 (sec) , antiderivative size = 8908, normalized size of antiderivative = 84.04

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth{(c+dx)}}{\sqrt{a+b\,\text{sech}{(c+dx)}}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth{(dx+c)}}{\sqrt{b\,\text{sech}{(dx+c)}+a}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth{(dx+c)}}{\sqrt{b\,\text{sech}{(dx+c)}+a}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = \int \frac{\coth(c+d\,x)}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(1/2), x)
```

3.137 $\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 262

$$\begin{aligned}\int \frac{\coth^3(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx &= \frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} \\ &+ \frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} - \frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} \\ &- \frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} - \frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4(a+b)d(1-\mathrm{sech}(c+dx))} \\ &- \frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4(a-b)d(1+\mathrm{sech}(c+dx))}\end{aligned}$$

```
[Out] 1/4*b*arctanh((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-1/4*b*arct
anh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/d+2*arctanh((a+b*sech(
d*x+c))^(1/2)/a^(1/2))/d/a^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2
))/d/(a-b)^(1/2)-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2))/d/(a+b)^(1/2)
-1/4*(a+b*sech(d*x+c))^(1/2)/(a+b)/d/(1-sech(d*x+c))-1/4*(a+b*sech(d*x+c))^
(1/2)/(a-b)/d/(1+sech(d*x+c))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3970, 912, 1252, 212, 205, 213}

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{3/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d\sqrt{a-b}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d\sqrt{a+b}} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{3/2}} - \frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)(1-\text{sech}(c+dx))} - \frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a-b)(\text{sech}(c+dx)+1)}$$

```
[In] Int[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(Sqrt[a]*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/(Sqrt[a - b]*d) + (b*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(3/2)*d) - (b*ArcTanh[Sqrt[a + b*Sech
[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]]/(Sqrt[a + b]*d) - Sqrt[a + b*Sech[c + d*x]]/(4*(a + b)*d
*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a - b)*d*(1 + Sech[c
+ d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule steps

$$\text{integral} = -\frac{b^4\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}(b^2-x^2)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{b^4(a-x^2)}+\frac{1}{4b^3(a+b-x^2)^2}+\frac{1}{2b^4(a+b-x^2)}-\frac{1}{4b^3(-a+b+x^2)^2}-\frac{1}{2b^4(-a+b+x^2)}\right)dx, x, \sqrt{a}\right.}{d}$$

$$
\begin{aligned}
&= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} \\
&+ \frac{\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} \\
&+ \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d} \\
&- \frac{b\text{Subst}\left(\int \frac{1}{(a+b-x^2)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{2d} \\
&+ \frac{b\text{Subst}\left(\int \frac{1}{(-a+b+x^2)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{2d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} \\
&- \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} - \frac{\sqrt{a+b\text{sech}(c+dx)}}{4(a+b)d(1-\text{sech}(c+dx))} \\
&- \frac{\sqrt{a+b\text{sech}(c+dx)}}{4(a-b)d(1+\text{sech}(c+dx))} - \frac{b\text{Subst}\left(\int \frac{1}{-a+b+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{4(a-b)d} \\
&- \frac{b\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{4(a+b)d}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}d} \\
&+ \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{3/2}d} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{3/2}d} \\
&- \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}d} - \frac{\sqrt{a+b\text{sech}(c+dx)}}{4(a+b)d(1-\text{sech}(c+dx))} \\
&- \frac{\sqrt{a+b\text{sech}(c+dx)}}{4(a-b)d(1+\text{sech}(c+dx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.07

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = -\frac{-\frac{b^2\arctan\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{-a+b}}\right)}{(-a+b)^{3/2}} - \frac{8b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{4b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b\text{se}}}{\sqrt{}}\right.}{(a+b)^{3/2}}}{}$$

[In] Integrate[Coth[c + d*x]^3/Sqrt[a + b*Sech[c + d*x]],x]

[Out] -1/4*(-((b^2*ArcTan[Sqrt[a + b*Sech[c + d*x]]/Sqrt[-a + b]])/(-a + b)^(3/2)
) - (8*b*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/Sqrt[a] + (4*b*ArcTanh
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (b^2*ArcTanh[Sqrt[a
+ b*Sech[c + d*x]]/Sqrt[a + b]])/(a + b)^(3/2) - (4*a*ArcTanh[Sqrt[a + b*Se
ch[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] + 4*Sqrt[a + b]*ArcTanh[Sqrt[a + b*S
ech[c + d*x]]/Sqrt[a + b]] - (b*Sqrt[a + b*Sech[c + d*x]])/((a + b)*(-1 + S
ech[c + d*x])) + (b*Sqrt[a + b*Sech[c + d*x]])/((a - b)*(1 + Sech[c + d*x])
))/(b*d)

Maple [F]

$$\int \frac{\coth\left(dx+c\right)^{3}}{\sqrt{a+b\,\text{sech}\left(dx+c\right)}}dx$$

[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)

[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1870 vs. 2(218) = 436.

Time = 4.30 (sec) , antiderivative size = 20300, normalized size of antiderivative = 77.48

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \text{Too large to display}$$

[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")

[Out] Too large to include

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth^3{(c+dx)}}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)**3/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth{(dx+c)}^3}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth{(dx+c)}^3}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^3/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\mathrm{coth}(c+d\,x)^3}{\sqrt{a+\frac{b}{\mathrm{cosh}(c+d\,x)}}}\,dx$$

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(1/2), x)
```

3.138 $\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 610

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx =$$

$$-\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{b^2d}$$

$$+\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a-}}}{15b^4d}$$

$$-\frac{4\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{bd}$$

$$+\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-}}{15b^3d}$$

$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+d}{a-b}}}{ad}$$

$$-\frac{8a\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{15b^2d}+\frac{2\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{5bd}$$

```
[Out] -4*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c)
)/(a-b))^(1/2)/b^2/d+2/15*(a-b)*(8*a^2+9*b^2)*coth(d*x+c)*EllipticE((a+b*se
ch(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*
x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/d-4*coth(d*x+c)*Ell
```

```
ipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)
*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2/15*
(8*a^2-2*a*b+9*b^2)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+
sech(d*x+c))/(a-b))^(1/2)/b^3/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^
(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c
))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-8/15*a*(a+b*sech(d*x+c
))^(1/2)*tanh(d*x+c)/b^2/d+2/5*sech(d*x+c)*(a+b*sech(d*x+c))^(1/2)*tanh(d*x
+c)/b/d
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3980, 3869, 3922, 3917, 4089, 3945, 4167, 4090}

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$= \frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}$$
$$+\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{15b^3d}$$
$$-\frac{4(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}$$
$$-\frac{4\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$
$$-\frac{8a\tanh(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{15b^2d}+\frac{2\tanh(c+dx)\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}}{5bd}$$

```
[In] Int[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (-4*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) + (2*(a - b)*Sqrt[a + b]*
(8*a^2 + 9*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sq
rt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sech[c + d*x]))/(a - b))])/(15*b^4*d) - (4*Sqrt[a + b]*Coth[c + d*x]
```

```
*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b
))]])/(b*d) + (2*Sqrt[a + b]*(8*a^2 - 2*a*b + 9*b^2)*Coth[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))]])/(15*b
^3*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))]])/(a*d) - (8*a*Sqrt[a +
b*Sech[c + d*x]]*Tanh[c + d*x])/(15*b^2*d) + (2*Sech[c + d*x]*Sqrt[a + b*Se
ch[c + d*x]]*Tanh[c + d*x])/(5*b*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3922

```
Int[csc[(e_.) + (f_.)*(x_)]^2/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x
_Symbol] :> -Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x] + Int[Csc[e + f*
x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f},
 x] && NeQ[a^2 - b^2, 0]
```

Rule 3945

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cos[e + f*x]*(d*Csc[e + f*x])^(n - 2)*(S
qrt[a + b*Csc[e + f*x]]/(b*f*(2*n - 3))), x] + Dist[d^3/(b*(2*n - 3)), Int[
((d*Csc[e + f*x])^(n - 3)/Sqrt[a + b*Csc[e + f*x]])*Simp[2*a*(n - 3) + b*(2
*n - 5)*Csc[e + f*x] - 2*a*(n - 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 2] && IntegerQ[2*n]
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
```

```
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{\sqrt{a + b\text{sech}(c+dx)}} - \frac{2\text{sech}^2(c+dx)}{\sqrt{a + b\text{sech}(c+dx)}} + \frac{\text{sech}^4(c+dx)}{\sqrt{a + b\text{sech}(c+dx)}} \right) dx$$

$$= -\left(2\int \frac{\text{sech}^2(c+dx)}{\sqrt{a + b\text{sech}(c+dx)}}\,dx\right) + \int \frac{1}{\sqrt{a + b\text{sech}(c+dx)}}\,dx + \int \frac{\text{sech}^4(c+dx)}{\sqrt{a + b\text{sech}(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a}}}{ad}$$

$$+\frac{2\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{5bd}$$

$$+2\int\frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx-2\int\frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}dx$$

$$+\frac{\int\frac{\text{sech}(c+dx)\left(2a+3b\text{sech}(c+dx)-4a\text{sech}^2(c+dx)\right)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{5b}$$

$$=$$

$$-\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{b^2d}$$

$$-\frac{4\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(}{a-b}}}{bd}$$

$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{ad}$$

$$-\frac{8a\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{15b^2d}$$

$$+\frac{2\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{5bd}$$

$$+\frac{2\int\frac{\text{sech}(c+dx)\left(ab+\frac{1}{2}(8a^2+9b^2)\text{sech}(c+dx)\right)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{15b^2}$$

$$= -\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+d}{a-b}}}{b^2d}$$
$$-\frac{4\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{bd}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$
$$-\frac{8a\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{15b^2d}$$
$$+\frac{2\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{5bd}$$
$$+\frac{1}{15}\left(9+\frac{8a^2}{b^2}\right)\int\frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$-\frac{(8a^2-2ab+9b^2)\int\frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{15b^2}$$

$$= -\frac{4(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+d}{a-b}}}{b^2d}$$
$$+\frac{2(a-b)\sqrt{a+b}(8a^2+9b^2)\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{15b^4d}$$
$$-\frac{4\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{bd}$$
$$+\frac{2\sqrt{a+b}(8a^2-2ab+9b^2)\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+a}{a+b}}}{15b^3d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$
$$-\frac{8a\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{15b^2d}$$
$$+\frac{2\text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{5bd}$$

Mathematica [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \$\text{Aborted}$$

```
[In] Integrate[Tanh[c + d*x]^4/Sqrt[a + b*Sech[c + d*x]],x]
[Out] $Aborted
```

Maple [F]

$$\int \frac{\tanh(dx+c)^4}{\sqrt{a+b\,\text{sech}(dx+c)}}dx$$

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)^4}{\sqrt{b\,\text{sech}(dx+c)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh^4(c+dx)}{\sqrt{a+b\,\text{sech}(c+dx)}}\,dx$$

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**4/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\tanh\left(dx+c\right)^4}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\tanh\left(dx+c\right)^4}{\sqrt{b\,\mathrm{sech}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^4/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \int \frac{\tanh(c+d\,x)^4}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(1/2), x)
```

3.139 $\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}} dx$

Optimal result

Integrand size = 23, antiderivative size = 310

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}} dx = -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{b^2d} - \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{bd} + \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+d}{a-b}}}{ad}$$

```
[Out] -2*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c)
)/(a-b))^(1/2)/b^2/d-2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*
(1+sech(d*x+c))/(a-b))^(1/2)/b/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))
^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+
c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3979, 4144, 4006, 3869, 3917, 4089}

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d} - \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{bd} + \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

```
[In] Int[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*d) - (2*Sqrt[a + b]*Coth[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\int \frac{-1+\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= -\int \frac{-1-\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx - \int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{b^2d} + \int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx + \int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+d}{a-b}}}{b^2d} - \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{bd} + \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$

Mathematica [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

```
[In] Integrate[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
[Out] Integrate[Tanh[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]], x]
```

Maple [F]

$$\int \frac{\tanh(dx+c)^2}{\sqrt{a+b\,\text{sech}\,(dx+c)}}dx$$

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(dx+c)^2}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh^2{(c+dx)}}{\sqrt{a+b\operatorname{sech}{(c+dx)}}}\,dx$$

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(tanh(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh{(dx+c)}^2}{\sqrt{b\operatorname{sech}{(dx+c)}+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh{(dx+c)}^2}{\sqrt{b\operatorname{sech}{(dx+c)}+a}}\,dx$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\tanh(c+d\,x)^2}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

3.140 $\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ad}$$

```
[Out] 2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)
/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+
c))/(a-b))^(1/2)/a/d
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3869}

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad}$$

```
[In] Int[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

integral

$$= \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ad}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$
$$= \frac{2b\sqrt{b+a\cosh(c+dx)}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a}\sqrt{b+a\cosh(c+dx)}}{\sqrt{a+b}\sqrt{a\cosh(c+dx)}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1+\text{sech}(c+dx))}{-a+b}}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a}\sqrt{a+b}d\sqrt{a\cosh(c+dx)}\sqrt{-\frac{b(-1+\text{sech}(c+dx))}{a+b}}\sqrt{a+b\text{sech}(c+dx)}}$$

```
[In] Integrate[1/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[b + a*Cosh[c + d*x]]*EllipticPi[(a + b)/a, ArcSin[(Sqrt[a]*Sqrt[b
 + a*Cosh[c + d*x]])/(Sqrt[a + b]*Sqrt[a*Cosh[c + d*x]])], (a + b)/(a - b)]
*Sqrt[(b*(1 + Sech[c + d*x]))/(-a + b)]*Tanh[(c + d*x)/2])/(Sqrt[a]*Sqrt[a
+ b]*d*Sqrt[a*Cosh[c + d*x]]*Sqrt[-((b*(-1 + Sech[c + d*x]))/(a + b))]*Sqrt
[a + b*Sech[c + d*x]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a+b\ \text{sech}\,(dx+c)}}dx$$

```
[In] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

```
[Out] int(1/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\text{sech}\,(c+dx)}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx=\int \frac{1}{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}\,dx$$

```
[In] int(1/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(1/(a + b/cosh(c + d*x))^(1/2), x)
```

3.141 $\int \frac{\coth^2(c+dx)}{\sqrt{a+b\,\mathrm{sech}(c+dx)}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 362

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\mathrm{sech}(c+dx)}}\,dx = \frac{\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{\sqrt{a+b}d}$$
$$-\frac{\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{\sqrt{a+b}d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{ad}$$
$$-\frac{\coth(c+dx)}{d\sqrt{a+b\mathrm{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\mathrm{sech}(c+dx)}}$$

```
[Out] coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/d/(a+b
)^(1/2)-coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2
)/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2
),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*
(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d-coth(d*x+c)/d/(a+b*sech(d*x+c))^(1/2)-
b^2*tanh(d*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3981, 3869, 3960, 3918, 21, 3914, 3917, 4089}

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx = -\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d\sqrt{a+b}} + \frac{\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d\sqrt{a+b}} + \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}}{ad} - \frac{\coth(c+dx)}{d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

```
[In] Int[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
```

```
[Out] (Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c +
 d*x]))/(a - b))])/(Sqrt[a + b]*d) - (Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*
x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(Sqrt[a + b]*d) + (
2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*d) - Coth[c + d*x]/(d*Sqrt[a
 + b*Sech[c + d*x]]) - (b^2*Tanh[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sech[c
 + d*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
```

```
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3914

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
```

```
f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\text{integral} = -\int \left(-\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}-\frac{\text{csch}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\right)dx$$

$$=\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx+\int \frac{\text{csch}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

$$=\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a}}}{ad}$$
$$-\frac{\coth(c+dx)}{d\sqrt{a+b\text{sech}(c+dx)}}+\frac{1}{2}b\int \frac{\text{sech}(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

$$=\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a}}}{ad}$$
$$-\frac{\coth(c+dx)}{d\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$
$$-\frac{b\int \frac{\text{sech}(c+dx)\left(-\frac{a}{2}-\frac{1}{2}b\text{sech}(c+dx)\right)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a^2-b^2}$$

$$=\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a}}}{ad}$$
$$-\frac{\coth(c+dx)}{d\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$
$$+\frac{b\int \text{sech}(c+dx)\sqrt{a+b\text{sech}(c+dx)}\,dx}{2(a^2-b^2)}$$

$$=\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sec}}{a}}}{ad}$$
$$-\frac{\coth(c+dx)}{d\sqrt{a+b\text{sech}(c+dx)}}-\frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$
$$+\frac{b\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{2(a+b)}+\frac{b^2\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{2(a^2-b^2)}$$

$$= \frac{\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{\sqrt{a+b}d}$$
$$- \frac{\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{\sqrt{a+b}d}$$
$$+ \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{ad}$$
$$- \frac{\coth(c+dx)}{d\sqrt{a+b\text{sech}(c+dx)}} - \frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$

Mathematica [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx$$

```
[In] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]],x]
[Out] Integrate[Coth[c + d*x]^2/Sqrt[a + b*Sech[c + d*x]], x]
```

Maple [F]

$$\int \frac{\coth(dx+c)^2}{\sqrt{a+b\,\text{sech}\,(dx+c)}}dx$$

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth(dx+c)^2}{\sqrt{b\,\text{sech}\,(dx+c)+a}}\,dx$$

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth^2\left(c+dx\right)}{\sqrt{a+b\operatorname{sech}\left(c+dx\right)}}\,dx$$

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(1/2),x)
[Out] Integral(coth(c + d*x)**2/sqrt(a + b*sech(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth\left(dx+c\right)^2}{\sqrt{b\operatorname{sech}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Giac [F]

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{\coth\left(dx+c\right)^2}{\sqrt{b\operatorname{sech}\left(dx+c\right)+a}}\,dx$$

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^2/sqrt(b*sech(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx = \int \frac{{\mathrm{coth}\left(c+d\,x\right)}^2}{\sqrt{a+\frac{b}{\mathrm{cosh}\left(c+d\,x\right)}}}\,dx$$

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2),x)
[Out] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(1/2), x)
```

3.142 $\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2(3a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d} + \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{b^4d} - \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*a*(a+b*sech(d*x+c))^
(3/2)/b^4/d-2/5*(a+b*sech(d*x+c))^(5/2)/b^4/d-2*(a^2-b^2)^2/a/b^4/d/(a+b*se
ch(d*x+c))^(1/2)-2*(3*a^2-2*b^2)*(a+b*sech(d*x+c))^(1/2)/b^4/d
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3970, 912, 1275, 212}

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2(3a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d} - \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d} + \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{b^4d}$$

```
[In] Int[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - (2*(a^2 - b^2)
^2)/(a*b^4*d*Sqrt[a + b*Sech[c + d*x]]) - (2*(3*a^2 - 2*b^2)*Sqrt[a + b*Sec
h[c + d*x]])/(b^4*d) + (2*a*(a + b*Sech[c + d*x])^(3/2))/(b^4*d) - (2*(a +
b*Sech[c + d*x])^(5/2))/(5*b^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{(b^2-x^2)^2}{x(a+x)^{3/2}}\,dx, x, b\text{sech}(c+dx)\right)}{b^4d}\\
&= -\frac{2\text{Subst}\left(\int \frac{(-a^2+b^2+2ax^2-x^4)^2}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4d}\\
&= -\frac{2\text{Subst}\left(\int \left(3a^2\left(1-\frac{2b^2}{3a^2}\right)-\frac{(a^2-b^2)^2}{ax^2}-3ax^2+x^4-\frac{b^4}{a(a-x^2)}\right)\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^4d}
\end{aligned}
$$

$$= -\frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2(3a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d} + \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{b^4d} - \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d} + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2(a^2-b^2)^2}{ab^4d\sqrt{a+b\text{sech}(c+dx)}} - \frac{2(3a^2-2b^2)\sqrt{a+b\text{sech}(c+dx)}}{b^4d} + \frac{2a(a+b\text{sech}(c+dx))^{3/2}}{b^4d} - \frac{2(a+b\text{sech}(c+dx))^{5/2}}{5b^4d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{2\left(5b^4\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\text{sech}(c+dx)}{a}\right)+a\left(4a(4a^2-5b^2)+2b(4a^2-5b^2)\,\text{sech}(c+dx)-2a\right.\right.}{5ab^4d\sqrt{a+b\text{sech}(c+dx)}}$$

```
[In] Integrate[Tanh[c + d*x]^5/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (-2*(5*b^4*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a] + a*(4*
a*(4*a^2 - 5*b^2) + 2*b*(4*a^2 - 5*b^2)*Sech[c + d*x] - 2*a*b^2*Sech[c + d*
x]^2 + b^3*Sech[c + d*x]^3)))/(5*a*b^4*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F]

$$\int \frac{\tanh(dx+c)^5}{(a+b\ \text{sech}(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1743 vs. 2(132) = 264.

Time = 0.69 (sec) , antiderivative size = 3745, normalized size of antiderivative = 25.30

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] [1/10*(5*(a*b^4*cosh(d*x + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x +
c)^5 + 3*a*b^4*cosh(d*x + c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x +
 c)^2 + 2*b^5*cosh(d*x + c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)
^5 + a*b^4 + (15*a*b^4*cosh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*si
nh(d*x + c)^4 + 4*(5*a*b^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^
4*cosh(d*x + c) + b^5)*sinh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5
*cosh(d*x + c)^3 + 18*a*b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^
4)*sinh(d*x + c)^2 + 2*(3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6
*a*b^4*cosh(d*x + c)^3 + 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^
5)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)
^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)*sinh(d*x + c)^3
+ 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (12*a^2*cosh(d*x +
c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*c
osh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (4*a*cosh(d*x + c)
 + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*co
sh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*a*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*
sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*
b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(co
sh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 4*((16*
a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^
4)*sinh(d*x + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b -
 10*a^2*b^3 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c
)^5 + 16*a^5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh
(d*x + c)^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5
*a*b^4)*cosh(d*x + c)^2 + 20*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^4 + 32*(a^4*b - a^2*b^3)*cosh(d*x + c)^3 + 4*(8*a^4*b - 8*a^2*b^3 + 5*
(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^3 + 10*(4*a^4*b - 5*a^2*b^3)*
cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^2 + (48*a^5 - 68*a^
3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^4 + 40*
(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^3 + 6*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*
cosh(d*x + c)^2 + 96*(a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(
4*a^4*b - 5*a^2*b^3)*cosh(d*x + c) + 2*(3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*c
osh(d*x + c)^5 + 8*a^4*b - 10*a^2*b^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x +
```

c)^4 + 2*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^3 + 48*(a^4*b - a^
2*b^3)*cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*si
nh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^4*d*cosh(d*x
 + c)^6 + a^3*b^4*d*sinh(d*x + c)^6 + 2*a^2*b^5*d*cosh(d*x + c)^5 + 3*a^3*b
^4*d*cosh(d*x + c)^4 + 4*a^2*b^5*d*cosh(d*x + c)^3 + 3*a^3*b^4*d*cosh(d*x +
 c)^2 + 2*a^2*b^5*d*cosh(d*x + c) + a^3*b^4*d + 2*(3*a^3*b^4*d*cosh(d*x + c
) + a^2*b^5*d)*sinh(d*x + c)^5 + (15*a^3*b^4*d*cosh(d*x + c)^2 + 10*a^2*b^5
*d*cosh(d*x + c) + 3*a^3*b^4*d)*sinh(d*x + c)^4 + 4*(5*a^3*b^4*d*cosh(d*x +
 c)^3 + 5*a^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5*d
)*sinh(d*x + c)^3 + (15*a^3*b^4*d*cosh(d*x + c)^4 + 20*a^2*b^5*d*cosh(d*x +
 c)^3 + 18*a^3*b^4*d*cosh(d*x + c)^2 + 12*a^2*b^5*d*cosh(d*x + c) + 3*a^3*b
^4*d)*sinh(d*x + c)^2 + 2*(3*a^3*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^5*d*cosh(d
*x + c)^4 + 6*a^3*b^4*d*cosh(d*x + c)^3 + 6*a^2*b^5*d*cosh(d*x + c)^2 + 3*a
^3*b^4*d*cosh(d*x + c) + a^2*b^5*d)*sinh(d*x + c)), -1/5*(5*(a*b^4*cosh(d*x
 + c)^6 + a*b^4*sinh(d*x + c)^6 + 2*b^5*cosh(d*x + c)^5 + 3*a*b^4*cosh(d*x
+ c)^4 + 4*b^5*cosh(d*x + c)^3 + 3*a*b^4*cosh(d*x + c)^2 + 2*b^5*cosh(d*x +
 c) + 2*(3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c)^5 + a*b^4 + (15*a*b^4*c
osh(d*x + c)^2 + 10*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^4 + 4*(5*a*b
^4*cosh(d*x + c)^3 + 5*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*s
inh(d*x + c)^3 + (15*a*b^4*cosh(d*x + c)^4 + 20*b^5*cosh(d*x + c)^3 + 18*a*
b^4*cosh(d*x + c)^2 + 12*b^5*cosh(d*x + c) + 3*a*b^4)*sinh(d*x + c)^2 + 2*(
3*a*b^4*cosh(d*x + c)^5 + 5*b^5*cosh(d*x + c)^4 + 6*a*b^4*cosh(d*x + c)^3 +
 6*b^5*cosh(d*x + c)^2 + 3*a*b^4*cosh(d*x + c) + b^5)*sinh(d*x + c))*sqrt(-
a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*c
osh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/co
sh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*((16*a^5 - 20*a^
3*b^2 + 5*a*b^4)*cosh(d*x + c)^6 + (16*a^5 - 20*a^3*b^2 + 5*a*b^4)*sinh(d*x
 + c)^6 + 4*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^5 + 2*(8*a^4*b - 10*a^2*b^3
 + 3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 16*a^
5 - 20*a^3*b^2 + 5*a*b^4 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^4
 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos
h(d*x + c)^2 + 20*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 32
*(a^4*b - a^2*b^3)*cosh(d*x + c)^3 + 4*(8*a^4*b - 8*a^2*b^3 + 5*(16*a^5 - 2
0*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x +
c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (4
8*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*
a*b^4 + 15*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c)^4 + 40*(4*a^4*b -
5*a^2*b^3)*cosh(d*x + c)^3 + 6*(48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x +
c)^2 + 96*(a^4*b - a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(4*a^4*b - 5
*a^2*b^3)*cosh(d*x + c) + 2*(3*(16*a^5 - 20*a^3*b^2 + 5*a*b^4)*cosh(d*x + c
)^5 + 8*a^4*b - 10*a^2*b^3 + 10*(4*a^4*b - 5*a^2*b^3)*cosh(d*x + c)^4 + 2*(
48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c)^3 + 48*(a^4*b - a^2*b^3)*cosh
(d*x + c)^2 + (48*a^5 - 68*a^3*b^2 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)
)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^4*d*cosh(d*x + c)^6 + a

```
^3*b^4*d*sinh(d*x + c)^6 + 2*a^2*b^5*d*cosh(d*x + c)^5 + 3*a^3*b^4*d*cosh(d
*x + c)^4 + 4*a^2*b^5*d*cosh(d*x + c)^3 + 3*a^3*b^4*d*cosh(d*x + c)^2 + 2*a
^2*b^5*d*cosh(d*x + c) + a^3*b^4*d + 2*(3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5
*d)*sinh(d*x + c)^5 + (15*a^3*b^4*d*cosh(d*x + c)^2 + 10*a^2*b^5*d*cosh(d*x
 + c) + 3*a^3*b^4*d)*sinh(d*x + c)^4 + 4*(5*a^3*b^4*d*cosh(d*x + c)^3 + 5*a
^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*cosh(d*x + c) + a^2*b^5*d)*sinh(d*x
+ c)^3 + (15*a^3*b^4*d*cosh(d*x + c)^4 + 20*a^2*b^5*d*cosh(d*x + c)^3 + 18*
a^3*b^4*d*cosh(d*x + c)^2 + 12*a^2*b^5*d*cosh(d*x + c) + 3*a^3*b^4*d)*sinh(
d*x + c)^2 + 2*(3*a^3*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^5*d*cosh(d*x + c)^4 +
 6*a^3*b^4*d*cosh(d*x + c)^3 + 6*a^2*b^5*d*cosh(d*x + c)^2 + 3*a^3*b^4*d*co
sh(d*x + c) + a^2*b^5*d)*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^5{(c+dx)}}{(a+b\operatorname{sech}{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)**5/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)**5/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh{(dx+c)}^5}{(b\operatorname{sech}{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^5(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh{(dx+c)}^5}{(b\operatorname{sech}{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^5/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^5/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(c+d\,x)^5}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^5/(a + b/cosh(c + d*x))^(3/2), x)
```

3.143 $\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}} dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d+2*(a^2-b^2)/a/b^2/d/(a
+b*sech(d*x+c))^(1/2)+2*(a+b*sech(d*x+c))^(1/2)/b^2/d
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3970, 912, 1275, 212}

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}} dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d}$$

```
[In] Int[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) + (2*(a^2 - b^2)
)/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*Sqrt[a + b*Sech[c + d*x]])/(b^2*
d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{b^2-x^2}{x(a+x)^{3/2}}\,dx, x, b\text{sech}(c+dx)\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \frac{-a^2+b^2+2ax^2-x^4}{x^2(-a+x^2)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d} \\
&= -\frac{2\text{Subst}\left(\int \left(-1+\frac{a^2-b^2}{ax^2}-\frac{b^2}{a(a-x^2)}\right)\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{b^2d} \\
&= \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2\sqrt{a+b\text{sech}(c+dx)}}{b^2d} \\
&\quad +\frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad}
\end{aligned}
$$

$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2(a^2-b^2)}{ab^2d\sqrt{a+b\operatorname{sech}(c+dx)}} + \frac{2\sqrt{a+b\operatorname{sech}(c+dx)}}{b^2d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \frac{2\left(-b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\operatorname{sech}(c+dx)}{a}\right)+a(2a+b\operatorname{sech}(c+d$$

$$ab^2d\sqrt{a+b\operatorname{sech}(c+dx)}$$

```
[In] Integrate[Tanh[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
[Out] (2*(-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a]) + a*(2*
a + b*Sech[c + d*x])))/(a*b^2*d*Sqrt[a + b*Sech[c + d*x]])
```

Maple [F]

$$\int \frac{\tanh(dx+c)^3}{(a+b\operatorname{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. 2(78) = 156.

Time = 0.65 (sec) , antiderivative size = 1107, normalized size of antiderivative = 12.58

$$\int \frac{\tanh^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] [1/2*((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3*cosh(d*x + c)
+ a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(a)*log(-(2*a^2*
cosh(d*x + c)^4 + 2*a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*
cosh(d*x + c) + a*b)*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*
cosh(d*x + c)^2 + (12*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 +
b^2)*sinh(d*x + c)^2 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b
*cosh(d*x + c)^3 + (4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x +
```

```
c)^2 + (6*a*cosh(d*x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b
*cosh(d*x + c) + (4*a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x
+ c) + b)*sinh(d*x + c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x +
c)) + 2*(4*a^2*cosh(d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b
^2)*cosh(d*x + c))*sinh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d
*x + c) + sinh(d*x + c)^2)) + 4*(2*a^2*b*cosh(d*x + c) + 2*a^3 - a*b^2 + (2
*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)*sinh(d*x + c)^2 + 2*(a^2*b
+ (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a*cosh(d*x + c) + b)/
cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^3*b^2*d*sinh(d*x + c)^2 + 2*
a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^2*d*cosh(d*x + c) + a^2*b^3*
d)*sinh(d*x + c)), -((a*b^2*cosh(d*x + c)^2 + a*b^2*sinh(d*x + c)^2 + 2*b^3
*cosh(d*x + c) + a*b^2 + 2*(a*b^2*cosh(d*x + c) + b^3)*sinh(d*x + c))*sqrt(
-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*a*
cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)/c
osh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x +
 c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) - 2*(2*a^2*b*cosh(d
*x + c) + 2*a^3 - a*b^2 + (2*a^3 - a*b^2)*cosh(d*x + c)^2 + (2*a^3 - a*b^2)
*sinh(d*x + c)^2 + 2*(a^2*b + (2*a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))
*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*b^2*d*cosh(d*x + c)^2 + a^
3*b^2*d*sinh(d*x + c)^2 + 2*a^2*b^3*d*cosh(d*x + c) + a^3*b^2*d + 2*(a^3*b^
2*d*cosh(d*x + c) + a^2*b^3*d)*sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^3{(c+dx)}}{(a+b\operatorname{sech}{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh{(dx+c)}^3}{(b\operatorname{sech}{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)^3}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

3.144 $\int \frac{\tanh(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{\tanh(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx=\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-2/a/d/(a+b*sech(d*x+c)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3970, 53, 65, 213}

$$\int \frac{\tanh(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx=\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{2}{ad\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

```
[In] Int[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - 2/(a*d*Sqrt[a
+ b*Sech[c + d*x]])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}}\,dx, x, b\text{sech}(c+dx)\right)}{d}\\
&= -\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+x}}\,dx, x, b\text{sech}(c+dx)\right)}{ad}\\
&= -\frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}} - \frac{2\text{Subst}\left(\int \frac{1}{-a+x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad}\\
&= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2}{ad\sqrt{a+b\text{sech}(c+dx)}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{\tanh(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = -\frac{2\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+\frac{b\mathrm{sech}(c+dx)}{a}\right)}{ad\sqrt{a+b\mathrm{sech}(c+dx)}}$$

```
[In] Integrate[Tanh[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a])/(a*d*Sqrt[a +
 b*Sech[c + d*x]])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{\frac{2}{a\sqrt{a+b\,\mathrm{sech}(dx+c)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	46
default	$-\frac{\frac{2}{a\sqrt{a+b\,\mathrm{sech}(dx+c)}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(dx+c)}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{d}$	46

```
[In] int(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(2/a/(a+b*sech(d*x+c))^(1/2)-2/a^(3/2)*arctanh((a+b*sech(d*x+c))^(1/2)
/a^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(46) = 92$.

Time = 0.64 (sec) , antiderivative size = 917, normalized size of antiderivative = 16.98

$$\int \frac{\tanh(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + 2*b*cosh(d*x + c) + 2*(a*cos
h(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(a)*log(-(2*a^2*cosh(d*x + c)^4 + 2*
a^2*sinh(d*x + c)^4 + 4*a*b*cosh(d*x + c)^3 + 4*(2*a^2*cosh(d*x + c) + a*b)
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + (4*a^2 + b^2)*cosh(d*x + c)^2 + (1
```

```
2*a^2*cosh(d*x + c)^2 + 12*a*b*cosh(d*x + c) + 4*a^2 + b^2)*sinh(d*x + c)^2
 + 2*a^2 + 2*(a*cosh(d*x + c)^4 + a*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 + (
4*a*cosh(d*x + c) + b)*sinh(d*x + c)^3 + 2*a*cosh(d*x + c)^2 + (6*a*cosh(d*
x + c)^2 + 3*b*cosh(d*x + c) + 2*a)*sinh(d*x + c)^2 + b*cosh(d*x + c) + (4*
a*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 4*a*cosh(d*x + c) + b)*sinh(d*x +
 c) + a)*sqrt(a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)) + 2*(4*a^2*cosh(
d*x + c)^3 + 6*a*b*cosh(d*x + c)^2 + 2*a*b + (4*a^2 + b^2)*cosh(d*x + c))*s
inh(d*x + c))/(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +
 c)^2)) - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d
*x + c)^2 + a)*sqrt((a*cosh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x +
 c)^2 + a^3*d*sinh(d*x + c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d*
cosh(d*x + c) + a^2*b*d)*sinh(d*x + c)), -((a*cosh(d*x + c)^2 + a*sinh(d*x
+ c)^2 + 2*b*cosh(d*x + c) + 2*(a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqr
t(-a)*arctan((a*cosh(d*x + c)^2 + a*sinh(d*x + c)^2 + b*cosh(d*x + c) + (2*
a*cosh(d*x + c) + b)*sinh(d*x + c) + a)*sqrt(-a)*sqrt((a*cosh(d*x + c) + b)
/cosh(d*x + c))/(a^2*cosh(d*x + c)^2 + a^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x
 + c) + a^2 + 2*(a^2*cosh(d*x + c) + a*b)*sinh(d*x + c))) + 2*(a*cosh(d*x +
 c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt((a*co
sh(d*x + c) + b)/cosh(d*x + c)))/(a^3*d*cosh(d*x + c)^2 + a^3*d*sinh(d*x +
c)^2 + 2*a^2*b*d*cosh(d*x + c) + a^3*d + 2*(a^3*d*cosh(d*x + c) + a^2*b*d)*
sinh(d*x + c))]
```

Sympy [F]

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh{(c+dx)}}{(a+b\operatorname{sech}{(c+dx)})^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh{(dx+c)}}{(b\operatorname{sech}{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)}{\left(b\operatorname{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(tanh(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(tanh(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\tanh(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}{\sqrt{a}}\right)}{a^{3/2}\,d} - \frac{2}{a\,d\,\sqrt{a+\frac{b}{\cosh(c+d\,x)}}}$$

[In] int(tanh(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)

[Out] (2*atanh((a + b/cosh(c + d*x))^(1/2)/a^(1/2)))/(a^(3/2)*d) - 2/(a*d*(a + b/cosh(c + d*x))^(1/2))

3.145 $\int \frac{\coth(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{\coth(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx = \frac{2\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b^2}{a\,(a^2-b^2)\,d\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-arctanh((a+b*sech(d*x+
c))^(1/2)/(a-b)^(1/2))/(a-b)^(3/2)/d-arctanh((a+b*sech(d*x+c))^(1/2)/(a+b)^
(1/2))/(a+b)^(3/2)/d+2*b^2/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3970, 912, 1301, 212}

$$\int \frac{\coth(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx = \frac{2\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{2b^2}{ad\,(a^2-b^2)\,\sqrt{a+b\,\mathrm{sech}(c+dx)}} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{d(a-b)^{3/2}} - \frac{\mathrm{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

```
[In] Int[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ArcTanh[Sqrt[a
 + b*Sech[c + d*x]]/Sqrt[a - b]]/((a - b)^(3/2)*d) - ArcTanh[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]]/((a + b)^(3/2)*d) + (2*b^2)/(a*(a^2 - b^2)*d*Sqrt[
a + b*Sech[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^2\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^2)\,\text{Subst}\left(\int \left(\frac{1}{a(a^2-b^2)x^2} - \frac{1}{ab^2(a-x^2)} + \frac{1}{2(a-b)b^2(a-b-x^2)} + \frac{1}{2b^2(a+b)(a+b-x^2)}\right)\,dx, x, \sqrt{a+b\text{sech}(c+\right.}{d}$$

$$= \frac{2b^2}{a(a^2-b^2)d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{(a-b)d} - \frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{(a+b)d}$$

$$= \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}d} - \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{2b^2}{a(a^2-b^2)d\sqrt{a+b\text{sech}(c+dx)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.49

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\text{sech}(c+dx)}{a-b}\right)}{(a-b)\sqrt{a+b\text{sech}(c+dx)}} + \frac{a\,\text{Hypergeon}}{}}{bd}$$

```
[In] Integrate[Coth[c + d*x]/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] -((-(ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]]/Sqrt[a - b]) + ArcTanh[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]]/Sqrt[a + b] - (a*Hypergeometric2F1[-
1/2, 1, 1/2, (a + b*Sech[c + d*x])/(a - b)])/((a - b)*Sqrt[a + b*Sech[c + d
*x]]) + (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sech[c + d*x])/(a + b)])/
((a + b)*Sqrt[a + b*Sech[c + d*x]]) + (2*b*Hypergeometric2F1[-1/2, 1, 1/2,
1 + (b*Sech[c + d*x])/a])/(a*Sqrt[a + b*Sech[c + d*x]]))/(b*d))
```

Maple [F]

$$\int \frac{\coth(dx+c)}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(122) = 244.
Time = 3.80 (sec) , antiderivative size = 14412, normalized size of antiderivative = 101.49

$$\int \frac{\coth(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(coth(c + d*x)/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth\left(dx+c\right)}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(c+d\,x)}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)/(a + b/cosh(c + d*x))^(3/2), x)
```

3.146 $\int \frac{\coth^3(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 316

$$
\begin{aligned}
\int \frac{\coth^3(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx &= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} \\
&- \frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} \\
&+ \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d} \\
&- \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} - \frac{2b^4}{a\,(a^2-b^2)^2\,d\sqrt{a+b\,\mathrm{sech}(c+dx)}} \\
&- \frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4(a+b)^2d(1-\mathrm{sech}(c+dx))} - \frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{4(a-b)^2d(1+\mathrm{sech}(c+dx))}
\end{aligned}
$$

```
[Out] 2*arctanh((a+b*sech(d*x+c))^(1/2)/a^(1/2))/a^(3/2)/d-1/2*(2*a-3*b)*arctanh(
(a+b*sech(d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d+1/4*b*arctanh((a+b*sech(
d*x+c))^(1/2)/(a-b)^(1/2))/(a-b)^(5/2)/d-1/4*b*arctanh((a+b*sech(d*x+c))^(1
/2)/(a+b)^(1/2))/(a+b)^(5/2)/d-1/2*(2*a+3*b)*arctanh((a+b*sech(d*x+c))^(1/2
)/(a+b)^(1/2))/(a+b)^(5/2)/d-2*b^4/a/(a^2-b^2)^2/d/(a+b*sech(d*x+c))^(1/2)-
1/4*(a+b*sech(d*x+c))^(1/2)/(a+b)^2/d/(1-sech(d*x+c))-1/4*(a+b*sech(d*x+c))
^(1/2)/(a-b)^2/d/(1+sech(d*x+c))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3970, 912, 1349, 212, 205}

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{2b^4}{ad\,(a^2-b^2)^2\sqrt{a+b\text{sech}(c+dx)}} + \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4d(a-b)^{5/2}} - \frac{b\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4d(a+b)^{5/2}} - \frac{(2a-3b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2d(a-b)^{5/2}} - \frac{(2a+3b)\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}} - \frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a+b)^2(1-\text{sech}(c+dx))} - \frac{\sqrt{a+b\text{sech}(c+dx)}}{4d(a-b)^2(\text{sech}(c+dx)+1)}$$

```
[In] Int[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a]])/(a^(3/2)*d) - ((2*a - 3*b)*A
rcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(2*(a - b)^(5/2)*d) + (b*Arc
Tanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/(4*(a - b)^(5/2)*d) - (b*ArcTa
nh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(4*(a + b)^(5/2)*d) - ((2*a + 3*
b)*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/(2*(a + b)^(5/2)*d) - (2
*b^4)/(a*(a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) - Sqrt[a + b*Sech[c + d
*x]]/(4*(a + b)^2*d*(1 - Sech[c + d*x])) - Sqrt[a + b*Sech[c + d*x]]/(4*(a
- b)^2*d*(1 + Sech[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 912

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*
(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 + a*e^2)/e^2 - 2*c*d*(x^
q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegersQ[n
, p] && FractionQ[m]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 3970

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_), x_Symbol] :> Dist[-(-1)^((m - 1)/2)/(d*b^(m - 1)), Subst[Int[(b^2 - x^2
)^((m - 1)/2)*((a + x)^n/x), x], x, b*Csc[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && IntegerQ[(m - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = -\frac{b^4\text{Subst}\left(\int \frac{1}{x(a+x)^{3/2}(b^2-x^2)^2}\,dx, x, b\text{sech}(c+dx)\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \frac{1}{x^2(-a+x^2)(-a^2+b^2+2ax^2-x^4)^2}\,dx, x, \sqrt{a+b\text{sech}(c+dx)}\right)}{d}$$

$$= -\frac{(2b^4)\,\text{Subst}\left(\int \left(-\frac{1}{a(a-b)^2(a+b)^2x^2} - \frac{1}{ab^4(a-x^2)} - \frac{1}{4(a-b)b^3(a-b-x^2)^2} + \frac{2a-3b}{4(a-b)^2b^4(a-b-x^2)} + \frac{1}{4b^3(a+b)(a+b-x^2}\right.\right.}{d}$$

$$= -\frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\operatorname{sech}(c+dx)}} + \frac{2\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{ad} - \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{2(a-b)^2 d} + \frac{b\text{Subst}\left(\int \frac{1}{(a-b-x^2)^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{2(a-b)d} - \frac{b\text{Subst}\left(\int \frac{1}{(a+b-x^2)^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{2(a+b)d} - \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{2(a+b)^2 d}$$

$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} - \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} - \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{4(a+b)^2 d(1-\operatorname{sech}(c+dx))} - \frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{4(a-b)^2 d(1+\operatorname{sech}(c+dx))} + \frac{b\text{Subst}\left(\int \frac{1}{a-b-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{4(a-b)^2 d} - \frac{b\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \sqrt{a+b\operatorname{sech}(c+dx)}\right)}{4(a+b)^2 d}$$

$$= \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{(2a-3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}d} + \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a-b}}\right)}{4(a-b)^{5/2}d} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{4(a+b)^{5/2}d} - \frac{(2a+3b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d} - \frac{2b^4}{a\left(a^2-b^2\right)^2 d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{4(a+b)^2 d(1-\operatorname{sech}(c+dx))} - \frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{4(a-b)^2 d(1+\operatorname{sech}(c+dx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.69 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\text{sech}(c+dx)}{a-b}\right)}{(a-b)\sqrt{a+b\text{sech}(c+dx)}}+\frac{2a\,\text{Hyperge}}{}}{}$$

```
[In] Integrate[Coth[c + d*x]^3/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((-2*ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a - b]])/Sqrt[a - b] + (2*
ArcTanh[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]])/Sqrt[a + b] - (2*a*Hypergeo
metric2F1[-1/2, 1, 1/2, (a + b*Sech[c + d*x])/(a - b)])/((a - b)*Sqrt[a + b
*Sech[c + d*x]]) + (2*a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sech[c + d*x
])/(a + b)])/((a + b)*Sqrt[a + b*Sech[c + d*x]]) + (4*b*Hypergeometric2F1[-
1/2, 1, 1/2, 1 + (b*Sech[c + d*x])/a])/(a*Sqrt[a + b*Sech[c + d*x]]) + (b^2
*Hypergeometric2F1[-1/2, 2, 1/2, (a + b*Sech[c + d*x])/(a - b)])/((a - b)^2
*Sqrt[a + b*Sech[c + d*x]]) - (b^2*Hypergeometric2F1[-1/2, 2, 1/2, (a + b*S
ech[c + d*x])/(a + b)])/((a + b)^2*Sqrt[a + b*Sech[c + d*x]]))/(b*d)
```

Maple [F]

$$\int \frac{\coth(dx+c)^3}{(a+b\,\text{sech}(dx+c))^{\frac{3}{2}}}\,dx$$

```
[In] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

```
[Out] int(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5984 vs. 2(266) = 532.

Time = 6.62 (sec) , antiderivative size = 53212, normalized size of antiderivative = 168.39

$$\int \frac{\coth^3(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)**3/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(coth(c + d*x)**3/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^3}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^3}{(b\operatorname{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)^3/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^3/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(c+d\,x)^3}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)^3/(a + b/cosh(c + d*x))^(3/2), x)
```

3.147 $\int \frac{\tanh^4(c+dx)}{(a+b\,\text{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 907

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = -\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{4a\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{b^2\sqrt{a+b}d}$$
$$-\frac{2a(8a^2-5b^2)\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{3b^4\sqrt{a+b}d}$$
$$+\frac{2\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{4\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{b\sqrt{a+b}d}$$
$$-\frac{2(2a+b)(4a+b)\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{3b^3\sqrt{a+b}d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a^2d}$$
$$-\frac{4a\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$
$$-\frac{2a^2\text{sech}(c+dx)\tanh(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2(4a^2-b^2)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3b^2\,(a^2-b^2)\,d}$$

```
[Out] -2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
/(a+b)^(1/2)+4*a*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),
((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a
-b))^(1/2)/b^2/d/(a+b)^(1/2)-2/3*a*(8*a^2-5*b^2)*coth(d*x+c)*EllipticE((a+b
*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+
b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b^4/d/(a+b)^(1/2)+2*coth(d*x+c)*
EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-se
ch(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d/(a+b)^(1/2)+4*
coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
```

```
2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/b/d/(a
+b)^(1/2)-2/3*(2*a+b)*(4*a+b)*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+se
ch(d*x+c))/(a-b))^(1/2)/b^3/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sec
h(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-
sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-4*a*tanh(d
*x+c)/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)+2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(
a+b*sech(d*x+c))^(1/2)-2*a^2*sech(d*x+c)*tanh(d*x+c)/b/(a^2-b^2)/d/(a+b*sec
h(d*x+c))^(1/2)+2/3*(4*a^2-b^2)*(a+b*sech(d*x+c))^(1/2)*tanh(d*x+c)/b^2/(a^
2-b^2)/d
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules

used = {3980, 3870, 4143, 4006, 3869, 3917, 4089, 3921, 4090, 3930, 4167}

$$\int \frac{\tanh^4(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = -\frac{2\operatorname{sech}(c+dx)\tanh(c+dx)a^2}{b(a^2-b^2)d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{4\tanh(c+dx)a}{(a^2-b^2)d\sqrt{a+b\operatorname{sech}(c+dx)}}$$
$$-\frac{2(8a^2-5b^2)\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}a}{3b^4\sqrt{a+b}d}$$
$$+\frac{4\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}a}{b^2\sqrt{a+b}d}$$
$$+\frac{2(4a^2-b^2)\sqrt{a+b\operatorname{sech}(c+dx)}\tanh(c+dx)}{3b^2(a^2-b^2)d}$$
$$-\frac{2(2a+b)(4a+b)\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}}{3b^3\sqrt{a+b}d}$$
$$+\frac{4\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}}{b\sqrt{a+b}d}$$
$$+\frac{2b^2\tanh(c+dx)}{(a^2-b^2)d\sqrt{a+b\operatorname{sech}(c+dx)}a}$$
$$-\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}}{\sqrt{a+b}da}$$
$$+\frac{2\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}}{\sqrt{a+b}da}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\operatorname{sech}(c+dx)+1)}{a-b}}}{da^2}$$

```
[In] Int[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*a*Coth[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
ch[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b^2*Sqrt[
a + b]*d) - (2*a*(8*a^2 - 5*b^2)*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*
Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a + b]*d) +
```

```
(2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
 + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (4*Coth[c + d*x]*EllipticF[ArcSin[
Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(b*Sqrt[a + b
]*d) - (2*(2*a + b)*(4*a + b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
h[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*d) + (2*
Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*
Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) - (4*a*Tanh[c + d*x])/((a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])/(a*(a^2 - b^
2)*d*Sqrt[a + b*Sech[c + d*x]]) - (2*a^2*Sech[c + d*x]*Tanh[c + d*x])/(b*(a
^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]]) + (2*(4*a^2 - b^2)*Sqrt[a + b*Sech[c
 + d*x]]*Tanh[c + d*x])/(3*b^2*(a^2 - b^2)*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[a*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(
a^2 - b^2))), x] - Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Cs
c[e + f*x])^(m + 1)*(b*(m + 1) - a*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{
```

```
a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 3930

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a^2)*d^3*Cot[e + f*x]*(a + b*Csc[e + f*x])^(
m + 1)*((d*Csc[e + f*x])^(n - 3)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[d^3/
(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^
(n - 3)*Simp[a^2*(n - 3) + a*b*(m + 1)*Csc[e + f*x] - (a^2*(n - 2) + b^2*(m
 + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[m, -1] && (IGtQ[n, 3] || (IntegersQ[n + 1/2, 2*m] && GtQ[n,
 2]))
```

Rule 3980

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Csc[c + d
*x]^2)^(m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && I
GtQ[m/2, 0] && IntegerQ[n - 1/2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{(a+b\text{sech}(c+dx))^{3/2}}-\frac{2\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}+\frac{\text{sech}^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\right)\,dx\\
&= -\left(2\int \frac{\text{sech}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx\right)\\
&\quad +\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx+\int \frac{\text{sech}^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx\\
&= -\frac{4a\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}+\frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}\\
&\quad -\frac{2a^2\text{sech}(c+dx)\tanh(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}-\frac{4\int \frac{\text{sech}(c+dx)\left(-\frac{b}{2}-\frac{1}{2}a\text{sech}(c+dx)\right)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a^2-b^2}\\
&\quad -\frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\frac{1}{2}ab\text{sech}(c+dx)+\frac{1}{2}b^2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\,(a^2-b^2)}\\
&\quad -\frac{2\int \frac{\text{sech}(c+dx)\left(a^2-\frac{1}{2}ab\text{sech}(c+dx)-\frac{1}{2}(4a^2-b^2)\text{sech}^2(c+dx)\right)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{b\,(a^2-b^2)}
\end{aligned}$$

$$= -\frac{4a\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$
$$- \frac{2a^2\text{sech}(c+dx)\tanh(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$
$$+ \frac{2(4a^2-b^2)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3b^2\,(a^2-b^2)\,d} - \frac{2\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a+b}$$
$$- \frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\,(a^2-b^2)} + \frac{(2a)\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a^2-b^2}$$
$$- \frac{4\int \frac{\text{sech}(c+dx)\left(\frac{1}{4}b(2a^2+b^2)+\frac{1}{4}a(8a^2-5b^2)\text{sech}(c+dx)\right)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{3b^2\,(a^2-b^2)}$$
$$- \frac{b^2\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\,(a^2-b^2)}$$

$$=$$
$$- \frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+ \frac{4a\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{b^2\sqrt{a+b}d}$$
$$+ \frac{4\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{b\sqrt{a+b}d}$$
$$- \frac{4a\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$
$$- \frac{2a^2\text{sech}(c+dx)\tanh(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\text{sech}(c+dx)}\tanh(c+dx)}{3b^2\,(a^2-b^2)\,d}$$
$$+ \frac{\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a} - \frac{b\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a(a+b)}$$
$$+ \frac{((2a+b)(4a+b))\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{3b^2(a+b)}$$
$$- \frac{(a(8a^2-5b^2))\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{3b^2\,(a^2-b^2)}$$

$$
\begin{aligned}
&= -\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} \\
&+ \frac{4a\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{b^2\sqrt{a+b}d} \\
&- \frac{2a(8a^2-5b^2)\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx)}{a-b}}}{3b^4\sqrt{a+b}d} \\
&+ \frac{2\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} \\
&+ \frac{4\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{b\sqrt{a+b}d} \\
&- \frac{2(2a+b)(4a+b)\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-b(}}{3b^3\sqrt{a+b}d} \\
&+ \frac{2\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{se}}{}}}{a^2d} \\
&- \frac{4a\tanh(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\mathrm{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\mathrm{sech}(c+dx)}} \\
&- \frac{2a^2\mathrm{sech}(c+dx)\tanh(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\mathrm{sech}(c+dx)}} + \frac{2(4a^2-b^2)\sqrt{a+b\mathrm{sech}(c+dx)}\tanh(c+dx)}{3b^2\,(a^2-b^2)\,d}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx
$$

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2),x]
[Out] Integrate[Tanh[c + d*x]^4/(a + b*Sech[c + d*x])^(3/2), x]
```

Maple [F]

$$\int \frac{\tanh(dx+c)^4}{(a+b\,\mathrm{sech}(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)^4}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^4/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^4(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)**4/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)**4/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\mathrm{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)^4}{(b\,\mathrm{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh\left(dx+c\right)^4}{\left(b\,\text{sech}\left(dx+c\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^4/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^4/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(c+d\,x)^4}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^4/(a + b/cosh(c + d*x))^(3/2), x)
```

3.148 $\int \frac{\tanh^2(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \frac{\tanh^2(c+dx)}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx = \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{ab^2d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{abd}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a^2d}$$
$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

```
[Out] 2*(a-b)*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))
/(a-b))^(1/2)/a/b^2/d+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)
^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b
*(1+sech(d*x+c))/(a-b))^(1/2)/a/b/d+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+
c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d
*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-2*tanh(d*x+c)/a/
d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3979, 4146, 4144, 4006, 3869, 3917, 4089}

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\text{arcs}\right.}{a^2d}$$
$$+\frac{2(a-b)\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ab^2d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{abd}$$
$$-\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

```
[In] Int[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d
*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*b^2*d) + (2*Sqrt[a + b]*Coth[c
+ d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/
(a - b))])/(a*b*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) -
 (2*Tanh[c + d*x])/(a*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3979

```
Int[cot[(c_.) + (d_.)*(x_)]^2*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_),
x_Symbol] :> Int[(-1 + Csc[c + d*x]^2)*(a + b*Csc[c + d*x])^n, x] /; FreeQ[
{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\text{integral} = -\int \frac{-1+\operatorname{sech}^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\operatorname{sech}(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(a^2-b^2)+\frac{1}{2}(a^2-b^2)\operatorname{sech}^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx}{a\,(a^2-b^2)}$$

$$= -\frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}} + \frac{\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a} + \frac{2\int \frac{\frac{1}{2}(a^2-b^2)-\frac{1}{2}(a^2-b^2)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a(a^2-b^2)}$$

$$= \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ab^2d}$$

$$- \frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}} + \frac{\int \frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a} - \frac{\int \frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a}$$

$$= \frac{2(a-b)\sqrt{a+b}\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{ab^2d}$$

$$+ \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+}{a-b}}}{abd}$$

$$+ \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{a^2d}$$

$$- \frac{2\tanh(c+dx)}{ad\sqrt{a+b\text{sech}(c+dx)}}$$

Mathematica [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
[Out] Integrate[Tanh[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2), x]
```

Maple [F]

$$\int \frac{\tanh(dx+c)^2}{(a+b\,\text{sech}(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)^2}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a)*tanh(d*x + c)^2/(b^2*sech(d*x + c)^2 + 2
*a*b*sech(d*x + c) + a^2), x)
```

Sympy [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh^2(c+dx)}{(a+b\,\text{sech}(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(tanh(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)^2}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(dx+c)^2}{(b\,\text{sech}(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(tanh(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(tanh(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\tanh(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(tanh(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)
```

3.149 $\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 347

$$\int \frac{1}{(a+b\,\mathrm{sech}(c+dx))^{3/2}}\,dx = -\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\coth(c+dx)\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\sqrt{a+b}\coth(c+dx)\,\mathrm{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\,\mathrm{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\mathrm{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\mathrm{sech}(c+dx))}{a-b}}}{a^2d} + \frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\,\mathrm{sech}(c+dx)}}$$

```
[Out] -2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^
(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a/d
/(a+b)^(1/2)+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b
))^(1/2)/a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2)/(
a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(a+b
))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d+2*b^2*tanh(d*x+c)/a/(a^2-b^
2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3870, 4143, 4006, 3869, 3917, 4089}

$$\int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticPi}\left(\frac{a+b}{a},\text{arcs}\right.}{a^2d}$$
$$+\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}$$
$$+\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$
$$-\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$

```
[In] Int[(a + b*Sech[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[
c + d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Coth[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*Sqrt[a +
b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x])
)/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a^2*d) + (2*b^2*Tanh[
c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
```

```
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\text{integral} = \frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\frac{1}{2}ab\operatorname{sech}(c+dx)+\frac{1}{2}b^2\operatorname{sech}^2(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx}{a(a^2-b^2)}$$

$$= \frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\operatorname{sech}(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2+b^2)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\operatorname{sech}(c+dx)}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx}{a(a^2-b^2)} - \frac{b^2\int \frac{\operatorname{sech}(c+dx)(1+\operatorname{sech}(c+dx))}{\sqrt{a+b\operatorname{sech}(c+dx)}}\,dx}{a(a^2-b^2)}$$

$$= -\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$

$$+ \frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{\int\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a} - \frac{b\int\frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a(a+b)}$$

$$=$$

$$-\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$

$$+\frac{2\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$

$$+\frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{a^2d}$$

$$+\frac{2b^2\tanh(c+dx)}{a(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}$$

Mathematica [F]

$$\int\frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int\frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Sech[c + d*x])^(-3/2),x]
[Out] Integrate[(a + b*Sech[c + d*x])^(-3/2), x]
```

Maple [F]

$$\int\frac{1}{(a+b\,\text{sech}\,(dx+c))^{\frac{3}{2}}}dx$$

```
[In] int(1/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(1/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(a + b\text{sech}(c + dx))^{3/2}} \, dx = \int \frac{1}{(b \operatorname{sech}(dx + c) + a)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sech(d*x + c) + a)/(b^2*sech(d*x + c)^2 + 2*a*b*sech(d*x +
c) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a + b\text{sech}(c + dx))^{3/2}} \, dx = \int \frac{1}{(a + b \operatorname{sech}(c + dx))^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral((a + b*sech(c + d*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b\text{sech}(c + dx))^{3/2}} \, dx = \int \frac{1}{(b \operatorname{sech}(dx + c) + a)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a + b\text{sech}(c + dx))^{3/2}} \, dx = \int \frac{1}{(b \operatorname{sech}(dx + c) + a)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*sech(d*x + c) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx=\int \frac{1}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(1/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(1/(a + b/cosh(c + d*x))^(3/2), x)
```

3.150 $\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 665

$$\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \frac{4a\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d}$$
$$-\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$-\frac{(3a-b)\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d}$$
$$+\frac{2\coth(c+dx)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d}$$
$$+\frac{2\sqrt{a+b}\coth(c+dx)\operatorname{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\operatorname{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\operatorname{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\operatorname{sech}(c+dx))}{a-b}}}{a^2d}$$
$$-\frac{\coth(c+dx)}{d(a+b\operatorname{sech}(c+dx))^{3/2}}-\frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d(a+b\operatorname{sech}(c+dx))^{3/2}}$$
$$-\frac{4ab^2\tanh(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\operatorname{sech}(c+dx)}}+\frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\operatorname{sech}(c+dx)}}$$

```
[Out] -coth(d*x+c)/d/(a+b*sech(d*x+c))^(3/2)+4*a*coth(d*x+c)*EllipticE((a+b*sech(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1
/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/d-(3*a-b)*coth(d*x+c
)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-
sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2
```

```
)/d-2*coth(d*x+c)*EllipticE((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/
a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticF((a+b*sech(d*x+c))^(1/2)/(a+b)^(1/2)
,((a+b)/(a-b))^(1/2))*(b*(1-sech(d*x+c))/(a+b))^(1/2)*(-b*(1+sech(d*x+c))/(
a-b))^(1/2)/a/d/(a+b)^(1/2)+2*coth(d*x+c)*EllipticPi((a+b*sech(d*x+c))^(1/2
)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sech(d*x+c))/(
a+b))^(1/2)*(-b*(1+sech(d*x+c))/(a-b))^(1/2)/a^2/d-b^2*tanh(d*x+c)/(a^2-b^2
)/d/(a+b*sech(d*x+c))^(3/2)-4*a*b^2*tanh(d*x+c)/(a^2-b^2)^2/d/(a+b*sech(d*x
+c))^(1/2)+2*b^2*tanh(d*x+c)/a/(a^2-b^2)/d/(a+b*sech(d*x+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3981, 3870, 4143, 4006, 3869, 3917, 4089, 3960, 3918, 4088, 4090}

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \frac{2\sqrt{a+b}\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticPi}\left(\frac{a+b}{a},\text{arcs}\right.}{a^2d}$$
$$+\frac{2b^2\tanh(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\text{sech}(c+dx)}}-\frac{4ab^2\tanh(c+dx)}{d\left(a^2-b^2\right)^2\sqrt{a+b\text{sech}(c+dx)}}$$
$$-\frac{b^2\tanh(c+dx)}{d\left(a^2-b^2\right)(a+b\text{sech}(c+dx))^{3/2}}$$
$$+\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$
$$-\frac{(3a-b)\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}$$
$$-\frac{2\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad\sqrt{a+b}}$$
$$+\frac{4a\coth(c+dx)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(\text{sech}(c+dx)+1)}{a-b}}E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d(a-b)(a+b)^{3/2}}$$
$$-\frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}}$$

```
[In] Int[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
```

```
[Out] (4*a*Coth[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech
[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) - (2*Coth[c + d*x]*Ellipti
```

```
cE[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/(a*
Sqrt[a + b]*d) - ((3*a - b)*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*d) + (
2*Coth[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c
+ d*x]))/(a - b))])/(a*Sqrt[a + b]*d) + (2*Sqrt[a + b]*Coth[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sech[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sech[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sech[c + d*x]))/(
a - b))])/(a^2*d) - Coth[c + d*x]/(d*(a + b*Sech[c + d*x])^(3/2)) - (b^2*Ta
nh[c + d*x])/((a^2 - b^2)*d*(a + b*Sech[c + d*x])^(3/2)) - (4*a*b^2*Tanh[c
+ d*x])/((a^2 - b^2)^2*d*Sqrt[a + b*Sech[c + d*x]]) + (2*b^2*Tanh[c + d*x])
/(a*(a^2 - b^2)*d*Sqrt[a + b*Sech[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3918

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*C
sc[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[
```

```
{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3960

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[Tan[e + f*x]*((a + b*Csc[e + f*x])^m/f), x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3981

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_
), x_Symbol] :> Int[ExpandIntegrand[(a + b*Csc[c + d*x])^n, (-1 + Sec[c + d
*x]^2)^(-m/2), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] &&
ILtQ[m/2, 0] && IntegerQ[n - 1/2] && EqQ[m, -2]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
```

```
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\int \left(-\frac{1}{(a+b\text{sech}(c+dx))^{3/2}}-\frac{\text{csch}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\right) dx\\
&= \int \frac{1}{(a+b\text{sech}(c+dx))^{3/2}}\,dx+\int \frac{\text{csch}^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx\\
&= -\frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}}+\frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}}\\
&\quad+\frac{1}{2}(3b)\int \frac{\text{sech}(c+dx)}{(a+b\text{sech}(c+dx))^{5/2}}\,dx\\
&\quad-\frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\text{sech}(c+dx)+\frac{1}{2}b^2\text{sech}^2(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)}\\
&= -\frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}}-\frac{b^2\tanh(c+dx)}{\left(a^2-b^2\right)d(a+b\text{sech}(c+dx))^{3/2}}\\
&\quad+\frac{2b^2\tanh(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\text{sech}(c+dx)}}-\frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)}\\
&\quad-\frac{b\int \frac{\text{sech}(c+dx)\left(-\frac{3a}{2}+\frac{1}{2}b\text{sech}(c+dx)\right)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx}{a^2-b^2}-\frac{b^2\int \frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}\,dx}{a\left(a^2-b^2\right)}
\end{aligned}
$$

$$= -\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} - \frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}} - \frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d(a+b\text{sech}(c+dx))^{3/2}} - \frac{4ab^2\tanh(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{\int\frac{1}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a} - \frac{b\int\frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{a(a+b)} + \frac{(2b)\int\frac{\text{sech}(c+dx)\left(\frac{1}{4}(3a^2+b^2)+ab\text{sech}(c+dx)\right)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{(a^2-b^2)^2}$$

$$= -\frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} + \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{}}}{a^2d} - \frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}} - \frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d(a+b\text{sech}(c+dx))^{3/2}} - \frac{4ab^2\tanh(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{((3a-b)b)\int\frac{\text{sech}(c+dx)}{\sqrt{a+b\text{sech}(c+dx)}}dx}{2(a-b)(a+b)^2} + \frac{(2ab^2)\int\frac{\text{sech}(c+dx)(1+\text{sech}(c+dx))}{\sqrt{a+b\text{sech}(c+dx)}}dx}{(a^2-b^2)^2}$$

$$
\begin{aligned}
&= \frac{4a\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d} \\
&\quad - \frac{2\coth(c+dx)E\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} \\
&\quad - \frac{(3a-b)\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{(a-b)(a+b)^{3/2}d} \\
&\quad + \frac{2\coth(c+dx)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a\sqrt{a+b}d} \\
&\quad + \frac{2\sqrt{a+b}\coth(c+dx)\,\text{EllipticPi}\left(\frac{a+b}{a},\arcsin\left(\frac{\sqrt{a+b\text{sech}(c+dx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\text{sech}(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{sech}(c+dx))}{a-b}}}{a^2d} \\
&\quad - \frac{\coth(c+dx)}{d(a+b\text{sech}(c+dx))^{3/2}} - \frac{b^2\tanh(c+dx)}{(a^2-b^2)\,d(a+b\text{sech}(c+dx))^{3/2}} \\
&\quad - \frac{4ab^2\tanh(c+dx)}{(a^2-b^2)^2\,d\sqrt{a+b\text{sech}(c+dx)}} + \frac{2b^2\tanh(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\text{sech}(c+dx)}}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx
$$

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2),x]
[Out] Integrate[Coth[c + d*x]^2/(a + b*Sech[c + d*x])^(3/2), x]
```

Maple [F]

$$
\int \frac{\coth(dx+c)^2}{(a+b\,\text{sech}(dx+c))^{\frac{3}{2}}}dx
$$

```
[In] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
[Out] int(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth^2(c+dx)}{(a+b\,\text{sech}\,(c+dx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)**2/(a+b*sech(d*x+c))**(3/2),x)
[Out] Integral(coth(c + d*x)**2/(a + b*sech(c + d*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^2}{(b\,\text{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\coth^2(c+dx)}{(a+b\text{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(dx+c)^2}{(b\,\text{sech}\,(dx+c)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(coth(d*x+c)^2/(a+b*sech(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(coth(d*x + c)^2/(b*sech(d*x + c) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{(a+b\operatorname{sech}(c+dx))^{3/2}}\,dx = \int \frac{\coth(c+d\,x)^2}{\left(a+\frac{b}{\cosh(c+d\,x)}\right)^{3/2}}\,dx$$

```
[In] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2),x)
[Out] int(coth(c + d*x)^2/(a + b/cosh(c + d*x))^(3/2), x)
```

3.151 $\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 191

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx = \frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^6} - \frac{192\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{5bc\left(1+e^{2c(a+bx)}\right)^5} + \frac{48\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} - \frac{64\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}$$

```
[Out] 32/3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^6-1
92/5*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^5+4
8*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4-64/3
*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6852, 2320, 12, 272, 45}

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx = -\frac{64\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3} + \frac{48\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4} - \frac{192\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{5bc\left(e^{2c(a+bx)}+1\right)^5} + \frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^6}$$

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
```

```
[Out] (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c*(a + b*
x)))^6) - (192*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(5*b*c*(1 + E^(
2*c*(a + b*x)))^5) + (48*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*
(1 + E^(2*c*(a + b*x)))^4) - (64*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2
])/(3*b*c*(1 + E^(2*c*(a + b*x)))^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^7(ac+bcx)\,dx\\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{128x^7}{(1+x^2)^7}\,dx,x,e^{c(a+bx)}\right)}{bc}
\end{aligned}
$$

$$= \frac{\left(128\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{x^7}{(1+x^2)^7}\,dx,x,e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\frac{x^3}{(1+x)^7}\,dx,x,e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\left(-\frac{1}{(1+x)^7}+\frac{3}{(1+x)^6}-\frac{3}{(1+x)^5}+\frac{1}{(1+x)^4}\right)\,dx,x,e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^6}-\frac{192\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{5bc\left(1+e^{2c(a+bx)}\right)^5}+\frac{48\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4}-\frac{64\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.44

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = -\frac{16\left(1+6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{15bc\left(1+e^{2c(a+bx)}\right)^6}$$

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(7/2),x]
[Out] (-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x))
)*Cosh[c*(a + b*x)]*Sqrt[Sech[c*(a + b*x)]^2])/(15*b*c*(1 + E^(2*c*(a + b*x
)))^6)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 127.39 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.45

method	result	size
default	$\frac{\operatorname{csgn}(\operatorname{sech}(c(bx+a)))\left(\frac{\tanh(c(bx+a))^6}{6}+\frac{\tanh(c(bx+a))^5}{5}-\frac{\tanh(c(bx+a))^4}{2}-\frac{2\tanh(c(bx+a))^3}{3}+\frac{\tanh(c(bx+a))^2}{2}+\tanh(c(bx+a))\right)}{cb}$	86
risch	$-\frac{16\left(20\,e^{6c(bx+a)}+15\,e^{4c(bx+a)}+6\,e^{2c(bx+a)}+1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}\,e^{-c(bx+a)}}{15bc\left(1+e^{2c(bx+a)}\right)^5}$	91

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(sech(c*(b*x+a)))/c/b*(1/6*tanh(c*(b*x+a))^6+1/5*tanh(c*(b*x+a))^5-1/2*
tanh(c*(b*x+a))^4-2/3*tanh(c*(b*x+a))^3+1/2*tanh(c*(b*x+a))^2+tanh(c*(b*x+a
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(173) = 346$.

Time = 0.29 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.08

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9+6\,bc\cosh\left(bcx+ac\right)^7\right.}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] -16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 +
 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c)
+ 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*c)*
sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)^7 +
 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh(b*c*
x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a*c)^2 +
 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*c*cosh(b
*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a*c))*sinh
(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*x + a*c)^4
 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 21*b*c*cosh(
b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*x + a*c)^5 +
 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c)^2
 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6 + 25*b*c*cosh(
b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(7/2),x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(173) = 346$.

Time = 0.21 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.02

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = -\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] -64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*
a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*
a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*b*c
*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b
*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 32/5
*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c)
+ 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c)
+ 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10
*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(
4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.34

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{7/2}\,dx = -\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc(e^{(2\,bcx+2\,ac)}+1)^6}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

```
[Out] -16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*
a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.12

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{7/2}\,dx = \frac{24\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^4} - \frac{32\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^3} - \frac{96\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{5\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^5} + \frac{16\sqrt{\frac{1}{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}\,\left(2\,e^{2\,a\,c+2\,b\,c\,x}+e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\,\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^6}$$

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(7/2),x)
```

```
[Out] (24*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c +
2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c +
3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^4) - (32*(1/(exp(a*c + b*c*x)/2 + exp(
- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*x) +
 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 + 1)^3) - (96*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*e
xp(2*a*c + 2*b*c*x) + exp(4*a*c + 4*b*c*x) + 1))/(5*b*c*(exp(a*c + b*c*x) +
 exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^5) + (16*(1/(exp(a*c + b*
c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)*(2*exp(2*a*c + 2*b*c*x) + exp(4*a*c
 + 4*b*c*x) + 1))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a
*c + 2*b*c*x) + 1)^6)
```

3.152 $\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx=-\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3}-\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

```
[Out] -4*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^4+32/
3*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^3-8*co
sh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c*(b*x+a)))^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 2320, 12, 272, 45}

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx=-\frac{8\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}+\frac{32\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{3bc\left(e^{2c(a+bx)}+1\right)^3}-\frac{4\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^4}$$

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 + E^(2*c*(a + b*x)
))^4) + (32*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(3*b*c*(1 + E^(2*c
*(a + b*x)))^3) - (8*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\text{integral} = \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^5(ac+bcx)\,dx$$

$$= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{32x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^5}{(1+x^2)^5}\,dx, x, e^{c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^2}{(1+x)^5}\,dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= \frac{\left(16\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int\left(\frac{1}{(1+x)^5}-\frac{2}{(1+x)^4}+\frac{1}{(1+x)^3}\right)dx, x, e^{2c(a+bx)}\right)}{bc}$$

$$= -\frac{4\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^4} + \frac{32\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{3bc\left(1+e^{2c(a+bx)}\right)^3} - \frac{8\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{5/2}\,dx = -\frac{4\left(1+4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\cosh(c(a+bx))\sqrt{\operatorname{sech}^2(c(a+bx))}}{3bc\left(1+e^{2c(a+bx)}\right)^4}$$

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] (-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)]*Sqrt[
Sech[c*(a + b*x)]^2])/(3*b*c*(1 + E^(2*c*(a + b*x)))^4)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 140.89 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.46

method	result	size
default	$-\frac{\operatorname{csgn}(\operatorname{sech}(c(bx+a)))\left(\frac{\tanh(c(bx+a))^4}{4}+\frac{\tanh(c(bx+a))^3}{3}-\frac{\tanh(c(bx+a))^2}{2}-\tanh(c(bx+a))\right)}{cb}$	65
risch	$-\frac{4\left(6\,e^{4c(bx+a)}+4\,e^{2c(bx+a)}+1\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}\,e^{-c(bx+a)}}{3bc\left(1+e^{2c(bx+a)}\right)^3}$	80

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -csgn(sech(c*(b*x+a)))/c/b*(1/4*tanh(c*(b*x+a))^4+1/3*tanh(c*(b*x+a))^3-1/2
*tanh(c*(b*x+a))^2-tanh(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(130) = 260$.

Time = 0.27 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.23

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}\right.}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*si
nh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)*s
inh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4 +
(15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b*c*x
 + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*sinh(b*
c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)^2 + 7
*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8*b*c*co
sh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.48

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx = -\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2*a
*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*
c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b
*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.36

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^4}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x +
 2*a*c) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.65

$$\int e^{c(a+bx)}\text{sech}^2(ac+bcx)^{5/2}\,dx = -\frac{2\,\mathrm{e}^{-a\,c-b\,c\,x}\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}\,\left(4\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+6\,\mathrm{e}^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] -(2*exp(- a*c - b*c*x)*(1/(exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1
/2)*(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*a*
c + 2*b*c*x) + 1)^3)
```

3.153 $\int e^{c(a+bx)} \mathbf{sech}^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx=\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}$$

```
[Out] 2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)*(sech(b*c*x+a*c)^2)^(1/2)/b/c/(1+exp(2*c
*(b*x+a)))^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 12, 270}

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx=\frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(e^{2c(a+bx)}+1\right)^2}$$

```
[In] Int[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] (2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x]*Sqrt[Sech[a*c + b*c*x]^2])/(b*c*(1 +
 E^(2*c*(a + b*x)))^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}^3(ac+bcx)\,dx \\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{8x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(8\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x^3}{(1+x^2)^3}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{2e^{4c(a+bx)}\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc\left(1+e^{2c(a+bx)}\right)^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = \frac{e^{3c(a+bx)}\sqrt{\operatorname{sech}^2(c(a+bx))}}{bc+bce^{2c(a+bx)}}$$

```
[In] Integrate[E^(c*(a + b*x))*(Sech[a*c + b*c*x]^2)^(3/2),x]
[Out] (E^(3*c*(a + b*x))*Sqrt[Sech[c*(a + b*x)]^2])/(b*c + b*c*E^(2*c*(a + b*x)))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\operatorname{csgn}\left(\operatorname{sech}\left(c \left(b x +a \right)\right)\right) \left(\frac{\tanh \left(c \left(b x +a \right)\right)^{2}}{2}+\tanh \left(c \left(b x +a \right)\right)\right)}{c b}$	38
risch	$-\frac{2 \left(2 \,{\mathrm e}^{2 c \left(b x +a \right)}+1\right) \sqrt{\frac{{\mathrm e}^{2 c \left(b x +a \right)}}{\left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)^{2}}}\, {\mathrm e}^{-c \left(b x +a \right)}}{b c \left(1+{\mathrm e}^{2 c \left(b x +a \right)}\right)}$	69

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] csgn(sech(c*(b*x+a)))/c/b*(1/2*tanh(c*(b*x+a))^2+tanh(c*(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(52) = 104.

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.14

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = -\frac{2\left(3\,\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)}{bc\cosh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{2}+bc\sinh\left(bcx+ac\right)^{3}+3\,bc\cosh\left(bcx+ac\right)+(3}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
[Out] -2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*b
*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*c*
cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = \text{Timed out}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(3/2),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)} - \frac{2}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
[Out] -4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}+1\right)}{bc(e^{(2\,bcx+2\,ac)}+1)^2}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
[Out] -2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.39

$$\int e^{c(a+bx)}\operatorname{sech}^2(ac+bcx)^{3/2}\,dx = -\frac{\mathrm{e}^{-a\,c-b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{\frac{1}{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}}{b\,c\,\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}$$

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(3/2),x)
[Out] -(exp(- a*c - b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*(1/(exp(a*c + b*c*x)/2 +
exp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1))
```

3.154 $\int e^{c(a+bx)}\sqrt{\text{sech}^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int e^{c(a+bx)}\sqrt{\text{sech}^2(ac+bcx)}\,dx=\frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)\sqrt{\text{sech}^2(ac+bcx)}}{bc}$$

```
[Out] cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))*(sech(b*c*x+a*c)^2)^(1/2)/b/c
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 12, 266}

$$\int e^{c(a+bx)}\sqrt{\text{sech}^2(ac+bcx)}\,dx=\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)\sqrt{\text{sech}^2(ac+bcx)}}{bc}$$

```
[In] Int[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[a*c + b*c*x]^2])/(b
*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\int e^{c(a+bx)}\operatorname{sech}(ac+bcx)\,dx \\
&= \frac{\left(\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\left(2\cosh(ac+bcx)\sqrt{\operatorname{sech}^2(ac+bcx)}\right)\operatorname{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^{c(a+bx)}\right)}{bc} \\
&= \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)\sqrt{\operatorname{sech}^2(ac+bcx)}}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \frac{\cosh(c(a+bx))\log\left(1+e^{2c(a+bx)}\right)\sqrt{\operatorname{sech}^2(c(a+bx))}}{bc}$$

```
[In] Integrate[E^(c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2],x]
[Out] (Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))]*Sqrt[Sech[c*(a + b*x)]^2])/(b
*c)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result	size
default	$\operatorname{csgn}\left(\operatorname{sech}\left(c(bx+a)\right)\right)\left(x+\frac{\ln(\cosh(c(bx+a)))}{cb}\right)$	29
risch	$\frac{\ln\left(\mathrm{e}^{2bcx}+\mathrm{e}^{-2ac}\right)\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}}}\,\mathrm{e}^{-c(bx+a)}}{bc}$	66

```
[In] int(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] csgn(sech(c*(b*x+a)))*(x+1/c/b*ln(cosh(c*(b*x+a))))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx=\frac{\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx=e^{ac}\int\sqrt{\operatorname{sech}^2\left(ac+bcx\right)}e^{bcx}\,dx$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(sqrt(sech(a*c + b*c*x)**2)*exp(b*c*x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \frac{\log\left(e^{(2\,bcx)}+e^{(-2\,ac)}\right)}{bc}$$

```
[In] integrate(exp(c*(b*x+a))*(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] log(e^(2*b*c*x) + e^(-2*a*c))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\sqrt{\operatorname{sech}^2(ac+bcx)}\,dx = \int \mathrm{e}^{c\,(a+b\,x)}\,\sqrt{\frac{1}{{\mathrm{cosh}\left(a\,c+b\,c\,x\right)}^2}}\,dx$$

```
[In] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))*(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

3.155 $\int \frac{e^{c(a+bx)}}{\sqrt{\text{sech}^2(ac+bcx)}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int \frac{e^{c(a+bx)}}{\sqrt{\text{sech}^2(ac+bcx)}} \, dx = \frac{e^{2c(a+bx)}\text{sech}(ac+bcx)}{4bc\sqrt{\text{sech}^2(ac+bcx)}} + \frac{x\text{sech}(ac+bcx)}{2\sqrt{\text{sech}^2(ac+bcx)}}$$

```
[Out] 1/4*exp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/2*x*se
ch(b*c*x+a*c)/(sech(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {6852, 2320, 12, 14}

$$\int \frac{e^{c(a+bx)}}{\sqrt{\text{sech}^2(ac+bcx)}} \, dx = \frac{e^{2c(a+bx)}\text{sech}(ac+bcx)}{4bc\sqrt{\text{sech}^2(ac+bcx)}} + \frac{x\text{sech}(ac+bcx)}{2\sqrt{\text{sech}^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
```

```
[Out] (E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(4*b*c*Sqrt[Sech[a*c + b*c*x]^2]) + (
x*Sech[a*c + b*c*x])/(2*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{sech}(ac+bcx)\int e^{c(a+bx)}\cosh(ac+bcx)\,dx}{\sqrt{\text{sech}^2(ac+bcx)}}\\
&= \frac{\text{sech}(ac+bcx)\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\text{sech}^2(ac+bcx)}}\\
&= \frac{\text{sech}(ac+bcx)\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\text{sech}^2(ac+bcx)}}\\
&= \frac{\text{sech}(ac+bcx)\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, e^{c(a+bx)}\right)}{2bc\sqrt{\text{sech}^2(ac+bcx)}}\\
&= \frac{e^{2c(a+bx)}\text{sech}(ac+bcx)}{4bc\sqrt{\text{sech}^2(ac+bcx)}}+\frac{x\text{sech}(ac+bcx)}{2\sqrt{\text{sech}^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{\left(e^{2c(a+bx)}+2bcx\right)\operatorname{sech}(c(a+bx))}{4bc\sqrt{\operatorname{sech}^2(c(a+bx))}}$$

```
[In] Integrate[E^(c*(a + b*x))/Sqrt[Sech[a*c + b*c*x]^2],x]
[Out] ((E^(2*c*(a + b*x)) + 2*b*c*x)*Sech[c*(a + b*x)])/(4*b*c*Sqrt[Sech[c*(a + b
*x)]^2])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\operatorname{csgn}(\operatorname{sech}(c(bx+a)))\left(\frac{\cosh(bcx+ac)^2}{2}+\frac{\sinh(bcx+ac)\cosh(bcx+ac)}{2}+\frac{bcx}{2}+\frac{ac}{2}\right)}{cb}$	60
risch	$\frac{x\,e^{c(bx+a)}}{2\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}}+\frac{e^{3c(bx+a)}}{4bc\left(1+e^{2c(bx+a)}\right)\sqrt{\frac{e^{2c(bx+a)}}{\left(1+e^{2c(bx+a)}\right)^2}}}$	106

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] csgn(sech(c*(b*x+a)))/c/b*(1/2*cosh(b*c*x+a*c)^2+1/2*sinh(b*c*x+a*c)*cosh(b
*c*x+a*c)+1/2*b*c*x+1/2*a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{(2\,bcx+1)\cosh(bcx+ac)-(2\,bcx-1)\sinh(bcx+ac)}{4\,(bc\cosh(bcx+ac)-bc\sinh(bcx+ac))}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
[Out] 1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\operatorname{sech}^2\left(ac+bcx\right)}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(1/2),x)
[Out] exp(a*c)*Integral(exp(b*c*x)/sqrt(sech(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \frac{\left(2\,bcxe^{(-ac)} + e^{(2\,bcx+ac)}\right)e^{(ac)}}{4\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
[Out] 1/4*(2*b*c*x*e^(-a*c) + e^(2*b*c*x + a*c))*e^(a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\,dx = \int \frac{\mathrm{e}^{c\,(a+b\,x)}}{\sqrt{\frac{1}{{\mathrm{cosh}\left(a\,c+b\,c\,x\right)}^2}}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2),x)
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(1/2), x)
```

3.156 $\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = -\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[Out] -1/16*sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+3/16*e
xp(2*c*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/32*exp(4*c*
(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+3/8*x*sech(b*c*x+a*c
)/(sech(b*c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6852, 2320, 12, 272, 45}

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = -\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] -1/16*Sech[a*c + b*c*x]/(b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]) +
 (3*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(16*b*c*Sqrt[Sech[a*c + b*c*x]^2])
```

```
 + (E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^2])
 + (3*x*Sech[a*c + b*c*x])/(8*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^3(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \frac{(1+x^2)^3}{8x^3}\,dx, x, e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, e^{c(a+bx)}\right)}{8bc\sqrt{\operatorname{sech}^2(ac+bcx)}}
\end{aligned}$$

$$= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx,x,e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= \frac{\operatorname{sech}(ac+bcx)\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx,x,e^{2c(a+bx)}\right)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

$$= -\frac{e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{16bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{3x\operatorname{sech}(ac+bcx)}{8\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.50

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(-\frac{1}{16}e^{-2c(a+bx)}+\frac{3}{16}e^{2c(a+bx)}+\frac{1}{32}e^{4c(a+bx)}+\frac{3bcx}{8}\right)\operatorname{sech}^3(c(a+bx))}{bc\operatorname{sech}^2(c(a+bx))^{3/2}}$$

```
[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(3/2),x]
```

```
[Out] ((-1/16*1/E^(2*c*(a + b*x)) + (3*E^(2*c*(a + b*x)))/16 + E^(4*c*(a + b*x))/
32 + (3*b*c*x)/8)*Sech[c*(a + b*x)]^3)/(b*c*(Sech[c*(a + b*x)]^2)^(3/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

method	result
default	$\frac{\operatorname{csgn}(\operatorname{sech}(c(bx+a)))\left(\frac{\cosh(bcx+ac)^4}{4}+\left(\frac{\cosh(bcx+ac)^3}{4}+\frac{3\cosh(bcx+ac)}{8}\right)\sinh(bcx+ac)+\frac{3bcx}{8}+\frac{3ac}{8}\right)}{cb}$
risch	$\frac{3x\,\mathrm{e}^{c(bx+a)}}{8\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+\frac{\mathrm{e}^{5c(bx+a)}}{32bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+\frac{3\,\mathrm{e}^{3c(bx+a)}}{16bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}-\frac{}{16bc(1+}$

```
[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] csgn(sech(c*(b*x+a)))/c/b*(1/4*cosh(b*c*x+a*c)^4+(1/4*cosh(b*c*x+a*c)^3+3/8
*cosh(b*c*x+a*c))*sinh(b*c*x+a*c)+3/8*b*c*x+3/8*a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = -\frac{\cosh\left(bcx+ac\right)^3+3\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2-3\,\sinh\left(bcx+ac\right)^3-6\left(2\,bcx+1\right)\cosh\left(bcx+ac\right)+}{32\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*si
nh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*cosh
(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sinh(b
*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\operatorname{sech}^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(3/2),x)
```

```
[Out] exp(a*c)*Integral(exp(b*c*x)/(sech(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{3\left(bcx+ac\right)}{8\,bc}+\frac{e^{\left(4\,bcx+4\,ac\right)}}{32\,bc}+\frac{3\,e^{\left(2\,bcx+2\,ac\right)}}{16\,bc}-\frac{e^{\left(-2\,bcx-2\,ac\right)}}{16\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
+ 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.51

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \frac{\left(12\,bcxe^{(-ac)} - 2\left(3\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-2\,bcx-3\,ac)} + \left(e^{(4\,bcx+9\,ac)} + 6\,e^{(2\,bcx+7\,ac)}\right)e^{(-6\,ac)}\right)e^{(ac)}}{32\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(12*b*c*x*e^(-a*c) - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x - 3*a*c
) + (e^(4*b*c*x + 9*a*c) + 6*e^(2*b*c*x + 7*a*c))*e^(-6*a*c))*e^(a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{3/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\left(\frac{1}{\cosh(a\,c+b\,c\,x)^2}\right)^{3/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(3/2), x)
```

3.157 $\int \frac{e^{c(a+bx)}}{\mathbf{sech}^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = -\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{6c(a+bx)}\operatorname{sech}(ac+bcx)}{192bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5x\operatorname{sech}(ac+bcx)}{16\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[Out] -1/128*sech(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)-5/64*
sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))/(sech(b*c*x+a*c)^2)^(1/2)+5/32*exp(2*c
*(b*x+a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/128*exp(4*c*(b*x+
a))*sech(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+1/192*exp(6*c*(b*x+a))*se
ch(b*c*x+a*c)/b/c/(sech(b*c*x+a*c)^2)^(1/2)+5/16*x*sech(b*c*x+a*c)/(sech(b*
c*x+a*c)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used

$= \{6852, 2320, 12, 272, 45\}$

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = -\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}} - \frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{e^{6c(a+bx)}\operatorname{sech}(ac+bcx)}{192bc\sqrt{\operatorname{sech}^2(ac+bcx)}} + \frac{5x\operatorname{sech}(ac+bcx)}{16\sqrt{\operatorname{sech}^2(ac+bcx)}}$$

```
[In] Int[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] -1/128*Sech[a*c + b*c*x]/(b*c*E^(4*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2])
- (5*Sech[a*c + b*c*x])/(64*b*c*E^(2*c*(a + b*x))*Sqrt[Sech[a*c + b*c*x]^2]
) + (5*E^(2*c*(a + b*x))*Sech[a*c + b*c*x])/(32*b*c*Sqrt[Sech[a*c + b*c*x]^
2]) + (5*E^(4*c*(a + b*x))*Sech[a*c + b*c*x])/(128*b*c*Sqrt[Sech[a*c + b*c*
x]^2]) + (E^(6*c*(a + b*x))*Sech[a*c + b*c*x])/(192*b*c*Sqrt[Sech[a*c + b*c
*x]^2]) + (5*x*Sech[a*c + b*c*x])/(16*Sqrt[Sech[a*c + b*c*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6852

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p])), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\operatorname{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^5(ac+bcx)\,dx}{\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{32x^5}\,dx,x,e^{c(a+bx)}\right)}{bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x^2)^5}{x^5}\,dx,x,e^{c(a+bx)}\right)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \frac{(1+x)^5}{x^3}\,dx,x,e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= \frac{\operatorname{sech}(ac+bcx)\operatorname{Subst}\left(\int \left(10+\frac{1}{x^3}+\frac{5}{x^2}+\frac{10}{x}+5x+x^2\right)\,dx,x,e^{2c(a+bx)}\right)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&= -\frac{e^{-4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}-\frac{5e^{-2c(a+bx)}\operatorname{sech}(ac+bcx)}{64bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5e^{2c(a+bx)}\operatorname{sech}(ac+bcx)}{32bc\sqrt{\operatorname{sech}^2(ac+bcx)}}\\
&\quad+\frac{5e^{4c(a+bx)}\operatorname{sech}(ac+bcx)}{128bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{e^{6c(a+bx)}\operatorname{sech}(ac+bcx)}{192bc\sqrt{\operatorname{sech}^2(ac+bcx)}}+\frac{5x\operatorname{sech}(ac+bcx)}{16\sqrt{\operatorname{sech}^2(ac+bcx)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(-\frac{1}{128}e^{-4c(a+bx)}-\frac{5}{64}e^{-2c(a+bx)}+\frac{5}{32}e^{2c(a+bx)}+\frac{5}{128}e^{4c(a+bx)}+\frac{1}{192}e^{6c(a+bx)}+\frac{5bcx}{16}\right)\operatorname{sec}}{bc\operatorname{sech}^2(c(a+bx))^{5/2}}$$

```
[In] Integrate[E^(c*(a + b*x))/(Sech[a*c + b*c*x]^2)^(5/2),x]
```

```
[Out] ((-1/128*1/E^(4*c*(a + b*x)) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b*
x)))/32 + (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 + (5*b*c*x)/16)
*Sech[c*(a + b*x)]^5)/(b*c*(Sech[c*(a + b*x)]^2)^(5/2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.67 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.35

method	result
default	$\frac{\operatorname{csgn}(\operatorname{sech}(c(bx+a)))\left(\frac{\cosh(bcx+ac)^6}{6}+\left(\frac{\cosh(bcx+ac)^5}{6}+\frac{5\cosh(bcx+ac)^3}{24}+\frac{5\cosh(bcx+ac)}{16}\right)\sinh(bcx+ac)+\frac{5bcx}{16}+\frac{5ac}{16}\right)}{cb}$
risch	$\frac{5x\,\mathrm{e}^{c(bx+a)}}{16\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+\frac{\mathrm{e}^{7c(bx+a)}}{192bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+\frac{5\,\mathrm{e}^{5c(bx+a)}}{128bc\left(1+\mathrm{e}^{2c(bx+a)}\right)\sqrt{\frac{\mathrm{e}^{2c(bx+a)}}{\left(1+\mathrm{e}^{2c(bx+a)}\right)^2}}}+\frac{}{32bc(}$

[In] int(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)

[Out] csgn(sech(c*(b*x+a)))/c/b*(1/6*cosh(b*c*x+a*c)^6+(1/6*cosh(b*c*x+a*c)^5+5/2
4*cosh(b*c*x+a*c)^3+5/16*cosh(b*c*x+a*c))*sinh(b*c*x+a*c)+5/16*b*c*x+5/16*a
*c)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx =$$
$$-\frac{\cosh(bcx+ac)^5+5\cosh(bcx+ac)\sinh(bcx+ac)^4-5\sinh(bcx+ac)^5-5\left(10\cosh(bcx+ac)^2+9\right)\sinh}{}$$

[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")

[Out] -1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*s
inh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 + 1
5*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*sin
h(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x + a
*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*c*cos
h(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(238) = 476.

Time = 86.60 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.12

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \begin{cases} x \\ \frac{xe^{ac}}{\left(\operatorname{sech}^2(ac)\right)^{\frac{5}{2}}} \\ x \\ -\frac{5xe^{ac}e^{bcx}\tanh^5(ac+bcx)}{16\left(\operatorname{sech}^2(ac+bcx)\right)^{\frac{5}{2}}} + \frac{5xe^{ac}e^{bcx}\tanh^4(ac+bcx)}{16\left(\operatorname{sech}^2(ac+bcx)\right)^{\frac{5}{2}}} + \frac{5xe^{ac}e^{bcx}\tanh^3(ac+bcx)}{8\left(\operatorname{sech}^2(ac+bcx)\right)^{\frac{5}{2}}} - \frac{5xe^{ac}e^{bcx}\tanh}{8\left(\operatorname{sech}^2(ac-\right.} \end{cases}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)**2)**(5/2),x)
```

```
[Out] Piecewise((x, Eq(b, 0) & Eq(c, 0)), (x*exp(a*c)/(sech(a*c)**2)**(5/2), Eq(b
, 0)), (x, Eq(c, 0)), (-5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**5/(16*(s
ech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**4
/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*x)*tanh(a*c + b*
c*x)**3/(8*(sech(a*c + b*c*x)**2)**(5/2)) - 5*x*exp(a*c)*exp(b*c*x)*tanh(a*
c + b*c*x)**2/(8*(sech(a*c + b*c*x)**2)**(5/2)) - 5*x*exp(a*c)*exp(b*c*x)*t
anh(a*c + b*c*x)/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 5*x*exp(a*c)*exp(b*c*
x)/(16*(sech(a*c + b*c*x)**2)**(5/2)) + 8*exp(a*c)*exp(b*c*x)*tanh(a*c + b*
c*x)**5/(15*b*c*(sech(a*c + b*c*x)**2)**(5/2)) - 53*exp(a*c)*exp(b*c*x)*tan
h(a*c + b*c*x)**4/(240*b*c*(sech(a*c + b*c*x)**2)**(5/2)) - 331*exp(a*c)*ex
p(b*c*x)*tanh(a*c + b*c*x)**3/(240*b*c*(sech(a*c + b*c*x)**2)**(5/2)) + 131
*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)**2/(240*b*c*(sech(a*c + b*c*x)**2)**
(5/2)) + 253*exp(a*c)*exp(b*c*x)*tanh(a*c + b*c*x)/(240*b*c*(sech(a*c + b*c
*x)**2)**(5/2)) - 11*exp(a*c)*exp(b*c*x)/(30*b*c*(sech(a*c + b*c*x)**2)**(5
/2)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.45

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{5\,(bcx+ac)}{16\,bc} + \frac{e^{(6\,bcx+6\,ac)}}{192\,bc} + \frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc} + \frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc} - \frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc} - \frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*c
*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2*a
*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \frac{\left(120\,bcxe^{(-ac)} - 3\left(30\,e^{(4\,bcx+4\,ac)} + 10\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-4\,bcx-5\,ac)} + \left(2\,e^{(6\,bcx+20\,a}\right.}{384\,bc}$$

```
[In] integrate(exp(c*(b*x+a))/(sech(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(120*b*c*x*e^(-a*c) - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2*a
*c) + 1)*e^(-4*b*c*x - 5*a*c) + (2*e^(6*b*c*x + 20*a*c) + 15*e^(4*b*c*x + 1
8*a*c) + 60*e^(2*b*c*x + 16*a*c))*e^(-15*a*c))*e^(a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\operatorname{sech}^2(ac+bcx)^{5/2}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{{\left(\frac{1}{{\mathrm{cosh}\left(a\,c+b\,c\,x\right)}^2}\right)}^{5/2}}\,dx$$

```
[In] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2),x)
```

```
[Out] int(exp(c*(a + b*x))/(1/cosh(a*c + b*c*x)^2)^(5/2), x)
```

3.158 $\int \frac{x^5}{\sqrt{\mathrm{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{x^5}{\sqrt{\mathrm{sech}(2\log(cx))}}\,dx = \frac{2x^2}{21c^4\sqrt{\mathrm{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\mathrm{sech}(2\log(cx))}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\mathrm{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{21c^5\left(c^4+\frac{1}{x^4}\right)x\sqrt{\mathrm{sech}(2\log(cx))}}$$

```
[Out] 2/21*x^2/c^4/sech(2*ln(c*x))^(1/2)+1/7*x^6/sech(2*ln(c*x))^(1/2)+1/21*(c^2+
1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arcc
ot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^5/(c^4+1/x^4)/x/s
ech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5670, 5668, 342, 283, 331, 226}

$$\int \frac{x^5}{\sqrt{\mathrm{sech}(2\log(cx))}}\,dx = \frac{2x^2}{21c^4\sqrt{\mathrm{sech}(2\log(cx))}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\mathrm{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{21c^5x\left(c^4+\frac{1}{x^4}\right)\sqrt{\mathrm{sech}(2\log(cx))}}+\frac{x^6}{7\sqrt{\mathrm{sech}(2\log(cx))}}$$

```
[In] Int[x^5/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (2*x^2)/(21*c^4*Sqrt[Sech[2*Log[c*x]]]) + x^6/(7*Sqrt[Sech[2*Log[c*x]]]) +
(Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*
x], 1/2])/(21*c^5*(c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^5}{\sqrt{\text{sech}(2\log(x))}}\,dx, x, cx\right)}{c^6}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^6\,dx, x, cx\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{x^6}{7\sqrt{\text{sech}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{2x^2}{21c^4\sqrt{\text{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\text{sech}(2\log(cx))}} + \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{21c^7\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{2x^2}{21c^4\sqrt{\text{sech}(2\log(cx))}} + \frac{x^6}{7\sqrt{\text{sech}(2\log(cx))}} + \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx), \frac{1}{2}\right)}{21c^5\left(c^4+\frac{1}{x^4}\right)x\sqrt{\text{sech}(2\log(cx))}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\left((1+c^4x^4)^{3/2} - \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -c^4x^4\right)\right)}{7c^6}$$

```
[In] Integrate[x^5/Sqrt[Sech[2*Log[c*x]]],x]
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(3/2) - H
ypergeometric2F1[-1/2, 1/4, 5/4, -(c^4*x^4)]))/(7*c^6)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{x^{2}\left(3c^{4}x^{4}+2\right)\sqrt{2}}{42c^{4}\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}-\frac{\sqrt{-ic^{2}x^{2}+1}\,\sqrt{ic^{2}x^{2}+1}\,\text{EllipticF}\left(x\sqrt{ic^{2}},i\right)\sqrt{2}\,x}{21c^{4}\sqrt{ic^{2}}\,\left(c^{4}x^{4}+1\right)\sqrt{\frac{c^{2}x^{2}}{c^{4}x^{4}+1}}}$	130

```
[In] int(x^5/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*x^2*(3*c^4*x^4+2)/c^4*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-1/21/c^4/(I*
c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*
(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{\sqrt{\text{sech}(2\log(cx))}}\,dx = -\frac{2\sqrt{2}\sqrt{c^4}c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\middle|-1\right)-\sqrt{2}\left(3\,c^8x^8+5\,c^4x^4+2\right)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{42\,c^6}$$

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/42*(2*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^(1/4
)/x), -1) - sqrt(2)*(3*c^8*x^8 + 5*c^4*x^4 + 2)*sqrt(c^2*x^2/(c^4*x^4 + 1))
)/c^6
```

Sympy [F]

$$\int \frac{x^5}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\text{sech}\,(2\log\,(cx))}}\,dx$$

```
[In] integrate(x**5/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^5/sech(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(sech(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^5}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

```
[In] int(x^5/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] int(x^5/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.159 $\int \frac{x^4}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{\left(c^4+\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/6*(c^4+1/x^4)*x^5/c^4/sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5670, 5668, 270}

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{x^5\left(c^4+\frac{1}{x^4}\right)}{6c^4\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[In] Int[x^4/Sqrt[Sech[2*Log[c*x]]],x]
[Out] ((c^4 + x^(-4))*x^5)/(6*c^4*Sqrt[Sech[2*Log[c*x]]])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^4}{\sqrt{\text{sech}(2\log(x))}}\,dx, x, cx\right)}{c^5}\\ &= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^5\,dx, x, cx\right)}{c^6\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}\\ &= \frac{\left(c^4+\frac{1}{x^4}\right)x^5}{6c^4\sqrt{\text{sech}(2\log(cx))}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{x^4}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{(1+c^4x^4)^2\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}{6c^6x}$$

```
[In] Integrate[x^4/Sqrt[Sech[2*Log[c*x]]],x]
[Out] ((1 + c^4*x^4)^2*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(6*c^6*x)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{\sqrt{2}\,x\left(c^4x^4+1\right)}{12\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,c^4}$	39

```
[In] int(x^4/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/12*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^4*x^4+1)/c^4

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.71

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\sqrt{2}(c^8x^8 + 2\,c^4x^4 + 1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{12\,c^6x}$$

[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="fricas")

[Out] 1/12*sqrt(2)*(c^8*x^8 + 2*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/(c^6*x)

Sympy [F]

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

[In] integrate(x**4/sech(2*ln(c*x))**(1/2),x)

[Out] Integral(x**4/sqrt(sech(2*log(c*x))), x)

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\left(\sqrt{2}c^4x^4 + \sqrt{2}\right)\sqrt{c^4x^4 + 1}}{12\,c^5}$$

[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="maxima")

[Out] 1/12*(sqrt(2)*c^4*x^4 + sqrt(2))*sqrt(c^4*x^4 + 1)/c^5

Giac [F]

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^4}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^4/sech(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(sech(2*log(c*x))), x)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{x^4}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\left(c^4\,x^4+1\right)^2\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{12\,c^6\,x}$$

```
[In] int(x^4/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] ((c^4*x^4 + 1)^2*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(12*c^6*x)
```

3.160 $\int \frac{x^3}{\sqrt{\mathbf{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 203

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2}{5c^4\left(c^2+\frac{1}{x^2}\right)x^2\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5c^3\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5c^3\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 2/5/c^4/sech(2*ln(c*x))^(1/2)-2/5/c^4/(c^2+1/x^2)/x^2/sech(2*ln(c*x))^(1/2)
+1/5*x^4/sech(2*ln(c*x))^(1/2)+2/5*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)
/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/
(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)-1/5*(c^2+1/x^2
)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*
x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)/x/sech(2
*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5670, 5668, 342, 283, 331, 311, 226, 1210}

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{2}{5c^4\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2}{5c^4x^2\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5c^3x\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[In] Int[x^3/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] 2/(5*c^4*Sqrt[Sech[2*Log[c*x]]]) - 2/(5*c^4*(c^2 + x^(-2))*x^2*Sqrt[Sech[2*
Log[c*x]]]) + x^4/(5*Sqrt[Sech[2*Log[c*x]]]) + (2*Sqrt[(c^4 + x^(-4))/(c^2
+ x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^
(-4))*x*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c
^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(5*c^3*(c^4 + x^(-4))*x*Sqrt[Se
ch[2*Log[c*x]]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^3}{\sqrt{\text{sech}(2\log(x))}}\,dx, x, cx\right)}{c^4}$$

$$= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^4\,dx, x, cx\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{x^4}{5\sqrt{\text{sech}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{2}{5c^4\sqrt{\text{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\text{sech}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{2}{5c^4\sqrt{\text{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\text{sech}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}} + \frac{2\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{5c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{2}{5c^4\sqrt{\text{sech}(2\log(cx))}} - \frac{2}{5c^4\left(c^2+\frac{1}{x^2}\right)x^2\sqrt{\text{sech}(2\log(cx))}} + \frac{x^4}{5\sqrt{\text{sech}(2\log(cx))}} + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5c^3\left(c^4+\frac{1}{x^4}\right)x\sqrt{\text{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5c^3\left(c^4+\frac{1}{x^4}\right)x\sqrt{\text{sech}(2\log(cx))}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.32

$$\int \frac{x^3}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{\left(\frac{c^2x^2}{1+c^4x^4}\right)^{3/2}\left(1+c^4x^4\right)^{3/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-c^4x^4\right)}{3\sqrt{2}c^4}$$

```
[In] Integrate[x^3/Sqrt[Sech[2*Log[c*x]]],x]
[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-1/2
, 3/4, 7/4, -(c^4*x^4)])/(3*Sqrt[2]*c^4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{\sqrt{2}\,x^4}{10\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{i\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ic^2},i\right)-\mathrm{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}\,x}{5\sqrt{ic^2}\,(c^4x^4+1)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	134

```
[In] int(x^3/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*2^(1/2)*x^4/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/5*I/(I*c^2)^(1/2)*(1-I*c^2*x
^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^2*(EllipticF(x*(I*c^2)^(1/2),I)
-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.59

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{2\sqrt{2}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\mid -1)-2\sqrt{2}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\mid -1)+\sqrt{2}(c^8x}{10\,c^6x^2}$$

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(2*sqrt(2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_e(arcsin((-1/c^4)^(
1/4)/x), -1) - 2*sqrt(2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_f(arcsin((
-1/c^4)^(1/4)/x), -1) + sqrt(2)*(c^8*x^8 + 3*c^4*x^4 + 2)*sqrt(c^2*x^2/(c^4
*x^4 + 1)))/(c^6*x^2)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x**3/sech(2*ln(c*x))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/sqrt(sech(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/sech(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to c
onvert to real %%{poly1[1.00000000000000000000000000000000,0.000000000000000
000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^3}{\sqrt{\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}}}\,dx$$

```
[In] int(x^3/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] int(x^3/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.161 $\int \frac{x^2}{\sqrt{\text{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{x^2}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{x^3}{4\sqrt{\text{sech}(2\log(cx))}} + \frac{\text{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

```
[Out] 1/4*x^3/sech(2*ln(c*x))^(1/2)+1/4*arctanh((1+1/c^4/x^4)^(1/2))/c^4/x/(1+1/c
^4/x^4)^(1/2)/sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5670, 5668, 272, 43, 65, 213}

$$\int \frac{x^2}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{\text{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\text{sech}(2\log(cx))}} + \frac{x^3}{4\sqrt{\text{sech}(2\log(cx))}}$$

```
[In] Int[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^3/(4*Sqrt[Sech[2*Log[c*x]]]) + ArcTanh[Sqrt[1 + 1/(c^4*x^4)]]/(4*c^4*Sqrt
[1 + 1/(c^4*x^4)]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{\text{sech}(2\log(x))}}\,dx, x, cx\right)}{c^3}\\ &= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^3\,dx, x, cx\right)}{c^4\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\text{sech}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\text{sech}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{x^3}{4\sqrt{\text{sech}(2\log(cx))}} + \frac{\text{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{x\left(c^2x^2\sqrt{1+c^4x^4}+\text{arcsinh}(c^2x^2)\right)}{4\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

```
[In] Integrate[x^2/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (x*(c^2*x^2*Sqrt[1 + c^4*x^4] + ArcSinh[c^2*x^2]))/(4*Sqrt[2]*c^2*Sqrt[(c^2
*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.45

method	result	size
risch	$\frac{\sqrt{2}\,x^3}{8\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\sqrt{2}\,x}{8\sqrt{c^4}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}\,\sqrt{c^4x^4+1}}$	97

```
[In] int(x^2/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*x^3/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/8*ln(c^4*x^2/(c^4)^(1/2)+(c^4
*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)/(c^4*x^4+1
)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{2\sqrt{2}(c^5x^5+cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}}+\sqrt{2}\log\left(-2\,c^4x^4-2\,(c^5x^5+cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)}{16\,c^3}$$

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] 1/16*(2*sqrt(2)*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) + sqrt(2)*log(-
2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) - 1))/c^3
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x**2/sech(2*ln(c*x))**(1/2),x)
[Out] Integral(x**2/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x^2/sech(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(sech(2*log(c*x))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x^2}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

```
[In] int(x^2/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] int(x^2/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.162 $\int \frac{x}{\sqrt{\text{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{x}{\sqrt{\text{sech}(2\log(cx))}}\,dx=\frac{x^2}{3\sqrt{\text{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{3c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\text{sech}(2\log(cx))}}$$

```
[Out] 1/3*x^2/sech(2*ln(c*x))^(1/2)-1/3*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/
cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(
c^2+1/x^2)^2)^(1/2)/c/(c^4+1/x^4)/x/sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5670, 5668, 342, 283, 226}

$$\int \frac{x}{\sqrt{\text{sech}(2\log(cx))}}\,dx=\frac{x^2}{3\sqrt{\text{sech}(2\log(cx))}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{3cx\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}}$$

```
[In] Int[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x^2/(3*Sqrt[Sech[2*Log[c*x]]]) - (Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^
2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(3*c*(c^4 + x^(-4))*x*Sqrt[Sech[
2*Log[c*x]]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{\text{sech}(2\log(x))}}\,dx, x, cx\right)}{c^2} \\
&= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x^2\,dx, x, cx\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}
\end{aligned}
$$

$$= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{3c^3\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

$$= \frac{x^2}{3\sqrt{\operatorname{sech}(2\log(cx))}} - \frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx), \frac{1}{2}\right)}{3c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{1}{4}, \frac{5}{4}, -c^4x^4\right)}{c^2}$$

```
[In] Integrate[x/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-1/2,
1/4, 5/4, -(c^4*x^4)])/c^2
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

method	result	size
risch	$\frac{\sqrt{2}\,x^2}{6\sqrt{\frac{c^2x^2}{c^4x^4+1}}} + \frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\text{EllipticF}\left(x\sqrt{ic^2}, i\right)\sqrt{2}\,x}{3\sqrt{ic^2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	114

```
[In] int(x/sech(2*ln(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*2^(1/2)*x^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/3/(I*c^2)^(1/2)*(1-I*c^2*x^2)
^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)
*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \frac{2\sqrt{2}\sqrt{c^4}c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F\left(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1\right)+\sqrt{2}(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{6\,c^2}$$

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^(1/4)/
x), -1) + sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/c^2
```

Sympy [F]

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x/sech(2*ln(c*x))**(1/2),x)
[Out] Integral(x/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(sech(2*log(c*x))), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/sech(2*log(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly exception caught Unable to c
onvert to real %%{poly1[1.00000000000000000000000000000000,0.000000000000000
000000
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{x}{\sqrt{\frac{1}{\cosh(2\ln(c\,x))}}}\,dx$$

```
[In] int(x/(1/cosh(2*log(c*x)))^(1/2),x)
```

```
[Out] int(x/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.163 $\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 59

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[Out] 1/2*x/sech(2*ln(c*x))^(1/2)-1/2*arccsch(c^2*x^2)/c^2/x/(1+1/c^4/x^4)^(1/2)/
sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {5664, 5662, 342, 281, 283, 221}

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx=\frac{x}{2\sqrt{\operatorname{sech}(2\log(cx))}}-\frac{\operatorname{csch}^{-1}(c^2x^2)}{2c^2x\sqrt{\frac{1}{c^4x^4}+1}\sqrt{\operatorname{sech}(2\log(cx))}}$$

```
[In] Int[1/Sqrt[Sech[2*Log[c*x]]],x]
```

```
[Out] x/(2*Sqrt[Sech[2*Log[c*x]]]) - ArcCsch[c^2*x^2]/(2*c^2*Sqrt[1 + 1/(c^4*x^4)
]*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5662

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Sech[d*(a
+ b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\text{sech}(2\log(x))}}\,dx, x, cx\right)}{c}\\
&= \frac{\text{Subst}\left(\int \sqrt{1+\frac{1}{x^4}}x\,dx, x, cx\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^3}\,dx, x, \frac{1}{cx}\right)}{c^2\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\text{sech}(2\log(cx))}}
\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{x}{2\sqrt{\text{sech}(2\log(cx))}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

$$= \frac{x}{2\sqrt{\text{sech}(2\log(cx))}} - \frac{\text{csch}^{-1}(c^2x^2)}{2c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\text{sech}(2\log(cx))}}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{x\left(\sqrt{1+c^4x^4} - \text{arctanh}\left(\sqrt{1+c^4x^4}\right)\right)}{2\sqrt{2}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

```
[In] Integrate[1/Sqrt[Sech[2*Log[c*x]]],x]
[Out] (x*(Sqrt[1 + c^4*x^4] - ArcTanh[Sqrt[1 + c^4*x^4]]))/(2*Sqrt[2]*Sqrt[(c^2*x
^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [F]

$$\int \frac{1}{\sqrt{\text{sech}\left(2\ln\left(cx\right)\right)}}dx$$

```
[In] int(1/sech(2*ln(c*x))^(1/2),x)
[Out] int(1/sech(2*ln(c*x))^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{\text{sech}(2\log(cx))}}\,dx = \frac{\sqrt{2}cx\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right) + 2\,\sqrt{2}(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{8\,c^2x}$$

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="fricas")
[Out] 1/8*(sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x
^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(
c^2*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(1/sech(2*ln(c*x))**(1/2),x)
[Out] Integral(1/sqrt(sech(2*log(c*x))), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}\,dx$$

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(sech(2*log(c*x))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \text{Timed out}$$

```
[In] integrate(1/sech(2*log(c*x))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\operatorname{sech}(2\log(cx))}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}\,dx$$

```
[In] int(1/(1/cosh(2*log(c*x)))^(1/2),x)
[Out] int(1/(1/cosh(2*log(c*x)))^(1/2), x)
```

3.164 $\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x}\,dx = -i\sqrt{\cosh(2\log(cx))}\,\text{EllipticF}(i\log(cx),2)\sqrt{\text{sech}(2\log(cx))}$$

```
[Out] -I*((1/2*c*x+1/2/c/x)^2)^(1/2)/(1/2*c*x+1/2/c/x)*EllipticF(I*(1/2*c*x-1/2/c
/x),2^(1/2))*cosh(2*ln(c*x))^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3856, 2720}

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x}\,dx = -i\sqrt{\text{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}\,\text{EllipticF}(i\log(cx),2)$$

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x,x]
```

```
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \sqrt{\text{sech}(2x)}\,dx, x, \log(cx)\right) \\ &= \left(\sqrt{\cosh(2\log(cx))}\sqrt{\text{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(2x)}}\,dx, x, \log(cx)\right) \\ &= -i\sqrt{\cosh(2\log(cx))}\,\text{EllipticF}(i\log(cx), 2)\sqrt{\text{sech}(2\log(cx))} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x}\,dx = -i\sqrt{\cosh(2\log(cx))}\,\text{EllipticF}(i\log(cx), 2)\sqrt{\text{sech}(2\log(cx))}$$

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x,x]
[Out] (-I)*Sqrt[Cosh[2*Log[c*x]]]*EllipticF[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(73) = 146.

Time = 0.67 (sec) , antiderivative size = 167, normalized size of antiderivative = 4.64

method	result	size
derivativedivides	$\frac{\sqrt{\left(2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2+1}\,\text{EllipticF}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)}{\sqrt{2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^4+\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1}}$	167
default	$\frac{\sqrt{\left(2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\sqrt{-2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2+1}\,\text{EllipticF}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)}{\sqrt{2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^4+\left(\frac{cx}{2}-\frac{1}{2cx}\right)^2}\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^2-1}}$	167

```
[In] int(sech(2*ln(c*x))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] ((2*(1/2*c*x+1/2/c/x)^2-1)*(1/2*c*x-1/2/c/x)^2)^(1/2)*(-(1/2*c*x-1/2/c/x)^2
)^(1/2)*(-2*(1/2*c*x+1/2/c/x)^2+1)^(1/2)/(2*(1/2*c*x-1/2/c/x)^4+(1/2*c*x-1/
2/c/x)^2)^(1/2)*EllipticF(1/2*c*x+1/2/c/x,2^(1/2))/(1/2*c*x-1/2/c/x)/(2*(1/
2*c*x+1/2/c/x)^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = -\frac{\sqrt{2}(-c^4)^{\frac{3}{4}}F\left(\arcsin\left((-c^4)^{\frac{1}{4}}x\right)\mid -1\right)}{c^3}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="fricas")
[Out] -sqrt(2)*(-c^4)^(3/4)*elliptic_f(arcsin((-c^4)^(1/4)*x), -1)/c^3
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x,x)
[Out] Integral(sqrt(sech(2*log(c*x)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(sech(2*log(c*x)))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x}\,dx$$

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x,x)
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x, x)
```

3.165 $\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^2}\,dx = -\frac{1}{2}c^2\sqrt{1+\frac{1}{c^4x^4}}x\text{csch}^{-1}(c^2x^2)\sqrt{\text{sech}(2\log(cx))}$$

```
[Out] -1/2*c^2*x*arccsch(c^2*x^2)*(1+1/c^4/x^4)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5670, 5668, 342, 281, 221}

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^2}\,dx = -\frac{1}{2}c^2x\sqrt{\frac{1}{c^4x^4}+1}\text{csch}^{-1}(c^2x^2)\sqrt{\text{sech}(2\log(cx))}$$

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
```

```
[Out] -1/2*(c^2*Sqrt[1 + 1/(c^4*x^4)]*x*ArcCsch[c^2*x^2]*Sqrt[Sech[2*Log[c*x]]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^2}\,dx, x, cx\right)\\
&= \left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^3}\,dx, x, cx\right)\\
&= -\left(\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\frac{1}{2}\left(c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)\right)\\
&= -\frac{1}{2}c^2\sqrt{1+\frac{1}{c^4x^4}}\,x\operatorname{csch}^{-1}\left(c^2x^2\right)\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = -\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\operatorname{arctanh}\left(\sqrt{1+c^4x^4}\right)}{x}$$

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^2,x]
[Out] -((Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*ArcTanh[Sqrt[1 + c^4*x
^4]])/x)
```

Maple [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\ln(cx))}}{x^2}\,dx$$

```
[In] int(sech(2*ln(c*x))^(1/2)/x^2,x)
[Out] int(sech(2*ln(c*x))^(1/2)/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \frac{1}{4}\sqrt{2}c\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="fricas")
[Out] 1/4*sqrt(2)*c*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c*x^5))
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**2,x)
[Out] Integral(sqrt(sech(2*log(c*x)))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^2}\,dx$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(sech(2*log(c*x)))/x^2, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^2}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x^2}\,dx$$

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^2,x)
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^2, x)
```

3.166 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = -\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}} + c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xE\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))} - \frac{1}{2}c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)/(c^2+1/x^2)+c*(c^2+1/x^2)*x*(cos(2*arcco
t(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2
))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)-1/2*c*(c^2+1/x^2
)*x*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(
c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5670, 5668, 342, 311, 226, 1210}

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = -\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}} - \frac{1}{2}cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right) + cx\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}E\left(2\cot^{-1}(cx)|\frac{1}{2}\right)$$

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -(((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]])/(c^2 + x^(-2))) + c*Sqrt[(c^4 + x
^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x*EllipticE[2*ArcCot[c*x], 1/2]*Sqr
t[Sech[2*Log[c*x]]] - (c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2
))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*Log[c*x]]])/2
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c^2\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^3}\,dx, x, cx\right)\\
&= \left(c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}x^4}\,dx, x, cx\right)\\
&= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&= -\left(\left(c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\right)\\
&\quad+\left(c^3\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)\\
&= -\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}}{c^2+\frac{1}{x^2}}\\
&\quad+c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)xE\left(2\cot^{-1}(cx)|\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}\\
&\quad-\frac{1}{2}c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = -\frac{c^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-c^4x^4\right)}{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}$$

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^3,x]
```

```
[Out] -((c^2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(c^4*x^4)])/(Sqrt[1 + c^4*x^4]*Sq
rt[(c^2*x^2)/(2 + 2*c^4*x^4)]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{(c^4x^4+1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{x^2}+\frac{ic^2\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\left(\text{EllipticF}\left(x\sqrt{ic^2},i\right)-\text{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{\sqrt{ic^2}\,x}$	134

```
[In] int(sech(2*ln(c*x))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(c^4*x^4+1)/x^2*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)+I*c^2/(I*c^2)^(1/2)*(1
-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*(EllipticF(x*(I*c^2)^(1/2),I)-Ellipti
cE(x*(I*c^2)^(1/2),I))*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx =$$
$$-\frac{\sqrt{2}(-c^4)^{\frac{3}{4}}\,cx^2E(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)|-1)-\sqrt{2}(-c^4)^{\frac{3}{4}}\,cx^2F(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)|-1)+\sqrt{2}(c^4x^4+1)\sqrt{\frac{c}{}}}{x^2}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(-c^4)^(3/4)*c*x^2*elliptic_e(arcsin((-c^4)^(1/4)*x), -1) - sqrt(
2)*(-c^4)^(3/4)*c*x^2*elliptic_f(arcsin((-c^4)^(1/4)*x), -1) + sqrt(2)*(c^4
*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/x^2
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^3}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**3,x)
[Out] Integral(sqrt(sech(2*log(c*x)))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^3}\,dx$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(sech(2*log(c*x)))/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^3}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x^3}\,dx$$

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^3,x)
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^3, x)
```

3.167 $\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^4}\,dx = -\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\sqrt{\text{sech}(2\log(cx))}$$

```
[Out] -1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5670, 5668, 267}

$$\int \frac{\sqrt{\text{sech}(2\log(cx))}}{x^4}\,dx = -\frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{sech}(2\log(cx))}$$

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
```

```
[Out] -1/2*((c^4 + x^(-4))*x*Sqrt[Sech[2*Log[c*x]]])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^3\text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^4}\,dx, x, cx\right)\\
&= \left(c^4\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^5}\,dx, x, cx\right)\\
&= -\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\sqrt{\operatorname{sech}(2\log(cx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{c^2}{2x\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}$$

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^4,x]
[Out] -1/2*c^2/(x*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

method	result	size
risch	$-\frac{(c^4x^4+1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{2x^3}$	38

```
[In] int(sech(2*ln(c*x))^(1/2)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/2*(c^4*x^4+1)/x^3*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{\sqrt{2}(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{2\,x^3}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="fricas")
[Out] -1/2*sqrt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1))/x^3
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^4}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**4,x)
[Out] Integral(sqrt(sech(2*log(c*x)))/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{1}{2}\,c^3\left(\frac{\sqrt{2}}{\sqrt{\frac{1}{c^4x^4}+1}} + \frac{\sqrt{2}}{c^4x^4\sqrt{\frac{1}{c^4x^4}+1}}\right)$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="maxima")
[Out] -1/2*c^3*(sqrt(2)/sqrt(1/(c^4*x^4) + 1) + sqrt(2)/(c^4*x^4*sqrt(1/(c^4*x^4)
 + 1)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^4}\,dx = -\frac{\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{2\,x^3} - \frac{c^4\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}}{2}$$

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^4,x)
```

```
[Out] - ((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)/(2*x^3) - (c^4*x*((2*c^2*x^2)/(c^4*x^4
+ 1))^(1/2))/2
```

3.168 $\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5} dx = -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}+\frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[Out] -1/3*(c^4+1/x^4)*sech(2*ln(c*x))^(1/2)+1/6*c^3*(c^2+1/x^2)*x*(cos(2*arccot(
c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))
*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)*sech(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5670, 5668, 342, 327, 226}

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5} dx = \frac{1}{6}c^3x\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\sqrt{\operatorname{sech}(2\log(cx))}\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

```
[In] Int[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*((c^4 + x^(-4))*Sqrt[Sech[2*Log[c*x]]]) + (c^3*Sqrt[(c^4 + x^(-4))/(c^
2 + x^(-2))^2]*(c^2 + x^(-2))*x*EllipticF[2*ArcCot[c*x], 1/2]*Sqrt[Sech[2*L
og[c*x]]])/6
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = c^4 \text{Subst}\left(\int \frac{\sqrt{\operatorname{sech}(2\log(x))}}{x^5}\,dx, x, cx\right)$$

$$= \left(c^5\sqrt{1+\frac{1}{c^4x^4}}\,x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{1}{x^4}}\,x^6}\,dx, x, cx\right)$$

$$= -\left(\left(c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int\frac{x^4}{\sqrt{1+x^4}}\,dx,x,\frac{1}{cx}\right)\right)$$

$$= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))} + \frac{1}{3}\left(c^5\sqrt{1+\frac{1}{c^4x^4}}x\sqrt{\operatorname{sech}(2\log(cx))}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+x^4}}\,dx,x,\frac{1}{cx}\right)$$

$$= -\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\operatorname{sech}(2\log(cx))} + \frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\sqrt{\operatorname{sech}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

$$\int\frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = -\frac{\sqrt{2}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-c^4x^4\right)}{3x^4}$$

```
[In] Integrate[Sqrt[Sech[2*Log[c*x]]]/x^5,x]
```

```
[Out] -1/3*(Sqrt[2]*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4]*Hypergeometri
c2F1[-3/4, 1/2, 1/4, -(c^4*x^4)])/x^4
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

method	result	size
risch	$-\frac{(c^4x^4+1)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3x^4}-\frac{c^4\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\ \text{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3\sqrt{ic^2}\,x}$	117

```
[In] int(sech(2*ln(c*x))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(c^4*x^4+1)/x^4*2^(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)-1/3*c^4/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)*EllipticF(x*(I*c^2)^(1/2),I)*2^
(1/2)*(c^2*x^2/(c^4*x^4+1))^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \frac{\sqrt{2}(-c^4)^{\frac{3}{4}}\,cx^4F(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1) - \sqrt{2}(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{3\,x^4}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="fricas")
[Out] 1/3*(sqrt(2)*(-c^4)^(3/4)*c*x^4*elliptic_f(arcsin((-c^4)^(1/4)*x), -1) - sq
rt(2)*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/x^4
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\log\left(cx\right)\right)}}{x^5}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(1/2)/x**5,x)
[Out] Integral(sqrt(sech(2*log(c*x)))/x**5, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\operatorname{sech}\left(2\,\log\left(cx\right)\right)}}{x^5}\,dx$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt(sech(2*log(c*x)))/x^5, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(1/2)/x^5,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(2\log(cx))}}{x^5}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(2\,\ln(c\,x))}}}{x^5}\,dx$$

```
[In] int((1/cosh(2*log(c*x)))^(1/2)/x^5,x)
[Out] int((1/cosh(2*log(c*x)))^(1/2)/x^5, x)
```

3.169 $\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/32*x/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/16*x^5/(c^4+1/x^4)/sech(2*ln
(c*x))^(3/2)+1/12*x^9/sech(2*ln(c*x))^(3/2)-1/32*arctanh((1+1/c^4/x^4)^(1/2
))/c^12/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5670, 5668, 272, 43, 44, 65, 213}

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{32c^{12}x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] x/(32*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(16*(c^4 + x^(-4))*S
ech[2*Log[c*x]]^(3/2)) + x^9/(12*Sech[2*Log[c*x]]^(3/2)) - ArcTanh[Sqrt[1 +
 1/(c^4*x^4)]]/(32*c^12*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^8}{\text{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^9}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{11}\,dx, x, cx\right)}{c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x^4}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x^9}{12\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^9}{12\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&\quad + \frac{x^9}{12\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{64c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&\quad + \frac{x^9}{12\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x}{32c^4\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{16\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&\quad + \frac{x^9}{12\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{32c^{12}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{c^3x^3\sqrt{1+c^4x^4}(3+14c^4x^4+8c^8x^8)-3cx\operatorname{arcsinh}(c^2x^2)}{192\sqrt{2}c^9\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

```
[In] Integrate[x^8/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(3 + 14*c^4*x^4 + 8*c^8*x^8) - 3*c*x*ArcSinh[c^2
*x^2])/(192*Sqrt[2]*c^9*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.99

method	result	size
risch	$\frac{x^3(8c^8x^8+14c^4x^4+3)\sqrt{2}}{384c^6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}-\frac{\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\sqrt{2}\,x}{128c^6\sqrt{c^4}\,\sqrt{c^4x^4+1}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	121

```
[In] int(x^8/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*x^3*(8*c^8*x^8+14*c^4*x^4+3)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/128/c^6*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*x/(
c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.89

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2\sqrt{2}(8\,c^{13}x^{13}+22\,c^9x^9+17\,c^5x^5+3\,cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}}+3\sqrt{2}\log\left(-2\,c^4x^4+2\,(c^5x^5+cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)}{768\,c^9}$$

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(2*sqrt(2)*(8*c^13*x^13 + 22*c^9*x^9 + 17*c^5*x^5 + 3*c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) + 3*sqrt(2)*log(-2*c^4*x^4 + 2*(c^5*x^5 + c*x)*sqrt(c^2*x
^2/(c^4*x^4 + 1)) - 1))/c^9
```

Sympy [F]

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**8/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**8/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^8/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^8/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^8}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

```
[In] int(x^8/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^8/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.170 $\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 141

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{77c^5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 4/77/c^4/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+6/77*x^4/(c^4+1/x^4)/sech(2*ln(c
*x))^(3/2)+1/11*x^8/sech(2*ln(c*x))^(3/2)+2/77*(c^2+1/x^2)*(cos(2*arccot(c*
x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*(
(c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^5/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2
)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

= {5670, 5668, 342, 283, 331, 226}

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{77c^5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 4/(77*c^4*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + (6*x^4)/(77*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^8/(11*Sech[2*Log[c*x]]^(3/2)) + (2*Sqrt[(c^4
 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/
(77*c^5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^7}{\text{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^8}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^{10}\,dx, x, cx\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^{12}}\,dx, x, \frac{1}{cx}\right)}{c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{x^8}{11\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^8}\,dx, x, \frac{1}{cx}\right)}{11c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^8}{11\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{77c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&\quad + \frac{x^8}{11\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{77c^{11}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&= \frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}\\
&\quad + \frac{x^8}{11\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx), \frac{1}{2}\right)}{77c^5\left(c^4+\frac{1}{x^4}\right)^2 x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.55

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\left(\left(1+c^4x^4\right)^{5/2}-\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-c^4x^4\right)\right)}{22c^8}$$

```
[In] Integrate[x^7/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*((1 + c^4*x^4)^(5/2) - H
ypergeometric2F1[-3/2, 1/4, 5/4, -(c^4*x^4)]))/(22*c^8)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{x^2(7c^8x^8+13c^4x^4+4)\sqrt{2}}{308c^6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}-\frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{ic^2},i\right)\sqrt{2}\,x}{77c^6\sqrt{ic^2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	138

```
[In] int(x^7/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/308*x^2*(7*c^8*x^8+13*c^4*x^4+4)/c^6*2^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)-
1/77/c^6/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*
EllipticF(x*(I*c^2)^(1/2),I)*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{4\sqrt{2}\sqrt{c^4}c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)|-1)-\sqrt{2}(7\,c^{12}x^{12}+20\,c^8x^8+17\,c^4x^4+4)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{308\,c^8}$$

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/308*(4*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^(1/
4)/x), -1) - sqrt(2)*(7*c^12*x^12 + 20*c^8*x^8 + 17*c^4*x^4 + 4)*sqrt(c^2*x
^2/(c^4*x^4 + 1)))/c^8
```

Sympy [F]

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

```
[In] integrate(x**7/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**7/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\operatorname{sech}(2\log(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^7/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^7/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^7}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^7}{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^7/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^7/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.171 $\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(c^4+\frac{1}{x^4}\right)x^7}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/10*(c^4+1/x^4)*x^7/c^4/sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5670, 5668, 270}

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x^7\left(c^4+\frac{1}{x^4}\right)}{10c^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^6/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] ((c^4 + x^(-4))*x^7)/(10*c^4*Sech[2*Log[c*x]]^(3/2))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^6}{\text{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^7} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^9\,dx, x, cx\right)}{c^{10}\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))} \\
&= \frac{\left(c^4+\frac{1}{x^4}\right)x^7}{10c^4\text{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$
\int \frac{x^6}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{(1+c^4x^4)^3\sqrt{\frac{c^2x^2}{2+2c^4x^4}}}{20c^8x}
$$

```
[In] Integrate[x^6/Sech[2*Log[c*x]]^(3/2),x]
[Out] ((1 + c^4*x^4)^3*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)])/(20*c^8*x)
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

method	result	size
risch	$\frac{\sqrt{2}\,x\left(c^8x^8+2c^4x^4+1\right)}{40c^6\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	47

```
[In] int(x^6/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/40*2^(1/2)/c^6*x/(c^2*x^2/(c^4*x^4+1))^(1/2)*(c^8*x^8+2*c^4*x^4+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{2}(c^{12}x^{12}+3\,c^8x^8+3\,c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{40\,c^8x}$$

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
[Out] 1/40*sqrt(2)*(c^12*x^12 + 3*c^8*x^8 + 3*c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4*x^4
+ 1))/(c^8*x)
```

Sympy [F]

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

```
[In] integrate(x**6/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**6/sech(2*log(c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{(\sqrt{2}c^4x^4+\sqrt{2})(c^4x^4+1)^{\frac{3}{2}}}{40\,c^7}$$

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] 1/40*(sqrt(2)*c^4*x^4 + sqrt(2))*(c^4*x^4 + 1)^(3/2)/c^7
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^6/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{x^6}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{(c^4x^4+1)^3\sqrt{\frac{2c^2x^2}{c^4x^4+1}}}{40c^8x}$$

```
[In] int(x^6/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] ((c^4*x^4 + 1)^3*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2))/(40*c^8*x)
```

3.172 $\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 251

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{15c^3\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{15c^3\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -4/15/c^4/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+4/15/c^4/(c^4+1
/x^4)/x^2/sech(2*ln(c*x))^(3/2)+2/15*x^2/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+
1/9*x^6/sech(2*ln(c*x))^(3/2)+4/15*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)
/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/
(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)-2/15*(c^2+
1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arcc
ot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c^3/(c^4+1/x^4)^2/x
^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5670, 5668, 342, 283, 331, 311, 226, 1210}

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4}{15c^4x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{4}{15c^4x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{15c^3x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{4\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{15c^3x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^5/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -4/(15*c^4*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 4/(1
5*c^4*(c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + (2*x^2)/(15*(c^4 + x^(-4
))*Sech[2*Log[c*x]]^(3/2)) + x^6/(9*Sech[2*Log[c*x]]^(3/2)) + (4*Sqrt[(c^4
+ x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcCot[c*x], 1/2])/(
15*c^3*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (2*Sqrt[(c^4 + x^(-4)
)/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/2])/(15*c^3*(
c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^6}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^8\,dx, x, cx\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^{10}}\,dx, x, \frac{1}{cx}\right)}{c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^6}\,dx, x, \frac{1}{cx}\right)}{3c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$+ \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$- \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4\text{Subst}\left(\int \frac{1-x^2}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{15c^9\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$+ \frac{4}{15c^4\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{2x^2}{15\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$+ \frac{x^6}{9\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{4\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{15c^3\left(c^4+\frac{1}{x^4}\right)^2 x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$
$$- \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{15c^3\left(c^4+\frac{1}{x^4}\right)^2 x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.26

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\left(\frac{c^2x^2}{1+c^4x^4}\right)^{3/2}(1+c^4x^4)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-c^4x^4\right)}{6\sqrt{2}c^6}$$

```
[In] Integrate[x^5/Sech[2*Log[c*x]]^(3/2),x]
[Out] (((c^2*x^2)/(1 + c^4*x^4))^(3/2)*(1 + c^4*x^4)^(3/2)*Hypergeometric2F1[-3/2
, 3/4, 7/4, -(c^4*x^4)])/(6*Sqrt[2]*c^6)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{x^4(5c^4x^4+11)\sqrt{2}}{180c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{i\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\left(\text{EllipticF}\left(x\sqrt{ic^2},i\right)-\text{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}\,x}{15\sqrt{ic^2}\,(c^4x^4+1)c^4\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	147

```
[In] int(x^5/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/180*x^4*(5*c^4*x^4+11)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/15*I/(I*
c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)/c^4*(Ellipti
cF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4
*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.51

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$
$$= \frac{12\sqrt{2}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}E(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)|-1) - 12\sqrt{2}\sqrt{c^4}cx^2\left(-\frac{1}{c^4}\right)^{\frac{3}{4}}F(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)|-1) + \sqrt{2}(5}{180\,c^8x^2}$$

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/180*(12*sqrt(2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_e(arcsin((-1/c^4)
^(1/4)/x), -1) - 12*sqrt(2)*sqrt(c^4)*c*x^2*(-1/c^4)^(3/4)*elliptic_f(arcsi
n((-1/c^4)^(1/4)/x), -1) + sqrt(2)*(5*c^12*x^12 + 16*c^8*x^8 + 23*c^4*x^4 +
 12)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/(c^8*x^2)
```

Sympy [F]

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

```
[In] integrate(x**5/sech(2*ln(c*x))**(3/2),x)
[Out] Integral(x**5/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\operatorname{sech}\left(2\log(cx)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^5/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^5}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^5/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^5/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.173 $\int \frac{x^4}{\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{x^4}{\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\operatorname{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 3/16*x/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/8*x^5/sech(2*ln(c*x))^(3/2)+3/16
*arctanh((1+1/c^4/x^4)^(1/2))/c^8/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(
3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5670, 5668, 272, 43, 65, 213}

$$\int \frac{x^4}{\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\operatorname{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{16c^8x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^5}{8\mathrm{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^4/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] (3*x)/(16*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^5/(8*Sech[2*Log[c*x]]^
(3/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(16*c^8*(1 + 1/(c^4*x^4))^(3/2)
*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^5}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^7\,dx, x, cx\right)}{c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x^3}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^5}{8\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{32c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{3x}{16\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^5}{8\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{16c^8\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{c^3x^3\sqrt{1+c^4x^4}(5+2c^4x^4)+3cx\text{arcsinh}(c^2x^2)}{32\sqrt{2}c^5\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

```
[In] Integrate[x^4/Sech[2*Log[c*x]]^(3/2),x]
[Out] (c^3*x^3*Sqrt[1 + c^4*x^4]*(5 + 2*c^4*x^4) + 3*c*x*ArcSinh[c^2*x^2])/(32*Sq
rt[2]*c^5*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.23

method	result	size
risch	$\frac{x^3(2c^4x^4+5)\sqrt{2}}{64c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}} + \frac{3\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\sqrt{2}\,x}{64\sqrt{c^4}\,c^2\sqrt{c^4x^4+1}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	113

```
[In] int(x^4/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

[Out] 1/64*x^3*(2*c^4*x^4+5)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+3/64*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)/c^2*x/(c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2\sqrt{2}(2\,c^9x^9 + 7\,c^5x^5 + 5\,cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}} + 3\sqrt{2}\log\left(-2\,c^4x^4 - 2\,(c^5x^5 + cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}} - 1\right)}{128\,c^5}$$

[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="fricas")

[Out] 1/128*(2*sqrt(2)*(2*c^9*x^9 + 7*c^5*x^5 + 5*c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) + 3*sqrt(2)*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^4*x^4 + 1)) - 1))/c^5

Sympy [F]

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

[In] integrate(x**4/sech(2*ln(c*x))**(3/2),x)

[Out] Integral(x**4/sech(2*log(c*x))**(3/2), x)

Maxima [F]

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="maxima")

[Out] integrate(x^4/sech(2*log(c*x))^(3/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^4}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^4/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^4/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.174 $\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 111

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{7c\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 2/7/(c^4+1/x^4)/sech(2*ln(c*x))^(3/2)+1/7*x^4/sech(2*ln(c*x))^(3/2)-2/7*(c^
2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*ar
ccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c/(c^4+1/x^4)^2/x
^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5670, 5668, 342, 283, 226}

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{7cx^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^4}{7\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^3/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 2/(7*(c^4 + x^(-4))*Sech[2*Log[c*x]]^(3/2)) + x^4/(7*Sech[2*Log[c*x]]^(3/2)
) - (2*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*Arc
Cot[c*x], 1/2])/(7*c*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^4}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^6\,dx, x, cx\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \operatorname{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}
$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^8}\,dx, x, \frac{1}{cx}\right)}{c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^4}{7\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^4}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{4\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \frac{1}{cx}\right)}{7c^7\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{2}{7\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^4}{7\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx), \frac{1}{2}\right)}{7c\left(c^4+\frac{1}{x^4}\right)^2 x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}, \frac{5}{4}, -c^4x^4\right)}{2c^4}$$

```
[In] Integrate[x^3/Sech[2*Log[c*x]]^(3/2),x]
[Out] (Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-3/2,
1/4, 5/4, -(c^4*x^4)])/(2*c^4)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

method	result	size
risch	$\frac{x^2(c^4x^4+3)\sqrt{2}}{28c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}} + \frac{\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\text{EllipticF}\left(x\sqrt{ic^2}, i\right)\sqrt{2}\,x}{7\sqrt{ic^2}\,(c^4x^4+1)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	129

```
[In] int(x^3/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x^2*(c^4*x^4+3)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+1/7/(I*c^2)^(1
/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*EllipticF(x*(I*c^2)
^(1/2),I)*2^(1/2)/c^2*x/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{4\,\sqrt{2}\sqrt{c^4}c\left(-\frac{1}{c^4}\right)^{\frac{3}{4}} F(\arcsin\left(\frac{\left(-\frac{1}{c^4}\right)^{\frac{1}{4}}}{x}\right)\,|\,-1)+\sqrt{2}(c^8x^8+4\,c^4x^4+3)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{28\,c^4}$$

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/28*(4*sqrt(2)*sqrt(c^4)*c*(-1/c^4)^(3/4)*elliptic_f(arcsin((-1/c^4)^(1/4)
/x), -1) + sqrt(2)*(c^8*x^8 + 4*c^4*x^4 + 3)*sqrt(c^2*x^2/(c^4*x^4 + 1)))/c
^4
```

Sympy [F]

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(x**3/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**3/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^3}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^3/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^3/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.175 $\int \frac{x^2}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{x^2}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{1}{2\left(c^4+\frac{1}{x^4}\right)x\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{csch}^{-1}(c^2x^2)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] 1/2/(c^4+1/x^4)/x/sech(2*ln(c*x))^(3/2)+1/6*x^3/sech(2*ln(c*x))^(3/2)-1/2*a
rccsch(c^2*x^2)/c^6/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5670, 5668, 342, 281, 283, 221}

$$\int \frac{x^2}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{1}{2x\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{csch}^{-1}(c^2x^2)}{2c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^3}{6\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x^2/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] 1/(2*(c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2)) + x^3/(6*Sech[2*Log[c*x]]^(3/
2)) - ArcCsch[c^2*x^2]/(2*c^6*(1 + 1/(c^4*x^4))^(3/2)*x^3*Sech[2*Log[c*x]]^
(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^3} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^5\,dx, x, cx\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}
\end{aligned}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^7}\,dx, x, \frac{1}{cx}\right)}{c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^3}{6\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{1}{2\left(c^4+\frac{1}{x^4}\right) x\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \frac{1}{c^2x^2}\right)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{1}{2\left(c^4+\frac{1}{x^4}\right) x\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^3}{6\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{\text{csch}^{-1}(c^2x^2)}{2c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{x\left(\sqrt{1+c^4x^4}(4+c^4x^4) - 3\text{arctanh}\left(\sqrt{1+c^4x^4}\right)\right)}{12\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

```
[In] Integrate[x^2/Sech[2*Log[c*x]]^(3/2),x]
[Out] (x*(Sqrt[1 + c^4*x^4]*(4 + c^4*x^4) - 3*ArcTanh[Sqrt[1 + c^4*x^4]]))/(12*Sq
rt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [F]

$$\int \frac{x^2}{\text{sech}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(x^2/sech(2*ln(c*x))^(3/2),x)
[Out] int(x^2/sech(2*ln(c*x))^(3/2),x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\sqrt{2}cx\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\sqrt{2}(c^8x^8+5\,c^4x^4+4)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{48\,c^4x}$$

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*sqrt(2)*c*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^
4*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*(c^8*x^8 + 5*c^4*x^4 + 4)*sqrt(c^2*x^2/(c
^4*x^4 + 1)))/(c^4*x)
```

Sympy [F]

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$$

```
[In] integrate(x**2/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(x**2/sech(2*log(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\operatorname{sech}\left(2\,\log(cx)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sech(2*log(c*x))^(3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x^2}{\left(\frac{1}{\cosh(2\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x^2/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x^2/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.176 $\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 214

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{12c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -12/5/(c^4+1/x^4)/(c^2+1/x^2)/x^4/sech(2*ln(c*x))^(3/2)+6/5/(c^4+1/x^4)/x^2
/sech(2*ln(c*x))^(3/2)+1/5*x^2/sech(2*ln(c*x))^(3/2)+12/5*c*(c^2+1/x^2)*(co
s(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticE(sin(2*arccot(c*x)),1
/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/(c^4+1/x^4)^2/x^3/sech(2*ln(c
*x))^(3/2)-6/5*c*(c^2+1/x^2)*(cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x)
)*EllipticF(sin(2*arccot(c*x)),1/2*2^(1/2))*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/
2)/(c^4+1/x^4)^2/x^3/sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5670, 5668, 342, 283, 311, 226, 1210}

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}dx = \frac{6}{5x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12}{5x^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{12c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)\middle|\frac{1}{2}\right)}{5x^3\left(c^4+\frac{1}{x^4}\right)^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[x/Sech[2*Log[c*x]]^(3/2),x]
```

```
[Out] -12/(5*(c^4 + x^(-4))*(c^2 + x^(-2))*x^4*Sech[2*Log[c*x]]^(3/2)) + 6/(5*(c^
4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2)) + x^2/(5*Sech[2*Log[c*x]]^(3/2)) +
(12*c*Sqrt[(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticE[2*ArcC
ot[c*x], 1/2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2)) - (6*c*Sqrt[
(c^4 + x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*EllipticF[2*ArcCot[c*x], 1/
2])/(5*(c^4 + x^(-4))^2*x^3*Sech[2*Log[c*x]]^(3/2))
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{x}{\text{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c^2}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^4\,dx, x, cx\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x^4)^{3/2}}{x^6}\,dx, x, \frac{1}{cx}\right)}{c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x^2}{5\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6\text{Subst}\left(\int \frac{\sqrt{1+x^4}}{x^2}\,dx, x, \frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3 \text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{12\text{Subst}\left(\int\frac{x^2}{\sqrt{1+x^4}}dx,x,\frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{12\text{Subst}\left(\int\frac{1}{\sqrt{1+x^4}}dx,x,\frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{12\text{Subst}\left(\int\frac{1-x^2}{\sqrt{1+x^4}}dx,x,\frac{1}{cx}\right)}{5c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x^2}{5\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{12c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)E\left(2\cot^{-1}(cx)|\frac{1}{2}\right)}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{6c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.30

$$\int\frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-c^4x^4\right)}{2\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\sqrt{1+c^4x^4}}$$

```
[In] Integrate[x/Sech[2*Log[c*x]]^(3/2),x]
[Out] -1/2*Hypergeometric2F1[-3/2, -1/4, 3/4, -(c^4*x^4)]/(Sqrt[2]*c^2*Sqrt[(c^2*
x^2)/(1 + c^4*x^4)]*Sqrt[1 + c^4*x^4])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{(c^8x^8-4c^4x^4-5)\sqrt{2}}{20(c^4x^4+1)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{3i\sqrt{-ic^2x^2+1}\,\sqrt{ic^2x^2+1}\,\left(\text{EllipticF}\left(x\sqrt{ic^2},i\right)-\text{EllipticE}\left(x\sqrt{ic^2},i\right)\right)\sqrt{2}\,x}{5\sqrt{ic^2}\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	159

[In] int(x/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/20*(c^8*x^8-4*c^4*x^4-5)/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1/2)+3/5*I/(I*c^2)^(1/2)*(1-I*c^2*x^2)^(1/2)*(1+I*c^2*x^2)^(1/2)/(c^4*x^4+1)*(EllipticF(x*(I*c^2)^(1/2),I)-EllipticE(x*(I*c^2)^(1/2),I))*2^(1/2)*x/(c^2*x^2/(c^4*x^4+1))^(1/2)

Fricas [F]

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="fricas")

[Out] integral(x/sech(2*log(c*x))^(3/2), x)

Sympy [F]

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

[In] integrate(x/sech(2*ln(c*x))**(3/2),x)

[Out] Integral(x/sech(2*log(c*x))**(3/2), x)

Maxima [F]

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="maxima")

[Out] integrate(x/sech(2*log(c*x))^(3/2), x)

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(x/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{x}{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}\,dx$$

```
[In] int(x/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(x/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.177 $\int \frac{1}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \frac{1}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{3\text{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[Out] -3/4/(c^4+1/x^4)/x^3/sech(2*ln(c*x))^(3/2)+1/4*x/sech(2*ln(c*x))^(3/2)+3/4*
arctanh((1+1/c^4/x^4)^(1/2))/c^4/(1+1/c^4/x^4)^(3/2)/x^3/sech(2*ln(c*x))^(3
/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5664, 5662, 272, 43, 52, 65, 213}

$$\int \frac{1}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\text{arctanh}\left(\sqrt{\frac{1}{c^4x^4}+1}\right)}{4c^4x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\text{sech}^{\frac{3}{2}}(2\log(cx))}-\frac{3}{4x^3\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}+\frac{x}{4\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

```
[In] Int[Sech[2*Log[c*x]]^(-3/2),x]
```

```
[Out] -3/(4*(c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2)) + x/(4*Sech[2*Log[c*x]]^(3
/2)) + (3*ArcTanh[Sqrt[1 + 1/(c^4*x^4)]])/(4*c^4*(1 + 1/(c^4*x^4))^(3/2)*x^
3*Sech[2*Log[c*x]]^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5662

```
Int[Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol] :> Dist[Sech[d*(a
+ b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*d*p)), Int[1/(x^(b*
d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p), x], x] /; FreeQ[{a, b, d, p}, x] &&
 !IntegerQ[p]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
```

```
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\text{sech}^{\frac{3}{2}}(2\log(x))}\,dx, x, cx\right)}{c}$$

$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}\right)^{3/2} x^3\,dx, x, cx\right)}{c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{\text{Subst}\left(\int \frac{(1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{c^4x^4}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= \frac{x}{4\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \frac{1}{c^4x^4}\right)}{8c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{sech}^{\frac{3}{2}}(2\log(cx))} - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

$$= -\frac{3}{4\left(c^4+\frac{1}{x^4}\right)x^3\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{x}{4\text{sech}^{\frac{3}{2}}(2\log(cx))} + \frac{3\text{arctanh}\left(\sqrt{1+\frac{1}{c^4x^4}}\right)}{4c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{1}{\text{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = -\frac{\sqrt{1+c^4x^4}\sqrt{\frac{c^2x^2}{2+2c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-c^4x^4\right)}{4c^4x^3}$$

```
[In] Integrate[Sech[2*Log[c*x]]^(-3/2),x]
[Out] -1/4*(Sqrt[1 + c^4*x^4]*Sqrt[(c^2*x^2)/(2 + 2*c^4*x^4)]*Hypergeometric2F1[-
3/2, -1/2, 1/2, -(c^4*x^4)])/(c^4*x^3)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42

method	result	size
risch	$\frac{(c^8x^8-c^4x^4-2)\sqrt{2}}{16x(c^4x^4+1)c^2\sqrt{\frac{c^2x^2}{c^4x^4+1}}}+\frac{3c^2\ln\left(\frac{c^4x^2}{\sqrt{c^4}}+\sqrt{c^4x^4+1}\right)\sqrt{2}\,x}{16\sqrt{c^4}\,\sqrt{c^4x^4+1}\,\sqrt{\frac{c^2x^2}{c^4x^4+1}}}$	131

```
[In] int(1/sech(2*ln(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(c^8*x^8-c^4*x^4-2)/x/(c^4*x^4+1)*2^(1/2)/c^2/(c^2*x^2/(c^4*x^4+1))^(1
/2)+3/16*c^2*ln(c^4*x^2/(c^4)^(1/2)+(c^4*x^4+1)^(1/2))/(c^4)^(1/2)*2^(1/2)*
x/(c^4*x^4+1)^(1/2)/(c^2*x^2/(c^4*x^4+1))^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \frac{3\,\sqrt{2}c^3x^3\log\left(-2\,c^4x^4-2\,(c^5x^5+cx)\sqrt{\frac{c^2x^2}{c^4x^4+1}}-1\right)+2\,\sqrt{2}(c^8x^8-c^4x^4-2)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{32\,c^4x^3}$$

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/32*(3*sqrt(2)*c^3*x^3*log(-2*c^4*x^4 - 2*(c^5*x^5 + c*x)*sqrt(c^2*x^2/(c^
4*x^4 + 1)) - 1) + 2*sqrt(2)*(c^8*x^8 - c^4*x^4 - 2)*sqrt(c^2*x^2/(c^4*x^4
+ 1)))/(c^4*x^3)
```

Sympy [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/sech(2*ln(c*x))**(3/2),x)
```

```
[Out] Integral(sech(2*log(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(sech(2*log(c*x))^(-3/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \text{Timed out}$$

```
[In] integrate(1/sech(2*log(c*x))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}\,dx = \int \frac{1}{{\left(\frac{1}{\cosh\left(2\,\ln\left(c\,x\right)\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(1/cosh(2*log(c*x)))^(3/2),x)
[Out] int(1/(1/cosh(2*log(c*x)))^(3/2), x)
```

3.178 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = i\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)\sqrt{\operatorname{sech}(2\log(cx))} + \sqrt{\operatorname{sech}(2\log(cx))}\sinh(2\log(cx))$$

```
[Out] sinh(2*ln(c*x))*sech(2*ln(c*x))^(1/2)+I*((1/2*c*x+1/2/c/x)^2)^(1/2)/(1/2*c*
x+1/2/c/x)*EllipticE(I*(1/2*c*x-1/2/c/x),2^(1/2))*cosh(2*ln(c*x))^(1/2)*sec
h(2*ln(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3853, 3856, 2719}

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \sinh(2\log(cx))\sqrt{\operatorname{sech}(2\log(cx))} + i\sqrt{\operatorname{sech}(2\log(cx))}\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)$$

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x,x]
```

```
[Out] I*Sqrt[Cosh[2*Log[c*x]]]*EllipticE[I*Log[c*x], 2]*Sqrt[Sech[2*Log[c*x]]] +
Sqrt[Sech[2*Log[c*x]]]*Sinh[2*Log[c*x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst}\left(\int \text{sech}^{\frac{3}{2}}(2x)\,dx, x, \log(cx)\right)\\
&= \sqrt{\text{sech}(2\log(cx))}\sinh(2\log(cx)) - \text{Subst}\left(\int \frac{1}{\sqrt{\text{sech}(2x)}}\,dx, x, \log(cx)\right)\\
&= \sqrt{\text{sech}(2\log(cx))}\sinh(2\log(cx))\\
&\quad - \left(\sqrt{\cosh(2\log(cx))}\sqrt{\text{sech}(2\log(cx))}\right)\text{Subst}\left(\int \sqrt{\cosh(2x)}\,dx, x, \log(cx)\right)\\
&= i\sqrt{\cosh(2\log(cx))}E(i\log(cx)|2)\sqrt{\text{sech}(2\log(cx))} + \sqrt{\text{sech}(2\log(cx))}\sinh(2\log(cx))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \frac{\frac{iE(i\log(cx)|2)}{\sqrt{\cosh(2\log(cx))}} + \tanh(2\log(cx))}{\sqrt{\text{sech}(2\log(cx))}}$$

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x,x]
[Out] ((I*EllipticE[I*Log[c*x], 2])/Sqrt[Cosh[2*Log[c*x]]] + Tanh[2*Log[c*x]])/Sq
rt[Sech[2*Log[c*x]]]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

method	result	size
derivativedivides	$\frac{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}+\text{EllipticE}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)\sqrt{-2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}-1}\,\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}}{\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}-1}}$	127
default	$\frac{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}+\text{EllipticE}\left(\frac{cx}{2}+\frac{1}{2cx},\sqrt{2}\right)\sqrt{-2\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}-1}\,\sqrt{-\left(\frac{cx}{2}-\frac{1}{2cx}\right)^{2}}}{\left(\frac{cx}{2}-\frac{1}{2cx}\right)\sqrt{2\left(\frac{cx}{2}+\frac{1}{2cx}\right)^{2}-1}}$	127

```
[In] int(sech(2*ln(c*x))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (2*(1/2*c*x+1/2/c/x)*(1/2*c*x-1/2/c/x)^2+EllipticE(1/2*c*x+1/2/c/x,2^(1/2))
*(-2*(1/2*c*x-1/2/c/x)^2-1)^(1/2)*(-(1/2*c*x-1/2/c/x)^2)^(1/2))/(1/2*c*x-1/
2/c/x)/(2*(1/2*c*x+1/2/c/x)^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \frac{\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^3x^2+\sqrt{2}(-c^4)^{\frac{3}{4}}E\left(\arcsin\left((-c^4)^{\frac{1}{4}}x\right)|-1\right)-\sqrt{2}(-c^4)^{\frac{3}{4}}F\left(\arcsin\left((-c^4)^{\frac{1}{4}}x\right)|-1\right)}{c}$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^3*x^2 + sqrt(2)*(-c^4)^(3/4)*ellipti
c_e(arcsin((-c^4)^(1/4)*x), -1) - sqrt(2)*(-c^4)^(3/4)*elliptic_f(arcsin((-
c^4)^(1/4)*x), -1))/c
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x,x)
```

```
[Out] Integral(sech(2*log(c*x))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="maxima")
[Out] integrate(sech(2*log(c*x))^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x}\,dx = \int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x}\,dx$$

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x,x)
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x, x)
```

3.179 $\int \frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^3\text{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] 1/2*(c^4+1/x^4)*x^3*sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5670, 5668, 267}

$$\int \frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \frac{1}{2}x^3\left(c^4+\frac{1}{x^4}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^2,x]
```

```
[Out] ((c^4 + x^(-4))*x^3*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
```

```
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c\text{Subst}\left(\int \frac{\text{sech}^{\frac{3}{2}}(2\log(x))}{x^2}\,dx, x, cx\right)\\
&= \left(c^4\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^5}\,dx, x, cx\right)\\
&= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^3\text{sech}^{\frac{3}{2}}(2\log(cx))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \sqrt{2}c^2x\sqrt{\frac{c^2x^2}{1+c^4x^4}}$$

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^2,x]
[Out] Sqrt[2]*c^2*x*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]
```

Maple [F]

$$\int \frac{\text{sech}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^2}dx$$

```
[In] int(sech(2*ln(c*x))^(3/2)/x^2,x)
[Out] int(sech(2*ln(c*x))^(3/2)/x^2,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^2x$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="fricas")
[Out] sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2*x
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**2,x)
[Out] Integral(sech(2*log(c*x))**(3/2)/x**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = c\left(\frac{\sqrt{2}}{\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}} + \frac{\sqrt{2}}{c^4x^4\left(\frac{1}{c^4x^4}+1\right)^{\frac{3}{2}}}\right)$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="maxima")
[Out] c*(sqrt(2)/(1/(c^4*x^4) + 1)^(3/2) + sqrt(2)/(c^4*x^4*(1/(c^4*x^4) + 1)^(3/
2)))
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^2}\,dx = c^2\,x\,\sqrt{\frac{2\,c^2\,x^2}{c^4\,x^4+1}}$$

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^2,x)
```

```
[Out] c^2*x*((2*c^2*x^2)/(c^4*x^4 + 1))^(1/2)
```

3.180 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3\,\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{4c}$$

```
[Out] 1/2*(c^4+1/x^4)*x^2*sech(2*ln(c*x))^(3/2)-1/4*(c^4+1/x^4)*(c^2+1/x^2)*x^3*(
cos(2*arccot(c*x))^2)^(1/2)/cos(2*arccot(c*x))*EllipticF(sin(2*arccot(c*x))
,1/2*2^(1/2))*sech(2*ln(c*x))^(3/2)*((c^4+1/x^4)/(c^2+1/x^2)^2)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5670, 5668, 342, 294, 226}

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \frac{1}{2}x^2\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{x^3\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\,\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)}{4c}$$

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^3,x]
```

```
[Out] ((c^4 + x^(-4))*x^2*Sech[2*Log[c*x]]^(3/2))/2 - ((c^4 + x^(-4))*Sqrt[(c^4 +
 x^(-4))/(c^2 + x^(-2))^2]*(c^2 + x^(-2))*x^3*EllipticF[2*ArcCot[c*x], 1/2]
*Sech[2*Log[c*x]]^(3/2))/(4*c)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= c^2\text{Subst}\left(\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(x))}{x^3}\,dx, x, cx\right)\\
&= \left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\operatorname{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^6}\,dx, x, cx\right)
\end{aligned}$$

$$= -\left(\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int\frac{x^4}{(1+x^4)^{3/2}}\,dx,x,\frac{1}{cx}\right)\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\text{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}\left(c^5\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+x^4}}\,dx,x,\frac{1}{cx}\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\text{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3\,\text{EllipticF}\left(2\cot^{-1}(cx),\frac{1}{2}\right)\text{sech}^{\frac{3}{2}}(2\log(cx))}{4c}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int\frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\left(1+\sqrt{1+c^4x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-c^4x^4\right)\right)$$

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^3,x]
[Out] Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*(1 + Sqrt[1 + c^4*x^4]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -(c^4*x^4)])
```

Maple [F]

$$\int\frac{\text{sech}\,(2\ln(cx))^{\frac{3}{2}}}{x^3}dx$$

```
[In] int(sech(2*ln(c*x))^(3/2)/x^3,x)
[Out] int(sech(2*ln(c*x))^(3/2)/x^3,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \frac{\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^3 - \sqrt{2}(-c^4)^{\frac{3}{4}}\,F\left(\arcsin\left((-c^4)^{\frac{1}{4}}\,x\right)\,|-1\right)}{c}$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="fricas")
[Out] (sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^3 - sqrt(2)*(-c^4)^(3/4)*elliptic_f(
arcsin((-c^4)^(1/4)*x), -1))/c
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}\left(2\log\left(cx\right)\right)}{x^3}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**3,x)
[Out] Integral(sech(2*log(c*x))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate(sech(2*log(c*x))^(3/2)/x^3, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^3,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^3}\,dx = \int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^3}\,dx$$

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^3,x)
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x^3, x)
```

3.181 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4} dx = \frac{1}{2}\left(c^4+\frac{1}{x^4}\right) x\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[Out] 1/2*(c^4+1/x^4)*x*sech(2*ln(c*x))^(3/2)-1/2*c^6*(1+1/c^4/x^4)^(3/2)*x^3*arc
csch(c^2*x^2)*sech(2*ln(c*x))^(3/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5670, 5668, 342, 281, 294, 221}

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4} dx = \frac{1}{2}x\left(c^4+\frac{1}{x^4}\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}c^6x^3\left(\frac{1}{c^4x^4}+1\right)^{3/2}\operatorname{csch}^{-1}\left(c^2x^2\right)\operatorname{sech}^{\frac{3}{2}}(2\log(cx))$$

```
[In] Int[Sech[2*Log[c*x]]^(3/2)/x^4,x]
```

```
[Out] ((c^4 + x^(-4))*x*Sech[2*Log[c*x]]^(3/2))/2 - (c^6*(1 + 1/(c^4*x^4))^(3/2)*
x^3*ArcCsch[c^2*x^2]*Sech[2*Log[c*x]]^(3/2))/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rule 5670

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p
_.), x_Symbol] :> Dist[(e*x)^(m + 1)/(e*n*(c*x^n)^((m + 1)/n)), Subst[Int[x
^((m + 1)/n - 1)*Sech[d*(a + b*Log[x])]^p, x], x, c*x^n], x] /; FreeQ[{a, b
, c, d, e, m, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rubi steps

$$\text{integral} = c^3\text{Subst}\left(\int \frac{\text{sech}^{\frac{3}{2}}(2\log(x))}{x^4}\,dx, x, cx\right)$$

$$= \left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2} x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{1}{x^4}\right)^{3/2}x^7}\,dx, x, cx\right)$$

$$= -\left(\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int\frac{x^5}{(1+x^4)^{3/2}}\,dx,x,\frac{1}{cx}\right)\right)$$

$$= -\left(\frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int\frac{x^2}{(1+x^2)^{3/2}}\,dx,x,\frac{1}{c^2x^2}\right)\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\text{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}\left(c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{sech}^{\frac{3}{2}}(2\log(cx))\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}\,dx,x,\frac{1}{c^2x^2}\right)$$

$$= \frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x\text{sech}^{\frac{3}{2}}(2\log(cx)) - \frac{1}{2}c^6\left(1+\frac{1}{c^4x^4}\right)^{3/2}x^3\text{csch}^{-1}\left(c^2x^2\right)\text{sech}^{\frac{3}{2}}(2\log(cx))$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int\frac{\text{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \frac{\sqrt{2}c^2\sqrt{\frac{c^2x^2}{1+c^4x^4}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},1+c^4x^4\right)}{x}$$

```
[In] Integrate[Sech[2*Log[c*x]]^(3/2)/x^4,x]
[Out] (Sqrt[2]*c^2*Sqrt[(c^2*x^2)/(1 + c^4*x^4)]*Hypergeometric2F1[-1/2, 1, 1/2,
1 + c^4*x^4])/x
```

Maple [F]

$$\int\frac{\text{sech}\left(2\ln\left(cx\right)\right)^{\frac{3}{2}}}{x^4}dx$$

```
[In] int(sech(2*ln(c*x))^(3/2)/x^4,x)
[Out] int(sech(2*ln(c*x))^(3/2)/x^4,x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \frac{\sqrt{2}c^3x\log\left(\frac{c^5x^5+2\,cx-2\,(c^4x^4+1)\sqrt{\frac{c^2x^2}{c^4x^4+1}}}{cx^5}\right)+2\,\sqrt{2}\sqrt{\frac{c^2x^2}{c^4x^4+1}}c^2}{2\,x}$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*c^3*x*log((c^5*x^5 + 2*c*x - 2*(c^4*x^4 + 1)*sqrt(c^2*x^2/(c^4
*x^4 + 1)))/(c*x^5)) + 2*sqrt(2)*sqrt(c^2*x^2/(c^4*x^4 + 1))*c^2)/x
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx$$

```
[In] integrate(sech(2*ln(c*x))**(3/2)/x**4,x)
[Out] Integral(sech(2*log(c*x))**(3/2)/x**4, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\operatorname{sech}\left(2\,\log\left(cx\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate(sech(2*log(c*x))^(3/2)/x^4, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate(sech(2*log(c*x))^(3/2)/x^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(2\log(cx))}{x^4}\,dx = \int \frac{\left(\frac{1}{\cosh(2\,\ln(c\,x))}\right)^{3/2}}{x^4}\,dx$$

```
[In] int((1/cosh(2*log(c*x)))^(3/2)/x^4,x)
[Out] int((1/cosh(2*log(c*x)))^(3/2)/x^4, x)
```

3.182 $\int \operatorname{sech}(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 63

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx = \frac{2e^a x(cx^n)^b \text{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+bn}$$

```
[Out] 2*exp(a)*x*(c*x^n)^b*hypergeom([1, 1/2*(b+1/n)/b],[3/2+1/2/b/n],-exp(2*a)*(
c*x^n)^(2*b))/(b*n+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5664, 5666, 269, 371}

$$\int \operatorname{sech}(a+b\log(cx^n))\,dx = \frac{2e^a x(cx^n)^b \text{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{bn+1}$$

```
[In] Int[Sech[a + b*Log[c*x^n]],x]
```

```
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1/(b*n))/2
, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1+e^{-2a}x^{-2b}}\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(2e^{-a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{e^{-2a}+x^{2b}}\,dx,x,cx^n\right)}{n}\\
&= \frac{2e^a x(cx^n)^b\,\text{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$
\int \operatorname{sech}(a+b\log(cx^n))\,dx = \frac{2e^a x(cx^n)^b\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right),\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+bn}
$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]],x]
[Out] (2*E^a*x*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1/(b*n))/2, -
(E^(2*a)*(c*x^n)^(2*b))])/(1 + b*n)
```

Maple [F]

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)dx$$

```
[In] int(sech(a+b*ln(c*x^n)),x)
[Out] int(sech(a+b*ln(c*x^n)),x)
```

Fricas [F]

$$\int \operatorname{sech}(a+b\log\left(cx^{n}\right))\,dx=\int \operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="fricas")
[Out] integral(sech(b*log(c*x^n) + a), x)
```

Sympy [F]

$$\int \operatorname{sech}(a+b\log\left(cx^{n}\right))\,dx=\int \operatorname{sech}\left(a+b\log\left(cx^{n}\right)\right)dx$$

```
[In] integrate(sech(a+b*ln(c*x**n)),x)
[Out] Integral(sech(a + b*log(c*x**n)), x)
```

Maxima [F]

$$\int \operatorname{sech}(a+b\log\left(cx^{n}\right))\,dx=\int \operatorname{sech}\left(b\log\left(cx^{n}\right)+a\right)\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="maxima")
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

Giac [F]

$$\int \operatorname{sech}(a + b\log(cx^n))\,dx = \int \operatorname{sech}(b\log(cx^n) + a)\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n)),x, algorithm="giac")
[Out] integrate(sech(b*log(c*x^n) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}(a + b\log(cx^n))\,dx = \int \frac{1}{\cosh(a + b\,\ln(c\,x^n))}\,dx$$

```
[In] int(1/cosh(a + b*log(c*x^n)),x)
[Out] int(1/cosh(a + b*log(c*x^n)), x)
```

3.183 $\int \text{sech}^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \text{sech}^2(a+b\log(cx^n))\,dx = \frac{4e^{2a}x(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right),\frac{1}{2}\left(4+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+2bn}$$

```
[Out] 4*exp(2*a)*x*(c*x^n)^(2*b)*hypergeom([2, 1+1/2/b/n],[2+1/2/b/n],-exp(2*a)*(
c*x^n)^(2*b))/(2*b*n+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5664, 5666, 269, 371}

$$\int \text{sech}^2(a+b\log(cx^n))\,dx = \frac{4e^{2a}x(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right),\frac{1}{2}\left(4+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{2bn+1}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (4*E^(2*a)*x*(c*x^n)^(2*b)*Hypergeometric2F1[2, (2 + 1/(b*n))/2, (4 + 1/(b*
n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^2(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1-2b+\frac{1}{n}}}{\left(1+e^{-2a}x^{-2b}\right)^2}\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(4e^{-2a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+2b+\frac{1}{n}}}{\left(e^{-2a}+x^{2b}\right)^2}\,dx,x,cx^n\right)}{n}\\
&= \frac{4e^{2a}x(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(2,\frac{1}{2}\left(2+\frac{1}{bn}\right),\frac{1}{2}\left(4+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+2bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.83

$$\int \operatorname{sech}^2(a+b\log(cx^n))\,dx$$
$$= \frac{x\left(-\frac{e^{2a}(cx^n)^{2b}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2bn},2+\frac{1}{2bn},-e^{2a}(cx^n)^{2b}\right)}{1+2bn}+\text{Hypergeometric2F1}\left(1,\frac{1}{2bn},1+\frac{1}{2bn},-e^{2a}(cx^n)^{2b}\right)\right)}{bn}$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2,x]
```

```
[Out] (x*(-((E^(2*a)*(c*x^n)^(2*b)*Hypergeometric2F1[1, 1 + 1/(2*b*n), 2 + 1/(2*b
*n), -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 2*b*n)) + Hypergeometric2F1[1, 1/(2*b*
n), 1 + 1/(2*b*n), -(E^(2*a)*(c*x^n)^(2*b))] + Tanh[a + b*Log[c*x^n]]))/(b*
n)
```

Maple [F]

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}dx$$

```
[In] int(sech(a+b*ln(c*x^n))^2,x)
[Out] int(sech(a+b*ln(c*x^n))^2,x)
```

Fricas [F]

$$\int \operatorname{sech}^2(a+b\log\left(cx^n\right))\,dx=\int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="fricas")
[Out] integral(sech(b*log(c*x^n) + a)^2, x)
```

Sympy [F]

$$\int \operatorname{sech}^2(a+b\log\left(cx^n\right))\,dx=\int \operatorname{sech}^2\left(a+b\log\left(cx^n\right)\right)dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**2,x)
[Out] Integral(sech(a + b*log(c*x**n))**2, x)
```

Maxima [F]

$$\int \operatorname{sech}^2(a+b\log\left(cx^n\right))\,dx=\int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^2\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="maxima")
[Out] -2*x/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n) + 4*integrate(1/2/(b*c^(2*b
)*n*e^(2*b*log(x^n) + 2*a) + b*n), x)
```

Giac [F]

$$\int \operatorname{sech}^2(a + b\log(cx^n))\,dx = \int \operatorname{sech}(b\log(cx^n) + a)^2\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^2,x, algorithm="giac")
[Out] integrate(sech(b*log(c*x^n) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^2(a + b\log(cx^n))\,dx = \int \frac{1}{\cosh(a + b\ln(c\,x^n))^2}\,dx$$

```
[In] int(1/cosh(a + b*log(c*x^n))^2,x)
[Out] int(1/cosh(a + b*log(c*x^n))^2, x)
```

3.184 $\int \operatorname{sech}^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = \frac{8e^{3a}x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

```
[Out] 8*exp(3*a)*x*(c*x^n)^(3*b)*hypergeom([3, 1/2*(3*b+1/n)/b],[5/2+1/2/b/n],-ex
p(2*a)*(c*x^n)^(2*b))/(3*b*n+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5664, 5666, 269, 371}

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = \frac{8e^{3a}x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{3bn+1}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (8*E^(3*a)*x*(c*x^n)^(3*b)*Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 +
1/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^3(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{(1+e^{-2a}x^{-2b})^3}\,dx,x,cx^n\right)}{n}\\
&= \frac{\left(8e^{-3a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{(e^{-2a}+x^{2b})^3}\,dx,x,cx^n\right)}{n}\\
&= \frac{8e^{3a}x(cx^n)^{3b}\,\text{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+3bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.44

$$\int \operatorname{sech}^3(a+b\log(cx^n))\,dx = \frac{x\left(2e^a(-1+bn)(cx^n)^b\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1+\frac{1}{bn}\right),\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)+\operatorname{sech}(a+b\log(cx^n$$
$$2b^2n^2$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3,x]
```

```
[Out] (x*(2*E^a*(-1 + b*n)*(c*x^n)^b*Hypergeometric2F1[1, (1 + 1/(b*n))/2, (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh
[a + b*Log[c*x^n]])))/(2*b^2*n^2)
```

Maple [F]

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{3}dx$$

```
[In] int(sech(a+b*ln(c*x^n))^3,x)
[Out] int(sech(a+b*ln(c*x^n))^3,x)
```

Fricas [F]

$$\int \operatorname{sech}^3(a+b\log\left(cx^n\right))\ dx = \int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^3\ dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="fricas")
[Out] integral(sech(b*log(c*x^n) + a)^3, x)
```

Sympy [F]

$$\int \operatorname{sech}^3(a+b\log\left(cx^n\right))\ dx = \int \operatorname{sech}^3\left(a+b\log\left(cx^n\right)\right)dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**3,x)
[Out] Integral(sech(a + b*log(c*x**n))**3, x)
```

Maxima [F]

$$\int \operatorname{sech}^3(a+b\log\left(cx^n\right))\ dx = \int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^3\ dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="maxima")
[Out] 8*(b^2*c^b*n^2 - c^b)*integrate(1/8*e^(b*log(x^n) + a)/(b^2*c^(2*b)*n^2*e^(
2*b*log(x^n) + 2*a) + b^2*n^2), x) + ((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(
x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(b*log(x^n) + a))/(b^2*c^(4*b)*n^2*e^(4*b
*log(x^n) + 4*a) + 2*b^2*c^(2*b)*n^2*e^(2*b*log(x^n) + 2*a) + b^2*n^2)
```

Giac [F]

$$\int \operatorname{sech}^3(a+b\log(cx^n))\ dx = \int \operatorname{sech}\left(b\log(cx^n)+a\right)^3\ dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^3,x, algorithm="giac")
[Out] integrate(sech(b*log(c*x^n) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^3(a+b\log(cx^n))\ dx = \int \frac{1}{\cosh\left(a+b\ \ln\left(c\,x^n\right)\right)^3}\,dx$$

```
[In] int(1/cosh(a + b*log(c*x^n))^3,x)
[Out] int(1/cosh(a + b*log(c*x^n))^3, x)
```

3.185 $\int \operatorname{sech}^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \frac{16e^{4a}x(cx^n)^{4b}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right),\frac{1}{2}\left(6+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{1+4bn}$$

```
[Out] 16*exp(4*a)*x*(c*x^n)^(4*b)*hypergeom([4, 2+1/2/b/n],[3+1/2/b/n],-exp(2*a)*
(c*x^n)^(2*b))/(4*b*n+1)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5664, 5666, 269, 371}

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \frac{16e^{4a}x(cx^n)^{4b}\,\text{Hypergeometric2F1}\left(4,\frac{1}{2}\left(4+\frac{1}{bn}\right),\frac{1}{2}\left(6+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{4bn+1}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] (16*E^(4*a)*x*(c*x^n)^(4*b)*Hypergeometric2F1[4, (4 + 1/(b*n))/2, (6 + 1/(b
*n))/2, -(E^(2*a)*(c*x^n)^(2*b))])/(1 + 4*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\text{sech}^4(a+b\log(x))\,dx, x, cx^n\right)}{n}\\
&= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1-4b+\frac{1}{n}}}{(1+e^{-2a}x^{-2b})^4}\,dx, x, cx^n\right)}{n}\\
&= \frac{\left(16e^{-4a}x(cx^n)^{-1/n}\right)\text{Subst}\left(\int \frac{x^{-1+4b+\frac{1}{n}}}{(e^{-2a}+x^{2b})^4}\,dx, x, cx^n\right)}{n}\\
&= \frac{16e^{4a}x(cx^n)^{4b}\,\text{Hypergeometric2F1}\left(4, \frac{1}{2}\left(4+\frac{1}{bn}\right), \frac{1}{2}\left(6+\frac{1}{bn}\right), -e^{2a}(cx^n)^{2b}\right)}{1+4bn}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 750 vs. $2(69) = 138$.

Time = 14.12 (sec) , antiderivative size = 750, normalized size of antiderivative = 10.87

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx$$
$$= \frac{(-1+4b^2n^2)\,x\operatorname{sech}(a+b(-n\log(x)+\log(cx^n)))\operatorname{sech}(a+bn\log(x)+b(-n\log(x)+\log(cx^n)))\sinh(bn\log}{6b^3n^3}$$
$$+\frac{x\operatorname{sech}(a+b(-n\log(x)+\log(cx^n)))\operatorname{sech}^3(a+bn\log(x)+b(-n\log(x)+\log(cx^n)))\sinh(bn\log(x))}{3bn}$$
$$+\frac{x\operatorname{sech}(a+b(-n\log(x)+\log(cx^n)))\operatorname{sech}^2(a+bn\log(x)+b(-n\log(x)+\log(cx^n)))\left(\cosh(a+b(-n\log\right.}{6b^2n^2}$$
$$+\frac{e^{-\frac{a+b(-n\log(x)+\log(cx^n))}{bn}}\operatorname{sech}(a+b(-n\log(x)+\log(cx^n)))\left(e^{\left(2+\frac{1}{bn}\right)(a+b\log(cx^n))}\cosh(a+b(-n\log(x)+\log(\right.}{}$$
$$-\frac{2e^{-\frac{a+b(-n\log(x)+\log(cx^n))}{bn}}\operatorname{sech}(a+b(-n\log(x)+\log(cx^n)))\left(e^{\left(2+\frac{1}{bn}\right)(a+b\log(cx^n))}\cosh(a+b(-n\log(x)+\log\right.}{}$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^4,x]
```

```
[Out] ((-1 + 4*b^2*n^2)*x*Sech[a + b*(-(n*Log[x]) + Log[c*x^n])]*Sech[a + b*n*Log
[x] + b*(-(n*Log[x]) + Log[c*x^n])]*Sinh[b*n*Log[x]])/(6*b^3*n^3) + (x*Sech
[a + b*(-(n*Log[x]) + Log[c*x^n])]*Sech[a + b*n*Log[x] + b*(-(n*Log[x]) + L
og[c*x^n])]^3*Sinh[b*n*Log[x]])/(3*b*n) + (x*Sech[a + b*(-(n*Log[x]) + Log[
c*x^n])]*Sech[a + b*n*Log[x] + b*(-(n*Log[x]) + Log[c*x^n])]^2*(Cosh[a + b*
(-(n*Log[x]) + Log[c*x^n])] + 2*b*n*Sinh[a + b*(-(n*Log[x]) + Log[c*x^n])])
)/(6*b^2*n^2) + (Sech[a + b*(-(n*Log[x]) + Log[c*x^n])]*(E^((2 + 1/(b*n))*(
a + b*Log[c*x^n]))*Cosh[a + b*(-(n*Log[x]) + Log[c*x^n])]*Hypergeometric2F1
[1, 1 + 1/(2*b*n), 2 + 1/(2*b*n), -E^(2*(a + b*Log[c*x^n]))] - E^(a/(b*n) +
 (-(n*Log[x]) + Log[c*x^n])/n)*(1 + 2*b*n)*x*(Cosh[a + b*(-(n*Log[x]) + Log
[c*x^n])]*Hypergeometric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), -E^(2*(a + b*n*Log
[x] + b*(-(n*Log[x]) + Log[c*x^n])))] + Sinh[a + b*(-(n*Log[x]) + Log[c*x^n
])])))/(6*b^3*E^((a + b*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*n^3*(1 + 2*b*n))
 - (2*Sech[a + b*(-(n*Log[x]) + Log[c*x^n])]*(E^((2 + 1/(b*n))*(a + b*Log[c
*x^n]))*Cosh[a + b*(-(n*Log[x]) + Log[c*x^n])]*Hypergeometric2F1[1, 1 + 1/(
2*b*n), 2 + 1/(2*b*n), -E^(2*(a + b*Log[c*x^n]))] - E^(a/(b*n) + (-(n*Log[x
]) + Log[c*x^n])/n)*(1 + 2*b*n)*x*(Cosh[a + b*(-(n*Log[x]) + Log[c*x^n])]*H
ypergeometric2F1[1, 1/(2*b*n), 1 + 1/(2*b*n), -E^(2*(a + b*n*Log[x] + b*(-(
n*Log[x]) + Log[c*x^n])))] + Sinh[a + b*(-(n*Log[x]) + Log[c*x^n])])))/(3*b
*E^((a + b*(-(n*Log[x]) + Log[c*x^n]))/(b*n))*n*(1 + 2*b*n))
```

Maple [F]

$$\int \operatorname{sech}\left(a+b\ln\left(c\,x^{n}\right)\right)^{4}dx$$

[In] int(sech(a+b*ln(c*x^n))^4,x)

[Out] int(sech(a+b*ln(c*x^n))^4,x)

Fricas [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^4\,dx$$

[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="fricas")

[Out] integral(sech(b*log(c*x^n) + a)^4, x)

Sympy [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}^4\left(a+b\log\left(cx^n\right)\right)dx$$

[In] integrate(sech(a+b*ln(c*x**n))**4,x)

[Out] Integral(sech(a + b*log(c*x**n))**4, x)

Maxima [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}\left(b\log\left(cx^n\right)+a\right)^4\,dx$$

[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="maxima")

[Out] 16*(4*b^2*n^2 - 1)*integrate(1/48/(b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) + b^3*n^3), x) + 1/3*((2*b*c^(4*b)*n + c^(4*b))*x*e^(4*b*log(x^n) + 4*a) - 2*(6*b^2*c^(2*b)*n^2 - b*c^(2*b)*n - c^(2*b))*x*e^(2*b*log(x^n) + 2*a) - (4*b^2*n^2 - 1)*x)/(b^3*c^(6*b)*n^3*e^(6*b*log(x^n) + 6*a) + 3*b^3*c^(4*b)*n^3*e^(4*b*log(x^n) + 4*a) + 3*b^3*c^(2*b)*n^3*e^(2*b*log(x^n) + 2*a) + b^3*n^3)

Giac [F]

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \operatorname{sech}(b\log(cx^n)+a)^4\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^4,x, algorithm="giac")
[Out] integrate(sech(b*log(c*x^n) + a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^4(a+b\log(cx^n))\,dx = \int \frac{1}{\cosh(a+b\ln(c\,x^n))^4}\,dx$$

```
[In] int(1/cosh(a + b*log(c*x^n))^4,x)
[Out] int(1/cosh(a + b*log(c*x^n))^4, x)
```

3.186 $\int \left(\left(1-b^2n^2\right)\mathbf{sech}(a+b\log\left(cx^n\right))+2b^2n^2\mathbf{sech}^3(a$

Optimal result

Integrand size = 44, antiderivative size = 40

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx \\ = x\operatorname{sech}(a+b\log\left(cx^n\right))+bnx\operatorname{sech}(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)$$

```
[Out] x*sech(a+b*ln(c*x^n))+b*n*x*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.48, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5664, 5666, 269, 371}

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx \\ = \frac{16e^{3a}b^2n^2x(cx^n)^{3b}\operatorname{Hypergeometric2F1}\left(3,\frac{3b+\frac{1}{n}}{2b},\frac{1}{2}\left(5+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)}{3bn+1} \\ +2e^ax(1-bn)\left(cx^n\right)^b\operatorname{Hypergeometric2F1}\left(1,\frac{b+\frac{1}{n}}{2b},\frac{1}{2}\left(3+\frac{1}{bn}\right),-e^{2a}(cx^n)^{2b}\right)$$

```
[In] Int[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c*x^n]]
^3,x]
```

```
[Out] 2*E^a*(1 - b*n)*x*(c*x^n)^b*Hypergeometric2F1[1, (b + n^(-1))/(2*b), (3 + 1
/(b*n))/2, -(E^(2*a)*(c*x^n)^(2*b))] + (16*b^2*E^(3*a)*n^2*x*(c*x^n)^(3*b)*
Hypergeometric2F1[3, (3*b + n^(-1))/(2*b), (5 + 1/(b*n))/2, -(E^(2*a)*(c*x^
n)^(2*b))])/(1 + 3*b*n)
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \left(2b^2n^2\right)\int \operatorname{sech}^3(a+b\log(cx^n))\,dx+\left(1-b^2n^2\right)\int \operatorname{sech}(a+b\log(cx^n))\,dx\\
&= \left(2b^2nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}^3(a+b\log(x))\,dx,x,cx^n\right)\\
&\quad+\frac{\left(\left(1-b^2n^2\right)x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int x^{-1+\frac{1}{n}}\operatorname{sech}(a+b\log(x))\,dx,x,cx^n\right)}{n}\\
&= \left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-3b+\frac{1}{n}}}{\left(1+e^{-2a}x^{-2b}\right)^3}\,dx,x,cx^n\right)\\
&\quad+\frac{\left(2e^{-a}\left(1-b^2n^2\right)x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1-b+\frac{1}{n}}}{1+e^{-2a}x^{-2b}}\,dx,x,cx^n\right)}{n}\\
&= \left(16b^2e^{-3a}nx(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+3b+\frac{1}{n}}}{\left(e^{-2a}+x^{2b}\right)^3}\,dx,x,cx^n\right)\\
&\quad+\frac{\left(2e^{-a}\left(1-b^2n^2\right)x(cx^n)^{-1/n}\right)\operatorname{Subst}\left(\int \frac{x^{-1+b+\frac{1}{n}}}{e^{-2a}+x^{2b}}\,dx,x,cx^n\right)}{n}
\end{aligned}
$$

$$= 2e^a(1-bn)x(cx^n)^b \text{Hypergeometric2F1}\left(1, \frac{b+\frac{1}{n}}{2b}, \frac{1}{2}\left(3+\frac{1}{bn}\right), -e^{2a}(cx^n)^{2b}\right) + \frac{16b^2e^{3a}n^2x(cx^n)^{3b} \text{Hypergeometric2F1}\left(3, \frac{3b+\frac{1}{n}}{2b}, \frac{1}{2}\left(5+\frac{1}{bn}\right), -e^{2a}(cx^n)^{2b}\right)}{1+3bn}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log(cx^n))+2b^2n^2\operatorname{sech}^3(a+b\log(cx^n))\right)\,dx = x\operatorname{sech}(a+b\log(cx^n))\left(1+bn\tanh\left(a+b\log(cx^n)\right)\right)$$

```
[In] Integrate[(1 - b^2*n^2)*Sech[a + b*Log[c*x^n]] + 2*b^2*n^2*Sech[a + b*Log[c
*x^n]]^3,x]
```

```
[Out] x*Sech[a + b*Log[c*x^n]]*(1 + b*n*Tanh[a + b*Log[c*x^n]])
```

Maple [A] (verified)

Time = 33.73 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{2x(bn\sinh(a+b\ln(cx^n))+\cosh(a+b\ln(cx^n)))}{\cosh\left(4b\ln\left(\sqrt{cx^n}\right)+2a\right)+1}$
risch	$\frac{2c^b(x^n)^b x\left(nb(x^n)^{2b}c^{2b}e^{3a}e^{\frac{3ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)^2}{2}}e^{-\frac{3ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(icx^n)\operatorname{csgn}(ic)}{2}}e^{-\frac{3ib\pi\operatorname{csgn}(icx^n)^3}{2}}e^{\frac{3ib\pi\operatorname{csgn}(icx^n)^2}{2}}\right.}{}$

```
[In] int((-b^2*n^2+1)*sech(a+b*ln(c*x^n))+2*b^2*n^2*sech(a+b*ln(c*x^n))^3,x,meth
od=_RETURNVERBOSE)
```

```
[Out] 2*x*(b*n*sinh(a+b*ln(c*x^n))+cosh(a+b*ln(c*x^n)))/(cosh(4*b*ln((c*x^n)^(1/2
))+2*a)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(40) = 80$.

Time = 0.25 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.72

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx = \frac{2\left((bn+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+2\left(bn+1\right)x\cosh\left(bn\log\left(x\right)\right.\right.}{\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^3+3\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2+\sinh}$$

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="fricas")
```

```
[Out] 2*((b*n + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(b*n + 1)*x*cosh(b*n*l
og(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + (b*n + 1)*x*sinh(b*
n*log(x) + b*log(c) + a)^2 - (b*n - 1)*x)/(cosh(b*n*log(x) + b*log(c) + a)^
3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
inh(b*n*log(x) + b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a) + 3*cosh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx = \int \left(2b^2n^2\operatorname{sech}^2\left(a+b\log\left(cx^n\right)\right)-b^2n^2+1\right)\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)dx$$

```
[In] integrate((-b**2*n**2+1)*sech(a+b*ln(c*x**n))+2*b**2*n**2*sech(a+b*ln(c*x**
n))**3,x)
```

```
[Out] Integral((2*b**2*n**2*sech(a + b*log(c*x**n))**2 - b**2*n**2 + 1)*sech(a +
b*log(c*x**n)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.40

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx = \frac{2\left(\left(bc^{3\,b}n+c^{3\,b}\right)xe^{\left(3\,b\log\left(x^n\right)+3\,a\right)}-\left(bc^{b}n-c^{b}\right)xe^{\left(b\log\left(x^n\right)+a\right)}\right)}{c^{4\,b}e^{\left(4\,b\log\left(x^n\right)+4\,a\right)}+2\,c^{2\,b}e^{\left(2\,b\log\left(x^n\right)+2\,a\right)}+1}$$

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="maxima")
```

```
[Out] 2*((b*c^(3*b)*n + c^(3*b))*x*e^(3*b*log(x^n) + 3*a) - (b*c^b*n - c^b)*x*e^(
b*log(x^n) + a))/(c^(4*b)*e^(4*b*log(x^n) + 4*a) + 2*c^(2*b)*e^(2*b*log(x^n
) + 2*a) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(40) = 80$.

Time = 0.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 5.38

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\frac{2\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}-\frac{2\,bc^{b}nxx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}+\frac{2\,c^{b}xx^{bn}e^{a}}{c^{4\,b}x^{4\,bn}e^{(4\,a)}+2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}$$

```
[In] integrate((-b^2*n^2+1)*sech(a+b*log(c*x^n))+2*b^2*n^2*sech(a+b*log(c*x^n))^
3,x, algorithm="giac")
```

```
[Out] 2*b*c^(3*b)*n*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^
(2*b*n)*e^(2*a) + 1) - 2*b*c^b*n*x*x^(b*n)*e^a/(c^(4*b)*x^(4*b*n)*e^(4*a) +
 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(c^(4*b)*
x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1) + 2*c^b*x*x^(b*n)*e^a/
(c^(4*b)*x^(4*b*n)*e^(4*a) + 2*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.65

$$\int \left(\left(1-b^2n^2\right)\operatorname{sech}(a+b\log\left(cx^n\right))+2b^2n^2\operatorname{sech}^3(a+b\log\left(cx^n\right))\right)\,dx$$
$$=\frac{2\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^b\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}-b\,n+b\,n\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}{{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}^2}$$

```
[In] int((2*b^2*n^2)/cosh(a + b*log(c*x^n))^3 - (b^2*n^2 - 1)/cosh(a + b*log(c*x
^n)),x)
```

```
[Out] (2*x*exp(a)*(c*x^n)^b*(exp(2*a)*(c*x^n)^(2*b) - b*n + b*n*exp(2*a)*(c*x^n)^
(2*b) + 1))/(exp(2*a)*(c*x^n)^(2*b) + 1)^2
```

3.187 $\int \operatorname{sech}^3\left(a+2 \log \left(c \sqrt{x}\right)\right)\, dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \operatorname{sech}^3\left(a+2 \log \left(c \sqrt{x}\right)\right)\, dx=\frac{2 c^6 e^{-a}}{\left(c^4+\frac{e^{-2 a}}{x^2}\right)^2}$$

```
[Out] 2*c^6/exp(a)/(c^4+1/exp(2*a)/x^2)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5664, 5666, 267}

$$\int \operatorname{sech}^3\left(a+2 \log \left(c \sqrt{x}\right)\right)\, dx=\frac{2 e^{-a} c^6}{\left(\frac{e^{-2 a}}{x^2}+c^4\right)^2}$$

```
[In] Int[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
```

```
[Out] (2*c^6)/(E^a*(c^4 + 1/(E^(2*a)*x^2))^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
```

```
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{2\text{Subst}\left(\int x\text{sech}^3(a+2\log(x))\,dx, x, c\sqrt{x}\right)}{c^2}$$
$$= \frac{(16e^{-3a})\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3 x^5}\,dx, x, c\sqrt{x}\right)}{c^2}$$
$$= \frac{2c^6e^{-a}}{\left(c^4+\frac{e^{-2a}}{x^2}\right)^2}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

$$\int \text{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx$$
$$= -\frac{2(\cosh(a)-\sinh(a))\left(2c^4x^2+\cosh^2(a)-2\cosh(a)\sinh(a)+\sinh^2(a)\right)}{c^2\left((1+c^4x^2)\cosh(a)+(-1+c^4x^2)\sinh(a)\right)^2}$$

```
[In] Integrate[Sech[a + 2*Log[c*Sqrt[x]]]^3,x]
[Out] (-2*(Cosh[a] - Sinh[a])*(2*c^4*x^2 + Cosh[a]^2 - 2*Cosh[a]*Sinh[a] + Sinh[a
]^2))/(c^2*((1 + c^4*x^2)*Cosh[a] + (-1 + c^4*x^2)*Sinh[a])^2)
```

Maple [F]

$$\int \text{sech}\left(a+2\ln\left(c\sqrt{x}\right)\right)^3\,dx$$

```
[In] int(sech(a+2*ln(c*x^(1/2)))^3,x)
[Out] int(sech(a+2*ln(c*x^(1/2)))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(23) = 46$.

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \operatorname{sech}^3\left(a + 2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(2\,c^4x^2e^{(2\,a)} + 1\right)}{c^{10}x^4e^{(5\,a)} + 2\,c^6x^2e^{(3\,a)} + c^2e^a}$$

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="fricas")
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)/(c^10*x^4*e^(5*a) + 2*c^6*x^2*e^(3*a) + c^2*e^a)
```

Sympy [F]

$$\int \operatorname{sech}^3\left(a + 2\log\left(c\sqrt{x}\right)\right)\,dx = \int \operatorname{sech}^3\left(a + 2\log\left(c\sqrt{x}\right)\right)dx$$

```
[In] integrate(sech(a+2*ln(c*x**(1/2)))**3,x)
[Out] Integral(sech(a + 2*log(c*sqrt(x)))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(23) = 46$.

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.96

$$\int \operatorname{sech}^3\left(a + 2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(\frac{2\,c^4x^2e^{(2\,a)}}{c^8x^4e^{(5\,a)} + 2\,c^4x^2e^{(3\,a)} + e^a} + \frac{1}{c^8x^4e^{(5\,a)} + 2\,c^4x^2e^{(3\,a)} + e^a}\right)}{c^2}$$

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="maxima")
[Out] -2*(2*c^4*x^2*e^(2*a)/(c^8*x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a) + 1/(c^8*
x^4*e^(5*a) + 2*c^4*x^2*e^(3*a) + e^a))/c^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \operatorname{sech}^3\left(a + 2\log\left(c\sqrt{x}\right)\right)\,dx = -\frac{2\left(2\,c^4x^2e^{(2\,a)} + 1\right)e^{(-a)}}{\left(c^4x^2e^{(2\,a)} + 1\right)^2c^2}$$

```
[In] integrate(sech(a+2*log(c*x^(1/2)))^3,x, algorithm="giac")
[Out] -2*(2*c^4*x^2*e^(2*a) + 1)*e^(-a)/((c^4*x^2*e^(2*a) + 1)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{sech}^3\left(a+2\log\left(c\sqrt{x}\right)\right)\,dx=-\frac{\frac{2\,e^{-a}}{c^2}+4\,c^2\,x^2\,e^a}{e^{4\,a}\,c^8\,x^4+2\,e^{2\,a}\,c^4\,x^2+1}$$

```
[In] int(1/cosh(a + 2*log(c*x^(1/2)))^3,x)
```

```
[Out] -((2*exp(-a))/c^2 + 4*c^2*x^2*exp(a))/(2*c^4*x^2*exp(2*a) + c^8*x^4*exp(4*a
) + 1)
```

3.188 $\int \text{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \text{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=\frac{2c^2e^{-3a}}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

```
[Out] 2*c^2/exp(3*a)/(exp(-2*a)+c^4/x^2)^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {5664, 5666, 269, 267}

$$\int \text{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=\frac{2e^{-3a}c^2}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}$$

```
[In] Int[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
[Out] (2*c^2)/(E^(3*a)*(E^(-2*a) + c^4/x^2)^2)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5666

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[2^p/E^(a*d*p), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*
d)))^p)), x], x] /; FreeQ[{a, b, d, e, m}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\left(2c^2\right)\text{Subst}\left(\int\frac{\text{sech}^3(a+2\log(x))}{x^3}\,dx,x,\frac{c}{\sqrt{x}}\right)\right)\\ &= -\left(\left(16c^2e^{-3a}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{e^{-2a}}{x^4}\right)^3x^9}\,dx,x,\frac{c}{\sqrt{x}}\right)\right)\\ &= -\left(\left(16c^2e^{-3a}\right)\text{Subst}\left(\int\frac{x^3}{\left(e^{-2a}+x^4\right)^3}\,dx,x,\frac{c}{\sqrt{x}}\right)\right)\\ &= \frac{2c^2e^{-3a}}{\left(e^{-2a}+\frac{c^4}{x^2}\right)^2}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.56

$$\int\text{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx = -\frac{2c^6\left(\left(c^4+2x^2\right)\cosh(a)+\left(c^4-2x^2\right)\sinh(a)\right)\left(\cosh(2a)+\sinh(2a)\right)}{\left(\left(c^4+x^2\right)\cosh(a)+\left(c^4-x^2\right)\sinh(a)\right)^2}$$

```
[In] Integrate[Sech[a + 2*Log[c/Sqrt[x]]]^3,x]
```

```
[Out] (-2*c^6*((c^4 + 2*x^2)*Cosh[a] + (c^4 - 2*x^2)*Sinh[a])*(Cosh[2*a] + Sinh[2
*a]))/((c^4 + x^2)*Cosh[a] + (c^4 - x^2)*Sinh[a])^2
```

Maple [F]

$$\int \operatorname{sech}\left(a+2\ln\left(\frac{c}{\sqrt{x}}\right)\right)^{3} dx$$

```
[In] int(sech(a+2*ln(c/x^(1/2)))^3,x)
[Out] int(sech(a+2*ln(c/x^(1/2)))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{sech}^{3}\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx = -\frac{2\left(c^{10}e^{(5a)}+2c^{6}x^{2}e^{(3a)}\right)}{c^{8}e^{(4a)}+2c^{4}x^{2}e^{(2a)}+x^{4}}$$

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="fricas")
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

Sympy [F]

$$\int \operatorname{sech}^{3}\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx = \int \operatorname{sech}^{3}\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx$$

```
[In] integrate(sech(a+2*ln(c/x**(1/2)))**3,x)
[Out] Integral(sech(a + 2*log(c/sqrt(x)))**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(23) = 46$.

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \operatorname{sech}^{3}\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right) dx = -\frac{2\left(c^{10}e^{(5a)}+2c^{6}x^{2}e^{(3a)}\right)}{c^{8}e^{(4a)}+2c^{4}x^{2}e^{(2a)}+x^{4}}$$

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="maxima")
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^8*e^(4*a) + 2*c^4*x^2*e^(2*a) + x^
4)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=-\frac{2\left(c^{10}e^{(5\,a)}+2\,c^6x^2e^{(3\,a)}\right)}{\left(c^4e^{(2\,a)}+x^2\right)^2}$$

```
[In] integrate(sech(a+2*log(c/x^(1/2)))^3,x, algorithm="giac")
[Out] -2*(c^10*e^(5*a) + 2*c^6*x^2*e^(3*a))/(c^4*e^(2*a) + x^2)^2
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \operatorname{sech}^3\left(a+2\log\left(\frac{c}{\sqrt{x}}\right)\right)\,dx=\frac{2\,c^2\,x^4\,\mathrm{e}^a}{\mathrm{e}^{4\,a}\,c^8+2\,\mathrm{e}^{2\,a}\,c^4\,x^2+x^4}$$

```
[In] int(1/cosh(a + 2*log(c/x^(1/2)))^3,x)
[Out] (2*c^2*x^4*exp(a))/(c^8*exp(4*a) + x^4 + 2*c^4*x^2*exp(2*a))
```

3.189 $\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*exp(2*a)*(2-p)*x*(1+(c*x^n)^(2/n/(2-p))/exp(2*a))*sech(a-ln(c*x^n)/n/(2
-p))^p/(1-p)/((c*x^n)^(2/n/(2-p)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5664, 5668, 267}

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx = \frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[In] Int[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (E^(2*a)*(2 - p)*x*(1 + (c*x^n)^(2/(n*(2 - p)))/E^(2*a))*Sech[a - Log[c*x^n
]/(n*(2 - p))]^p)/(2*(1 - p)*(c*x^n)^(2/(n*(2 - p))))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\text{sech}^p\left(a+\frac{\log(x)}{n(-2+p)}\right)\,dx,x,cx^n\right)}{n}$$

$$= \frac{\left(x(cx^n)^{-\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1+e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\text{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1+e^{-\right.\right.}{n}$$

$$= \frac{e^{2a}(2-p)x(cx^n)^{-\frac{2}{n(2-p)}}\left(1+e^{-2a}(cx^n)^{\frac{2}{n(2-p)}}\right)\text{sech}^p\left(a-\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] (verified)

Time = 2.53 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.28

$$\int \text{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx = -\frac{2^{-1+p}(-2+p)x\left(\frac{e^a(cx^n)^{\frac{1}{2n-np}}}{e^{2a}+(cx^n)^{-\frac{2}{n(-2+p)}}}\right)^p\left(-1+e^{2a}(cx^n)^{\frac{2}{n(-2+p)}}\left(-1+\left(1+e^{-2a}(cx^n)^{-\frac{2}{n(-2+p)}}\right)^p\right)\right)}{-1+p}$$

```
[In] Integrate[Sech[a + Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] -((2^(-1 + p)*(-2 + p)*x*((E^a*(c*x^n)^(2*n - n*p)^(-1))/(E^(2*a) + (c*x^n)
^(-2/(n*(-2 + p)))))^p*(-1 + E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))*(-1 + (1 + 1/
(E^(2*a)*(c*x^n)^(2/(n*(-2 + p)))))^p)))/(-1 + p))
```

Maple [F]

$$\int \operatorname{sech}\left(a+\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

```
[In] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(sech(a+ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(76) = 152$.

Time $= 0.27$ (sec) , antiderivative size $= 474$, normalized size of antiderivative $= 5.33$

$$\int \operatorname{sech}^{p}\left(a+\frac{\log\left(cx^{n}\right)}{n(-2+p)}\right)\,dx = \frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\right)}{\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)^{2}+2\,\cosh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)\sinh\left(\frac{anp-2\,an+n\log(x)+\log(c)}{np-2\,n}\right)+\sinh\left(\frac{anp-2}{}\right.}\right.\right.}{}$$

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] ((p - 2)*x*cosh(p*log(2*(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*
n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)))/(cosh((a*n*p -
 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2*cosh((a*n*p - 2*a*n + n*log(
x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2
*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 1)))*cosh(
(a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)) + (p - 2)*x*cosh((a*n*p -
2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh((a*n*p - 2*a*n +
 n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c))
/(n*p - 2*n)))/(cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))^2 + 2
*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n))*sinh((a*n*p - 2*a*n
+ n*log(x) + log(c))/(n*p - 2*n)) + sinh((a*n*p - 2*a*n + n*log(x) + log(c)
)/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh((a*n*p - 2*a*n + n*log(x) + log(c))/
(n*p - 2*n)) - (p - 1)*sinh((a*n*p - 2*a*n + n*log(x) + log(c))/(n*p - 2*n)
))
```

Sympy [F]

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(p-2)}\right)dx$$

```
[In] integrate(sech(a+ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(sech(a + log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a+\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

```
[In] integrate(sech(a+log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
[Out] integrate(sech(a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\cosh\left(a+\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

```
[In] int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/cosh(a + log(c*x^n)/(n*(p - 2))))^p, x)
```

3.190 $\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\frac{(2-p)x\left(1+e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[Out] 1/2*(2-p)*x*(1+1/exp(2*a)/((c*x^n)^(2/n/(2-p))))*sech(a+ln(c*x^n)/n/(2-p))^
p/(1-p)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5664, 5668, 270}

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\frac{(2-p)x\left(e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}+1\right)\operatorname{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

```
[In] Int[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] ((2 - p)*x*(1 + 1/(E^(2*a)*(c*x^n)^(2/(n*(2 - p))))))*Sech[a + Log[c*x^n]/(n
*(2 - p))]^p)/(2*(1 - p))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5664

```
Int[Sech[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> D
ist[x/(n*(c*x^n)^(1/n)), Subst[Int[x^(1/n - 1)*Sech[d*(a + b*Log[x])]^p, x]
, x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1])
```

Rule 5668

```
Int[((e_.)*(x_))^(m_.)*Sech[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_.), x_Symbol]
 :> Dist[Sech[d*(a + b*Log[x])]^p*((1 + 1/(E^(2*a*d)*x^(2*b*d)))^p/x^((-b)*
d*p)), Int[(e*x)^m*(1/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p)), x], x]
/; FreeQ[{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\left(x(cx^n)^{-1/n}\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}}\text{sech}^p\left(a-\frac{\log(x)}{n(-2+p)}\right)\,dx,x,cx^n\right)}{n}$$

$$=\frac{\left(x(cx^n)^{-\frac{1}{n}-\frac{p}{n(-2+p)}}\left(1+e^{-2a}(cx^n)^{\frac{2}{n(-2+p)}}\right)^p\text{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\right)\text{Subst}\left(\int x^{-1+\frac{1}{n}+\frac{p}{n(-2+p)}}\left(1+e^{-2\right.\right.}{n}$$

$$=\frac{(2-p)x\left(1+e^{-2a}(cx^n)^{-\frac{2}{n(2-p)}}\right)\text{sech}^p\left(a+\frac{\log(cx^n)}{n(2-p)}\right)}{2(1-p)}$$

Mathematica [A] (warning: unable to verify)

Time = 1.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int \text{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx$$

$$=\frac{2^{-1+p}e^{-a}(-2+p)x(cx^n)^{\frac{1}{n(-2+p)}}\left(\frac{e^{\frac{a(2+p)}{-2+p}}(cx^n)^{\frac{1}{n(-2+p)}}}{e^{\frac{2ap}{-2+p}}+e^{\frac{4a}{-2+p}}(cx^n)^{\frac{2}{n(-2+p)}}}\right)^{-1+p}}{-1+p}$$

```
[In] Integrate[Sech[a - Log[c*x^n]/(n*(-2 + p))]^p,x]
```

```
[Out] (2^(-1 + p)*(-2 + p)*x*(c*x^n)^(1/(n*(-2 + p)))*((E^((a*(2 + p))/(-2 + p))*
(c*x^n)^(1/(n*(-2 + p))))/(E^((2*a*p)/(-2 + p)) + E^((4*a)/(-2 + p))*(c*x^n
)^(2/(n*(-2 + p)))))^(-1 + p))/(E^a*(-1 + p))
```

Maple [F]

$$\int \operatorname{sech}\left(a-\frac{\ln\left(c\,x^{n}\right)}{n\left(-2+p\right)}\right)^{p}dx$$

```
[In] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

```
[Out] int(sech(a-ln(c*x^n)/n/(-2+p))^p,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(55) = 110$.

Time = 0.27 (sec) , antiderivative size = 538, normalized size of antiderivative = 8.28

$$\int \operatorname{sech}^p\left(a-\frac{\log\left(cx^n\right)}{n(-2+p)}\right)\,dx = \frac{(p-2)x\cosh\left(p\log\left(\frac{2\left(\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\right)}{\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)^2+2\,\cosh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)\sinh\left(-\frac{anp-2\,an-n\log(x)-\log(c)}{np-2\,n}\right)+\sinh\left(-\cdots\right.}\right.\right.}{\cdots}$$

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="fricas")
```

```
[Out] ((p - 2)*x*cosh(p*log(2*(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2
*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)))/(cosh(-(a*n*
p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*
log(x) - log(c))/(n*p - 2*n))*sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*
p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))^2 + 1)))
*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + (p - 2)*x*cosh(-(
a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(p*log(2*(cosh(-(a*n*p
- 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*log(x)
 - log(c))/(n*p - 2*n)))/(cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p -
2*n))^2 + 2*cosh(-(a*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n))*sinh(-(a
*n*p - 2*a*n - n*log(x) - log(c))/(n*p - 2*n)) + sinh(-(a*n*p - 2*a*n - n*l
og(x) - log(c))/(n*p - 2*n))^2 + 1))))/((p - 1)*cosh(-(a*n*p - 2*a*n - n*lo
g(x) - log(c))/(n*p - 2*n)) - (p - 1)*sinh(-(a*n*p - 2*a*n - n*log(x) - log
(c))/(n*p - 2*n)))
```

Sympy [F]

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(p-2)}\right)dx$$

```
[In] integrate(sech(a-ln(c*x**n)/n/(-2+p))**p,x)
[Out] Integral(sech(a - log(c*x**n)/(n*(p - 2)))**p, x)
```

Maxima [F]

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a-\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="maxima")
[Out] integrate(sech(-a + log(c*x^n)/(n*(p - 2)))^p, x)
```

Giac [F]

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \operatorname{sech}\left(a-\frac{\log(cx^n)}{n(p-2)}\right)^p\,dx$$

```
[In] integrate(sech(a-log(c*x^n)/n/(-2+p))^p,x, algorithm="giac")
[Out] integrate(sech(a - log(c*x^n)/(n*(p - 2)))^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \operatorname{sech}^p\left(a-\frac{\log(cx^n)}{n(-2+p)}\right)\,dx=\int \left(\frac{1}{\cosh\left(a-\frac{\ln(c\,x^n)}{n\,(p-2)}\right)}\right)^p\,dx$$

```
[In] int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p,x)
[Out] int((1/cosh(a - log(c*x^n)/(n*(p - 2))))^p, x)
```

3.191 $\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx=\frac{\arctan(\sinh(a+b\log(cx^n)))}{bn}$$

```
[Out] arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3855}

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx=\frac{\arctan(\sinh(a+b\log(cx^n)))}{bn}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\operatorname{Subst}\left(\int \operatorname{sech}(a+bx)\,dx,x,\log(cx^n)\right)}{n}\\&=\frac{\arctan(\sinh(a+b\log(cx^n)))}{bn}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(a+b\log(cx^n)))}{bn}$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]/x,x]
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(b*n)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{\arctan(\sinh(a+b\ln(c\,x^n)))}{bn}$
default	$\frac{\arctan(\sinh(a+b\ln(c\,x^n)))}{bn}$
parallelrisch	$-\frac{i\left(\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)-i\right)-\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)+i\right)\right)}{bn}$
risch	$\frac{i\ln\left(c^b(x^n)^b\mathrm{e}^a\mathrm{e}^{\frac{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)^2}{2}}\mathrm{e}^{-\frac{ib\pi\ \mathrm{csgn}(ix^n)\,\mathrm{csgn}(ic\,x^n)\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{-\frac{ib\pi\,\mathrm{csgn}(ic\,x^n)^3}{2}}\mathrm{e}^{\frac{ib\pi\,\mathrm{csgn}(ic\,x^n)^2\,\mathrm{csgn}(ic)}{2}}+\right.}{bn}$

```
[In] int(sech(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
[Out] arctan(sinh(a+b*ln(c*x^n)))/b/n
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = \frac{2\arctan(\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a))}{bn}$$

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="fricas")
[Out] 2*arctan(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))
/(b*n)
```

Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{\operatorname{sech}(a + b\log(cx^n))}{x}\,dx = -\begin{cases} -\log(x)\operatorname{sech}(a) & \text{for } b = 0 \\ -\log(x)\operatorname{sech}(a + b\log(c)) & \text{for } n = 0 \\ -\frac{2\,\text{atan}\left(\tanh\left(\frac{a}{2} + \frac{b\log(cx^n)}{2}\right)\right)}{bn} & \text{otherwise} \end{cases}$$

```
[In] integrate(sech(a+b*ln(c*x**n))/x,x)
[Out] -Piecewise((-log(x)*sech(a), Eq(b, 0)), (-log(x)*sech(a + b*log(c)), Eq(n,
0)), (-2*atan(tanh(a/2 + b*log(c*x**n)/2))/(b*n), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(a + b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(b\log(cx^n) + a))}{bn}$$

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="maxima")
[Out] arctan(sinh(b*log(c*x^n) + a))/(b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{sech}(a + b\log(cx^n))}{x}\,dx = \frac{2\arctan\left(\frac{c^{2b}x^{bn}e^a}{c^b}\right)}{bn}$$

```
[In] integrate(sech(a+b*log(c*x^n))/x,x, algorithm="giac")
[Out] 2*arctan(c^(2*b)*x^(b*n)*e^a/c^b)/(b*n)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{\operatorname{sech}(a+b\log(cx^n))}{x}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,{\left(c\,x^n\right)}^b}\right)}{\sqrt{b^2\,n^2}}$$

```
[In] int(1/(x*cosh(a + b*log(c*x^n))),x)
[Out] -(2*atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(b^2*n^2)^(1/2)
```

3.192 $\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x} \, dx = \frac{\tanh(a+b\log(cx^n))}{bn}$$

```
[Out] tanh(a+b*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3852, 8}

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x} \, dx = \frac{\tanh(a+b\log(cx^n))}{bn}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^2/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{sech}^2(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{i\text{Subst}\left(\int 1\,dx, x, -i\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\
&= \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = \frac{\tanh(a+b\log(cx^n))}{bn}$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^2/x,x]
[Out] Tanh[a + b*Log[c*x^n]]/(b*n)
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\tanh(a+b\ln(c\,x^n))}{bn}$
default	$\frac{\tanh(a+b\ln(c\,x^n))}{bn}$
parallelrisch	$\frac{2\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)}{bn\left(1+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2\right)}$
risch	$-\frac{2}{bn\left(\left(x^n\right)^{2b}c^{2b}e^{2a}e^{ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}e^{-ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)}e^{-ib\pi\operatorname{csgn}(ic\,x^n)^3}e^{ib\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(ic)}\ldots\right.}$

```
[In] int(sech(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
[Out] tanh(a+b*ln(c*x^n))/b/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.89

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2}{bn\cosh(bn\log(x)+b\log(c)+a)^2+2\,bn\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)}$$

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

```
[Out] -2/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c
) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**2/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2}{bc^{2\,b}ne^{(2\,b\log(x^n)+2\,a)}+bn}$$

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

```
[Out] -2/(b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2}{(c^{2b}x^{2bn}e^{(2a)}+1)bn}$$

```
[In] integrate(sech(a+b*log(c*x^n))^2/x,x, algorithm="giac")
[Out] -2/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)*b*n)
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\operatorname{sech}^2(a+b\log(cx^n))}{x}\,dx = -\frac{2}{b\,n+b\,n\,e^{2\,a}\,(c\,x^n)^{2\,b}}$$

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^2),x)
[Out] -2/(b*n + b*n*exp(2*a)*(c*x^n)^(2*b))
```

3.193 $\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(a+b\log(cx^n)))}{2bn} + \frac{\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}$$

```
[Out] 1/2*arctan(sinh(a+b*ln(c*x^n)))/b/n+1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x
^n))/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3853, 3855}

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(a+b\log(cx^n)))}{2bn} + \frac{\tanh(a+b\log(cx^n))\operatorname{sech}(a+b\log(cx^n))}{2bn}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^3/x,x]
```

```
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \text{sech}^3(a+bx)\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn} + \frac{\text{Subst}\left(\int \text{sech}(a+bx)\,dx, x, \log(cx^n)\right)}{2n} \\
&= \frac{\arctan(\sinh(a+b\log(cx^n)))}{2bn} + \frac{\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{\arctan(\sinh(a+b\log(cx^n)))}{2bn} + \frac{\text{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{2bn}$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^3/x,x]
[Out] ArcTan[Sinh[a + b*Log[c*x^n]]]/(2*b*n) + (Sech[a + b*Log[c*x^n]]*Tanh[a + b
*Log[c*x^n]])/(2*b*n)
```

Maple [A] (verified)

Time = 19.64 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{\operatorname{sech}(a+b\ln(c\,x^{n}))\tanh(a+b\ln(c\,x^{n}))}{2}+\arctan\left(\mathrm{e}^{a+b\ln(c\,x^{n})}\right)}{nb}$
default	$\frac{\frac{\operatorname{sech}(a+b\ln(c\,x^{n}))\tanh(a+b\ln(c\,x^{n}))}{2}+\arctan\left(\mathrm{e}^{a+b\ln(c\,x^{n})}\right)}{nb}$
parallelrisch	$\frac{i(-1-\cosh(2b\ln(c\,x^{n})+2a))\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)-i\right)+i(\cosh(2b\ln(c\,x^{n})+2a)+1)\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)+i\right)+2\sin}{2bn\left(\cosh\left(4b\ln\left(\sqrt{c\,x^{n}}\right)+2a\right)+1\right)}$
risch	$\frac{c^{b}(x^{n})^{b}\left((x^{n})^{2b}c^{2b}\mathrm{e}^{3a}\mathrm{e}^{\frac{3ib\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}{2}}\mathrm{e}^{-\frac{3ib\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})\,\mathrm{csgn}(ic)}{2}}\mathrm{e}^{-\frac{3ib\pi\,\mathrm{csgn}(ic\,x^{n})^{3}}{2}}\mathrm{e}^{\frac{3ib\pi\,\mathrm{csgn}(ic\,x^{n})^{2}}{2}}\right.}{bn\left((x^{n})^{2b}c^{2b}\mathrm{e}^{2a}\mathrm{e}^{ib\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(ic\,x^{n})^{2}}\mathrm{e}^{-ib\pi\ \mathrm{csgn}(ix^{n})\,\mathrm{csgn}(i\right.}$

```
[In] int(sech(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*(1/2*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x^n))+arctan(exp(a+b*ln(c*x^n)
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 452, normalized size of antiderivative = 8.22

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{\cosh(bn\log(x)+b\log(c)+a)^3+3\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^2+\sinh}{}$$

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

```
[Out] (cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a)*sinh
(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) + b*log(c) + a)^3 + (cosh(b
*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*lo
g(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 2*(3*cosh(b*n*
log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*
n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*
n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + (3*cosh(b*n
*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x) + b*log(c) + a) - cosh(b*n*l
og(x) + b*log(c) + a))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(
b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n
*log(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*
b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a
)^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x)
+ b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx=\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**3/x,x)
[Out] Integral(sech(a + b*log(c*x**n))**3/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx=\int \frac{\operatorname{sech}(b\log(cx^n)+a)^3}{x}\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
[Out] 8*c^b*integrate(1/8*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a) +
x), x) + (c^(3*b)*e^(3*b*log(x^n) + 3*a) - c^b*e^(b*log(x^n) + a))/(b*c^(4*
b)*n*e^(4*b*log(x^n) + 4*a) + 2*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*a) + b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(51) = 102$.

Time = 0.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.09

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = c^{3b}\left(\frac{\arctan\left(\frac{c^{2b}x^{bn}e^{a}}{c^b}\right)e^{(-3a)}}{bc^{2b}c^bn}+\frac{\left(c^{2b}x^{3bn}e^{(2a)}-x^{bn}\right)e^{(-2a)}}{\left(c^{2b}x^{2bn}e^{(2a)}+1\right)^2bc^{2b}n}\right)e^{(3a)}$$

```
[In] integrate(sech(a+b*log(c*x^n))^3/x,x, algorithm="giac")
[Out] c^(3*b)*(arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-3*a)/(b*c^(2*b)*c^b*n) + (c^(2
*b)*x^(3*b*n)*e^(2*a) - x^(b*n))*e^(-2*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^
2*b*c^(2*b)*n))*e^(3*a)
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^3(a+b\log(cx^n))}{x}\,dx = \frac{2\,\mathrm{e}^{-a}}{(c\,x^n)^b\,\left(b\,n+\frac{2\,b\,n\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{b\,n\,\mathrm{e}^{-4\,a}}{(c\,x^n)^{4\,b}}\right)} - \frac{\mathrm{e}^{-a}}{(c\,x^n)^b\,\left(b\,n+\frac{b\,n\,\mathrm{e}^{-2\,a}}{(c\,x^n)^{2\,b}}\right)} - \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,(c\,x^n)^b}\right)}{\sqrt{b^2\,n^2}}$$

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^3),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4
*a))/(c*x^n)^(4*b))) - exp(-a)/((c*x^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^(2
*b))) - atan((exp(-a)*(b^2*n^2)^(1/2))/(b*n*(c*x^n)^b))/(b^2*n^2)^(1/2)
```

3.194 $\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{\tanh(a+b\log(cx^n))}{bn} - \frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

```
[Out] tanh(a+b*ln(c*x^n))/b/n-1/3*tanh(a+b*ln(c*x^n))^3/b/n
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3852}

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{\tanh(a+b\log(cx^n))}{bn} - \frac{\tanh^3(a+b\log(cx^n))}{3bn}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^4/x,x]
```

```
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{sech}^4(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{i\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -i\tanh\left(a+b\log\left(cx^n\right)\right)\right)}{bn} \\
&= \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{\tanh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\tanh^3\left(a+b\log\left(cx^n\right)\right)}{3bn}$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^4/x,x]
[Out] Tanh[a + b*Log[c*x^n]]/(b*n) - Tanh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Maple [A] (verified)

Time = 18.95 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(a+b\ln(c\,x^n))^2}{3}\right)\tanh(a+b\ln(c\,x^n))}{nb}$
default	$\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(a+b\ln(c\,x^n))^2}{3}\right)\tanh(a+b\ln(c\,x^n))}{nb}$
parallelrisch	$\frac{6\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^5+4\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^3+6\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)}{3bn\left(1+\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^n}\right)\right)^2\right)^3}$
risch	$-\frac{4\left(3(x^n)^{2b}c^{2b}e^{2a}e^{ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}e^{-ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)}e^{-ib\pi\operatorname{csgn}(ic\,x^n)^3}e^{ib\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(ic)}+\right.}{3bn\left((x^n)^{2b}c^{2b}e^{2a}e^{ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)^2}e^{-ib\pi\ \operatorname{csgn}(ix^n)\operatorname{csgn}(ic\,x^n)\operatorname{csgn}(ic)}e^{-ib\pi\operatorname{csgn}(ic\,x^n)^3}e^{ib\pi\operatorname{csgn}(ic\,x^n)^2\operatorname{csgn}(ic)}+\right.}$

```
[In] int(sech(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
[Out] 1/n/b*(2/3+1/3*sech(a+b*ln(c*x^n))^2)*tanh(a+b*ln(c*x^n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(40) = 80.

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 6.48

$$\int \frac{\operatorname{sech}^4(a + b\log(cx^n))}{x}\,dx =$$

$$-\frac{}{3\left(bn\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)^5 + 5\,bn\cosh\left(bn\log\left(x\right) + b\log\left(c\right) + a\right)\sinh\left(bn\log\left(x\right) + b\log\left(c\right) \right.\right.}$$

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/
(b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*b*n*cosh(b*n*log(x) + b*log(c) +
 a)*sinh(b*n*log(x) + b*log(c) + a)^4 + b*n*sinh(b*n*log(x) + b*log(c) + a)
^5 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + (10*b*n*cosh(b*n*log(x) + b*
log(c) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^3 + 4*b*n*cosh(b*n*l
og(x) + b*log(c) + a) + (10*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 9*b*n*c
osh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^2 + (5*b*n*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*log(c) + a)^2
 + 2*b*n)*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^4\left(a + b\log\left(cx^n\right)\right)}{x}\,dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**4/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**4/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(40) = 80.

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.17

$$\int \frac{\operatorname{sech}^4(a + b\log(cx^n))}{x}\,dx = -\frac{4\left(3\,c^{2\,b}e^{\left(2\,b\log\left(x^n\right) + 2\,a\right)} + 1\right)}{3\left(bc^{6\,b}ne^{\left(6\,b\log\left(x^n\right) + 6\,a\right)} + 3\,bc^{4\,b}ne^{\left(4\,b\log\left(x^n\right) + 4\,a\right)} + 3\,bc^{2\,b}ne^{\left(2\,b\log\left(x^n\right) + 2\,a\right)} + bn\right)}$$

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

```
[Out] -4/3*(3*c^(2*b)*e^(2*b*log(x^n) + 2*a) + 1)/(b*c^(6*b)*n*e^(6*b*log(x^n) +
6*a) + 3*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 3*b*c^(2*b)*n*e^(2*b*log(x^n)
 + 2*a) + b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = -\frac{4\left(3\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)}{3\,\left(c^{2\,b}x^{2\,bn}e^{(2\,a)}+1\right)^3bn}$$

```
[In] integrate(sech(a+b*log(c*x^n))^4/x,x, algorithm="giac")
[Out] -4/3*(3*c^(2*b)*x^(2*b*n)*e^(2*a) + 1)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^3*b
*n)
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{\operatorname{sech}^4(a+b\log(cx^n))}{x}\,dx = \frac{4\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}\,\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+3\right)}{3\,b\,n\,{\left(\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}+1\right)}^3}$$

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^4),x)
[Out] (4*exp(4*a)*(c*x^n)^(4*b)*(exp(2*a)*(c*x^n)^(2*b) + 3))/(3*b*n*(exp(2*a)*(c
*x^n)^(2*b) + 1)^3)
```

3.195 $\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 89

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{3\arctan(\sinh(a+b\log(cx^n)))}{8bn} + \frac{3\operatorname{sech}(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{8bn} + \frac{\operatorname{sech}^3(a+b\log(cx^n))\tanh(a+b\log(cx^n))}{4bn}$$

```
[Out] 3/8*arctan(sinh(a+b*ln(c*x^n)))/b/n+3/8*sech(a+b*ln(c*x^n))*tanh(a+b*ln(c*x
^n))/b/n+1/4*sech(a+b*ln(c*x^n))^3*tanh(a+b*ln(c*x^n))/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3853, 3855}

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{3\arctan(\sinh(a+b\log(cx^n)))}{8bn} + \frac{\tanh(a+b\log(cx^n))\operatorname{sech}^3(a+b\log(cx^n))}{4bn} + \frac{3\tanh(a+b\log(cx^n))\operatorname{sech}(a+b\log(cx^n))}{8bn}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Sech[a + b*Log[c*x^n]]*Tanh
[a + b*Log[c*x^n]])/(8*b*n) + (Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^
n]])/(4*b*n)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{sech}^5(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{\operatorname{sech}^3(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)}{4bn}+\frac{3\text{Subst}\left(\int \operatorname{sech}^3(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{4n}\\
&= \frac{3\operatorname{sech}(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)}{8bn}\\
&\quad+\frac{\operatorname{sech}^3(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)}{4bn}\\
&\quad+\frac{3\text{Subst}\left(\int \operatorname{sech}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{8n}\\
&= \frac{3\arctan\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}+\frac{3\operatorname{sech}(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)}{8bn}\\
&\quad+\frac{\operatorname{sech}^3(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)}{4bn}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \frac{\operatorname{sech}^5(a+b\log\left(cx^n\right))}{x}\,dx &= \frac{3\arctan\left(\sinh\left(a+b\log\left(cx^n\right)\right)\right)}{8bn}\\
&\quad+\frac{3\operatorname{sech}(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)}{8bn}\\
&\quad+\frac{\operatorname{sech}^3(a+b\log\left(cx^n\right))\tanh\left(a+b\log\left(cx^n\right)\right)}{4bn}
\end{aligned}
$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^5/x,x]
```

```
[Out] (3*ArcTan[Sinh[a + b*Log[c*x^n]]])/(8*b*n) + (3*Sech[a + b*Log[c*x^n]]*Tanh
[a + b*Log[c*x^n]])/(8*b*n) + (Sech[a + b*Log[c*x^n]]^3*Tanh[a + b*Log[c*x^
n]])/(4*b*n)
```

Maple [A] (verified)

Time = 233.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\left(\frac{\operatorname{sech}(a+b\ln(c\,x^{n}))^{3}}{4}+\frac{3\,\operatorname{sech}(a+b\ln(c\,x^{n}))}{8}\right)\tanh(a+b\ln(c\,x^{n}))+\frac{3\arctan\left(e^{a+b\ln(c\,x^{n})}\right)}{4}}{nb}$
default	$\frac{\left(\frac{\operatorname{sech}(a+b\ln(c\,x^{n}))^{3}}{4}+\frac{3\,\operatorname{sech}(a+b\ln(c\,x^{n}))}{8}\right)\tanh(a+b\ln(c\,x^{n}))+\frac{3\arctan\left(e^{a+b\ln(c\,x^{n})}\right)}{4}}{nb}$
parallelrisch	$\frac{3i(-\cosh(4b\ln(c\,x^{n})+4a)-4\cosh(2b\ln(c\,x^{n})+2a)-3)\ln\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)-i\right)+3i(\cosh(4b\ln(c\,x^{n})+4a)+4\cosh(2}{8bn(\cosh(4b\ln(c\,x^{n})+4a)+4\cosh(2b\ln}$
risch	Expression too large to display

```
[In] int(sech(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n/b*((1/4*sech(a+b*ln(c*x^n))^3+3/8*sech(a+b*ln(c*x^n)))*tanh(a+b*ln(c*x^
n))+3/4*arctan(exp(a+b*ln(c*x^n))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. 2(83) = 166.

Time = 0.28 (sec) , antiderivative size = 1326, normalized size of antiderivative = 14.90

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \text{Too large to display}$$

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(b*n*log(x) + b*log(c) + a)^7 + 21*cosh(b*n*log(x) + b*log(c) +
a)*sinh(b*n*log(x) + b*log(c) + a)^6 + 3*sinh(b*n*log(x) + b*log(c) + a)^7
+ (63*cosh(b*n*log(x) + b*log(c) + a)^2 + 11)*sinh(b*n*log(x) + b*log(c) +
a)^5 + 11*cosh(b*n*log(x) + b*log(c) + a)^5 + 5*(21*cosh(b*n*log(x) + b*log
(c) + a)^3 + 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c)
 + a)^4 + (105*cosh(b*n*log(x) + b*log(c) + a)^4 + 110*cosh(b*n*log(x) + b*
log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(c) + a)^3 - 11*cosh(b*n*log(x)
+ b*log(c) + a)^3 + (63*cosh(b*n*log(x) + b*log(c) + a)^5 + 110*cosh(b*n*lo
g(x) + b*log(c) + a)^3 - 33*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^2 + 3*(cosh(b*n*log(x) + b*log(c) + a)^8 + 8*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + sinh(b*n*log(x) + b
*log(c) + a)^8 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^2 + 1)*sinh(b*n*log(x
) + b*log(c) + a)^6 + 4*cosh(b*n*log(x) + b*log(c) + a)^6 + 8*(7*cosh(b*n*l
og(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x
) + b*log(c) + a)^5 + 2*(35*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*cosh(b*n
*log(x) + b*log(c) + a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a)^4 + 6*cosh(b
```

```
*n*log(x) + b*log(c) + a)^4 + 8*(7*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*c
osh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log(x) + b*log(c) + a))*sinh(
b*n*log(x) + b*log(c) + a)^3 + 4*(7*cosh(b*n*log(x) + b*log(c) + a)^6 + 15*
cosh(b*n*log(x) + b*log(c) + a)^4 + 9*cosh(b*n*log(x) + b*log(c) + a)^2 + 1
)*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)^2 +
 8*(cosh(b*n*log(x) + b*log(c) + a)^7 + 3*cosh(b*n*log(x) + b*log(c) + a)^5
 + 3*cosh(b*n*log(x) + b*log(c) + a)^3 + cosh(b*n*log(x) + b*log(c) + a))*s
inh(b*n*log(x) + b*log(c) + a) + 1)*arctan(cosh(b*n*log(x) + b*log(c) + a)
+ sinh(b*n*log(x) + b*log(c) + a)) + (21*cosh(b*n*log(x) + b*log(c) + a)^6
+ 55*cosh(b*n*log(x) + b*log(c) + a)^4 - 33*cosh(b*n*log(x) + b*log(c) + a)
^2 - 3)*sinh(b*n*log(x) + b*log(c) + a) - 3*cosh(b*n*log(x) + b*log(c) + a)
)/(b*n*cosh(b*n*log(x) + b*log(c) + a)^8 + 8*b*n*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a)^7 + b*n*sinh(b*n*log(x) + b*log(c) +
a)^8 + 4*b*n*cosh(b*n*log(x) + b*log(c) + a)^6 + 4*(7*b*n*cosh(b*n*log(x) +
 b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^6 + 6*b*n*cosh(b*n*
log(x) + b*log(c) + a)^4 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b
*n*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a)^5 + 2*(
35*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 30*b*n*cosh(b*n*log(x) + b*log(c
) + a)^2 + 3*b*n)*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*log(x)
 + b*log(c) + a)^2 + 8*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 10*b*n*co
sh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a))*si
nh(b*n*log(x) + b*log(c) + a)^3 + 4*(7*b*n*cosh(b*n*log(x) + b*log(c) + a)^
6 + 15*b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 9*b*n*cosh(b*n*log(x) + b*lo
g(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n + 8*(b*n*cosh(b*
n*log(x) + b*log(c) + a)^7 + 3*b*n*cosh(b*n*log(x) + b*log(c) + a)^5 + 3*b*
n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x) + b*log(c) + a))*
sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(a + b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}^5(a + b\log(cx^n))}{x}\,dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**5/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**5/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}(b\log(cx^n)+a)^5}{x}\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

```
[Out] 96*c^b*integrate(1/128*e^(b*log(x^n) + a)/(c^(2*b)*x*e^(2*b*log(x^n) + 2*a)
 + x), x) + 1/4*(3*c^(7*b)*e^(7*b*log(x^n) + 7*a) + 11*c^(5*b)*e^(5*b*log(x
^n) + 5*a) - 11*c^(3*b)*e^(3*b*log(x^n) + 3*a) - 3*c^b*e^(b*log(x^n) + a))/
(b*c^(8*b)*n*e^(8*b*log(x^n) + 8*a) + 4*b*c^(6*b)*n*e^(6*b*log(x^n) + 6*a)
+ 6*b*c^(4*b)*n*e^(4*b*log(x^n) + 4*a) + 4*b*c^(2*b)*n*e^(2*b*log(x^n) + 2*
a) + b*n)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{1}{4}c^{5b}\left(\frac{3\arctan\left(\frac{c^{2b}x^{bn}e^a}{c^b}\right)e^{(-5a)}}{bc^{4b}c^bn} + \frac{\left(3c^{6b}x^{7bn}e^{(6a)} + 11c^{4b}x^{5bn}e^{(4a)} - 11c^{2b}x^{3bn}e^{(2a)} - 3x^{bn}\right)e^{(-4a)}}{\left(c^{2b}x^{2bn}e^{(2a)}+1\right)^4bc^{4b}n}\right)e^{(5a)}$$

```
[In] integrate(sech(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

```
[Out] 1/4*c^(5*b)*(3*arctan(c^(2*b)*x^(b*n)*e^a/c^b)*e^(-5*a)/(b*c^(4*b)*c^b*n) +
 (3*c^(6*b)*x^(7*b*n)*e^(6*a) + 11*c^(4*b)*x^(5*b*n)*e^(4*a) - 11*c^(2*b)*x
^(3*b*n)*e^(2*a) - 3*x^(b*n))*e^(-4*a)/((c^(2*b)*x^(2*b*n)*e^(2*a) + 1)^4*b
*c^(4*b)*n))*e^(5*a)
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.53

$$\int \frac{\operatorname{sech}^5(a+b\log(cx^n))}{x}\,dx = \frac{2\,e^{-a}}{(c\,x^n)^b\,\left(b\,n+\frac{3\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{3\,b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}+\frac{b\,n\,e^{-6\,a}}{(c\,x^n)^{6\,b}}\right)} - \frac{3\,\operatorname{atan}\left(\frac{e^{-a}\,\sqrt{b^2\,n^2}}{b\,n\,(c\,x^n)^b}\right)}{4\,\sqrt{b^2\,n^2}} - \frac{3\,e^{-a}}{4\,(c\,x^n)^b\,\left(b\,n+\frac{b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}\right)} + \frac{4\,e^{-3\,a}}{(c\,x^n)^{3\,b}\,\left(b\,n+\frac{4\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{6\,b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}+\frac{4\,b\,n\,e^{-6\,a}}{(c\,x^n)^{6\,b}}+\frac{b\,n\,e^{-8\,a}}{(c\,x^n)^{8\,b}}\right)} - \frac{e^{-a}}{2\,(c\,x^n)^b\,\left(b\,n+\frac{2\,b\,n\,e^{-2\,a}}{(c\,x^n)^{2\,b}}+\frac{b\,n\,e^{-4\,a}}{(c\,x^n)^{4\,b}}\right)}$$

```
[In] int(1/(x*cosh(a + b*log(c*x^n))^5),x)
```

```
[Out] (2*exp(-a))/((c*x^n)^b*(b*n + (3*b*n*exp(-2*a))/(c*x^n)^(2*b) + (3*b*n*exp(
-4*a))/(c*x^n)^(4*b) + (b*n*exp(-6*a))/(c*x^n)^(6*b))) - (3*atan((exp(-a)*(
b^2*n^2)^(1/2))/(b*n*(c*x^n)^b)))/(4*(b^2*n^2)^(1/2)) - (3*exp(-a))/(4*(c*x
^n)^b*(b*n + (b*n*exp(-2*a))/(c*x^n)^(2*b))) + (4*exp(-3*a))/((c*x^n)^(3*b)
*(b*n + (4*b*n*exp(-2*a))/(c*x^n)^(2*b) + (6*b*n*exp(-4*a))/(c*x^n)^(4*b) +
 (4*b*n*exp(-6*a))/(c*x^n)^(6*b) + (b*n*exp(-8*a))/(c*x^n)^(8*b))) - exp(-a
)/(2*(c*x^n)^b*(b*n + (2*b*n*exp(-2*a))/(c*x^n)^(2*b) + (b*n*exp(-4*a))/(c*
x^n)^(4*b)))
```

3.196 $\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{3bn} + \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn}$$

```
[Out] 2/3*sech(a+b*ln(c*x^n))^(3/2)*sinh(a+b*ln(c*x^n))/b/n-2/3*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticF(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3853, 3856, 2720}

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sinh(a+b\log(cx^n))\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{3bn} - \frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\operatorname{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sech[a + b*Log[c*x^n]]^(3/2)*
Sinh[a + b*Log[c*x^n]])/(3*b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \text{sech}^{\frac{5}{2}}(a+bx)\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn} + \frac{\text{Subst}\left(\int \sqrt{\text{sech}(a+bx)}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= \frac{2\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn} + \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(cx^n))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)), 2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{3bn} + \frac{2\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{3bn}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))\left(-i\cosh^{\frac{3}{2}}(a+b\log(cx^n))\operatorname{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)+\sinh(a+b\log(cx^n\right.}{3bn}$$

[In] Integrate[Sech[a + b*Log[c*x^n]]^(5/2)/x,x]

[Out] (2*Sech[a + b*Log[c*x^n]]^(3/2)*((-I)*Cosh[a + b*Log[c*x^n]]^(3/2)*Elliptic F[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(3*b*n)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(123) = 246.

Time = 119.73 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.04

method	result
derivativedivides	$\frac{2\left(2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2}\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2-1}\operatorname{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right),\sqrt{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2+2\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\right.}}$
default	$\frac{2\left(2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2}\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2-1}\operatorname{EllipticF}\left(\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right),\sqrt{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2+2\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\right.}}$

[In] int(sech(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)

[Out] 2/3/n*(2*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x ^n))^2-1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*sinh(1/2*a+1 /2*b*ln(c*x^n))^2+2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n)) ^2+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2 -1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))*((-1+2*cosh(1/2*a +1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(2*sinh(1/2*a+1/2 *b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(-1+2*cosh(1/2*a+1/2*b *ln(c*x^n))^2)^(3/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.25

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{2\left(\sqrt{2}(\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)\right.}{}$$

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log
(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^
2 - 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) +
 a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1))
+ (sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b
*log(c) + a)^2 + sqrt(2))*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*lo
g(c) + a) + sinh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(
c) + a)^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c
) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(sech(a+b*ln(c*x**n))**(5/2)/x,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\operatorname{sech}(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

```
[In] integrate(sech(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{5/2}}{x}\,dx$$

```
[In] int((1/cosh(a + b*log(c*x^n)))^(5/2)/x,x)
[Out] int((1/cosh(a + b*log(c*x^n)))^(5/2)/x, x)
```

3.197 $\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn} + \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{bn}$$

```
[Out] 2*sinh(a+b*ln(c*x^n))*sech(a+b*ln(c*x^n))^(1/2)/b/n+2*I*(cosh(1/2*a+1/2*b*l
n(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticE(I*sinh(1/2*a+1/2*b
*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2)/b/
n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3853, 3856, 2719}

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sinh(a+b\log(cx^n))\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn} + \frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

```
[In] Int[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
```

```
[Out] ((2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]*
Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sqrt[Sech[a + b*Log[c*x^n]]]*Sinh[
a + b*Log[c*x^n]])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \operatorname{sech}^{\frac{3}{2}}(a+bx)\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{2\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sinh\left(a+b\log\left(cx^n\right)\right)}{bn} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\operatorname{sech}(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{2\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sinh\left(a+b\log\left(cx^n\right)\right)}{bn} \\
&\quad - \frac{\left(\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n} \\
&= \frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|\,2\right)\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn} \\
&\quad + \frac{2\sqrt{\operatorname{sech}\left(a+b\log\left(cx^n\right)\right)}\sinh\left(a+b\log\left(cx^n\right)\right)}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\sqrt{\operatorname{sech}(a+b\log(cx^n))}\left(i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(a+b\log(cx^n))\right)}{bn}$$

```
[In] Integrate[Sech[a + b*Log[c*x^n]]^(3/2)/x,x]
[Out] (2*Sqrt[Sech[a + b*Log[c*x^n]]]*(I*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(
I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]))/(b*n)
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

method	result
derivativedivides	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+2\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \mathrm{EllipticE}\left(\cosh\left(\frac{a}{2}+\right.\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,b}$
default	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+2\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \mathrm{EllipticE}\left(\cosh\left(\frac{a}{2}+\right.\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,b}$

```
[In] int(sech(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 2/n*(2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2+(-2*sinh(1
/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*Ell
ipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/sinh(1/2*a+1/2*b*ln(c*x^n))/(-
1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{2\left(\sqrt{2}\sqrt{\frac{\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)}{\cosh(bn\log(x)+b\log(c)+a)^2+2\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a)^2+1}}(\cosh(bn\right.}{}$$

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2*(sqrt(2)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(
c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
+ a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 +
1))*(cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)) + s
qrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log(x) +
b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**(3/2)/x,x)
```

```
[Out] Integral(sech(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{\operatorname{sech}(b\log(cx^n)+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sech(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx=\text{Timed out}$$

```
[In] integrate(sech(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{3/2}}{x}\,dx$$

```
[In] int((1/cosh(a + b*log(c*x^n)))^(3/2)/x,x)
[Out] int((1/cosh(a + b*log(c*x^n)))^(3/2)/x, x)
```

3.198 $\int \frac{\sqrt{\text{sech}(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{2i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log\left(cx^n\right)),2\right)\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticF(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech
(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3856, 2720}

$$\int \frac{\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}}{x}\,dx = -\frac{2i\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log\left(cx^n\right)),2\right)}{bn}$$

```
[In] Int[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \sqrt{\text{sech}(a+bx)}\,dx, x, \log(cx^n)\right)}{n} \\
&= \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(cx^n))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log(cx^n)\right)}{n} \\
&= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)), 2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{bn}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\text{sech}(a+b\log(cx^n))}}{x}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)), 2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{bn}$$

```
[In] Integrate[Sqrt[Sech[a + b*Log[c*x^n]]]/x,x]
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(90) = 180.

Time = 1.22 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.16

method	result
derivativedivides	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$

```
[In] int(sech(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^
2+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln(c*
x^n))/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \frac{2\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log(x)+b\log(c)+a))}{bn}$$

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx$$

```
[In] integrate(sech(a+b*ln(c*x**n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sech(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\operatorname{sech}(b\log(cx^n)+a)}}{x}\,dx$$

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(sech(b*log(c*x^n) + a))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \text{Timed out}$$

```
[In] integrate(sech(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{x}\,dx = \int \frac{\sqrt{\frac{1}{\cosh(a+b\,\ln(c\,x^n))}}}{x}\,dx$$

```
[In] int((1/cosh(a + b*log(c*x^n)))^(1/2)/x,x)
[Out] int((1/cosh(a + b*log(c*x^n)))^(1/2)/x, x)
```

3.199 $\int \frac{1}{x\sqrt{\mathbf{sech}(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{bn}$$

```
[Out] -2*I*(cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*Elli
pticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech
(a+b*ln(c*x^n))^(1/2)/b/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3856, 2719}

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = -\frac{2i\sqrt{\operatorname{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

```
[In] Int[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
```

```
[Out] ((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2]
*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{\text{sech}(a+bx)}}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(cx^n))}\right)\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{bn}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\text{sech}(a+b\log(cx^n))}}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\text{sech}(a+b\log(cx^n))}}$$

```
[In] Integrate[1/(x*Sqrt[Sech[a + b*Log[c*x^n]]]),x]
[Out] ((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n*Sqrt[Cosh[a + b*Log[c*
x^n]]]*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(90) = 180.

Time = 1.59 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.16

method	result
derivativedivides	$-\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+1}\ \mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\ \sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.}}$
default	$-\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+1}\ \mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\ \sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c}{2}\right.}}$

```
[In] int(1/x/sech(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))
^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+1/
2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c
*x^n))/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 4.28

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx = -\frac{\sqrt{2}\left(\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^{2}+2\,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right.}{}$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c)
 + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 +
 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)
)/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*si
nh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)) + 2
*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log(x)
 + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))))/(b*n*cosh(b*n*log(x)
+ b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx=\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx$$

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(1/2),x)
[Out] Integral(1/(x*sqrt(sech(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx=\int \frac{1}{x\sqrt{\operatorname{sech}(b\log(cx^n)+a)}}\,dx$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(x*sqrt(sech(b*log(c*x^n) + a))), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\operatorname{sech}(a+b\log(cx^n))}}\,dx=\int \frac{1}{x\,\sqrt{\frac{1}{\cosh(a+b\,\ln(c\,x^n))}}}\,dx$$

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)),x)
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(1/2)), x)
```

3.200 $\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{3bn} + \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\text{sech}(a+b\log(cx^n))}}$$

```
[Out] 2/3*sinh(a+b*ln(c*x^n))/b/n/sech(a+b*ln(c*x^n))^(1/2)-2/3*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticF(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3854, 3856, 2720}

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\text{sech}(a+b\log(cx^n))}} - \frac{2i\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn}$$

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]
```

```
[Out] (((-2*I)/3)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(3*b*
n*Sqrt[Sech[a + b*Log[c*x^n]]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(a+bx)}\,dx, x, \log(cx^n)\right)}{n}$$

$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\operatorname{sech}(a+b\log(cx^n))}} + \frac{\text{Subst}\left(\int \sqrt{\operatorname{sech}(a+bx)}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\operatorname{sech}(a+b\log(cx^n))}} + \frac{\left(\sqrt{\cosh(a+b\log(cx^n))}\sqrt{\operatorname{sech}(a+b\log(cx^n))}\right)\text{Subst}\left(\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx, x, \log(cx^n)\right)}{3n}$$

$$= -\frac{2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)\sqrt{\operatorname{sech}(a+b\log(cx^n))}}{3bn} + \frac{2\sinh(a+b\log(cx^n))}{3bn\sqrt{\operatorname{sech}(a+b\log(cx^n))}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.78

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{\sqrt{\text{sech}(a+b\log(cx^n))}\left(-2i\sqrt{\cosh(a+b\log(cx^n))}\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)+\sinh(2(a+b\log(\right.}{3bn}$$

[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(3/2)),x]

[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-2*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[2*(a + b*Log[c*x^n])]))/(3*b*n)

Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.44

method	result
derivativedivides	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^3+\sqrt{-\sinh\left(\frac{a}{2}+\right.}\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}\right.}$
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^3+\sqrt{-\sinh\left(\frac{a}{2}+\right.}\right.}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}\right.}$

[In] int(1/x/sech(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)

[Out] 2/3/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.81

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$
$$= \frac{\sqrt{2}\left(\cosh(bn\log(x)+b\log(c)+a)^4+4\cosh(bn\log(x)+b\log(c)+a)^3\sinh(bn\log(x)+b\log(c)+a)+\right.}{}$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log
(c) + a)^3*sinh(b*n*log(x) + b*log(c) + a) + 6*cosh(b*n*log(x) + b*log(c) +
 a)^2*sinh(b*n*log(x) + b*log(c) + a)^2 + 4*cosh(b*n*log(x) + b*log(c) + a)
*sinh(b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 - 1)
*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))/(
cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2 + 1)) + 4*(s
qrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x) + b*log
(c) + a)^2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + si
nh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*
b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b*n*s
inh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F]

$$\int \frac{1}{x\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\,\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(3/2),x)
```

```
[Out] Integral(1/(x*sech(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\mathrm{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\,\mathrm{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\mathrm{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\mathrm{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\,{\left(\frac{1}{\mathrm{cosh}\left(a+b\,\ln\left(c\,x^n\right)\right)}\right)}^{3/2}}\,dx$$

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)),x)
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(3/2)), x)
```

3.201 $\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{6i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)\sqrt{\text{sech}(a+b\log(cx^n))}}{5bn} + \frac{2\sinh(a+b\log(cx^n))}{5bn\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))}$$

```
[Out] 2/5*sinh(a+b*ln(c*x^n))/b/n/sech(a+b*ln(c*x^n))^(3/2)-6/5*I*(cosh(1/2*a+1/2
*b*ln(c*x^n))^2)^(1/2)/cosh(1/2*a+1/2*b*ln(c*x^n))*EllipticE(I*sinh(1/2*a+1
/2*b*ln(c*x^n)),2^(1/2))*cosh(a+b*ln(c*x^n))^(1/2)*sech(a+b*ln(c*x^n))^(1/2
)/b/n
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3854, 3856, 2719}

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\sinh(a+b\log(cx^n))}{5bn\text{sech}^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{6i\sqrt{\text{sech}(a+b\log(cx^n))}\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn}$$

```
[In] Int[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]
```

```
[Out] (((-6*I)/5)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n])
, 2]*Sqrt[Sech[a + b*Log[c*x^n]]])/(b*n) + (2*Sinh[a + b*Log[c*x^n]])/(5*b*
n*Sech[a + b*Log[c*x^n]]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{1}{\text{sech}^{\frac{5}{2}}(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{n}\\
&= \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)} + \frac{3\text{Subst}\left(\int \frac{1}{\sqrt{\text{sech}(a+bx)}}\,dx, x, \log\left(cx^n\right)\right)}{5n}\\
&= \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}\\
&\quad + \frac{\left(3\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}\right)\text{Subst}\left(\int \sqrt{\cosh(a+bx)}\,dx, x, \log\left(cx^n\right)\right)}{5n}\\
&= -\frac{6i\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}E\left(\frac{1}{2}i\left(a+b\log\left(cx^n\right)\right)\middle|2\right)\sqrt{\text{sech}\left(a+b\log\left(cx^n\right)\right)}}{5bn}\\
&\quad + \frac{2\sinh\left(a+b\log\left(cx^n\right)\right)}{5bn\text{sech}^{\frac{3}{2}}\left(a+b\log\left(cx^n\right)\right)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{\sqrt{\text{sech}(a+b\log(cx^n))}\left(-12i\sqrt{\cosh(a+b\log(cx^n))}E\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sinh(a+b\log(cx^n))+\text{si}\right.}{10bn}$$

[In] Integrate[1/(x*Sech[a + b*Log[c*x^n]]^(5/2)),x]

[Out] (Sqrt[Sech[a + b*Log[c*x^n]]]*((-12*I)*Sqrt[Cosh[a + b*Log[c*x^n]]]*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]] + Sinh[3*(a + b*Log[c*x^n])]))/(10*b*n)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(123) = 246.

Time = 2.64 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.64

method	result
derivativedivides	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\frac{b}{}\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n}{2}\right.}}$
default	$\frac{2\sqrt{\left(-1+2\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2\right)\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\frac{b}{}\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(cx^n}{2}\right.}}$

[In] int(1/x/sech(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/5/n*((-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10*cosh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(-1+2*cosh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/b

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 602, normalized size of antiderivative = 6.21

$$\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*(sqrt(2)*(cosh(b*n*log(x) + b*log(c) + a)^6 + 6*cosh(b*n*log(x) + b*lo
g(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^5 + sinh(b*n*log(x) + b*log(c) +
a)^6 + (15*cosh(b*n*log(x) + b*log(c) + a)^2 - 11)*sinh(b*n*log(x) + b*log(
c) + a)^4 - 11*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*(5*cosh(b*n*log(x) + b
*log(c) + a)^3 - 11*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*lo
g(c) + a)^3 + (15*cosh(b*n*log(x) + b*log(c) + a)^4 - 66*cosh(b*n*log(x) +
b*log(c) + a)^2 - 13)*sinh(b*n*log(x) + b*log(c) + a)^2 - 13*cosh(b*n*log(x
) + b*log(c) + a)^2 + 2*(3*cosh(b*n*log(x) + b*log(c) + a)^5 - 22*cosh(b*n*
log(x) + b*log(c) + a)^3 - 13*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log
(x) + b*log(c) + a) - 1)*sqrt((cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*l
og(x) + b*log(c) + a))/(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(
x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*lo
g(c) + a)^2 + 1)) - 24*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*sqrt(
2)*cosh(b*n*log(x) + b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*sq
rt(2)*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + s
qrt(2)*sinh(b*n*log(x) + b*log(c) + a)^3)*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(
c) + a))))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*b*n*cosh(b*n*log(x) +
 b*log(c) + a)^2*sinh(b*n*log(x) + b*log(c) + a) + 3*b*n*cosh(b*n*log(x) +
b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n*sinh(b*n*log(x) + b*l
og(c) + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\operatorname{sech}^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/sech(a+b*ln(c*x**n))**(5/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x\,\text{sech}\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
[Out] integrate(1/(x*sech(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/sech(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\text{sech}^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx = \int \frac{1}{x\left(\frac{1}{\cosh(a+b\,\ln(c\,x^n))}\right)^{5/2}}\,dx$$

```
[In] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)),x)
[Out] int(1/(x*(1/cosh(a + b*log(c*x^n)))^(5/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
      m1 = max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):

   #print ("Enter grade_antiderivative for sagemath")
   #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

   leaf_count_result  = leaf_count(result)
   leaf_count_optimal = leaf_count(optimal)

   #print("leaf_count_result=",leaf_count_result)
   #print("leaf_count_optimal=",leaf_count_optimal)

   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)

   if str(result).find("Integral") != -1:
      grade = "F"
      grade_annotation =""
   else:
      if expnType_result <= expnType_optimal:
         if result.has(I):
            if optimal.has(I): #both result and optimal complex
               if leaf_count_result <= 2*leaf_count_optimal:
                  grade = "A"
                  grade_annotation =""
               else:
                  grade = "B"
                  grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
            else: #result contains complex but optimal is not
               grade = "C"
               grade_annotation ="Result contains complex when optimal does not."
         else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
               grade = "A"
               grade_annotation =""
            else:
               grade = "B"
               grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
      else:
         grade = "C"
         grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```python
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```